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The Optimal Packet Duration of ALOHA and CSMA in Ad Hoc Wireless Networks

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Problem Description

Resource allocation is one of the greatest challenges in the design of wireless networks. The need for resource allocation is primarily due to the scarcity of available resources, such as transmission power and frequency spectrum, and to the constraints that the intended application imposes, such as transmission rate and delay. A popular way to approach these issues is through MAC (medium access control) layer design.

In this project we focus on the two MAC protocols, ALOHA and CSMA (carrier sensing multiple access) in ad hoc wireless networks. In ALOHA, signal packets are transmitted regardless of the channel conditions, while in CSMA, packets are only transmitted if the channel is sensed to have an interference and noise level below a predefined accepted threshold. We investigate the performance of these protocols, when the transmission rate
used by the nodes in the network is varied.

We base our work on previous research done is this area, and analyze the performance of these protocols in terms of outage probability. We wish to understand whether bursty transmission, i.e., transmitting a fixed amount of information during a short time interval, is more advantageous than sending the information distributed over a longer time period. The objective is to find an optimal transmission time that minimizes the outage probability of packet transmissions in our network.

Assignment given: 22. January 2009
Supervisor: Geir Egil Øien, IET

Preface

The following report is the result of my Master's thesis work for the Norwegian University of Science and Technology (NTNU). The project was carried out at the department of Electronics and Telecommunications at NTNU in Trondheim, Norway, and addresses the challenges present in the MAC layer design of wireless communication systems. In particular, this project considers ”The Optimal Packet Duration of ALOHA and CSMA in Ad Hoc Wireless Networks”.

I would like to thank my advisors, Professor Geir Øien and PhD student Mariam Kaynia, for their guidance and support. Also, I would like to thank fellow student Martin Carlsen, for useful discussions, and for providing an excellent coffee service.

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Abstract

In this thesis the optimal transmission rate in ad hoc wireless networks is analyzed. The performance metric used in the analysis is probability of outage. In our system model, users/packets arrive randomly in space and time according to a Poisson point process, and are thereby transmitted to their intended destinations using either ALOHA or CSMA as the MAC protocol. Our model is based on an SINR requirement, i.e., the received SINR must be above some predetermined threshold value, for the whole duration of a packet, in order for the transmission to be considered successful. If this is not the case an outage has occurred.

In order to analyze how the transmission rate affects the probability of outage, we assume packets of K bits, and let the packet duration, T, vary. The nodes in the network then transmit packets with a requested transmission rate of $R_{req} = K/T$ bits per second.

We incorporate transmission rate into already existing lower bounds on the probability of outage of ALOHA and CSMA, and use these expressions to

find the optimal packet duration that minimizes the probability of outage. For the ALOHA protocol, we derive an analytic expression for the optimal spectral efficiency of the network as a function of path loss, which is used to find the optimal packet duration T_{opt}. For the CSMA protocol, the optimal packet duration is observed through simulations.

We find that in order to minimize the probability of outage in our network, we should choose our system parameters such that our requested transmission rate divided by system bandwidth is equal to the optimal spectral efficiency of our network.

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Abbreviations

AWGN	Additive White Gaussian Noise
CSMA	Carrier Sense Multiple Access
CSMA-TX	Carrier Sense Multiple Access with Transmitter-sensing
CSMA-RX	Carrier Sense Multiple Access with Reciever-sensing
CSI	Channel State Information
dB	deciBel
MAC	Medium Access Control
NTNU	Norwegian University of Science and Technology
OSI	Open Systems Interconnection
PPP	Poisson Point Process
RA	Random Access
RX	Receiver
SINR	Signal to Interference plus Noise Ratio
SNR	Signal to Noise Ratio
TX	Transmitter

Chapter 1

Introduction

Ad hoc wireless networks consist of a collection of mobile nodes dynamically forming a network without the use of any establised infrastructure or centralized administration. The nodes in the network are free to move randomly and organize themselves arbitrarily. Thus, the network topology may change rapidly and in an unpredictable manner.

Ad hoc wireless networks have many desired features. They avoid the cost, installation, and maintenance of network infrastructure. The nodes can be rapidly deployed and reconfigured. They also exhibit great robustness due to their distributed nature, node redundancy, and lack of single points of failure.

The properties that make ad hoc wireless networks so desirable also introduce many challenges in the design of such networks. Despite many advances over the last decades in wireless communications in general, ad hoc wireless networks still remain poorly understood. Many questions about the performance of such networks still remain unanswered, which makes ad hoc wireless networks a popular field of research.

When the different nodes in an ad hoc network wish to communicate over a common medium, some kind of control over how and when the different users may access the channel becomes necessary. This control is defined by a *Medium Access Control* (MAC) protocol, which provides control over the channel access, to make it possible for several network nodes to simultaneously communicate within the network. The channel access can be done in a number of different ways, depending on application-specific requirements. In applications that require continuous data, like voice and video, a dedicated channel for each user is often used. However, most data applications do not require continuous transmission, i.e., data is generated at random time instances. In such cases, dedicated channel assignment can be extremely inefficient. *Random Access* (RA) strategies are used in such systems to efficiently assign channels to active users [2]. In the most basic form of random access, a node simply sends data onto the channel whenever it has data to send. This introduces the problem that different users may transmit at the same time, causing interference and possibly erroneous reception of packets.

Whenever the interference is so severe that the errors can not be corrected by the receiver, the packet is said to be received in outage. Packets that are received in outage must be retransmitted by the transmitter, resulting in a lower throughput and increased power consumption. This makes outage an important metric when evaluating the performance of the network.

There are numerous known techniques that can be used to decrease the probability of outage in random access protocols, e.g. by introducing time slots (which decreases the time packets may overlap) or by sensing the channel before transmission.

1.1 Problem Statement

In this thesis we will investigate how the choice of transmission rate affects the outage probability metric of the two random access MAC protocols, ALOHA and CSMA. We wish to understand whether sending a packet of information over a short time interval is more advantageous than sending the same packet of information over a longer period of time, in terms of outage probability. Our objective is to find an optimal packet duration that minimizes the outage probability of packet transmissions.

1.2 Structure of the Thesis

This thesis is structured as follows. In chapter 2, we give an overview of some important background material related to our work, as well as presenting some of the previous research done in our field of research. In chapter 3, the system model in which our analysis is performed, is introduced. In chapter 4, we start off by explaining the underlying fundamentals of our analysis. Further on, we derive lower bounds for the outage probability of ALOHA and CSMA as a function of the packet duration. We also derive an analytic expression for the optimal packet duration of the ALOHA protocol.

Chapter 5 contains our results, obtained through simulations. Here, we first introduce the model used for our simulations. Then, a section that compares our obtained lower bounds with simulations, is presented. We also perform a comparison of the performance of our considered MAC protocols. Finally, in chapter 6, we present some concluding remarks about our results.

Chapter 2

Background and Related Works

In this chapter, we will briefly present some important background material, which is needed in order to better understand our work in subsequent chapters. A more detailed version of this background information, can be found in [6]. We also present a selection of related works, in order to get an overview of previous research done in this field.

2.1 Ad Hoc Wireless Networks

An *ad hoc wireless network* is a collection of wireless mobile nodes that self-configure themselves to form a network without the aid of any established infrastructure [2]. Without the inherent infrastructure, the nodes in the network must be able to perform the necessary control and networking tasks by themselves. This is generally carried out through the use of distributed control algorithms [1].

In ad hoc wireless networks, the nodes are free to move randomly and organize themselves arbitrarily, thus the network's topology may change rapidly and unpredictably. This mobility, together with large network size, and bandwidth and power constraints, makes the design of adequate networking protocols a major challenge [3].

Despite its challenges, wireless ad hoc networks have many appealing features. They avoid the cost, installation and maintenance of network infrastructure. The nodes can be rapidly deployed and reconfigured. Ad hoc networks also exhibit great robustness due to their distributed nature, node redundancy and lack of single points of failure.

The self-configuring nature and the lack of any inherent infrastructure makes ad hoc wireless networks highly desirable for low-cost commercial systems, since they obviate the need for a large investment to get the network up and running, and deployment cost may scale with the economic success of the system. The lack of infrastructure is also highly desirable for military applications, as it allows for fast deployment and configuration, once the need has arisen.

[1] Distributed control algorithms are algorithms which are designed to run on distributed networks, where the network nodes cooperate on solving a given networking problem, e.g. routing.

2.2 Medium Access Control

In ad hoc wireless networks, the network architecture is an important aspect of the network design. It has, through the OSI-model, become normal to divide the network architecture into seven layers [1]. The different layers, from top to bottom, are the Application, Presentation, Session, Transport, Network, Data-Link, and the Physical layer.

This project will focus on a sublayer of the Data-Link layer, called the Medium Access Control (MAC) layer. The MAC layer is responsible for channel control mechanisms that make it possible for different nodes in a network to communicate over a common media. This control is carried out in MAC protocols, whose task is to ensure that the channel is utilized in the most effective way possible. In this project we will focus on the type of MAC protocols called Random Access MAC protocols.

2.2.1 Random Access Protocols

In most data applications, data are generated at random time instances, and the total number of users in the network is often much higher than what can be accommodated simultaneously. In this case dedicated channel allocation can be extremely inefficient. Random access strategies are used in such systems to efficiently assign channels to the active users [2].

All random access techniques are based on the premise of packetized data, i.e., user data is collected into packets, of a given number of bits, and is sent over the channel once a packet is formed. Each of the packets are transmitted over the channel independently. This random access implies one big drawback: Different users may transmit at the same time, causing interference and possibly erroneous reception of packets.

There are numerous techniques that can be used to decrease the probability of simultaneous transmissions in random access protocols, e.g. by using a slotted system, which decreases the time packets may overlap in time, or by sensing the channel before transmission and only transmit if the channel is idle. We will focus on two of the most basic random access schemes; ALOHA and CSMA, which will be presented in subsequent chapters. These protocols, with modifications, are the most widespread random access MAC protocols in use today.

2.2.2 Unslotted and Slotted ALOHA

The first random access system was the ALOHA system, pioneered by Norman Abramson at the University of Hawaii in 1970, and was used to connect computer terminals on different parts of this cluster of islands to a central computer stationed at Honolulu.

