



Norwegian University of  
Science and Technology

# Amplifier for optimal reflection Coefficient of ultrasound transducer

A study of op amp based circuits for ultrasound transducers,  
targeted for low reflection Coefficient, high gain, and low noise

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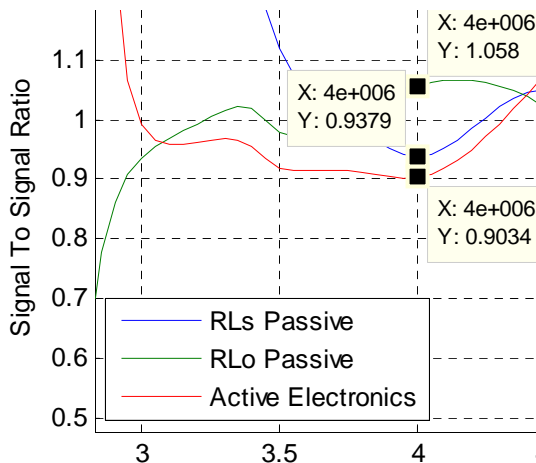
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## II. - ABSTRACT:

Reverberation is defined as equally-spaced, bright linear echoes resulting from reflection from specular-type interfaces. They are provoked by the acoustic Impedance change between the tissue and transducer front surface. B. Angelsen developed a mathematical approach to correct this ultrasound artifact by coupling the ultrasound transducer with an ideal electrical load in order to obtain zero reflection coefficients on the transducer from face [1]. However, when analyzing Impedance spectroscopy this approach cannot be achieved by using passive electronics in most simulated cases. Active impedance may be seen as a trend to improve image quality. This research therefore implements and simulates methods for active impedance synthesis [5] and applies Particle Swarm Optimization to the proposed electronics. Finally, the simulations are compared through a LAB experiment.

### Material, Methods and Results:



“ $Z_i$ ”, the electrical impedance of a specific transducer is simulated using X-Trans and Matlab software. Given  $Z_i$ , the ideal electrical Impedance of the matching network called  $Z_0$  is calculated to cancel the reverberation. The PSO algorithm is applied to the total input impedance of passive and active electronics to approach  $Z_0$  and minimize the reflection coefficients. The simulation is tested with a laboratory experiment where two transducers face each other and the pulse echo is

measured at transducer 1, amplitude is decreased when the electronics are added to transducer 2, which proves that the front face of the second transducer becomes more absorptive due to the connected load. The graph shows the signal-to-signal ratio, the division of the power energy spectrum of the measured signals when transducer two is connected to the electronics and a reference signal. This reference signal is the pulse echo when the same transducer 2 is not connected to a load. Because a reduction exists, the ratio is below to 1 on y-axis. X-axis is given in MHz and the simulated transducer has a central frequency of 4 MHz.

**Conclusions:** Reduction of the Ultrasound Transducer surface reflection coefficient is possible with active electronic. The general approach towards Impedance synthesis utilizes a filter design to affix the values of the Active Impedance.

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## VII.-LIST OF ABBREVIATIONS

1. SSR: Signal-to-Signal Ratio
2. TXD: Transducer
3. OPAM: Operational Amplifier
4. PSO: Particle Swarm Optimization
5. RX: Reception
6. TX: Transmission
7. Z0: Electrical Impedance for non-reverberation model.

## 8) REVIEW LITERATURE

### 8.1 ULTRASOUND IMAGING BASES

In this section the principles of the pulse echo image will be superficially treated, the study problem is introduced as an image artifact and formulas for simulation will be also written.

#### 8.1.1 WAVE PROPAGATION

When a propagating wave “ $V_{pe}$ ” experiences an acoustical impedance change, i.e. it faces a different medium, part of its energy will experience different physical phenomenon such as: transmission, reflection, diffraction and scattering. If  $V_{pe}$  is normal to the target plane, transmission and reflection will only appear provoking two new waves: “ $V_{pe\_t}$ ” and “ $V_{pe\_r}$ ” known as the transmitted and reflected waves respectively.

To illustrate, let’s consider that a transducer faces in water thank to a disk reflector, part of the energy will be absorbed by the target which is the transmitted wave, and part of the energy will be captured back because of the reflection. Those pulses, reflected and transmitted contain information of the sent pulse  $V_{pe}$  and obey formula 1.

$$V_{PE} = V_{PE\_T} + V_{PE\_R} \quad \text{Eq. 1}$$

If the acoustical impedance property of the reflector “ $Z_{Object}$ ” is well known, the reflection “R” and the transmission “T” coefficients can be found by formula 2. For the water tank, the reflection or transmission on the target surface is:

$$R = \frac{Z_{object} - Z_{water}}{Z_{object} + Z_{water}} \Leftrightarrow T = \frac{2 \cdot Z_{object}}{Z_{object} + Z_{water}} \quad \text{Eq. 2}$$

In the same way, it is also possible to know the coefficients and multiple pulses in a changeable medium, by using formula 2 for each one. Chapter 3 in [1] introduces some methods for computing these multiple reflections and their contributions.

## 8.1.2 IMAGE FORMATION AND DISTURBANCES

### 8.1.2.1 PULSE ECHO

To compute and simulate the transmitted pulse that is expulsed on the transducer front face,  $P_{tt}$ , equation 3 is used.

$$P_{tt} = H_{tt} \cdot Z_L \cdot H_{e\_tx} \cdot V_g \quad \text{Eq. 3}$$

This outcome pressure on the surface is product of the transfer function  $H_{e\_tx}$  of the electrical network coupled to the electrical terminals of the transducer, the transducer transfer function  $H_{tt}$ , the acoustical tissue or water impedance  $Z_L$  and the used excitation voltage signal  $V_g$ .

The pulse echo  $V_{pe}$  captured at the received electrical termination that in some cases is displayed using an oscilloscope follows equation 4.

$$V_{pe} = 2 \cdot H_{tt} \cdot H_{ac} \cdot H_{e\_rx} \cdot R_t \cdot P_{tt} \quad \text{Eq. 4}$$

$H_{e\_rx}$  is the transfer function of the electrical network used in reception, in some cases the same circuit is used for transmission and reception.  $R_t$  is the reflection coefficient caused by the reflector, and finally  $H_{ac}$  as called in [4] is the acoustical transfer function for the signal channel.

Because of the transducer surface reflection coefficients, in a practical experiment of the transducer radiating to a flat disk reflector in a normal direction in a water tank, there will be reverberation  $V_{rev1}$  that can be said as an image artifact. This reverberation can be computed with equation 5. However, if both the pulse and the reflection coefficients are big there will be more than only one reverberation echo  $V_{rev2}$ , as formula 6. More and more echoes will noise our image until being attenuated by the signal channel  $H_{ac}$ .

$$V_{rev1} = 2 \cdot R_{TXD} \cdot H_{ac} \cdot 2 \cdot R_t \cdot V_{pe} \quad \text{Eq. 5}$$

$$V_{rev2} = 2 \cdot R_{TXD} \cdot H_{ac} \cdot 2 \cdot R_t \cdot V_{rev1} \quad \text{Eq. 6}$$

By using formulas 4, 5 and 6, if the reflection on the surface  $R_{TXD}$  is desired to be found, the ratio between formula 5 and 4 or 5 and 6 is used and should be exactly the same.

$$R_{TXD} = \frac{V_{rev1}}{V_{pe}} \cdot \frac{1}{R_t \cdot H_{ac}} \quad \text{Eq. 7}$$

When the transducer is placed as close as possible to the flat disk reflector, the signal channel  $H_{ac}$  is neglected or computed as one.

### 8.1.2.2 REVERBERATION

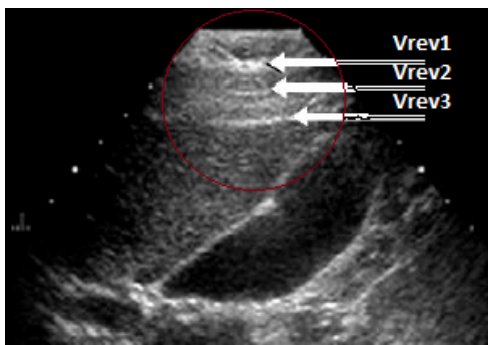


Figure 1: reverberation artifacts

Reverberation is defined as equally spaced, bright linear echoes resulting from reflection from specular-type interface [2] as figure 1. Due to the impedance change discussed earlier, the pulse echo or incoming wave will hit the transducer front surface, which has

different acoustical impedance to the tissue or water, provoking partial transmission of the wave and partial reflection due to the T and R coefficients.

This reflection is sent back to the target and again will hit the reflector object causing a second pulse echo with double the distance of the first one and with a much lower amplitude signal. This is called the first reverberation echo and again will cause other reverberation forward and backwards until they disappear because of the medium absorption. Even though there is not an object at double or three times the distance of the target, one experiences those false images with the reflection coefficients at the front face of the ultrasound transducer.

This can be easily demonstrated when doing the pulse echo measurement with a water tank experiment as shown in figure 2.

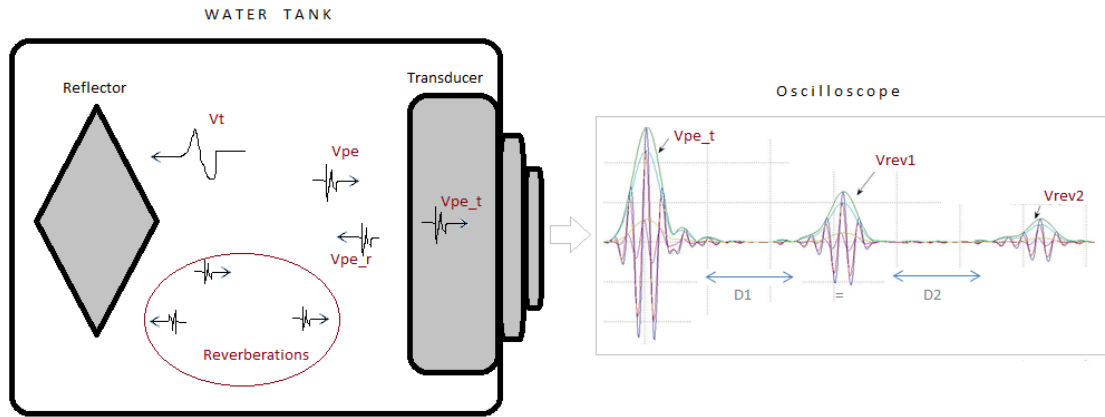


Figure 2: Water tank experiment and wave propagation

## 8.2 TRANSDUCER DESIGN, ELECTRICAL AND ACOUSTICAL PERSPECTIVES

A piezo-electric material vibrates when an excitation signal  $V_g$  flows over its electrical electrodes, since it is constructed by placing two plaques together, it behaves as a capacitor and therefore has a resonant frequency  $f_0$  at a characteristic impedance  $Z_0$ , this impedance is typically around 34MRayls [1]. To have a higher signal more power transmission is vital in both acoustical and electrical means. To exemplify this, the tissue has a characteristic impedance of 1.6MRayls and this high contrast will cause much more reflection than transmission, as mentioned earlier. The same happens in the electrical side; the transducer also has electrical impedance  $Z_i$  and either the source generator or the receiver amplifiers has their own resistance value that will cause power dissipation.

### 8.2.1 ACOUSTICAL LAYERS

There are three main types of acoustical layers in an ultrasound transducer: the matching layer, the piezo-electric material and the backing. Both the backing and matching layer are used to shorten the pulse when transmitting, because the length of the pulse is inversely proportional to the bandwidth. One or two matching layers are typically preferred to meet the maximal power transfer criterion, and in some cases backing material is only air. The length of the matching layer when only one is used is a quarter of the wavelength for maximal transfer energy and resonance frequency considerations as mentioned in [3].

**8.2.2 ELECTRICAL MODEL AND ELECTRICAL CONSIDERATIONS**

Before discussing about the relevance of the electrical network design for both Rx and Tx is important to discuss that when the piezo-electric material and the number as well as the type of matching layer and backing is known, the transducer can be modeled using lumped elements [1]. Lumped circuits help to understand of how this mechanical-acoustical compression and decompression will cause an electrical impedance  $Z_i$  dependent on frequency. This calculation will definitely influence the strategy for the electrical network to design not only for matching or transmitting more power as is discussed now, but also because this will influence the reflection on the front face of the transducer as it will be discussed later.

**8.2.2.1 THE TRANSDUCER ELECTRICAL CIRCUIT MODEL**

Figure 3 [3], can be used to compute the electrical impedance in the electrical termination of a given transducer. There,  $Z_m$  is the mechanical impedance and both a mechanical and an electrical description will be used to describe the transducer electro-mechanical behavior.

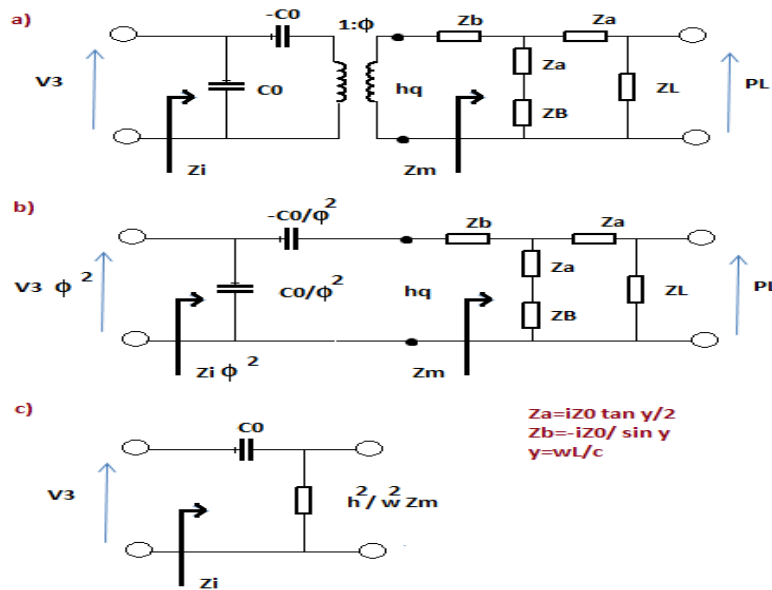


Figure 3: Electrical Model of the Transducer



Both circuit analysis in formulas 8 and 9 are derived. And the speed of sound is expressed with  $c$ .

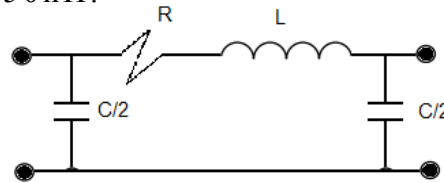
$$Z_m = \frac{Z_0(Z_L + Z_B) \cos \gamma + i(Z_0^2 + Z_L Z_B) \sin \gamma}{2Z_0 \cos \gamma + i(Z_L + Z_B) \sin \gamma - 2Z_0} \quad \text{Eq. 8}$$

$$Z_i = \frac{1}{i\omega C_0} \left( 1 + i \frac{h^2 C_0}{\omega Z_m} \right) \quad \text{Eq. 9}$$

### 8.2.2.2 MATCHING AND TUNING NETWORKS

Any passive or active circuit element placed on the electrical path to pulser, including the cable effect, in the transmission case or to the receiver amplifier will be considered as the electrical network. This is used mainly for tuning the capacitive effect of the transducer and for matching the power transfer criterion but later it is also related to the reflection on the front face of the transducer.

Typically the cable model is figure 4, however when the impedance of the transducer is below 100 ohms the capacitor can be shorted doing an RL effect. The cable resistance is 50 ohms when is normal coaxial and the inductor values in 250nH.



A suitable model of the cable

**Figure 4: Cable Electrical Model**

As mentioned in the pulse echo section, this network will strongly influence the pulse shape because it will basically filtered provoking a change in magnitude, phase and pulse length. If and only if the total thevening  $Z_T$  or equivalent impedance of the electrical matching-tuning network is known, then formula 10 and 11 computed how these filters behave in transmission tx and reception rx respectively.

$$H_{e\_rx} = \frac{Z_T \cdot Z_i}{Z_T + Z_i} \quad \text{Eq. 10}$$

$$H_{e_{-tx}} = \frac{Z_r}{Z_T + Z_i} \quad \text{Eq. 11}$$

### 8.2.2.3 ADMITTANCE MATRIX

It is a representation of the transducer as a two or more ports, the number depend on how many matching layers and piezo electric material are place. This is a multiport model that is used to simulated transducers parameters as those in the formula 12.

$$\begin{pmatrix} I \\ U \end{pmatrix} = \begin{pmatrix} y_m & H_u \\ H_u^T & y_e \end{pmatrix} \begin{pmatrix} V \\ 2p_i \end{pmatrix} \quad \text{Eq. 12}$$

Ym and Ye are the mechanical and electrical admittance which are inversely proportional to the mechanical and electrical impedance of the transducer.

### 8.2.2.4 FRONT FACE REFLECTION COEFFICIENTS

When using the admittance matrix, the reflection on the transducer surface is computed as:

$$R_{TXD} = Z_L \cdot U_{TXD} - 1 \quad \text{Eq. 13}$$

The particle velocity  $U_{TXD}$  will vary with two cases, causing changes in the reflection also:

Case 1: When the transducer is short circuit, this is the voltage V in the matrix is zero.

$$U_{TXD} = 2y_m \quad \text{Eq. 14}$$

Case 2: When the transducer is open circuit, this is the pressure P in the matrix is zero.

$$U_{TXD} = 2 \left( \frac{H_u^2}{y_e} + y_m \right) \quad \text{Eq. 15}$$

#### 8.2.2.4.1 RELEVANCE OF THE ELECTRICAL LOAD

Using Formula 13, the electrical Admittance load in reception  $Y_{Tx}$  will affect the particle velocity and thus will influence the transducer reflection coefficients at the front face.

$$U_{TXD} = 2 \left( \frac{H_{tt}^2}{y_e + y_{Tx}} + y_m \right) \quad \text{Eq. 16}$$

### 8.3 A MATHEMATICAL APPROACH FOR NULL FRONT FACE REFLECTION

From the maximal power consideration, if the transducer is normal to the target, a formulation can be derived for ideal impedance  $Z_0$  to be the total or the thevening impedance  $Z_T$  in the received network. However, this approach is not valid if the transducer front face has an angle which is not normal i.e. 90 degrees in single elements and if there are more than one matching layer. Formula 17 is found in [1].

$$Z_0 = \frac{-R_s}{R_o} Z_i \quad \text{Eq. 17}$$

$R_s$  and  $R_o$  are the reflection coefficients in short and open circuit.

