NTNU – Trondheim
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Generic Framework for Stochastic Modeling of Reinforced Concrete Deterioration Caused by Corrosion

Jürgen Hackl

Civil and Environmental Engineering (2 year)
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Supervisor: Jochen Kohler, KT
Co-supervisor: Gerhard Schickhofer, Graz University of Technology

Norwegian University of Science and Technology
Department of Structural Engineering

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Master's Thesis

Jürgen Hackl

Norwegian University of Science and Technology
Department of Structural Engineering
Jochen Köhler

Graz University of Technology
Faculty for Civil Engineering
Gerhard Schickhofer

Trondheim, August 2013

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Preface

This master's thesis is the result of my research in the field of structural engineering. It was written in cooperation with the Norwegian University of Science and Technology (NTNU) and the Graz University of Technology (TUGraz).

The principal supervisor of the master project was Jochen Köhler from the NTNU. He was also the one who enabled me the opportunity to do the research in Norway. Gerhard Schickhofer and Reinhard Brandner from the TUGraz acted as co-supervisors and made this cooperation possible.

This thesis is intended to be a summary of my research with the topic *generic framework for stochastic modeling of reinforced concrete deterioration caused by corrosion.* As the title suggests, is this work situated in the area of *structural reliability* and *concrete structures.* Hence, a variety of different topics are covered here, among others: Bayesian networks, computational mechanics, decision theory, design of concrete structures, material science, probability theory, and structural reliability analysis, just to name a few.

For me it is a matter of concern to publish a well written thesis, such that the reader can easily follow my thoughts and conclusions. In this sense - *have fun.*

Jürgen Hackl

Trondheim, August 2013

Statutory Declaration

I declare that I have authored this thesis independently, that I have not used other than the declared sources/resources, and that I have explicitly marked all material which has been quoted either literally or by content from the used sources.

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Graz, am ....

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Acknowledgment

First of all, I would like to thank my supervisor Jochen Köhler, who offered me the possibility to go to Norway and write my thesis at the Norwegian University of Science and Technology. Furthermore, he joined me at this unique educational journey, had always helpful advice just when I needed it and gave me so much academic freedom to explore and discover the field of Bayesian networks, structural reliability analysis and concrete technology. Here, I would like to express my deepest gratitude to him.

Additionally, I would like to thank Gerhard Schikhofer and Reinhard Brandner, who have selflessly offered their support on concerning the part of Graz University of Technology, even though my thesis does not fit so properly into their portfolio.

Special thanks goes also to Maung Min-Oo from the McMaster University, for his amazing course about probability. His classes and meetings have proved to be one of my best learning experiences at university. Furthermore, he motivated me to think out of the box.

Moreover, I would like to thank my friends and colleagues at Graz University of Technology, with whom I have had the pleasure of working together over the years. This includes especially Christine Markt, Daniel Bichler, Stefan Voithofer and all the members of the FVBau and ZSGeo.

Finally, I would like to thank my parents Annemarie and Erich, my sister Melanie and my brother Michael. Without you, I would not be where I am today. Thank you for all your support and encouragement.

Abstract

Reinforced concrete structures constitute an important fraction of the building infrastructure. This infrastructure is aging and a large number of structures will exceed the prescribed service period in the next decades. The aging of concrete structures is often accompanied by correspondent deterioration mechanisms. One of the major deterioration mechanisms is the corrosion of the reinforcing steel, caused by chloride ions or carbon dioxide exposure.

The decisions, made in connection to possible repair or renewals of these structures, have major implications on safety and cost efficiency in a societal dimension. Public authorities, entitled to administrate the infrastructure, are in need of schemes and methodologies that facilitate the optimal management of the already existing stock of structures, especially in regard to repair and maintenance planning.

In this thesis a generic framework for a stochastic modeling of reinforced concrete deterioration caused by corrosion is presented. This framework couples existing probabilistic models for chloride and carbonation initiation with models for the propagation and the effects of corrosion. For this purpose, a combination of structural reliability analysis and Bayesian networks is used for the reliability assessment of the reinforced concrete structure. This approach allows to compute probabilities of rare events for complex structures in an efficient way to update the model with new information from measurements, monitoring and inspection results.

This framework enables, for the first time, a holistic view of the current service life models, with corresponding sensitivity studies and finding optimal decisions for treating deteriorated reinforced concrete structures. The temporal evolvement of structures can also be represented and analyzed within this framework.

Keywords: Bayesian networks, corrosion, degradation, probabilistic modeling, probability, reinforced concrete, structural reliability analysis;

Zusammenfassung

Konstruktionen aus Stahlbeton bilden einen wichtigen Bestandteil im Bauwesen. Vieler dieser Ingenieurbauwerke wurden in den letzten Jahrzehnten errichtet und erreichen nun das Ende ihres geplanten Lebenszyklusses. Dieser Prozess der Alterung geht oft mit einer Verschlechterung der Eigenschaften des Stahlbetons einher. In vielen Fällen ist die Korrosion von Bewerungsstählen ein Grund dafür. Dieser Schädigungsprozess tritt vor allem dann auf, wenn die Konstruktion Chloriden oder Kohlendioxiden ausgesetzt ist.

Die Entscheidungen, über Reparatur oder Erneuerung dieser Bauwerke haben großen Einfluss auf die Sicherheit und Kosten. Einrichtungen die mit diesen Aufgaben betraut sind, benötigen ein Werkzeug, welches es auf einfache Art und Weise ermöglicht, zukünftige aber auch bereits bestehende Bauwerke diesbezüglich zu beurteilen.

Diese Arbeit präsentiert einen Rahmen für eine stochastische Modellierung des Schädigungsprozesses von Stahlbeton verursacht durch Korrosion der Stahlbewehrung. Wobei hier bereits bestehende probabilistische Modelle für die durch Chloride oder Kohlendioxid initiierte Korrosion mit Modellen für die Ausbreitung und die Effekte dieser, verknüpft werden. Zu diesem Zweck wird eine Kombination aus struktureller Zuverlässigkeit und Bayesian Netzwerken für eine Zuverlässigkeitsanalyse des Stahlbetonbauwerkes verwendet.

Dieser Ansatz erlaubt es, einerseits die Versagenswahrscheinlichkeit von komplexen Bauteilen effektiv zu ermitteln und andererseits können neue Informationen, die durch Messungen, Monitoring und Inspektionen ermittelt werden, in das Modell mit aufgenommen werden.

Ebenso ermöglicht dieses Modell zum ersten Mal eine ganzheitliche Betrachtung des Lebenszyklusses im Kontext der strukturellen Zuverlässigkeit, basierend auf den existierenden Modellen und erlaubt es, neben einer detaillierten Betrachtung der Parametersensitivität, optimale Entscheidungen für die Handhabung von Korrosion beschädigter Stahlbetonbauteile zu finden.

Schlüsselwörter: Bayesian Netzwerk, Korrosion, probabilistische Modellierung, Schädigungsprozess, Stahlbeton, strukturellen Zuverlässigkeit;

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Nomeclature

A_{pit}	cross sectional area of a pit
A_s	bar area
$A_{s,\text{nom}}$	nominal bar area
$A_{s,\%}$	loss of bar area in [%]
a	lower bound value of random variable
a	binding capacity for CO_2
α	shape parameter for a gamma random variable
α	shape parameter for a Gumbel random variable
α_y	empirical coefficient
$\boldsymbol{\alpha}$	normal vector
B	any set of real numbers
$\mathcal{B}$	Bayesian Network
$B(q,r)$	beta function
Beta	beta random variable
BN	Bayesian Network
b	upper bound value of random variable
b	beam width
β	rate parameter for a gamma random variable
β	reliability index
β_{HL}	Hasofer and Lind reliability index
β_t	target reliability index
C	concentration of some material
C_{cl}	chloride concentration at depth x_{cl} after time t
C_{crit}	critical chloride concentration
$C_{s,\text{ca}}$	surface concentration of carbon dioxide
$C_{s,\text{cl}}$	surface concentration of chloride
CI	Conditional Independent
CM	Coupled Model
CMC	Crude Monte Carlo
CoRe	Concrete Reliability
CO_2	carbon dioxide
CoV	coefficient of variation
Cov	covariance
CPD	Conditional Probability Distribution
cdf	cumulative distribution function
D	diffusion coefficient
D_o	chloride diffusion coefficient

D_{eff}	effective diffusion coefficient
DAG	Directed Acyclic Graph
DBN	Dynamic Bayesian Network
DCM	Dynamic Coupled Model
$\mathrm{d-sep}$	d-separated
d	effective depth
d_c	concrete cover depth
d_s	bar diameter
$d_{s,o}$	nominal bar diameter
Δ	Difference
E	Event (subset of S)
$\mathbb{E}$	expectation (expected value, mean)
$\mathcal{E}$	Edges
e	evidence (state of the random variable)
e_c	exposure class
e_e	exposure environment
erf	error function
$\emptyset$	Empty set
F_{Cl}	chloride corrosion rate factor
F_{Galv}	galvanic effect factor
F_{O_2}	oxygen availability factor
F_X	cumulative distribution function of X
FORM	First Order Reliability Method
f_c	compressive strength of concrete
f_X	probability density function of X
$f_{X,Y}$	joint probability density function of X and Y
f_Y	probability density function of Y
f_y	yield strength
$f_{y,o}$	nominal yield strength
$f_{y,\%}$	loss of yield strength in [%]
G	random variable for the permanent load
$\mathcal{G}$	graph
Gumbel	Gumbel random variable
GeNIe	Graphical Network Interface
GM	Graphical Model
g	limit state function
Γ	gamma random variable
$\Gamma(\alpha)$	gamma function
γ	partial safety factor
HCPM	Half-Cell Potential Measurement
h_X	importance sampling probability density function of X
I	indicator function
I_{corr}	corrosion current
IS	Importance Sampling
i_{corr}	corrosion current density
J	diffusion flux
$\mathbf{J}$	Jacobian matrix

$k_{c,\text{ca}}$	execution parameter for carbonation
$k_{c,\text{cl}}$	execution parameter for chloride penetration
$k_{c,\text{r}}$	execution parameter for concrete resistivity
$k_{Cl,\text{r}}$	chloride factor for concrete resistivity
$k_{e,\text{ca}}$	environmental parameter for carbonation
$k_{e,\text{cl}}$	environmental parameter for chloride penetration
$k_{RH,\text{r}}$	humidity factor for concrete resistivity
$k_{T,\text{r}}$	temperature factor for concrete resistivity
$k_{t,\text{ca}}$	test method parameter for carbonation
$k_{t,\text{cl}}$	test method parameter for chloride penetration
$k_{t,\text{r}}$	test method parameter for concrete resistivity
$\mathbf{L}$	Cholesky decomposition
L_o	reference length of reinforcement bar
L_U	uniform capacity length of reinforcement bar
LAB	Laboratory environment
$\ln \mathcal{N}$	lognormal random variable
λ	location parameter for a lognormal random variable
LRFD	Load and Resistance Factor Design
M	safety margin
M_{nom}	nominal flexural capacity
M_u	ultimate flexural capacity
m_o	constant for corrosion rate versus resistivity
μ	mean
μ	location parameter for a Gumbel random variable
$\mathcal{N}$	normal random variable
n	number of experiments
n_{ca}	age factor for carbonation
n_{cl}	age factor for chloride penetration
n_{r}	age factor of the concrete resistivity
∇	nabla operator
OS	Outdoor Sheltered environment
OUS	Outdoor Unsheltered environment
P	probability
PyBN	Python Bayesian Networks
PyRe	Structural Reliability Analysis with Python
p	obtained probability
p_{av}	average penetration
p_{corr}	probability of corrosion
p_f	probability of failure

$p_{f,\mathrm{ca}}$	probability of carbonation induced corrosion
$p_{f,\mathrm{cl}}$	probability of chloride induced corrosion
$p_{\max}$	pitting depth
p_X	probability mass function of X
$p_{X,Y}$	joint probability mass function of X and Y
p_Y	probability mass function of Y
pa	parent set of a node
pdf	probability density function
pH	pH-value
pmf	probability mass function
ϕ	probability density function of a standard normal random variable
Φ	cumulative distribution function of a standard normal random variable
Q	random variable for the variable load
Q_{corr}	percentage corrosion loss
q	shape parameter for a beta random variable
R	random variable for the resistance
R_{nom}	random variable for the nominal resistance
$\mathbf{R}$	vector of random variables for the resistance
$\mathbf{R}$	correlation matrix
R_{ca}	effective resistance of concrete to carbonation
R_{pit}	pitting factor
r	value of random variable R
r	shape parameter for a beta random variable
RC	Reinforced Concrete
RH	relative humidity
ρ	correlation coefficient
ρ	concrete resistivity
ρ_o	portential concrete resistivity
S	random variable for the internal strength or stress
$\mathbf{S}$	vector of random variables for the internal strength or stress
$\mathcal{S}$	sample space
SM	Single Model
SMILE	Structural Modeling, Inference, and Learning Engine
SORM	Second Order Reliability Method
SRA	Structural Reliability Analysis
s	value of random variable S
σ	standard deviation
σ^2	variance
T	temperature

$\mathcal{T}$	time space
t	time
t	exposure time
t_o	reference time
t_{corr}	time from the beginning of the corrosion
t_{cur}	curing period
t_{Hydr}	time of hydration
t_{sp}	time till loss of structural performance
θ	parameter
$\mathcal{U}$	continuous uniform random variable
$\mathcal{V}$	vertices or nodes
V_{corr}	corrosion rate
V_u	ultimate shear capacity
Var	variance
VI	Visual Inspection
w/c	water/cement ratio
X	Random variable
$\mathbf{X}$	vector of a random variable
x	Value of random variable
$\mathbf{x}$	value of a random vector
x_{ca}	depth of carbonation fornt
x_{cl}	depth of chloride penetration
Y	random variable
$\mathbf{Y}$	vector of a random variable
y	value of random variable
$\mathbf{y}$	value of a random vector
Z	random variable
$\mathbf{Z}$	vector of a random variable
z	value of random variable
$\mathbf{z}$	value of a random vector
$\mathbf{z}^*$	design point
z_e	environmental zones
ζ	scale parameter for a lognormal random variable

1 Introduction

"ἓν οἶδα ὅτι οὐδὲν οἶδα hèn oîda hóti oudèn oîda" (Socrates)[a]

[a]I know that I know nothing.

1.1 Context

1.1.1 Reinforced Concrete Structures

Reinforced concrete (RC) is a versatile and widely used building construction material. In many countries RC is a dominant structural material in engineered structures. The universal nature of RC structures is based on the wide availability of the constitutes of *concrete* and *reinforcing bars* on the simple skills, required on concrete construction, and on the economy of reinforced concrete, compared with other forms of construction. Hence, plain concrete and RC are widely used in all kinds of engineered structures. For example, in buildings of all sorts, underground structures, water tanks, wind turbine foundations and towers, offshore structures, dams, bridges and even ships. (Wight and MacGregor, 2012, p.1)

In terms of structural design, RC represents a composite material, because concrete has a high compressive strength, but a low tensile strength. Therefore, reinforcing bars are embedded in the concrete such that the tension stress can developed in the bars. For the RC composite material, concrete cracking is required in order to fully engage the tensile capacity of the reinforcement and to ensure a safe structural response of the RC structure to external influences. (Pease, 2010, p.3)

Concrete itself is also a composite material composed of aggregates, generally sand and gravel, chemically bound together by hydrated cement. The reinforcement is typically provided by high strength steel reinforcing bars. (Wight and MacGregor, 2012, p.43)

1.1.2 Degradation of Concrete Structures

Under certain circumstances deterioration of a RC structure leads to a a loss of structural functionality. One of the major deterioration mechanisms is *corrosion* of the reinforcing steel. The process of corrosion caused effects such, as cracking, spalling, or delamination of the concrete and also leads a reduction in the reinforcement cross-section and a loss of bond strength. (Bertolini et al., 2004, p.75). These changes are accompanied by a decrease of structural reliability, and lead to an increase of the probability of a failure event.

To avoid such an event, maintenance and repair work are performed on existing structures, providing a certain level of structural integrity. Unfortunately, the available economical budget is limited; nevertheless, the rate of structural deterioration appears to be increasing. (Stewart and Rosowsky, 1998a)

For instance, BRIME (2001) estimates that for France 39 %, Germany 37 %, Norway 26 % and United Kingdom 30 % of the concrete highway bridges are affected by deterioration and thus considered to be substandard. The annual amount of money spent for maintenance in Europe is located in a three-digit billion Euro range.

Also the U.S. Department of Transportation declares more than 30 % of the American highway bridges as deficient. (FHWA, 2012) To eliminate all bridge deficiencies an amount of $ 9.4 billion annually has to be spent over a period of 20 years. (W. Liu et al., 2011)

Hence, the aim of the responsible *decision makers* is to maintain and manage the portfolio of RC structures efficiently and within the economical budges available. The decision maker has the authority over the resources being allocated, but is also responsible for the consequences of the decision to third parties. (JCSS, 2008) However, finding the *optimal* decision to deal with deterioration of RC structures is not a trivial process; especially, when the problem is related to complex physical and chemical phenomena, which are hardly predictable and accompanied by high financial costs.

Therefore, a huge amount of research has been done over the last decades and is still going on. At the one hand side, physical models have been developed to understand the process of corrosion and to provide a tool to estimate a period of time (*service life*) during which a RC structure maintains a desired level of safety. On the other side, the development of new and enhanced methods for rational and efficient *risk assessment* and decision making has been driven.

1.1.3 Service Life Models

The period time a desired level of functionality is achieved, called service life. The end of the service life is defined by a *limit state* which is determined by the decision maker. Frequently used limit states are for example, the initiation of corrosion, appearance of visual corrosion, damages caused by corrosion, such as cracking or spalling, and the failure of the RC structure.

Beside frequently used standards and codes like ACI (2011), CSA (2004) or Eurocode 2 (2004), which are often based on simplified assumptions and do not take the process of corrosion into account, service life models , such as DuraCrete (2000b), LIFECON (2003), fib Bulletin 34 (2006), etc. exist, which consider those phenomena.

The basic approach of such a service live model is based on Tuutti (1982), where the service life is subdivided to two phases, *initiation* and *propagation.*

During the initiation phase the RC structure is exposed to environmental and mechanical effects. Especially, *chloride ions* and *carbon dioxide* can lead to steel corrosion, when they penetrate into the concrete and reach a critical depth considering the embedded reinforcement. If the onset of corrosion has started, the initial phase ends and the propagation phase starts.

During the propagation phase the process of corrosion proceeds, which leads to a reduction in the reinforcement cross-section and realization of corrosion products ("rust"). The reduction of the cross-section affects the capacity of the RC structure, which may lead to structural failure. The expanded volume of corrosion products may cause cracking and spalling of the covering concrete. (Pease, 2010, p.7)

While the models for the initiation phase are well documented, there is a lack of information for the propagation phase. Additionally, models for both phases are developed separately, such that connections from the initiation phase to the propagation phase can not be made in terms of a unified model. But in scope of a holistic view of the service life and the findings of optimal decisions for treating deteriorated RC structures this is unsatisfactory.

1.1.4 Risk Assessment in Engineering

In order to find an optimal decision, many parameters have to be taken into account. Not only physical models of reinforcement corrosion or the costs of maintenance and repair work have to be considered but also organizational structures, laws and regulations, expectations of the society, etc. may influence the *decision making process.* To fulfill all these conditions, a systematic procedure, as proposed in JCSS (2008), should be chosen.

A suitable manner to handle such a complex problem is to analyze the states of the real world and describe the problem in terms of an idealized system. This includes a justification of the parts that are not considered in the analysis. Hence, the system representation will have consequences for the level of detail in the risk analysis. (Faber, 2009, p.1.13)

The risk assessment of a given system is facilitated by a generic representation, where the exposure events, the induced damages, failures and consequences are represented.

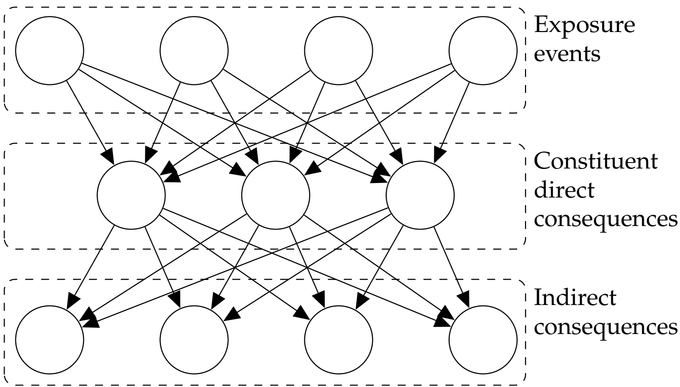


Figure 1.1: A generic representation of the system. With exposure events, constituent failure events and direct consequences onto follow-up or indirect consequences. Based on JCSS (2008).

The *exposure* of the system is represented as different exposure events acting on the constituents of the system. The damage of the system, caused by failure of the constituents is considered, to be associated with *direct consequences.* These consequences do not consider a loss of system functionality. However, based on the combination of events of constituent failures and the corresponding direct consequences, *indirect consequences* may occur. Indirect consequences describe any consequences associated with the loss of functionality of the system related to the direct consequences. Besides direct consequences, indirect consequences are very important for a risk assessments. (JCSS, 2008)

Any constituent in a system can be modeled as a system itself. For example, such a system could be a road network. The constituents here can be the roadways, tunnels, bridges, etc. The bridges themselves can also be considered as a system, where the constituents are the bridge deck, beams, arcs or columns. Furthermore, the system beam may include the subsystem reinforcement and concrete, as shown in Figure 1.2.

In a practical application the definition of exposures, constituents, direct and indirect consequences are generally given by the decision problem itself. In case of the optimal allocation of reliability concerning concrete bridges in a road network in, regarding the event of reinforcement corrosion, the exposures would be chloride ions and carbon dioxide and the constituents would be a single reinforcing steel with a direct consequence of corrosion onset followed by a reduction in the bar cross-section. This may lead to a failure of the reinforcement bar, regarding any operational or environmental load, followed by the indirect consequences of damage or failure of structural elements, and so on.

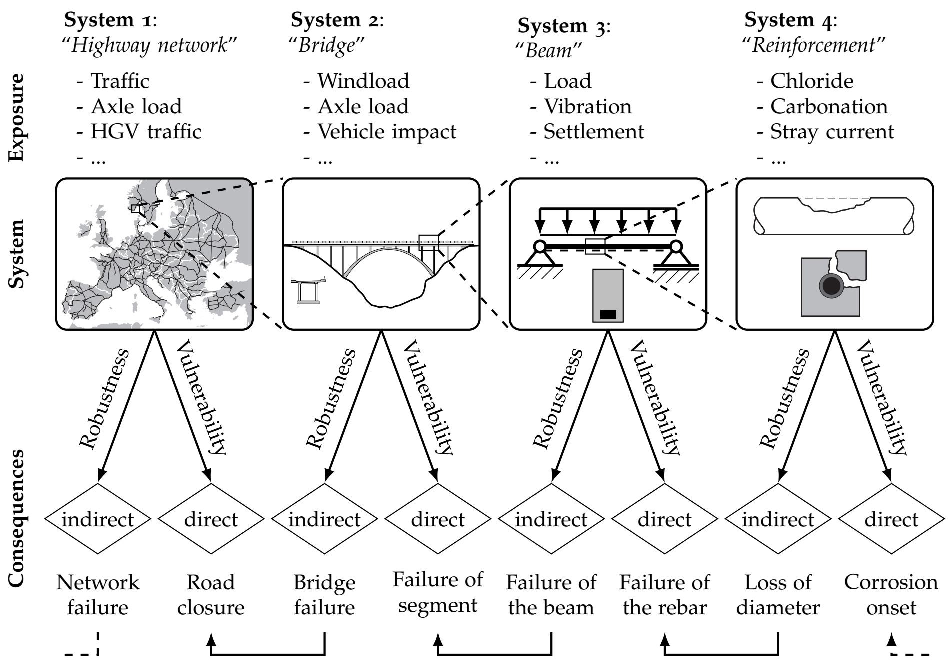


Figure 1.2: A generic system characterization of a road network and an infrastructure at different scales in terms of exposure, direct and indirect consequences. The vulnerability is related to the risk associated with the direct consequences. Additionally the lack of robustness is related to the degree of the total risk that is increased beyond the direct consequences. Based on JCSS (2008)

Even if only one bridge was damaged, there would be an indirect consequence in form of network functionalities as well as consequences due to possible budget overruns, caused by loss of individual bridges or other surrounding assets.

The constituents for the discussed road network and bridge may include, beside structural elements, passive and active protection and monitoring system as well, which serves to provide the structure with a sufficient safety. (JCSS, 2008)

As shown in Figure 1.2, a system change that leads to direct consequences, is denoted by *vulnerability.* They are related to risk over time in terms of expected potential future losses, considering all possible events they may lead to such. System changes that lead to indirect consequences are denoted by *robustness* and can be understood as a structure that will not loose functionality at a rate or extent disproportional to the cause of the change in the state variables. JCSS (2008)

Both, vulnerability and robustness, can be expressed in mathematical term. Therefore, a generic and indicator based risk assessment framework is used. This framework facilitates a *Bayesian* approach to risk assessment and full utilization of risk indicators.

Using such a framework, the risk associated with one particular event can be expressed as the probability that the event occurs and the consequences are associated with the event. Furthermore, the initial decision problem can be mathematical treated with the so-called *decision theory,* which is related to the *probability theory.*

1.2 Scope and Objectives

1.2.1 Objectives

This thesis is meant to develop a generic framework for stochastic modeling of reinforced concrete deterioration caused by corrosion. Thereby, the combination of structural reliability analysis and Bayesian networks provides a powerful tool for a computationally efficient and robust method, computing probabilities of rare events in complex structures, and also allows Bayesian updating of the model with measurements, monitoring and inspection results.

Although the largest part of this project was to program such a framework, the major part of this thesis deals with the theoretical principals and considerations about the implementation.

1.2.2 Research Approach

To achieve this aim, the following issues are identified and treated in this thesis:

1. Identification of the main phenomena for degradation of concrete and a discussion of existing probabilistic modeling approaches for those. Selecting the most appropriate models for implementation in a Bayesian network.
2. Implementation of the probabilistic models into a framework of structural reliability analysis and Bayesian networks.
3. Restructuring the Bayesian networks to provide a coupled model, treating both the initiation and the propagation phase.
4. Optimizing the coupled model to expand it over time.
5. Analysis of the developed framework in terms of sensitivity, initial condition and evolving over time.

1.2.3 Limitation and Assumptions

This thesis covers several different topics: Bayesian networks, decision theory, design of concrete structures, material science, probability theory, structural reliability analysis, etc.

For all these topics a large amount of research has been done. Countless research papers and books have been published; nevertheless, still more research needs to be done.

Even if more accurate and precise (probabilistic) models for the process of concrete degradation caused by corrosion are available, this thesis is restricted on the DuraCrete (2000b) model. Hence, the representation of exposure events is limited, to the level of detail of this model. However, the proposed framework itself, can be used for any probabilistic model.

The phenomenon of corrosion is associated with spatial and temporal variability. In this thesis only the temporal effects are treated. Spatial variability of deterioration can also be embedded in the probabilistic framework but is not done yet.

Dealing with continuous random variables, in context of Bayesian networks, leads often to the discretization of these variables. Furthermore, the size of Bayesian networks strongly depends on the size of the used variables. Hence, a fine discretization leads to a huge Bayesian networks. Unfortunately, the power of computers are limited, such that the amount of intervals for discretization is limited. This issue is treated in chapter 4.

1.3 Outline of the Thesis

This thesis is organized in seven chapters, after an introduction about the context of this work, the remaining chapters can be summarized as follows:

- *Chapter 2* provides a brief introduction into the fields of probability theory, structural reliability analysis and Bayesian networks. The issues discussed in that chapter are theoretical in nature and should review, summarize and enrich existing knowledge.
- *Chapter 3* deals with degradation of concrete structures. Especially, concrete deterioration caused by corrosion is discussed in detail. Therefore, probabilistic models for the initiation and propagation phases of corrosion are introduced and quantified.
- *Chapter 4* describes the modeling approaches and the implementation of the probabilistic models into a Bayesian network framework. Beyond that, several issues related with Bayesian networks, structural reliability analysis, continuous random variables, time dependency, and simplification approaches are discussed in this chapter.
- *Chapter 5* focuses on the analysis of the single, coupled and dynamic coupled models proposed in chapter 4. Beside a sensitivity analysis over the major parameter from the probabilistic models, an analysis for the initial model and the behavior in service is performed. This includes also Bayesian updating of the models when new information becomes available.
- *Chapter 6* discusses some specific results in detail. While chapter 5 analyzes more general properties of the models, chapter 6 is focuses on some particular case studies.
- *Chapter 7* summarizes and concludes the main work and provides recommendation for future investigations.

2 Theoretical Background

> *"Do not worry about our difficulties in mathematics, I assure you that mine are greater."*
> (Albert Einstein)

To provide a consistent nomenclature in the thesis, this chapter is a review of some standard results and definitions from probability theory, Bayes network theory and reliability analysis. All of this material is intended to focus only on the minimal subset of ideas, required to understand most of the discussion in the remainder of the master's thesis, rather than to provide a comprehensive overview.

2.1 Fundamentals of Probability Theory

2.1.1 Definition of Probability

The word "probability" is used in everyday life to refer to a degree of confidence that an *event* of an uncertain nature will occur. Probability theory deals with the formal foundation of discussing, for example estimates and rules. (Koller and Friedman, 2009, p.15)

There are at least two different ways to define the probability of an event.

One is called the *frequentistic* interpretation. In this view, probabilities represent long run frequencies of events. (Murphy, 2012, p.27) Here $P(E)$, the probability of the Event E is defined as

$$P(E) = \lim_{n \to \infty} \frac{n(E)}{n} \tag{2.1}$$

where $n(E)$ is the number of times in the first n repetitions of the experiment that the event E occurs. It is thus the *limiting frequency* of E. (Ross, 2009, p.26)

The other interpretation is called the *Bayesian* interpretation of probability. In this view, probability is used to quantify the *uncertainty* about something; hence it is fundamentally related to information rather than repeated trails. (Jaynes,

2003) In this interpretation the probability $P(E)$ of the Event E is formulated as degree of *belief* that E will occur.

$$P(E) = \text{degree of belief that } E \text{ will occur} \tag{2.2}$$

The degree of belief is a reflection of the state of mind of the individual person in terms of experience, expertise and preferences. In this respect the Bayesian interpretation of probability is subjective or more precisely person-dependent. (Faber, 2007, p.B-3) This demonstrates the possibility that it can be used to model uncertainty about events that do not have long term frequencies. (Murphy, 2012, p.27)

2.1.2 Sample Space and Events

For both views on probability, which were described above, a set of all possible outcomes of an experiment comprises a *sample space* of the experiment and is denoted by S. Here the term "experiment" refers to any type of process with uncertain outcome. For example, the throw of a dice or the failure of a bridge.

Any subset E of the sample space is known as *event*. There are two special types of events. The *certain event* is defined by the entire sample space. The implication of this definition is that a certain event will definitely occur. An *impossible event* is defined as an outcome that cannot occur. Therefore, the subset is empty, denoted by $\emptyset$. (Nowak and Collins, 2000, p.7)

Any two events E_1 and E_2 of a sample space S, define a new event $E_1 \cup E_2$ for instance, that all outcomes are either in E_1, in E_2 or in both. The event $E_1 \cup E_2$ is called *union* of the events E_1 and E_2.