In the *pure* or *unslotted* ALOHA protocol, users transmit their packets as soon as they are formed. This implies that the transmitter (TX) chooses its transmission time completely at random, and does not take into consideration

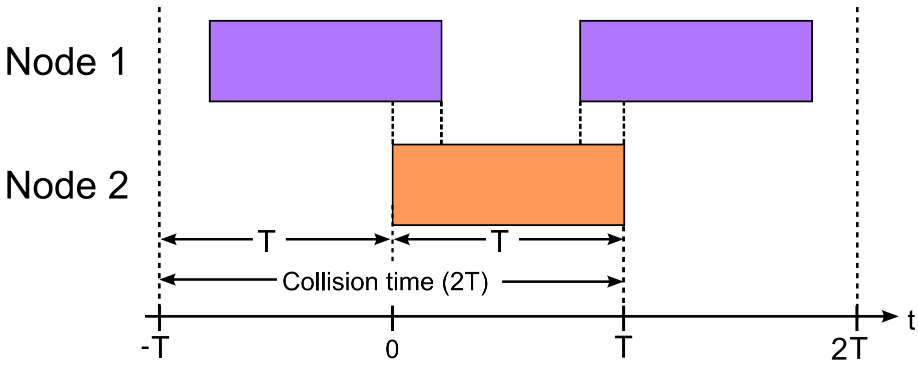

Figure 2.1: Possible collision time in unslotted ALOHA

that the channel may already be occupied by another user. This can lead to a situation where multiple users want to transmit information simultaneously, and packets may overlap in time. An overlap of transmissions will cause interference between the users. Figure 2.1 shows that any node (in this case node 1) starting its transmission in the interval $[-T, T]$ will interfere with the packet of node 2. If this interference is severe enough, a received packet may be unusable for the receiver and has to be retransmitted. Such ”collisions” of packets will decrease the effective data rate of the system. For networks with a moderate to high traffic load, unslotted ALOHA is extremely inefficient, because the probability of simultaneous transmissions becomes large.

In a *slotted ALOHA* system, time is assumed to be slotted in time slots of length T, and users can only start their transmission at the beginning of the next time slot after its packet has been formed. This removes the partial overlap of packets and increases the throughput of the system.

2.2.3 Carrier Sense Multiple Access

In the *Carrier Sense Multiple Access* (CSMA) protocol, the transmitter senses the channel, and delays its transmission if it detects that its transmission will be unsuccessful, e.g., if the SINR at the receiver is expected to be below a certain threshold value. The transmission will then be delayed a random time before it is retransmitted. This is called *random backoff* and avoids having multiple users simultaneously transmitting their packet once the channel is free. CSMA only works when all users can detect each other's transmission and the propagation delays are small. Wired LANs exhibit these characteristics, and CSMA is used as the access method in the Ethernet protocol.

However, in wireless networks, the nature of the wireless channel may prevent a given user from detecting all signals transmitted by all other users. It is often the case that a user can only hear transmissions from its immediate neighbors. This gives rise to the *hidden node problem*, illustrated by Figure 2.2. In the figure, node 5 and node 3 each wish to transmit to node 4. Suppose node 5 starts its transmission. Since node 3 is too far away to detect this transmission, it assumes the channel is idle and carries out its own transmission, causing a collision with node 5's transmission. Node 3 is said to be "hidden" from node 5 because it cannot detect node 5's transmission.

Another problem with CSMA is inefficiencies in channel utilization from the *exposed node problem*, also illustrated in Figure 2.2. Assume that the exposed node, node 2, wishes to send a packet to node 1 at the same time as

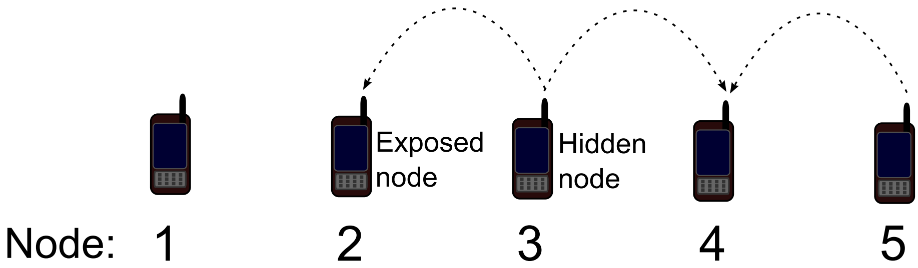

Figure 2.2: The hidden and exposed node problem. Node 3 is "hidden" from node 5 and cannot hear node 3's transmission to node 4. Node 2 is "exposed", in the sense that it will not start a transmission to node 1 when sensing node 2's transmission to node 4.

node 3 is sending to node 4. When node 2 senses the channel, it will detect node 3's transmission and assume the channel is busy, even though node 3's transmission does not interfere with reception of node 2's transmission at node 1. Thus node 2 will not transmit to node 1 even though its transmission would have been successful.

2.3 Related Works

There has been an extensive amount of research involving the choice of transmission rate in ad hoc networks. A key concern in these networks is energy efficiency, because of the often limited power available at the transceivers. Most of the research involving transmission rate has its focus on minimizing power or energy consumption in the network.

It has been shown that, for many coding schemes, the energy needed to transmit a fixed amount of information is a monotonically decreasing, convex function of the transmission time [7] [8]. That is, the energy required to send a packet can be reduced by transmitting the packet with a lower bit rate encoding. Hence, an energy-conserving transmitter should attempt to transmit packets at the slowest possible rate. This technique is known as *lazy packet scheduling* [8].

The lazy packet scheduling approach is optimal on a per-node basis, but might be heavily sub-optimal in a network perspective. This is because each node tries to maximize its own timeshare of the channel, while the other nodes contending for the channel will have to delay their transmission or speed it up if they have to meet a deadline.

Also, lazy packet scheduling only minimizes the contribution of the electronics whose power consumption is a function of the transmit power. In ad hoc networks, an important part of the power dissipation is the contribution of the frequency synthesizer, the mixers, and the filters, which are not proportional to the transmit power [9]. This motivates the approaches based on radio shutdown that tends to minimize the duty cycle of the radio circuitry, and therefore transmit the information as fast as possible. As a result, they give other nodes maximum timeshare of the channel, as opposed to the selfish behavior in the lazy scheduling approach. Some work utilizing the shut down approach to minimizing power consumption can be found in [10], [11], and [12]. In [13] the authors propose a transmission strategy that optimally mixes the shut down approach and the lazy scheduling approach in a clustered ad hoc network.

The related work referenced so far has been related to minimizing power consumption in ad hoc networks, subject to some constraint, such as for instance delay or packet life time. In this thesis we will, instead of minimizing power, minimize the probability of outage in the network. Even though the probability of outage is a performance metric mostly associated with network throughput, it can also be an important metric when the goal is to minimize power consumption in ad hoc networks.

We will now present some research more closely related to our problem description, utilizing basically the same system model for analyzing the performance ALOHA and CSMA in ad hoc networks.

The work most closely related to ours, is the work done by Kaynia and Jindal in [14]. They have analysed the performance of ALOHA and CSMA in spatially distributed wireless networks. In their model, packets arrive randomly in space and time according to a Poisson point process. Each packet is then transmitted to its intended destination through a fully-distributed ALOHA or CSMA protocol. They assume a fixed distance, R, between the transmitter and the dedicated receiver. The transmission power ρ is constant for all transmitters, and only path loss attenuation effects (with $\alpha > 2$) are considered. The channel noise is denoted η.

They consider a stochastic SINR requirement, and develop accurate bounds for the probability of outage of ALOHA and CSMA as a function of the transmitter density. From the SINR requirement they define a distance s to be the distance between the receiver under observation and and its closest interferer that causes the SINR to fall just below the SINR threshold β. This distance s was first defined by Hasan and Andrews in their work on "guard zones" in wireless ad hoc networks [15]. The derivation of s is obtained by considering only one interferer in the SINR model and letting s be the distance between the receiver and the interferer.

$$\frac{\rho R^{-\alpha}}{\eta + \rho s^{-\alpha}} \leq \beta, \tag{2.1}$$

where the left side of the equation is the expression for the SINR at the intended receiver. Solving for s gives:

$$s = \left(\frac{R^{-\alpha}}{\beta} - \frac{\eta}{\rho} \right)^{-\frac{1}{\alpha}}. \tag{2.2}$$

Recall that for slotted ALOHA, transmitters can only start to send packets at the beginning of the next time slot after the packet is formed, and that a time slot is equal to a packet length. Thus, a receiver can only experience interference from transmitters transmitting in the same time slot, resulting in a vulnerable period of T seconds, where the receiver can go into outage. Consider the area of B(R1,s) which is given by a circle of radius s around the receiver under observation. One situation that would cause the receiver to go into outage is if at least one interfering transmitter falls within B(R1,s), while the receiver under observation is receiving a packet. The probability of this event can be found using stochastic geometry and is presented in [17]. Note that, since the event where the signal power of multiple interfering transmitters outside the area B(R1,s) add up to cause an outage, is not considered, the expressions derived are lower bounds on the probability of outage.

From the expression for the probability of the first event, the following lower bound for slotted ALOHA is presented:

$$P_{\text{out}}^{\text{LB}}(\text{Slotted ALOHA}) = 1 - e^{-\lambda\pi s^2}. \tag{2.3}$$

In the unslotted version of ALOHA, packets are transmitted once they are formed, regardless of the channel conditions. Hence, packets are transmitted continuously in time, which results in a period of twice the packet length where packets may overlap. Now, the packet of any transmitter that started its transmission less than T seconds before the arrival of our transmitter-receiver pair, will overlap with our packet, and thus contribute to the outage probability. We now have a vulnerability time of $2T$ seconds where packets may overlap. The lower bound for the outage probability of unslotted ALOHA can be derived by requiring that there are no active interferers inside a circle of radius s of the receiver under observation, denoted $B(R1, s)$, during the period $[-T, T]$, as derived in [14]:

$$\begin{aligned} &P_{\text{out}}^{\text{LB}}(\text{Unslotted ALOHA}) \\ &\quad = P(\text{outage in } [-T,0] \cup \text{outage in } [0,T]) \\ &\quad = 2 \cdot (1 - e^{-\lambda\pi s^2}) - (1 - e^{-\lambda\pi s^2}) \cdot (1 - e^{-\lambda\pi s^2}) \\ &\quad = 1 - e^{-2\lambda\pi s^2}. \end{aligned} \tag{2.4}$$

This derivation is valid because the number of packet arrivals in [-T,0] is independent of the number of packet arrivals in [0,T].