## 8.4 FREQUENCY RESPONSE OF NETWORK FUNCTIONS

Frequency response is the steady state response of a system to a sinusoidal input [4]. This approach allows easy computations of how network function that can be a transfer function, impedance/ admittance functions or any other type, behave on the frequency domain. In some network functions, there are huge and well defined mathematical tools for prediction or controlling their response as with the transfer function i.e. the ratio between the outputs of the system and its input. But with impedance synthesis, there is still a lot of effort for better mathematical tools.

### 8.4.1 GRAPHICAL REPRESENTATION

Any network function frequency analysis is mostly done using bode and Nyquist graphs. Network functions are expressed by two polynomial expressions, one as numerator and other as denominator. The degree of the numerator varies for every network function. Formula 18 is the general network function form. When factorizing, the number of root that the polynomial expression contains is found and we call the roots at the numerator Zeros and those at the denominator we call Poles.

$$H(p) = \frac{\sum_{k=0}^N A_k p^k}{\sum_{k=0}^M B_k p^k} \quad \text{Eq. 18}$$

#### 8.4.1.1 NYQUIST DIAGRAM

The Nyquist diagram is basically the plot of the real part of a given function. An example of this is equation 18 versus the imaginary part of the same one. Usually the important information of this mathematical plot is to note in which side of the Nyquist plane a transfer function is located because it determinates many crucial factors such as stability and reliability. In Appendix A is a chart that contains some examples of basic network functions and its plot in Nyquist domain.

### 8.4.1.2 BODE DIAGRAM

Any transfer function has information, phase and magnitude as described in equations 19 and 20 respectively.

$$|H(p)| = \sqrt{[\text{Re}\{H(p)\}]^2 + [\text{Im}\{H(p)\}]^2} \quad \text{Eq. 19}$$

$$\angle H(p) = \text{tg}^{-1} \frac{\text{Im}\{H(p)\}}{\text{Re}\{H(p)\}} \quad \text{Eq. 20}$$

Thus formulas are the basis for bode plots which are two sub plots:

1. Magnitude in decibels vs. frequency
2. Phase in degrees vs. frequency

The main advantage of using bode plot is that multiplication of magnitudes can be converted into addition and the sketching follows a behavior such as the one in figure 5. For every single pole or zero their magnitude and phase are:

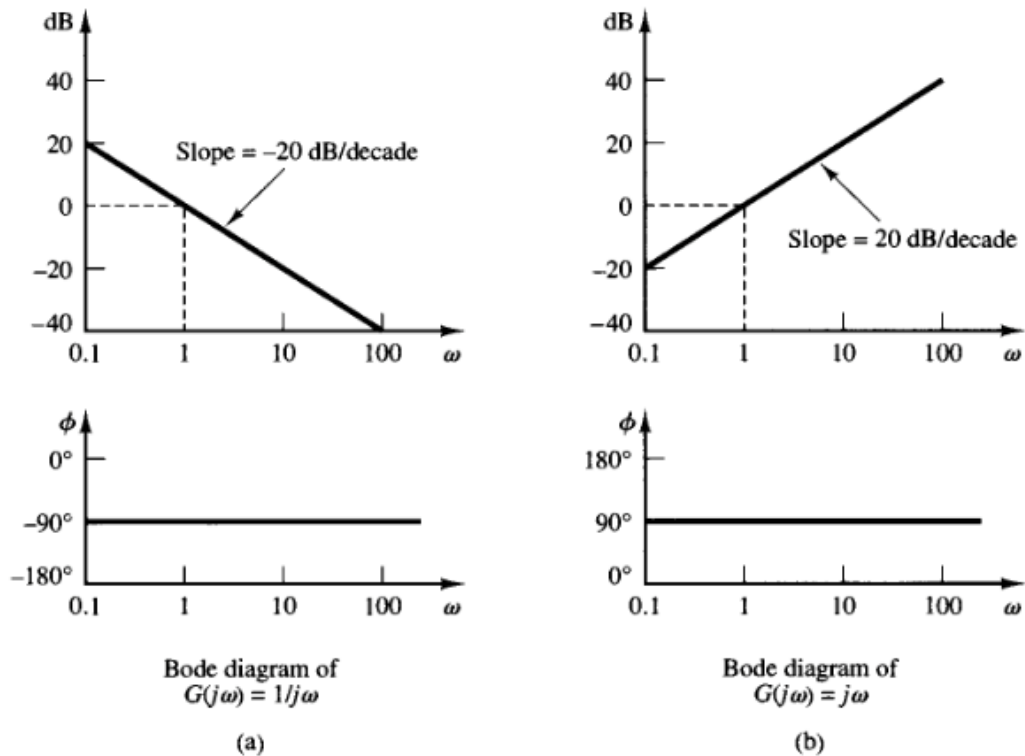


FIGURE 5: BODE PLOT FOR POLE AND ZEROS

### 8.4.2 TRANSFER FUNCTION OR IMPEDANCE FUNCTION

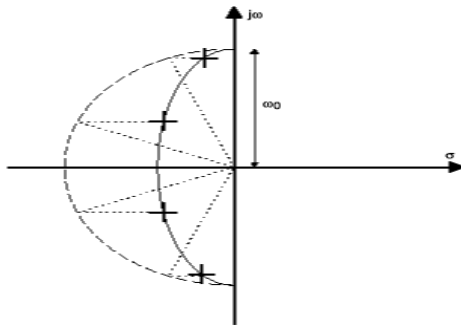


Figure 6: Pole and Zeros Plot

A simple and very essential property of any network function is that by plotting a Pole and Zero diagram, poles and zeros are symmetric with regard to the real axis of the complex plane because they are complex conjugated pairs or real. Other than that the network function cannot be realizable with passive elements.

In order to identify whether this is realizable impedance or admittance function we must take into account the following: if the maximum or minimum exponential of the denominator and numerator differ in more than one exponential degree, this will be a transfer function and not impedance or admittance function. It could be also said to be Active impedance [5].

Impedance or admittance functions meet all the criteria of the transfer function but they are divided in some special class or type to know if they can be achieved or not.

### 8.5 ACTIVE FILTER DESIGN

The main advantage of working with active filters is that in this way we avoid the necessity of having inductors. Inductors tend to be problematic elements not just because of their physical size, but also because they always have resistive notable effects and at high frequency they also have capacitive effect, because the spins may be to close. Using Operational Amplifiers or OPAMS the need for inductors is avoided.

The operational amplifier “OPAM” is a controlled voltage source and ideally they have some important properties:

- CMRR  $\rightarrow \infty$
- Input resistance  $\rightarrow \infty$

- Output resistance  $\rightarrow$  null
- Voltage Gain  $\rightarrow \infty$
- Bandwidth  $\rightarrow \infty$

### 8.5.1 FILTER TYPE

The filter type is a classification based on the frequency response of a given filter transfer function. There are 5 types: low pass LP, high pass HP, band pass BP, stop band SB, and notch filter. Using OPAMS structures LP and HP filter basically can form all the other types. By OPAMS structures we mean the configuration, for example differential, integral, sum, etc. see [6, 7]

The filter order mainly determines the slope of its cut off limits and how fast they decay or raise. For every OPAM used we could have a maximum order of 2, if we want to implement a filter order 5 or 6 we need 3 OPAMs.

### 8.5.2 RAUCH FILTER

The Rauch structure in figure 7, the transfer function is found as formula 21.

Depending on the impedance type of the filter, it can be a LP, HP or any other type.

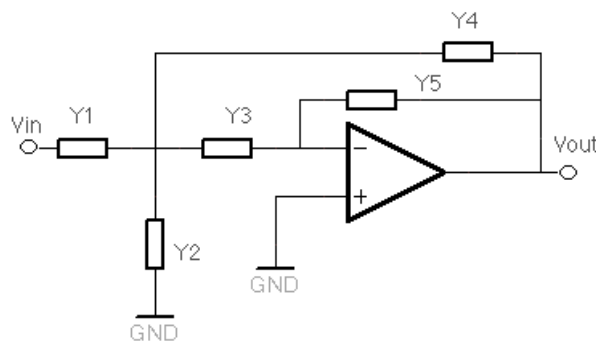


Figure 7: Rauch Filter

$$H_{rauch} = \frac{-Y_1 Y_3}{Y_5 (Y_1 Y_2 Y_3 Y_4) + Y_3 Y_4} \text{ Eq. 21}$$

To compute a LP or HP filter, formulas 22 and 23 are used. Q determines the damping in the limit or cut off slope, the biggest Q the highest the magnitude of the ripple in the slope. Figures 8 and 9 are the typical frequency response of high and low pass filter.

$$H_{LP}(\omega) = -k \frac{\omega_L^2}{S^2 + 2\alpha_L S + \omega_L^2}$$

$$Q_L = \frac{\omega_L}{2\alpha_L} = \frac{1}{2\xi_L}$$

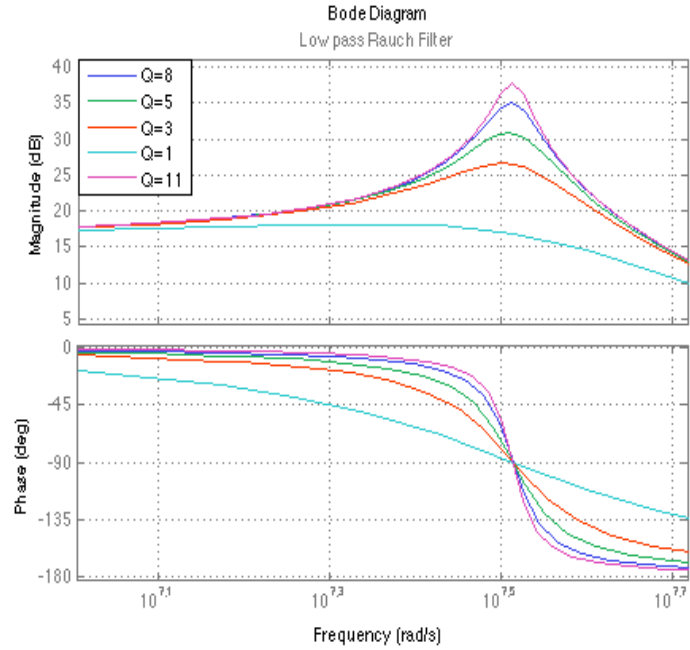


Figure 8: Low Pass Filter Frequency Response

$$H_{HP}(\omega) = -k \frac{S^2}{S^2 + 2\alpha_H S + \omega_H^2}$$

$$Q_H = \frac{\omega_H}{2\alpha_H} = \frac{1}{2\xi_H}$$

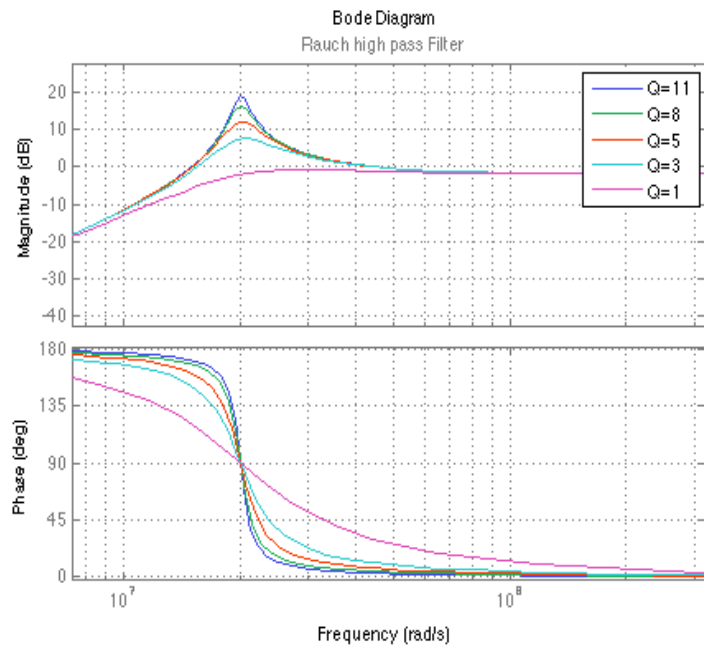


Figure 9 : Low Pass Filter Frequency Response



## **8.6 IMPEDANCE SYNTHESIS**

Cauer, Foster, Brune, Bott-Duffin, Darlington developed powerful and general approaches for Impedance or admittance synthesis of networks, and they are still used to classify Impedance reliability condition for simple networks. But interest in the topic lost momentum due to the growing importance of integrated circuits [5,8]. Active impedance then has also taken advantage of the chip size reduction and the advantage for substituting the inductors.

### **8.6.1 IMPEDANCE TYPE**

The Impedance methods for synthesis are strongly dependent of the classification of the impedance type. There are four types: resistive-capacitive RC, resistive-Inductive RL, Inductive-capacitive LC and the combination of all of them RCL.

RC impedance will have resistive and capacitive behavior so that the numerator will never exceed the denominator in terms of exponential degree or in the case of having the same degree there must be a zero with null root and the smallest bode frequency will correspond to a pole. In controversy, RL impedance will follow an inductive behavior with higher degree in the numerator and the smallest frequency will be a zero in bode plot.

LC impedance does not have a resistive behavior. Therefore, the maximum and minimum degree terms in numerator and denominator will be obligatorily different in no more than one, and the transitory behavior is constantly present in the frequency axis because there is no resistive dissipation. On the other hand, LRC will have some large non-transitory response because of the resistor, and numerator degree terms can be the same as the denominator.

### 8.6.2 PASSIVE IMPEDANCE

For RC, RL and LC networks synthesis foster approach and Cauers method are quite friendly, but for RLC circuit, there is still a lot to investigate.

“To identify whatever impedance is passive; the maximum or minimum exponential of the denominator and numerator differ in more than one; other than that it is Active impedance”.

#### 8.6.2.1 FOSTER AND CAUERIER APPROACHES

In this section, the synthesis of RL circuits will only be treated. RC or RL, however,, are treated in the same way by replacing the inductor with capacitors, or by replacing the resistor with capacitors to and from an LC circuit.

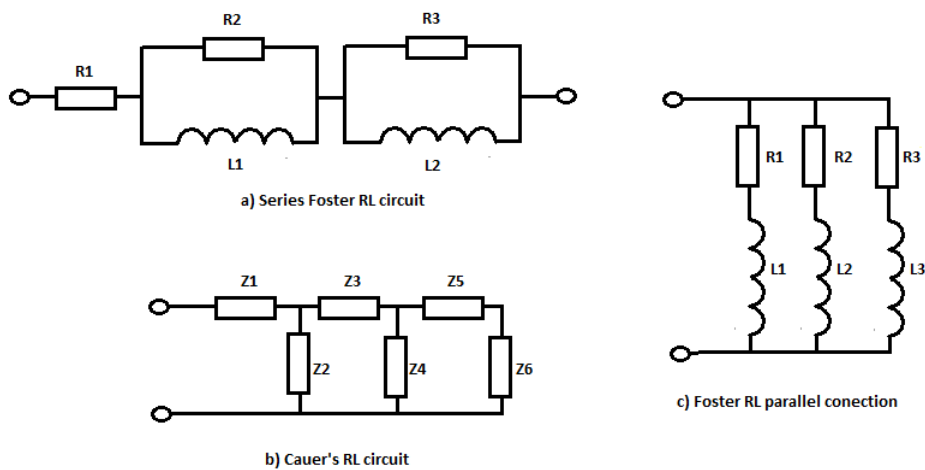


Figure 10: Passive Foster and Cauer Circuits

Given the series RL foster’s circuit of figure 10a, the method consists of the use of partial fraction expansion or decomposition. Any function can be expressed as formula 24

$$Z(p) = \sum_k \frac{A_k}{p + a_k} \tag{Eq. 24}$$

The more poles and zeros, the more RL series stage are added to figure 10a, the component values for resistor and coils are:

$$R_k = A_k \Leftrightarrow L_k = \frac{A_k}{a_k} \tag{Eq. 25}$$

To give an example let's consider the transfer function in formula 26 and by using the fraction, expansion 27 is found.

$$Z(p) = \frac{p^2 + 31p + 30}{p^2 + 110p + 1000} = \frac{(p+1)(p+30)}{(p+10)(p+100)} \tag{Eq. 26}$$

$$Z(p) = \frac{A_0}{p} + \frac{A_1}{p+10} + \frac{A_2}{p+100} \tag{Eq. 27}$$

Where  $A_0=0.03$   $A_1=0.2$   $A_2=0.77$ .

To convert this foster RL circuit into a parallel as in figure 10c, eq.24 is converted as in 28 for admittance. This is a convenient circuit if we need the inductor to be grounded. For example, when gyrators are needed to simulate coils.

$$Y(p) = \sum_k \frac{B_k}{p + b_k} \tag{Eq. 28}$$

$$R_k = \frac{b_k}{B_k} \Leftrightarrow L_k = \frac{1}{b_k} \tag{Eq. 29}$$

The Cauer's method consists in the admittance or impedance decomposition as eq. 30 and by using Euclides algorithm; we calculate the components of figure 10b.

$$Z(p) = \frac{1}{Z_1 + \frac{1}{Z_2 + \frac{1}{Z_3 + \frac{1}{Z_4 + \frac{1}{Z_5}}}}} \tag{Eq. 30}$$

### 8.6.2.2 RLC METHOD

A recompilation of theory for converting a broadband impedance matching problem into a purely filter design hitch is found in [9]. From it, Darlington’s theorem applied by Fano, “Any realizable impedance function is discomposed into a purely reactive lossless network terminated into a  $1\Omega$  resistor by including a transformer into the reactive lossless network”. This states that the load to match in a typical matching problem as in figure 11.A, can be converted as in 11.B. and since the source resistance can be also replaced by a  $1\Omega$  resistor, the diagram changed to figure 11.C taking the problem of the matching network into a filter design.