Similar, the event $E_1 \cap E_2$ is called *intersection* and comprises all outcomes which are in E_1 and in E_2. A special case occurs when $E_1 \cap E_2 = \emptyset$. Here the events are *disjoint*, which is called *mutually exclusive.*

2.1.3 Axioms of Probability

To measure the degree of uncertainty of an experiment, a probability $P(E)$ is assigned to each event $E \subseteq S$. This probability must obey the following three axioms (Jensen and Nielsen, 2007, p.2):

Axiom 1 *Any event E must have a non-negative probability between* 0 *and* 1.

$$0 \leq P(E) \leq 1 \tag{2.3}$$

Axiom 2 *The probability of occurrence of an event corresponding to the entire sample space is equal to* 1.

$$P(S) = 1 \tag{2.4}$$

Axiom 3 *For any sequence of mutually exclusive events* $E_1, E_2, \ldots,$ *the probability of the combined event is the sum of the probabilities for the individual events.*

$$P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i) \tag{2.5}$$

2.1.4 Conditional Probabilities

Whenever a statement about a probability $P(E)$ of an event E is given, it is also implicitly given conditioned on other known factors. In this way any statement on probabilities is a statement conditioned on what else is known. These types of probabilities are called *conditional probabilities* (Jensen and Nielsen, 2007, p.4) and are generally denoted by

$$P(E_1|E_2) = p \tag{2.6}$$

Which means that the obtained probability p is the conditional probability that E_1 occurs, given that E_2 has occurred.

Definition 2.1 (Conditional Probability) *For two events* E_1 *and* E_2, *with* $P(E_2) > 0$, *the conditional probability for* E_1 *given* E_2 *is:*

$$P(E_1|E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)} \tag{2.7}$$

After multiplying both sides of the Equation (2.7) by $P(E_2)$, it follows the *product rule.*

Theorem 2.1 (Product Rule)

$$P(E_1 \cap E_2) = P(E_2)P(E_1|E_2) \tag{2.8}$$

A generalization of Equation (2.8), which provides an expression for the probability of the intersection of an arbitrary number of events is sometimes referred to as the *chain rule.* (Ross, 2009, p.62)

Theorem 2.2 (Chain Rule)

$$P(E_1 \cap \cdots \cap E_k) = P(E_1)P(E_2|E_1)\cdots P(E_k|E_1 \cap \cdots \cap E_{k-1}) \tag{2.9}$$

2.1.5 Bayes' Rule

From $P(E_1 \cap E_2) = P(E_2 \cap E_1)$ follows that $P(E_2)P(E_1|E_2) = P(E_1 \cap E_2) = P(E_1)P(E_2|E_1)$ is according to the product rule. This yields the *Bayes' rule.*

Theorem 2.3 (Bayes' Rule)

$$P(E_2|E_1) = \frac{P(E_2|E_1)P(E_1)}{P(E_2)} \tag{2.10}$$

This rule allows updating the beliefs about an event E_1 given, that an information about another event E_2 can be obtained. For this reason, $P(E_1)$ is called the *prior* probability of E_1, whereas $P(E_1|E_2)$ is called the *posterior* probability of E_1 given E_2. The probability $P(E_1|E_2)$ is called the *likelihood* of E_1 given E_2. (Jensen and Nielsen, 2007, p.5)

2.1.6 Independence and Conditional Independence

Sometimes information about one event E_2 does not change the belief about the occurrence of another event E_1, in this case E_1 and E_2 are *independent.*

Definition 2.2 (Independent) *An event E_1 is independent of event E_2 in P, denoted $P \models (E_1 \perp E_2)$, if*

$$P(E_1|E_2) = P(E_1) \tag{2.11}$$

The concept of independence also appears when there is conditioning on several events. Specifically, if information about the event E_2 does not change the belief about the event E_1 when event E_3 is already known. Than E_1 and E_2 are *conditional independent* given E_3.(Jensen and Nielsen, 2007, p.6)

Definition 2.3 (Conditional Independent) *An event E_1 is conditional independent of event E_2 given E_3 in P, denoted $P \models (E_1 \perp E_2 | E_3)$, if*

$$P(E_1|E_2 \cap E_3) = P(E_1|E_3) \tag{2.12}$$

Furthermore, when two events are conditional independent, the probability that both events occur can be calculated with the product rule.

$$P(E_1 \cap E_2|E_3) = P(E_1|E_3)P(E_2|E_3) \tag{2.13}$$

2.1.7 Random Variables

A *random variable* is defined as a function that maps events onto intervals on the axis of real numbers. Usually uppercase letters like $X, Y, Z, \ldots$ are used to denote random variables. Lowercase letters like $x, y, z, \ldots$ denote real numbers, which refer to a numerical value of a random variable.

Definition 2.4 (Random Variable) *A random variable is a real-valued function on the sample space S*

$$X : S \to \mathbb{R} \tag{2.14}$$

Random variables can take different sets of values. A random variable is called *discrete* if its *range* [1] is finite or at most countably infinite. It is called *continuous* if the random variable can take infinitely many values. (Bertsekas and Tsitsiklis, 2008, p.73)

The most important way to characterize a random variable is through the probabilities of the values that it can take. (Bertsekas and Tsitsiklis, 2008, p.74) In this context a *probability function* can be defined.

2.1.8 Basic Probability Function

A discrete random variable X, is captured by the *probability mass function* (pmf) of X, denoted p_X. In particular, $p_X(x)$ is the probability that a discrete random variable X is equal to a specific value x where x is a real number.

$$p_X(x) = P(X = x) \tag{2.15}$$

[1] the set of values that it can take

The probability law for a continuous random variable X can be described in terms of a non-negative function f_X, called the *probability density function* (pdf) of X, which satisfies

$$P(X \in B) = \int_B f_X(x)dx \tag{2.16}$$

for any set B of real numbers. (Ross, 2009, p.186)

The *cumulative distribution function* (cdf) is defined for both discrete and continuous random variables. The cdf of a random variable X is denoted by F_X and provides the probability $P(X \leq x)$. In particular, for every x follows, (Bertsekas and Tsitsiklis, 2008, p.148)

$$F_X(x) = P(X \leq x) = \begin{cases} \sum_{k \leq x} p_X(k) & X: \text{ discrete} \\ \int_{-\infty}^{x} f_X(t)dt & X: \text{ continuous} \end{cases} \tag{2.17}$$

To illustrate the relationship between pdf (pmf) and cdf, a continuous random variable X should be considered. The pdf and cdf functions might look like those shown in Figure 2.1. Equation (2.17) represents the shaded area under the pdf for the case $x = x_i$.

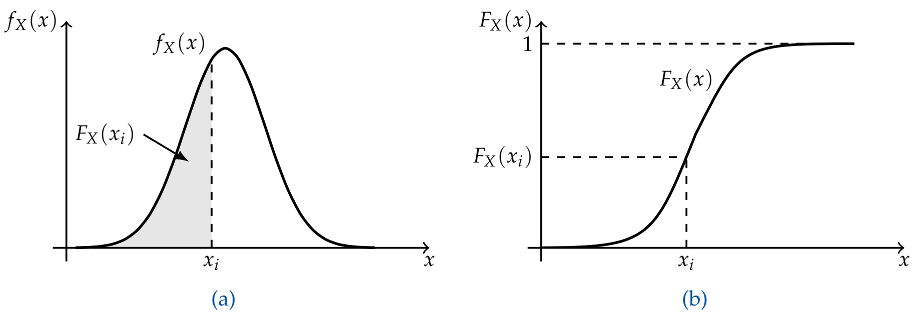


Figure 2.1: Example of (a) probability density function and (b) cumulative distribution function.

2.1.9 Parameters of a Random Variable

Although the value of a random variable X is uncertain, there are certain parameters which help to mathematically describe the properties of the variable. (Nowak and Collins, 2000, p.13)

One of the most important concepts in probability theory is the one of the expectation of a random variable. For a discrete random variable with the probability mass function p_X, the *expectation* (also called the *expected value, mean,* or the *first moment*) of X, denoted by $\mathbb{E}[X]$ is given by

$$\mu_X = \mathbb{E}[X] = \sum_x x p_X(x) \tag{2.18}$$

For a continuous random variable with the probability density function f_X, the *expected value* of X is given by

$$\mu_X = \mathbb{E}[X] = \int_{-\infty}^{\infty} x f_X(x) dx \tag{2.19}$$

There are many other quantities that can be associated with a random variable and its probability function. For example, the *2nd moment* of the random variable X is defined as the expected value of the random variable X^2. More generally, the *nth moment* is defined as $\mathbb{E}[X^n]$ and the expected value of the random variable X^n.

An other important quantity beside the mean is the *variance*, which is denoted by $\mathrm{Var}(X)$ and defined as

$$\mathrm{Var}(X) = \mathbb{E}\left[(X - \mathbb{E}[X])^2\right] \tag{2.20}$$

Thus, the variance is the expectation of the squared difference between X and its expected value. It indicates the spread of values of X around the expected value. (Koller and Friedman, 2009, p.33)

An alternative formulation of the variance is

$$\mathrm{Var}(X) = \mathbb{E}[\mathrm{X}^2] - (\mathbb{E}[\mathrm{X}])^2 \tag{2.21}$$

The variance of a discrete random variable is given by

$$\sigma_X^2 = \mathrm{Var}[X] = \sum_x (x - \mu)^2 p_X(x) \tag{2.22}$$

And for a continuous random variable the variance is

$$\sigma_X^2 = \mathrm{Var}[X] = \int_{-\infty}^{\infty} (x - \mu)^2 f_X(x) dx \tag{2.23}$$

Another technique of measuring the dispersion is σ_X, the *standard deviation* of X, which is defined as the square root of the variance

$$\sigma_X = \sqrt{\mathrm{Var}(X)} \tag{2.24}$$

The ratio between the standard derivation σ_X and the mean μ_X of a random variable X is called the *coefficient of variation* denoted by $\mathrm{CoV}[X]$

$$\mathrm{CoV}[X] = \frac{\sigma_X}{\mu_X} \tag{2.25}$$

The coefficient of variation provides a useful description of the variability of a random variable around its mean value. (Faber, 2007, p.D-7) This parameter is always taken to be positive by convention even if the mean may be negative. (Nowak and Collins, 2000, p.14)

2.1.10 Joint Probability Distributions

Often probability statements involve two or more random variables. All of which are associated with the same experiment, sample space, probability law and their values may relate to each other in some way. (Bertsekas and Tsitsiklis, 2008, p.92) In order to deal with such probabilities, a *joint probability mass function* (or *joint probability density function*) and a *joint cumulative probability distribution function* can be defined for any two (or more) random variables X and Y.

If X and Y are both discrete random variables, the joint pmf of X and Y is given by

$$p_{X,Y}(x,y) = P(X = x, Y = y) \tag{2.26}$$

The joint pmf determines the probability of any event that can be specified in terms of the random variables X and Y. In fact p_X, the pmf of X can be obtained from $p_{X,Y}(x,y)$ by (Bertsekas and Tsitsiklis, 2008, p.92)

$$p_X(x) = P(X = x) = \sum_y P(X = x, Y = y) = \sum_y p_{X,Y}(x,y) \tag{2.27}$$

The formula for p_Y, the pmf of Y, can be verified in a similar way. The mass functions p_X and p_Y are sometimes referred to as the *marginal* mass functions of X and Y.

If X and Y are both continuous random variables, the joint pdf of X and Y is a non-negative function $f_{X,Y}$ that satisfies

$$P((X,Y) \in B) = \iint_{(x,y)\in B} f_{X,Y}(x,y)dxdy \tag{2.28}$$

for every subset B in a two-dimensional plane. In the same manner as previously with the pmf, a *marginal* density function f_X of X can be given by

$$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)dy \tag{2.29}$$

2.1.11 Conditional Probability Distributions

Following the same considerations as in subsection 2.1.4, it is natural for two discrete random variables X and Y to define the *conditional probability mass function* of X given that $Y = y$, by

$$p_{X|Y}(x|y) = \frac{P(X = x, Y = y)}{P(Y = y)} = \frac{p_{X,Y}(x,y)}{p_Y(y)} \tag{2.30}$$

for all values of y such as $p_Y(y) > 0$. The conditional pmf is often convenient for the calculation of the joint pmf, using the similar approach of Equation (2.8), the product rule. (Bertsekas and Tsitsiklis, 2008, p.101)

$$p_{X,Y}(x,y) = p_Y(y)p_{X|Y}(x|y) \tag{2.31}$$

The conditional pmf can also be used to calculate the marginal pmfs.

$$p_X(x) = \sum_y p_{X,Y}(x,y) = \sum_y p_Y(y)p_{X|Y}(x|y) \tag{2.32}$$

If X and Y have a joint probability density function $f_{X,Y}(x,y)$, then the *conditional probability density function* of X given that $Y = y$, is defined for all values of y such as $f_Y(y) > 0$, by (Ross, 2009, p.266)

$$f_{X|Y}(x|y) = \frac{f_{X,Y}(x,y)}{f_Y(y)} \tag{2.33}$$

2.1.12 Covariance and Correlation

The *covariance* between two random variables X and Y measures the degree to which X and Y are linearly related. (Murphy, 2012, p.44)

Definition 2.5 (Covariance) *The covariance between X and Y, denoted by* $\text{Cov}(X,Y)$, *is defined by*

$$\text{Cov}(X,Y) = \mathbb{E}\left[(X - \mathbb{E}[X])(Y - \mathbb{E}[Y])\right] \tag{2.34}$$

If X and Y are *independent*, meaning $P(X,Y) = P(X)P(Y)$ then $\text{Cov}(X,Y) = 0$. If $\text{Cov}(X,Y) = 0$ then X and Y are *uncorrelated*. However, the converse is not true.[2]

If $\mathbf{x}$ is a d-dimensional random vector, its *covariance matrix* is defined to be the following symmetric, positive definite matrix:

$$\text{Cov}(\mathbf{x}) := \mathbb{E}\left[(\mathbf{x} - \mathbb{E}[\mathbf{x}])(\mathbf{x} - \mathbb{E}[\mathbf{x}])^T\right] \tag{2.35}$$

The covariance can be between 0 and infinity. A normalized measure with a lower bound of -1 and a upper bound of 1 is $\rho(X,Y)$, which is called the (Perason) *correlation coefficient* of two random variables X and Y

$$\rho(X,Y) = \frac{\text{Cov}(X,Y)}{\sqrt{\text{Var}(X)\text{Var}(Y)}} \tag{2.36}$$

A *correlation matrix* has the form

$$\mathbf{R} = \begin{pmatrix} \rho(X_1,X_1) & \rho(X_1,X_2) & \cdots & \rho(X_1,X_n) \\ \vdots & \vdots & \ddots & \vdots \\ \rho(X_n,X_1) & \rho(X_n,X_2) & \cdots & \rho(X_n,X_n) \end{pmatrix} \tag{2.37}$$

[2] uncorreltion does not imply independence!

2.1.13 Common Random Variables

The behavior of any random variable is defined by its cumulative distribution function $F_X(x)$ and in case of a discrete random variable X, also by its probability mass function $p_X(x)$. The probability density function $f_X(x)$ for a continuous random variable X is the first derivative of $F_X(x)$.

Physical phenomena or material properties in structural engineering applications are commonly modeled by those random variables. Many different types of probability distributions are available to model either discrete or continuous random variables. (Malioka, 2009, p.10)

The most important random variables used in this work are as following: uniform, normal, lognormal, gamma and beta. Each of these is briefly described in the following sections. A larger selection of some common probability distribution is summarized in the appendix.

Continuous Uniform Random Variable

A random variable X, where the pdf has a constant value for all possible values of X on an interval $[a, b]$, is called *uniform* or *uniformly distributed.* The pdf of $X \sim \mathcal{U}(a, b)$ is given by

$$\mathcal{U}(x; a, b) := f_X(x) = \begin{cases} \dfrac{1}{b-a} & \text{if } a \leq x \leq b \\ 0 & \text{otherwise} \end{cases} \tag{2.38}$$

where a and b define the lower and upper bounds of the random variable. The mean and the variance of the uniform random variable X are

$$\mu_X = \mathbb{E}[X] = \frac{a+b}{2} \tag{2.39}$$

$$\sigma_X^2 = \text{Var}(X) = \frac{(b-a)^2}{12} \tag{2.40}$$

Normal Random Variable

The *normal random variable* is probably one of the most important distributions used for engineering problems. For instance, for probabilistic modeling of uncertain phenomena which may be a result from a cumulative effect of several uncertain contributions. (Faber, 2007, p.D-19)

A random variable X is a *normal random variable,* or simply *Gaussian* or *normally distributed* with mean μ and variance σ^2 denoted by $X \sim \mathcal{N}(\mu, \sigma^2)$, if the pdf is given by

$$\mathcal{N}(x; \mu, \sigma^2) := f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right) \tag{2.41}$$

Here $\mu = \mathbb{E}[X]$ and $\sigma^2 = \text{Var}(X)$ are two *scalar parameters* that characterize the shape of the pdf. If $\mu = 0$ and $\sigma^2 = 1$, the random variable X follows a *standard normal* distribution. The pdf of a standard normal random variable is customary denoted by $\phi(x)$ and defined by

$$\phi(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}(x)^2\right) \tag{2.42}$$

The cdf of the standard normal variable is denoted by $\Phi(x)$. Negative values of x can be obtained from the symmetric relationship

$$\Phi(-x) = 1 - \Phi(x) \tag{2.43}$$

An important property of a normal random variable X is that Z a linear transformation of X is also normally distributed. This property can be used to *standardize* X by defining a new random variable Z given by

$$Z = \frac{X-\mu}{\sigma} \tag{2.44}$$

Furthermore, Z is a standard normal random variable. This fact allows to calculate the probability of any event defined in terms of X by redefining the event in terms of Z, which makes it possible to solve the problem. (Bertsekas and Tsitsiklis, 2008, p.156)

$$P(X \leq x) = P\left(\frac{X-\mu}{\sigma} \leq \frac{x-\mu}{\sigma}\right) = P\left(Z \leq \frac{x-\mu}{\sigma}\right) = \Phi\left(\frac{x-\mu}{\sigma}\right) \tag{2.45}$$

In particular, $\Phi(x)$ can be computed in terms of the *error function* (erf) (Murphy, 2012, p.38)

$$\Phi(x; \mu, \sigma) = \frac{1}{2}\left[1 + \text{erf}\left(\frac{z}{\sqrt{2}}\right)\right] \tag{2.46}$$

where z comes from Equation (2.44) and

$$\text{erf}(x) := \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2) dt \tag{2.47}$$

Lognormal Random Variable

A random variable X is a *lognormal random variable* whereas its logarithm Y is normally distributed. The lognormally distributed random variable $X = \exp(Y)$ is defined on the interval $[0, \infty]$. (Benjamin and Cornell, 1970, p.264)

In contrast to the normal distribution, the lognormal distribution is not simply described through the mean and the standard deviation of the lognormal random variable X, but the parameters can be expressed as function of the normal random variable $Y = \ln(X)$.

$$\lambda = \mu_Y = \mu_{\ln(X)} = \ln(\mu_X) - \frac{1}{2}\zeta^2 \tag{2.48}$$

$$\zeta^2 = \sigma_Y^2 = \sigma_{\ln(X)}^2 = \ln(\mathrm{CoV}_X^2 + 1) \tag{2.49}$$

The pdf for a lognormal random variable $X \sim \ln\mathcal{N}(\lambda, \zeta)$ is given by

$$\ln\mathcal{N}(x; \lambda, \zeta) := f_X(x) = \frac{1}{\sqrt{2\pi} x\zeta} \exp\left(-\frac{1}{2}\left(\frac{\ln(x) - \lambda}{\zeta}\right)^2\right) \tag{2.50}$$

The mean and the variance of the lognormal random variable X are

$$\mu_X = \mathbb{E}[X] = \exp\left(\lambda + \frac{\zeta^2}{2}\right) \tag{2.51}$$

$$\sigma_X^2 = \mathrm{Var}(X) = \exp\left(2\lambda + \zeta^2\right)\exp(\zeta^2) - 1 \tag{2.52}$$

Gamma Distribution

A random variable X that is *gamma distributed* with shape parameter $\alpha > 0$ and rate parameter $\beta > 0$ is denoted by $X \sim \Gamma(\alpha, \beta)$ and the pdf is given by

$$\Gamma(x; \alpha, \beta) := f_X(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} \exp(-\beta x) x^{\alpha-1} \qquad \forall x \geq 0 \tag{2.53}$$

$\Gamma(\alpha)$ is called the *gamma function*, which is defined as

$$\Gamma(\alpha) := \int_0^\infty e^{-y} y^{\alpha-1} dy \tag{2.54}$$

The mean and variance of a gamma random variable X can be calculated as followed:

$$\mu_X = \mathbb{E}[X] = \frac{\alpha}{\beta} \tag{2.55}$$

$$\sigma_X^2 = \mathrm{Var}(X) = \frac{\alpha}{\beta^2} \tag{2.56}$$

Beta Distribution

A random variable X is said to have a *beta distribution* if its pdf is given by

$$\mathrm{Beta}(x; q, r, a, b) = f_X(x) = \frac{1}{B(q,r)} \frac{(x-a)^{q-1}(b-x)^{r-1}}{(b-a)^{q+r-1}} \tag{2.57}$$

In which $B(q,r)$ is the *beta function* defined by

$$B(q,r) := \frac{\Gamma(q)\Gamma(r)}{\Gamma(q+r)} \tag{2.58}$$

A beta distributed random variable $X \sim \mathrm{Beta}(q,r,a,b)$ with the shape parameters $q > 0$ and $r > 0$, can model phenomena which set of possible values, is some finite interval $[a,b]$.

The mean and variance of a beta random variable X are

$$\mu_X = \mathbb{E}[X] = a + q\frac{b-a}{q+r} \tag{2.59}$$

$$\sigma_X^2 = \mathrm{Var}(X) = \frac{b-a}{q+r}\sqrt{\frac{qr}{q+r+1}} \tag{2.60}$$

2.2 Bayesian Networks

In this section *graphical models* (GMs), which are the basic graphical feature for *Bayesian networks* (BNs), will be introduced. (Jensen and Nielsen, 2007, p.23) This theory is implemented in the Python library, *Python Bayesian Networks* (PyBN) by Hackl (2013a) and is an important part for the subsequent work.

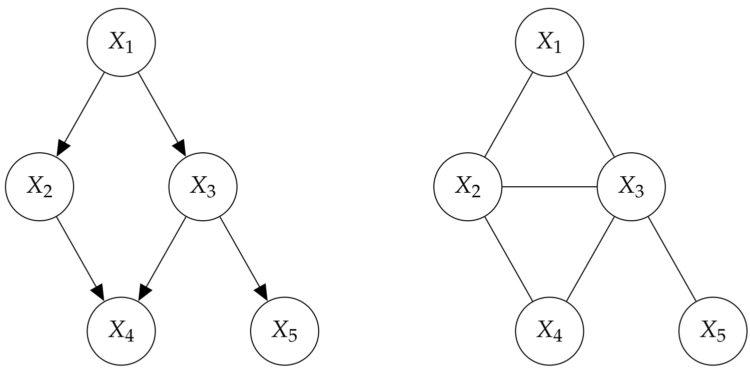
2.2.1 Graphical Notation and Terminology

Graphical models (GMs) are tools used to visually illustrate and work with conditional independence (CI) among variables in given problems. (Stephenson, 2000) In particular, a *graph* consists of a set $\mathcal{V}$ *vertices* (or *nodes*) and a set $\mathcal{E}$ of *edges* (or *links*). The vertices correspond to random variables and the edges will denote a certain relationship between two variables. (Pearl, 2000, p.12)

$$\mathcal{G} := \text{graph } G = (\mathcal{V}, \mathcal{E}) \tag{2.61}$$

Whit $\mathcal{V} = \{X_1, X_1, \dots X_n\}$ and $\mathcal{E} = \{(X_i, X_j) : i \neq j\}$.

A pair of nodes X_i, X_j can be connected by a *direct edge* $X_i \rightarrow X_j$ or an *undirected edge* $X_i - X_j$. A graph is called *directed graph* if all edges are either $X_i \rightarrow X_j$ or $X_i \leftarrow X_j$ and called *undirected graph* if all edges are $X_i - X_j$. (Koller and Friedman, 2009, p.34)



(a) (b)

Figure 2.2: (a) A simple acyclic directed graph (DAG) numbered in topological order. (b) A simple undirected graph. Based on Murphy (2012)

Two variables connected by an edge are called *adjacent*. A *path* consists of a series of nodes, where each one is connected to the previous one by an edge. If a path in a graph is a sequence of edges in order that each edge has a directionality going in the same direction, then it is called *directed path*. For example, $X_1 \rightarrow X_2 \rightarrow X_4$ in Figure 2.2(a). A directed graph may include *direct cycles* when a direct part starts and ends at the same node, for instance $X \rightarrow Y \rightarrow X$, but this includes no *self-loops* ($X \rightarrow X$). A graph that contains no directed cycles is called *acyclic*, whereas a graph that is directed and acyclic is called *directed acyclic graph* (DAG) (Pearl, 2000, p.12). This kind of graph is one of the central concepts which underlies Bayesian networks. (Koller and Friedman, 2009, p.37)

To denote the relationships in a graph, the terminology of kinship is used. A *parent* to *child* relationship in a directed graph occurs in case there is an edge from $X_1 \rightarrow X_2$. X_1 is called the parent of X_2 and X_2 the child of X_1. If X_4 is a child of X_2 than X_1 is its *ancestor* and X_4 is X_1 *descendant.* A *family* is the set of vertices composed of X and the parents of X; for example, $\{X_2, X_3, X_4\}$ in Figure 2.2(a). The term *adjacent* (or *neighbor*) is used to describe the relationship between two nodes connected in an undirected graph. (Stephenson, 2000)

Furthermore, the notation of a *forest* is used to define some properties of a directed graph. So a forest is a DAG where each node has either one parent or none at all. A *tree* is a forest where only one node, called the *root*, has no parent. However, a node without any parents is called *leaf.* (Murphy, 2012, p.309)

2.2.2 Sturcture of Bayesian Networks

Formally BNs are DAG in which each node represents a random variable, or uncertain quantity, which can take on two or more possible values. The edges signify the existence of direct causal influences between linked variables. The strengths of these influences are quantified by conditional probabilities.

In other words, each variable X_i is a stochastic function of its parents, denoted by $P(X_i|\text{pa}(X_i))$. It is called *conditional probability distribution* (CPD), when $\text{pa}(X_i)$ is the parent set of a variable X_i. The conjunction of these local estimates specifies a complete and consistent global model (joint probability distribution) on the basis of which all probability queries can be answered. A representing joint probability distribution for all variables is expressed by the chain rule for Bayesian networks (Pearl, 1988, p.51)

Theorem 2.4 (Chain Rule for Bayesian Networks) *Let $\mathcal{G}$ be a DAG over the variables $\mathcal{V} = \{X_1, \ldots, X_n\}$. Then $\mathcal{G}$ specifies a unique joint probability distribution $P(X_1, \ldots, X_n)$ given by the product of all CPDs*

$$P(X_1, \ldots, X_n) = \prod_{i=1}^{n} P(X_i|\text{pa}(X_i)) \tag{2.62}$$

This process is called *factorization* and the individual factors $\text{pa}(X_i)$ are called CPDs or local *probabilistic models.* (Koller and Friedman, 2009, p.62) This properties are used to define a Bayesian network in a formal way.

Definition 2.6 (Bayesian Network) *A Bayesian Network $\mathcal{B}$ is a tuple $\mathcal{B} = (\mathcal{G}, P)$, where $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is a DAG, each node $X_i \in \mathcal{V}$ corresponds to a random variable and P is a set of CPDs associated with $\mathcal{G}$'s nodes. The Bayesian Network $\mathcal{B}$ defines the joint probability distribution $P_{\mathcal{B}}(X_1, \ldots, X_n)$ according to Equation (2.62).*

For example, is the joint probability distribution corresponding to the network in Figure 2.2(a) given by

$$P(X_1, X_2, X_3, X_4, X_5) = P(X_1)P(X_2|X_1)P(X_3|X_1)P(X_4|X_2, X_3)P(X_5|X_3) \quad (2.63)$$

This structure of a BN can be used to determine the marginal probability or likelihood of each node holding on of its state. This procedure is called *marginalisation.*

2.2.3 Inserting Evidence

A major advantage of BNs comes by calculating new probabilities, for example, if new information is observed. The effects of the observation are propagated throughout the network and in every propagation step the probabilities of a different node are updated.

New information in a BN are denoted as *evidence* and defined by a subset E of random variables in the model and an instantiation e to these variables.

The task is to compute $P(X|E = e)$, the *posterior probability distribution* over the values x of X, conditioned on the fact that $E = e$. This expression can also be viewed as the marginal over X in the distribution that obtains by conditioning on e. (Koller and Friedman, 2009, p.26)

Theorem 2.5 *Let $\mathcal{B}$ be a Bayesian network over the variables $\mathcal{V} = \{X_1, \ldots, X_n\}$ and $e = \{e_1, \ldots, e_m\}$ some observations. Then*

$$P(\mathcal{V}, e) = \prod_{X \in \mathcal{V}} P(X|\text{pa}(X)) \cdot \prod_{i=1}^{m} e_i \quad (2.64)$$

and for $X \in \mathcal{V}$ follows

$$P(X|e) = \frac{\sum_{\mathcal{V} \setminus \{X\}} P(\mathcal{V}, e)}{Pe} \quad (2.65)$$

If X_1 and X_2 are *d-separated* in a BN with evidence $E = e$ entered, then $P(X_1|X_2, e) = P(X_1|e)$, this means that X_1 and X_2 are conditional independent given E, denoted $P \models (X_1 \perp X_2|E)$. (Pearl, 1988, p.117)

2.2.4 Network Models

At the core of any graphical model is a set of conditional independence assumptions. The aim is to understand when an independence $(X_1 \perp X_2|X_3)$ can be guaranteed. In other words, is it possible that X_1 can influence X_2 given X_3? (Koller and Friedman, 2009, p.70)

Deriving these independencies for DAGs is not always easy because of the need to respect the orientation of the directed edges. (Murphy, 2012, p.324) However, a separability criterion, which takes the directionality of the edges in the graph into consideration, is called *d-separation* [3]. (Pearl, 1988, p.117)

Definition 2.7 (d-separation) *If X_1, X_2 and X_3 are three subsets of nodes in a DAG $\mathcal{G}$, then X_1 and X_2 are d-separated given X_3, denoted $\mathrm{d-sep}_{\mathcal{G}}(X_1; X_2|X_3)$, if there is no path between a node X_1 and a node X_2 along with the following two conditions hold:*

1. *the connection is serial of diverging and the state of X_3 is observed, or*
2. *the connection is converging and neither the state of X_3 nor the state of any descendant of X_3 is observed.*

If a path satisfies the d-separation condition above, it is said to be *active,* otherwise it is said to be *blocked* by X_3.

Networks are categorized according to their configuration. The underlying concept can be illustrated by three simple graphs and thereby conditional independencies can be implemented. (Pernkopf et al., 2013)

Serial Connection

The BN illustrated in Figure 2.3 is a so called *serial connection.* Here X_1 has an influence on X_3, which in turn has an influence on X_2. Evidence about X_1 will influence the certainty of X_3, which influences the certainty of X_2, and vice versa by observing X_2. However, if the state of X_3 is known, then the path is blocked and X_1 and X_2 become independent. Now X_1 and X_2 are d-separated given X_3. (Jensen and Nielsen, 2007, p.26)

[3] The notation d-separated stands for "directed separation"

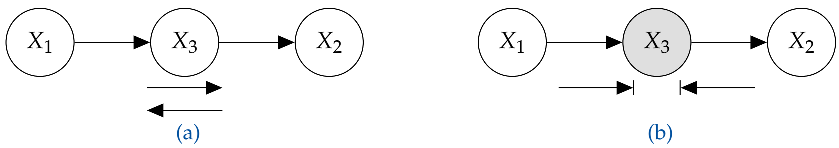


Figure 2.3: Serial connection. (a) When X_3 is not observed the path is active, X_1 and X_2 are marginally dependent. (b) When X_3 is observed (node is shaded), then the path is blocked, X_1 and X_2 are conditionally independent. Based on Jordan (2007)

Diverging Connection

In Figure 2.4 a so called *diverging connection* for a BN is illustrated. Here influence can pass between all the children of X_3, unless the state of X_3 is known. When X_3 is observed, then variables X_1 and X_2 are conditional independent given X_3, while, when X_3 is not observed they are dependent in general. (Jensen and Nielsen, 2007, p.27)

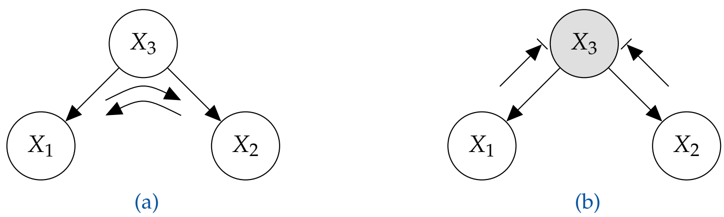


Figure 2.4: Diverging connection. (a) When X_3 is not observed the path is active, X_1 and X_2 are marginally dependent. (b) When X_3 is observed (node is shaded), then the path is blocked, X_1 and X_2 are conditionally independent. Based on Jordan (2007)

Converging Connection

A *converging connection,* illustrated in Figure 2.5 is more sophisticated than the two previous cases. As far nothing is known about X_3 except what may be inferred from knowledge of its parents X_1 and X_2, the parents are independent. This means that an observation of one parent cannot influence the certainties of the other. However, if anything is known about the common child X_3, then the information on one possible cause may tell something about the other cause. (Jensen and Nielsen, 2007, p.20)

In other words, variables which are marginally independent become conditional dependent when a third variable is observed. (Jordan, 2007)

This important effect is known as *explaining away* or *Berkson's paradox*. (Berkson, 1946)

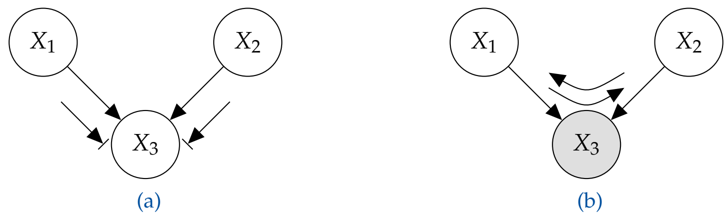


Figure 2.5: Converging connection. (a) When X_3 is not observed the path is blocked, X_1 and X_2 are conditionally independent. (b) When X_3 is observed (node is shaded), then the path is active, X_1 and X_2 are marginally dependent. Based on Jordan (2007)

2.2.5 Dynamic Bayesian Networks

A *dynamic Bayesian network* (DBN) is just another way to represent stochastic processes using a DAG. To model domains that evolve over time, the *system state* represents the system at time t and is an assignment of some set of random variables $\mathcal{V}$. Thereby the random variable X_i itself is instantiated at different points in time t, represented by X_i^t and called *template variable*. To simplify the problem, the timeline is discretized into a set of *time slices* with a predetermined time interval Δ. This leads to a set of random variables in form of $\mathcal{V}^0, \mathcal{V}^1, \ldots, \mathcal{V}^t, \ldots, \mathcal{V}^T$ with a joint probability distribution $P(\mathcal{V}^0, \mathcal{V}^1, \ldots, \mathcal{V}^t, \ldots, \mathcal{V}^T)$ over the time $\mathcal{T}$, abbreviated by $P(\mathcal{V}^{0:T})$. This distribution can be reparameterized by using Equation (2.9); the chain rule for probabilities. (Koller and Friedman, 2009, p.201)

$$P(\mathcal{V}^{0:T}) = \prod_{t=0}^{T-1} P(\mathcal{V}^{t+1}|\mathcal{V}^{0:t}) \tag{2.66}$$

This is the product of conditional distributions, for the variables in each time slice are given by the previous ones. Assuming conditional independence, the formulation can be simplified in the same way as discussed in section 2.2.2.

Definition 2.8 (Markov assumption) *If the present is known, then the past has no influence on the future.*

$$(\mathcal{V}^{t+1} \perp \mathcal{V}^{0:(t+1)}|\mathcal{V}^t) \tag{2.67}$$

This *Markov assumption* allows to define a compact representation of a DBN

$$P(\mathcal{V}^0, \dots, \mathcal{V}^T) = \prod_{t=0}^{T-1} P(\mathcal{V}^{t+1} | \mathcal{V}^t) \tag{2.68}$$

Plate Notation

A short graphical representation of template-based models are *plate models*. Additional to nodes and edges are object called *plates* used. Objects in the plate share the same set of attributes. By convention the objects within the plate will get repeated when the model is *unrolled*.

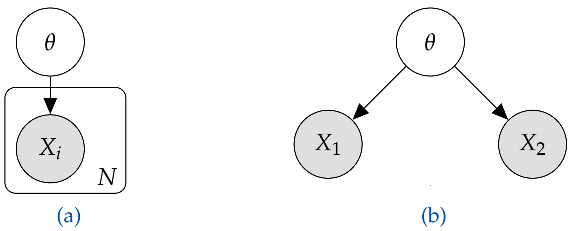


Figure 2.6: (a) Plate notation. This represents the same model as the right one, except the repeated nodes are inside the plate. The number in the right hand corner N specifies the number of repetitions of the node X_i. (b) It is the unrolled model of the system. Based on Murphy (2012)

2.3 Structural Reliability Analysis

Structural reliability analysis (SRAs) is an important part to handle structural engineering applications. This section provides a brief introduction to this topic and is also the theoretical background for the Python library, *Structural Reliability Analysis with Python* (PyRe) by Hackl (2013c).

2.3.1 Limit States

The word *structural reliability* refers to the meaning "how much reliable is a structure in terms of fulfilling its purpose" (Malioka, 2009, p.7). The performance of structures and engineering systems was based on *deterministic parameters* even for a long time, even if it was known that all stages of the system involve uncertainties. SRA provides a method to take those *uncertainties* into account in a consistent manner. In this content the term *probability of failure* is more common than *reliability.* (Malioka, 2009, p.7)

In general, the term " *failure* " is a vague definition because it means different things in different cases. For this purpose the concept of *limit state* is used to define failure in the context of SRA. (Nowak and Collins, 2000, p.91)

Definition 2.9 (Limit State) *A limit state represents a boundary between desired and undesired performance of a structure.*

This boundary is usually interpreted and formulated within a mathematical model for the *functionality* and *performance* of a structural system, and expressed by a *limit state function.* (Ditlevsen and Madsen, 2007, p.13)

Theorem 2.6 (Limit State Function) *Let* $\mathbf{X}$ *describe a set of random variables* $\{X_1, \ldots, X_n\}$ *which influence the performance of a structure. Then the functionality of the structure is called limit state function, denoted by g and given by*

$$g(\mathbf{X}) = g(X_1, \ldots, X_n) \tag{2.69}$$

The boundary between desired and undesired performance would be given when $g(\mathbf{X}) = 0$. If $g(\mathbf{X}) > 0$, it implies a *desired performance* and the structure is *safe.* An *undesired performance* is given by $g(\mathbf{X}) \leq 0$ and it implies an unsafe structure or *failure* of the system. (J. Baker, 2010)

The *probability of failure* p_f is equal to the probability that an undesired performance will occur. It can be mathematical expressed as

$$p_f = P(g(\mathbf{X}) \leq 0) = \int_{g(\mathbf{X}) \leq 0} \cdots \int f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \tag{2.70}$$

assuming that all random variables $\mathbf{X}$ are continuous. However, there are three major issues related to the Equation (2.70), proposed by J. Baker (2010):

1. There is not always enough information to define the complete joint probability density function $f_X(\mathbf{x})$.
2. The limit state function $g(\mathbf{X})$ may be difficult to evaluate.
3. Even if $f_X(\mathbf{x})$ and $g(\mathbf{X})$ are known, numerical computing of high dimensional integrals is difficult.

For this reason various methods have been developed to overcome these challenges. The most common ones are the *Monte Carlo simulation method* and the *First Order Reliability Method* (FORM).

2.3.2 The Classical Approach

Before discussing more general methods, the principles are shown on a "historical" and simplified limit state function.

$$g(R, S) = R - S \tag{2.71}$$

Where R is a random variable for the *resistance* with the outcome r and S represents a random variable for the *internal strength* or *stress* with the outcome of s. (Lemaire et al., 2010, p.39) The probability of failure is according to Equation (2.70):

$$p_f = P(R - S \leq 0) = \iint_{r \leq s} f_{R,S}(r, s) dr ds \tag{2.72}$$

If R and S are independent the Equation (2.72) can be rewritten as a convolution integral, where the probability of failure p_f can be (numerical)[4] computed. (Schneider, 2007, p.72)

$$p_f = P(R - S \leq 0) = \int_{-\infty}^{\infty} F_R(x) f_S(x) dx \tag{2.73}$$

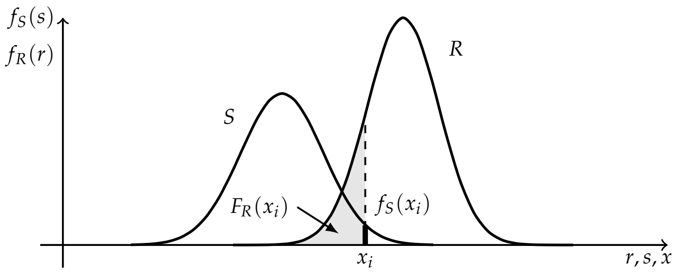


Figure 2.7: Classical Approach $R - S$. Where R and S are any random variables and $x_i \in [-\infty, +\infty]$. An integration over the whole domain computes the probability of failure p_f. Based on Schneider (2007)

[4] Only simple cases can be performed analytically

If R and S are independent and $R \sim \mathcal{N}(\mu_R, \sigma_R)$ as well as $S \sim \mathcal{N}(\mu_S, \sigma_S)$ are normally distributed, the convolution integral (2.73) can be evaluated analytically.

$$M = R - S \tag{2.74}$$

where M is the *safety margin* and also normal distributed $M \sim \mathcal{N}(\mu_M, \sigma_M)$ with the parameters

$$\mu_M = \mu_R - \mu_S \tag{2.75}$$

$$\sigma_M = \sqrt{\sigma_R^2 + \sigma_S^2} \tag{2.76}$$

The probability of failure p_f can be determined by the use of the standard normal distribution function (2.45)

$$p_f = \Phi\left(\frac{0 - \mu_m}{\sigma_M}\right) = \Phi(-\beta) \tag{2.77}$$

Where β is the so called *Cornell reliability index,* named after Cornell (1969), and is equal to the number of the standard derivation σ_M by which the mean values μ_M of the safety margin M are zero. (Faber, 2009, p.5.14)

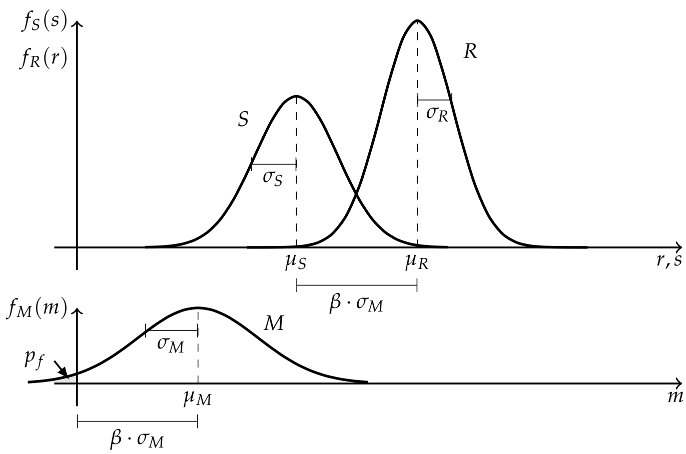


Figure 2.8: Are the random variables R and S normally distributed also the safety margin M is a normal random variable. The reliability index β provides the information how often σ_M has space between the origin and μ_M. Based on Schneider (2007)

2.3.3 Hasofer and Lind Reliability Index

The reliability index can be interpreted as a measure of the distance to the *failure surface*, as shown in Figure 2.8. In the one dimensional case the standard deviation of the safety margin was used as scale. To obtain a similar scale in the case of more basic variables, Hasofer and Lind (1974) proposed a non-homogeneous linear mapping of a set of random variables $\mathbf{X}$ from a physical space into a set of normalized and uncorrelated random variables $\mathbf{Z}$ in a normalized space. (Madsen et al., 2006, p.50)

Definition 2.10 (Hasofer and Lind Reliability Index) *The Hasofer and Lind reliability index, denoted by β_{HL}, is the shortest distance $\mathbf{z}^*$ from the origin to the failure surface $g(\mathbf{Z})$ in a normalized space.*

$$\beta_{HL} := \beta = \boldsymbol{\alpha}^T \mathbf{z}^* \tag{2.78}$$

The shortest distance to the failure surface $\mathbf{z}^*$ is also known as *design point* and $\boldsymbol{\alpha}$ denotes the normal vector to the failure surface $g(\mathbf{Z})$ and is given by

$$\boldsymbol{\alpha} = -\frac{\nabla g(\mathbf{z}^*)}{|\nabla g(\mathbf{z}^*)|} \tag{2.79}$$

where $\nabla g(\mathbf{z})$ is the gradient vector, which is assumed to exist: (Madsen et al., 2006, p.53)

$$\nabla g(\mathbf{z}) = \left(\frac{\partial g}{\partial z_1}(\mathbf{z}), \ldots, \frac{\partial g}{\partial z_n}(\mathbf{z}) \right) \tag{2.80}$$

Finding the reliability index β is therefore an optimization problem

$$\min_x |\mathbf{z}| \; : \; g(\mathbf{z}) = 0 \tag{2.81}$$

The calculation of β can be undertaken in a number of different ways. In the general case where the failure surface is non-linear, an iterative method must be used. (Thoft-Christensen and M. Baker, 1982, p.89)

2.3.4 Probability Transformation

Due to the reliability index β_{HL}, being only defined in a normalized space, the basic random variables $\mathbf{X}$ have to be transformed into standard normal random variables $\mathbf{Z}$. Additionally, the basic random variables $\mathbf{X}$ can be correlated and those relationships should also be transformed.

Transformation of Dependent Random Variables using Nataf Approach

One method to handle this is using the *Nataf* joint distribution model, if the marginal cdfs are known. (J. Baker, 2010)

The correlated random variables $\mathbf{X} = (X_1, \ldots, X_n)$ with the correlation matrix $\mathbf{R}$ can be transformed by

$$y_i = \Phi^{-1}\left(F_{X_i}(x_i)\right) \qquad i = 1, \ldots, n \tag{2.82}$$

into normally distributed random variables $\mathbf{Y}$ with zero means and unit variance, but still correlated with $\mathbf{R}_0$. Nataf's distribution for $\mathbf{X}$ is obtained by assuming that $\mathbf{Y}$ is jointly normal. (P.-L. Liu and Der Kiureghian, 1986)

The correlation coefficients for $\mathbf{X}$ and $\mathbf{Y}$ are related by

$$\begin{aligned}\rho_{X_i,X_j} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left(\frac{x_i - \mu_{X_i}}{\sigma_{X_i}}\right)\left(\frac{x_j - \mu_{X_j}}{\sigma_{X_j}}\right) \\ \frac{1}{2\pi\sqrt{1-\rho^2_{Y_i,Y_j}}} \exp\left(-\frac{y_i^2 - 2\rho_{Y_i,Y_j} y_i y_j + y_j^2}{2(1-\rho^2_{Y_i,Y_j})}\right) dz_i dz_j\end{aligned} \tag{2.83}$$

Once this is done, the transformation from the correlated normal random variables $\mathbf{Y}$ to uncorrelated normal random variables $\mathbf{Z}$ is addressed. Hence, the transformation is

$$\mathbf{z} = \mathbf{L}_0^{-1}\mathbf{y} \quad \Leftrightarrow \quad \mathbf{y} = \mathbf{L}_0\mathbf{z} \tag{2.84}$$

where $\mathbf{L}$ is the *Cholesky decomposition* of the correlation matrix $\mathbf{R}$ of $\mathbf{Y}$. The *Jacobian matrix,* denoted by $\mathbf{J}$, for the transformation is given by

$$\mathbf{J}_{ZX} = \frac{\partial \mathbf{z}}{\partial \mathbf{x}} = \mathbf{L}_0^{-1}\text{diag}\left(\frac{f_{X_i}(x_i)}{\phi(z_i)}\right) \tag{2.85}$$

This approach is useful when the marginal distribution for the random variables $\mathbf{X}$ is known and the knowledge about the variables dependence is limited to correlation coefficients. (J. Baker, 2010)

Transformation of Dependent Random Variables using Rosenblatt Approach

An alternative to the Nataf approach is to consider the joint pdf of $\mathbf{X}$ as a product of conditional pdfs.

$$f_{\mathbf{X}}(\mathbf{x}) = f_{X_1}(x_1) f_{X_2|X_1}(x_2|x_1) \dots f_{X_n|X_1,\dots,X_{n-1}}(x_n|x_1,\dots,x_{n-1}) \tag{2.86}$$

As a result of the sequential conditioning in the pdf, the conditional cdfs are given for $i \in [1, n]$

$$F_{X_i|X_1,\dots,X_{i-1}}(x_i|x_1,\dots,x_{i-1}) = \int_{-\infty}^{x_i} f_{X_i|X_1,\dots,X_{i-1}}(x_i|x_1,\dots,x_{i-1}) dx_i \tag{2.87}$$

These conditional distributions for the random variables $\mathbf{X}$ can be transformed into standard normal marginal distributions for the variables $\mathbf{Z}$, using the so called *Rosenblatt transformation* (Rosenblatt, 1952), suggested by Hohenbichler and Rackwitz (1981).

$$\begin{aligned} z_1 &= \Phi^{-1}\left(F_{X_1}(x_1)\right) \\ z_2 &= \Phi^{-1}\left(F_{X_2|X_1}(x_2|x_1)\right) \\ &\vdots \\ z_n &= \Phi^{-1}\left(F_{X_n|X_1,\dots,X_{n-1}}(x_n|x_1,\dots,x_{n-1})\right) \end{aligned} \tag{2.88}$$

The Jacobian of this transformation is a lower triangular matrix having the elements (J. Baker, 2010)

$$[\mathbf{J}_{ZX}]_{i,j} = \frac{\partial z_i}{\partial x_j} = \begin{cases} \dfrac{1}{\phi(u_i)} \dfrac{\partial}{\partial x_j} F_{X_i|X_1,\dots,X_{i-1}}(x_i|x_1,\dots,x_{i-1}) & i \geq j \\ 0 & i < j \end{cases} \tag{2.89}$$

In some cases the Rosenblatt transformation cannot be applied, because the required conditional pdfs cannot be provided. In this case other transformations may be useful, for example Nataf transformation. (Faber, 2009, p.6.14)

2.3.5 First Order Reliability Method

Let $\mathbf{Z}$ be a set of uncorrelated and standardized normally distributed random variables $(Z_1, \ldots, Z_n)$ in the normalized z-space, corresponding to any set of random variables $\mathbf{X} = (X_1, \ldots, X_n)$ in the physical x-space, then the limit state surface in x-space is also mapped on the corresponding limit state surface in z-space.

According to Definition 2.10, the reliability index β is the minimum distance from the z-origin to the failure surface. This distance β can directly be mapped to a probability of failure

$$p_f \approx p_{f1} = \Phi(-\beta) \tag{2.90}$$

this corresponds to a linearization of the failure surface. The linearization point is the design point $\mathbf{z}^*$. This procedure is called *First Order Reliability Method* (FORM) and β is the *First Order Reliability Index*. (Madsen et al., 2006, p.73)

Better results can be obtained by higher order approximations of the failure surface. The *Second Order Reliability Method* (SORM) uses; for example, a quadratic approximation of the failure surface. (J. Baker, 2010)

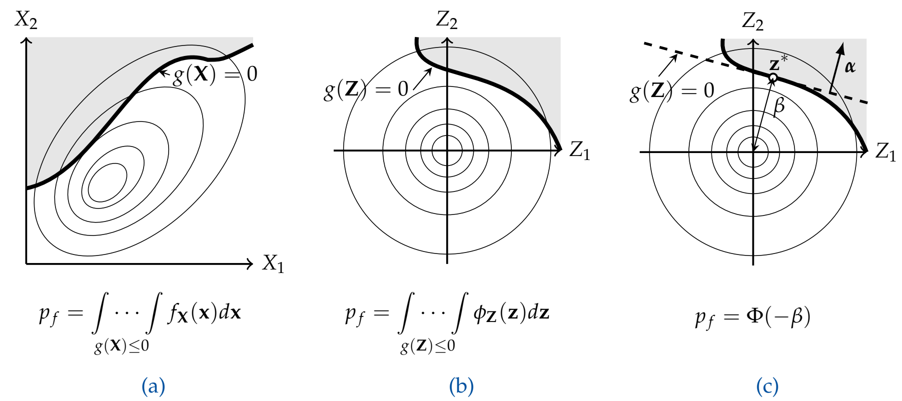


Figure 2.9: (a) Representation of a physical space with a set $\mathbf{X}$ of any two random variables. The shaded area denotes the failure domain and $g(\mathbf{X}) = 0$ the failure surface. (b) After transformation in the normalized space, the random variables $\mathbf{Z}$ are now uncorrelated and standardized normally distributed, also the failure surface is transformed into $g(\mathbf{Z}) = 0$. (c) FORM corresponds to a linearization of the failure surface $g(\mathbf{Z}) = 0$. Performing this method, the design point $\mathbf{z}^*$ and the reliability index β can be computed. Based on J. Baker (2010)

2.3.6 Simulation Methods

The preceding sections describe some methods for determining the reliability index β for some common forms of the limit state function. However, it is sometimes extremely difficult or impossible to find β. (Nowak and Collins, 2000, p.138)

In this case, Equation (2.70) may also be estimated by *numerical simulation methods*. A large variety of simulation techniques can be found in the literature, indeed, the most commonly used method is the *Monte Carlo* method. (Faber, 2009, p.6.18)

The principle of simulation methods is to carry out random sampling in the physical (or standardized) space. For each of the samples the limit state function is evaluated to figure out, whether the configuration is desired or undesired. The probability of failure p_f is estimated by the number of undesired configurations, respected to the total numbers of samples. (Lemaire et al., 2010, p.232)

For this analysis Equation (2.70) can be rewritten as

$$p_f = P(g(\mathbf{X}) \leq 0) = \int \cdots \int_{g(\mathbf{X}) \leq 0} I(g(\mathbf{X}) \leq 0) f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \tag{2.91}$$

where I is an *indicator function* that is equals to 1 if $g(\mathbf{X}) \leq 0$ and otherwise 0. Equation (2.91) can be interpreted as expected value of the indicator function. Therefore, the probability of failure can be estimated such as (Malioka, 2009, p.24)

$$\tilde{p}_f = \mathbb{E}\left[I(g(\mathbf{X}) \leq 0)\right] = \frac{1}{n} \sum_{i=1}^{n} I(g(\mathbf{X}) \leq 0) \tag{2.92}$$

Crude Monte Carlo Simulation

The *Crude Monte Carlo* simulation (CMC) is the most simple form and corresponds to a direct application of Equation (2.92). A large number n of samples are simulated for the set of random variables $\mathbf{X}$. All samples that lead to a failure are counted n_f and after all simulations the probability of failure p_f may be estimated by (Faber, 2009, p.6.18)

$$\tilde{p}_f = \frac{n_f}{n} \tag{2.93}$$

Theoretically, an infinite number of simulations will provide an exact probability of failure. However, time and the power of computers are limited; therefore, a suitable amount of simulations n are required to achieve an acceptable level of accuracy. One possibility to reach such a level is to limit the coefficient of variation CoV for the probability of failure. (Lemaire et al., 2010, p.251)

$$\text{CoV} = \sqrt{\frac{1-p_f}{np_f}} \approx \frac{1}{\sqrt{np_f}} \qquad \text{for} \quad p_f \to 0 \tag{2.94}$$

For an objective $\text{CoV} = 0.1$ and a probability of failure $p_f = 10^{-k}$, are $n = 10^{k+2}$ simulations required.

Importance Sampling

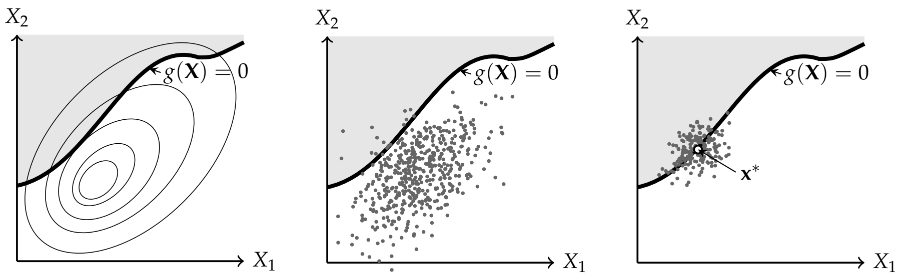
To decrease the number of simulations and the coefficient of variation, other methods can be performed. One commonly applied method is the *Importance Sampling* simulation method (IS). Here the prior information about the failure surface is added to Equation (2.91)

$$p_f = P(g(\mathbf{X}) \leq 0) = \int \cdots \int_{g(\mathbf{X}) \leq 0} I(g(\mathbf{X}) \leq 0) \frac{f_{\mathbf{X}}(\mathbf{x})}{h_{\mathbf{X}}(\mathbf{x})} h_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \tag{2.95}$$

where $h_X(\mathbf{X})$ is the *importance sampling probability density function* of $\mathbf{X}$. Consequently Equation (2.92) is extended to (Faber, 2009, p.6.20)

$$\tilde{p}_f = \mathbb{E}\left[I(g(\mathbf{X}) \leq 0) \frac{f_{\mathbf{X}}(\mathbf{x})}{h_{\mathbf{X}}(\mathbf{x})} \right] = \frac{1}{n} \sum_{i=1}^{n} I(g(\mathbf{X}) \leq 0) \frac{f_{\mathbf{X}}(\mathbf{x})}{h_{\mathbf{X}}(\mathbf{x})} \tag{2.96}$$

The key to this approach is to choose $h_X(\mathbf{X})$ so that samples are obtained more frequently from the failure domain. For this reason, often a FORM (or SORM) analysis is performed to find a prior design point. (J. Baker, 2010)



$$p_f = \int \cdots \int_{g(\mathbf{X}) \leq 0} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}$$

(a)

$$\tilde{p}_f = \frac{1}{n} \sum_{i=1}^{n} I(g(\mathbf{X}) \leq 0)$$

(b)

$$\tilde{p}_f = \frac{1}{n} \sum_{i=1}^{n} I(g(\mathbf{X}) \leq 0) \frac{f_{\mathbf{X}}(\mathbf{x})}{h_{\mathbf{X}}(\mathbf{x})}$$

(c)

Figure 2.10: (a) Representation of a physical space with a set $\mathbf{X}$ of any two random variables. The shaded area denotes the failure domain and $g(\mathbf{X}) = 0$ the failure surface. (b) For the CMC method every dot corresponds to one configuration of the random variables $\mathbf{X}$. Dots in shaded areas lead to a failure. (c) The IS simulation method uses a distribution centered on the design point $\mathbf{x}^*$, is obtained from a FORM (or SORM) analysis. More dots in the failure domain can be observed. Based on J. Baker (2010)

3 Probabilistic Models for Degradation of Concrete

"*sapientia aedificabitur domus et prudentia roborabitur*" (Solomon, Proverb 24:3)[a]

[a] Through wisdom is an house builded; and by understanding it is established.

Designing new structures or accepting existing ones as adequately safe, is from a probabilistic point of view, the result of a *decision making process* based on some optimality criteria. This process links requirements and expectations of a structure, load and actions, geometry and material properties and also expectations of society. (JCSS, 2002)

Such *decision problems* are mathematical treated with the so-called *decision theory.* An important aspect of the decision theory is the assessment of consequences and probabilities, which are depict in *probabilistic models.* (Faber, 2009, p.1.8)

Definition 3.1 (Probabilistic Model (Benjamin and Cornell, 1970)) *A probabilistic model remains an abstraction until it has been related to observations of physical phenomenon.*

This requires the collection and progression of input data as well as determination of statistical distribution, corresponding statistical parameters and possible correlations.

For the degradation of concrete structures, several models have been developed to provide methods to estimate the length of time during which RC structures maintain a desired level of functionality. *Service life* models such as DuraCrete (1999), DuraCrete (2000a,b), LIFECON (2003), and fib Bulletin 34 (2006) provide valuable information about the durability characteristics of concrete structures.

The proposed method of this work is not limited to any of those models. However, in the remainder of this thesis only the DuraCrete (2000b) model is going to be treated. This chapter provides an overview of this probabilistic model. Hence, there are no explicit references cited for the assumptions of the DuraCrete (2000b) model.

3.1 Material Properties

The description of each material property consists of a mathematical model and random variables. Functional relationships between the variables may be part of the material model. (JCSS, 2002)

3.1.1 Concrete

Concrete is a composite material made of aggregates and reaction products of cement and mixing water. The reference property is the *compressive strength* of standard specimens tested according to standard conditions and at a standard age of 28 days, denoted by f_c and approximated as a lognormal random variable. (JCSS, 2002)

$$f_c \sim \ln \mathcal{N}(\mu_{f_c}, \sigma_{f_c}) \tag{3.1}$$

The binding capacity of concrete for substances such as chlorides or carbon dioxide will increase with a higher cement content. Also the type of cement will affect those properties. However, the properties of concrete are also governed in large parts by the *water/cement ratio* (w/c). For example, a lower w/c reduces the porosity of the hardened concrete. (Wight and MacGregor, 2012, p.51) Consequently, the volume of the capillary pores can increase with the amount of water. Therefore, it may cause an acceleration of the corrosion process. (Malioka, 2009, p.36)

Other influences on the concrete properties depend on the initial *curing* and *moisture* conditions. By increasing the curing time, a reduction of the porosity follows too. (Bertolini et al., 2004, p.7)

In addition the thickness of the *concrete cover* denoted by d_c, is an important parameter for the RC structure. Beside the effect to bond the reinforcement to the concrete, the cover protects steel bars against corrosion. (Wight and MacGregor, 2012) So the time at which corrosion initiates is closely affected by this parameter, which can be simplified and approximated as a lognormal random variable with a CoV of 0.3.

$$d_c \sim \ln \mathcal{N}(\mu_{d_c}, \sigma_{d_c}) \tag{3.2}$$

3.1.2 Reinforcement

Because of the low *tensile strength* of concrete, it is *reinforced* with steel bars or wires that resist the tensile stresses. The reference property is the *yield strength*, denoted by f_y and approximated as normal random variable. (JCSS, 2002)

$$f_y \sim \mathcal{N}(\mu_{f_y}, \sigma_{f_y}) \tag{3.3}$$

The nominal bar area $A_{s,\text{nom}}$ with the nominal diameter $d_{s,0}$ is also normal distributed with CoV $= 0.02$.

$$d_0 \sim \mathcal{N}(\mu_{d_0}, \sigma_{d_0}) \tag{3.4}$$

3.2 Environmental Actions

The degradation process of concrete is closely related with the *environment*. For example, the risk of chloride induced corrosion is higher than in coastal environments as in the interior of the country. (Stewart and Rosowsky, 1998b)

In the model of DuraCrete (2000b) environmental parameters are used as initial conditions for the degradation models. In this context, these parameters are more or less represented as deterministic variables, so that the user of the model has to decide which assumption is accurate for the present situation of the structure.

The statistical quantification of the parameters is based on a limited amount of data that are available in the databases from various countries. (DuraCrete, 2000b, p.45) As a result of this quantification the variables, which are representative for specific initial condition indicators, are: exposure environment, exposure class, temperature and humidity.

3.2.1 Exposure Environment

For corrosion due to chloride DuraCrete (1999) and DuraCrete (2000b) use two different *exposure environments:*

Marine Environment

Areas that are influenced by the vicinity to an ocean, including coastal areas, are said to have a *marine environment*. Within these, different *zones* depending on the position compared to the water level, are identified as shown in Figure 3.1.