In the CSMA protocol, a transmitter backs off if the accumulated interference from all the other transmitters results in a SINR lower than the threshold value β at the beginning of a packet. The probability of this happening is denoted the *backoff probability*, P_b. Since no retransmissions are allowed, a backoff is considered an outage. Also, if the transmitter measures a SINR higher than β and decides to transmit, a packet will be received in outage if the SINR falls below β any time during the transmission. Kaynia and Jindal [14] derives these probabilities, and present analytical lower bound expressions for the outage probability of CSMA, both for transmitter-sensing and receiver-sensing. In the transmitter-sensing configuration the *transmitter* senses its own SINR and decides whether or not to transmit, i.e, if an interfering transmitter falls within the distance s of an already active transmitter, the new transmitter backs off. Because of the backoff property of CSMA the number of transmitters on the plane no longer follows an exact PPP. However, as an approximation, it is assumed that the nodes are still Poisson distributed. Simulations prove that this assumption is reasonable.

For CSMA with transmitter-sensing, referred to as CSMA-TX, the following bound on the probability of outage is presented:

$$\begin{aligned} P_{\text{out}}^{\text{LB}}(\text{CSMA-TX}) &= P_b + (1 - P_b) P_{\text{out}}^{\text{LB}}(\text{CSMA} \mid \text{no backoff}) \\ &+ P_b[1 - P_{\text{out}}^{\text{LB}}(\text{CSMA} \mid \text{no backoff})][1 - P_{\text{out}}^{\text{LB}}(\text{RX beg.} \mid \text{backoff})], \end{aligned} \tag{2.5}$$

where P_b is the probability of backoff and is given in terms of the Lambert function as:

$$P_b = 1 - \frac{W_0(\lambda \pi s^2)}{\lambda \pi s^2}. \tag{2.6}$$

$P_{\text{out}}^{\text{LB}}(\text{CSMA} \mid \text{no backoff})$ is the probability that a packet is received in outage given an active transmitter-receiver pair, and is given by:

$$\begin{aligned} &P_{\text{out}}^{\text{LB}}(\text{CSMA} \mid \text{no backoff}) \\ &= \int_{(s-R)^2}^{s^2} \left[1 - \frac{1}{\pi}\cos^{-1}\left(\frac{d^2 + R^2 - s^2}{2Rd}\right)\right] \pi\lambda e^{-\pi\lambda d^2} d(d^2). \end{aligned} \tag{2.7}$$

Finally, $P_{\text{out}}^{\text{LB}}(\text{RX beg.} \mid \text{backoff})$ is the probability that the closest interferer, which is given to be inside $B(T_1, s)$, is also inside $B(R_1, s)$. That is:

$$P_{\text{out}}^{\text{LB}}(\text{RX beg.} \mid \text{backoff}) = \frac{2}{\pi}\cos^{-1}\left(\frac{R}{2s}\right) - \frac{R}{\pi}\sqrt{1 - \left(\frac{R}{2s}\right)^2}. \tag{2.8}$$

In the receiver sensing configuration the receiver senses the channel and informs its transmitter over a control channel whether to start its transmission. This adds an extra factor to the expression for the probability of outage, namely the relative position of the receiver of an incoming transmitter-receiver pair with respect to the active transmitter and the transmitter of the incoming transmitter-receiver pair.

The authors present the following bound on the probability of outage for

CSMA with receiver-sensing, referred to as CSMA-RX:

$$P_{\text{out}}^{\text{LB}}(\text{CSMA-RX}) = P_b + (1 - P_b) P_{\text{out}}^{\text{LB}}(\text{CSMA} \mid \text{no backoff}), \tag{2.9}$$

where P_b is the same as for the transmitter-sensing case, and $P_{\text{out}}^{\text{LB}}(\text{CSMA} \mid \text{no backoff})$ is the probability that an ongoing packet is received in outage:

$$\begin{aligned} &P_{\text{out}}^{\text{LB}}(\text{CSMA} \mid \text{no backoff}) \\ &= \int_0^{s^2} \int_{\alpha(d)}^{\gamma(d)} \frac{1}{2\pi} P(\text{active} \mid d, \phi) \pi \lambda e^{-\pi\lambda d^2} d\phi d(d^2). \end{aligned} \tag{2.10}$$

where $P(\text{active} \mid d, \phi)$, $\alpha(d)$ and $\gamma(d)$ are given as:

$$\begin{aligned} P(\text{active} \mid d, \phi) &= 1 - \frac{1}{\pi} \cos^{-1}\left(\frac{d^2 + 2R^2 - s^2 - 2Rd\cos\phi}{2R\sqrt{d^2 + R^2 - 2Rd\cos\phi}} \right), \\ \alpha(d) &= \cos^{-1}\left(\frac{d^2 + 2Rs - s^2}{2Rd} \right), \ \gamma(d) = 2\pi - \alpha(d). \end{aligned} \tag{2.11}$$

Their results show that slotted ALOHA performs better than unslotted ALOHA by a factor of two in terms of outage probability, which is consistent with results obtained with the conventional model for the slotted and unslotted ALOHA protocol [5]. Simulations of both ALOHA and CSMA are presented together with the obtained lower bounds. From their results, Kaynia and Jindal show that for low densities, CSMA-TX actually performs worse than unslotted ALOHA, having about 10% more outage probability. As the density increases, the use of CSMA becomes more advantageous. They also show that when the receiver is allowed to sense the channel before transmission, and decide whether to back off or not, the performance of the

CSMA protocol can be increased by approximately 23%.

In [16] Kaynia et al. consider the performance of ALOHA and CSMA in wireless ad hoc networks, where the total system bandwidth may be divided into smaller subbands. They consider generally the same network model as the work in [14], with the addition that each transmitter randomly selects a subband to transmit across. Given a fixed system and a requested transmission rate, they wish to find how many subbands the system bandwidth should be divided into, in order to minimize the probability of outage.

By incorporating subbands into the lower bound formulas for the outage probability of ALOHA and CSMA presented in [14], they find optimal values for the number of subbands that minimize the probability of outage. For the ALOHA protocol, an analytic expression for the optimal number of subbands N_{opt} is obtained, and is given as:

$$\begin{aligned} N_{\text{opt}} &= \frac{W}{2R_{\text{req}}\ln(2)}\left[\alpha + 2W_0\left(-\frac{1}{2}\alpha e^{-\alpha/2}\right)\right] \\ &= \frac{W}{2R_{\text{req}}\ln(2)}\left[\alpha + 2\sum_{n=1}^{\infty}\frac{(-n)^{n-1}}{n!}\left(-\frac{1}{2}\alpha e^{-\alpha/2}\right)^{n}\right], \end{aligned} \tag{2.12}$$

where W is the total system bandwidth, R_{req} is the requested transmission rate per link, α is the path loss exponent, and $W_0(\cdot)$ is the Lambert function.

Note that given a total system bandwidth and requested rate, the optimal number of subbands is only dependent on the path loss exponent α. For example, if $\alpha = 3$ and $R_{\text{req}}/W = 1/3$, the closest integer value for the optimal number of subbands is 4.

For the CSMA protocols (both transmitter-sensing and receiver-sensing) the optimal number of subbands are found through simulations. For $\alpha = 3$ and $R_{\text{req}} = 1/3$, the integer values of the optimal number of subbands for CSMA-TX and CSMA-RX are found to be 7 and 6, respectively.

In [22], Jindal et al. explore the tradeoff between bandwidth and SINR in ad hoc networks. Their model assume a total system bandwidth of W Hertz, and a fixed data rate R bps for each transmission. The total system bandwidth is divided into N subbands of size W/N Hertz, and they wish to answer the following question: How many subbands should the band be partitioned into to maximize the number of simultaneous transmissions in the network?

Their system model assumes that transmitting node locations are a realization of a homogeneous spatial Poisson process with intensity λ. Each transmitter communicates with a single receiver that is located a distance d meters away. All transmissions are constrained to have an absolute rate of R bps regardless of bandwidth. Furthermore, all multiuser interference is treated as noise. The channel is frequency flat, reflects path loss and possibly fast and/or slow fading, and is constant over the duration of a transmission. Their MAC protocol is in ALOHA fashion, where transmissions are independent and random. The transmitters have no CSI, and no transmission scheduling is performed.

They use a SINR based model, where the SINR of the receiver under observation is given by:

$$\text{SINR} = \frac{\rho d^{-\alpha} \mid h_0 \mid}{\eta + \sum_{k\in\Pi(\lambda)} \rho X_k^{-\alpha} \mid h_k \mid}, \tag{2.13}$$

where ρ is the transmission power, α is the path loss exponent ($\alpha > 2$), η is the noise power, X_k is the distance between the k-th interferer and the receiver under observation and h_k is the distance independent fading coefficient for the k-th interferer to the receiver under observation.

They use an outage-based transmission capacity framework, where an outage occurs whenever the SINR falls below a threshold value β, which in their model is equivalent with the mutual information falling below $\log_2(1+\beta)$. If the maximum intensity of *attempted* transmissions is $\lambda(\epsilon)$ such that the outage probability, for a fixed β, is no larger than the *outage constraint* ϵ, then the transmission capacity is defined as $c(\epsilon) = \lambda(\epsilon)(1-\epsilon)b$, which is the maximum density of *successful* transmissions times the spectral efficiency b of each transmission. Using results from [17], the maximum spatial intensity $\lambda(\epsilon)$ for small values of ϵ is given as:

$$\lambda(\epsilon) = \frac{c}{\pi d^2}\left(\frac{1}{\beta} - \frac{\eta}{\rho d^{-\alpha}}\right)^{\frac{2}{\alpha}} \epsilon + O\left(\epsilon^2\right). \tag{2.14}$$

Manipulating Shannon's channel capacity formula, β can be expressed as a function of the number of subbands, N:

$$\beta(N) = 2^{\frac{NR}{W}} - 1. \tag{2.15}$$

Plugging $\beta(N)$ into (2.14) with noise power $\eta = \frac{W}{N}N_0$ gives the maximum spatial intensity *per subband* for a particular value of N. Dropping the second term of (2.14) yields:

$$\lambda(\epsilon, N) = N\left(\frac{\epsilon}{\pi d^2}\right)\left(\frac{1}{\beta(N)} - \frac{1}{N \cdot SNR}\right)^{\frac{2}{\alpha}}, \tag{2.16}$$

where the constant $SNR = \frac{\rho d^{-\alpha}}{N_0 W}$ is the signal-to-noise ratio in the absence of interference when the entire band is used.