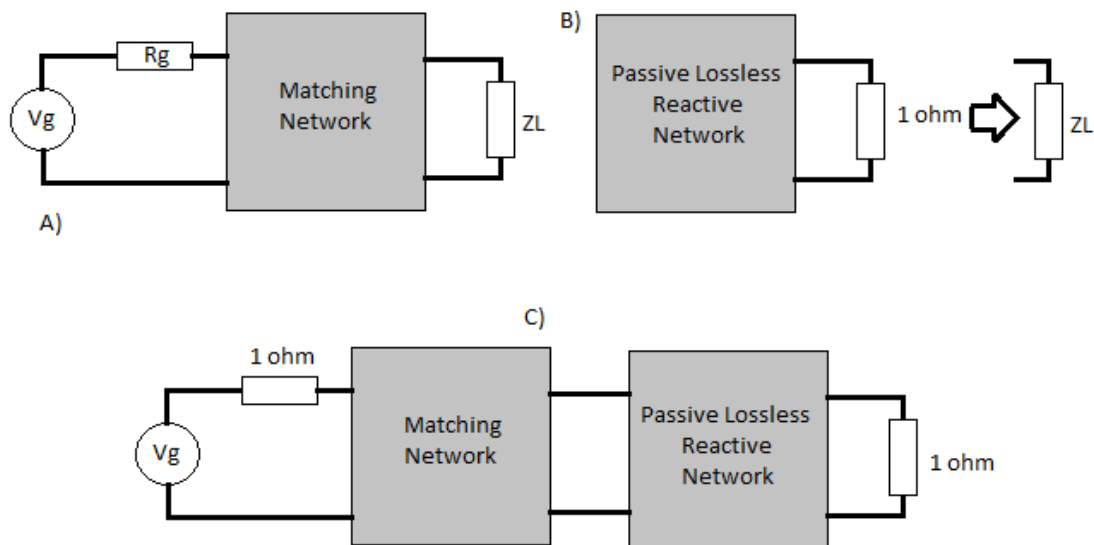


Figure 11: Filter Design towards Impedance Matching Principle

This was at a certain point a partial filter design consideration for the matching network block, but the passive lossless network is given by the Darlington’s theorem. This approach is quite versatile after optimization. But it still requires better approximation when more degrees are presented in the polynomial expression of the impedance to obtain. It is also important to highlight the need of a transformer. Further in the mentioned literature source, some people implement RLC

lumped circuits using equalizers, instead of only using Reactive elements.

To back it up [8] introduce some LRC networks which are transformer less to realize non regular but positive and real biquadratic functions by canonical means as equation 31.

$$Z_n = \frac{S^2 + 2U\sqrt{W}S + W}{S^2 + 2\frac{V}{\sqrt{W}}S + \frac{1}{W}} \quad \text{Eq. 31}$$

$W, V, U > 0$

This is based on the idea that any positive real function could be realized as the driving point impedance of a network consisting of resistors, capacitors and inductors only (referring the work done by Bott and Duffin). The concept of regular positive real function is a class transformer less network containing only reactive elements.

One of the circuits presented in the mentioned article will be used here, because it tends to be more inductive than capacitive, which is contrary to the electrical impedance  $Z_i$  of the transducer as studied before. Figure 12, shows the circuit and equation 32 gives it frequency response.

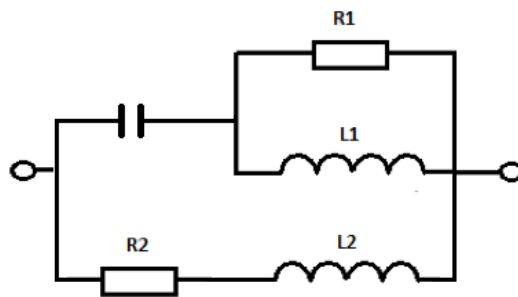


Figure 12: RLC approach

$$Z_n = \frac{(SL_2 + R_2)(S^2L_1R_1C_1 + SL_1 + R_1)}{S^3L_2C_1L_2 + ((R_1 + R_2)L_1 + L_2R_1)C_1S^2 + (L_1 + R_2C_1R_1)S + R_1} \quad \text{Eq. 32}$$

### 8.6.3 ACTIVE IMPEDANCE

Active impedance has polynomial functions that have roots in the right side of the Nyquist plot and they are only obtained by using controlled voltage sources that may be seen as power injection to realize complex functions. The operational amplifier “OPAM” is a controlled voltage source example and, therefore, different approaches to active impedance synthesis are based on it like gyrators and the general formulation of negative impedance as will be seen soon.

#### 8.6.3.1 GYRATORS

An active circuit called gyrator is an OPAM based model for emulating inductive circuits by only using capacitors and resistors; however, much consideration should be taken when designing a gyrator. For example, the OPAM integrated circuits are not ideal in practice. Therefore, they are bandwidth limited, one must carefully choose the integrated circuit according to the specific task. The Bruton gyrator [5] is widely used for emulation of inductors and other non-conventional impedance, because the sub-impedances placed in its electrical diagram interact as filter design form when computing its total impedance given in equation 33.

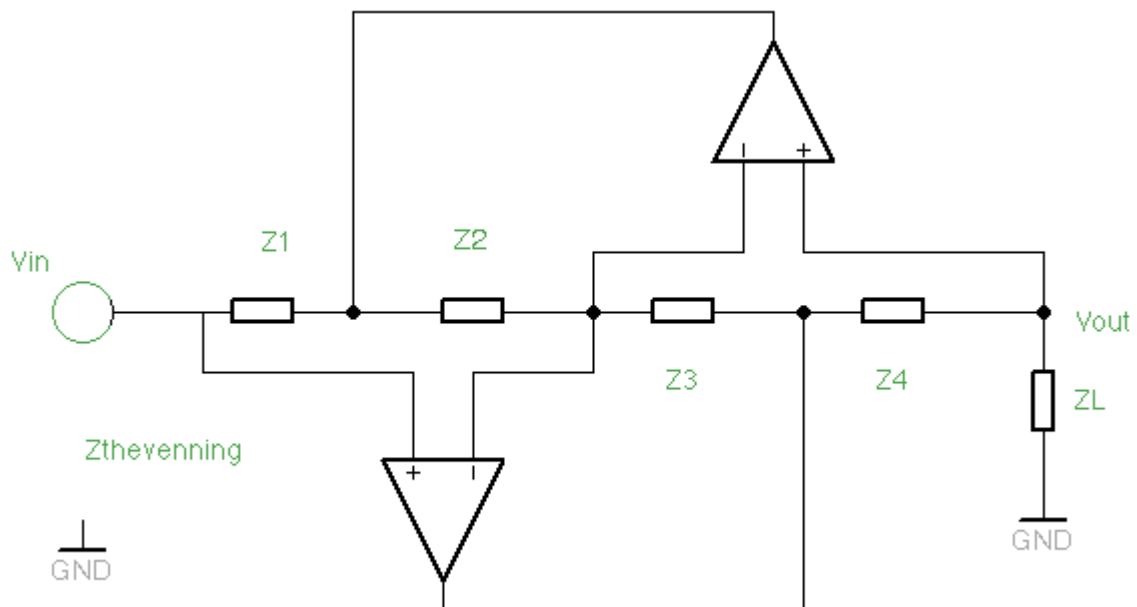


Figure 13: Bruton Gyrator

$$Z_T = \frac{Z_1 Z_3}{Z_4 Z_2} R_L \quad \text{Eq. 33}$$

### 8.6.3.2 ACTIVE IMPEDANCE GENERAL FORM

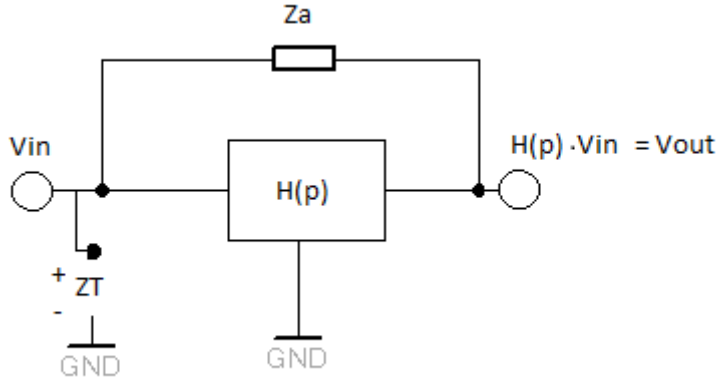


Figure 14: Active Impedance Principle

If and only if  $H(p)$  is a controlled voltage source, the input current of figure 14 is:

$$I_{in}(\omega) = \frac{V_{in}(\omega) - V_{in}(\omega)H(p)}{Z_a(\omega)} \quad \text{Eq. 34}$$

Because the input impedance  $Z_{in}$  is the ratio between the input voltage  $V_{in}$  and the input current in:

$$Z_{in}(\omega) = \frac{Z_a(\omega)}{1 - H(p)} \quad \text{Eq. 35}$$

It also allows us to convert the impedance matching design problem into a purely a filter design if  $Z_a(\omega)$  is known:

$$H(p) = \frac{Z_{in}(\omega) - Z_a(\omega)}{Z_{in}(\omega)} \quad \text{Eq. 36}$$

This implies that a filter with a frequency response  $H_p(\omega)$  will emulate an impedance  $Z_{in}(\omega)$  when connected in parallel to an impedance  $Z(a)$  which can be chosen strategically for convenience.

## 8.7 PARTICLE SWARM OPTIMIZATION

In this section Particle Swarm Optimization algorithm PSO is introduced for adjusting the values of electrical circuit parameters within impedance spectroscopy. Further in this Master thesis, a computer based transducer simulation to achieve an ideal impedance, in order to have non-reflection coefficients in the transducer surface, is proposed for that specific model. Since there is not a mathematical formulation of the model because the given ideal impedance comes from the admittance matrix previously introduced, some electrical circuits will intend to fit the given response by Optimization means.

### 8.7.1 ADVANTAGE AND USE JUSTIFICATION

The Optimization task for our study is a non-linear system problem for fitting curves, because given a model we force to our circuit to behave as the model. The Levenberg-Marquardt Algorithm is one of the most important optimization codes for fitting models which localize the best local minimum of a multivariable function; however, when the range of individual parameters is large, this algorithm loses steadfastness because its best solution may be the best local solution, but not the best global solution. Particle Swarm Optimization on the other hand is a genetic evaluative algorithm that finds the best optimal global solution because it uses mutation and recombination of the different parameter values. PSO is much less computer time consuming [10] and does not need the user to restring the space range at all as long as the number of iterations is high enough.

### 8.7.2 PSO ALGORITHM

The PSO algorithm and a matlab example can be found in [11]:

$$v_{id}^{n+1} = v_{id}^n + cr_1^n (p_{id}^n - x_{id}^n) + cr_2^n (p_{gd}^n - x_{id}^n) \quad \text{Eq. 37}$$

$$x_{id}^{n+1} = v_{id}^{n+1} + x_{id}^n \quad \text{Eq. 38}$$



Suppose that the search space is denoted by  $D$ , and a given particle  $i$ -th of the swarm is denoted as  $X_i=(x_{i1},x_{i2},\dots,x_{iD})^T$ . The velocity or change rate is  $V_i=(v_{i1},v_{i2},\dots,v_{iD})^T$  for the  $i$ -th particle. Since it works by mutation and combination, the best previous position or solution is given by  $P_i=(p_{i1},p_{i2},\dots,p_{iD})^T$ .  $D$  and  $d$  are integer and real numbers 1,2...until  $N$  which is the size of the Swarm.  $C$  is a positive constant associated with the acceleration and  $r_1$  and  $r_2$  are random numbers between 0 and 1, finally  $n$  determinates the interaction number.

To improve the optimization of the best solution in the whole space and also to find the best local minimum the necessity of 3 new parameter in the above algorithm more intelligence was essential and thus the final algorithm is:

$$v_{id}^{n+1} = \chi(wv_{id}^n + c_1r_1^n(p_{id}^n - x_{id}^n) + c_2r_2^n(p_{gd}^n - x_{id}^n)) \quad \text{Eq. 39}$$

$$x_{id}^{n+1} = v_{id}^{n+1} + x_{id}^n \quad \text{Eq. 40}$$

Where  $w$  is called inertia weight,  $C_1$  and  $C_2$  cognitive and social positive parameters respectively and  $\chi$  is a contrition factor. [11] Recommends  $C_1=C_2=0.5$  while  $r_1$  and  $r_2 = (0,1)$

## 9) AIM, GOAL AND HYPOTHESIS

### 9.1 THE AIM:

The objective of this master thesis is the reduction of the ultrasound transducer front face reflection coefficients by electrical and/or electronics interaction prioritizing the pulse echo considerations mentioned in the review literature section.

### 9.2 THE GOAL:

The goal is to use active electronics for emulating electrical impedance and reduce the reflection coefficients. More specifically, the goal is to introduce some OPAMS circuits for active impedance synthesis and demonstrate their effect on simulation and Lab work experiments. In this way, physical dimension problems from inductors are also avoided.

### 9.3 HYPOTHESIS:

1. - A Computer based transducer simulation will behave closer to one of the lab manufactured transducers at the university, in this way, being good enough for experimenting and achieving the goal and aim.
2. – The computed based electrical impedance for having null reverberation on the front face of the transducer for the specific transducer simulated in hypothesis 1, is possible to be achieved only by active elements within a limited bandwidth close to the central frequency of the transducer, without destroying the pulse echo.
3. - The Impedance synthesis problem may be forced to be a purely filter synthesis when connecting a specific R or RC impedance parallel to the filter.
4. – The aim and goal are expected to be achieved by active elements using no more than one OPAMS and a couple of elements for noise exclusion.
5. - It is possible to realize lab work experiments by using two transducers set up for power protection consideration and see the amount of reflection reduction .

## 10) METHODS

### 10.1 STATISTICAL SOFTWARE FOR ANALYSIS AND SIMULATIONS:

1. - MATLAB® is a high-level language and interactive environment that enables us to perform computationally tasks. Under NTNU license for students, this thesis work bases all simulation on MATLAB platform.
2. – X-TRANS® is a MATLAB toolbox designed at our home university NTNU for simulating 1D Transducers.
3. – MAPLE® is a computer algebra system used here for factorization and equation synthesis.
4. – PROBELAB is a MATLAB toolbox design for data acquisition of different equipment in our LAB.

### 10.2 MATERIAL. EQUIPMENT AND SET UP FOR LAB EXPERIMENTS

1. – Impedance Analyzer
2. – Digital Oscilloscope
3. – Signal Generator
4. - two ultrasound transducers similar to the computed model
5. – a PC with the required software for data analysis

The set Up for experiments

The transducer 1 is faced as close as possible to a second transducer and using water as the propagation medium.

The pulse echoes measurement is done using an oscilloscope and PC connected for obtaining data in matlab files. Note: Using a signal generator, a sine pulse with a central frequency as the transducer central frequency is transmitted.

Once the measurements are recorded and stored, we repeat the first step but connecting impedance load (passive and Active) to the second transducer instead, then measure the pulse echo experiment as step 2 for these cases and storage data. Once the different data is saved, it may need to be filtered using matlab post-processing code or the oscilloscope's filter tools. To see the effect of the electrical circuit for reducing the reverberation, we process the data as next:

Based on the Signal to Signal ratio, we take as a reference signal the pulse echo in transducer 1, given transducer two as reflector without any load in transducer two.

We capture again the pulse echo in transducer 1, but this time for each connected load to the second transducer.

We calculate the energy power spectrum of all the measured signals and finally divide the spectrum of each signal by the spectrum of the reference signal.

We do a Plot of the amount of reduction given by each load to prove that the electrical load increases or decreases the reverberation.

Here some pictures and sketch of the set up:

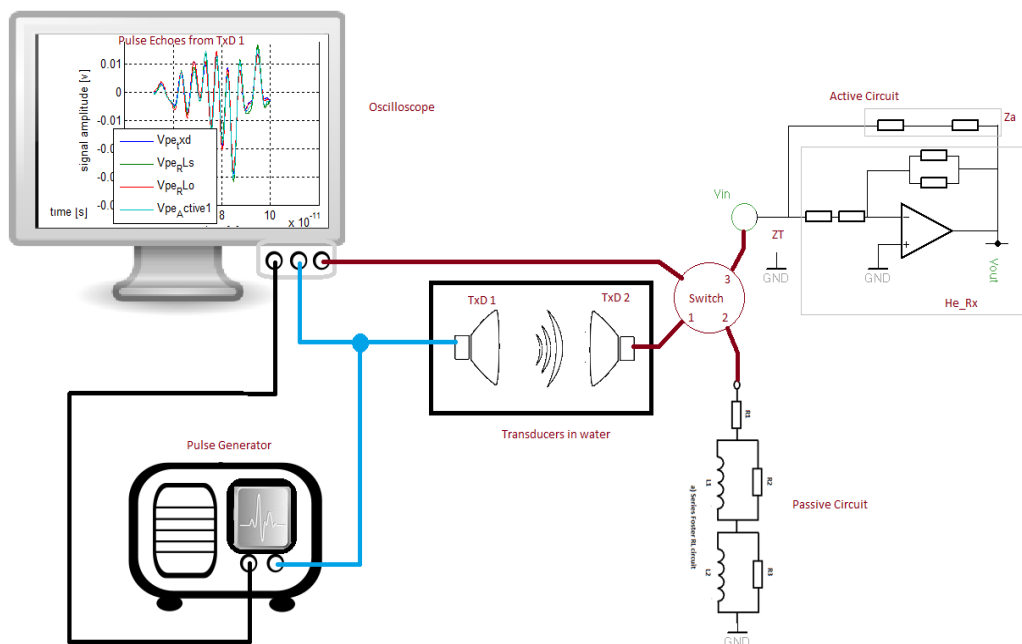
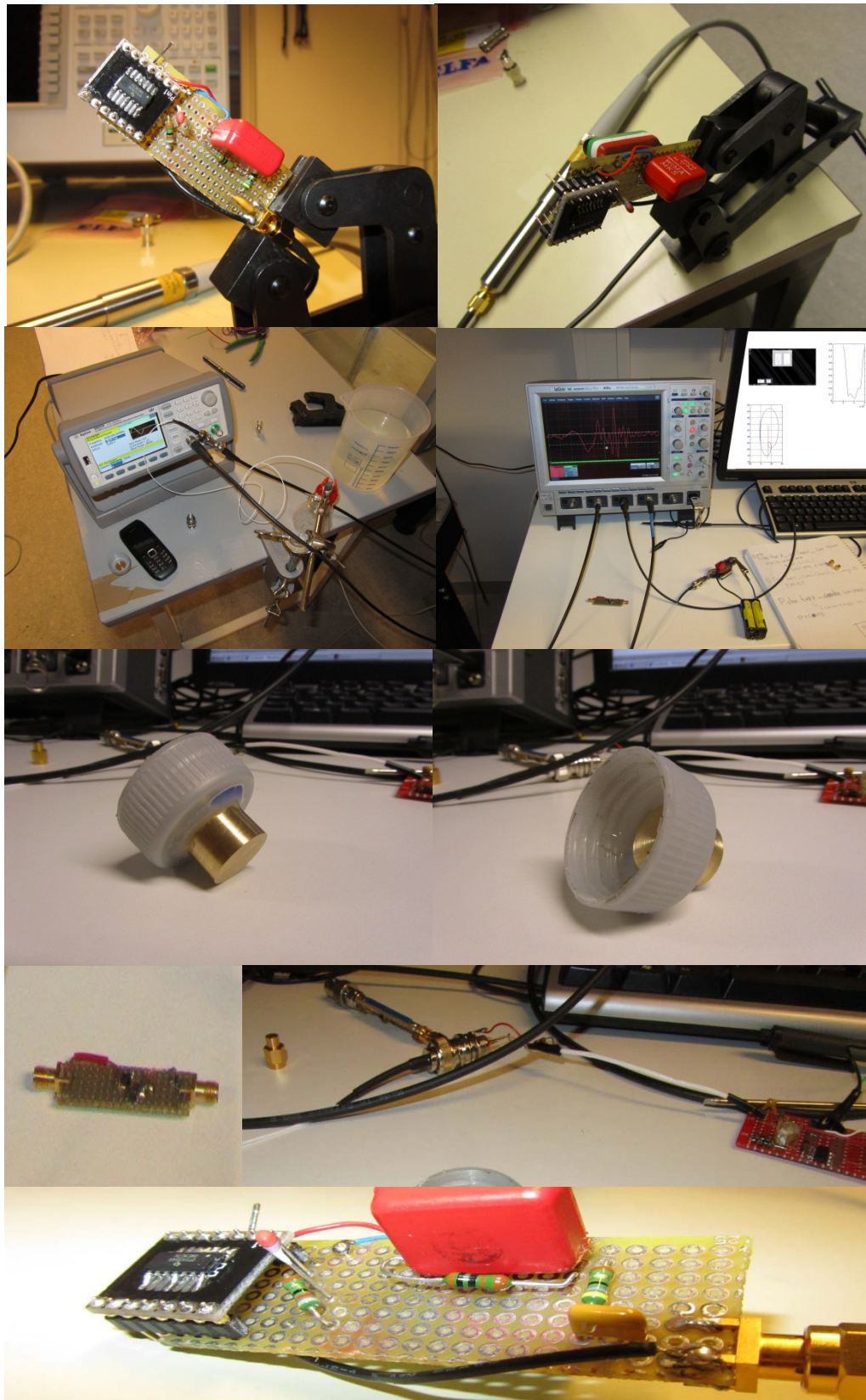


Figure 15: Lab Experiment Setup



### 10.3 PROCEDURES

#### 10.3.1 TRANSDUCER MODEL

As mentioned previously, the simulations in this work are held by using X-Trans software that contains the admittance matrix model of the review literature section, the corresponding parameters were as figure 16.