- *Atmospheric zone.* The temperature and humidity conditions in this zone are normally assumed to be equal to the regional climate.
- *Splash zone* and *tidal zone.* The temperature and humidity conditions in these zones are a mix between the conditions in the atmospheric and submerged zone. The tidal actions are caused by the gravity of the moon and the range of this zone can vary from about 0.5 m to as much as 15 m.
- *Submerged zone.* The temperature and humidity conditions in this zone are equal to the conditions in the water.

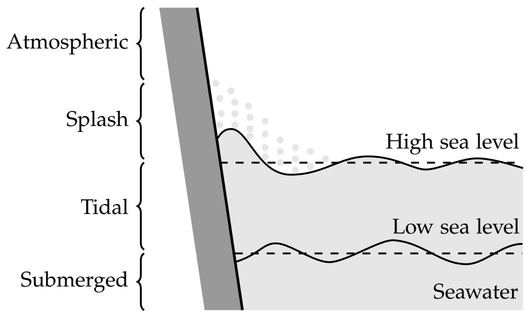


Figure 3.1: The marine environment can be subdivided into four different zones, depending on the relative position with respect to the water level, wave height, tidal cycle length and so on. Based on Bertolini et al. (2004)

Road Environment

The environment surrounding a road bridge could be divided into two principal zones, a dry zone and a wet zone.

- The *dry zone* is not exposed to direct rain and therefore the concrete has a low relative humidity.
- The *wet zone* is directly exposed to rain and therefore the concrete can reach a high level of relative humidity. The wet environment is also exposed to radiation, which could lead to extreme temperature variations in the concrete.

3.2.2 Exposure Class

For corrosion due to carbonation DuraCrete (1999) and DuraCrete (2000b) have proposed three different *exposure classes* [1]:

- *Laboratory environment* (LAB) has a constant temperature and relative humidity in the air. The surface is protected against precipitation.
- *Outdoor sheltered environment* (OS), where the air temperature and the relative humidity in the air changes over the year. The surface is protected against precipitation.
- *Outdoor unsheltered environment* (OUS), where the air temperature and the relative humidity in the air changes over the year. The surface is not protected against precipitation.

3.2.3 Temperature and Humidity

The influence of *temperature* (T) and *humidity* (RH) affects the degradation of concrete in a strong manner. For example, an increase in temperature will raise the rate of carbonation (Bertolini et al., 2004, p.82); beside the rate of chloride penetration and corrosion depends on the temperature. (Gjørv, 2009, p.104)

3.3 Degradation of Concrete

Concrete structures are exposed to environmental conditions, which may lead to deterioration of the RC structure. Bertolini et al. (2004) classified these degradation processes as: *mechanical, physical, structural, biological* and *chemical.* In practice these processes may occur simultaneously. Degradation of concrete and corrosion of reinforcement are closely connected as shown in Figure 3.2. This thesis is going to mention the *carbonation* and *chloride penetration* , which causes corrosion of reinforcement, and structural degradation, too.

[1]Note: In the original DuraCrete (2000b) report, this parameter is also called "exposure environment", but for the purpose of better clearness it will be denoted by "exposure classes" in this document.

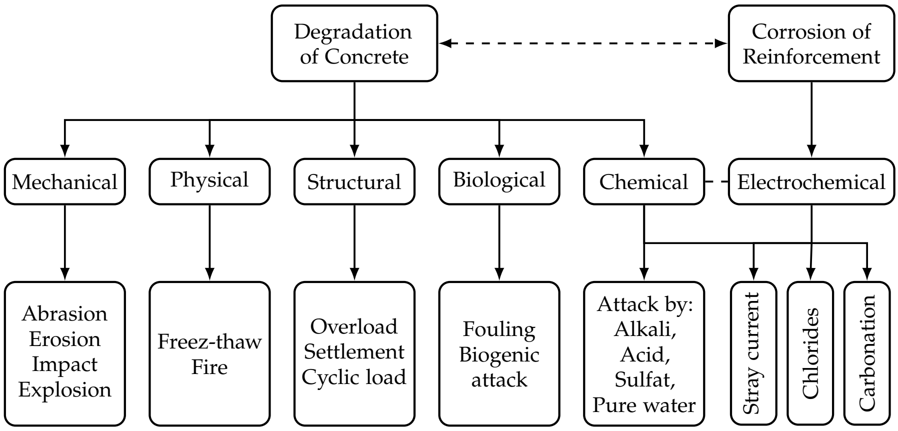


Figure 3.2: The degradation process of concrete can be classified as: mechanical, physical, structural, biological and chemical, depending on the exposed environment. Chemical and electrochemical reactions may lead to corrosion of reinforcement. Degradation of concrete and corrosion of reinforcement are closely connected. Based on Bertolini et al. (2004)

3.3.1 Corrosion of Reinforcement

Under normal conditions concrete protects embedded reinforcement against corrosion. This is caused by a highly alkaline environment that is provided by the concrete pore solution with a pH value between 13 and 14. These circumstances cause a thin protective *oxide film*, also called *passive film*, on the surface of the steel. (Arup, 1983) The protective action of the passive film is immune to mechanical damage of the steel surface. However, it can be destroyed by carbonation of concrete or by the presence of chloride ions. Then the reinforcement is *depassivated*. (Bertolini et al., 2004, p.71)

The classical concept which was developed by Tuutti (1982), divides the service life of RC structures in two distinct phases:

1. The fist phase is the *initiation of corrosion*. During which carbonation or chloride penetration in the concrete cover takes place and may lead to initiation of steel corrosion.
2. The second phase is *propagation of corrosion*. It begins when corrosion is induced and ends when a certain level of limit state is reached.

The DuraCrete (2000b) model and the other current service life models subdivide the propagation phase into four parts

1. After the *depassivation of reinforcement* the initiation phase ends and the propagation phase starts either when the carbonation front reaches the reinforcement or when the chloride concentration at the reinforcement reaches a critical threshold value.
2. The second event is the *cracking of the concrete cover* due to the expansive forces generated by the corrosion products.
3. If corrosion is continuing after cracking it may lead to *spalling of the concrete cover.*
4. Finally, the RC structure will *collapse,* if the load carrying capacity of the element is reduced sufficiently due to ongoing corrosion, in form of cross sectional loss of the concrete and steel or loss of bond.

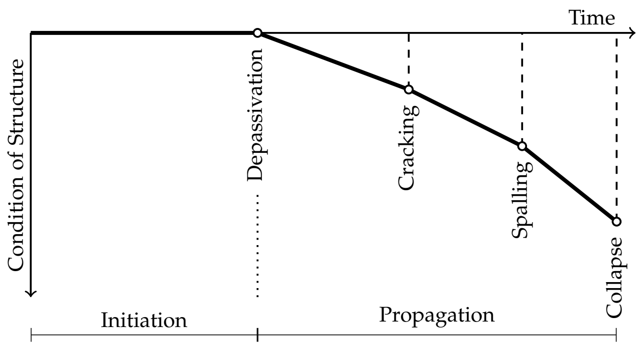


Figure 3.3: Two phases service life model for deterioration of a concrete structure due to steel corrosion. The propagation phase includes four points of interest: depassivation, cracking, spalling and collapse. The first three points may be used as limit state. Based on Tuutti (1982)

In the current service life models, the limit state of a RC structure is defined by one of these four points. For example, DuraCrete (2000a) recommends that depassivation or cracking is related to the serviceability of the structure and spalling is related to serviceability and ultimate failure. However, the fib Bulletin 34 (2006) model code is even more strictly in these assumptions, here serviceability will be classified as depassivation and ultimate failure as cracking or spalling.

3.4 Carbonation Induced Corrosion

Carbonation describes the process of neutralizing the alkalinity of concrete by *carbon dioxide* (CO_2). Hereby CO_2 diffuses from the atmosphere into the concrete and reacts with the hydrated cement paste. The reaction, which takes place in aqueous solution, can be written schematically as: (Bertolini et al., 2004; Taylor, 1997, p.79, p.114)

$$CO_2 + Ca(OH)_2 \xrightarrow{H_2O} CaCO_3 + H_2O \tag{3.5}$$

Carbonation itself does not cause any damage to the concrete, but the consequence is that the pH value of the pore solution drops from its normal level to values approaching neutrality.

The carbonation process starts at the surface and penetrates into the concrete. The *diffusion* of CO_2 into concrete can be described by *Fick's first law* of diffusion. (Schiessl, 1997)

$$J = -D \frac{\partial C}{\partial x_{\text{ca}}} \tag{3.6}$$

where J is the *diffusion flux* measuring the amount of substances that will flow through a small area during a small time interval ($\text{kg}/\text{m}^2\text{s}$). C is the concentration of CO_2 per unit volume, D the *diffusion coefficient* (m/s) and x_{ca} is the distance from the concrete surface to the carbonation front. These coefficients can change, as a function of position, time, variation in the pore structure, humidity or temperature.

3.4.1 Deterioration Model for Carbonation Induced Corrosion

In order to take the above mentioned parameters into account, several models have been developed and proposed. DuraCrete (2000b) proposed the following model for the progress of the carbonation front:

$$x_{\text{ca}} = \sqrt{2 k_{c,\text{ca}} k_{e,\text{ca}} k_{t,\text{ca}} C_{s,\text{ca}} R_{\text{ca}}^{-1}} \sqrt{t} \left(\frac{t_o}{t} \right)^{n_{\text{ca}}} \tag{3.7}$$

with

$$R_{\text{ca}} = \frac{a}{D_{\text{eff}}} \tag{3.8}$$

Material Variables

D_{eff}	effective diffusion coefficient of dry carbonated concrete for CO_2 at defined compaction, curing and environmental conditions.	$[\text{mm}^2/\text{yr}]$
a	binding capacity for CO_2	$[\text{kgCO}_2/\text{m}^3]$

R_{ca} effective resistance of concrete to carbonation, taking binding into account, at defined compaction, curing and environmental conditions. [$\text{kgCO}_2/\text{m}^3/\text{mm}^2/\text{yr}$]

Environmental Variables

$C_{s,\text{ca}}$ surface concentration of CO_2 [kgCO_2/m^3]
$k_{e,\text{ca}}$ environmental parameter that considers the influence of environment. [-]
n_{ca} age factor that considers the influence of climatic conditions. [-]

Execution and Test Variables

$k_{c,\text{ca}}$ execution parameter takes the influence of curing into account. [-]
$k_{t,\text{ca}}$ test method parameter considers the influence of test methods. [-]

In- and Output Variables

t exposure time [yr]
t_0 reference time (1 yr) [yr]
x_{ca} depth of carbonation front at time t [mm]

3.4.2 Material Parameter for Carbonation Induced Corrosion

Effective Resistance of Concrete to Carbonation

The material parameter R_{ca} expresses the *effective resistance* of concrete to carbonation and is strongly influenced by the concrete composition. Especially, the w/c ratio and cement content have a strong influence on the parameter. R_{ca} can be approximated as normal random variable.

$$R_{\text{ca}}^{-1} \sim \mathcal{N}(\mu_{R_{\text{ca}}^{-1}}, \sigma_{R_{\text{ca}}^{-1}}) \tag{3.9}$$

Table 3.1: Distributions for the effective resistance of concrete to carbonation R_{ca}^{-1} in [$10^{-11}\ \text{m}^2/\text{s}/\text{kgCO}_2/\text{m}^3$] and the value of grade in [N/mm^2].

Grade	w/c ratio	Distribution	$\mu_{R_{\text{ca}}^{-1}}$	$\sigma_{R_{\text{ca}}^{-1}}$
45	0.45	Normal	25	2.23
40	0.50	Normal	5	0.38
25	0.55	Normal	35	1.75
35	0.55	Normal	15	0.89

3.4.3 Environmental Parameters for Carbonation Induced Corrosion

Surface Concentration of Carbon Dioxide

The DuraCrete (2000b) model assumes the *surface concentration of carbon dioxide* as constant and equal to $C_{s,\mathrm{ca}} = 5 \cdot 10^{-4}\,\mathrm{kg/m^3}$. This means initially that this parameter is not considered as a random variable. However, Parrott (1987) observed that the surface concentration of carbon dioxide has a significant influence on the rate of carbonation. Li (2004), Defaux et al. (2006) and Sudret (2008) proposed some values for the surface concentration. In this work $C_{s,\mathrm{ca}}$ is approximated by a normal random variable with $\mu_{C_{s,\mathrm{ca}}} = 6 \cdot 10^{-4}$ and $\mathrm{CoV} = 0.17$. (Li, 2004)

$$C_{s,\mathrm{ca}} \sim \mathcal{N}(\mu_{C_{s,\mathrm{ca}}}, \sigma_{C_{s,\mathrm{ca}}}) \tag{3.10}$$

Environmental Parameter for Carbonation

The *environmental parameter* represents a ratio between observations made in an actual climate and observations made in a reference climate. The value $k_{e,\mathrm{ca}}$ can be approximated as lognormal random variable.

$$k_{e,\mathrm{ca}} \sim \ln\mathcal{N}(\lambda_{k_{e,\mathrm{ca}}}, \zeta_{k_{e,\mathrm{ca}}}) \tag{3.11}$$

Table 3.2: Distributions for the environmental parameter $k_{e,\mathrm{ca}}$.

Environment	**Distribution**	$\lambda_{k_{e,\mathrm{ca}}}$	$\zeta_{k_{e,\mathrm{ca}}}$	$\mu_{k_{e,\mathrm{ca}}}$	$\sigma_{k_{e,\mathrm{ca}}}$
LAB	Deterministic	-	-	1.00	-
OS	Lognormal	-0.207	0.299	0.85	0.26
OUS	Lognormal	-0.198	0.266	0.85	0.23

Age Factor for Carbonation

The material property for carbonation is represented as R_{ca} the *effective carbonation resistance,* which is a combination of binding capacity a and the effective diffusion coefficient D_{eff} as shown in Equation (3.8). The diffusion coefficient will increase with age, what is expressed by the age factor n_{ca}, which follows a beta distribution.

$$n_{\mathrm{ca}} \sim \mathrm{Beta}(q_{n_{\mathrm{ca}}}, r_{n_{\mathrm{ca}}}, a_{n_{\mathrm{ca}}}, b_{n_{\mathrm{ca}}}) \tag{3.12}$$

Table 3.3: Distributions for the age factor n_{ca}. Based on Malioka (2009)

Environment	Distribution	$q_{n_{\mathrm{ca}}}$	$r_{n_{\mathrm{ca}}}$	$a_{n_{\mathrm{ca}}}$	$b_{n_{\mathrm{ca}}}$	$\mu_{n_{\mathrm{ca}}}$	$\sigma_{n_{\mathrm{ca}}}$
LAB	Deterministic	-	-	-	-	0.000	-
OS	Beta	0.850	1.290	0.0	0.5	0.199	0.138
OUS	Beta	0.554	0.491	0.0	0.5	0.265	0.175

3.4.4 Execution and Test Parameters for Carbonation Induced Corrosion

Execution Parameter for Carbonation

The *execution parameter,* denoted by $k_{c,\mathrm{ca}}$, takes executional influences upon the effective diffusion coefficient into account. This parameter is determined empirically and depends not only on the material, but also upon the surrounding environmental conditions. The final results based on observations of concrete elements are listed in Table 3.4

Table 3.4: Distributions for the execution parameter $k_{c,\mathrm{ca}}$. With curing period in days.

Curing	Distribution	$q_{k_{c,\mathrm{ca}}}/\lambda_{k_{c,\mathrm{ca}}}$	$r_{k_{c,\mathrm{ca}}}/\zeta_{k_{c,\mathrm{ca}}}$	$a_{k_{c,\mathrm{ca}}}$	$b_{k_{c,\mathrm{ca}}}$
1	shifted Lognormal	2.52	0.84	0.46	-
3	shifted Lognormal	0.87	1.03	0.88	-
7	Deterministic	1.00	-	-	-
28	Beta	1.86	1.10	0.35	1.00

Test Method Parameter for Carbonation

The *test method parameter,* denoted by $k_{t,\mathrm{ca}}$, can only be evaluated if alternative test methods are applicable to measure comparable material parameters and represent a functional relationship between different test methods.

$$k_{t,\mathrm{ca}} \sim \mathcal{N}(\mu_{k_{t,\mathrm{ca}}}, \sigma_{k_{t,\mathrm{ca}}}) \tag{3.13}$$

The statistical quantification of the test method parameter assumes a $\mu_{k_{t,\mathrm{ca}}} = 0.983$ and a $\sigma_{k_{t,\mathrm{ca}}} = 0.023$.

3.4.5 Limit State for Carbonation Induced Corrosion

For the onset of corrosion the limit state can be assumed as the probability that the carbonation front x_{ca} reaches or is larger than the concrete cover depth d_c beyond the reinforcement.

$$p_f = P[d_c - x_{\text{ca}}(t) \leq 0] \tag{3.14}$$

3.5 Chloride Induced Corrosion

A frequent cause of reinforcement steel corrosion is contamination by *chloride.* (Arup, 1983) To initiate the corrosion process, the chloride content at the surface of the reinforcement has to reach a certain *threshold value.* (Bertolini et al., 2004, p.93) However, the chloride transport in concrete is a rather complicate process, which involves inter alia iron diffusion and convection. (Tang et al., 2012, p.8) This complex transport mechanism can be simplified and estimated by use of *Fick's second law* of diffusion. (Collepardi et al., 1970)

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x_{\text{cl}}^2} \tag{3.15}$$

where C is the concentration of chloride ions at distance x_{cl} from the concrete surface after time t of exposure to chlorides and D the *chloride diffusion coefficient.* To solve Equation (3.15) boundary and initial conditions are needed.

The boundary conditions are described by the assumption that the concentration of the diffusing ion, measured on the surface of the concrete, is constant in time and equal to $C_{s,\text{cl}}$. Also it is assumed that the coefficient of diffusion D does not vary in time and through the thickness of concrete. The initial condition is that there is no chloride in the concrete at the beginning. (Bertolini et al., 2004, p.29)

$$C(x_{\text{cl}} = 0, t > 0) = C_{s,\text{cl}} \tag{3.16}$$

$$C(x_{\text{cl}} > 0, t = 0) = 0 \tag{3.17}$$

Thus, the obtained solution is:

$$C(x_{\text{cl}}, t) = C_{s,\text{cl}} \left(1 - \text{erf}\left(\frac{x_{\text{cl}}}{2\sqrt{Dt}}\right)\right) \tag{3.18}$$

3.5.1 Deterioration Model for Chloride Induced Corrosion

The proposed DuraCrete (2000b) model for the chloride penetration includes material and environmental parameters. Additionally, the change of the diffusion coefficient is covered by a time dependent exponential function including an age factor to reflect the decrease of the coefficient due to material and environmental influences.[2]

$$C_{\text{cl}}(x_{\text{cl}}, t) = C_{s,\text{cl}} \left(1 - \text{erf} \left(\frac{x_{\text{cl}}}{2\sqrt{k_{e,\text{cl}} k_{t,\text{cl}} k_{c,\text{cl}} D_o \left(\frac{t_o}{t}\right)^{n_{\text{cl}}} t}} \right) \right) \tag{3.19}$$

Material Variables

D_o chloride diffusion coefficient at defined compaction, curing and environmental conditions, at a reference time t_o. [mm^2/yr]

Environmental Variables

$k_{e,\text{cl}}$ environmental parameter that considers the influence of environment. [-]

Environmental and Material Variables

$C_{s,\text{cl}}$ surface concentration of chloride [%wb]

n_{cl} age factor that considers the influence of material and environmental conditions. [-]

Execution and Test Variables

$k_{c,\text{cl}}$ execution parameter takes the influence of curing into account. [-]

$k_{t,\text{cl}}$ test method parameter considers the influence of test methods. [-]

In- and Output Variables

t exposure time [yr]

t_o reference time [yr]

x_{cl} depth of chloride penetration [mm]

C_{cl} chloride concentration at depth x_{cl} after time t [%wb]

[2]Note: DuraCrete (2000b) uses the same variable name for carbonation and chloride induced corrosion, but these variables refer to different phenomena!

3.5.2 Material Parameter for Chloride Induced Corrosion

Chloride Diffusion Coefficient

The *chloride diffusion coefficient* D_o is strongly influenced by the concrete mix, curing and compaction. Additionally, the diffusion coefficient increases significantly with the w/c ratio. D_o can be approximated as normal random variable.

$$D_o \sim \mathcal{N}(\mu_{D_o}, \sigma_{D_o}) \tag{3.20}$$

Table 3.5: Distributions for the chloride diffusion coefficient D_o in [mm^2/yr]. Based on Malioka (2009)

w/c ratio	**Distribution**	μ_{D_o}	σ_{D_o}
0.40	Normal	220.92	25.41
0.45	Normal	315.60	32.51
0.50	Normal	473.40	43.24

3.5.3 Environmental Parameter for Chloride Induced Corrosion

Environmental Parameter for Chloride Penetration

The *environmental parameter* $k_{e,\mathrm{cl}}$ has been introduced in the model to make the diffusion coefficient applicable for different environmental conditions. The quantification is made in the four different exposure zones of a marine environment. Here $k_{e,\mathrm{cl}}$ is approximated as Gamma distribution.

$$k_{e,\mathrm{cl}} \sim \Gamma(\alpha_{k_{e,\mathrm{cl}}}, \beta_{k_{e,\mathrm{cl}}}) \tag{3.21}$$

For the road environment a reasonable approach is the quantification of the environmental parameters from the splash zone in a marine environment.

Table 3.6: Distributions for the environmental parameter $k_{e,\mathrm{cl}}$.

Environment	**Distribution**	$\alpha_{k_{e,\mathrm{cl}}}$	$\beta_{k_{e,\mathrm{cl}}}$	$\mu_{k_{e,\mathrm{cl}}}$	$\sigma_{k_{e,\mathrm{cl}}}$
Submerged	Gamma	35.3038	26.6444	1.325	0.223
Tidal	Gamma	35.5370	38.4599	0.924	0.155
Splash	Gamma	34.6790	130.8642	0.265	0.045
Atmospheric	Gamma	35.1628	52.0160	0.676	0.114

3.5.4 Environmental and Material Parameter for Chloride Induced Corrosion

The surface concentration $C_{s,\mathrm{cl}}$ and the age factor n_{cl} are two parameters in the model which are influenced by the environmental conditions but also by the material properties.

Surface Concentration of Chloride

The *surface chloride concentration*, denoted by $C_{s,\mathrm{cl}}$, is described as a function of the w/c ratio and an error term.

$$C_{s,\mathrm{cl}} = A \cdot (w/c) + \varepsilon \tag{3.22}$$

where A is a normally distributed regression parameter and ε a normally distributed error term.

Table 3.7: Distributions for the regression parameters A and ε for the surface concentration $C_{s,\mathrm{cl}}$.

Environment	Distribution	μ_A	σ_A	σ_ε
Submerged	Normal	10.348	0.714	0.580
Tidal	Normal	7.758	1.360	1.059
Splash	Normal	7.758	1.360	1.105
Atmospheric	Normal	2.565	0.356	0.405

Because of the linear Equation (3.22) and the normal distributed regression parameters, the surface chloride concentration $C_{s,\mathrm{cl}}$ can also be assumed as a normal random variable.

$$C_{s,\mathrm{cl}} \sim \mathcal{N}(\mu_{C_{s,\mathrm{cl}}}, \sigma_{C_{s,\mathrm{cl}}}) \tag{3.23}$$

with

$$\mu_{C_{s,\mathrm{cl}}} = \mathbb{E}[C_{s,\mathrm{cl}}] = \mathbb{E}[A \cdot (w/c) + \varepsilon] = (w/c) \cdot \mu_A \tag{3.24}$$

$$\sigma^2_{C_{s,\mathrm{cl}}} = \mathrm{Var}(C_{s,\mathrm{cl}}) = \mathrm{Var}(A \cdot (w/c) + \varepsilon) = (w/c)^2 \cdot \sigma_A^2 + \sigma_\varepsilon^2 \tag{3.25}$$

For the road environment only limited data for the dry zone are available, where $C_{s,\mathrm{cl}}$ is a lognormal random variable with $\lambda_{C_{s,\mathrm{cl}}} = -1.611$ and $\zeta_{C_{s,\mathrm{cl}}} = 0.606$ ($\mu_{C_{s,\mathrm{cl}}} = 0.24$, $\sigma_{C_{s,\mathrm{cl}}} = 0.16$).

Age Factor for Chloride Penetration

The *age factor*, denoted by n_{cl}, depends on the exposure environment and the used binder. It takes into account that the diffusion coefficient D_0 decreases with increasing age of the concrete. (LIFECON, 2003, p.78)

$$n_{\mathrm{cl}} \sim \mathrm{Beta}(q_{n_{\mathrm{cl}}}, r_{n_{\mathrm{cl}}}, a_{n_{\mathrm{cl}}}, b_{n_{\mathrm{cl}}}) \tag{3.26}$$

Table 3.8: Distributions for the age factor n_{cl}.

Environment	**Distribution**	$p_{n_{\mathrm{cl}}}$	$r_{n_{\mathrm{cl}}}$	$a_{n_{\mathrm{cl}}}$	$b_{n_{\mathrm{cl}}}$	$\mu_{n_{\mathrm{cl}}}$	$\sigma_{n_{\mathrm{cl}}}$
Submerged	Beta	24.90	58.10	0.0	1.0	0.30	0.05
Tidal	Beta	17.23	29.34	0.0	1.0	0.37	0.07
Splash	Beta	17.23	29.34	0.0	1.0	0.37	0.07
Atmospheric	Beta	29.53	15.90	0.0	1.0	0.65	0.07

3.5.5 Execution and Test Parameters for Chloride Induced Corrosion

Execution Parameter for Chloride Penetration

The *execution parameter*, denoted by $k_{c,\mathrm{cl}}$, takes executional influences upon the effective diffusion coefficient into account. This parameter is determined empirically and depends not only on the material, but also upon the surrounding environmental condition. $k_{c,\mathrm{cl}}$ can be approximated as Beta distribution.

$$k_{c,\mathrm{cl}} \sim \mathrm{Beta}(q_{k_{c,\mathrm{cl}}}, r_{k_{c,\mathrm{cl}}}, a_{k_{c,\mathrm{cl}}}, b_{k_{c,\mathrm{cl}}}) \tag{3.27}$$

Table 3.9: Distributions for the execution parameter $k_{c,\mathrm{cl}}$. With curing period in days.

Curing	**Distribution**	$q_{k_{c,\mathrm{cl}}}$	$r_{k_{c,\mathrm{cl}}}$	$a_{k_{c,\mathrm{cl}}}$	$b_{k_{c,\mathrm{cl}}}$	$\mu_{k_{c,\mathrm{cl}}}$	$\sigma_{k_{c,\mathrm{cl}}}$
1	Beta	1.667	1.905	1.0	4.0	2.4	0.7
3	Beta	2.148	10.741	1.0	4.0	1.5	0.3
7	Deterministic	1.000	-	-	-	-	-
28	Beta	4.445	2.333	0.4	1.0	0.8	0.1

Test Method Parameter for Chloride Penetration

The *test method parameter*, denoted by $k_{t,\text{cl}}$, can only be evaluated if alternative test methods are applicable to measure comparable material parameters and represent a functional relationship between different test methods.

$$k_{t,\text{cl}} \sim \mathcal{N}(\mu_{k_{t,\text{cl}}}, \sigma_{k_{t,\text{cl}}}) \tag{3.28}$$

The statistical quantification of the test method parameter assumes a $\mu_{k_{t,\text{cl}}} = 0.832$ and a $\sigma_{k_{t,\text{cl}}} = 0.024$.

Critical Chloride Concentration

As mentioned before, chloride induced corrosion can only take place if a certain threshold value is reached. This value is called *critical chloride concentration* and is denoted by C_{crit}. Actually, there are two definitions for these values. The first critical chloride concentration refers to a threshold value at which a depassivation of the steel surfaces begins. The second definition suggests that the critical chloride concentration is reached when cracking occurs.

C_{crit} is assumed to be normally distributed and influenced by the w/c ratio and the humidity of the environment.

$$C_{\text{crit}} \sim \mathcal{N}(\mu_{C_{\text{crit}}}, \sigma_{C_{\text{crit}}}) \tag{3.29}$$

Table 3.10: Distributions for the critical chloride concentration C_{crit} in [%wb] for the 1st definition.

w/c ratio	Distribution	$\mu_{C_{\text{crit}}}$	$\sigma_{C_{\text{crit}}}$
0.30	Normal	0.48	0.15

Table 3.11: Distributions for the critical chloride concentration C_{crit} in [%wb] for the 2nd definition.

w/c ratio	Distribution	$\mu_{C_{\text{crit}}}^{\text{saturated}}$	$\sigma_{C_{\text{crit}}}^{\text{saturated}}$	$\mu_{C_{\text{crit}}}^{\text{changing}}$	$\sigma_{C_{\text{crit}}}^{\text{changing}}$
0.30	Normal	2.30	0.20	0.90	0.15
0.40	Normal	2.10	0.20	0.80	0.10
0.50	Normal	1.60	0.20	0.50	0.10

3.5.6 Limit State for Chloride Induced Corrosion

For the onset of corrosion, the limit state can be assumed as the probability that the critical chloride concentration C_{crit} is reached at the reinforcement.

$$p_f = P[C_{\text{crit}} - C_{\text{cl}}(x_{\text{cl}} = d_c, t) \leq 0] \quad (3.30)$$

3.6 Propagation of Corrosion

After the depassivation of the reinforcement has occurred so that the passive layer broke down, the so-called *propagation phase* starts and the reinforcement steel starts to "rust," which is nothing else as the products of corrosion.

The chemical reactions are the same whether corrosion occurs by carbonation or by chloride attack. Furthermore, it can be described by the same electrochemical process as the corrosion of a metal in an electrolyte. (Broomfield, 2007, p.7)

Corrosion occurs as two half-cell reactions, an *anodic* reaction (*oxidation*) and a *cathodic* reaction (*reduction*). This mechanism is referred as the fundamental mechanism of corrosion and schematically summarized with the following reaction: (Küter, 2009, p.143)

$$\text{iron} + \text{oxygen} + \text{water} \rightarrow \text{product of corrosion} \quad (3.31)$$

More specific, when steel corrodes in concrete it dissolves in the pore water and gives up electrons. This is called the *anodic reaction.*

$$Fe \longrightarrow Fe^{2+} + 2\,e^- \quad (3.32)$$

The two electrons $2\,e^-$, that were created in this process must be consumed elsewhere on the steel surface to preserve electrical neutrality. This means that large amounts of electrical charge cannot build up at one place on the steel. For this reason, an other chemical reaction consumes these electrons. This so-called *cathodic reaction* consumes water, oxygen, and electrons and produces alkalinity. (Broomfield, 2007, p.7)

$$2\,e^- + H_2O + \tfrac{1}{2}\,O_2 \longrightarrow 2\,OH^- \quad (3.33)$$

These two reactions are only the fist steps in the process of creating "rust". One way to express such a process is when ferrous hydroxide (3.34) becomes ferric hydroxide (3.35) and then hydrated ferric oxide or "rust" (3.36). (Broomfield, 2007, p.7)

$$Fe^{2+} + 2\,OH^- \longrightarrow Fe(OH)_2 \tag{3.34}$$
$$4\,Fe(OH)_2 + O_2 + 2\,H_2O \longrightarrow 4\,Fe(OH)_3 \tag{3.35}$$
$$2\,Fe(OH)_3 \longrightarrow Fe_2O_3 \cdot H_2O + 2\,H_2O \tag{3.36}$$

In this process the volume of the corrosion products increases six to ten times in comparison to the starting material. This leads to the effects of cracking and spalling of concrete.

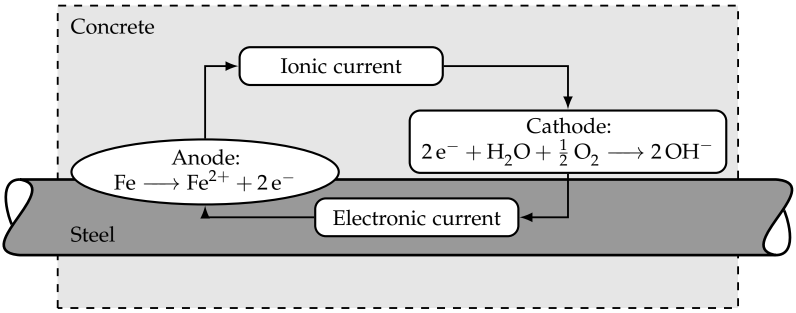


Figure 3.4: After the breakdown of the passive film an anode and a cathode will be developed. Within the metal electrons are transported away from the anodic regions where they become available to the cathodic regions, where they are consumed. A nominal electrical current flowing in the opposite direction. To close the circle an ionic current flows from the anode to the cathode. This is based on the transport of ions in the pore solution. Based on Broomfield (2007)

The electrical current flow I_{corr} and the generation and consumption of electrons in the anode and cathode reactions are used to determine the *corrosion rate*. (Broomfield, 2007, p.7)

The corrosion rate V_{corr} is usually expressed as the penetration rate and is measured in [mm/yr]. It can also be expressed in terms of the *current density*, denoted by i_{corr}, which represents the corrosion current I_{corr} related to the steel surface area [µA/cm²]. (LIFECON, 2003, p.98)

In case of reinforcement steel the relationship between current density and penetration rate is approximately: (Stewart, 2004)

$$1\ \mu A/cm^2 \approx 11.6\ \mu m/yr \tag{3.37}$$

However, probabilistic modeling of the propagation phase is still a challenge. While for the initiation phase (carbonation and chloride propagation) compatible design models are available, there are no suitable design models existing for the propagation phase of reinforcement corrosion. (Schießl et al., 2012) So further development is needed to fill this gap of knowledge and provide a comparable design model for the propagation phase.

3.6.1 Deterioration Model for Propagation of Corrosion

To stay consistent with the previous sections, a simplified propagation model based on Nilsson and Gehlen (1998) chloride corrosion rate factor F_{Cl} and proposed in the DuraCrete (2000b) model is used to predict the corrosion rate.

The corrosion rate V_{corr} can be modeled empirically. Hereby the basic assumption is to represent the corrosion rate as a product of local influencing factors and two material parameters

$$V_{\text{corr}} = \frac{m_o}{\rho} F_{Cl} F_{\text{Galv}} F_{O_2} \tag{3.38}$$

with

$$\rho = \rho_o \left(\frac{t_{\text{Hydr}}}{t_o} \right)^{n_{\text{r}}} k_{t,\text{r}} k_{c,\text{r}} k_{T,\text{r}} k_{RH,\text{r}} k_{Cl,\text{r}} \tag{3.39}$$

where ρ is the *concrete resistivity* and:

Unspecified Variables

m_o	parameter which represents a constant for corrosion rate versus resistivity based on Faraday's law.	[µmΩm/yr]
F_{Cl}	chloride corrosion rate factor, taking the chloride content in the concrete in account.	[-]
F_{Galv}	galvanic effect factor, considers the micro-galvanic actions in the concrete.	[-]
F_{O_2}	oxygen availability factor is expressing the availability of oxygen to the corrosion process.	[-]

Material Variables

ρ_o	potential concrete resistivity for a reference environment.	[Ωm]
n_{r}	age factor of the concrete resistivity.	[-]

Environmental Variables

$k_{T,\mathrm{r}}$	temperature factor for concrete resistivity.	[-]
$k_{RH,\mathrm{r}}$	humidity factor for concrete resistivity.	[-]
$k_{Cl,\mathrm{r}}$	chloride factor for concrete resistivity.	[-]

Execution and Test Variables

$k_{c,\mathrm{r}}$	execution parameter, takes the influence of curing into account.	[-]
$k_{t,\mathrm{r}}$	test method parameter, considers the influence of test methods.	[-]

In- and Output Variables

t_{Hydr}	time of hydration	[yr]
t_0	reference time (1 yr)	[yr]
V_{corr}	corrosion rate	[mm/yr]

3.6.2 Unspecified Parameter for Propagation of Corrosion

Corrosion Rate versus Resistivity

The model bias can be considered by including in the factor m_0 that relates the resistivity with the corrosion current. Nilsson and Gehlen (1998) propose to use a constant $m_0 = 822$ for the time being. However, if m_0 is quantified better, it is simple to change this constant into a stochastic parameter.

Chloride Corrosion Rate Factor

The *chloride corrosion rate factor,* denoted by F_{Cl} expresses the relation between the corrosion rate in concrete with and without chloride, for the same resistivity. Nilsson and Gehlen (1998) estimated this factor as shifted lognormal distributed

$$F_{Cl} \sim s\ln\mathcal{N}(\lambda_{F_{Cl}}, \zeta_{F_{Cl}}, a_{F_{Cl}}) \tag{3.40}$$

Table 3.12: Distributions for the chloride corrosion rate factor F_{Cl}. With the states true or false for chloride induced corrosion.

Chloride	Distribution	$\lambda_{F_{Cl}}$	$\zeta_{F_{Cl}}$	$a_{F_{Cl}}$
true	shifted Lognormal	0.62	1.35	1.09
false	Deterministic	1.00	-	-

Galvanic Effect Factor

Further studies are needed to quantify the galvanic effect factor. If the model is restricted to ordinary cases of carbonation with microcell corrosion, the galvanic factor is not relevant. Consequently

$$F_{\text{Galv}} = 1 \tag{3.41}$$

Oxygen Availability Factor

F_{O_2} is a factor expressing the effect of availability of oxygen to the corrosion process. Since data is very limited, a statistical quantification cannot be made. However, since the supply of oxygen usually not believed to be a limiting factor, except in submerged concrete, a crude alternative could be used:

$$F_{O_2} = \begin{cases} 0 & \text{submerged} \\ 1 & \text{otherwise} \end{cases} \tag{3.42}$$

3.6.3 Material Parameter for Propagation of Corrosion

Potential Concrete Resistivity

The *concrete resistivity* ρ a main material parameter, which influencing the corrosion rate. This property can be measured by several methods. The output of a standardized so-called compliance test is the potential concrete resistivity ρ_o, which can be approximated as a normal random variable with a mean of 77 Ωm and a standard deviation of 12 Ωm.

$$\rho_o \sim \mathcal{N}(\mu_{\rho_o}, \sigma_{\rho_o}) \tag{3.43}$$

Aging Factor for Resistivity

The aging factor n_{r} takes into account that the resistivity changes over time. This parameter is determined as a normal random variable with a mean of 0.23 and a standard deviation of 0.04.

$$n_{\text{r}} \sim \mathcal{N}(\mu_{n_{\text{r}}}, \sigma_{n_{\text{r}}}) \tag{3.44}$$

3.6.4 Environmental Parameter for Propagation of Corrosion

Temperature Factor for Resistivity

The temperature factor $k_{T,\mathrm{r}}$ can be approximated by

$$k_{T,\mathrm{r}} = \frac{1}{1 + K(T - 20)} \tag{3.45}$$

in which parameter K is the temperature dependence of the conductivity and approximated by

Table 3.13: Distributions for the parameter K. With the temperature in [°C].

Temperature	Distribution	μ_K	σ_K
$T > 20$	Normal	0.073	0.015
$T = 20$	Deterministic	1.000	-
$T < 20$	Normal	0.025	0.001

Humidity Factor for Resistivity

The humidity factor $k_{RH,\mathrm{r}}$ is different for OS and OUS conditions. For an OS environment the humidity factor depends on the RH, which can be estimated from the annual average air humidity for the location of the structure. Due to the limitation of data the humidity factor for an OUS environment is assumed to be not dependent on the RH.

Table 3.14: Distributions for the humidity Factor $k_{RH,\mathrm{r}}$, with the relative humidity in [%].

Environment	Humidity	Distribution	$\lambda_{k_{RH,\mathrm{r}}}$	$\zeta_{k_{RH,\mathrm{r}}}$	$a_{k_{RH,\mathrm{r}}}$
OS	50	shifted Lognormal	6.70	1.20	3.23
OS	65	shifted Lognormal	2.11	1.14	2.41
OS	80	shifted Lognormal	1.43	0.72	1.33
OS	95	Lognormal	1.07	0.14	-
OS	100	Deterministic	1.00	-	-
OUS	-	shifted Lognormal	0.62	0.33	0.79

Chloride Factor for Resistivity

The chloride factor $k_{Cl,\mathrm{r}}$ might be expressed as a normal random variable.

$$k_{Cl,\mathrm{r}} \sim \mathcal{N}(\mu_{k_{Cl,\mathrm{r}}}, \sigma_{k_{Cl,\mathrm{r}}}) \tag{3.46}$$

with $\mu_{k_{Cl,\mathrm{r}}} = 0.72$ and $\sigma_{k_{Cl,\mathrm{r}}} = 0.11$.

3.6.5 Execution and Test Parameters for Propagation of Corrosion

Execution Parameter for Propagation of Corrosion

The curing factor $k_{c,\mathrm{r}}$ is taken as a constant due to the lack of available data.

$$k_{c,\mathrm{r}} = 1 \tag{3.47}$$

Test Method Parameter for Propagation of Corrosion

The test method factor $k_{t,\mathrm{r}}$ takes into account, if another test method is used, to determine the potential concrete resistivity ρ_0. Due to a lack of available data the test method factor is assumed to be constant.

$$k_{t,\mathrm{r}} = 1 \tag{3.48}$$

3.7 Effects of Corrosion

If corrosion is initiated, the consequences are a reduction in the cross section of the reinforcement steel, increase in bar diameter resulting from the volumetric expansion of the corrosion products, and a change in the mechanical properties of the reinforcement and the concrete. (Cabrera, 1996) These effects do not only involve serviceability, but may also affect its structural reliability and therefore the safety of the structure. Correspondingly the corrosion affects the reinforcement itself and the surrounding concrete, as shown in Figure 3.5. In the remainder of the work only the effects of corrosion on the reinforcement steel will be treated.

As mentioned previously, a breakdown of the passive film is necessary for the initiation of corrosion. Once this protective layer is destroyed, corrosion will occur only if water and oxygen are present on the surface of the reinforcement, as shown in Equation (3.33).

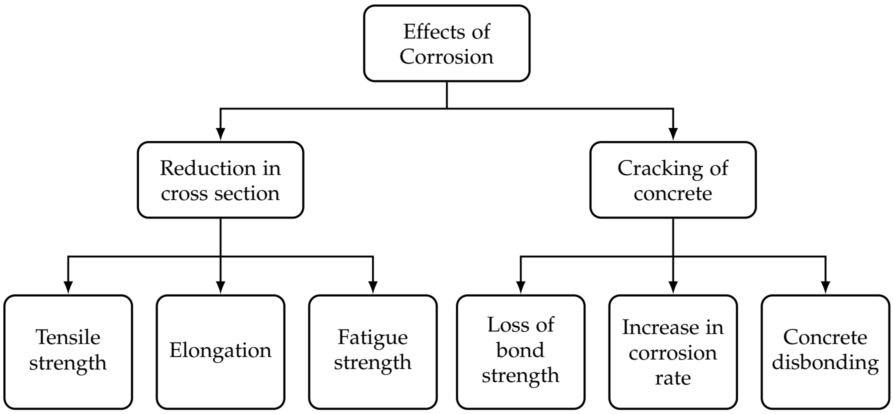


Figure 3.5: Corrosion causes two main effects for RC structures: A reduction in the cross section of the reinforcing bar, which leads to decreasing steel properties. It also causes corrosion cracking (and spalling) of concrete which influences the concrete properties. Based on Bertolini et al. (2004)

Carbonation of concrete may lead to a complete depassivation of the reinforcement. Instead chloride induced corrosion causes localized breakdowns, unless chlorides are present in very large amounts. (Bertolini et al., 2004, p.73)

If corrosion takes place on the whole surface of the steel, it is called *general corrosion*. In contrast, when only limited areas, so-called *pits*, are corroded and surrounded by non corroded areas, then this phenomenon is called *pitting corrosion*.

3.7.1 General Corrosion

General or *uniform corrosion* is caused by very high levels of chlorides or carbonation of concrete. It is typically associated with "rust" over the entire steel surface, which occupying a greater volume than the parent material. This can lead to cracking and spalling of the concrete cover. A loss of bond strength, caused by reinforcement slip that is initiated by corroded steel surfaces, and loss of reinforcement cross section may be the consequences of general corrosion, too. (Osterminski and Schießl, 2012)

3.7.2 Pitting Corrosion

Local or *pitting corrosion* is only associated with chloride induced corrosion. The area of the anode (*active zone*) may be relative small, but once corrosion has been initiated the resulting electrical field attracts negative chloride ions towards the pit. Hence, the corrosion rate can be relative high which leads to an extreme loss of steel cross section. (LIFECON, 2003, p.114)

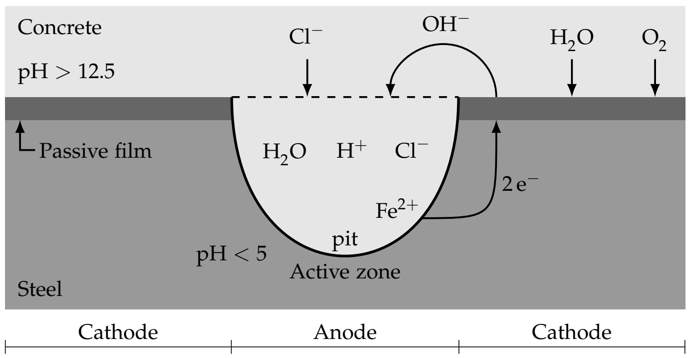


Figure 3.6: Pitting corrosion is a localized kind of corrosion. After a local breakdown of the passive film an anodic reaction $Fe \longrightarrow Fe^{2+} + 2e^-$ begins. Here electrons ($2e^-$) move from the anode to the cathode, where a cathodic reaction $2e^- + H_2O + \frac{1}{2}O_2 \longrightarrow 2OH^-$ takes place. Based on CEB (1992)

While general corrosion produces "rust" and leads to cracking and spalling, which can be easily detected by inspection. On the contrary part, pitting corrosion occurs only over small areas of reinforcement and often does not cause disruption of the concrete cover. Therefor, it is more difficult to detect these events. (Val and Melchers, 1997)

3.7.3 Probabilistic Model for Effects of Corrosion

The DuraCrete (2000b) model provides only a very simplified model for the effects of pitting corrosion. For this reason, a hemispherical model of a pit, suggested by Val and Melchers (1997) and modified by Stewart (2004, 2009, 2012), is used in this context.

The corrosion penetration can be modeled in general with t_i being the time of corrosion initiation.

$$p(t) = \int_{t_i}^{t_e} V_{\text{corr}}(t)dt \tag{3.49}$$

For general corrosion a uniform loss of steel diameter can be assumed

$$d_s(t) = d_{s,o} - V_{\text{corr}} t_{\text{corr}} \tag{3.50}$$

where $d_{s,o}$ is the initial reinforcement diameter, V_{corr} the corrosion rate in [mm/yr] and t_{corr} the time since corrosion has been started in [yr]. The product $V_{\text{corr}} \cdot t_{\text{corr}}$ is also called the *average penetration* and denoted by p_{av}.

3.7.4 Pitting Factor

According to González et al. (1995) is the corrosion rate for pitting corrosion four to eight times higher than the average penetration p_{av} on the surface of a reinforcement bar. This ratio between maximum and average corrosion penetration is called *pitting factor* and denoted by $R_{\text{pit}} = p_{\max}/p_{av}$. The pitting factor can be treated as a random variable modeled by a Gumbel distribution. (Stewart, 2004)

$$R_{\text{pit}} \sim \text{Gumbel}(\mu_{R_{\text{pit}}}, \alpha_{R_{\text{pit}}}) \tag{3.51}$$

where the Gumbel statistical parameters can be modified as (Sheikh et al., 1990)

$$\mu_{R_{\text{pit}}} = \mu_o + \frac{1}{\alpha_o} \ln\left(\frac{L_U}{L_o}\right) \tag{3.52}$$

$$\alpha_{R_{\text{pit}}} = \alpha_o \tag{3.53}$$

Table 3.15: Distributions for the Gumbel parameters of the pitting factor R_{pit}, with the bar diameter and the reference length L_o in [mm]. Based on Stewart (2012)

Diameter	L_o	**Distribution**	μ_o	α_o
10	100	Gumbel	5.08	1.02
16	100	Gumbel	5.56	1.16
27	100	Gumbel	6.55	1.07

where the Gumbel parameters μ_o and α_o are derived from statistical analysis of pitting data for reinforcement of *reference length* L_o. Besides, L_U is the *uniform capacity length*, referring to the distance along a structural member in which pitting corrosion will have a detrimental effect on structural capacity. (Stewart, 2009) The values of L_U are varying from 100 mm to over 1000 mm. (Stewart, 2004; Stewart and Al-Harthy, 2008)

3.7.5 Pit Configuration

For simplicity, a hemispherical form of pits is assumed. The radius of the pit, $p_{\max}$, at time t, can be estimated as

$$p_{\max}(t) = V_{\text{corr}} t_{\text{corr}} R_{\text{pit}} \tag{3.54}$$

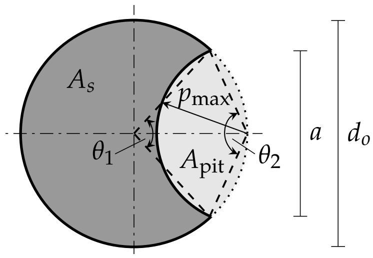


Figure 3.7: Pit Configuration. It is assumed that the pit has a hemispherical form with the radius $p_{\max}$. The dark shaded area A_s is the remaining steel area after t years of corrosion. Therefore, the light shaded area A_{pit} is the loss of area. Based on Val and Melchers (1997)

The pit configuration shown in Figure 3.7 is used to predict the cross sectional area of the pit, denoted by A_{pit}, which can be expressed as (Val and Melchers, 1997)

$$A_{\text{pit}}(t) = \begin{cases} A_1 + A_2 & p_{\max}(t) \le \dfrac{d_o}{\sqrt{2}} \\ \dfrac{\pi d_{s,o}^2}{4} - A_1 + A_2 & \dfrac{d_{s,o}}{\sqrt{2}} < p_{\max}(t) \le d_{s,o} \\ \dfrac{\pi d_{s,o}^2}{4} & p_{\max}(t) \ge d_{s,o} \end{cases} \tag{3.55}$$

where

$$A_1 = \frac{1}{2}\left(\theta_1\left(\frac{d_{s,o}}{2}\right)^2 - a\left|\frac{d_{s,o}}{2} - \frac{p_{\max}(t)^2}{d_{s,o}}\right|\right) \tag{3.56}$$

$$A_2 = \frac{1}{2}\left(\theta_2 p_{\max}(t)^2 - a\frac{p_{\max}(t)^2}{d_{s,o}}\right) \tag{3.57}$$

with

$$a = 2p_{\max}(t)\sqrt{1 - \left(\frac{p_{\max}(t)}{d_{s,o}}\right)^2} \tag{3.58}$$

$$\theta_1 = 2\arcsin\left(\frac{a}{d_{s,o}}\right) \tag{3.59}$$

$$\theta_2 = 2\arcsin\left(\frac{a}{2p_{\max}(t)}\right) \tag{3.60}$$

The cross section of an uncorroded reinforcement bar is

$$A_{s,\text{nom}} = \frac{\pi d_{s,o}^2}{4} \tag{3.61}$$

so the cross section area of a reinforcing bar after t years of pitting corrosion is denoted as A_s and estimated as

$$A_s(t) = A_{s,\text{nom}} - A_{\text{pit}}(t) \tag{3.62}$$

3.7.6 Reinforcement Properties Influenced by Pitting Corrosion

In addition to the decrease in cross-section, also the strength and ductility of reinforcing steel is affected. According to Du et al. (2005), the yield strength reduces linearly with corrosion loss so that

$$f_y(t) = (1 - \alpha_y Q_{\text{corr}}(t))f_{y,o} \tag{3.63}$$

where $f_{y,o}$ is the yield strength of an non corroded reinforcing bar, α_y is an empirical coefficient and Q_{corr} is the percentage corrosion loss in [%], which can be measured in terms of reduced cross section area or weight loss.

$$Q_{\text{corr}}(t) = \frac{A_{\text{pit}}(t)}{A_{s,\text{nom}}} \cdot 100 \tag{3.64}$$

The empirical coefficient α_y can be expressed as a Beta distributed random variable in the range from $a = 0.0$ to $b = 0.017$ and a mean $\mu_{\alpha_y} = 0.005$ and $\mathrm{CoV} = 0.2$.

4 Modeling Bayesian Networks

“ *Be honest,*
be brave,
be smart.” (Maung Min-Oo)

The deterioration process of RC structures is highly complex and additionally evolves in time, as shown in chapter 3. The modeling of those processes is subjected to significant uncertainties that are based on a simplistic representation of the actual physical process and limited information on material, environmental and loading characteristics. (Straub, 2009)

On the other hand, new information about the system may change the assessment of the structure and lead to new decisions and a verification of the used probabilistic model. (Attoh-Okine and Bowers, 2006)

In this context a combination of classical *structural reliability analysis* (SRA) and *Bayesian networks* (BNs) seems to be appropriate to model this problem.

SRA enables the accurate assessment of probabilities of failure events represented by computationally demanding physically based models. BNs are efficient in representing and evaluating probabilistic dependence structures. Additionally, BNs are able to update the model when new information becomes available. (Straub and Der Kiureghian, 2010a,b)

4.1 Modeling Approaches

The development of a consistent model, starts by the edification of the RC structure and ends by reaching a critical limit state but evolving over time is quite complicated, too. The modeling is broken down into several steps, which are discussed in more detail in the following sections.

4.1.1 Single Modeling

First, each physical model, explained in chapter 3 is transformed in a BN. Here every parameter of the model is represented as a node in the *single model* (SM) network. Also the input parameters are represented as nodes; for example, environmental or curing parameters, but only for one defined period of time.

A major issue of BNs is the limitation by dealing with continuous random variables. One common solution to avoid this issue is discretizing all continuous variables. Unfortunately the structure that characterizes the variable often gets lost by discretization of a continuous random variable. It is generally not the case that each discrete value can be associated with an arbitrary probability. (Koller and Friedman, 2009, p.185)

Beyond this, there is a lack of information about discretization of continuous random variables in the context of physical models related to decision problems. (Straub and Der Kiureghian, 2010b)

For the purpose to model and analyze the previous physical models, the continuous random variables get discretizatied in the first steps. Later on the model will be rewritten so that the discretization is limited to the smallest possible amount.

4.1.2 Coupled Modeling

After modeling the single events, *shared* and *dependent* parameters in each BN are singled out and connected with the other BNs. Where shared variables or shared parameters are denoted in this work these random variable that influences different nodes in different BNs, but it does not have any parents itself; for example, the concrete cover d_c. Compared to that, dependent variables classify the same type of nodes but with parents; for example, the corrosion rate V_{corr}. This procedure leads to a *coupled model* (CM).

4.1.3 Dynamic Coupled Modeling

The last step is to expand the CM over time. Therefore, the properties of DBNs are used, that lead to a so-called *dynamic coupled model* (DCM). Key of this procedure is to model the DBN efficiently, which includes methodologies to decrease the size of the network. The amount of nodes is decreased, but there is also an increase in accuracy by using less discretizations.

4.2 Bayesian Network for Carbonation Induced Corrosion

According to Equations (3.7) and (3.14) the limit state function of carbonation induced corrosion $g_{\mathrm{ca}}(\mathbf{X})$ is given by

$$g_{\mathrm{ca}}(\mathbf{X}) = g_{\mathrm{ca}}(x_{\mathrm{ca}}, k_{c,\mathrm{ca}}, k_{e,\mathrm{ca}}, k_{t,\mathrm{ca}}, n_{\mathrm{ca}}, C_{s,\mathrm{ca}}, R_{\mathrm{ca}}^{-1}, d_c, f_c, t_{\mathrm{cur}}, e_c) \tag{4.1}$$

Each random variable in Equation (4.1) can be expressed as a node in a BN. Taking into account the dependencies between the individual random variables, a BN for carbonation induced corrosion may be represented as in Figure 4.1.

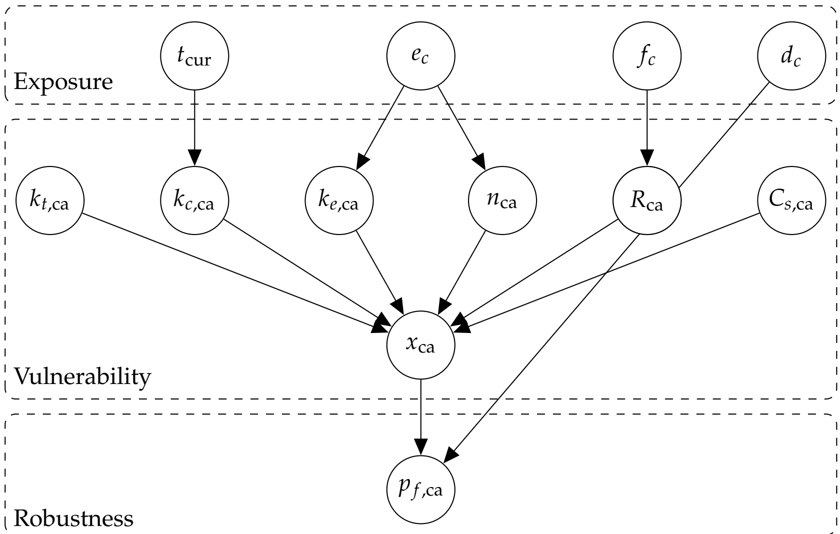


Figure 4.1: The BN for carbonation induced corrosion according to the DuraCrete (2000b) model consists of four input parameters, which influence the physical model. The probability of failure, in this case the probability if carbonation induced corrosion occurs or not, is represented in node $p_{f,\mathrm{ca}}$.

The random variable for the event of carbonation induced corrosion $p_{f,\mathrm{ca}}$ is represented as a *binomial* random variable (or *Bernoulli trial*) where only two possible outcomes can occur: " *success* " or " *failure* ".

Here the input parameters t_{cur}, e_c, f_c and d_c are the *exposure* to the SM and represent all possible endogenous and exogenous effects with potential consequences for the considered system.

The *vulnerability* of the SM is related to the direct consequences caused by the damages that follow by the given exposure events. In this case the direct consequences are related to x_{ca}, the depth of the carbonation front.

The *robustness* of the SM is related to indirect consequences that are depend not only on the damage state but also on the exposure of the damaged system. Here the robustness is assessed to the probability of carbonation induced corrosion as an expected value over all possible damage states and exposure events. (Faber, 2009, p 4.11)

As mentioned previously, discretization of random variables have some disadvantages; for example, assuming a discretization of 100 intervals per random variable, the node x_{ca} will have $100 \cdot 100^6$ different stages, which is almost impossible to calculate with commercial software for BNs. An additional difficulty here is to achieve an acceptable level of accuracy for the probability of failure if only 100 configurations are available. This problem will be treated at the end of this chapter.

4.3 Bayesian Network for Chloride Induced Corrosion

Based on the Equations (3.19) and (3.30) the limit state function $g_{\mathrm{cl}}(\mathbf{X})$ for chloride induced corrosion can be written as

$$g_{\mathrm{cl}}(\mathbf{X}) = g_{\mathrm{cl}}(k_{c,\mathrm{cl}}, k_{e,\mathrm{cl}}, k_{t,\mathrm{cl}}, n_{\mathrm{cl}}, D_o, C_{\mathrm{cl}}, C_{s,\mathrm{cl}}, C_{crit}, d_c, t_{\mathrm{cur}}, w/c, e_e) \qquad (4.2)$$

The output of the limit state function is represented as $p_{f,\mathrm{cl}}$ a random variable that expressed the probability of chloride induced corrosion. This variable can have two states: "yes", chloride induced corrosion occurs with a probability of p_f or "no" there is no chloride induced corrosion, the probability therefore is $1 - p_f$.

The representation as a BN is shown in Figure 4.2. Here, the network is also subdivided into exposure events, vulnerability and robustness, which follows the same assumption as discussed previously.

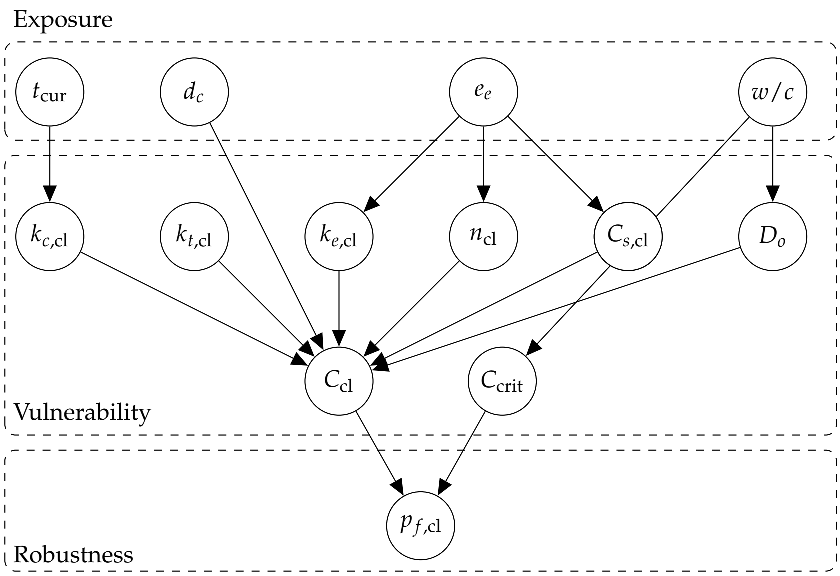


Figure 4.2: The BN for chloride induced corrosion according to the DuraCrete (2000b) model consists of four input parameters that influences the physical model. The probability of failure, in this case the probability if chloride induced corrosion occurs or not, is represented in node $p_{f,\text{cl}}$.

4.4 Bayesian Network for Propagation of Corrosion

Using the DuraCrete (2000b) model for the propagation of corrosion, the corrosion rate V_{corr} is a function of a set $\mathbf{X}$ random variables, which is given by

$$V_{\text{corr}}(\mathbf{X}) = V_{\text{corr}}(m_0, \rho, F_{Cl}, F_{\text{Galv}}, F_{O_2}, \rho_0, n_{\text{r}}, k_{t,\text{r}}, k_{c,\text{r}}, k_{T,\text{r}}, k_{RH,\text{r}}, k_{Cl,\text{r}}, RH, T, p_{f,\text{cl}}) \tag{4.3}$$

Different from the output of the SM of carbonation or chloride induced corrosion, the corrosion rate V_{corr} may take the output of any positive real value[1].

An other difference to both models discussed before is that not all variables are random. This is based on a lack of data in the physical model, as explained in section 3.6. So the variables m_0, F_{Galv}, F_{O_2}, $k_{c,\text{r}}$ and $k_{t,\text{r}}$ are assumed to be constant with only one state or outcome.

[1] A negative value can be excluded, because this means that the diameter is increasing.

Nevertheless, to model a complete BN for the propagation of corrosion, these variables are represented as uniform random variables in the range of ±10% of the initial value. For evaluation of the BN evidences are set at the assumed constant values.

In the SM for propagation of corrosion the first dependent random variable occurs in form of the probability of chloride induced corrosion $p_{f,\mathrm{cl}}$. This means that the random variable depends on the outcome of a different BN. In the first step to model the propagation independent, of any other models, values for $p_{f,\mathrm{cl}}$ are assumed and later on both models get coupled on this node.

A graphical expression of the BN for propagation of corrosion is illustrated in Figure 4.3, where the constant values are constituted as observed variables.

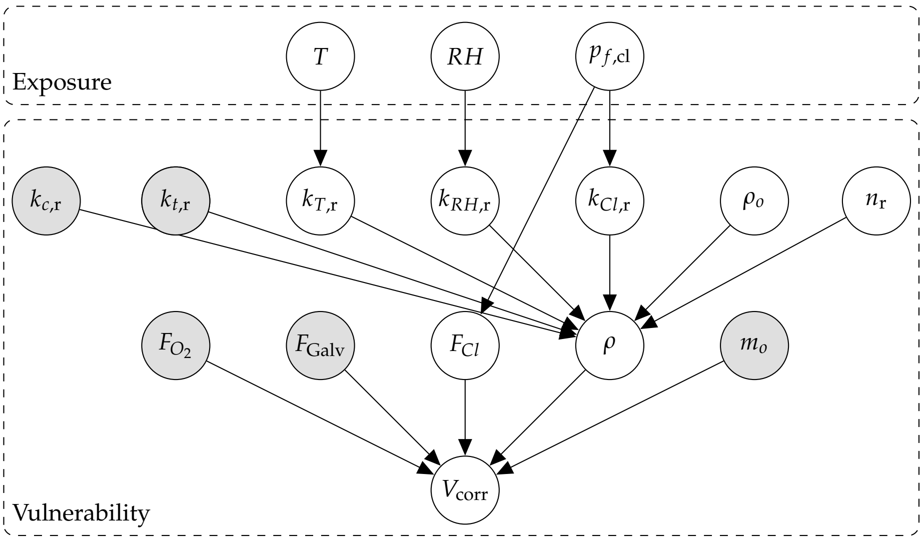


Figure 4.3: The BN for propagation of corrosion according to the DuraCrete (2000b) model consists of three input parameters, which influence the physical model. Thereby the parameter $p_{f,\mathrm{cl}}$ depends on the BN for chloride induced corrosion. The output of this SM is no probability of failure, instead a positive real value for the corrosion rate V_corr can be observed. Constant variables from the physical model are modeled as uniform distributed nodes and set evidence at the initial value, here illustrated as shaded nodes.

As shown in Figure 4.3 no state of robustness is available in the SM. This is caused by the fact that the propagation is the first process of corrosion, which is followed by the effects of corrosion. Therefore, the SM includes no indirect consequences, the output variable V_corr, denotes a physical variable with unknown distribution.

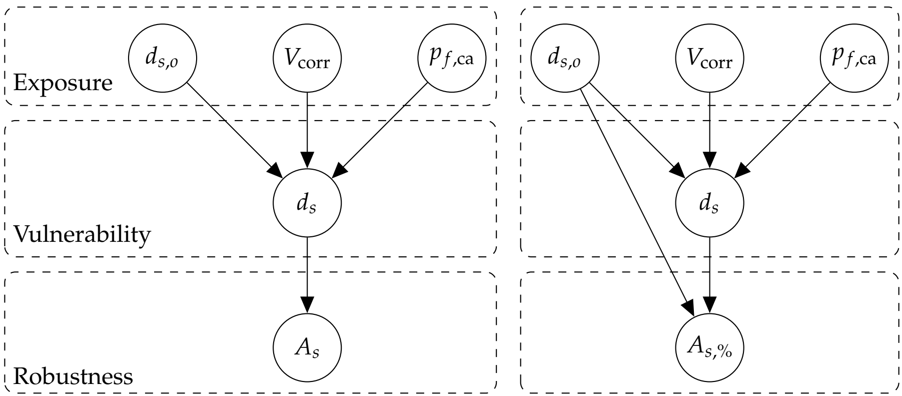
4.5 Bayesian Network for the Effects of Corrosion

4.5.1 Bayesian Network for General Corrosion

Under the assumption that general corrosion occurs if carbonation induced corrosion occurs and the amount on general corrosion, related to high levels of chloride is negligible, then the loss of bar area A_s is given as a function of

$$A_s(\mathbf{X}) = A_s(d_s, d_{s,o}, V_{\text{corr}}) \tag{4.4}$$

Since a SM represents only one defined period of time, the time t_{corr} from Equation (3.50) is implied in the BN, which is shown in Figure 4.4(a).



(a) (b)

Figure 4.4: (a) The BN for general corrosion include of five nodes. The corrosion rate V_{corr} and the probability of carbonation induced corrosion $p_{f,\text{ca}}$ are depending on other BNs. (b) A modified BN for general corrosion is where $A_{s,\%}$ is the percentual loss of bar area.

If A_s is also influenced by the information of $d_{s,o}$, it is possible to express the loss of bar area denoted by $A_{s,\%}$, as a ratio between current bar area A_s and nominal bar area $A_{s,\text{nom}}$. This modification brings the advantage that the bar area can be expressed by a random variable on the interval from 0 to 1, instead of an interval for any positive number limited by the initial diameter. This property becomes very useful by discretized continuous random variables.

4.5.2 Bayesian Network for Pitting Corrosion

Also the model of Val and Melchers (1997) for pitting corrosion can be expressed as a BN, shown in Figure 4.5. Here, in addition to the loss of bar area A_s, also a decrease of yield strength f_y occurs. Both are represented in the same BN and can be expressed as a function of

$$A_s(\mathbf{X}) = A_s(A_{\text{pit}}, A_{s,\text{nom}}, p_{\max}, R_{\text{pit}}, L_U, d_{s,o}, V_{\text{corr}}) \tag{4.5}$$

and

$$f_y(\mathbf{X}) = f_y(Q_{\text{corr}}, A_{\text{pit}}, A_{s,\text{nom}}, p_{\max}, R_{\text{pit}}, L_U, \alpha_y, d_{s,o}, V_{\text{corr}}, f_{y,o}) \tag{4.6}$$

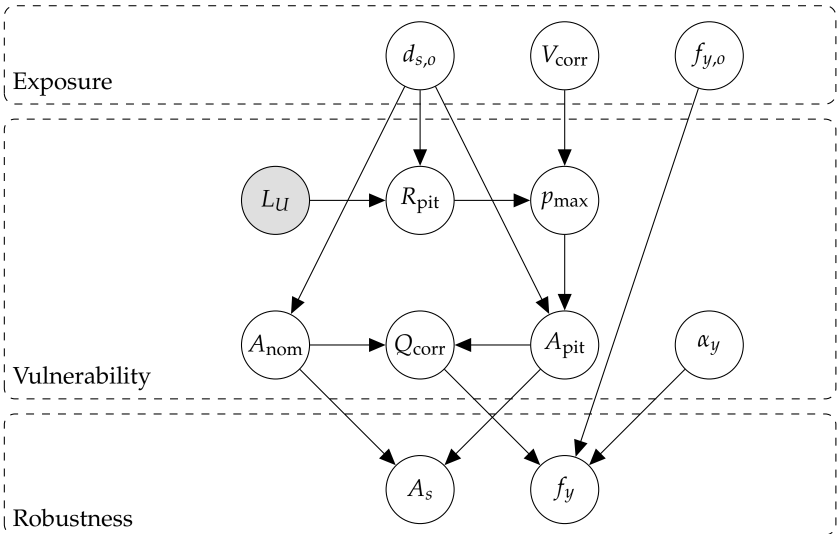


Figure 4.5: The BN for pitting corrosion according to the model of Val and Melchers (1997) and Stewart (2004, 2009, 2012). The model consists of three input parameters, which influencing the physical model. Thereby the parameter V_{corr} depends on the BN for propagation of corrosion. The constant parameter L_U, from the physical model, is modeled as uniform distributed node and sets the evidence at the initial value, here illustrated as shaded nodes. The output of this SM is A_s and f_y at a defined period of time.

This model contains a constant parameter for the uniform capacity length L_U, which is modeled as a uniform random variable in the range form 100 mm to 1000 mm. (Stewart, 2009)

The output of the SM for pitting corrosion is the bar area A_s and the yield strength f_y after a defined period of time exposed to corrosion. In the same manner as previously shown, these variables can be modified to $A_{s,\%}$ and $f_{y,\%}$, which describing the loss of area or yield strength according to the initial values in the range from 0 to 1.

4.6 Coupled Bayesian Network

To get a model over the whole physical process, the SMs have to be connected. Therefore, shared and dependent random variables from the different BNs must be singled out and replaced with the corresponding nodes or BNs. In certain circumstances some additional nodes must be added to the CM.

4.6.1 Shared Parameters

Shared parameters are denoted parameters that occur in multiple places across one or more networks. Where the notation *parameter* describes a node in which the values are not determined by others.

In general, it can be distinguished between *general* and *local* parameters. Global structures occur when the same CPD is used across multiple variables in the network. This type of sharing arises naturally from the plate model and is used in the previous BNs. A Local structure is finer grained and allows parameters to be shared even within a single CPD. (Koller and Friedman, 2009, p.754). Later on, *parameter estimation* for a BN can be performed on this basis. In this work only general parameters are used.

The parameters for the corrosion problem are already defined in section 3.1 and section 3.2. In a condensed form, there are:

- Material parameters are the following: compressive strength f_c, water cement ration w/c, curing period t_{cur}, concrete cover depth d_c, yield strength f_y and bar diameter d_s.
- Environmental parameters are the following: exposure environment e_e, exposure class e_c, temperature T and relative humidity RH.

These shared parameters occur in the different SMs as discussed previously. Not only within a BN the parameters are shared but also with other networks. For example, the concrete cover depth d_c or the nominal bar diameter $d_{s,0}$ occur in different BNs and influence different nodes.

An additional property of the shared parameter is the time dependency. Some parameters do not change over time; for example, the initial bar diameter $d_{s,0}$, these parameters are called *initial parameters.* Other parameters, such as temperature T, vary over time and are denoted as *template parameters* or *time dependent parameters.*

4.6.2 Dependent Parameters

Dependent parameters act in the same way as shared parameters for a SM, but with the difference that the parameter itself depends on an other BN. In a CM these parameters are part of the coupled BN and link the different models to an entire consistent physical model, starting by the edification of the RC structure and ending by reaching a critical limit state.

Definition 4.1 (Dependent Parameter) *Let $\mathcal{B}_i$ and $\mathcal{B}_j$ be Bayesian networks over the variables $\mathcal{V}_i$ and $\mathcal{V}_j$. Then θ specifies a dependent parameter if*

$$\theta \in \mathcal{V}_i \cap \mathcal{V}_j \tag{4.7}$$

In other words, dependent parameters transmit information from one SM to an other. Hence, the quality of the information is an important value, which gets a very strongly influence by discretization of the node. A major goal is to optimize these links, so that no or only limited loss of information occurs.

Are there no dependent parameters, then the models are not able to get linked, which also means there no physical (or logical) relationship between the different SMs exist. In this case other models have to be chosen.

However, in the case of concrete degradation, caused by corrosion of reinforcing steel, such a "path" of dependent parameters is available. The BNs of carbonation and chloride induced corrosion are connected by the probability of failure for the onset of corrosion with the BN for propagation of corrosion. This network is linked to the BN for effects of corrosion by the shared parameter V_{corr}.

4.6.3 Additional Nodes

In some cases it is beneficial to add extra nodes to the existing BNs, to extract some extra information from the network and on the other hand to simplify the existent model.

One example for getting additional information is to add a node corrosion to the network, as it is schematically shown in Figure 4.6(a). Like the probability of carbonation induced corrosion $p_{f,\mathrm{ca}}$ or chloride induced corrosion $p_{f,\mathrm{cl}}$ contains the node corrosion p_{corr} to states of outcome: "yes" if corrosion occurs or "no" if there is no corrosion. However, this time it takes the carbonation and chloride penetration into account.

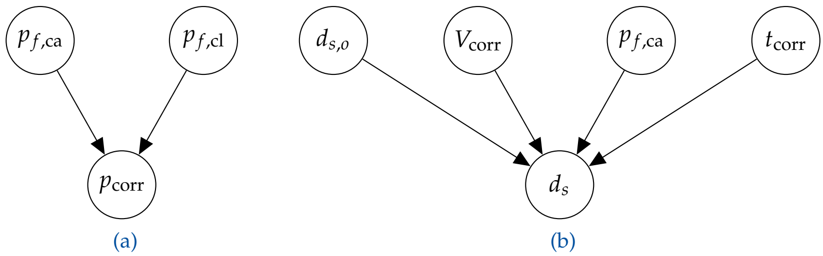


Figure 4.6: (a) The event of corrosion p_{corr} is an additional node with the parents $p_{f,\mathrm{ca}}$ and $p_{f,\mathrm{cl}}$ and provides new information about the state of corrosion. (b) A simplification of the BN for general corrosion is adding the time t_{corr}. This allows to couple and expand different BNs over time.

An *additional node* to simplify the CM is shown in Figure 4.6(b). Here the time t_{corr}, which describes the time where the reinforcement is exposed to corrosion, is added to the BN. In the SM discussed in section 4.5, it was not necessary to add this node, because the time is implied in the BN itself. However, in the CM two different types of time occurs: the service time t of the structure and the time t_{corr} where corrosion propagates. Only one of them can be represented in a plate model.

Because of the fact $t_{\mathrm{corr}} \leq t$ it is appropriate to express t in a plate model and t_{corr} as random variable. Since t_{corr} is represented as a random variable, this value of time is not longer a discrete value, it rather represents the probability of corrosion for a defined time period.

4.6.4 Coupled Bayesian Network for Concrete Degradation Caused by Corrosion

Using previous assumed properties, the SMs can be coupled to one consistent CM, which is shown in Figure 4.7. This model represents the degradation of concrete, caused by corrosion for one defined period of time and one element[2] of the RC structure.

[2] According to Stewart (2004) is the process of pitting corrosion highly related with spatial and temporal variables. However, in this work only the temporal effects are treated.

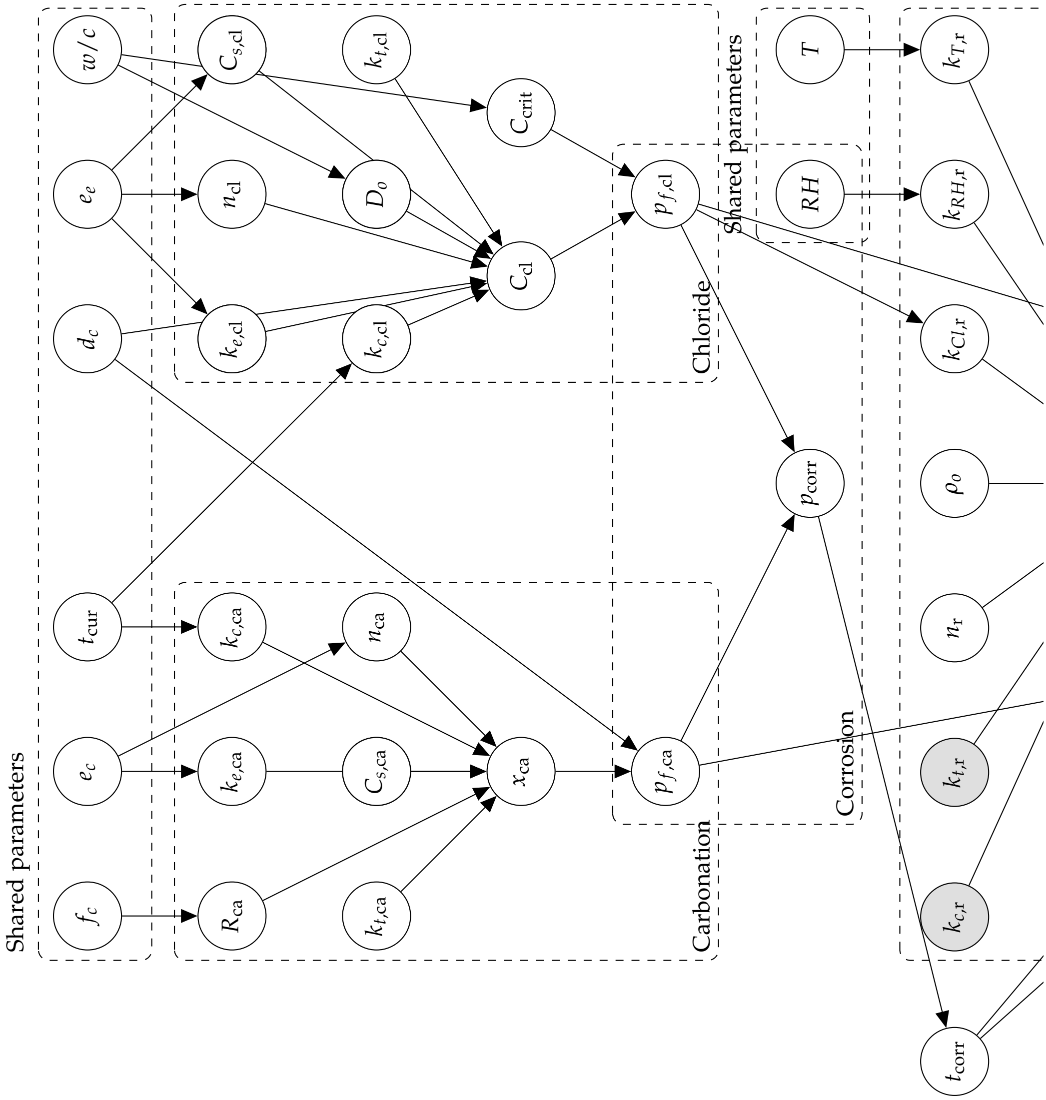
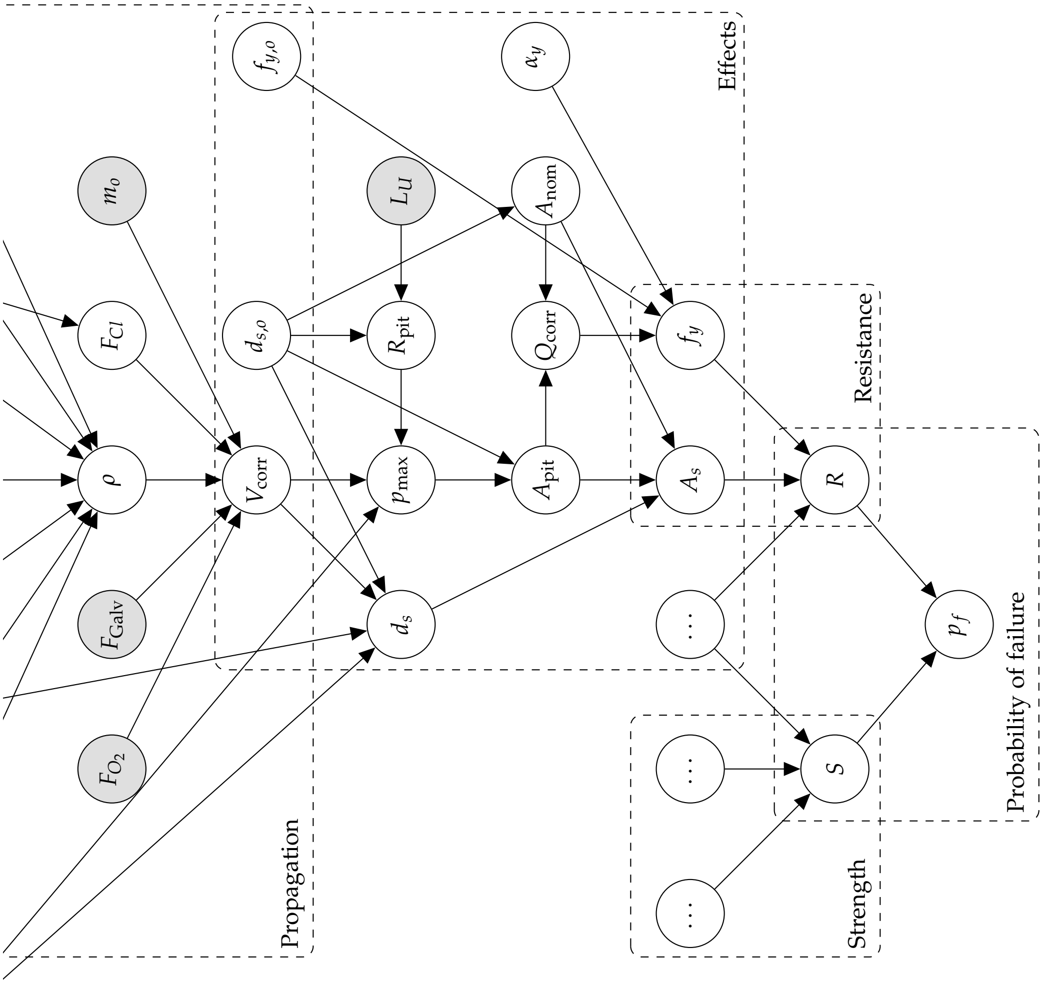


Figure 4.7: The BN for a CM. Additional indicated are the resistance and the strength for the determination of the probability of failure. Where the nodes "..." represent additional nodes that are necessary to compute the structural reliability.


Propagation
Effects
Resistance
Strength
Probability of failure
m_o
F_{Cl}
ρ
F_{Galv}
F_{O_2}
$f_{y,o}$
$d_{s,o}$
V_{corr}
L_U
R_{pit}
$p_{\max}$
d_s
α_y
A_{nom}
Q_{corr}
A_{pit}
f_y
A_s
R
S
p_f
…

As shown in Figure 4.7 it is quite complex to represent such a CM as a graphical network. To shrink the size and make the representation more clearly, a schematically notation of a BN is used. Here the nodes do not represent mainly random variables, but more the physical meaning beyond.

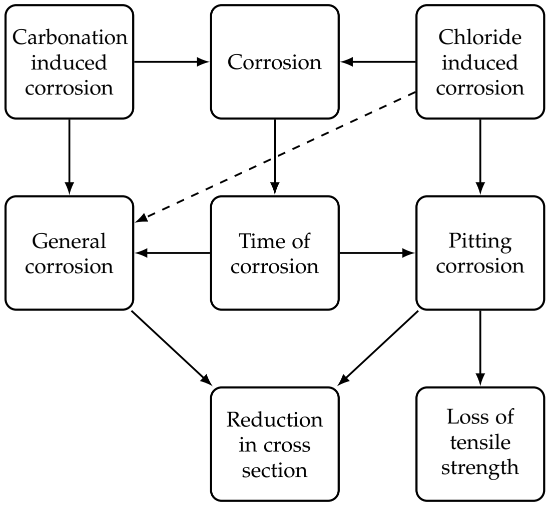


Figure 4.8: A simplified representation of the CM for degradation of concrete caused by corrosion. Here the nodes represent the based physical model. Chloride induced corrosion can also caused general corrosion as indicated by the dashed arrow. This effect is not considered in the treated model.

4.7 Dynamic Coupled Bayesian Network

The last step to provide a sophisticated model for the deterioration process of RC structures evolving in time is to unroll the previous discussed CM over the service life of the structure. Unfortunately, this process is not so easy to implement, especially with discretized nodes where the loss of information grows by each time step.

4.7.1 Simplification of the Model

The goal of the final model is to describe the structural reliability of a RC structure; therefore, it is not necessary to represent every random variable in the model as explicit as a node in the BN. Instead parts of the BN can be reduced to a single random variable, containing the information of the other ones.

Simplification of the Bayesian Network for Carbonation Induced Corrosion

In case of the BN for carbonation induced corrosion, the random variable of interest is the probability of corrosion $p_{f,\text{ca}}$. All other random variables are not explicitly necessary to describe the structural reliability. So the information of the material, environmental execution, and test variables are collected and treated in node for the probability of failure. Only the input parameters have to be modeled beside $p_{f,\text{ca}}$.

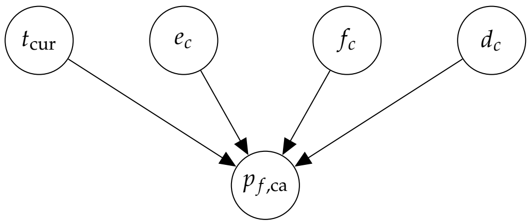


Figure 4.9: The simplified BN for carbonation induced corrosion includes only the input parameters and the output parameter $p_{f,\text{ca}}$. The other values of the models are included in the probability of failure.

The disadvantages of this simplification is that information of the model values, such as the age factor n_{ca}, are no longer available. On the other hand, the huge advantage is that those values need no longer to be modeled and so a discretization of those continuous random variables is omitted. Beyond this, SRA can be used to calculate the probability of failure, which leads to more accurate values for the node $p_{f,\text{ca}}$ as any discretization of the unsimplified BN would be able to. Also the property that the node $p_{f,\text{ca}}$ only can reach two states, allows to transmit those outcome values without any loss of information to an other model (or node).

The only variables that must be discretized are the input variables. Here it is the grade f_c and the cover depth d_c. These discretizations depend on the decision making process and may be different for each RC structure.

Simplification of the Bayesian Network for Chloride Induced Corrosion

The same principal as discussed previously can be used to simplify the BN for chloride induced corrosion. The simplification is shown in Figure 4.10.

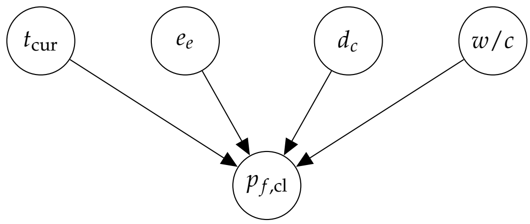


Figure 4.10: The simplified BN for chloride induced corrosion includes only the input parameters and the output parameter $p_{f,\text{cl}}$. The other values of the models are included in the probability of failure.

Simplification of the Bayesian Network for Propagation of Corrosion

Also the BN for propagation of corrosion can be simplified this way. However, here is the output value V_{corr} a random variable where the distribution is not defined.

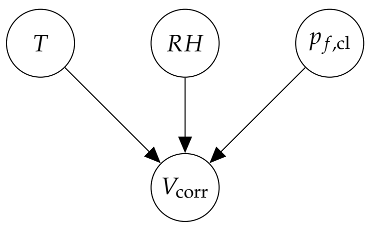


Figure 4.11: The simplified BN for propagation of corrosion includes only the input parameters and the output parameter V_{corr}. The other values of the models are included in the output node.

To use this variable for subsequent calculations, the discretized values of V_{corr} must be fitted to a distribution of a random variable. For this fitting a *Kolmogorov–Smirnov test*, *maximum likelihood estimator* or some other estimators can be used.[3]

[3] In this work the two mentioned methods are used.

An other option is to use a different model; for example, Stewart (2004) or Osterminski and Schießl (2012), in which the corrosion rate can be defined as a random variable.

Simplification of the Bayesian Network for the Effects of Corrosion

The BN for general corrosion is all ready in a very simple form and the BN for pitting corrosion can be simplified as the models before.

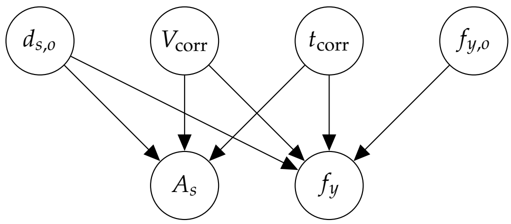


Figure 4.12: The simplified BN for pitting corrosion includes only the input parameters and the output parameters A_s and f_y. The other values of the models are included in the output nodes.

Here the same problem as in the simplified model for propagation of corrosion occurs. The output variables A_s and f_y can take any positive value or any value in the range of 0 to 1 after modification of the BN.

Simplification of the Bayesian Network for the Propagation and the Effects of Corrosion

However, V_{corr}, A_s and f_y are also only intermediate results for the structural reliability. Using the idea of simplification as before, these values can be included in R, the random variable for the resistance of a RC structure or even included in p_f, the probability of failure of the structure. This leads to a two-phase model, where the first phase is the failure of corrosion and the second phase the failure of the system according to Tuutti (1982), but now coupled through a probabilistic model based on a BN.

4.7.2 Plate Model

After the simplification of the CM and the subdivision of the shared parameters into initial and temporal values, the BN can be expanded over time. Therefor, plate notation from subsection 2.2.5 is used. So the model for degradation of concrete, caused by corrosion, can be expressed as a DBN, which is shown in Figure 4.13.

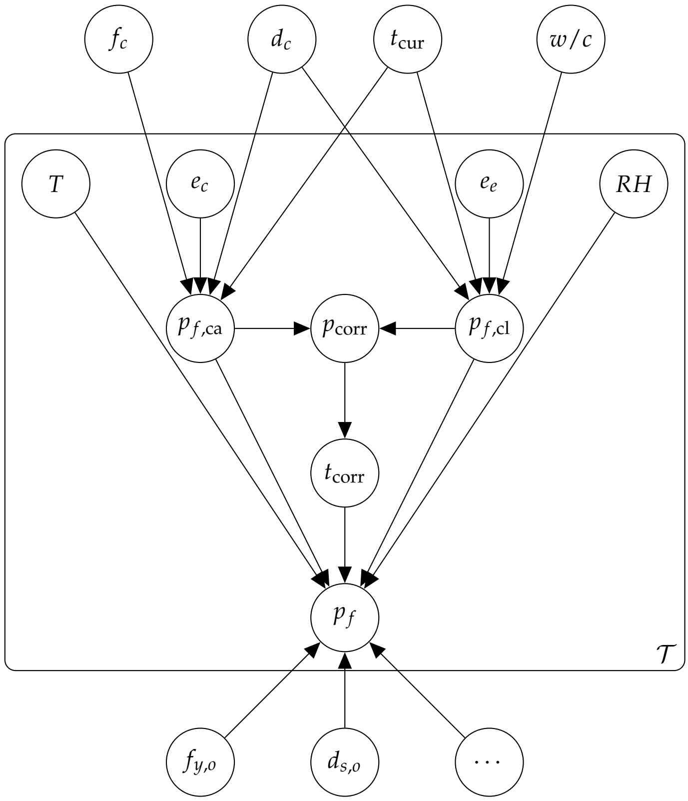


Figure 4.13: The DBN for degradation of concrete caused by corrosion is expressed as a plate model. Nodes inside the plate are evolving over time, outside are the initial parameters. The node "…" denotes other random variables that are necessary to compute the probability of failure; for example, geometric or load parameters.

To provide a very general model, the focus of this DCM is on the effects of corrosion. Geometric and load properties are included in the probability of failure[4], which makes it possible to use the model for any kind of RC structure and load situation.

[4] A more detailed explanation can be found in section 4.8.

4.7.3 Dynamic Coupled Model for Degradation of Corrosion Caused by Chloride Induced Corrosion

Chloride induced corrosion is one of the most common causes for structural deterioration, related to corrosion and the models are quite often discussed in literature. Hence, the remainder of the work will focus on this event and neglect the event of carbonation induced corrosion. However, the theory and all results later on can also be achieved by using the DCM represented in Figure 4.13, but to make the model simpler and comparable with other results of the literature the DCM shown in Figure 4.14 is used from now on.

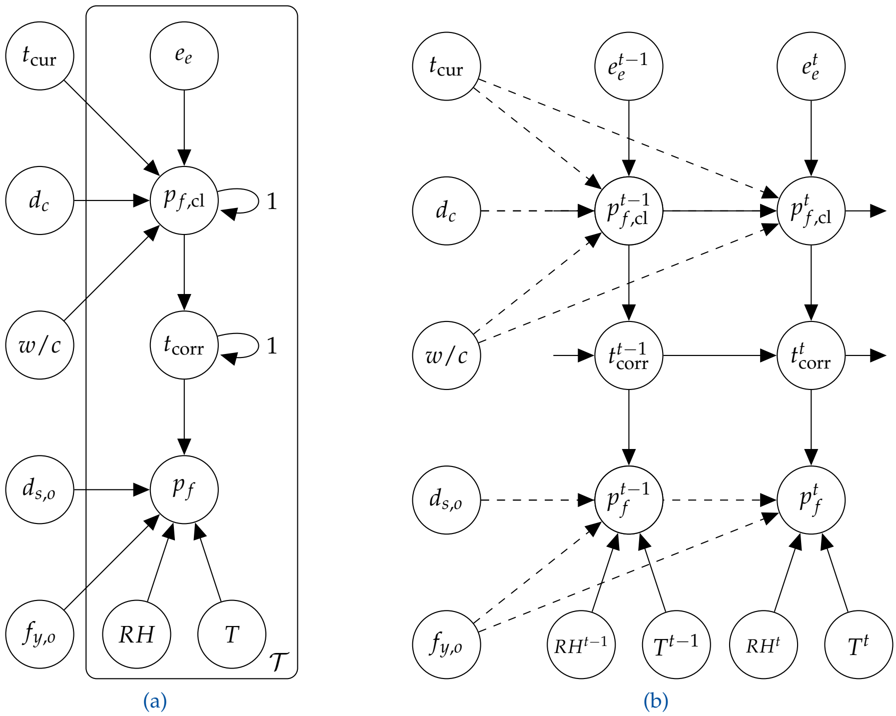


Figure 4.14: (a) The DBN for degradation of concrete caused by corrosion is expressed as a plate model. Nodes inside the plate are evolving over time, outside are the initial parameters. (b) Is an unrolled part of the model in the range of $t-1$ to t. The initial parameter occurs only once and their connections are indicated here with the dashed arcs.

4.8 Modeling Structural Reliability

The principals of SRA are discussed previously in section 2.3. However, to provide a computationally model for degradation of concrete caused by corrosion, these ideas have to be implemented in the CM and DCM.

Principally, structural reliability can be expressed as a set **R** of random variables, describing the resistance of the structure, and a set **S** to describe the internal stress. This principle be used for any kind of structure and design situation. Hence, it is difficult to make general statements for RC structures, because of their different geometries and load situations.

One way to take these properties into account, is to normalize the random variables with respect to their nominal values. This makes the model applicable to a wide range of design situations.

$$X = \frac{X}{X_{\text{nom}}} X_{\text{nom}} \tag{4.8}$$

The random variable for the resistance can be taken as the strength of the RC structure. Likewise, can the random variable for the internal strength or stress be taken as the load effect (moment, shear, etc.), which is considered as dimensionally consistent with the resistance. Both of them can be used directly to formulate the limit state function. (Ellingwood et al., 1980, p.36)

4.8.1 Model of Resistance

The load carrying capacity R of a RC structure depends on the resistance of their components and connections; for example, material strength, section geometry and dimensions. This resistance R can be considered as a product of the nominal resistance R_{nom} and a random variable X_m, which describes the model uncertainties. (Nowak and Collins, 2000, p.182)

$$R = R_{\text{nom}} X_m \tag{4.9}$$

In the reminder of this work the structural configuration is assumed to be a simple supported RC beam as shown in Figure 4.15. Therefore, the ultimate flexural capacity M_u and the ultimate shear capacity V_u of the RC beam can be used to describe the resistance R of the structure. An other simplification is made by neglecting shear failure for the structural reliability estimation. This assumption leads to the result that the resistance R can be expressed in terms of ultimate flexural capacity M_u for a simple supported RC beam.

$$R = M_u = A_s f_y \left(d - \frac{A_s f_y}{1.7 f_c b} \right) X_m \tag{4.10}$$

where A_s is the cross-sectional area of the reinforcement, f_y the yield strength, d the effective depth, f_c the compressive strength of concrete, b the beam with and X_m describes the model uncertainties.

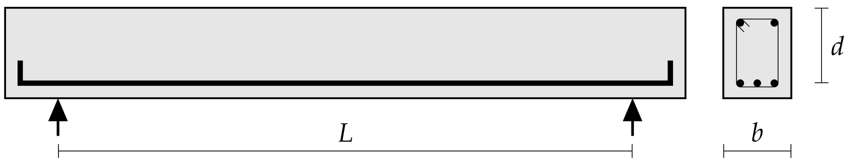


Figure 4.15: Simple supported RC for the evaluation of the structural reliability. Where L is the clear span, d the effective length and b the beam width.

Since pitting corrosion only affects A_s and f_y, a near linear relationship exists between the ultimate flexural capacity and these variables. (Stewart, 2009)

This makes it possible to analyze different design situations by normalizing the flexural capacity in terms of an un-corroded RC beam, denoted by M_{nom}, and comparing it with the changes caused by corrosion.

According to Ellingwood et al. (1980), the ultimate-to-flexural resistance M_u / M_{nom} can be described by a normal random variable with a mean of $\mu = 1.05$ and a coefficient of variation of $\text{CoV} = 0.11$. These statistics include the random variability of the variables in Equation (4.10).

4.8.2 Structural Load Model

To design any RC structure, different types and magnitudes of the loads that are expected to act on the structure during its lifetime have to be considered. In the scope of this work, a relative simple model is used to describe the random variable S.

The random variable G describes the *permanent* or *dead load* and is assumed to be normal distributed with a mean $\mu_G = 1.0$ and a $\text{CoV}_G = 0.1$. (Nowak and Collins, 2000, p.148)

$$G \sim \mathcal{N}(\mu_G, \sigma_G) \tag{4.11}$$

The *variable* or *live load*, denoted by Q, describes a random variable that considers a load varying over time. Q is assumed to follow a Gumbel distribution with $\mu_Q = 1.0$ and a $\text{CoV}_Q = 0.4$. (Köhler et al., 2012)

$$Q \sim \text{Gumbel}(\mu_q, \alpha_Q) \tag{4.12}$$

Hence, the random variable S for the internal strength or stress can be described as the load effect.

$$S = G + Q \tag{4.13}$$

4.8.3 Model of the Limit State Function

The probability of failure p_f can be expressed according to Equation (2.71) in combination with Equation (4.13) as

$$pf = P[g(R, G, Q) \leq 0] \tag{4.14}$$

where

$$g(R, G, Q) = R - G - Q = 0 \tag{4.15}$$

After solving Equation (4.14) returns the probability of failure p_f that can be related to a reliability index β as discussed in section 2.3. However, this reliability index β should reach a target value β_t, which represents the general requirement to the safety of the RC structure.

For design purpose, each random variable in the limit state function corresponds to a nominal design value specified by a structural design code, in case of a RC structure; for example, the ACI, CSA or Eurocode 2.

Therefore *partial safety factors,* denoted by γ, are used. Those factors are nothing else than scaling factors that allow the designer to convert a nominal design value x_{nom} of a variable to a value x^*, needed to satisfy Equation (4.13) for the target β_t. (Nowak and Collins, 2000, p.229)

$$\gamma_i = \frac{x_i^*}{x_{i,\text{nom}}} \tag{4.16}$$

In this case, Equation (4.15) can be transformed into a so-called *load and resistance factor design* (LRFD). (Köhler et al., 2012)

$$z_i \frac{R_{\text{nom}}}{\gamma_m} - \alpha_i \gamma_G G_{\text{nom}} - (1 - \alpha_i) \gamma_Q Q_{\text{nom}} = 0 \tag{4.17}$$

Where γ_m, γ_G and γ_Q are the corresponding partial safety factors for the resistance and for the load. z_i is the design variable that is defined by the chosen dimension of the structural component. In additional α_i describes the ratio between permanent and variable load and is defined in the range from 0 to 1. Within the Eurocode[5] the nominal values are so-called characteristic values and correspond to fractile values of the random variables. (Köhler et al., 2012)

According to Eurocode 0 (2012) a target β of $\beta_t = 4.7$ ($p_f \approx 10^{-6}$) after 1 year and $\beta_t = 3.8$ ($p_f \approx 10^{-4}$) after 50 years should be reached, assuming no deterioration of the RC structure. This can be ensured by using a suitable set of partial safety factors ($\gamma_m = 1.4, \gamma_G = 1.35, \gamma_Q = 1.5$) and a permanent to variable load ratio of $\alpha = 0.5$.

[5] In case of concrete structures published under Eurocode 2 (2004).

5 Implementation and Analysis

> *"If we knew what it was we were doing,*
> *it would not be called research, would it?"*
> (Albert Einstein)

To work with the previous proposed DCM for the degradation of concrete, caused by chloride induced corrosion, it must be implemented in a computational model. Therefore, the high-level programming languages Python and C++ are used.

5.1 Computational Model

Performing BNs and SRA requires a complex mathematical algorithm as shown in chapter 2 that has to be implemented in an appropriate programming language. For both BNs and SRA different software packages are available; for example, HUGIN, GeNIe and SMILE, STRUREL or FERUM. However, none of these are capable to handle BNs and SRA at once. Furthermore, the majority of the software packages are proprietary, hence the source code is not available.

Under these aspects new software packages, in form of Python libraries, are developed within the scope of this work. The code and the documentation is published under the GNU General Public Licence[1] and is online available. A detailed description is beyond this work, so only an overview will be provided.

5.1.1 Reinforced Concrete Structural Reliability - CoRe

Main part of the library is the module *Concrete Reliability* (CoRe)(Hackl, 2013b). Here the physical model from chapter 3 is implemented. Beside the exposure to carbonation or chloride penetration, the propagation and effects of corrosion, also the geometric and load properties can be defined. CoRe includes also methods for summarizing output and plotting.

[1] More information about GNU General Public Licence is available under `http://www.gnu.org/licenses/`

5.1.2 Structural Reliability Analysis with Python - PyRe

PyRe (*Python Reliability*) is a Python module for structural reliability analysis. Its flexibility and extensibility make it applicable to a large suite of problems. Along with core reliability analysis functionality, PyRe is also used to define different random variables, which are used in CoRe.

PyRe provides the functionalities, which are described in section 2.3; for example, FORM, SORM, CM and IS analysis. In addition, the correlations between the random variables can be modeled. (Hackl, 2013c)

5.1.3 Simple Bayesian Network with Python - PyBN

To handle BNs, the module PyBN (*Python Bayesian Networks*) is developed. The theoretical background, which is covered in the program code, is explained in section 2.2. However, the field of Bayesian networks and decision graphs contain a lot more as provided in PyBN, here only the major features are implemented. To use analysis tools, graphical representation or other advantage functions, PyBN is capable to export data to GeNIe and SMILE. (Hackl, 2013a)

5.1.4 Discretization of a Continuous Random Variables

In order to provide a computational model, continuous random variables have to be discretized. For the SM all nodes which occur in the BN, have to be expressed as discrete variables. Due to the limited power of computers, a discretization with only seven intervals for the random variables of chloride induced corrosion is possible.

Performing some simplifications as explained in chapter 4, the amount of continuous random variables can be decreased to two. Based on the boundary conditions of the DuraCrete (2000b) model, only the concrete cover depth d_c and the temperature T have to be discretized. Beneficially, these two random variables are shared parameters and subjects of the decision making process. Therefore, the accuracy and the size of the discretization interval is part of the decision problem.

For instance, if the cover depth d_c is known from construction plans, the input variable can be eliminated to a constant variable. Furthermore, the variation of the initial depth is taken into account. In the reminder of this work the cover depth is discretized into eight values from 15 mm to 85 mm with a step size of

10 mm. This approximation covers the majority of the values that are suggest in the current design codes.

The discretization of the temperature T is more complicated, because an infinite variety of values is possible, which is related with the location of the RC structure. If distributions for the temperature are available, then the temperature can be included in the probability of failure just as the other random variable. Is no information available, an useful level of discretization has to be chosen. Here is the temperature assumed in the interval from $-5\,°C$ to $30\,°C$ with a step size of $5\,°C$.

5.2 Sensitivity Analysis

After modeling the previously discussed BNs in CoRe, different analysis methods can be performed on the SMs and the CM.

To investigate the robustness of the output probabilities of a BN, a sensitivity analysis can be performed. Therefore, Castillo et al. (1997) and Kjaerulff and Gaag (2000) proposed a method for sensitivity analysis that are implemented in GeNIe.

In a general mathematical context, *sensitivity analysis* is the computation of the effects when changing the input parameters or assumptions on the output values. (Morgan and Small, 1992, p.39) For BNs this means, more specifically, how sensitive CPDs of a target node are to small changes in the parameters and evidences values. (Castillo et al., 1997)

Briefly explained, each entry of a CPD has its own sensitivity, defined as the value of derivative at x of a function $y = P(x)$, where y is the posterior of the target node and x the value of the specific parameter in the CPD. Since targets can have more than two states and more than one target can be enabled for sensitivity calculation, each CPD entry may have multiple values for each pair of target and target outcome.

The maximum sensitivity value per node can be divided by the maximum sensitivity value per network, so that a normalized value can be determined over the network. Since sensitivity depends on a set of evident nodes, every modification changes the sensitivity.

5.2.1 Sensitivity Analysis for Carbonation Induced Corrosion

A sensitivity analysis on the BN for carbonation induced corrosion, shown in section 4.2, should provide information about how the state of each model parameter impacts on the target node, in this case the probability of carbonation induced corrosion $p_{f,\text{ca}}$. Due to an evaluation prediction of importance and influence of each variable can be made. This information may influence future interpretations of the model or may show critical variables for the decision making process.

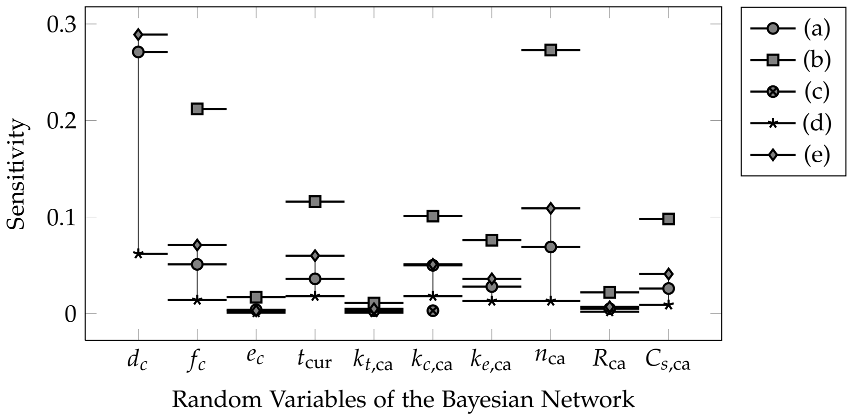


Figure 5.1: Sensitivity analysis for the BN of carbonation induced corrosion after 50 yr with the target node $p_{f,\text{ca}}$. (a) No evidence is observed. (b) A low concrete cover depth d_c is observed. (c) A high concrete cover depth d_c is observed. (d) Evaluation after 10 yr with no evidence. (e) Evaluation after 100 yr with no evidence.

As shown in Figure 5.1(a), with no observation and an exposure time of 50 yr, the concrete cover depth d_c has a huge impact on the SM. This correlates with the major fact, that the cover is a important parameter to protect the steel bars against corrosion.

With measurements, information from construction plans, or other methods the concrete cover d_c can be estimated (or observed). Is a (b) low concrete cover observed, the impact of the age factor n_{ca} and the grade f_c increases. In contrast, a (c) high concrete cover can be obtained, then the impact of the execution parameter $k_{e,\text{ca}}$ is leading. However, the value of the sensitivity index rapidly decreases caused by the fact that the probability of carbonation induced corrosion from a high concrete cover is quite small ($p_{f,\text{ca}} \approx 6.86 \cdot 10^{-6}$ or $\beta = 4.3$ with $d_c \approx 85\,\text{mm}$).

Evolving over time represented in Figure 5.1(a), (d) and (e), the impact of most parameters increases. Beside the concrete cover d_c, the age factor n_{ca} becomes the second most important parameter.

As it can easily be noticed, that there is a huge variety of different possible settings which can be analyzed. For the development of a physical model for carbonation induced corrosion, it may be interesting how individual model parameters influence the model. However, from the perspective of a decision maker, the scatter of the input parameters become more important. Therefore, the simplified model, shown in Figure 4.9 can be used. Here only the shared parameters and the probability of carbonation induced corrosion are represented as nodes for the BN.

In the same manner, as shown previously, a sensitivity analysis can be performed. GeNIe offers also the possibility to show the results of a sensitivity analysis in form of a *Tornado Diagram.* The uncertainty in the parameter associated with the largest bar, the one at the top of the chart, has the maximum impact on the result, with each successive lower bar having a lesser impact. This arrangement is why the result is called a Tornado Diagram. In Figure 5.2 the sensitivity Tornado for the probability of failure, caused by carbonation after 50 yr, is represented.

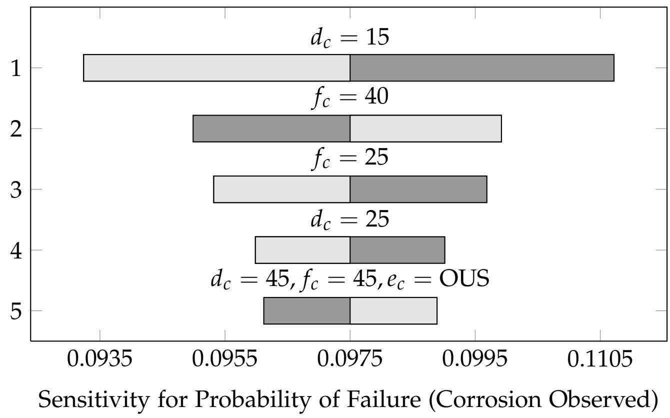


Figure 5.2: Sensitivity Tornado for the simplified BN of carbonation induced corrosion after 50 yr with the target node $p_{f,\mathrm{ca}}$ and a parameter spread of 10 % of the current value. The bar at the top of the chart has the maximum impact on the probability of failure. Only the top five settings are represented.

As shown in Figure 5.2, it is not surprising that the case where the concrete cover is observed as 15 mm has the largest impact on the SM. But on the second and third place are the influences of the concrete grade. Not only individual observed parameters have a huge impact on the model, but also combinations of different observations can have influence, as it is shown in the fifth setting of the Tornado Diagram.

5.2.2 Sensitivity Analysis for Chloride Induced Corrosion

Performing a sensitivity analysis for the parameters of chloride induced corrosion based on the BN represented in Figure 4.2, leads to the selected results shown in Figure 5.3.

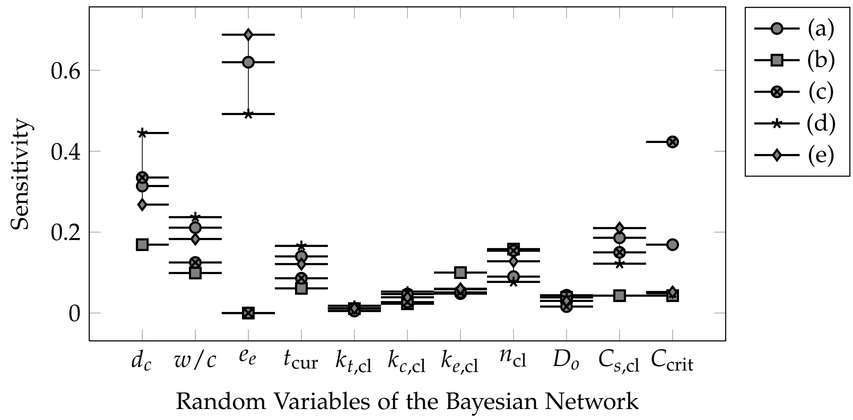


Figure 5.3: Sensitivity analysis for the BN of chloride induced corrosion after 50 yr with the target node $p_{f,\mathrm{cl}}$. (a) No evidence is observed. (b) A submerged zone is observed. (c) A atmospheric zone is observed. (d) Evaluation after 10 yr with no evidence. (e) Evaluation after 100 yr with no evidence.

Without an evidence (a) not the concrete cover d_c but the environmental exposure e_e has the highest impact on the probability of chloride induced corrosion. After observing, for example, a submerged zone (b), the impact of the children of node e_e increases. Especially, the impact of the age factor n_{cl} and the environmental parameter $k_{e,\mathrm{cl}}$; moreover, the effects of the concrete cover d_c decreases. By observing an atmospheric zone (c), the sensitivity index of the age factor n_{cl} increases again. However, now also the influence of the chloride surface concentration $C_{s,\mathrm{cl}}$, the concrete cover d_c and the critical chloride concentration C_{crit} grows.

Considering an evolvement over time, represented in Figure 5.3(a), (d) and (e), it can be observed that the impact of the environmental exposure e_e increases, while the importance of the concrete cover d_c decreases.

In the Tornado representation of the simplified BN, shown in Figure 4.10, becomes the fact apparent that the environmental exposure e_e is the leading parameter for chloride induced corrosion for the DuraCrete (2000b) proposed model. Only on place five comes the effect of a low concrete cover, as shown in Figure 5.4.

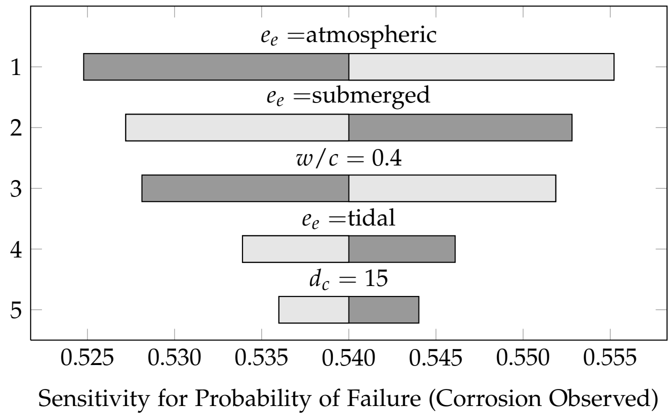


Figure 5.4: Sensitivity Tornado for the simplified BN of chloride induced corrosion after 50 yr with the target node $p_{f,\mathrm{cl}}$ and a parameter spread of 10 % of the current value. The bar at the top of the chart has the maximum impact on the probability of failure. Only the top five settings are represented.

This leads to the conclusion that for a decision making process the information of the current environmental exposure e_e is more valuable than any information of the other parameters. While a precise description of the concrete cover d_c, the w/c ratio and the curing period can be made, the definition of the environmental exposure e_e; for example, the different zones of the marine environment is quite difficult to make, as explained in section 3.2. Also a variation of the environmental exposure over time is not excluded. In order to develop a more accurate model of chloride induced corrosion, these effects should be taken into account. Therefore, see section 5.3 and section 5.4.

5.2.3 Sensitivity Analysis for Propagation of Corrosion

Analyzing the behavior of the propagation of corrosion based on the model from Nilsson and Gehlen (1998), the present of chlorides is an important fact as shown in Figure 5.5. There are also model parameters, which are defined as constant values. To evaluate the impact of these variables on the physical models, values in the range of ± 10 % are assumed.

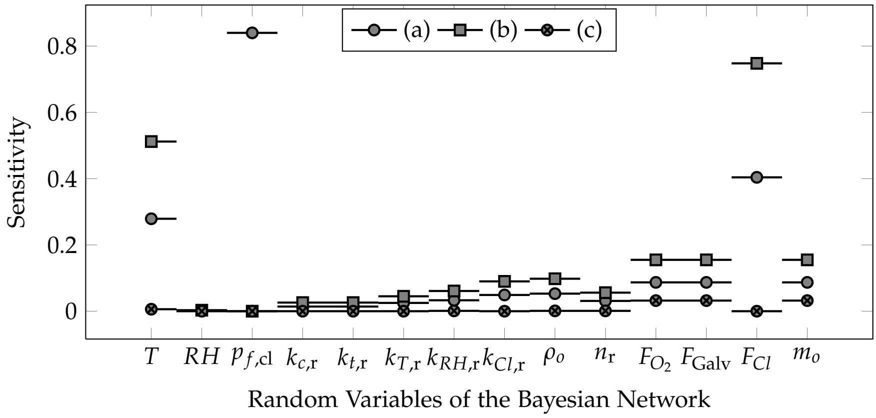


Figure 5.5: Sensitivity analysis for the BN of propagation of corrosion after 50 yr of chloride penetration with the target node V_corr. (a) No evidence is observed. (b) Chloride induced corrosion $p_{f,\mathrm{cl}}$ occurs. (c) No chloride induced corrosion $p_{f,\mathrm{cl}}$ is observed.

Apart from the probability of chloride induced corrosion $p_{f,\mathrm{cl}}$, the chloride corrosion rate factor F_{Cl} and the temperature parameter T have a great impact on the SM. Since the value $p_{f,\mathrm{cl}}$ evolves over time, this value should be, for further analyzes, either (b) "yes", meaning that chloride induced corrosion occurs or (c) "no", meaning chloride corrosions.

If (b) chloride induced corrosion occurs, the impact of the chloride corrosion rate factor F_{Cl} and the temperature parameter T increase rapidly. Humidity RH and the humidity factor $k_{RH,\mathrm{r}}$ only play a minor role. Can no chloride induced corrosion be observed, then also the chloride factor $k_{Cl,\mathrm{r}}$ and the chloride corrosion rate factor F_{Cl} are evident. In general, the impact of all variables on the physical model decreases and the influence of the constant variables m_o, F_{O_2} and F_Galv are determining the state of V_corr.

Also an evaluation of the simplified BN from Figure 4.11 results in the fact that the parameters $p_{f,\mathrm{cl}}$ and T have the major impact on the model. Therefore, the effect of Humidity RH can be neglected.

5.2.4 Sensitivity Analysis for Pitting Corrosion

The sensitivity analysis for pitting corrosion per se is quite complicated, because the corrosion rate V_corr and the time, since the structure is exposed to corrosion t_corr, are depending on various other assumptions. If it is assumed that these values are observed; for example, a low corrosion rate over a fix period of time, then the nominal bar diameter $d_{s,o}$ is almost the only impact on the physical model. If also the bar diameter is known, then the pitting factor R_{pit} and the uniform capacity length L_U gain more influence.

5.2.5 Sensitivity Analysis for Concrete Degradation Caused by Corrosion

Considering a sensitivity analysis for the CM as shown in Figure 4.7, (a) in the case of no evidence the nominal bar diameter $d_{s,o}$ and the yield strength $f_{y,o}$ are the most dominant parameters for the probability of failure p_f. This is a plausible result according to the fact that only the consequences caused by corrosion are considered to have influence on the structural reliability, allowing to neglected geometrical and load properties.

Assuming that the bar diameter and the yield strength are observed (b), here for example, a medium diameter of $d_{s,o} = 16\,\text{mm}$ and a yield strength of $f_{y,o} = 500\,\text{N/mm}^2$, then the results of the sensitivity analysis change completely as shown in Figure 5.6. The concrete cover d_c, the environmental exposure e_e and the chloride corrosion rate factor F_{Cl} become the major impact factors, as already obtained previously in the SM.

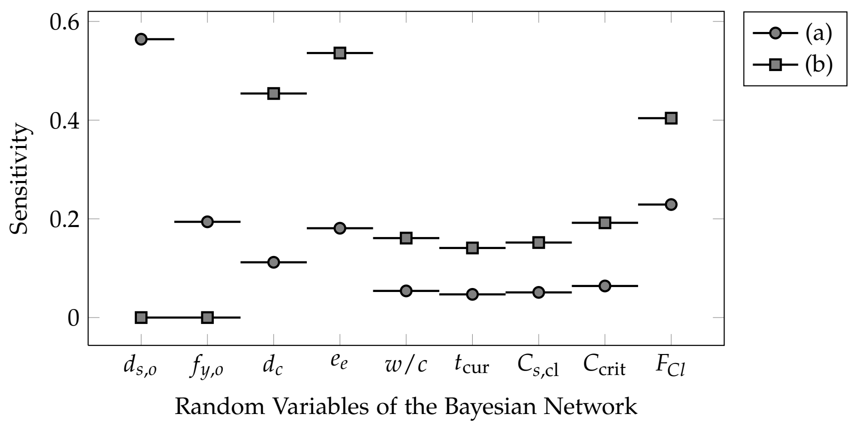


Figure 5.6: Sensitivity analysis for the BN of concrete degradation caused by corrosion after a service life of 50 yr. (a) No evidence is observed. (b) A medium bar diameter of $d_{s,o} = 16\,\text{mm}$ and a yield strength of $f_{y,o} = 500\,\text{N/mm}^2$ is observed.

5.2.6 Comments to the Sensitivity Analysis

However, the proposed results in Figure 5.6 should be treated qualitatively and not quantitatively, because of the fact that the BN is rather huge and the software GeNIe is limited by the size of the CPDs. So only a discretization of five to seven intervals for the model parameters are possible in order to perform a sensitivity analysis.

Comparing the sensitivity analysis from the SMs with the simplified models, the same trend can be observed. It also follows the analysis of the CM trend of the SM.

Already mentioned several times, the appropriated discretization is the problem of. Beside the limitation on the amount of intervals, it is also the question of the optimal interval size; especially, by dealing with different kinds of random variables.

For example, the corrosion rate V_{corr} changes from a likely normal distributed random variable with a mean of approximately $0.004\,\text{mm}/\text{yr}$ and a $\text{CoV} = 0.2$, to a likely lognormal distributed random variable with a mean around $0.011\,\text{mm}/\text{yr}$ ($\text{CoV} = 0.2$), if chloride induced corrosion is observed. Increasing the amount of intervals is limited on the one hand side by the size of input parameters ρ, m_o,F_{Cl}, F_{O_2} and F_{Galv}, which are itself limited by the size of their parents. On the other side they are limited by the size of the other input parameters that influence the child p_{max} of V_{corr}. To represent different random variables in this node, with the limitation of the intervals, an optimal interval size must be found, which is a complicated process considering the complex relationships in the BN.

This problem occurs several times in the BN, which leads, beyond a poor approximation of the random variable, to an error propagation over the total BN. To circumvent this problem simplifications of the BN are performed as explained in section 4.7. However, to keep a "complete" BN further researches have to be done on discretization of continuous random variables in the context of physical models and reliability analysis.

5.3 Initial Condition Analysis

The probabilistic models for the case of corrosion initiation and propagation, caused by carbonation or chloride penetration, are functions of a number of random variables, discussed in chapter 3. The simplified models in chapter 4 have reduced the amount of random variables to a set of shared parameters.

These parameters can also be called *initial condition indicators* and are used in the probabilistic model as prior estimations for the probability that the RC structure is in a condition state at some defined time period during its service life. (Malioka, 2009, p.75)

Initial condition indicators, mainly based on design specifications, are used for the decision making process, as mentioned previously. Current service life models like DuraCrete (2000b), LIFECON (2003), fib Bulletin 34 (2006) and others use such parameters for their probabilistic model.

In the DuraCrete (2000b) model for degradation of concrete, caused by chloride induced corrosion, these conditions are the following: concrete cover d_c, w/c ratio, curing time t_{cur}, environmental exposure e_e, nominal bar diameter $d_{s,0}$, relative humidity RH, temperature T and the time since when the RC structure is exposed to corrosion t_{corr}. A graphical representation of these structures is given in Figure 4.14.

However, current models and research papers assume that these initial condition indicators can be exactly specified for the purpose of analyzing the corrosion process. For example, Malioka (2009) assumed a curing time t_{cur} of 7 days, Osterminski and Schießl (2012) used a w/c ratio of 0.50 and an environment exposed to splash water and Stewart (2012) used an uniform capacity length of reinforcement bar L_U of 500 mm, beside some other assumptions, for evaluating their models.

The question is if it is right to make such strong assumptions on the input parameters of a probabilistic model? Especially, in the context that those models are used to evaluate already existing RC structures. Uncertainties, lack of information or evolving in time, are not considered here. For example, the behavior of the whole model changes if a w/c ratio is 0.40 instead of 0.50 or a tidal zone changes to a splash zone event.

To make existing service life models more accurate and realistic, the framework of BNs can be added to the classical SRA, as shown in chapter 4. This allows, on the one hand, to deal with uncertainties by the input parameter of the model and on the other hand to consider dependency in time.

5.3.1 Assumptions for the Initial Condition Analysis

To use a BN the initial condition indicators have to be discretized. The values and the amount of discretization is explained for each parameter in the according section. Thereby a uniform distribution for the values is assumed, which can be interpreted as if there is no information about the parameter. If exact information is available or a prediction can be made, then the probabilities for the single values can be adapted. For example, according to current standards the concrete cover is around a mean value $d_c \approx 10\,\text{mm}$.

Analyzing one specific parameter, it is assumed that all other input variables are unknown, such that there is no evidence in the BN. Furthermore, the analysis extends over a period $\mathcal{T}$ of 50 yr with a step size t of 1 yr.

The DCM for degradation of corrosion caused by chloride induced corrosion, which is discussed in section 4.7.3 is used for the analysis. Here the results are subdivided into two parts, according to the basic model proposed by Tuutti (1982). The first one covers the probability of chloride induced corrosion $p_{f,\text{cl}}$ and is related to the initiation phase. The second one deals with the probability of failure of the RC structure, which corresponds to the propagation phase.

The following sections provide a brief overview over the major input parameters. A discussion over the relevance in context of structural reliability, code calibration or optimal maintenance strategies, can be found in chapter 6.

5.3.2 Concrete Cover

The importance of the concrete cover d_c has been discussed previously and the sensitivity analysis confirms this assumption. As an initial condition indicator the concrete cover d_c can be used to provide a prior estimate of the structural performance.

Normally, the value of this variable is defined in the design process as the nominal concrete cover depth and represented by a single value. (Malioka, 2009, p.76) However, an analysis with BNs allows to define a set of these parameters and assigns a probability of occurrence to each value. This property may be useful if there is no evidence of the real nominal concrete cover. As it will be described later, concrete cover depth measurements can be performed during the service life or the RC structure. When such new information becomes available this prior probability estimation can be updated.

For the following analysis a set of eight input values, in the range between 15 mm and 85 mm with a step size of 10 mm, is chosen. The effects of the concrete cover d_c on the probability of chloride induced corrosion $p_{f,\text{cl}}$ and on the probability of failure p_f are shown in figure Figure 5.7.

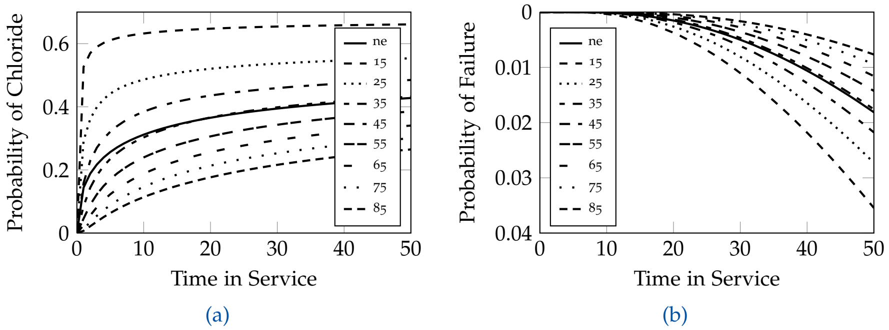


Figure 5.7: Analyzing the influence of the concrete cover d_c on the: (a) probability of chloride induced corrosion (b) probability of failure.

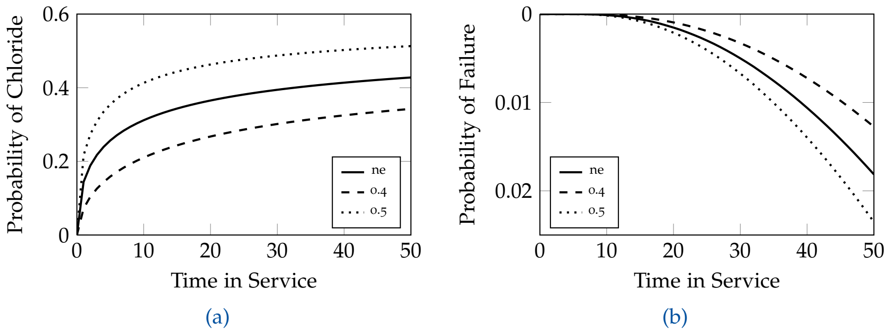


Figure 5.8: Analyzing the influence of the w/c ratio on the: (a) probability of chloride induced corrosion (b) probability of failure.

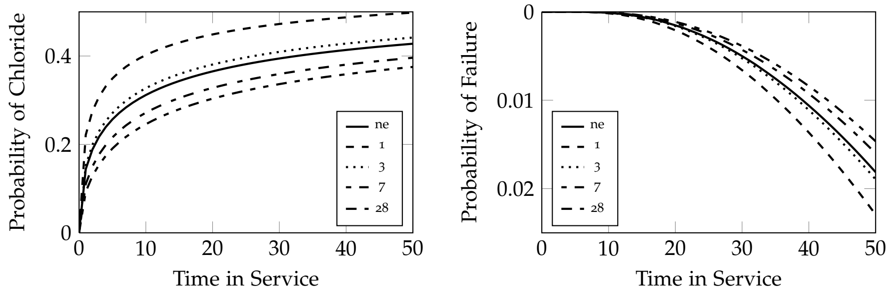
5.3.3 Water-Cement Ratio

The w/c ratio affects the rate of ingress of harmful substances into the concrete. The ratio can be defined based on the design specifications for the RC structure. However, using the proposed data from the (DuraCrete, 2000b) model, only a limited set of two w/c ratios are available, which restricts the significance of the model enormously. However, using a BN with the opportunity to choose any probability for occurrence of one value increases the possibilities immeasurably.

5.3.4 Curing Period

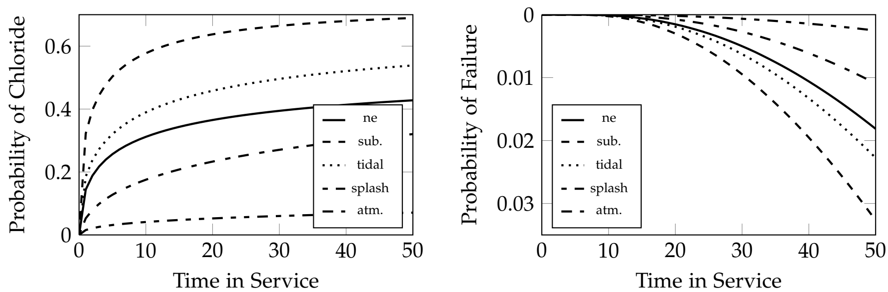
The curing time parameter t_{cur} affects the execution variable $k_{c,cl}$ in the probabilistic model for chloride induced corrosion. A limited set of data is provided in the DuraCrete (2000b) model, in which 1, 3, 7 or 28 days can be chosen. Furthermore, experimental data shows that $t_{cur} > 28$ can be represented by $t_{cur} = 28$.

The exact estimation of the curing period t_{cur} for an existent RC structure is not always possible. Hence, using a BN framework allows to take those uncertainties into account.



(a) (b)

Figure 5.9: Analyzing the influence of the curing period t_{cur} on the: (a) probability of chloride induced corrosion (b) probability of failure.



(a) (b)

Figure 5.10: Analyzing the influence of the exposure environment e_e on the: (a) probability of chloride induced corrosion (b) probability of failure.

5.3.5 Exposure Environment

As pointed out in the sensitivity analysis, has the environmental exposure e_e a major impact on the physical model, both the SM and the CM. The exposure environment is required as an input parameter for the surface concentration of chloride $C_{s,\text{cl}}$, the environmental parameter $k_{e,\text{cl}}$ and the age factor n_{cl}.

Even if the exposure environment e_e is a major parameter for the probabilistic model of chloride induced corrosion, the definition is not very obvious. For example, the marine environment is subdivided in four different zones, each of which is depending on the sea level, as explained in section 3.2.1.

It is hard to evaluate where the borders between atmospheric, splash, tidal and submerged zones are. In the context of future spatial evaluations, it is not realistic to assume that the zone changes abruptly between two different elements. As well it is likely that the zone can changes during the service life of the RC structure by natural events or human intervention.

5.3.6 Bar Diameter

Given a RC structure, then the bar diameter d_s is, beside the compressive strength of concrete f_c and the yield strength f_y, a major component. Considering degradation of the RC structure caused by chloride induced corrosion, the bar diameter becomes the most important parameter. This is shown in previous section and is caused by the fact that pitting corrosion decreases the steel area of the reinforcement.

A limited set of data allows only to evaluate three kinds of bar diameter exactly (Stewart and Al-Harthy, 2008). BN reduces this limitation as explained previously.

5.3.7 Relative Humidity and Temperature

Relative humidity RH and temperature T are supposed to influence the corrosion rate V_{corr}, which in turn influences the bar diameter. The determination of these parameters is easier and exacter than all the other ones. However, the high degree of time dependency makes it difficult to include the humidity and the temperature in a standard model.

A DBN allows to model those evolvings in time as shown in Figure 5.12. A limitation is done by the size of the CPD, which will be necessary by discretization of a continuous variable such as the temperature T. Here the temperature is modeled from $-5\,°C$ to $30\,°C$ with a step size of $5\,°C$. The relative humidity is limited by the DuraCrete (2000b) model to five values.

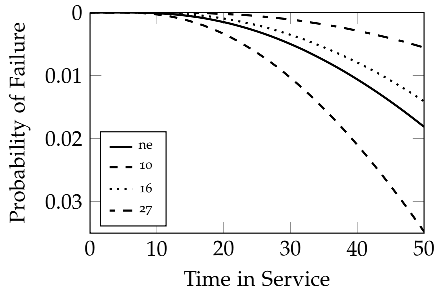
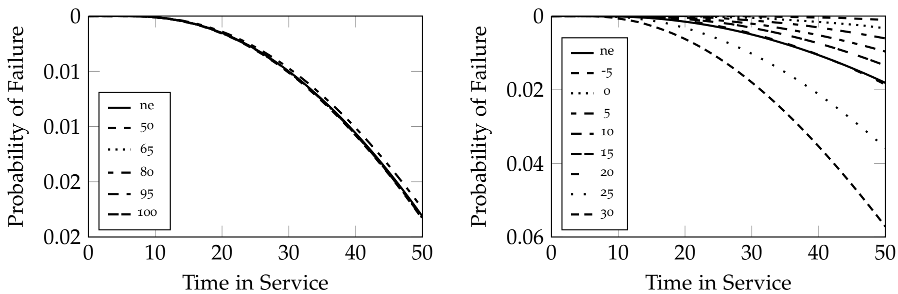


Figure 5.11: Analyzing the influence of the bar diameter $d_{s,0}$ on the probability of failure.



(a) (b)

Figure 5.12: (a) Analyzing the influence of the relative humidity RH on the probability of failure. (b) Analyzing the influence of the temperature T on the probability of failure.

5.4 In-Service Condition Analysis

As the name *initial condition analysis* indicates, those parameters represent initial conditions for the probabilistic model of concrete degradation caused by chloride induced corrosion. The knowledge provided by them can be seen as prior information. During service life several parameters may change and/or the RC structure evolves differently in time as expected.

Hence, the information about the condition of the RC structure can be collected during the service life. One way is to use *monitoring* for the RC structure and collecting continuously data about the condition states. An other option is to do selective *inspections* of the condition of interest during the service life.

To collect information about the current condition of the RC structure, several different methods have been developed. A brief overview for some of the available techniques is proposed by Broomfield (2007).

However, typical non-destructive methods that can be performed for degradation of concrete caused by chloride induced corrosion, are: (Faber, Straub, et al., 2006)

- visual inspection
- cover depth measurement
- half-cell potential measurement

Based on the concept proposed by Faber and Sorensen (2002), the parameters for the physical model, which represents prior information, can be updated if new information based on inspections or monitoring becomes available. This can be modeled by the following two probabilities:

$$P(X_i|Y_j) = p_k \tag{5.1}$$

Where p_k represents the probability that the inspected component is in any condition state X_i given the indication Y_j.

$$P(X_i|\bar{Y}_j) = p_l \tag{5.2}$$

Where p_l represents the probability that the inspected component is in any condition state X_i given non-indication $\bar{Y}_j$.

5.4.1 Visual Inspection

The *visual inspection* (VI) is the first step in any investigation. It should give a first indication of the condition of the RC structure. The accuracy of a visual inspection is inbetween the range from a simple general impression of the RC structure, to an identification of every defect that can be seen on the concrete surface. (Bertolini et al., 2004, p.273)

Visual inspections are relative simple to perform and can provide accurate information about the visible condition states, which are related to corrosion. For example, “rust” stains, cracking and spalling are some possible results of what can be observed.

However, visual inspection is limited by several tasks. For instance, the surfaces of the structural components have to be accessible, but the main limitation is the skill of the inspector. Some defects can be mistaken for others; for example, different types of cracking can be attributed to different causes. (Broomfield, 2007, p.37)

Based on these assumptions, Faber, Straub, et al. (2006) proposed that whether the probability of corrosion given by visual inspection, is observed or not, is stated:

$$P(p_{f,\text{cl},y}|I_{\text{vi}}) = 1 \tag{5.3}$$

$$P(p_{f,\text{cl},y}|\bar{I}_{\text{vi}}) = 0 \tag{5.4}$$

Where $p_{f,\text{cl},y}$ is the conditional state of “yes”, chloride induced corrosion occurs and I_{vi} denotes indication of visible corrosion detected by visual inspection.

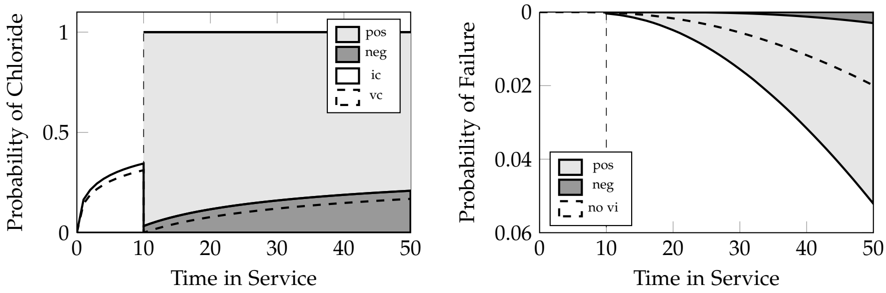
This approximation assumes that visual inspection is perfect. (Faber, Straub, et al., 2006) This statement might be right for Equation (5.3) where it says that if corrosion can be observed during an inspection, then corrosion occurs. But the statement of (5.4), stating that if no corrosion can be observed, no corrosion occurred so far, should be treated with caution. Especially, in the case of pitting corrosion, which often does not cause disruptions of the concrete cover, as discussed in section 3.7.

5.4.2 Cover Depth Measurement

As discussed in section 5.2 and shown in Figure 5.7, sensitivity analysis the concrete cover depth d_c has huge impact on the probabilistic model for degradation of concrete caused by corrosion.

In simple terms, low cover will favor the inset of corrosion because carbonation and chloride reach the reinforcement more rapidly. (Bertolini et al., 2004, p.274) A determination of the concrete cover may help to explain why the RC structure is corroding and shows which areas are most susceptible to corrosion due to low cover. (Broomfield, 2007, p.42)

Measurements of concrete cover can be easily combined with visual inspections. The treatment of the new information is already discussed in section 5.3.2.



(a) (b)

Figure 5.13: Influence of visual inspection on the RC structure. The inspection is performed after 10 yr. The light shaded area denotes a positive indication and the dark shaded area a negative indication of corrosion. (a) Probability of visible chloride induced corrosion (vc) and initiated corrosion (ic). (b) Probability of failure caused by chloride induced corrosion by a positive or negative indication or no visual inspection (no vi).

5.4.3 Half-Cell Potential Measurement

A widely recognized and standardized non-destructive method for assessing the corrosion state of the reinforcement in RC structure is the *half-cell potential measurement* (HCPM).

Because corroding and passive reinforcement bars show a different electrical potential, as explained in section 3.6, showing current flows between these two areas. The electrical field, coupled with the corrosion current I_{corr} between corroding and passive area of the reinforcing steel, can be measured experimentally with a suitable reference electrode (half-cell) placed on the concrete surface. The result is a potential field that allows to estimate the location of corroding reinforcement. Thereby areas with the most negative values are more likely related with corrosion. (Bertolini et al., 2004, p.277) Typically indicate more negative values than −350 mV the probability of corrosion with a value of 90 %. (Johnsen et al., 2003)

The quality of the HCPM is affected by different factors. For example, the moisture of the concrete surface, cracks of the concrete surface, stray currents, or electrochemical treatments, such as cathodic protection, electrochemical chloride extraction and electrochemical re-alkalization, can influence the results of a HCPM. (Broomfield, 2007, p.54)

Hence, the interpretation of a HCPM should be combined with other measurements and information. The quality of HCPM can be modeled in a general case according to Faber, Straub, et al. (2006) as:

$$P(I_{\text{hc}}|p_{f,\text{cl},y}) = p_{i,\text{hc}} \tag{5.5}$$

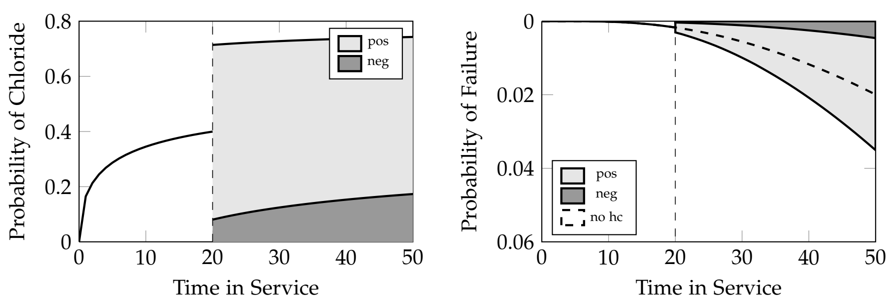
$$P(I_{\text{hc}}|\bar{p}_{f,\text{cl},y}) = p_{j,\text{hc}} \tag{5.6}$$

Where the inspection is expressed through the probability of an indication of corrosion initiation I_{hc}, given that corrosion occurs $p_{f,\text{cl},y}$, or given that corrosion does not occur $\bar{p}_{f,\text{cl},y}$.

Lentz et al. (2002) and Johnsen et al. (2003) have proposed some representative value for the probabilities p_i and p_j by using HCPMs. The values are related to different structural components as shown in Table 5.1.

Table 5.1: Quality of the HCPM for different types of structural components. Where U_{lim} is the corresponding limit potential in [mV] and p_i and p_j the obtained probabilities in [%]. Based on Lentz et al. (2002).

Components	U_{lim}	$p_{i,\text{hc}}$	$p_{j,\text{hc}}$
Joint data	-207	90	24.0
Bridge arcs	-238	90	1.3
Bridge decks	-259	90	18.1
Columns next to retreets	-193	90	33.1



(a) (b)

Figure 5.14: Influence of HCPM on the RC structure. The measurement is performed after 10 yr. The light shaded area denotes a positive indication and the dark shaded area a negative indication of corrosion. (a) Probability of chloride induced corrosion. (b) Probability of failure caused by chloride induced corrosion by a positive or negative indication or no HCPM (no hc).

6 Case Studies

"Science is nothing but perception." (Plato)

In the previous chapter 5 some analysis methods are discussed and results are shown. However, these results only give a rough overview over the capability of a DCM. For example, it is assumed that only one parameter is observed or that values do not change over time. The model can be used for code calibration or finding optimal maintenance strategies, too.

6.1 Comparison of Service Life Models

Several service life models have been developed to estimate the service life of RC structures where a desired level of structural reliability is assured. Service life models such as DuraCrete (2000b), LIFECON (2003), fib Bulletin 34 (2006), or Life-365 (2013) are based on the approach proposed by Tuutti (1982). This approach subdivides the service life of a RC structure into the following two phases: the initiation and propagation phase, as discussed in section 3.3, which are discussed in Figure 3.2.

Hence, different models are developed to describe the different phases of the service life. However, these models are developed separately without taking models for the other phase into account, as for example the herein treated DuraCrete (2000b) model shows. Furthermore, the limit state for such service life models is defined as the occurrence of a special event; for example, depassivation, cracking or spalling. The "real" structural collapse, in terms of structural reliability, is not treated.

Coupling those different models allows an analysis for the whole process of corrosion. In other words the initiation process, the propagation process and the mechanical performance of a RC structure are dependent on each other. This leads to the result that the qualitative service life model proposed by Tuutti (1982) actually can be represented as quantitative model.

In the DCM, a conventional initial phase is no longer outlined, which describes the period during the depassivation of reinforcement, because of the fact that even

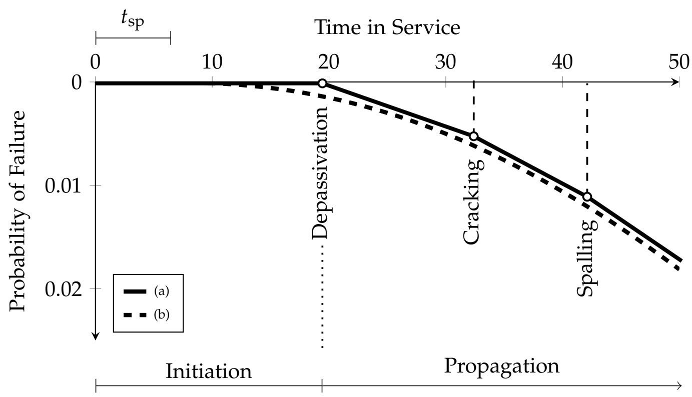


Figure 6.1: (a) Qualitative two phases service life model for deterioration of a concrete structure due to steel corrosion, based on Tuutti (1982). (b) Quantitative dynamic coupled model, in case of no evidence.

in the first weeks the probability of corrosion onset is considered. This assumption has been confirmed under experimental conditions and field conditions. (Pease et al., 2011)

Instead, the initiation phase in the DCM describes the period of time where no significant loss of structural performance can be expected, here denoted by t_{sp}. This criteria is related to the general requirements concerning the safety of the RC structure, in terms of a reliability index β as shown in Figure 6.2(b).

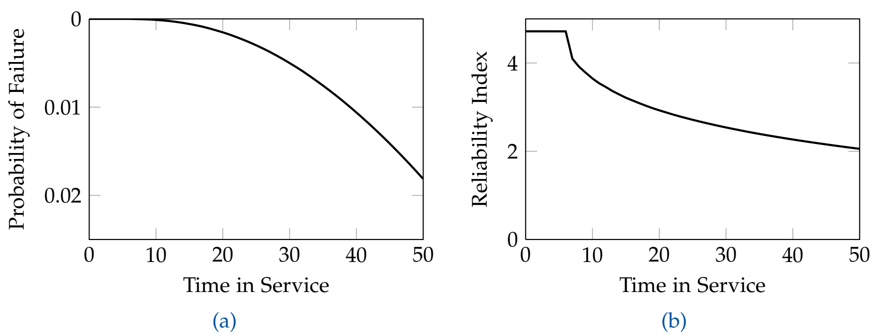


Figure 6.2: (a) Probability of failure p_f caused by chloride induced corrosion (b) Reliability index β over the service life.

The period of time t_{sp} depends not only on the models for carbonation and chloride induced corrosion but rather on the whole DCM, which includes also the propagation and the effects of corrosion.

6.2 Structural Codes and Concrete Cover

Since corrosion of reinforcing bars leads to a decrease of the structural performance, this issue is treated in structural codes for the design of concrete structures. Therefore, the concrete cover is a variable that takes the protection of the reinforcement against corrosion into account.

However, most structural codes assume a minimum concrete cover depending on the exposure conditions in terms of chemical and physical conditions, to which the RC structure is exposed in addition to the mechanical actions. (Eurocode 2, 2004)

The range of this minimum cover varies from the range of 50 mm to 75 mm, depending on the code. An overview for some structural codes is given in Table 6.1. Additionally, the specification, performance and other properties of concrete are recommended in specific codes. In case of concrete degradation caused by corrosion using the DuraCrete (2000b) proposed approaches like the w/c ratio and the cement content are therefore parameters of interest. Here the upper limits of the w/c ratio should be in the range of 0.4 to 0.5, according to the different codes.

Table 6.1: Durability requirements of some structural codes for corrosion of reinforcing bars. With the maximum w/c ratio in [-] and the minimum concrete cover d_c is in [mm].

Code	**w/c ratio**	**cover**
ACI (2011)	0.4	50
Eurocode 2 (2004),EN 206-1 (2000)	0.4	55
CSA (2004)	0.4	60
FIP (1973)	0.4	75

Where ACI, CSA and Eurocode 2 describe rather the durability requirements for land based RC structures, the proposed values from FIP initial are recommended for offshore concrete structures.

In comparison with the results from the DCM none of these codes fulfill the requirements of a $\beta_t = 3.8$ after 50 years, if nothing else is known then the concrete cover d_c and the w/c ratio as shown in Figure 6.3. Even if a lower value of β_t is assumed (3.3), according to JCSS (2002) or (3.51) Stewart (2009), none of the codes will reach them, except the FIP code, under the assumption that the required service life is only 25 to 30 years for offshore concrete structures. (Gjørv, 2009, p.86)

Additional, if it is assumed that the values in Table 6.1 are for a splash zone of an marine environment according to Gjørv (2009, p.91), then the FIP code will fulfill all requirements.

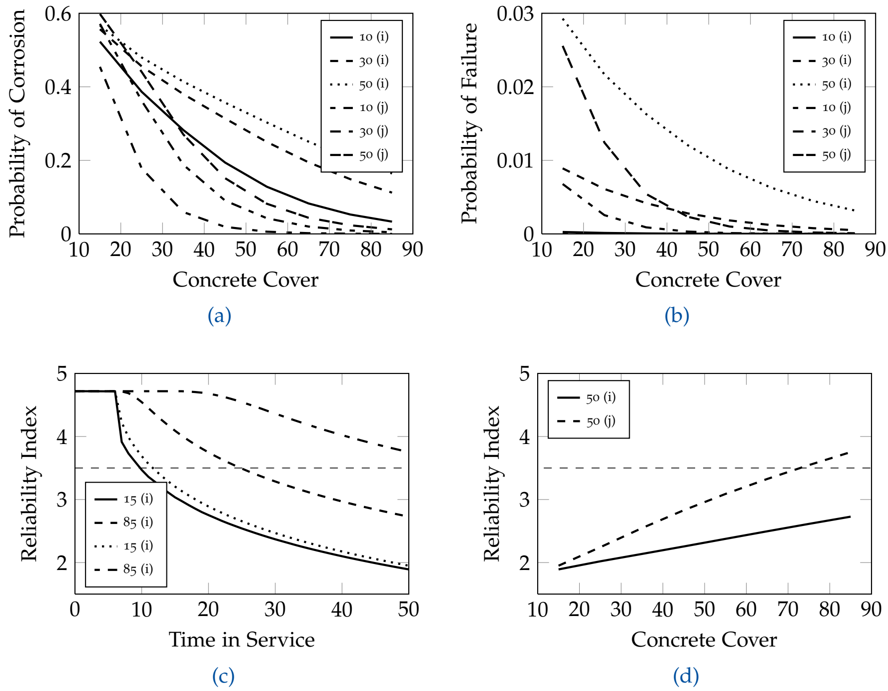


Figure 6.3: Influence of the concrete cover d_c related to the structural codes. In case (i) the concrete cover d_c and the w/c ratio are observed. In case (j) also a splash zone is given. (a) Probability of chloride induced corrosion $p_{f,\mathrm{cl}}$ dependent on the cover depth after 10, 30 and 50 yr. (b) Probability of failure p_f. (c) Reliability index β over a period of 50 yr for a concrete cover d_c of 15 and 85 mm. (d) Reliability index β dependent on the cover depth after 50 yr.

7 Conclusion

> “*Nichts ist getan, wenn noch etwas zu tun übrig ist*” (Carl Friedrich Gauß)[a]
>
> [a]Nothing is done, if there is still something left to do.

7.1 Overview

The purpose of this work was to develop a generic framework for stochastic modeling of reinforced concrete deterioration caused by corrosion. Thereby, the ideas of structural reliability analysis and Bayesian networks were combined.

7.1.1 Dynamic Coupled Model Framework

The concrete deterioration caused by corrosion is a complex physical, chemical and mechanical process. The modeling of this process is subjected to significant uncertainties, which are based on a simplistic representation of the actual physical process and limited information on material, environmental and loading characteristics.

During the last decades several service life models have been developed to estimate the length of time during which RC structures maintain a desired level of functionality. All of those probabilistic models are based on the classical concept developed by Tuutti (1982). Where the service life is divided in two distinct phases: the initiation and the propagation of corrosion.

However, for each phase several (independent) models are developed. Hence, the probabilistic models for the initiation phase (carbonation and chloride propagation) can not be combined with the models for the propagation phase (propagation and effects of corrosion) in terms of a unified model, which is necessary to provide a consistent model, starting by the edification of the RC structure and ending by reaching a critical limit state but also evolving over time.

The present work proposes a DCM framework, based on SRA and BNs, which allows to couple the probabilistic models for the initiation and propagation of corrosion. Thereby uncertainties by model parameters, but also additional information, provided by measurements, monitoring and inspection results, can be considered.

Using the presented simplification and optimization of the SMs, the critical part of discretizing a continuous random variable for BNs can be reduced to a minimum amount or even eliminated. This allows to expanding the CM over a period of time and updating the model when new information becomes available.

The treated DCM framework can be used for any kind of probabilistic model that, describing the phenomena of corrosion. This allows to use different models even for the same physical process and if new enhanced models are available, they also can be simply embedded in this framework.

7.1.2 Analysis

This thesis shows that not only a coupling of different probabilistic models is feasible, but performing different analyzes to a previously unknown scale are also possible.

Current service life models assume that the limit state of a RC structure is defined as the occurrence of a special event; for example, depassivation, cracking or spalling, but the structural safety in terms of structural reliability is not treated.

The proposed framework allows, beside the sensitivity analysis of each parameter over all used probabilistic models, also the estimation of the structural safety in terms of reliability index and probability of failure.

Suggestions from design codes and information from the design process can be implemented in the model and used for the analysis of future or existing RC structures. The framework is also capable to consider new information in any point of time by the analysis.

This allows, beyond a standard analysis, also the analysis of measurements, monitoring and inspection results and their influence on the RC structure.

7.2 Outlook

In this thesis several different topics are covered. The range reaches from mathematical over physical and chemical to engineering problems. During the research process, open questions and new ideas have arisen.

7.2.1 Probabilistic Models

As mentioned previously the DuraCrete (2000b) model is implemented in the framework and represents only a limited set of data. Other enhanced probabilistic models can be added to the DCM. This allows apart from dealing with more specific input parameter also a comparison of different models which describing the same physical phenomena.

Because the proposed framework is not limited in terms of probabilistic models, such that it can be used for other physical problems; for example, corrosion of steal structures or pipelines, but maybe also the deterioration process of timber, caused by fungal infestation.

7.2.2 Bayesian Framework

The BNs themselves offer huge opportunities that are not treated in this thesis so far. Beside Bayesian updating of the model with new information, a BN can also be used for learning on real data or estimating parameters.

As discussed in chapter 3 the provided data sets often have missing observations, usually due to some logistical problem during the data collection process. The easiest way of dealing with observations that contain missing values is simply to exclude them from the analysis. However, this results in loss a of information if an excluded observation contains valid values for other quantities, accordingly it can bias results. An alternative is to impute the missing values, based on information in the rest of the model. In a Bayesian modeling framework missing data are accommodated simply by treating them as unknown model parameters. This allows to estimate missing data and increases the scope and the validity of future calculations.

Discretization of continuous random variables in the context of SRA is still a problem. If BNs should be applied on a wide range of structural problems, than a reasonable approach for discretization of continuous random variables is needed for further research.

7.2.3 Spatial Analysis

Keeping it feasable for the beginning, this work only considers one element of a RC structure that is exposed to the process of corrosion. However, the deterioration of concrete caused by corrosion is strongly related to spatial and temporal variability. This property can be modeled by different approaches and will be necessary for a holistic contemplation of the system.

7.2.4 Using Sensitivity Analysis

Beside the information how individual parameter influence the results of a model, sensitivity analysis can be used in a wider framework. For example, performing laboratory experiments to evaluate existing or new physical models. The informative value can be increased while the costs can be decreased by using sensitivity analysis to find the parameter of interest.

7.2.5 Optimal Maintenance Strategies

Schematically indicated in chapter 5 and chapter 6, information from measurements, monitoring and inspection results can be included in the DCM. This allows to develop optimal maintenance strategies in context of a given level of structural safety, especially by adding decision and utility nodes to the BNs. In combination with costs of maintenance, repair and expectations of the society, a support tool for decision makers may be created.

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