Assuming infinite SNR, they obtain:

$$\lambda(\epsilon, N) \approx \left(\frac{\epsilon}{\pi d^2}\right) N \cdot \beta(N)^{\frac{2}{\alpha}} \tag{2.17}$$

$$= \left(\frac{\epsilon}{\pi d^2}\right) N \cdot \left(2^{\frac{NR}{W}} - 1\right)^{\frac{2}{\alpha}}. \tag{2.18}$$

Maximizing this function with respect to the *per subband spectral efficiency* $\frac{NR}{W}$, yields the optimal spectral efficiency. Its solution is only dependent on the path loss exponent α:

$$\frac{NR}{W} = \log_2(e)\left[\frac{\alpha}{2} + W_0\left(-\frac{\alpha}{2}e^{-\alpha/2}\right)\right]. \tag{2.19}$$

The optimal spectral efficiency is very small for α close to 2 but then increases nearly linearly with α; for example, the optimal spectral efficiency for $\alpha = 3$ is 1.26 bps/Hz and for $\alpha = 4$ it is 2.3 bps/Hz. A network can operate at the optimal point by dividing the total available bandwidth into subbands sized such that the optimal spectral efficiency is reached on each subband.

As a result the optimal number of subbands is simply the optimal spectral efficiency divided by the the normalized (by total bandwidth) transmission rate.

Chapter 3

System Model

For analyzing the tradeoff between transmission rate and probability of outage in ALOHA and CSMA, we need a model for our random access network. In this chapter, we look at two different, but equivalent, models with randomly located users and random transmission times. Both models have previously been presented in [20], and shown to yield the same network performance. These random access models come close to representing a real wireless ad hoc network.

3.1 Model Specifications

We consider a model where transmitters are located on an infinite 2-D plane according to a homogeneous 2-D Poisson point process (PPP) with spatial density λ^s [nodes / m^2]. The Poisson distribution is given by:

$$f(k;\lambda^s) = \frac{(\lambda^s)^k}{k!}e^{-\lambda^s}, \tag{3.1}$$

which gives the probability that the event k occurs given the expected spatial density λ^s.

Each of the transmitters on plane receive packets in time according to an independent 1-D PPP with temporal density λ^t [packets / s], which is the expected density of packet arrivals at each node. Each packet is then transmitted to its own dedicated receiver, meaning that each receiver gets its packets from a single transmitter. The distance between each transmitter-receiver pair, denoted R, is fixed and equal for all pairs in the network. For a system with a Poisson arrival rate, the interarrival times are exponentially distributed with rate parameter equal to λ^t. The exponential distribution is given by

$$f\left(k;\lambda^t\right) = \lambda^t \cdot e^{-\lambda^t k}, \tag{3.2}$$

and yields the probability that the interarrival time between two packets is $1/k$, given that the expectation value for the interarrival time is $1/\lambda^s$. The packets are assumed to have a fixed duration T $[s]$. The density of receivers that have received a packet in the last T seconds is then $\lambda(T) = \lambda^s \cdot \lambda^t \cdot T$.

Although this model is easy to comprehend, it is difficult to analyze in terms of outage probability, because we would have to average over both the temporal and spatial statistics.

Let us look at an equivalent model, which will greatly simplify the analysis. We assume that packets arrive at a random point in space and time, and then disappear after the packet is sent on to the channel, regardless of whether the transmission is successful or not. In the model above, node locations are first fixed and then traffic is generated, while in this model, traffic is first generated, and with the arrival of each packet it is assigned to a transmitter-receiver pair, which is then randomly placed on the plane. This simplifies analysis, as we can now describe both spatial and temporal variations by a single process. In the following we describe this model in more details.

We consider a finite area A, and let packet arrivals be modeled by a 1-D PPP with arrival rate $(A/T)\lambda(T)$. Each packet is assigned to a random transmitter location (uniformly distributed on A), with its corresponding receiver located a distance R away, with random orientation. The parameter used in the Poisson distribution for this new model, $(A/T) \cdot \lambda(T) = A \cdot \lambda^s \cdot \lambda^t$, will indicate the temporal density of packet arrivals for *all* nodes on the plane. Let us look at this in more details. Recall that $\lambda(T) = \lambda^s \cdot \lambda^t \cdot T$, from the first model. This $\lambda(T)$ is a measure of active packets per unit area given packet duration T. Now, when we introduce a plane of area A, we can express the number of active packets on the plane as $A\lambda$. This corresponds to the number of receivers on the plane that have received a packet in the last T seconds. The packet arrival rate will then simply be $(A/T) \cdot \lambda(T)$. That is, if we place all possible packet transmissions between all transmitter-receiver

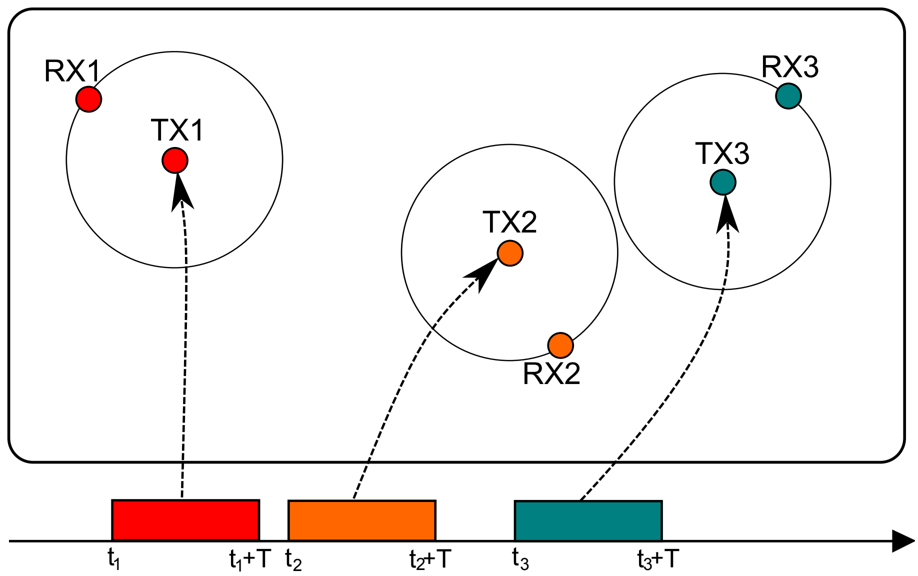

Figure 3.1: Packets are distributed according to a 1-D PPP with arrival rate $A \cdot \lambda/T$ and assigned to a randomly chosen transmitter-receiver pair on the plane with area A.

pair in a queue, the rate of these packet arrivals will be $(A/T) \cdot \lambda(T)$. Note that the number of packet arrivals during a time interval of T seconds follows Poisson$(A\lambda(T))$. When A is made large, this translates to a spatial density of $\lambda(T)$, which is the same as in the model with fixed position of nodes, which was initially discussed. Therefore, results generated with this model can be fairly compared to the first network model with density $\lambda(T)$.

All transmitters are assumed to transmit with equal signal power ρ over a bandwidth, W. We also assume that the channel is constant over the duration of a transmission (ignore fading) and only consider path loss attenuation effects (with $\alpha > 2$). The propagation delay is assumed to be negligibly small relative to the packet duration. Both the transmitters and receivers are assumed to use omni-directional antennas. Each receiver sees interference from all the other transmitters. These interference powers add at the receiver, together with channel noise η. We may then express the signal to interference plus noise ratio (SINR) for a given receiver as:

$$\text{SINR} = \frac{\rho R^{-\alpha}}{\eta + \sum_k \rho r_k^{-\alpha}}, \tag{3.3}$$

where r_k is the distance between the receiver under observation and the k-th interfering transmitter.

If this SINR falls below a certain threshold β, at any time during the packet transmission, the packet is received in outage. In practical systems, when a packet is received in outage, the transmitter will try to retransmit the same packet at a time later. No such retransmissions are applied in this model, as retransmissions will increase the complexity of the analysis. We can write the probability that a packet is received in outage as follows:

$$P_{out} = \Pr\left(\frac{\rho R^{-\alpha}}{\eta + \sum_k \rho r_k^{-\alpha}} < \beta\right). \tag{3.4}$$

That is, in order for a packet to be received correctly, we require that the received SINR is above the threshold β.

This outage definition can be transformed from an SINR requirement to a rate requirement through Shannon's capacity formula for AWGN channels, given by $C = W \log_2(1 + \text{SINR}_i)$, where C is the acheivable rate of transmission (also known as the capacity) for link i, W is the system bandwidth and SINR_i is the instantaneous SINR for link i.

This allows us to write the probability of outage as:

$$P_{out} = \Pr\left(C < R_{\text{req}}\right) \tag{3.5}$$
$$= \Pr\left[W\ \log_2(1 + \text{SINR}_i) < R_{req}\right]. \tag{3.6}$$

Outage is now defined in the following way: A packet transmission is considered to be in outage if the achievable rate of transmission C for link i is less than the requested rate of transmission R_{req}. In our network, all nodes communicate with the same transmission rate, R_{req}, determined by the number of bits per packet K divided by the packet duration T. The probability of outage can then be expressed as:

$$P_{out} = \Pr\left(C \leq \frac{K}{T}\right) \tag{3.7}$$
$$= \Pr\left(W\ \log_2(1 + \text{SINR}_i) < \frac{K}{T}\right) \tag{3.8}$$
$$= \Pr\left(\text{SINR}_i < 2^{\frac{K}{TW}} - 1\right). \tag{3.9}$$

That is, the definition of outage is equivalent to the SINR falling below the SINR threshold $\beta = 2^{\frac{K}{TW}} - 1$.

In the unslotted ALOHA protocol, the transmitter starts transmitting once a node has been placed on the plane, regardless of the channel condition. In a slotted ALOHA protocol, a transmitter starts its transmission in the next time slot after it has been placed on the plane. For the CSMA protocol, either the receiver (as in CSMA-RX) or the transmitter (as in CSMA-TX) senses the channel at the beginning of the packet, and if the SINR is below

β (equivalent to $C < R_{\text{req}}$), the transmitter cancels its transmission, i.e., the node backs off, if the measured SINR is below $\beta(T)$. Since no retransmissions are allowed in our model, this backoff is considered as an outage event.

Chapter 4

Outage probability: Analysis

In this chapter we will start off by clarifying the fundamental aspects of our analysis. We will then proceed by analyzing the impact of transmission rate in terms of probability of outage. We base our work on already existing formulas, and derive lower bounds on the outage probability of ALOHA and CSMA as a function of transmission rate. We also find an analytical expression for the optimal transmission rate that minimizes the probability of outage for ALOHA.