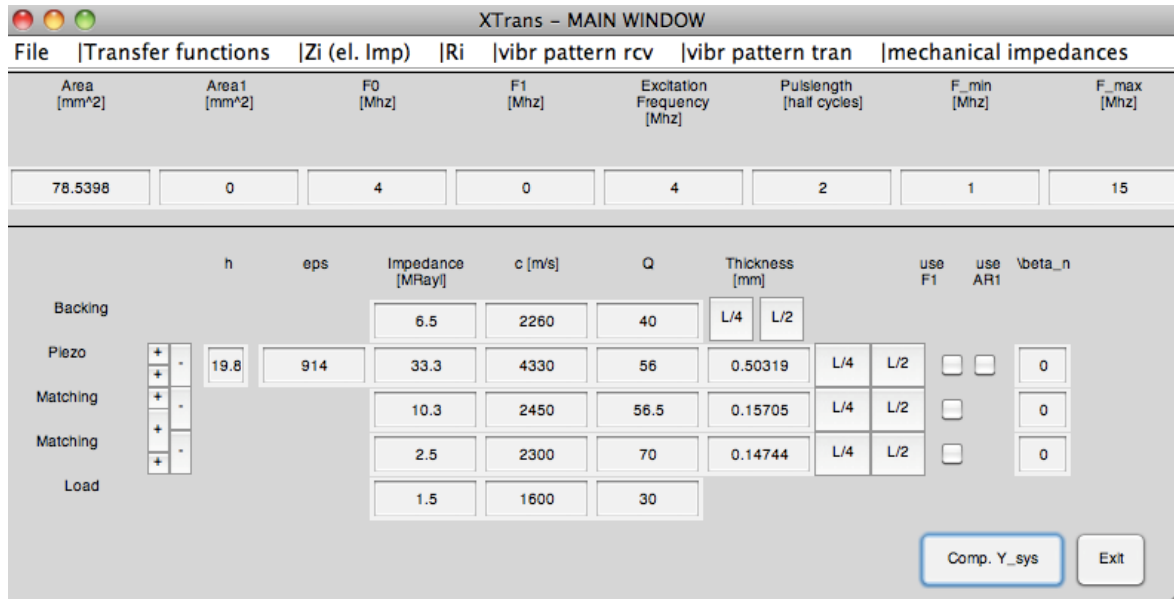


Figure 16: Xtrans Set Up Window

#### 10.3.1.1 SPECTROSCOPY OF IDEAL ELECTRICAL MATCHING IMPEDANCE “Z0” FOR THE GIVEN MODEL

By applying the formulation of sections 8.2.1.3 and 8.3, it is possible to compute  $Z_0$ ; it should be the total impedance from the receiver internal resistance, including cabling and any transmission line until the electrodes of the transducer. In this research the impedance synthesis is given only within bandwidth close to the transducer central frequency  $k=2-6$  MHz, because realization outside of  $k$  range would be useless, due to the band limit transducer modelled, and usually one may only send pulses around the central frequency.

*The Analysis:* For frequency response analysis, figure 17 and 18 are Bode and Nyquist plot respectively.  $Z_i$  in the plot is the electrical impedance of the model transducer while  $Z_0$  is the matching impedance



that must be placed at the electric termination of the transducer for having non reverberation artifacts for that particular model.

From the bode plot, a band pass response is shown in the magnitude side, while the phases plot indicates the inductive response that this impedance needs to have. An odd phase shift near the central frequency would be a challenge because it shifts up and down quickly and in a short frequency range. The Nyquist plot clearly shows the instability criterion if this were a controlled system. It also shows that it would never be achieved with only passive elements due to location on the right side and negative resistance values that may be interpreted as a sort of power injection. Active elements may need to be used at some specific frequency values; however, the central frequency can be achieved by only passive elements.

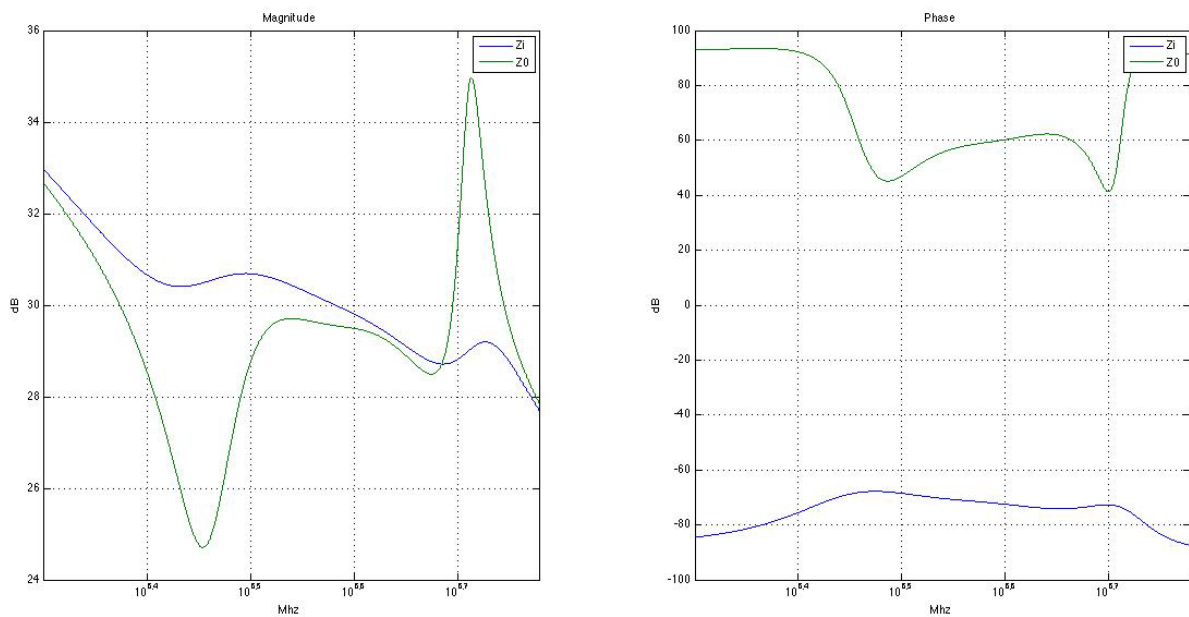


Figure 17: Bode Plot Analysis for the impedance model to achieve, Z0.

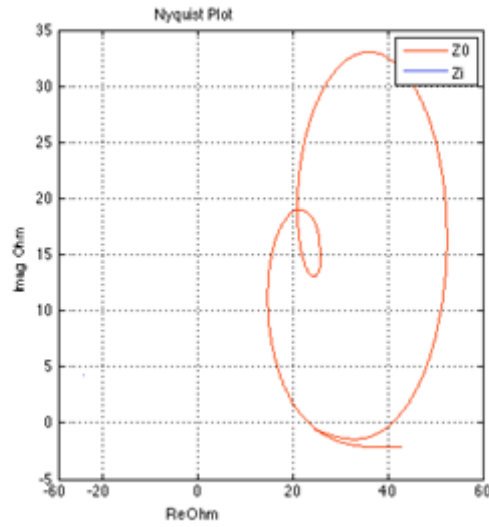


Figure 18: Nyquist Plot for Z0

To have an idea of a mathematical expression that defines Z0 performance, we approach a model by filter means and see how close with cold we get to define an equation for it. Using a low and high pass filter with a network function as formula A.1, the plot 19 shows how it approximates to Z0 in the magnitude, but not in phase. For filter's design experts, the use of a phase shifter is evident after using any filter design type.

$$H_{Bp}(\omega) = \frac{5.6S^2}{9.36^{-16}S^4 + 1^{-8}S^3 + 1.4S^2 + 8.249^6S + 4^{14}} \quad \text{FORMULA A.1}$$

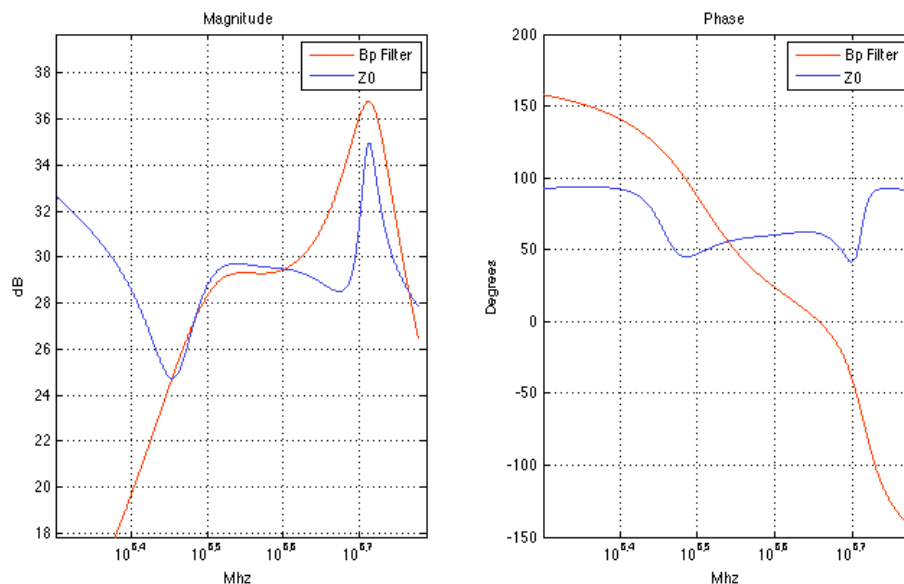


Figure 19: fitting Z0 with Rauch filters

When a down-shifter equation is added to the band-pass filter, the new network function becomes eq. A.2.



$$H_{Bp}(\omega) = \frac{5.6S^3 + 1.05^{-8}S^2}{9.36^{-16}S^5 + 7.55^{-9}S^4 + 1.214S^3 + 1.8249^7S^2 + 2.4^{14}S - 7.62^{21}} \quad \text{FORMULA A.2}$$

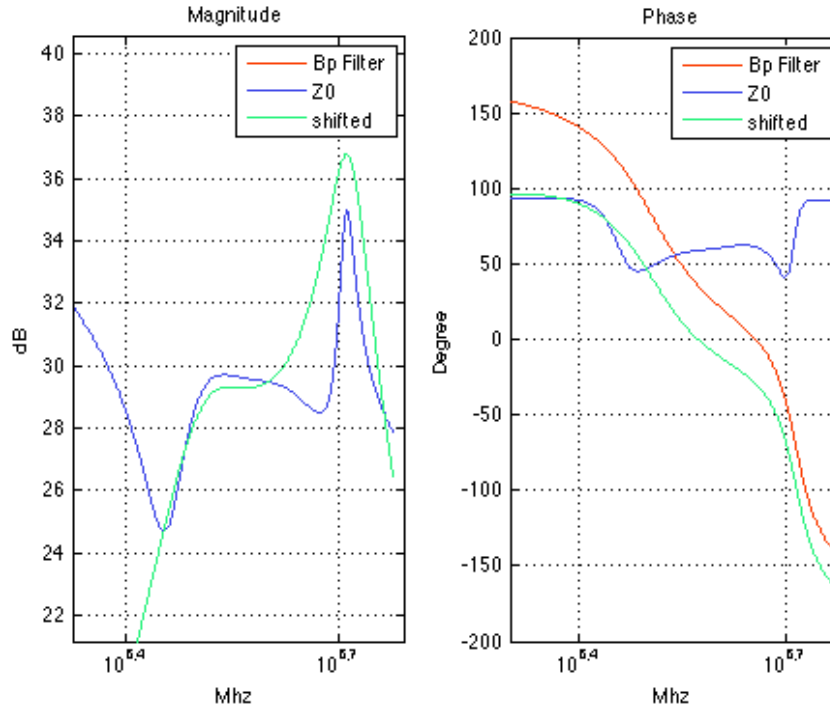
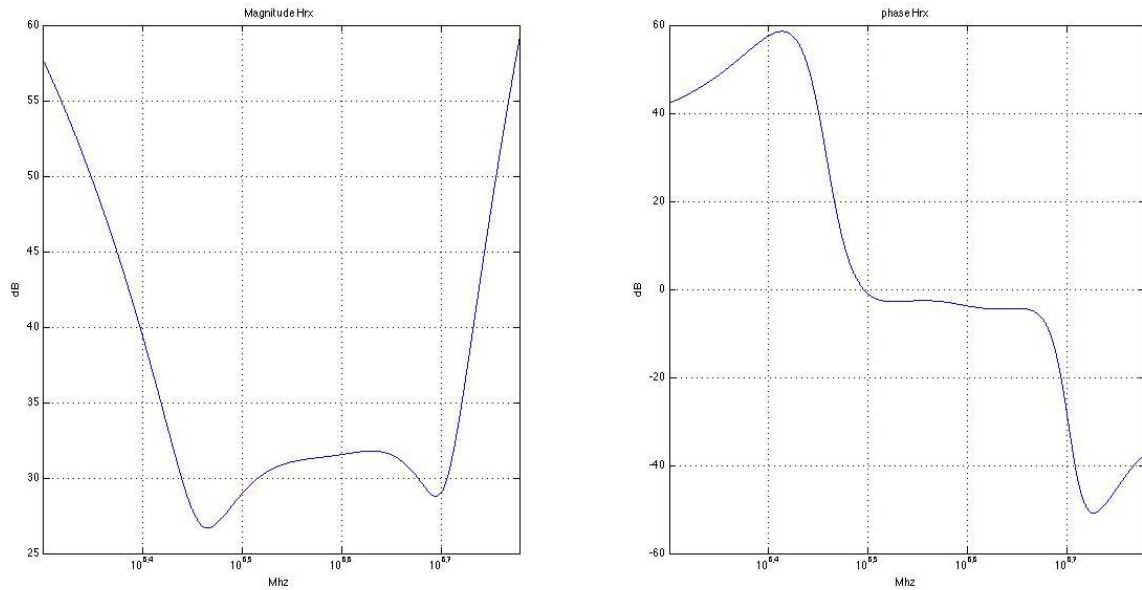


Figure 20: The effect of shifting the network function

The need for an intelligent algorithm to sweep the poles and zeros along the frequency axis that fits Z0 better is clear, but the aim here is to show that Z0 is a polynomial expression with high degree in both numerator and denominator, and to achieve this expression with passive element would require many semiconductors.

### 10.3.1.2 SPECTROSCOPY OF THE ELECTRICAL NETWORK UNDER PULSE ECHO RESTRICTIONS

Because the pulse echo formation introduced in section 8.1.2.1 depends on the electrical matching network and its transfer function He in both Tx and Rx modes, and the reflection only depends on the He in Rx mode; If the Thevenning total electrical impedance of the receiving network is as planted in section 8.2.12, then its electrical transfer function is the filter response plot in figure 21.



**Figure 21: Bode Plot of  $H_{e_{rx}}$**

The filter then does not attenuate the pulse echo; it neither has ripple in the band frequencies nor outside that increases the pulse length. If this impedance is achieved as the model  $Z_0$ , the pulse echo would be not destroyed and on the contrary, would be benefited because it would be amplified.

### ***10.3.2 PROPOSED PASSIVE CIRCUIT FOR ACHIEVING “ $Z_0$ ”***

It is relevant to mention two important factors that influenced the decision on the chosen circuit that will be introduced:

- 1) It is not possible to achieve with only passive elements an impedance that fits the curve model  $Z_0$  perfectly for this proposed transducer case based on previous spectroscopy analysis, but a good approximation is expected with passive circuits within the central frequency of the transducer and its surrounding frequencies for minimizing the reflection.
- 2) Typically, a medical ultrasound transducer consists of around 150 and 200 elements that each need to be matched with an electrical network, and thus, we cannot accept passive circuit that requires more than two inductors due to the large space that they would demand.

