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Models for Punching Shear Capacity in Concrete Slabs

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TITLE:

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Beregningsmodeller for konsentrerte laster på betongkonstruksjoner

BY:
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SUMMARY:
Model Code 2010 was published in the spring of 2012 by the International Federation for Structural Concrete (*fib*), and presents a whole new model for punching shear design. Whereas the method for punching shear design in Eurocode 2 is mainly empirical, the method in Model Code 2010 is grounded in a physical model called the Critical Shear Crack Theory. The method for punching shear design in Model Code 2010 makes the foundation for the punching shear design procedure in the next edition of Eurocode 2.

In the first part of the thesis the background to shear and cracking, an introduction to punching shear and a presentation of the punching shear design procedure in Eurocode 2 is presented. Then a literature study is carried out, where the physical model behind the punching shear design in Model Code 2010 is presented in detail. This leads to the final formulations for the punching shear design in Model Code 2010. The next part of the thesis consists of design examples according to Model Code 2010 on an existing project in Oslo, originally designed according to Eurocode 2. The purpose of the calculations is to demonstrate the use of Model Code 2010 and to compare the results from this method to the results by Eurocode 2. Further on, linear analyses in a FEM program are performed, in terms of verifying some of the models used in the design in Eurocode 2. The models investigated are the shear distribution at the basic control perimeter defined in Eurocode 2 and the effect of openings in the slab close to the column edge.

The results from the design examples show how Eurocode 2 often underestimates the punching shear capacity. For slabs with large spans however, the capacity seems to be overestimated by Eurocode 2 compared to the results by Model Code 2010. The chance of obtaining too high capacities and therefore risk using a too low amount of shear reinforcement is one of the reasons why further research should be done on slender slabs in the future. The results from the linear analyses are satisfying concerning both models investigated. The shear stresses are conservative in Eurocode 2 compared to the analyses, but the values are quite close and the results by Eurocode 2 are on the safe side.

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CARRIED OUT AT: Department of Structural Engineering, NTNU

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NORGES TEKNISK NATURVITENSKAPELIGE
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Institutt for konstruksjonsteknikk

Masteroppgave i konstruksjonsteknikk 2013
for
Ingeborg Skarholt Bølviken

Beregningsmodeller for konsentrerte laster på betongkonstruksjoner

Models for Punching Shear Capacity in Concrete Slabs

OPPGAVE

Modeller og regler for beregning av konsentrerte laster på plater er ofte en kombinasjon mellom empiriske og teoretiske modeller. I 2010 ble den nye Model Code fra *fib* utgitt. Der behandles konsentrerte laster med en helt ny beregningsmodell.

Oppgaven skal først og fremst avdekke bakgrunnen og øke forståelsen for beregningsmodeller for dimensjonering av punktlast på plate. En viktig del av oppgaven er å sammenligne Model Code 2010 med Eurokode 2.

Oppgaven kan deles opp som følger:

- Grunnleggende om konsentrert last på plate
- Formelverk og bakgrunn for konsentrert last med fokus på Model Code 2010
- Sammenligninger med Eurokode 2
- Lineære FEM analyser som verifikasjon av modeller

Oppgaven skal være gjennomført innen den 10. juni 2013.

Trondheim den 31.01.2013

Jan Arve Øverli
Førsteamanuensis / Faglærer

Preface

This report is written as a Master Thesis at the Department of Structural Engineering at the Faculty of Engineering Science and Technology at the Norwegian University of Science and Technology. The thesis is written over a period of 20 weeks during the spring semester of 2013.

The report covers the phenomenon of punching shear, a failure mechanism as a result of concentrated loads on concrete slabs. The main focus has been on the theoretical background to the design procedure for punching shear in the Model Code from 2010, which forms the foundation for the next edition of Eurocode 2. From previous courses in concrete structures, I have seen how today's design rules in the Eurocode are difficult to use, and that the formulas are based on experiments. The punching shear design procedure in the Model Code is based on a physical model, and may therefore present more accurate results and at the same time give the designer a better understanding of the calculations during the design.

The report is divided in two parts, where the first part is a literature study that covers the background to shear and cracking, presents the calculation method used in today's Eurocode and the theoretical background to the design procedure for punching shear in Model Code 2010. Part two consists of calculation examples based on the method in the Model Code and verifications of some of the models used in the design in Eurocode 2 by modelling in the FEM-program DIANA.

I would like to thank my supervisor Jan Arve Øverli for good guidance throughout the semester and for always taking the time to answer my questions. I would also like to thank Max Hendrix for help with the part concerning the FEM-program DIANA. In addition to this I would like to thank Svein Barstad and Hans Auver Lahus in Multiconsult AS for the help of finding an appropriate project to base my calculations on and for general assistance during the semester.

Ingeborg Skarholt Bølviken

Summary

Model Code 2010 was published in the spring of 2012 by the International Federation for Structural Concrete (*fib*), and presents a whole new model for punching shear design. Whereas the method for punching shear design in Eurocode 2 is mainly empirical, the method in Model Code 2010 is grounded in a physical model called the Critical Shear Crack Theory. The method for punching shear design in Model Code 2010 makes the foundation for the punching shear design procedure in the next edition of Eurocode 2.

The first part of the thesis consists of the background to shear and cracking in 2D, an introduction to punching shear and a presentation of the punching shear design procedure in Eurocode 2. The main part of the assignment contains of a literature study, where the physical model behind the punching shear design in Model Code 2010, the Critical Shear Crack theory, is presented in detail. This leads to the final formulations for the punching shear design in Model Code 2010.

The next part of the thesis consists of design examples according to the formulations in Model Code 2010 on an existing project in Bjørvika in Oslo. The design of the building in this project has already been performed by Multiconsult AS, and the design is done according to the approach in Eurocode 2. The calculations in this master thesis are performed in terms of demonstrating the use of the design procedure in Model Code 2010 and to compare the results from this method to the results by the design approach in Eurocode 2.

Further on, linear analyses are performed on models of a slab-column connection in a FEM program, in terms of verifying some of the models used in the design in Eurocode 2. The models investigated are the shear distribution at the basic control perimeter defined in Eurocode 2 and the effect of openings in the slab close to the column edge.

The results from the design examples show how Eurocde 2 often underestimates the punching shear capacity, which confirms theories described in the literature study. For slabs with large spans however, the capacity seems to be overestimated by Eurocode 2 compared to the results by Model Code 2010. The design of all concrete structures in Norway today has to follow the design procedure in Eurocode 2, and the chance of obtaining too high capacities and therefore risk using a too low amount of shear reinforcement is one of the reasons why further research should be done on slender slabs in the future.

The results from the linear analyses in the FEM program are satisfying concerning both the shear stress distribution and the effect of slab openings. The values of the shear stresses are a little conservative in Eurocode 2 compared to the calculated values from the analyses, but the values are quite close and the results by Eurocode 2 are on the safe side. Still, more analyses should be performed in terms of concluding on anything here, although the results seem to be satisfying for the particular slab-column connection modelled in this thesis.

Sammendrag

Model Code 2010 ble publisert våren 2012 av the International Federation for Structural Concrete (*fib*), og presenterer en helt ny metode for dimensjonering for gjennomlokking. Mens beregningsmetoden for gjennomlokking i Eurocode 2 hovedsakelig er empirisk, baserer metoden i Model Code 2010 seg på en fysisk modell, the Critical Shear Crack Theory. Dimensjoneringsmetoden i Model Code 2010 legger grunnlaget for beregningsmetodene for konsentrerte laster som skal inkluderes i den neste utgaven av Eurocode 2.

Den første delen av oppgaven inneholder bakgrunnsteorien til skjær og riss i 2D, en introduksjon til begrepet gjennomlokking og presenterer metoden for dimensjonering for konsentrerte laster på betongplater i Eurocode 2. Hoveddelen av oppgaven består av en litteraturstudie, der den fysiske modellen bak metoden for gjennomlokkingsberegninger i Model Code 2010 er beskrevet i detalj. Denne modellen leder frem til de endelige formuleringene for dimensjoneringsprosedyren i Model Code 2010.

Neste del av oppgaven består av beregningseksempler basert på metodene i Model Code 2010 på et eksisterende prosjekt i Bjørvika i Oslo. Prosjekteringen av bygningen har blitt utført av Multiconsult AS, og dimensjoneringen er gjort i henhold til fremgangsmåten i Eurocode 2. Beregningene i denne masteroppgaven er utført for å demonstrere bruken av dimensjoneringsmetoden i Model Code 2010 og for å kunne sammenligne de oppnådde resultatene med resultatene etter Eurocode 2.

Videre er lineære analyser i et elementmetodeprogram utført på modeller av en forbindelse mellom søyle og dekke, for å kunne vurdere om et utvalg av modellene som er brukt til gjennomlokkingsdimensjonering i Eurocode 2 er rimelige. Modellene som undersøkes er fordelingen av skjærspenninger rundt den aktuelle kontrollomkretsen definert i Eurocode 2, samt effekten av åpninger i dekket i nærheten av søylekanten.

Resultatene fra beregningseksemplene viser hvordan Eurocode 2 ofte undervurderer gjennomlokkingskapasiteten, hvilket bekrefter teorier beskrevet i litteraturstudien. For dekker med store spennvidder derimot, kan det ofte virke som Eurocode 2 overestimerer kapasiteten sammenlignet med resultatene basert på metoden i Model Code 2010. Alle betongkonstruksjoner i Norge må i dag dimensjoneres etter Eurocode 2, og da sjansen er tilstede for at kapasiteten i noen tilfelles kan bli overestimert og man dermed risikerer å

tilføre for lite skjærarmering til systemet, er det nødvendig med mer forskning på slanke betongdekker i fremtiden.

Resultatene fra de lineære analysene i elementprogrammet er tilfredsstillende, både når det gjelder fordelingen av skjærspenninger langs den aktuelle kontrollomkretsen og effekten av åpninger i dekket nær søylekanten. De beregnede skjærspenningene etter Eurocode 2 er noe konservative sammenlignet med resultatene fra analysene, men verdiene avviker ikke så mye fra hverandre og resultatene fra Eurocode 2 er på den sikre siden. Før man kan konkludere med noe er det allikevel nødvendig å gjøre ytterligere analyser, selv om resultatene i dette tilfellet er av tilfredsstillende art.

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Chapter 1

Introduction

The use of concrete slabs supported by columns is today very common in buildings in Norway. Both for design purposes, economy reasons and to maximize the use of a storey in a building, it has become more and more requested that the slabs are as thin as possible and that the columns have small cross-sections. This causes the phenomenon of punching shear to become important, as the chance of this failure mechanism occurring increases as the column and slab dimensions decreases. Earlier, capitals were often used for load transfer from the slab to the column, but as the use of these capitals decreased, a lot of research has been done on the topic of punching shear.

All over the world today, different design codes are used for designing for punching shear. In Norway the design of concrete structures has to follow the design rules presented in Eurocode 2, where special rules for punching shear design are presented. Eurocode 2 is based on the Model Code from 1990, and most of the formulas presented are based on experiments and therefore empirical.

Model Code 2010 was published in the spring of 2012, and presents new methods for the punching shear calculations on concrete structures. The punching shear design method in this design code is based on a physical model, and gives the designer a better understanding of the phenomenon behind the calculation approach compared to using the empirical formulas in Eurocode 2. The punching shear approach in Model Code 2010 makes the foundation for the new design approach that is going to be included in the next version of Euroode 2.

The thesis is divided in two parts, and the first part covers the background to shear and cracking in beams, a brief introduction to the phenomenon of punching shear and a presentation of the calculation method used in Eurocode 2. After the background is presented, the Critical Shear Crack Theory and how this theory can be applied to slabs with and without shear reinforcement is described. The Critical Shear Crack Theory forms the background to the design approach used in Model Code 2010, so after the theoretical background and the use of the method are described, the final formulations in

Model Code 2010 are presented.

In the second part of the thesis calculations are performed according to the design approach in Model Code 2010, on a slab in a building in Oslo designed by the Norwegian consulting engineer firm Multiconsult AS. The design by Multiconsult AS follows the design procedures in Eurocode 2, as all buildings in Norway should, and the calculations done by the approach in Model Code 2010 are therefore compared to the results obtained by Multiconsult AS and the calculation procedure in Eurocode 2. The second part of the thesis also consists of verifications of two models used in the punching shear design in Eurocode 2. The models of interest are the model for the shear distribution at the basic control perimeter and the model used for punching shear control for loaded areas near slab openings. For the verifications of the two models in Eurocode 2, the FEM program DIANA is used for the modelling of a slab-column connection.

Chapter 2

Background

A general introduction to punching shear will be given in Section 2.3. However, the concept of punching shear is a complex problem in three dimensions. Before describing this problem in more detail, it is reasonable to start by describing the two dimensional theory of shear and cracks. In Sections 2.1 and 2.2 the general theories of shear and cracks in 2D are therefore presented. In Section 2.4 the design methods used in Eurocode 2 will be presented.

2.1 Shear in Beams

To illustrate the effect of 2D shear, a simply supported beam with a uniformly distributed load is considered, as shown in Figure 2.1.

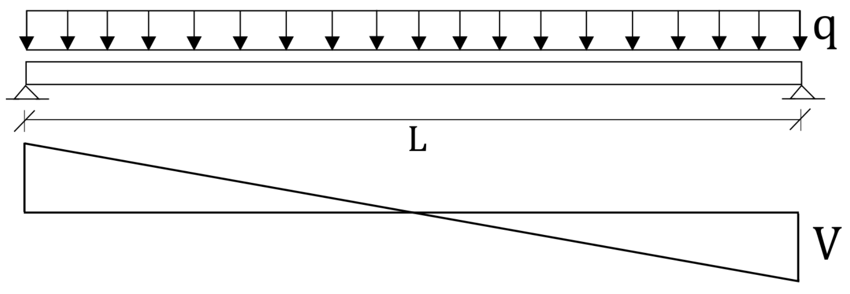


Figure 2.1: Simply supported beam with uniformly distributed load

The general theory used to describe the effect of shear on a material is the same for all homogeneous materials. Concrete however, is a non-homogeneous material, where the tension capacity is about ten percent of the compression capacity. This is the reason why cracks often develop in areas with tension in the concrete. The shear distribution in the cracked zones will therefore differ from the shear distribution in the uncracked zones [10].

The simply supported beam in Figure 2.1 is assumed to be reinforced only on the lower edge of the beam. If an assumption that the concrete has no ability to transfer tensile forces is made, all the tensile forces will be transferred by the reinforcement, and a distribution of forces as shown in Figure 2.2 is gotten.

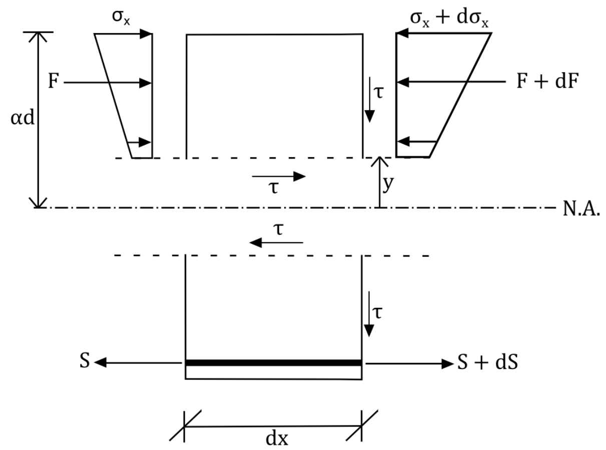


Figure 2.2: Horizontal section of the lower and upper part of the beam

From horizontal equilibrium of the cross-section below the neutral axis, the following relation is obtained:

$$\tau \cdot b \cdot dx = dS = \frac{dM}{z} \tag{2.1}$$

$$\tau = \frac{dM}{dx \cdot z \cdot b} = \frac{V}{z \cdot b} \tag{2.2}$$

After establishing the expression in Equation (2.2), the area above the neutral axis is evaluated. Horizontal equilibrium of the forces acting on the cross-section gives the relation given in Equation (2.3).

$$\tau \cdot b \cdot dx = dF \tag{2.3}$$

$$dF = \int_{A_2} d\sigma dA = \int_{A_2} \frac{dM}{I_c} y dA = \frac{dM}{I_c} \int_{A_2} y dA \tag{2.4}$$

In Equation (2.4), A_2 is the area of the cross-section above the dotted line in the upper edge of the beam and I_c is the second moment of area of the uncracked cross-section with reinforcement. Rewriting of Equation (2.3) gives the expression for τ given in Equation (2.5).

$$\tau = \frac{dM \cdot S_M}{dx \cdot I_c \cdot b} = \frac{V \cdot S_M}{I_c \cdot b} \tag{2.5}$$

$$S_M = \int_{A_2} y dA = y_{st} \cdot A_2 \tag{2.6}$$

Here, S_M is the static moment of the cross-section and y_{st} is the distance from the neutral axis to the centre of gravity of the area A_2.

For a rectangular cross-section the expressions for y_{st}, A_2 and S_M become as given in Equations (2.7), (2.8) and (2.9).

$$y_{st} = y + \frac{\alpha d - y}{2} = \frac{1}{2}(\alpha d + y) \tag{2.7}$$

$$A_2 = b(\alpha d - y) \tag{2.8}$$

$$S_M = \frac{1}{2}(\alpha d + y)(\alpha d - y)\, b = \frac{1}{2}\left(\alpha^2 d^2 - y^2\right) b \tag{2.9}$$

The distance αd is shown in Figure 2.2 and defined as the distance from the neutral axis to the top of the compression zone of the beam. For a rectangular cross-section, the following expression for τ is obtained:

$$\tau = \frac{V}{I_c} \cdot \frac{1}{2}\left(\alpha^2 d^2 - y^2\right) \tag{2.10}$$

Further on, an expression for the second moment of area of the uncracked cross-section with reinforcement, I_c, must be found. From Figure 2.3 the value of I_c can be found, with the assumptions that there are no stresses in the concrete in the tension zone.

From the material mechanics and Figure 2.3, the following relations are obtained:

$$\sigma_c = \frac{M}{I_c} \alpha d \tag{2.11}$$

$$M = T_c \cdot z = \frac{1}{2} \sigma_c b \alpha d \left(1 - \frac{\alpha}{3}\right) d \tag{2.12}$$

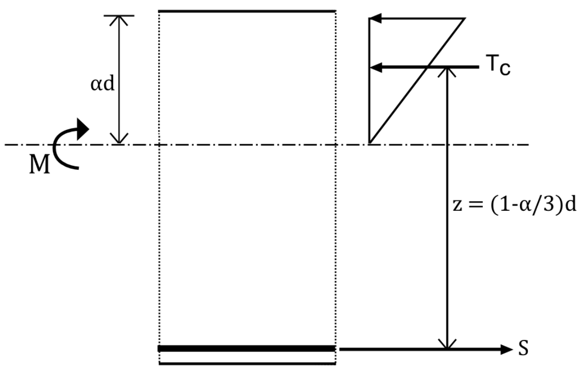


Figure 2.3: Cracked cross-section

$$\sigma_c = \frac{\frac{1}{2}\sigma_c \alpha \left(1 - \frac{\alpha}{3}\right) bd^2}{I_c} \cdot \alpha d \tag{2.13}$$

Restructuring of Equation (2.13) gives the expression for the second moment of area of the uncracked cross-section, I_c, given in Equation (2.14) [10].

$$I_c = \frac{1}{2}\alpha^2 \left(1 - \frac{\alpha}{3}\right) bd^3 \tag{2.14}$$

2.2 Cracking of Beams

In the following section, a brief introduction to the theory behind cracking of beams will be presented. Figure 2.4 shows both the diagonal cracks from the shear forces and the vertical cracks due to bending moment in the beam from Figure 2.1.

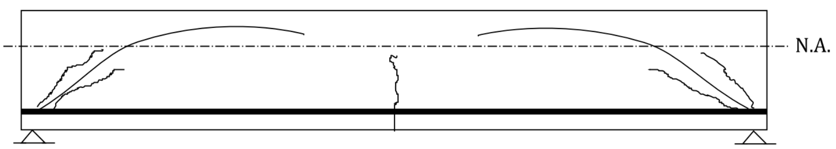


Figure 2.4: Cracking of beam

When the largest principal stress, σ_1, reaches the value for the characteristic tension capacity of the material, cracks in the concrete may propagate with an angle normal to σ_1. The direction of σ_1 depends on the sizes of the normal stress σ_x and the shear stress τ [10].

First, an element situated in the tension zone of the beam is evaluated. With the assumptions made in Section 2.1, the normal stresses in x- and y-direction, σ_x and σ_y, are both equal to zero. Figure 2.5 shows the stress distribution and the crack angle in a Mohr's circle for an element in the tension zone.

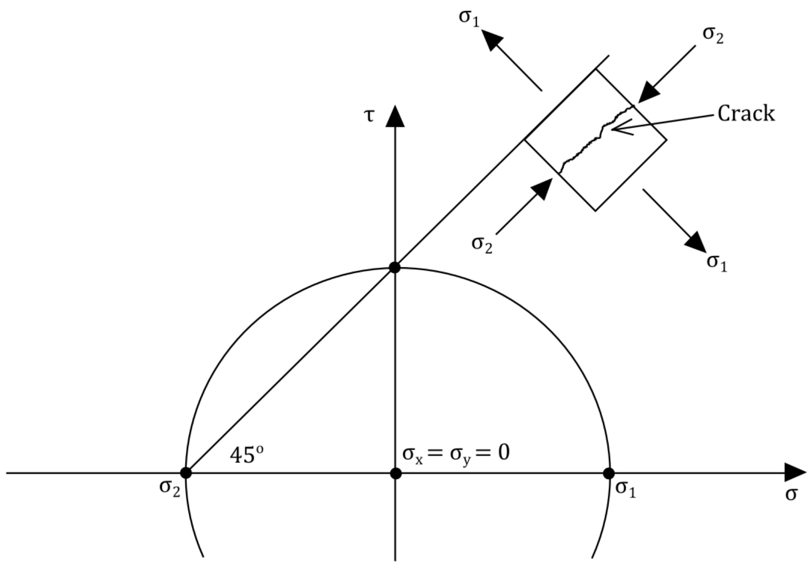


Figure 2.5: Mohr's circle for an element in the tension zone

As is seen in Figure 2.5, the cracks form an angle of 45°.

Then, an element situated in the compression zone of the beam is evaluated. Here, σ_x and τ are both unlike zero, and σ_y is equal to zero. This gives the Mohr's circle given in Figure 2.6 [10].

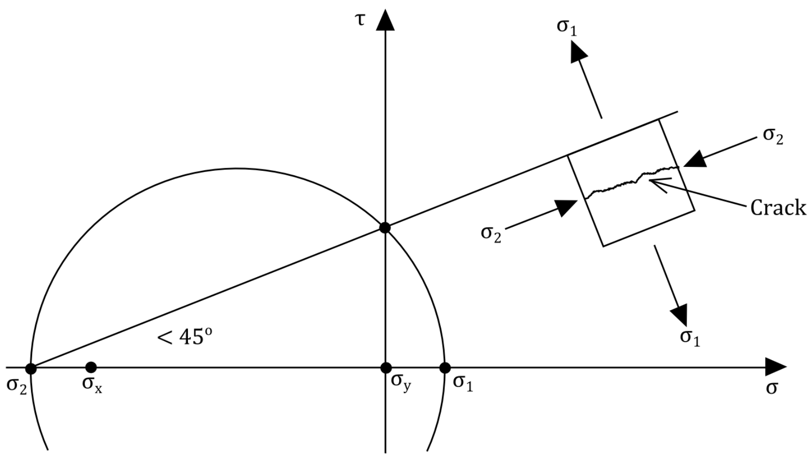


Figure 2.6: Mohr's circle for an element in the compression zone

The crack angle is now smaller than 45°, and the crack angle will continue to decrease as the value of σ_x increases. This explains why the crack angle becomes more and more horizontal as we move from the starting point of the compression zone and upwards in direction of the upper part of the beam, as shown in Figure 2.4.

2.3 Punching Shear in General

When a flat slab is exposed to a concentrated load larger than the capacity, the effect on the slab is referred to as punching shear. In these slabs, the shear force per unit length can become high close to the area of loading. If the capacity for shear punching in the slab is exceeded, a punching shear failure may occur within the discontinuity regions (D-regions) of the flat slab. This type of failure is a brittle failure mechanism, and may cause a global failure of the structure. Punching shear failure is a typical failure for slab-column connections [11]. Figure 2.7 shows an example of a global failure of a structure due to punching shear.

Figure 2.7: Global collapse of structure due to punching shear failure

Punching shear failure is a local failure mechanism, where diagonal tensile cracks form a failure surface around the loaded area of the slab. The failure occurs along a truncated cone shape in the structure, as shown in Figure 2.8 [11].

Concrete slabs supported by columns were first introduced in the US and Europe in the beginning of the 20th century. Capitals were often used to transfer the forces from the slab to the column. In the mid 50's, columns without these capitals became more demanded, since this simplified both the construction and the use of the building. Slabs supported directly on columns without capitals presented new problems concerning punching shear, and over the last 60 years many different methods for calculating the punching shear effect

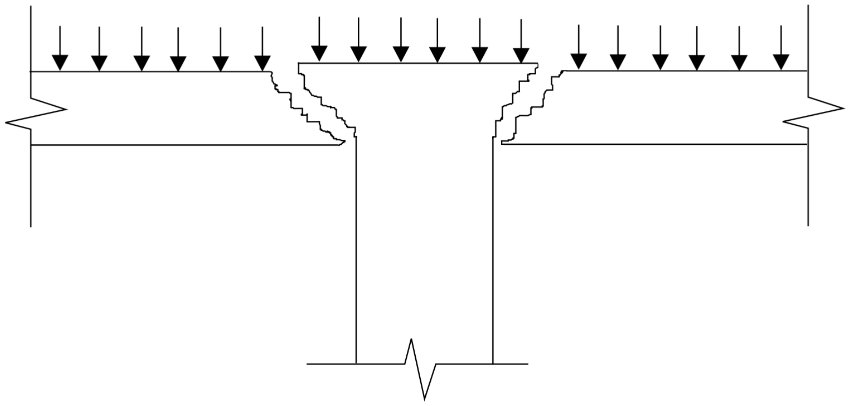


Figure 2.8: Typical shear failure in form of a truncated cone

on slabs have been presented [4].

A control perimeter at some distance from the loaded area defines the section for punching shear calculations. This control perimeter varies in the different methods for calculating punching shear [11]. Figure 2.9 shows basic control perimeters for two different design codes, Eurocode 2 and Model Code 2010.

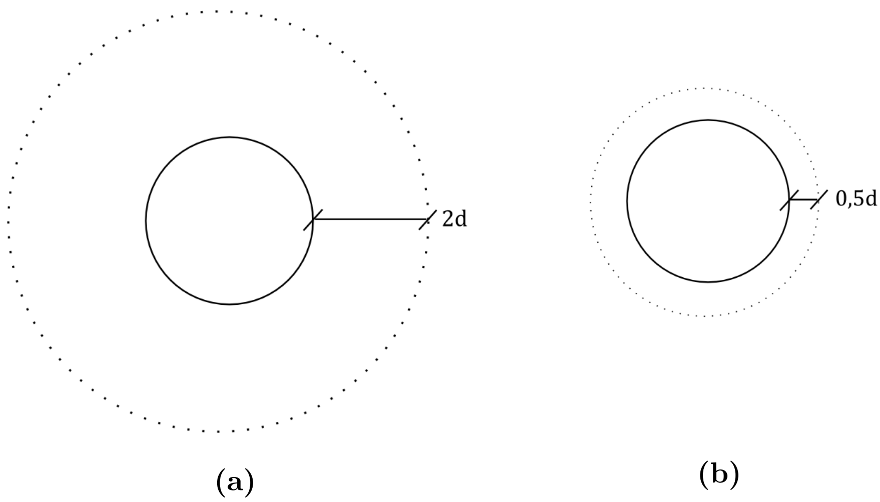


Figure 2.9: Basic control perimeter used for punching shear control in (a) Eurocode 2 and (b) Model Code 2010

Normal design practice is to always control for punching shear in cases where the structure functions as a flat slab. This means that the column has such dimensions that a shear control must be performed for the structure. If the critical section for punching shear for example cuts into a neighbouring beam structure, a shear control that differs from the pure punching shear check might be the designing control. In this case a punching shear check combined with a normal shear control in the beam might be necessary.

2.4 Punching Shear Design in Eurocode 2

The method used in designing for shear punching in "Eurocode 2: Design of concrete structures - Part 1-1: General rules and rules for buildings", from now on referred to as EC2, is based on the Model Code from 1990. The method in EC2 is based on experiments, and most of the formulas are therefore empirical. The next sections will briefly present the calculation method used in EC2. The formulas given in this section are gotten from Sections 6.4 and 9.4.3 in EC2 [1], and are mainly given for cases with uniformly distributed loading. This section will only cover the effect of punching shear on slabs, and punching shear on foundations will not be discussed.

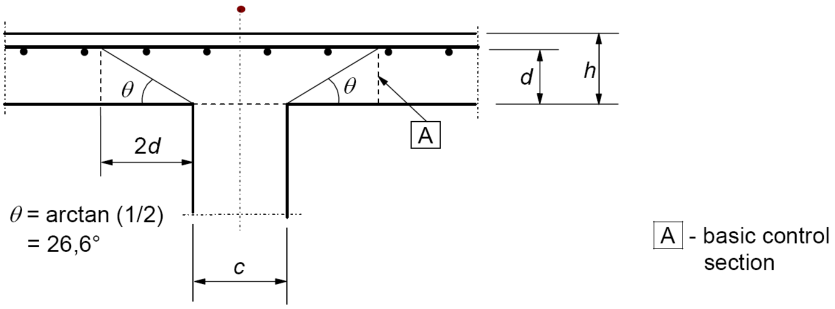


Figure 2.10: Critical control section for punching shear design [1]

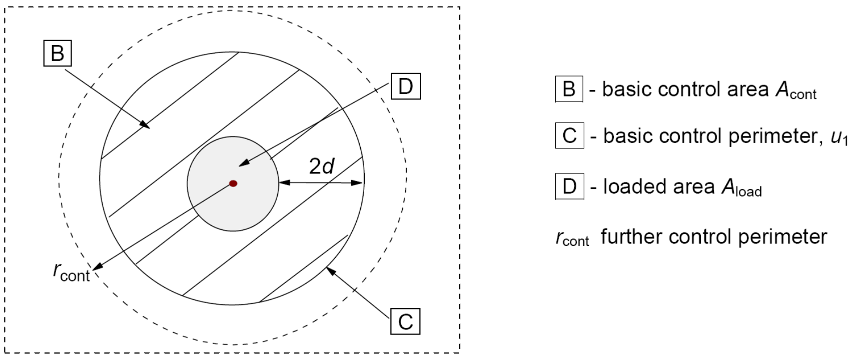


Figure 2.11: Control area for punching shear design [1]

A calculation model for control of the punching shear capacity in the ultimate limit state is shown in Figures 2.10 and 2.11, where A_{cont} is the basic control area, u_1 is the basic control perimeter, A_{load} is the loaded area and r_{cont} is the further control perimeter. The shear capacity is to be controlled at the edge of the column and at the basic control perimeter u_1.

2.4.1 Critical Control Perimeter

The critical control perimeter u_1 can normally be evaluated at the distance $2d$ from the loaded area, and should be constructed to minimize the length of the perimeter. Figure 2.12 shows examples of typical critical control perimeters around loaded areas.

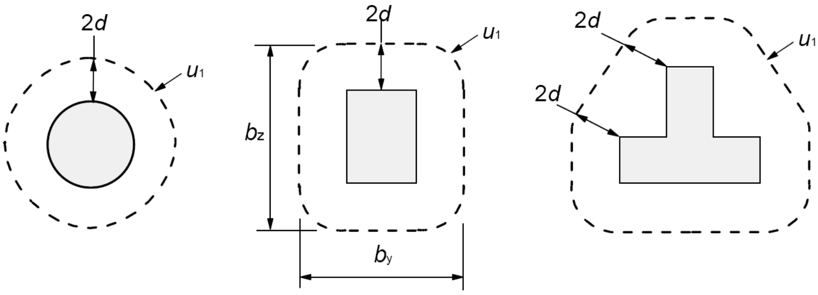


Figure 2.12: Typical critical control perimeters around loaded areas [1]

The effective thickness of the slab, d_{eff}, is assumed to be constant and is normally given by the expression in Equation (2.15).

$$d_{eff} = \frac{d_y + d_z}{2} \tag{2.15}$$

In Equation (2.15), d_y and d_z are the effective thicknesses for the reinforcement in two orthogonal directions.

If the loaded area is close to an opening in the slab, and the distance between the edge of the loaded area and the edge of the opening does not exceed $6d$, the critical control perimeter must be reduced. This reduction is done by assuming the part of the control perimeter contained between two tangents drawn from the centre of the loaded area to the outline of the opening to be ineffective, as shown in Figure 2.13.

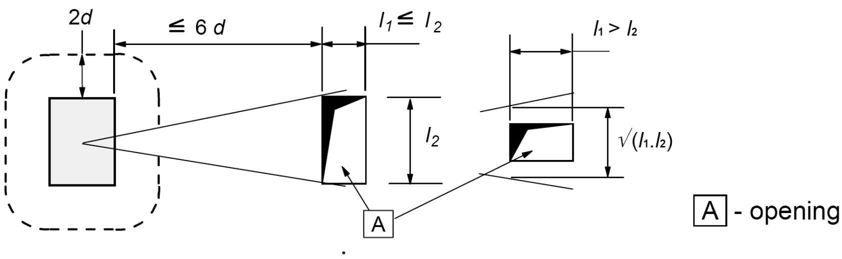


Figure 2.13: Critical control perimeter around loaded area close to slab opening [1]

For columns situated near a corner or an edge, this must be taken into account when defining the critical control perimeter. Figure 2.14 shows the critical control sections for different situations where a column is situated near a corner or an edge. The section including continuation to the edge of the slab must be smaller than the one defined by Figure 2.12.

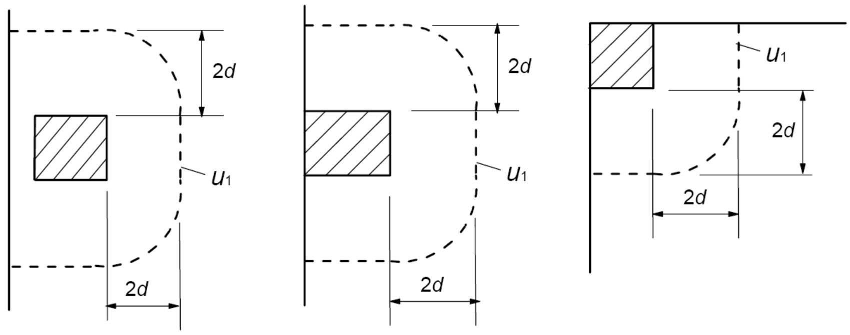


Figure 2.14: Critical control perimeters around loaded areas close to edges or corners [1]

Figure 2.15 shows the location of the control perimeter for a column with an enlarged circular column head. For $l_H < 2h_H$, as shown in Figure 2.15, the punching shear capacity must only be controlled at a critical control section outside the column head. The distance from the centre of the cross-section to the critical section, r_{cont}, can be defined as given in Equation (2.16) for a circular column.

$$r_{cont} = 2d + l_H + 0.5c \tag{2.16}$$

In Equation (2.16), l_H is the distance from the column edge to the edge of the column head and c is the diameter of a circular column.

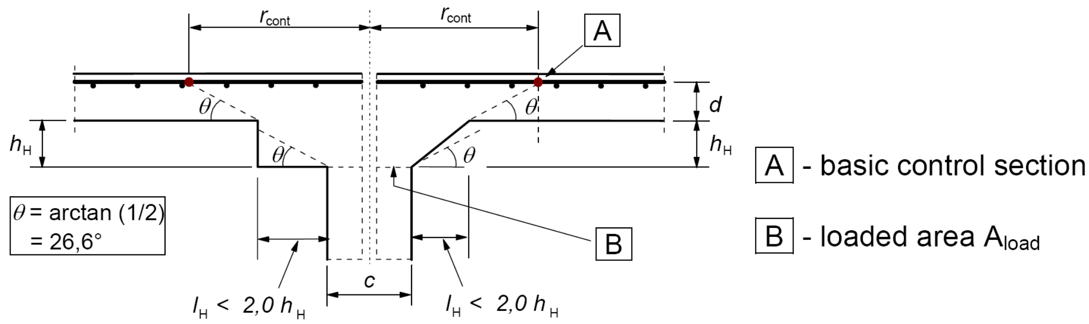


Figure 2.15: Slab supported by column with enlarged column head, for $l_H < 2h_H$ [1]

For a rectangular column with a rectangular column head, $l_H < 2h_H$ and edges with lengths l_1 and l_2, the value of r_{cont} can be given as the smallest of the values given in Equations (2.17) and (2.18).

$$r_{cont} = 2d + 0.56\sqrt{l_1 l_2} \tag{2.17}$$

$$r_{cont} = 2d + 0.69 l_1 \tag{2.18}$$

In Equations (2.17) and (2.18), $l_1 = c_1 + 2l_{H1}$, $l_2 = c_2 + 2l_{H2}$ and $l_1 \leq l_2$.

For $l_H > 2h_H$, as shown in Figure 2.16, the punching shear capacity must be controlled at a critical section outside the column head as well as inside the column head.

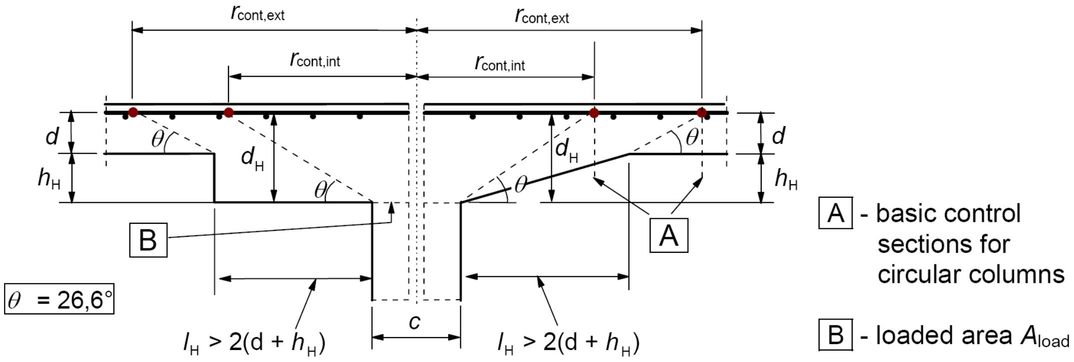


Figure 2.16: Slab supported by column with enlarged column head, for $l_H > 2h_H$ [1]

For circular columns the distance from the centre of the cross-section to the control section within the column head can be assumed as given in Equation (2.19).

$$r_{cont,int} = 2(d + h_H) + 0.5c \tag{2.19}$$

The distance from the centre of the cross section to the control section outside the column head can be assumed as in Equation (2.20).

$$r_{cont,ext} = l_H + 2d + 0.5c \tag{2.20}$$

2.4.2 Shear Force from Concentrated Loading

The controls given in Equations (2.21) and (2.22) must be performed for a slab with concentrated loading.

$$v_{Ed} \leq v_{Rd,max} \tag{2.21}$$

$$v_{Ed} \leq v_{Rd,c} \tag{2.22}$$

In Equations (2.21) and (2.22), v_{Ed} is the maximum shear stress from the concentrated loading, $v_{Rd,c}$ is the design value of the punching shear stress resistance along the control section for a slab without punching shear reinforcement and $v_{Rd,max}$ is the design value of the maximum punching shear stress resistance along the cross-section. If the condition in Equation (2.22) is not fulfilled, shear reinforcement has to be added to the section according to Section 2.4.4.

For an eccentric support reaction force, V_{Ed}, the value of v_{Ed} can be defined as given in Equation (2.23) for a rectangular column.

$$v_{Ed} = \beta \frac{V_{Ed}}{u_i d} \tag{2.23}$$

Here, u_i is the length of the considered control section, and β is given by Equation (2.24).

$$\beta = 1 + k \frac{M_{Ed}}{V_{Ed}} \frac{u_1}{W_1} \tag{2.24}$$

In Equation (2.24), u_1 is the length of the critical control perimeter, k is a coefficient that depends on the sizes of the column edges, as shown in Table 2.1, and W_1 corresponds to a shear distribution, as shown in Figure 2.17. W_1 is given by Equation (2.25), where dl is a length increment of the perimeter and e is the distance from dl to the axis where the moment M_{Ed} acts.

Table 2.1: Values of k for rectangular loaded areas

$\mathbf{c_1/c_2}$	≤ 0.5	1.0	2.0	≥ 3.0
k	0.45	0.60	0.70	0.80

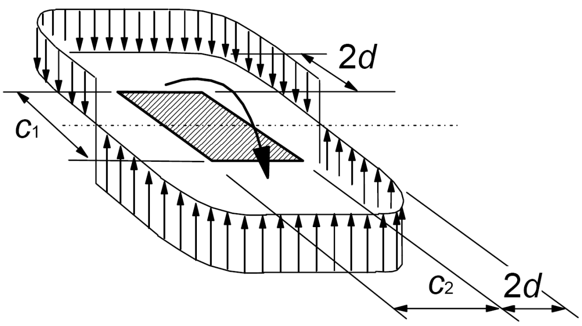


Figure 2.17: Shear distribution from an unbalanced moment at connection between slab and inner column [1]

$$W_1 = \int_0^{u_i} |e| \, dl \tag{2.25}$$

The expression for W_1, and therefore β, for rectangular columns differs from the expression for β for circular columns. The expressions also depend on whether there is eccentricity in one or two directions, and whether the column is an inner column, en edge column or a corner column. These expressions will not be given in detail here.