4.1 Packet Duration Trade-Offs

From our system model we have that, if the achievable rate on a transmitter-receiver link drops below the requested transmission rate R_{req}, any time

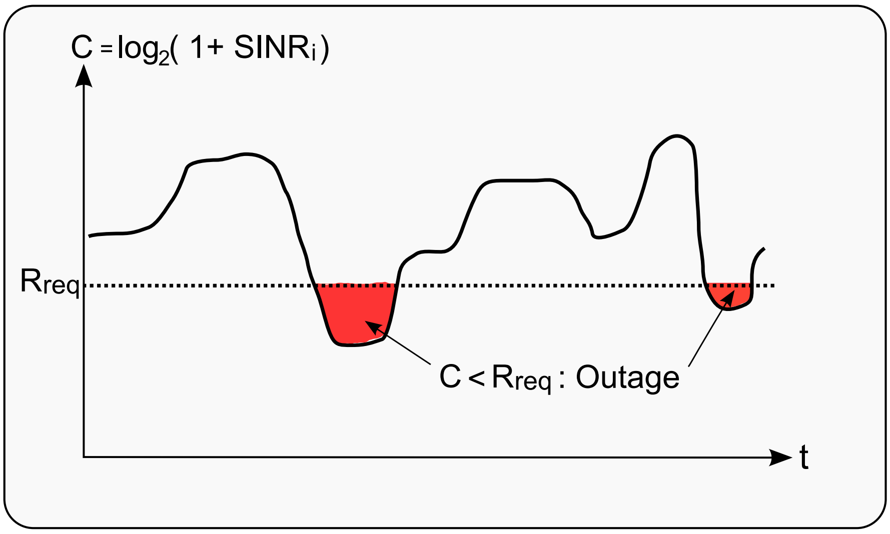

Figure 4.1: Instantaneous achievable rate for link i during a packet transmission. If C falls below R_{req} at any time during the transmission of the packet, the packet is considered to be received in outage.

during the transmission of a packet, the packet is received in outage.

The maximum achievable transmission rate C in which reliable communication is possible (known as the channel capacity) is, according to Shannon's capacity formula for AWGN channels, directly determined by the system bandwidth and the link SINR through:

$$C = W \log_2(1 + SINR). \tag{4.1}$$

By manipulating (4.1) we can, for any requested transmission rate R_{req}, determine the minimum required SINR, β, needed in order to communicate with an arbitrarily low bit error rate. We wish to transmit information packets of K bits over a system bandwidth of W Hertz. The duration of a packet is T seconds. The SINR threshold β as a function of packet duration is then given as:

$$\beta(T) = 2^{\frac{K}{TW}} - 1. \tag{4.2}$$

Since the SINR threshold β is directly linked to the required transmission rate, it is reasonable to assume that the probability of outage will be affected by the choice of R_{req}.

From (4.2) it is clear that as the packet duration T decreases, i.e., the requested transmission rate increases, the SINR threshold β also increases. An increase in β implies that a receiver can handle less interference before a packet is received in outage. Thus, intuitively increasing the probability of outage.

However, decreasing the packet duration will impact the density of active nodes on the plane. Recall that $\lambda(T) = \lambda_s \lambda_t T$ from our system model, which is the density of transmitters that have received a packet during the last T seconds. Assuming ALOHA, this is equivalent to the density of active transmitters on the plane, at a snapshot in time. Active transmitters are the source of the aggregate interference for a given receiver. Thus, reducing $\lambda(T)$, by decreasing the packet duration, will result in less aggregate interference and intuitively reduce the probability of outage.

From this, it becomes evident that the choice of packet duration will impact the network performance, and that there exists a tradeoff between aggregate interference and the SINR threshold β. We will examine this tradeoff by modifying already existing analytical formulas for the probability of outage of ALOHA and CSMA, to incorporate transmission rate and packet duration.

4.2 ALOHA

In [14], Kaynia and Jindal presented lower bounds for the outage probability of ALOHA and CSMA. These lower bounds are dependent on the SINR threshold β, through the radius s (see chapter 2.3). By inserting our expression for the SINR threshold, $\beta(T) = 2^{\frac{K}{TW}} - 1$ into (2.2), we can express s as a function of the packet duration T:

$$s(T) = \left(\frac{R^{-\alpha}}{2^{\frac{K}{T \cdot W}} - 1} - \frac{\eta}{\rho} \right)^{-\frac{1}{\alpha}}. \tag{4.3}$$

If we look at this formula in detail, we see that an increase of the packet duration T, i.e., reducing the SINR threshold, will lead to a reduction in the radius s. If we assume that there is only one interfering transmitter on the plane, s(T) is a measure of how close this interferer may be situated to the receiver under observation without the SINR at the receiver falling below $\beta(T)$. Since the interference power at the receiver is an decreasing function of the distance between the interferer and the receiver, a small s(T) corresponds to the receiver being able to handle higher levels of interference, compared to a large $s(T)$.

Note that, $s(T)$ must obtain real values, making the expression for $s(T)$ only valid when $\frac{R^{-\alpha}}{2^{\frac{K}{T\cdot W}}-1} > \frac{\eta}{\rho}$. Or equivalently, (4.3) is valid when $\beta < \frac{\rho R^{-\alpha}}{\eta}$. The expression $\frac{\rho R^{-\alpha}}{\eta}$ is the SINR in the absence of interference, which is also known as the signal to noise ratio (SNR). Even though the analytic expression for s(T) fails when it obtains a complex value, it is easy to interpret its physical result on the probability on outage. If $\beta(T)$ is larger than the SNR, the SINR will always be smaller than the threshold $\beta(T)$, which through our definition of outage leads to a probability of outage equal to one.

Inserting s(T) into the lower bounds for the outage probability of both slotted and unslotted ALOHA given in [14], we get:

$$\begin{aligned} &P_{\text{out}}^{\text{LB}}(\text{Slotted ALOHA}) \\ &\quad = 1 - e^{-\pi\lambda(T)s^2(T)}, \qquad \text{for } T > \frac{K}{W\log_2(1+\frac{\rho R^{-\alpha}}{\eta})}. \qquad (4.4) \\ &P_{\text{out}}^{\text{LB}}(\text{Slotted ALOHA}) \\ &\quad = 1, \qquad \text{for } T \leq \frac{K}{W\log_2(1+\frac{\rho R^{-\alpha}}{\eta})} \qquad (4.5) \end{aligned}$$

$$\begin{aligned} &P_{\text{out}}^{\text{LB}}(\text{Unslotted ALOHA}) \\ &\quad = 1 - e^{-2\pi\lambda(T)s^2(T)}, \qquad \text{if } T > \frac{K}{W\log_2(1+\frac{\rho R^{-\alpha}}{\eta})} \qquad (4.6) \\ &P_{\text{out}}^{\text{LB}}(\text{Unslotted ALOHA}) \\ &\quad = 1, \qquad \text{if } T \leq \frac{K}{W\log_2(1+\frac{\rho R^{-\alpha}}{\eta})} \qquad (4.7) \end{aligned}$$

Note that, if we ignore the noise in the network, (4.4) and (4.6) will be valid for all positive values of T. This is because the right side of $T > \frac{K}{W\log_2(1+\frac{\rho R^{-\alpha}}{\eta})}$ becomes zero, which makes the equations valid for $T > 0$.

4.2.1 Derivation of the Optimal Packet Duration

Now that we have found expressions for the outage probability as a function of the packet length, our next step is to minimize these expressions to find whether there exists an optimal packet duration. Firstly, let us introduce the term *spectral efficiency* $\mu(T) = \frac{K}{WT}$, which refers to the information rate in bits/s, that can be transmitted over given bandwidth. The spectral efficiency is in our work a measure of how efficiently a limited spectrum is utilized by our MAC protocols. In the following we assume a strictly interference-limited network, and set the noise power η to zero ($SNR = \infty$), resulting in a simplified expression for the radius s:

$$s(T) = R(2^{\frac{K}{WT}} - 1)^{1/\alpha}. \tag{4.8}$$

Inserting s(T) and $\mu(T)$ into (4.4), we obtain:

$$\begin{aligned} P_{\text{out}}^{\text{LB}}(\text{Slotted ALOHA}) &= 1 - e^{-\pi\lambda(T)s^2(T)} \\ &= 1 - e^{-\pi\lambda_s\lambda_t\frac{KR^2}{W\mu(T)}(2^{\mu(T)}-1)^{2/\alpha}}. \end{aligned} \tag{4.9}$$

Note that K and W remain constant, and that the dependence of the probability of outage on T is apparent through $\mu(T)$. To minimize the probability of outage, we differentiate (4.9) with respect to $\mu(T)$. Setting the derivative equal to 0 and solving for $\mu(T)$ we get, through some manipulations, the following optimal spectral efficiency:

$$\mu_{\text{opt}} = \frac{1}{2\ln(2)} \left[\alpha + 2W_0 \left(-\frac{1}{2} \alpha e^{-\alpha/2} \right) \right], \tag{4.10}$$

where $W_0(\cdot)$ is the principal branch of the Lambert W function. Note that (4.10) is equal to (4.10), and that the μ_{opt} is only dependent on the path loss exponent α.

The optimal packet duration T_{opt} as a function of K, W and μ_{opt} is then given by:

$$T_{\text{opt}} = \frac{K}{W} \cdot \frac{1}{\mu_{\text{opt}}} \tag{4.11}$$

$$= \frac{K}{W} \cdot \frac{2\ln(2)}{\left[\alpha + 2W_0 \left(-\frac{1}{2} \alpha e^{-\alpha/2} \right) \right]} \tag{4.12}$$

$$= \frac{K}{W} \cdot \frac{2\ln(2)}{\left[\alpha + 2 \sum_{n=1}^{\infty} \frac{(-n)^{n-1}}{n!} \left(-\frac{1}{2} \alpha e^{-\alpha/2} \right)^n \right]}. \tag{4.13}$$

Since the only difference in the expressions for unslotted and slotted ALOHA is the constant two in the exponent, which does not impact the result, the formulas for μ_{opt} and T_{opt}, in the equations (4.10) and (4.12), are valid for both slotted and unslotted ALOHA.

4.3 CSMA

In this section we will present lower bounds on the probability of outage, as a function of the packet duration, for our two versions of the CSMA random access protocol. As for the ALOHA protocols, the derived lower bound expressions are valid whenever $\beta(T)$ is less than the SNR of the system. If the T is too small, the value of $\beta(T)$ becomes larger than the SNR, and the probability of outage becomes one. This will not be expressed explicitly in the expressions presented below.

In the CSMA protocol, a transmitter backs off if the accumulated interference from all the other transmitters results in a SINR lower than the threshold value β at the beginning of a packet. The probability of this happening is called the backoff probability, P_b.