### 10.3.2.1 THE FOSTER RL MODEL:

The circuit to compute will be the same as figure 10a; PSO algorithm will be applied but to improve computing time and initialization of the algorithm, Formula 42 is derived.

$$Z_{RL} = R_1 + \frac{SR_2L_1}{SL_1 + R_2} + \frac{SR_3L_2}{SL_2 + R_3} = \frac{S^2 + 2\alpha S + \omega_z^2}{(S + \omega_1)(S + \omega_2)} \quad \text{Formula 42}$$

To characterize this circuit and approach to Z0, the two poles roots “w<sub>1</sub>” and “w<sub>2</sub>” of formula 42 are set up to be at 3 and 6 MHz. Since the zeros roots depend of the poles and R1 value the characterization strategy is:

**EXPERIMENT 1:** *From Formula 42, W1 and W2 are set to interact at the frequencies 3 and 6 MHz respectively, the resistor R2 and R3 to be the same value of 10 Ohms. We considered six cases to plot:*

*Case 1: R1=0.1*

*Case 2: R1=10*

*Case 3: R1=1000*

*Moving W1 and W2 at the frequencies of 6 and 12 MHz:*

*Case 4: R3=R4= 100*

*Case 5: R3=10R4*

*Case 6: R3= 0.1R4*

**EXPERIMENT 1a:** *Given the results of Experiment 1, we rearrange the six mentioned cases close to the values of the best case.*

**EXPERIMENT 1b:** *We initialize the PSO algorithm with width range of values and using the objective function in formula 43, we compare the results with those achieved in experiment 1a:*

$$F(x_i) = \sqrt{\sum_{i=1}^k \frac{(\text{Re}\{Z_0(i)\} - \text{Re}\{Z_{RL}(i)\})^2 + (\text{Im}\{Z_0(i)\} - \text{Im}\{Z_{RL}(i)\})^2}{(\text{Re}\{Z_0(i)\})^2 + (\text{Im}\{Z_0(i)\})^2}} \quad \text{FORMULA 43}$$

*Note: this formula finds the best values of RL foster circuit elements for fitting the curve Z0 (The model).*

**EXPERIMENT 1c:** We initialize the PSO algorithm with width range of values and using the objective function in formula 44, we compare the results with those achieved in experiment 1a and 1b:

$$F(x_i) = \sum_{i=1}^k R_{TXD}(i) = \sum_{i=1}^k Z_L \cdot U_{TXD}(i) - 1 \quad \text{FORMULA 44}$$

*Note: this formula finds the best values of RL foster circuit elements for minimizing the reflection coefficients at the front face of the transducer.*

### 10.3.2.1 THE RLC MODEL:

The circuit to compute will be the same as figure 12; PSO algorithm will be applied and to improve computing time and initialization, Formula 45 is derived.

$$Z_{RLC} = \frac{(S + \omega_1)(S^2 L_1 R_1 C_1 + S L_1 + R_1)}{S^3 L_2 C_1 L_1 + S^2 C_1 (L_2 R_2 + L_1 (R_1 + R_2)) + S(L_1 + R_1 R_2 C_1) + R_1} \quad \text{FORMULA 45}$$

It is quite difficult to foresee the roots of the poles in equation 45 because factorization launches complex roots in both numerator and denominator. We simplify 45 to become equation 46 and set up a new experiment.

$$Z_{RLC} = \frac{(S + \omega_1)(S + \beta_a)(S + \beta_b)}{S^3 L_2 C_1 L_1 + S^2 C_1 (L_2 R_2 + L_1 (R_1 + R_2)) + S(L_1 + R_1 R_2 C_1) + R_1} \quad \text{FORMULA 46}$$

$$\beta_b^a = -\frac{1}{2} \frac{L_1^+ \sqrt{L_1^2 - 4R_1^2 C_1 L_1}}{R_1 C_1 L_2}; \rightarrow \omega_1 = \frac{R_2}{L_2}$$

**EXPERIMENT 2:** From Formula 46, we set  $\beta_a$  to take values within the interesting bandwidth (2-6 MHz), resistor R2 to be 10 ohms and vary L2 to move W2 as the six cases to plot:

Table 1

Case:	$\omega_2$
1	1 Mhz
2	1.5 Mhz
3	2 Mhz
4	3 Mhz
5	4 Mhz
6	5.5 Mhz

**EXPERIMENT 2a:** Given the results of Experiment 2, we rearrange the six mentioned cases close to the values of the best case.

**EXPERIMENT 2b:** we initialize the PSO algorithm with close values to best case in the previous experiment using the objective function in equation 43:

$$F(x_i) = \sqrt{\sum_{i=1}^k \frac{(\text{Re}\{Z_0(i)\} - \text{Re}\{Z_{RLC}(i)\})^2 + (\text{Im}\{Z_0(i)\} - \text{Im}\{Z_{RLC}(i)\})^2}{(\text{Re}\{Z_0(i)\})^2 + (\text{Im}\{Z_0(i)\})^2}} \quad \text{FORMULA 47}$$

**EXPERIMENT 2c:** we initialize the PSO algorithm with close values to the best case in the previous experiment using the objective function

$$F(x_i) = \sum_{i=1}^k R_{TXD}(i) \quad \text{FORMULA 48}$$

### 10.3.3 PROPOSED ACTIVE CIRCUITS FOR ACHIEVING “Z0”

#### 10.3.3.1 BRUTON’S GYRATOR:

The input impedance of the Antonius gyrator is given by eq. 33. We believe in section 10.3.1 that in order to achieve electrical matching impedance as Z0, the polynomial expression that may describe it, should be quite similar to that used for a BP filter and a shifter circuit or expressions.

**EXPERIMENT 3:** Centered on that and using the gyrator circuits proposed in section 8.6.3.1, we propose a combination of 4 types of RC circuit as figure 22.  $Z_a$ ,  $Z_b$ ,  $Z_c$  and  $Z_d$  substitute impedances  $Z_1$ ,  $Z_2$ ,  $Z_3$ , and  $Z_4$  in formula 33, and  $Z_5$  as a load  $R_7$ . In this way 2 combinations will be studied as part of the experiment.

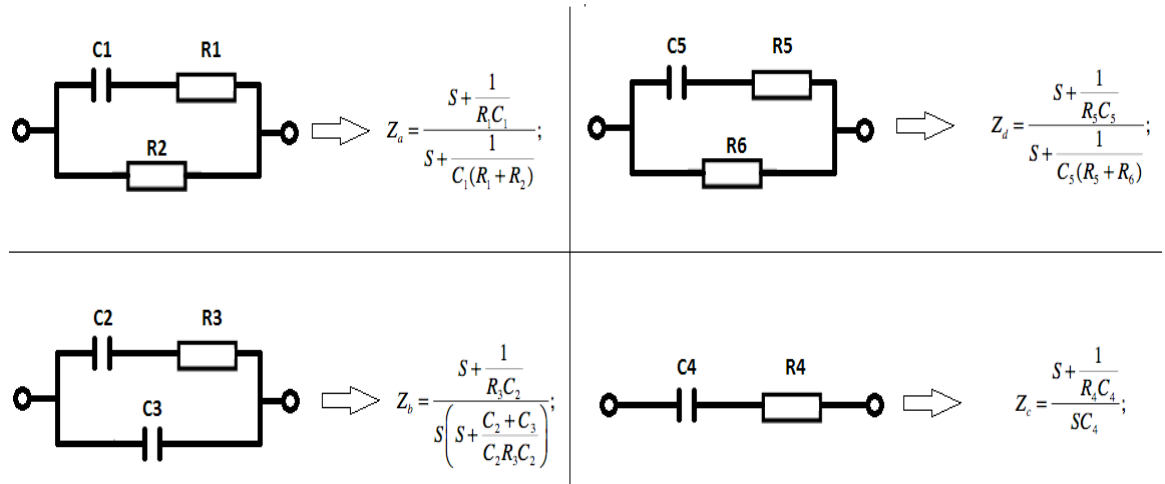


Figure 22: 4 RC circuits to Integrate the Bruton's Gyrator

**Experiment 3a:** Given the prior circuits and its equations, insert them in formula 33; use the PSO algorithm to minimize the reflection coefficients on the transducer front face and to find the best optimal values for each component in formula 49.

Combination 1:

$$Z_{gyrator} = R_7 \frac{Z_b Z_c}{Z_a Z_d} \quad \text{FORMULA 49}$$

**Experiment 3b:** Use the PSO algorithm to minimize the reflection coefficients on the transducer front face and to find the best optimal values for each component given the combination 2.

Combination 2:

$$Z_{gyrator} = R_7 \frac{Z_a Z_d}{Z_b Z_c} \quad \text{FORMULA 50}$$

**10.3.3.2 ACTIVE IMPEDANCE APPROACHES USING THE GENERAL FORM:**

The next experiment 4 consists of using equation 36 given the connection of a filter H0 in parallel to impedance  $Z_a$  in figure 14, and see how H0 should be given  $Z_a$  if:

**EXPERIMENT 4a)**  $Z_a$  is purely resistive.

**EXPERIMENT 4b)**  $Z_a$  is purely capacitive.

**EXPERIMENT 4c)**  $Z_a$  is a series RC circuit. The dependency in both R and C values is analyzed.

**EXPERIMENT 4d)**  $Z_a$  is a parallel RC circuit. The dependency in both R and C values is analyzed.

**10.3.4 PROPOSED CIRCUITS FOR MATCHING THEORY AND LAB RESULTS.**

**EXPERIMENT 5:** based on the RL foster approach and given the best values in experiment 1a-c and figure10a. Construct an electrical circuit with commercial values as those to the optimal; this will be our passive network. Computes how the reflection coefficients vary if:

A) The transducer has not loaded.

B) The RL circuit is ground. This is parallel to the transducer electrical terminals.

C) The RL circuit is connected to 1MHz resistor as it is when connected to the oscilloscope.

D) The electrical circuit of the next figure 23 is connected as load.

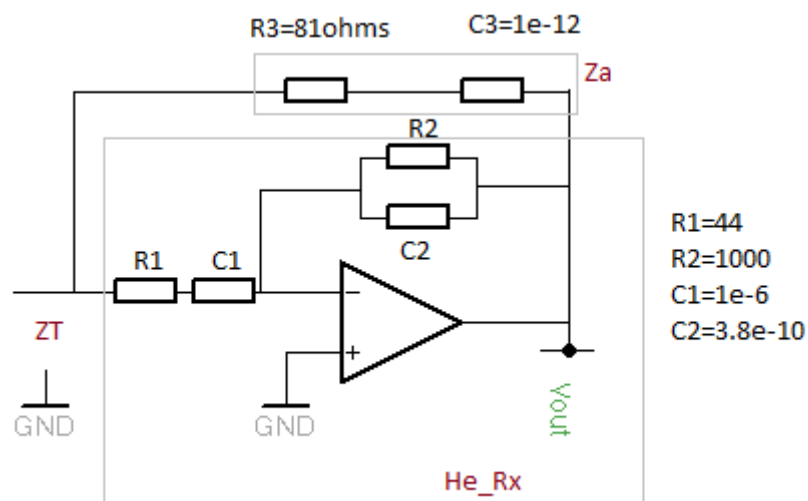


Figure 23: Active Circuit for the Lab Experiment



- E) Use the R.C of the transducer as reference and compare the ratio between the R.C for each load situation and the reference. In this way, we measure the reduction given by the added load.
- F) Realize the set Up for experiments described in section 10.2 and compare the simulation with the practical information.



## 11) RESULTS:

### 11.1 SIMULATIONS:

#### 11.1.1 Passive Elements:

##### Analysis for the Foster Circuit Approach on Experiments 1 and 1a-c

- ❖ The aim of the experiment was to find a mathematical expression for impedance synthesis of this circuit by using spectroscopy of the poles and root of the impedance polynomial expression. Poles roots were set to a specific cut off frequency to make behave and supervise the effect of the resistor's values on the zeros roots.
- ❖ Two Objective functions for the PSO algorithm with huge initialization values were tested to see how powerful it was. The results are next and exposed

- 1) **Experiment 1**, see the bode plot of figure 24 and the Reflection Coefficients R.C in figure 25 :

The bod magnitude of the RL foster circuit was expected to approach the Z0 plot (Ideal Model) by setting the poles root to be 3 and 6 MHZ for cases 1, 2 and 3. R1 was randomly to see its upshot. The computation was certain in magnitude giving Case 2 as the best of these three cases because R1 had a low of 10 ohms value. However, the phase was completely odd because zeros also play a crucial role that we tried to guess. The optimum case was to shift the poles roots cut off frequency and keep R1 even lower. This is case 5. Thus the consequences of case 2 and 5 on R.C are slightly but appreciably reduced.

- 2) **Experiment 1a**: see figure 26 and figure 27:

1a tryout played with the best case of experiment 1, and recalculated six new cases. Keeping the poles roots but increasing or reducing the resistors values to find out how they impacted the zeros expression was the focus of this new experiment. We found a notable reduction by decreasing the resistors R2 and R3. The best outcome was Case 5 as can be seen.

- 3) **Experiment 1b and c**: see figures 28 and 29.

Ideally the Optimization tool (PSO algorithm) was chosen for its great flexibility for fitting curve shapes where non initialization is needed or cannot be predicible. However, we have the feeling that the objective function should rather focus on minimization of the reflection coefficients equation by using the "DJong's function" thus we proceed to compare this with 1b and 1c trials. We also tested the mathematical best calculation of experiment 1 and 1a to see if the PSO has or not a better solution.

The outcomes: tremendous R.C reduction is achieved by using the PSO algorithm even though a limited value of initialization was not introduced and when converting the fitting curve problem into a minimization using DJong's function for R. Coefficients. Mathematical approaches cannot however be neglected, but computer means show great impact.

