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Reassessment of the integrity of a partially failed glulam structure

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Submission date: June 2014
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Abstract

Background: This study will seek to determine if the partial failure of a glulam roof in 2011 was only due to a measured accidental overload of $1,6kN/m^2$ more than planned for in the design, and also whether the surviving roof structure received significant permanent damage. Monte Carlo methods are used to calculate probabilities of failure, and there is a heavy focus on the so-called "duration of load"-effect (DOL) in timber, first described by Wood (1947). Two different models for the DOL-effect are used in parallel; the Foschi & Yao method (Foschi et al. 1986), and the Gerhards method (Gerhards 1979), the results are compared to each other.
Results: The simulations show a low probability of failure of a single beam overloaded for 10 years, for both models. $p_{fail} = 8,94 \cdot 10^{-4}$ and $p_{fail} = 3,10 \cdot 10^{-5}$ for the Foschi & Yao method and the Gerhards method, respectively.
Conclusion: This study finds it unlikely that the failure was exclusively due to the overload. It also finds that the structural integrity of the remaining structure is not weakened by the overload.

Bakgrunn: Denne studien ønsker å bestemme om den delvise kollapsen av en limtrekonstruksjon i 2011 var utelukkende på grunn av en målt overbelastning på $1,6kN/m^2$ mer enn planlagt, og også om den resterende konstruksjonen ble påført betydelig permanent skade. Monte Carlo simuleringer blir brukt til å beregne sannsynligheter for brudd og det er et stort fokus på den såkalte "duration of load"-effekten (DOL) for treelementer, (Wood 1947). To forskjellige modeller for bestemmelse av DOL-effekten blir brukt; Foschi & Yao's metode (Foschi et al. 1986), og Gerhards metode (Gerhards 1979), resultatene fra disse to metodene blir sammenlignet.
Resultater: Simuleringene viser en lav sannsynlighet for brudd for en enkel bjelke under overbelastning over 10 år, for begge modeller. $p_{fail} = 8,94 \cdot 10^{-4}$ og $p_{fail} = 3,10 \cdot 10^{-5}$ for Foschi & Yao's metode og Gerhards' metode, henholdsvis.
Konklusjon: Denne studien finner det usannsynlig at kollapsen var utelukkende på grunn av overbelastningen. Den finner også at den resterende konstruksjonens holdbarhet ikke burde være betydelig svekket av overbelastningen.

Contents

1 Introduction

1.1 Problem description

May 31st 2011 a glued laminated beam, that was part of the secondary load bearing structure for the roof of Bauhaus Schlieren in Zurich, failed in bending (Fink et al. 20011). Investigations done on the roof showed that the load from the soil on the green roof was up to $2.4kN/m^2$ in the area around the failed beam, significantly more than the the $0.8kN/m^2$ that was used in the design as a characteristic value. In this case characteristic load refers to a mean value.

This study will seek to determine if this failure was to be expected given the unplanned overloading, and the assumed material parameters. And whether significant permanent damage has been done to the remaining structure, based on the long-term overload.

The damage caused by increased loading in the time after the failure happened until temporary supports were installed, is not a part of this study.

Two different methods for calculating damage from the so-called "duration of load effect" (DOL) in timber will be used in parallel and will be compared to each other.

1.2 Bending resistance in timber

In this study bending resistance in timber will be referred to as the maximum stress value occurring in the upper and lower end of the member exposed to bending, before failure happens. This is in accordance with the Eurocode, and the general literature on timber engineering. In reality this representation is misleading since any timber member should be considered as a system of many parts that work together, and failure happens when enough parts of the system are stressed to their point of failure. i.e. failure does not necessarily begin at the upper or lower edge. The maximum bending stress is thus a representation of the moment load where the whole system is brought to the point of failure.

"Bending strength" and "bending resistance" both refer to this value and are in this study used interchangeably.

Name	Definition area	Scale	Location	Probability density function
Normal	$a < x < b$	σ	μ	$\frac{1}{\sigma\sqrt{2\pi}} exp(-\frac{(x-\mu)^2}{2\sigma^2})$
Log-normal	$0 \leq a < x < b$	σ_{log}	μ_{log}	$\frac{1}{x\sqrt{2\pi\sigma}} exp(-\frac{ln(x-\mu)^2}{\sqrt{2\sigma}})$
Gumbel max.	$a < x < b$	M_o	β	$\frac{1}{\beta} exp(-\frac{x-M_o}{\beta} + exp(-\frac{x-M_o}{\beta}))$
Exponential	$0 \leq a < x < b$	$\frac{1}{\lambda}$	N/A	$\lambda \cdot exp(-\lambda \cdot x)$

Table 1: Probability distributions and their parameters.

1.3 Distributions

The probability distributions used in this study are listed in table 1. The normal distribution is used for values that were determined by using regression analysis of empirical data. The log-normal distribution is chosen for variables where negative values are not applicable, e.g. bending resistance or permanent loads. The Gumbel max. distribution is for variables were extremely high values have to be taken into account, e.g. snow load. The exponential distribution is used at points where the time between two events is modelled.

2 Design according to the Eurocode

Since the building in question was designed according to the Swiss building codes that were valid at the time of construction, 2001, it was necessary to determine whether the design would have been appropriate with regards to the Eurocode or not. In this section the Norwegian National Annex (NA) has been used whenever values from an NA are required (EN-1990, EN-1991, and EN-1995). Characteristic snow load is taken from the Swiss national building code (SIA 216), since this is a parameter determined by local climate conditions.

2.1 Case outline

The design is for a single simply supported beam, glued laminated timber with characteristic bending strength of $32N/mm^2$, with a span of $17.62m$ and load width of $4.5m$. Beam dimensions are rectangular $1120 \cdot 179mm^2$ (Fink et al. 2011). Due to the nature of the connection between the roof and beam, the beam is considered restrained against lateral torsional buckling.

Considering the long span ($17,62m$) it is likely that the limit for maximum deflection would be the deciding factor in determining the dimensions of the beam. This study however, will only consider the ultimate limit states, not the serviceability- or any other limit states.

The moisture levels are in the Eurocode taken into account by selecting one out of three service classes. The beam was chosen to be in service class 1, due to being in a heated ventilated space, protected against precipitation. This assumes a relative air moisture content of below 65%, and a corresponding wood moisture content of 12% or lower.

To include the duration of load effect in the design calculations, the Eurocode uses the resistance reduction factor k_{mod}. k_{mod} is chosen based on the assumed cumulative duration of the load case that is being considered, and the relative air moisture levels around the component in question (service class). Type of timber material is also a factor in determining k_{mod}, glued laminated- and solid timber are considered by the code to be less weakened by this effect than for instance particle boards. If there are multiple relevant load cases, k_{mod} has to be determined for each one. Values for k_{mod} are determined by table 3.1 in EN-1995-1-1.

Two load cases were considered to be relevant to the design: Case A, snow load is dominant: Load is considered for a medium term, $k_{mod} = 0.8$. Case B, self-weight is dominant: Load is assumed to be permanent, $k_{mod} = 0.6$. Values for k_{mod} are found in EN 1995-1-1 tab 3.1. A third load case where an imposed load (maintenance work etc.) is dominant could also be considered, but the imposed load has lower value ($0.75kN/m^2$), and shorter duration (short term) than the snow load, and is not to be combined with other variable loads (EN-1991-1-1 3.3.1(1)). It will therefore not have an impact on the design. Wind is excluded for a similar reason. The wind load would be negative (suction), and would be mostly countered by the permanent loads, which on green roofs tend to be considerably high.

2.2 Loads

There was no easily available data for the self-weight of the roof-structure. Because of this, the load was chosen based on a standard table value for steel roofs of $0,3kN/m^2$ (Bautabellen fűr ingenieure). Self-weight of the beam was determined using an apparent density of $4.0kN/m^3$, (tab A.3 EN 1991-1) . Dimensions of the beam are $1.120m \cdot 0.179m$ (Fink et al. 2011). Load from the beam is then $1.120m \cdot 0.179m \cdot 4.0kN/m^3 = 0.80192kN/m$.

Snow load	$Q_S = 0.9$
Self-weight, roof	$G_R = 0.3$
Filling, green roof	$G_F = 0.8$
Self-weight, beam	$g_B = 0.8019$

Table 2: Loads. Capital letters signify load per unit area (kN/m^2), lower case is for load per unit length (kN/m)

Weight of the filling in a green roof is something that varies between different green roof products, to such a degree that it should be determined on a per project basis. The value that is used here is the same that was chosen for the actual design of the building, $0.8kN/m^2$.

Characteristic snow load for this area of Switzerland, the canton of Zurich, is determined by the national building code to be $0,9kN/m^2$ (SIA 261). Since the roof is flat, and in a relatively open space, there will be no further factors modifying this value.

2.3 Bending

2.3.1 Bending resistance

Design resistance stress for a beam under bending is $f_{mk}/\gamma_M \cdot k_{mod} \cdot k_{sys}$. Where f_{mk} is the characteristic bending stress ($32N/mm^2$), k_{mod} is the duration/moisture reduction factor, and γ_M is the partial safety factor related to the material. $\gamma_M = 1.15$ is used, taken from EN 1995-1-1 tab NA.2.3. The system modification factor k_{sys} is set to 1,1 after EN-1995-1-1 6.6. This factor incorporates the positive effects due to load sharing.

For case A this leads to a design resistance stress of $R_A = 32 \cdot 0,8/1,15 \cdot 1,1 = 24,48N/mm^2$. For case B this leads to a design resistance stress of $R_B = 32 \cdot 0,6/1,15 \cdot 1,1 = 18,36N/mm^2$.

2.3.2 Bending stress

It is assumed that loads are evenly shared between the parallel beams. Loads are therefore converted from area loads to line loads using the effective load width of 4.5m, which is the center to center distance between the parallel

Snow load	$q_s = 4.05$
Self-weight, roof	$g_r = 1.35$
Filling, green roof	$g_f = 3.6$
Self-weight, beam	$g_b = 0.8019$

Table 3: Line loads. All values are in kN/m

Snow load	$\sigma_s = 4.200$
Self-weight, roof	$\sigma_r = 1.400$
Filling, green roof	$\sigma_f = 3.733$
Self-weight, beam	$\sigma_b = 0.8316$

Table 4: Load effects. All values are in N/mm^2

beams. Loads on the beam are then as in table 3.

Maximum moment will happen in the middle of the beams length. Because it is simply supported the moment will be

$$M = \frac{qL^2}{8}$$

Where q is the load per unit length and L is the span length $L = 17.62m$. The stress in the top and bottom of the cross-section will then be represented by: $\sigma_{max} = M/W$. Where M is the moment and W is the section modulus which is determined by the beams cross-sectional geometry. The beam has a rectangular cross-section, which leads to:

$$W = \frac{h^2 \cdot b}{6} = 0.03742m^3$$

A factor that converts line loads to maximum stress in the beam can then be written as:

$$cf = \frac{L^2}{W \cdot 8 \cdot 1000} = 1.0370$$

Where the factor 1/1000 serves to convert from kN/m^2 to N/mm^2. Characteristic stresses from the loads are then as shown in table 4.

2.3.3 Load combination

In case A, the snow load is dominant. It will then be given a load factor of $\gamma_{Q,1} = 1,5$, while the permanent loads are assigned a factor of $\xi \cdot \gamma_G = 0,89 \cdot$

$1,35 \approx 1,2$ in accordance with table NA.A1.2(B) in EN-1990:2002/NA:2008. Design stress is then:

$$S_A = 4,2 \cdot 1,5 + 1,2 \cdot (1,4 + 3,733 + 0,8316) = 13,46 N/mm^2$$

In case B, the permanent loads are dominant and are assigned the load factor $\gamma_G = 1,35$, while the snow load is factor is modified to be $\gamma_{Q,1} \cdot \psi_0$. $\psi_0 = 0,7$ for commercial buildings (table NA.A1.1) so the load factor is $1,5 \cdot 0,7 = 1,05$. Design stress is then:

$$S_B = 4,2 \cdot 1,05 + 1,35 \cdot (1,4 + 3,733 + 0,8316) = 12,46 N/mm^2$$

2.3.4 Ultimate limit state

The limit state equation can be written as:

$$R_d - S_d > 0$$

Case A: Design resistance is $22.26 N/mm^2$, versus a design bending stress of $13.46 N/mm^2$.

Case B: Design resistance is $16.69 N/mm^2$, versus a design bending stress of $12.46 N/mm^2$.

It is clear that the duration of load effect, through k_{mod}, makes the permanent-only load case for bending, the most significant one.

2.4 Shear forces

2.4.1 Shear resistance

Shear stress resistance for a glulam member can be expressed as

$$R_{Vd} = f_{vk} \cdot k_{mod} / \gamma_M$$

Where f_{vk} is the characteristic shear resistance, k_{mod} is the duration/moisture reduction factor, and γ_M is the partial safety factor for the material. $f_v k$ for a GL32c beam is $3,2 N/mm^2$. γ_M is still 1.15.

We then have for case A and B:

$$R_{AV} = 3.2 \cdot 0.8 / 1.15 = 2.26 N/mm^2$$

$$R_{BV} = 3.2 \cdot 0.6 / 1.15 = 1.67 N/mm^2$$

Snow load	$\tau_s = 0,3983$
Self-weight, roof	$\tau_r = 0,1328$
Filling, green roof	$\tau_f = 0,3540$
Self-weight, beam	$\tau_b = 0,07886$

Table 5: Shear stresses. All values are in N/mm^2

2.4.2 Shear stress

The maximum shear force from a line load on a simply supported beam is.

$$V = qL/2$$

This occurs at the ends of the beam right next to both of the supports. Maximum design shear stress in the cross-section for a given shear force is

$$\tau_d = 1.5 \cdot V/(0.67 \cdot A_S)$$

Where A_S is the effective shear area. A_S for a rectangular cross-section in a glulam member, can be considered equal to the entirety of the cross-sectional area.

$$A_S = h \cdot w = 1.12 \cdot 0.179 = 0.2005m^2$$

The conversion factor cf2 between line loads (kN/m) and maximum shear stresses (N/mm^2) can then be written as:

$$cf2 = \frac{1.119 \cdot L}{A_S \cdot 1000} = 0.09834$$

The shear stresses from the different loads are then as shown in table 5

Shear stresses are combined using the same combination and load factors as for bending.

Case A: Resistance is $2.44N/mm^2$, versus a design shear stress of $1.28N/mm^2$.

Case B: Resistance is $1.83N/mm^2$, versus a design shear stress of $1.19N/mm^2$.

We then see that shear is less of an issue than bending. B is still the most significant case.

2.5 Optimized dimensions

When comparing the different load cases, case B in bending has the highest utilization of the resistance allowed by the Eurocode. At $13,46/22,26 = 0,6047 \Rightarrow 60,47\%$. Using the load case and k_{mod} for case B in bending. The minimum allowed height of the beam's cross-section, when rounded up to the nearest millimetre, was found to be $911mm$, versus the actual height of $1120mm$. This optimized height will be used later in the study.

It is likely that the significantly larger dimensions of the actual beam are to avoid large deflections. The deflection in the middle of the beam w can for a simply supported beam be expressed as:

$$w = \frac{5}{384} \cdot \frac{q \cdot L^4}{E \cdot I} \tag{1}$$

Where q is the load per length unit, L is the span length, and E and I are the beam's modulus of elasticity and second moment of area, respectively. Compare this with the equivalent equation for bending moment, M:

$$M = \frac{q \cdot L^2}{8} \tag{2}$$

From the L^4 part in equation (1) it is clear that the longer the span, the more severe the deflection becomes compared to the bending moment.

Since this study does not consider the serviceability limit state, this will not be investigated further.

2.6 Conclusion

This design would have been appropriate according to the Eurocode, for the ultimate limit state when using the values from the Norwegian National Annex.

3 Probabilistic modelling of snow loads

3.1 Model

To properly perform the duration of load calculations, it is necessary to determine, not only the maximum possible snow load, but also the magnitude and duration of every snow event, here referred to as "snow packs", in the relevant time periods. The time periods to be considered are 1 year, 10 years and 50 years. 10 years was the age of the structure at the time of failure. 50 years is the standard service lifetime the Eurocode assumes for most regular buildings.

The model for the snow load's time history is the one presented in Sørensen et al. (2002). Snow loads are modeled as triangularly shaped "packs" when plotted over time.

3.2 Distribution parameters

The peak magnitude of the snow packs are modelled as a gumbel max distribution. The one year 98% fractile for maximum load is $Q_{sk} = 0,9kN/m^2$, this is the characteristic load used in the design. This means that there is a 98% probability that this value will not be exceeded in a single year. It is assumed that the coefficient of variation (cov) for this load is 0,21.

The parameters of the gumbel distribution can be written as $q = M_o + \beta \cdot z_p$. Where q is the snow load, the location parameter M_o is the mode, i.e. the peak value of the probability density function, β is the scale factor which can be written as $\beta = \sqrt{6} \cdot \sigma/\pi$, where σ is the standard deviation. z_p is the standard gumbel variate that corresponds to the probability that is being considered. z_p is determined using the inverse of the standard gumbel distribution where $M_o = 0$ and $\beta = 1$, and the probability is p. The inverse gumbel max function can be written as:

$$q = M_o - \beta ln(-ln(p))$$

z_p is then found using the standard inverse gumbel distribution:

$$z_p = ln(-ln(p))$$

The relation between M_o and the mean is $M_o = \mu - \beta \cdot \gamma$, where μ is the mean and γ is the Euler Mascheroni-constant, $\gamma \approx 0,5772$. The one year mean can then be calculated as:

$$\mu_1 = \frac{Q_{sk}}{1 + cv \cdot \sqrt{6}/\pi \cdot (z_{098} - \gamma)} = 0,5828kN/m^2$$

Since the coefficient of variation is known for this time period, the standard deviation is $0,1224kN/m^2$. β is then $0,0954kN/m^2$. The standard deviation remains constant regardless of the duration of the time period that is being considered.

Based on climate data from 1981 to 2010 (MeteoSwiss 2010) it was assumed that the yearly expected number of snow packs in Zurich is $\lambda_s = 10,13$, this is the number of snowfall days divided by 1,5. i.e. the assumption we are making is that two out of three snowfall days are connected to the same snow pack. The probability that Q_{sk} will not be exceeded in a single snow pack is then $p_{single} = 0,981/\lambda_s \approx 0,9980$. The mode for a single pack is then $M_{o0} = Q_{sk} - z_{p0998} \cdot \beta_0 \approx 0,3067$. The two parameters, β_0 and M_{o0}, needed to model a single snow packs' maximum magnitude, called Pm, are now known.

3.3 Duration between packs

The time between the starting point of two packs is modelled as an exponential distribution with the mean being the inverse of the expected number of snow packs, i.e. $1/\lambda_s$. The realizations of this distribution are called t1. t1(1) is then the time from the starting point to when the first snow pack occurs. t1(2) is the time between the start of the first pack to the start of the second pack, as shown in figure 1. This means that snow packs are spread evenly out over the course of a year. Since snowfall in Zurich occurs mainly between November and April, this modelling is inaccurate, but should not negatively impact the accuracy of the duration of load calculations since the mean for the cumulative time with, and without snow is accurate.

3.4 Duration of the packs

The snow packs are modelled with triangularly shaped load histories with the given maximum, P_m as shown in figure 1. The duration of each pack will be modelled as an exponential distribution. The average number of snow cover days is 33,2. The assumed number of snow packs is 10,13. Mean duration is then $m_t = 33,2/10,13 \approx 3,28$ days. Duration of a snow pack is closely related to the magnitude, i.e. larger accumulations take longer time to form, and thereby also to disperse. The duration will therefore be calculated as $P_m \cdot X_t$ where P_m is the realization of the aforementioned gumbel distribution and X_t is an exponentially distributed factor with units $d/(kN/m^2)$. Mean for X_t is $m_{X_t} = mt/m_{Pm} = 9,056$ where m_{Pm} is mean snow magnitude.

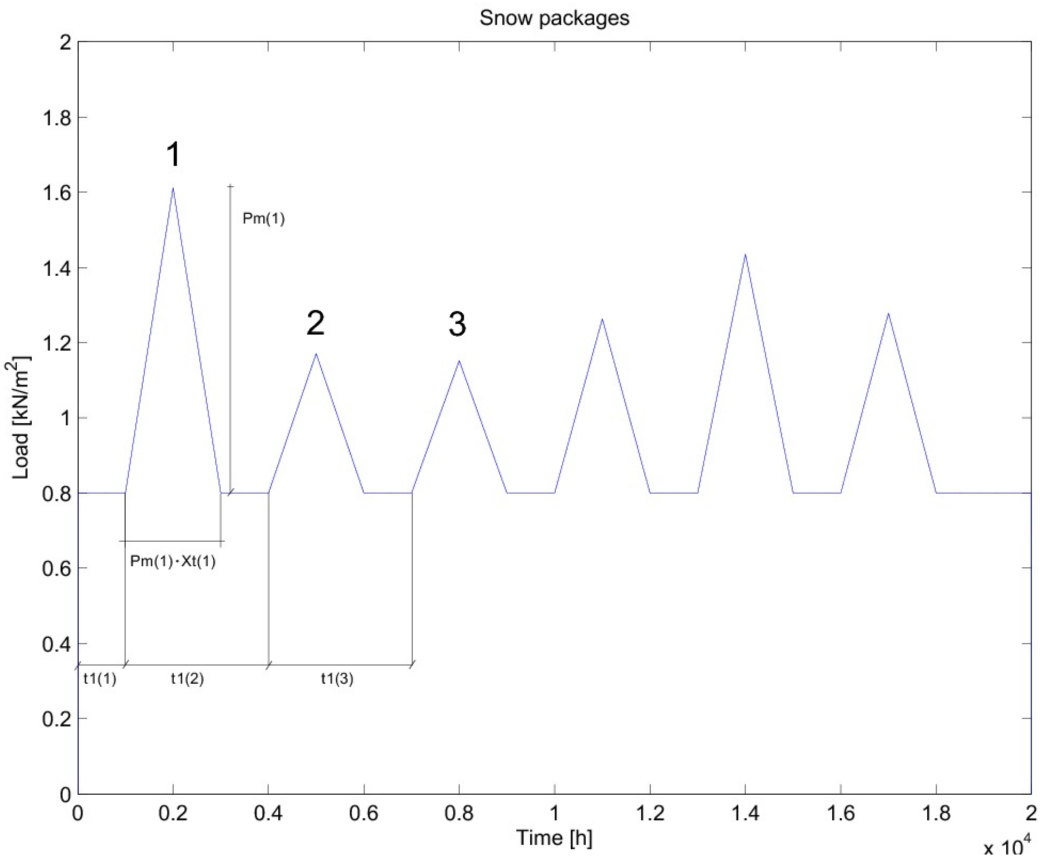

Figure 1: Example for a realization of a load history. Total load is the permanent load plus the snow pack.

$m_{Pm} = M_{o0} + \beta_0 \cdot \gamma.$

3.5 Shape factor

The snow loads, and thereby also their duration, are modified by the Gumbel max. distributed shape factor sf. This factor indicates the difference between snow load on the ground, and the roof. This factor has a mean of 1 and coefficient of variation of 0,35. As suggested by Kőhler (2007) table 4-19.

4 Duration of load damage modelling

4.1 Duration of load effect

Key to this study is the so-called "duration of load" (DOL) effect. This refers to timbers tendency to lose strength when heavily loaded over time, even during static load conditions. This effect was first described by Wood (1947) based on experiments performed on clear wood, and has since been confirmed by multiple other experiments to be valid for structural timber as well.

The effect was later also shown to be connected to the moisture contents of the structural component (Hoffmeyer, 1978). Higher moisture levels leads to greater strength reduction. The DOL-effect is then not just dependant on the load and its duration, but also on the environment around the timber.

The physical reasons for this behaviour are not known exactly. Since this effect has been clearly proven to take place during constant loads, it cannot be exclusively due to fatigue, though fatigue is assumed to greatly enhance this effect. The prevailing theory is that creep deformation cause local fibre buckling, and this combined with propagation of cracks, that are almost always present in timber elements, will eventually cause failure.

There exists several different models designed with the intention of accurately describing this effect as a function of time. In this study we will be using two of them. They are referred to here as Foschi and Yao's method (Foschi et al. 1986) and Gerhards' method (Gerhards 1979), both named after their creators. They are both empirical models that represent the DOL-effect as "damage" accumulated in the timber element. i.e. they are based on regression of experiment data, and do not try to explain the reasons behind this behaviour. They only try to predict its effects.

In this study damage is represented by the value α, where $\alpha = 0$ indicates no damage, and $\alpha = 1$ indicates failure. The following expressions are based on those found in Kőhler (2007).

The parameters used for both models assume an average timber moisture content of 11%, which is in line with the design assumptions made in section 2.

4.2 Foschi & Yaos method

The expression for damage accumulation over time is:

$$\frac{d\alpha}{dt} = A(S(t)/R_0 - \eta)^B + C(S(t)/R_0 - \eta)^D \cdot \alpha(t) \tag{3}$$

Where S(t) is the current bending stress, R_0 is the short term bending resistance (stress). η is the so-called damage threshold, which is the ratio $(S(t)/R_0)$ below which damage does not accumulate, chosen to be 0,5. We then have no accumulation if $S(t) < R0 \cdot \eta$.

B, C, and D are normal distributed model parameters determined by regression analysis of empirical data, A is expressed as:

$$A = k\frac{B+1}{R_0(1-\eta)(B+1)}$$

Where k is the initial rate of loading. This is because the models are calibrated to laboratory tests where the rate of loading should be considered a factor. In this study k is set to a nominal value of $k = 1N/(mm^2 \cdot h)$ for calculating damage from constant loading in both models. Details for the parameters are found in table 6. For the snow packs:

$$k = \frac{Pm \cdot 2}{\Delta t}$$

where Pm is the snow load (stress), and Δt is the duration of the pack. This is the case for both models.

For a constant loading equation 3 is solved as:

$$\alpha(t) = \kappa\alpha_0 + \kappa\lambda - \lambda \tag{4}$$

Where

$$\kappa = exp\left(C\left(\frac{S_c}{R0} - \eta\right)^D \Delta t\right)$$

$$\lambda = \frac{k(B+1)}{C \cdot R0(1-\eta)^{(B+1)}}\left(\frac{S_c}{R0} - \eta\right)^{B-D}$$

Δt is here the duration of the period of constant loading. S_c is the magnitude of the constant load (stress).

After the damage has been calculated the residual strength R_r can be calculated using the following expression:

$$\frac{R_r}{R_0} = \eta + (1-\eta)(1-\alpha)^{1/(1+B)} \tag{5}$$

In this model, damage accumulation from the triangle loads is calculated numerically based on equation (3) with 1000 steps per pack.

As can be seen in equation 3. The accumulated damage is dependant on the current damage. Suggesting a non-linear relationship between duration of load, and damage.

4.3 Gerhards method

The damage accumulation according to the Gerhards method is defined as:

$$\frac{d\alpha}{dt} = exp(-a + b \cdot S(t)/R_0) \tag{6}$$

Where b is a normal distributed model parameter and:

$$a = ln\left(\frac{R_0}{bk}(exp(b) - 1)\right)$$

Where k is the rate of loading at $k = 1N/(mm^2 \cdot h)$. Details for the model parameters are found in table 6.

For a constant load equation 6 is solved as:

$$\alpha(t) = \left(\frac{bk \cdot exp(b \cdot S_c/R0)}{R0 \cdot (exp(b) - 1)}\right)\Delta t + \alpha_0 \tag{7}$$

For the triangle loads equation 6 is solved as:

$$\alpha_{tri} = \frac{R0 \cdot t_2}{b \cdot Sr}\left(exp\left(\frac{b \cdot Sr}{R0 \cdot t_2}t_1 + v\right) - exp\left(\frac{b \cdot Sr}{R0 \cdot t_2}t_0 + v\right)\right) \cdot 2 + \alpha_0 \tag{8}$$

Where:

$$v = -a - \frac{b}{R0}\left(\frac{Sr \cdot t_0}{t_2} + S_c\right)$$

t_0 is the time from the start of the simulation to the start of the load pack. t_1 is the total time from the start of the simulation to the top of the triangle. $t_2 = (t_1 - t_0)/2 = \Delta t/2$, i.e. half the duration of the pack, or the time from the start of the pack to the top of the triangle. Sr is the total load at the top of the triangle, $Sr = Pm + S_c$.

This model also assumes that duration of load damage causes a permanent reduction in short term strength. The residual strength can be expressed as:

$$\frac{R_r}{R_0} = \frac{1}{b}ln(1 + (1 - \alpha)(exp(b) - 1)) \tag{9}$$

As opposed to Foschi & Yao's method, this method has a linear relationship between duration of load and accumulated damage. It should be noted however, that both models treat the α-value as a theoretical size not corresponding to any physical parameter. Considering this, the most important aspect to these models is the point in time when failure happens. However, there is also the assumed relationship between α-values and residual strength which is expressed in equations [5] and [9].

Also unlike the Foschi & Yao method. There is no threshold value where damage is not accumulated below. In practice this alone does not lead to large differences in calculated damage. Since load levels of $\frac{S(t)}{R_0} < 0.5$ will only lead to minuscule damage increases.

Parameters	Mean	Standard deviation
Foschi & Yao		
B	30,03	0.92
C	17,05	12,30
D	5,69	0,45
Gerhards		
b	51,41	0,01

Table 6: Parameters for the different models, as found in Kőhler (2007). All parameters are normal distributed.

5 Proof load effect

5.1 Proof Loading

Proof loading is a method used in structural engineering to improve the known characteristics of the resistance of a set of load bearing members. The method consists of a controlled loading of the set, e.g. glulam beams, up to a certain fractile of the assumed probability distribution of the relevant resistance, e.g. bending. At this point some members may fail. These are the lower realizations of this sample.

It can then be safely assumed that none of the surviving members had a short term resistance lower than the proof load level. Based on this information, a new probability distribution for the resistance may be formulated as proposed by Faber et al. (2005).

$$F''_R(r) = \frac{F'_R(r) - F'_R(\sigma_l)}{1 - F'_R(\sigma_l)} \tag{10}$$

Where $F'_R(r)$ is the prior distribution for the short term strength, $F_R(r)''$ is the posterior distribution, and $F'_R(\sigma_l)$ is the fractile of the resistance that the proof load represents, i.e. the probability of failure at this load for any given member pertaining to this distribution. An example of this udating is shown in figure 2.

The reason this method is particularly useful in timber engineering is that the variability of structural timber populations can be very high. This is due to the natural variability of organic materials, combined with the uncertainties in determining timber strength with non-destructive methods. Proof loading does destroy some members, but in the process direct information is gained about the survivors.

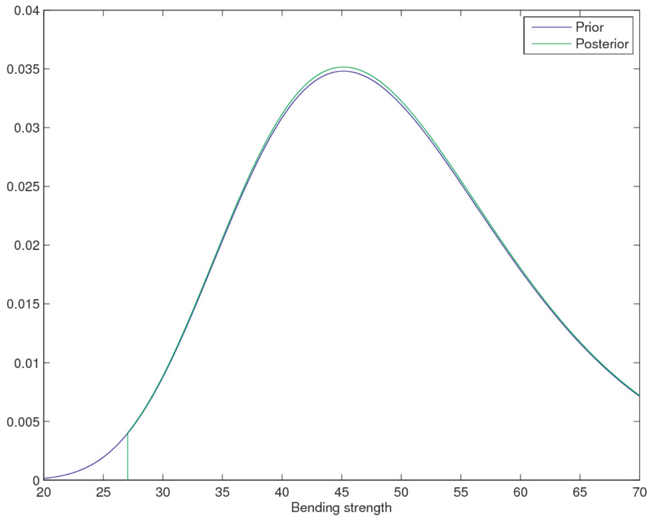

Figure 2: Probability density function of bending strength, before and after a proof load at the 10%-fractile.

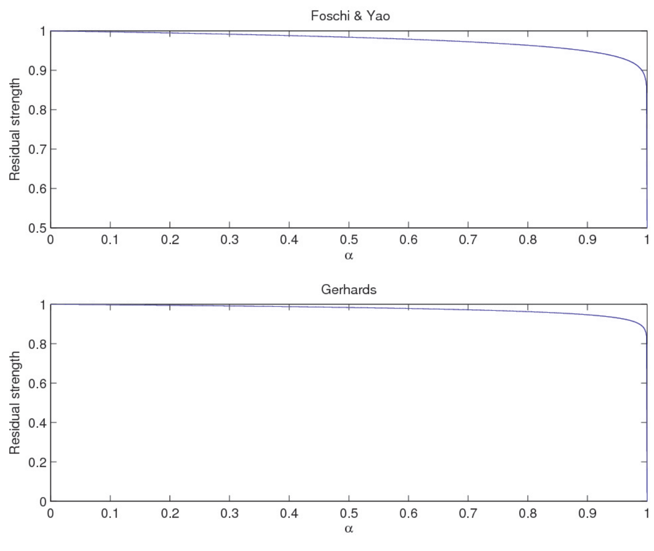

Figure 3: Relationship between damage and residual short-term strength for both methods. Equations [5] and [9] respectively.

Estimating the strength of any material always has some amount of uncertainty connected to the results, but with e.g. steel or concrete, one can do destructive tests of some amount of the same lot, and obtain somewhat accurate data from this. With timber, the individual variations are of a greater degree. Even members made from the same specimen will have different properties due to factors such as knot occurrences, grain direction, or local variations in density. Proof loading sidesteps parts of this problem by defining an absolute minimum strength for the tested population.

It should be kept in mind that even though some members are expected to be destroyed in, for example a bending proof load test, since most strength parameters for timber are positively correlated with each other, these are most likely the members of the lowest quality overall. Not just for bending strength.

If the failure rate during testing is considerably higher than expected, this may be a reason to believe that the initial strength grading is flawed, and more investigations may be needed. A conclusion like this will obviously require a sufficiently large sample.

5.2 Using the overload as a proof load

It is interesting to note about equation [10] that the magnitude of the proof load is irrelevant. What matters is the probability of failure at this magnitude. In this case it is not a single load at all, but the damage caused by the combination of the loads and their duration.

This fact is exploited in this study. We are assuming that whether it is a short term test, or a long term accidental overload, a members' "rank" within a population remains the same. Both of the DOL-models have this assumption as is indicated by the R_0-terms in equations [3] and [6].

In this case study, the equivalent for the laboratory proof-loading is the load history recieved during the 10-year period. Since the beams might receive permanent damage from the overloading, the reduction of strength from this damage has to be taken into account. The relationship between damage and residual strength is shown in figure 4.

To obtain the probability distribution for the remaining resistances from a

Monte Carlo simulation, one can simply plot an array of the remaining resistances, sorted from lowest to highest.

A study by Lam et al. (2003) showed, by using Foschi & Yao's method in Monte Carlo simulations, that the negative influence on the assumed strength distribution from of the reduction in strength due to short term proof loading, was greatly outweighed by the positive effects of removing the weakest members. The study also showed that this fact was even more pronounced at a 15% fractile level than at 5% or 3%. This would indicate that larger probabilities of failure will lead to better improvements in the reliability for the survivors.

6 Monte Carlo simulation

6.1 Monte Carlo methods

Many of the probability calculations in this study are performed using Monte Carlo simulations. This is a general term for any simulation where an experiment with a random outcome is performed a large number of times, with the intention of determining a probability distribution for the outcome. One of the simplest examples of a Monte Carlo simulation is flipping a coin n times and counting the number of heads that occurred. The calculated probability of heads being the result for a single coin flip is then:

$$p_{heads} = \frac{n_{heads}}{n}$$

If $n \Rightarrow \infty$, p_{heads} should be exact.

In this study Monte Carlo simulations will be performed by using MatLab's "rand()" function together with inverse versions of the relevant probability distributions. An example of a simulation that will be performed often is realizing a beam's bending strength. In MatLab this may look like:

$$R_0 = logninv(rand, \mu_{log}, \sigma_{log})$$

Where $logninv()$ is the inverse lognormal distribution function, and μ_{log} and σ_{log} are the logarithmic mean and standard deviations for the bending strength, respectively.

The main advantage of using Monte Carlo methods to determine probabilities, is that expressions which combine multiple probability distributions can be very hard, and in many cases impossible, to solve analytically. The main disadvantages are that, like with many numerical methods, they are never exact, and higher accuracy comes at the cost of longer computation times.

6.2 Beam parameters

Short term bending resistance of the beam (R_0) is modeled with characteristic value (5% fractile) of $R_k = 32N/mm^2$. The assumed coefficient of variation is 0,25. This value assumes a large insecurity with regards to the quality of the beams used for this structure. The arithmetic mean is calculated using the same method as for the snow loads, but for a log-normal distribution. The relationships between arithmetic and logarithmic means and standard deviations are:

$$\mu_{log} = ln(\frac{\mu^2}{\sqrt{\sigma^2 + \mu^2}})$$

$$\sigma_{log} = \sqrt{ln(1 + \frac{\sigma^2}{\mu^2})}$$

Since $\sigma = cov \cdot \mu$, it can be derived from the above expression that the lognormal standard deviation, σ_{log} is independent of the mean when the coefficient of variation, is known:

$$\sigma_{log} = \sqrt{ln(cov^2 + 1)} \approx 0,2462$$

From the expression:

$$X = exp(\mu_{log} + \sigma_{log} \cdot z_p)$$

Which defines the log-normal distribution, the logarithmic mean is calculated as:

$$\mu_{log} = ln(R_k) - \sigma_{log} \cdot z_{005} \approx 3,8707$$

Where z_{005} is the standard normal deviate for the 5%-fractile, $z_{005} \approx -1,645$. From this the arithmetic mean is found to be:

$$\mu = exp(\mu_{log}) \cdot \sqrt{cov^2 + 1} = 49,4541N/mm^2$$

6.3 Permanent load parameters

In the simulation, none of the permanent loads will vary over time. They are variables that will be realized once per realization. This includes the green roof filling.

The weight of the green roof filling is considered to be lognormal distributed with a mean of 0,8 kN/m^2, for the ideal load case, and $2,4kN/m^2$ for the overload case. And an assumed coefficient of variation of 0,1 for both cases.

The weight of the green roof will fluctuate as rain falls, runs off, and evaporates. Unlike ordinary roofs which usually only hold precipitation in the form of snow, green roofs will also hold rain for some time after it falls. However, the purpose of this study is to determine the probability of failure at these specific weights for the filling, the first of which was assumed in the design process, the second which was measured after the failure. The weight will thus not vary over time.

It should be kept in mind that climate data shows that the monthly rainfall in Zurich can be around $115mm$ in the summer season, which is when the heaviest rain falls. This equates to a load of $1.17kN/m^2$. How much of this water that is retained by the soil at the same time, and for how long, is a hard question to answer without multiple on-site measurements over time, and is not a part of this study.

The other two permanent loads; self-weight for the roof, and the beam itself, are assumed to be lognormal distributed, the design values mentioned in section 2.2 are used as means, with coefficients of variation both assumed to be 0,1. The means are $0,3kN/m^2$ and $0,8019kN/m$ for the rooftop and glued laminated beam respectively.

All loads and their parameters are listed in table 7.

6.4 Methodology

For the calculations, only the snow loads vary over time. The permanent loads are being realized once per realization. A set of snow loads will be realized for the relevant duration, once per realization. As described in chapter 3.

The parameters for the DOL models are assumed to vary between different members, but are constant when regarding a single member. They are thus

Load	Distribution	Mean (kN/m^2)	cov
Snow	Gumbel max.	0,9	0,21
Green roof filling (ideal)	Log-normal	0,8	0,1
Green roof filling (overload)	Log-normal	2,4	0,1
Roof	Log-normal	0,3	0,1
Beam	Log-normal	0,2	0,1

Table 7: Loads and distributions for the MC-simulation. The beams self-weight is here converted to an area load, and approximated.

1:	1 year, ideal load, optimized height.
2:	10 years, ideal load, optimized height.
3:	50 years, ideal load, optimized height.
4:	10 years, ideal load.
5:	50 years, ideal load.
6:	10 years, overload.
7:	50 years, overload.

Table 8: List of the cases that are being considered.

realized before the MC simulation itself, and will not vary within a given simulation.

For the DOL-damage calculations, snow packs are generated for the time period corresponding to the case that is being calculated, one year, ten years, and 50 years.

There are seven cases that are being considered, they are described in table 8.

Ideal load refers to the mean weight of the green roof being $0,8kN/m^2$. Overload refers to the mean weight of the green roof being $2,4kN/m^2$.

Case 6 is the closest case to the actual situation. The failed beam had been holding for ten years, and was overloaded at the time of failure.

We are here assuming that the measured overloading of the green roof was present from the moment the roof was constructed. This is based only on the measurements performed after the failure.

The optimized cases are performed using the minimum allowed dimensions to compare our probabilistic model to the required minimum reliabilities in the

Eurocode. Optimized refers to a heigth of the beam of $911mm$, as opposed to the actual height of $1120mm$, as mentioned in section 2.5. The building is considered to be in consequence class 2 (CC2) after table B1 in EN-1990. For the corresponding reliability class (RC2) in table B2, the one year and 50 year reliability indexes are 4,7 and 3,8 respectively.

6.5 Reliability

In reliability theory, the probability of failure is often represented as the negative of the standard normal variate for that probability. This value is referred to as the reliability index and is usually given the symbol β.

$$p_{failure} = \Phi(-\beta) \tag{11}$$

Where $\Phi()$ represents the standard normal distribution with $\mu = 0$ and $\sigma = 1$.

β is a way of representing very low probabilities of failure by using a more practical number. For example: $p_{failure} = 5 \cdot 10^{-6}$ has a reliability index of $\beta \approx 4,42$.

6.6 Simulation

The simulation process is as follows:

1. Duration of load model parameters are realized.

2. n. snow load histories are realized for the period being simulated, this entails duration of the periods between packs, duration of the packs, magnitude of the packs, and the shape factor. n is the number of realizations. Permanent loads are realized once per realization.

3. Duration of load damage is calculated based on the snow load histories, this entails simulating damage between snow packs using the constant load equations [4] and [7], and simulating damage caused by the snow packs using the triangle equation [8] for Gerhards' method, and integrating [3] step-by-step for Foschi & Yao's method.

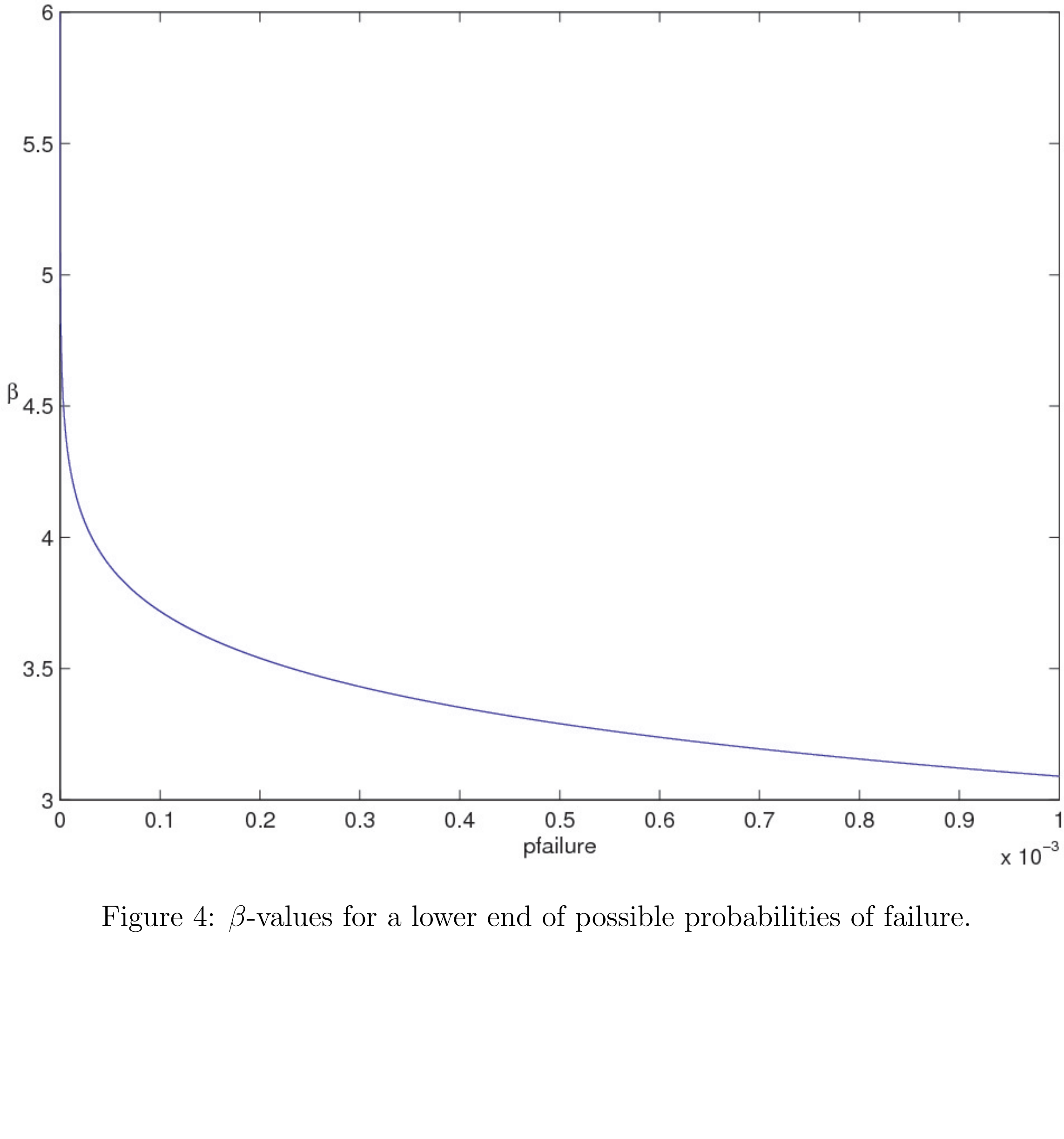

Figure 4: β-values for a lower end of possible probabilities of failure.

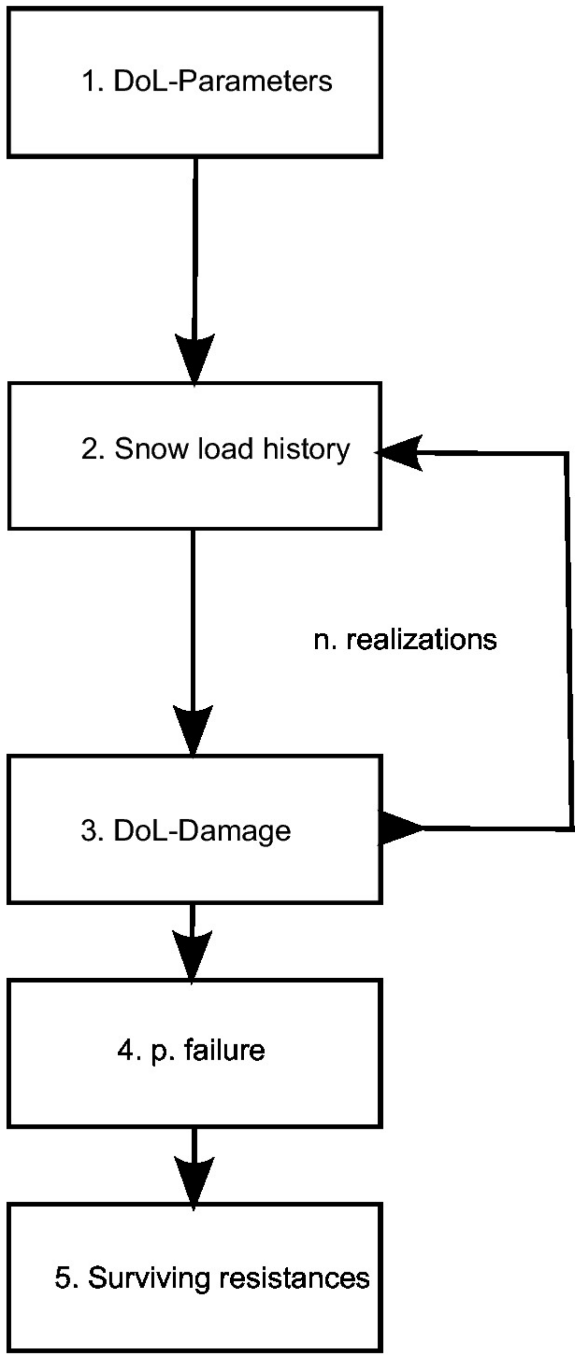

Figure 5: Program structure diagram of the Monte Carlo simulation.

4. Both methods define failure as α-values of 1 or above. Failures are thus counted, and probability of failure using each method is calculated as:

$$p_{failure} = \frac{c}{n}$$

   Where c is the number of failures, and n is the number of realizations.

5. Reduction in short term strength is calculated for the surviving realizations based on equations [5] and [9]. The surviving resistances are the results of this.

This sequence is visualized in figure 5.

6.7 Results

6.7.1 Probabilities of failure

The results from the Monte Carlo simulation are found in table 9

As can be seen in table 9, in the time before the beam failed, the calculated probability of failure for any beam exposed to the overload is 0,0894% for the Foschi & Yao method, and 0,0031% for the Gerhards method.

Basic probability theory gives us the means to scale up these probabilities of failure based on the number of beams, n, covered by the overload. The likelihood that at least one beam will fail is one minus the probability that a single beam will not fail, n. times in a row. The probability that a single beam will not fail is $1 - p_1$. We then have:

$$p_{failure} = 1 - (1 - p_1)^n \tag{12}$$

Unfortunately data surrounding the propagation of the overload is insufficient. We only know that it covered a larger area than this one beam, but not the exact number.

6.7.2 Updated reliability

As the results in table 9 show, the probability of failure for the 10 year overload case is too low for both methods to effectively use the method described

Load situation	n. realizations.	$p_{failF\&Y}$	$\beta_{F\&Y}$	p_{failG}	β_G
1 year ideal optimized	10^7	$1,48 \cdot 10^{-5}$	$4,1765$	$3,00 \cdot 10^{-7}$	$4,9912$
10 year ideal optimized	10^6	$1,45 \cdot 10^{-4}$	$6,6241$	$3,00 \cdot 10^{-5}$	$3,9505$
50 year ideal optimized	10^6	$5,98 \cdot 10^{-4}$	$3,2398$	$8,00 \cdot 10^{-5}$	$3,7750$
10 year ideal	10^7	$2,00 \cdot 10^{-7}$	5,0690	$1,00 \cdot 10^{-7}$	5,1993
50 year ideal	$5 \cdot 10^6$	$8,00 \cdot 10^{-7}$	$4,7983$	$4,00 \cdot 10^{-7}$	$4,9354$
10 year overload	10^6	$8,94 \cdot 10^{-4}$	3,1234	$3,10 \cdot 10^{-5}$	4,0051
50 year overload	10^6	$4,60 \cdot 10^{-3}$	2,6051	$4,00 \cdot 10^{-5}$	3,9444

Table 9: Results from Monte Carlo simulation. Foschi & Yao (F&Y) and Gerhards (G) methods. For different loads and durations.

in section 5.2 to update the reliability.

The new characteristic value is $32,0733 N/mm^2$ and $32,0147 N/mm^2$ for the Foschi & Yao method and the Gerhards method, respectively. Compared to the prior value of $32 N/mm^2$. The change in resistances is shown in figure 6, but as expected the change is barely visible.

While this points toward no beneficial effect of the overload, on the other hand there is no negative effect either. The slight positive from the proof load effect, i.e. removing the failed members, has negated the slight negative effect from the DOL-damage. As is witnessed by the slightly increased characteristic values.

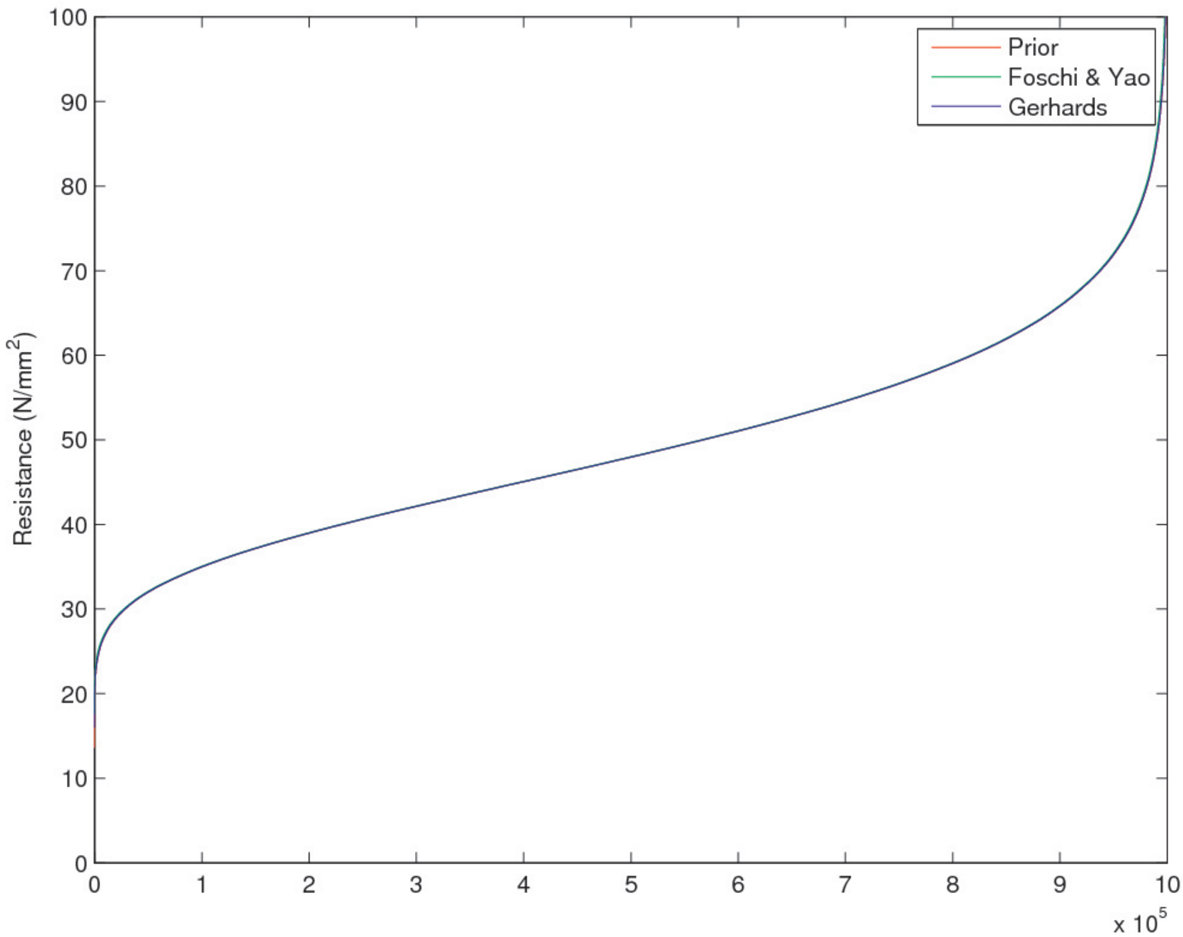

Figure 6: Plot of initial distribution for the assumed bending resistance versus the updated distribution for both methods. As expected from the results, there is almost no difference due to a low calculated probability of failure.

7 Discussion

The results in table 9 show lower probabilities of failure than initially expected, and they point toward the overload not being the only reason for the failure, though it is highly unlikely that the failure would have happened without the overload. Any alternative reasons thus come in addition to the overload.

7.1 Possible explanations for the results

- These results could be accurate and the failure is the result of an extremely low realization of the timbers' resistance. This would explain why the failure was limited to this one beam, even though the surrounding beams received an increased load in the time directly after the collapse. Since they would have to carry the load previously held by the failed beam.

- The number of beams covered by the overload could have been high enough to skew the probability of failure to a more likely value. As shown by equation (12). Combined with the first point in this list, this is a very likely reason for the low results. As mentioned earlier the number of beams covered by the overload remains unknown.

- The failed beam could have been damaged prior to, or under construction. Leading to a significantly lower bending resistance than our assumed resistance distribution would indicate.

- The parameters used for the DOL models, may be poorly suited to describe this lot of glulam timber. The parameters were taken from Kőhler (2007), calibrated based on results from Hoffmeyer (1990). There may be differences based on the origin of the timber that make the parameters less accurate, or our assumptions with regards to the moisture content are wrong.

- Our assumptions about the variation of the loads may be inaccurate. The variations of the load over time due to rain may be a critical factor. This study only considered snow loads to vary over time.

Load situation	n. realizations.	$p_{failF\&Y}/p_{failG}$	$n_{failF\&Y}$	n_{failG}
1 year ideal optimized	10^7	49,33	148	3
10 year ideal optimized	10^6	4,83	145	30
50 year ideal optimized	10^6	7,475	598	80
10 year ideal	10^7	2	1	2
50 year ideal	$5 \cdot 10^6$	2	4	1
10 year overload	10^6	28,83	894	31
50 year overload	10^6	115,25	4581	40

Table 10: Comparing the results from the different methods.

- Our assumptions about the bending resistance of the timber members could be inaccurate. Characteristic value, coefficient of variation, or both could be more or less than the assumed values of $32N/mm^2$ and $0,25$, respectively.

7.2 Other observations

The Foschi & Yao method yields consistently higher probabilities of failure than the Gerhards for all cases. This could be a consequence of the differences in the DOL-models, or due to an error in our programming. Table 10 shows the differences.

It should be noted that some cases in the simulation have low failure occurrences, i.e. $n_{fail} < 10$, due to the low probabilities of failure combined with an insufficiently high n, as shown in table 10. This is especially true for the two unoptimized ideal cases, but also 1 year optimized for the Gerhards method. Redoing these simulations with a sufficiently high n. would take prohibitive amounts of time with the resources available. These results should be therefore be considered less reliable than the rest.

The results for optimized cases for the Gerhards method are very close to the reliability indexes prescribed by EN-1990. 4,9912 for one year and 3,7750 for 50 years. EN-1990 requires 4,7 for one year and 3,8 for 50 years. Though as mentioned in the previous paragraph, the one year case is somewhat unreliable.

8 Conclusion

The calculated probability of failure for a single beam under our assumed circumstances is $0,0894\%$ for the Foschi & Yao method and $0,00310\%$ for the Gerhards method. Based on these results, it is unlikely, but not impossible, that the overload was the sole reason the beam failed. Additional explanations are listed in section 7.

According to the models used to determine DOL-damage, the overload has caused no significant permanent damage to the remaining roof structure.

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Appendices

MatLab code. The following scripts are also added as a digital attachment.

A Design Eurocode

```
clear all
close all

g=9.8;  %Gravity acceleration

%Beam data
wb=0.179; %width
hb=1.120; %heigth
L=17.620; %Span length (m)
rho_b=4.0; %Apparent density of the beam kN/m^3 (tab A.3 EN 1991-1)
fmk=32;  %Characteristic bending resistance (N/mm^2), (5% fractile).
fvk=3.2; %Characteristic shear resistance. NS-EN 1194 (Obsolete norwegian
         %code, should check a more recent source, but the number is
         %probably identical.)

ls=4.5; %Load width (m). The beam in question is fairly close to the middle of the roof structure which has a high number of equally sized beams. We will therefore assume that the load will be distributed evenly between this beam and the adjacent ones.
gam0=1.15; %Partial safety factor for glued laminated timber after
           %EN 1995-1-1 tab NA.2.3 (norwegian national annex)


kmodA=0.8; %Modification factors for resistance after EN 1995-1-1 tab 3.1.
kmodB=0.6; %Takes into account load duration and relative moisture levels.
           %Assuming service class 1

RdA=fmk/gam0*kmodA; %Design resistance, bending.
RdB=fmk/gam0*kmodB;

RvdA=fvk*kmodA/gam0; %Design resistance, shear.
RvdB=fvk*kmodB/gam0;
```

```
%Loads are assessed according to Eurocode (EN 1990 and EN 1991)
%Category 4, regular building, nominal service life: 50 years.

%Reliability class 2


%In this document capital letters signify characteristic load per unit of
%area (kN/m^2), lower case is characteristic load per unit of length
%(kN/m).

%%Variable actions (Q,q)%%

%Imposed load Ql (maintenance work etc.)
Qi=0.75;  %Chosen from EN 1991 1-1 tab NA.6.10 (Norwegian national annex),
qi=Qi*ls;

%Snow load Qs (Zurich)
Qs=0.9; %As per the swiss standard
qs=Qs*ls;
%Elevation in Zurich is about 400 m.o.h.


%%Permanent actions (G)%%

%Self weigth Gs (Roof)
Gr=0.3; %Guess
gr=Gr*ls;


gb=hb*wb*rho_b; %Self weight of the beam (kN/m).

%Dead load Gf (Filling, green roof)
Gf=0.8;
gf=Gf*ls;


%%Load combinations:

%Combination factors for variable loads:
%psi0, basic combination factor. psi1, often occuring. psi2, approximately
%permanent.
psi0_s=0.7; %Snow load
psi0_i=0; %Imposed load
```

```
CombA=1.5*(qs+psi0_i*qi)+1.2*(gr+gb+gf); %Combination A, snow is dominant
CombB=1.5*(psi0_s*qs+psi0_i*qi)+1.35*(gr+gb+gf); %Combination B, self weight is dominant

VmaxA=CombA*L/2; %Maximum design shear force (kN)
VmaxB=CombB*L/2;

TaudA=1.5/0.67*VmaxA/hb/wb/10^3 %Design shear stress
TaudB=1.5/0.67*VmaxB/hb/wb/10^3

RdVA=fvk/gam0*kmodA %Design shear resistance
RdVB=fvk/gam0*kmodB

MmaxA=CombA*L^2/8; %Maximum design moment (kNm)
MmaxB=CombB*L^2/8;

SdA=MmaxA/(wb*hb^2/6*10^9)*10^6 %Maximum design stress due to bending (MPa)
SdB=MmaxB/(wb*hb^2/6*10^9)*10^6

RdA %Design bending resistance (MPa)
RdB
```

B DOL, constant load, Foschi & Yao

```
function [alph]=DoL_damage1_C(B,C,D,t0,t,Sc,R0,alph0,k)

%Foschi & Yao model (1986), constant load between t0 and t.

eta=0.5;
alph=alph0;

if Sc>eta*min(R0)

    kap=exp(C*(Sc./R0-eta).^D*(t-t0));
    lam=k*(B+1)./(C*R0*(1-eta)^(B+1)).*(Sc./R0-eta).^(B-D);

    alph=kap.*alph+kap.*lam-lam; %Equation solved for constant
        load

end

if isnan(alph)==1 %causes failure if 0/0 happens
    alph=1.1;
end
```

C DOL, triangle load, Foschi & Yao

```
function [alph]=DoL_damage1_tri(B,C,D,t0,t,Sc,Sc_tri,R0,alph0)

%Foschi & Yao model (1986)

%Damage accumulation for load over time shaped as an isosceles triangle
%Duration is t-t0, max stress is Sc_tri+Sc at time=(t-t0)/2

eta=0.5; %Damage threshold

k=Sc_tri/((t-t0)/2); %Rate of loading

A=(k*(B+1))/(R0*(1-eta)^(B+1));

n_it=1000; %Number of iterations for numerical solution.
alph=alph0; %Initial damage
dt=(t-t0)/n_it; %Time segment

Sct=zeros(n_it,1);

if Sc+Sc_tri>eta*R0 %Skips iteration completely if max load is below the threshold
    if Sc+Sc_tri>R0 %Designates failure if load is above R0
        alph=1.1;
    else

%First half
for n=1:n_it/2
    if alph>1
        break
    end
    Sct(n)=Sc_tri*2*n/n_it+Sc; %Load is increasing up to the half-way point
    if Sct(n)>eta*R0 %No damage accumulation below the threshold
        alph=alph+(A*(Sct(n)/R0-eta)^B+C*(Sct(n)/R0-eta)^D*alph)*dt;
    end
end

for n=n_it/2:n_it
%Second half
    if alph>1
        break
    end
    Sct(n)=Sc_tri*2*(1-n/n_it)+Sc; %Load is decreasing towards zero at the end
    if Sct(n)>eta*R0 %No damage accumulation below the threshold
```

```
        alph=alph+(A*(Sct(n)/R0-eta)^B+C*(Sct(n)/R0-eta)^D*alph)
           *dt;
    end
end
    end

end
```

D DOL, constant load, Gerhards

```
function [alph]=DoL_damage2_C(b,t0,t,Sc,R0,alph0,k)

%Gerhards model (1987), constant load between t0 and t.

alph=alph0;

alph=(b*k./R0.*exp(b.*Sc./R0)./(exp(b)-1)).*(t-t0)+alph;

end
```

E DOL, triangle load, Gerhards

```
function [alph]=DoL_damage2_tri(b,t0,t,Sc,Sc_tri,R0,alph0)

%Gerhards model (1987)

%Damage accumulation for load over time shaped as an isosceles triangle
%Duration is t-t0, max stress is Sc_tri+Sc at time=(t-t0)/2

t2=(t-t0)./2; %Time to peak load
k=Sc_tri./t2; %Rate of loading

a=log(R0./b./k.*(exp(b)-1));

v=-a-b./R0.*Sc_tri./t2.*t0+b.*Sc./R0;

if Sc+Sc_tri>R0 %Designates failure if load is above R0
alph=1.1;
else

alph=R0.*t2./b./Sc_tri.*(exp(b.*Sc_tri./R0./t2.*t/2+v)...
    -exp(b.*Sc_tri./R0./t2.*t0+v)).*2+alph0; %Solved for ramp load,
                                              %integrated from t0 to t/2.
                                    %Doubled to account for the way down.

end
end
```

F Main Monte Carlo script

```
clear all
close all

%In this script the subscript "1" refers to Foschi & Yao's method,
%"2" refers to Gerhards method

%Depentant on special functions:
%DoL_damage1_C -> Foschi & Yao's method for constant load
%DoL_damage2_C -> Gerhards' method for constant load
%DoL_damage1_tri -> Foschi & Yao's method for triangle load
%DoL_damage2_tri -> Gerhards' method for triangle load

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Variable parameters
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

k=1*365*24;     %Rate of loading (MPa/year)
Gf=0.8;         %Load from filling (kN/m^2). 0.8 for ideal, 2.4 for overload
t_sim=10;       %Time to consider (years)
nb=10^2;        %Number of realizations
hb=1.12;        %Cross-sectional heigth (m), 1.12m for actual, 0.911m for optimized

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Calculating lognormal mean based on known 5% fractile and cov:
my=0; %Standard lognormal mean
sig=1; %Standard lognormal sd.
z005=norminv(0.05,my,sig); %normal variate for 5% fractile

Rk=32; %Characteristic bending resistance (N/mm^2), (5% fractile).
covR=0.25; %Coefficient of variation for bending resistance
sig=sqrt(log(covR^2+1)); %lognormal sd, is independent of mean

myR0=log(Rk)-sig*z005; %Logarithmic mean for R0
mR0=exp(myR0)*sqrt(covR^2+1); %Arithmetic mean for R0

%Beam data
wb=0.179; %width (m)
%hb=1.12; %heigth (m), defined at the top
L=17.620; %Span length (m)
rho_b=4.0; %Apparent density of the beam kN/m^3 (tab A.3 EN 1991-1)
```

```
W=hb^2*wb/6; %Section modulus (m^3)
sdR0=mR0*covR; %Standard deviation

myR=log(mR0^2/sqrt(sdR0^2+mR0^2)); %Logarithmic mean
sigR=sqrt(log(1+sdR0^2/mR0^2));   %Logarithmic sd

ls=4.5; %Load width (m).

cf=L^2/(W*8*1000); %Converts line loads to max. stress in the beam.
cf2=W/ls*8/L^2*1000; %Conversion factor, max. MPa in the beam to kN/m^2


%%Variable action%%

%Snow load Qs (Zurich)
gam=0.5772; %Euler Mascheroni constant
SFD=15.2; %Snowfall days (yearly)
SCD=33.2; %Snow cover days (yearly)
Qsk=0.9; % 98% fractile for yearly ground snow load. Determined by the Swiss standard

cvs=0.21; %Coefficient of variation (yearly)
lam_s=SFD/1.5; %Expected number of yearly snow packages. Assumption is based on SFD.
mean_s=1/lam_s; %Mean for exponential dist. time between packages

%One year parameters
p1=0.98; %p. for no exceedance of Qsk over 1 year.
zp1=-log(-log(p1)); %z value for p1
m1=Qsk/(1+cvs*sqrt(6)/pi*(-gam+zp1)); %1 year mean
sd=m1*cvs; %Standard deviation, constant
beta=sqrt(6)*sd/pi; %Beta, Gumbel value, constant
my1=m1-beta*gam; %One year mode

%Determining parameters for a single snow package:
p0=0.98^(1/lam_s); %p. for no exceedance of Qsk in a single package
zp0=-log(-log(p0)); %z value for p1
my0=Qsk-zp0*beta; %Mode for one package
m0=my0+beta*gam; %Mean
mSn=m0*cf2; %Mean stress on the beam, snow (MPa)

%Duration of snow pack is Pm*Xt
%Pm is the magnitude of a snow pack (kN/m^2)
%Xt is an exponentially distributed relation between tpack and Pm
```

```
mt=SCD/lam_s; %Mean pack duration (days)
mXt=mt/m0; %Mean for Xt (days/MPa)


%Roof shape factor: Determines the relation between load on the
    ground, and the roof.
msf=1; %mean
cvsf=0.35; %coefficient of variation
sdsf=msf*cvsf;
betasf=sqrt(6)/pi*sdsf; %Beta
mysf=msf-betasf*gam; %mode


%%Permanent actions (G)%%

%Self weigth Gs (Roof)
Gr=0.3;                           %Assumed
gr=Gr*ls;                         %Beam load
cov_gr=0.1;                       %Assumed coefficient of
    variation
sdrf=cov_gr*gr;                   %Standard deviation
siggr=sqrt(log(1+sdrf^2/gr^2));  %Logarithmic sd
grl=log(gr^2/sqrt(sdrf^2+gr^2)); %Logarithmic mean

%Self weight of the beam
gb=hb*wb*rho_b; %Self weight of the beam (kN/m).
cov_gb=0.1; %PMC part II, tab 2.1.1, all regular species of
    timber.
sdb=cov_gb*gb;
sigb=sqrt(log(1+sdb^2/gb^2));   %Logarithmic sd
gbl=log(gb^2/sqrt(sdb^2+gb^2)); %Logarithmic mean

%Dead load Gf, filling, green roof
%Gf=Gf; Mean load for the filling. Defined at the top.

gf=Gf*ls;                        %Beam load
cov_gf=0.1;                      %Assumed cov.
sdf=cov_gf*gf;                   %Standard deviation
sigf=sqrt(log(1+sdf^2/gf^2));   %Logarithmic sd
gfl=log(gf^2/sqrt(sdf^2+gf^2)); %Logarithmic mean

%Means for permanent stresses:
m_gr=grl*L^2/8/W/1000; %Logarithmic mean stress roof
m_gb=gbl*L^2/8/W/1000; %Logarithmic mean stress beam
m_gf=gfl*L^2/8/W/1000; %Logarithmic mean stress filling

%Log. sd for permanent stresses
siggrs=siggr*cf; %Roof
sigbs=sigb*cf; %Beam
```

```
sigfs=sigf*cf; %Filling

%DoL parameters

alph0=0;%Initial damage
t0=0; %Starting time, (years)
%k=k; %Rate of loading up to Sc, (MPa/hour). Defined at the top

%1: Foschi & Yao's model

eta=0.5; %Damage threshold. No damage is accumulated at loads
    below eta*R0.

%Model parameters B, C and D:
mB=30.03; %Mean
sdB=0.92; %SD
B=norminv(rand,mB,sdB);

mC=17.05; %Mean
sdC=12.30; %SD
C=norminv(rand,mC,sdC)*365*24; %Unit is converted from 1/hour to
     1/year (Kohler 2011)

mD=5.69; %Mean
sdD=0.45; %SD
D=norminv(rand,mD,sdD);

%2: Gerhards' model

%Model parameter b:
mb=51.41;
sdb=0.01;
b=norminv(rand,mb,sdb)+log(365*24); %unit is converted from ln(h
    ^-1) to ln(y^-1) (Kohler 2011)


%%Determining snow packages%%
%t_sim=t_sim; %Time to consider (years). Defined at the top
n_int=1000; %Number of intervals between packages. Size is
    irrelevant as long as it is large enough to span more than 50
     years.
t1=zeros(n_int,1); %Time between snow packages (years)
Pm=zeros(n_int,1);
Xt=zeros(n_int,1);


%%Monte Carlo simulation%%
%nb=nb; %Number of realizations. Defined at the top
c1=0; %Counter for failures
```

```
c2=0;

R0v=zeros(nb,1); %Vector for short term strengths
alph1_vector=zeros(nb,1); %Vectors for alpha-values
alph2_vector=zeros(nb,1);
ML=zeros(nb,1); %Vector for maximum load



tic
for n2=1:nb
t=0; %Counter for time
%Generating snow packages:
sf=(mysf-betasf*log(-log(rand))); %Roof shape factor
n_sim=0; %Counter for snowfall
for n=1:n_int
t1(n)=-log(1-rand)*mean_s; %Time between starting time of the
    packages (years), t1(1) is then the time before the first
    package.
Pm(n)=my0-beta*log(-log(rand))*sf; %Max magnitude per package (
    kN/m^2)
Xt(n)=-log(1-rand)*mXt/365; %Duration factor (y/kN*m^2)
t=t+t1(n); %Total time passed (y)
n_sim=n_sim+1;
if t>=t_sim
     break %Stops iteration after the relevant amount of time
         have passed n_sim is now the number of packages in the
         relevant period. Xt(n).*Pm(n) is the duration of snow
         package n.
end

end


R0=logninv(rand,myR,sigR); %Short term resistance
R0v(n2)=R0; %Storing R0
Sg=logninv(rand,m_gr,siggrs)+logninv(rand,m_gb,sigbs)+logninv(
    rand,m_gf,sigfs);
%Permanent stress
ML(n2)=max(Pm)/cf2+Sg;

t0=0; %Starting time

%Calculating damage from DoL-effect:
alph0=0;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%1: Foschi & Yao
alph1=alph0; %Initial damage
check1=0; %Checks for failure
```

```
t2=t0; %Starting time
for n=1:n_sim
alph1=DoL_damage1_C(B,C,D,t2,t1(n),Sg,R0,alph1,k); %Damage accumulation
                                                    %between snow packages

t2=t1(n)+Pm(n).*Xt(n); %End time for this cycle/starting time for the next.

alph1=DoL_damage1_tri(B,C,D,t1(n),t2,Sg,Pm(n)/cf2,R0,alph1);
%Damage accumulation during snow package
if alph1>=1
    break %Ends calculations if failure happens.
end
end

alph1_vector(n2)=alph1; %stores damage value
if alph1>=1
    c1=c1+1; %Counts failure
end


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%2: Gerhards
alph2=alph0; %Initial damage
check2=0; %Checks for failure
t2=t0; %Starting time
for n=1:n_sim
alph2=DoL_damage2_C(b,t2,t1(n),Sg,R0,alph2,k); %Damage accumulation
                                               %between snow packages

t2=t1(n)+Pm(n).*Xt(n); %End time for this cycle/starting time for the next.

alph2=DoL_damage2_tri(b,t1(n),t2,Sg,Pm(n)/cf2,R0,alph2);
%Damage accumulation during snow package

if alph2>1
    break %Ends calculations if failure happens.
end
end

alph2_vector(n2)=alph2;
if alph2>=1
    c2=c2+1; %Counts failure
end
```

```
end

toc


pfailure1=c1/nb %p. failure for a beam using this method
beta1=-norminv(pfailure1,0,1); %Beta, z-value for pfailure (standard normal distribution)
pfailure2=c2/nb %p. failure for a beam using this method
beta2=-norminv(pfailure2,0,1); %Beta, z-value for pfailure (standard normal distribution)


save roof_sim_latest.mat %Safety copy of latest results
% save 10year_overload.mat


c19=0; %Counter for survivors
c29=0; %Counter for survivors

%Sorting surviving beams from the broken ones.
for n=1:nb
    if alph1_vector(n)<1
        c19=c19+1; %Counts number of survivors for Foschi & Yao
        alph1_vectorb(c19,1)=alph1_vector(n); %Stores surviving damage values
        R0v1(c19,1)=R0v(n); %Stores surviving bending strengths
    end

    if alph2_vector(n)<1
        c29=c29+1; %Counts number of survivors for Gerhards
        alph2_vectorb(c29,1)=alph2_vector(n); %Stores surviving damage values
        R0v2(c29,1)=R0v(n); %Stores surviving bending strengths
    end
end

if c1==0 %Prevents error if no failures
    alph1_vectorb=alph1_vector*0;
end
if c2==0 %prevents error if no failures
    alph2_vectorb=alph2_vector*0;
end

ep=60; %Endpoint for plots
r=0:0.01:ep; %Field variable for resistance
```

```
%Strength reductions
red1=eta+(1-eta)*(1-alph1_vectorb).^(1/(1+B));
red2=1/b*log(1+(1-alph2_vectorb)*(exp(b)-1));

%Residual strengths:
R0r1=R0v1.*red1; %Vectors for residual strength
R0r2=R0v2.*red2;

mR0r1=mean(R0r1); %Means for residual strengths
mR0r2=mean(R0r2);

sdR0r1=std(R0r1); %Sd for resiudal strengths
sdR0r2=std(R0r2);

myR1=log(mR0r1^2/sqrt(sdR0r1^2+mR0r1^2)); %Reduced logarithmic mean
myR2=log(mR0r2^2/sqrt(sdR0r2^2+mR0r2^2)); %Reduced logarithmic mean

sigR1=sqrt(log(1+sdR0r1^2/mR0r1^2)); %Reduced logarithmic sd
sigR2=sqrt(log(1+sdR0r2^2/mR0r2^2)); %Reduced logarithmic sd


%Updated cdfs for reduced resistances
cdfR0r1=(logncdf(r,myR1,sigR1)-pfailure1)/(1-pfailure1);
cdfR0r2=(logncdf(r,myR2,sigR2)-pfailure2)/(1-pfailure2);

% cdfR0=logncdf(r,myR,sigR); %Cdf for initial resistance
% cdfR01=(cdfR0-pfailure1)/(1-pfailure1); %Unreduced updated resistances
% cdfR02=(cdfR0-pfailure2)/(1-pfailure2);


%Pdfs
r1=logninv(pfailure1,myR,sigR):0.01:ep; %Variables for pdfs, starting point
r2=logninv(pfailure2,myR,sigR):0.01:ep; %is the zero point for each plot.

lr=length(r);   %Lengths of field variables
lr1=length(r1);
lr2=length(r2);

cdf1=cdfR0r1(lr-lr1+1:lr); %Cdfs cut at the zero point
cdf2=cdfR0r2(lr-lr2+1:lr);
PDF1=gradient(cdf1)/0.01; %Pdfs
PDF2=gradient(cdf2)/0.01;

%Variables for vertical lines
```

```
r3=[logninv(pfailure1,myR,sigR)-0.01,logninv(pfailure1,myR,sigR)];
r4=[logninv(pfailure2,myR,sigR)-0.01,logninv(pfailure2,myR,sigR)];

%Draws vertical lines to the starting point of the pdfs
vl1=[0,PDF1(1,1)];
vl2=[0,PDF2(1,1)];

%Plots all three pdfs
figure
plot(sort(R0v),'r')
hold on
plot(sort(R0v1),'g')
plot(sort(R0v2),'b')


time=toc; %Saves time to completion
```