This probability of backoff P_b can be expressed, as a function of T, in terms of the Lambert W function as [14]:

$$P_b(T) = 1 - \frac{W_0(\pi\lambda(T)s^2(T))}{\pi\lambda(T)s^2(T)}. \tag{4.14}$$

We will consider two versions of the CSMA protocol. In the first protocol, referred to as CSMA-TX, the *transmitter* senses the channel upon a new packet arrival, and makes its decision on whether to transmit or back off based on its own SINR. In the second protocol, CSMA-TX, the *receiver* senses the channel and informs the transmitter whether or not to transmit, through a dedicated control channel.

Modifying the outage probability expressions derived in [14], we get the following expression for the outage probability of CSMA-TX:

$$P_{\text{out}}^{\text{LB}}(\text{CSMA-TX}) = P_b(T) + (1 - P_b(T))P_{\text{out}}^{\text{LB}}(\text{CSMA} \mid \text{no backoff}) \tag{4.15}$$
$$+ P_b(T)[1 - P_{\text{out}}^{\text{LB}}(\text{CSMA} \mid \text{no backoff})][1 - P_{\text{out}}^{\text{LB}}(\text{RX beg.} \mid \text{backoff})].$$

$P_{\text{out}}^{\text{LB}}(\text{CSMA} \mid \text{no backoff})$ is the probability that a packet is received in outage given an active transmitter-receiver pair, and is given by:

$$P_{\text{out}}^{\text{LB}}(\text{CSMA} \mid \text{no backoff}) \tag{4.16}$$
$$= \int_{(s(T)-R)^2}^{s^2(T)} \left[1 - \frac{1}{\pi}\cos^{-1}\left(\frac{d^2 + R^2 - s^2(T)}{2Rd}\right)\right] \pi\lambda(T)e^{-\pi\lambda(T)d^2}\, d(d^2)$$

Note that $P_{\text{out}}^{\text{LB}}(\text{CSMA} \mid \text{no backoff})$ is derived for values of $s(T) > R/2$. For the case where $s(T) < R/2$, there will be some minor changes to the expression, which can be found in [21]. However, we find the optimal packet duration to occur for the case when $s(T) > R/2$. Thus, we find it sufficient to only present the expression valid for $s(T) > R/2$.

Finally, $P_{\text{out}}^{\text{LB}}(\text{RX beg.} \mid \text{backoff})$ is the probability that the closest interferer, which is given to be inside $B(T_1, s(T))$, is also inside $B(R_1, s(T))$. That is:

$$P_{\text{out}}^{\text{LB}}(\text{RX beg.} \mid \text{backoff}) = \frac{2}{\pi}\cos^{-1}\left(\frac{R}{2s(T)}\right) - \frac{R}{\pi s(T)}\sqrt{1 - \left(\frac{R}{2s(T)}\right)^2} \tag{4.17}$$

Similarly, by modifying the outage probability expressions derived in [14], we get the following lower bound for the outage probability of CSMA-RX, as a function of packet duration:

$$P_{\text{out}}^{\text{LB}}(\text{CSMA-RX}) = P_b(T) + (1 - P_b(T))P_{\text{out}}^{\text{LB}}(\text{CSMA} \mid \text{no backoff}), \tag{4.18}$$

where $P_b(T)$ is the same as for the transmitter-sensing case, and $P_{\text{out}}^{\text{LB}}(\text{CSMA} \mid \text{no backoff})$ is the probability that an ongoing packet is received in outage:

$$\begin{aligned} &P_{\text{out}}^{\text{LB}}(\text{CSMA} \mid \text{no backoff}) \\ &= \int_0^{s(T)^2} \int_{\alpha(d)}^{\gamma(d)} \frac{1}{2\pi} P(\text{active} \mid d, \phi) \pi \lambda(T) e^{-\pi\lambda(T)d^2} d\phi d(d^2) \end{aligned} \tag{4.19}$$

where $P(\text{active} \mid d, \phi)$, $\alpha(d)$ and $\gamma(d)$ are given as:

$$\begin{aligned} &P(\text{active} \mid d, \phi) = 1 - \frac{1}{\pi} \cos^{-1}\left(\frac{d^2 + 2R^2 - s(T)^2 - 2Rd\cos\phi}{2R\sqrt{d^2 + R^2 - 2Rd\cos\phi}}\right), \\ &\alpha(d) = \cos^{-1}\left(\frac{d^2 + 2Rs(T) - s^2(T)}{2Rd}\right), \ \gamma(d) = 2\pi - \alpha(d) \end{aligned} \tag{4.20}$$

Note that these expressions are generally the same as the expressions found in [14]. Detailed derivations of these expressions can be found in [20], with the only difference being that we express the radius s and the density λ explicitly as a function of the packet duration T. These modifications are done in order to clearly point out where the packet duration T plays a part in the probability of outage. We have not managed to find analytic expressions for the optimal packet duration of CSMA-TX and CSMA-RX. Fortunately, these optimal values may be observed through the simulations presented in the following chapter.

Chapter 5

Outage Probability: Numerical Results

In order to validate our analytic work we will in this chapter present our analytic expressions for ALOHA and CSMA together with simulation results. We also perform a comparison of the performance differences of the four protocols considered in chapter 4. Firstly, let us take a look at the simulation model.

5.1 Simulation model

Our simulations are based on the MATLAB code found in [20]. The simulations follow the system model closely. Packets arrive according to a PPP in time. Each packet is then assigned to a transmitter, which is randomly placed on a finite plane of area A. The number of transmitters on the plane follow a PPP with the expected number of nodes on the entire plane equal to $\lambda^s \cdot \lambda^t \cdot T \cdot A$. Active nodes start their transmission once they are placed on the plane. In the ALOHA protocol the nodes are set active once they have received a packet, while in the CSMA protocol only nodes that have received a packet and sensed their SINR to be above β, are considered active. The active nodes remain on the plane for the duration of a packet, T, before they disappear.

Since we assumed, in parts of our analytic work, an interference-limited network, we have in our simulations set the noise power $\eta = -150$ [dBW], which allows us to compare our analytical expressions with our simulation results. The transmit power ρ and the distance between the transmitter and receiver are chosen such that the received power is 1 at the distance 1 unit away from the transmitter. In our numerical results we have fixed the packet length K [bits] and system bandwidth W [Hz] and let the packet duration T [seconds] vary. The SINR threshold, β, is then determined from T. Table 5.1 shows the system parameters used for the simulations, unless otherwise specified.

Table 5.1: Simulation parameters

Parameter	Value	Unit	Description
α	[2.5, 6]	-	Path loss exponent
R	1	Meters	Distance between TX and designated RX
L	100	Meters	Length of each side in a LxL plane
ρ	0	dBW	Transmit power
W	1	Hertz	System bandwidth
K	100	Bits	Packet length
T	[0 250]	Seconds	Packet duration
η	-150	dBW	Noise power
λ^s	10^{-3}	Nodes/m^2	Spatial density of nodes
λ^t	0.1	Packets/s	Temporal density of packets

5.2 ALOHA

5.2.1 Outage Performance vs. Packet Duration

In Figure 5.1, we have plotted the analytical lower bound expressions for unslotted and slotted ALOHA, with path loss exponent $\alpha = 3$, as a function of the packet duration T, together with simulation results. We see that the analytic expressions follow the simulation results tightly. Also, we observe that slotted ALOHA outperforms unslotted ALOHA by a factor two, which is consistent with results obtained with the conventional model for the slotted and unslotted ALOHA protocol [5].

Let us try to explain how the packet duration affects the probability of outage in our network. Recall that the density of active transmitters on the plane, which are the sources of interference, is given by $\lambda(T) = \lambda_s \lambda_t T$. Thus, the

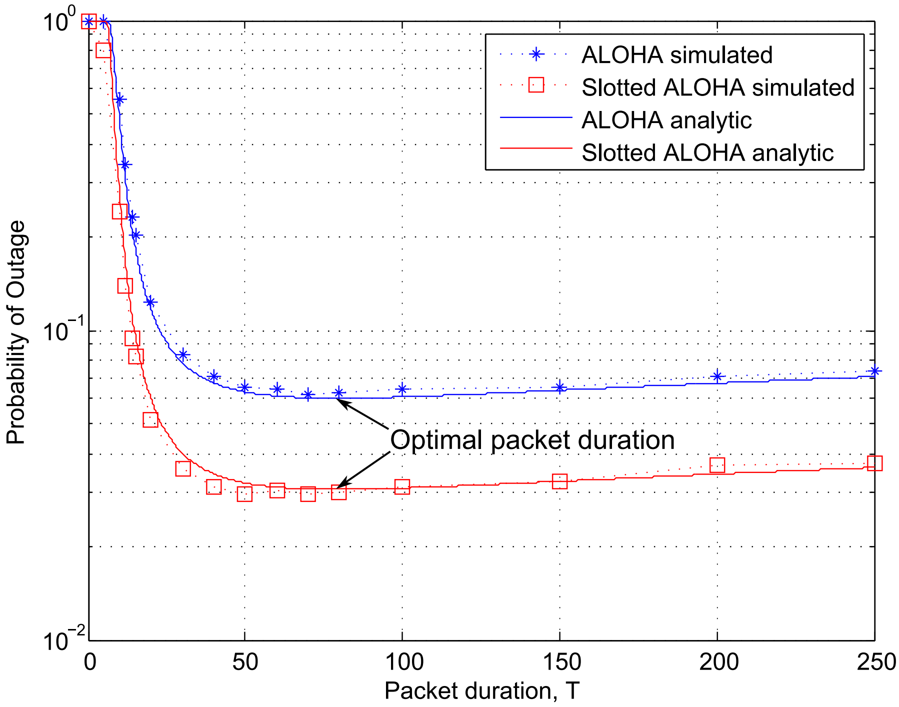

Figure 5.1: Probability of outage versus packet duration for the ALOHA protocols, with a fixed spatial node density $\lambda_s = 10^{-3}$ and $\alpha = 3$.

level of interference in the network, is directly linked to the choice of T. It is also clear that $\lambda(T)$ is a linearly increasing function of T. The SINR threshold is given by $\beta(T) = 2^{\frac{K}{TW}} - 1$, and is a monotonically decreasing function of T.

The tradeoff between $\lambda(T)$ and $\beta(T)$ can be recognized in the exponent of our analytical lower bound for the probability of outage, namely through the product $\lambda(T) \cdot s^2(T)$ (see (4.4) and (4.6)). The first factor represents the interference in the network, and the second factor represents the systems sensitivity to interference. The optimal tradeoff between the two quantities is obtained when the product of the two factors is minimized.

