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Visco-elastic response of thermoplastics

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TITLE:

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Viskoelastisk respons av plastmaterialer

BY: Vegard Berge Kristensen

SUMMARY:
In this study a recently developed visco-elastic visco-plastic material model has been evaluated with the intention of improving the simulated behaviour of polymers. In order for polymers to become a more reliable construction material the behaviour has to be rendered realistically in simulations. A set of eleven experimental tests have been conducted to establish a database for further simulations. By use of some of these experimental tests the visco-elastic visco-plastic material model has been calibrated. In addition a purpose developed Matlab program based on a uni-axial tension case of the material model served to evaluate the response of the experimental tests. Although a visco-elastic visco-plastic material model was employed, it was impossible to recreate the magnitude of the relaxation and creep in the experimental tests for large deformations. A modified visco-elastic visco-plastic material model with the network stress as a function of plastic strain instead of total strain, was therefore proposed and implemented in Matlab. This new model was able to simulate the response for large deformations. However, new calibrations had to be carried out for each of the different stress-strain domains in order to give an accurate response. Introducing more Maxwell-elements in the visco-elastic part of the model did not help significantly, as more elements just gave a smoother response. None of the material models presented was able to simulate the unloading procedure for all the different parts of the stress-strain domain based on one optimized calibration. A calibration of the models fitted to the elastic domain was too stiff in the early plastic domain, and too soft for large deformations.

RESPONSIBLE TEACHER: Professor Arild H. Clausen

SUPERVISOR(S): Arild H. Clausen, Marius Andersen

CARRIED OUT AT: SIMLab, NTNU

NORGES TEKNISK-NATURVITENSKAPELIGE UNIVERSITET
INSTITUTT FOR KONSTRUKSJONSTEKNIKK

MASTER THESIS 2013

Vegard Kristensen

Visco-elastic response of thermoplastics

(Viskoelastisk respons av plastmaterialer)

As a part of the development of the finite element method, significant effort has been devoted to propose new material models which are able to represent the material behaviour at different conditions. Relevant parameters involve strain level, strain rate, temperature, and the material at hand may also exhibit anisotropy, visco-elasticity etc. Today, material models for metals are in general more accurate than models for e.g. polymers. A model describing the behaviour of ductile thermoplastics, thus representing the response at large deformations, has been developed at SIMLab. On the other hand, this model does not capture the visco-elastic response, but a first version of a model doing so is ready for evaluation. This calls for an experimental database, which (i) Exhibits the physical phenomena that have to be captured by the model, and (ii) Provides information for identification of the material coefficients of the model.

A major challenge for polymers is to distinguish between elastic, visco-elastic and visco-plastic response. Relaxation and unloading tests are useful for this purpose. The candidate shall carry out a test series in the laboratory. Subsequently, the tests should be analyzed with the preliminary version of the visco-elastic visco-plastic model, leading to suggestions for possible improvements of the model.

Possible keywords for activities in this master thesis research work may include:

- Literature: Polymers in general, material models, visco-elastic response
- Experimental tests: Material tests. Presentation of test results
- Calibration: Identify the coefficients of a material model
- Numerical modelling: Simulation of experimental tests. Evaluation of the model

The candidate may agree with the supervisors to pay particular attention to specific parts of the investigation, or include other aspects than those already mentioned.

The thesis is to be organized as a research report, recognising the guidelines provided by Department of Structural Engineering.

Supervisors: Arild Holm Clausen and Marius Andersen

The report is to be handed in not later than 10 June 2013.

NTNU, 15 January 2013

Arild Holm Clausen

Abstract

In this study a recently developed visco-elastic visco-plastic material model has been evaluated with the intention of improving the simulated behaviour of polymers. In order for polymers to become a more reliable construction material the behaviour has to be rendered realistically in simulations. A set of eleven experimental tests have been conducted to establish a database for further simulations. By use of some of these experimental tests the visco-elastic visco-plastic material model has been calibrated. In addition a purpose developed Matlab program based on a uni-axial tension case of the material model served to evaluate the response of the experimental tests. Although a visco-elastic visco-plastic material model was employed, it was impossible to recreate the magnitude of the relaxation and creep in the experimental tests for large deformations. A modified visco-elastic visco-plastic material model with the network stress as a function of plastic strain instead of total strain, was therefore proposed and implemented in Matlab. This new model was able to simulate the response for large deformations. However, new calibrations had to be carried out for each of the different stress-strain domains in order to give an accurate response. Introducing more Maxwell-elements in the visco-elastic part of the model did not help significantly, as more elements just gave a smoother response. None of the material models presented was able to simulate the unloading procedure for all the different parts of the stress-strain domain based on one optimized calibration. A calibration of the models fitted to the elastic domain was too stiff in the early plastic domain, and too soft for large deformations.

Sammendrag

I dette studiet har en nylig utviklet viskoelastisk viskoplastisk materialmodell blitt evaluert med en intensjon om å forbedre den simulerte oppførselen av polymerer. For at polymerer skal kunne bli et mer pålitelig konstruksjonsmateriale bør en realistisk oppførselen kunne bli simulert. Totalt elleve eksperimentelle forsøk har blitt gjennomført for å etablere en database for videre simuleringer. Den viskoelastiske viskoplastiske materialmodellen har blitt kalibrert ved bruk av noen av disse eksperimentelle testene. Et en-aksielt strekk tilfelle har blitt utviklet i et Matlab program for å hjelpe til i evalueringen av responsen til de eksperimentelle testene. Selv om en viskoelastisk viskoplastisk material modell ble brukt var det umulig å gjenskape størrelsen på relaksasjonen og krypet i de eksperimentelle testene for store deformasjoner. Det har blitt foreslått en modifisert viskoelastisk viskoplastisk material modell med nettverks spenningen som en funksjon av den plastiske tøyningen istedenfor den totale tøyningen. Denne nye modellen klarte å simulere responsen for store deformasjoner. For at responsen skulle være nøyaktig måtte nye kalibreringer til for hvert nytt område av spenning-tøynings domenet. Å introdusere flere Maxwellelementer i den viskoelastiske delen av modellen hjalp ikke nevneverdig når det gjaldt dette problemet, men responsen ble glattere. Ingen av materialmodellene presentert i dette studiet klarte å simulere avlastningen for alle de forskjellige delene av spenning-tøynings domenet basert på en optimalisert kalibrering. Kalibreringer som passet for det elastiske domenet gav for stiv respons i den tidlige delen av det plastiske området, og for myk respons for store deformasjoner.

Preface

This report is a master thesis carried out at Structural Impact Laboratory at Norwegian University of Science and Technology, NTNU. The report is part of an ongoing study regarding polymers and polymer behaviour and takes on the visco-elastic part of a new developed visco-elastic visco-plastic material model.

A special thanks is given to my supervisor Professor Arild Holm Clausen and tutor Ph.D. candidate Marius Andersen for excellent guidance and mentoring through the entire study.

I would also like to thank my fellow students Heine Røstum and Olve Winjum for complementary results in their specialization project on polymer behaviour.

In addition credits goes to Egil Fagerholt for a great introduction course to his digital image correlation program cCorr, Trond Auestad for carrying out all the experimental tests in the laboratory and Torodd Berstad for implementing the visco-elastic visco-plastic material model in LS-DYNA.

Trondheim, 7th June 2013

Vegard Kristensen

Contents

1 Introduction

Polymers are widely used materials within many industries. Earlier, polymers have been thought of as weak materials more useful in packaging rather than construction. However, nowadays many industries seek lighter construction materials and the research on polymers has gradually increased. An example is the car industry which seeks lighter construction materials in order to keep the fuel consumption down.

This thesis is part of a bigger study organised by NTNU's Structural Impact Laboratory in order to improve the behaviour of polymeric material models. These material models are an important basis in the matter of implementing material behaviour into numerical codes. Program software like like LS-DYNA and Abaqus CAE uses these numerical codes in order to simulate behaviour of components and other three dimensional structures.

The main objective of this master thesis was to study the visco-elastic part of a new material model developed at SIMLab. In order to do so a set of eleven experiments have been conducted by use of the thermoplastic material high-density poly-ethylene. The experiments covered creep as well as relaxation, and were designed with the intention of putting visco-elastic effects in focus.

In Chapter 2 some basic theory of visco-elastic behaviour along with general theory about polymers are presented. The visco-elastic visco-plastic material model is further explained in Chapter 3. In Chapter 4 the experiments are explained and a selection of the results are presented. The reduction of the material model to a uni-axial case and the implementation of this case in Matlab is described in Chapter 5. As a supplement to some analytical calibrations, this Matlab program is further used to calibrate the three dimensional LS-DYNA model. Chapter 6 takes on the simulations of the experimental tests by use of the calibrated LS-DYNA model. In the end of Chapter 6 the results are discussed. Improvements to the material model are proposed in Chapter 7. These improvements are thereafter implemented in the uni-axial Matlab program. In the end new quantitative simulations are performed by use of Matlab and the results from these simulations are discussed.

2 Theory

In this chapter theory about visco-elastic models are presented in Section 2.1. Polymers in general are presented in Section 2.2. The polymer part is directed towards material behaviour concerning visco-elasticity and visco-plasticity in particular.

2.1 Viscoelastic material models

The two most common rheological elements to represent visco-elasticity is known as the Maxwell-element and the Kelvin-Voigt-element. These are two basic elements with respective basic properties. However, by a model consisting of these elements in combinations with other elements an adequate response concerning visco-elasticity may be obtained.

2.1.1 Maxwell-element

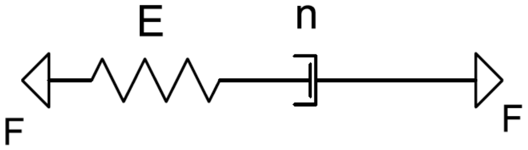


Figure 2.1: Maxwell-element

The Maxwell-element seen in Figure 2.1 consists of two parts. These are a spring and a dash-pot arranged in series. The stress in the spring and in the dash-pot are given as

$$\sigma_{spring} = E\varepsilon \tag{2.1}$$

$$\sigma_{dashpot} = \eta\dot{\varepsilon} \tag{2.2}$$

Here E represent the spring constant while η is a viscosity constant. The arrangement in series leads to equal force and stress in the spring and in the dash-pot. The dash-pot will acquire more and more of the strain as time goes by. This will reduce the strain in the spring, and therefore also reduce the total stiffness. Assuming that the total strain in the element is the sum of the strain in the spring and in the dash-pot the stress-strain relation can be given as

$$\dot{\varepsilon} = \dot{\varepsilon}_{spring} + \dot{\varepsilon}_{dashpot} = \frac{\dot{\sigma}}{E} + \frac{\sigma}{\eta} \tag{2.3}$$

A solution to the differential equation with stress as a function of time is

$$\sigma(t) = Ae^{-tE/\eta} \tag{2.4}$$

In a relaxation test strain will be kept constant after an initial loading procedure Assuming constant strain from t=0 an expression for a relaxation test is given as

$$\sigma(t) = E\varepsilon_0 e^{-t/\tau} \tag{2.5}$$

Here $\tau = \eta/E$ and represents the relaxation time of the element. The dash-pot in the element will eventually claim all the strain and the stress will approach zero.

In a creep test the force will be kept constant. This will give a constant increase in strain.

$$\varepsilon(t) = \sigma_0/E(1 + t/\tau) \tag{2.6}$$

2.1.2 Kelvin-Voigt-element

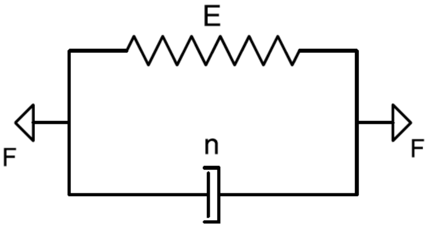


Figure 2.2: Kelvin-Voigt-element

The Kelvin-Voigt-element in Figure 2.2 also consists of a spring and a dash-pot, but here they are arranged in parallel. This also leads to a time dependent total stiffness but here the total stress is the sum of the stress in the spring and in the dash-pot.

$$\sigma = E\varepsilon + \eta\dot{\varepsilon} \tag{2.7}$$

In a creep test the strain will increase towards σ_0/E as time goes towards infinity. Here σ_0 represent a constant stress. An expression for a creep situation is given in Equation (2.8).

$$\varepsilon(t) = \frac{\sigma_0}{E}(1 - e^{-t/\tau}) \tag{2.8}$$

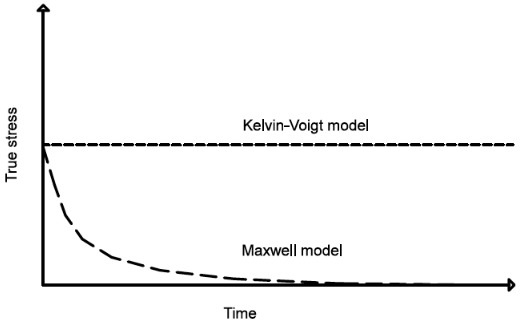
The model will however not be able to handle any relaxation. If the strain is kept constant,

$$\dot{\varepsilon} = 0 \tag{2.9}$$

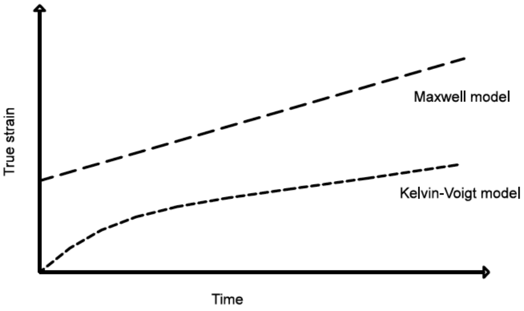
the stress will also be constant.

$$\sigma(t) = E\varepsilon \tag{2.10}$$

Figures of creep and relaxation principles for Maxwell- and Kelvin-Voigt-elements are shown in Figure 2.3.



(a)



(b)

Figure 2.3: Relaxation test (a) and creep test(b)

2.2 Polymers

The chapter about polymers contains information from various sources but is mainly based on the book Mechanical Behaviour of Engineering Materials by Rösler et al. (2007).

2.2.1 General

A polymer is a material consisting of molecules called monomers arranged in chains. In each chain approximately 10^3 - 10^5 of these monomers are linked together with covalent bonds. Between the polymer chains there are weaker links. Van der Waal's-, ionic- and hydrogen-bonds are examples of such links.

The process where these polymer chains are made is called polymerization and can be looked further in to by reading Polymer Engineering by Ram (1997). It is worth to mention that the average number of monomers in each chain is a measure on the degree of polymerization. Longer polymer chains have a tendency of becoming more twisted and entangled with other polymer chains. This arrangement prevents the polymer chains to slide past each other, and thereby decreases mobility for plastic sliding

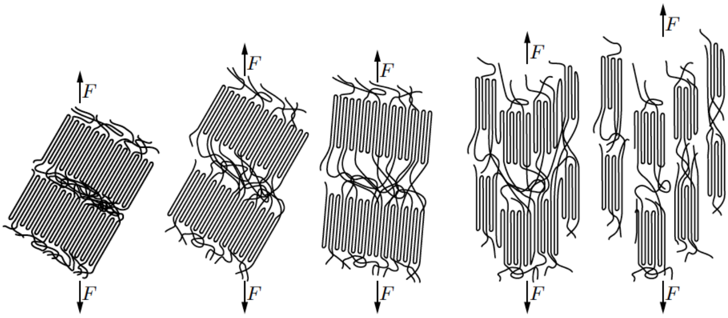
A wide spectrum of molecules are capable of acting as monomers and thereby being linked together in polymeric chains. Depending on the polymerization technique each of these polymer chains can be given different properties. Thus, lots of different polymers with different abilities exists. Three main groups of polymers are however possible to distinguish between. These are thermoplastics, elastomers and thermosets. Elastomers and thermosets both have cross-links between the polymer chains preventing them to slide past each other. This leads to a rubbery effect. Thermoplastics on the other hand has no cross-links and therefore experience a more plastic kind of behaviour.

2.2.2 Amorphus vs crystalline thermoplastics

Crystallinity is a property where the chains are strongly bound through folding. Cross-links prevent this arrangement and crystallinity can therefore only be found in thermoplastics. Crystallinity makes the bond length smaller and more dense. Most of the relaxation processes are impeded by this folding, but not entirely.

A polymer can never be completely crystalline. There is always some part of the material structure that is not perfectly folded. For this reason a thermoplastic which contains a crystalline part, is given a degree of crystallinity based on how much of the material that is crystalline. Polymers with a crystalline part are called semi-crystalline polymers. The part of the polymer that has not been arranged through folding is called the amorphous part. Here there are no regular arrangements between the polymer chains. The mechanical properties of polymers are mainly determined by the degree of mobility within and between the polymer chains in the amorphous part of the material.

The bond strength between the polymer chains is higher in the crystalline part of the material so at first the amorphous part is being stretched out. This leads to an orientation change in the crystalline part. Eventually the crystalline part will break up and separate into smaller blocks. The amorphous part of the material is where the impurities are found and therefore also where the cracks initiate. An illustration is shown in Figure 2.4.



(a) Initial stage (b) Lengthening amorphous regions (c) Rearrangement of crystalline regions (d) Separation into blocks (e) Formation of microscopic fibres (microfibrils)

Figure 2.4: Draw out of a semi-crystalline polymer (Rösler et al., 2007)

2.2.3 Activation energy and relaxation processes

The mobility between each chain is relaying on the weak bonds connecting them. Increasing the amount of side groups increases the total strength of these bonds. This happens due to the side-groups' ability to create stronger bonds with connecting chains. However, these bonds may break if sufficient amount of energy is provided. This barrier breaching energy is called the activation energy and has to come through a thermally activated process.

The mobility between each molecule in the polymer chains depends on the covalent bond connecting them. A double covalent bond between two molecules makes it impossible for the molecules to rotate relatively to each other. A single covalent bond should in theory be able to rotate freely, but the connecting hydrogen atoms and the chain itself prevents it from doing so. In order to rotate molecules within each chain, an energy barrier has to be broken. This can happen through a thermal activation, bringing the rotation angle to one of the two other local minima seen in Figure 2.5. This breaching energy is also termed activation energy.

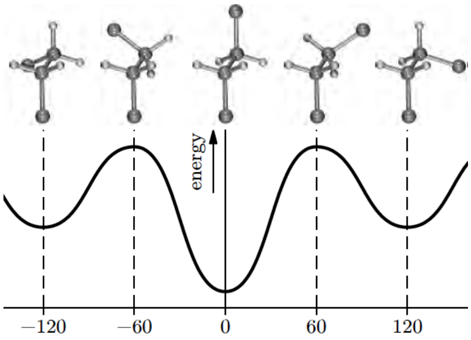


Figure 2.5: Rotational activation energy (Rösler et al., 2007)

Thermally activated processes which cause rearrangements within and between polymeric chains are called relaxation processes. The motivation for the term ”relaxation processes” is that a thermal activation is time dependent.

In thermoplastics both the elastic and plastic behaviour are influenced by thermally activated processes. In other words they are both visco-elastic and visco-plastic.

2.2.4 Elastic and visco-elastic behaviour

For thermoplastics, much larger strains can be experienced without moving into the plastic phase compared to metals. This has to do with the ratio between Young’s modulus and yield strength of the materials and give polymers comparatively a larger elastic deformation. Elasticity in thermoplastics is mainly based on the strength of the intermolecular bonds between the chains, and not the covalent bonds between the monomers.

Above glass transition temperature thermoplastics experience a visco-elastic effect. There is a distinct relation between temperature and time concerning visco elasticity. At very low temperatures below the glass transition where not all of the relaxation processes have the ability to affect the material this relation is no longer so obvious. The same goes for high temperatures near and above the glass transition temperature where sliding between polymer chains dominates the elastic behaviour. Visco-elastic effects may be negligible if the time is short and the temperature is low.

The activation energy of different relaxation processes differs. Therefore the visco-elastic effects can not be simulated by a simple Kelvin-Voigt or Maxwell model. Coupling of several elements, each describing individual processes would be necessary to give a quality analysis.

Semi crystalline polymers are dependent on two different elasticity processes above the glass transition temperature. The amorphous part experience entropy elasticity, while the crystalline part experience energy elasticity. The crystalline part remains in the energy elastic state due to the stronger connections between the chains.

Energy Elasticity

The process of returning atoms to their original position during unloading is called energy elasticity. The intermolecular bonds are being strained and have to be brought back to their original position.

Even below glass transition temperature some visco-elastic effects caused by movements of molecule segments are found. This movement is called secondary transition and are caused by a relaxation processes in which a required activation energy is needed. Due to the thermal activation needed it gets more probable when loading time increases.

Entropy elasticity

Strong entanglements of the molecular chains prevent the amorphous part of the polymer to exhibit viscosity if the temperature is below glass transition temperature. Above the glass temperature the distance between the molecules are larger and the process of sliding and rotation are easier to overcome by thermal activation. The relaxation time is therefore severely shortened. At unloading the molecules experience an entropy effect caused by stochastic thermal movement which prevent the molecule chains to stay straight, thus making them return to a coiled geometry. This property is called entropy elasticity.

2.2.5 Plastic and visco-plastic behavior

Plastic behaviour is caused by chains moving relative to each other. The behaviour is strongly dependent on temperature, because temperature controls the specific volume, and therefore also the size of the tunnels in which the polymer chains slide past each other.

Crazes

Crazes are lens-shaped cavities held together by bridging fibrils. These fibrils consist of several polymer chains. The cavities are formed by impurities within the material. The crazes only cause a slight reduction in strength due to the straightening of molecules around them. The thickness of the crazes increases with increasing temperature. When the polymer is strained the deformation of these crazes eventually lead to the so-called necking of the specimen. This happens through a meniscus instability growth mechanism. Eventually the fibrils at the edges grow together, and fibrils in the center break. This loss of strength causes the material to deform continuously at a constant level of force. The plasticity is thereby time-dependent and visco-plastic.

Deformation pattern

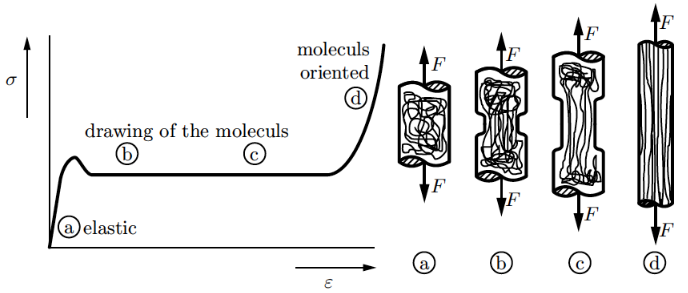


Figure 2.6: Principles of deformation for theromplastics (Rösler et al., 2007)

As deformation continues, more and more chains are drawn out and aligned in parallel within the section of where necking has taken place. Eventually a local hardening occurs when the interaction between the polymer chains ceases. Then the covalent bonds are loaded more heavily and the strength increases. When this overcompensates for the reduction in cross section the necking stop in this region. This effect spreads from this first local hardening, and the neck is drawn out until it comprises the entire specimen.

Shear bands

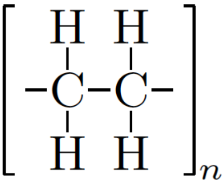
Also shear bands may occur but this is most relevant when it comes to compression. Shear strength is forcing molecules either to straighten up or form two kinks, aligning them in a angle of 45 to 60 degrees compared to the loading direction. This severely increases the fracture strength in compression compared to tension.

3 Material

In this chapter the polymer used in the experimental tests will be presented in Section 3.1, and the hypo-visco-elastic visco-plastic material model used in later simulations will be explained in Section 3.2. The hypo-visco-elastic visco-plastic material model is the main topic of this study. In Chapter 7 two more material models will be presented and compared to this original one.

3.1 High-density poly-ethylene (HDPE)

High-density poly-ethylene(HDPE) is a product of ethylene molecules fabricated by an addition polymerization. HDPE is a polymer consisting of linear chains and has compared to low-density poly-ethylene(LDPE) lower branching. This lead to a crystallinity of about 80 to 90% which again gives a higher persistence temperature, higher resistance to permeability, as well as higher stiffness and tensile strength. However the ductility and toughness has dropped compared to LDPE. HDPE is often used in rigid packaging, bottles, grocery bags etc.(Ram, 1997).

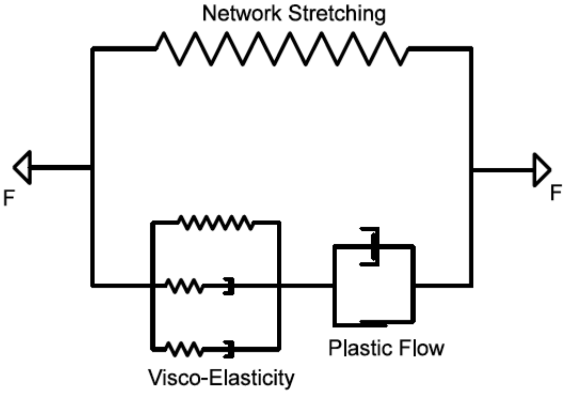


$$\left[\begin{array}{c} \text{H}\ \ \text{H} \\ -\text{C}-\text{C}- \\ \text{H}\ \ \text{H} \end{array}\right]_n$$

Figure 3.1: Chemical structure of poly ethylene

3.2 Hypo-visco-elastic visco-plastic material model

The model presented is based on an earlier developed model proposed by Boyce et al. (1999) and further developed by Polanco-Loria et al. (2010). It is, however, extended with a visco-elastic part. Moreover, the present model is hypo-elastic rather than hyper-elastic. The term hypo comes from the fact that the model is formulated on a rate form. Elastic energy may be dissipated due to this (Du Bois et al., 2006). The principle of the model is described in the Figure 3.2. As seen the model is divided into two parts. The intermolecular resistance (part A) and the stretching of the polymer network (part B). The idea of this division originally came from Haward and Thackray (1968).



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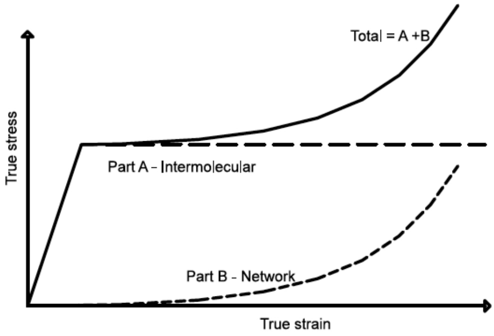


(b)

Figure 3.2: Rheological material model (a) and principles of the stress strain relation (b)

3.2.1 Part A- The intermolecular behavior

Part A covers the intermolecular behaviour of the polymer, consisting of a visco-elastic part in series with a plastic part.

Visco-elastic part

The visco-elastic part is put together by two linear Maxwell-elements and one spring in parallel. The total response is given as the sum of the contributions from each of the three elements.

$$\boldsymbol{\sigma}_{Tot} = \boldsymbol{\sigma}_{spring} + \boldsymbol{\sigma}_{Maxwell1} + \boldsymbol{\sigma}_{Maxwell2} \tag{3.1}$$

Complementary equations are found in Wang (2001) and Hopperstad and Børvik (2013).

Contributions from a spring and a dash-pot are governed by the following equations respectively.

$$\boldsymbol{\sigma} = \mathbf{c}^e \boldsymbol{\varepsilon} \tag{3.2}$$

$$\boldsymbol{\sigma} = \boldsymbol{\eta} \dot{\boldsymbol{\varepsilon}} \tag{3.3}$$

By use of Equation (3.2) and Equation (3.3) the governing equation for the Maxwell-element is derived.

$$\frac{\boldsymbol{\sigma}}{\boldsymbol{\eta}} + \frac{\dot{\boldsymbol{\sigma}}}{\mathbf{c}^e} = \dot{\boldsymbol{\varepsilon}} \tag{3.4}$$

Here $\mathbf{c}^e$ is the fourth order elasticity tensor and given by

$$\mathbf{c}^e = 3K\mathbf{I}_{vol} + 2G\mathbf{I}_{dev} \tag{3.5}$$

while the fourth order viscosity tensor $\boldsymbol{\eta}$ is given by

$$\boldsymbol{\eta} = 3\eta^m \mathbf{I}_{vol} + 2\eta^D \mathbf{I}_{dev} \tag{3.6}$$

η^m and η^D are the bulk viscosity and shear viscosity respectively and can be written as

$$\eta^m = \frac{\nu\eta}{(1+\nu)(1-2\nu)} \tag{3.7}$$

$$\eta^D = \frac{\eta}{2(1+\nu)} \tag{3.8}$$

$\mathbf{I}_{vol}$ is the fourth order volumetric tensor and given by $\mathbf{I}_{vol} = \frac{1}{3}\mathbf{I} \times \mathbf{I}$. $\mathbf{I}_{dev}$ is the fourth order deviatoric tensor $\mathbf{I}_{dev} = \mathbf{I}_4 - \frac{1}{3}\mathbf{I} \times \mathbf{I}$. Here $\mathbf{I}$ is the second order unit tensor and $\mathbf{I}_4$ is the fourth order unit tensor. Assuming no viscous effect in the volumetric response $\eta^m = \infty$ and that the shear modulus G and bulk modulus K are given by the relations

$$G = \frac{E}{2(1+\nu)} \tag{3.9}$$

$$K = \frac{E}{3(1-2\nu)} \tag{3.10}$$

six parameter constants must be determined for the visco-elastic part of the material model. The stiffness contribution E_0 of the elastic spring. The stiffness contributions E_1 and E_2 of the Maxwell-elements. The shear viscosities η_1 and η_2 of the Maxwell-elements, and Poisson's ratio ν which is constant for all elements.

3.2.2 Plastic part

As mentioned, the visco-elastic part is coupled together in series with a plastic part. This plastic part is composed of a friction element in parallel with a dashpot. The friction element represents the yield criterion f_A=0 and a non-associated flow potential $g_A(\beta)$. Both functions representing the yield criterion and the flow rule are based on earlier studies by Raghava et al. (1973).

For polymers an excellent agreement has been found between experiments and a theory regarding a mean stress dependency in polymers (Raghava et al., 1973). This mean stress dependency involves differences in polymeric behaviour in compression compared to tension. A yield stress ratio α was introduced, and inplemented in the original von Mises yield criterion. In its simplest form this yield criterion can be written as

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 + 2(|\sigma_C| - |\sigma_T|) = 2|\sigma_T \sigma_C| \tag{3.11}$$

By introducing the hydrostatic invariant and the deviatoric invariant

$$I_1 = \sigma_1 + \sigma_2 + \sigma_3 \tag{3.12}$$

$$J_2 = \frac{1}{6}[\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] \tag{3.13}$$

and the stress ratio

$$\alpha = \frac{\sigma_C}{\sigma_T} \tag{3.14}$$

a concise expression for the Raghava yield criterion can be given as

$$f_A = \bar{\sigma}_A - \sigma_T = 0 \tag{3.15}$$

Here the Raghava equivalent stress $\bar{\sigma}_A$ reads

$$\bar{\sigma}_A = \frac{(\alpha - 1)I_1 + \sqrt{(\alpha - 1)^2 \cdot I_1^2 + 12\alpha \cdot J_2}}{2\alpha} \tag{3.16}$$

As seen from the Equation (3.14), $\alpha > 1$ represents a compression sensitive material.

Setting $\alpha = 1$, corresponding to a material with same yield stress in tension as in compression, reduces the Raghava equivalent stress to the von Mises equivalent stress.

$$\bar{\sigma}_A = \sqrt{3 \cdot J_2} \tag{3.17}$$

An isotropic hardening function R has been introduced in the yield criterion altering the original Raghava yield criterion. This expression improves the representation of the transition stage between elastic and plastic behaviour.

$$R(\varepsilon_p) = (\sigma_S - \sigma_T)[1 - e^{-H\varepsilon_p}] \tag{3.18}$$

Here H is the isotropic hardening parameter, σ_T represents the yield stress in tension while σ_S represent the saturated yielding stress. In theory by use of a strain rate close to zero, σ_S would represent the yield strength found by Concidéres construction (Pelleg, 2013). See Figure 3.3 for the principles of the function.

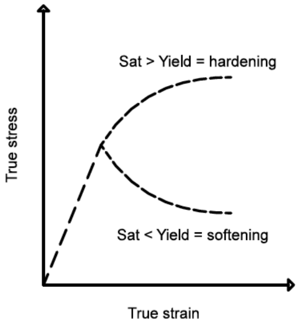


Figure 3.3: Principle of isotropic hardening function, where "Yield" corresponds to σ_T and "Sat" corresponds to σ_S

The yield criterion including isotropic hardening is given as

$$f_A = \bar{\sigma}_A - (\sigma_T + R) = 0 \tag{3.19}$$

Earlier studies (Clausen et al., 2011) has shown that the volume during plastic flow is inconsistent. A non-associated plastic flow rule has therefore been introduced.

$$g_A(\beta) = \frac{(\beta - 1)I_1 + \sqrt{(\beta - 1)^2 \cdot I_1^2 + 12\beta \cdot J_2}}{2\beta} \tag{3.20}$$

From the equation it is obvious that introducing β equal to α reduces the flow rule to an associative flow rule. Setting β equal to one leaves no volume changes.

The strain rate dependency is represented by the dash-pot in the model. This strain rate dependency is composed by a reference strain rate $\dot{\varepsilon_0}$, the equivalent Raghava stress $\bar{\sigma}_A$, the theoretical yield stress σ_T, the isotropic hardening function R, and a magnitude of strain rate constant C.

$$\dot{\bar{\varepsilon_p}} = \dot{\varepsilon_0}[e^{\frac{1}{C}\left(\frac{\bar{\sigma}_A}{\sigma_T+R}-1\right)} - 1] \tag{3.21}$$

as a function of equivalent stress it can be written as

$$\bar{\sigma}_A = (\sigma_T + R)\left(1 + C \cdot ln\left(\frac{\dot{\bar{\varepsilon_p}}}{\dot{\varepsilon_0}} + 1\right)\right) \tag{3.22}$$

It has been shown through experiments that it represents the behavior of thermoplastics in a satisfactory manner.(Polanco-Loria et al., 2010)

Part B- The intramolecular behaviour

The intramolecular behaviour is represented by the part B spring. The behaviour of this spring is dependent on two material constants: A locking stretch $\bar{\lambda}_L$ and the initial stiffness C_R. The principle of this network stretching is shown in Figure 2.6. After a local hardening in the necking area of the specimen, the neck propagates due to inability to stretch beyond a certain locking stretch. The initial stiffness determines the magnitude of stress needed for the network to harden. The stress distribution this represents is described in Equation (3.23).

$$\boldsymbol{\sigma}_B = \frac{C_R}{3J} \cdot \frac{\bar{\lambda}_L}{\bar{\lambda}} \cdot L^{-1}(\frac{\bar{\lambda}}{\bar{\lambda}_L}) \cdot [\mathbf{B}_B^* - \bar{\lambda}^2\mathbf{I}] \tag{3.23}$$

Here L^{-1} is the inverse Langevin function, where $L = coth(x) - \frac{1}{x}$. The distortional left Cauchy Green tensor is $\mathbf{B}_B^* = \mathbf{F}_B^*(\mathbf{F}_B^*)^T$, where the deformation tensor $\mathbf{F}_B^*$ is given as $\mathbf{F}_B^* = \mathbf{F}_B/\sqrt[3]{J_B}$. The Jacobean is $J = det(\mathbf{F})$ and the effective distortional stretch is $\bar{\lambda} = \sqrt{tr(\mathbf{B}_B^*)/3}$.

3.3 Summary of material parameters

A total of fifteen parameters are used to describe the material model and are given in Table 3.1.

PART A	
E_0	Elastic spring stiffness
E_1	Stiffness in first Maxwell-element
η_1	Viscosity constant in first Maxwell-element
E_2	Stiffness in second Maxwell-element
η_2	Viscosity constant in second Maxwell-element
v_0	Poisson's ratio
σ_T	Yield stress in tension
$\dot{\varepsilon}_{0,A}$	Reference strain rate
σ_S	Saturated yielding stress
C	Magnitude of strain rate dependency
H	Isotropic hardening parameter
α	Yield stress ratio
β	Dilatation parameter
PART B	
C_R	Initial network stiffness
$\bar{\lambda}_L$	Locking stretch

Table 3.1: Material parameters

4 Experimental Tests

In this chapter selected experimental results are presented. Section 4.1 takes on the procedure of the different tests. Then the test set up and recording systems are explained in Section 4.2. Next, Section 4.3 covers post processing. Finally, test results are presented in Sections 4.4-4.7. More comprehensive results from each individual test are found in Appendix A.

4.1 Procedure

In the laboratory eleven different tests were conducted. Two monotonic tension tests were done to give a backbone result used for later calibration. Then three different kind of experiments were carried out for three different stress-strain regimes.

The first experiment was a simple relaxation test. Here the specimen was pulled with constant speed until it reached the desired force. Then the displacement was freezed for 1000 seconds. This led to a relaxation process within the specimen.

The second experiment was a special kind of a creep test. Here the specimen also got pulled until it reached the desired force. But instead of freezing the displacement, it was unloaded until no force was left in the specimen. This force was kept at zero for about 1000 seconds, allowing the specimen to creep.

The third one was a double relaxation test, meaning that after about 200 seconds of relaxation similar to the simple relaxation test, the force and deformation was dropped down to a new level. The deformation was freezed at this level allowing yet another relaxation process to take place. This time the test specimen was held for 1000 seconds.