**The Experiment 1 was the use mathematical formulas for Impedance syntheses of the RL foster circuit proposed.**

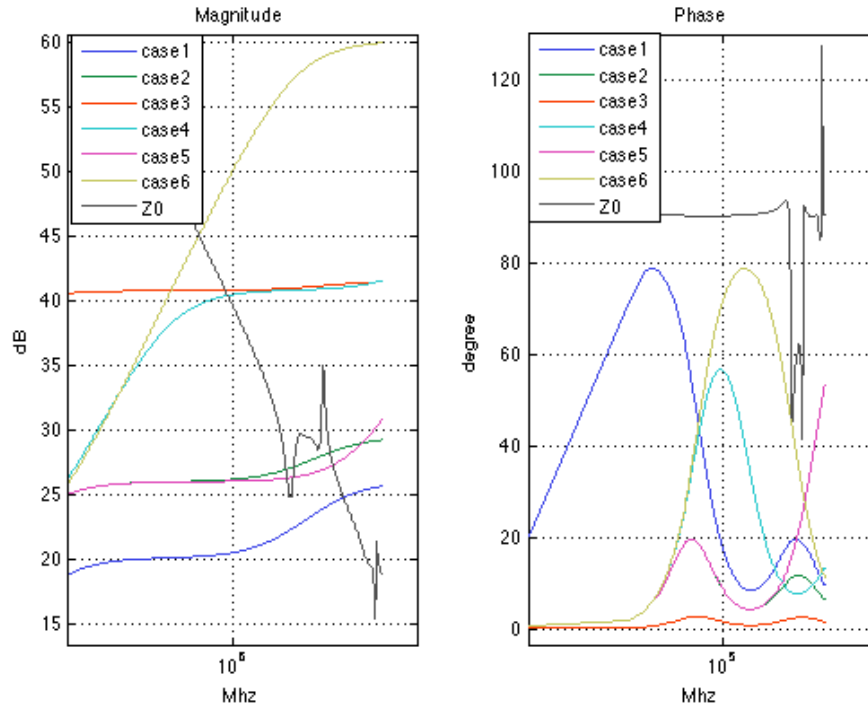


Figure 24: BODE PLOT OF THE RESULTS OF EXPERIMENT 1

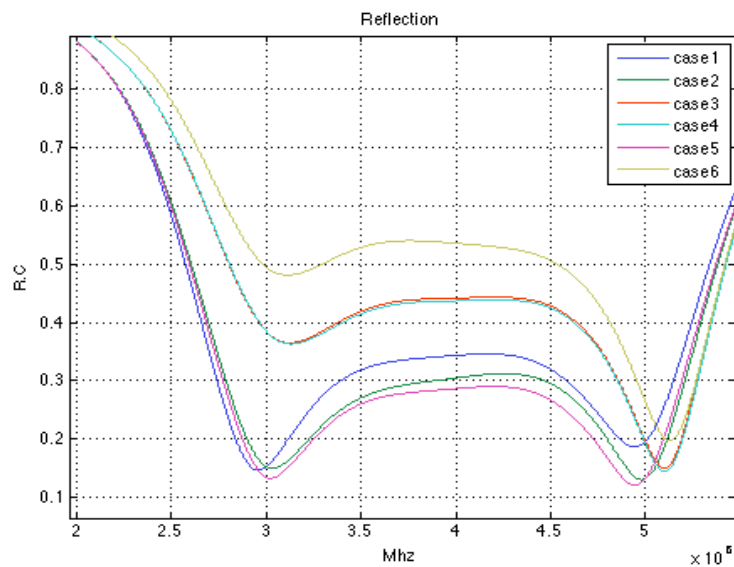


Figure 25: Reflection C. of Experiment 1

**Experiment 1a repeated best case in experiment 1 but moving its pole roots:**

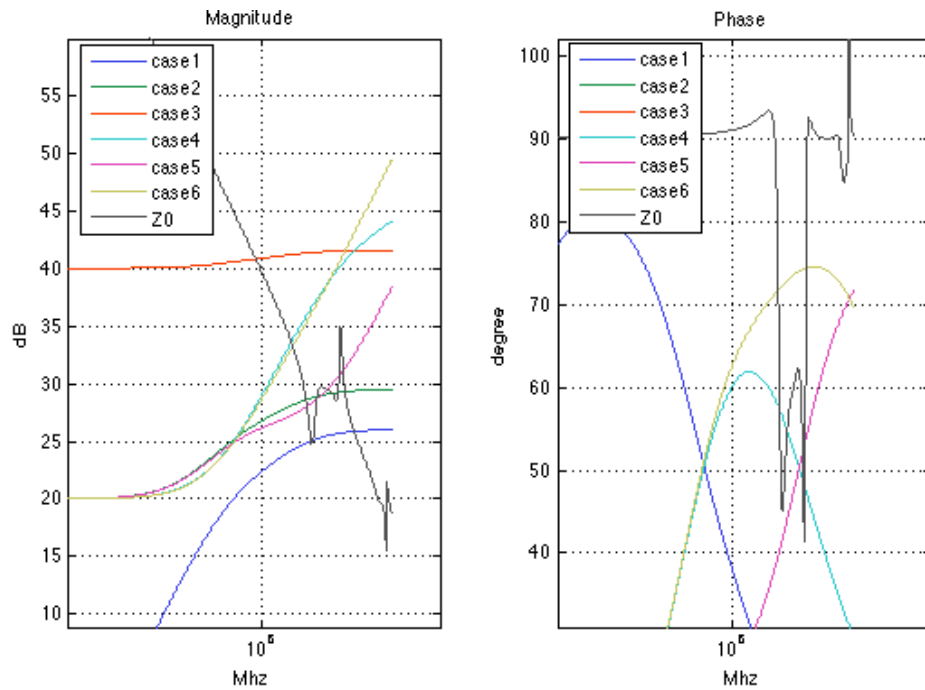


Figure26: BODE PLOT OF THE RESULTS OF EXPERIMENT 1a

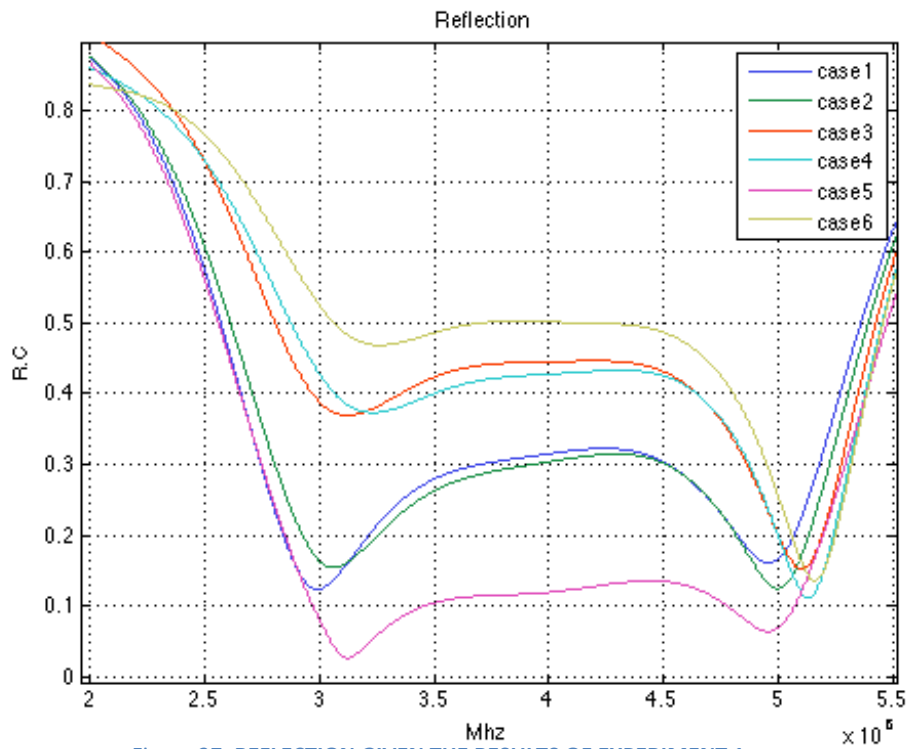


Figure 27: REFLECTION GIVEN THE RESULTS OF EXPERIMENT 1a

For 1b experiment the optimization consisted of fitting a curve, this is the total impedance of the RL foster circuit to be identical to the  $Z_0$  which was the ideal impedance for achieving non R.C. In contrast, experiment 1c used minimization of an objective function that computes the reflection coefficients. The plot for compering all experiments of the foster RL circuit is pasted here:

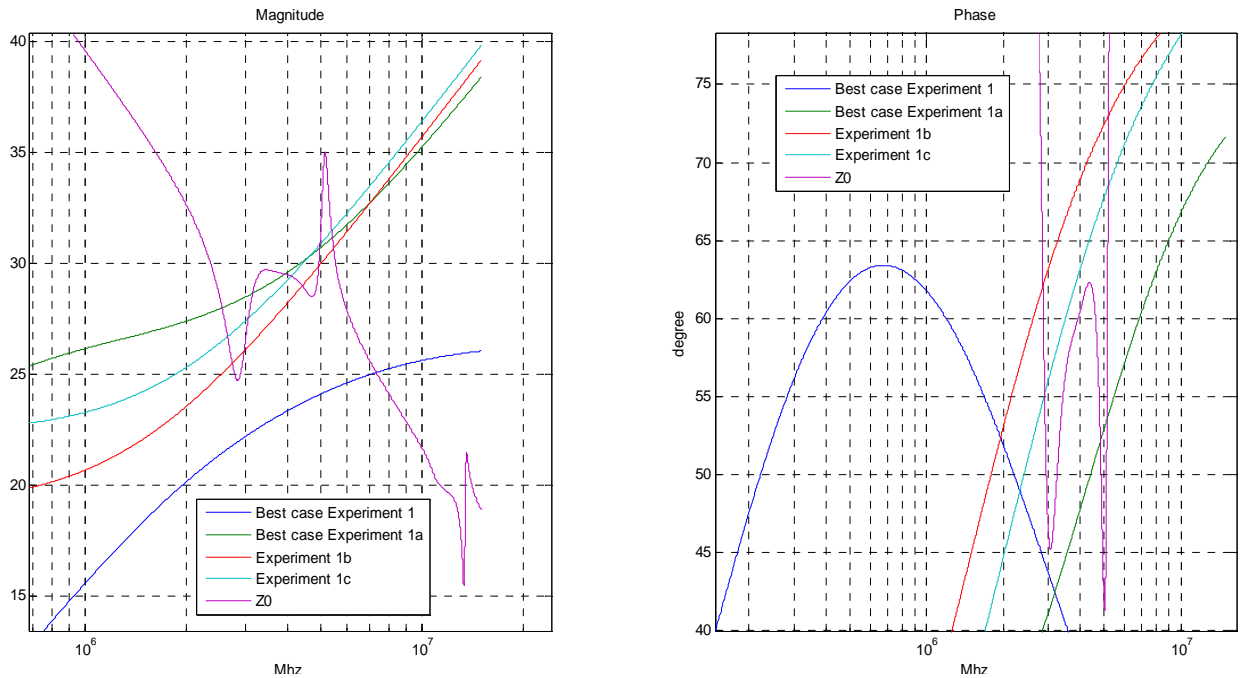


Figure: 28 BODE PLOT OF THE RESULTS OF EXPERIMENT 1-1c

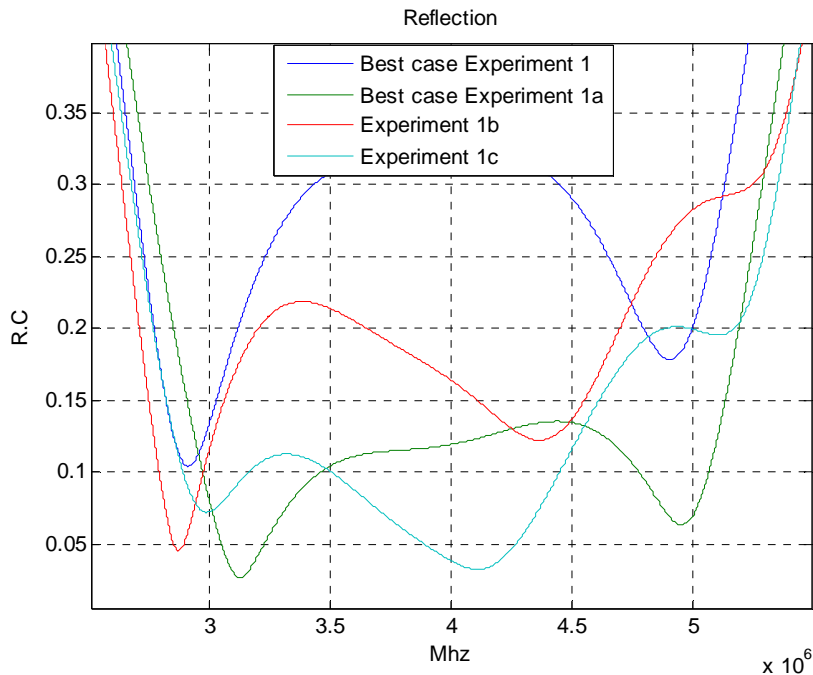


Figure 29: REFLECTION GIVEN THE RESULTS OF EXPERIMENT 2-2c

**Analysis for the RLC Circuit Approach on Experiments 2 and 2a-c**

- ❖ A canonical experiment RLC circuit was chosen because it was believed that by introducing a Capacitor component in the foster RL circuit, more flexibility in phase shift would be attained.
- ❖ The experiments 2 and 2a-c consisted of explaining mathematically how the RLC circuit behaves in frequency domain and used the PSO algorithm for comparing once again the importance of choosing the Objective Function and optimization problem carefully.

Experiment 2, see the bode plot of figure 30 and the Reflection Coefficients R.C in figure 31: Paying attention to the formulation in equation 45 and 46, a factorization to control only the complex root on the numerator containing most of the element in the circuit was done. This allowed the controlling of the action of the circuit only in the desired bandwidth 1-5.5 MHz and letting R2 be a fixed value and thus allowing the complex root to influence the root called “W2”. The best case in the bode plots and R.C was achieved by case 3 where the complex root reached 2 MHz.

Experiment 1a: see figure 32 and figure 33:

Once we saw that a good approaching was obtained by our factorization process, we evaluated the influence of moving the value of the capacitor and the resistor 2 to control the equation, results were interesting but not improved, thus we proceeded to the next experiment.

Experiment 1b and c: see figures 34 and 35.

Once again we have proved the importance of choosing the right objective function to work with, using the minimization of RC by Djong’s equation must then be the objective function for futures optimization.

*Observation: It is crucial to mentioned that the hypothesis based on the experiment 2 of introducing a capacitor for controlling the phase shift of the circuit was proven.*

**The Experiment 2 was the use mathematical formulas for Impedance syntheses of the proposed RLC circuit by canonical approach.**

The dashed ellipse, in the Bode Plot for the results of experiment 2, points the vulnerable region that depends on  $\omega_2$ .

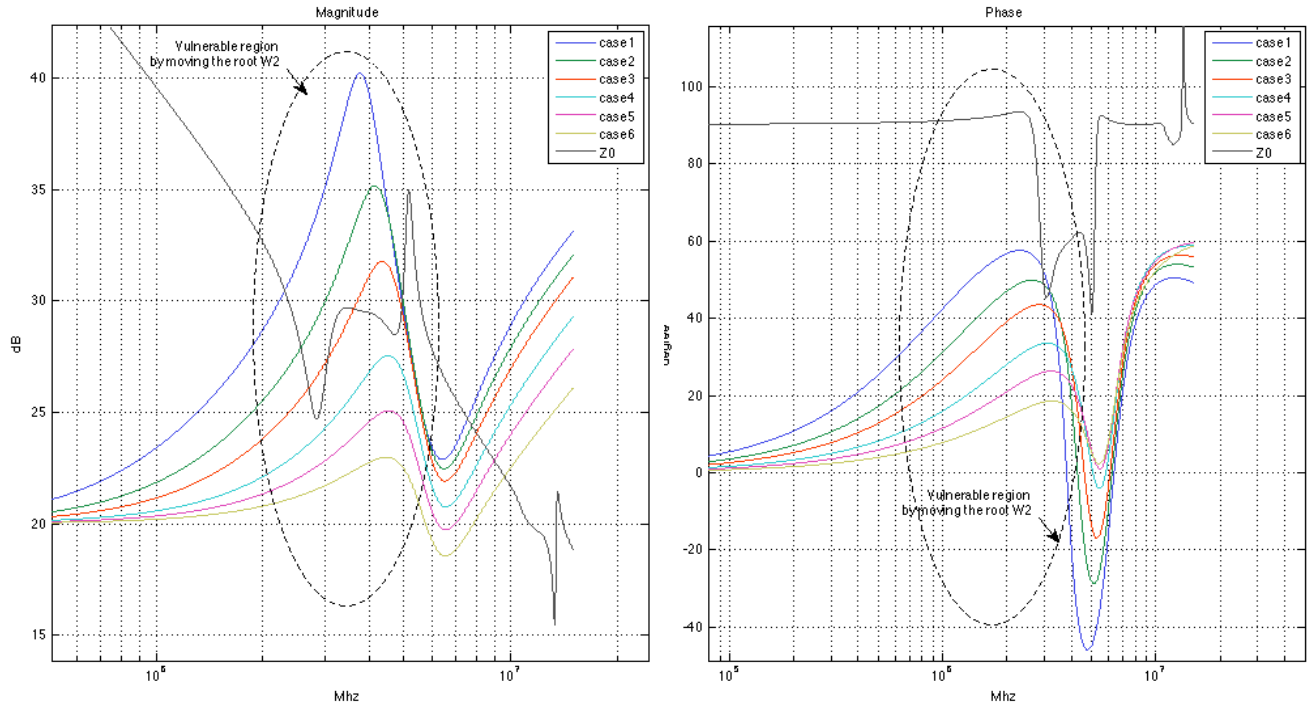


Figure 30: BODE PLOT OF THE RESULTS OF EXPERIMENT 2

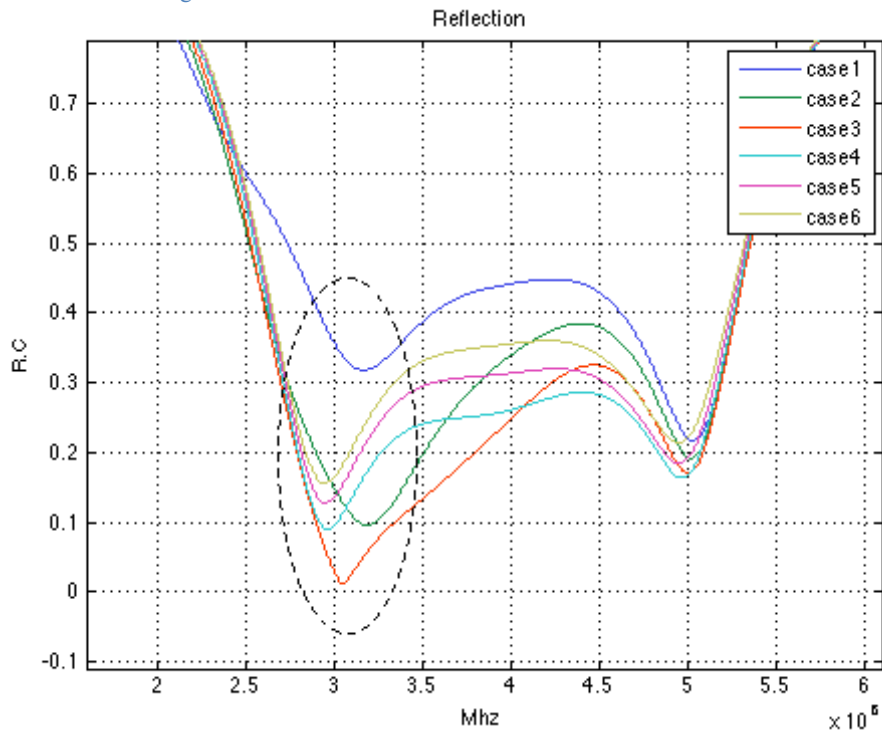


Figure 31: REFLECTION GIVEN THE RESULTS OF EXPERIMENT 2

**The Experiment 2a improved the result in experiment 2 by moving the capacitor and the pole root of the best case result on it.**

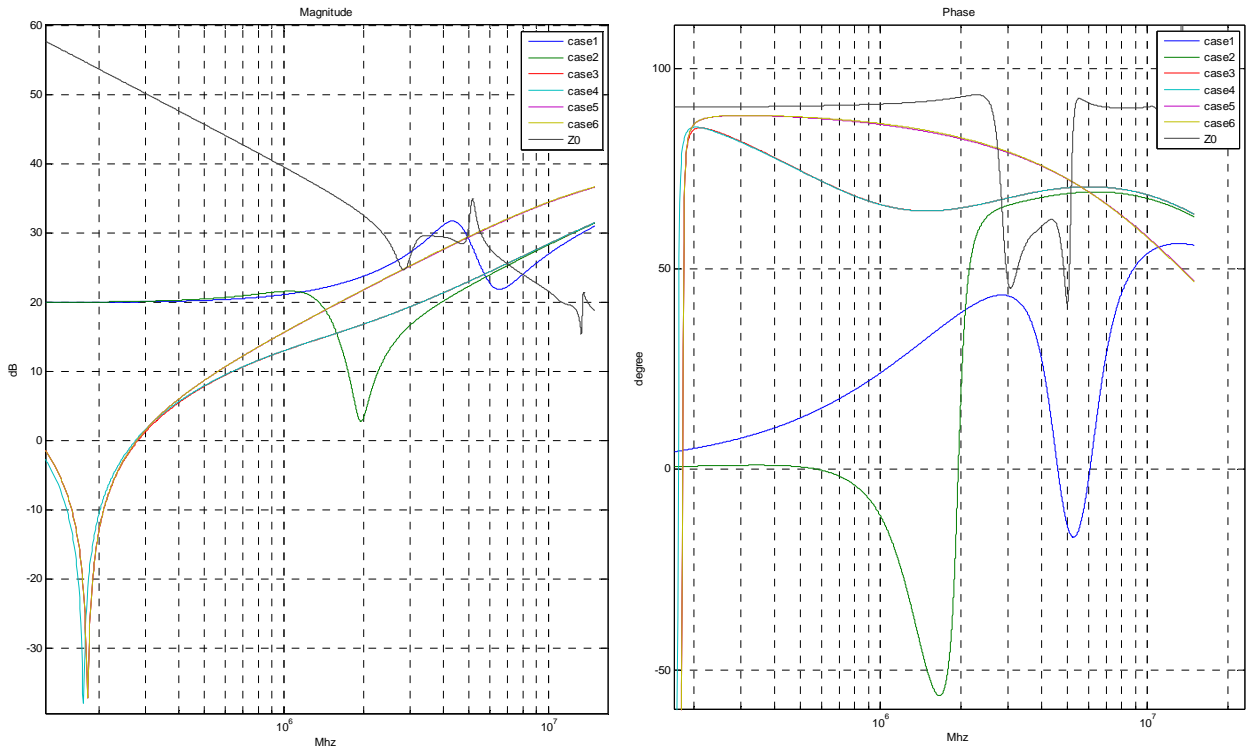


Figure 32: BODE PLOT OF THE RESULTS OF EXPERIMENT 2a

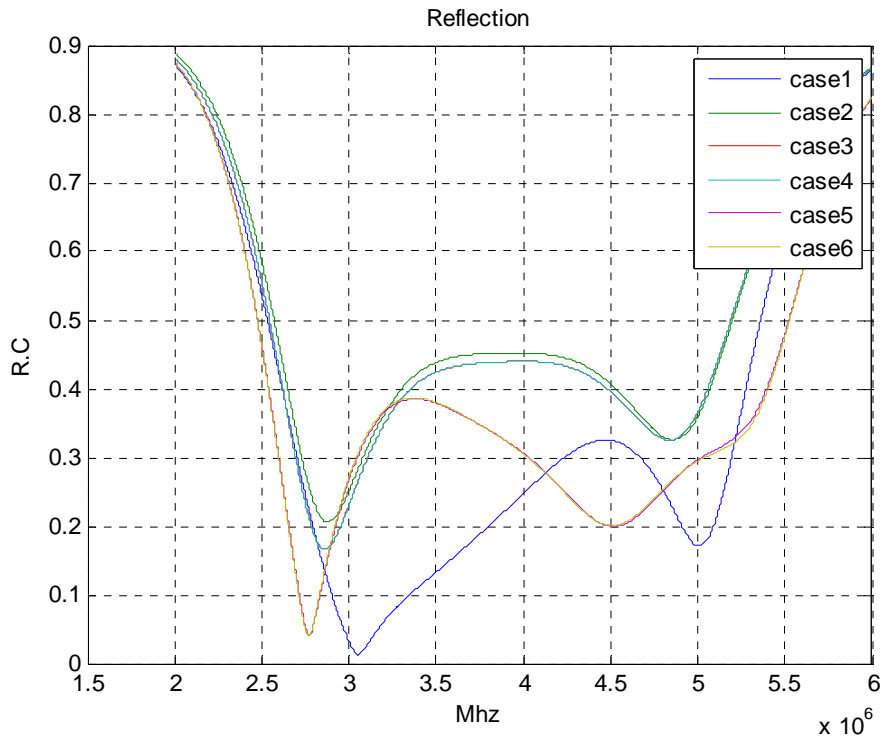


Figure 33: REFLECTION GIVEN THE RESULTS OF EXPERIMENT 2

**Experiment 2b and 2c used optimization with different objective function.**

For 2b experiment the optimization consisted of fitting a curve, this is the total impedance of the RL foster circuit to be identical to the  $Z_0$  which was the ideal impedance for achieving non R.C. On the contrary, experiment 2c used minimization of an objective function that computes the reflection coefficients. The plot for compering all experiments of the foster RL circuit is pasted here:

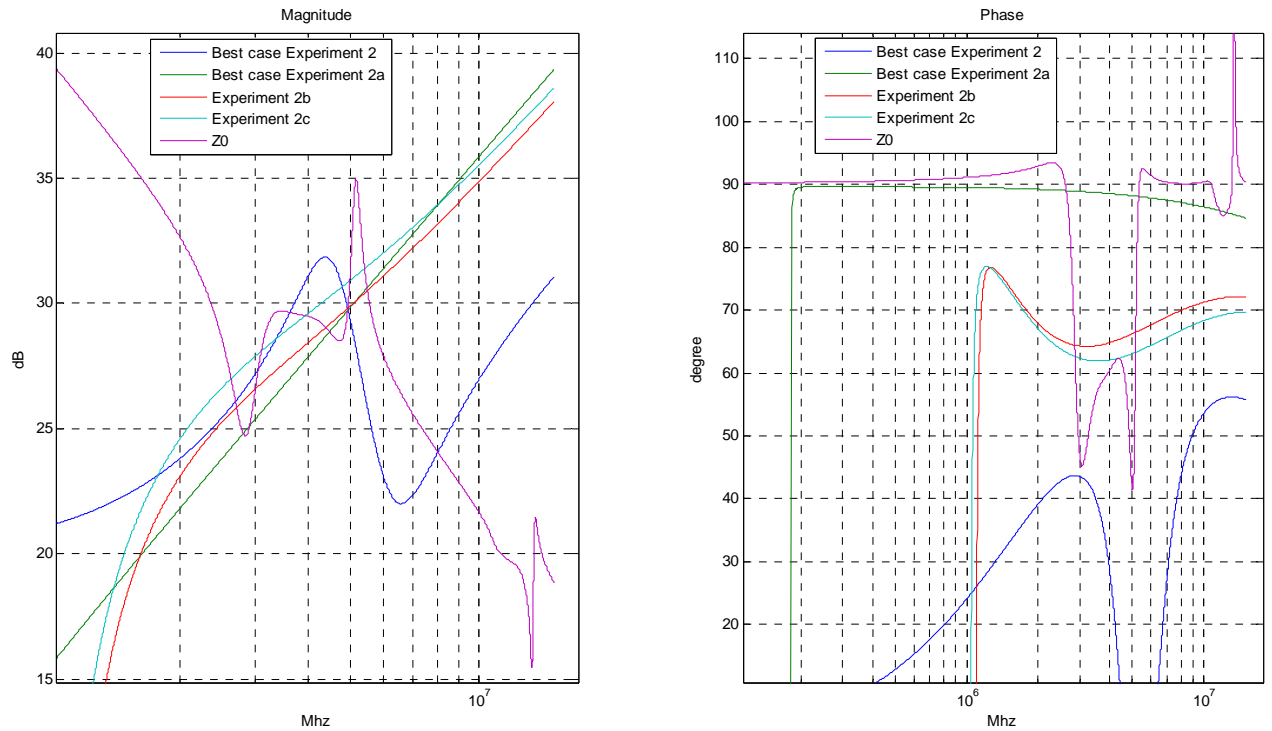


Figure 34: BODE PLOT OF THE RESULTS OF EXPERIMENT 2-2c

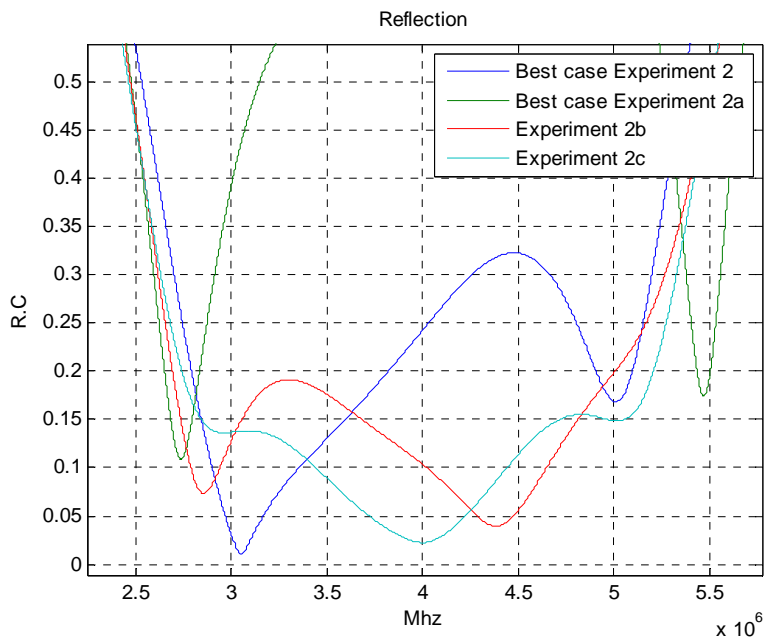


Figure 35: REFLECTION GIVEN THE RESULTS OF EXPERIMENT 2-2c



### 11.1.2 Active Elements:

#### **Gyrator circuits Experimental results (3)**

The higher the exponential degree in both numerator and denominator in any polynomial expression, the bigger the needed effort for characterizing or controlling the roots of poles and zeros in that network function independent from each other. The propose of this master thesis was not to find a mathematical method for characterizing the impedance synthesis, rather we tried to find a mathematical expression to characterize the X-Trans simulated model for this specific transducer case that allowed us to achieve zero reflection caused by our transducer front face by spectroscopy analysis (see section 10.3.1.1), and then to find passive or active circuits that may fit  $Z_0$  (the model) or have a similar mathematical expression as the model. Thus, Experiment 3 was to apply the PSO algorithm to the equation of two different gyrator combinations and see how far they could approach the target.

For the outcomes here see figures 36 for Bode graphs and 37 for Reflection coefficients caused when using the two gyrator cases.

*“Using nonspecific values for initialization the PSO algorithm, the Obtained R.C in both cases are quite convincing in this simulation section for orientating the Matching network to Gyrtors characterization. Both of them tend to reduce dramatically the reflection at the central frequency of the simulated transducer, this is 4 MHz*

**Experiment 3 used optimization for two different Gyrator circuits.**

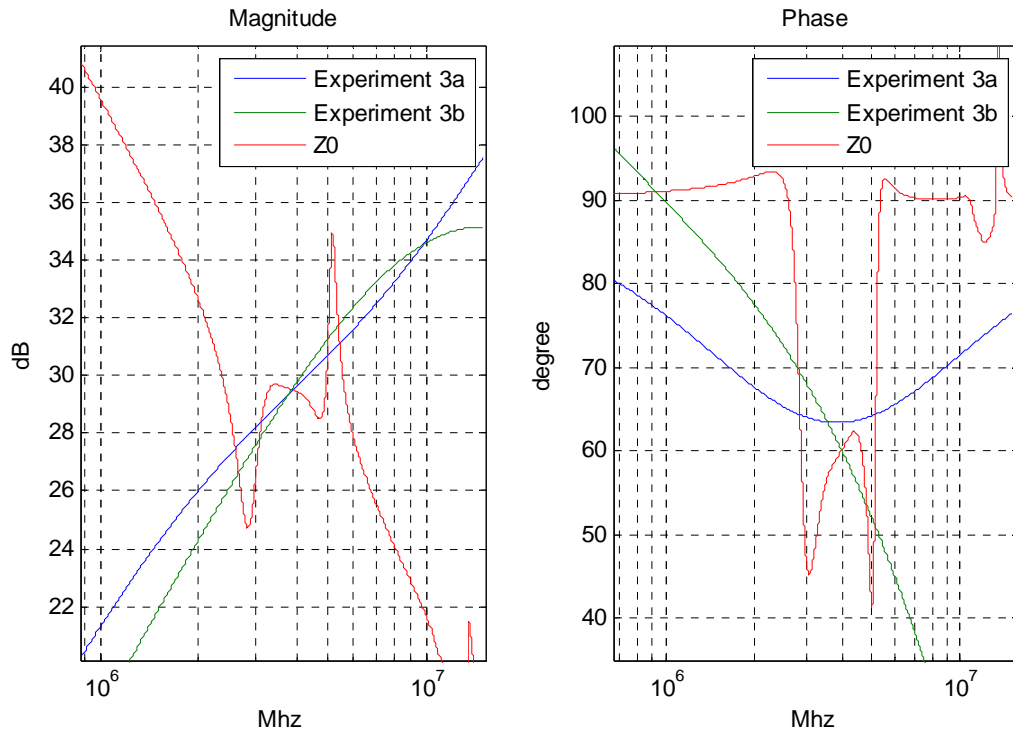


Figure 36: Gyrator Combination 1 vs 2. Bode analysis

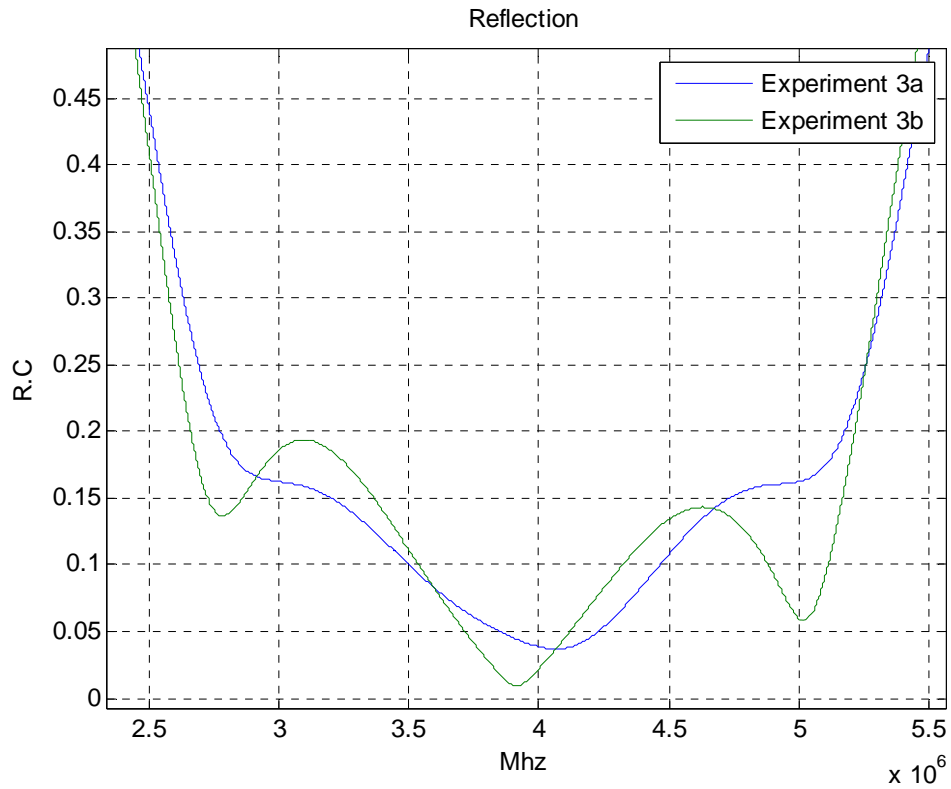


Figure 37: R.C of the Gyrator

#### **The Experiment 4:**

Let's remember the theory in section 8.6.2.2, it describes the general principle for active impedance synthesis, the approach is illustrated using figure 14. It states that any active circuit connected in parallel to given passive impedance, called  $Z_a$ , will emulate an active impedance at the input terminal or junction between the transfer function of the active element  $H(p)$  and the impedance  $Z_a$ . This emulated active impedance is written as  $Z_T$  in formula 35.

#### **The Description:**

The computed research in this section consisted of considering  $Z_0$ , the impedance X-Trans model, as  $Z_T$  in formula 36 and discovering how  $H(p)$  would be if  $Z_a$  is as in the next 4 cases. In this way if  $Z_a$  is predefined and  $Z_0$  is known, we convert the impedance synthesis into a filter synthesis, this is a tremendous approach because there is an amount of literature for filter synthesis in frequency.

- a)  $Z_a$  is purely Resistive, this is  $Z_a$  is a resistor.
- b)  $Z_a$  is purely Capacitive, this is  $Z_a$  is a capacitor.
- c)  $Z_a$  is a RC series circuit.
- d)  $Z_a$  is a RC parallel circuit.

The results are shown in figures 38 for a), 39 for b), and figure 40 has the comparison of the 4 cases.

- IF the case is to have  $Z_a$  only as a resistor, the filter is strongly dependent on the values of  $R$ , if the resistor were small, the magnitude of the filter needed would be almost 0dB and flat in the whole frequency axis. The phase shift is a bit more sensible to  $R$  but still it can still be said to be.

-If the case is to have a capacitor, the filter reacts similarly to being a resistor but the phase shift and magnitude of the simulated filter is closest to zero when the capacitor value is as big as possible for our simulation the red line in figure 39 is when  $C$  is 1 $\mu$ F and yellow if it is 1 pf.

If the circuit were an RC type, no matter whether it is series or parallel, we'd only need a band pass filter that behaves as the plots in figure 41, but this requires having more than one semiconductor.

*The best and further work is to investigate if by using only a buffer OPAM in parallel to a big capacitor would dramatically reduce the transducer reflection in its front.*