Where the adjacent spans do not differ in length by more than 25%, and the lateral stability does not depend on the frame action between the slab and the column, simplified expressions for β, as presented in Figure 2.18, may be used.

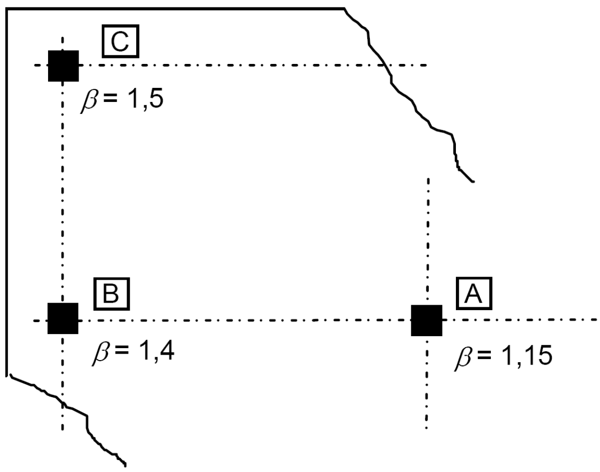


Figure 2.18: Recommended simplified values of β for internal column, edge column and corner column [1]

2.4.3 Punching Shear Capacity in Slabs Without Punching Shear Reinforcement

The punching shear capacity in slabs without punching shear reinforcement, v_{Ed}, is to be controlled at the critical section, as described in Section 2.4.2, and can be calculated by Equation (2.26).

$$v_{Rd,c} = C_{Rd,c}\, k\, \left(100\, \rho_l\, f_{ck}\right)^{1/3} + k_1\, \sigma_{cp} \geq \left(v_{min} + k_1\, \sigma_{cp}\right) \tag{2.26}$$

The value given for $C_{Rd,c}$ varies for different countries, and is in Norway set to be as in Equation (2.27).

$$C_{Rd,c} = \frac{0.18}{\gamma_c} \tag{2.27}$$

The values for k, ρ_l, σ_{cp} and v_{min} are given in Equations (2.28), (2.29), (2.30) and (2.31).

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2d \tag{2.28}$$

$$\rho_l = \sqrt{\rho_{ly} \cdot \rho_{lz}} \leq 0.02 \tag{2.29}$$

$$\sigma_{cp} = \frac{(\sigma_{cy} + \sigma_{cz})}{2} = \frac{\dfrac{N_{Ed,y}}{A_{cy}} + \dfrac{N_{Ed,z}}{A_{cz}}}{2} \tag{2.30}$$

$$v_{min} = 0.035\, k^{3/2}\, f_{ck}^{1/2} \tag{2.31}$$

ρ_{ly} and ρ_{lz} are the reinforcement ratios in y- and z-direction. σ_{cy} and σ_{cz} are the normal stresses in the concrete in y- and z-direction, resulting from the normal forces over the concrete areas A_{cy} and A_{cz}. f_{ck} is the characteristic compressive strength of the concrete.

The derivation of the formulation for the minim value of the punching shear capacity, v_{min}, given in Equation (2.31), is not presented in EC2. The derivation of this expression may be done by looking at a simply supported beam in 2D with an applied shear load V, as shown in Figure 2.19.

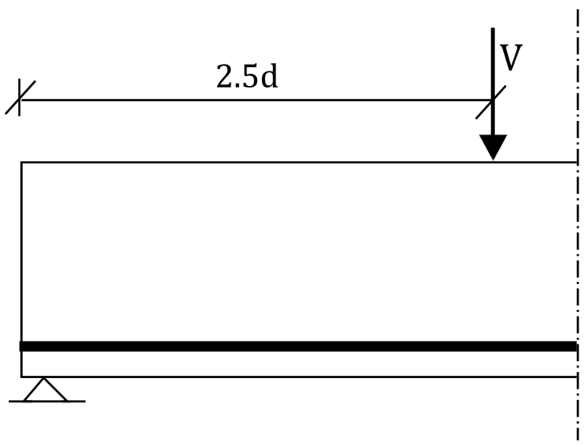


Figure 2.19: Simply supported beam with applied shear load at the distance $2.5d$ from the support

For the beam in Figure 2.19, the shear force is applied at the distance $2.5d$ from the support. The shear force is placed here, as it may be shown that the shear capacity in a beam is smallest at this particular distance from the support. The shear capacity will increase as the shear force moves closer to the support, as a larger fraction of the force will be transferred directly to the support.

At the distance $2.5d$ from the support, the moment may be given as:

$$M = V \cdot 2.5 \cdot d \tag{2.32}$$

As the distance $2.5d$ is the distance for which the shear capacity is smallest, this will also be the loading situation that in theory could give shear failure and failure due to lack of sufficient moment capacity at the same time. By inserting the expressions for the moment capacity M_{Rd} and the shear capacity $V_{Rd,c}$ into Equation (2.32), it is possible to find the reinforcement ratio that will result in this situation. Expressions for the moment capacity and the shear capacity are presented in Equations (2.33) and (2.34). The value of M_{Rd} is an approximated value, as the value of the compression arm of the cross-section is not given and assumed equal to $0.9d$.

$$M_{Rd} = A_s \cdot f_{yk} \cdot 0.9 \cdot d \tag{2.33}$$

$$V_{Rd,c} = C_{Rd,c}\, k\ (100\, \rho_l\, f_{ck})^{1/3}\ b\, d \tag{2.34}$$

The expression in Equation (2.34) is gotten from Equation (6.2.a) in EC2. In the commentary to EC2 it is described how the value of $C_{Rd,c}$ changes for different tests on concrete with different strengths, effective depths, reinforcement ratios and column diameters. Based on the test results described in the commentary, the value of 0.15 may be chosen as a lower bound for $C_{Rd,c}$ [12]. It should be noted that the material factor has not been included in this value. Based on this the following relation is obtained:

$$\begin{aligned} & V_{Rd,c} \cdot 2.5 \cdot d = M_{Rd} \\ & \rightarrow 0.15 \cdot k \cdot\ (100 \cdot \rho_l \cdot f_{ck})^{1/3}\ \cdot b \cdot d \cdot 2.5 \cdot d = A_s \cdot f_{yk} \cdot 0.9 \cdot d \\ & \rightarrow k \cdot\ (100 \cdot \rho_l \cdot f_{ck})^{1/3} = \frac{A_s}{b \cdot d} \cdot f_{yk} \cdot 2.4 \end{aligned} \tag{2.35}$$

Knowing that the reinforcement ratio $\rho_l = A_s / \,(b \cdot d)$, the expression may be rewritten as presented in Equation (2.36).

$$k \cdot 100^{1/3} \cdot \rho_l^{1/3} \cdot f_{ck}^{1/3} = \rho_l \cdot f_{yk} \cdot 2.4 \tag{2.36}$$

By separating ρ_l to one side of the equality sign, the following relation is obtained:

$$
\begin{aligned}
\rho_l^{2/3} &= \frac{100^{1/3}}{2.4} \cdot \frac{k \cdot f_{ck}^{1/3}}{f_{yk}} \\
\rho_l &= \frac{10}{2.4^{3/2}} \cdot \frac{k^{3/2} \cdot f_{ck}^{1/2}}{f_{yk}^{3/2}}
\end{aligned}
\tag{2.37}
$$

Inserting the final obtained expression for ρ_l into the expression for the punching shear capacity given in Equation (2.34), and now using the relation $C_{Rd,c} = 0.18/\gamma_c$, the final expression for v_{min} presented in Equation (2.31) is obtained as follows:

$$
\begin{aligned}
v_{min} &= \frac{0.18}{\gamma_c} \cdot k \cdot \left(100 \cdot \rho_l \cdot f_{ck}\right)^{1/3} \\
&= \frac{0.18}{1.5} \cdot k \cdot \left(100 \cdot \frac{10}{2.4^{3/2}} \cdot \frac{k^{3/2} \cdot f_{ck}^{1/2}}{f_{yk}^{3/2}} \cdot f_{ck}\right)^{1/3} \\
&= 0.12 \cdot k \cdot \left(\frac{1000}{2.4^{3/2} \cdot 500^{3/2}} \cdot k^{3/2} \cdot f_{ck}^{3/2}\right)^{1/3} \\
&= 0.035 \cdot k^{3/2} \cdot f_{ck}^{1/2}
\end{aligned}
\tag{2.38}
$$

Here, the value of 500 MPa is used at the yield strength of reinforcement steel, as this is the common steel quality used in Norway.

2.4.4 Punching Shear Capacity in Slabs With Punching Shear Reinforcement

In cases where punching shear reinforcement is shown to be necessary, the design value of the punching shear stress resistance for a slab with punching shear reinforcement, $v_{Rd,cs}$, can be calculated by the expression given in Equation (2.39).

$$
v_{Rd,cs} = 0.75 v_{Rd,c} + 1.5 \frac{d}{s_r} A_{sw} f_{ywd,ef} \frac{1}{u_1 d} \sin\alpha \tag{2.39}
$$

In Equation (2.39) A_{sw} is the area of one perimeter of shear reinforcement around the column, s_r is the radial spacing of perimeters of shear reinforcement, d is the mean value of the effective depth in orthogonal directions, α is the angle between the plane of the slab and the shear reinforcement and $f_{ywd,ef}$ is the effective design strength of the punching shear reinforcement, given by Equation (2.40).

$$f_{ywd,ef} = 250 + 0.25d \leq f_{ywd} \tag{2.40}$$

At the edge of a column, the punching shear capacity must be smaller than $v_{Rd,max}$, as given in Equation (2.41).

$$v_{Ed} = \frac{\beta V_{Ed}}{u_0\, d} \leq v_{Rd,max} \tag{2.41}$$

In Equation (2.41), β is as given in Section 2.4.2 and u_0 is the control perimeter at the edge of the column. The value of $v_{Rd,max}$ varies for different countries, and is in Norway set to be like the expression given in Equation (2.42).

$$v_{Rd,max} = min\left(0.4 \cdot \nu \cdot f_{cd};\ 1.6 \cdot v_{Rd,c} \cdot \frac{u_1}{\beta\, u_0}\right) \tag{2.42}$$

The control section where shear reinforcement is not necessary is given by Equation (2.43).

$$u_{out,ef} = \frac{\beta V_{Ed}}{v_{Rd,c}\, d} \leq 1.6 \cdot v_{Rd,c} \cdot \frac{u_1}{\beta\, u_0} \tag{2.43}$$

2.4.5 Punching Shear Reinforcement

If punching shear reinforcement is shown to be necessary, the punching shear reinforcement should be placed between the loaded area and the length kd within the perimeter where shear reinforcement is not necessary, $u_{out,ef}$. The value of k varies for different countries, and can in Norway be set equal to 1.0.

Link legs should be provided in at least two perimeters, and the spacing of the link leg perimeters should not exceed $s_{r,max} = 0.75d$. The spacing of link legs around a perimeter, $s_{t,max}$, should not exceed $1.5d$ within the first perimeter and should not exceed $2d$ outside the first perimeter. The criteria for spacing of link legs are shown in Figure 2.20.

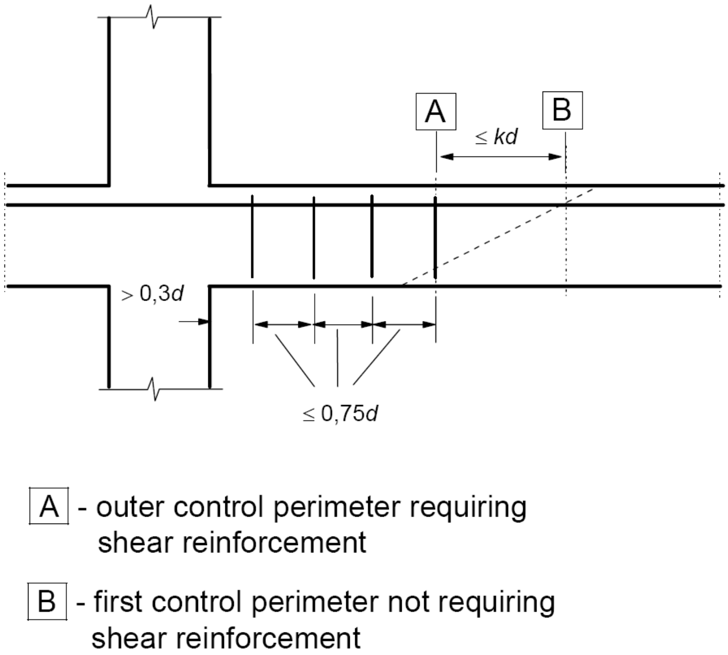


Figure 2.20: Spacing of link legs [1]

Where punching shear reinforcement is provided, the area of a link leg is given by Equation (2.44).

$$A_{sw,min} \cdot (1.5 \cdot \sin\alpha + \cos\alpha) \,/\, (s_r \cdot s_t) \geq 0.008 \frac{\sqrt{f_{ck}}}{f_{yk}} \tag{2.44}$$

Here, α is the angle between the shear reinforcement and the main reinforcement, s_r is the spacing of shear links in radial direction and s_t is the spacing of links in tangential direction.

The spacing of bent-up bars will not be described in this thesis.

Chapter 3

The Critical Shear Crack Theory

When designing new structures by looking at the strength of existing ones, a levels-of-approximation (LoA) approach have often been used by engineers over the world. This makes it possible to do very simple and not too time consuming analyses for preliminary design, and more time consuming analyses in more detailed design [2]. These methods are on the other hand not always possible to follow, as they require refining of parameters used in the design, which in some cases are impossible. Examples of this are empirical formulas, like the ones used in the punching shear design in EC2.

Although empirical formulas often give satisfying results in structural design, a mechanical representation of a phenomenon is often wanted, as it gives the designer a better physical understanding of the problem. In addition to this, mechanical models are often more consistent than the empirical methods [3].

In 1960 in Sweeden, *Kinnunen and Nylander* developed the first rational approach based on a physical model for designing for punching shear. Although this approach described the behaviour of punching shear in concrete well, the formulas presented for the calculations were somewhat complicated. This resulted in a low degree of implementation of this procedure in different designing codes over the world. Thorough research has been done on the topic since *Kinnunen and Nylander* presented their theory in 1960, and today physical models rather than empirical models have been implemented to different codes over the world, including "Model Code 2010 - Final draft - Volume 1 & 2", from now on referred to as MC2010 [2].

The method for designing for punching shear in MC2010 is based on a physical model called the Critical Shear Crack Theory (CSCT), considering punching shear behaviour in structures with and without transverse reinforcement. The principles of the CSCT in the design for punching shear were introduced by *Muttoni and Schwarts* in 1991 [2]. A lot of research has been done on the use of this method, and it is shown to be satisfying both in terms of calculation simplicity and precision [3].

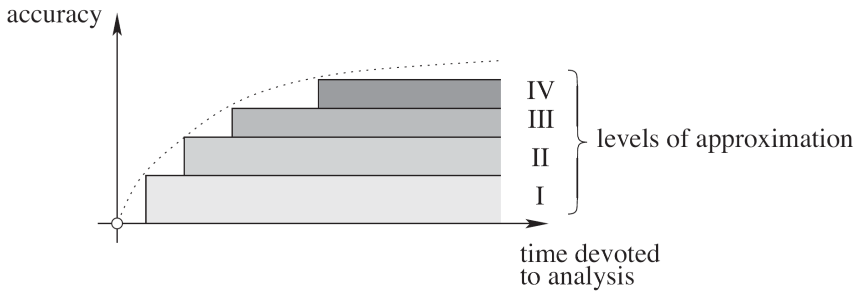


Figure 3.1: Accuracy as a function of time for the different levels of approximation [2]

Since the accuracy of the strength estimated by the CSCT depends on the levels of approximation of the hypothesis used in the model, MC2010 uses a LoA approach in calculating the shear strength. Figure 3.1 shows the different LoAs in a graph, with time devoted to the analysis represented on the x-axis and the accuracy represented on the y-axis. As an example, LoA I would require shorter calculation time but would also give a lower degree of accuracy, compared to LoAs II, III and IV. LoA I would therefore be quite effective in for example preliminary design, where a lower degree of accuracy is often requested. As the project evolves from preliminary design to a construction project, more time is often allowed as the precision is more important [2].

3.1 The Fundamentals of the CSCT

The CSCT is based on the assumption that the shear strength in a concrete member without transverse reinforcement is governed by the roughness and the opening of the critical crack developed in a compression strut in the structure, as is shown in Figure 3.2 [2] [3].

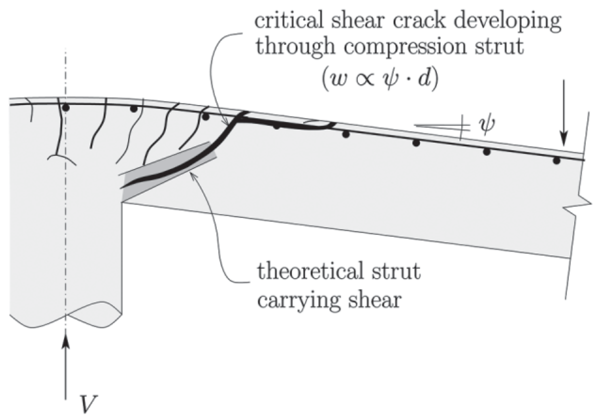


Figure 3.2: Position of critical shear crack developing through compression strut [2]

Further on, by assuming a free-body with kinematics at failure defined by the rotation of the slab, the shear strength can be calculated. With this assumption we get a development of tensile stresses and stresses due to aggregate interlock along the critical shear crack. Aggregate interlock can be described as the aggregate's effect on load transfer in compression and shear, and the size of the aggregate interlock effect is affected by the roughness of the aggregate. The concept of aggregate interlock is show in Figure 3.3. The shear strength is found by integrating the contributions of the tensile stresses and the stresses due to aggregate interlock along the critical shear surface [2].

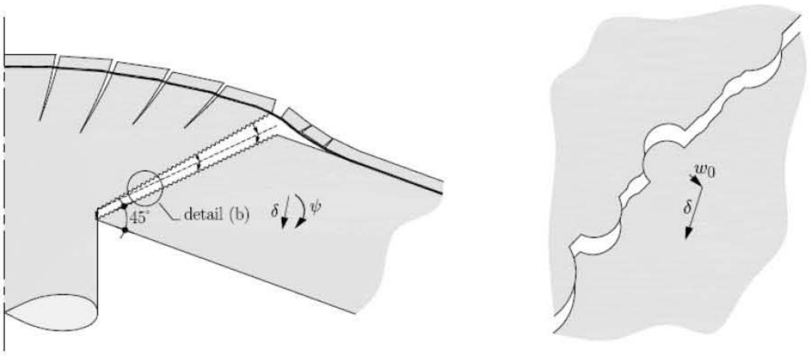


Figure 3.3: Aggregate interlock activation [3]

3.2 Punching Shear Strength by the CSCT

The theory that critical cracks in the slabs play an important role in the punching shear strength of a slab has been widely supported in the literature. It has been shown that, after reaching a maximum for a certain load level, the radial compressive strain in the bottom of the slab near the column begins to decrease again. Further on, a development of an elbow-shaped strut with a tensile member along the bottom of the member is caused by the development of the critical shear crack. This is the reason why tensile strains may be observed shortly before punching [4]. The development of the elbow-shaped strut and the radial compressive strain as a function of load level is shown in Figures 3.4 and 3.5.

The scheme of integrating the contributions of the tensile stresses and the stresses due to aggregate interlock along the critical shear surface to find the shear strength, with different variations of the mechanical parameters, was done by *Guidotti*. Figure 3.6a shows the results from the numerical integration procedure [13]. In Figure 3.6b the results from 99 punching shear tests by *Muttoni* are shown, and in Figure 3.6c the failure band from Figure 3.6a and the results presented in Figure 3.6b are compared to each other.

The shear strength decreases as the crack angle, shown in Figures 3.2 and 3.3, increases. This is logical, since both the tensile stresses in the concrete and the aggregate interlock effect is reduced as the angle increases. An increased opening of the critical crack will have the same effect; the shear strength will decrease as the crack opening increases.

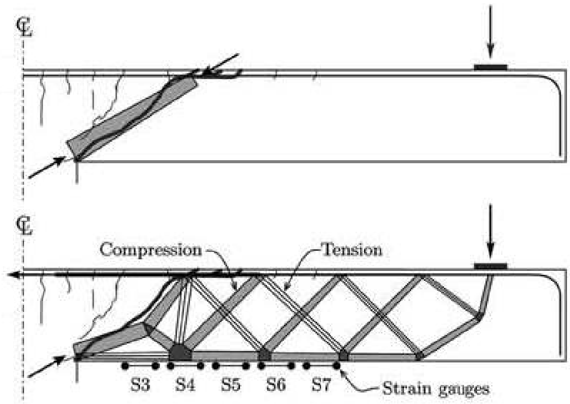


Figure 3.4: Development of elbow-shaped strut [4]

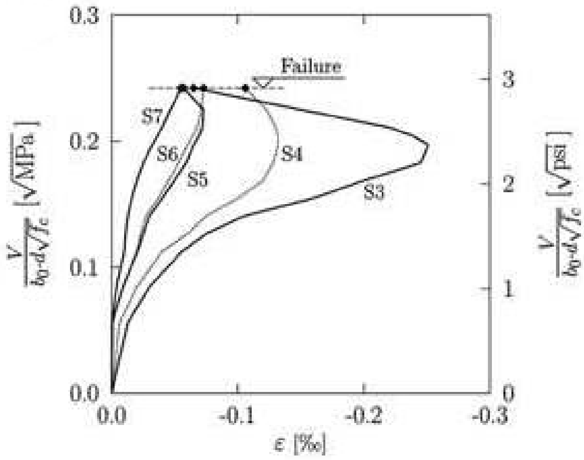


Figure 3.5: Radial strains in slab as a function of applied load [4]

From Figure 3.6a it is observed that the failure occurs in a narrow band for all cases. In many cases, the somewhat complicated and time consuming integration scheme is therefore not necessary. Based on this, *Muttoni* proposed the simplified failure criterion given in Equation (3.1) [3].

$$\frac{V_R}{b_0 \cdot d_v} = \sqrt{f_c} \cdot f\left(w, d_g\right) \tag{3.1}$$

In Equation (3.1) the punching shear strength is a function of the opening and roughness of the critical shear crack. Further on, V_R is the shear strength, d_v the shear-resisting effective depth of the member, f_c the compressive strength of the concrete, w the width of the critical shear crack, d_g the maximum size of aggregate accounting for the roughness of the cracks' lips and b_0 is the shear-resisting control perimeter. b_0 is set at the distance $d_v/2$ from the edge of the support region [5].

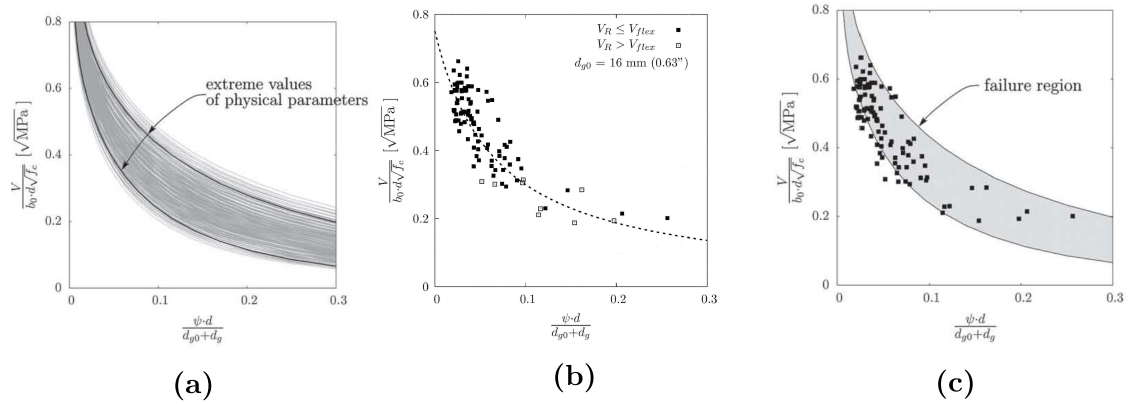


Figure 3.6: (a) Results from tests by *Guidotti* [2], (b) 99 experimental results by *Muttoni*, based on figure from [4], and (c) comparison of failure band and results from 99 punching shear tests [2]

In terms of evaluating the size of the critical shear crack, w, *Muttoni and Schwartz* assumed it to be proportional to the rotation of the slab, ψ, multiplied with the depth of the member, d, as shown in Figure 3.2.

$$w \propto \psi \cdot d \tag{3.2}$$

The assumptions that lead to Equation (3.2), lead to the semi empirical failure formulation given in Equation (3.3).

$$\frac{V_E}{b_0 \cdot d_v^3 \sqrt{f_c}} = \frac{1}{1 + \left(\dfrac{\psi \cdot d}{4\,\mathrm{mm}}\right)^2} \tag{3.3}$$

In Equation (3.3) all the parameters are given in SI-units. The amount of shear that can be transferred over a critical shear crack depends on the size and the roughness of the crack, which again is a result of the size of the aggregate. According to *Walraven* and *Vecchio and Collins*, this can be accounted for by dividing the crack width to the sum of the maximum aggregate size, d_g, and the aggregate reference size, d_{g0}. These assumptions led to *Muttoni's* formulation of the criterion for punching shear failure for members without transverse reinforcement given in Equation (3.4) [4].

$$\frac{V_{Rd}}{b_0 \cdot d_v \sqrt{f_c}} = \frac{3/4}{1 + 15\dfrac{\psi \cdot d}{d_{g0} + d_g}} \tag{3.4}$$

In Equation (3.4), d_{g0} is the reference aggregate size equal to 16 mm, and all the units are input in the equation as SI-units [5].

In Figure 3.6b the dotted line symbolizes the simplified failure criterion given in Equation (3.4), and the criterion makes a good match with the test results from the 99 punching shear tests.

3.3 Load-rotation Relation

Figure 3.7a was presented in Section 3.2 and presents the results from 99 punching shear tests done by *Muttoni*. The dotted line symbolizes the punching shear failure criterion given in Equation (3.4). Figure 3.7b presents the load-rotation relation for punching test by *Kinnunen and Nylander* done on slabs for different reinforcement ratios [4].

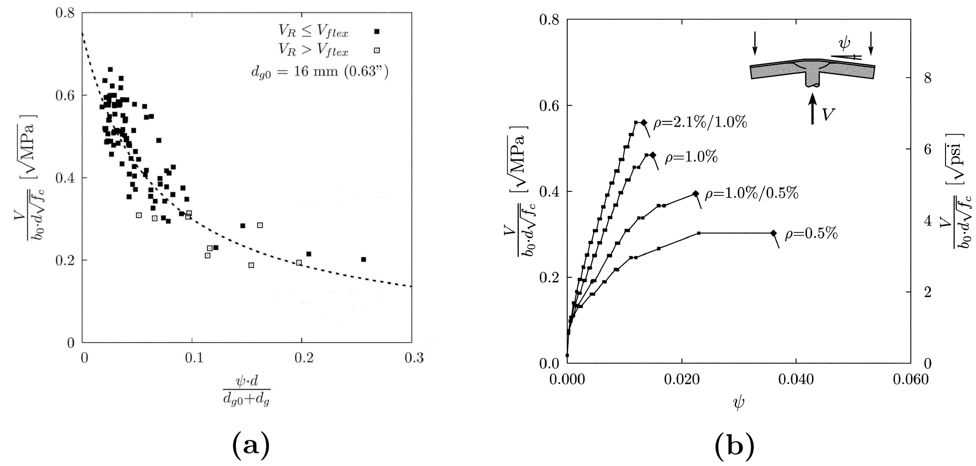


Figure 3.7: (a) 99 experimental results by *Muttoni*, based on figure from [4], and (b) load-rotation curves for tests by *Kinnunen and Nylander* [4]

The behaviour for the different reinforcement ratios varied. In Figure 3.7b the horizontal line for the reinforcement ratio of $\rho = 0.5\%$ shows the ductile behaviour of the slab, with yielding of the entire flexural reinforcement. This causes the strength to be dominated by the flexural capacity, and punching failure occurs after large plastic deformations, and at the end of the plastic area.

For reinforcement ratios up to $\rho = 1.0\%$, punching shear failure occurs before yielding of the entire flexural reinforcement, and for reinforcement ratios up to $\rho = 2.0\%$, punching occurs before any reinforcement has started to yield.

Figure 3.7b clearly shows that the punching shear capacity is influenced by the reinforcement ratio, and increasing the reinforcement causes a higher punching shear strength. On the other hand, an increased reinforcement ratio reduces the ductility of the slab and therefore reduces the deformation capacity [4].

Figure 3.7 shows that punching shear failure occurs at the intersection point of the failure criterion and the load-rotation curve. To evaluate the punching shear strength, the connection between the applied load V and the rotation ψ must be defined. The load-rotation relation can often be found by a numerical simulation of the slab, and in axis-symmetric cases an integration of the relationship between the moment and curvature can be done directly numerically [4].

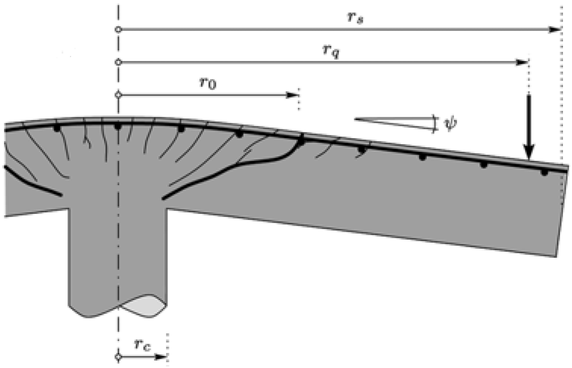


Figure 3.8: Rotation of slab with geometrical parameters [4]

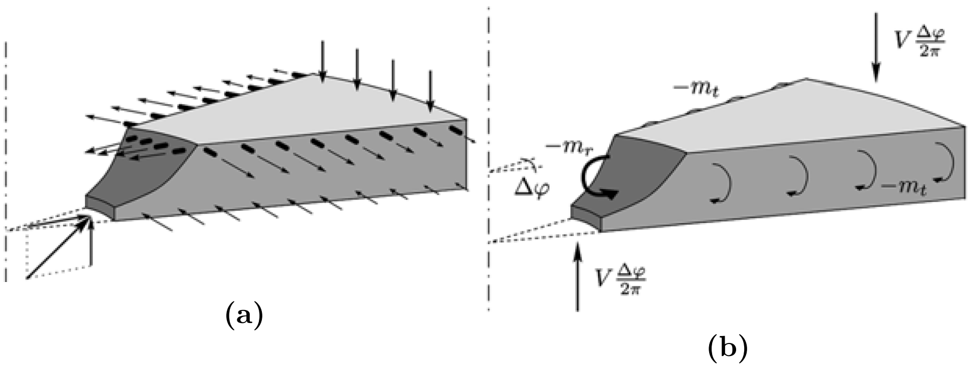


Figure 3.9: (a) External and (b) internal forces acting on slab [4]

Numerical integration can in many cases be relatively time consuming and is often not necessary. For the use in design codes, simplifications may be done such that the load-rotation relation can be calculated without the use of numerical integration [4].

Figure 3.8 shows a flat slab with geometrical parameters and the rotation ψ, and Figure 3.9 shows the forces acting on the slab. r_0 is assumed to be at a distance d from the column and r_s is the radius of the slab. Near the column, the radial curvature and the tangential cracks are concentrated, and as we move further away from r_0, the radial curvature and moment decreases, as shown in Figure 3.10a.

For the tangential distribution of moments, Figure 3.10b shows both the quadrilinear moment-curvature relation and the bilinear moment-curvature relation (dashed line). The two models used in describing the moment-curvature relation is shown in more detail

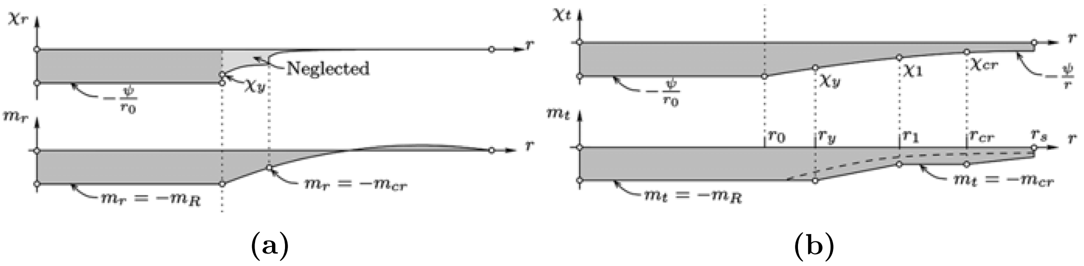


Figure 3.10: (a) Distribution of radial curvature and radial moment [4] and (b) distribution of tangential curvature and tangential moment [4]

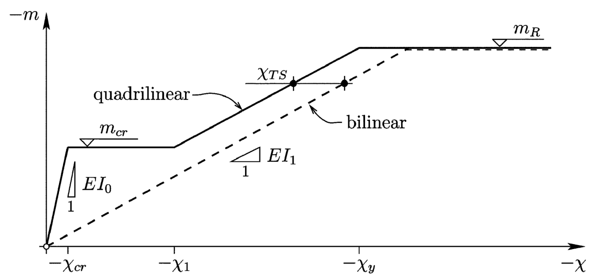


Figure 3.11: Quadrilinear and bilinear moment-curvature relation [4]

in Figure 3.11. As is seen in Figure 3.11, the bilinear approach is simpler than the quadrilinear approach. The derivation of the expressions for the quadrilinear and the bilinear moment-curvature relation will be described in detail in the next two sections [4].

3.3.1 The Quadrilinear Expression

First, the curvature distribution from Figure 3.10b is being evaluated. In terms of being able to evaluate the load-rotation relation, the deflected slab outside the critical shear crack is assumed to be conically shaped. The curvature in tangential direction then follows the expression given in Equation (3.5), for $r > r_0$ [4].

$$\chi_t = -\frac{\psi}{r} \tag{3.5}$$

Along the cross-sections defined by the inclining cracks in the slab, the forces in the reinforcement remain constant. Equilibrium is obtained along these cross-sections, and the curvatures and moments are therefore constant within r_0 in both directions. On these assumptions, the expression in Equation (3.6) is obtained for $r \leq r_0$ [4].

$$\chi_r = \chi_t = -\frac{\psi}{r_0} \tag{3.6}$$

In Figure 3.11 two different stiffnesses are defined, EI_0 before cracking and EI after cracking. In addition to this, the figure shows the cracking moment m_{cr}, the moment capacity m_R and the tension stiffening effect χ_{TS}.

The expressions in Equations (3.7), (3.8) and (3.9) are obtained according to Figure 3.11, neglecting the effect of the reinforcement before cracking.

$$m_{cr} = \frac{f_{ct} \cdot h^2}{6} \tag{3.7}$$

$$EI_0 = \frac{E_c \cdot h^3}{12} \tag{3.8}$$

$$-\chi_{cr} = \frac{m_{cr}}{EI_0} = \frac{2 \cdot f_{ct}}{h \cdot E_c} \tag{3.9}$$

In Equations (3.7), (3.8) and (3.9), f_{ct} is the tensile strength of the concrete, h is the slab thickness and E_c is the Young's modulus of the concrete.

After cracking a linear-elastic behaviour of the reinforcement and concrete is assumed, and the relation in Equation (3.10) is obtained.

$$EI_1 = \rho \cdot \beta \cdot E_s \cdot d^3 \cdot \left(1 - \frac{c}{d}\right) \cdot \left(1 - \frac{c}{3d}\right) \tag{3.10}$$

$$c = \rho \cdot \beta \cdot \frac{E_s}{E_c} \cdot d \cdot \left(\sqrt{1 + \frac{2 \cdot E_c}{\rho \cdot \beta \cdot E_s}} - 1\right) \tag{3.11}$$

In Equations (3.10) and (3.11), ρ is the reinforcement ratio, β is the efficiency factor, E_s is the Young's modulus of the reinforcement steel, d is the distance from the extreme compression element to the centre of the longitudinal tensile reinforcement and c is the depth of the compression zone. The efficiency factor β accounts for the reduction in the ratio of bending and torsional stiffness after cracking and the layout of the reinforcement [4].

After defining the cracking moment, the two stiffnesses and the curvature, the moment capacity m_R must be defined. After yielding, the reinforcement is assumed to have perfect plastic behaviour. In addition to this, compressive stresses in the reinforcement are neglected and a rectangular stress block is assumed for the concrete in the compression zone *c*. The expression for m_R given in Equation (3.12) is then obtained under the assumptions given in [4].

$$m_R = \rho \cdot f_y \cdot d^2 \cdot \left(1 - \frac{\rho \cdot f_y}{2 \cdot f_c}\right) \tag{3.12}$$

In Equation (3.12), f_y is the yield strength of the reinforcement steel and f_c is the average compressive cylinder strength of the concrete.

Further on, tension stiffening makes the behaviour stiffer and therefore decreases the curvature in the slab. Tension stiffening is activated by bond slip between the concrete and reinforcement, and refers to the capacity of intact concrete to carry a limited amount of tensile forces between neighbouring cracks [14]. The decrease in curvature caused by tension stiffening can be approximated by the constant contribution given in Equation (3.13).

$$\chi_{TS} = \frac{f_{ct}}{\rho \cdot \beta \cdot E_s} \cdot \frac{1}{6 \cdot h} \approx 0.5 \cdot \frac{m_{cr}}{EI_1} \tag{3.13}$$

The expressions for the curvatures at the beginning of the stabilized cracked area, χ_1, and at yielding, χ_y, can then be expressed by Equations (3.14) and (3.15).

$$-\chi_1 = \frac{m_{cr}}{EI_1} - \chi_{TS} \tag{3.14}$$

$$-\chi_y = \frac{m_R}{EI_1} - \chi_{TS} \tag{3.15}$$

The different curvatures given in the Equations (3.9), (3.14) and (3.15) represent a zone in Figure 3.10b, and have corresponding radii delimiting the zones, r_{cr}, r_1 and r_y. These radii can be calculated by the expressions given in Equations (3.16), (3.17) and (3.18).

$$r_{cr} = -\frac{\psi}{\chi_{cr}} = \frac{\psi \cdot EI_0}{m_{cr}} \leq r_s \tag{3.16}$$

$$r_1 = -\frac{\psi}{\chi_1} = \frac{\psi}{\frac{m_{cr}}{EI_1} - \chi_{TS}} \leq r_s \tag{3.17}$$

$$r_y = -\frac{\psi}{\chi_y} = \frac{\psi}{\frac{m_R}{EI_1} - \chi_{TS}} \leq r_s \tag{3.18}$$

r_{cr} represents the zone up to where the concrete is cracked, r_1 is the zone for which cracking is stabilized and r_y represents the plastic radius for which the reinforcement is yielding [4].

Equilibrium of the part of the slab shown in Figure 3.9b gives the expression given in Equation (3.19).

$$V \cdot \frac{\Delta\varphi}{2\pi} \cdot (r_q - r_c) = -m_r \cdot \Delta\varphi \cdot r_0 - \Delta\varphi \cdot \int_{r_0}^{r_s} m_q \cdot dr \tag{3.19}$$

r_c is the radius of a circular column, r_q is the radius of the load introduction, m_r is the radial moment at $r = r_0$ and m_q is the moment at $r = r_q$.

Finally, the expression in Equation (3.20) is obtained, where the operator $\langle x \rangle$ is x for $x \geq 0$ and 0 for $x < 0$ [4].

$$V = \frac{2\pi}{r_q - r_c} \begin{pmatrix} -m_r \cdot r_0 + m_R \cdot \langle r_y - r_0 \rangle + EI_1 \cdot \psi \cdot \langle \ln(r_1) - ln(r_y) \rangle + \\ EI_1 \cdot \chi_{TS} \cdot \langle r_1 - r_y \rangle + m_{cr} \cdot \langle r_{cr} - r_1 \rangle + EI_0 \cdot \psi \cdot \langle \ln(r_s) - \ln(r_{cr}) \rangle \end{pmatrix} \tag{3.20}$$

3.3.2 The Bilinear Expression

The bilinear expression for the load-rotation relation is a simpler expression than the quadrilinear relation, and is obtained by neglecting the effect of tension stiffening and the tensile strength of the concrete. The moment-curvature relation for the bilinear law is shown in Figure 3.12.

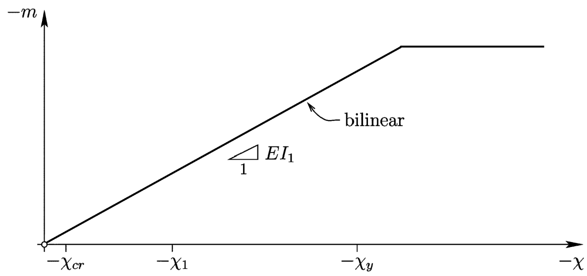


Figure 3.12: Bilinear moment-curvature relation, based on figure from [4]

For the elastic zone, $r_y \leq r_0$, the expression in Equation (3.21) describes the relation between load and curvature in the slab.

$$V = \frac{2\pi}{r_q - r_c} \cdot EI_1 \cdot \psi \cdot \left(1 + \ln \frac{r_s}{r_0}\right) \tag{3.21}$$

For the elastic-plastic zone, $r_0 \leq r_y \leq r_s$, we get the expression given in Equation (3.22).

$$V = \frac{2\pi}{r_q - r_c} \cdot EI_1 \cdot \psi \cdot \left(1 + \ln \frac{r_s}{r_y}\right) \tag{3.22}$$

When the yielding zone of the slab has the same radius as the slab, the flexural strength of the slab is reached. Almost all the operators in Equation (3.20) is then equal to zero, and the expression for V_{flex} becomes as given in Equation (3.23) [4].

$$V_{flex} = 2\pi \cdot m_R \frac{r_s}{r_q - rc} \tag{3.23}$$

3.3.3 Comparison of Expressions for the Load-rotation Relation

Figure 3.13 shows a comparison of the punching shear tests by *Kinnunen and Nylander* presented in Figure 3.7b and the proposed analytical expressions presented in Equations (3.20), (3.21), (3.22) and (3.23). The dotted line symbolizes the failure criterion given in Equation (3.4).

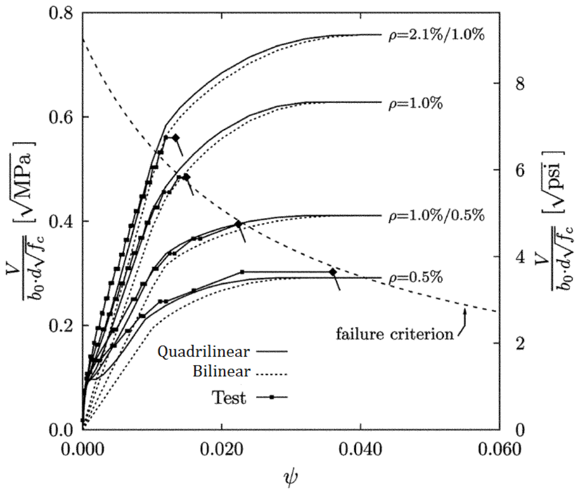
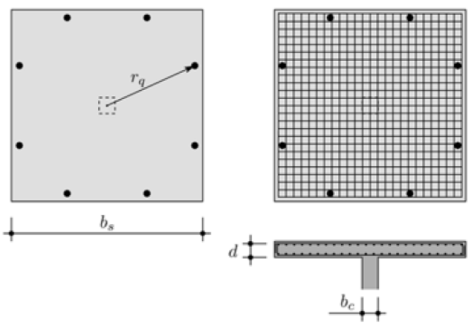


Figure 3.13: Comparison of load-rotation curves for tests and for proposed expressions, based on figure from [4]

As is seen in Figure 3.13, both the quadrilinear and the bilinear expression predict the punching shear failure load with good accuracy for all the different reinforcement ratios. The two different expressions give almost the same graphs for large reinforcement ratios, but for smaller reinforcement ratios the two solutions differ more from one another for smaller loads. The quadrilinear expression still gives a good accuracy for small loads, while the bilinear expression is less accurate. This can be explained by the fact that the effect of tension stiffening and the tensile strength of the concrete is neglected in the bilinear expression, and these two effects have a bigger impact on the total strength for smaller reinforcement ratios [4].

Even though the quadrilinear approach is more accurate for all general cases, both methods describe the actual rotation capacity in the slab, and the punching shear strength can be obtained by substituting the expressions for the quadrilinear or the bilinear case into Equation (3.4).

Further on, the thickness of the slab affects the precision of the quadrilinear and the bilinear approach. *Guandalini and Muttoni* performed tests on slabs with various thicknesses to study the effect of the thickness of the slab. The geometry as well as the geometrical and mechanical parameters for the slabs PG-3 and PG-10 in the tests are given in Figure 3.14.



(a)

	PG − 3	PG − 10
b_s [mm (in)]	6000 (236)	3000 (118)
b_c [mm (in)]	520 (20.4)	260 (10.2)
r_q [mm (in)]	2845 (112)	1423 (56)
d [mm (in)]	456 (17.9)	210 (8.26)
ρ [%]	0.33	0.33
f_c [MPa (psi)]	32.4 (4700)	34.7 (5000)
f_y [MPa (ksi)]	520 (75.3)	577 (83.6)

(b)

Figure 3.14: (a) Geometry of the specimens PG-3 and PG-10 and (b) geometric and mechanical parameters for the specimens PG-3 and PG-10 [4]

Both tests had the same reinforcement ratio $\rho = 0.33\%$ and the same maximum aggregate size $d_g = 16\,\text{mm}$, but different dimensions, as defined in Figure 3.14b. Figure 3.15 shows the load-rotation curves for these two tests, with the actual slab rotation without correction for aggregate size and size effect on the x-axis. The failure criteria calculated according to Equation (3.4) for the two different slab thicknesses are also defined in the figure.

While the load-rotation relation for the two test specimens is quite similar, the failure criterion differs as the thicknesses are not the same for the two specimens. This causes the quadrilinear and the bilinear approaches to achieve different preciseness in defining the punching shear capacity. The bilinear expression underestimates the capacity for both test specimens, but especially for the thickest specimen. The quadrilinear expression on the other hand describes the behaviour for all loading stages in a precise and satisfying manner [4].

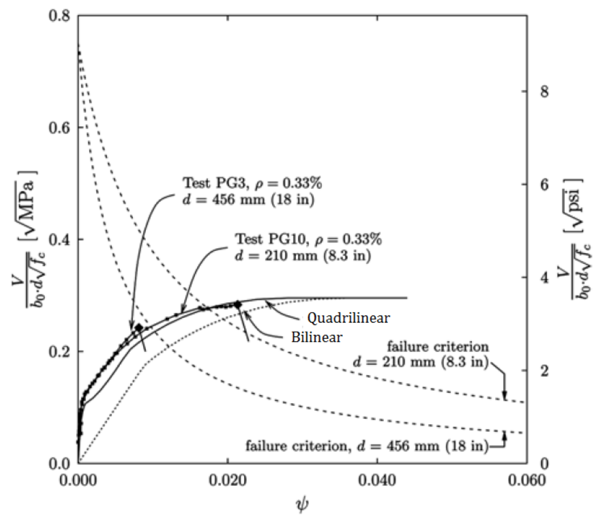


Figure 3.15: Quadrilinear and bilinear load-rotation curves, and load-rotation curve and failure criteria for test specimens PG-3 and PG-10, based on figure from [4]

Figure 3.15 also shows how the size of the slab affects the type of failure and the general behaviour of the slab. The size effect causes the thick slab to have a lower rotation capacity than the thin slab, and the thick slab will therefore experience a more brittle failure. The thin slab will experience a much more ductile behaviour.

Figure 3.16 shows the load-rotation relation according to the quadrilinear expression given in Equation (3.20) for various reinforcement ratios along with the failure criteria for many different slab thicknesses.

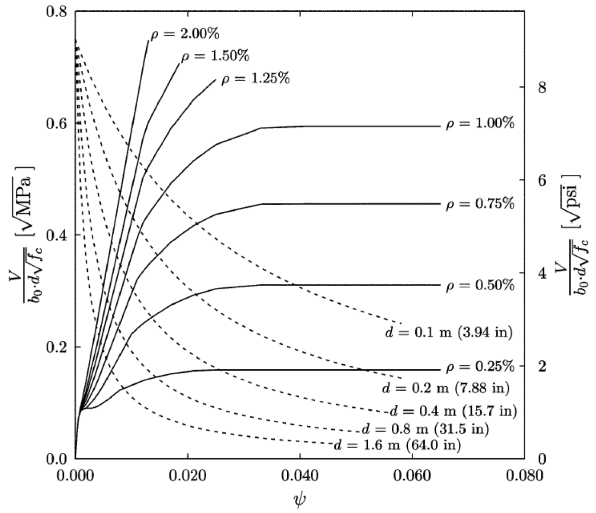


Figure 3.16: Load-rotation relations based on equation (3.20) for various reinforcement ratios and failure criteria for various slab thicknesses, based on figure from [4]

Figure 3.16 shows that the behaviour of the slab becomes more brittle as the reinforcement ratio increases, as described earlier in this section. This gives that although thicker slabs experience a more brittle failure in general, brittle failure may also occur for thinner slabs for large reinforcement ratios. This coincides well with the results from the tests by *Kinnunen and Nylander* presented in Figure 3.7b.

3.4 Application of the CSCT to Punching of Flat Slabs

In this section the application of the CSCT to reinforced concrete flat slabs will be presented. The section will cover the application to slabs both with and without shear reinforcement.

3.4.1 Application to Slabs Without Shear Reinforcement

The method for defining the punching shear strength of a slab by looking at the intersection between the failure criterion and the load-rotation relation coincide well with the actual behaviour of the slab, as described in earlier sections. However, the expressions used in the calculation are somewhat complicated and may become too time consuming for general use. Some simplifications to the bilinear expressions may be done in terms of simplifying the calculation method.

The value of the plastic radius for which the reinforcement is yielding, r_y, was in Section 3.3.1 defined by Equation (3.18). By neglecting the effect of tension stiffening χ_{TS}, a simplified expression for the bilinear relation is defined according to Equation (3.24).

$$r_y = -\frac{\psi}{\chi_y} = \frac{\psi}{\frac{m_R}{EI_1}} \leq r_s \tag{3.24}$$

The expression for EI_1 under the assumption of linear-elastic behaviour of the reinforcement and concrete is given in Equation (3.10), and the expression for m_R presented in Equation (3.12) is assumed. In addition to these two assumptions it is assumed that the flexural strength, V_{flex}, is reached for a radius of the yielded zone equal to 75% of the radius of the isolated slab element, as given in Equation (3.25).

$$r_y = 0,75\, r_s \tag{3.25}$$

Rewriting of Equation (3.24) then gives the expression for ψ given in Equation (3.26).

$$\psi = r_y \cdot \frac{m_R}{EI_1} = 0.75\, r_s \cdot \frac{m_R}{EI_1} \tag{3.26}$$

By inserting the expressions for m_R and EI_1 given in Equations (3.12) and (3.10), we get the expression for ψ given in Equation (3.27).

$$\psi = 0.75\, r_s \cdot \frac{\rho f_y d^2 \cdot \left(1 - \dfrac{\rho \cdot f_y}{2 \cdot f_c}\right)}{\rho \beta E_s d^3 \cdot \left(1 - \dfrac{c}{d}\right) \cdot \left(1 - \dfrac{c}{3d}\right)} = \frac{0.75\, r_s f_y}{\beta d E_s} \cdot \frac{\left(1 - \dfrac{\rho \cdot f_y}{2 \cdot f_c}\right)}{\left(1 - \dfrac{c}{d}\right) \cdot \left(1 - \dfrac{c}{3d}\right)} \tag{3.27}$$

The expression obtained for ψ is still somewhat complicated and to simplify the expression further, β is assumed to have the value 0.5. Further on, the assumption that the rotation ψ is proportional to the ratio V/V_{flex} with the exponent 3/2 is made, and finally the expression for ψ given in Equation (3.28) is obtained [4].

$$\psi = 1,5 \cdot \frac{r_s f_y}{d E_s} \cdot \left(\frac{V}{V_{flex}}\right)^{3/2} \tag{3.28}$$

A comparison of the four tests by *Kinnunen and Nylander* presented in Figure 3.7b, the quadrilinear load-rotation relation presented in Equation (3.20) and the simplified expression in Equation (3.28) are presented in Figure 3.17.

The simplified expression predicts a smaller punching failure load than the quadrilinear expression and is therefore conservative. Considering the simple formulation of the simplified expression, the predicted punching failure load is close to the real failure load. The value for the punching shear load for the simplified equation is most conservative for large reinforcement ratios. The simplicity of the expression makes it more suitable in many general cases.

Figures 3.18a and 3.18b show plots of test results by various researchers for a total of 87 tests compared to the solutions of Equations (3.20) and (3.28). In addition to this, Figure 3.18c compares the test results to the results obtained by the formulations in EC2.

The y-axis shows the safety factors obtained in the different calculation methods gotten from dividing the results from the tests by the theoretical value, and the x-axis represents test results divided by the flexural strength.

A safety factor of 1.0 gives that the results from the tests are equal to the ones obtained by the chosen formulation. All values larger than 1.0 describe conservative results, and

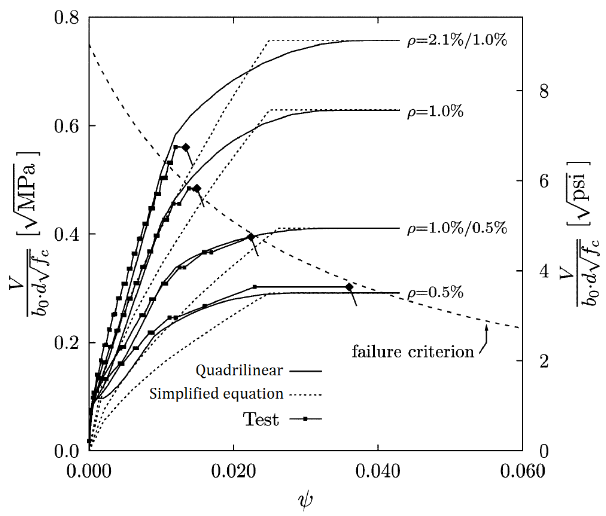


Figure 3.17: Load-rotation relations for tests by *Kinnunen and Nylander* compared to equations (3.20) and (3.28), based on figure from [4]

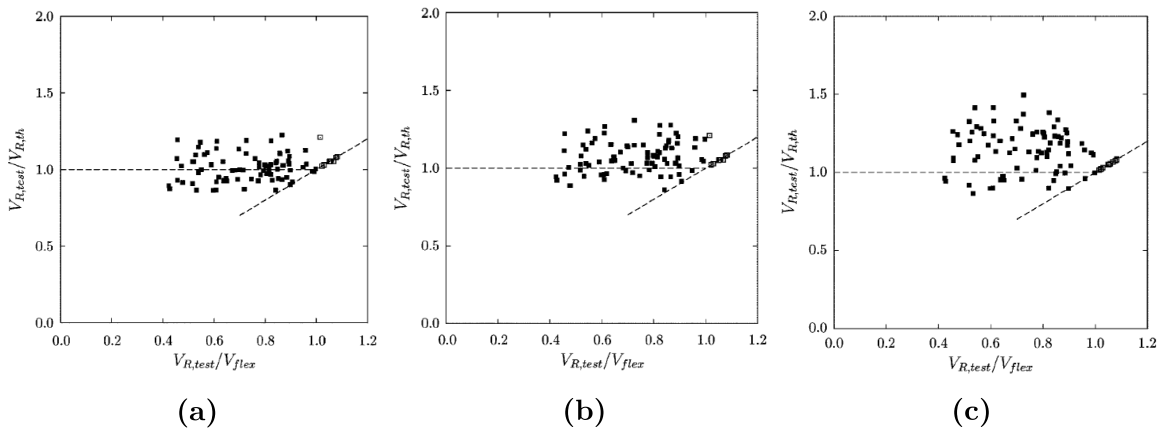


Figure 3.18: Comparison of test results by various researchers to (a) the combination of equations (3.4) and (3.20), (b) the combination of equations (3.4) and (3.28) and (c) the formulations in EC2 [4]

all values smaller than 1.0 describe non-conservative results. A non-conservative ratio smaller than 1.0 means that the actual strength can be lower than the one predicted by the different formulations [4].

The safety factors gotten from the tests are in general close to the value of 1.0, and the smallest safety factor for all the three plots have the value of 0.86.

Based on the presented equations for calculating the rotation of the slab and the failure criterion presented in Equation (3.4), a simple design approach in terms of checking the punching shear capacity of a slab may be done as shown in Figure 3.19.

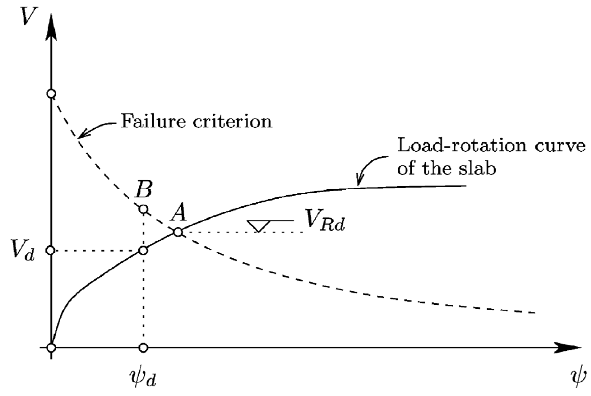


Figure 3.19: Design approach for control of the punching shear capacity [4]

First, the acting shear force on the slab, V_d, must be found. Based on this value, the rotation of the slab may be found by the simplified formulation in Equation (3.28). Inserting this value of ψ into Equation (3.4) gives the corresponding punching shear strength of the slab. If this value is larger than the shear force acting on the slab, the slab has sufficient punching shear capacity. If the value of the calculated punching shear strength is smaller than the shear force acting on the slab, the slab has insufficient capacity. If the latter turned out to be the case, the slab thickness, flexural reinforcement amount or the size of the column needs to be increased.

The punching shear strength found according to the method described will give the shear force value at point B in Figure 3.19. This is a conservative value and is in most cases sufficient enough. If the actual punching shear capacity represented at point A is desired found, iterative methods can be used [4].

Figure 3.20 shows how the different parameters affect the punching shear capacity for the refined method based on Equations (3.4) and (3.20), the simplified method based on Equations (3.4) and (3.28), the formulations in EC2 and for various tests [4].

An increase of the reinforcement ratio ρ will, as described earlier, increase the punching shear capacity. This effect is also included in the formulations in EC2, but the solution is a little conservative compared to the other solution methods presented, as shown in Figure 3.20a. The results from the different methods presented in Figures 3.20b, 3.20c and 3.20d also make a good match with the test results available.

The effect of the yield strength of the steel to the punching shear capacity is as expected, as increasing the strength of the steel increases the capacity, as is shown in Figure 3.20e. The results from the formulations in EC2 show the same tendency, but again EC2 is a little conservative compared to the other methods.

Figure 3.20f presents the relation between the span-depth ratio and the punching shear capacity of the slab. The method in EC2 does not take this into account, and in this

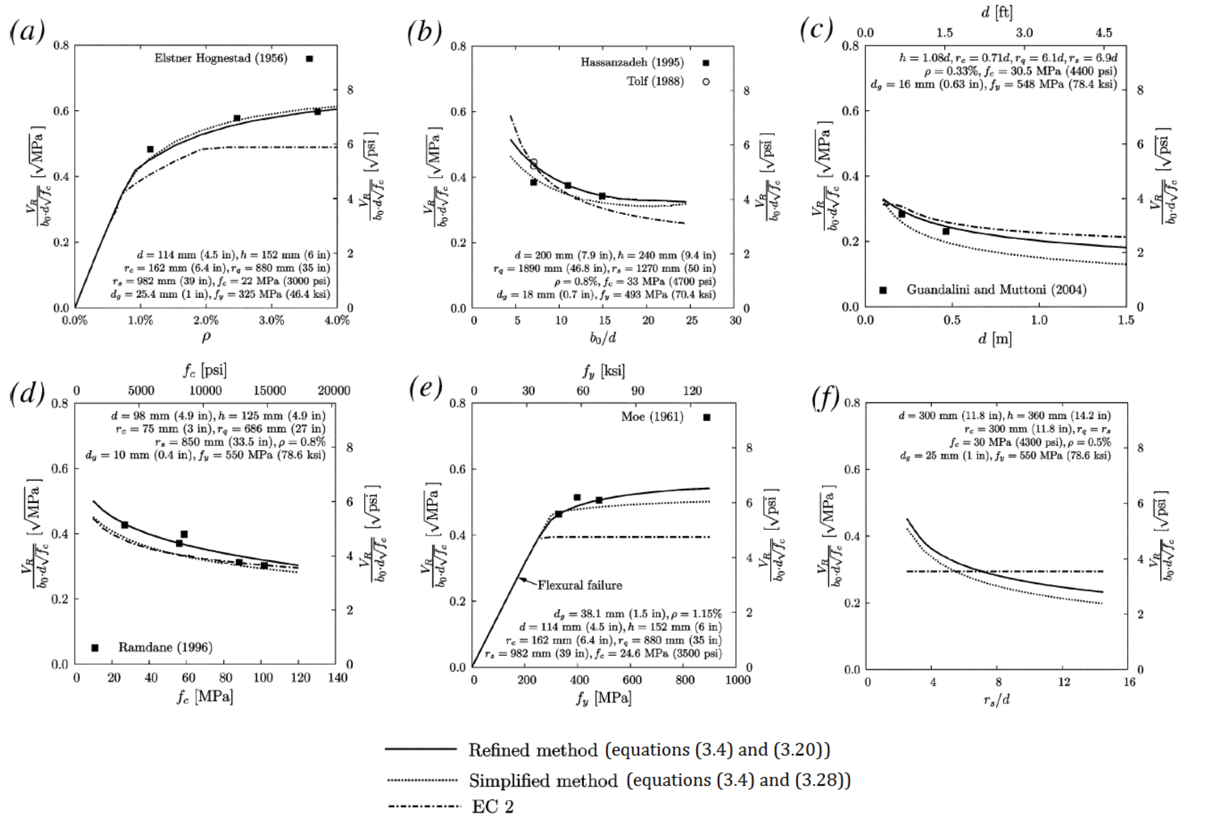


Figure 3.20: Comparison of how the different methods and results from various tests are affected by (a) the reinforcement ratio, (b) the punching shear perimeter, (c) the effective depth of the slab, (d) the concrete strength, (e) the yield strength of the steel and (f) the slenderness of the slab, based on figure from [4]

method the punching shear capacity seams unaffected by the span-depth ratio. This is a problem, since the curve representing the results from EC2 presents larger capacities for slender slabs than the curves for both the refined and the simplified method, which makes it seem like EC2 in these cases can be non-conservative.

3.4.2 Application to Slabs With Shear Reinforcement

For a slab with distributed shear reinforcement, a punching shear failure may occur by either crushing of the compression struts in the concrete near the column, punching shear failure within the shear reinforced area, punching shear failure outside the shear reinforced area, delamination of concrete core or flexural yielding. These five failure mechanisms are shown in Figure 3.21 [5].

Crushing of compression struts, shown in Figure 3.21a, may occur in systems with a large amount of bending and transverse reinforcement, where large compressive struts develop

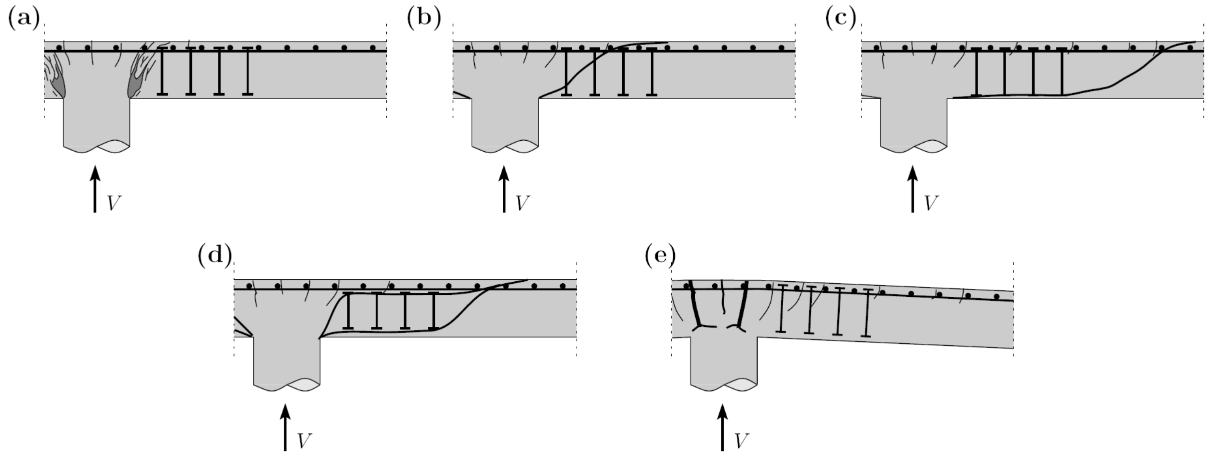


Figure 3.21: (a) Crushing of compression struts in the concrete, (b) failure within the shear reinforced area, (c) failure outside the shear reinforced area, (d) delamination of concrete core and (e) flexural yielding [5]

in the concrete near the column [5].

Punching shear failure within the shear reinforced area, shown in Figure 3.21b, happens for low amounts of shear reinforcement by yielding of the shear reinforcement with crack localization of the strain in the shear reinforced zone [5].

Punching shear failure outside the shear reinforced area, shown in Figure 3.21c, may be governing when the shear reinforcement is arranged over a small region outside the column [5].

Delamination of the concrete core, shown in Figure 3.21d, is a punching shear failure that may occur if the shear reinforcement is arranged such that it does not enclose around the flexural reinforcement. This failure mechanism is not common, as shear reinforcement that does not enclose the flexural reinforcement is not used in most design codes [5].

The last failure mechanism, shown in Figure 3.21e, is not really a punching shear failure mechanism. This type of failure occurs for a low amount of flexural reinforcement, and the strength of the system is therefore determined by the bending strength [5].

Since delamination is prevented in most codes of practice, and flexural yielding is not a punching shear failure mechanism, only the alternatives (a), (b) and (c) form the expression for the minimum strength of the slab, presented in Equation (3.29) [6].

$$V_R = min\left(V_{R,crush};\, V_{R,in};\, V_{R,out}\right) \tag{3.29}$$

The crushing strength is in most codes usually evaluated by either considering the compressive strength near the column to be reduced or by defining that the maximum shear strength should not be greater than the punching shear strength of a slab without shear reinforcement [6].

When controlling the punching shear strength outside the shear reinforced area it is common to use a similar formulation as the one defined for the punching shear strength without shear reinforcement. In this case, the control perimeter and the shear strength should be modified to appropriate values for the case of a shear reinforced system [6].

The punching shear strength within the shear reinforced zone is often determined on the basis that both the concrete and the reinforcement contribute to the total capacity, as formulated in Equation (3.30).

$$V_{R,in} = \eta_c \cdot V_{c0} + \eta_s \cdot V_{s0} \tag{3.30}$$

Here, $\eta_c \cdot V_{c0}$ is the contribution of the concrete, $\eta_s \cdot V_{s0}$ is the contribution of the shear reinforcement and the η-parameters are factors of a value of 1.0 or lower. V_{c0} is the punching shear strength without shear reinforcement and V_{s0} is the strength of the reinforcement within the punching cone [6].

In most design codes the ratio of the contribution of the concrete to the punching shear capacity, η_c, is assumed constant. This assumption is in many codes made independent of the flexural reinforcement ratio, the amount of shear reinforcement and the bond condition of the shear reinforcement. Figure 3.22 shows a comparison of the design approach in EC2 and the actual behaviour [6].

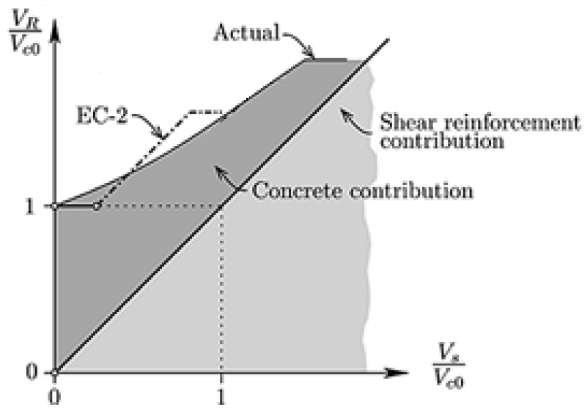


Figure 3.22: Comparison of the design approach in EC2 and the actual behaviour, based on figure from [6]

In the following section, the design approaches based on the CSCT for the failure mechanisms (a), (b) and (c) in Figure 3.21 will be presented in more detail.

Crushing Shear Failure

The crushing shear strength of a slab is mainly influenced by the compressive strength of the concrete and the transverse strains.

Figure 3.23 shows different types of transverse cracks that may develop in the compression zone near the column. These cracks may be caused by either bending, shear or delamination of the concrete core [5].

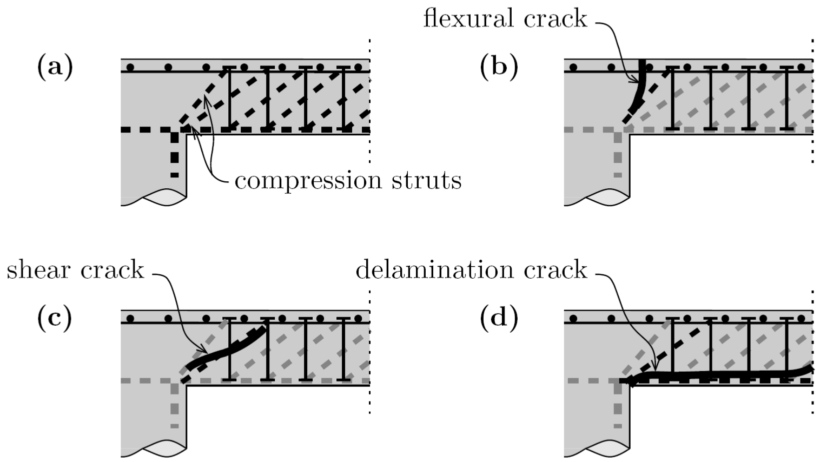


Figure 3.23: (a) Compression struts near the supported area, (b) development of flexural crack, (c) development of shear crack and (d) development of delamination crack [5]

The transverse cracks influence the crushing shear strength of the concrete, as they reduce the strength in the critical region for crushing. Therefore, the type of shear reinforcement used plays an important part in the crushing shear strength, as different types of shear reinforcement influence the position, development and size of the cracks in different ways [5].

The equation for the crushing shear strength obtained based on the CSCT can be estimated as given in Equation (3.31).

$$V_{R,crush} = \lambda \cdot \frac{3}{4} \cdot \frac{b_{0,col} \cdot d \cdot \sqrt{f_c}}{1 + 15 \dfrac{\psi \cdot d}{d_{g0} + d_g}} \tag{3.31}$$

This expression is equal to the one proposed for the failure criterion in Equation (3.4), but multiplied with a factor λ. This factor depends on the type of shear reinforcement used. It is set equal to 3.0 for well-anchored shear reinforcement, and is limited to 2.0 for all types of reinforcement [6].

The strength of the slab is, as in previous described cases, found where the load-rotation relation intersects with the failure criterion.

Failure Within the Shear Reinforced Zone

The punching shear strength within the shear reinforced zone can be given as in Equation (3.32).

$$V_{R,in} = V_c + V_s \tag{3.32}$$

The shear reinforcement is activated as the rotations of the slab increase and the critical shear cracks open. A fraction of the shear force can then still be carried by the concrete, limited by the roughness and the opening of the shear cracks. The rest of the shear force has to be carried by the reinforcement, as given in Equation (3.32). Figure 3.24 shows the localization of the strains within the shear reinforced zone and the contributions from the concrete and reinforcement, and Figure 3.25 shows the total shear strength and the strength carried by the concrete and the reinforcement respectively [6].

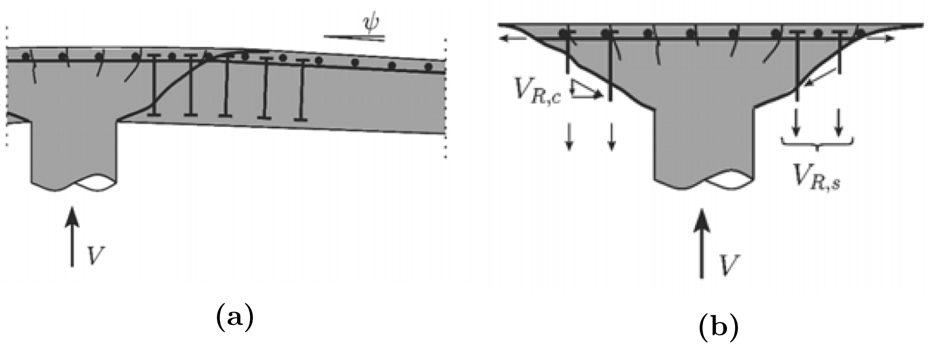


Figure 3.24: (a) Localization of strains within the shear reinforced zone and (b) contributions from the concrete and shear reinforcement [6]

Assuming that a single crack develops in the failure zone, the concrete contribution can be estimated according to the CSCT, which gives the formulation given in Equation (3.33).

$$V_c = \frac{3}{4} \cdot \frac{b_{0,int} \cdot d \cdot \sqrt{f_c}}{1 + 15\dfrac{\psi \cdot d}{d_{g0} + d_g}} \tag{3.33}$$

The control perimeter $b_{0,int}$ is defined at the distance $d/2$ beyond the tip of the crack, and the rest of the parameters are as defined for Equation (3.4).

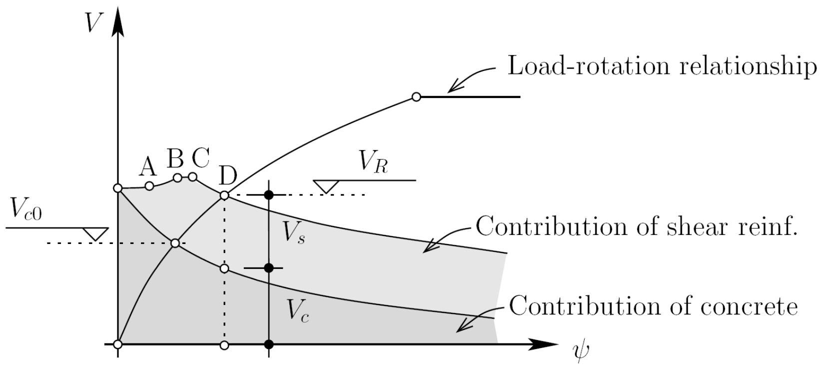


Figure 3.25: Shear strength contribution from the concrete and the reinforcement [5]

The assumptions made in Section 3.2 that the crack opening is proportional to the rotation of the slab multiplied with the effective depth, given in Equation (3.2), can be rewritten into:

$$w = \kappa \cdot \psi \cdot d \tag{3.34}$$

The constant κ is in the background to MC2010 proposed to have the value 0.5. Assuming that the cracks are straight and that the crack's rotation centre is located at the tip of the crack, Equations (3.35) and (3.36) describe the relative displacements of the crack lips.

$$w_{bi} = \kappa \cdot \psi \cdot h_i \cdot \cos \cdot \left(\alpha + \beta_i - \frac{\pi}{2}\right) \tag{3.35}$$

$$\delta_{bi} = \kappa \cdot \psi \cdot h_i \cdot \sin \cdot \left(\alpha + \beta_i - \frac{\pi}{2}\right) \tag{3.36}$$

Here, w_{bi} and δ_{bi} are the relative displacements parallel and perpendicular to the shear reinforcement respectively, h_i is the vertical distance between the point where the reinforcement crosses the shear crack and the crack tip, α is the critical shear crack angle and β is the angle between the shear reinforcement and the slab. The parameters in Equations (3.35) and (3.36) are defined in Figure 3.26 [6].

For neglected dowel action, σ_{si} may be evaluated as a function of ψ. As is given in Figure 3.26c, the value of the axial force in the shear reinforcement is given by the expression in Equation (3.37).

$$N_{si} = \sigma_{si}\left(\psi\right) \cdot A_{si} \tag{3.37}$$

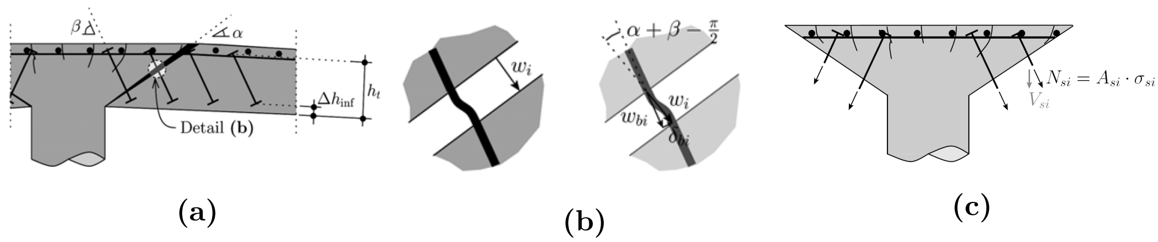


Figure 3.26: (a) Parameters of the critical shear crack and the shear reinforcement [6], (b) crack opening and relative displacements of crack lips [6] and (c) contribution from the shear reinforcement [5]

Here, A_{si} is the cross-sectional area of one shear reinforcement bar. The vertical component of N_{si} represents the contribution of the shear reinforcement to the punching shear strength, and is given by Equation (3.38).

$$V_{si} = \sigma_{si}(\psi) \cdot A_{si} \cdot \sin(\beta_i) \tag{3.38}$$

For all the reinforcement bars crossing the critical crack, the expressions of N_s and V_s can be given according to Equations (3.39) and (3.40).

$$N_s = \sum_{i=1}^{n} \sigma_{si}(\psi) \cdot A_{si} \tag{3.39}$$

$$V_s = \sum_{i=1}^{n} \sigma_{si}(\psi) \cdot A_{si} \cdot \sin(\beta_i) \tag{3.40}$$

As in earlier cases, the punching shear strength is now found at the intersection of the load-rotation curve and the failure criterion. The failure criterion has now contributions from both the concrete and the shear reinforcement, given in Equations (3.33) and (3.40). This intersection point is given as point D in Figure 3.25 [5].

Failure Outside the Shear Reinforced Zone

Failure outside the shear reinforced zone occurs by the development of a single critical crack localizing strains. The expression for the punching shear strength outside the shear reinforced zone is similar to the expression for the punching shear capacity contribution from the concrete given in Equation (3.33), and is given by Equation (3.41).

$$V_{R,out} = \frac{3}{4} \cdot \frac{b_{0,out} \cdot d \cdot \sqrt{f_c}}{1 + 15 \dfrac{\psi \cdot d}{d_{g0} + d_g}} \tag{3.41}$$

Equations (3.33) and (3.41) differ from each other in the definition of the control perimeter b_0. While $b_{0,int}$ is defined at the distance $d/2$ beyond the tip of the crack, $b_{0,out}$ is defined at the distance $d/2$ beyond the outer layer of shear reinforcement. $4d$ is considered as the maximum distance between two shear reinforcement bars.

The effective depth of the slab d_v considers that the cracks develop around the shear reinforcement, and the type and geometry of the shear reinforcement bars influence the size of the effective depth, as shown in Figure 3.27.

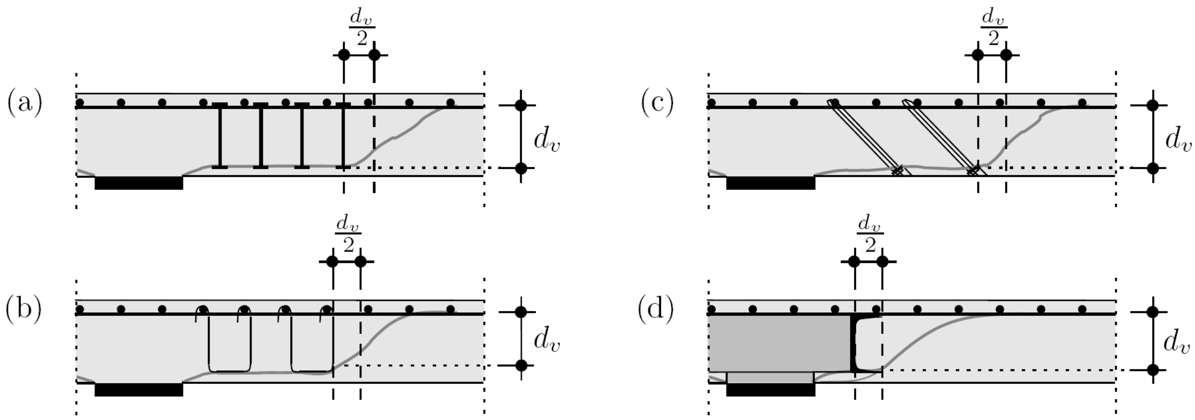


Figure 3.27: Effective depth and control perimeter outside the shear reinforced zone for (a) studs, (b) stirrups, (c) bonded reinforcement with anchorage plates and (d) shear-heads [5]

The expression in Equation (3.41) makes a good match with test results, although the results from this method are somewhat conservative.

Chapter 4

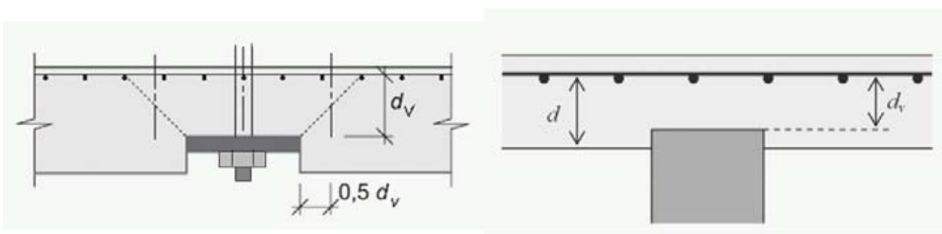
Punching Shear in Model Code 2010

"Model Code 2010 - Final draft - Volume 1 & 2", referred to in this thesis as MC2010, was published in the spring of 2012 by The International Federation for Structural Concrete (*fib*). Previous versions of the Model Code were published in 1978 and 1990 [7]. In MC2010 the punching shear design method is grounded in a physical model based on the CSCT, and the calculation models presented in MC2010 are used as a foundation in the work of developing a new edition of EC2. In the following section the final formulations in MC2010 Sections 7.3.5 and 7.13.5.3 will be presented [7].

4.1 Formulations for Slabs Without Shear Reinforcement

In this section, the final formulations in MC2010 for slabs without shear reinforcement will be presented. The formulations are based on the CSCT, described in Chapter 3.

4.1.1 Shear-resisting Effective Depth and Control Perimeter



(a)

(b)

Figure 4.1: Effective depth for (a) support penetration and (b) bending calculations [7]

The shear-resisting effective depth of a slab, d_v, is defined as the distance from the supported area to the centre of the reinforcement layers, as shown in Figure 4.1. The effective depth for bending calculations, d, is also defined in the figure. In many cases these two values are the same.

The basic control perimeter b_1 is usually taken at the distance $0.5d_v$ from the edges of the supported area. The length of the perimeter is defined such that the length is minimized, depending on the type of cross-section. If the supported area is close to an edge or a corner, the basic perimeter is limited by these edges. Basic control perimeters for different types of columns are shown in Figure 4.2. Basic control perimeters around walls are shown in Figure 4.3.

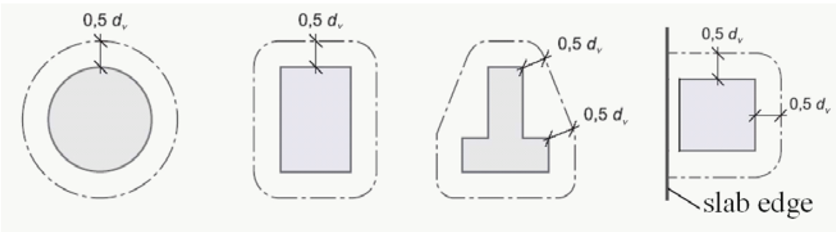


Figure 4.2: Basic control perimeters [7]

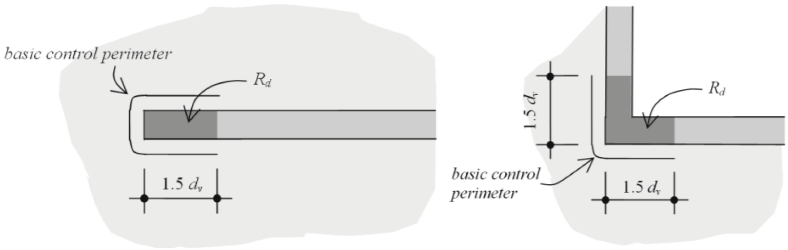


Figure 4.3: Basic control perimeters around walls [7]

If the slab does not have a constant thickness, the control perimeter may be defined at a section at a greater distance from the supported area, as shown in Figure 4.4.

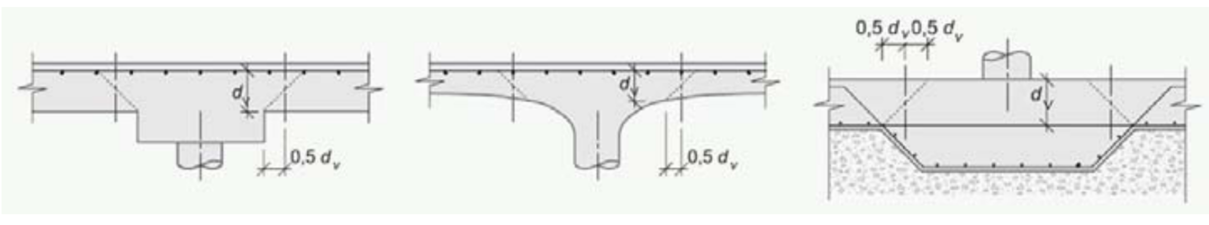


Figure 4.4: Basic control perimeters for slabs with various depth [7]

The shear-resisting control perimeter b_0 is defined for a non-uniform distribution of shear forces along the basic control perimeter and is defined according to Equation (4.1), where

$v_{perp,d,max}$ is the shear force per unit length perpendicular to the control perimeter, as shown in Figure 4.5.

$$b_0 = \frac{V_{Ed}}{v_{perp,d,max}} \tag{4.1}$$

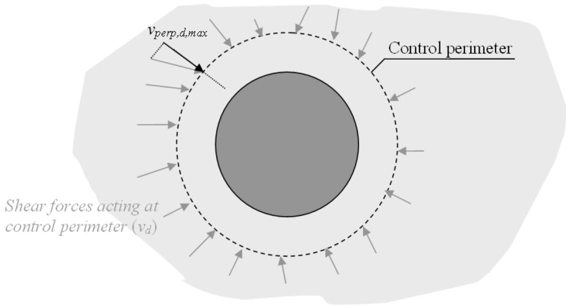


Figure 4.5: Maximum value of the shear force per unit length perpendicular to the control perimeter [7]

For concentrations of shear forces at the corners of the supported areas, the effect can be approximated by reducing the basic control perimeter. The reduced control perimeter $b_{1,red}$ is then obtained. The length of its straight areas should not exceed $3d_v$, as shown in Figure 4.6.

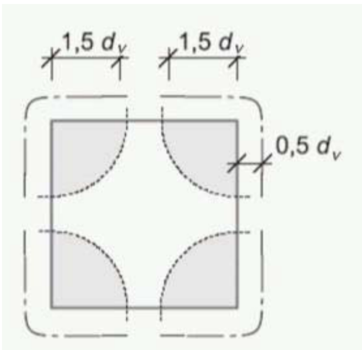
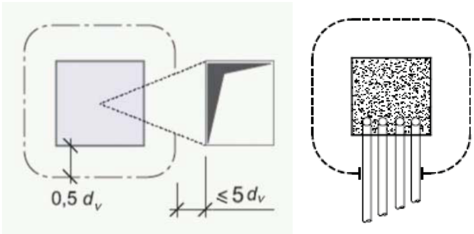


Figure 4.6: Reduction of basic control perimeter, $b_{1,red}$ [7]

For discontinuities near the loaded area, for example cast-in pipes, pipe bundles or slab openings, the basic control perimeter is to be reduced according to Figure 4.7, given that the discontinuity is at a distance less than $5d_v$ form the loaded area.

If the loaded area is subjected to a moment, the shear-resisting control perimeter can be approximated by introducing the eccentricity coefficient k_e, as given in Equation (4.2).

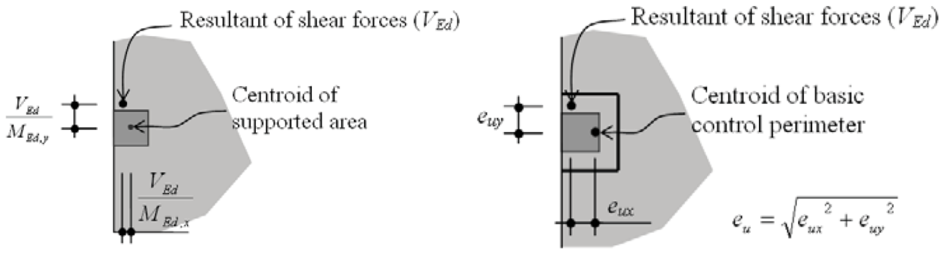
$$b_0 = k_e \cdot b_{1,red} \tag{4.2}$$



(a) **(b)**

Figure 4.7: Reduction of basic control perimeter due to (a) slab openings or (b) pipes [7]

The value of the eccentricity coefficient may be approximated according to Figure 4.8.



(a) **(b)**

Figure 4.8: Resultant of shear forces with respect to (a) position of centre of loaded area and (b) approximated basic control perimeter for calculation of centre position [7]

The eccentricity coefficient may be approximated by the expression given in Equation (4.3), where b_u is the diameter of a circle with the same surface as the area inside the basic control perimeter and e_u is defined according to Figure 4.8.

$$k_e = \frac{1}{1 + e_u/b_u} \tag{4.3}$$

Approximated values for the eccentricity coefficient may be adapted in cases where the lateral stability does not depend on frame actions of slabs and columns. In addition to this, adjacent spans must not differ in length by more than 25%. Values for different types of columns and wall corners are given in Table 4.1.

Table 4.1: Values of k_e for different types of columns and wall corners [7]

Column type	$\mathbf{k_e}$
Inner columns	0.90
Edge columns	0.70
Corner columns	0.65
Corners of walls	0.75

4.1.2 Punching Shear Strength

The punching shear resistance for slabs without shear reinforcement is given by Equation (4.4).

$$V_{Rd} = k_\psi \frac{\sqrt{f_{ck}}}{\gamma_c} b_0 d_v \tag{4.4}$$

The parameter k_ψ depends on the rotations of the slab and γ_c is the material factor for concrete. k_ψ is given by Equation (4.5).

$$k_\psi = \frac{1}{1.5 + 0.9 k_{dg} \psi d} \leq 0.6 \tag{4.5}$$

In Equation (4.5) k_{dg} is a factor that takes into account the maximum aggregate size d_g. If the value of d_g is not less than 16 mm, the value of k_{dg} can be set equal to 1.0. If the value of d_g on the other hand is less than 16 mm, the value of k_{dg} can be estimated according to Equation (4.6).

$$k_{dg} = \frac{32}{16 + d_g} \geq 0.75 \tag{4.6}$$

4.1.3 LoAs for Calculation of Rotations

As described in the introduction to Chapter 3, the CSCT uses a levels-of-approximation (LoA) approach in the design. There are four different LoAs defined in MC2010, where LoA I is the simplest and LoA IV is the most intricate. All the different LoAs are based on the simplified expression given in Equation (3.28), but the relation between the acting shear force and the flexural shear strength is replaced with the relation between the average moment per unit length, m_{Ed}, and the moment capacity per unit length, m_{Rd}. For all LoAs, the design yield strength of the reinforcement steel f_{yd} is used.

LoA I

For the first level, the rotation of the slab may be determined by the expression given in Equation (4.7).

$$\psi = 1.5 \cdot \frac{r_s}{d} \frac{f_{yd}}{E_s} \tag{4.7}$$

Here, the relation between the average moment and the moment capacity is set equal to 1. The value r_s defines where the radial bending moment is equal to zero, and can be approximated as $0.22L_x$ or $0.22L_y$ for the two directions of the slab, for ratios $0.5 \leq L_x/L_y \leq 2.0$. Here, L_x and L_y are the lengths of the slab span in x- and y-direction respectively. The maximum value of r_s has to be considered in Equation (4.7).

This level is primarily used in preliminary design, as it gives an indication of whether the capacity is sufficient, but is highly simplified and therefore not very accurate.

LoA II

For the second level, the rotation of the slab may be determined by the expression given in Equation (4.8).

$$\psi = 1.5 \cdot \frac{r_s}{d} \frac{f_{yd}}{E_s} \cdot \left(\frac{m_{Ed}}{m_{Rd}}\right)^{3/2} \tag{4.8}$$

The value of the average moment per unit length for calculation of the flexural reinforcement in the support strip varies for different column types and different reinforcement directions, and has to be calculated along the two main directions of the reinforcement. For inner columns with top reinforcement in each direction the expression for m_{Ed} is given by Equation (4.9), for edge columns considering the tension reinforcement parallel to the edge the expression is given by Equation (4.10), for edge columns considering the tension reinforcement perpendicular to the edge the expression is given by Equation (4.11) and for corner columns with tension reinforcement in each direction the expression is given by Equation (4.12).

$$m_{Ed} = V_{Ed} \cdot \left(\frac{1}{8} + \frac{|e_{u,i}|}{2 \cdot b_s}\right) \tag{4.9}$$

$$m_{Ed} = V_{Ed} \cdot \left(\frac{1}{8} + \frac{|e_{u,i}|}{2 \cdot b_s}\right) \geq \frac{V_{Ed}}{4} \tag{4.10}$$

$$m_{Ed} = V_{Ed} \cdot \left(\frac{1}{8} + \frac{|e_{u,i}|}{b_s} \right) \tag{4.11}$$

$$m_{Ed} = V_{Ed} \cdot \left(\frac{1}{8} + \frac{|e_{u,i}|}{b_s} \right) \geq \frac{V_{Ed}}{2} \tag{4.12}$$

In Equations (4.9), (4.10), (4.11) and (4.12), $e_{u,i}$ is the eccentricity of the resultant shear force in the two main reinforcement directions and b_s is defined according to Equation (4.13).

$$b_s = 1.5 \cdot \sqrt{r_{s,x} \cdot r_{s,y}} \leq L_{min} \tag{4.13}$$

The width of the strip is defined according to Figure 4.9, and $r_{s,x}$ and $r_{s,y}$ are the positions where the radial bending moment is zero in x- and y-direction respectively. The same values for r_s as in LoA I may be used.

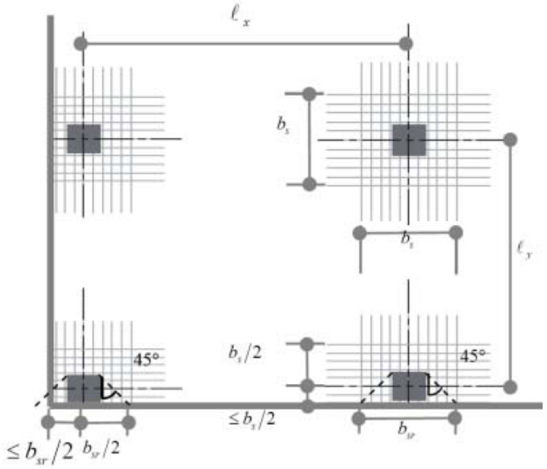


Figure 4.9: Support strip dimensions [7]

For prestressed slabs, the rotation of the slab may be expressed according to Equation (4.14). Effects due to shrinkage, creep and relaxation must be taken into account.

$$\psi = 1.5 \cdot \frac{r_s}{d} \frac{f_{yd}}{E_s} \cdot \left(\frac{m_{Ed} - m_{Pd}}{m_{Rd} - m_{Pd}} \right)^{3/2} \tag{4.14}$$

Here, m_{Pd} expresses the average decompression moment due to prestressing over b_s.

LoA III

In LoA III a linear elastic analysis of an uncracked model of the slab must be performed. This analysis will present values for r_s and m_{Ed}, and m_{Ed} is determined at the edge of the loaded area. Since this level is more accurate than LoA II, the factor 1.5 may be replaced with 1.2. This gives the expressions for the rotation of the slab given in Equations (4.15) and (4.16), for regular and prestressed reinforcement respectively.

$$\psi = 1.2 \cdot \frac{r_s}{d}\frac{f_{yd}}{E_s} \cdot \left(\frac{m_{Ed}}{m_{Rd}}\right)^{3/2} \tag{4.15}$$

$$\psi = 1.2 \cdot \frac{r_s}{d}\frac{f_{yd}}{E_s} \cdot \left(\frac{m_{Ed} - m_{Pd}}{m_{Rd} - m_{Pd}}\right)^{3/2} \tag{4.16}$$

The value of r_s is limited by Equation (4.17), and may be calculated as in LoA II.

$$r_s \geq 0.67\, b_{sr} \tag{4.17}$$

An example of sections for integration of support moments is shown in Figure 4.10.

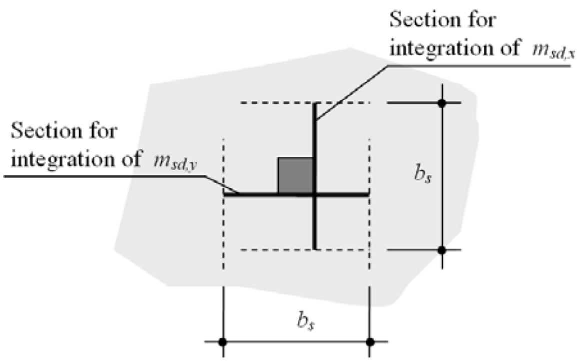


Figure 4.10: Example of sections for integration of support moments [7]

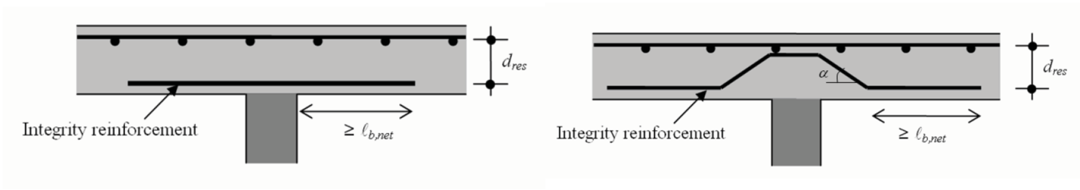
LoA III is recommended in cases where the ratio L_x/L_y is not between 0.5 and 2.0.

LoA IV

The last LoA, LoA IV, is performed by a non-linear analysis of the slab in terms of determining the slab rotation. All relevant non-linear behaviour must be included, for example cracking and yielding of the reinforcement.

4.1.4 Integrity Reinforcement

If shear reinforcement is not required, the slab still needs to be reinforced with integrity reinforcement in terms of avoiding collapse of the structure. Integrity reinforcement with straight and bent-up bars is shown in Figure 4.11.



(a) **(b)**

Figure 4.11: Integrity reinforcement of (a) straight bars and (b) bent-up bars [7]

The resistance of the integrity reinforcement after punching is given by Equation (4.18).

$$V_{Rd,int} = \sum A_s f_{yd} \left(f_t/f_y\right)_k \sin \alpha_{ult} \leq \frac{0.5\sqrt{f_{ck}}}{\gamma_c} d_{res} b_{int} \tag{4.18}$$

Here, A_s is the total cross-sectional area of the integrity reinforcement, f_{yd} is the design yield strength of the integrity reinforcement, $(f_t/f_y)_k$ is a factor depending on the ductility class of the reinforcement, α_{ult} is the angle between the integrity reinforcement and the slab after failure, d_{res} is shown in Figure 4.11 and is the distance between the integrity reinforcement and the flexural reinforcement and b_{int} is the control perimeter after punching activated by the integrity reinforcement.

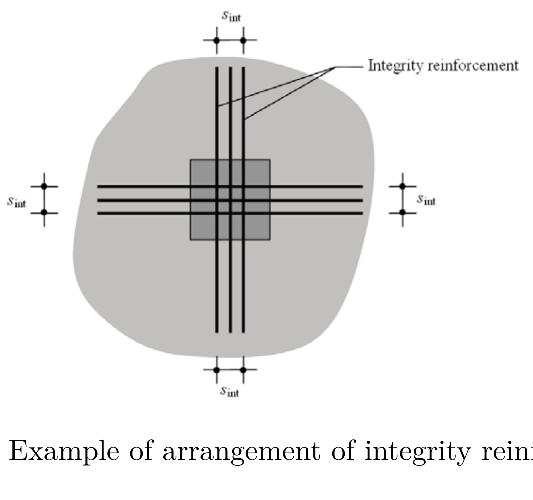


Figure 4.12: Example of arrangement of integrity reinforcement [7]

The value of b_{int} is given by Equation (4.19), where s_{int} is as defined in Figure 4.12 and equal to the width of the group of bars.

$$b_{int} = \sum \left(s_{int} + \frac{\pi}{2} d_{res} \right) \tag{4.19}$$

4.2 Formulations for Slabs With Shear Reinforcement

In this section, the final formulations in MC2010, based on the CSCT, for slabs with shear reinforcement will be presented. The formulations cover the calculations of the crushing shear capacity, the capacities within and outside the shear reinforced zone and the requirements for the arrangement of the shear reinforcement bars.

4.2.1 Crushing Shear Capacity

The capacity of the slab is limited by the crushing of the compression struts in the concrete. This capacity may be determined by the expression in Equation (4.20).

$$V_{Rd,max} = k_{sys} k_{\psi} \frac{\sqrt{f_{ck}}}{\gamma_c} b_0 d_v \leq \frac{\sqrt{f_{ck}}}{\gamma_c} b_0 d_v \tag{4.20}$$

This expression is somewhat similar to the expression in Equation (3.31), except that in Equation (4.20) the rotation is implemented in the factor k_{ψ}, and the material factor γ_c and the factor k_{sys} are introduced. In many cases the value of 2.0 may be used for the factor k_{sys}, and the value may be increased to 2.4 for stirrups with sufficient development length at the compression side of the slab and no compression length at the tension side and to 2.8 for studs.

4.2.2 Punching Shear Capacity Within the Shear Reinforced Zone

The punching shear strength is, as described in Section 3.4.2, defined as a combination of the strength carried by the concrete and the strength carried by the shear reinforcement. The contribution from the concrete is the same as the total shear strength for a slab without shear reinforcement as defined in Equation (4.4) in Section 4.1.2. This gives:

$$V_{Rd,c} = k_{\psi} \frac{\sqrt{f_{ck}}}{\gamma_c} b_0 d_v \tag{4.21}$$

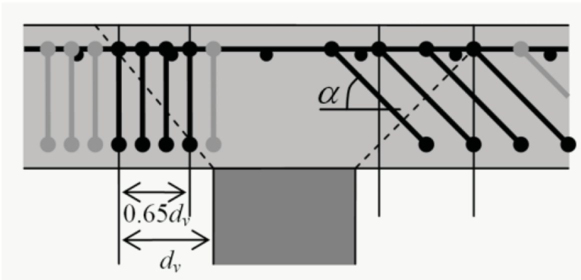


Figure 4.13: Shear reinforcement [7]

The shear reinforcement contribution may be calculated by the expression given in Equation (4.22) for inclined shear reinforcement or bent-up bars, as shown in Figure 4.13.

$$V_{Rd,s} = \sum A_{sw} k_e \sigma_{swd} \sin\alpha \tag{4.22}$$

This expression is similar to the expression presented in Equation (3.40) in Section 3.4.2, but in Equation (4.22) the factor k_e is introduced. The value for k_e is as presented in Table 4.1. Note that the angle previously defined as β is now called α. The angle is, as shown in Figure 4.13, the angle between the shear reinforcement and the slab. $\sum A_{sw}$ is the sum of the cross-sectional area of the reinforcement from the distance $0.35d_v$ to d_v from the loaded area. The stress activated in the shear reinforcement, σ_{swd}, can be calculated by the expression in Equation (4.23).

$$\sigma_{swd} = \frac{E_s \psi}{6} (\sin\alpha + \cos\alpha) \cdot \left(\sin\alpha + \frac{f_{bd}}{f_{ywd}} \cdot \frac{d}{\phi_w} \right) \leq f_{ywd} \tag{4.23}$$

ϕ_w is the diameter of the reinforcement bars, f_{ywd} is the yield strength of the shear reinforcement and f_{bd} is the bond strength. In many design cases the value of f_{bd} may be set equal to 3 MPa.

For vertical stirrups the expressions for the shear reinforcement contribution and σ_{swd} become as given in Equations (4.24) and (4.25).

$$V_{Rd,s} = \sum A_{sw} k_e \sigma_{swd} \tag{4.24}$$

$$\sigma_{swd} = \frac{E_s \psi}{6} \left(1 + \frac{f_{bd}}{f_{ywd}} \cdot \frac{d}{\phi_w} \right) \leq f_{ywd} \tag{4.25}$$

As a minimum amount of shear reinforcement, the expression in Equation (4.26) is used.

$$\sum A_{sw} k_e f_{ywd} \geq 0.5 V_{Ed} \tag{4.26}$$

4.2.3 Punching Shear Capacity Outside the Shear Reinforced Zone

The punching shear capacity outside the shear reinforced zone is calculated as described in Section 3.4.2, but with some modifications. The control perimeter is defined at a distance $d/2$ beyond the outer layer of the reinforcement, and $3d$ is considered as the maximum distance between two shear reinforcement bars, see Figure 4.14.

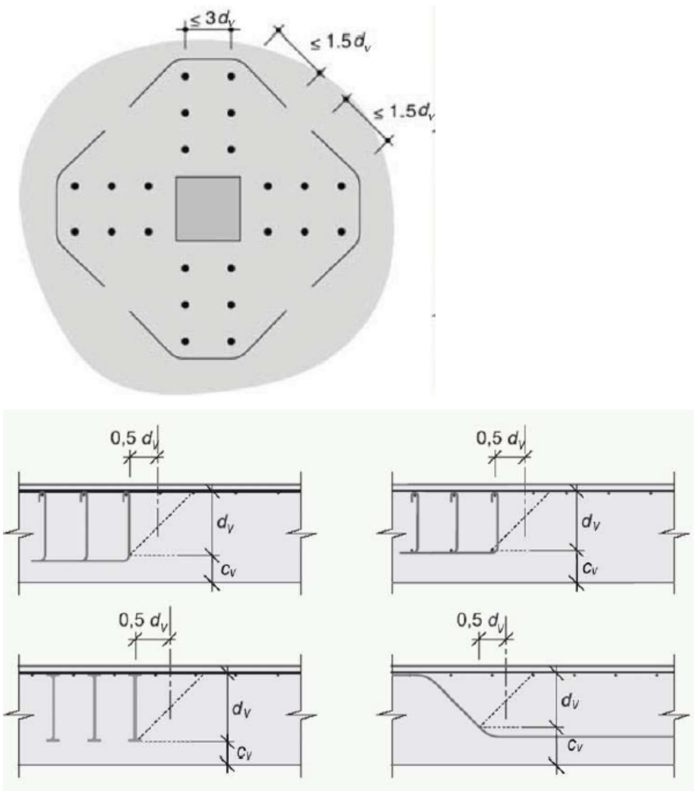


Figure 4.14: Reduced control perimeter and effective depth for punching outside the shear reinforced zone [7]

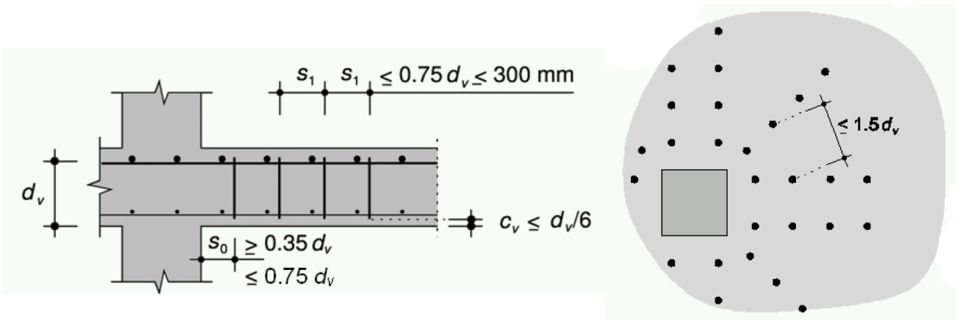
The capacity is hence given by Equation (4.27), where b_2 defines the new control perimeter outside the shear reinforced zone.

$$V_{Rd,out} = k_\psi \frac{\sqrt{f_{ck}}}{\gamma_c} b_2 d_{v,out} \tag{4.27}$$

4.2.4 Punching Shear Reinforcement

There are several requirements given in MC2010 for the arrangement of the reinforcement. First, the development length of the flexural reinforcement should not be located closer than at the minimum of the distance of $2.5d_v$ from the control perimeter of the loaded area and the distance to which the radial bending moment is zero.

There has to be a minimum of two reinforcement bars in radial direction, and the type of shear reinforcement must be chosen such that sufficient anchorage at both ends is obtained. The distance from the edge of the loaded area to the first reinforcement bar should be larger than $0.35d_v$ and smaller than $0.75d_v$, as shown in Figure 4.15a.



(a) **(b)**

Figure 4.15: Arrangement rules for the punching shear reinforcement in (a) radial and (b) tangential direction [7]

Figure 4.15a also shows how the maximum spacing of the shear reinforcement bars in radial direction should not exceed the smallest value of $0.75d_v$ and 300 mm, and that the maximum cover at the compression side should not exceed the value of $d_v/6$. The maximum distance in tangential direction between the reinforcement bars in the second radial row should not exceed $1.5d_v$, as shown in Figure 4.15b.

Table 4.2: Maximum diameter ϕ_{max} as a function of the effective slab thickness [7]

$\mathbf{d_v}$	$\phi_{\mathbf{max}}$
< 160	-
160-180	14
181-220	16
221-260	18
261-340	20
341-600	25
> 600	30

Table 4.2 presents the maximum diameters allowed as a function of d_v.

Chapter 5

Design Examples

The Norwegian consulting engineer firm Multiconsult AS has performed the design of the slabs in the building "Barcode B13", from now on referred to as B13, in Bjørvika in Oslo. This design includes the punching shear design of the slabs by the design procedure given in EC2. A 3D model of the B13 building is shown in Figure 5.1.

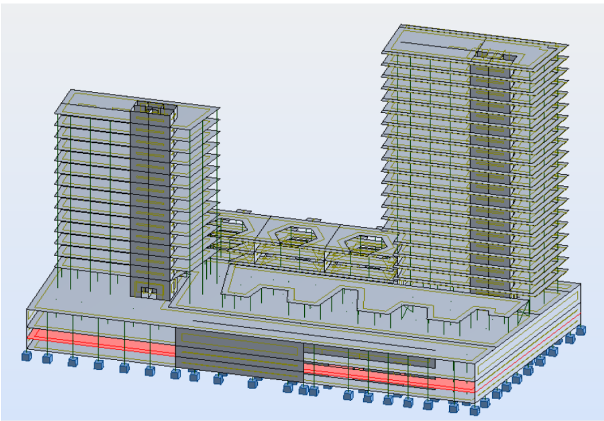


Figure 5.1: 3D model of the Barcode B13 building [8]

In this chapter, the results from the punching shear design by Multiconsult AS of the slab-column connection for selected columns in the slab over the lowest basement floor in the B13 building is presented and compared to the results from the design by the procedures for punching shear design in MC2010. The slab is highlighted in red in Figure 5.1. Figure 5.2 shows the slab with attached columns and load-carrying walls.

All the necessary information about the design by Multiconsult AS is gotten from the design report "Barcode B13 - kjeller vest: Dekke over K3" [8].

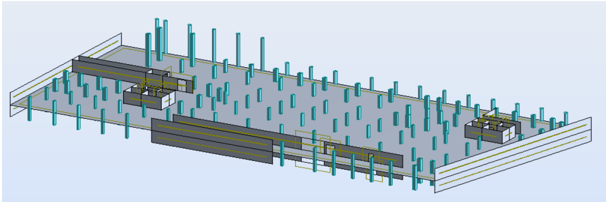


Figure 5.2: 3D model of the slab over the lowest basement floor [8]

5.1 Properties of the Slab-column Connections

Three different column cross-sections are used in the building. All the columns are rectangular, but they differ from one another in the dimensions. The cross-section dimensions of the columns are presented in Table 5.1.

Table 5.1: Cross-section dimensions of the columns [8]

Type	width · height [mm^2]
1	$700 \cdot 700$
2	$450 \cdot 700$
3	$400 \cdot 550$

An overview of the columns that were proven to need shear reinforcement and the type they represent is shown in Figure 5.3. The figure also shows corners of slab openings that need to be shear reinforced, but as punching shear is not the case for these areas, it will not be discussed in more detail.

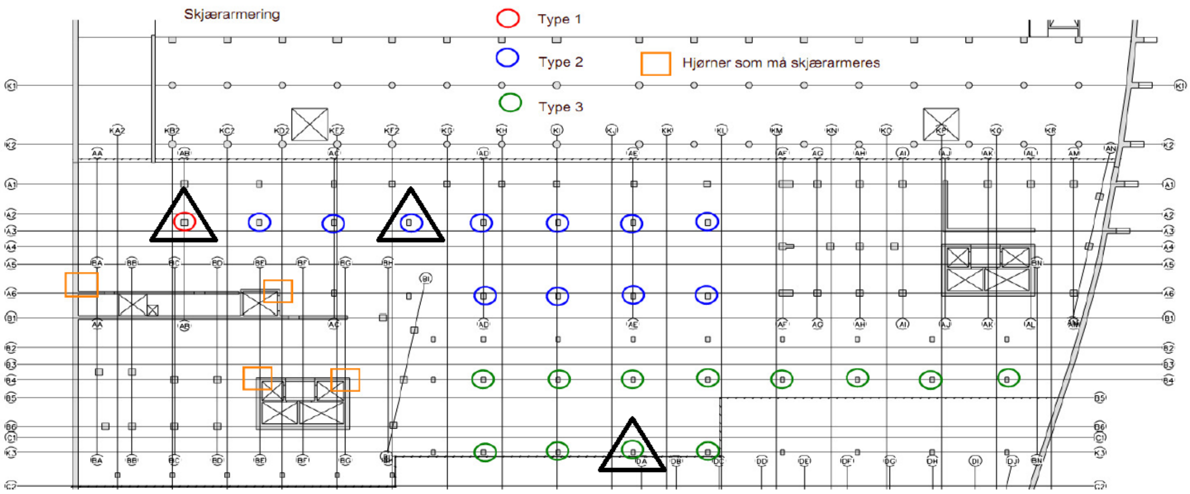


Figure 5.3: Columns with type specifications [8]

The calculations presented in the following sections are only presented for one column from each of the three column types. These columns are marked with triangles in Figure 5.3.

The slab has the same material parameters for the concrete and the reinforcement for all the column connections. The slab is reinforced with Ø16cc150 in both x- and z-direction, and in addition to this, extra reinforcement of Ø16cc150 in both directions is added around the three columns marked with triangles in Figure 5.3. This gives the total reinforcement amount of Ø16cc75 around all the three columns. All material parameters are given in Table 5.2.

Table 5.2: Material parameters for the concrete and the reinforcement [8]

Parameter	**Value**	**Unit**
Characteristic concrete strength, f_{ck}	35	MPa
Design concrete strength, f_{cd}	19.83	Mpa
Material factor concrete, γ_c	1.5	-
Mean E-modulus concrete, E_{cm}	34 077	MPa
Maximum aggregate size, d_g	20	mm
Mean tensile concrete strength, f_{ctm}	3.21	MPa
Design tensile concrete strength, f_{ctd}	1.27	MPa
Characteristic yield strength of steel, f_{yk}	500	MPa
Design yield strength of steel, f_{yd}	434.8	MPa
Material factor steel, γ_s	1.15	-
E-modulus steel, E_s	200 000	MPa
Slab thickness, t	300	mm
Effective depth, d_v	239	mm
Flexural reinforcement per unit length in x-direction, $A_{s,x}$	2680.8	mm^2
Flexural reinforcement per unit length in z-direction, $A_{s,z}$	2680.8	mm^2
Reinforcement ratio in x-direction, ρ_x	0.01122	-
Reinforcement ratio in z-direction, ρ_z	0.01122	-
Reiforcement ratio, ρ	0.01122	-

The punching shear tests were done in two different computer programs, G-PROG and FEM-design. Both programs use EC2 in the punching shear controls.

In FEM-design the building is modelled in 3D with all relevant load combinations. This program then finds the most critical load combination and performs a punching shear test based on the shear force acting on the slab-column connection. This force includes the contribution from the moment acting on the slab-column connection.

In the analyses in G-PROG, different combinations of shear force and moment are implemented, and the different loads are gotten from the model in FEM-design. Based on the forces implemented, G-PROG performs a punching shear design check. Only the loads implemented in the G-PROG analyses will be used in the calculations in Section 5.3.

The spans in x- and z-direction for the three columns are shown in Figure 5.4. These are

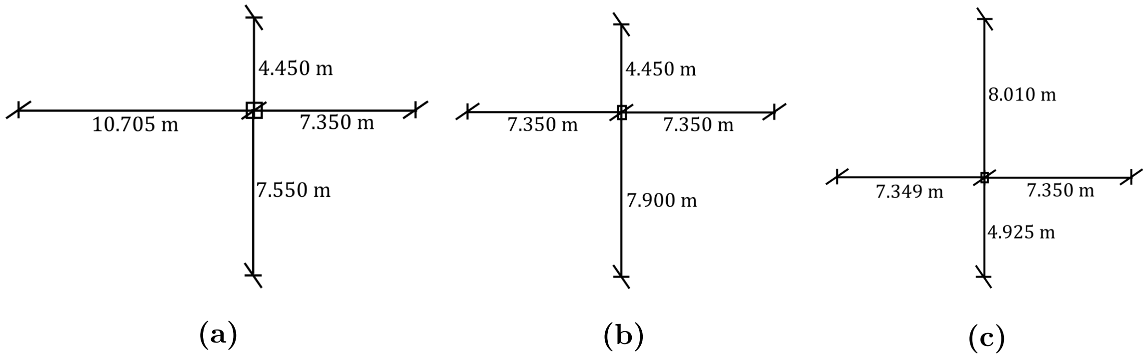


Figure 5.4: Spans in x- and z- direction for (a) column type 1, (b) column type 2 and (c) column type 3

taken from design sketches of the slab in the Multiconsult AS report.

The load combinations used in the G-PROG analyses for the three columns are shown in Tables 5.3, 5.4 and 5.5. The value of M_{Ed} represents the vector addition of the moment in x- and z-direction.

Table 5.3: Load combinations for column type 1 [8]

Load combination	$\mathbf{V_{Ed}}$ [kN]	$\mathbf{M_{Ed,x}}$ [kNm]	$\mathbf{M_{Ed,z}}$ [kNm]	$\mathbf{M_{Ed}}$ [kNm]
1	-850	0	0	0
2	-860	0	0	0
3	-1027	34	24	41.62
4	-1260	0	0	0

Table 5.4: Load combinations for column type 2 [8]

Load combination	$\mathbf{V_{Ed}}$ [kN]	$\mathbf{M_{Ed,x}}$ [kNm]	$\mathbf{M_{Ed,z}}$ [kNm]	$\mathbf{M_{Ed}}$ [kNm]
1	-785	0	0	0
2	-790	0	0	0
3	-969	1	52	52.01
4	-1035	0	0	0

Table 5.5: Load combinations for column type 3 [8]

Load combination	$\mathbf{V_{Ed}}$ [kN]	$\mathbf{M_{Ed,x}}$ [kNm]	$\mathbf{M_{Ed,z}}$ [kNm]	$\mathbf{M_{Ed}}$ [kNm]
1	-725	0	0	0
2	-730	0	0	0
3	-1038	0	0	0
4	-963	7	46	46.53

5.2 Results by Multiconsult AS

In this section, the results in the report by Multiconsult AS for the three columns will be presented. It should be noted that for the punching shear analyses i G-PROG the reinforcement amount implemented was Ø16cc150. The reason why Ø16cc75 was not used, although this is the correct reinforcement amount around the columns, was that the punching shear controls were performed before the additional reinforcement amount was shown necessary. The capacities presented in this section are therefore a little lower than they should. Calculations with the correct flexural reinforcement amount can be found in Appendix A. The appendix does not include calculations by EC2 of the necessary punching shear reinforcement amount, so the values from the Multiconsult AS report are used for that case.

5.2.1 Punching Shear Compression Capacity

At the edge of the columns, the shear compression capacity must be sufficient according to the criterion presented in Equation (2.21). Tables 5.6, 5.7 and 5.8 show the calculated capacities and the shear force acting on the perimeter at the edge of the three columns for the different loading combinations.

Table 5.6: Shear compression capacity for column type 1 [8]

Load combination	$\mathbf{u_0}$ [mm]	$\mathbf{v_{Ed,c}}$ [MPa]	$\mathbf{v_{Rd,max}}$ [MPa]	Utilization
1	2800	1.27	2.06	0.62
2	2800	1.29	2.06	0.63
3	2800	1.60	1.97	0.81
4	2800	1.88	2.06	0.92

Table 5.7: Shear compression capacity for column type 2 [8]

Load combination	$\mathbf{u_0}$ [mm]	$\mathbf{v_{Ed,c}}$ [MPa]	$\mathbf{v_{Rd,max}}$ [MPa]	Utilization
1	2300	1.43	2.29	0.62
2	2300	1.44	2.29	0.63
3	2300	1.87	2.15	0.87
4	2300	1.88	2.29	0.82

Table 5.8: Shear compression capacity for column type 3 [8]

Load combination	u_0 [mm]	$v_{Ed,c}$ [MPa]	$v_{Rd,max}$ [MPa]	Utilization
1	1900	1.60	2.56	0.62
2	1900	1.61	2.56	0.63
3	1900	2.29	2.56	0.89
4	1900	2.25	2.41	0.93

Here, $v_{Ed,c}$ is the shear force for shear compression failure control and $v_{Rd,max}$ is the maximum shear compression capacity at the edge of the columns. The shear compression capacity is, as is shown in the tables, sufficient for the three columns.

5.2.2 Punching Shear Tension Capacity

The shear tension capacity must also be sufficient according to EC2. In Tables 5.9, 5.10 and 5.11, the critical shear force, $v_{Ed,t}$, at a perimeter at the distance $2d$ from the edge of the column is presented for the three columns for the four different loading combinations. The concrete contribution to the punching shear capacity, $v_{Rd,c}$, is calculated by Equation (2.26). Based on the maximum shear force and the concrete contribution, the value of the amount of shear that must be carried by the shear reinforcement, $v_{Rd,s}$, is calculated according to Equation (2.39).

Table 5.9: Shear tension capacity for column type 1 [8]

Load combination	$v_{Ed,t}$ [MPa]	$v_{Rd,c}$ [MPa]	$v_{Rd,s}$ [MPa]	$A_{sw,tot,1}$ [mm^2]
1	0.61	0.62	-	-
2	0.62	0.62	0.16	1208
3	0.77	0.62	0.31	1359
4	0.91	0.62	0.44	1963

Table 5.10: Shear tension capacity for column type 2 [8]

Load combination	$v_{Ed,t}$ [MPa]	$v_{Rd,c}$ [MPa]	$v_{Rd,s}$ [MPa]	$A_{sw,tot,1}$ [mm^2]
1	0.62	0.62	-	-
2	0.62	0.62	0.16	1286
3	0.81	0.62	0.35	1746
4	0.82	0.62	0.35	1746

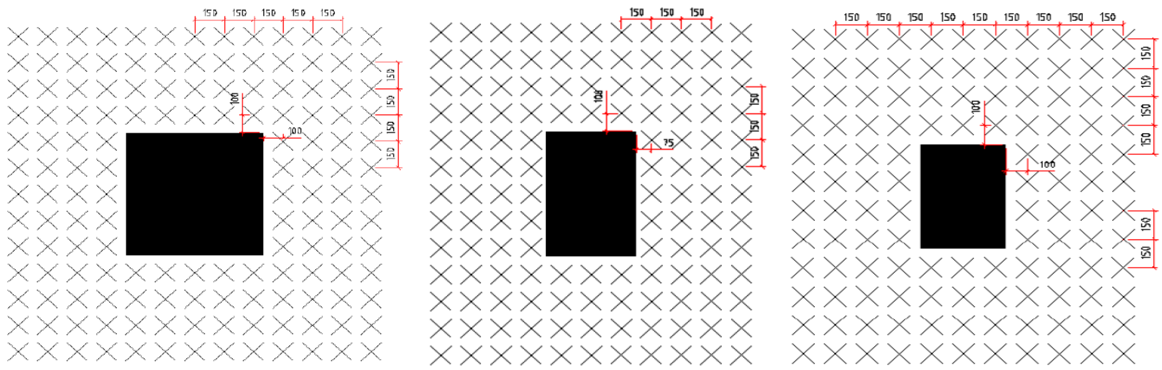
Table 5.11: Shear tension capacity for column type 3 [8]

Load combination	$v_{Ed,t}$ [MPa]	$v_{Rd,c}$ [MPa]	$v_{Rd,s}$ [MPa]	$A_{sw,tot,1}$ [mm^2]
1	0.62	0.62	-	-
2	0.62	0.62	0.16	1441
3	0.89	0.62	0.42	2328
4	0.87	0.62	0.41	2217

$A_{sw,tot,1}$ is the necessary total amount of shear reinforcement. This is not the same as the actual amount of shear reinforcement, as there are several requirements for maximum centre distances. The outer perimeter at which shear reinforcement is not necessary, $u_{out,ef}$, is equal to the perimeter at a distance $2d$ from the edge of the column for all three columns, and is 5803 mm for column type 1, 5303 mm for column type 2 and 4903 mm for column type 3.

Shear reinforcement is shown to be necessary for all load combinations except load combination 1.

For practical reasons with respect to the arrangement of the longitudinal reinforcement, the three columns are shear reinforced as shown in Figure 5.5, where each X represents a reinforcement bar. All the reinforcement bars have diameters of 10 mm.



(a) (b) (c)

Figure 5.5: Reinforcement arrangement of Ø10 reinforcement bars for (a) column type 1, (b) column type 2 and (c) column type 3 [8]

As is seen in Figure 5.5, the actual amount of punching shear reinforcement applied to each slab-column connection is much larger than the calculated necessary amount shown in Tables 5.9, 5.10 and 5.11. The reason for this is the requirements for maximum centre distances for the reinforcement bars. The total necessary amount of punching shear reinforcement accounting for requirements for maximum centre distances of the reinforcement bars along with the actual applied shear reinforcement area according to Figure 5.5 for the critical load combination for each column is shown in Table 5.12.

Table 5.12: Required and applied amount of shear reinforcement for the critical load combinations for the three columns [8]

Column type	**Load combination**	$\mathbf{A_{sw,tot,2}}$ **[mm^2]**	$\mathbf{A_{sw,applied}}$ **[mm^2]**	**SR [mm]**
1	4	5958	11310	828
2	4	5756	10053	828
3	3	6368	9425	753

In Table 5.12, $A_{sw,tot,2}$ denotes the required amount of shear reinforcement accounting for requirements, $A_{sw,applied}$ is the actual applied amount of shear reinforcement and SR the distance from the column centre to where shear reinforcement is not required.

One of the reasons why the applied amount of reinforcement is so much larger than the required value is that the value of the centre distance between the reinforcement bars is chosen such that it matches the centre distance between the flexural reinforcement bars, which is a convenient and preferable way to arrange the shear reinforcement bars.

5.3 Design According to MC2010

In the following sections, calculations on the slab-column connections in the report by Multiconsult AS will be done according to the design approach in MC2010, presented in Chapter 4. All the calculations are also presented in detail in Appendix B.

Since LoA III depends on a linear elastic analysis to present the value of r_s, this method will not be used in the calculations in this section. LoA IV depends on a non-linear analysis of the slab, and is therefore not discussed further either. It might be noted that since the span lengths are not symmetric on each side of the columns, as presented in Figure 5.4a, LoA III or LoA IV might give more accurate results. Nevertheless, for calculation simplicity reasons the calculations in this section will focus on LoA I and LoA II.

Since the slab-column connection has the same material parameters, presented in Table 5.2, both the flexural strength m_{Rd} and the aggregate size factor k_{dg} is the same for all the three column types. k_{dg} is calculated according to Equation (7.3-62) in MC2010:

$$k_{dg} = \frac{32}{16 + d_g} = \frac{32}{16 + 20} = 0.89 \tag{5.1}$$

Since the value of d_g is larger than 16 mm the value 1.0 could have been used as well, which would have given more conservative results. Here however, the calculated value of k_{dg} will be used.

The flexural strength is calculated according to Equation (3.12) in Section 3.3.1, assuming a rigid-plastic behaviour of concrete and steel, which gives:

$$
\begin{aligned}
m_{Rd} &= \rho \cdot f_{yd} \cdot d^2 \cdot \left(1 - \frac{\rho \cdot f_{yd}}{2 \cdot f_{cd}}\right) \\
&= 0.01122 \cdot 434.8 \cdot 239^2 \cdot \left(1 - \frac{0.01122 \cdot 434.8}{2 \cdot 19.83}\right) \\
&= 244.3\,\text{kNm/m}
\end{aligned} \tag{5.2}
$$

For all the three columns, both the punching shear resistance and the maximum punching shear resistance limited by crushing of concrete struts must be larger than the applied load. The punching shear resistance is calculated according to Equation (5.3)/MC2010 (7.3-69) and the maximum punching shear resistance is limited by crushing according to Equation (5.4)/MC2010 (7.3-60).

$$
V_{Rd,max} = k_{sys} k_{\psi} \frac{\sqrt{f_{ck}}}{\gamma_c} b_0 d_v \leq \frac{\sqrt{f_{ck}}}{\gamma_c} b_0 d_v \tag{5.3}
$$

$$
V_{Rd} = V_{Rd,c} + V_{Rd,s} = k_{\psi} \frac{\sqrt{f_{ck}}}{\gamma_c} b_0 d_v + \sum A_{swi} k_e \sigma_{swd} \sin\alpha \tag{5.4}
$$

For LoA I the capacity of the concrete contribution will be controlled against the total applied loads for the different load combinations. If the contribution from the concrete is not large enough to carry the shear force alone, punching shear reinforcement is necessary. A more accurate approach is in this case appropriate in terms of determining the amount of punching shear reinforcement, as the LoA I approach is highly conservative. The punching shear reinforcement detailing will therefore only be performed according to the calculated capacities by LoA II. In LoA II the capacity is determined by looking at acting moments and the moment capacity on a unit length in the support strip.

Since it is obvious from Tables 5.3, 5.4 and 5.5 that load combination 1 and 2 will not be governing, calculations will only be performed on load combination 3 and 4 for LoA I and LoA II.

5.3.1 Column Type 1

The lengths of the spans in x- and z-direction for the column are presented in Figure 5.4a. In terms of maximizing the value of r_s, as suggested in MC2010, the largest values of the spans in x- and z-direction respectively are used as the span lengths. The column dimensions, the span lengths and the design loads for column type 1 are presented in Table 5.13.

Table 5.13: Parameters for column type 1 [8]

Parameter	**Value**	**Unit**
Column dimensions, $w_c \cdot h_c$	$700 \cdot 700$	mm^2
Span length in x-direction, L_x	10.705	m
Span length in y-direction, L_z	7.550	m
Design shear force in LC3, $V_{Ed,3}$	1027	kN
Design moment in LC3, $M_{Ed,3}$	41.62	kNm
Design shear force in LC4, $V_{Ed,4}$	1260	kN
Design moment in LC4, $M_{Ed,4}$	0	kNm

LoA I for Load Combination 3

In terms of determining the punching shear capacity, first the control section b_0 must be defined. This is calculated according to Section 4.1.1, and is dependent of the value of the eccentricity coefficient k_e. The approximated values of k_e in Table 4.1 may be used if the adjacent spans do not differ in length by more than 25%. In this case the adjacent spans differ more in length than 25%, as is shown in Figure 5.4a. The value of k_e must therefore be determined by Equation (4.3)/MC2010 (7.3-59).

The value of e_u in the expression for k_e may be determined by Equation (5.5).

$$e_u = \frac{M_{Ed}}{V_{Ed}} = \frac{41.62 \cdot 10^6}{1027 \cdot 10^3} = 40.5\,\text{mm} \tag{5.5}$$

The value of b_u in the expression for k_e is the diameter of a circle with the same surface as the region inside the basic control perimeter, A_{per}. A_{per}, b_u and k_e are calculated in Equations (5.6), (5.7) (5.8).

$$\begin{aligned} A_{per} &= w_c \cdot h_c + 2 \cdot 0.5 \cdot d \cdot w_c + 2 \cdot 0.5 \cdot d \cdot h_c + \pi \cdot (0.5 \cdot d)^2 \\ &= 700 \cdot 700 + 2 \cdot 0.5 \cdot 239 \cdot 700 + 2 \cdot 0.5 \cdot 239 \cdot 700 + \pi \cdot (0.5 \cdot 239)^2 \\ &= 869462.7\,\text{mm}^2 \end{aligned} \tag{5.6}$$

$$b_u = \sqrt{\frac{4 \cdot A_{per}}{\pi}} = \sqrt{\frac{4 \cdot 869462.7}{\pi}} = 1052.2\,\text{mm} \tag{5.7}$$

$$k_e = \frac{1}{1 + \dfrac{e_u}{b_u}} = \frac{1}{1 + \dfrac{40.5}{1052.2}} = 0.96 \tag{5.8}$$

Now, the length of the shear-resisting control perimeter may be calculated by Equation (5.9).

$$b_0 = k_e \cdot (2 \cdot (w_c + h_c) + \pi \cdot d) = 0.96 \cdot (2 \cdot (700 + 700) + \pi \cdot 239) = 3419\,\text{mm} \tag{5.9}$$

According to MC2010 7.3.5.4, the value of r_s can be approximated as the largest value of $0.22\,L_x$ and $0.22\,L_z$, which is given by Equations (5.10), (5.11) and (5.12).

$$r_{s,x} = 0.22 \cdot L_x = 0.22 \cdot 10.705 = 2.36\,\text{m} \tag{5.10}$$

$$r_{s,z} = 0.22 \cdot L_z = 0.22 \cdot 7.55 = 1.66\,\text{m} \tag{5.11}$$

$$r_s = max\,(r_{s,x};\, r_{s,z}) = 2.36\,\text{m} \tag{5.12}$$

Then, the angle is calculated according to Equation (7.3-70) in MC2010, which gives the value of the angle given in Equation (5.13).

$$\psi = 1.5 \cdot \frac{r_s}{d} \frac{f_{yd}}{E_s} = 1.5 \cdot \frac{2360}{239} \frac{434.8}{200000} = 0.0321 \tag{5.13}$$

Further on, the value of k_ψ given in Equation (7.3-63) in MC2010, is calculated as given in Equation (5.14).

$$k_\psi = \frac{1}{1.5 + 0.9 \cdot k_{dg} \cdot \psi \cdot d} = \frac{1}{1.5 + 0.9 \cdot 0.89 \cdot 0.0321 \cdot 239} = 0.13083 \tag{5.14}$$

The concrete contribution to the punching shear capacity according to Equation (5.4) is then obtained as:

$$V_{Rd,c} = k_\psi \frac{\sqrt{f_{ck}}}{\gamma_c} b_0 d_v = 0.13083 \cdot \frac{\sqrt{35}}{1.5} \cdot 3419 \cdot 239 = 421.7\,\text{kN} \tag{5.15}$$

Since the contribution from the concrete to the punching shear capacity is smaller than the design shear force for load combination 3, punching shear reinforcement is necessary.

The punching shear compression capacity is expressed by Equation (5.3)/MC2010 (7.3-69). It is possible to check what type of reinforcement system that can be used by evaluating the factor k_{sys}, which accounts for the performance of punching shear reinforcement systems.

$$V_{Rd,max} \geq V_{Ed} \rightarrow k_{sys} \geq \frac{V_{Ed}\gamma_c}{k_\psi \sqrt{f_{ck}} b_0 d_v} = \frac{V_{Ed}}{V_{Rd,c}} = \frac{1027}{421.7} = 2.44 \tag{5.16}$$

According to MC2010 7.3.5.3 the value $k_{sys} = 2.0$ may be adopted, but higher values up to $k_{sys} = 2.8$ may be used if more restrictive detailing rules are adopted and if the arrangement of the shear reinforcement is checked at the construction site.

LoA I for Load Combination 4

The calculation procedure for load combination 4 at LoA I is naturally the same as for load combination 3. Since the design moment for load combination 4 is zero, there is no eccentricity for this load combination, and the value of e_u is zero. This gives the expressions for k_e and b_0 given in Equations (5.17) and (5.18).

$$k_e = \frac{1}{1 + \frac{e_u}{b_u}} = \frac{1}{1 + \frac{0}{1052.2}} = 1.0 \tag{5.17}$$

$$b_0 = k_e \cdot (2 \cdot (w_c + h_c) + \pi \cdot d) = 1.0 \cdot (2 \cdot (700 + 700) + \pi \cdot 239) = 3551\,\mathrm{mm} \tag{5.18}$$

Further on, the value of r_s is the same as for load combination 3, which also gives the same values for the rotation ψ and the rotation factor k_ψ. The final expression for the concrete contribution to the punching shear capacity by LoA I for load combination 4 then becomes as given in Equation (5.19).

$$V_{Rd,c} = k_\psi \frac{\sqrt{f_{ck}}}{\gamma_c} b_0 d_v = 0.13083 \cdot \frac{\sqrt{35}}{1.5} \cdot 3551 \cdot 239 = 437.9\,\mathrm{kN} \tag{5.19}$$

This value is slightly larger than the value for load combination 3, which is a result of the fact that there is no eccentricity for load combination 4. The larger design shear force V_{Ed} for load combination 4 however, causes larger compressive shear forces at the shear-resisting control perimeter, which might be critical in comparison to the punching shear compression capacity.

The expression for k_{sys} for load combination 4 becomes as given in Equation (5.20).

$$k_{sys} \geq \frac{V_{Ed}}{V_{Rd,c}} = \frac{1260}{437.9} = 2.88 \tag{5.20}$$

This value of k_{sys} is slightly larger than the allowed value of 2.8, but as LoA I is only for preliminary design, this value is sufficient enough for this approximation level. If the punching shear compression capacity determined by LoA II is larger than the design shear load, the punching shear compression capacity is sufficient for the slab, as this approximation level is much more accurate as it accounts for the moment capacity of the slab in the calculation of the rotation.

LoA II for Load Combination 3

The calculation method in LoA II is quite similar to the LoA I approach. The main difference in the two methods is that the critical rotation angle is calculated based on the design moment per unit length of the support strip and the moment capacity, given in Equation (5.2).

The expression for calculation of the rotation of the slab according to LoA II is given by Equation (4.8)/MC2010 (7.3-75). In terms of calculating the value of the rotation according to this expression, the design moment per unit length of the support strip must be calculated. For inner columns, this expression was presented in Equation (4.9)/MC2010 (7.3-71). This value is dependent of the width of the support strip b_s and the eccentricity coefficient e_u.

Since the eccentricity coefficient is only dependent of the ratio between the moment and the shear force acting on the system, the value will be equal to the one obtained for LoA I. The width of the support strip is dependent of the values of $r_{s,x}$ and $r_{s,z}$, and they will also be the same in LoA II as in LoA I, as they only depend on the span lengths of the slab. The width of the support strip may be calculated by Equation (5.21).

$$b_s = 1.5 \cdot \sqrt{r_{s,x} \cdot r_{s,z}} = 1.5 \cdot \sqrt{2.36 \cdot 1.66} = 2.97\,\mathrm{m} \tag{5.21}$$

Now that the value of b_s is known, the value of the design moment per unit length may be calculated by Equation (5.22).

$$m_{Ed} = V_{Ed} \cdot \left(\frac{1}{8} + \frac{|e_u|}{2 \cdot b_s}\right) = 1027 \cdot \left(\frac{1}{8} + \frac{40.5}{2 \cdot 2970}\right) = 135.4\,\text{kNm/m} \tag{5.22}$$

The values of the rotation angle and the rotation factor may then be determined by Equations (5.23) and (5.24).

$$\psi = 1.5 \cdot \frac{r_s}{d}\frac{f_{yd}}{E_s} \cdot \left(\frac{m_{Ed}}{m_{Rd}}\right)^{3/2} = 1.5 \cdot \frac{2360}{239}\frac{434.8}{200000} \cdot \left(\frac{135.4}{244.3}\right)^{3/2} = 0.0133 \tag{5.23}$$

$$k_\psi = \frac{1}{1.5 + 0.9 \cdot k_{dg} \cdot \psi \cdot d} = \frac{1}{1.5 + 0.9 \cdot 0.89 \cdot 0.0133 \cdot 239} = 0.24787 \tag{5.24}$$

The concrete contribution to the punching shear capacity according to Equation (5.4) is then obtained as:

$$V_{Rd,c} = k_\psi \frac{\sqrt{f_{ck}}}{\gamma_c} b_0 d_v = 0.24787 \cdot \frac{\sqrt{35}}{1.5} \cdot 3419 \cdot 239 = 798.9\,\text{kN} \tag{5.25}$$

This value is much larger than then value obtained by LoA I, but it is still not large enough to carry the applied shear force without shear reinforcement, which means that punching shear reinforcement is necessary.

The punching shear compression capacity must be sufficient according to Equation (5.3). The value of k_{sys} may, as earlier described, be set equal to 2.0 unless it is proven that a larger value may be used. In this case the value of 2.0 is adopted. This gives the value of the punching shear compression capacity given in Equation (5.26).

$$\begin{aligned} V_{Rd,max} &= 2.0 \cdot 0.24787 \cdot \frac{\sqrt{35}}{1.5} \cdot 3419 \cdot 239 \leq \frac{\sqrt{35}}{1.5} \cdot 3419 \cdot 239 \\ &\rightarrow 1597.8\,\text{kN} \leq 3223.0\,\text{kN} \\ &\rightarrow V_{Rd,max} = 1597.8\,\text{kN} \end{aligned} \tag{5.26}$$

$$V_{Rd,max} \geq V_{Ed} = 1027\,\text{kN} \tag{5.27}$$

As is shown in Equation (5.27), the punching shear compression capacity is sufficient according to LoA II. This means that the slab will have sufficient capacity with sufficient amount of shear reinforcement. The utilization of the punching shear compression capacity is given by Equation (5.28).

$$u_{Rd,max} = \frac{V_{Ed}}{V_{Rd,max}} = \frac{1027}{1597.8} = 0.64 \tag{5.28}$$

The amount of shear force that must be taken by the shear reinforcement is given in Equation (5.29).

$$V_{Rd,s} = V_{Ed} - V_{Rd,c} = 1027 - 798.9 = 228.1\,\mathrm{kN} \tag{5.29}$$

LoA II for Load Combination 4

The same values for k_e and b_0 are obtained for LoA II as for LoA I for load combination 4, and the same value as for LoA II on load combination 3 is obtained for the width of the support strip b_s.

With LoA II on load combination 4, the expressions for m_{Ed} in Equation (5.30), the rotation ψ in Equation (5.31) and the rotation factor k_ψ in Equation (5.32) are obtained.

$$m_{Ed} = V_{Ed} \cdot \left(\frac{1}{8} + \frac{|e_u|}{2 \cdot b_s}\right) = 1260 \cdot \left(\frac{1}{8} + \frac{0}{2 \cdot 2970}\right) = 157.5\,\mathrm{kNm/m} \tag{5.30}$$

$$\psi = 1.5 \cdot \frac{r_s}{d}\frac{f_{yd}}{E_s} \cdot \left(\frac{m_{Ed}}{m_{Rd}}\right)^{3/2} = 1.5 \cdot \frac{2360}{239}\frac{434.8}{200000} \cdot \left(\frac{157.5}{244.3}\right)^{3/2} = 0.0166 \tag{5.31}$$

$$k_\psi = \frac{1}{1.5 + 0.9 \cdot k_{dg} \cdot \psi \cdot d} = \frac{1}{1.5 + 0.9 \cdot 0.89 \cdot 0.0166 \cdot 239} = 0.21368 \tag{5.32}$$

The concrete contribution to the punching shear capacity for load combination 4 becomes as given in Equation (5.33).

$$V_{Rd,c} = k_\psi \frac{\sqrt{f_{ck}}}{\gamma_c} b_0 d_v = 0.21368 \cdot \frac{\sqrt{35}}{1.5} \cdot 3551 \cdot 239 = 715.2\,\mathrm{kN} \tag{5.33}$$

The slab will have sufficient capacity with punching shear reinforcement if the punching shear compression capacity is sufficient. The expression for the compression capacity is given by Equation (5.34). The value of $k_{sys} = 2.0$ is adopted, as for load combination 3.

$$\begin{aligned} V_{Rd,max} &= 2.0 \cdot 0.21368 \cdot \frac{\sqrt{35}}{1.5} \cdot 3551 \cdot 239 \leq \frac{\sqrt{35}}{1.5} \cdot 3551 \cdot 239 \\ &\rightarrow 1430.4\,\text{kN} \leq 3347.1\,\text{kN} \\ &\rightarrow V_{Rd,max} = 1430.4\,\text{kN} \end{aligned} \tag{5.34}$$

$$V_{Rd,max} \geq V_{Ed} = 1260\,\text{kN} \tag{5.35}$$

The compression capacity is sufficient, and the slab will have sufficient punching shear capacity with the necessary amount of shear reinforcement applied. The utilization of the punching shear compression capacity is given by Equation (5.36).

$$u_{Rd,max} = \frac{V_{Ed}}{V_{Rd,max}} = \frac{1260}{1430.4} = 0.88 \tag{5.36}$$

The amount of shear force that must be taken by the punching shear reinforcement is given by Equation (5.37).

$$V_{Rd,s} = V_{Ed} - V_{Rd,c} = 1260 - 715.2 = 544.8\,\text{kN} \tag{5.37}$$

Required Punching Shear Reinforcement by LoA II

Since the value of the necessary punching shear reinforcement contribution to the capacity is much larger for load combination 4, this load combination will be governing for the choice of a proper shear reinforcement amount. Therefore, only the results for load combination 4 will be presented.

The expression for the shear reinforcement contribution presented in Equation (5.4)/MC2010 (7.3-64) gives the following expression for vertical reinforcement bars, i.e. $\alpha = 90°$:

$$V_{Rd,s} = \sum A_{swi} k_e \sigma_{swd} \sin(90°) = \sum A_{swi} k_e \sigma_{swd} \tag{5.38}$$

The required value of the amount of shear that must be taken by the punching shear reinforcement, $V_{Rd,s}$, inserted into Equation (5.38) gives the expression for the required amount of shear reinforcement within the zone bounded by $0.35d_v$ and d_v given in Equation (5.39).

$$A_{sw} = \sum A_{swi} = \frac{V_{Rd,s}}{k_e \sigma_{swd}} \tag{5.39}$$

Equation (7.3-67) in MC2010 presents a requirement for the minimum amount of reinforcement given in Equation (5.40).

$$\sum A_{swi} k_e f_{ywd} \geq 0.5V_{Ed} \rightarrow A_{sw,min} = \frac{0.5V_{Ed}}{k_e f_{ywd}} \tag{5.40}$$

The value of σ_{swd} is given by Equation (7.3-65) in MC2010, which gives the following expression with vertical stirrups:

$$\sigma_{swd} = \frac{E_s \psi}{6} \left(1 + \frac{f_{bd}}{f_{ywd}} \cdot \frac{d_v}{\phi_w}\right) \leq f_{ywd} \tag{5.41}$$

Here, the bond strength f_{bd} may be set equal 3 MPa according to MC2010 7.3.5.3. ϕ_w is the diameter of the shear reinforcement bars, which is 10 mm, and f_{ywd} is the yield strength of the reinforcement bars, equal to 434.8 MPa. The value of σ_{swd} is then calculated in Equation (5.42).

$$\sigma_{swd} = \frac{200000 \cdot 0.0166}{6} \left(1 + \frac{3}{434.8} \cdot \frac{239}{10}\right) = 645.8\,\text{MPa} \tag{5.42}$$

This value is larger than the value of f_{ywd}, which gives that the final value of σ_{swd} becomes:

$$\sigma_{swd} = f_{ywd} = 434.8\,\text{MPa} \tag{5.43}$$

After determining the value of σ_{swd}, the value of the required amount of reinforcement may be calculated by Equation (5.44).

$$\begin{aligned} A_{sw,tot,1} &= max\left(A_{sw};\ A_{sw,min}\right) \\ &= max\left(\frac{V_{Rd,s}}{k_e\sigma_{swd}};\ \frac{0.5V_{Ed}}{k_e f_{ywd}}\right) \\ &= max\left(\frac{544.8}{1.0\cdot 434.8};\ \frac{0.5\cdot 1260}{1.0\cdot 434.8}\right) \\ &= max\left(1253\,\mathrm{mm}^2;\ 1449\,\mathrm{mm}^2\right) \\ &= 1449\,\mathrm{mm}^2 \end{aligned} \tag{5.44}$$

This is the amount of shear reinforcement that must be placed within the zone bounded by $0.35d_v$ and d_v. In addition to this, the slab must be shear reinforced according to requirements for minimum reinforcement amounts out to the outer perimeter b_2. The calculation of b_2 is done according to Equation (4.27) in Section 4.2.3, where $d_{v,out}$ is the effective depth of the slab minus the cover at the lower edge of the slab. The cover at the lower edge should according to MC2010 7.13.5.3 not exceed $d_v/6 = 239/6 = 39.8$ mm. The cover in the Multiconsult AS report is 45 mm, and with shear reinforcement bars of 10 mm included, this gives a cover of 35 mm, given that the shear reinforcement bars are placed outside the flexural reinforcement. This gives the value of b_2 given in Equation (5.45).

$$b_2 = \frac{V_{Ed}\gamma_c}{k_\psi\sqrt{f_{ck}}d_{v,out}} = \frac{1260\cdot 1.5}{0.21368\cdot\sqrt{35}\cdot 204} = 7328.7\,\mathrm{mm} \tag{5.45}$$

Since there is no eccentricity of loading in load combination 4, the factor k_e is still equal to 1.0. This gives the expression for the outer perimeter where shear reinforcement is not necessary given in Equation (5.46).

$$b_{out} = \frac{b_2}{k_e} = \frac{7328.7}{1.0} = 7328.7\,\mathrm{mm} \tag{5.46}$$

The centre distance between the shear reinforcement bars was in the Multiconsult AS report chosen as 150 mm in both directions. This is an appropriate value, as it matches the centre distance of the flexural reinforcement, which is common for design purposes. The same value of the centre distance is therefore chosen here. The reinforcement ratio with the chosen centre distances and shear reinforcement diameters becomes as given in Equation (5.47), where $cc_{w,x}$ and $cc_{w,z}$ denotes the centre distances of the reinforcement bars in x-and z-direction.

$$\rho_{sw} = \frac{\frac{\pi \cdot \phi_w^2}{4}}{cc_{w,x} \cdot cc_{w,z}} = \frac{\frac{\pi \cdot 10^2}{4}}{150 \cdot 150} = 0.35\% \tag{5.47}$$

The area within the zone bounded by $0,35d_v$ and d_v and the reinforcement amount within this zone with the calculated reinforcement ratio become as given in Equations (5.48) and (5.49).

$$\begin{aligned} A_{within} &= 2 \cdot d \cdot (w_c + h_c) + \pi \cdot d^2 - 2 \cdot 0.35 \cdot d \cdot (w_c + h_c) - \pi \cdot (0.35 \cdot d)^2 \\ &= 2 \cdot 239 \cdot (700 + 700) + \pi \cdot 239^2 - 2 \cdot 0.35 \cdot 239 \cdot (700 + 700) - \pi \cdot (0.35 \cdot 239)^2 \\ &= 592448\,\mathrm{mm}^2 \end{aligned} \tag{5.48}$$

$$A_{sw,within} = \rho_{sw} \cdot A_{within} = 0.0035 \cdot 592448 = 2068\,\mathrm{mm}^2 \tag{5.49}$$

The calculated area within the zone is larger than the required area calculated in Equation (5.44).

In addition to the amount given in Equation (5.49), the slab must be reinforced out to the perimeter b_{out}, calculated in Equation (5.46). In addition to this, the requirements presented in Section 4.2.4 must be fulfilled. After applying the detailing requirements of this section, and using a rectangular reinforcement arrangement, a total amount of 160 reinforcement bars is shown to be necessary. This is shown in Figure 5.6, and gives the total reinforcement area given in Equation (5.50).

$$A_{sw,applied} = 160 \cdot \frac{\pi \cdot 10^2}{4} = 12566\,\mathrm{mm}^2 \tag{5.50}$$

As is seen in Figure 5.6, within the zone bounded by $0.35d_v$ and d_v, the area marked in grey, there is a total of 24 reinforcement bars, which gives an area of 1885 mm^2, which is higher than the required amount of 1449 mm^2. This reinforcement area is the area that should be included in the calculations.

It should be noted that the total reinforcement amount of 12566 mm^2 and the amount of 1885 mm^2 that should be included in calculations are approximate values after the most important requirements are fulfilled. A detailed design of the reinforcement bars

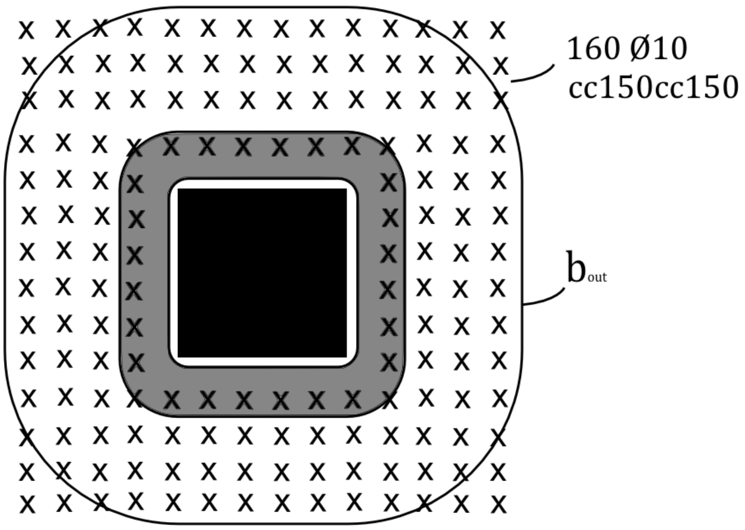


Figure 5.6: Applied shear reinforcement

will not be performed as it is not necessary in terms of performing a simple comparison of the method in EC2 and the method in MC2010. The design presented in Figure 5.6 is sufficient enough for this purpose.

5.3.2 Column Type 2 and 3

Since the calculation procedure for column type 2 and 3 is the same as for column type 1, the calculations of the strengths and the required amount of punching shear reinforcement for column type 2 and 3 will not be presented in detail. These are shown in detail in Appendix B. The important governing results from the calculations on column type 2 and 3 are shown in Table 5.14.

Table 5.14: Results for column type 2 and 3

Parameter	Column type 2	Column type 3
$V_{Rd,s}$, LoA I	558.4 kN	628.2 kN
$u_{Rd,max}$, LoA II	0.59	0.68
$A_{sw,tot,1}$, LoA II	1190 mm^2	1194 mm^2
$A_{sw,within}$, LoA II	1797 mm^2	1580 mm^2
SR, LoA II	665 mm	673 mm

In Table 5.14, $A_{sw,tot,1}$ is the calculated required amount of shear reinforcement within the zone bounded by $0.35d_v$ and d_v and $A_{sw,within}$ is the area of reinforcement bars within this zone when the centre distance of 150 mm is adopted. $u_{Rd,max}$ is the utilization of the punching shear compression capacity, $V_{Rd,s}$ is the amount of shear that must be taken by the shear reinforcement for LoA I and SR is the distance from the centre of the column to the perimeter where shear reinforcement is not necessary.

Both these two columns have a lower utilization of the punching shear compression capacity than column type 1. This is caused by the fact that column type 2 and 3 are subjected to smaller loads, which makes the rotations around these slab-column connections smaller, and the capacities larger.

5.4 Comparison of EC2 and MC2010 for Punching Shear Design

In this section, the results from the Multiconsult AS report, based on EC2, and the calculations done according to MC2010 are compared, with respect to punching shear compression capacity and amount of reinforcement required.

5.4.1 Punching Shear Compression Capacity

The calculated maximum utilizations of the punching shear compression capacity according to EC2 and LoA II in MC2010 are shown in Table 5.15.

Table 5.15: Punching shear compression capacity utilization according to EC2 and MC2010

Design approach	Column type 1	Column type 2	Column type 3
EC2	0.73	0.70	0.74
MC2010, LoA II	0.88	0.59	0.68

The utilizations according to EC2 in Table 5.15 have been updated according to the correct amount of flexural reinforcement presented in calculations in Appendix A, and are therefore smaller than the values presented in Tables 5.6, 5.7 and 5.8. This is caused by the fact that the governing value of the punching shear compression capacity in this case is dependent of the concrete contribution to the punching shear capacity, which again is dependent of the reinforcement ratio in the slab. How the reinforcement ratio influences the punching shear capacity was thoroughly described in Section 3.3. The influence of the reinforcement ratio according to EC2 is more conservative than in MC2010.

For column type 1 the approach in MC2010 gives a higher utilization than the approach in EC2. For the other two column types, the utilizations are smaller for the approach in MC2010. The reason for this might be that the punching shear compression capacity in MC2010 depends a lot on the value of the rotation factor, which again depends on the rotation of the slab. This rotation is almost twice as large for column 1 as for column type 2 and 3. The rotation depends on the size of the largest span length, and since the span length in x-direction is much larger for column type 1, this might be one of the reasons for the large rotation value. The approach in EC2 does not account for the length of the spans. The only factors in the governing expression in this example for the punching shear compression capacity in EC2 is the length of the control perimeter and the factor β, and these two values do not differ too much in size for the different column types. In other words, the influence of the length of the span in the calculation approach in MC2010 might be the reason why the punching shear compression capacity varies much more in value for the different column types by the MC2010 approach, while it is almost the same for all the columns according to EC2. For large span lengths, as for column type 1, the slab may be defined as slender. EC2 might in some cases become non-conservative for slender slabs, as shown in Figure 3.20f in Section 3.4.1. This will be discussed later, in section 5.4.2.

For column type 2 and 3, where the span lengths are not so large, the capacities calculated according to EC2 are more conservative than the ones calculated by MC2010.

If the value 1.0 for k_{dg} had been used, the utilizations obtained for all the columns from the calculations by MC2010 had become slightly larger, but they would still be larger for column type 1 and smaller for column type 2 and 3 than the utilizations calculated by EC2.

It should be noted that the two different approaches uses different control perimeters for calculation of the punching shear compression capacity, as the method in EC2 uses the edge around the column, while MC2010 uses a section at a distance of $0.5d_v$ from the edge of the column.

In addition to this, a change in the formulas for calculating the punching shear compression capacity according to EC2 is under progress. Too much focus on the comparison of the results for the punching shear compression capacity according to EC2 is therefore not necessary.

5.4.2 Required Amount of Punching Shear Reinforcement

Tables 5.16, 5.17 and 5.18 show the maximum distances from the centre of the column to the perimeter where shear reinforcement is not necessary, SR, the calculated necessary reinforcement amount that should be included in the calculations, A_{sw}, and the actual applied reinforcement amount, $A_{sw,applied}$, for the three column types.

Table 5.16: Distance to outer reinforcement perimeter, required reinforcement amount and applied reinforcement amount according to EC2 and MC2010 for column type 1

Design approach	**SR** [mm]	$\mathbf{A_{sw}}$ [$\mathbf{mm^2}$]	$\mathbf{A_{sw,applied}}$ [$\mathbf{mm^2}$]
EC2	828	5958	11310
MC2010, LoA II	1166	1449	12566

Table 5.17: Distance to outer reinforcement perimeter, required reinforcement amount and applied reinforcement amount according to EC2 and MC2010 for column type 2

Design approach	**SR** [mm]	$\mathbf{A_{sw}}$ [$\mathbf{mm^2}$]	$\mathbf{A_{sw,applied}}$ [$\mathbf{mm^2}$]
EC2	828	5756	10053
MC2010, LoA II	665	1190	-

Table 5.18: Distance to outer reinforcement perimeter, required reinforcement amount and applied reinforcement amount according to EC2 and MC2010 for column type 3

Design approach	**SR** [mm]	$\mathbf{A_{sw}}$ [$\mathbf{mm^2}$]	$\mathbf{A_{sw,applied}}$ [$\mathbf{mm^2}$]
EC2	753	6368	9425
MC2010, LoA II	673	1194	-

All the values according to EC2 are the same as the ones given in Section 5.2, and they have not been corrected for a higher flexural reinforcement amount.

The value of A_{sw} for EC2 is the value after requirements of maximum centre distances are fulfilled, defined as $A_{sw,tot,2}$ in Section 5.2.2. The reason why the values for A_{sw} differ as much as they do for the two different design approaches is that in MC2010 this is merely the required amount that has to be placed within the zone bounded by $0.35d_v$ and d_v. These two are therefore not comparable.

The distance from the centre of the column to where shear reinforcement is not necessary is larger for the results by MC2010 than for EC2 for column type 1. This is why the total applied amount of shear reinforcement, $A_{sw,applied}$, is larger for MC2010 than EC2. The reason for this might be that the large length of the span in x-direction for column type 1 causes the calculated rotation to become very large. The large rotation causes a small rotation factor, which causes the required outer perimeter to become very large. This is not accounted for in EC2.

The total amount of punching shear reinforcement used after all requirements in MC2010 are fulfilled has not been calculated for column 2 and 3, as the calculation procedure would be the same as for column type 1. However, the distance from the centre of the column out to the point where punching shear reinforcement is no longer necessary has been calculated, and are shown in Tables 5.16, 5.17 and 5.18. The calculations are shown in detail in Appendix B. The distance out to the point where shear reinforcement is no longer necessary was for both column types 2 and 3 calculated to be smaller than the value of the distance to the outer reinforcement bar in the Multiconsult AS report. This gives that the total reinforcement amount used around column type 2 and 3 would become smaller by the requirements in MC2010 than the calculated amount by Multiconsult AS.

It is in most cases expected that the required amount of reinforcement would be smaller by the approach in MC2010, as the required outer perimeters often become smaller because the value of the capacity is larger, as for column type 2 and 3. This is also documented in the Master's thesis by Bjørnar Foldøy Byberg [15]. For column type 1 this was not the case, because of large rotations of the slab.

As shown in Figure 3.20f in Section 3.4.1, EC2 might become non-conservative in cases with large span lengths, as the slab then is defined as more slender. In this case, it might mean that the results obtained by the approach in EC2 are non-conservative, which means that EC2 might have overestimated the punching shear capacity. As non-conservative results are highly unwanted in design, this is a topic that requires further research in the future.

In this case, fortunately, the total amount of reinforcement around column type 1 for the two methods is not that different, which means that the applied reinforcement amount by Multiconsult AS is satisfying. Both methods require 4 reinforcement bars i radial direction and the centre distance of 150 mm is used in both x- and z-direction.

If it had not been for the rectangular arrangement of the reinforcement bars for both methods, the applied reinforcement area would have been much lower, as both design approaches suggests a circular arrangement of the reinforcement bars.

Chapter 6

Verification of Models in EC2

EC2 uses different approximated models in the design for punching shear. In this chapter, slabs are modelled in the finite element program DIANA in terms of checking whether the assumptions made in some of the models in EC2 are reasonable. The models examined are the shear distribution at the perimeter at the distance $2d$ from the edge of a column, presented in Figure 2.17, and the assumptions for calculation of the reduced control perimeter for columns close to a slab opening, presented in Figure 2.13. The input files from DIANA are presented in Appendix C.

6.1 Shear Distribution at the Control Perimeter

The distribution of shear stresses in a slab due to a moment in the column for a slab-column connection is in EC2 assumed to have a constant value at the perimeter $2d$ from the column edge, both in compression and tension, as is shown in Figure 2.17 in Section 2.4. The same figure is presented in Figure 6.1.

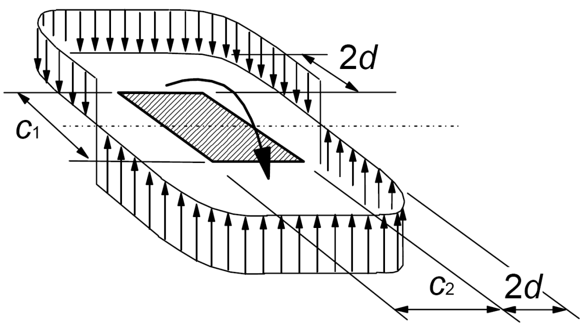


Figure 6.1: Shear distribution from an unbalanced moment at connection between slab and inner column [1]

It is interesting to find out whether this distribution at the perimeter $2d$ from the column edge in fact corresponds to the correct distribution of shear stresses at this perimeter caused by an unbalanced moment in the column. By modelling a slab-column connection with an acting moment on the column in a finite element method program, it is possible to find the distribution of the shear stresses at this particular perimeter. This section will describe the results obtained by a linear analysis of a slab-column connection with a given thickness and effective depth.

6.1.1 Structure of the Model in DIANA

A model of the slab-column connection is shown in Figure 6.2.

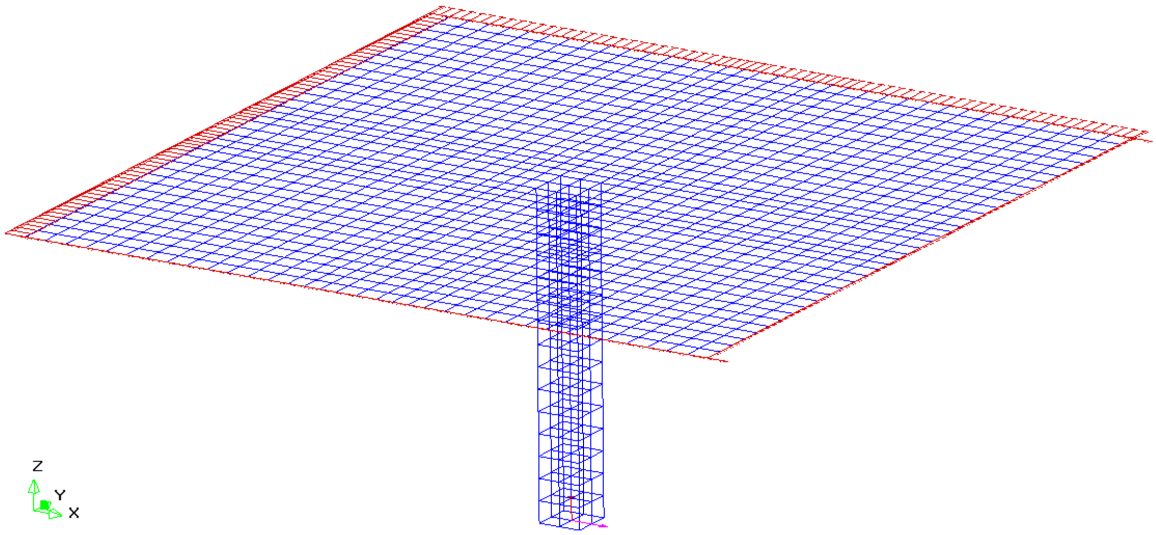


Figure 6.2: Model of slab-column connection in DIANA with loading and constraints

The model is constrained against rotations at all the edges, which makes the slab equal to a slab with twice as large span lengths between the columns. Two of the edges are constrained against translations in the x- and y- directions respectively, and the column is constrained against translations in the z-direction, in terms of making the model statically determined.

The bottom of the column is subjected to a horizontal load with the size of 1 kN in the x-direction, which along with the height of the column of 3 meters, gives a moment of 3 kNm acting on the slab-column connection.

In the model, eight-node quadrilateral isoparametric curved shell elements of the type CQ40S are used for the slab, and twenty-node isoparametric solid brick elements of the type CHX60 are used for the column [9]. Both element types are shown in Figure 6.3.

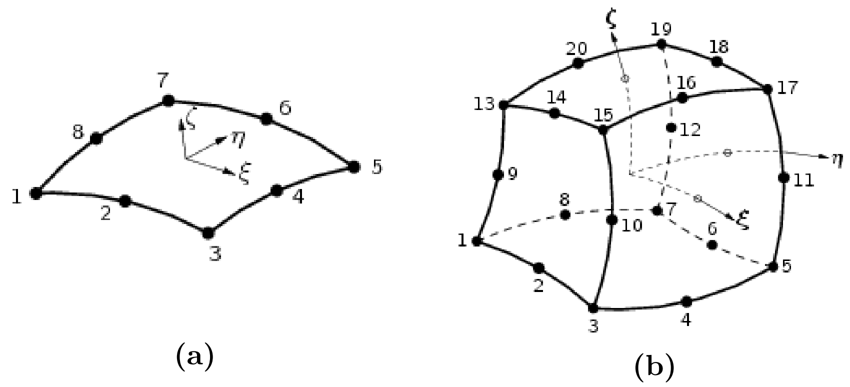


Figure 6.3: (a) CQ40S curved shell element and (b) CHX60 solid brick element [9]

Since only linear analyses are run on the model, and the stress distribution at the perimeter $2d$ from the edge of the column is the only requested result from the analyses, the theoretical meaning of the choice of elements will not be discussed any further.

The model is meshed using automatic mesh divisions, but with defined points around the perimeter $2d$ from the column, in terms of making it simpler to contract the values of the shear stresses in these points. The model contains a total of 1356 elements.

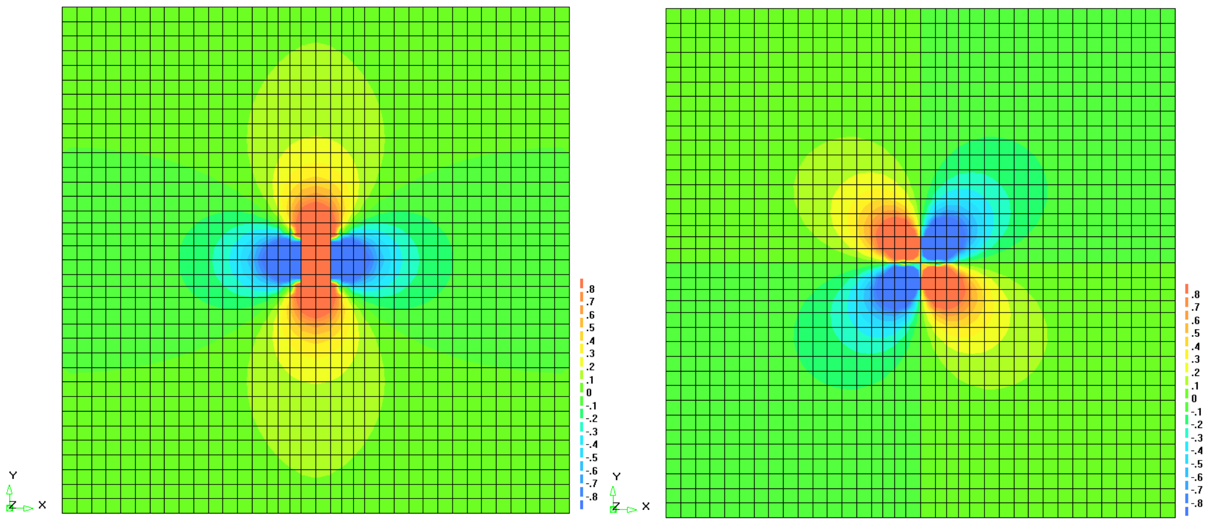
The model is defined with the parameters given in Table 6.1.

Table 6.1: Material parameters for the DIANA model

Parameter	**Value**	**Unit**
Length of slab in x-direction, L_x	7.0	m
Length of slab in x-direction, L_y	7.0	m
Length of column, L_c	3.0	m
Moment acting on slab, M_{Ed}	3.0	kNm
Column dimensions, $c_1 \cdot c_2$	$400 \cdot 400$	mm^2
Thickness of slab, t	300	mm
Effective depth of slab, d	239	mm
E-modulus concrete, E_c	34 000	MPa
Poisson's ratio, ν	0.2	-

6.1.2 Results from the Analysis

First, a regular linear analysis was run on the slab, in terms of obtaining an idea of the distribution of shear stresses around the control perimeter. Figure 6.4 shows the contours of the shear stresses Q_{xz} and Q_{yz} in the slab, where x denotes the shear stresses at the surface perpendicular to the x-axis and y at the surface perpendicular to the y-axis.



(a) (b)

Figure 6.4: Distribution of shear stresses (a) Q_{xz} and (b) Q_{yz}

The distributions in Figure 6.4 clearly show a similarity to the distribution presented in Figure 6.1, as the shear force Q_{xz} is almost constant along the vertical part of the perimeter close to the column and the shear force Q_{yz} goes from certain values in tension to the same values in compression along the horizontal part of the perimeter.

To obtain more accurate values for the shear distribution around the column, the analysis was then run again, this time as *tabulated*, which means that tabulated values of the requested parameters are obtained. The analysis presented values for both Q_{xz} and Q_{yz} for all the nodes in the slab. The only interesting nodal values however are the values at the nodes along the perimeter $2d$ from the column edge.

For the circular areas of the perimeter, there are not any nodes in the model situated exactly on the perimeter. For these cases, the average value of the stresses in the two nodes situated closest to the perimeter is used. Since these nodes are arranged with an angle to the axis, the sums of $Q_{xz} \sin\alpha + Q_{yz} \cos\alpha$ for the average value of the two nodes closest to the parameter are calculated. For the nodes situated along the y-axis, the values of Q_{xz} are collected, and the values of Q_{yz} are collected for the nodes situated along the x-axis.

In Figure 6.5 the points along the perimeter for which the values for the shear stresses are collected are given values from 1 to 32. Table 6.2 shows the corresponding values of the shear stresses in the points 1 to 32. Since the slab has a thickness of 300 mm, the values gotten from the analysis are divided by this thickness in terms of obtaining the value for the shear stress v. All the values are scaled by the factor 1000. Since the loading causes a symmetric distribution, many points will have the same value of v.

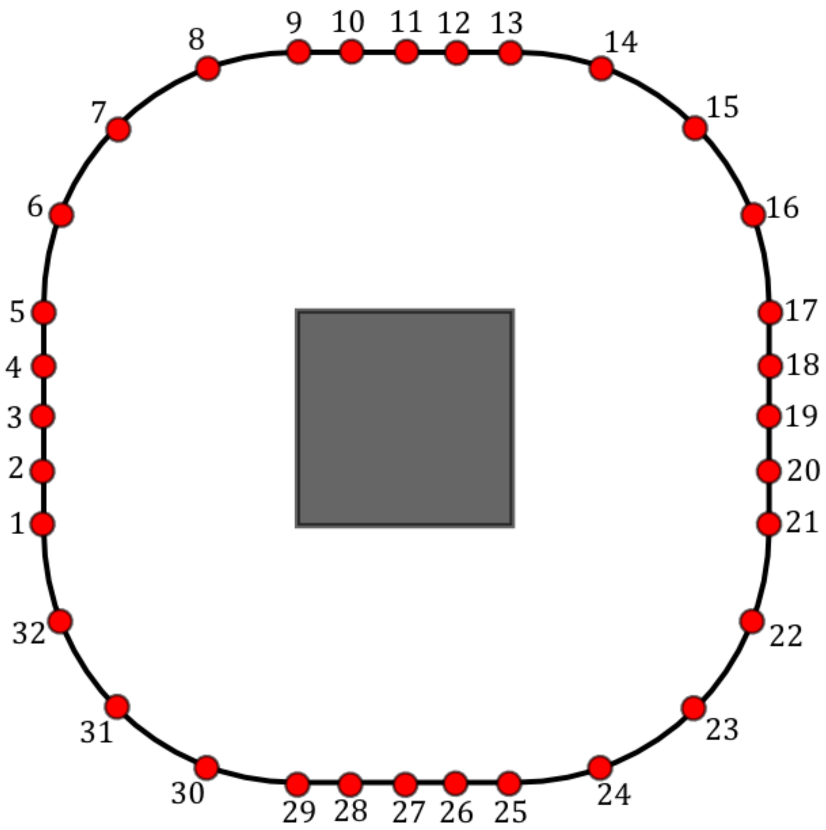


Figure 6.5: Points along the perimeter with given numbers from 1 to 32

Table 6.2: Shear stresses at the points around the perimeter

Point numbers	v [MPa] · 1000
3	-3.2
2, 4	-2.9
1, 5	-2.5
6, 32	-2.3
7, 31	-2.1
8, 30	-3.1
9, 29	-1.8
10, 28	-0.9
11, 27	0
12, 26	0.9
13, 25	1.8
14, 24	3.1
15, 23	2.1
16, 22	2.3
17, 21	2.5
18,20	2.9
19	3.2

The final result for the shear stress distribution at the perimeter $2d$ from the edge of the column is presented in Figure 6.6.

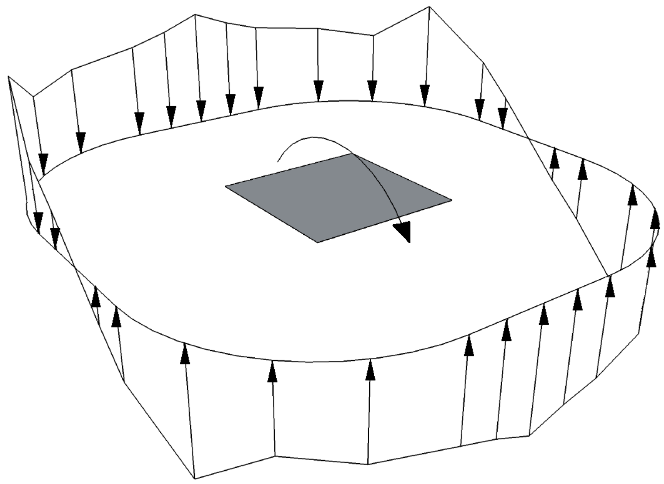


Figure 6.6: Shear stress distribution at the control perimeter

6.1.3 Comparison to Results by EC2

Figure 6.6 shows that the calculated shear stress distribution at the perimeter makes a good match with the presented distribution in Figure 6.1. The values of the shear stresses by the results from the analysis are not constant around the perimeter, but a constant distribution is a good approximation as the values do not differ that much from each other. Along the x-axis, the shear stress according to the analysis decreases linearly from one value in tension to the same value in compression and opposite. This differs from Figure 6.1, as the shear stress here goes directly from one value in tension to the same value in compression without the gradually linear decrease. The distribution in Figure 6.1 is only an approximated distribution, and the linear decrease shown in Figure 6.6 is more likely in reality.

As presented in Section 2.4, and given in Equation (6.38) in EC2, the expression for the shear stress at the perimeter u_1 at the distance $2d$ from the edge of the column can be given by the expression in Equation (6.1).

$$v_{Ed} = \beta \frac{V_{Ed}}{u_1 d} \tag{6.1}$$

In Equation (6.1), β is given by the expression in Equation (6.2).

$$\beta = 1 + k \cdot \frac{M_{Ed}}{V_{Ed}} \cdot \frac{u_1}{W_1} \tag{6.2}$$

The value of k is in Table 6.1 in EC2 given the value of 0.6 for the quadratic cross-sectional area of the column. The value of W_1 is given in Equation (6.41) in EC2 and is calculated in Equation (6.3).

$$\begin{aligned} W_1 &= \frac{c_1^2}{2} + c_1 c_2 + 4c_2 d + 16d^2 + 2\pi d c_1 \\ &= \frac{400^2}{2} + 400 \cdot 400 + 4 \cdot 400 \cdot 239 + 16 \cdot 239^2 + 2\pi \cdot 239 \cdot 400 \\ &= 2137008.5\,\mathrm{mm}^2 \end{aligned} \tag{6.3}$$

By inserting Equation (6.2) into Equation (6.1) and knowing that the value of V_{Ed} is zero in the model, the value of v_{Ed} at the control perimeter is obtained in Equation (6.4).

$$\begin{aligned} v_{Ed} &= \left(1 + k \cdot \frac{M_{Ed}}{V_{Ed}} \cdot \frac{u_1}{W_1}\right) \cdot \frac{V_{Ed}}{u_1 d} = \frac{V_{Ed}}{u_1 d} + k \cdot \frac{M_{Ed}}{W_1 d} \\ &= k \cdot \frac{M_{Ed}}{W_1 d} = 0.6 \cdot \frac{3 \cdot 10^6}{2137008.5 \cdot 239} \\ &= 3.5 \cdot 10^{-3}\,\mathrm{MPa} \end{aligned} \tag{6.4}$$

If this value is scaled by the factor 1000 like the results from the analysis, the value becomes equal to 3.5. This value is larger than the largest value obtained in points 3 and 19 in Figure 6.5, which means that the value from EC2 is on the conservative side. The values of 3.2 and 3.5 are quite close, which means that the expression in EC2 gives a nice approximation to the actual values of the shear stresses around the perimeter at the distance $2d$ from the column edge. If the values in the analysis had been divided by 239 mm instead of 300 mm, the maximum shear stress would have become 4.0, which is in fact larger than the value calculated by EC2.

6.2 Loaded Areas Close to Slab Openings

The model for calculations of the reduced control perimeter due to an opening in the slab near the column was presented in Figure 2.13 in Section 2.4. A similar model is used in MC2010 for columns near slab openings. The figure from EC2 is presented again in Figure 6.7.

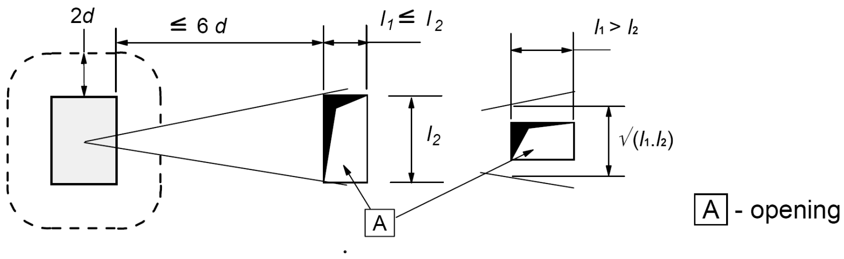


Figure 6.7: Critical control perimeter around loaded area close to slab opening [1]

According to Figure 6.7 the effect of openings in the slab closer than $6d$ from the column edge may be accounted for by assuming that the part of the perimeter between two tangent lines drawn from the centre of the column to the corner of the slab opening is ineffective. This causes the total shear stress along the perimeter to become larger.

6.2.1 Structure of the Model in DIANA

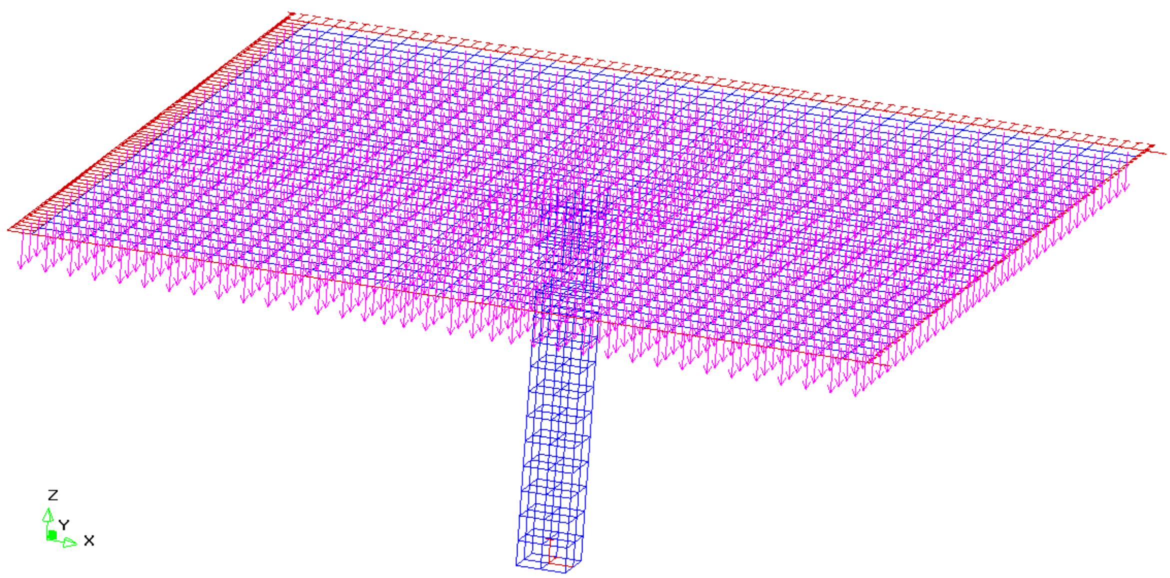


Figure 6.8: Model of slab-column connection in DIANA with loading and constraints

To find out whether the effect of a slab opening can be approximated as suggested in EC2, a slab-column connection is modelled. Both the slab and the column have the same dimensions as the slab presented in Section 6.1, and the same element types are used. The model is shown in Figure 6.8.

This model is, in addition to the constraints described in Section 6.1, also constrained against translations in x- and y-direction at the bottom of the column. In addition to this, the loading situation is different. Where the model in Section 6.1 was loaded with a point load at the bottom of the column, this model has a uniformly distributed load over the entire slab surface, with the size of 0.1 MPa. The slab in Figure 6.8 has also two openings located at each side of the column in x-direction. The dimensions of the slab openings and the distances from the openings to the column edges are shown in Figure 6.9.

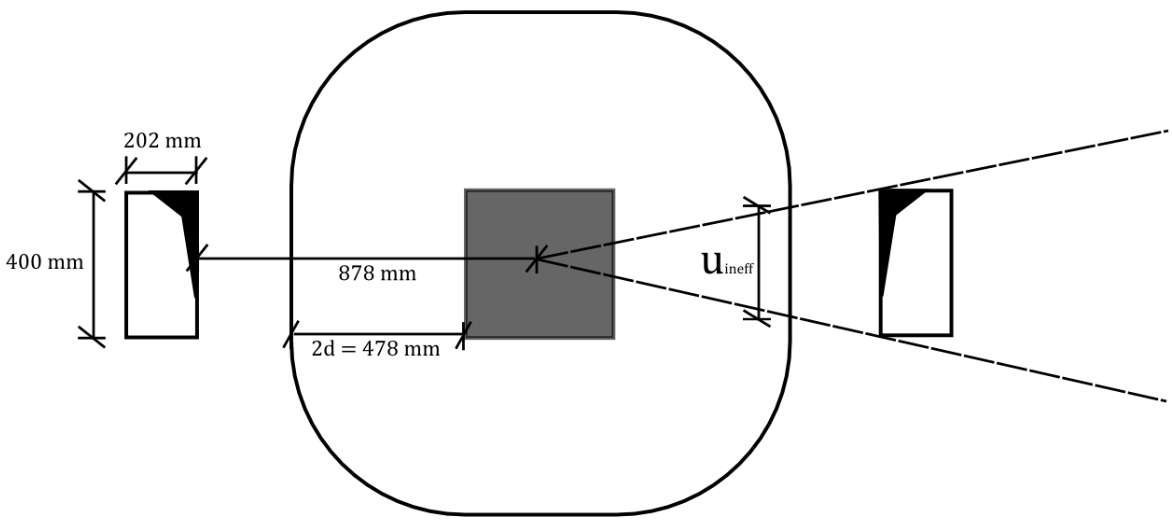


Figure 6.9: Dimensions of slab openings and ineffective parts of the perimeter

6.2.2 Results from the Analysis

For a tabulated linear analysis on the model, the results for the shear stresses along the perimeter are obtained. These values are collected in the same way as in Section 6.1, but since the loading now is much larger, the values are not scaled by a factor 1000, as they were in Section 6.1. The values for the shear stresses in the points given in Figure 6.5 are presented in Table 6.3.

Table 6.3: Shear stresses at the points around the perimeter

Point numbers	v [MPa]
3, 19	4.1
2, 4, 18, 20	3.4
1, 5, 17, 21	3.0
6, 16, 22, 32	3.6
7, 15, 23, 31	3.3
8, 14, 24, 30	3.5
9, 13, 25, 29	3.4
10, 12, 26, 28	3.6
11, 27	3.7

Based on the values given in Table 6.3, the final result for the shear stress distribution at the perimeter $2d$ from the column edge is presented in Figure 6.10.

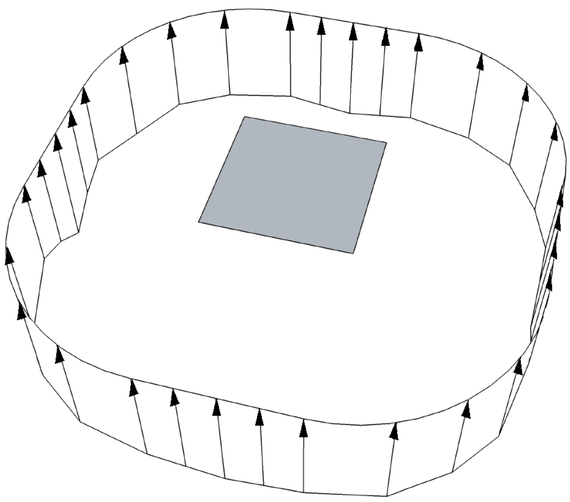


Figure 6.10: Shear stress distribution at the control perimeter

As expected, the analysis presents largest values at the points 3 and 19, which are situated closest to the slab opening.

6.2.3 Comparison to Results by EC2

It is obvious that the values of the shear stresses along the perimeter become larger when there are openings in the slab near the column edge. To compare the values from the analysis to the method described in EC2, the shear stress according to the rules for slabs with openings in EC2 must be calculated.

The part of the perimeter that can be assumed ineffective in the calculations of the shear stress, given as u_{ineff} in Figure 6.9, can be calculated with some simple triangle equalities. The calculations of the ineffective part of the perimeter u_{ineff} and the reduced perimeter $u_{1,red}$ are presented in Equations (6.5) and (6.6). It should be noted that two times the value of u_{ineff} should be subtracted from the perimeter, as there are two openings in the slab.

$$\begin{aligned}
&\frac{u_{ineff}/2}{678} = \frac{200}{878} \\
&\rightarrow \frac{u_{ineff}}{2} = 678 \cdot \frac{200}{878} = 154.4\,\text{mm} \\
&\rightarrow u_{ineff} = 2 \cdot 154.4 = 308.9\,\text{mm}
\end{aligned} \tag{6.5}$$

$$u_{1,red} = u_1 - 2 \cdot u_{ineff} = 4 \cdot 400 + 2 \cdot \pi \cdot 478 - 2 \cdot 308.9 = 3985.6\,\text{mm} \tag{6.6}$$

The loading on the column, V_{Ed}, may easily be calculated according to Equation (6.7). Then the final value for the shear stress with the reduced perimeter taken into account, $v_{Ed,red}$, may be calculated according to Equation (6.8).

$$V_{Ed} = q \cdot A_{slab} = 0.1 \cdot (7000 \cdot 7000 - 202 \cdot 400 \cdot 2) = 4883840\,\text{N} = 4883.8\,\text{kN} \tag{6.7}$$

$$v_{Ed,red} = \beta \cdot \frac{V_{Ed}}{u_{1,red} \cdot d} = 1.0 \cdot \frac{4883840}{3985.6 \cdot 239} = 5.1\,\text{MPa} \tag{6.8}$$

The calculated value of the shear stress along the perimeter is larger than the maximum of the values in Table 6.3. Again, the method in EC2 is conservative compared to the value gotten from the analysis. The value from EC2 is, however, not that much larger than the value obtained in the analysis.

To be able to compare the results in more detail, the results for a slab without slab openings, but with the same loading situation as the slab in this section, are evaluated. Table 6.4 shows the values for the shear stresses at the points around the perimeter for the slab without openings.

Table 6.4: Shear stresses at the points around the perimeter

Point numbers	**v [MPa]**
3, 11, 19, 27	3.7
2, 4, 10, 12, 18, 20, 26, 28	3.6
1, 5, 9, 13, 17, 21, 25, 29	3.4
6, 8, 14, 16, 22, 24, 30, 32	3.4
7, 15, 23, 31	3.3

Table 6.4 shows that the values are more evenly distributed around the perimeter than for the model with openings in the slab. The maximum value around the perimeter is 3.7 MPa. This value must be compared to the value calculated according to EC2. The calculated values of the perimeter u_1, the shear force in the column, $V_{Ed,without}$, and the shear stress around the perimeter, $v_{Ed,without}$, for the case without openings in the slab are given in Equations (6.9), (6.10) and (6.11).

$$u_1 = 4 \cdot 400 + 2 \cdot \pi \cdot 478 = 4603.4\,\text{mm} \tag{6.9}$$

$$V_{Ed,without} = q \cdot A_{slab} = 0.1 \cdot 7000 \cdot 7000 = 4900000\,\text{N} = 4900.0\,\text{kN} \tag{6.10}$$

$$v_{Ed,without} = \beta \cdot \frac{V_{Ed,without}}{u_1 \cdot d} = 1.0 \cdot \frac{4900000}{4603.4 \cdot 239} = 4.5\,\text{MPa} \tag{6.11}$$

The maximum value from Table 6.4 is approximately 82% of the value calculated according to EC2. For the slab with openings near the column edge the maximum value from the analysis was approximately 80% of the value calculated by EC2. This means that EC2 overestimates the size of the maximum shear stress with almost the same percentage both for the slab with and the slab without openings. This means that the assumptions made in EC2 make a good match with the results from the analyses.

The differences in the values from EC2 and the analyses may be caused by the fact that the values obtained from the analyses are divided by the slab thickness of 300 mm, while the method in EC2 divides the shear force on the effective depth of 239 mm.

By dividing the results from the analyses by 239 mm instead of 300 mm, we get the values in Equations (6.12) and (6.13) for the maximum shear stress at the perimeter for the slabs with and without opening respectively.

$$v_{Ed,d,with} = 4.1 \cdot \frac{300}{239} = 5.1\,\text{MPa} \tag{6.12}$$

$$v_{Ed,d,without} = 3.7 \cdot \frac{300}{239} = 4.6\,\mathrm{MPa} \tag{6.13}$$

Now, by distributing the shear force over the effective depth instead of the whole thickness of the slab, the results from EC2 and the analyses become almost identical for both cases; with and without slab openings. For the slab without openings, the maximum value from the analysis even becomes slightly larger than the calculated value according to EC2. The assumptions made in EC2 are therefore satisfying. It may be noted that EC2 assumes an approximately even distribution of shear stresses around the perimeter, whereas in the analyses, the distribution is not constant, especially not for the situation with slab openings.

Chapter 7

Discussion

The method in EC2 for punching shear design is empirical, and it usually presents conservative results. This was among others shown in Section 3.4.1, where it was described how most of the results in the tests described presented higher capacities than the ones calculated by EC2. In most cases, the results by the approach that MC2010 is based on also gave higher capacities than the approach in EC2. The punching shear design of a structure according to the approach in EC2 should therefore be conservative and safe in most cases.

The method in MC2010 is based on a physical model, and the results are therefore assumed to become more accurate for this method than for the method in EC2. This is confirmed in Section 3.4.1, where the results from both the refined method and the simplified method according to the CSCT match the test results in a satisfying manner.

Design examples according to MC2010 were presented in Chapter 5 and the results from the FEM analyses were described in Chapter 6, in terms of comparing the results by EC2 to the results by MC2010 and to find out whether EC2 in fact always present accurate and conservative calculation models.

7.1 Punching Shear Capacity and Required Shear Reinforcement

The simplified expression based on the rotation of the slab obtained in Section 3.4.1 gave more accurate results than EC2 for most cases. This simplified expression is very similar to the expression for the rotation of the slab in LoA II in MC2010. The results by the simplified expression for the rotation were in most cases conservative compared to the test results, but not as conservative as the results according to EC2.

These observations cause the assumptions that MC2010 would give more accurate and less conservative results for the design examples presented in Chapter 5 than the results according to EC2. For column type 2 and 3 this turned out to be the case. For column type 1 on the other hand, the utilization of the punching shear capacity turned out to be higher according to the method in MC2010.

As shown in Figure 3.20f in Section 3.4.1, there were no relevant test results available for the comparison of how the slenderness of a slab affects the capacity. In this case it is therefore assumed that the results from the refined method give accurate enough results for comparison with EC2. In EC2, the slenderness of the slab is not included in the calculations of the capacity, and it will therefore not affect the results. Figure 3.20f clearly shows how the capacity according to the refined and simplified methods decreases as the slab becomes more slender. This causes the calculated capacity according to EC2 to become larger than for the two other methods for sufficient slender slabs.

The chance of EC2 overestimating the capacity in some cases is therefore present and the design example on column type 1 in Chapter 5 certified this theory further. Column type 1 had large span lengths, much larger than column type 2 and 3.

Luckily, the requirements for spacing are more conservative in EC2 than in MC2010. This caused the total applied reinforcement amount for column type 1 not to differ too much for the two different approaches, although EC2 originally required a shorter distance to the outer shear reinforcement perimeter than MC2010.

In the case of column type 1, the total applied reinforcement amount did not differ too much for the two different approaches. However, for another given case with a structure with a very slender slab, the difference in the applied amount for the two methods might have become larger. The idea that EC2 might sometimes underestimate the required amount of shear reinforcement makes it clear that this is a topic that needs to be further investigated in the future. It is necessary to perform punching shear tests on slender slabs, in terms of finding out how non-conservative EC2 in reality is for these types of slabs. These tests would also help verifying if the simplifications in the approach in MC2010 are reasonable and the results satisfying.

As for the LoA I approach in MC2010, this method is overly conservative as it predicts a very large rotation of the slab. This is logical as the amount of flexural reinforcement applied to the system is not accounted for in this LoA. The large amount of flexural reinforcement used in the design examples in Chapter 5 causes the slab to have decreased rotations, and the capacity predicted by LoA I for the system without shear reinforcement is therefore too low. However, this level of approximation gives a nice prediction of whether shear reinforcement is necessary or not for preliminary design. In terms of finding the required amount of shear reinforcement, LoA II will be more appropriate in terms of not exaggerating the amount of reinforcement, as this is neither practical nor economical.

Although the LoA II approach in the design examples gave quite satisfying results, more slab-column connections should be examined before drawing any conclusions on whether the method is accurate enough. By examining more slab-column connections it would also be possible to find out for which cases EC2 presents non-conservative results.

7.2 Shear Distribution and the Effect of Openings in the Slab

When it comes to the shear stress distribution around a column that is subjected to a moment, EC2 assumes it to be constant at the basic control perimeter $2d$ from the column edge. The results from the analysis presented in Section 6.1 showed that the shear stress distribution at the basic control perimeter is not exactly constant. However, a constant distribution is an appropriate simplification, as the values do not vary too much around the perimeter.

According to EC2 the value of the shear stress goes directly from one value in tension to the same value in compression and opposite along the sides parallel to the x-axis when the moment is applied about the y-axis. The results from the linear analysis gives that the shear stress distribution in x-direction has a linear decrease from tension to compression and opposite. This is a more realistic distribution, and it is reasonable to assume that the model in EC2 is just approximate and that this linear decrease is not taken into account.

The calculated value of the shear stress along the basic control perimeter according to EC2 gave a value quite close to the maximum value obtained from the analysis. This means that, although the distribution does not appear to be constant in reality, the maximum value is almost the same for the analysis and for the method in EC2, and the procedure in EC2 may therefore be satisfying for design purposes.

The effect of openings near the column edge is according to EC2 being accounted for by assuming that a part of the perimeter is ineffective for carrying shear stresses. The loading situation described in Section 6.2 gave a value of the maximum shear stress in the analysis of approximately 80% of the value calculated based on the assumptions in EC2. These values are very close, and the assumption in EC2 is therefore quite satisfying.

The calculated values according to EC2 were in both cases in Chapter 6 higher than the values obtained by the analyses. This means that the design procedure in EC2 is on the safe side and that the results will be conservative when using this approach.

It must be noted that for the shear distribution in Section 6.1 only one model was used for the comparison to EC2. The results might have become different for other column dimensions, slab thicknesses and spans. These are examples of analyses that should be performed in the future in terms of making additional verifications to the models in EC2. The same variations in the dimensions of the slab and the column could have been done for the analysis in Section 6.2. For this analysis it might also have been interesting to look at the effect of placing the slab openings closer or further away from the column edge, but within the distance of $6d$ from the column edge, as described in EC2.

Chapter 8

Conclusions

The design approach for punching shear in EC2 is based on empirical formulations. The design approach in MC2010 on the other hand is based on a physical model, with some simplifications for the use in design. The method in MC2010 also presents different LoAs, where LoA I is easiest to use but also the least accurate level, and LoA IV is most intricate to use and very accurate.

Results from the theory that the punching shear design in MC2010 is based on are shown to make a good match to the results by different tests on slabs. LoA I is not as accurate as LoA II and underestimates the capacity, especially in the cases where a large amount of flexural reinforcement causes the moment capacity of the slab to become large, as the moment capacity is not included in the calculation of the rotation in LoA I.

Design examples on column type 2 and 3 according to LoA II in MC2010 presented higher values for the concrete contribution to the capacity than EC2, and would therefore also have given a lower required shear reinforcement amount. For column type 1, with large span lengths, the result was opposite, as EC2 presented a lower required shear reinforcement amount. This is probably caused by the fact that the span length is not included in the calculations of the punching shear capacity in EC2. EC2 might therefore predict too high capacities for cases where the slab has large spans. Before concluding on anything concerning the punching shear capacity and the required reinforcement amount, further analyses and tests on slender slabs should be performed, as it is clear that EC2 both overestimates and underestimates the capacities for different slab dimensions. Where is the limit of the span length for which EC2 goes from underestimating the capacity to overestimating it?

LoA II and higher LoAs in MC2010 take the slenderness of the slab into account, and are therefore more reliable for design cases. Still, more tests should be done on slender slabs in terms of verifying the method in MC2010 as there are not sufficient test results available on these types of slabs.

When it comes to the shear stress distribution at the perimeter $2d$ from the column edge, the model in EC2 that assumes a constant distribution of shear stresses makes a good match to the results obtained by a linear analysis. The distribution in the analysis is not precisely constant, but a constant distribution is a good approximation. In addition to this, the calculated value of the shear stress around the perimeter according to the method in EC2 and the maximum value from the analysis were quite close.

The effect of openings in the slab near the column edge may according to EC2 be accounted for by assuming a part of the perimeter to be ineffective. This approximation turned out to fit the real stress distribution situation according to a linear analysis quite well. The value of the shear stress around the perimeter according to EC2 was, also in this case, close to the maximum value gotten from the analysis.

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Appendices

Appendix A

Design Examples by EC2

In this appendix, the design of the three column types according to EC2 is shown. The calculations are only presented for load combinations 3 and 4, as the two other combinations are clearly not governing.

Only the calculations without applied punching shear reinforcement are attached, in terms of finding an updated punching shear compression capacity with the correct flexural reinforcement amount. The calculations with applied shear reinforcement has been performed by Multiconsult AS, and these calculations have not been updated, as the required reinforcement amount will not increase much with the updated amount of flexural reinforcement.

Column 1, EC2

Load Combination 3

Material factor, concrete	y_c	1,5		
Characteristic concrete strength	f_{ck}	35	N/mm^2	
Design yield strength of steel	f_{yd}	434,8	N/mm^2	
Young's modulus steel	E_s	200000	N/mm^2	
Slab thickness	h	300	mm	
Effective depth	d_v	239	mm	
Column dimension	w_c	700	mm	
Column dimension	h_c	700	mm	
	c_1/c_2	1		
Dimension coefficient	k	0,6		EC2 Table 6.1
Flexural reinforcement per unit lenght, x-direction	$A_{s,x}$	2680,82	mm	
Flexural reinforcement per unit lenght, z-direction	$A_{s,y}$	2680,82	mm	
Design shear force	V_{Ed}	1027	kN	
Design moment, x-direction	$M_{Ed,x}$	34	kNm	
Design moment, z-direction	$M_{Ed,z}$	24	kNm	
Critical control section	u_1	5803,362577	mm	EC2 Fig. 6.13
Basic control section	u_0	2800	mm	EC2 6.4.5(3)
Eccentricity, x-direction	e_x	33,10613437	mm	EC2 6.4.3(3)
Eccentricity, z-direction	e_z	23,36903603	mm	EC2 6.4.3(3)
Dimension control section, x-direction	b_x	1656	mm	EC2 6.4.3(3)
Dimension control section, z-direction	b_z	1656	mm	EC2 6.4.3(3)
Factor for incresed loadin	β	1,044046933		EC2 (6.43)
Size for calculation of $V_{Rd,max,1}$	υ	0,516		EC2 (6.6N)
Reinforcement ratio, x-direction	ρ_x	0,01121682		EC2 6.4.4(1)
Reinforcement ratio, z-direction	ρ_z	0,01121682		EC2 6.4.4(1)
Reinforcement ratio	ρ	0,01121682		EC2 6.4.4(1)
Value of k	$k_{calculated}$	1,914778707		EC2 6.4.4(1)
Value of k calculated against maximum value	k	1,914778707		EC2 6.4.4(1)
Punching shear tension capacity, calculated	$v_{Rd,c,calculated}$	0,78	N/mm^2	EC2 (6.47)
Punching shear tension capacity, min	$v_{Rd,c,min}$	0,32	N/mm^2	EC2 (6.47)
Punching shear tension capacity	$v_{Rd,c}$	0,78	N/mm^2	EC2 (6.47)
Punching shear compression capacity, max	$v_{Rd,max,1}$	4,09	N/mm^2	EC2 N.A.6.4.5
Punching shear compression capacity	$v_{Rd,max,2}$	2,48	N/mm^2	EC2 N.A.6.4.5
Capacity at the section u_1	$V_{Rd,u1}$	1037,456793	kN	EC2 (6.38)
Capacity at the section u_0	$V_{Rd,u0}$	1589,900623	kN	EC2 (6.53)

			Utilization	
Punching shear compression capacity OK?	**1589,90**	kN	0,65	**OK**
Punching shear tension capacity OK?	**1037,46**	kN	0,99	**OK**

Load Combination 4

Material factor, concrete	y_c	1,5		
Characteristic concrete strength	f_{ck}	35	N/mm^2	
Design yield strength of steel	f_{yd}	434,8	N/mm^2	
Young's modulus steel	E_s	200000	N/mm^2	
Slab thickness	h	300	mm	
Effective depth	d_v	239	mm	
Column dimension	w_c	700	mm	
Column dimension	h_c	700	mm	
	c_1/c_2	1		
Dimension coefficient	k	0,6		EC2 Table 6.1
Flexural reinforcement per unit lenght, x-direction	$A_{s,x}$	2680,82	mm	
Flexural reinforcement per unit lenght, z-direction	$A_{s,y}$	2680,82	mm	
Design shear force	V_{Ed}	1260	kN	
Design moment, x-direction	$M_{Ed,x}$	0	kNm	
Design moment, z-direction	$M_{Ed,z}$	0	kNm	
Critical control section	u_1	5803,362577	mm	EC2 Fig. 6.13
Basic control section	u_0	2800	mm	EC2 6.4.5(3)
Eccentricity, x-direction	e_x	0	mm	EC2 6.4.3(3)
Eccentricity, z-direction	e_z	0	mm	EC2 6.4.3(3)
Dimension control section, x-direction	b_x	1656	mm	EC2 6.4.3(3)
Dimension control section, z-direction	b_z	1656	mm	EC2 6.4.3(3)
Factor for incresed loadin	β	1		EC2 (6.43)
Size for calculation of $V_{Rd,max,1}$	υ	0,516		EC2 (6.6N)
Reinforcement ratio, x-direction	ρ_x	0,01121682		EC2 6.4.4(1)
Reinforcement ratio, z-direction	ρ_z	0,01121682		EC2 6.4.4(1)
Reinforcement ratio	ρ	0,01121682		EC2 6.4.4(1)
Value of k	$k_{calculated}$	1,914778707		EC2 6.4.4(1)
Value of k calculated against maximum value	k	1,914778707		EC2 6.4.4(1)
Punching shear tension capacity, calculated	$v_{Rd,c,calculated}$	0,78	N/mm^2	EC2 (6.47)
Punching shear tension capacity, min	$v_{Rd,c,min}$	0,32	N/mm^2	EC2 (6.47)
Punching shear tension capacity	$v_{Rd,c}$	0,78	N/mm^2	EC2 (6.47)
Punching shear compression capacity, max	$v_{Rd,max,1}$	4,09	N/mm^2	EC2 N.A.6.4.5
Punching shear compression capacity	$v_{Rd,max,2}$	2,59	N/mm^2	EC2 N.A.6.4.5
Capacity at the section u_1	$V_{Rd,u1}$	1083,153583	kN	EC2 (6.38)
Capacity at the section u_0	$V_{Rd,u0}$	1733,045733	kN	EC2 (6.53)

			Utilization	
Punching shear compression capacity OK?	**1733,05**	kN	0,73	**OK**
Punching shear tension capacity OK?	**1083,15**	kN	1,16	**NOT OK**

Column 2, EC2

Load Combination 3

Material factor, concrete	y_c	1,5		
Characteristic concrete strength	f_{ck}	35	N/mm^2	
Design yield strength of steel	f_{yd}	434,8	N/mm^2	
Young's modulus steel	E_s	200000	N/mm^2	
Slab thickness	h	300	mm	
Effective depth	d_v	239	mm	
Column dimension	w_c	450	mm	
Column dimension	h_c	700	mm	
	c_1/c_2	0,642857143		
Dimension coefficient	k	0,492857143		EC2 Table 6.1
Flexural reinforcement per unit lenght, x-direction	$A_{s,x}$	2680,81	mm	
Flexural reinforcement per unit lenght, z-direction	$A_{s,y}$	2680,81	mm	
Design shear force	V_{Ed}	969	kN	
Design moment, x-direction	$M_{Ed,x}$	1	kNm	
Design moment, z-direction	$M_{Ed,z}$	52	kNm	
Critical control section	u_1	5303,362577	mm	EC2 Fig. 6.13
Basic control section	u_0	2300	mm	EC2 6.4.5(3)
Eccentricity, x-direction	e_x	1,031991744	mm	EC2 6.4.3(3)
Eccentricity, z-direction	e_z	53,66357069	mm	EC2 6.4.3(3)
Dimension control section, x-direction	b_x	1406	mm	EC2 6.4.3(3)
Dimension control section, z-direction	b_z	1656	mm	EC2 6.4.3(3)
Factor for incresed loadin	β	1,068710741		EC2 (6.43)
Size for calculation of $V_{Rd,max,1}$	υ	0,516		EC2 (6.6N)
Reinforcement ratio, x-direction	ρ_x	0,011216778		EC2 6.4.4(1)
Reinforcement ratio, z-direction	ρ_z	0,011216778		EC2 6.4.4(1)
Reinforcement ratio	ρ	0,011216778		EC2 6.4.4(1)
Value of k	$k_{calculated}$	1,914778707		EC2 6.4.4(1)
Value of k calculated against maximum value	k	1,914778707		EC2 6.4.4(1)
Punching shear tension capacity, calculated	$v_{Rd,c,calculated}$	0,78	N/mm^2	EC2 (6.47)
Punching shear tension capacity, min	$v_{Rd,c,min}$	0,32	N/mm^2	EC2 (6.47)
Punching shear tension capacity	$v_{Rd,c}$	0,78	N/mm^2	EC2 (6.47)
Punching shear compression capacity, max	$v_{Rd,max,1}$	4,09	N/mm^2	EC2 N.A.6.4.5
Punching shear compression capacity	$v_{Rd,max,2}$	2,70	N/mm^2	EC2 N.A.6.4.5
Capacity at the section u_1	$V_{Rd,u1}$	926,1918205	kN	EC2 (6.38)
Capacity at the section u_0	$V_{Rd,u0}$	1386,630503	kN	EC2 (6.53)

			Utilization	
Punching shear compression capacity OK?	**1386,63**	kN	0,70	**OK**
Punching shear tension capacity OK?	**926,19**	kN	1,05	**NOT OK**

Load Combination 4

Material factor, concrete	y_c	1,5		
Characteristic concrete strength	f_{ck}	35	N/mm^2	
Design yield strength of steel	f_{yd}	434,8	N/mm^2	
Young's modulus steel	E_s	200000	N/mm^2	
Slab thickness	h	300	mm	
Effective depth	d_v	239	mm	
Column dimension	w_c	450	mm	
Column dimension	h_c	700	mm	
	c_1/c_2	0,642857143		
Dimension coefficient	k	0,492857143		EC2 Table 6.1
Flexural reinforcement per unit lenght, x-direction	$A_{s,x}$	2680,82	mm	
Flexural reinforcement per unit lenght, z-direction	$A_{s,y}$	2680,82	mm	
Design shear force	V_{Ed}	1035	kN	
Design moment, x-direction	$M_{Ed,x}$	0	kNm	
Design moment, z-direction	$M_{Ed,z}$	0	kNm	
Critical control section	u_1	5303,362577	mm	EC2 Fig. 6.13
Basic control section	u_0	2300	mm	EC2 6.4.5(3)
Eccentricity, x-direction	e_x	0	mm	EC2 6.4.3(3)
Eccentricity, z-direction	e_z	0	mm	EC2 6.4.3(3)
Dimension control section, x-direction	b_x	1406	mm	EC2 6.4.3(3)
Dimension control section, z-direction	b_z	1656	mm	EC2 6.4.3(3)
Factor for incresed loadin	β	1		EC2 (6.43)
Size for calculation of $V_{Rd,max,1}$	υ	0,516		EC2 (6.6N)
Reinforcement ratio, x-direction	ρ_x	0,01121682		EC2 6.4.4(1)
Reinforcement ratio, z-direction	ρ_z	0,01121682		EC2 6.4.4(1)
Reinforcement ratio	ρ	0,01121682		EC2 6.4.4(1)
Value of k	$k_{calculated}$	1,914778707		EC2 6.4.4(1)
Value of k calculated against maximum value	k	1,914778707		EC2 6.4.4(1)
Punching shear tension capacity, calculated	$v_{Rd,c,calculated}$	0,78	N/mm^2	EC2 (6.47)
Punching shear tension capacity, min	$v_{Rd,c,min}$	0,32	N/mm^2	EC2 (6.47)
Punching shear tension capacity	$v_{Rd,c}$	0,78	N/mm^2	EC2 (6.47)
Punching shear compression capacity, max	$v_{Rd,max,1}$	4,09	N/mm^2	EC2 N.A.6.4.5
Punching shear compression capacity	$v_{Rd,max,2}$	2,88	N/mm^2	EC2 N.A.6.4.5
Capacity at the section u_1	$V_{Rd,u1}$	989,8323776	kN	EC2 (6.38)
Capacity at the section u_0	$V_{Rd,u0}$	1583,731804	kN	EC2 (6.53)

			Utilization	
Punching shear compression capacity OK?	**1583,73**	kN	0,65	**OK**
Punching shear tension capacity OK?	**989,83**	kN	1,05	**NOT OK**

Column 3, EC2

Load Combination 3

Material factor, concrete	y_c	1,5		
Characteristic concrete strength	f_{ck}	35	N/mm^2	
Design yield strength of steel	f_{yd}	434,8	N/mm^2	
Young's modulus steel	E_s	200000	N/mm^2	
Slab thickness	h	300	mm	
Effective depth	d_v	239	mm	
Column dimension	w_c	400	mm	
Column dimension	h_c	550	mm	
	c_1/c_2	0,727272727		
Dimension coefficient	k	0,518181818		EC2 Table 6.1
Flexural reinforcement per unit lenght, x-direction	$A_{s,x}$	2680,82	mm	
Flexural reinforcement per unit lenght, z-direction	$A_{s,y}$	2680,82	mm	
Design shear force	V_{Ed}	1038	kN	
Design moment, x-direction	$M_{Ed,x}$	0	kNm	
Design moment, z-direction	$M_{Ed,z}$	0	kNm	
Critical control section	u_1	4903,362577	mm	EC2 Fig. 6.13
Basic control section	u_0	1900	mm	EC2 6.4.5(3)
Eccentricity, x-direction	e_x	0	mm	EC2 6.4.3(3)
Eccentricity, z-direction	e_z	0	mm	EC2 6.4.3(3)
Dimension control section, x-direction	b_x	1356	mm	EC2 6.4.3(3)
Dimension control section, z-direction	b_z	1506	mm	EC2 6.4.3(3)
Factor for incresed loadin	β	1		EC2 (6.43)
Size for calculation of $V_{Rd,max,1}$	υ	0,516		EC2 (6.6N)
Reinforcement ratio, x-direction	ρ_x	0,01121682		EC2 6.4.4(1)
Reinforcement ratio, z-direction	ρ_z	0,01121682		EC2 6.4.4(1)
Reinforcement ratio	ρ	0,01121682		EC2 6.4.4(1)
Value of k	$k_{calculated}$	1,914778707		EC2 6.4.4(1)
Value of k calculated against maximum value	k	1,914778707		EC2 6.4.4(1)
Punching shear tension capacity, calculated	$v_{Rd,c,calculated}$	0,78	N/mm^2	EC2 (6.47)
Punching shear tension capacity, min	$v_{Rd,c,min}$	0,32	N/mm^2	EC2 (6.47)
Punching shear tension capacity	$v_{Rd,c}$	0,78	N/mm^2	EC2 (6.47)
Punching shear compression capacity, max	$v_{Rd,max,1}$	4,09	N/mm^2	EC2 N.A.6.4.5
Punching shear compression capacity	$v_{Rd,max,2}$	3,22	N/mm^2	EC2 N.A.6.4.5
Capacity at the section u_1	$V_{Rd,u1}$	915,1754132	kN	EC2 (6.38)
Capacity at the section u_0	$V_{Rd,u0}$	1464,280661	kN	EC2 (6.53)

			Utilization	
Punching shear compression capacity OK?	**1464,28**	kN	0,71	**OK**
Punching shear tension capacity OK?	**915,18**	kN	1,13	**NOT OK**

Load Combination 4

Material factor, concrete	y_c	1,5		
Characteristic concrete strength	f_{ck}	35	N/mm^2	
Design yield strength of steel	f_{yd}	434,8	N/mm^2	
Young's modulus steel	E_s	200000	N/mm^2	
Slab thickness	h	300	mm	
Effective depth	d_v	239	mm	
Column dimension	w_c	400	mm	
Column dimension	h_c	550	mm	
	c_1/c_2	0,727272727		
Dimension coefficient	k	0,518181818		EC2 Table 6.1
Flexural reinforcement per unit lenght, x-direction	$A_{s,x}$	2680,82	mm	
Flexural reinforcement per unit lenght, z-direction	$A_{s,y}$	2680,82	mm	
Design shear force	V_{Ed}	963	kN	
Design moment, x-direction	$M_{Ed,x}$	7	kNm	
Design moment, z-direction	$M_{Ed,z}$	46	kNm	
Critical control section	u_1	4903,362577	mm	EC2 Fig. 6.13
Basic control section	u_0	1900	mm	EC2 6.4.5(3)
Eccentricity, x-direction	e_x	7,268951194	mm	EC2 6.4.3(3)
Eccentricity, z-direction	e_z	47,76739356	mm	EC2 6.4.3(3)
Dimension control section, x-direction	b_x	1356	mm	EC2 6.4.3(3)
Dimension control section, z-direction	b_z	1506	mm	EC2 6.4.3(3)
Factor for incresed loadin	β	1,064000479		EC2 (6.43)
Size for calculation of $V_{Rd,max,1}$	υ	0,516		EC2 (6.6N)
Reinforcement ratio, x-direction	ρ_x	0,01121682		EC2 6.4.4(1)
Reinforcement ratio, z-direction	ρ_z	0,01121682		EC2 6.4.4(1)
Reinforcement ratio	ρ	0,01121682		EC2 6.4.4(1)
Value of k	$k_{calculated}$	1,914778707		EC2 6.4.4(1)
Value of k calculated against maximum value	k	1,914778707		EC2 6.4.4(1)
Punching shear tension capacity, calculated	$v_{Rd,c,calculated}$	0,78	N/mm^2	EC2 (6.47)
Punching shear tension capacity, min	$v_{Rd,c,min}$	0,32	N/mm^2	EC2 (6.47)
Punching shear tension capacity	$v_{Rd,c}$	0,78	N/mm^2	EC2 (6.47)
Punching shear compression capacity, max	$v_{Rd,max,1}$	4,09	N/mm^2	EC2 N.A.6.4.5
Punching shear compression capacity	$v_{Rd,max,2}$	3,03	N/mm^2	EC2 N.A.6.4.5
Capacity at the section u_1	$V_{Rd,u1}$	860,1268811	kN	EC2 (6.38)
Capacity at the section u_0	$V_{Rd,u0}$	1293,423299	kN	EC2 (6.53)

			Utilization	
Punching shear compression capacity OK?	**1293,42**	kN	0,74	**OK**
Punching shear tension capacity OK?	**860,13**	kN	1,12	**NOT OK**

Appendix B

Design Examples by MC2010

In this appendix the calculations according to the design procedure in MC2010 are shown for all the three column types. The calculations only include load combination 3 and 4 as the other two load combinations are obviously not governing.

The calculations include both LoA I and LoA II for all the three column types, and for LoA II the calculations of the required shear reinforcement amount are also performed. The final applied amount of shear reinforcement after all requirements of maximum distances are fulfilled is not included in the calculations, as this has to be done manually.

Column 1, MC2010 LoA I

Load Combination 3

Material factor, concrete	γ_c	1,5		
Material factor, steel	γ_s	1,15		
Characteristic concrete strength	f_{ck}	35	N/mm^2	
Design concrete strength	f_{cd}	19,83	N/mm^2	
Mean tensile concrete strength	f_{ctm}	3,21	N/mm^2	
Design tensile concrete strength	f_{ctd}	1,27	N/mm^2	
Maximal aggregate size	d_g	20	mm	
Aggregate size factor	k_{dg}	0,89		MC2010 7.3-62
Characteristic yield strength of steel	f_{yk}	500	N/mm^2	
Design yield strength of steel	f_{yd}	434,8	N/mm^2	
Young's modulus steel	E_s	200000	N/mm^2	
Slab thickness	h	300	mm	
Effective depth	d_v	239	mm	
Column dimension	w_s	700	mm	
Column dimension	h_s	700	mm	
Diameter of flexural reinforcement	Ø	16	mm	
Centering of flexural reinforcement bars	cc	75	mm	
Flextural reinforcement per unit length, x-direction	$A_{s,x}$	2680,83	mm^2	
Flextural reinforcement per unit length, z-direction	$A_{s,z}$	2680,83	mm^2	
Reinforcement ratio, x-direction	ρ_x	0,01122		
Reinforcement ratio, z-direction	ρ_z	0,01122		
Reinforcement ratio	ρ	0,01122		
Span in x-direction	L_x	10,705	m	
Span in y-direction	L_y	7,55	m	
Design shear force	V_{Ed}	1027	kN	
Design moment	M_{Ed}	41,62	kNm	
Area within control section	A_{per}	869462,7	mm^2	MC2010 7.3.5.2
Eccentricity of resultant forces	e_u	40,5	mm	MC2010 7.3.5.2
Diameter of circle with same surface A_c	b_u	1052,2	mm	MC2010 7.3.5.2
Eccentricity coefficient, approximated	k_e	0,96		MC2010 (7.3-59)
Shear-resisting control perimeter	b_0	3419	mm	MC2010 7.3.5.2
Distance to zero moment, x-direction	$r_{s,x}$	2,36	m	MC2010 7.3.5.4
Distance to zero moment, z-direction	$r_{s,z}$	1,661	m	MC2010 7.3.5.4
Distance to zero moment, design value	r_s	2,36	m	MC2010 7.3.5.4
Rotation, x-direction	ψ_x	0,0321		MC2010 (7.3-70)
Rotation, y-direction	ψ_z	0,0227		MC2010 (7.3-70)
Rotation	ψ	0,0321		MC2010 (7.3-70)
Rotation factor, x-direction	$k_{\psi,x}$	0,13083		MC2010 (7.3-63)
Rotation factor, z-direction	$k_{\psi,z}$	0,17144		MC2010 (7.3-63)
Rotation factor	k_{ψ}	0,13083		MC2010 (7.3-63)
Shear capacity, concrete contribution	$V_{Rd,c}$	**421,65**	**kN**	MC2010 (7.3-61)
Shear reinforcement necessary?		**YES**		
Minimum value of reinforcement system parameter	k_{sys}	**2,44**		MC2010 (7.3-69)
Amount that must be taken by shear reinforcement	$V_{Rd,s}$	605,35	kN	

Load Combination 4

Material factor, concrete	γ_c	1,5		
Material factor, steel	γ_s	1,15		
Characteristic concrete strength	f_{ck}	35	N/mm^2	
Design concrete strength	f_{cd}	19,83	N/mm^2	
Mean tensile concrete strength	f_{ctm}	3,21	N/mm^2	
Design tensile concrete strength	f_{ctd}	1,27	N/mm^2	
Maximal aggregate size	d_g	20	mm	
Aggregate size factor	k_{dg}	0,89		MC2010 7.3-62
Characteristic yield strength of steel	f_{yk}	500	N/mm^2	
Design yield strength of steel	f_{yd}	434,8	N/mm^2	
Young's modulus steel	E_s	200000	N/mm^2	
Slab thickness	h	300	mm	
Effective depth	d_v	239	mm	
Column dimension	w_s	700	mm	
Column dimension	h_s	700	mm	
Diameter of flexural reinforcement	Ø	16	mm	
Centering of flexural reinforcement bars	cc	75	mm	
Flextural reinforcement per unit length, x-direction	$A_{s,x}$	2680,83	mm^2	
Flextural reinforcement per unit length, z-direction	$A_{s,z}$	2680,83	mm^2	
Reinforcement ratio, x-direction	ρ_x	0,01122		
Reinforcement ratio, z-direction	ρ_z	0,01122		
Reinforcement ratio	ρ	0,01122		
Span in x-direction	L_x	10,705	m	
Span in y-direction	L_y	7,55	m	
Design shear force	V_{Ed}	1260	kN	
Design moment	M_{Ed}	0	kNm	
Area within control section	A_{per}	869462,7	mm^2	MC2010 7.3.5.2
Eccentricity of resultant forces	e_u	0,0	mm	MC2010 7.3.5.2
Diameter of circle with same surface A_c	b_u	1052,2	mm	MC2010 7.3.5.2
Eccentricity coefficient, approximated	k_e	1,00		MC2010 (7.3-59)
Shear-resisting control perimeter	b_0	3551	mm	MC2010 7.3.5.2
Distance to zero moment, x-direction	$r_{s,x}$	2,36	m	MC2010 7.3.5.4
Distance to zero moment, z-direction	$r_{s,z}$	1,661	m	MC2010 7.3.5.4
Distance to zero moment, design value	r_s	2,36	m	MC2010 7.3.5.4
Rotation, x-direction	ψ_x	0,0321		MC2010 (7.3-70)
Rotation, y-direction	ψ_z	0,0227		MC2010 (7.3-70)
Rotation	ψ	0,0321		MC2010 (7.3-70)
Rotation factor, x-direction	$k_{\psi,x}$	0,13083		MC2010 (7.3-63)
Rotation factor, z-direction	$k_{\psi,z}$	0,17144		MC2010 (7.3-63)
Rotation factor	k_{ψ}	0,13083		MC2010 (7.3-63)
Shear capacity, concrete contribution	$V_{Rd,c}$	**437,89**	**kN**	MC2010 (7.3-61)
Shear reinforcement necessary?		**YES**		
Minimum value of reinforcement system parameter	k_{sys}	**2,88**		MC2010 (7.3-69)
Amount that must be taken by shear reinforcement	$V_{Rd,s}$	822,11	kN	

Column 2, MC2010 LoA I

Load Combination 3

Material factor, concrete	γ_c	1,5		
Material factor, steel	γ_s	1,15		
Characteristic concrete strength	f_{ck}	35	N/mm^2	
Design concrete strength	f_{cd}	19,83	N/mm^2	
Mean tensile concrete strength	f_{ctm}	3,21	N/mm^2	
Design tensile concrete strength	f_{ctd}	1,27	N/mm^2	
Maximal aggregate size	d_g	20	mm	
Aggregate size factor	k_{dg}	0,89		MC2010 7.3-62
Characteristic yield strength of steel	f_{yk}	500	N/mm^2	
Design yield strength of steel	f_{yd}	434,8	N/mm^2	
Young's modulus steel	E_s	200000	N/mm^2	
Slab thickness	h	300	mm	
Effective depth	d_v	239	mm	
Column dimension	w_s	450	mm	
Column dimension	h_s	700	mm	
Diameter of flexural reinforcement	Ø	16	mm	
Centering of flexural reinforcement bars	cc	75	mm	
Flextural reinforcement per unit length, x-direction	$A_{s,x}$	2680,83	mm^2	
Flextural reinforcement per unit length, z-direction	$A_{s,z}$	2680,83	mm^2	
Reinforcement ratio, x-direction	ρ_x	0,01122		
Reinforcement ratio, z-direction	ρ_z	0,01122		
Reinforcement ratio	ρ	0,01122		
Span in x-direction	L_x	7,35	m	
Span in y-direction	L_y	7,9	m	
Design shear force	V_{Ed}	969	kN	
Design moment	M_{Ed}	52,01	kNm	
Area within control section	A_{per}	634712,7	mm^2	MC2010 7.3.5.2
Eccentricity of resultant forces	e_u	53,7	mm	MC2010 7.3.5.2
Diameter of circle with same surface A_c	b_u	899,0	mm	MC2010 7.3.5.2
Eccentricity coefficient, approximated	k_e	0,94		MC2010 (7.3-59)
Shear-resisting control perimeter	b_0	2879	mm	MC2010 7.3.5.2
Distance to zero moment, x-direction	$r_{s,x}$	1,62	m	MC2010 7.3.5.4
Distance to zero moment, z-direction	$r_{s,z}$	1,738	m	MC2010 7.3.5.4
Distance to zero moment, design value	r_s	1,74	m	MC2010 7.3.5.4
Rotation, x-direction	ψ_x	0,0221		MC2010 (7.3-70)
Rotation, y-direction	ψ_z	0,0237		MC2010 (7.3-70)
Rotation	ψ	0,0237		MC2010 (7.3-70)
Rotation factor, x-direction	$k_{\psi,x}$	0,17488		MC2010 (7.3-63)
Rotation factor, z-direction	$k_{\psi,z}$	0,16573		MC2010 (7.3-63)
Rotation factor	k_ψ	0,16573		MC2010 (7.3-63)
Shear capacity, concrete contribution	$V_{Rd,c}$	**449,75**	**kN**	MC2010 (7.3-61)
Shear reinforcement necessary?		**YES**		
Minimum value of reinforcement system parameter	k_{sys}	**2,15**		MC2010 (7.3-69)
Amount that must be taken by shear reinforcement	$V_{Rd,s}$	519,25	kN	

Load Combination 4

Material factor, concrete	γ_c	1,5		
Material factor, steel	γ_s	1,15		
Characteristic concrete strength	f_{ck}	35	N/mm^2	
Design concrete strength	f_{cd}	19,83	N/mm^2	
Mean tensile concrete strength	f_{ctm}	3,21	N/mm^2	
Design tensile concrete strength	f_{ctd}	1,27	N/mm^2	
Maximal aggregate size	d_g	20	mm	
Aggregate size factor	k_{dg}	0,89		MC2010 7.3-62
Characteristic yield strength of steel	f_{yk}	500	N/mm^2	
Design yield strength of steel	f_{yd}	434,8	N/mm^2	
Young's modulus steel	E_s	200000	N/mm^2	
Slab thickness	h	300	mm	
Effective depth	d_v	239	mm	
Column dimension	w_s	450	mm	
Column dimension	h_s	700	mm	
Diameter of flexural reinforcement	Ø	16	mm	
Centering of flexural reinforcement bars	cc	75	mm	
Flextural reinforcement per unit length, x-direction	$A_{s,x}$	2680,83	mm^2	
Flextural reinforcement per unit length, z-direction	$A_{s,z}$	2680,83	mm^2	
Reinforcement ratio, x-direction	ρ_x	0,01122		
Reinforcement ratio, z-direction	ρ_z	0,01122		
Reinforcement ratio	ρ	0,01122		
Span in x-direction	L_x	7,35	m	
Span in y-direction	L_y	7,9	m	
Design shear force	V_{Ed}	1035	kN	
Design moment	M_{Ed}	0	kNm	
Area within control section	A_{per}	634712,7	mm^2	MC2010 7.3.5.2
Eccentricity of resultant forces	e_u	0,0	mm	MC2010 7.3.5.2
Diameter of circle with same surface A_c	b_u	899,0	mm	MC2010 7.3.5.2
Eccentricity coefficient, approximated	k_e	1,00		MC2010 (7.3-59)
Shear-resisting control perimeter	b_0	3051	mm	MC2010 7.3.5.2
Distance to zero moment, x-direction	$r_{s,x}$	1,62	m	MC2010 7.3.5.4
Distance to zero moment, z-direction	$r_{s,z}$	1,738	m	MC2010 7.3.5.4
Distance to zero moment, design value	r_s	1,74	m	MC2010 7.3.5.4
Rotation, x-direction	ψ_x	0,0221		MC2010 (7.3-70)
Rotation, y-direction	ψ_z	0,0237		MC2010 (7.3-70)
Rotation	ψ	0,0237		MC2010 (7.3-70)
Rotation factor, x-direction	$k_{\psi,x}$	0,17488		MC2010 (7.3-63)
Rotation factor, z-direction	$k_{\psi,z}$	0,16573		MC2010 (7.3-63)
Rotation factor	k_ψ	0,16573		MC2010 (7.3-63)
Shear capacity, concrete contribution	$V_{Rd,c}$	**476,61**	**kN**	MC2010 (7.3-61)
Shear reinforcement necessary?		**YES**		
Minimum value of reinforcement system parameter	k_{sys}	**2,17**		MC2010 (7.3-69)
Amount that must be taken by shear reinforcement	$V_{Rd,s}$	558,39	kN	

Column 3, MC2010 LoA I

Load Combination 3

Material factor, concrete	γ_c	1,5		
Material factor, steel	γ_s	1,15		
Characteristic concrete strength	f_{ck}	35	N/mm^2	
Design concrete strength	f_{cd}	19,83	N/mm^2	
Mean tensile concrete strength	f_{ctm}	3,21	N/mm^2	
Design tensile concrete strength	f_{ctd}	1,27	N/mm^2	
Maximal aggregate size	d_g	20	mm	
Aggregate size factor	k_{dg}	0,89		MC2010 7.3-62
Characteristic yield strength of steel	f_{yk}	500	N/mm^2	
Design yield strength of steel	f_{yd}	434,8	N/mm^2	
Young's modulus steel	E_s	200000	N/mm^2	
Slab thickness	h	300	mm	
Effective depth	d_v	239	mm	
Column dimension	w_s	400	mm	
Column dimension	h_s	550	mm	
Diameter of flexural reinforcement	Ø	16	mm	
Centering of flexural reinforcement bars	cc	75	mm	
Flextural reinforcement per unit length, x-direction	$A_{s,x}$	2680,83	mm^2	
Flextural reinforcement per unit length, z-direction	$A_{s,z}$	2680,83	mm^2	
Reinforcement ratio, x-direction	ρ_x	0,01122		
Reinforcement ratio, z-direction	ρ_z	0,01122		
Reinforcement ratio	ρ	0,01122		
Span in x-direction	L_x	7,35	m	
Span in y-direction	L_y	8,01	m	
Design shear force	V_{Ed}	1038	kN	
Design moment	M_{Ed}	0	kNm	
Area within control section	A_{per}	491912,7	mm^2	MC2010 7.3.5.2
Eccentricity of resultant forces	e_u	0,0	mm	MC2010 7.3.5.2
Diameter of circle with same surface A_c	b_u	791,4	mm	MC2010 7.3.5.2
Eccentricity coefficient, approximated	k_e	1,00		MC2010 (7.3-59)
Shear-resisting control perimeter	b_0	2651	mm	MC2010 7.3.5.2
Distance to zero moment, x-direction	$r_{s,x}$	1,62	m	MC2010 7.3.5.4
Distance to zero moment, z-direction	$r_{s,z}$	1,7622	m	MC2010 7.3.5.4
Distance to zero moment, design value	r_s	1,76	m	MC2010 7.3.5.4
Rotation, x-direction	ψ_x	0,0221		MC2010 (7.3-70)
Rotation, y-direction	ψ_z	0,0240		MC2010 (7.3-70)
Rotation	ψ	0,0240		MC2010 (7.3-70)
Rotation factor, x-direction	$k_{\psi,x}$	0,17488		MC2010 (7.3-63)
Rotation factor, z-direction	$k_{\psi,z}$	0,16401		MC2010 (7.3-63)
Rotation factor	k_{ψ}	0,16401		MC2010 (7.3-63)
Shear capacity, concrete contribution	$V_{Rd,c}$	**409,83**	**kN**	MC2010 (7.3-61)
Shear reinforcement necessary?		**YES**		
Minimum value of reinforcement system parameter	k_{sys}	**2,53**		MC2010 (7.3-69)
Amount that must be taken by shear reinforcement	$V_{Rd,s}$	628,17	kN	

Load Combination 4

Material factor, concrete	γ_c	1,5		
Material factor, steel	γ_s	1,15		
Characteristic concrete strength	f_{ck}	35	N/mm^2	
Design concrete strength	f_{cd}	19,83	N/mm^2	
Mean tensile concrete strength	f_{ctm}	3,21	N/mm^2	
Design tensile concrete strength	f_{ctd}	1,27	N/mm^2	
Maximal aggregate size	d_g	20	mm	
Aggregate size factor	k_{dg}	0,89		MC2010 7.3-62
Characteristic yield strength of steel	f_{yk}	500	N/mm^2	
Design yield strength of steel	f_{yd}	434,8	N/mm^2	
Young's modulus steel	E_s	200000	N/mm^2	
Slab thickness	h	300	mm	
Effective depth	d_v	239	mm	
Column dimension	w_s	400	mm	
Column dimension	h_s	550	mm	
Diameter of flexural reinforcement	Ø	16	mm	
Centering of flexural reinforcement bars	cc	75	mm	
Flextural reinforcement per unit length, x-direction	$A_{s,x}$	2680,83	mm^2	
Flextural reinforcement per unit length, z-direction	$A_{s,z}$	2680,83	mm^2	
Reinforcement ratio, x-direction	ρ_x	0,01122		
Reinforcement ratio, z-direction	ρ_z	0,01122		
Reinforcement ratio	ρ	0,01122		
Span in x-direction	L_x	7,35	m	
Span in y-direction	L_y	8,01	m	
Design shear force	V_{Ed}	963	kN	
Design moment	M_{Ed}	46,53	kNm	
Area within control section	A_{per}	491912,7	mm^2	MC2010 7.3.5.2
Eccentricity of resultant forces	e_u	48,3	mm	MC2010 7.3.5.2
Diameter of circle with same surface A_c	b_u	791,4	mm	MC2010 7.3.5.2
Eccentricity coefficient, approximated	k_e	0,94		MC2010 (7.3-59)
Shear-resisting control perimeter	b_0	2498	mm	MC2010 7.3.5.2
Distance to zero moment, x-direction	$r_{s,x}$	1,62	m	MC2010 7.3.5.4
Distance to zero moment, z-direction	$r_{s,z}$	1,7622	m	MC2010 7.3.5.4
Distance to zero moment, design value	r_s	1,76	m	MC2010 7.3.5.4
Rotation, x-direction	ψ_x	0,0221		MC2010 (7.3-70)
Rotation, y-direction	ψ_z	0,0240		MC2010 (7.3-70)
Rotation	ψ	0,0240		MC2010 (7.3-70)
Rotation factor, x-direction	$k_{\psi,x}$	0,17488		MC2010 (7.3-63)
Rotation factor, z-direction	$k_{\psi,z}$	0,16401		MC2010 (7.3-63)
Rotation factor	k_{ψ}	0,16401		MC2010 (7.3-63)
Shear capacity, concrete contribution	$V_{Rd,c}$	**386,25**	**kN**	MC2010 (7.3-61)
Shear reinforcement necessary?		**YES**		
Minimum value of reinforcement system parameter	k_{sys}	**2,49**		MC2010 (7.3-69)
Amount that must be taken by shear reinforcement	$V_{Rd,s}$	576,75	kN	

Column 1, MC2010 LoA II

Load Combination 3				
Material factor, concrete	γ_c	1,5		
Material factor, steel	γ_s	1,15		
Characteristic concrete strength	f_{ck}	35	N/mm^2	
Design concrete strength	f_{cd}	19,83	N/mm^2	
Mean tensile concrete strength	f_{ctm}	3,21	N/mm^2	
Design tensile concrete strength	f_{ctd}	1,27	N/mm^2	
Maximal aggregate size	d_g	20	mm	
Aggregate size factor	k_{dg}	0,89		MC2010 (7.3-62)
Characteristic yield strength of steel	f_{yk}	500	N/mm^2	
Design yield strength of steel	f_{yd}	434,8	N/mm^2	
Young's modulus steel	E_s	200000	N/mm^2	
Slab thickness	h	300	mm	
Effective depth	d_v	239	mm	
Column dimension	w_s	700	mm	
Column dimension	h_s	700	mm	
Diameter of flexural reinforcement	Ø	16	mm	
Centering of flexural reinforcement bars	cc	75	mm	
Cover outer reinforcement layer	c	45		
Flextural reinforcement per unit length, x-direction	$A_{s,x}$	2680,83	mm^2	
Flextural reinforcement per unit length, z-direction	$A_{s,z}$	2680,83	mm^2	
Reinforcement ratio, x-direction	ρ_x	0,01122		
Reinforcement ratio, z-direction	ρ_z	0,01122		
Reinforcement ratio	ρ	0,01122		
Span in x-direction	L_x	10,705	m	
Span in y-direction	L_y	7,55	m	
Design shear force	V_{Ed}	1027	kN	
Design moment	M_{Ed}	41,62	kNm	
Flexural strength per unit length	m_{Rd}	244,32	kNm/m	Eq. (3.12)
Area within control section	A_{per}	869462,7	mm^2	MC2010 7.3.5.2
Eccentricity of resultant forces, LC4	e_u	40,5	mm	MC2010 7.3.5.2
Diameter of circle with same surface A_c	b_u	1052,2	mm	MC2010 7.3.5.2
Eccentricity coefficient, approximated	k_e	0,96		MC2010 (7.3-59)
Shear-resisting control perimeter	b_0	3419	mm	MC2010 7.3.5.2
Distance to zero moment, x-direction	$r_{s,x}$	2,36	m	MC2010 7.3.5.4
Distance to zero moment, z-direction	$r_{s,z}$	1,661	m	MC2010 7.3.5.4
Distance to zero moment, design value	r_s	2,36	m	MC2010 7.3.5.4
Width of support strip	b_s	2,97	m	MC2010 (7.3-76)
Design moment per unit length	m_{Ed}	135,39	kNm	MC2010 (7.3-71)
Rotation	ψ	0,0133		MC2010 (7.3-75)
Rotation factor	k_ψ	0,24787		MC2010 (7.3-63)
Performance coefficient of reinforcement system	k_{sys}	2,0		MC2010 7.3.5.3
Shear capacity, concrete contribution	$V_{Rd,c}$	**798,89**	**kN**	MC2010 (7.3-61)
Shear reinforcement necessary?		**YES**		
Maximum punching shear compression capacity, 1	$V_{Rd,max1}$	1597,78	kN	MC2010 (7.3-69)
Maximum punching shear compression capacity, 2	$V_{Rd,max2}$	3222,99	kN	MC2010 (7.3-69)
Maximum punching shear compression capacity	$V_{Rd,max}$	**1597,78**	**kN**	MC2010 (7.3-69)
Sufficient cunching shear compression capacity?		**YES**		
Utilization of punching shear compression capacity	$u_{Rd,max}$	**0,64**		
Amount that must be taken by shear reinforcement	$V_{Rd,s}$	228,11	kN	
Design bond strength	f_{bd}	3	N/mm^2	MC2010 7.3.5.3
Diameter of shear reinforcement bars	$Ø_w$	10		
Shear reinforcement angle to the slab plane	α	90	degrees	
Centering of shear reinforcement bars, in both directions	cc	150		
Cover of shear reinforcement	c	35	mm	
Activated stress in shear reinforcement, with bond	$\sigma_{swd,1}$	514,69	N/mm^2	MC2010 (7.3-67)
Activated stress in the shear reinforcement, without bond	$\sigma_{swd,2}$	441,83	N/mm^2	MC2010 (7.3-67)
Design activated stress in shear reinforcement, with bond	σ_{swd}	434,78	N/mm^2	MC2010 (7.3-67)

Design activated stress in shear reinforcement, without bond	σ_{swd}	434,78	N/mm^2	MC2010 (7.3-67)
Calculated necessary shear reinforcement outside 0,35 d_v and within 1d_v	A_{sw}	544,86	mm^2	MC2010 (7.3-64)
Minimun shear reinforcement outside 0,35 d_v and within 1d_v	$A_{sw,min}$	1226,54	mm^2	MC2010 (7.3-68)
Design shear reinforcement outside 0,35 d_v and within 1d_v	A_{sw}	1226,54	mm^2	
Effective depth, outer	$d_{v,out}$	204,00	mm	MC2010 7.3.5.5
Length of outer perimeter for calculation of k_e	b_2	5149,54	mm	MC2010 7.3.5.5
Radius of a circle with perimeter b_2	r_{out}	819,58	mm	MC2010 7.3.5.5
Approximate value of k_e for outer perimeter	k_e	0,98		MC2010 7.3.5.5
Length of outer perimeter where shear reinforcement is not needed	b_{out}	5276,86	mm	MC2010 7.3.5.5
Reinforcement density	ρ_{sw}	0,35 %		
Area outside 0,35 d_v and withing 1d_v	$A_{reinforce}$	592448,18	mm^2	
Calculated shear reinforcement amount with chosen centering included	$A_{sw,calc}$	**2068,03**	**mm^2**	
Sufficient shear reinforcement within the area		**YES**		

Load Combination 4

Material factor, concrete	γ_c	1,5		
Material factor, steel	γ_s	1,15		
Characteristic concrete strength	f_{ck}	35	N/mm^2	
Design concrete strength	f_{cd}	19,83	N/mm^2	
Mean tensile concrete strength	f_{ctm}	3,21	N/mm^2	
Design tensile concrete strength	f_{ctd}	1,27	N/mm^2	
Maximal aggregate size	d_g	20	mm	
Aggregate size factor	k_{dg}	0,89		MC2010 (7.3-62)
Characteristic yield strength of steel	f_{yk}	500	N/mm^2	
Design yield strength of steel	f_{yd}	434,8	N/mm^2	
Young's modulus steel	E_s	200000	N/mm^2	
Slab thickness	h	300	mm	
Effective depth	d_v	239	mm	
Column dimension	w_s	700	mm	
Column dimension	h_s	700	mm	
Diameter of flexural reinforcement	Ø	16	mm	
Centering of flexural reinforcement bars	cc	75	mm	
Cover outer reinforcement layer	c	45		
Flextural reinforcement per unit length, x-direction	$A_{s,x}$	2680,83	mm^2	
Flextural reinforcement per unit length, z-direction	$A_{s,z}$	2680,83	mm^2	
Reinforcement ratio, x-direction	ρ_x	0,01122		
Reinforcement ratio, z-direction	ρ_z	0,01122		
Reinforcement ratio	ρ	0,01122		
Span in x-direction	L_x	10,705	m	
Span in y-direction	L_y	7,55	m	
Design shear force	V_{Ed}	1260	kN	
Design moment	M_{Ed}	0	kNm	
Flexural strength per unit length	m_{Rd}	244,32	kNm/m	Eq. (3.12)
Area within control section	A_{per}	869462,7	mm^2	MC2010 7.3.5.2
Eccentricity of resultant forces, LC4	e_u	0,0	mm	MC2010 7.3.5.2
Diameter of circle with same surface A_c	b_u	1052,2	mm	MC2010 7.3.5.2
Eccentricity coefficient, approximated	k_e	1,00		MC2010 (7.3-59)
Shear-resisting control perimeter	b_0	3551	mm	MC2010 7.3.5.2
Distance to zero moment, x-direction	$r_{s,x}$	2,36	m	MC2010 7.3.5.4
Distance to zero moment, z-direction	$r_{s,z}$	1,661	m	MC2010 7.3.5.4
Distance to zero moment, design value	r_s	2,36	m	MC2010 7.3.5.4
Width of support strip	b_s	2,97	m	MC2010 (7.3-76)
Design moment per unit length	m_{Ed}	157,50	kNm	MC2010 (7.3-71)
Rotation	ψ	0,0166		MC2010 (7.3-75)
Rotation factor	k_ψ	0,21368		MC2010 (7.3-63)
Performance coefficient of reinforcement system	k_{sys}	2,0		MC2010 7.3.5.3
Shear capacity, concrete contribution	$V_{Rd,c}$	**715,22**	**kN**	MC2010 (7.3-61)
Shear reinforcement necessary?		**YES**		
Maximum punching shear compression capacity, 1	$V_{Rd,max1}$	1430,44	kN	MC2010 (7.3-69)
Maximum punching shear compression capacity, 2	$V_{Rd,max2}$	3347,12	kN	MC2010 (7.3-69)
Maximum punching shear compression capacity	$V_{Rd,max}$	**1430,44**	**kN**	MC2010 (7.3-69)
Sufficient cunching shear compression capacity?		**YES**		
	$u_{Rd,max}$	**0,88**		
Amount that must be taken by shear reinforcement	$V_{Rd,s}$	544,78	kN	
Design bond strength	f_{bd}	3	N/mm^2	MC2010 7.3.5.3
Diameter of shear reinforcement bars	$Ø_w$	10		
Shear reinforcement angle to the slab plane	α	90	degrees	
Centering of shear reinforcement bars, in both directions	cc	150		
Cover of shear reinforcement	c	35	mm	
Activated stress in shear reinforcement, with bond	$\sigma_{swd,1}$	645,79	N/mm^2	MC2010 (7.3-67)
Activated stress in the shear reinforcement, without bond	$\sigma_{swd,2}$	554,37	N/mm^2	MC2010 (7.3-67)

Design activated stress in shear reinforcement, with bond	σ_{swd}	434,78	N/mm^2	MC2010 (7.3-67)
Design activated stress in shear reinforcement, without bond	σ_{swd}	434,78	N/mm^2	MC2010 (7.3-67)
Calculated necessary shear reinforcement outside 0,35 d_v and within 1d_v	A_{sw}	1252,99	mm^2	MC2010 (7.3-64)
Minimun shear reinforcement outside 0,35 d_v and within 1d_v	$A_{sw,min}$	1449,00	mm^2	MC2010 (7.3-68)
Design shear reinforcement outside 0,35 d_v and within 1d_v	A_{sw}	1449,00	mm^2	
Effective depth, outer	$d_{v,out}$	204,00	mm	MC2010 7.3.5.5
Length of outer perimeter for calculation of k_e	b_2	7328,74	mm	MC2010 7.3.5.5
Radius of a circle with perimeter b_2	r_{out}	1166,41	mm	MC2010 7.3.5.5
Approximate value of k_e for outer perimeter	k_e	1,00		MC2010 7.3.5.5
Length of outer perimeter where shear reinforcement is not needed	b_{out}	7328,74	mm	MC2010 7.3.5.5
Reinforcement density	ρ_{sw}	0,35 %		
Area outside 0,35 d_v and withing 1d_v	$A_{reinforce}$	592448,18	mm^2	
Calculated shear reinforcement amount with chosen centering included	$A_{sw,calc}$	**2068,03**	**mm^2**	
Sufficient shear reinforcement within the area		**YES**		

Column 2, MC2010 LoA II

Load Combination 3

Material factor, concrete	γ_c	1,5		
Material factor, steel	γ_s	1,15		
Characteristic concrete strength	f_{ck}	35	N/mm^2	
Design concrete strength	f_{cd}	19,83	N/mm^2	
Mean tensile concrete strength	f_{ctm}	3,21	N/mm^2	
Design tensile concrete strength	f_{ctd}	1,27	N/mm^2	
Maximal aggregate size	d_g	20	mm	
Aggregate size factor	k_{dg}	0,89		MC2010 (7.3-62)
Characteristic yield strength of steel	f_{yk}	500	N/mm^2	
Design yield strength of steel	f_{yd}	434,8	N/mm^2	
Young's modulus steel	E_s	200000	N/mm^2	
Slab thickness	h	300	mm	
Effective depth	d_v	239	mm	
Column dimension	w_s	450	mm	
Column dimension	h_s	700	mm	
Diameter of flexural reinforcement	Ø	16	mm	
Centering of flexural reinforcement bars	cc	75	mm	
Cover outer reinforcement layer	c	45		
Flextural reinforcement per unit length, x-direction	$A_{s,x}$	2680,83	mm^2	
Flextural reinforcement per unit length, z-direction	$A_{s,z}$	2680,83	mm^2	
Reinforcement ratio, x-direction	ρ_x	0,01122		
Reinforcement ratio, z-direction	ρ_z	0,01122		
Reinforcement ratio	ρ	0,01122		
Span in x-direction	L_x	7,35	m	
Span in y-direction	L_y	7,9	m	
Design shear force	V_{Ed}	969	kN	
Design moment	M_{Ed}	52,01	kNm	
Flexural strength per unit length	m_{Rd}	244,32	kNm/m	Eq. (3.12)
Area within control section	A_{per}	634712,7	mm^2	MC2010 7.3.5.2
Eccentricity of resultant forces, LC4	e_u	53,7	mm	MC2010 7.3.5.2
Diameter of circle with same surface A_c	b_u	899,0	mm	MC2010 7.3.5.2
Eccentricity coefficient, approximated	k_e	0,94		MC2010 (7.3-59)
Shear-resisting control perimeter	b_0	2879	mm	MC2010 7.3.5.2
Distance to zero moment, x-direction	$r_{s,x}$	1,62	m	MC2010 7.3.5.4
Distance to zero moment, z-direction	$r_{s,z}$	1,738	m	MC2010 7.3.5.4
Distance to zero moment, design value	r_s	1,74	m	MC2010 7.3.5.4
Width of support strip	b_s	2,51	m	MC2010 (7.3-76)
Design moment per unit length	m_{Ed}	131,47	kNm	MC2010 (7.3-71)
Rotation	ψ	0,0094		MC2010 (7.3-75)
Rotation factor	k_ψ	0,30399		MC2010 (7.3-63)
Performance coefficient of reinforcement system	k_{sys}	2,0		MC2010 7.3.5.3
Shear capacity, concrete contribution	$V_{Rd,c}$	**824,96**	**kN**	MC2010 (7.3-61)
Shear reinforcement necessary?		**YES**		
Maximum punching shear compression capacity, 1	$V_{Rd,max1}$	1649,93	kN	MC2010 (7.3-69)
Maximum punching shear compression capacity, 2	$V_{Rd,max2}$	2713,78	kN	MC2010 (7.3-69)
Maximum punching shear compression capacity	$V_{Rd,max}$	**1649,93**	**kN**	MC2010 (7.3-69)
Sufficient cunching shear compression capacity?		**YES**		
Utilization of punching shear compression capacity	$u_{Rd,max}$	**0,59**		
Amount that must be taken by shear reinforcement	$V_{Rd,s}$	144,04	kN	
Design bond strength	f_{bd}	3	N/mm^2	MC2010 7.3.5.3
Diameter of shear reinforcement bars	$Ø_w$	10		
Shear reinforcement angle to the slab plane	α	90	degrees	
Centering of shear reinforcement bars, in both directions	cc	150		
Cover of shear reinforcement	c	35	mm	
Activated stress in shear reinforcement, with bond	$\sigma_{swd,1}$	363,44	N/mm^2	MC2010 (7.3-67)

Activated stress in the shear reinforcement, without bond	$\sigma_{swd,2}$	311,99	N/mm^2	MC2010 (7.3-67)
Design activated stress in shear reinforcement, with bond	σ_{swd}	363,44	N/mm^2	MC2010 (7.3-67)
Design activated stress in shear reinforcement, without bond	σ_{swd}	311,99	N/mm^2	MC2010 (7.3-67)
Calculated necessary shear reinforcement outside 0,35 d_v and within 1d_v	A_{sw}	419,97	mm^2	MC2010 (7.3-64)
Minimun shear reinforcement outside 0,35 d_v and within 1d_v	$A_{sw,min}$	1180,88	mm^2	MC2010 (7.3-68)
Design shear reinforcement outside 0,35 d_v and within 1d_v	A_{sw}	1180,88	mm^2	
Effective depth, outer	$d_{v,out}$	204,00	mm	MC2010 7.3.5.5
Length of outer perimeter for calculation of k_e	b_2	3961,78	mm	MC2010 7.3.5.5
Radius of a circle with perimeter b_2	r_{out}	630,54	mm	MC2010 7.3.5.5
Approximate value of k_e for outer perimeter	k_e	0,96		MC2010 7.3.5.5
Length of outer perimeter where shear reinforcement is not needed	b_{out}	4130,40	mm	MC2010 7.3.5.5
Reinforcement density	ρ_{sw}	0,35 %		
Area outside 0,35 d_v and withing 1d_v	$A_{reinforce}$	514773,18	mm^2	
Calculated shear reinforcement amount with chosen centering included	$A_{sw,calc}$	**1796,90**	**mm^2**	
Sufficient shear reinforcement within the area		**YES**		

Load Combination 4

Material factor, concrete	γ_c	1,5		
Material factor, steel	γ_s	1,15		
Characteristic concrete strength	f_{ck}	35	N/mm^2	
Design concrete strength	f_{cd}	19,83	N/mm^2	
Mean tensile concrete strength	f_{ctm}	3,21	N/mm^2	
Design tensile concrete strength	f_{ctd}	1,27	N/mm^2	
Maximal aggregate size	d_g	20	mm	
Aggregate size factor	k_{dg}	0,89		MC2010 (7.3-62)
Characteristic yield strength of steel	f_{yk}	500	N/mm^2	
Design yield strength of steel	f_{yd}	434,8	N/mm^2	
Young's modulus steel	E_s	200000	N/mm^2	
Slab thickness	h	300	mm	
Effective depth	d_v	239	mm	
Column dimension	w_s	450	mm	
Column dimension	h_s	700	mm	
Diameter of flexural reinforcement	Ø	16	mm	
Centering of flexural reinforcement bars	cc	75	mm	
Cover outer reinforcement layer	c	45		
Flextural reinforcement per unit length, x-direction	$A_{s,x}$	2680,83	mm^2	
Flextural reinforcement per unit length, z-direction	$A_{s,z}$	2680,83	mm^2	
Reinforcement ratio, x-direction	ρ_x	0,01122		
Reinforcement ratio, z-direction	ρ_z	0,01122		
Reinforcement ratio	ρ	0,01122		
Span in x-direction	L_x	7,35	m	
Span in y-direction	L_y	7,9	m	
Design shear force	V_{Ed}	1035	kN	
Design moment	M_{Ed}	0	kNm	
Flexural strength per unit length	m_{Rd}	244,32	kNm/m	Eq. (3.12)
Area within control section	A_{per}	634712,7	mm^2	MC2010 7.3.5.2
Eccentricity of resultant forces, LC4	e_u	0,0	mm	MC2010 7.3.5.2
Diameter of circle with same surface A_c	b_u	899,0	mm	MC2010 7.3.5.2
Eccentricity coefficient, approximated	k_e	1,00		MC2010 (7.3-59)
Shear-resisting control perimeter	b_0	3051	mm	MC2010 7.3.5.2
Distance to zero moment, x-direction	$r_{s,x}$	1,62	m	MC2010 7.3.5.4
Distance to zero moment, z-direction	$r_{s,z}$	1,738	m	MC2010 7.3.5.4
Distance to zero moment, design value	r_s	1,74	m	MC2010 7.3.5.4
Width of support strip	b_s	2,51	m	MC2010 (7.3-76)
Design moment per unit length	m_{Ed}	129,38	kNm	MC2010 (7.3-71)
Rotation	ψ	0,0091		MC2010 (7.3-75)
Rotation factor	k_ψ	0,30797		MC2010 (7.3-63)
Performance coefficient of reinforcement system	k_{sys}	2,0		MC2010 7.3.5.3
Shear capacity, concrete contribution	$V_{Rd,c}$	**885,67**	**kN**	MC2010 (7.3-61)
Shear reinforcement necessary?		**YES**		
Maximum punching shear compression capacity, 1	$V_{Rd,max1}$	1771,34	kN	MC2010 (7.3-69)
Maximum punching shear compression capacity, 2	$V_{Rd,max2}$	2875,81	kN	MC2010 (7.3-69)
Maximum punching shear compression capacity	$V_{Rd,max}$	**1771,34**	**kN**	MC2010 (7.3-69)
Sufficient cunching shear compression capacity?		**YES**		
	$u_{Rd,max}$	**0,58**		
Amount that must be taken by shear reinforcement	$V_{Rd,s}$	149,33	kN	
Design bond strength	f_{bd}	3	N/mm^2	MC2010 7.3.5.3
Diameter of shear reinforcement bars	$Ø_w$	10		
Shear reinforcement angle to the slab plane	α	90	degrees	
Centering of shear reinforcement bars, in both directions	cc	150		
Cover of shear reinforcement	c	35	mm	
Activated stress in shear reinforcement, with bond	$\sigma_{swd,1}$	354,80	N/mm^2	MC2010 (7.3-67)
Activated stress in the shear reinforcement, without bond	$\sigma_{swd,2}$	304,57	N/mm^2	MC2010 (7.3-67)

Design activated stress in shear reinforcement, with bond	σ_{swd}	354,80	N/mm^2	MC2010 (7.3-67)
Design activated stress in shear reinforcement, without bond	σ_{swd}	304,57	N/mm^2	MC2010 (7.3-67)
Calculated necessary shear reinforcement outside 0,35 d_v and within $1d_v$	A_{sw}	420,88	mm^2	MC2010 (7.3-64)
Minimun shear reinforcement outside 0,35 d_v and within $1d_v$	$A_{sw,min}$	1190,25	mm^2	MC2010 (7.3-68)
Design shear reinforcement outside 0,35 d_v and within $1d_v$	A_{sw}	1190,25	mm^2	
Effective depth, outer	$d_{v,out}$	204,00	mm	MC2010 7.3.5.5
Length of outer perimeter for calculation of k_e	b_2	4176,90	mm	MC2010 7.3.5.5
Radius of a circle with perimeter b_2	r_{out}	664,77	mm	MC2010 7.3.5.5
Approximate value of k_e for outer perimeter	k_e	1,00		MC2010 7.3.5.5
Length of outer perimeter where shear reinforcement is not needed	b_{out}	4176,90	mm	MC2010 7.3.5.5
Reinforcement density	ρ_{sw}	0,35 %		
Area outside 0,35 d_v and withing $1d_v$	$A_{reinforce}$	514773,18	mm^2	
Calculated shear reinforcement amount with chosen centering included	$A_{sw,calc}$	**1796,90**	**mm^2**	
Sufficient shear reinforcement within the area		**YES**		

Column 3, MC2010 LoA II

Load Combination 3

Material factor, concrete	γ_c	1,5		
Material factor, steel	γ_s	1,15		
Characteristic concrete strength	f_{ck}	35	N/mm^2	
Design concrete strength	f_{cd}	19,83	N/mm^2	
Mean tensile concrete strength	f_{ctm}	3,21	N/mm^2	
Design tensile concrete strength	f_{ctd}	1,27	N/mm^2	
Maximal aggregate size	d_g	20	mm	
Aggregate size factor	k_{dg}	0,89		MC2010 (7.3-62)
Characteristic yield strength of steel	f_{yk}	500	N/mm^2	
Design yield strength of steel	f_{yd}	434,8	N/mm^2	
Young's modulus steel	E_s	200000	N/mm^2	
Slab thickness	h	300	mm	
Effective depth	d_v	239	mm	
Column dimension	w_s	400	mm	
Column dimension	h_s	550	mm	
Diameter of flexural reinforcement	Ø	16	mm	
Centering of flexural reinforcement bars	cc	75	mm	
Cover outer reinforcement layer	c	45		
Flextural reinforcement per unit length, x-direction	$A_{s,x}$	2680,83	mm^2	
Flextural reinforcement per unit length, z-direction	$A_{s,z}$	2680,83	mm^2	
Reinforcement ratio, x-direction	ρ_x	0,01122		
Reinforcement ratio, z-direction	ρ_z	0,01122		
Reinforcement ratio	ρ	0,01122		
Span in x-direction	L_x	7,35	m	
Span in y-direction	L_y	8,01	m	
Design shear force	V_{Ed}	1038	kN	
Design moment	M_{Ed}	0	kNm	
Flexural strength per unit length	m_{Rd}	244,32	kNm/m	Eq. (3.12)
Area within control section	A_{per}	491912,7	mm^2	MC2010 7.3.5.2
Eccentricity of resultant forces, LC4	e_u	0,0	mm	MC2010 7.3.5.2
Diameter of circle with same surface A_c	b_u	791,4	mm	MC2010 7.3.5.2
Eccentricity coefficient, approximated	k_e	1,00		MC2010 (7.3-59)
Shear-resisting control perimeter	b_0	2651	mm	MC2010 7.3.5.2
Distance to zero moment, x-direction	$r_{s,x}$	1,62	m	MC2010 7.3.5.4
Distance to zero moment, z-direction	$r_{s,z}$	1,7622	m	MC2010 7.3.5.4
Distance to zero moment, design value	r_s	1,76	m	MC2010 7.3.5.4
Width of support strip	b_s	2,53	m	MC2010 (7.3-76)
Design moment per unit length	m_{Ed}	129,75	kNm	MC2010 (7.3-71)
Rotation	ψ	0,0093		MC2010 (7.3-75)
Rotation factor	k_ψ	0,30496		MC2010 (7.3-63)
Performance coefficient of reinforcement system	k_{sys}	2,0		MC2010 7.3.5.3
Shear capacity, concrete contribution	$V_{Rd,c}$	**762,03**	**kN**	MC2010 (7.3-61)
Shear reinforcement necessary?		**YES**		
Maximum punching shear compression capacity, 1	$V_{Rd,max1}$	1524,07	kN	MC2010 (7.3-69)
Maximum punching shear compression capacity, 2	$V_{Rd,max2}$	2498,76	kN	MC2010 (7.3-69)
Maximum punching shear compression capacity	$V_{Rd,max}$	**1524,07**	**kN**	MC2010 (7.3-69)
Sufficient cunching shear compression capacity?		**YES**		
Utilization of punching shear compression capacity	$u_{Rd,max}$	**0,68**		
Amount that must be taken by shear reinforcement	$V_{Rd,s}$	275,97	kN	
Design bond strength	f_{bd}	3	N/mm^2	MC2010 7.3.5.3
Diameter of shear reinforcement bars	$Ø_w$	10		
Shear reinforcement angle to the slab plane	α	90	degrees	
Centering of shear reinforcement bars, in both directions	cc	150		
Cover of shear reinforcement	c	35	mm	
Activated stress in shear reinforcement, with bond	$\sigma_{swd,1}$	361,31	N/mm^2	MC2010 (7.3-67)

Activated stress in the shear reinforcement, without bond	$\sigma_{swd,2}$	310,16	N/mm^2	MC2010 (7.3-67)
Design activated stress in shear reinforcement, with bond	σ_{swd}	361,31	N/mm^2	MC2010 (7.3-67)
Design activated stress in shear reinforcement, without bond	σ_{swd}	310,16	N/mm^2	MC2010 (7.3-67)
Calculated necessary shear reinforcement outside 0,35 d_v and within 1d_v	A_{sw}	763,80	mm^2	MC2010 (7.3-64)
Minimun shear reinforcement outside 0,35 d_v and within 1d_v	$A_{sw,min}$	1193,70	mm^2	MC2010 (7.3-68)
Design shear reinforcement outside 0,35 d_v and within 1d_v	A_{sw}	1193,70	mm^2	
Effective depth, outer	$d_{v,out}$	204,00	mm	MC2010 7.3.5.5
Length of outer perimeter for calculation of k_e	b_2	4230,34	mm	MC2010 7.3.5.5
Radius of a circle with perimeter b_2	r_{out}	673,28	mm	MC2010 7.3.5.5
Approximate value of k_e for outer perimeter	k_e	1,00		MC2010 7.3.5.5
Length of outer perimeter where shear reinforcement is not needed	b_{out}	4230,34	mm	MC2010 7.3.5.5
Reinforcement density	ρ_{sw}	0,35 %		
Area outside 0,35 d_v and withing 1d_v	$A_{reinforce}$	452633,18	mm^2	
Calculated shear reinforcement amount with chosen centering included	$A_{sw,calc}$	**1579,99**	**mm^2**	
Sufficient shear reinforcement within the area		**YES**		

Load Combination 4

Material factor, concrete	γ_c	1,5		
Material factor, steel	γ_s	1,15		
Characteristic concrete strength	f_{ck}	35	N/mm^2	
Design concrete strength	f_{cd}	19,83	N/mm^2	
Mean tensile concrete strength	f_{ctm}	3,21	N/mm^2	
Design tensile concrete strength	f_{ctd}	1,27	N/mm^2	
Maximal aggregate size	d_g	20	mm	
Aggregate size factor	k_{dg}	0,89		MC2010 (7.3-62)
Characteristic yield strength of steel	f_{yk}	500	N/mm^2	
Design yield strength of steel	f_{yd}	434,8	N/mm^2	
Young's modulus steel	E_s	200000	N/mm^2	
Slab thickness	h	300	mm	
Effective depth	d_v	239	mm	
Column dimension	w_s	400	mm	
Column dimension	h_s	550	mm	
Diameter of flexural reinforcement	Ø	16	mm	
Centering of flexural reinforcement bars	cc	75	mm	
Cover outer reinforcement layer	c	45		
Flextural reinforcement per unit length, x-direction	$A_{s,x}$	2680,83	mm^2	
Flextural reinforcement per unit length, z-direction	$A_{s,z}$	2680,83	mm^2	
Reinforcement ratio, x-direction	ρ_x	0,01122		
Reinforcement ratio, z-direction	ρ_z	0,01122		
Reinforcement ratio	ρ	0,01122		
Span in x-direction	L_x	7,35	m	
Span in y-direction	L_y	8,01	m	
Design shear force	V_{Ed}	963	kN	
Design moment	M_{Ed}	46,53	kNm	
Flexural strength per unit length	m_{Rd}	244,32	kNm/m	Eq. (3.12)
Area within control section	A_{per}	491912,7	mm^2	MC2010 7.3.5.2
Eccentricity of resultant forces, LC4	e_u	48,3	mm	MC2010 7.3.5.2
Diameter of circle with same surface A_c	b_u	791,4	mm	MC2010 7.3.5.2
Eccentricity coefficient, approximated	k_e	0,94		MC2010 (7.3-59)
Shear-resisting control perimeter	b_0	2498	mm	MC2010 7.3.5.2
Distance to zero moment, x-direction	$r_{s,x}$	1,62	m	MC2010 7.3.5.4
Distance to zero moment, z-direction	$r_{s,z}$	1,7622	m	MC2010 7.3.5.4
Distance to zero moment, design value	r_s	1,76	m	MC2010 7.3.5.4
Width of support strip	b_s	2,53	m	MC2010 (7.3-76)
Design moment per unit length	m_{Ed}	129,56	kNm	MC2010 (7.3-71)
Rotation	ψ	0,0093		MC2010 (7.3-75)
Rotation factor	k_ψ	0,30532		MC2010 (7.3-63)
Performance coefficient of reinforcement system	k_{sys}	2,0		MC2010 7.3.5.3
Shear capacity, concrete contribution	$V_{Rd,c}$	**719,03**	**kN**	MC2010 (7.3-61)
Shear reinforcement necessary?		**YES**		
Maximum punching shear compression capacity, 1	$V_{Rd,max1}$	1438,06	kN	MC2010 (7.3-69)
Maximum punching shear compression capacity, 2	$V_{Rd,max2}$	2354,98	kN	MC2010 (7.3-69)
Maximum punching shear compression capacity	$V_{Rd,max}$	**1438,06**	**kN**	MC2010 (7.3-69)
Sufficient cunching shear compression capacity?		**YES**		
	$u_{Rd,max}$	**0,67**		
Amount that must be taken by shear reinforcement	$V_{Rd,s}$	243,97	kN	
Design bond strength	f_{bd}	3	N/mm^2	MC2010 7.3.5.3
Diameter of shear reinforcement bars	$Ø_w$	10		
Shear reinforcement angle to the slab plane	α	90	degrees	
Centering of shear reinforcement bars, in both directions	cc	150		
Cover of shear reinforcement	c	35	mm	
Activated stress in shear reinforcement, with bond	$\sigma_{swd,1}$	360,53	N/mm^2	MC2010 (7.3-67)
Activated stress in the shear reinforcement, without bond	$\sigma_{swd,2}$	309,49	N/mm^2	MC2010 (7.3-67)

Design activated stress in shear reinforcement, with bond	σ_{swd}	360,53	N/mm^2	MC2010 (7.3-67)
Design activated stress in shear reinforcement, without bond	σ_{swd}	309,49	N/mm^2	MC2010 (7.3-67)
Calculated necessary shear reinforcement outside 0,35 d_v and within 1d_v	A_{sw}	718,03	mm^2	MC2010 (7.3-64)
Minimun shear reinforcement outside 0,35 d_v and within 1d_v	$A_{sw,min}$	1175,06	mm^2	MC2010 (7.3-68)
Design shear reinforcement outside 0,35 d_v and within 1d_v	A_{sw}	1175,06	mm^2	
Effective depth, outer	$d_{v,out}$	204,00	mm	MC2010 7.3.5.5
Length of outer perimeter for calculation of k_e	b_2	3920,08	mm	MC2010 7.3.5.5
Radius of a circle with perimeter b_2	r_{out}	623,90	mm	MC2010 7.3.5.5
Approximate value of k_e for outer perimeter	k_e	0,96		MC2010 7.3.5.5
Length of outer perimeter where shear reinforcement is not needed	b_{out}	4071,87	mm	MC2010 7.3.5.5
Reinforcement density	ρ_{sw}	0,35 %		
Area outside 0,35 d_v and withing 1d_v	$A_{reinforce}$	452633,18	mm^2	
Calculated shear reinforcement amount with chosen centering included	$A_{sw,calc}$	**1579,99**	**mm^2**	
Sufficient shear reinforcement within the area		**YES**		

Appendix C

DIANA Input Files

This appendix includes the input files for the DIANA model used in Section 6.1 and the DIANA model used for the analysis with slab openings in Section 6.2. For the analysis in Section 6.2 without slab openings the input file is almost equal, the only difference is that some elements are removed in the model with slab openings.

The dotted lines in the input files symbolize where some lines are removed from the appendix in terms of not using too much space. The complete input files, the output files and the tabulated files for the values of the shear stresses for all the analyses in Chapter 6 are presented in the electronic appendices.

DAT-file for model in Section 6.1

```
FEMGEN MODEL      : SLAB7M
ANALYSIS TYPE     : Structural 3D
'UNITS'
LENGTH   MM
TIME     SEC
TEMPER   KELVIN
FORCE    N
'COORDINATES'
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    2      2.015714E+02     0.000000E+00     0.000000E+00
---------------------------------------------------------------------------
 4482      3.600000E+03     3.700000E+03    -4.000000E+02
 4483      3.600000E+03     3.700000E+03    -2.000000E+02
'ELEMENTS'
CONNECTIVITY
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    2 CQ40S  2 227 3 242 18 256 17 241
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 1295 CQ40S  3625 4004 3626 4018 3640 4032 3639 4017
 1296 CQ40S  3626 4005 3627 4019 3641 4033 3640 4018
 1297 CHX60  4069 4098 4055 4141 4127 4184 4170 4213 4113 4084 4156 4199
             1754 1781 1775 1998 1997 1999 1990 1993
 1298 CHX60  4068 4097 4054 4140 4126 4183 4169 4212 4112 4083 4155 4198
             4069 4098 4055 4141 4127 4184 4170 4213
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             4115 4306 4263 4442 4428 4471 4325 4339
 1356 CHX60  4037 4218 4215 4425 4424 4426 4320 4322 4142 4290 4455 4352
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MATERIALS
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GEOMETRY
/ 1-1296 /  1
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     POISON    2.000000E-01
'GEOMETRY'
   1 THICK     3.000000E+02
'GROUPS'
ELEMEN
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NODES
   2 SLAB1_N / 1-1613 /
ELEMEN
   3 SLAB2 / 505-612 /
NODES
   4 SLAB2_N / 211-225 632-645 688-690 775-777 792 821 836 865 908-910
               995-997 1194-1207 1600-1943 /
ELEMEN
   5 SLAB3 / 613-648 /
NODES
   6 SLAB3_N / 1644-1658 1732-1745 1752-1754 1770-1772 1775 1781
               1784 1790 1797-1799 1815-1817 1846-1859 1930-2053 /
ELEMEN
   7 SLAB4 / 649-684 /
NODES
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   8 SLAB4_N / 1944-1958 1974-1990 1994-1997 1999 2000 2002-2005
               2009-2025 2040-2163 /
ELEMEN
   9 SLAB5 / 685-792 /
NODES
  10 SLAB5_N / 2054-2068 2084-2100 2104-2107 2109 2110 2112-2115
               2119-2135 2150-2493 /
ELEMEN
  11 SLAB6 / 793-1296 /
NODES
  12 SLAB6_N / 2194-2208 2282-2295 2302-2304 2320-2322 2325 2331
               2334 2340 2347-2349 2365-2367 2396-2409 2480-4033 /
ELEMEN
  13 SLAB / 1-1296 /
NODES
  14 SLAB_N / 1-4033 /
ELEMEN
  15 COLUMN / 630 631 666 667 1297-1356 /
NODES
  16 COLUMN_N / 1754 1775 1781 1784 1790 1990 1993 1997-2002 2100
                2103 2107-2112 4034-4483 /
'SUPPORTS'
 / 1 16 31 46 61 76 91 106 121 136 151 166 181 196 211 240 269 298
   327 356 385 414 443 472 501 530 559 588 617 1614 1629 1644 1659
   1688 1717 1944 1959 2054 2069 2164 2179 2194 2209 2238 2267 2494
   2509 2524 2539 2554 2569 2584 2599 2614 2629 2644 2659 2674 2689
   2704 2733 2762 2791 2820 2849 2878 2907 2936 2965 2994 3023 3052
   3081 /   TR     1
 / 1-15 226-239 646-648 691-693 778 793 822 837 866-868 911-913
   998-1011 1208-1221 2689-2703 3096-3109 3149-3151 3233-3235 3249
   3277 3291 3319 3359-3361 3443-3445 3628-3641 4020-4033 /   RO     1
 / 2689-2703 3096-3109 3149-3151 3233-3235 3249 3277 3291 3319 3359-3361
   3443-3445 3628-3641 4020-4033 /   TR     2
 / 1 16 31 46 61 76 91 106 121 136 151 166 181 196 211 240 269 298
   327 356 385 414 443 472 501 530 559 588 617 1011 1025 1039 1053
   1067 1081 1095 1109 1123 1137 1151 1165 1179 1193 1207 1235 1263
   1291 1319 1347 1375 1403 1431 1459 1487 1515 1543 1571 1599 1614
   1629 1644 1659 1688 1717 1831 1845 1859 1873 1901 1929 1944 1959
   2025 2039 2054 2069 2135 2149 2164 2179 2194 2209 2238 2267 2381
   2395 2409 2423 2451 2479 2494 2509 2524 2539 2554 2569 2584 2599
   2614 2629 2644 2659 2674 2689 2704 2733 2762 2791 2820 2849 2878
   2907 2936 2965 2994 3023 3052 3081 3459 3473 3487 3501 3515 3529
   3543 3557 3571 3585 3599 3613 3627 3641 3655 3683 3711 3739 3767
   3795 3823 3851 3879 3907 3935 3963 3991 4019 /   RO     2
 / 4037 /   TR     3
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    2   0.000000E+00   1.000000E+00   0.000000E+00
    3   0.000000E+00   0.000000E+00   1.000000E+00
'END'
```

DAT-file for model in Section 6.2

```
FEMGEN MODEL       : SLAB7MOPENING
ANALYSIS TYPE      : Structural 3D
'UNITS'
LENGTH   MM
TIME     SEC
TEMPER   KELVIN
FORCE    N
'COORDINATES'
    1      0.000000E+00     0.000000E+00     0.000000E+00
    2      2.015714E+02     0.000000E+00     0.000000E+00
----------------------------------------------------------------------------
 4482      3.600000E+03     3.700000E+03    -4.000000E+02
 4483      3.600000E+03     3.700000E+03    -2.000000E+02
'ELEMENTS'
CONNECTIVITY
    1 CQ40S  1 226 2 241 17 255 16 240
    2 CQ40S  2 227 3 242 18 256 17 241
----------------------------------------------------------------------------
 1295 CQ40S  3625 4004 3626 4018 3640 4032 3639 4017
 1296 CQ40S  3626 4005 3627 4019 3641 4033 3640 4018
 1297 CHX60  4069 4098 4055 4141 4127 4184 4170 4213 4113 4084 4156 4199
             1754 1781 1775 1998 1997 1999 1990 1993
 1298 CHX60  4068 4097 4054 4140 4126 4183 4169 4212 4112 4083 4155 4198
             4069 4098 4055 4141 4127 4184 4170 4213
----------------------------------------------------------------------------
 1355 CHX60  4114 4305 4262 4441 4427 4470 4324 4338 4143 4291 4456 4353
             4115 4306 4263 4442 4428 4471 4325 4339
 1356 CHX60  4037 4218 4215 4425 4424 4426 4320 4322 4142 4290 4455 4352
             4114 4305 4262 4441 4427 4470 4324 4338
MATERIALS
/ 1-1356 /  1
GEOMETRY
/ 1-1296 /  1
'MATERIALS'
   1 YOUNG     3.400000E+04
     POISON    2.000000E-01
'GEOMETRY'
   1 THICK     3.000000E+02
'GROUPS'
ELEMEN
   1 SLAB1 / 1-504 /
NODES
   2 SLAB1_N / 1-1613 /
ELEMEN
   3 SLAB2 / 505-612 /
NODES
   4 SLAB2_N / 211-225 632-645 688-690 775-777 792 821 836 865 908-910
               995-997 1194-1207 1600-1943 /
ELEMEN
   5 SLAB3 / 613-624 626-635 637-648 /
NODES
   6 SLAB3_N / 1644-1658 1732-1745 1752-1754 1770-1772 1775 1781
               1784 1790 1797-1799 1815-1817 1846-1859 1930-2053 /
ELEMEN
   7 SLAB4 / 649-660 662-671 673-684 /
NODES
```

```
   8 SLAB4_N / 1944-1958 1974-1990 1994-1997 1999 2000 2002-2005
               2009-2025 2040-2163 /
ELEMEN
   9 SLAB5 / 685-792 /
NODES
  10 SLAB5_N / 2054-2068 2084-2100 2104-2107 2109 2110 2112-2115
               2119-2135 2150-2493 /
ELEMEN
  11 SLAB6 / 793-1296 /
NODES
  12 SLAB6_N / 2194-2208 2282-2295 2302-2304 2320-2322 2325 2331
               2334 2340 2347-2349 2365-2367 2396-2409 2480-4033 /
ELEMEN
  13 SLAB / 1-1296 /
NODES
  14 SLAB_N / 1-4033 /
ELEMEN
  15 COLUMN / 630 631 666 667 1297-1356 /
NODES
  16 COLUMN_N / 1754 1775 1781 1784 1790 1990 1993 1997-2002 2100
                2103 2107-2112 4034-4483 /
'SUPPORTS'
 / 1 16 31 46 61 76 91 106 121 136 151 166 181 196 211 240 269 298
   327 356 385 414 443 472 501 530 559 588 617 1614 1629 1644 1659
   1688 1717 1944 1959 2054 2069 2164 2179 2194 2209 2238 2267 2494
   2509 2524 2539 2554 2569 2584 2599 2614 2629 2644 2659 2674 2689
   2704 2733 2762 2791 2820 2849 2878 2907 2936 2965 2994 3023 3052
   3081 4037 /   TR     1
 / 1-15 226-239 646-648 691-693 778 793 822 837 866-868 911-913
   998-1011 1208-1221 2689-2703 3096-3109 3149-3151 3233-3235 3249
   3277 3291 3319 3359-3361 3443-3445 3628-3641 4020-4033 /   RO     1
 / 2689-2703 3096-3109 3149-3151 3233-3235 3249 3277 3291 3319 3359-3361
   3443-3445 3628-3641 4020-4033 4037 /   TR     2
 / 1 16 31 46 61 76 91 106 121 136 151 166 181 196 211 240 269 298
   327 356 385 414 443 472 501 530 559 588 617 1011 1025 1039 1053
   1067 1081 1095 1109 1123 1137 1151 1165 1179 1193 1207 1235 1263
   1291 1319 1347 1375 1403 1431 1459 1487 1515 1543 1571 1599 1614
   1629 1644 1659 1688 1717 1831 1845 1859 1873 1901 1929 1944 1959
   2025 2039 2054 2069 2135 2149 2164 2179 2194 2209 2238 2267 2381
   2395 2409 2423 2451 2479 2494 2509 2524 2539 2554 2569 2584 2599
   2614 2629 2644 2659 2674 2689 2704 2733 2762 2791 2820 2849 2878
   2907 2936 2965 2994 3023 3052 3081 3459 3473 3487 3501 3515 3529
   3543 3557 3571 3585 3599 3613 3627 3641 3655 3683 3711 3739 3767
   3795 3823 3851 3879 3907 3935 3963 3991 4019 /   RO     2
 / 4037 /   TR     3
'LOADS'
CASE 1
ELEMEN
 / 1-196 /
       FACE
       FORCE     -0.100000E+00
       DIRECT      3
 / 197-238 /
       FACE
       FORCE     -0.100000E+00
       DIRECT      3
```

```
/ 239-252 /
      FACE
      FORCE     -0.100000E+00
      DIRECT      3
/ 253-266 /
      FACE
      FORCE     -0.100000E+00
      DIRECT      3
/ 267-308 /
      FACE
      FORCE     -0.100000E+00
      DIRECT      3
/ 309-504 /
      FACE
      FORCE     -0.100000E+00
      DIRECT      3
/ 613-624 626 /
      FACE
      FORCE     -0.100000E+00
      DIRECT      3
/ 627-629 /
      FACE
      FORCE     -0.100000E+00
      DIRECT      3
/ 630 /
      FACE
      FORCE     -0.100000E+00
      DIRECT      3
/ 631 /
      FACE
      FORCE     -0.100000E+00
      DIRECT      3
/ 632-634 /
      FACE
      FORCE     -0.100000E+00
      DIRECT      3
/ 635 637-648 /
      FACE
      FORCE     -0.100000E+00
      DIRECT      3
/ 649-660 662 /
      FACE
      FORCE     -0.100000E+00
      DIRECT      3
/ 663-665 /
      FACE
      FORCE     -0.100000E+00
      DIRECT      3
/ 666 /
      FACE
      FORCE     -0.100000E+00
      DIRECT      3
/ 667 /
      FACE
      FORCE     -0.100000E+00
      DIRECT      3
```

```
/ 668-670 /
      FACE
      FORCE     -0.100000E+00
      DIRECT      3
/ 671 673-684 /
      FACE
      FORCE     -0.100000E+00
      DIRECT      3
/ 685-726 /
      FACE
      FORCE     -0.100000E+00
      DIRECT      3
/ 727-735 /
      FACE
      FORCE     -0.100000E+00
      DIRECT      3
/ 736-738 /
      FACE
      FORCE     -0.100000E+00
      DIRECT      3
/ 739-741 /
      FACE
      FORCE     -0.100000E+00
      DIRECT      3
/ 742-750 /
      FACE
      FORCE     -0.100000E+00
      DIRECT      3
/ 751-792 /
      FACE
      FORCE     -0.100000E+00
      DIRECT      3
/ 793-988 /
      FACE
      FORCE     -0.100000E+00
      DIRECT      3
/ 989-1030 /
      FACE
      FORCE     -0.100000E+00
      DIRECT      3
/ 1031-1044 /
      FACE
      FORCE     -0.100000E+00
      DIRECT      3
/ 1045-1058 /
      FACE
      FORCE     -0.100000E+00
      DIRECT      3
/ 1059-1100 /
      FACE
      FORCE     -0.100000E+00
      DIRECT      3
/ 1101-1296 /
      FACE
      FORCE     -0.100000E+00
      DIRECT      3
```

```
 / 505-546 /
       FACE
       FORCE     -0.100000E+00
       DIRECT      3
 / 547-555 /
       FACE
       FORCE     -0.100000E+00
       DIRECT      3
 / 556-558 /
       FACE
       FORCE     -0.100000E+00
       DIRECT      3
 / 559-561 /
       FACE
       FORCE     -0.100000E+00
       DIRECT      3
 / 562-570 /
       FACE
       FORCE     -0.100000E+00
       DIRECT      3
 / 571-612 /
       FACE
       FORCE     -0.100000E+00
       DIRECT      3
'DIRECTIONS'
    1   1.000000E+00   0.000000E+00   0.000000E+00
    2   0.000000E+00   1.000000E+00   0.000000E+00
    3   0.000000E+00   0.000000E+00   1.000000E+00
'END'
```