The following name tagging was used to recognize and separate the different tests. Letters describe what kind of test, while numbers describe what part of the stress-strain domain the specimen was tested for. Besides the monotonic tension tests in all three different kind of tests was carried out in three different parts of the stress-strain domain. The first test within each test group was kept within the elastic domain. The second test was carried to the the first part of the plastic domain, while the third test was taken far into the plastic domain. See Figure 4.1

T	tension test
R	simple relaxation test
C	creep test
DR	double relaxation test
1	only the elastic domain
2	the first part of the plastic domain
3	further into the plastic domain

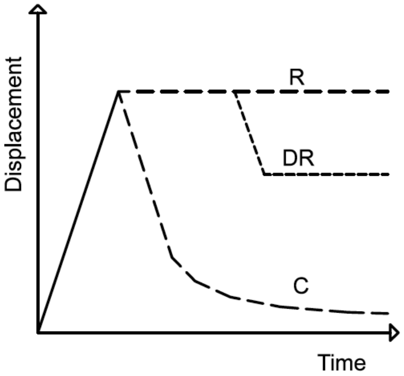
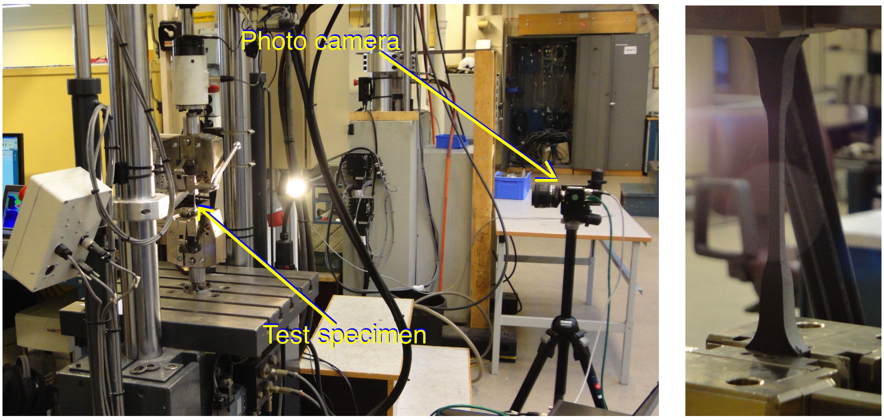


Figure 4.1: Principles of experimental test procedures

4.2 Test set up

All tests were performed at SIMLab, NTNU, and by use of the hydraulic Dartec M1000 RK machine. This machine had a load capacity of about $20kN$ and was therefore capable of carrying out all the desired tests. The test results were logged by two different digital recorders. A camera was installed taking pictures of the specimen during the test. A force and displacement correlated with each of these pictures were also gathered. The second recorder tracked the displacement and force at a frequency of $0.1Hz$. The dog-bone test specimens were attached to the machine by use of clamps. One side of the specimen was kept steadily during the entire test, while the other was given a velocity. For all of the tests, the initial velocity was set to $0.33mm/s$, which corresponds to a nominal strain rate of $10^{-2}s^{-1}$. A picture of the test set up is seen in Figure 4.2(a) along with a specimen rigged up to the machine during testing in Figure 4.2(b).



(a) (b)

Figure 4.2: Test set up (a) and specimen during experiment (b)

4.2.1 Geometry

All the tests was conducted by use of dog bone specimens like the one in Figure 4.3(b), with the same initital geometry. Figure 4.3(a) shows geometrical measures of the specimens.

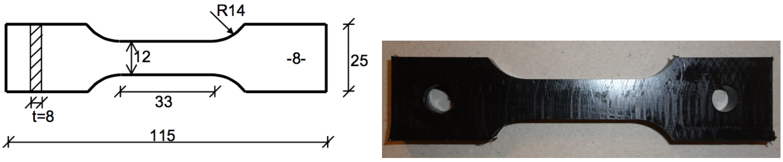


Figure 4.3: Dog bone specimen geometry (a) and a real specimen sample (b)

4.2.2 Image recording

A camera was placed normal to the width-length-plane of the specimen, recording deformation in the width and length direction. This camera is found on the right side of Figure 4.2(a). The pictures was taken at a predetermined frequency, and later used in a digital image correlation program. The deformation in the thickness direction was though never recorded. A force-displacement history came along correlated with each picture. The displacement was given by the distance between the clamps holding the specimen.

4.2.3 Force-displacement recording

In addition to the force displacement recorder connected to the camera, a force-displacement recorder was connected to the Dartec machine giving an accurate tracking of the force and displacement at a frequency of $0.1Hz$. If recordings of higher frequency was needed at critical stages of the experimental tests, this log could be necessary. Also, this recording machine was directly connected to the test procedure, starting a new time line at zero for every test stage determined through the Dartec machine.

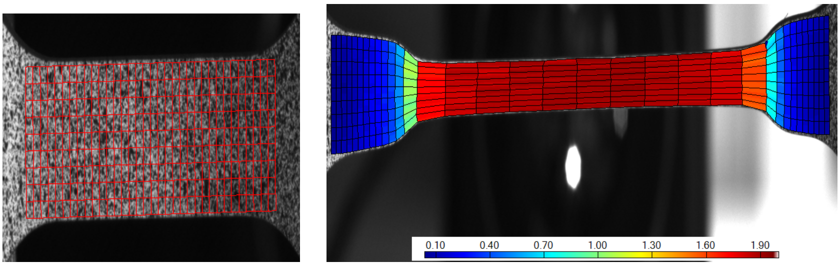
4.2.4 Simple meassurements

As seen in Figure 4.4(b) below, the specimen cross section got heavily altered in some of the tests. A manual measurement was carried out of the cross section geometry by use of a calliper. By help of these measurements it was possible to compare the strains in the width and thickness direction. This could determine whether additional thickness measurements are necessary in such experiments.

4.3 Post processing

4.3.1 Digital Image Correlation

By use of a new program eCorr developed by Flataker (2013), the true strains of the specimens was extracted by use of digital image correlation. A true strain-time history was found for each of the different tests.



(a) (b)

Figure 4.4: Specimen given an initial mesh (a) and DPE-T test given a strain field (b)

At first the specimen was covered by white paint. This gave a high contrast to a set of random black spots which were sprayed on right after. This had to be done just before the experiment was conducted so that the paint would not crack up during deformation. Figure 4.4(a) shows the speckled pattern. All pictures taken during deformation was opened in eCorr. The first picture was given a mesh where each mesh quadrant was set to 20x20 pixels. A more refined mesh would produce more noise without adding any accuracy. This mesh was then able to follow the entire deformation by correlating to the black spots sprayed on to the specimen. Some of the transitions between two pictures was however too abrupt for the program to handle. In these cases a helping tool which limited the degrees of freedom of the deforming mesh was added. This increased the computational time, but produced great results. In order to collect the desired strain history, each test had to be examined accurately. Through testing and visual observation the section of where the necking first took place was exposed. This was a fairly easy procedure due the relative large mesh size. If a more refined mesh had been used a similar program to the one developed by Torgrim Østen (Østen, 2012), that tracks this information, could be of more help. When the correct cross section was found the strain-history was extracted. Some strange behaviour was observed close to the edges of the strain area. To get accurate results the mesh quadrants closest to the edges was left out and only about 75% of the cross-section was used.

For the three tests which remained in the elastic domain, and thereby never experienced any necking, an avarage over a large area of 25x7 mesh quadrants was chosen to give accurate strain results. In the elastic domain a lot of noise was observed in each element, but by doing this trick the noise got averaged out and a smooth curve was thereby obtained.

The camera was set to take pictures at a frequency of 2Hz for the reaxation and creep tests, and 0.5Hz for the monotonic tension tests. Knowing this a strain-time history was obtained from the results.

4.3.2 Force record

Some problem occurred when the the force-time was extracted from the machine correlated with the pictures. The force log was highly inaccurate. Some major oscillations with a period of 500 seconds was observed at every force-time curve. See Figure 4.5 where the force measured by the Dartec machine is compared to the force from the image machine for test HDPE-R3.

By this problem no direct and useful correlation between the strain and force was present. The force graph directly from the Dartec machine was as seen in Figure 4.5 of much better quality. This force history was however not correlated with the pictures. In order to calibrate these results some curve-fitting was necessary. The first part of the force curve logged from the photos was of decent quality so by fitting the Dartec force to this part gave a good force-time correlation. The second problem was the number of recorded values. Every fifth force value from the tension

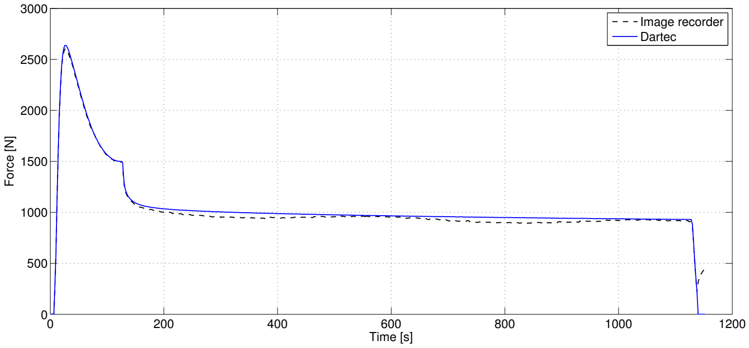


Figure 4.5: Force from Dartec machine vs force from image recorder

tests and every twentieth value for the restoring tests had to be extracted. In order to deal with this a simple excel program was developed.

The fact that the results from the relaxation and creep tests were recorded only every other second gave some concerns when it came to the accuracy this would produce in a simulation. This could be a problem, especially for transition phases like for instance the end of a force drop or the start of a relaxation process. Also the force peak could be lost due to the lack of successive recording. In order to deal with this a linear regression was carried out for the peak force and each of the transition phases in every test. An example of the regression procedures performed on the history data for test HDPE-R3 is given below.

4.3.3 Linear regression on test HDPE-R3

When the force-time history from the Dartec machine was aligned with the force-time history from the image recorder, each value extracted from the Dartec log corresponded to the correct picture. By tracking the original force measured by the Dartec Machine, the force peak was found in between two pictures. Through a linear regression in time, approximated strain values was found for this force-value, and added into the data history. The onset of relaxation was also found in between two pictures. Here the strain was approximated to be the same as the strain given by the latter of the two pictures. This was done because it corresponds to the nature of a relaxation test. Also here time was taken into consideration. As for the beginning of the test this was also found in between pictures and had to be approximated as a linear elongation of the history data from picture two and three.

For the remaining tests similar regressions have been carried out for every transition phase.

4.4 Monotonic tension test

As mentioned two monotonic experiments were conducted. One to give a backbone result, and one to either confirm or disconfirm the results from the first test. A problem occurred at the second tension test when the photo camera stopped recording early in the test. No strain data was possible to extract from these results and this test was therefore set aside the report. However, the force-time curve produced from this test was very similar to the force-time curve from the first monotonic test. The first test was therefore deemed reliable and used as the backbone curve for this study.

The true stress has been calculated by use of Equation (4.1)

$$\sigma = \frac{F}{A_0 \cdot e^{\varepsilon_b} \cdot e^{\varepsilon_b}} \tag{4.1}$$

where A_0 is the initial cross section area, F is force and ε_b is logarithmic strain in the width direction. As seen the strain in thickness direction is assumed to be similar to the strain in the width direction. This uniform strain distribution was a necessary aproximation due to the lack of strain data in the thickness direction. This assumption is based on findings in earlier tests of HDPE (Moura et al., 2009).

The yield stress was found by use of Considére's construction (Pelleg, 2013) which is based on Equation (4.2).

$$f = \lambda - \frac{d\lambda}{d\sigma} \cdot \sigma = 0 \tag{4.2}$$

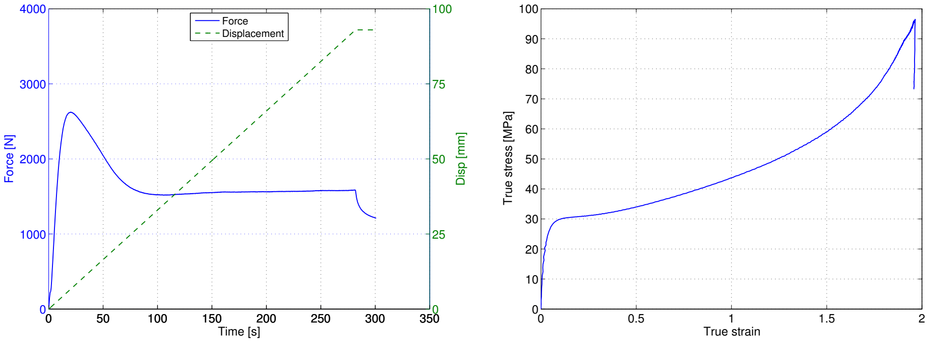
Here lambda is

$$\lambda = e^{\varepsilon} \tag{4.3}$$

The stress this relation gives was taken as the yield stress.

4.4.1 Results

A force-displacement and stress-strain curve for the first monotonic tension test is shown in Figure 4.6.



(a) Force-time & disp-time T (b) Stress-strain T

Figure 4.6: Tension test results

As the specimen was stretched a distinct necking occurred when the force peaked. The neck area hardened as can be seen from the stress-strain curve. From here on the neck was drawn out consuming more of the specimen. This supports the theory of molecules being stretch out until the molecular chains are all aligned as explained in Section 2.2.5. The force peak is evident in the force graph and the drop right after is due to the local contraction of the area in the neck. As the area stabilizes, the force also stabilizes. None of the tension tests were stretched to failure, so no data or observation concerning this behaviour was recorded. Some basic results regarding the tension test are shown in Table 4.1.

Max Force $[N]$	2625
Yield stress [MPa]	29.7
End strain	1.96
Max stress [MPa]	96
Initial area A_0 $[mm^2]$	8.05
Deformed Area A $[mm^2]$	3.37
Initial width b_0 $[mm]$	11.94
Deformed width b $[mm]$	5.25
Initial thickness t_0 $[mm]$	8.05
Deformed thickness t $[mm]$	3.37
End strain in width direction ε_b	0.822
End strain in thickness direction ε_t	0.871

Table 4.1: General results

A correction of the stress due to the geometry alterations in the sections where necking occurs was suggested by Bridgeman (1944) and further developed by Hill (1950) and Le Roy et al. (1981). This correction has though not been taken into account in this study based on findings by Hovden (2010).

In Table 4.1 a distinct difference in end strain is seen for the strain in the thickness direction compared to the width direction. A difference in strain ratio (DSR) has been calculated by use of Equation (4.4).

$$DSR = \frac{ln\frac{t}{t_0}}{ln\frac{b}{b_0}} = 1.06 \tag{4.4}$$

By including this ratio in the calculation of stress from Equation (4.1) yields

$$\sigma = \frac{F}{A_0 \cdot e^{\varepsilon_b} \cdot e^{\varepsilon_b \cdot DSR}} \tag{4.5}$$

The stress strain response is seen in Figure 4.7. Here in comparison with the original response.

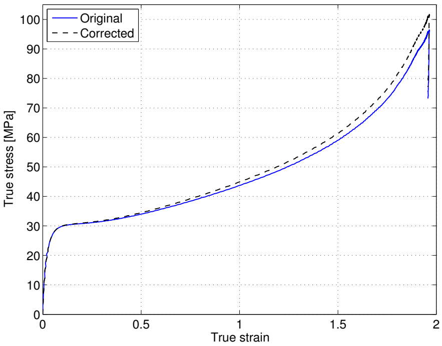


Figure 4.7: Corrected vs original stress-strain response

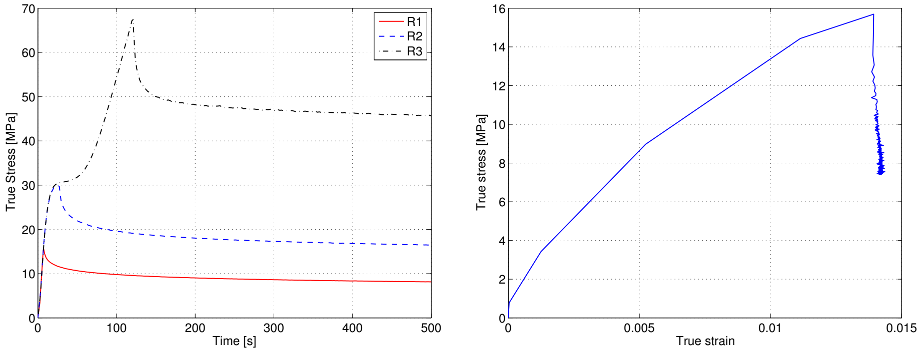
As seen in Figure 4.7 this has a certain impact on the response. However, due to the fact that only one reliable tension test was conducted, with no test to either confirm or disconfirm the result, the correction has been disregarded in this study.

4.5 Simple relaxation tests

The relaxation tests were conducted in this manner. At first a constant initial speed was set to one of the clamps. The specimens were then stretched until they reached the desired deformation level. Then the clamps were stopped and held for 1000 seconds. This gave us a simple test that could give answers regarding the nature of the relaxation processes in the material, and the behaviour concerning the stress-strain regime.

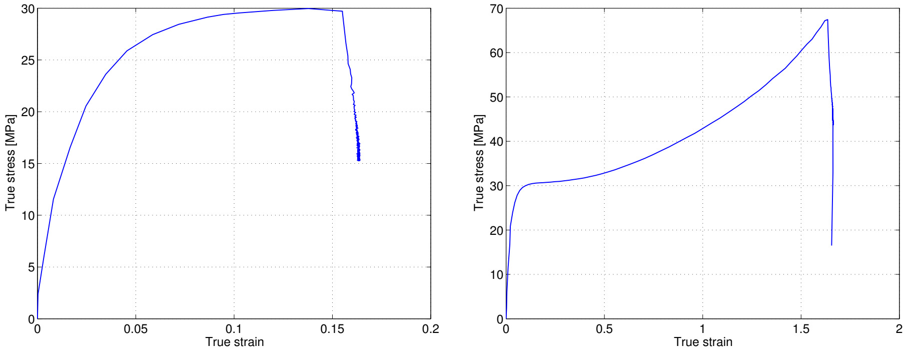
4.5.1 Results

The stress response for all of the relaxation tests are seen in Figure 4.8



(a) Stress-time R

(b) Stress-strain R1



(c) Stress-strain R2

(d) Stress-strain R3

Figure 4.8: Simple relaxation test results

As mentioned in Section 2.2.4 several relaxation processes may take place at once. This may describe the non-linearity of the relaxation curves seen in Figure 4.8(a). The shape implies that some of the relaxation processes is quite rapid while others are more tedious.

The drop in stress due to relaxation is increasing as the specimens has been taken further into the strain regime. In the HDPE-R3 test the stress is decreasing significantly more than for the R2 and R1 tests in the relaxation process. The steep slope of this stress decrease indicates that something within the material that produces a rapid relaxation effect still is present in this part of the domain.

The relaxation process seems to stabilize at a specific level depending on how far the specimen has been stretched. However, as the tests did not run for longer than 1000 seconds it is difficult to establish this exact level. Whether the stress would actually go all the way to zero and act as a true visco-elastic material like the Maxwell element described in Section 2.1.1, is difficult to envision but can not be alleged based on the HDPE-R test results.

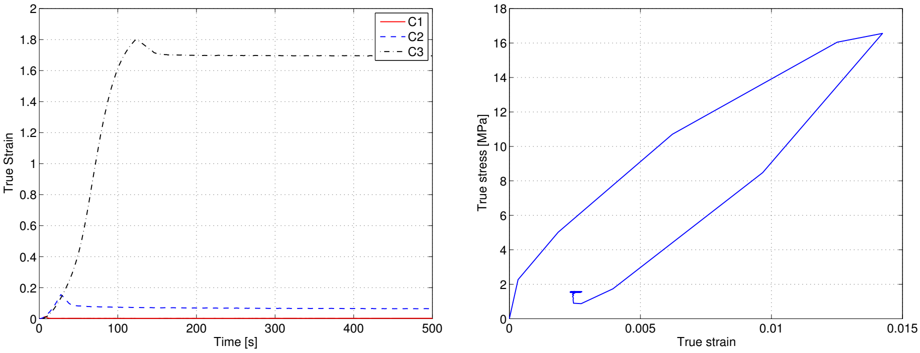
4.6 Creep tests

The creep tests started in the same way that the relaxation tests. A constant initial speed was set to one of the clamps, and the specimens were pulled into different stress-strain domains. However this time instead of holding the clamps, they were reversed immediately given the same speed but in the other direction until the force and stress level was approximately equal to zero. From here the force-level was kept constant and the specimen was free to creep. They were held like this for approximately 1000 seconds. These experiments was carried out to test the material creep behaviour after a rapid drop in stress. The creep behaviour depends on the same material behaviour as the relaxation explained in Section 2.2.3. However, instead of letting the thermally activated processes relax the intermolecular tension, the material expands so that the intermolecular processes are prevented.

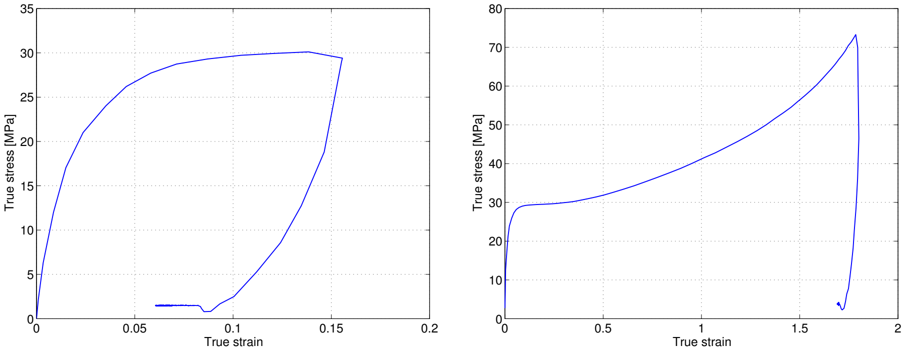
4.6.1 Results

From Figure 4.9 it was observed that the HDPE-C2 test experienced most creep and seems to yield during unloading. The C1 test exhibited almost no creep. The reason for this was probably due to the test procedure that did not allow for it. This has been further explained below. In the C3 test the section that was tracked had experienced a lot of hardening. Taking the tangent to the stress-strain curve at a point right before the drop in stress, the slope is found to be pretty steep compared to for instance the C2 test. This indicate that stress changes a lot with relatively small alterations in strain in this part of the domain. One can also say that the network stretch as explained in 3.2.2 is closer to locking. In other words, not a lot of creep is possible at this stage. The lack of creep in terms of alterations in strain, for the C3 test did therefore not come as a surprise.

Looking back it became obvious that better results could have been obtained by holding the displacement or force for a period of time before the stress was dropped. At least for the HDPE-C1 test that remained in the elastic domain. Here most of the sought material behaviour got eliminated by the rapid reversed stress as it did not allow much time dependent material behaviour to initiate.



(a) Stress-time C (b) Stress-strain C1



(c) Stress-strain C2 (d) Stress-strain C3

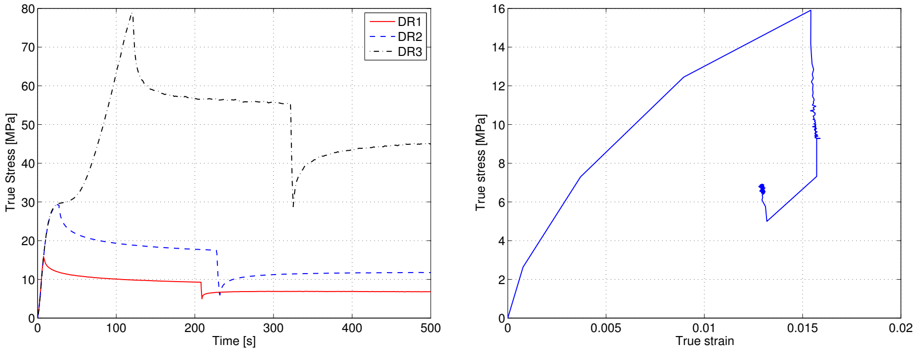
Figure 4.9: Creep test results

4.7 Double relaxation tests

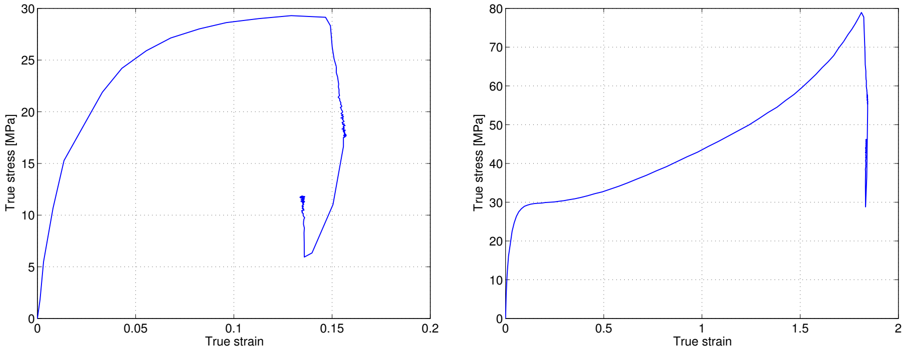
The double relaxation tests started in the exact same manner that the simple relaxation tests, but the relaxation was only held for 200 seconds. Thereafter an unloading procedure took place. Approximately half of the force was removed through a reversed displacement of the clamps. Then the specimen was held yet another time and relaxed for a 1000 seconds. These test were done so that it would be easier to explain and distinguish between different relaxation processes within the material. According to Section 2.2.4 more than one relaxation process may take place in the material behaviour. Through this experiment it was expected to get a more distinguished separation of the rapid relaxation processes from the slow. The rapid relaxation processes should be relaxed before the stress drop and therefore restart in compression, while the slow ones should still be in progress from the first relaxation procedure

4.7.1 Results

In Firgure 4.10 some selected results are shown for the DR tests.



(a) Stress-time DR (b) Stress-strain DR1



(c) Stress-strain DR2 (d) Stress-strain DR3

Figure 4.10: Double relaxation test results

As seen in Figure 4.10 the reversed relaxation increase is proportional to the stress-strain regime the tests are in, similar to the first relaxation process. A severely larger relaxation increase in stress is observed for test DR3 than for DR2 and DR1. Interesting though is that the slope of the stress curve becomes negative a certain amount of time after the reversed displacement. This indicates, as was expected in advance, that relaxation processes with various relaxation times take place. The first part is the rapid relaxation procedures. These are completed when the reversed displacement occurs. This allows the relaxation process to work in the other direction giving a distinct increase in stress right after this reversed displacement. The other relaxation processes are however not completed and the drop in stress only seems to slow them down. Therefore as the rapid relaxation processes are completed for the second time, only the slow relaxation processes are in progress and the slope of the curve turns negative yet another time.

4.8 Sources of Error

Due to sources of error such as mentioned in Section 4.3.2 with the inaccurate recording system, and Section 4.4.1 where differences were found for strains in the thickness and width direction, the experimental test results have to be treated carefully. Other sources of error may be:

- Inaccuracy in the geometry
- Imperfections or inconsistency within the material
- The clamps does not provide a totally rigid boundary condition and the stiffness of the specimen may therefore seem weaker than it actually is
- Sliding between the specimen and the clamps may occur

However, by performing a number of similar experiments some sources of error may be averaged out. In this this study there was though not performed more than one experiment on each test type. By comparing the different tests with each other and the expectations of each test, none of them did though seem unreasonable.

5 Calibration

In this chapter the calibration procedure for the numerical model is described. In Section 5.1 the material model is reduced to a uni-axial case from which a Matlab program has been designed. Then in Section 5.2 and 5.3 some analytical calibration has been done to keep the variables within proper limits. Finally in Section 5.4 som finishing touches have been made to the LS-DYNA model through inverse modelling and by help from the Matlab program.

5.1 Matlab-model

In order to calibrate the material model for later use in numerical simulations and to easily implement simple alterations, a uni-axial tension program in Matlab has been developed for the visco-elastic-visco-plastic model. This program features most of the parameters found in the material model. Some parameters are however lacking due to the fact that it is a uni-axial tension case. The program takes in a given strain-time history, and produces a stress curve.

5.1.1 Inital conditions

In order for the model to be functional some parameters have to be specified. These are the Young's modulus E_0, the visco-elastic parameters E_1, E_2, η_1 and η_2, the reference strain rate $\dot{\varepsilon}_{0,A}$, the magnitude of strain rate dependency C, the isotropic hardening parameter H, the initial stiffness for the network spring C_R, the locking stretch $\bar{\lambda}_L$, and the saturation and theoretical yield stress σ_S and σ_T respectively.

The stress in the model is initially set to zero at time equal to zero and stress equal to zero.

5.1.2 Part A-Elastic Domain

The elastic part of the model consists of three elements. One elastic spring and two Maxwell-elements, all in parallel. The total stress in this visco-elastic part of the model is assumed to be the sum of the stresses in each of these elements. In uni-axial tension Equation (3.2) and Equation (3.3) yields the following relations

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{bmatrix} = \frac{\sigma_1}{E} \begin{bmatrix} 1 \\ -\nu \\ -\nu \end{bmatrix} \tag{5.1}$$

$$\begin{bmatrix} \dot{\varepsilon}_1 \\ \dot{\varepsilon}_2 \\ \dot{\varepsilon}_3 \end{bmatrix} = \frac{\sigma_1}{\eta} \begin{bmatrix} 1 \\ -\nu \\ -\nu \end{bmatrix} \tag{5.2}$$

In the Matlab model the stress in the elastic domain only depends on the strain distribution in the first principle direction. Here the first principle direction is defined as the longitudinal direction. Knowing this the two governing equations for the Matlab program yields

$$\dot{\varepsilon_1} = \frac{\dot{\sigma_1}}{E} + \frac{\sigma_1}{\eta} \tag{5.3}$$

$$\sigma_1 = E\varepsilon_1 \tag{5.4}$$

By use of Equation (5.3) and Equation (5.4) explicit expressions is derived for the Matlab program.

$$\sigma_{i,n+1} = \sigma_{i,n} + E_i dt_{n+1} \left(\frac{d\varepsilon_{n+1}}{dt_{n+1}} - \frac{\sigma_n}{\eta} \right) \tag{5.5}$$

Here n+1 refers to strain and time step and i refers to the Maxwell-element number. From this yields

$$\sigma_{i,n+1} = \sigma_{i,n} \left(1 - dt_{n+1} \frac{E_i}{\eta_i} \right) + E_i d\varepsilon_{n+1} \tag{5.6}$$

The stress situation in the linear spring is given by the explicit numerical equation

$$\sigma_{0,n+1} = \sigma_{0,n} + E_0 d\varepsilon_{n+1} \tag{5.7}$$

The sum of these three stresses gives us a stress-strain-time relation in the elastic domain.

$$\sigma_{n+1} = \sigma_{0,n+1} + \sigma_{1,n+1} + \sigma_{2,n+1} \tag{5.8}$$

5.1.3 Part A-Plastic Domain

Yield criterion

Reducing the expression for Raghava equivalent stress in Equation (3.16) to a case of uni-axial tension yields

$$I_1 = \sigma_1 \tag{5.9}$$

$$J_2 = \frac{\sigma_1^2}{3} \tag{5.10}$$

$$\bar{\sigma}_A = \frac{(\alpha-1)\sigma_1 + \sqrt{(\alpha-1)^2\sigma_1^2 + 12\alpha\frac{\sigma_1^2}{3}}}{2\alpha} = \frac{(\alpha-1)\sigma_1 + (\alpha+1)\sigma_1}{2\alpha} = \sigma_1 \tag{5.11}$$

This results in a reduced Raghava yield criterion

$$f_A = \bar{\sigma}_1 - (\sigma_T + R) = 0 \tag{5.12}$$

which is independent of α and similar to the von Mises criterion for uni-axial stress. Here $\bar{\sigma}_1$ is the equivalent stress in the first principle direction.

Flow rule

The plastic flow on the yield surface can be written in matrix notation as

$$\bar{\mathbf{D}}^p = \dot{\lambda}\frac{\delta g}{\delta \boldsymbol{\sigma}} = \dot{\lambda}\left[\frac{\delta g}{\delta I_1}\frac{\delta I_1}{\delta \boldsymbol{\sigma}} + \frac{\delta g}{\delta J_2}\frac{\delta J_2}{\delta \boldsymbol{\sigma}}\right] \tag{5.13}$$

Here $\bar{\mathbf{D}}^p$ is the plastic deformation strain rate in matrix format. In a uni-axial tension case and by use of Equation (5.9) and Equation (5.10) the partial derivatives yield

$$\frac{\delta g}{\delta I_1} = \frac{\beta-1}{2\beta}\mathbf{I} + \frac{(\beta-1)^2}{2\beta(1+\beta)}\mathbf{I} \tag{5.14}$$

$$\frac{\delta I_1}{\delta \sigma} = \mathbf{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{5.15}$$

$$\frac{\delta g}{\delta J_2} = \frac{3\beta}{\sigma_1(1+\beta)} \tag{5.16}$$

$$\frac{\delta J_2}{\delta \sigma} = \sigma^{dev} = \frac{\sigma_1}{3}\begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \tag{5.17}$$

σ^{dev} is the deviatoric stress matrix. To decide the plastic multiplier $\dot{\lambda}$ in a uni-axial case a relation is given as (Hopperstad and Børvik, 2013)

$$\dot{\lambda} = \frac{\sigma_1 \dot{\varepsilon}_1^p}{\bar{\sigma}_1} = \dot{\varepsilon}_1^p \tag{5.18}$$

By inserting Equation (5.14)-(5.18) in Equation (5.13) yields

$$\bar{\mathbf{D}}^p = \dot{\varepsilon}_1^p \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1-\frac{3}{\beta+1} & 0 \\ 0 & 0 & 1-\frac{3}{\beta+1} \end{bmatrix} \tag{5.19}$$

As seen from Equation (5.19) the plastic flow in the first principle direction is described by $\dot{\varepsilon}_1^p$ and is independent of the β value.

Friction plates

The start of each step in time and strain begins with a trial stress state represented by only elastic behaviour. If this trial stress step exceeds the yield criterion, the test is in the plastic domain. Assuming a uni-axial tension case the yield criterion that defines this limit is based on Equation (3.19) and given as

$$f = |\sigma_{trial}| - (\sigma_T + R_{trial}) = 0 \tag{5.20}$$

Here σ_{trial} is the trial stress state in the first principle direction. The isotropic hardening of the trial step R_{trial} is always assumed to be similar to the earlier calculated R_n of the previous step.

The isotropic hardening function in Equation (3.18) can be written as

$$R_{trial} = (\sigma_S - \sigma_T)(1 - e^{-H\varepsilon_{p,n}}) \tag{5.21}$$

However, due to calculation reasons the yield criterion has been rewritten in the Matlab program. By adding Equation (5.21) in Equation (5.20) yields

$$f = |\sigma_{trial}| - (\sigma_T + (\sigma_S - \sigma_T)(1 - e^{-H\varepsilon_{p,n}})) = 0 \tag{5.22}$$

$$f = |\sigma_{trial}| - \sigma_S - (\sigma_S - \sigma_T)e^{-H\varepsilon_{p,n}} = 0 \tag{5.23}$$

Redefining the isotropic hardening function

$$R_{trial} = -(\sigma_S - \sigma_T)e^{-H\varepsilon_{p,n}} \tag{5.24}$$

leads to

$$f = |\sigma_{trial}| - (\sigma_S + R_{trial}) = 0 \tag{5.25}$$

This new yield criterion is exactly the same as the one in Equation (5.20). But the isotropic hardening function R is easier to write in an explicit manner.

Dashpot

Assuming uni-axial tension for the strain rate dependency described in Equation (3.21) reads

$$\bar{\dot{\varepsilon}}_{p,n+1} = \begin{cases} 0 & f <= 0 \\ \Phi_{n+1} & f > 0 \end{cases} \tag{5.26}$$

$$\Phi_{n+1} = \dot{\varepsilon_0}[e^{\frac{1}{C}\left(\frac{|\sigma_{trial}|}{\sigma_T + R_{trial}} - 1\right)} - 1] \tag{5.27}$$

The strain rate dependency becomes a vital part of the iteration procedure with regard to the size of the plastic strain increment.

Iteration procedure

An iteration procedure is necessary to decide how much of the strain that is elastic and how much that is plastic in strain step n+1. At first no plastic strain is assumed as the algorithm enters the iteration procedure. This comes from the trial state as mentioned earlier. In order to get the iteration procedure to work smoothly, and be as straightforward as possible, all elements in the strain rate function Φ is established explicitly as functions of plastic strain magnitude change *dp*. This plastic strain magnitude change of iteration step $k+1$ is represented by dp^{k+1}.

$$|\sigma^{k+1}_{trial}| = |\sigma^{k}_{trial}| - (E_0 + E_1 + E_2)dp^{k+1} \tag{5.28}$$

$$R^{k+1}_{trial} = R^{k}_{trial}e^{-Hsgn(|\sigma^{k+1}_{trial}|)dp^{k+1}} \tag{5.29}$$

This gives us $\Phi^{k+1} = \Phi^{k+1}(dp^{k+1})$.

In order to find this dp^{k+1} some new relations have to be established.

$$dp^{k+1} = \Phi^{k+1} dt \tag{5.30}$$

is reformulated into

$$\Psi^{k+1} = \Phi^{k+1} dt - dp^{k+1} \tag{5.31}$$

This Ψ^{k+1} represents the restoring unbalance or difference in strain that needs to be taken care of in order to assume a correct value for dp^{k+1}. See Figure 5.1.

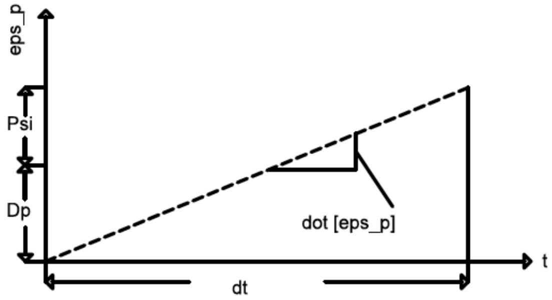


Figure 5.1: Iteration step. Here Psi is Ψ, dot$[eps_p]$ is $\dot{\varepsilon}_p$ and $[eps_p]$ is ε_p

By extracting the Jacobean of Ψ^{k+1} with respect to plastic strain magnitude change δdp^{k+1}, and by use of Newtons method, dp^{k+1} is established

$$\delta dp^{k+1} = -\Psi^{k+1} / J^{k+1} \tag{5.32}$$

$$dp^{k+1} = dp^{k} + \delta dp^{k+1} \tag{5.33}$$

This new established dp^{k+1} can be evaluated on different terms. In this program a lower limit of difference between each plastic strain estimate δdp is used as the iteration criterion. If this lower limit is surpassed the iteration procedure ends. However if it is not surpassed, the iteration starts over again with the new dp as the starting point.

when finally the fully iterated value of dp_{n+1} is established, corrected values of $\sigma_{0,n+1}$, $\sigma_{1,n+1}$ and $\sigma_{2,n+1}$ are calculated.

$$\varepsilon_{p,n+1} = sgn(\sigma_{trial}) dp_{n+1} \tag{5.34}$$

$$\sigma_{0,n+1} = \sigma_{0,trial} - E_0\varepsilon_{p,n+1} \tag{5.35}$$

$$\sigma_{1,n+1} = \sigma_{1,trial} - E_1\varepsilon_{p,n+1} \tag{5.36}$$

$$\sigma_{2,n+1} = \sigma_{2,trial} - E_2\varepsilon_{p,n+1} \tag{5.37}$$

5.1.4 Part B- The Langevin spring

The part B stress in a three degree of freedom system is calculated based on the matrix in Equation (3.23).

$$\boldsymbol{\sigma}_B = \frac{C_R}{3J}\frac{\bar{\lambda_L}}{\bar{\lambda}}L^{-1}(\frac{\bar{\lambda}}{\bar{\lambda_L}})[\mathbf{B}_B^* - \bar{\lambda}^2\mathbf{I}] \tag{5.38}$$

Based on a uni-axial case for this stress-matrix a function for the first principle stress is derived.

The deformation gradient is given by

$$\mathbf{F} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \tag{5.39}$$

where λ_i is

$$\lambda_i = e^{\varepsilon_i} \tag{5.40}$$

The left Cauchy Green tensor is

$$\mathbf{B}_B^* = J_B^{-2/3} \begin{bmatrix} \lambda_1^2 & 0 & 0 \\ 0 & \lambda_2^2 & 0 \\ 0 & 0 & \lambda_3^2 \end{bmatrix} \tag{5.41}$$

where the Jacobean is

$$J = det(\mathbf{F}) = \lambda_1\lambda_2\lambda_3 \tag{5.42}$$

and the equivalent stretch is given as

$$\bar{\lambda} = \sqrt{\frac{J^{-2/3}(\lambda_1^2 + \lambda_2^2 + \lambda_3^2)}{3}} \tag{5.43}$$

As the inverse Langevin function has yet to be solved analytically, an approximation by Padé has been used in this Matlab code.

$$L^{-1}\left(\frac{\bar{\lambda}}{\bar{\lambda}_L}\right) = \left(\frac{\bar{\lambda}}{\bar{\lambda}_L}\right)\frac{3-\left(\frac{\bar{\lambda}}{\bar{\lambda}_L}\right)^2}{1-\left(\frac{\bar{\lambda}}{\bar{\lambda}_L}\right)^2} \tag{5.44}$$

From equation (5.38) the following can be derived as a function for the uni-axial stress.

$$\sigma_{B,n+1} = Cr\bar{\lambda}_L L^{-1}_{n+1}\left(\frac{\bar{\lambda}_{n+1}}{\bar{\lambda}_L}\right)\frac{\lambda^2_{1,n+1}-\bar{\lambda}^2_{n+1}}{3\bar{\lambda}_{n+1}} \tag{5.45}$$

In order to calculate the stress in the first principle direction, strains in all three degrees of freedom are necessary. In the experiments both longitudinal and transversal strain in the width direction were extracted. As no strain data was extracted in the thickness direction the strain in the thickness direction is approximated to be similar to the strain in the width direction.

The total stress in the Matlab model is given as

$$\sigma_{tot,n+1} = \sigma_{0,n+1} + \sigma_{1,n+1} + \sigma_{2,n+1} + \sigma_{B,n+1} \tag{5.46}$$

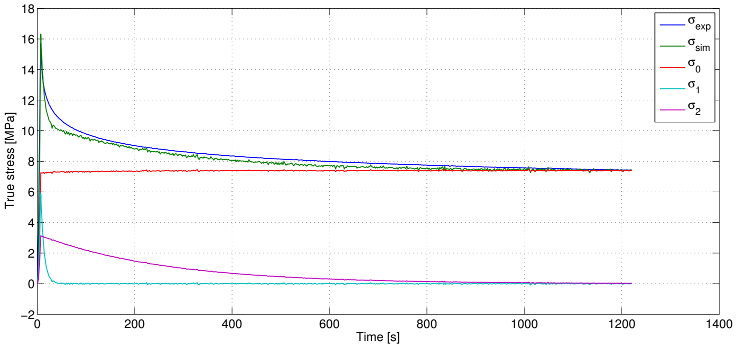
5.2 Calibration of Part A

The visco elastic part of the model has been calibrated based on the three tests in which the test specimen was kept in the elastic domain. This was the first relaxation test HDPE-R1, first creep test HDPE-C1 and first double relaxation test HDPE-DR1.

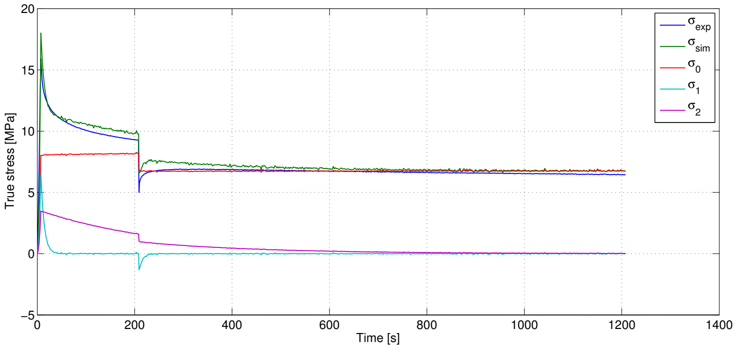
The plastic calibration is based on the result from the simple uni-axial tensile tests, HDPE-T. The fact that good results from only one tensile test was conducted for this study restricted some of the analytical calibration possibilities. Some material constants and parameter relations were still possible to extract. This limited some of the randomness to the curve-fitting procedure in Matlab concerning the possibility of different parameter value combinations yielding same response.

5.2.1 Visco-elastic part

In order to calibrate the elastic and visco-elastic parameters, E_0, E_1, η_1, E_2 and η_2, a non-linear least square fit method was performed by use of Matlab. A reduced version of the Matlab model described above, containing only the elastic and visco-elastic parts, was used to fit the stress function to the experimental stress history. By altering the parameters within a user subscribed domain good results were obtained for each of the three tests. However the composition of the parameter values was slightly different between each of the three tests. In order to enhance the calibration an improved program was created, taking in several strain-time history curves and calibrating them all at once. Calibrating more than two curves at once proved to be very time consuming so the HDPE-R1 and HDPE-DR1 test was eventually chosen as the basis of the visco-elastic calibration. The result of the curve fitting is seen in Figure 5.2 with corresponding parameters in Table 5.1. For full Matlab script, see Appendix C.



(a)



(b)

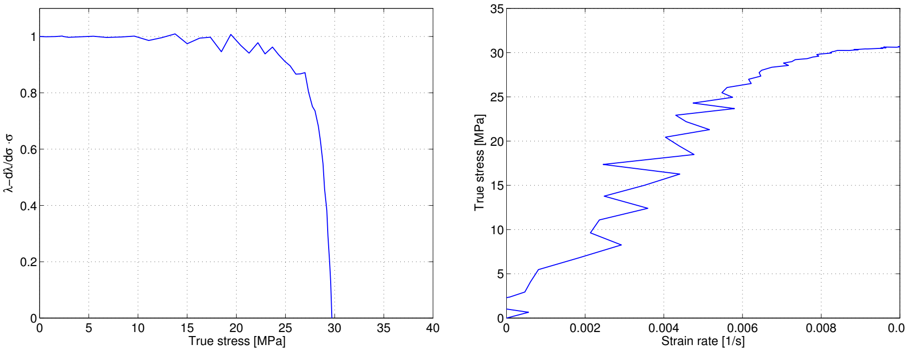
Figure 5.2: Calibration of visco-elastic parameters

5.2.2 Plastic part

The plastic part was calibrated by use of the same method as the visco-elastic part, but now the entire Matlab model described in Section 5.1 was used. In the Matlab script there is no limit to how many of the parameters that can be calibrated at the same time. Keeping the most physical parameters at a decent level should give a more trustworthy calibration. In this study an analytical calibration has been carried out regarding some of the parameters while others have been kept as variables to fit the curve in a best possible manner.

Strength and strainrate at yield

The yield strength was found by use of Considére's construction as mentioned in Section 4.4. Plotting stress against $\lambda - \frac{d\lambda}{d\sigma} \cdot \sigma$ gave the curve represented in Figure 5.3(a). As seen, it surpasses the x-axis at a stress level of approximately $29.7 MPa$. Examining the strain rate at this stress level also gave the corresponding strain rate at yielding. This was found to be about $0.7\dot{e}$, where $\dot{e}$ represents the relative strain rate. Earlier studies (Hovden, 2010) have shown that a good approximation of the reference strain rate, $\dot{\varepsilon}_0$ is $0.7 \cdot 10^{-3}$.



(a) (b)

Figure 5.3: Yield strength found by Considére's construction (a) and strainrate at function of stress (b)

Yield strength in tension

Yield stress in tension must not be mistaken as the yield stress found by Considére's construction. By physical means it can be seen as the onset of yielding in tension. This value is taken to be the stress given at 0.2% plastic strain-stress. In order to find this, a representing E-modulus has to be established. Since the spring in the model is accompanied by two Maxwell-elements the E-modulus is taken to be the sum of the spring stiffness and the two modulus's in the Maxwell-elements. An alternative method would be to do a linear regression over the elastic domain. See Figure 5.4.

Yield stress ratio -α

The yield stress ration α was found by comparing the yield stress in tension, which was found above, by the yield stress in compression which is found by use of the

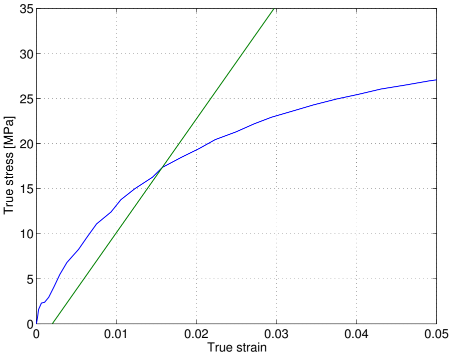


Figure 5.4: Onset of yielding found by 0.2% plastic strain

same method for a compression test. The relation used is given in Equation (3.14)

$$\alpha = \frac{\sigma_C}{\sigma_T} \tag{5.47}$$

No compression tests were carried out in this study, but a parallel specialization project (Røstum and Winjum, 2013) taking on the same material found $\alpha = 1.00$.

Strain rate dependency

The strain rate dependency of the material model is given by

$$\bar{\sigma}_A = (\sigma_T + R)\left(1 + Cln\left(\frac{\dot{\bar{\varepsilon}}_p}{\dot{\varepsilon}_0} + 1\right)\right) \tag{5.48}$$

as described in chapter 3. At yielding

$$\sigma_T + R = \sigma_S \tag{5.49}$$

Keeping this in mind, and knowing the strength and the strain rate at yielding, there is a direct relation between the saturation stress and the variable C.

$$\sigma_{yield} = \sigma_S\left(1 + Cln\left(\frac{\dot{\bar{\varepsilon}}_p}{\dot{\varepsilon}_0} + 1\right)\right) \tag{5.50}$$

Rearranging gives

$$\sigma_S = \frac{\sigma_{yield}}{\left(1 + Cln\left(\frac{\dot{\bar{\varepsilon}}_p}{\dot{\varepsilon}_0} + 1\right)\right)} \tag{5.51}$$

Knowing that $\sigma_{yield} = 29.7MPa$ and $\frac{\dot{\bar{\varepsilon}}_p}{\dot{\varepsilon}_0} = 10$ for $\sigma = 29.7MPa$ the following relation arises in Equation (5.52).

$$\sigma_S = \frac{29.7}{Cln(11) + 1} \tag{5.52}$$

This relation has been implemented in the Matlab script and used during the calibration to keep the relation between C and σ_S at an appropriate level.

Isotropic hardening parameter H

It is difficult to find a physical relation that corresponds to the isotropic hardening parameter H. An approximation can however be found through setting a target stress that should be present when the stress curve is through the ramping part of the curve. An approximation for the plastic strain is found through the relation

$$\varepsilon_p = \varepsilon_{tot} - \frac{\sigma}{E_0 + E_1 + E2} \tag{5.53}$$

and seen in Figure 5.5.

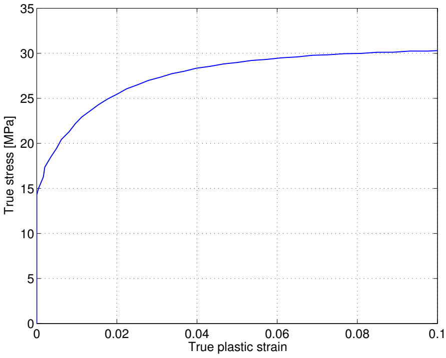


Figure 5.5: Stress as a function of plastic strain

Saying that the target stress should be at least 95% at yielding gives

$$\left(1 - e^{-H \cdot \varepsilon_p}\right) > 0,95 \tag{5.54}$$

which rewritten and by setting $\varepsilon_p = 0.07$ reads

$$H > -\frac{ln(0,05)}{0.07} = 42.8 \tag{5.55}$$

Retraction ratio and Poisson's ratio

The retraction Ratio, $R = -\varepsilon_t/\varepsilon_l$, where ε_l is longitudinal strain and ε_t is transversal strain, is plotted against ε_l. From this plot the Poisson's ratio ν can be estimated by the initial retraction ratio in the elastic domain, and R is chosen as a constant mean retraction ratio over the plastic part of the test.

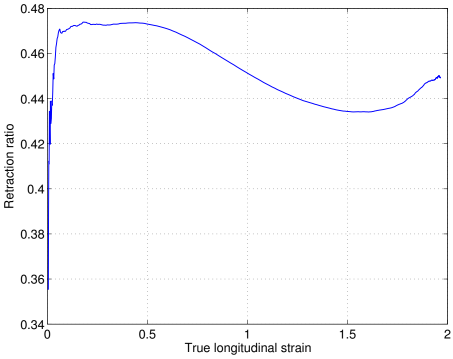


Figure 5.6: Retraction ratio as function of longitudinal strain

A simple relation between the dilation parameter β and retraction ratio R can be found through the uni-axial stress case presented in Section 5.1.3

Assuming a constant retraction ratio and isotropic transverse strains the plastic deformation strain rate $\bar{\mathbf{D}}^p$ is also given as

$$\bar{\mathbf{D}}^p = \dot{\varepsilon}^p \begin{bmatrix} 1 & 0 & 0 \\ 0 & -R & 0 \\ 0 & 0 & -R \end{bmatrix} \tag{5.56}$$

Setting Equation (5.19) up against Equation (5.56) yields the following relation

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -R & 0 \\ 0 & 0 & -R \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & (1-\frac{3}{\beta+1}) & 0 \\ 0 & 0 & (1-\frac{3}{\beta+1}) \end{bmatrix} \tag{5.57}$$

Rewriting the last two equations gives

$$\beta = \frac{2-R}{1+R} \tag{5.58}$$

from which the dilatation parameter can be calibrated based on a constant retraction ratio R.

Bulk and shear modules

The bulk modulus K and the shear-modulus G is found through the well known relations

$$K = \frac{E}{3(1-2\nu)} \tag{5.59}$$

$$G = \frac{E}{2(1+\nu)} \tag{5.60}$$

E, the total Young's modulus can be found by adding the visco-elastic contributions or through linear regression.

5.3 Calibration of part B

5.3.1 Locking stretch lDL

In order to approximate the locking stretch $\bar{\lambda}_L$ a relation between the locking stress and locking strain has to be established. Introducing $\lambda_i = \lambda_{L,i}$ and $\lambda_{L,i} = e^{\varepsilon_{L,i}}$ in Equation (5.43), yields the following equation

$$\bar{\lambda}_L = \sqrt{\frac{1}{3}\left[e^{\frac{4}{3}(\varepsilon_{L,1}-\varepsilon_{L,2})} + 2e^{\frac{2}{3}(\varepsilon_{L,2}-\varepsilon_{L,1})}\right]} \tag{5.61}$$

Using the already established relation $R = -\frac{\varepsilon_t}{\varepsilon_l}$

gives a final expression

$$\bar{\lambda}_L = \sqrt{\frac{1}{3}\left[e^{\frac{4}{3}\varepsilon_{L,1}(1+R)} + 2e^{-\frac{2}{3}\varepsilon_{L,1}(1+R)}\right]} \tag{5.62}$$

The locking strain ε_L is assumed to be the vertical asymptote of the strain when stress is given as a function of strain. In the experimental HDPE-T test the maximum stretch reached was 1.96. With this in mind the locking strain should be at least this value and probably not higher than 2.5 by looking at the behaviour of the curve. These two values correspond to locking stretch of 3.85 and 6.48 respectively. For this reason the locking stretch is kept within this range.

5.3.2 Initial stiffness Cr

The initial stiffness Cr has simply been fitted through least square method so that the curve represents the actual response.

5.4 Calibration of LS-DYNA model

By help of the Matlab program and the analytical calibration above, a LS-DYNA calibration was carried out for the HDPE-T test.

The model behaviour was not entirely similar in the simulations done in Matlab to the simulations carried out in in LS-DYNA. However, the overall behaviour was reflected in a good manner. By doing a calibration by help of the Matlab program at first, good initial parameters were established for the LS-DYNA calibration. Each simulation in LS-DYNA was severely more time consuming than those performed in Matlab. In order to speed up the calibration process a simple parameter study was conducted in the uni-axial tension program to help understand how each of the parameters affected the behaviour. This is not further explained.

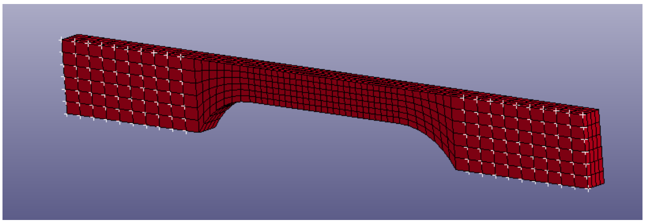
5.4.1 Model

No 1 elements, meaning reduced eight nodes solid elements with a structured mesh was chosen for the LS-DYNA model. A quarter of the total model was simulated and given symmetry boundary conditions in both width and thickness direction. Every simulation was checked for the amount of kinetic energy. This had to be small in comparison with the total energy, so that inertia forces could be disregarded. The LS-DYNA model is implemented in a hypo format, meaning that the model is formulated on a rate form. In the simulations both density and mesh-size had to be chosen carefully in order to keep the simulation response converged.

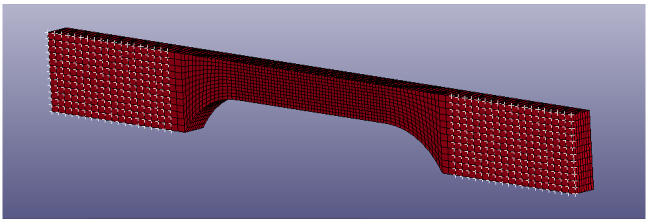
Density

The density was checked against differences in general behaviour, oscillations due to inertia forces and kinetic energy within the model. Scaling with a factor of 10^9 gave good results concerning the HDPE-T test. Scaling any less did not improve the result significantly.

Mesh



(a)



(b)

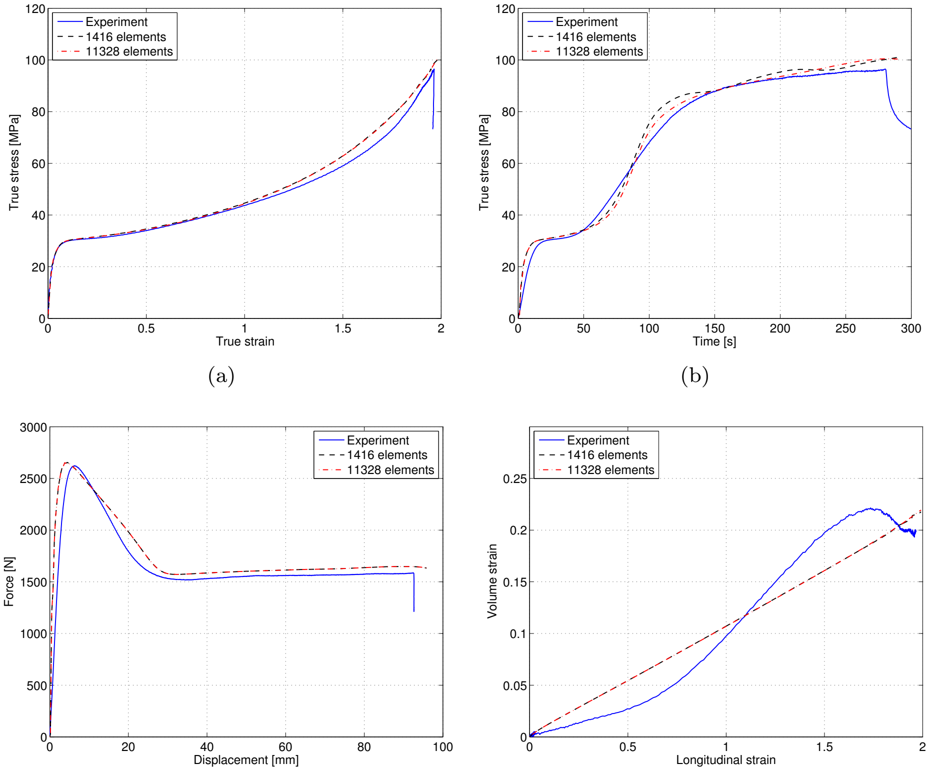
Figure 5.7: Comparison of small and large mesh

The mesh was first of all checked against general behaviour. Reducing the mesh size would improve the results, however the computational time was also a factor that had to be taken into concern. A mesh size that gave good result without bringing the computational time up too much was preferred.

A comparison of two different meshed models can be seen in Figure 5.7. The different element sizes chosen in these models are 0.5x0.5x0.5 mm^3 and 1x1x1 mm^3.

5.4.2 Final calibration

The bold written parameters in Table 5.1 was kept unchanged through the inverse LS-DYNA calibration along with the relation between σ_S and C in order to keep the calibration realistic. With the high number of different parameters, more than one combination of values can extract equivalent results. Locking of the most physical parameters should therefore help preventing false solutions.



(a) (b)

(c) (d)

Figure 5.8: Comparison of 11328 elements ($0.5x0.5x0.5mm^3$) *and 1416 elements* ($1x1x1mm^3$)

As can be seen from Figure 5.8 the calibration could have been better. Here a comparison of the two different meshed models are presented. Due to that the purpose of this model was to later simulate visco-elastic effects such as creep and relaxation, the time domain played a more important role than it normally does for standard calibrations as described by Polanco-Loria et al. (2012). A reasonable stress-time behaviour was therefore prioritised on the expense of strains and force.

$E_0 [MPa]$	$E_1 [MPa]$	$\eta_1 [MPa \cdot s]$	$E_2 [MPa]$	$\eta_2 [MPa \cdot s]$	v_0	$\sigma_T [MPa]$	$\dot{\varepsilon}_{0,A} [1/s]$
520.2	**516.8**	**4023.4**	**226.4**	**56980**	**0.41**	**16**	**0.7** $\cdot 10^{-3}$

$\sigma_S [MPa]$	C	H	α	β	C_R	$\bar{\lambda}_L$	κ	$G [MPa]$	$K [MPa]$
24.95	0.794	66.94	**1.0**	**1.07**	2.00	5.5	**0.0**	**448**	**2340**

Table 5.1: Calibrated parameters for LS-DYNA model

It is seen in Figure 5.8(b) that some oscillations occur in the stress-time domain for the model with element size 1x1x1 mm^3. These oscillation starts approximately 150 seconds into the simulation.

A comparison of the deformed LS-DYNA model with the largest mesh size and the deformed specimen for HDPE-T is shown in Figure 5.9. As seen, the experimental and simulated deformation patterns are similar.

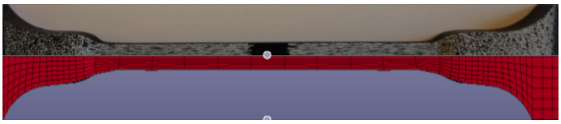


Figure 5.9: Comparison of LS-DYNA simulation and experimental test of HDPE-T

6 LS-DYNA-Simulations

Each of the experimental tests described in Chapter 4 has been simulated by use of the LS-DYNA model calibrated in Chapter 5 (LSTC, 2007). In Section 6.1 the procedures to perform the simulations are explained. Selected results from the different simulations are presented in Section 6.2.

6.1 Procedure

The experimental tests were all monitored globally by measuring clamp displacement and force. As mentioned in Section 4.8 there are some sources of error regarding both the stiffness of the boundary conditions, and the friction between the surface of the clamps and the surface of the specimens. In the LS-DYNA model the clamps have been simulated as rigid nodes at the area where the clamps were attached in the experimental tests. This gave a much stiffer initial part of the simulation compared to the experiments. The simulated force-displacement compared to the experimental force-displacement of HDPE-R2 is seen in Figure 6.1.

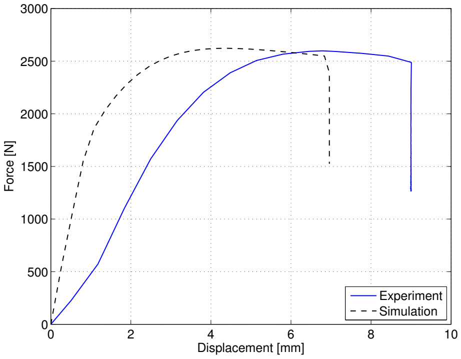


Figure 6.1: Force displacement curves of HDPE-R2

In order to keep the material behaviour in the simulations as similar as possible to the material behaviour in the experimental specimens the simulations are based on the experimental strain-time history on contrary to the displacement-time history. This gave good comparisons. Displacement control was used to recreate this equivalent strain, so the simulated displacement was still linear as in the experimental tests.

Both the relaxation tests and the double relaxation tests were simulated exclusively in terms of the strain-time history. In the creep tests the purpose of the experiment was to bring the force down to approximately zero after an initial displacement procedure, and from here on let the specimen be allowed to freely creep. In the experimental tests this force relief was carried out through a force control sequence. A force control sequence was therefore chosen to simulate this behaviour in LS-DYNA, hence not the strain-time history.

6.1.1 LS-DYNA model

The LS-DYNA model used for the virtual simulation of the experimental tests is explained in Section 5.4 and shown in Figure 5.7(a). The larger mesh was chosen due to the shorter computational time. The oscillations described in Section 5.4.2 should not be a problem because the rest of the experimental tests are not exposed to those kind of deformations. Corresponding parameter values are given in Table 5.1. The density had to be lowered to a scaling factor of 10^7 to prevent oscillations after the force drop and in the relaxations. This model is used for all the different creep and relaxation tests so that all simulations could be compared to each other.

6.2 Results

At the initial constant velocity stage all simulations behave accordingly to the calibrated model. Some minor differences are seen in comparison with the experimental tests. The reason for this could be minor material differences between each test specimen, or sources of error due to the test set up as explained in Section 4.8. For each of the different simulation only some selected data have been presented. For complementary data see Appendix B. For keyword file of the HDPE-DR3 simulation see Appendix D.

6.2.1 Simple Relaxation Simulations

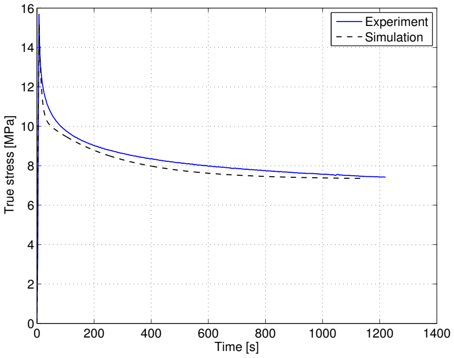
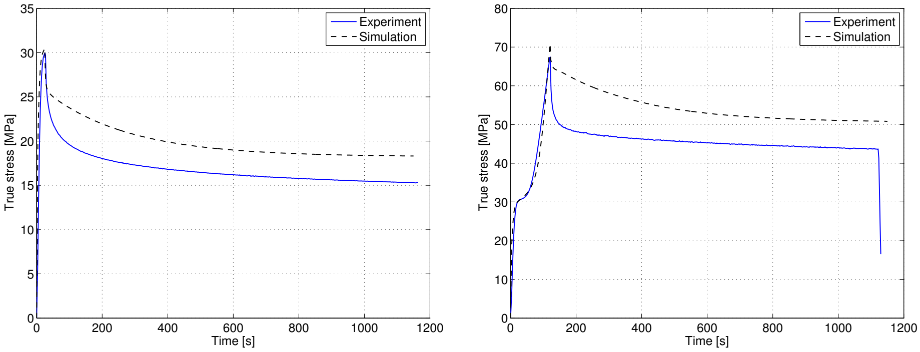


Figure 6.2: HDPE R1 simulation

In the simple relaxation simulations especially the HDPE-R1 simulation shows similar relaxation behaviour and relaxation stress magnitude to the experimental test. The fact that the visco-elastic part was partly calibrated by use of the R1 test (Section 5.2.1) could be a contributing factor to this good result.



(a) Stress-time R2

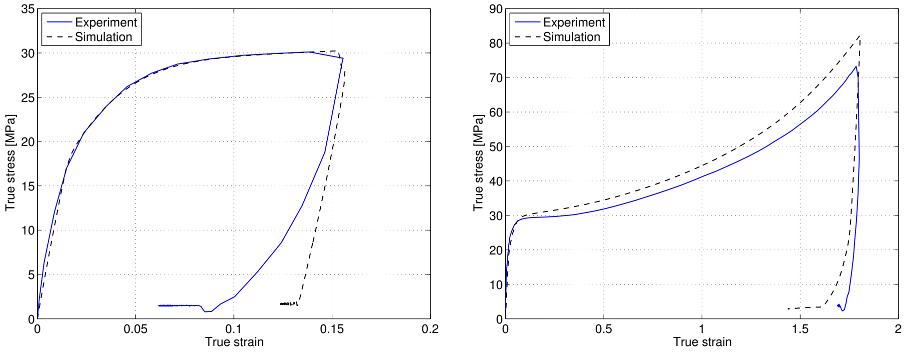
(b) Stress-Strain R3

Figure 6.3: HDPE R2 and HDPE R3 simulation

Both the HDPE-R2 and R3 simulations in Figure 6.3 differ more in terms of the stress magnitude of the relaxation. The difference is larger in the R3 simulation compared to the R2 simulation. From Figure 6.3(a) and Figure 6.3(b) a distinct kink is observed in the relaxation response only seconds after the start of the relaxation procedures in both simulations. This may be due to rapid Maxwell-element with the relaxation time of approximately seven seconds. This element should be more or less fully relaxed at this stage. From here on, the simulated response separates more and more from the experimental response.

6.2.2 Creep simulations

The LS-DYNA model did not entirely manage to handle the rapid reversed displacement procedure from the unloading sequence in the creep tests. The mesh got partly disrupted in the boundary condition and the displacement started to grow uncontrollably. However, the force remained unaltered through the entire specimen despite this problem. After a while the disruption in the boundary condition started to migrate across the specimen. Until this happened the results remained good. The few selected results presented in Figure 6.4 and the remaining results in Appendix B should therefore be of good quality.



(a) Stress-strain C2 (b) Stress-strain C3

Figure 6.4: HDPE C2 and HDPE C3 simulation

As mentioned in Section 4.6.1, the HDPE-C2 experimental test seems to yield when the stress is getting close to zero. This does however not seem to happen in the simulation where the model remains in the elastic domain. Based on the coefficients and yield strength chosen in the model this was expected and is seen on the linearity of the stress relief in the stress-strain domain. On the other hand the C3 experimental test looks as though only minor plastic deformations is experienced in the stress relief. The C3 simulation does on contrary get further into the plastic domain. This can be seen in Figure 6.5. Here the yield strain represents the limit that the strain must surpass in order to yield. When the limit gets pushed it means that the material yields. This can be seen in the later parts of the stress relief, corresponding to the strain relief which is seen in this figure.

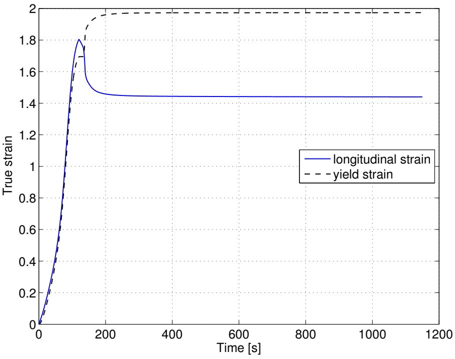


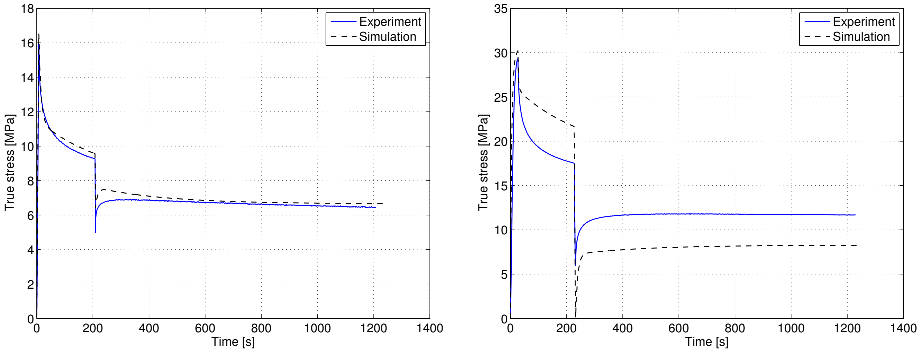
Figure 6.5: Longitudinal strain and yield strain limit as function of time in HDPE-C3 simulation

A similar behaviour pattern is seen the double relaxation simulations. However, the deformation in the simulations are smaller and the plastic yield is never reached for the DR3 simulation during the stress relief.

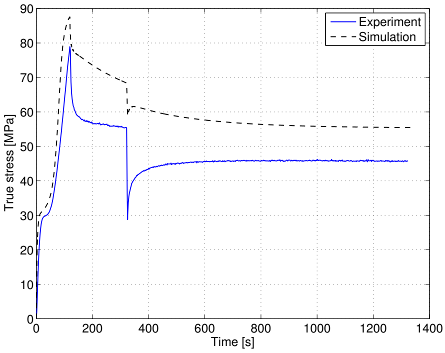
In terms of creep in the simulations, a significant creep behaviour is seen in the C2 simulation, while little is observed in the C1 and C3 simulation. This corresponds to the results in the experimental tests in Section 4.6.1. However, the magnitude of the creep is not that precise. This may be due to the stiffness of the model which is mentioned earlier.

6.2.3 Double relaxation simulations

From the HDPE-DR simulations in Figure 6.6 it becomes clear that the two Maxwell-elements contribute to the relaxing behaviour in different manners, especially when it comes to the second relaxation procedure. The Maxwell-element with the shortest relaxation time gets fully relaxed by the first relaxation procedure in the simulation. When the stress is relieved the rapid Maxwell-element goes in compression contributing with a stress increase in all three simulations as seen in Figure 6.6. The first relaxation procedure in the double relaxation simulations gives similar results to the simple relaxation simulations as expected.



(a) Stress-time DR1 (b) Stress-time DR2



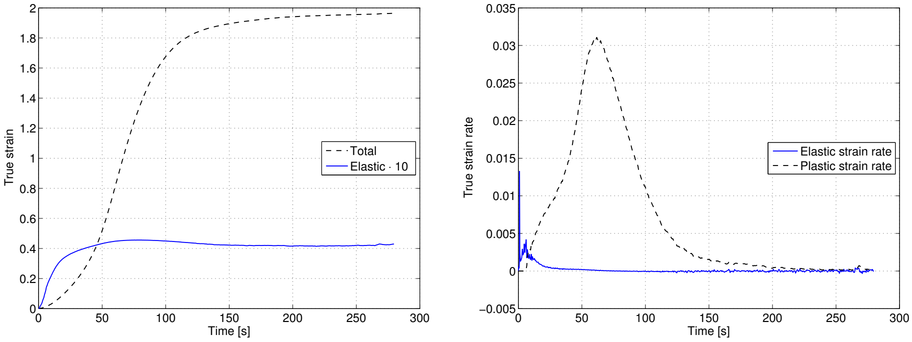
(c) Stress-time DR3

Figure 6.6: HDPE-DR1, HDPE-DR2 and HDPE-DR3 simulation

The behaviour of the slow Maxwell-element is on the other hand depending on how much it has relaxed in the first relaxation procedure. In the HDPE-DR1 simulation the element has had too little time to fully relax. The stress relief is therefore only partly stopping the relaxation due to the first relaxation procedure. Hence, the Maxwell-element does not experience any compression, and the stress contribution from this element only continues to decrease. However, this happens at a slower rate than before the stress relief. The same Maxwell-element behaviour is seen in the DR3 simulations as well. In the DR2 simulation the stress relief seems to eliminate the entire contribution from the slow Maxwell-element as the curve is almost flatting out after the stress increase given by the Maxwell-element with the shortest relaxation time. The reason why the DR2 simulation stops entirely while the DR1 and DR3 simulations does not, may be due to the stiffness of the model as explained in Section 6.2.2. The relief in strain corresponds to a greater stress relief in comparison to the experimental tests and therefore manages to stop the slow Maxwell-element.

6.2.4 Discussion

The visco-elastic parameters were calibrated through the experimental tests that was kept in the elastic domain so that the visco-plastic effects could be neglected. The simulations that also was kept in the elastic regime are those which in terms of the relaxation and creep processes show most similarities to the experimental tests. The simulations taken further into the plastic domain are struggling to keep up with the magnitude of the stress relaxation. In the material model this magnitude is decided by the condition of the Maxwell-elements at the start of the relaxation procedure, and where the specimen is located in the stress strain domain at this point. A high elastic strain rate, which keeps the Maxwell-elements prestressed, and being far into the plastic regime, due to the slope of the stress-strain curve, is factors that should increase the magnitude of the relaxation procedure. A problem concerning the material model is the lack of producing elastic strain or elastic strain rate in the plastic regime. After the initial elastic part of the simulation the plastic part takes almost entirely over the strain distribution due to the composition of the material model. An example is shown of the HDPE-T test by use of Matlab in Figure 6.7. The figure shows that the elastic strain stops and elastic strain rate goes to zero early in the plastic domain and stays there for the rest of the deformation. The elastic strain is multiplied with a magnitude of ten.



(a) (b)

Figure 6.7: Elastic strain (a) and Strain rate (b) in HDPE-T simulation in Matlab

This means that the Maxwell-elements with a short relaxation time is already fully relaxed when the relaxation procedure start far out in the plastic regime. This is seen in Figure 6.3(b) and Figure 6.6(c). Here the rapid relaxation is absent which made it difficult for the simulation to follow the experimental results. Any linear Maxwell-element that still has the opportunity to relax at this stage must though be a slow Maxwell-element. A slow Maxwell-element also gives a slow relaxation response. Thus the material model does not seem to be able to simulate the relaxation response in this part of the stress-strain regime.

In terms of the unloading sequence in the creep and double relaxation simulations an interesting response is observed. While the stiffness in the DR1 and C1 simulation are equivalent to the experimental tests, the DR2 and C2 simulations are too stiff compared to the tests. Interesting then is that both the DR3 and C3 simulations gave a much too soft response. The reason for this may have something to to with a non-linear visco-elastic effect or the definition of the yield criteria in the model.

In the relaxation procedure a kink was observed in many of the simulations. This is probably due to the fact that only two Maxwell-elements are present in the material model. By introducing more elements with different relaxation times, the individual processes should end at different times and thereby creating a smoother response curve. This may especially help in the case of the double relaxation simulation. Introducing more Maxwell-elements should however not have much effect regarding the magnitude of the relaxation. More elements yields less stiffness contribution for each of the elements. Increasing the magnitude of these stiffness contributions may create a more similar relaxation response for the HDPE-R3 and HDPE-DR3 tests, but the stiffness in the elastic regime would become too large.

7 Modified material models

In Section 7.1 two new material models are presented and compared to the original material model from Chapter 3. Further on the new implementation is described for Matlab, and a simple calibration has been carried out. In Section 7.2 these new models have then been tested for different attributes by use of Matlab and further compared with experimental data. The simulations are mainly quantitative and are meant to reveal the new material models possibilities and limitations rather than to recreate the actual response of the experimental tests.

7.1 New material models

The lack of elastic strain rate in the plastic domain, see Section 6.2.4, makes it difficult for the material model presented in Chapter 3 to handle the relaxation in the plastic domain. The new material models presented in this Chapter may improve the response concerning this problem.

7.1.1 Elastic visco-plastic model with part B as back-stress

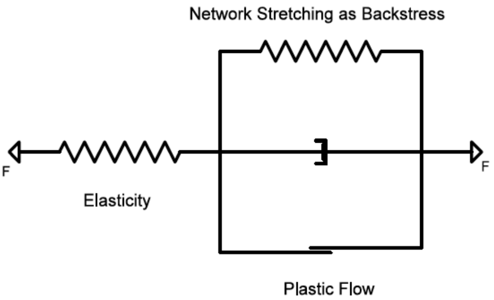


Figure 7.1: Elastic visco-plastic model with network stretch as backstress

As seen in Figure 7.1, the main difference between this model and the original model presented in Chapter 3, is the part B stress. Instead of keeping part B in parallel with the total strain, the part B stress is now only affected by the plastic strain contribution. From a practical point of view, this entails that the elastic part of the strain will acquire more of the total strain when the strain becomes large.

The plastic strain ceases when the locking stretch is reached and the material will once again behave elastically. On a molecular level this represents a state where all the polymer chains are aligned and can not slide past each other any more as explained in Section 2.2.5. This is contrary to the original model where the elastic strain only occurs in the initial stage of the stress-strain regime.

This model is not visco-elastic. The two Maxwell-elements have been removed, and what remaining is a single spring with a Young's modulus and a Poisson's ratio. The reason for including this model is to observe how the plastic part of the model is behaving without the visco-elastic part. The model is also, like the original model presented in Chapter 3, implemented in LS-DYNA, but this has not been taken advantage of in this study other than simple confirmation tests performed to validate the response of the Matlab model.

7.1.2 Visco-elastic visco-plastic model with part B as backstress

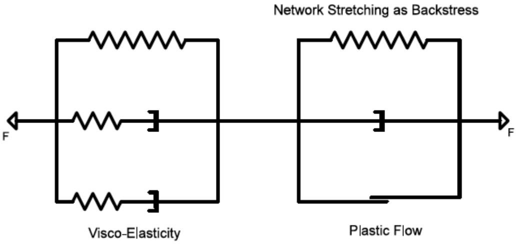


Figure 7.2: Visco-elastic visco-plastic material model with network stretch as backstress

The only difference between this model and the elastic visco-plastic model is the reintroduction of the visco-elastic part. A rheological model is seen in Figure 7.2.

Compared to the original visco-elastic model, this material model might give a better representation of the material behavior. As mentioned in Section 6.2.4 the lack of elastic strain rate far into the plastic regime prevents rapid relaxation processes to take place at this stage. The locking of plastic strain and reintroduction of elastic strain rate may thereby increase the magnitude of the relaxation response in the rapid Maxwell-element. This again may yield a better representation of the material.

A visco-elastic visco-plastic material model with the part B stress in parallel only with the plastic strain has not yet been implemented in LS-DYNA.

7.1.3 Matlab coding

The two new material models have been implemented in Matlab and are as the original model based on a uni-axial tension case. The new altered part B stress, from now on referred to as the back-stress, has to be implemented in both new models. In terms of the visco-elasticity this has simply been removed from the elastic visco-plastic model by clearing the visco-elastic stress contributions from the script. In order to increase the stability of the iteration procedures an interpolation script has been implemented. This adds an extra strain history point in between every two original strain history points through simple linear interpolation. For full Matlab scripts see Appendix C.

Back-stress

The back-stress χ has to become part of the trial state, and is therefore always assumed to be similar to the back-stress of the previous step.

$$\chi_{trial} = \chi_n \tag{7.1}$$

In order to get the back-stress as a function of only the plastic strain it has to be implemented in the yield criterion and be part of the plastic strain iteration procedure. To do so first a new yield criterion has been established by introducing the back-stress χ in Equation (5.20).

$$f = |\sigma_{trial} - \chi_{trial}| - (\sigma_T + R_{trial}) = 0 \tag{7.2}$$

For the back-stress to take part of the plastic strain iteration it has to be, like the rest of the contributing stress parts, rewritten as a function of plastic strain magnitude as explained in Section 5.1.3.

The longitudinal plastic strain used in the back stress update in the iteration procedure is therefore written as

$$\varepsilon_{p,n+1}^{k+1} = \varepsilon_{p,n} + dp_{n+1}^{k+1} sgn(\sigma_{trial} - \chi_{trial}) \tag{7.3}$$

Here n+1 refers to strain step while k+1 refers to iteration step. As mentioned in Section 5.1.4 the part B spring is dependent on the strains in the longitudinal direction as well as the two transverse directions. Assuming a constant retraction ratio and isotropic transverse strains the plastic strain in the transverse directions yields

$$\varepsilon_{p,t,n+1} = R\varepsilon_{p,l,n+1} \tag{7.4}$$

R is found rewriting Equation (5.58) and given as

$$R = \frac{2-\beta}{1+\beta} \tag{7.5}$$

This implementation introduces the dilatation parameter β that has to be calibrated.

The plastic strain rate will also be dependent on the back-stress. By introducing χ in Equation (5.27) yields

$$\Phi_{n+1} = \dot{\varepsilon_0}[e^{\frac{1}{C}\left(\frac{|\sigma_{trial}-\chi_{trial}|}{\sigma_T+R_{trial}}-1\right)} - 1] \tag{7.6}$$

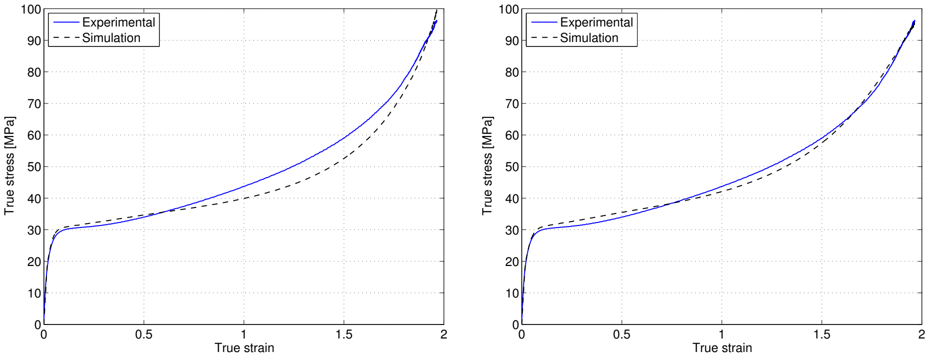
The plastic strain contribution does not only rely on the sign of σ_{trial} as in Equation (5.34) but the difference between σ_{trial} and χ_{trial}. The plastic strain contribution of step n+1 is written as in Equation (7.7) when the iteration procedure has been completed.

$$d\varepsilon_{p,n+1} = dp_{n+1} sgn(\sigma_{trial} - \chi_{trial}) \tag{7.7}$$

Apart from this the iteration procedure is kept similar to the iteration procedure in the original Matlab model.

7.1.4 Calibration

The new material models are calibrated in Matlab. Only C_R and $\bar{\lambda}_L$ has been altered in the calibration apart from the Young's modulus for the elastic visco-plastic model which also was kept as a free variable. Calibrated results for the new elastic visco-plastic and visco-elastic visco-plastic Matlab models are seen in Figure 7.3(a).



(a) (b)

Figure 7.3: Elastic visco-plastic calibration (a) and visco-elastic visco-plastic calibration (b)

Calibrated parameters for the two new models are seen in Table 7.1.

Material model	**Elastic visco-plastic with backstress**	**Visco-elastic visco-plastic with backstress**
$\mathbf{E}_0 [MPa]$	1139.7	520.2
$\mathbf{E}_1 [MPa]$		226.4
$\boldsymbol{\eta}_1 [MPa \cdot s]$		4023.4
$\mathbf{E}_2 [MPa]$		226.4
$\boldsymbol{\eta}_2 [MPa \cdot s]$		56980
$\boldsymbol{\sigma}_T [MPa]$	16	16
$\boldsymbol{\sigma}_S [MPa]$	24.95	24.95
$\dot{\boldsymbol{\epsilon}}_{0,A} [1/s]$	$0.7 \cdot 10^{-3}$	$0.7 \cdot 10^{-3}$
C	0.794	0.794
H	66.94	66.94
$\mathbf{C}_R$	1.788	2.448
$\bar{\boldsymbol{\lambda}}_L$	4.599	4.972
$\boldsymbol{\beta}$	1.07	1.07

Table 7.1: Calibrated parameters for new Matlab models

7.2 Matlab simulations

In this section quantitative simulations have been carried out to test the limitations and possibilities of the two new material models. The main purpose of the simulations in this Section is to see if the introduction of part B as a back-stress yields advantages or disadvantages concerning the material behaviour in the plastic domain.

At first some simulations are carried out with the elastic visco-plastic model where it is easier to single out what happens in the plastic part of the model, especially concerning viscosity. Then some more simulations are performed with the new visco-elastic visco-plastic model to study the interaction between the elastic and plastic part of the model, and the influence of the new back-stress. In the end some new improvements have been made to the visco-elastic visco-plastic model regarding both calibration method and the visco elastic part of the model.

7.2.1 Elastic visco-plastic simulations

Plasticity

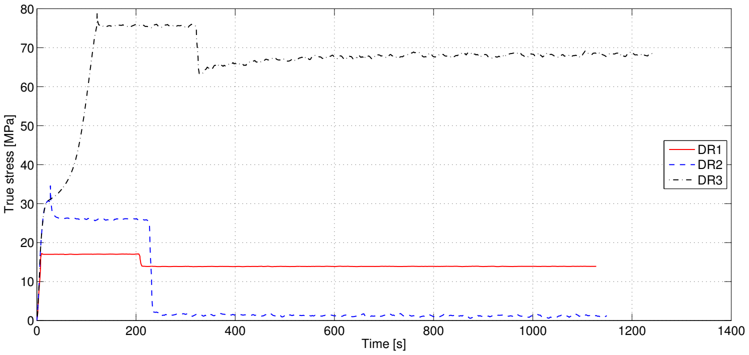
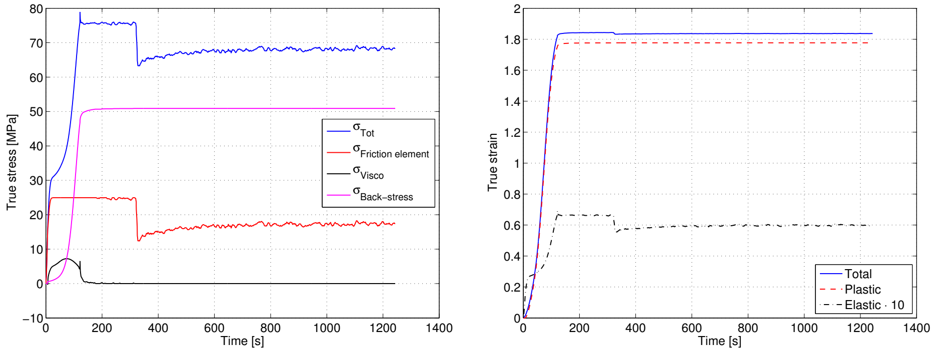


Figure 7.4: Comparison of HDPE DR simulations with elastic visco-plastic model

Even though no visco-elastic part is implemented in the model a certain relaxation still takes place as seen in Figure 7.4 for the HDPE-DR simulations. The relaxation is largest for the DR2 simulation but still present in both the DR1 and DR3 simulations. By separating the stress in the different parts of the model as is seen for the DR3 simulation in Figure 7.5(a), it becomes clear that this relaxation happens due to a relaxation of the dash-pot in the plastic part of the model. The stress in this dash-pot is controlled by the strain rate in the plastic part of the model, and goes to zero as the deformation stops and the plastic strain rate drops to zero.

Creep in experimental test



(a)

(b)

Figure 7.5: Different stress contribution (a) and different strain contributions (b) of HDPE-DR3 simulation

Comparing the material model to the simple Kelvin-Voigt model described in Section 2.1.2 it becomes evident that the model should experience no relaxation unless the dash-pot is in motion. It is therefore peculiar that the DR3 simulation seems as though it relaxes after the force relief seen at about 320 seconds. Both the DR1 and DR2 simulation behaves as expected with no relaxation in this second relaxation procedure. From Figure 7.5(a) where the stress is parted up between the dash-pot, friction plates and spring, it is seen that the increase in stress comes from the friction plates. The only possible way stress alterations can come in this part of the model at this stage of the Stress-Strain regime is through alterations of strain in the elastic part of the model.

It is seen in Figure 7.5(b) where the elastic strain is plotted against total strain and plastic strain, that the elastic strain curve follows a similar path as the slight stress increase in Figure 7.5(a). As mentioned in Section 4.6.1 the stress is very strain sensitive in the domain this simulation is carried out in. Even though the elastic strain shown in Figure 7.5(b) is the elastic strain multiplied with a magnitude of ten, this is still enough to give this slight increase in stress observed in the response. In a perfect relaxation test this should however not be possible as the total strain rate and the elastic strain rate would be zero in the elastic regime. It is likely to assume that this slight alteration in strain comes from creeping within the material in the experimental test. Even though the deformation was maintained some creep may possibly have taken place across the specimen length.

Influence of λ_L and C on the plasticity

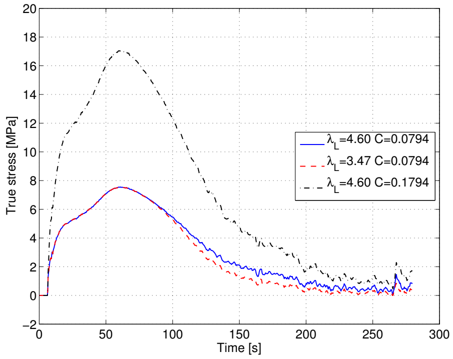


Figure 7.6: σ_{visco} in HDPE-T simulation for different values of λ_L and C

As has been observed in Figure 7.5(a) the relaxation in this elastic visco-plastic model is governed by the strain rate dependent stress in the dash-pot. A simple parameter study has though been conducted regarding how some material parameters influences this dash-pot. In addition to the original parameters given in Table 7.1, simulations have been carried out with a locking stretch of 3.47 corresponding to a locking strain of $\varepsilon_L = 1.85$. Also a slightly higher value for strain rate dependency magnitude parameter C has been tested out. As seen in Figure 7.6 the locking stretch alteration has little impact on the visco-plasticity early in the simulation. However, for large deformations a certain but small decrease in stress is seen. The value change for the C parameter gives a significant increase in stress, proportional to the increase of the parameter value itself. This was expected and shows how this parameter can influence the visco-plasticity in the relaxation of the model.

7.2.2 Visco-elastic visco-plastic simulations

The visco-elastic contribution has a distinct influence on the relaxation processes as can be seen in Figure 7.7 in comparison to Figure 7.4. In the first relaxation process both DR1 and DR2 gets a significant relaxation due to contributions from both the slow Maxwell-element, and the rapid Maxwell-element.

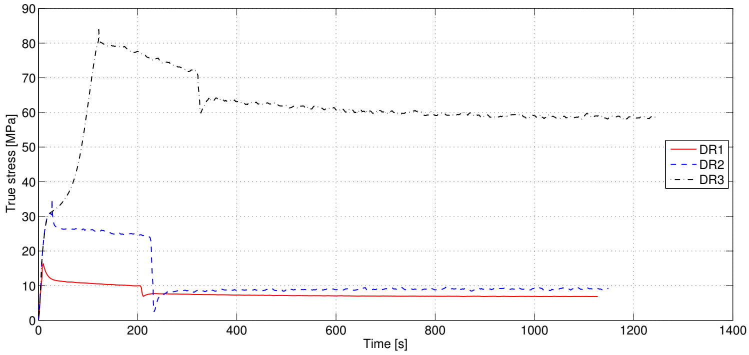
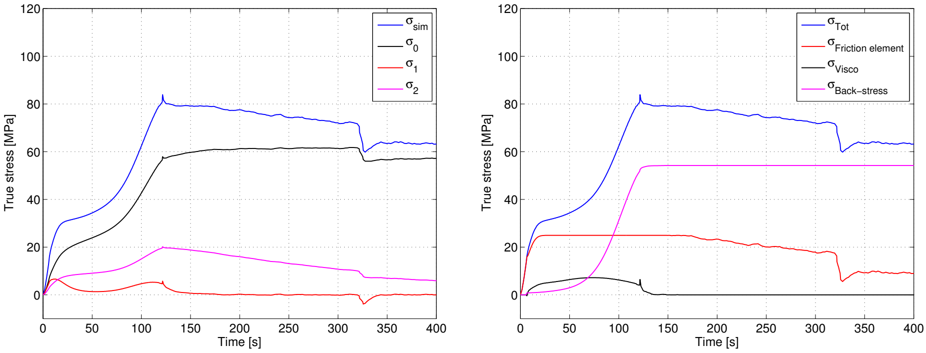


Figure 7.7: Comparison of HDPE DR simulations with visco-elastic visco-plastic model

The DR3 simulation also relaxes in the two Maxwell-elements. However, due to creep as seen in Figure 7.8(a) for σ_0 early in the relaxation process, explained in Section 7.2.1, the stress decrease in the quick Maxwell-element is equalized from the total response. The real experiment also experience creep, but the relaxation in the experimental tests is probably larger compared to the relaxation in the simulations.

Elastic vs Plastic behaviour

In Figure 7.8 the elastic behaviour of the model is compared to the plastic behaviour of the model. In Section 7.2.1 a plastic relaxation was observed for the first relaxation procedures in the DR simulations. An interesting feature in this visco-elastic visco-plastic model is to see how the visco-plasticity interacts with the visco-elasticity. As seen in Figure 7.8(b) the initial stress decrease in the first relaxation process is carried out by the dash-pot in the plastic part of the model. In Figure 7.8(a) it can be seen that the rapid Maxwell-element in the elastic part of the model carries out this same initial stress decrease. However a small part is also carried by the slow Maxwell-element and the elastic spring. Thus the viscosity is both visco-plastic and visco-elastic in this part of the simulation. The initial stress increase in the second relaxation process is still carried out by the rapid Maxwell-element in the elastic part of the model as seen in Figure 7.8(a). However, in the plastic part the stress increase is due to an increase of stress between the friction elements. In other words the second relaxation process is only visco-elastic in the simulation.

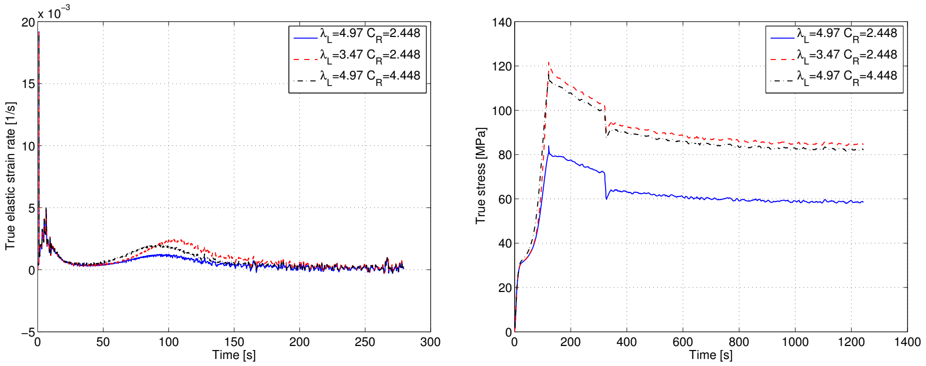


(a) (b)

Figure 7.8: Elastic contributions(a) and plastic contributions (b) in HDPE-DR3 simulation

Locking stretch and initial network stiffness

The idea of placing the part B stress as back-stress instead of a parallel contribution gives the opportunity of letting the elastic strain acquire more of the total strain when the simulation is moving further into the plastic domain. Simulations have been carried out with the intention of figuring out what happens to the elastic strain and elastic contributions in the model when the locking stretch or the initial network stiffness is altered. The results are seen in Figure 7.9.



(a) (b)

Figure 7.9: Elastic strain rate for different $\bar{\lambda}_L$ and C_R combinations on HDPE-T simulations (a) and this influence on stress-strain curves for HDPE-DR3 simuations (b)

With the locking stretch lowered to 3.47 corresponding to a locking strain of about 1.85 the elastic strain rate gets severely increased when the total strain is surpassing the locking strain. By letting elasticity take over the strain as seen in Figure 7.10, the stiffness of the model increases a lot. This is shown in Figure 7.9(b). Similar effects are seen when increasing the initial stiffness. However, here the elastic strain rate is increased for the entire plastic domain. Not just the part after locking stretch is reached. The relaxation process of the model is highly dependent on the elastic strain rate. The higher the elastic strain rate and further the elastic strain is taken, the larger the magnitudes of the relaxation in the Maxwell-elements becomes.

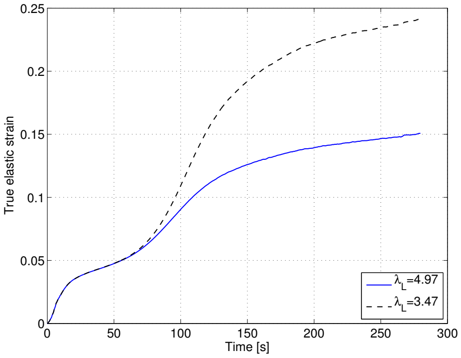


Figure 7.10: Elastic strain contribution with different locking stretches

Calibration of visco-elasticity based on behaviour in the plastic domain

In order to improve the relaxation response in the plastic regime, a visco-elastic non-linear square calibration was carried out for the HDPE-DR3 test. Here all the plastic parameters was kept as in the original model, while the elastic parameters was free to change. The result of this calibration is seen in Figure 7.11. This kind of calibration was not possible for the original material model presented in Chapter 3 due to lack of elastic strain rate for large deformations.

E_0 [MPa]	286.6
E_1 [MPa]	2092
η_1	11535
E_2 [MPa]	151.1
η_2	18014

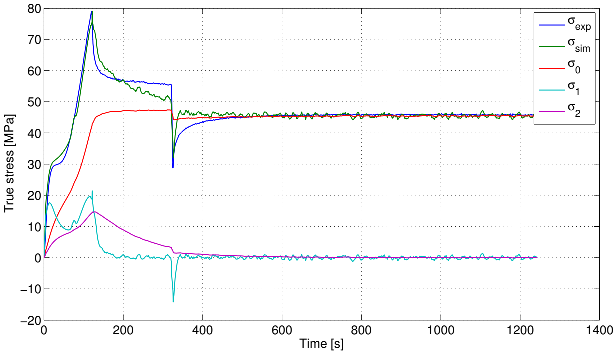
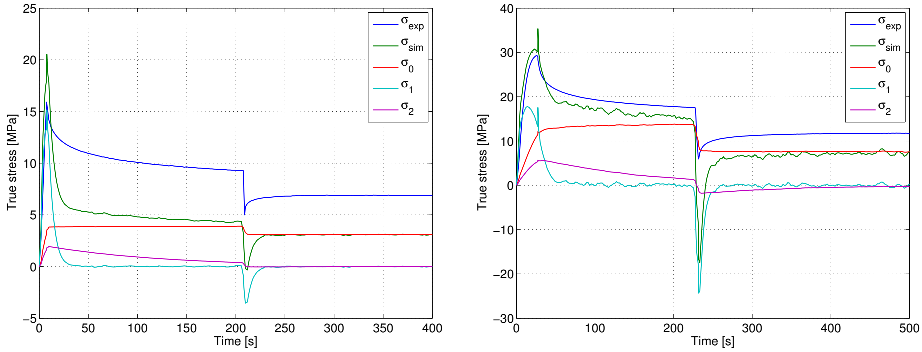


Figure 7.11: Calibrating visco elastic parameters based on the relaxation for the DR 3 test

A more similar response curve was as seen in Figure 7.11 obtained for the HDPE-DR3 test. The simulated relaxation stress decrease is though a bit larger then the experimental stress decrease in the first relaxation process, and a bit too small in the second process. The model is too soft regarding the stress drop, but all in all the calibration is much better than the original calibration. These new calibrated parameters was used to simulate the DR1 and DR2 tests. The results are seen in Figure 7.12. The simulation of the DR2 shows good similarities to the DR2 experimental test but here the second relaxation process has a rather larger stress increase than in the experiment. Also the model acts slightly stiffer than the real specimen. When it comes to the DR1 simulation the second relaxation process is very similar to the experiment. The first relaxation process is though too large, and the model is too soft when it comes to the stress relief.



(a) (b)

Figure 7.12: New visco-elastic parameters for DR1(a) and DR2(b)

E_0 [MPa]	197.6
E_1 [MPa]	664.7
η_1	4203
E_2 [MPa]	91.8
η_2	10954

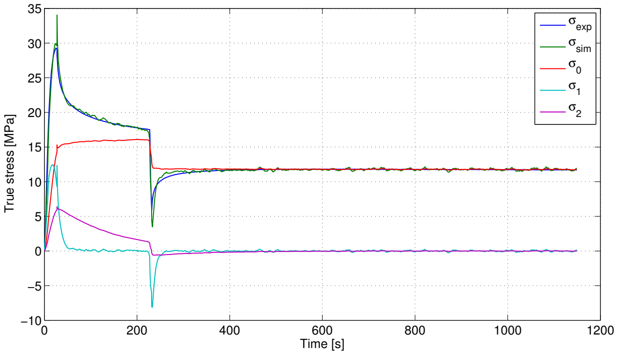
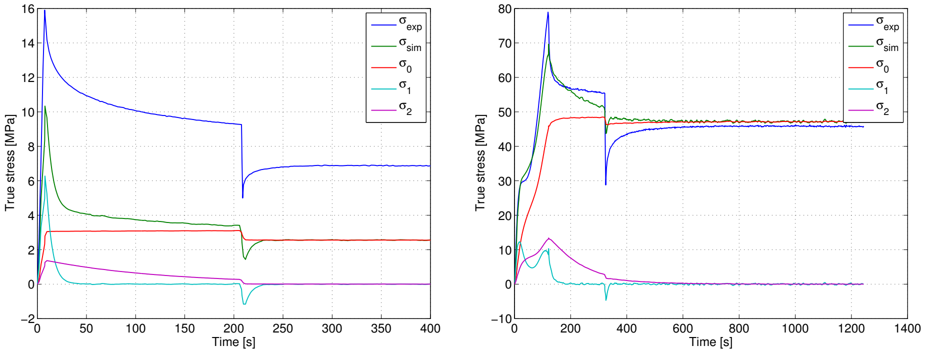


Figure 7.13: Calibrating visco elastic parameters based on the relaxation for the DR 2 test

A new calibration was carried out in the early plastic domain as seen in Figure 7.13. The calibration fits almost perfectly with the experimental results for the DR2 test. However by implementing them into the DR1 an DR3 simulation seen in Figure 7.14 the results was just as bad as for the DR3 calibration.



(a) (b)

Figure 7.14: New visco-elastic from DR2 implemented in DR1(a) and DR3(b) simulations

More Maxwell-elements in the material model

An other way of improving the quality of the material response was to add an extra Maxwell-element to the visco-elastic part of the material model. By doing so a new calibration had to be done regarding the visco-elastic part of the model, similar to the one in Section 5.2.1. The relaxation times in this calibration was

E_0	521.7 MPa
E_1	350.3 MPa
η_1	1050.9 MPa · s
E_2	172.1 MPa
η_2	4302.4 MPa · s
E_3	218.8 MPa
η_3	52512 MPa · s

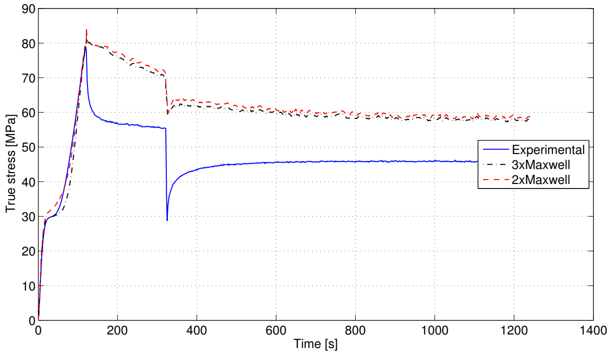


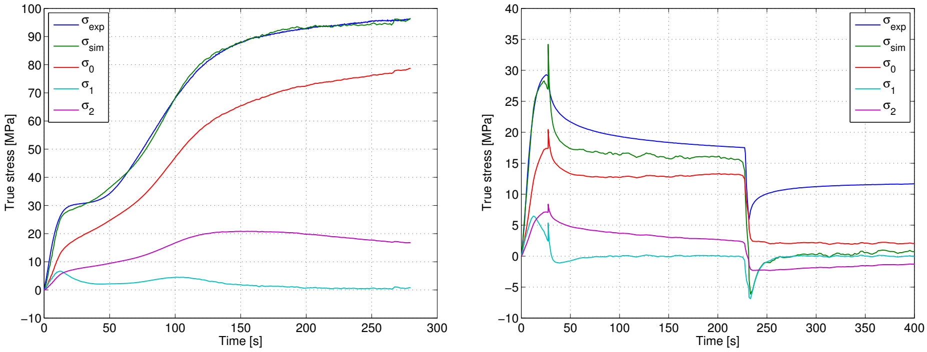
Figure 7.15: Two vs Three Maxwell-elements in model

predetermined and chosen to be 3 , 25 and 240 seconds. This was done in order to keep the parameters from producing equal relaxation times which proved to be a problem when they were free. Otherwise the material parameters were kept as in the first calibration.

Introducing more Maxwell-elements as seen in Figure 7.15 gave a smoother response. The relaxation process remained similar to the model with two Maxwell-elements however the initial part of the plastic stress behaves more like the experimental test.

Model stiffness in different domains

As mentioned in Section 6.2.4 the model stiffness has not been accurately calibrated for each domain compared to the experimental results. In the C2 and DR2 simulations a contraction in strain was observed during the stress relief which looked as though the specimen was yielding. Therefore a calibration by changing the yield criteria parameters was carried out. By setting down the onset of yield, plastic strain should take place earlier in the simulation. The idea was that the model then also would yield earlier for the stress relief and follow the stress strain curve of the HDPE C2 and HDPE DR2 experimental results. The yield in tension parameter was forced down to a value of 5MPa and the saturated yielding stress was set to 15MPa while the rest of the plastic parameters were free during the calibration. A decent calibration was then obtained for the HDPE-T as seen in Figure 7.16(a) However, no significant change was observed for the stiffness of the model in the different stress-strain domains. An example is seen for the HDPE-DR2 simulation in Figure 7.16(b) The model was still too stiff in the start of the plastic domain and too soft further into the plastic domain. The visco-plastic contribution did however increase severely which gave a much larger first relaxation stress decrease even though the visco-elastic parameters was kept unaltered. The spike at the onset of relaxation after about 30 seconds may be due to the linear regression applied for the strain history explained in Section 4.3.3 or instability in the Matlab program.



(a)

(b)

Figure 7.16: Calibration with low onset of yield(a) and simulation of DR2 test with new parameters(b)

7.3 Discussion

The implemented plasticity plays an important role in the material models. By highlighting the plastic behaviour which was done in the elastic visco-plastic model it became obvious that a certain visco-plastic behaviour is present at the initial part of the relaxation. However, the model must yield for this visco-plastic relaxation to be present. In yielding due to a meniscus instability growth mechanism a polymer deforms continuously at a constant level of force as explained in Section 2.2.5. So, if the deformation is kept constant, the force will have to drop in order to stop this meniscus instability growth mechanism. This must happen almost immediately, something that is a behaviour well represented by the dash-pot in the plastic part of the model.

The elastic and plastic part of the visco-elastic visco-plastic material model deals with relaxation in different manners. The relaxation that happens while the material is in yield is taken care of by the dash-pot in the plastic part. However, if the material is not in yield the relaxation happens as a response to the relaxation in the elastic part and takes place in the friction elements. For the elastic part most of the relaxation takes place in the Maxwell-elements. A small but certain relaxation response in the elastic spring is though seen in Figure 7.8(a) due to the visco-plastic relaxation. In Figure 7.16(b) it is also seen that the total relaxation is increased significantly by increasing the visco-plastic contribution in the material model. By observing the second relaxation procedure in the DR simulations, where no visco-plasticity takes place still a certain stress relaxation takes place in both parts of the model. Based on this there is good reason to think that the visco-elasticity and visco-plasticity in the visco-elastic visco-plastic material model happens on different terms and maintain their contribution despite what happens in the other part. Hence, they do not offset each other.

Redefining the part B stress as a back-stress does as expected make the elastic part of the model acquire more of the total strain. This again leads to a model with more elastic strain and a higher elastic strain rate in the plastic domain in the HDPE-T simulation. The higher elastic strain rate gave larger relaxation response in the Maxwell-elements for the tests taken far into the plastic domain. However, the difference was not significant for reasonable values of $\bar{\lambda}_L$. In order for the rest of the material response to still stay tuned in the calibration the locking strain could not be altered too much. It is seen that if the $\bar{\lambda}_L$ is taken down to 3.47 in Figure 7.9(b) a significant impact is given to the stress response. Through calibration procedures where the locking stretch was predetermined and the rest of the parameters were kept free, the lowest value to produce a decent calibration was approximately 4 for the $\bar{\lambda}_L$. Decreasing the $\bar{\lambda}_L$ value beyond this point gave a bad representation of the material behaviour.

Through findings in Section 7.2.2 there is reason to think that more Maxwell-elements improves the response in terms of smoothness. However as mentioned in Section 6.2.4 these extra Maxwell-elements does not have much impact on the magnitude of the relaxations. The same stiffness contribution has to be divided between more elements in order for the model not to become too stiff in the elastic domain.

Calibration of the visco-elastic parameters in each of the plastic domains tested for improved the response concerning relaxation and general behaviour in this exact domain. The response is neither accurate for the simulation in the elastic regime or the simulation in the other part of the plastic regime. The simulations in the plastic domain was though more similar to each other than to the simulation in the elastic domain. It has been shown in Section 7.2.1 that the strain dependency magnitude parameter C also has a great influence on the relaxation in yield so this should probably be taken into concern when calibrating the visco-plastic parameters based on the plastic domain. However, based on findings above and in Chapter 6 it becomes hard to imagine that the visco-elastic part can be simulated by simple linear Maxwell-elements in the material model. There is reason to think that visco-elasticity is of a more non-linear character. However, keeping part B as back-stress seems favourable to the visco-elastic behaviour.

When it comes to the stiffness of the model in the different parts of the domain no good solution was found through the visco-plastic calibration of the C2 test DR2 test or the alterations made to the yield parameters. The results given through experimental tests are not always one hundred percent trustworthy. For instance some creep may have taken place over the specimen length as mentioned in Section

7.2.1. A set of nine experimental tests has been examined in this study and a larger set should certainly be examined to confirm the behaviour seen for these experimental tests.

The Matlab models used in this chapter lack many of the material features given in a 3-D simulation like for example by use of LS-DYNA. They are however good at distinguishing each parameter and each part of the model contributions from the total response and great at performing calibrations for those parameters that are implemented. Also, where a LS-DYNA simulation may take hours, each test in Matlab only takes seconds.

Simulating the Creep tests by use of the Matlab program is difficult. As mentioned in Section 6.1 the drop in force has to be simulated by use of force control. This however, is not possible with the Matlab program, which is only based on strain-time history as input. In order to improve the creep simulation results LS-DYNA should be used and the simulations should be carried out like those in Section 6.2.2.

8 Conclusion

The objective of this study was to calibrate and thereafter evaluate the new visco-elastic material model developed at SIMLab, NTNU, by use of the program software LS-DYNA. By examining the behaviour new improvements have been made to the material model and a set of quantitative simulations have been performed by use of a uni-axial stress program implemented in Matlab as a part of this thesis.

The visco-elastic visco-plastic material model was found incapable of dealing with the large rapid stress relaxation which was observed far out in the plastic domain. This allegation is based on the lack of elastic strain rate in the plastic domain for the material model. The cause of this lack of elastic strain rate was that no prestressed rapid Maxwell-elements were left to deal with the relaxation at this stage. An increase of the plastic magnitude parameter C increased the visco-plastic response. However, the change was not significant for values of C that still kept the rest of the material behaviour reasonable.

A problem with all the material models presented in this study was that for simulations based on calibrations in the elastic domain, the unloading was found too stiff in the early plastic domain and too soft for large deformations. Different calibration techniques have been carried out without a significant improvement to the behaviour in this manner.

Where the first presented visco-elastic visco-plastic model failed to simulate realistic relaxation behaviour for large deformations, the new developed visco-elastic visco-plastic model with the part B stress introduced as a function of plastic strain succeeded in doing so. Adapting the elastic stiffness to the relaxation behaviour in the plastic domain, which was done to give reasonable relaxation response for large deformations, led to a too stiff model in the elastic domain.

Introducing more Maxwell elements gave a smoother response for all material models, but did not help regarding the magnitude of the relaxations. In order to still be able to represent the behaviour in the elastic domain, the total stiffness contribution can not be increased. In other words, more Maxwell-elements yields the same total contribution, just divided between more elements.

Calibration attempts was also carried out by reducing the locking stretch so that elastic strain would initiate at an earlier stage. This should lead to a stiffer model in the elastic domain. However, no good calibration results were obtained. This does though seem to be the right way to move further in order to improve this material model where the visco-elasticity is described by linear Maxwell-elements.

Suggestions for further work

The introduction of the part B stress as function of plastic strain should be implemented in LS-DYNA and examined further

A more comprehensive database must be conducted for visco-elastic response.

A non-linear visco-elastic material response beyond the introduction of more Maxwell-elements could be looked further into.

Implementing an α parameter in the Matlab script should make it possible for the program to handle compression as well as tension

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Appendices

A Experimental results

In Appendix A some results from the different tests are shown. In the table, key information about the geometry and cross-head velocity is given. In the figures, results concerning force, stress, strain, strain rate, displacement and time are illustrated in graphs.

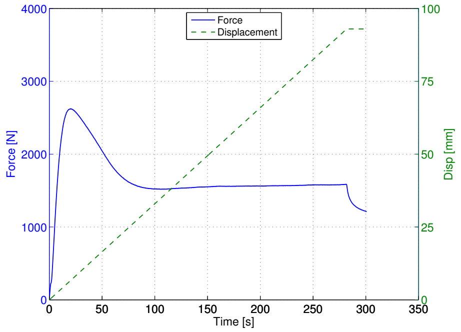
The letters and numbers used in the headline describes what kind of test, and how far into the elastic and plastic domain it was initially taken. A short description is given in the table below.

T	tension test
R	simple relaxation test
C	creep test
DR	double relaxation test
1	only the elastic domain
2	the first part of the plastic domain
3	further into the plastic domain

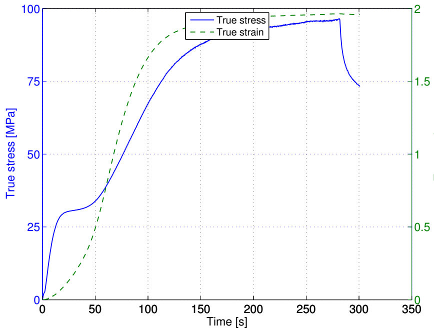
A description of the key information regarding geometry is given in the table below

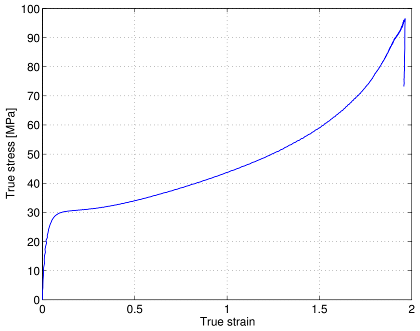
b0	initial width
t0	initial thickness
A0	initial area
b	width after deformation
t	thickness after deformation
A	area after deformation

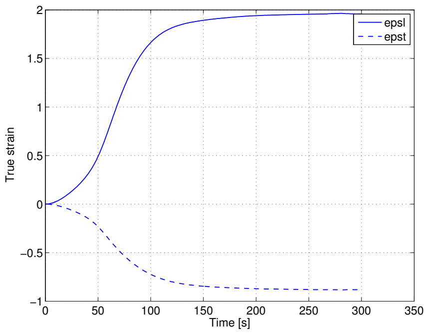
HDPE-T

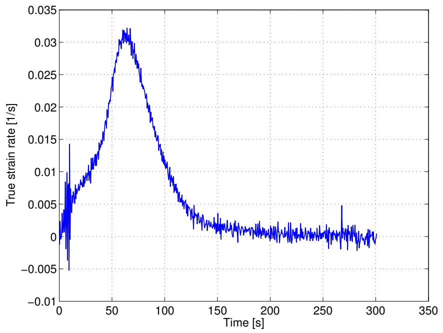


Specimen Dimensions	
b0	$11.94 mm$
t0	$8.05 mm$
A0	$96.12 mm^2$
b	$5.25 mm$
t	$3.37 mm$
A	$17.69 mm^2$
Crosshead Velocity	$0.01 mm/s$

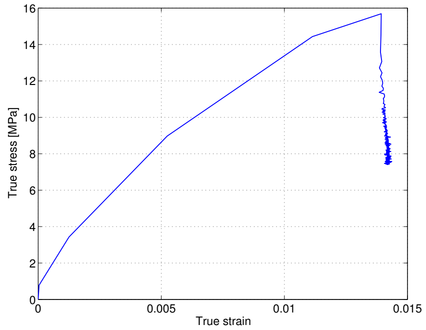








HDPE-R-1



Specimen Dimensions	
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A0	$96.12mm^2$
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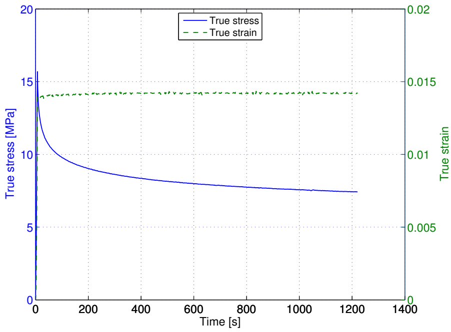
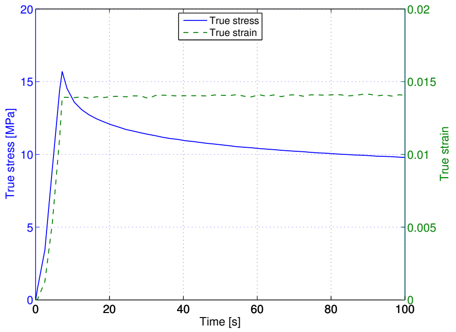
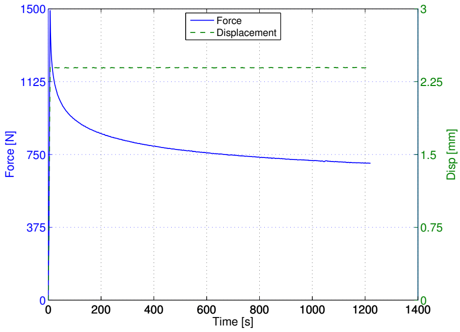
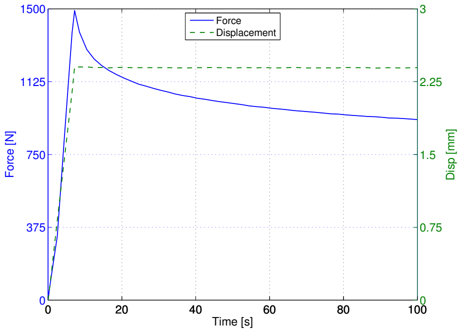
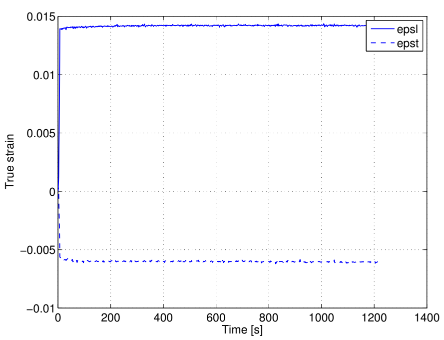
t	$8.08mm$
A	$96.48mm^2$
Crosshead Velocity	$0.01mm/s$

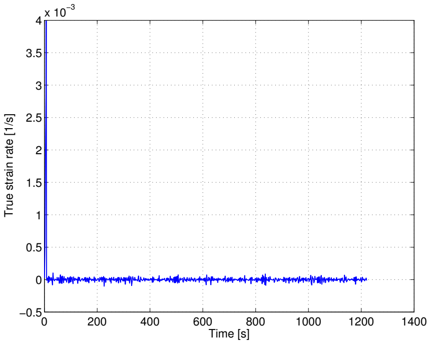




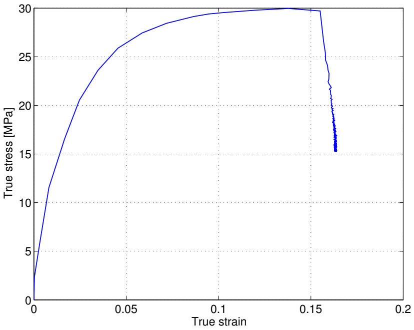








HDPE-R-2



Specimen Dimensions	
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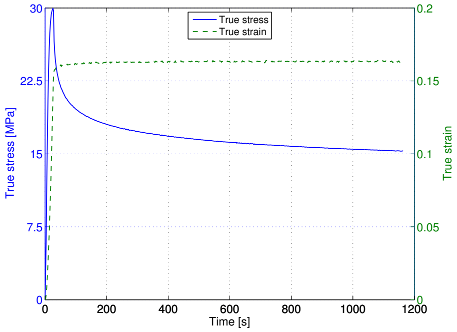
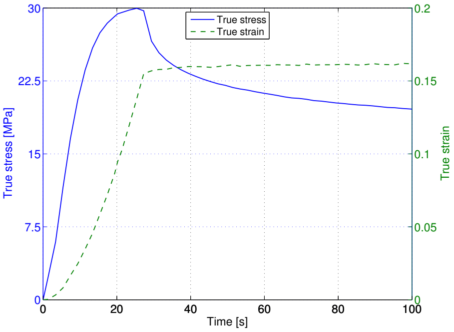
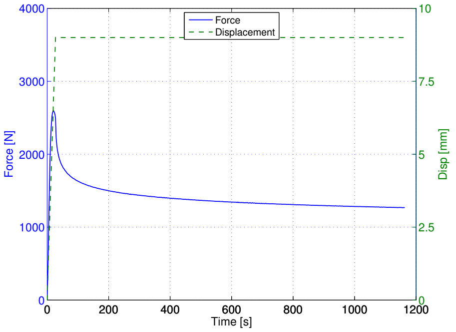
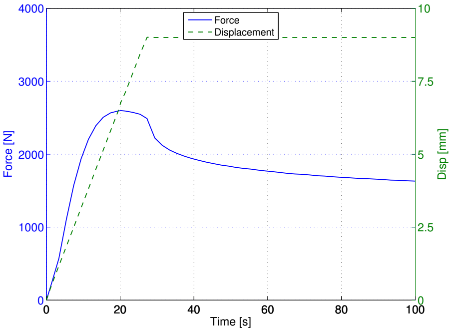
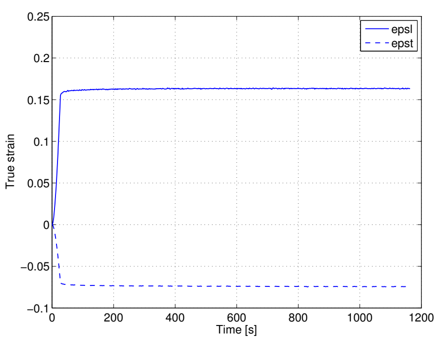
A0	$96.24mm^2$
b	$11.74mm$
t	$7.80mm$
A	$91.57mm^2$
Crosshead Velocity	$0.01mm/s$

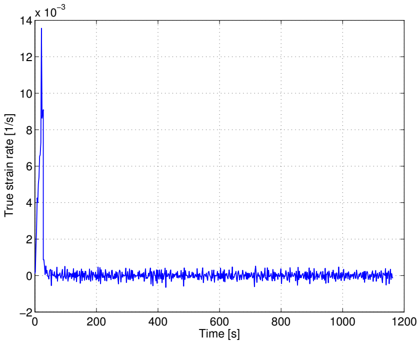




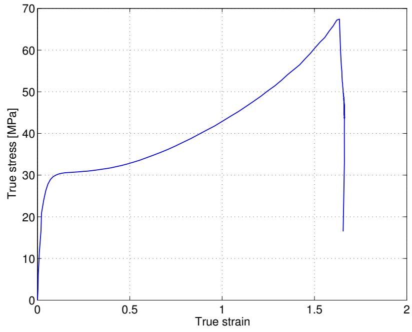








HDPE-R-3



Specimen Dimensions	
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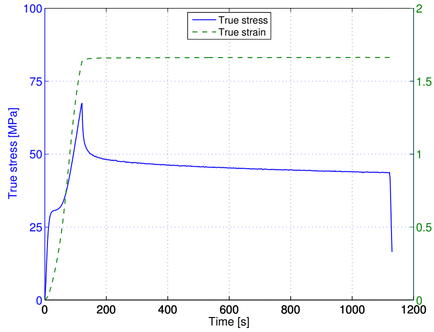
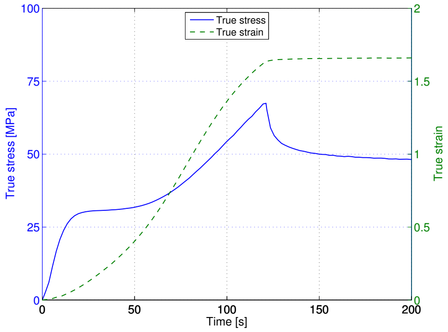
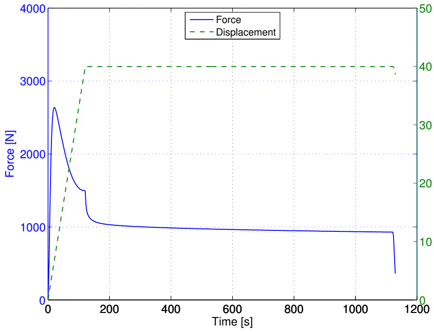
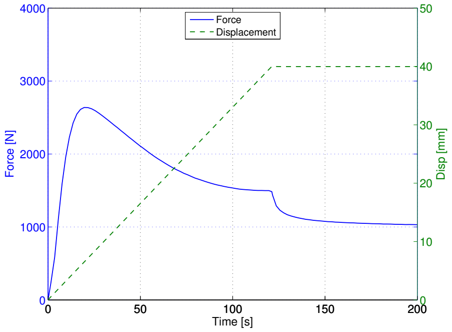
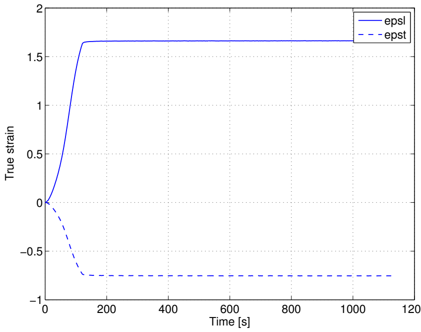
A0	96.12mm^2
b	6.23mm
t	4.13mm
A	25.73mm^2
Crosshead Velocity	0.01mm/s

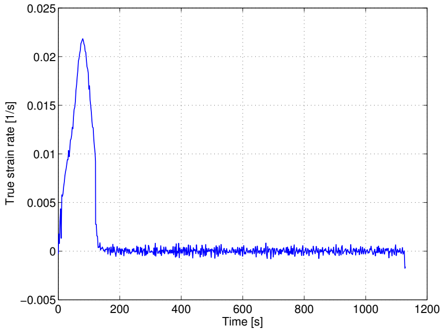






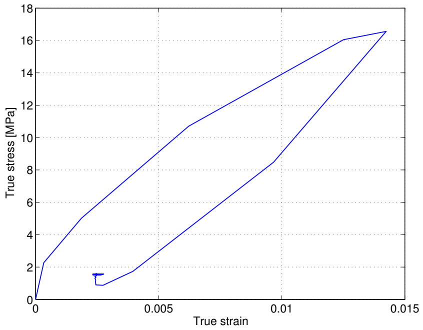


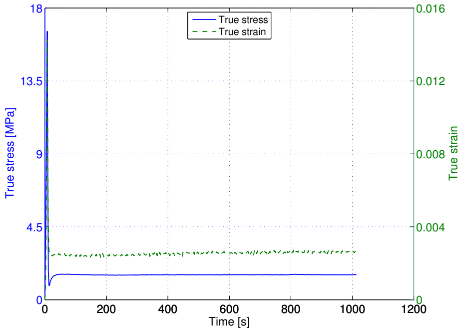


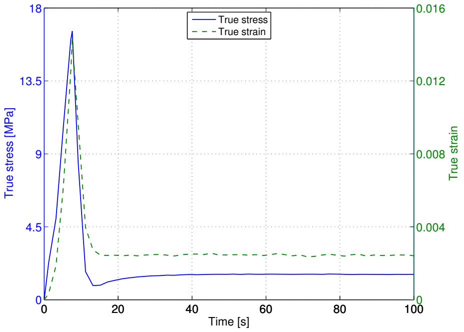


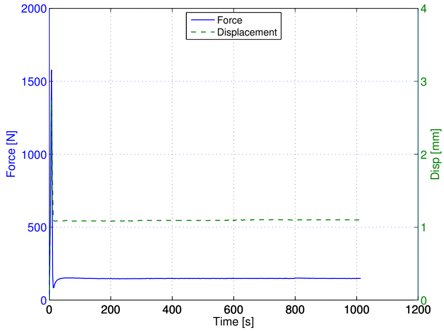
HDPE-C-1

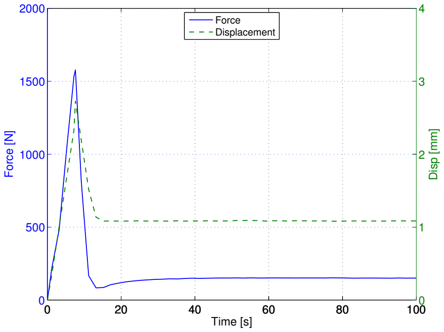
Specimen Dimensions	
b0	11.98mm
t0	8.05mm
A0	96.44mm^2
b	11.94mm
t	8.08mm
A	96.48mm^2
Crosshead Velocity	0.01mm/s

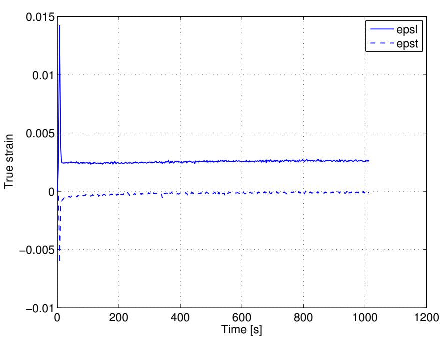


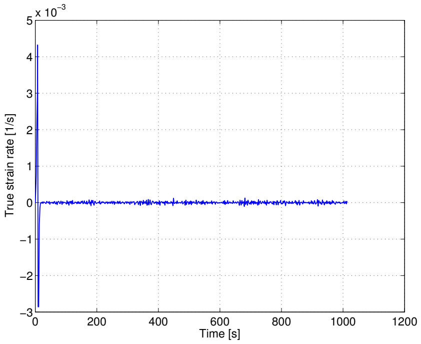




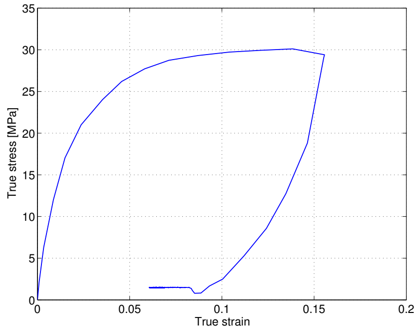




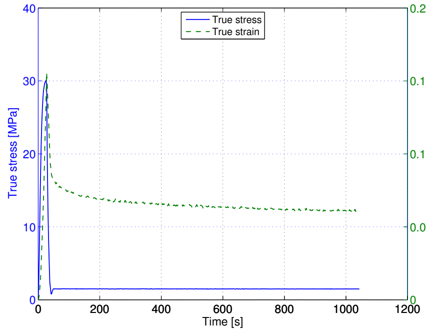


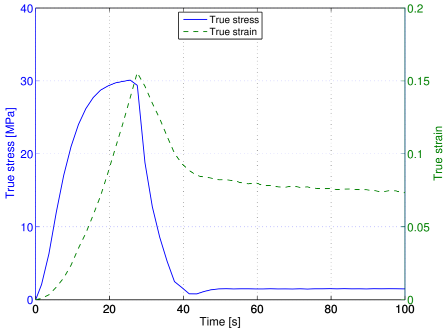


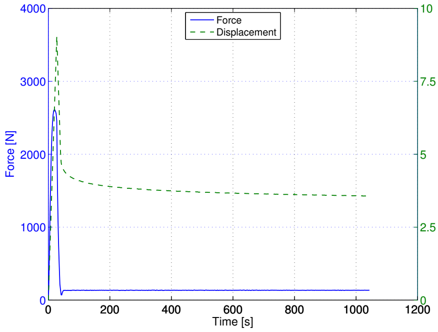
HDPE-C-2

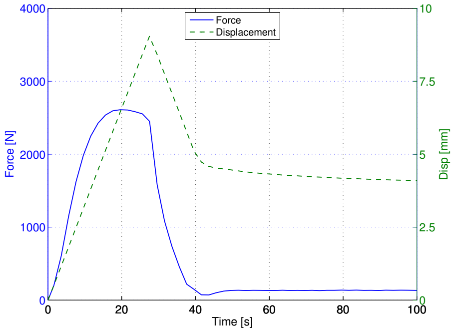


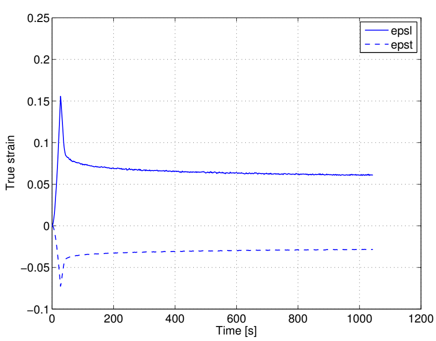
Specimen Dimensions	
b0	11.98mm
t0	8.05mm
A0	96.44mm^2
b	11.82mm
t	7.97mm
A	94.21mm^2
Crosshead Velocity	0.01mm/s

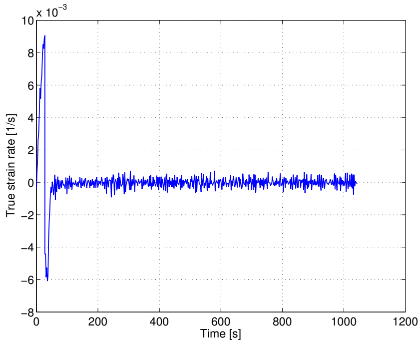




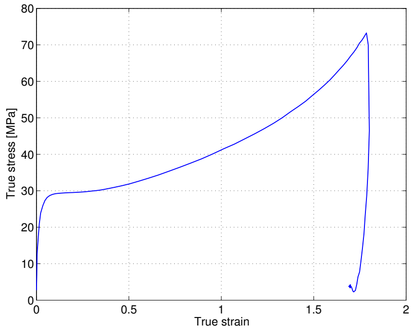




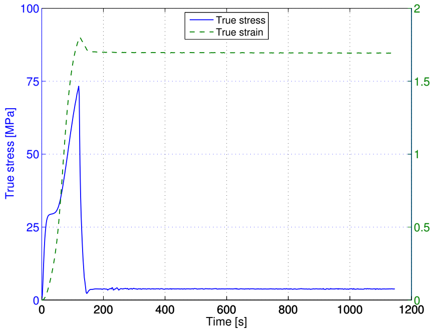


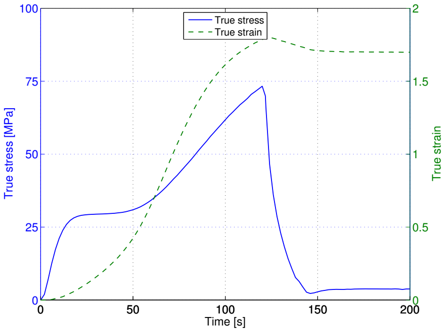


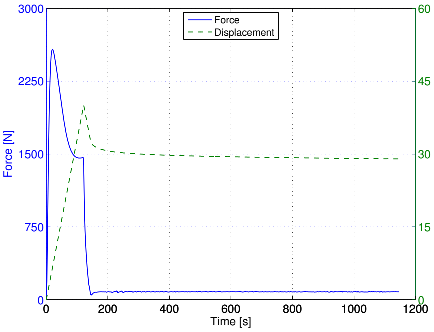
HDPE-C-3

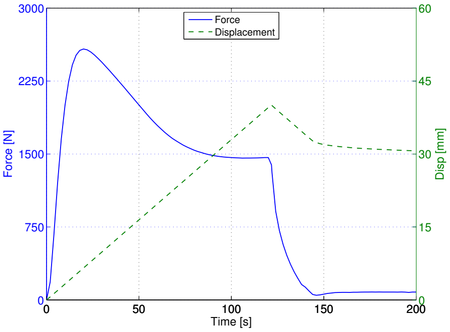


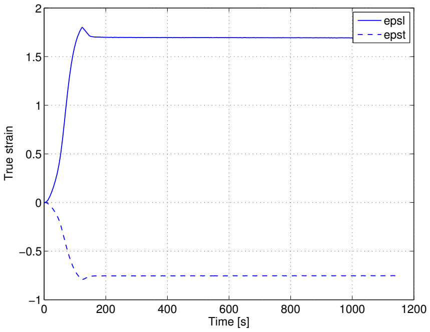
Specimen Dimensions	
b0	11.95mm
t0	8.04mm
A0	96.08mm^2
b	5.83mm
t	3.79mm
A	22.10mm^2
Crosshead Velocity	0.01mm/s

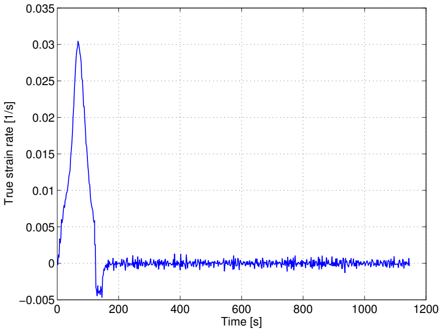




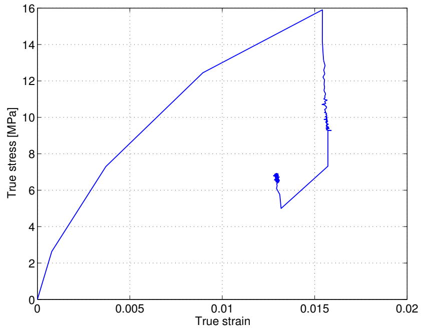




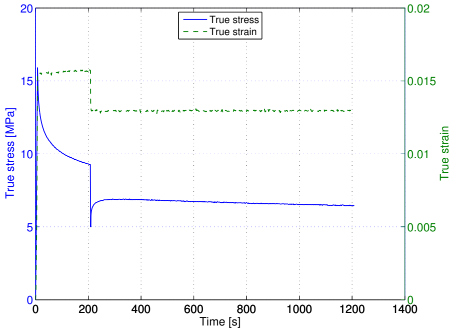


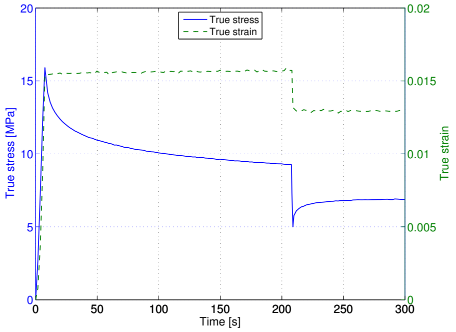


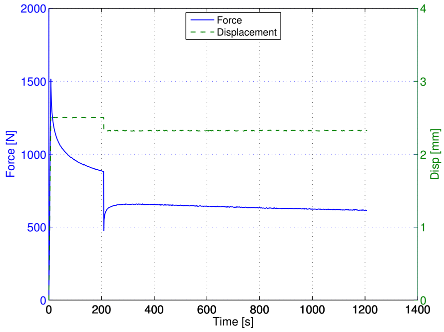
HDPE-DR-1

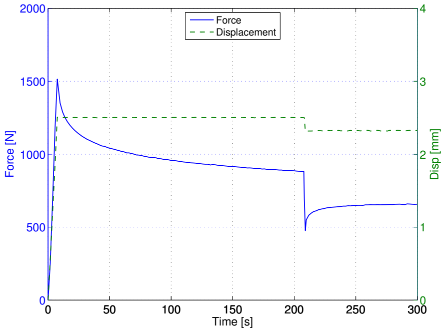


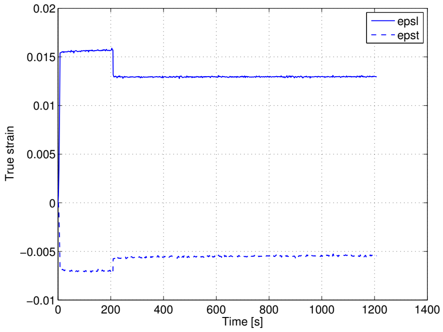
Specimen Dimensions	
b0	11.99mm
t0	8.05mm
A0	96.52mm^2
b	11.94mm
t	8.09mm
A	96.59mm^2
Crosshead Velocity	0.01mm/s

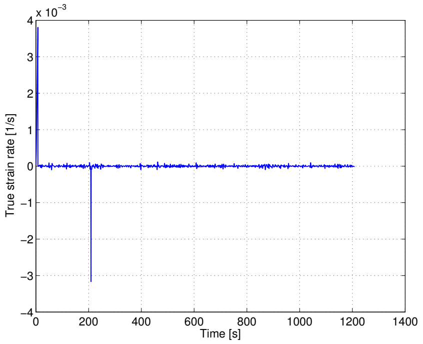




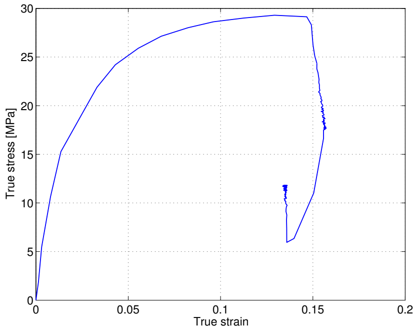




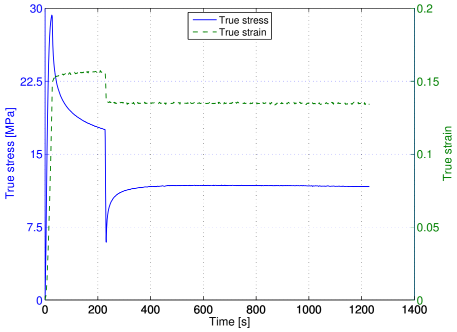


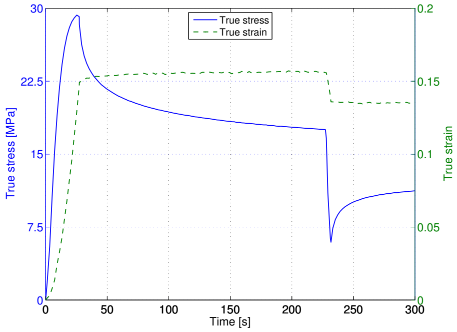


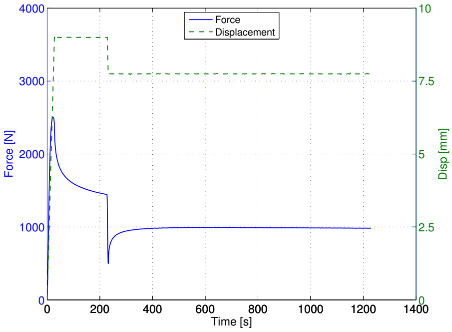
HDPE-DR-2

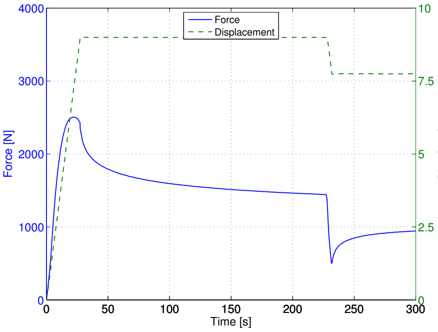


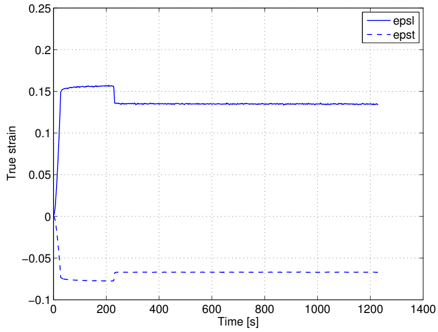
Specimen Dimensions	
b0	$11.95mm$
t0	$8.05mm$
A0	$96.20mm^2$
b	$11.78mm$
t	$7.97mm$
A	$93.89mm^2$
Crosshead Velocity	$0.01mm/s$

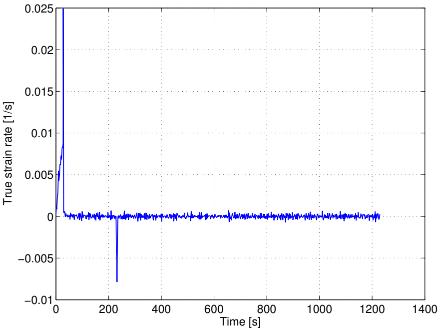




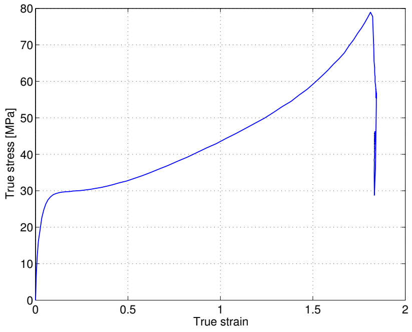




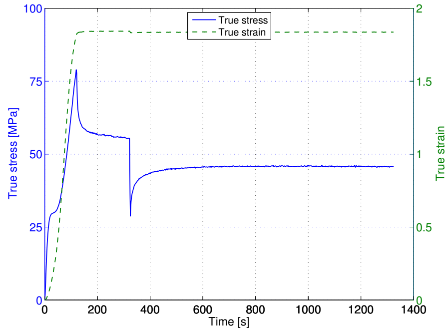


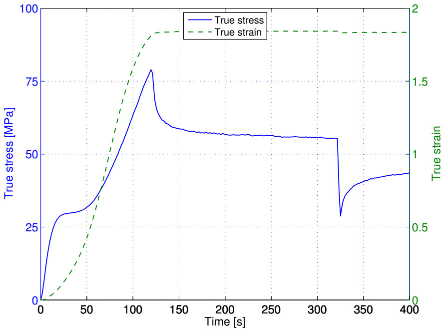


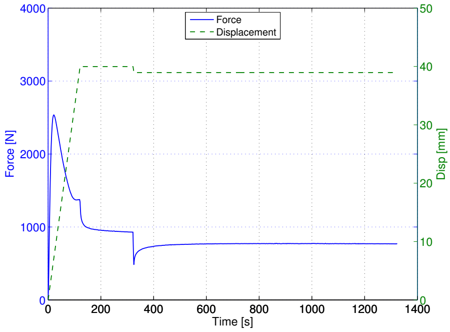
HDPE-DR-3

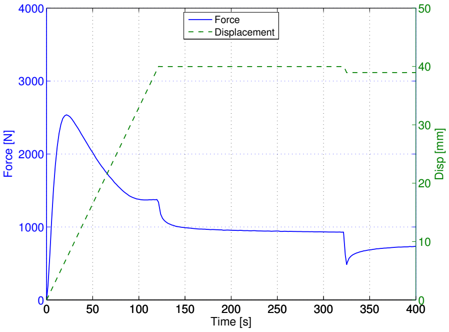


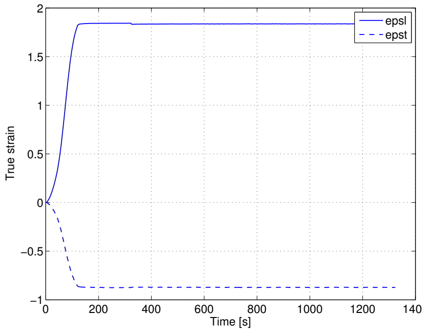
Specimen Dimensions	
b0	11.99mm
t0	8.04mm
A0	96.40mm^2
b	5.30mm
t	3.64mm
A	19.29mm^2
Crosshead Velocity	0.01mm/s

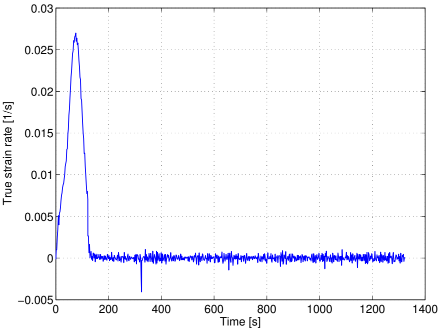












B Simulation results

In Appendix B some results from the different simulations in LS-DYNA is shown in comparison with the experimental results .

In order to simulate the experimental test, some choices have been made regarding force, time and displacement.

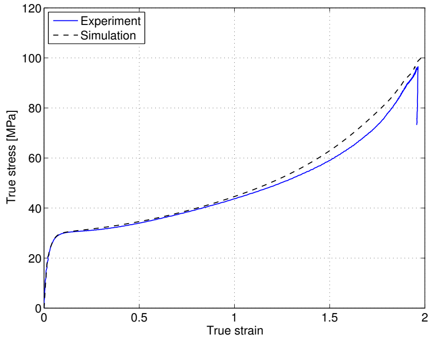
The relaxation and double relaxation simulations are performed by keeping strain and time similar to the strain and time in the experimental test

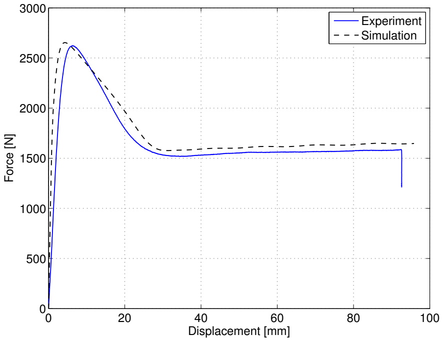
I the creep simulations strain and time is controlling the first part of the simulation. The drop in force is however force and time controlled.

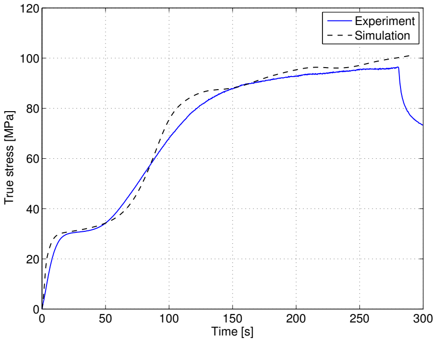
The letters and numbers used in the headline describes what kind of test, and how far into the elastic and plastic domain it was initially taken. A short description is given in the table below.

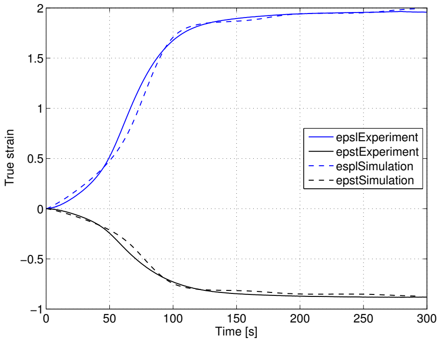
T	tension test
R	simple relaxation test
C	creep test
DR	double relaxation test
1	only the elastic domain
2	the first part of the plastic domain
3	further into the plastic domain

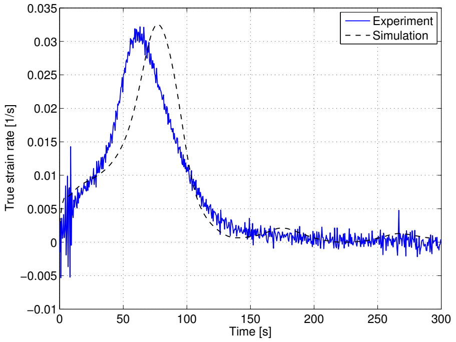
HDPE-T-1416-elements LS-DYNA-Simulation

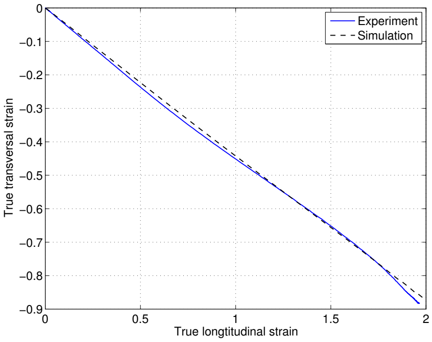




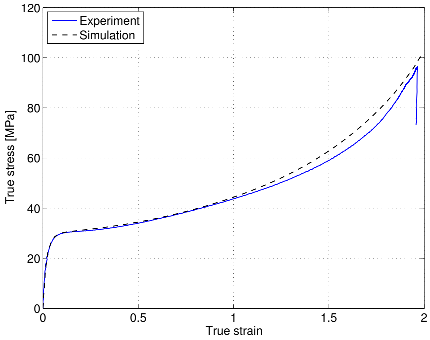


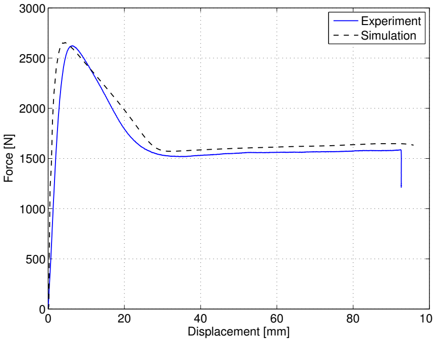


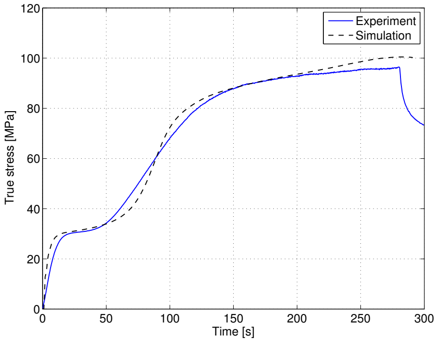


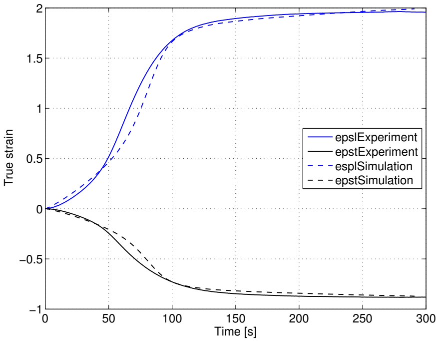


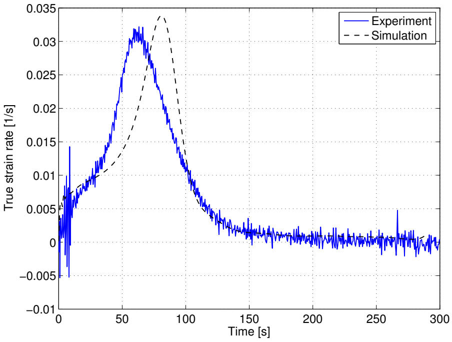
HDPE-T-11328-elements LS-DYNA-Simulation

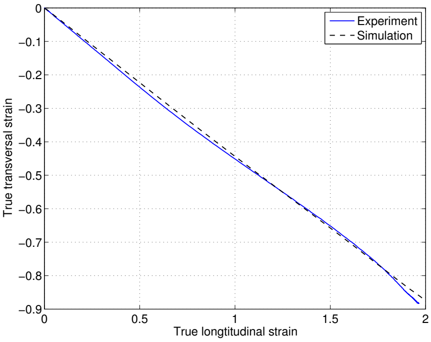






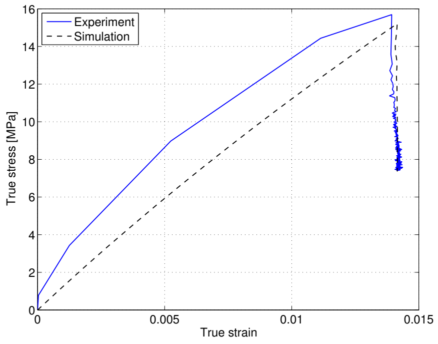


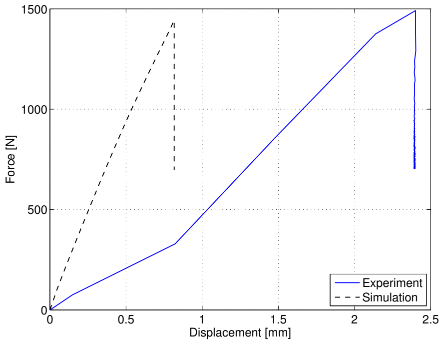


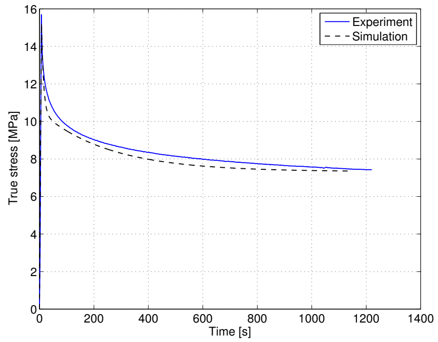


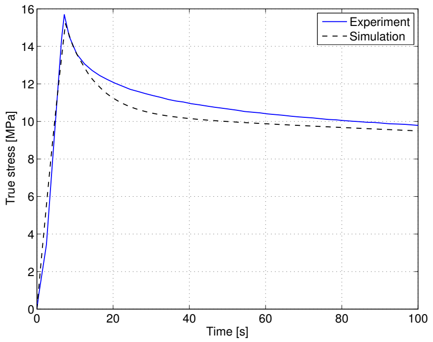
HDPE-R-1

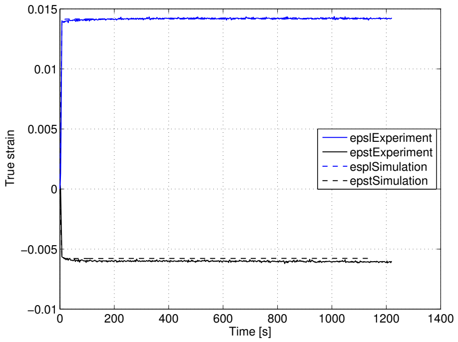
LS-DYNA-Simulation

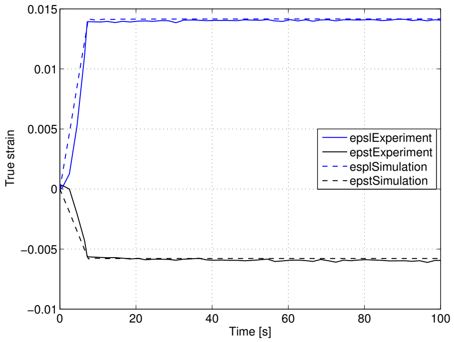


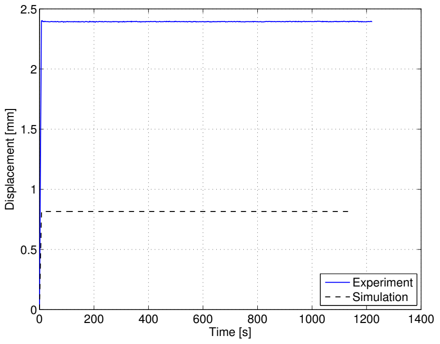


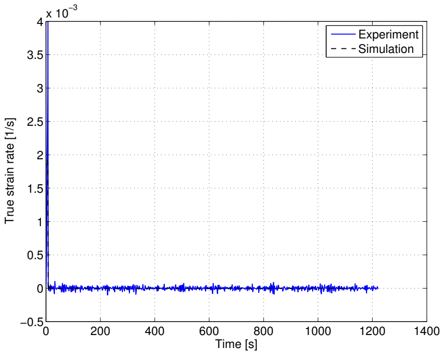




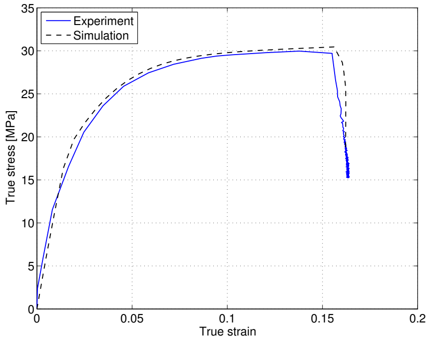


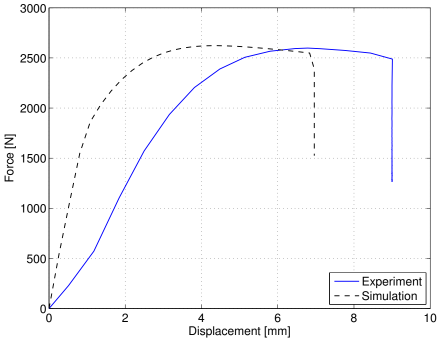


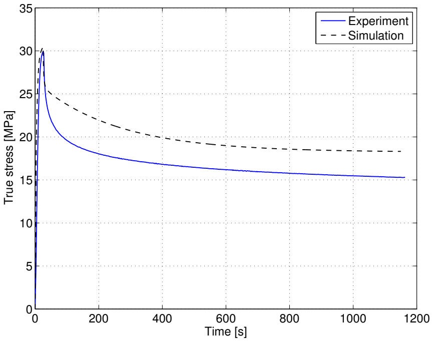


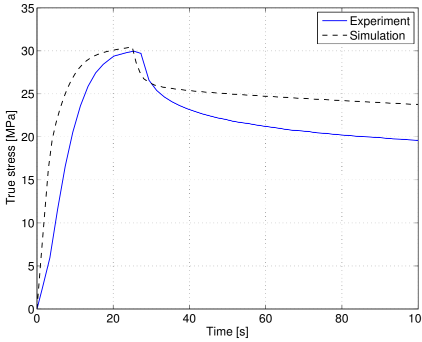


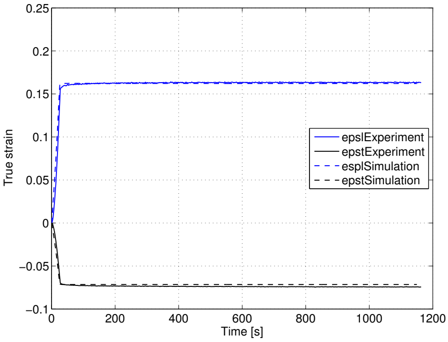
HDPE-R-2 LS-DYNA-Simulation

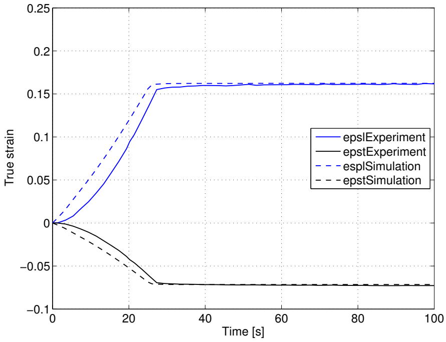


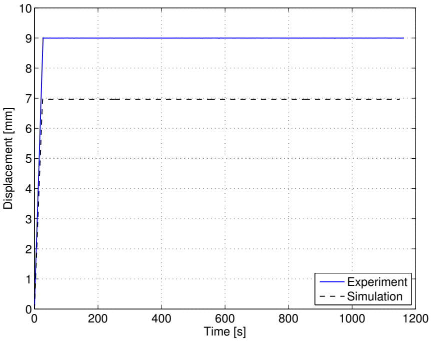


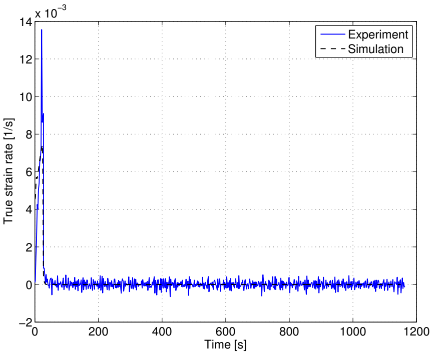






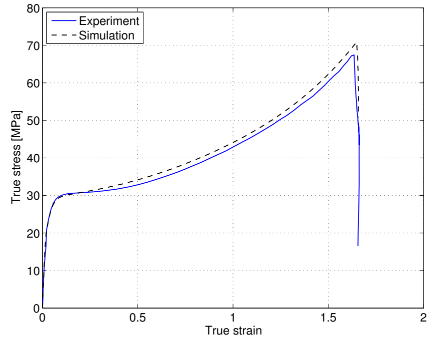


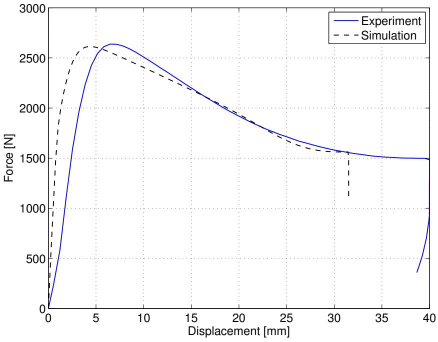


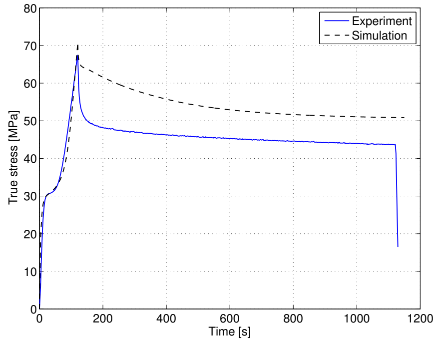


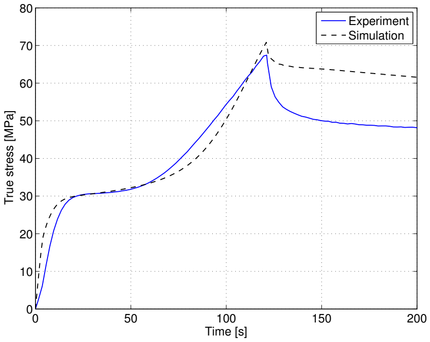
HDPE-R-3

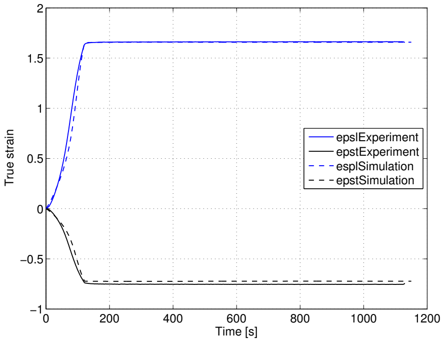
LS-DYNA-Simulation

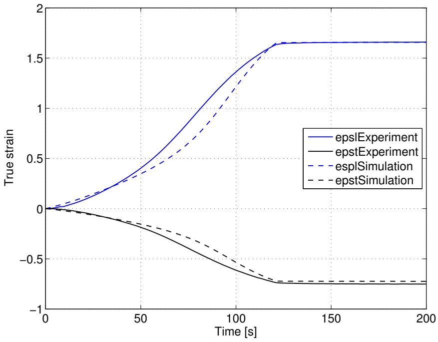


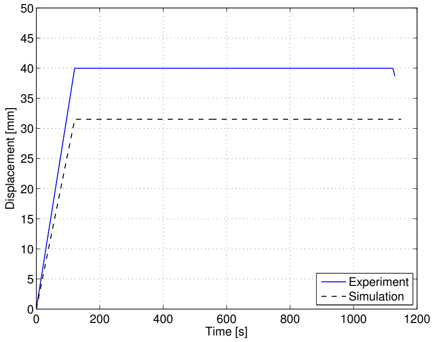


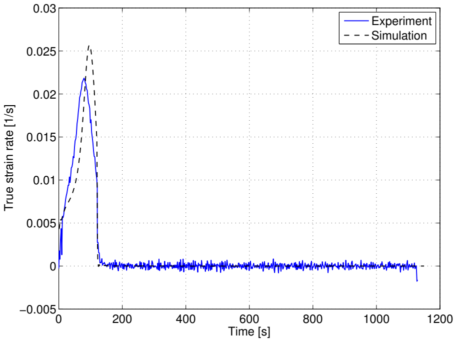




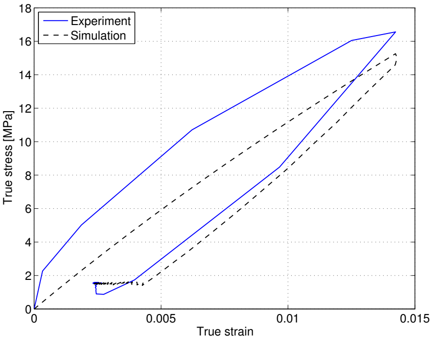


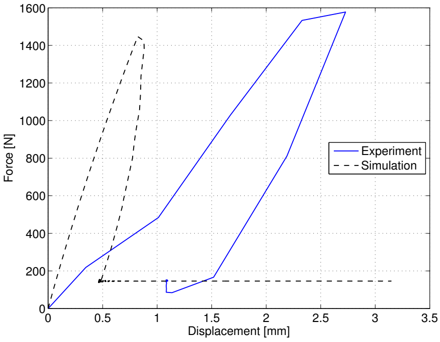


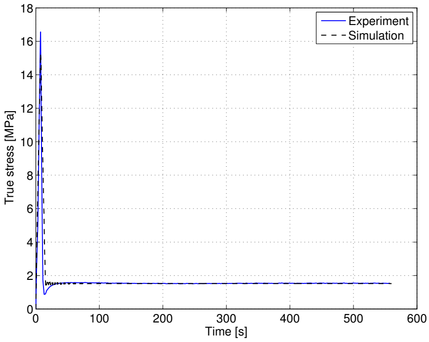


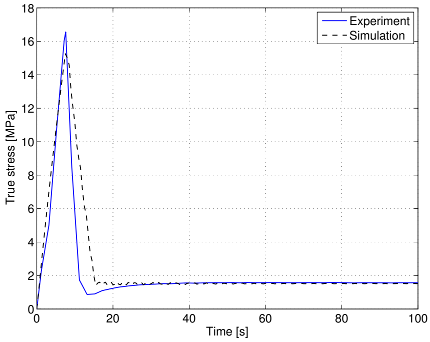


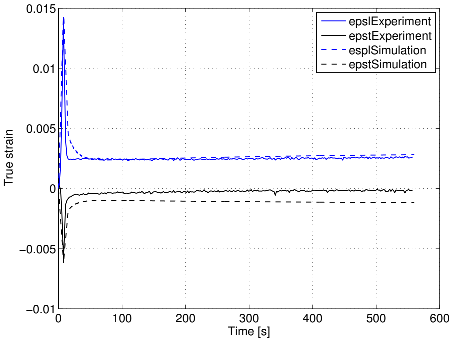
HDPE-C-1 LS-DYNA-Simulation

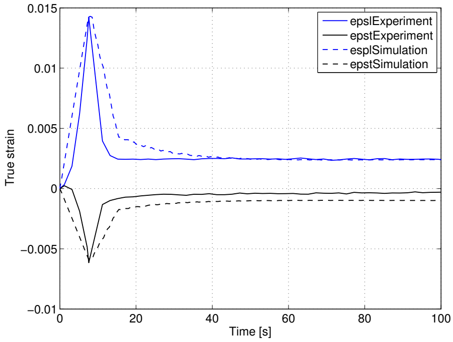


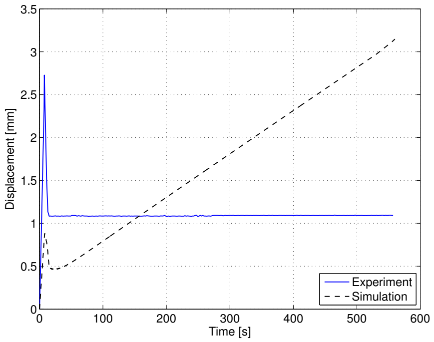


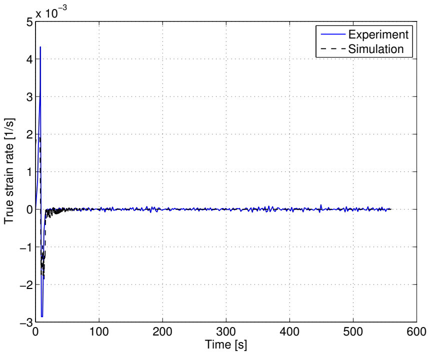






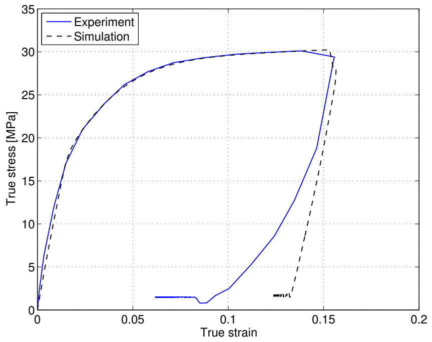


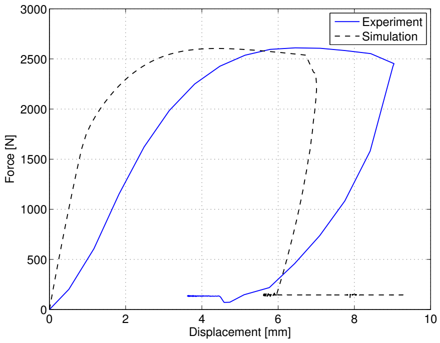


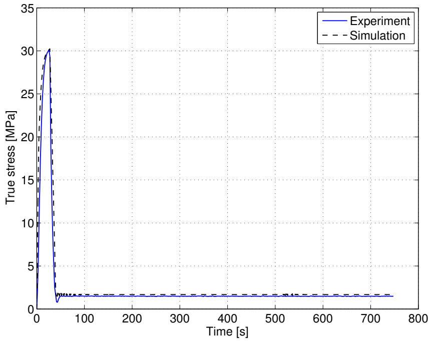


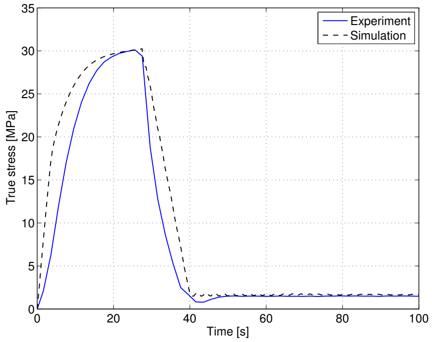
HDPE-C-2

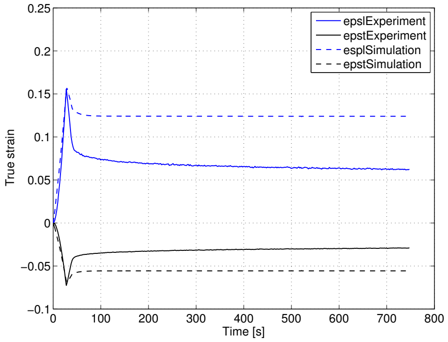
LS-DYNA-Simulation

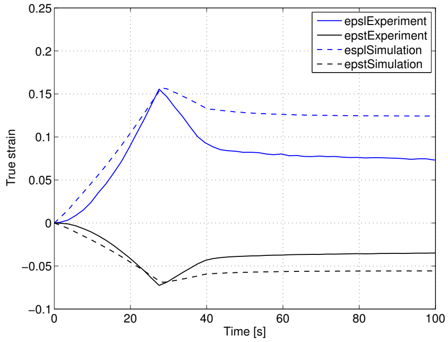


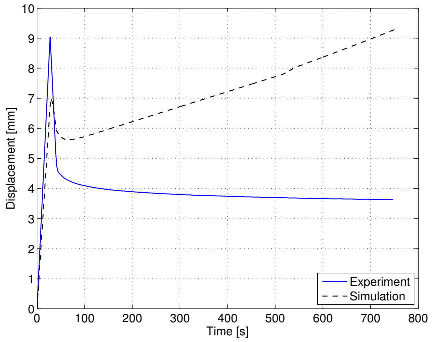


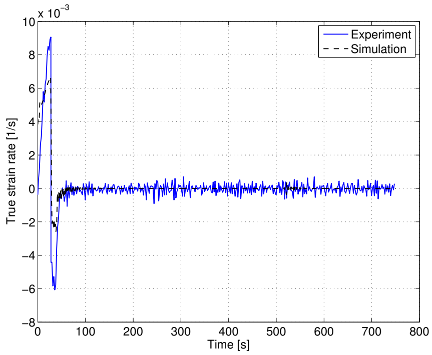




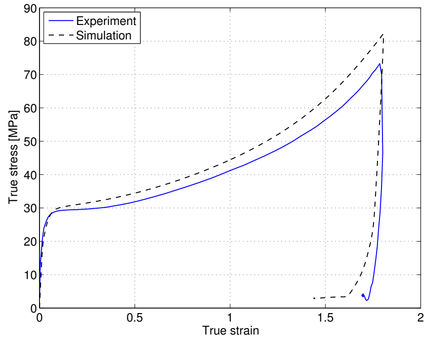


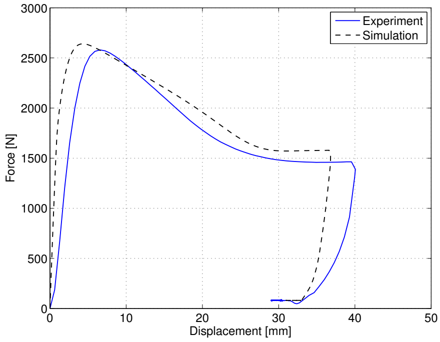


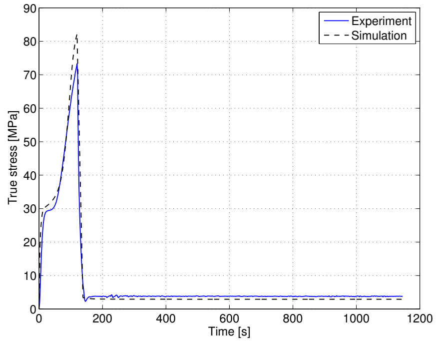


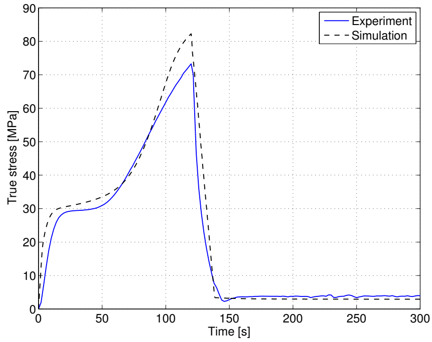


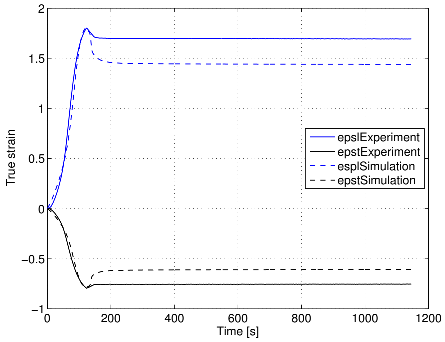
HDPE-C-3 LS-DYNA-Simulation

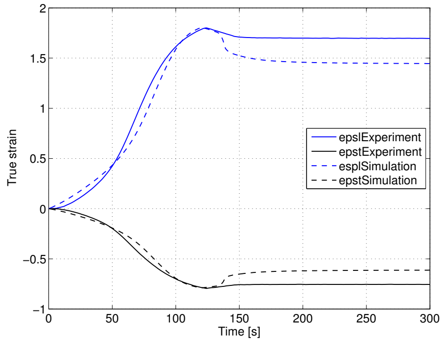


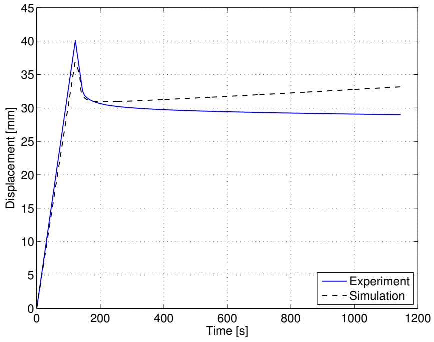


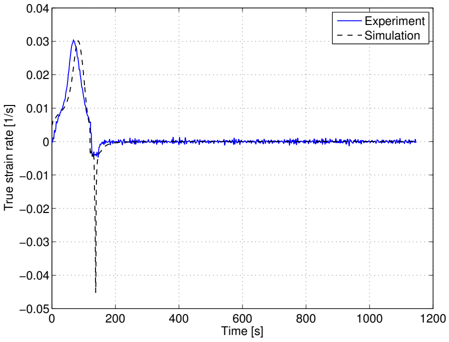




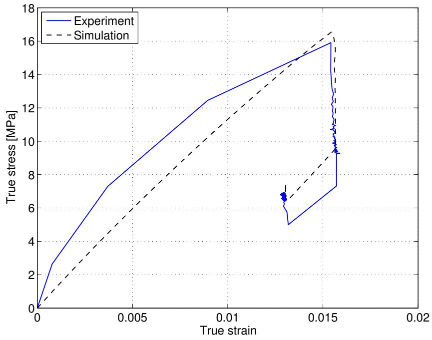


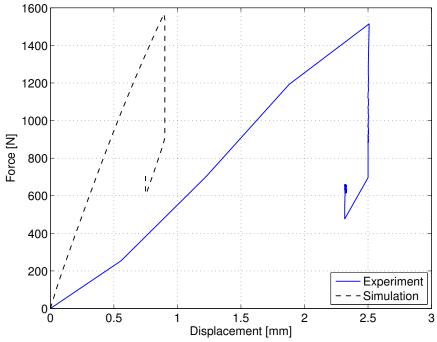


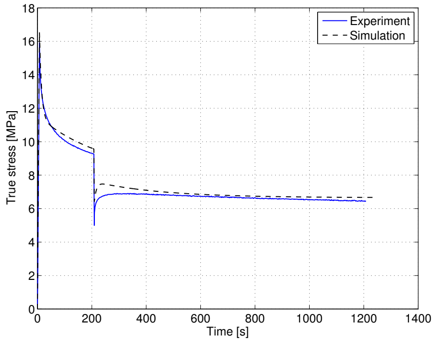


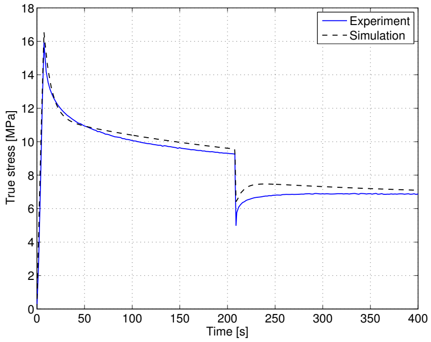


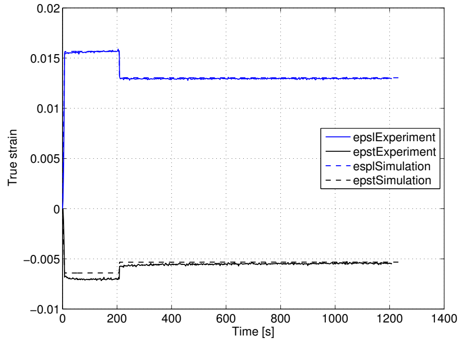
HDPE-DR-1 LS-DYNA-Simulation

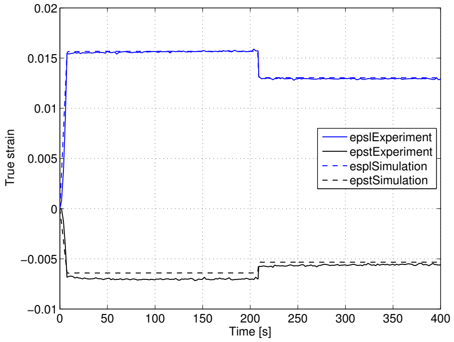


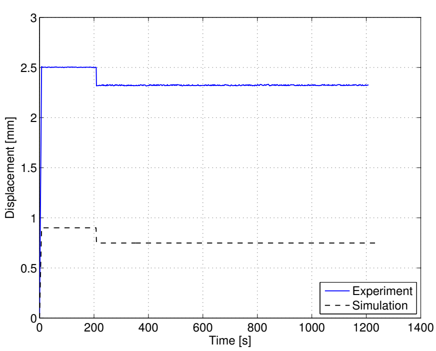


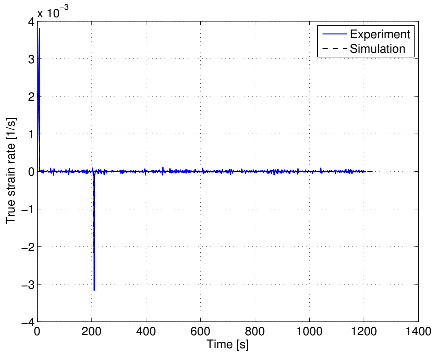




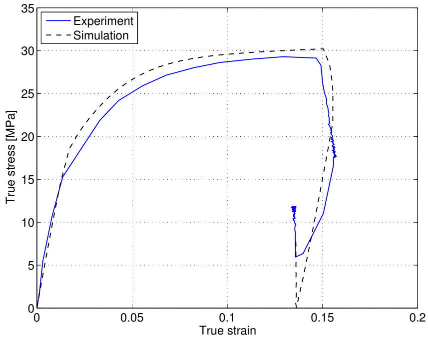


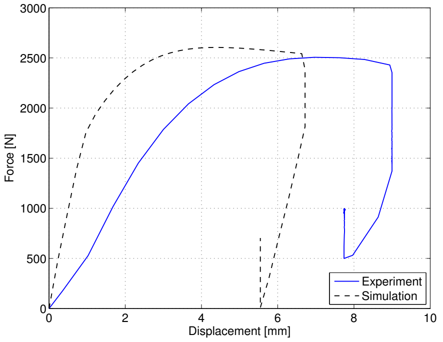


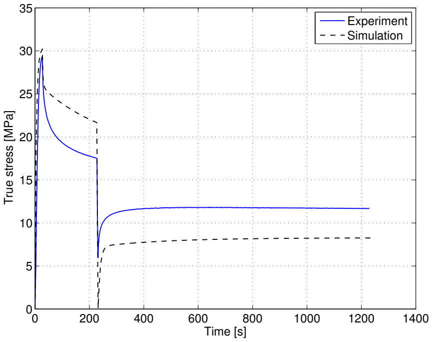


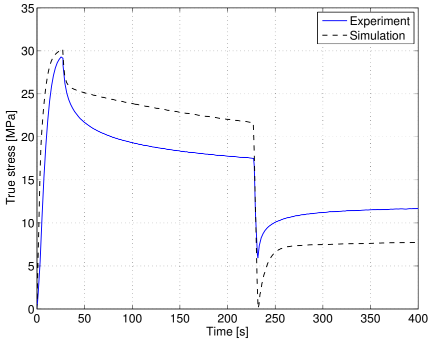


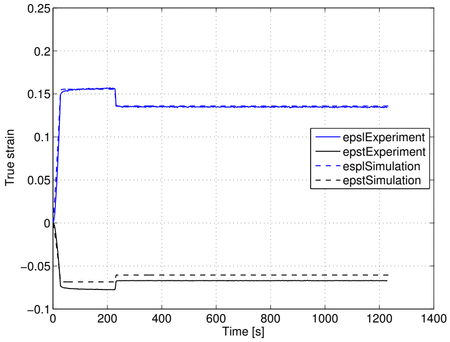
HDPE-DR-2 LS-DYNA-Simulation

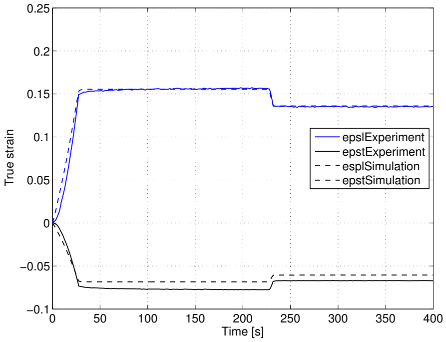


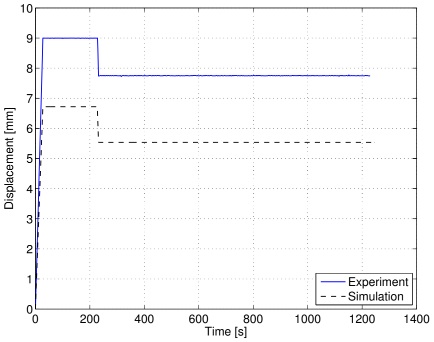


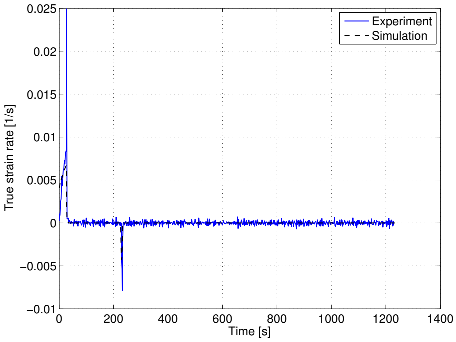






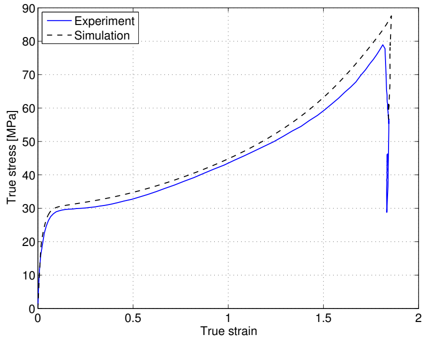


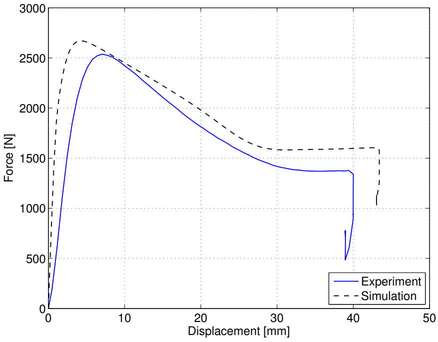


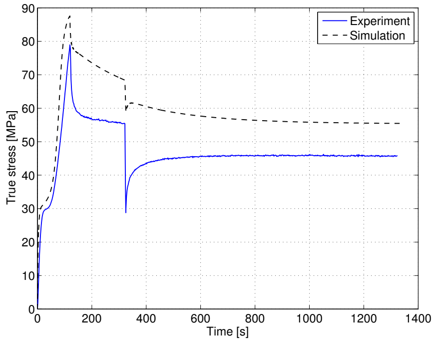


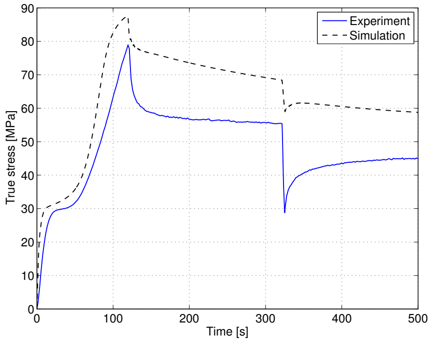
HDPE-DR-3

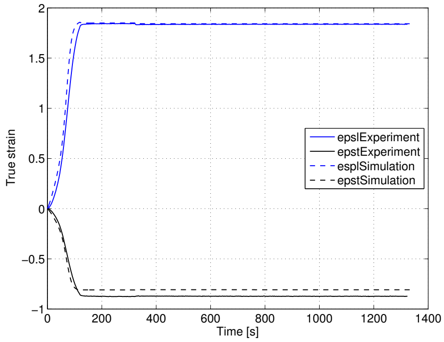
LS-DYNA-Simulation

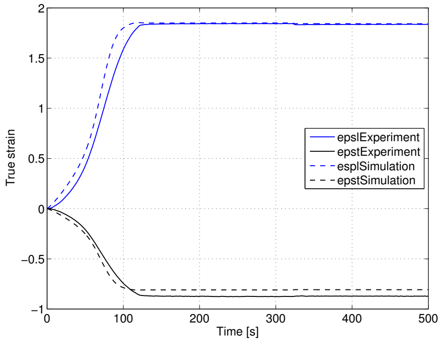


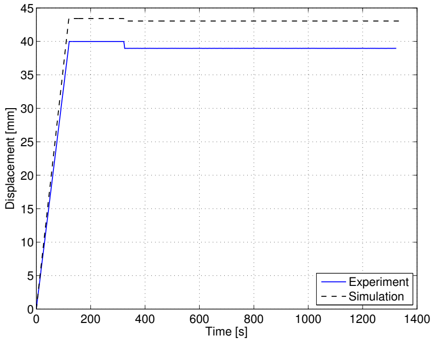


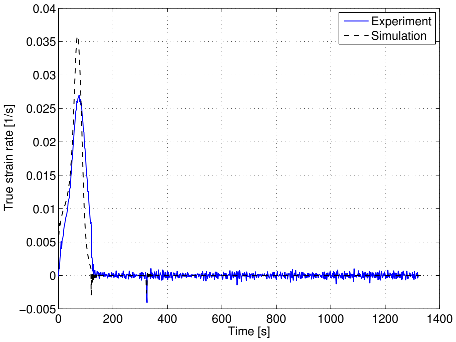












C Matlab scripts

In Appendix C the a total of five Matlab programs and corresponding functions used in this study is put forward.

The first program takes on the calibration of the visco-elastic parameters in the elastic domain based on the HDPE-R1 and HDPE-DR1. As seen in the function one can switch between the three different tests in the elastic domain.

The rest of the models are the three different material models presented in this study, and a version of the visco-elastic visco plastic model with part B as back-stress, with three Maxwell-elements.

Each model can either calibrate or run for predetermined variables. This is decided by either keeping the solver part in the program, or by commenting it out. Further details are explained the report and in the Matlab codes.

Visco-Elastic Calibration

Program

```
clc
clear all
close all
k=1;
for j=1:13
if  j==3 || j==8 || j==11
for i=1:7

%% Import the datak
[~, ~, raw] = xlsread(['/Users/vegardkristensen/Documents/
    Masteroppgave/Matrialdata/Nytt/Rawdata/Test' num2str(j)
    '/HDPE-T' num2str(j) '-DICmatlab.xls'],'MAT');
raw = raw(6:end,i);

%% Create output variable
t = cell2mat(raw);
if      i==1
    T{k,:}= t;
elseif  i==4
    eps{k,:}=t;
elseif  i==6
    sig{k,:}=t;

end

%% Clear temporary variables
clearvars raw;

end
k=k+1;
end
end

%--Create vectors--%
eps1=smooth(eps{1,:},1);
sig1=sig{1,:};
t1=T{1,:};
epsORG1=eps1;

eps2=smooth(eps{2,:},1);
```

```
sig2=sig{2,:};
t2=T{2,:};
epsORG2=eps2;

eps3=smooth(eps{3,:},1);
sig3=sig{3,:};
t3=T{3,:};
epsORG3=eps3;



%E=  [E0                    E1                    n1                    E2                    n2                    ]
Eny= [520.237539293667, 516.831120240936, 4023.35841909114, 226.431457276094, 56979.8304340666];      %input values if lsqnonlin is commented out
Eorg=[595.016645702997, 721.129195200939, 5080.92865737939, 249.077609823313, 63020.7456260707];      %Initial value of parameters
Emin=[10                    10                    10                    10                    10                    ];    %Minimum value of parameters
Emax=[1000                  10000                 100000                1000                  1000000               ];    %Maximum value of parameters

FsigRelax=@(E)viscoelasticMultipleFUNC(t1,t2,t3,eps1,eps2,eps3,sig1,sig2,sig3,E);              %Establish function used in solver
options=optimset('TolFun',1e-1000,'TolX',1e-1000,'maxFunevals',150,'MaxIter',50);              %Simple limitations to the solver
%[Eny,ll,fval]=lsqnonlin(FsigRelax,Eorg,Emin,Emax,options);                                    %Solver

%Extract stress data from function
[sigfun,s01,s11,s21,Sny1,s02,s12,s22,Sny2,s03,s13,s23,Sny3]=viscoelasticMultipleFUNC(t1,t2,t3,eps1,eps2,eps3,sig1,sig2,sig3,Eny);

%Plotting
scrsz = get(0,'ScreenSize');
hFig = figure(1);
set(gca,'fontsize',14)
set(hFig, 'OuterPosition', [1 scrsz(4)/2 1000 500])
```

```matlab
plot(t1,sig1,t1,Sny1,t1,s01,t1,s11,t1,s21,'LineWidth',1.1)
grid on
legend('\sigma_{exp}','\sigma_{sim}','\sigma_{0}','\sigma_{1}','\sigma_{2}')
%axis([0 1 0 1])
xlhand = get(gca,'xlabel');
set(xlhand,'string','Time [s]','fontsize',14)
xlhand = get(gca,'ylabel');
set(xlhand,'string','True stress [MPa]','fontsize',14)

scrsz = get(0,'ScreenSize');
hFig = figure(2);
set(gca,'fontsize',14)
set(hFig, 'OuterPosition', [1 scrsz(4)/2 1000 500])
plot(t2,sig2,t2,Sny2,t2,s02,t2,s12,t2,s22,'LineWidth',1.1)
grid on
legend('\sigma_{exp}','\sigma_{sim}','\sigma_{0}','\sigma_{1}','\sigma_{2}')
%axis([0 1 0 1])
xlhand = get(gca,'xlabel');
set(xlhand,'string','Time [s]','fontsize',14)
xlhand = get(gca,'ylabel');
set(xlhand,'string','True stress [MPa]','fontsize',14)

scrsz = get(0,'ScreenSize');
hFig = figure(3);
set(gca,'fontsize',14)
set(hFig, 'OuterPosition', [1 scrsz(4)/2 1000 500])
plot(t3,sig3,t3,Sny3,t3,s03,t3,s13,t3,s23,'LineWidth',1.1)
grid on
legend('\sigma_{exp}','\sigma_{sim}','\sigma_{0}','\sigma_{1}','\sigma_{2}')
%axis([0 1 0 1])
xlhand = get(gca,'xlabel');
set(xlhand,'string','Time [s]','fontsize',14)
xlhand = get(gca,'ylabel');
set(xlhand,'string','True stress [MPa]','fontsize',14)
```

Function

```
function [sigfun,sig01,sig11,sig21,Ssum1,sig02,sig12,sig22,Ssum2,sig03,sig13,sig23,Ssum3] = viscoelasticMultipleFUNC(t1,t2,t3,eps1,eps2,eps3,SIG1,SIG2,SIG3,Eorg)


%---Input parameters to calibrate---%
E0=Eorg(1);
E1=Eorg(2);
n1=Eorg(3);
E2=Eorg(4);
n2=Eorg(5);

%---Establish strain and time steps---%
%--R1--%
for i=1:length(t1)-1
deps1(i)=eps1(i+1)-eps1(i);
dt1(i)=t1(i+1)-t1(i);
end
%--C1--%
for i=1:length(t2)-1
deps2(i)=eps2(i+1)-eps2(i);
dt2(i)=t2(i+1)-t2(i);
end
%--DR1--%
for i=1:length(t3)-1
deps3(i)=eps3(i+1)-eps3(i);
dt3(i)=t3(i+1)-t3(i);
end




%%%%%     R-1     %%%%%%%%%

%---Initial conditions---%
sig01=0;
sig11=0;
sig21=0;


sig02=0;
sig12=0;
```

```
sig22=0;
Ssum2=0;

sig03=0;
sig13=0;
sig23=0;

%---Calculation procedure---%
%--R1--%
for i=2:length(t1)
        sig01(i)=sig01(i-1)+deps1(i-1)*E0;
        sig11(i)=sig11(i-1)*(1-(E1/n1)*dt1(i-1))+E1*deps1(i-1);
        sig21(i)=sig21(i-1)*(1-(E2/n2)*dt1(i-1))+E2*deps1(i-1);

Ssum1=sig01+sig11+sig21;


%--C1--%


% for j=2:length(t2)
%         sig02(j)=sig02(j-1)+(eps2(j)-eps2(j-1))*E0;
%         sig12(j)=sig12(j-1)*(1-(E1/n1)*dt2(j-1))+E1*deps2(j-1);
%         sig22(j)=sig22(j-1)*(1-(E2/n2)*dt2(j-1))+E2*deps2(j-1);

%Ssum2=sig02+sig12+sig22;


%--DR1--%

for k=2:length(t3)
        sig03(k)=sig03(k-1)+(eps3(k)-eps3(k-1))*E0;
        sig13(k)=sig13(k-1)*(1-(E1/n1)*dt3(k-1))+E1*deps3(k-1);
        sig23(k)=sig23(k-1)*(1-(E2/n2)*dt3(k-1))+E2*deps3(k-1);
Ssum3=sig03+sig13+sig23;




```

```



%--Calculates the restoring unbalance of both R1 and DR1--%
sigfun(i,k)=abs(sig03(k)+sig13(k)+sig23(k)-SIG3(k))+abs(
    sig01(i)+sig11(i)+sig21(i)-SIG1(i));

end
end
end
```

Visco-elastic visco-plastic model with part B in parallel with entire model

Program

```
clc
clear all
%close all
k=1;
for j=1:13;
if j==1                  %Numbers 1-13 decides which test to run
    for i=1:6

%% Import the datak
[~, ~, raw] = xlsread(['/Users/vegardkristensen/Documents/
    Masteroppgave/Matrialdata/Nytt/Rawdata/Test' num2str(j)
    '/HDPE-T' num2str(j) '-DICmatlab.xls'],'MAT');
raw = raw(6:end-40,i);

%% Create output variable
t = cell2mat(raw);
if i==1
    T{k,:}= t;
elseif i==4
    eps1{k,:}=t;
elseif i==5
    eps2{k,:}=t;
elseif i==6
    sig{k,:}=t;

end

%% Clear temporary variables
clearvars raw;
end
k=k+1;
end
end

%--Create vectors--%
eps=smooth(eps1{1,:},8);
epst=smooth(eps2{1,:},8);
sig=sig{1,:};
```

```
t=T{1,:};


%PLPA=    [P0          C            H            lDL          Cr          sigy         sigs    ];
PLPAny=   [0.0007,    0.079395,    66.94,       5.800,       2.080,      16,          24.9566];       %input values if lsqnonlin is commented out
PLPAorg=  [0.0007,    0.079395,    66.94,       5.000,       2.081,      16,          24.9566];       %Initial value of parameters
PLPAmin=  [0.0006999,0.0793,       66.93,       3.85,        1.5,        15.999,      24.94    ];      %Minimum value of parameters
PLPAmax=  [0.0007001,0.0795,       66.95,       10.48,       3,          16.001,      24.95    ];      %Maximum value of parameters



FsigRelax=@(PLPA)viscoelasticviscoplasticVer1FUNC(t,PLPA,sig,eps,epst);                     %Establish function used in solver
options=optimset('TolFun',1e-1000,'TolX',1e-1000,'maxFunevals',100,'MaxIter',50);                 % Simple limitations to the solver
%[PLPAny,ll,fval]=lsqnonlin(FsigRelax,PLPAorg,PLPAmin,PLPAmax,options);                     %Solver


%Extract stress data from function
[s,s0,s1,s2,sL,epsp,sre,srp,Sny]=viscoelasticviscoplasticVer1FUNC(t,PLPAny,sig,eps,epst);

%Plotting
scrsz = get(0,'ScreenSize');
hFig = figure(1);
set(hFig, 'OuterPosition', [1 scrsz(4)/2 500 500])
plot(t,sig,t,Sny,t,s0,t,s1,t,s2,t,sL)
grid on
legend('\sigma_{exp}','\sigma_{sim}','\sigma_{0}','\sigma_{1}','\sigma_{2}','\sigma_{B}')
%axis([0 1 0 1])
xlhand = get(gca,'xlabel');
set(xlhand,'string','Time [s]','fontsize',14)
```

```
xlhand = get(gca,'ylabel');
set(xlhand,'string','True stress [MPa]','fontsize',14)
```

Function

```
function [sigdiff,sig0,sig1,sig2,sigLang,epsp,sre,srp,sig] = viscoelasticviscoplasticVer1FUNC(t,PLPA,SIG,eps,epst)



%----------Elastic parameters----------%
EP=[520.237539293667, 516.831120240936, 4023.35841909114, 226.431457276094, 56979.8304340666];

E0=EP(1);
E1=EP(2);
n1=EP(3);
E2=EP(4);
n2=EP(5);
%--------------------------------------------------%


%----------Plastic paramters for first part of plastic domain----------%
sigyield=PLPA(6);
sigsat=PLPA(7);
sigCon=29.7;                                        % yieldstress by consideres criterion
p0_dot=PLPA(1);
p0=0.007;                                           % strainrate at yield
C=sigCon/(log(1+p0/p0_dot)*sigsat)-1/log(1+p0/p0_dot);   % PLPA(2);
H=PLPA(3);
%----------------------------------------------------------------------------------------%



%----------Plastic paramters for second part of plastic domain---------%
ldL=PLPA(4);
Cr=PLPA(5);
%----------------------------------------------------------------------------------------%



```

```
%————%Establish timestep (dt) & total strain step (deps)———%
deps=zeros(1,length(t-1));
dt=zeros(1,length(t-1));
for i=1:length(t)-1
deps(i)=eps(i+1)-eps(i);
dt(i)=t(i+1)-t(i);
end
%————————————————————————————————————————————————————————————————————%



%—————Initial conditions—————%

                                    %Yield stress
%sigsat=25.51;                                %Saturation stress
sig0=zeros(1,length(eps));                  %Elastic string stress
sig1=zeros(1,length(eps));                  %Maxwell element 1 stress
sig2=zeros(1,length(eps));                  %Maxwell element 2 stress
sig=sig0+sig1+sig2;                         %Total stress set to zero
sig_tr=sig0 + sig1 + sig2;                  %Total trial stress set to zero

epsp=zeros(1,length(eps));                  %Plastic strain
depsp=zeros(1,length(eps)-1);               %Plastic strain step
p=zeros(1,length(eps));                     %plastic strain magnitude
dp=zeros(1,length(eps)-1);                  %Plastic strain magnitude step

sigLang=zeros(1,length(eps));               %Langevin stress set to zero
R=-(sigsat-sigyield)*exp(-H*epsp);          %Ramping stress set to zero
R_tr=R;                                     %Ramping trial stress set to zero
%————————————————————————————————————%


```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%---------------Calulation procedure----------------%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

sigdiff=zeros(1,length(eps));             %Difference in stresses between experimental and test (Output for LSQNonlin)
z=1;                                      %Simple counter to track plastic strain

for i=2:length(t)


                %--Part B in parallel---%
                eps1=eps(i);
                eps2=epst(i);
                eps3=epst(i); %eps2*0.871/0.822;
                %eps2=log(1/exp(0.5*eps1));
                %eps3=eps2;

                ld1=exp(eps1); ld2=exp(eps2) ; ld3=exp(eps3);
                J=ld1*ld2*ld3;
                ld=sqrt((ld1^2+ld2^2+ld3^2)/3*J^(-2/3));
                langInv=(ld/ldL)*(3-(ld/ldL)^2)/(1-(ld/ldL)^2);             %Pades approximate Langevin inverse
                sigLang(i)=J^(-5/3)*Cr/3*ldL/ld*langInv*(ld1^2-1/3*(ld1^2+ld2^2*ld3^2));
                %-------------------------%


    %Establish stresses for trial step (assume all is elastic)
    sig0(i)=sig0(i-1)+E0*deps(i-1);
    sig1(i)=sig1(i-1)*(1-(E1/n1)*dt(i-1))+E1*deps(i-1);
    sig2(i)=sig2(i-1)*(1-(E2/n2)*dt(i-1))+E2*deps(i-1);

    sig_tr(i)=sig0(i)+sig1(i)+sig2(i);        % Trial stress
    R_tr(i)=R(i-1);                           % Trial ramping stress



```

```
    %-----Yield criterion-----%
    f= abs(sig_tr(i)) - (sigsat+R_tr(i));

    if f < 0                                 % Elastic domain


        dp(i)=0;                             % No plastic contribution

        %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
        %Paramters after elastic step i%
        %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
        sre(i)=deps(i-1)/dt(i-1);           %Elastic strain rate
        srp(i)=0;                           %Plastic strain rate
        R(i)=R_tr(i);
        epsp(i)=epsp(i-1);
        p(i)=p(i-1);
        sig(i)=sig_tr(i)+sigLang(i);
        sigdiff(i)=sig(i)-SIG(i);


    else                                     % Elastoplastic domain

        %Algorithm set of constitutive equations for the backward euler
        %method.

        %Iteration procedure to decide portion of strain to be plastic.


        %Inital conditions%
        maxiter=50;                                                    %Maximum nuber of iterations
```

```
Tol=10e-9;

    %Acceptance tolerance to end iterations
Dp=zeros(1,length(maxiter));
                                                  %
    Initial plastic strain magnitude in step set
     to zero
dDp=zeros(1,length(maxiter));
                                                 %
    Establish plastic strain differ matrix
dDp(1)=0.0000001;

    %Intitial plastic strain magnitude differ
    set to small number, but higher than Tol to
    enter iteration procedure
Phi=zeros(1,length(maxiter));
                                                 %
    Establish plastic strain rate matrix
Phi(1)=p0_dot*(exp(1/C*(((abs(sig_tr(i)))/(
    sigsat+R_tr(i)))-1))-1); %Initial plastic
    strain rate
Psi=zeros(1,length(maxiter));
                                                 %
    Establish Psi matrix (Reisdual from
    viscoplastic constitutive relation). Should
    be equal to zero if plastic strain magnitude
     in step is correct
Psi(1)=Phi(1)*dt(i-1)-Dp(1);
                                                  %
    Initial Psi
J=zeros(1,length(maxiter));
                                                   %
    Establish Jacobean matrix




k=2;

while abs(dDp(k-1))>=Tol


        Dp(k)=Dp(k-1)+dDp(k-1);
        sigIter=sig_tr(i)-(E0+E1+E2)*Dp(k);
        Riter=R_tr(i)*exp(-H*sign(sigIter)*Dp(k
```

```
                        ));
                    Phi(k)=p0_dot*(exp(1/C*((abs(sigIter)/(
                        sigsat+Riter))-1))-1);
                    Psi(k)=Phi(k)*dt(i-1)-Dp(k);
                    J(k)=(Psi(k)-Psi(k-1))/(Dp(k)-Dp(k-1));
                    dDp(k)=-Psi(k)/J(k);


            k=k+1;
            if k==maxiter
                break
            end
            end
            dp(i)=Dp(k-1);                                   %
                Plastic strain magnitude contribution
            depsp(i)=dp(i)*sign(sigIter);                    %
                Plastic strain contribution

            %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
            %Parameters after elastoplastic step i%
            %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

            sig0(i)=sig0(i)-E0*depsp(i);
            sig1(i)=sig1(i)-E1*depsp(i);
            sig2(i)=sig2(i)-E2*depsp(i);
            sig(i)=sig0(i)+sig1(i)+sig2(i)+sigLang(i);
                %Total stress update
            sigdiff(i) =sig(i)-SIG(i);


            epsp(i)=epsp(i-1)+depsp(i);
            p(i)=p(i-1)+dp(i);
            R(i)=R(i-1)*exp(-H*depsp(i));
            sre(i)=(deps(i-1)-depsp(i))/dt(i-1);             %
                Elastic strain rate
            srp(i)=depsp(i)/dt(i-1);                         %
                Plastic strain rate
    end


end
end
```

Elastic visco-plastic model with part B as back-stress

Program

```
clc
clear all
%close all
k=1;
for j=1:13;
if j==1      %Numbers 1-13 decides which test to run
    for i=1:6

%% Import the datak
[~, ~, raw] = xlsread(['/Users/vegardkristensen/Documents/Masteroppgave/Matrialdata/Nytt/Rawdata/Test' num2str(j) '/HDPE-T' num2str(j) '-DICmatlab.xls'],'MAT');
raw = raw(6:end-40,i);

%% Create output variable
t = cell2mat(raw);
if i==1
    T{k,:}= t;
elseif i==4
    eps1{k,:}=t;
elseif i==5
    eps2{k,:}=t;
elseif i==6
    sigma{k,:}=t;

end

%% Clear temporary variables
clearvars raw;
end
k=k+1;
end
end

%%%%----Interpolates extra history input values----%%%%%
t=T{12};
EPS1=eps1{12};
EPS2=eps2{12};
```

```

t2(1)=t(1);
epsl(1)=EPS1(1);
epst(1)=EPS2(1);
for i=2:2*length(t)-1
        if mod(i,2)==1
        t2(i)=t((i+1)/2);
        epsl(i)=EPS1((i+1)/2);
    epst(i)=EPS1((i+1)/2);
    else
        t2(i)=(t(i/2)+t(i/2+1))/2;
        epsl(i)=(EPS1(i/2)+EPS1(i/2+1))/2;
    epst(i)=(EPS2(i/2)+EPS2(i/2+1))/2;
    end
end
epsl(2*length(t))=EPS1(length(t));
epst(2*length(t))=EPS2(length(t));
t2(2*length(t))=t(length(t));
%------------------------------------------------------------------%

%PLPA=   [p0          C           H          lDL        Cr
              sigy        sigs        E0         B       ]
PLPAny= [0.0007,    0.1794,     66.94,    5.96,     1.833,
     16,          24.95,       916.10     1.07    ]; %input
    values if lsqnonlin is commented out
PLPAorg=[0.0007,    0.0794,     66.94,    3.47,     1.62,
      16,          24.95,       1263.5,    1.07     ]; %initial
    value
PLPAmin=[0.0006999, 0.0793,     66.93,    3.0,      0.1,
        15.99,       24.94,      863.4,     1.06999]; %minimum
     value
PLPAmax=[0.0007001, 0.0795,     66.95,    6.00,     5,
          16.01,       24.96,       1263.6     1.07001]; %
    maximum value



FsigRelax=@(PLPA)elasticviscoplasticFUNC(t,PLPA,sig,eps,
    epst);
options=optimset('TolFun',1e-1000,'TolX',1e-1000,'
    maxFunevals',50,'MaxIter',25);
%[PLPAny,ll,fval]=lsqnonlin(FsigRelax,PLPAorg,PLPAmin,
    PLPAmax,options);                                     %
    solver

```

```
%Plotting
[s,s0,sL,sV,epsp,Sny]=elasticviscoplasticFUNC(t,PLPAny,sig,eps,epst);

%Plotting stress vs time
scrsz = get(0,'Screensize');
hFig = figure(1);
set(hFig, 'Outerposition', [1 scrsz(4)/2 500 500])
set(gca,'fontsize',14)
plot(t,sig,t,Sny,t,sL)
grid on
legend('\sigma_{exp}','\sigma_{sim}','\sigma_{b}')
set(legend,'location','east','fontsize',14)
%axis([0 1 0 1])
xlhand = get(gca,'xlabel');
set(xlhand,'string','Time [s]','fontsize',14)
xlhand = get(gca,'ylabel');
set(xlhand,'string','True stress [MPa]','fontsize',14)
```

Function

```
function [sigdiff,sig0,sigLang,sigvisco,epsp,sig] = elasticviscoplasticFUNC(t,PLPA,SIG,eps,epst)



%--------Elastic parameters---------%

E0=PLPA(8);

%-----------------------------------%


%--------Plastic paramters for first part of plastic domain--------%
p0_dot=PLPA(1);
C=PLPA(2);
H=PLPA(3);
%------------------------------------------------------------------%



%--------Plastic paramters for second part of plastic domain-------%
ldL=PLPA(4);
Cr=PLPA(5);
B=PLPA(8);
%------------------------------------------------------------------%



%--------%Establish timestep (dt) & total strain step (deps)-------%
deps=zeros(1,length(t-1));
dt=zeros(1,length(t-1));
for i=1:length(t)-1
deps(i)=eps(i+1)-eps(i);
dt(i)=t(i+1)-t(i);
end
%------------------------------------------------------------------%

```

```


%----------Initial conditions----------%

sigyield=PLPA(6);                             %Yield stress
sigsat=PLPA(7);                               %Saturation stress
sig0=zeros(1,length(eps));                    %Elastic string stress
sig=sig0;                                     %Total stress set to zero
sig_tr=sig0;                                  %Total trial stress set to zero

epsp=zeros(1,length(eps));                    %Plastic strain
depsp=zeros(1,length(eps)-1);                 %Plastic strain step
epspt=zeros(1,length(eps));
depspt=zeros(1,length(eps)-1);
p=zeros(1,length(eps));                       %plastic strain magnitude
dp=zeros(1,length(eps)-1);                    %Plastic strain magnitude step

sigLang=zeros(1,length(eps));                 %Langevin stress set to zero (backstress)
sigLang_tr=sigLang;                           %Langevin trial stress set to zero (backstress)
R=-(sigsat-sigyield)*exp(-H*epsp);            %Ramping stress set to zero
R_tr=R;                                       %Ramping trial stress set to zero
%--------------------------------------%


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%---------------Calulation procedure---------------%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

sigdiff=zeros(1,length(eps));                 %Difference in stresses between experimental and test (Output for LSQNonlin)
z=1;                                          %Simple counter to track plastic strain

for i=2:length(t)
```

```



    %Establish stresses for trial step (assume all is elastic)
    sig0(i)=sig0(i-1)+E0*deps(i-1);
    sig_tr(i)=sig0(i);                          % Trial stress
    R_tr(i)=R(i-1);                             % Trial ramping stress
    sigLang_tr(i)=sigLang(i-1);                 % Trail langevin stress(backstress)


    %------Yield criterion------%
    f= abs(sig_tr(i)-sigLang_tr(i)) - (sigsat+R_tr(i));

    if f < 0                                    % Elastic domain


        dp(i)=0;                                % No plastic contribution

        %%%%%%%%%%%%%%%%%%%%%%%%%
        %Paramters after elastic step i%
        %%%%%%%%%%%%%%%%%%%%%%%%%

        R(i)=R_tr(i);
        epsp(i)=epsp(i-1);
        epspt(i)=epspt(i-1);
        p(i)=p(i-1);
        sigLang(i)=sigLang_tr(i);
        sig(i)=sig_tr(i);
        sigdiff(i)=sig(i)-SIG(i);
        sigvisco(i)=0;

    else                              % Elastoplastic domain

        %Algorithm set of constitutive equations for the backward euler
        %method.

        %Iteration procedure to decide portion of strain to be plastic.

```

```

        %Inital conditions%
        maxiter=50;                                                      %Maximum nuber of iterations
        Tol=10e-9;                                                       %Acceptance tolerance to end iterations
        Dp=zeros(1,length(maxiter));                                     %Initial plastic strain magnitude in step set to zero
        dDp=zeros(1,length(maxiter));                                    %Establish plastic strain differ matrix
        dDp(1)=0.0000001;                                                %Intitial plastic strain magnitude differ set to small number, but higher than Tol to enter iteration procedure
        langInv=zeros(1,length(maxiter));                                %Establish inverse langevin matrix
        Fi=zeros(1,length(maxiter));                                     %Establish plastic strain rate matrix
        Fi(1)=p0_dot*(exp(1/C*(((abs(sig_tr(i)-sigLang_tr(i)))/(sigsat+R_tr(i)))-1))-1); %Initial plastic strain rate
        Psi=zeros(1,length(maxiter));                                    %Establish Psi  matrix (Reisdual from viscoplastic constitutive relation). Should be equal to zero if plastic strain magnitude in step is correct
        Psi(1)=Fi(1)*dt(i-1)-Dp(1);                                      %Initial Psi
        J=zeros(1,length(maxiter));                                      %Establish Jacobean matrix
        sigIter=sig_tr(i);



        k=2;

        while abs(dDp(k-1))>=Tol
```

```

                    srfactor=1-3/(B+1);          %Retraction ratio

                    Dp(k)=Dp(k-1)+dDp(k-1);

                    %Langevin%%%%%%%%%%%%%%%%%(Backstress)
                    epsp1=epsp(i-1)+Dp(k)*sign(sig_tr(i)-sigLang_tr(i));
                    epsp2=epspt(i-1)+srfactor*Dp(k)*sign(sig_tr(i)-sigLang_tr(i));
                    epsp3=epsp2;


                    ld1=exp(epsp1); ld2=exp(epsp2)  ; ld3=ld2;
                    J=ld1*ld2*ld3;
                    ld=sqrt((ld1^2+ld2^2+ld3^2)/3*J^(-2/3));
                    langInv=(ld/ldL)*(3-(ld/ldL)^2)/(1-(ld/ldL)^2);  %Pades approximate Langevin inverse
                    sigLangIter=J^(-5/3)*Cr/3*ldL/ld*langInv*(ld1^2-1/3*(ld1^2+ld2^2*ld3^2));
                    %----------------%

                    sigIter=sig_tr(i)-(E0)*Dp(k);
                    Riter=R_tr(i)*exp(-H*sign(sig_tr(i)-sigLang_tr(i))*Dp(k));



                    Fi(k)=p0_dot*(exp(1/C*((abs(sigIter-sigLangIter)/(sigsat+Riter))-1))-1);
                    Psi(k)=Fi(k)*dt(i-1)-Dp(k);
                    J(k)=(Psi(k)-Psi(k-1))/(Dp(k)-Dp(k-1));
                    dDp(k)=-Psi(k)/J(k);






            k=k+1;
```

```
            if k==maxiter
                break
            end
            end
            dp(i)=Dp(k-1);                                      %Plastic strain magnitude contribution
            depsp(i)=dp(i)*sign(sigIter-sigLangIter);           %Plastic strain contribution
            depspt(i)=srfactor*dp(i)*sign(sigIter-sigLangIter);
            %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
            %Parameters after elastoplastic step i%
            %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

            sig0(i)=sig0(i)-E0*depsp(i);                        %Stress update
            sig(i)=sig0(i);
            sigdiff(i) =sig(i)-SIG(i);


            epsp(i)=epsp(i-1)+depsp(i);
            epspt(i)=epspt(i-1)+depspt(i);
            p(i)=p(i-1)+dp(i);
            R(i)=R(i-1)*exp(-H*depsp(i));
            sigLang(i)=sigLangIter;
            sigvisco(i)=sig(i)-sigLang(i)-(sigsat+R(i));
            z=z+1;                                              %Plastic step counter update
    end


end
end
```

Visco-elastic visco-plastic model with part B as back-stress

Program

```
clc
clear all
%close all
k=1;
for j=1:13;
if j==1      %Numbers 1-13 decides which test to run
    for i=1:6

%% Import the datak
[~, ~, raw] = xlsread(['/Users/vegardkristensen/Documents/Masteroppgave/Matrialdata/Nytt/Rawdata/Test' num2str(j) '/HDPE-T' num2str(j) '-DICmatlab.xls'],'MAT');
raw = raw(6:end-40,i);

%% Create output variable
t = cell2mat(raw);
if i==1
    T{k,:}= t;
elseif i==4
    eps1{k,:}=t;
elseif i==5
    eps2{k,:}=t;
elseif i==6
    sigma{k,:}=t;

end

%% Clear temporary variables
clearvars raw;
end
k=k+1;
end
end

%%%%-----Interpolates extra history input values-----%%%%%
t=T{12};
EPS1=eps1{12};
EPS2=eps2{12};
```

```

t2(1)=t(1);
epsl(1)=EPS1(1);
epst(1)=EPS2(1);
for  i=2:2*length(t)-1
        if mod(i,2)==1
        t2(i)=t((i+1)/2);
        epsl(i)=EPS1((i+1)/2);
    epst(i)=EPS1((i+1)/2);
    else
        t2(i)=(t(i/2)+t(i/2+1))/2;
        epsl(i)=(EPS1(i/2)+EPS1(i/2+1))/2;
    epst(i)=(EPS2(i/2)+EPS2(i/2+1))/2;
    end
end
epsl(2*length(t))=EPS1(length(t));
epst(2*length(t))=EPS2(length(t));
t2(2*length(t))=t(length(t));
%----------------------------------------------------------------%


%PLPA=[        P0            C          H          lDL          Cr
    sigy          sigs            B        E0          E1            n1
        E2          n2]
PLPAny= [0.0007,     0.0794,      66.94,    4.972,    2.448,
    16,          24.95,       1.07,       520.2,    516.8,
    4023.4, 226.4,   56979.8]; %input values if lsqnonlin is
    commented out
PLPAorg=[0.0007,     0.0794,      66.94,    4.949,    2.495,
    16,          24.95,       1.07,       520.2,    516.8,
    4023.4, 226.4,   56979.8]; %initial value
PLPAmin=[0.0006999, 0.0393,       16.93,    3,         1,
    4.99,        14.99,       1.06999,  520.1,    516.7,
    4023.3, 226.3,   56979.7]; %minimum value
PLPAmax=[0.0007001, 0.53095,      166.95,   8.0,       5,
    5.01,        15.01,       1.07001,  520.3,    516.9,
    4023.5, 226.5,   56979.9]; %maximum value



FsigRelax=@(PLPA)viscoelasticviscoplasticVer2FUNC(t,PLPA,
    sig,eps,epst);
options=optimset('TolFun',1e-10000,'TolX',1e-1000,'
    maxFunevals',100,'MaxIter',50);
%[PLPAny,ll,fval]=lsqnonlin(FsigRelax,PLPAorg,PLPAmin,
```

```
    PLPAmax,options); %solver

%Plotting
[s,s0,s1,s2,sL,sV,sre,srp,epsp,Sny]=viscoelasticviscoplasticVer2FUNC(t,PLPAny,sig,eps,epst);


%Plotting stress vs time
scrsz = get(0,'Screensize');
hFig = figure(1);
set(gca,'fontsize',14)
set(hFig, 'Outerposition', [1 scrsz(4)/2 500 500])
plot(t,sig,t,Sny,t,s0,t,s1,t,s2,'LineWidth',1.1)
grid on
legend('\sigma_{exp}','\sigma_{sim}','\sigma_{0}','\sigma_{1}','\sigma_{2}')
set(legend,'location','Northeast','fontsize',14)
axis([0 400 -10 40])
xlhand = get(gca,'xlabel');
set(xlhand,'string','Time [s]','fontsize',14)
xlhand = get(gca,'ylabel');
set(xlhand,'string','True stress [MPa]','fontsize',14)
print(gcf, '-depsc2', ['/Users/vegardkristensen/Documents/Masteroppgave/Rapport/files/figures/chap7/lowyieldDR2.eps'],'-r1000');
```

Function

```
function [sigdiff,sig0,sig1,sig2,sigLang,sigvisco,sre,srp,epsp,sig] = viscoelasticviscoplasticVer2FUNC(t,PLPA,SIG,eps,epst)



%--------Elastic parameters---------%
E0=PLPA(9);
E1=PLPA(10);
n1=PLPA(11);
E2=PLPA(12);
n2=PLPA(13);
%------------------------------------------%


%--------Plastic paramters for first part of plastic domain--------%
p0_dot=PLPA(1);
C=PLPA(2);
H=PLPA(3);
%--------------------------------------------------------------------------%



%--------Plastic paramters for second part of plastic domain-------%
ldL=PLPA(4);
Cr=PLPA(5);
B=PLPA(8);
%--------------------------------------------------------------------------%



%--------%Establish timestep (dt) & total strain step (deps)------%
deps=zeros(1,length(t-1));
dt=zeros(1,length(t-1));
for i=1:length(t)-1
deps(i)=eps(i+1)-eps(i);
dt(i)=t(i+1)-t(i);
end
```

```
%-----------------------------------------------------------------------------%



%---------Initial conditions---------%

sigyield=PLPA(6);                        %Yield stress
sigsat=PLPA(7);                          %Saturation stress
sig0=zeros(1,length(eps));               %Elastic string stress
sig1=zeros(1,length(eps));               %Maxwell element 1 stress
sig2=zeros(1,length(eps));               %Maxwell element 2 stress
sig=sig0+sig1+sig2;                      %Total stress set to zero
sig_tr=sig0 + sig1 + sig2;               %Total trial stress set to zero

epsp=zeros(1,length(eps));               %Plastic strain
depsp=zeros(1,length(eps)-1);            %Plastic strain step
epspt=zeros(1,length(eps));
depspt=zeros(1,length(eps)-1);
p=zeros(1,length(eps));                  %plastic strain magnitude
dp=zeros(1,length(eps)-1);               %Plastic strain magnitude step

sigLang=zeros(1,length(eps));            %Langevin stress set to zero (backstress)
sigLang_tr=sigLang;                      %Langevin trial stress set to zero (backstress)
R=-(sigsat-sigyield)*exp(-H*epsp);       %Ramping stress set to zero
R_tr=R;                                  %Ramping trial stress set to zero
%--------------------------------------%


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%-------------Calulation procedure-------------%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

```

```
sigdiff=zeros(1,length(eps));             %Difference in stresses between experimental and test (Output for LSQNonlin)
z=1;                                      %Simple counter to track plastic strain

for i=2:length(t)



    %Establish stresses for trial step (assume all is elastic)
    sig0(i)=sig0(i-1)+E0*deps(i-1);
    sig1(i)=sig1(i-1)*(1-(E1/n1)*dt(i-1))+E1*deps(i-1);
    sig2(i)=sig2(i-1)*(1-(E2/n2)*dt(i-1))+E2*deps(i-1);
    sig_tr(i)=sig0(i)+sig1(i)+sig2(i);      % Trial stress
    R_tr(i)=R(i-1);                         % Trial ramping stress
    sigLang_tr(i)=sigLang(i-1);             % Trail langevin stress


    %------Yield criterion------%
    f= abs(sig_tr(i)-sigLang_tr(i)) - (sigsat+R_tr(i));

    if f < 0                                % Elastic domain


        dp(i)=0;                            % No plastic contribution

        %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
        %Paramters after elastic step i%
        %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
        sre(i)=deps(i-1)/dt(i-1);           %Elastic strain rate
        srp(i)=0;                           %Plastic strain rate
        R(i)=R_tr(i);
        epsp(i)=epsp(i-1);
        epspt(i)=epspt(i-1);
        p(i)=p(i-1);
        sigLang(i)=sigLang_tr(i);
        sig(i)=sig_tr(i);
```

```
            sigdiff(i)=sig(i)-SIG(i);
            sigvisco(i)=0;

        else                                        % Elastoplastic domain

            %Algorithm set of constitutive equations for the backward euler
            %method.

            %Iteration procedure to decide portion of strain to be plastic.



            %Inital conditions%
            maxiter=50;                                                 % Maximum nuber of iterations
            Tol=10e-9;                                                  % Acceptance tolerance to end iterations
            Dp=zeros(1,length(maxiter));                        %Initial plastic strain magnitude in step set to zero
            dDp=zeros(1,length(maxiter));                       %Establish plastic strain differ matrix
            dDp(1)=0.0000001;                                          %Intitial plastic strain magnitude differ set to small number, but higher than Tol to enter iteration procedure
            langInv=zeros(1,length(maxiter));                 %Establish inverse langevin matrix
            Fi=zeros(1,length(maxiter));                        %Establish plastic strain rate matrix
            Fi(1)=p0_dot*(exp(1/C*(((abs(sig_tr(i)-sigLang_tr(i)))/(sigsat+R_tr(i)))-1))-1); %Initial plastic strain rate
            Psi=zeros(1,length(maxiter));                       %Establish Psi  matrix (Reisdual from viscoplastic constitutive relation). Should be equal to zero if
```

```
plastic strain magnitude in step is correct
    Psi(1)=Fi(1)*dt(i-1)-Dp(1);                     %Initial Psi
    J=zeros(1,length(maxiter));                     %Establish Jacobean matrix
    sigIter=sig_tr(i);



    k=2;

    while abs(dDp(k-1))>=Tol


        srfactor=1-3/(B+1);          %Retraction ratio
        Dp(k)=Dp(k-1)+dDp(k-1);

        %Langevin%%%%%%%%%%%%%%%%%(Back-stress)
        epsp1=epsp(i-1)+Dp(k)*sign(sig_tr(i)-sigLang_tr(i));
        epsp2=epspt(i-1)+srfactor*Dp(k)*sign(sig_tr(i)-sigLang_tr(i));
        epsp3=epsp2;



        ld1=exp(epsp1); ld2=exp(epsp2) ; ld3=ld2;
        J=ld1*ld2*ld3;
        ld=sqrt((ld1^2+ld2^2+ld3^2)/3*J^(-2/3));
        langInv=(ld/ldL)*(3-(ld/ldL)^2)/(1-(ld/ldL)^2); %Pades approximate Langevin inverse
        sigLangIter=J^(-5/3)*Cr/3*ldL/ld*langInv*(ld1^2-1/3*(ld1^2+ld2^2*ld3^2));
        %----------------%
        sigIter=sig_tr(i)-(E0+E1+E2)*Dp(k);
        Riter=R_tr(i)*exp(-H*sign(sig_tr(i)-sigLang_tr(i))*Dp(k));


```

```

                Fi(k)=p0_dot*(exp(1/C*((abs(sigIter-sigLangIter)/(sigsat+Riter))-1))-1);
                Psi(k)=Fi(k)*dt(i-1)-Dp(k);
                J(k)=(Psi(k)-Psi(k-1))/(Dp(k)-Dp(k-1));
                dDp(k)=-Psi(k)/J(k);






        k=k+1;

        if k==maxiter
            break
        end
        end
        dp(i)=Dp(k-1);                                        %Plastic strain magnitude contribution
        depsp(i)=dp(i)*sign(sigIter-sigLangIter);             %Plastic strain contribution
        depspt(i)=srfactor*dp(i)*sign(sigIter-sigLangIter);
        %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
        %Parameters after elastoplastic step i%
        %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

        sig0(i)=sig0(i)-E0*depsp(i);
        sig1(i)=sig1(i)-E1*depsp(i);
        sig2(i)=sig2(i)-E2*depsp(i);
        sig(i)=sig0(i)+sig1(i)+sig2(i);                       %Updated stress
        sigdiff(i) =sig(i)-SIG(i);


        epsp(i)=epsp(i-1)+depsp(i);
        epspt(i)=epspt(i-1)+depspt(i);
        p(i)=p(i-1)+dp(i);
        R(i)=R(i-1)*exp(-H*depsp(i));
        sigLang(i)=sigLangIter;
        sigvisco(i)=sig(i)-sigLang(i)-(sigsat+R(i));
        z=z+1;                                                %Plastic step counter update
```

```
                sre(i)=(deps(i-1)-depsp(i))/dt(i-1);                %
                    Elastic strain rate
                srp(i)=depsp(i)/dt(i-1);                            %
                    Plastic strain rate
        end


end
end
```

Visco-elastic visco-plastic model with 3 Maxwell elements and part B as back-stress

Program

```
clc
clear all
%close all
k=1;
for j=1:13;
if j==13        %Numbers 1-13 decides which test to run
    for i=1:6

%% Import the datak
[~, ~, raw] = xlsread(['/Users/vegardkristensen/Documents/Masteroppgave/Matrialdata/Nytt/Rawdata/Test' num2str(j) '/HDPE-T' num2str(j) '-DICmatlab.xls'],'MAT');
raw = raw(6:end-40,i);

%% Create output variable
t = cell2mat(raw);
if i==1
    T{k,:}= t;
elseif i==4
    eps1{k,:}=t;
elseif i==5
    eps2{k,:}=t;
elseif i==6
    sigma{k,:}=t;

end

%% Clear temporary variables
clearvars raw;
end
k=k+1;
end
end
%%%%-----Interpolates extra history input values-----%%%%%
t=T{12};
EPS1=eps1{12};
EPS2=eps2{12};

```

```
t2(1)=t(1);
epsl(1)=EPS1(1);
epst(1)=EPS2(1);
for  i=2:2*length(t)-1
        if mod(i,2)==1
        t2(i)=t((i+1)/2);
        epsl(i)=EPS1((i+1)/2);
    epst(i)=EPS1((i+1)/2);
    else
        t2(i)=(t(i/2)+t(i/2+1))/2;
        epsl(i)=(EPS1(i/2)+EPS1(i/2+1))/2;
    epst(i)=(EPS2(i/2)+EPS2(i/2+1))/2;
    end
end
epsl(2*length(t))=EPS1(length(t));
epst(2*length(t))=EPS2(length(t));
t2(2*length(t))=t(length(t));
%-------------------------------------------------------------------%

%PLPA=[   P0          C           H         lDL       Cr         sigy          sigs         B          E0         E1          n1          E2         n2           E3        n3]
PLPAny= [0.0007,      0.0794,      66.94,   4.972,   2.448,   16,          24.95,      1.07,      521.7,    350.3,      1050.9,  172.1,   4302.5       218.8     52512  ];%input values if lsqnonlin is commented out
PLPAorg=[0.0007,      0.0795,      63.0,    2,       1.62,    16,          24.96,      1.07,      520.2,    516.8,      4023.4,  226.4,   56979.8      100       3000   ]; %initial value
PLPAmin=[0.0006999,  0.03,        10,      3.0,     0.1,     5,           16          1.06999,   520.1,    516.7,      4023.3,  226.3,   56979.7      99.99     2999.9]; %minimum value
PLPAmax=[0.0007001,  0.2,         100,     6.00,    5,       20,          30          1.07001,   520.3,    516.9,      4023.5,  226.5,   56979.9      100.01    3000.1]; %maximum value



FsigRelax=@(PLPA)viscoelasticviscoplasticVer23xMFUNC(t,PLPA,sig,eps,epst);
options=optimset('TolFun',1e-1000,'TolX',1e-1000,'maxFunevals',50,'MaxIter',25);
```

```
%[PLPAny,ll,fval]=lsqnonlin(FsigRelax,PLPAorg,PLPAmin,
    PLPAmax,options); %solver

%Plotting
[s,s0,s1,s2,s3,sL,sV,epsp,Sny]=
    viscoelasticviscoplasticVer23xMFUNC(t,PLPAny,sig,eps,
    epst);


%Plotting stress vs time
scrsz = get(0,'ScreenSize');
hFig = figure(1);
set(hFig, 'OuterPosition', [1 scrsz(4)/2 500 500])
plot(t,sig,t,Sny,t,s0,t,s1,t,s2,t,s3,t,sL)
grid on
legend('\sigma_{exp}','\sigma_{sim}','\sigma_{0}','\sigma_
    {1}','\sigma_{2}','\sigma_{3}','\sigma_{B}')
%axis([0 1 0 1])
xlhand = get(gca,'xlabel');
set(xlhand,'string','Time [s]','fontsize',14)
xlhand = get(gca,'ylabel');
```

Function

```
function [sigdiff,sig0,sig1,sig2,sig3,sigLang,sigvisco,epsp,sig] = viscoelasticviscoplasticVer23xMFUNC(t,PLPA,SIG,eps,epst)



%--------Elastic parameters---------%
E0=PLPA(9);
E1=PLPA(10);
n1=PLPA(11);
E2=PLPA(12);
n2=PLPA(13);
E3=PLPA(14);
n3=PLPA(15);
%----------------------------------------------%


%--------Plastic paramters for first part of plastic domain --------%
p0_dot=PLPA(1);
C=PLPA(2);
H=PLPA(3);
%-----------------------------------------------------------------------------------------%



%--------Plastic paramters for second part of plastic domain -------%
ldL=PLPA(4);
Cr=PLPA(5);
B=PLPA(8);
%-----------------------------------------------------------------------------------------%



%--------%Establish timestep (dt) & total strain step (deps) ------%
deps=zeros(1,length(t-1));
dt=zeros(1,length(t-1));
for i=1:length(t)-1
deps(i)=eps(i+1)-eps(i);
```

```
dt(i)=t(i+1)-t(i);
end
%----------------------------------------------------------------------------------%



%----------Initial conditions----------%

sigyield=PLPA(6);                            %Yield stress
sigsat=PLPA(7);                              %Saturation stress
sig0=zeros(1,length(eps));                   %Elastic string stress
sig1=zeros(1,length(eps));                   %Maxwell element 1 stress
sig2=zeros(1,length(eps));                   %Maxwell element 2 stress
sig3=zeros(1,length(eps));                   %Maxwell element 3 stress
sig=sig0+sig1+sig2+sig3;                     %Total stress set to zero
sig_tr=sig0 + sig1 + sig2+sig3;              %Total trial stress set to zero

epsp=zeros(1,length(eps));                   %Plastic strain
depsp=zeros(1,length(eps)-1);                %Plastic strain step
epspt=zeros(1,length(eps));
depspt=zeros(1,length(eps)-1);
p=zeros(1,length(eps));                      %plastic strain magnitude
dp=zeros(1,length(eps)-1);                   %Plastic strain magnitude step

sigLang=zeros(1,length(eps));                %Langevin stress set to zero (backstress)
sigLang_tr=sigLang;                          %Langevin trial stress set to zero (backstress)
R=-(sigsat-sigyield)*exp(-H*epsp);           %Ramping stress set to zero
R_tr=R;                                      %Ramping trial stress set to zero
%----------------------------------------%


```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%------------Calulation procedure---------------%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

sigdiff=zeros(1,length(eps));            %Difference in stresses between experimental and test (Output for LSQNonlin)
z=1;                                     %Simple counter to track plastic strain

for i=2:length(t)



    %Establish stresses for trial step (assume all is elastic)
    sig0(i)=sig0(i-1)+E0*deps(i-1);
    sig1(i)=sig1(i-1)*(1-(E1/n1)*dt(i-1))+E1*deps(i-1);
    sig2(i)=sig2(i-1)*(1-(E2/n2)*dt(i-1))+E2*deps(i-1);
    sig3(i)=sig3(i-1)*(1-(E3/n3)*dt(i-1))+E3*deps(i-1);
    sig_tr(i)=sig0(i)+sig1(i)+sig2(i)+sig3(i);      % Trial stress
    R_tr(i)=R(i-1);                                 % Trial ramping stress
    sigLang_tr(i)=sigLang(i-1);                     % Trail langevin stress


    %----Yield criterion----%
    f= abs(sig_tr(i)-sigLang_tr(i)) - (sigsat+R_tr(i));

    if f < 0                                        % Elastic domain


        dp(i)=0;                                    % No plastic contribution

        %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
        %Paramters after elastic step i%
        %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

        R(i)=R_tr(i);
        epsp(i)=epsp(i-1);
        epspt(i)=epspt(i-1);
```

```
            p(i)=p(i-1);
            sigLang(i)=sigLang_tr(i);
            sig(i)=sig_tr(i);
            sigdiff(i)=sig(i)-SIG(i);
            sigvisco(i)=0;

        else                                        % Elastoplastic domain

            %Algorithm set of constitutive equations for the backward euler
            %method.

            %Iteration procedure to decide portion of strain to be plastic.


            %Inital conditions%
            maxiter=50;                                             % Maximum nuber of iterations
            Tol=10e-9;                                              % Acceptance tolerance to end iterations
            Dp=zeros(1,length(maxiter));                            %Initial plastic strain magnitude in step set to zero
            dDp=zeros(1,length(maxiter));                           %Establish plastic strain differ matrix
            dDp(1)=0.0000001;                                       %Intitial plastic strain magnitude differ set to small number, but higher than Tol to enter iteration procedure
            langInv=zeros(1,length(maxiter));                       %Establish inverse langevin matrix
            Fi=zeros(1,length(maxiter));                            %Establish plastic strain rate matrix
            Fi(1)=p0_dot*(exp(1/C*(((abs(sig_tr(i)-sigLang_tr(i)))/(sigsat+R_tr(i)))-1))-1); % Initial plastic strain rate
            Psi=zeros(1,length(maxiter));
```

```
                                        %Establish Psi matrix (Reisdual from viscoplastic constitutive relation). Should be equal to zero if plastic strain magnitude in step is correct
        Psi(1)=Fi(1)*dt(i-1)-Dp(1);                                          %Initial Psu
        J=zeros(1,length(maxiter));                                         %Establish Jacobean matrix
        sigIter=sig_tr(i);



        k=2;

        while abs(dDp(k-1))>=Tol


                Dp(k)=Dp(k-1)+dDp(k-1);
                srfactor=1-3/(B+1);    %Retraction ratio

                %Langevin(Backstress)%%%%%%%%%%%%%%%%%
                epsp1=epsp(i-1)+Dp(k)*sign(sig_tr(i)-sigLang_tr(i));
                epsp2=epspt(i-1)+srfactor*Dp(k)*sign(sig_tr(i)-sigLang_tr(i));
                epsp3=epsp2;


                ld1=exp(epsp1); ld2=exp(epsp2) ; ld3=ld2;
                J=ld1*ld2*ld3;
                ld=sqrt((ld1^2+ld2^2+ld3^2)/3*J^(-2/3));
                langInv=(ld/ldL)*(3-(ld/ldL)^2)/(1-(ld/ldL)^2);  %Pades approximate Langevin inverse
                sigLangIter=J^(-5/3)*Cr/3*ldL/ld*langInv*(ld1^2-1/3*(ld1^2+ld2^2*ld3^2));
                %------------------------%

                sigIter=sig_tr(i)-(E0+E1+E2)*Dp(k);
                Riter=R_tr(i)*exp(-H*sign(sig_tr(i)-sigLang_tr(i))*Dp(k));
```

```



                Fi(k)=p0_dot*(exp(1/C*((abs(sigIter-sigLangIter)/(sigsat+Riter))-1))-1);
                Psi(k)=Fi(k)*dt(i-1)-Dp(k);
                J(k)=(Psi(k)-Psi(k-1))/(Dp(k)-Dp(k-1));
                dDp(k)=-Psi(k)/J(k);






        k=k+1;

        if k==maxiter
            break
        end
        end
        dp(i)=Dp(k-1);                                                  %Plastic strain magnitude contribution
        depsp(i)=dp(i)*sign(sigIter-sigLangIter);               %Plastic strain contribution
        depspt(i)=srfactor*dp(i)*sign(sigIter-sigLangIter);
        %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
        %Parameters after elastoplastic step i%
        %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

        sig0(i)=sig0(i)-E0*depsp(i);
        sig1(i)=sig1(i)-E1*depsp(i);
        sig2(i)=sig2(i)-E2*depsp(i);
        sig3(i)=sig3(i)-E3*depsp(i);
        sig(i)=sig0(i)+sig1(i)+sig2(i)+sig3(i);              %Updated stress

        sigdiff(i) =sig(i)-SIG(i);


        epsp(i)=epsp(i-1)+depsp(i);
        epspt(i)=epspt(i-1)+depspt(i);
        p(i)=p(i-1)+dp(i);
        R(i)=R(i-1)*exp(-H*depsp(i));
```

```
            sigLang(i)=sigLangIter;
            sigvisco(i)=sig(i)-sigLang(i)-(sigsat+R(i));
            z=z+1;    %Plastic step counter update
        end


end
end
```

D LS-DYNA keyword file

In Appendix D the LS-DYNA keyword file of the HDPE-DR3 simulation is put forward. Element and node information are included through own keyword files. The rest of the LS-DYNA keyword files have been delivered to the supervisors.

HDPE-DR3 keyword file

`quartermodelDR3stort.k`

```
$# LS-DYNA Keyword file created by LS-PrePost 4.0 - 11Jan201316:00
$# Created on Apr-08-2013 13:07:32
*KEYWORD
*TITLE
$# title
LS-DYNA keyword deck by LS-PrePost
*CONTROL_TERMINATION
$#  endtim    endcyc     dtmin    endeng    endmas
 1350.000         0     0.000     0.000     0.000
*CONTROL_TIMESTEP
$#  dtinit    tssfac      isdo    tslimt     dt2ms      lctm     erode     ms1st
     0.000  0.900000         0     0.000     0.000         0         0         0
$#  dt2msf   dt2mslc     imscl    unused    unused     rmscl
     0.000         0         0         0         0     0.000
*DATABASE_ELOUT
$#      dt    binary      lcur     ioopt   option1   option2   option3   option4
  0.500000         1         0         1         0         0         0         0
*DATABASE_GLSTAT
$#      dt    binary      lcur     ioopt
 50.000000         1         0         1
*DATABASE_NODOUT
$#      dt    binary      lcur     ioopt   option1   option2
  1.000000         1         0         1  0.001000         0
*DATABASE_SECFORC
$#      dt    binary      lcur     ioopt
  1.000000         1         0         1
*DATABASE_BINARY_D3PLOT
$#      dt      lcdt      beam     npltc    psetid
 50.000000         0         0         0         0
$#   ioopt
         0
*DATABASE_EXTENT_BINARY
$#   neiph     neips    maxint    strflg    sigflg    epsflg    rltflg    engflg
        36         0         3         1         1         1         1         1
$#  cmpflg    ieverp    beamip     dcomp      shge     stssz    n3thdt   ialemat
         0         0         0         1         1         1         2         1
$# nintsld   pkp_sen      sclp    unused     msscl     therm    intout    nodout
         0         0  1.000000         0         0         0STRESS    STRESS
$#    dtdt    resplt
         0         0
*DATABASE_CROSS_SECTION_SET_ID
$#    csid                                                                 title
         1CrossSection Force
$#    nsid      hsid      bsid      ssid      tsid      dsid        id     itype
         5         1         0         0         0         0         0         0
*SET_NODE_LIST_TITLE
Cross section
$#     sid       da1       da2       da3       da4    solver
         5     0.000     0.000     0.000     0.000MECH
$#    nid1      nid2      nid3      nid4      nid5      nid6      nid7      nid8
      1604       147       173       199       225       251       277       303
```

```
      1049      1050      1051      1052      1053      1054      1055      1056
      1201      1202      1203      1204      1301      1302      1303      1304
      1401      1402      1403      1404      1501      1502      1503      1504
      1601      1602      1603         0         0         0         0         0
*SET_NODE_LIST_TITLE
MoveTracking
$#     sid       da1       da2       da3       da4    solver
         6     0.000     0.000     0.000     0.000MECH
$#    nid1      nid2      nid3      nid4      nid5      nid6      nid7      nid8
      2128         0         0         0         0         0         0         0
*DATABASE_HISTORY_NODE_SET
$#     id1       id2       id3       id4       id5       id6       id7       id8
         6         0         0         0         0         0         0         0
*DATABASE_HISTORY_SOLID_SET
$#     id1       id2       id3       id4       id5       id6       id7       id8
         1         0         0         0         0         0         0         0
*BOUNDARY_SPC_SET
$#    nsid       cid      dofx      dofy      dofz     dofrx     dofry     dofrz
         1         0         0         0         1         0         0         0
*SET_NODE_LIST_TITLE
Zsymmetry
$#     sid       da1       da2       da3       da4    solver
         1     0.000     0.000     0.000     0.000MECH
$#    nid1      nid2      nid3      nid4      nid5      nid6      nid7      nid8
         1         2         3         4         5         6         7         8
         9        10        11        12        13        14        15        16
        17        18        19        20        21        22        23        24
        25        26        27        28        29        30        31        32
        33        34        35        36        37        38        39        40
        41        42        43        44        45        46        47        48
        49        50        51        52        53        54        55        56
        57        58        59        60        61        62        63        64
        65        66        67        68        69        70        71        72
        73        74        75        76        77        79        80        81
        82        83        84        85        87        88        89        90
        91        92        93        95        96        97        98        99
       100       101       103       104       105       106       107       108
       109       111       112       113       114       115       116       117
       119       120       121       122       123       124       125       127
       128       129       130       131       132       133       135       136
       137       138       139       140       141       142       143       144
       145       146       147       148       149       150       151       152
       153       154       155       156       157       158       159       161
       162       163       164       165       166       167       168       169
       170       171       172       173       174       175       176       177
       178       179       180       181       182       183       184       185
       187       188       189       190       191       192       193       194
       195       196       197       198       199       200       201       202
       203       204       205       206       207       208       209       210
       211       213       214       215       216       217       218       219
       220       221       222       223       224       225       226       227
       228       229       230       231       232       233       234       235
       236       237       239       240       241       242       243       244
       245       246       247       248       249       250       251       252
       253       254       255       256       257       258       259       260
       261       262       263       265       266       267       268       269
```

```
       270       271       272       273       274       275       276       277
       278       279       280       281       282       283       284       285
       286       287       288       289       291       292       293       294
       295       296       297       298       299       300       301       302
       303       304       305       306       307       308       309       310
       311       312       313       314       315       317       318       319
       320       321       322       323       325       326       327       328
       329       330       331       333       334       335       336       337
       338       339       341       342       343       344       345       346
       347       349       350       351       352       353       354       355
       357       358       359       360       361       362       363       365
       366       367       368       369       370       371       373       374
       375       376       377       378       379       380       381       382
       384       385       386       387       388       389       390       391
       392       393       395       396       397       398       399       400
       401       402       403       404       406       407       408       409
       410       411       412       413       414       415       417       418
       419       420       421       422       423       424       425       426
       428       429       430       431       432       433       434       435
       436       437       439       440       441       442       443       444
       445       446       447       448         0         0         0         0
*BOUNDARY_SPC_SET
$#    nsid       cid      dofx      dofy      dofz     dofrx     dofry     dofrz
         2         0         0         1         0         0         0         0
*SET_NODE_LIST_TITLE
Ysymmetry
$#     sid       da1       da2       da3       da4    solver
         2     0.000     0.000     0.000     0.000MECH
$#    nid1      nid2      nid3      nid4      nid5      nid6      nid7      nid8
        67        68        69        70        71        72        73        74
        75        76        77       127       128       129       130       131
       132       133       291       292       293       294       295       296
       297       298       299       300       301       302       303       304
       305       306       307       308       309       310       311       312
       313       314       315       365       366       367       368       369
       370       371       439       440       441       442       443       444
       445       446       447       448       713       714       715       716
       717       718       719       720       721       722       723       724
       725       726       727       728       729       730       731       732
       733       734       735       736       737       738       739       740
       741       742       743       744       745       746       747       748
       749       750       751       752       753       754       755       756
       925       926       927       928       929       930       931       932
       933       934       935       936       937       938       939       940
       941       942       943       944       945       946       947       948
       949       950       951       952      1553      1554      1555      1556
      1557      1558      1559      1560      1561      1562      1563      1564
      1565      1566      1567      1568      1569      1570      1571      1572
      1573      1574      1575      1576      1577      1578      1579      1580
      1581      1582      1583      1584      1585      1586      1587      1588
      1589      1590      1591      1592      1593      1594      1595      1596
      1597      1598      1599      1600      1601      1602      1603      1604
      1605      1606      1607      1608      1609      1610      1611      1612
      1613      1614      1615      1616      1617      1618      1619      1620
      1621      1622      1623      1624      1625      1626      1627      1628
      1629      1630      1631      1632      1633      1634      1635      1636
```

```
      1637      1638      1639      1640      1641      1642      1643      1644
      1645      1646      1647      1648      1649      1650      1651      1652
      1821      1822      1823      1824      1825      1826      1827      1828
      1829      1830      1831      1832      1833      1834      1835      1836
      1837      1838      1839      1840      1841      1842      1843      1844
      1845      1846      1847      1848      2089      2090      2091      2092
      2093      2094      2095      2096      2097      2098      2099      2100
      2101      2102      2103      2104      2105      2106      2107      2108
      2109      2110      2111      2112      2113      2114      2115      2116
      2117      2118      2119      2120      2121      2122      2123      2124
      2125      2126      2127      2128         0         0         0         0
*BOUNDARY_SPC_SET
$#    nsid       cid      dofx      dofy      dofz     dofrx     dofry     dofrz
         3         0         1         1         1         0         0         0
*SET_NODE_LIST_TITLE
Rigid part
$#     sid       da1       da2       da3       da4    solver
         3     0.000     0.000     0.000     0.000MECH
$#    nid1      nid2      nid3      nid4      nid5      nid6      nid7      nid8
       461       462       463       464       471       472       479       480
       487       488       495       496       503       504       511       512
       519       520       527       528       543       544       548       552
       556       560       564       568       572       576       587       588
       592       596       600       604       608       612       616       620
       631       632       636       640       644       648       652       656
       660       664       675       676       680       684       688       692
       696       700       704       708       719       720       724       728
       732       736       740       744       748       752         0         0
*BOUNDARY_SPC_SET
$#    nsid       cid      dofx      dofy      dofz     dofrx     dofry     dofrz
         4         0         0         1         1         0         0         0
*SET_NODE_LIST_TITLE
Moving part
$#     sid       da1       da2       da3       da4    solver
         4     0.000     0.000     0.000     0.000MECH
$#    nid1      nid2      nid3      nid4      nid5      nid6      nid7      nid8
      1855      1856      1863      1864      1871      1872      1879      1880
      1887      1888      1895      1896      1903      1904      1911      1912
      1919      1920      1927      1928      1932      1936      1940      1944
      1948      1952      1956      1960      1964      1968      1972      1976
      1980      1984      1988      1992      1996      2000      2004      2008
      2012      2016      2020      2024      2028      2032      2036      2040
      2044      2048      2052      2056      2060      2064      2068      2072
      2076      2080      2084      2088      2092      2096      2100      2104
      2108      2112      2116      2120      2124      2128         0         0
*PART
$# title
Plast
$#     pid     secid       mid     eosid      hgid      grav    adpopt      tmid
         1         1         1         0         0         0         0         0
*SECTION_SOLID_TITLE
Plastsection
$#   secid    elform       aet
         1         1         0
*MAT_USER_DEFINED_MATERIAL_MODELS
$#     mid        ro        mt       lmc       nhv    iortho     ibulk        ig
```

```
         1  940.E-05        44        24        34         1        15        16
$#   ivect     ifail    itherm    ihyper      ieos      lmca    unused    unused
         1         0         0         0         0
$#    aopt      mafc        xp        yp        zp        a1        a2        a3
  2.000000  1.000000     0.000     0.000     0.000  1.000000  0.000000     0.000
$#      v1        v2        v3        d1        d2        d3      beta
     0.000     0.000     0.000     0.000  1.000000     0.000     0.000
$        E        PR      Eps0        Ct        St        Cr       Lam     Alpha
 520.2E$     beta              Sigsat         H   Epsfail                 K         G
  1.07000      0.00     24.95     66.94      10.0          2339.8E$    E1/G1        t1        G2
516.8      7.785      226.4      251.64
$
*BOUNDARY_PRESCRIBED_MOTION_SET
$#    nsid       dof       vad      lcid        sf       vid     death     birth
         4         1         0         7  0.330000         0 19050.000     0.000
*DEFINE_CURVE
$#    lcid      sidr       sfa       sfo      offa      offo    dattyp
         7         0  0.100000   1.10000     0.000     0.000         0
$#                a1                  o1
                 0.000               0.000
                 1.000               1.000
             1196.0000               1.000
             1197.0000               0.000
              3217.000               0.000
              3218.000             -0.3000
              3251.000             -0.3000
              3252.000               0.000
             12500.000               0.000
*SET_SOLID_TITLE
Cross section
$#     sid    solver
         1MECH
$#      k1        k2        k3        k4        k5        k6        k7        k8
       811       812       813       814       911       912       913       914
      1011      1012      1013      1014      1111      1112      1113      1114
      1211      1212      1213      1214      1311      1312      1313      1314
*INCLUDE elementsolid.k
*INCLUDE node.k
*END
```