Experiment 4

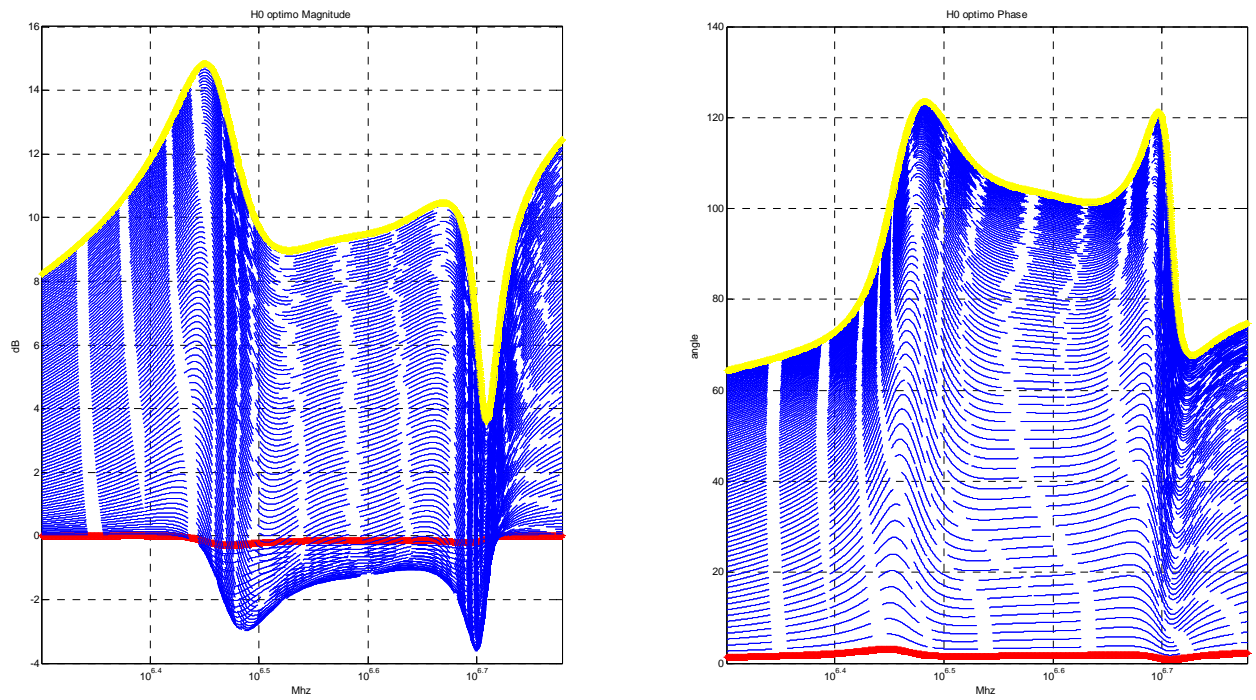


Figure 38: The filter design if  $Z_a$  were only resistive

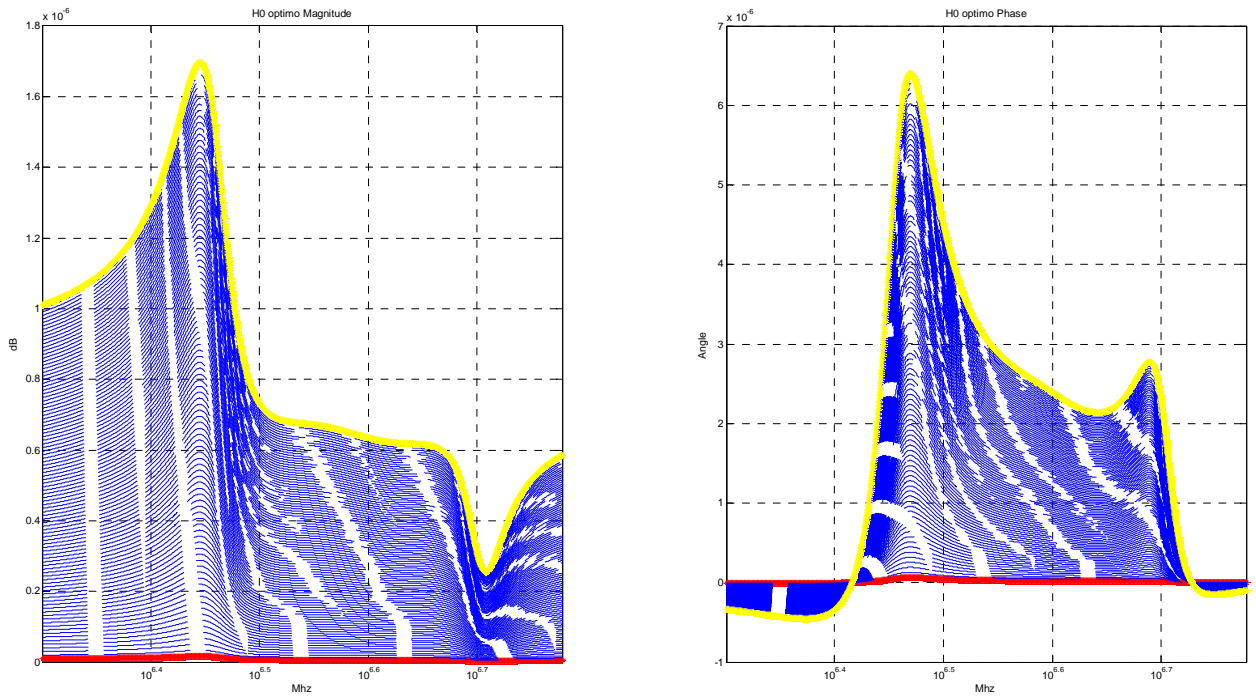


Figure 39: The filter design if  $Z_a$  were only capacitive

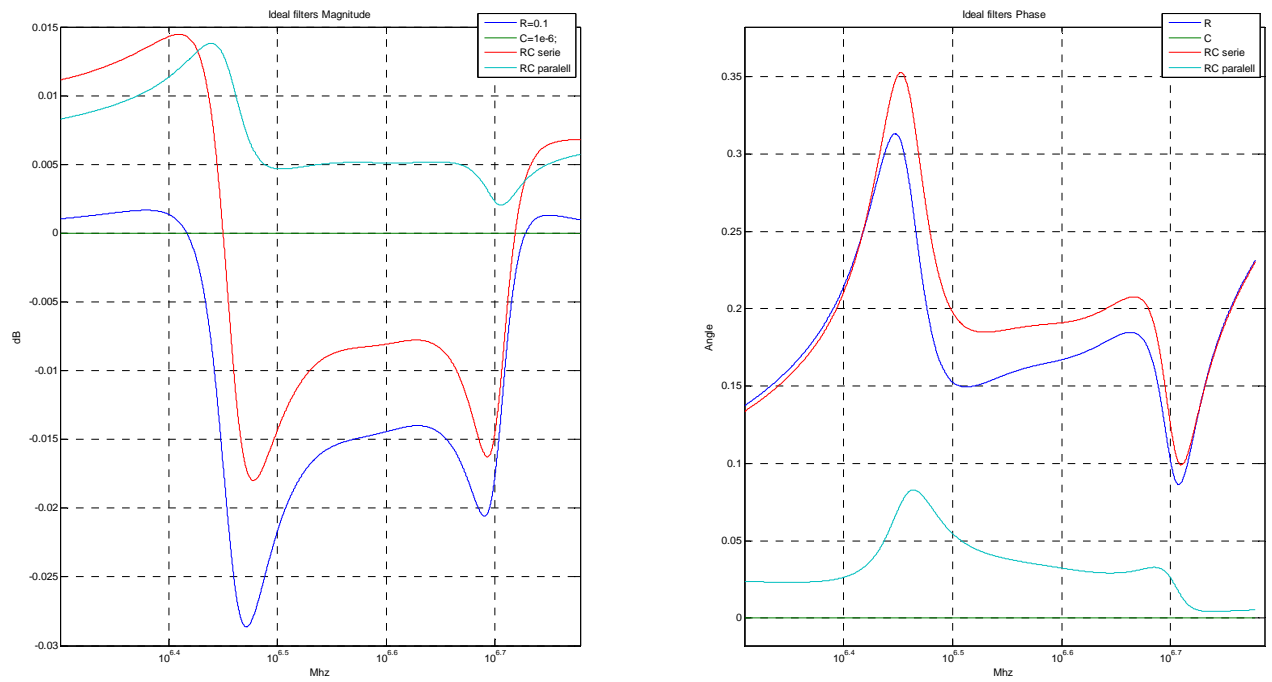


Figure 40: The filter design if  $Z_a$  were as the legend in the figure.

## 11.2 LAB EXPERIMENTS:

### *The Experiment 5:*

Here we tried to verify theory with simulation and experimental lab. The main Objective was only to verify if a Reduction of the pulse echo exist due to the load i.e. electrical circuits.

❖ Processed data from real test:

Figure 41 shows the pulse echoes obtained by measuring the signal coming from transducer 1 given the transducer 2 as target with and without load.

The Label VpeRLs points the pulse echo in transducer one facing transducer two connected to the RL foster circuit when the circuit is shorted. VpeRLo by contrariety is when the same circuit is open. VpeTxd is the same pulse echo but when there is not load in transducer two.

Figure 42 it illustrates the Signal-to-Signal Ratio for all tested impedance. The graph shows the signal-to-signal ratio, the division of the power energy spectrum of the measured signals when transducer two is connected to the electronics and a reference signal. This reference signal is the pulse echo when the same transducer 2 is not connected to a load. **Because a reduction exists, the ratio is below to 1 on y-axis.** The simulated transducer has a central frequency of 4 MHz

The Label RLs points the result given the RL foster circuit when the circuit is shorted. RLo by contrariety is when the same circuit is open. Finally Active 1 indicates the results or amount of reduction when the active circuit of figure 40 is connected to the transducer 2.

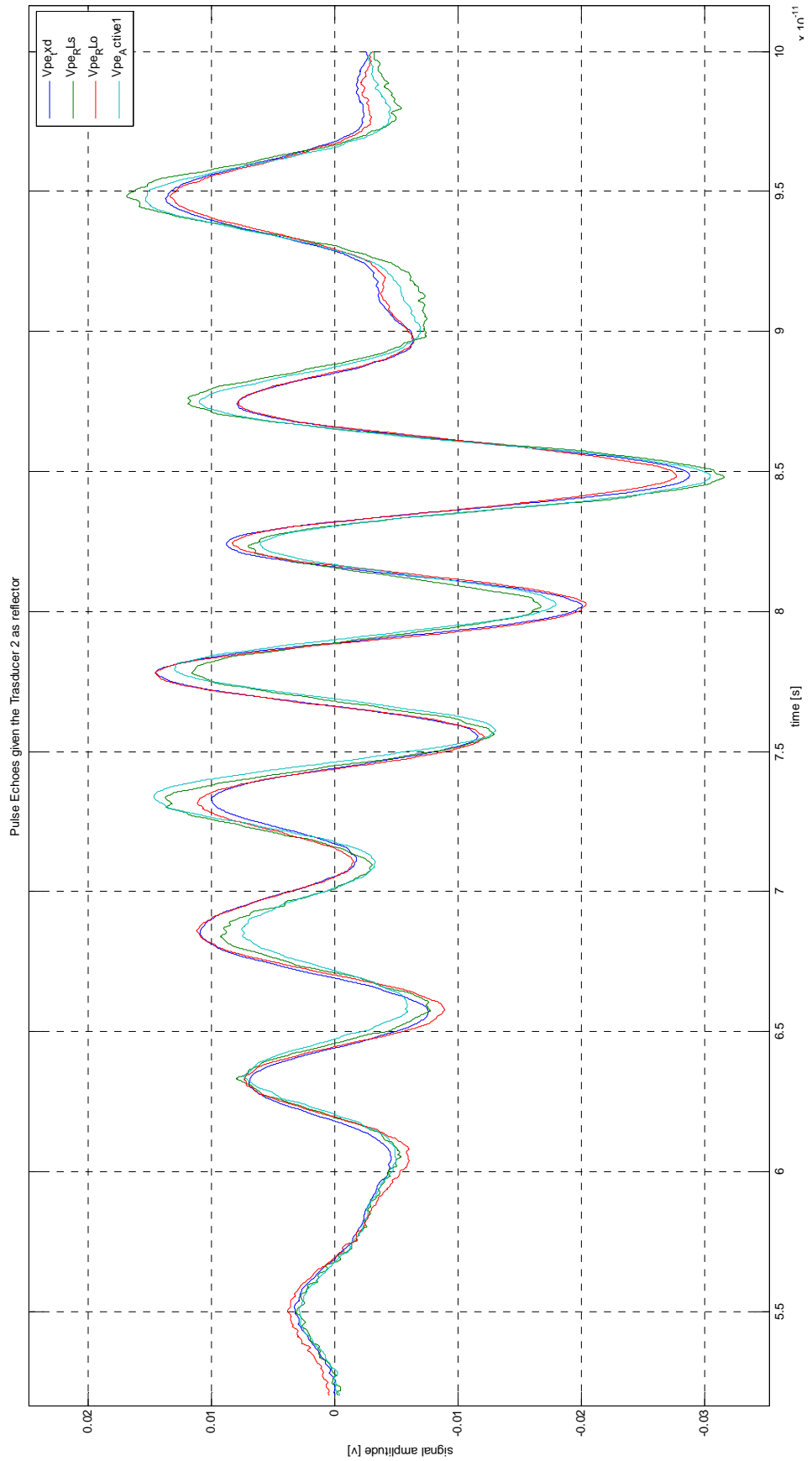


Figure 42: Pulse Echoes of the LAB Experiment

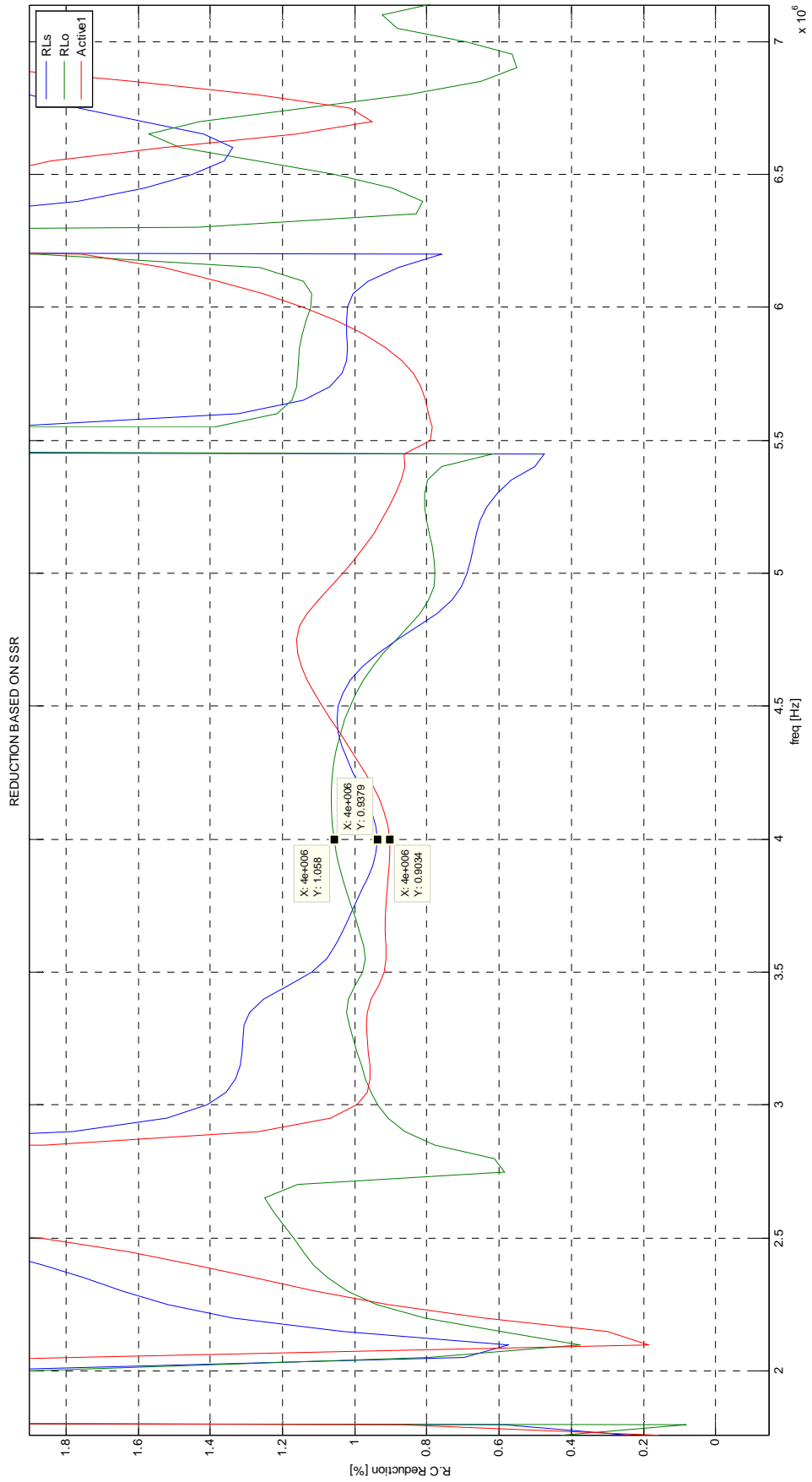


Figure 42: Signal-to-Signal Spectra Ratio



## 12 VALIDITY AND RELIABILITY

All simulation presented in here were done base mathematical expression of different circuits, this equations were obtained either from the references or by circuit analysis and confirmed by using Maple for calculating roots. The PSO was tested before implemented for the task. Simulations are high reliable while Lab work was not really supervised by an expert of the matter.

It should have spent more time testing different circuits through the Lab experiment. One had also to work with materials that were availed at the department of medical circulation and not close to those optimal in simulation. Better layout or circuit board and characterization of noise are needed to match simulation with experimental work.

Beside all the above, the Lab Experiment results and post processing was done using the power spectrum signal-to-signal ratio to illustrate how the active circuit and also the passive circuit decrease or increase the peak-to-peak voltage of the pulse echo measured when the two transducer are faced. This experiment is high reliable. It points the reduction of the voltage which means that the transducer two becomes absorptive given the electrical load.

We feel happy to show that OPAMS met the aim and also that the circuit was really noise robust.

## 13 CONCLUSIONS

To find mathematical methods for impedance synthesis it is possible for any purely passive impedance circuit using the methods presented in the reviewed literature. However, the ideal impedance simulated model  $Z_0$ , which will not provoke reflection on the transducer front face for the specific simulated transducer here, requires better approximation or estimation methods to find a polynomial mathematical equation that describes its behavior such as  $Z_0$  does in the frequency axis.

Particle Swarm Optimization (PSO) algorithm showed robustness to the initialization of the same algorithm. This means that using the PSO algorithm, the range of values that a given individual semiconductor in the electric circuit to work do not need to be delimited to approach the best global solution. However, wide initialization ranges will decrease the compute speed and increase the required time to optimize.

The PSO algorithm was chosen because it has been applied to the fitting curve optimization task in many others scientific fields, but we compared how it would work if the optimization task focus only in minimization of the formula for computing the reflection coefficients derived by the admittance matrix and in section 8.2.1.3 by using the objective called “Djong’s equation” and we achieved incredible results. For this research PSO algorithm and the mentioned optimization objective were vital to achieve the hypothesis.

Restricting to the optimization task was not included as the part of the objective function to use. However section 10.3.1.2 indicates that if and only if the total impedance of any electric circuit passive or active is measured at the transducer’s electrical terminals, then the pulse echo is benefited as mentioned there.

The passive electric circuits that we have proposed here approached the aim in a good - modest manner because they reduce the reflection but slightly reduce the pick to pick voltage level of the pulse echo signal.

The active impedance emulated using gyrators move toward the goal but still cannot be optimal if and only if they are not close enough to the impedance model. In order to achieve that for these cases was  $Z_0$  because it may also attenuate  $Z_0$  if they are not designed carefully.

On the other hand, the Active impedance general formula referred to in section 8.6.2.2 brings a possible trend for transducer electrical coupling design, because the general approach towards Impedance synthesis utilizes a filter design to affix the values of the Active Impedance circuit. In other words, we could take this property to define any kind of Analogue active filter to improve or process the pulse echo and reduce the reflection coefficients by adding a parallel passive impedance to the desired filter.

The Lab experiment was successfully implemented and the set up was useful to test the simulation.

## 14 REFERENCES

- [1] Bjørn A. J. Angelsen. Ultrasound Imaging: waves, signals and signal processing. EMATEC (2000), Norway.
- [2] Jim Baun. Chapter 14: Image Artifacts, Physical principles of general vascular sonography. USA.
- [3] Tonni F. Johansen, Tarjei Rommetveit. Characterization of ultrasound transducers. NTNU (2010), Norway.
- [4] Katsuhiko Ogata. Chapter 11: Frequency Analysis. Modern Control Engineering 3rd Edition. USA
- [5] Jose Espi, Gustavo Campus and Rafael Magdalena. Síntesis de Redes: Impedancias y Filtros. DELTA (2008), Spain.
- [6] Ron Mancini. OP AMPS for everyone. Texas Instrument (2002). USA
- [7] Jose Espi, Gustavo Campus and Rafael Magdalena. Síntesis de Redes: Impedancias y Filtros. DELTA (2008), Spain.
- [8] Jason Zheng Jiang and Malcolm C. Smith. Synthesis of Positive-Real Functions with Low-Complexity Series-Parallel Networks.
- [9] Vishwanath Iyer. Broadband Impedance Matching of Antenna Radiators. Worcester Polytechnic Institute (2010), USA.
- [10] Carlos Restrepo. Javier Calvete Alejandro Garcés. Algoritmo evolutivo para la optimización de parámetros de un modelo circuital en el análisis de la espectroscopia de impedancia eléctrica. Spain.
- [11] K.E. Parsopoulos and M.N VRahatis. Recent approaches to global optimization problems through particle swarm optimization.

APENDIX: A.1

Nyquist plots:

