

Hybrid Control to Improve Transient Response of Integral Action in Dynamic Positioning of Marine Vessels ^{*}

Svenn Are Tuttüren ^{*} Roger Skjetne ^{*}

^{*} *Centre for Autonomous Marine Operations (AMOS), Department of Marine Technology, Norwegian University of Science and Technology (NTNU), Otto Nielsens vei 10, 7491 Trondheim, Norway (e-mail: svenn.are.tutturen@ntnu.no, roger.skjetne@ntnu.no)*

Abstract: A hybrid control approach for integral action in the PID control law for dynamically positioned marine vessels is considered. The proposed method is essentially a resetting of the integration gain when the control performance deteriorates. The method allows for a flexible tuning, and could be useful when there are long periods of normal operating conditions, but abnormal events may occur. In that case the hybrid controller will have a low tuning in the normal regime and switch to a more aggressive tuning in the abnormal regime. Stability of the hybrid system is investigated, and a simulation case is performed.

Keywords: Dynamic positioning; Integral action; Hybrid dynamical systems

1. INTRODUCTION

Dynamically positioned (DP) vessels normally experience wave loads, wind loads, and currents. The loads the integral action part of the controller compensate for are slowly varying forces, almost constant for long periods of time. Because of this the integral action is normally tuned very low, such that it does not induce unnecessary oscillation in the closed loop system. Also, the tuning could be low to avoid that the integral action compensates for motion due to 1st order wave loads. Even though this motion is filtered out with a wave filter [Fossen 2011], the exact knowledge of the peak frequency of the wave spectrum is uncertain, and perfect filtering is difficult. Therefore, there is some oscillatory motion left that the integral action ideally should not compensate for.

One issue with a low tuning of the integral action is that it will spend some time building up to the correct value. This is especially the case at initialization, or when large changes in force occur. This could for instance be caused by ice forces, a tension line that breaks, or a sudden wave train. In these instances, it is of interest to improve the transient response of the integral action.

Hybrid control of DP vessels has been considered in several papers in the literature. A framework with several continuous controller and observer-pairs based on the work in Hespanha [2001] was proposed for hybrid control of DP vessels in Nguyen et al. [2007]. The operational window of a DP vessel is extended by switching between different observer-controller pairs depending on the sea state. Using the same type of continuous controller and observer-pair methodology as in Nguyen et al. [2007], a hybrid control

approach was proposed to combine dynamic positioning, maneuvering, and transit operation in Nguyen et al. [2008], and also to be applied for switching control for position mooring in Nguyen and Sørensen [2009]. See [Sørensen 2013] for an overview. In Brodtkorb et al. [2014] the control problem considered in Nguyen et al. [2007] is analyzed in the framework of Goebel et al. [2012], which is the framework used in this paper as well.

The main contribution of the paper is a novel control structure that allows for increased flexibility in integral action in the PID control law, for dynamically positioned marine vessels. This is achieved by a hybrid global/local controller approach. A hybrid control framework is used, and particularly the methods analyzed in [Goebel et al. 2012, Ch. 3] have motivated the method presented here.

The idea of the proposed hybrid controller is that close to the desired position the nominal local integrator is active. When the vessel is far off target, for instance due to a rapid disturbance, the global and more aggressive integral action is turned on. The benefit is that the aggressive integral action will give a faster response to a disturbance. When close to the desired values, this aggressive part is turned off, and the system is back to the nominally tuned integral action. Stability of the hybrid system is analyzed, and a simulation study is performed to demonstrate the benefit of the approach.

Notation: The time derivative is denoted by dot notation, such that \dot{x} is the time derivative of x . The minimum and maximum eigenvalue of a matrix P are denoted by $\lambda_{min}(P)$ and $\lambda_{max}(P)$, respectively.

2. PROBLEM STATEMENT

Given the 3 degree of freedom (DOF) control design model of a DP system [Fossen 2011],

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$$\dot{\eta} = R(\psi)\nu \quad (1a)$$

$$M\dot{\nu} = -D\nu + R(\psi)^\top b + \tau, \quad (1b)$$

where $\eta = [N, E, \psi]^\top \in \mathbb{R}^3$ is a vector containing the North/East positions and heading, and $\nu = [u, v, r]^\top \in \mathbb{R}^3$ contains the surge/sway velocities and yaw rate, $\tau \in \mathbb{R}^3$ is the control input, and $R(\psi) \in \mathbb{R}^{3 \times 3}$ is the 3 DOF rotation matrix,

$$R(\psi) = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (2)$$

The mass matrix is $M = M^\top > 0$, and $D > 0$ is the damping matrix. The disturbance or bias vector $b \in \mathbb{R}^3$ contains all the remaining forces affecting the vessel, such as current, second order wave forces, and unmodeled dynamics [Sørensen 2013]. It is common to assume b is constant. The task of the integral action is to compensate this bias.

Due to saturation limits in the thrusters and bounded environmental loads, we make the following assumption.

Assumption 1 The yaw rate $r := \dot{\psi}$ is bounded, with $|r| \leq r_{max} < \infty$.

In the following, the position and velocity are assumed measured. The velocity is normally found through a state observer, but to simplify the analysis, velocity is assumed known. The proposed integral action integrates the position error.

3. HYBRID CONTROLLER APPROACH

In hybrid control, continuous and discrete dynamics are combined [Goebel et al. 2012]. The continuous dynamics is called "flow", which is allowed on a flow set \mathcal{C} . The discrete dynamics is called "jump", which is allowed on a jump set \mathcal{D} .

Consider a case with two different controllers for the same system dynamics. One controller works locally, and has good performance around the equilibrium. The other "global" controller is used when the states are far from the equilibrium.

In the following, this controller structure will be used for PID control of a DP plant. Under normal conditions the local integral action will be active. The global integral action will first activate under large disturbance events that deteriorate the control performance. The global integral action will then be more aggressive in response to the disturbance.

The system setup is similar to the local/global control structure of Goebel et al. [2012], but here both controllers are globally stable. Another difference is that Goebel et al. [2012] assumes full state knowledge. Here, knowledge of the position and the velocity is assumed known, but the bias force is not known.

4. FLOW DYNAMICS

The integrator state ξ is given the dynamics

$$\dot{\xi} = L\eta, \quad (3)$$

where the properties that the matrix $L \in \mathbb{R}^{3 \times 3}$ needs to satisfy will be elaborated later.

The controller considered is the standard DP PID-control law

$$\tau = -K_p R(\psi)^\top \eta - K_d \nu - K_i R(\psi)^\top \xi, \quad (4)$$

where $K_p, K_d, K_i \in \mathbb{R}^{3 \times 3}$ are all positive definite matrices, and K_i commutes with the rotation matrix, that is, $K_i R(\psi) = R(\psi) K_i$. Let the integral action error be $\tilde{\xi} = \xi - K_i^{-1} b$. Then the resulting error dynamics becomes

$$\dot{\tilde{\xi}} = L\eta \quad (5a)$$

$$\dot{\eta} = R(\psi)\nu \quad (5b)$$

$$M\dot{\nu} = -K_p R(\psi)^\top \eta - (D + K_d)\nu - K_i R(\psi)^\top \tilde{\xi}. \quad (5c)$$

Collecting the error states in a state vector $x \in \mathbb{R}^9$,

$$x = \begin{bmatrix} \tilde{\xi} \\ \eta \\ \nu \end{bmatrix}, \quad (6)$$

the error dynamics becomes

$$\dot{x} = F_0(\psi)x, \quad (7)$$

where

$$F_0(\psi) = \begin{bmatrix} 0 & L & 0 \\ 0 & 0 & R(\psi) \\ -M^{-1}K_i R(\psi)^\top & -M^{-1}K_p R(\psi)^\top & -M^{-1}(D + K_d) \end{bmatrix}. \quad (8)$$

Consider the global diffeomorphism, similar to the one proposed by Lindegaard [2003],

$$x = T(\psi)z, \quad (9)$$

where

$$T(\psi) = \text{diag}\{R(\psi), R(\psi), I\}. \quad (10)$$

The z -dynamics becomes

$$\begin{aligned} \dot{z} &= \dot{T}(\psi)^\top x + T(\psi)^\top \dot{x} \\ &= \dot{T}^\top(\psi)T(\psi)z + T(\psi)^\top F_0(\psi)T(\psi)z \end{aligned} \quad (11)$$

First, consider the term $T(\psi)^\top F_0(\psi)T(\psi)$,

$$T(\psi)^\top F_0(\psi)T(\psi) = \begin{bmatrix} 0 & R(\psi)^\top L R(\psi) & 0 \\ 0 & 0 & I \\ -M^{-1}K_i & -M^{-1}K_p & -M^{-1}(D + K_d) \end{bmatrix} \quad (12)$$

Given that L commutes with the rotation matrix $R(\psi)$, that is, $R(\psi)L = LR(\psi)$, then $A_0 := T(\psi)^\top F_0 T(\psi)$ becomes

$$A_0 = \begin{bmatrix} 0 & L & 0 \\ 0 & 0 & I \\ -M^{-1}K_i & -M^{-1}K_p & -M^{-1}(D + K_d) \end{bmatrix}. \quad (13)$$

For a specification of what the K_i and L matrix needs to satisfy to commute with $R(\psi)$, see [Fossen 2011, ch. 11].

Now, consider the second term $\dot{T}(\psi)^\top T(\psi)z$. Since $\dot{R}(\psi) = R(\psi)Sr$, where

$$S = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad (14)$$

the term $\dot{T}(\psi)^\top$ can be written as

$$\begin{aligned} \dot{T}(\psi)^\top &= \text{diag}\{\dot{R}(\psi)^\top, \dot{R}(\psi)^\top, 0\} \\ &= \text{diag}\{-\dot{\psi}SR(\psi)^\top, -\dot{\psi}SR(\psi)^\top, 0\} \\ &= -rS_T T(\psi)^\top, \end{aligned} \quad (15)$$

where

$$S_T = \text{diag}\{S, S, 0\} = -S_T^\top. \quad (16)$$

This gives

$$\dot{T}(\psi)^\top T(\psi)z = -rS_T T(\psi)^\top T(\psi)z = -rS_T z, \quad (17)$$

which finally gives the z -dynamics as

$$\dot{z} = A_0 z - rS_T z. \quad (18)$$

Let A_0 be Hurwitz, and choose a Lyapunov function candidate as

$$V(z) = z^\top P z, \quad (19)$$

where $P = P^\top > 0$ and

$$A_0^\top P + P A_0 = -G_0 < 0. \quad (20)$$

The time derivative of V along the trajectories of z becomes

$$\dot{V} = z^\top [A_0^\top P + P A_0 - r(S_T^\top P + P S_T)]z \quad (21a)$$

$$\leq [-\lambda_{\min}(G_0) + 2r_{\max}\lambda_{\max}(P)]|z|^2. \quad (21b)$$

For (21b) to be negative definite, r_{\max} needs to be very small, due to the structure of A_0 . A less conservative estimate of r_{\max} is found in Lindegaard [2003], and is based on (21a). Given A_0 , r_{\max} , P and G are found from an LMI optimization problem, such that

$$\begin{aligned} \dot{V} &= z^\top [A_0^\top P + P A_0 - r(S_T^\top P + P S_T)]z \\ &\leq -G|z|^2 < 0 \quad \forall |r| \leq r_{\max}. \end{aligned} \quad (22)$$

For how to find this r_{\max} , see [Lindegaard 2003, Corollary 5.1].

5. HYBRID SYSTEM ANALYSIS

The condition for switching between the two controllers is the norm of the position error, $|\eta|$. Two scalar quantities $\eta_1^* = 0$ and $\eta_2^* > 0$ are defined, and switching is based on an idea that is similar to what was proposed in Brodtkorb et al. [2014], where switching between different DP controllers was decided based on the estimated peak frequency of the wave spectrum. In the following, the switching will be based on the position error. When $|\eta|$ is closer to η_1^* than η_2^* , that is, $\text{abs}(|\eta| - \eta_1^*) < \text{abs}(|\eta| - \eta_2^*)$, the local controller is used, and similarly the global controller is used when $|\eta|$ is closer to η_2^* .

For the hybrid system the closed loop dynamics can be described by

$$\dot{z} = f(z, L_q), \quad (23)$$

where $f(z, L_q)$ is given by (18), and L_q is the L -matrix from (5a). This L -matrix is the only difference between the local and global controller. The variable $q \in \{1, 2\} =: \mathcal{Q}$ is a switching variable between the two controllers, where L_1 is the L -matrix used in the local controller, and L_2 is used by the global controller ($\|L_2\| \geq \|L_1\|$).

The system includes dwell-time switching, and the timer variable τ has continuous dynamics $\dot{\tau} = 1$. When $\tau = T$ jumps are allowed, and the preferred controller is decided based on a check of whether $|\eta|$ is closer to η_1^* than η_2^* , and vice versa. This can be formally written as $q = \text{argmin}_{\alpha \in \mathcal{Q}} [\text{abs}(|\eta| - \eta_\alpha^*)]$ [Brodtkorb et al. 2014].

Define the augmented state space as

$$z_e := \begin{bmatrix} z \\ \tau \end{bmatrix} \in \mathbb{R}^{11}. \quad (24)$$

Then the hybrid system becomes

$$\left. \begin{aligned} \dot{z} &= f(z, L_q) \\ \dot{q} &= 0 \\ \dot{\tau} &= 1 \end{aligned} \right\} \tau \in [0, T] \quad (25)$$

$$\left. \begin{aligned} z^+ &= z \\ q^+ &= \text{argmin}_{\alpha \in \mathcal{Q}} [\text{abs}(|\eta| - \eta_\alpha^*)] \\ \tau^+ &= 0 \end{aligned} \right\} \tau = T, \quad (26)$$

where the flow set is

$$\mathcal{C} := \mathbb{R}^8 \times [-r_{\max}, r_{\max}] \times \mathcal{Q} \times [0, T], \quad (27)$$

and the jump set is

$$\mathcal{D} := \mathbb{R}^8 \times [-r_{\max}, r_{\max}] \times \mathcal{Q} \times \{T\}. \quad (28)$$

The goal is to prove that the set

$$\mathcal{A} = \{z_e : z = 0, q \in \mathcal{Q}, \tau \in [0, T]\} \quad (29)$$

is uniformly globally (pre-) asymptotically stable. The relevant theorem from Goebel et al. [2012] is given in Appendix A. Note that the distance to the set $|z_e|_{\mathcal{A}} = |z|$.

For the two controllers let their A_0 matrices from (18) be Hurwitz. In addition, the controllers need to satisfy (19)-(22) with a common quadratic (Lyapunov) function $W(z)$,

$$W(z) = z^\top P z, \quad P = P^\top > 0, \quad (30)$$

such that the time derivative of $W(z)$ along the trajectories of $f(z, L_q)$ is given by

$$\langle \nabla W, f(z, L_q) \rangle \leq -G|z|^2 \quad \forall z \in \mathbb{R}^8 \times [-r_{\max}, r_{\max}], \quad (31)$$

for both the controllers.

For any scalar $\mu > 0$, consider the following Lyapunov function

$$V(z_e) = e^{\mu\tau} W(z), \quad (32)$$

and notice that

$$\lambda_{\min}(P)|z|^2 \leq V(z_e) \leq \lambda_{\max}(P)e^{\mu T}|z|^2,$$

such that condition(A.1) is satisfied for $\alpha_1(|z|) := \lambda_{\min}(P)|z|^2$ and $\alpha_2(|z|) := \lambda_{\max}(P)e^{\mu T}|z|^2$.

Consider next the flow dynamics of $V(z_e)$,

$$\begin{aligned} \langle \nabla V(z_e), f(z, L_q) \rangle &= e^{\mu\tau} [\mu W(z) + \langle \nabla W(z), f(z, L_q) \rangle] \\ &\leq e^{\mu\tau} [\mu \lambda_{\max}(P) - \lambda_{\min}(G)]|z|^2, \end{aligned} \quad (33)$$

and let $\mu = \varepsilon \frac{\lambda_{\min}(G)}{\lambda_{\max}(P)}$, where $\varepsilon < 1$, such that the flow dynamics becomes

$$\langle \nabla V(z_e), f(z, L_q) \rangle \leq -\rho_1 |z|^2, \quad \forall z_e \in \mathcal{C}, f \in F(x), \quad (34)$$

where $\rho_1 = (1-\varepsilon)\lambda_{\min}(G)e^{\mu T} > 0$, such that the Lyapunov function decreases in flow, and condition (A.2) is satisfied.

Let $V(g)$ be the value of $V(z_e)$ after a jump, and $V(z_e)$ the value right before a jump. Looking at the jump dynamics, $V(g) - V(z_e)$ becomes

$$\begin{aligned} V(g) - V(z_e) &= W(z) - e^{\mu T} W(z) \\ &= -(e^{\mu T} - 1)W(z) \\ &\leq -\rho_2 |z|^2, \quad \forall z_e \in \mathcal{D}, g \in G(D) \end{aligned} \quad (35)$$

where $\rho_2 > 0$ since $e^{\mu T} > 1$ for $T > 0$, and condition (A.3) is satisfied.

Note that in the stability proof there was only a demand for the timer variable T to be strictly larger than zero. However, the main restriction is that both controllers need to satisfy (31) with the same quadratic (Lyapunov) function $W(z)$.

6. SIMULATION CASE STUDY

As a case study a vessel simulated in MATLAB/Simulink, using the MSS Toolbox [MSS 2010] is considered. The simulated vessel is a model ship called Cybership III (CS3), which is used for experiments in the Marine Cybernetics Lab (MC-lab) at the Norwegian University of Science and Technology.

The vessel model used is given by (1), where the mass and damping matrices are given in Appendix B.

The case study will compare the hybrid local/global controller implementation to a non-switching PID control law. Good tuning rules are stated in [Fossen 2011, Ch. 12]. A damping ratio corresponding to critical damping is chosen, and design time constants for the closed loop system and the controller gains are given in Appendix B.

The comparison is performed between the hybrid controller, a non-switching controller with nominal integral action (named "low gain"), and a non-switching controller with aggressive integral action (named "high gain"). For the local controller, we use $L = I_{3 \times 3}$, and for the global controller we use $L = 3I_{3 \times 3}$. For the hybrid controller, other relevant parameters are the timer variable T that is set to $T = 2s$, and $\eta_1^* = 0$, and $\eta_2^* = 1.0$.

The vessel is initialized in $(\eta, \nu) = 0$. The bias force b is acting in the NED frame, and in both North and East direction there is a sine wave of amplitude $0.3N$ with a frequency of $0.1Hz$. In North direction there is also a ramp component equal to $0.001t$ to illustrate a slowly varying bias. In both North, East, and yaw there is a step disturbance at $t = 500s$ of magnitude $5N$ in North, $3N$ in East, and $1Nm$ in yaw. In yaw there is also a white noise component with variance 0.001 . See Figure 3.

The results are shown in figures 1 to 4. In Figure 1 the error in position is shown, and also the switching signal q . From the plot of q it is observed that the global controller is active right after the step inputs, and some seconds later the controller switches back to the nominal integral action. Note that all controllers show similar ability to maintain position in normal conditions, but after the step disturbance the low tuned controller is slower to get back to position. This is illustrated more clearly in Figure 2 where the cumulative error in position is shown.

In Figure 3 the bias force is shown. Also, the integral action of the three controllers are plotted. Note that in North and East up to $t = 500s$ the high gain controller has a higher integral action amplitude, and therefore spends more control action trying to compensate for the sine waves. This is also illustrated in Figure 4 where the cumulative control action is shown between $t = 200s$ and $t = 300s$. This figure illustrates that the high gain controller spends more control effort than the hybrid and the low gain controllers (those two are equal in this time interval), whereas Figure 2 shows that this does not give a considerable gain in position performance. Note that the hybrid solution gives a trade-off between the two controllers, where the integral action is relaxed in the normal regime, and responsive when there is a step change in b . This ensures that the position offset is maintained

close to the level of the high gain integral action controller, without the additional control effort in the normal regime.

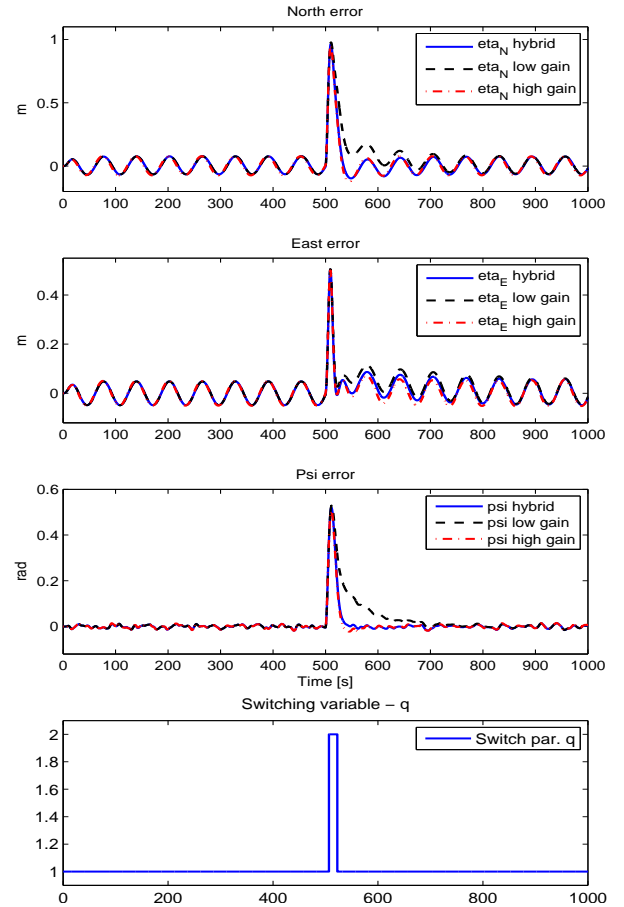


Fig. 1. North position error (top), East position error (second), yaw angle error (third), q (bottom).

7. CONCLUSION

In the following a hybrid control approach for integral action in the PID control law for dynamically positioned marine vessels is considered. The proposed approach allows the integral action to work aggressively when there is a large change is external force affecting the vessel, and to work slowly in normal conditions. The controller is shown to be uniformly globally asymptotically stable, and the benefit of the design is illustrated through simulations.

Appendix A. STABILITY THEOREM

Theorem 1. (Goebel et al. [2012] Theorem 3.18). (Sufficient Lyapunov conditions)

Let $\mathcal{H} = (\mathcal{C}, F, \mathcal{D}, G)$ be a hybrid system and let $\mathcal{A} \subset \mathbb{R}^n$ be closed. If V is a Lyapunov function candidate for \mathcal{H} and there exists $\alpha_1, \alpha_2 \in K_\infty$, and a continuous positive definite function ρ such that

$$\alpha_1(|x|_{\mathcal{A}}) \leq V(x) \leq \alpha_2(|x|_{\mathcal{A}}) \quad \forall x \in \mathcal{C} \cup \mathcal{D} \cup G(\mathcal{D}) \quad (\text{A.1})$$

$$\langle \nabla V(x), f \rangle \leq -\rho(|x|_{\mathcal{A}}) \quad \forall x \in \mathcal{C}, f \in F(x) \quad (\text{A.2})$$

$$V(g) - V(x) \leq -\rho(|x|_{\mathcal{A}}) \quad \forall x \in \mathcal{D}, g \in G(\mathcal{D}) \quad (\text{A.3})$$

then \mathcal{A} is uniformly globally pre-asymptotically stable for \mathcal{H} .

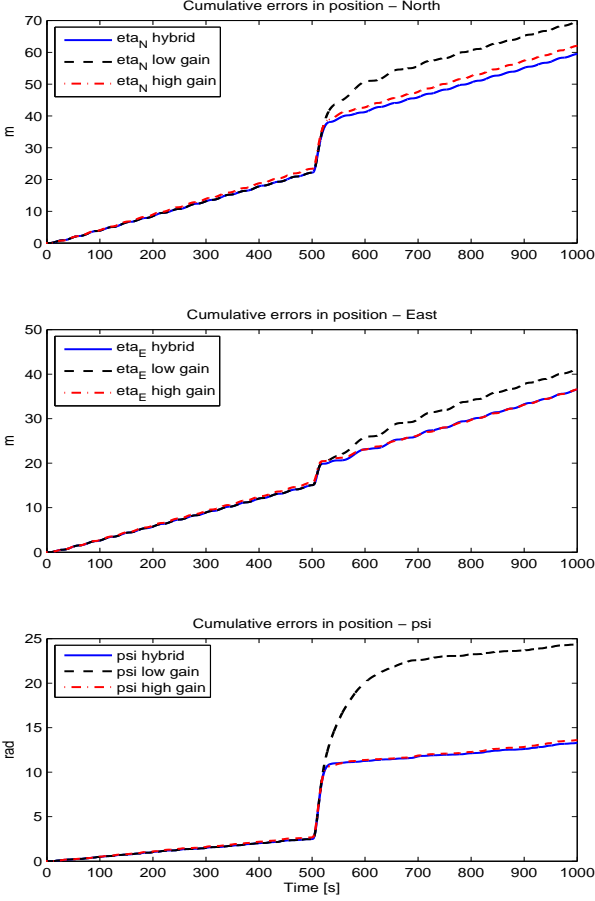


Fig. 2. Cumulative position error. North (top), East (middle), yaw (bottom).

Note on pre-asymptotically [Goebel et al. 2012, p. 45]: Pre-asymptotically indicates the possibility of a maximal solution that is not complete, even though it may be bounded. By including "pre-" this phenomena is included. Lyapunov functions do not guarantee existence or completeness of solutions, so this inclusion is reasonable.

Appendix B. CASE STUDY - DIMENSIONS AND TUNING

Parameters Cybership III

Mass matrix M ,

$$M = \begin{bmatrix} 76.88 & 0 & 0 \\ 0 & 149.58 & -1.07 \\ 0 & -1.07 & 34.10 \end{bmatrix},$$

and damping matrix D ,

$$D = \begin{bmatrix} 12.20 & 0 & 0 \\ 0 & 11.87 & 0.59 \\ 0 & 0.59 & 4.37 \end{bmatrix}.$$

Controller tuning

Critical damping is chosen for all degrees of freedom (DOF), such that the damping ratio ζ becomes

$$\zeta = \text{diag}\{1.0, 1.0, 1.0\}.$$

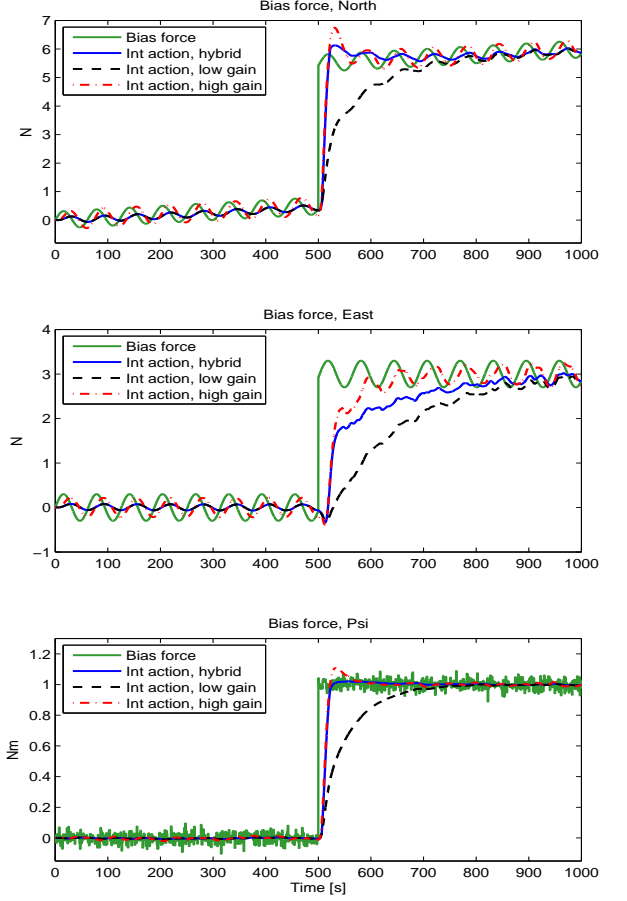


Fig. 3. Bias force and integral action. North (top), East (middle), yaw (bottom).

The design time constants

$$T_n = [T_{n_{surge}}, T_{n_{sway}}, T_{n_{yaw}}] = [15, 15, 20] [s],$$

and corresponding design natural frequencies

$$\omega_n = \text{diag}\left\{\left[\frac{2\pi}{T_{n_{surge}}}, \frac{2\pi}{T_{n_{sway}}}, \frac{2\pi}{T_{n_{yaw}}}\right]\right\},$$

and from Table 12.2 in [Fossen 2011, p. 374], the P and D-tuning become

$$K_p = \text{diag}\{M\omega_n^2\} = \text{diag}\{13.49, 26.24, 3.36\}$$

$$K_d = \text{diag}\{2M\zeta\omega_n - D\} = \text{diag}\{52.20, 113.44, 17.06\},$$

and K_i is assigned as follows

$$K_i = \text{diag}\{0.16, 0.16, 0.05\}.$$

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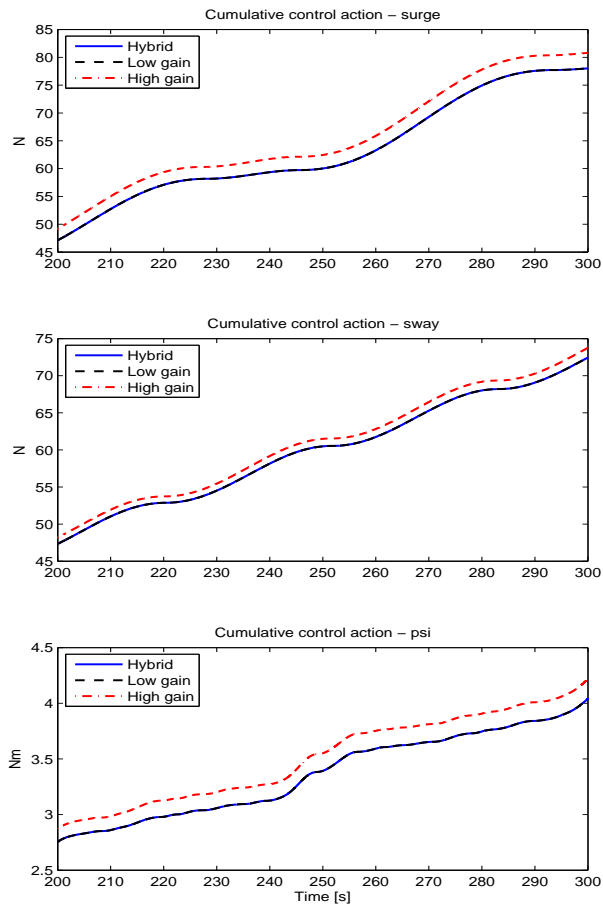


Fig. 4. Cumulative control action. Surge (top), sway (middle), yaw (bottom).

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