As T approaches zero, the probability of outage approaches one. For the case where noise is present in the system (i.e., $SNR < \infty$), we already know that if $T \to 0$, $\beta(T)$ approaches infinity, making $\beta(T)$ larger than the SNR. Thus, the probability of outage is equal to one, by definition. For the case where we ignore the noise (i.e., we set the $SNR = \infty$), the probability of outage can be shown to be equal to one by taking the limit of the product $\lambda(T) \cdot s^2(T)$, as T goes towards zero.

$$\begin{aligned} &\lim_{T \to 0} \lambda(T) \cdot s^2(T) \\ &= \lim_{T \to 0} \lambda_s \lambda_t T R^2 (2^{\frac{K}{WT}} - 1)^{2/\alpha} \; = \infty, \qquad \text{for } \alpha > 2. \end{aligned} \tag{5.1}$$

An infinite value for the product $\lambda(T) \cdot s^2(T)$ in the exponent of (4.4) or (4.6), will result in a probability of outage equal to one. This may be explained by the fact that as the packet duration goes towards zero, the increase in required SINR, determined by $\beta(T)$, dominates the benefit that a decreased packet duration does for the interference levels in the network, making the probability of the SINR on the channel being below $\beta(T)$ approach one.

On the other hand, as T increases above the optimal point, the increase of interference in the network, caused by the increasing density of interfering transmitters on the plane, dominates the receiver's ability to handle higher levels of interference (as $\beta(T)$ decreases). Again, the probability of the SINR on the channel being below the required SINR will approach one. This can be shown analytically, by taking the limit of the product $\lambda(T) \cdot s^2(T)$, as T

goes towards infinity.

$$\begin{aligned} &\lim_{T\to\infty} \lambda(T)\cdot s^2(T) \\ &= \lim_{T\to\infty} \lambda_s\lambda_t T \left(\frac{R^{-\alpha}}{2^{\frac{K}{T\cdot W}}-1} - \frac{\eta}{\rho} \right)^{-\frac{2}{\alpha}} = \infty, \qquad \text{for } \alpha > 2. \end{aligned} \tag{5.2}$$

Inserting the result of the limit into the exponent in the lower bound expressions for the two ALOHA protocols gives us an outage probability equal to one.

The optimal packet duration occurs when there is an optimal tradeoff between interference in the network and the the minimum SINR requirement β, and is presented in the following section.

5.2.2 The Optimal Packet Duration

From Figure 5.1, we see that the simulation results verify that there exists an optimal packet duration where the probability of outage is minimized. For both protocols the probability of outage minimized when $T = 79.3$ s. Inserting the system parameters used in the simulations into (4.12), we find the optimal packet duration of the ALOHA protocols to be $T_{opt} = 79.288$ s. The reason why the optimal value of slotted ALOHA coincides with unslotted ALOHA, is that the introduction of slots does not change the interference levels in the network, which is dependent on $\lambda(T)$, nor the SINR threshold $\beta(T)$. It will simply halve the period of time where a packet is vulnerable to interference, and thus halving the probability of outage.

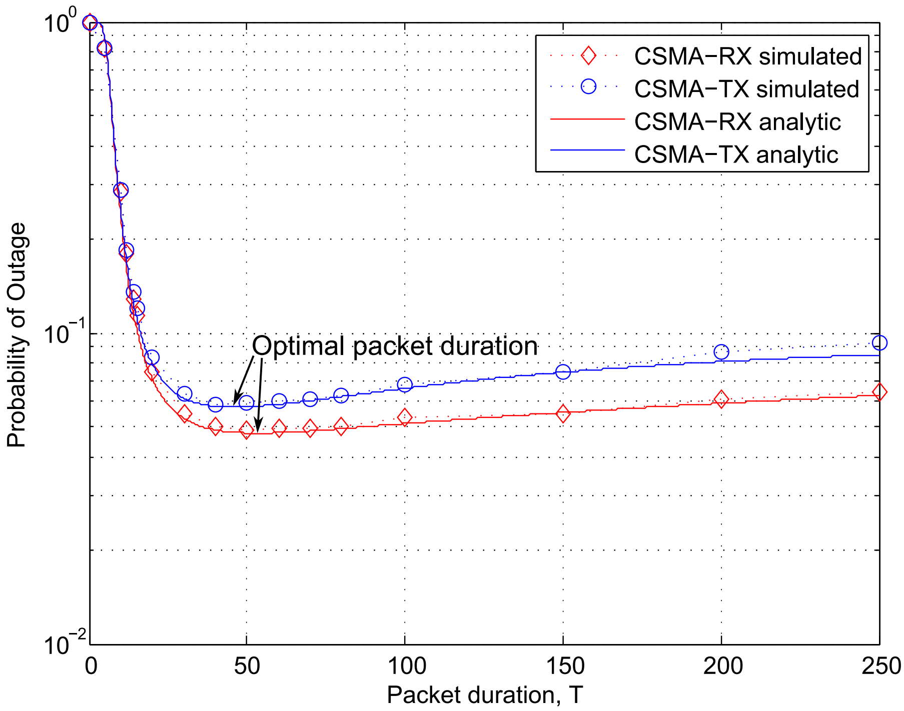

Figure 5.2: Probability of outage versus packet duration for CSMA-RX and CSMA-TX, for a fixed spatial node density $\lambda_s = 10^{-3}$ and $\alpha = 3$.

5.3 CSMA

5.3.1 Outage Performance vs. Packet Duration

In Figure 5.2, the analytical lower bound expressions for CSMA-TX and CSMA-RX are plotted as a function of packet duration T, together with the simulation results. We see that the analytical lower bound for the two versions of the CSMA protocol follow the simulation results tightly. From the figure, we also observe that CSMA-RX achieves a lower probability of outage compared to CSMA-TX, which is consistent with the results obtained in [14].

In CSMA-TX protocol, the transmitter senses the channel and decides to back off based on its own SINR, which might not be a good estimation of the SINR at the receiver. This may lead the case where the transmitter backs off, even though the packet may have been received correctly at the receiver. The CSMA-RX protocol on the other hand, only drops a packet transmission if the *actual* SINR at the receiver is too low for the packet to be received correctly. Thus, CSMA-RX outperforms CSMA-TX in terms of probability of outage.

5.3.2 The Optimal Packet Duration

From Figure 5.2, we observe that the optimal packet duration for CSMA-TX is $T = 46$ seconds, while CSMA-RX achieves its optimal packet duration at $T = 53.5$ seconds. This corresponds to optimal spectral efficiencies of 2.17 bit/s/Hz and 1.86 bit/s/Hz, respectively.

5.4 Comparing ALOHA and CSMA

In Figure 5.3 the probability of outage is plotted as a function of packet duration, for both the ALOHA and CSMA MAC protocols, with system parameters $\alpha = 3$, $\lambda_s = 10^{-3}$, $K = 100$ bits and $W = 1$ Hz. We see that the slotted ALOHA protocol performs a factor two better that the pure ALOHA protocol, and is superior to both versions of the CSMA protocol.

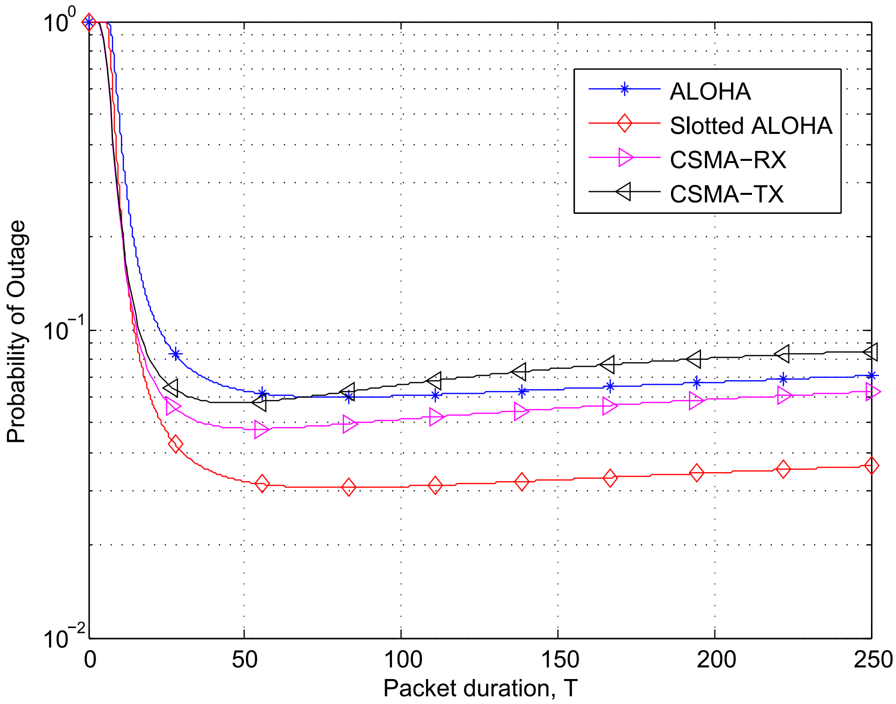

Figure 5.3: Probability of outage vs. transmission time with $\alpha = 3$ for all four MAC-protocols.

Both versions of the CSMA protocols obtains their minimum values at lower packet durations, hence obtaining higher optimal spectral efficiencies than ALOHA. Of the two versions of the CSMA protocol, CSMA-RX achieves the lowest probability of outage.

It is interesting to notice that for higher transmission times, CSMA-TX performs worse than the ALOHA protocol, as was also concluded in [14] and [16]. This can be explained by the "hidden node problem" and the "exposed node problem" described in chapter 2.2.3. The hidden node problem can be reduced by the channel sensing mechanism of the CSMA protocol, and is the reason why the CSMA protocol is an improvement over the ALOHA protocol. A large sensing radius, $s(T)$, will reduce the number of hidden nodes in the network. However, when the transmitter senses the channel, as in the CSMA-TX protocol, it introduces the "exposed node problem". This problem occurs when the transmitter decides to back off, even though its transmission would have been received correctly, In our model, where a back off is equivalent to outage, this event increases the overall outage probability. When T is small, corresponding to a large $s(T)$, this event is rather rare. However, as T increases the number of exposed nodes in the network increases. When T get sufficiently large, the increase in exposed nodes dominates decrease of hidden nodes in the network. This results in an increased probability of outage, even past the point where the CSMA-TX protocol is outperformed by ALOHA protocol. From the figure, we see that in order for the CSMA-TX protocol to be advantageous over ALOHA, it must be operated close to its optimal packet duration.

As we have seen, the key factor to minimizing the probability of outage, is the choice of spectral efficiency for the system. That is, if we want to transmit a packet of a certain length (in bits) over an ad hoc network, and our goal is to minimize the outage probability, we should choose the packet duration and system bandwidth such that the spectral efficiency equals the optimal spectral efficiency for the considered MAC protocol. It is also apparent from our results, that choosing the packet duration too small, involves a much larger penalty than choosing the packet duration too large.

In Figure 5.4 we have plotted the optimal spectral efficiency as a function

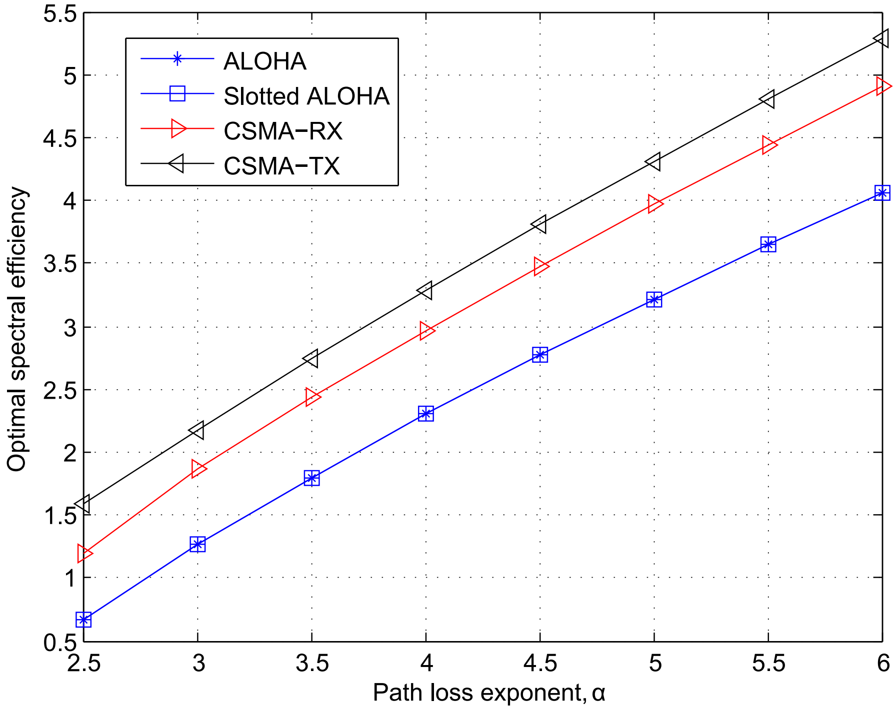

Figure 5.4: Optimal spectral efficiency vs. path loss exponent. The optimal spectral efficiency is an approximately linear function of the path loss exponent.

of the path loss exponent α, for our four MAC protocols. For all four MAC protocols, we see that the optimal spectral efficiency is an approximately linear function of the path loss exponent α. We also see that CSMA obtains a higher optimal spectral efficiency than the ALOHA protocol, for a given path loss exponent. The highest spectral efficiency is obtained for CSMA-TX, but as we have seen, at a higher probability of outage than CSMA-RX.

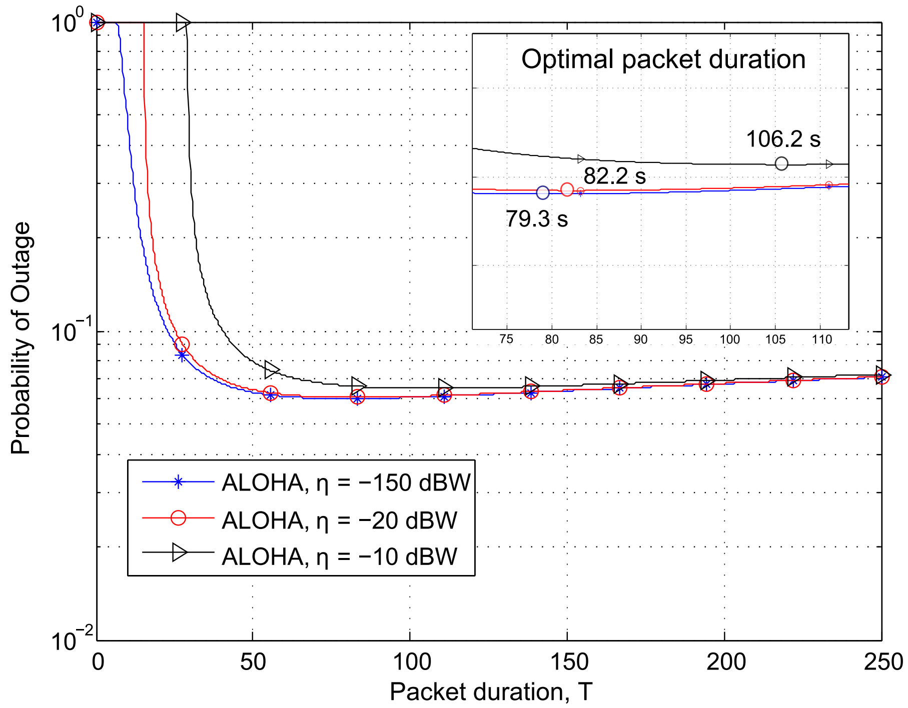

Figure 5.5: Probability of outage vs. packet duration for the ALOHA protocol, for three different values of noise power. As the noise power increases, the optimal packet duration also increases.

5.5 The Impact of Non-Ignorable Noise

The optimal packet durations and optimal spectral efficiencies presented in our results, is for the case where we have set the $SNR = \infty$. When the noise power of the system is not ignorable, the values for the optimal spectral efficiencies will be lower. In Figure 5.5, we have plotted the probability of outage for the ALOHA protocol, with three different values for the noise power η. We observe that the optimal packet duration remains almost unchanged when the noise power increases from $\eta = -150$ dBW to $\eta = -20$ dBW. However, as the noise power increases further, up to $\eta = -10$ dBW, the optimal packet duration changes significantly. Fortunately, unless we approach the area where $\beta(T)$ gets comparable with the SNR, the penalty of the increasing noise levels is rather small in terms of probability of outage.

Chapter 6

Conclusions

6.1 Main Findings

In this thesis, we have investigated how the choice of transmission rate in ad hoc wireless networks, utilizing either ALOHA or CSMA as the underlying MAC protocol, affects the probability of outage. We use an SINR based model, where packets arrive randomly in space and time according to a Poisson point process. Packets are transmitted with a requested rate R_{req}, determined by the packet length K [bits] divided by the packet duration T [seconds]. A packet is considered successfully received if the channels capacity does not fall below R_{req} during the transmission of the packet. Otherwise, the packet is considered to be received in outage.

We incorporate transmission rate into already existing lower bounds on the probability of outage for ALOHA and CSMA, and use these expressions to find the optimal packet duration that minimizes the probability of outage in our network. For the ALOHA protocol, we derive an analytic expression for the optimal spectral efficiency of the network, which we use to find the optimal packet duration (given packet length and system bandwidth) that minimizes the probability of outage.

We find that in order to minimize the probability of outage we should choose our system parameters such that our requested transmission rate divided by the system bandwidth is equal to the optimal spectral efficiency of our network. For a purely interference-limited network, this optimal spectral efficiency is only dependent on the choice of MAC-protocol and the channels path loss exponent. For the ALOHA protocol we find an analytic expression for the optimal spectral efficiency to be:

$$\mu_{\text{opt}} = \frac{1}{2\ln(2)}\left[\alpha + 2W_0\left(-\frac{1}{2}\alpha e^{-\alpha/2}\right)\right]. \tag{6.1}$$

We were not able to find equivalent analytic expressions for the two versions of the CSMA protocol. However, through simulations, we find optimal spectral efficiencies for both CSMA-RX and CSMA-TX.

6.2 Future Work

A natural continuation of this project, would be to derive expressions for the optimal spectral efficiency and the optimal packet duration of our two versions of the CSMA protocol.

Some other potential research topics for further investigation could be to:

- Incorporate retransmissions in the model. This would alter the density of interferers on the plane, possibly impacting the optimal packet duration.
- Let the packet duration, T, adapt to the quality of the channel, and investigate the results. That is, let each transmitter transmit with a rate given by the channel capacity, based on SINR measurements at the beginning of a packet transmission. This may possibly improve the probability of outage for the network.
- In a power-constrained setting, adapt the transmission power, ρ, to the quality of the channel, i.e., transmit with lower power when the channel condition is good, and if too much interference is present, use more transmit power. This would effect the interference distributions in the network, and possibly also the optimal packet duration.

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Appendix

A MATLAB Code

The following sections include the MATLAB codes for simulating the ALOHA and CSMA MAC protocols, resulting in the graphs presented in Chapter 5.

A.1 MATLAB code for P_{out} vs. T

```
%******Initializing system parameters******************

P_TX = 1;                                % Transmitted power
R = 1;                  % Distance between RX and TX pair
noise =10^-15;                                % Noise power;
alpha = 3;                % Path loss exponent (alpha>2)
W=1;                                    % System bandwidth
L = 100;                                  % Length of area LxL
```

```
T =[0.1 5 10:2:14 15 20:10:100 150:50:250];%Packet lengths
lambda_t = 0.1;                    % Poisson rate in time
lambdas_vec = 10^-3;           % Spatial density of nodes
PL=100;                           % Packet length in bits

%------------ALOHA and CSMA network simulation---------------
packets = 20000;                          % Number of packets

K = length( lambdas_vec );
max_inst = 2;
outage_csma_rx = zeros(1,K);
outage_csma_tx = zeros(1,K);
outage_aloha = zeros(1,K);
outage_saloha = zeros(1,K);
SINRo=zeros(1,length(T));
s=zeros(1,length(T));

for q=1:length(T)
    SINRo(q) = 2^(PL/(T(q)*W))-1;
    s(q) = ( (R^-alpha)/SINRo(q) - noise/P_TX )^(-1/alpha);

for l = 1:K

 outage_csma_inst_rx = zeros(1,max_inst);
 outage_csma_inst_tx = zeros(1,max_inst);
 outage_aloha_inst = zeros(1,max_inst);
 outage_saloha_inst = zeros(1,max_inst);

 for inst = 1:max_inst
  TX_X = zeros(1,packets);
```

```
TX_Y = zeros(1,packets);
RX_X = zeros(1,packets);
RX_Y = zeros(1,packets);
arrival_time = zeros(1,packets);
arrival_slot = zeros(1,packets);
transmit_rx = ones(1,packets);
transmit_tx = ones(1,packets);

interf_csma_rx = zeros(1,packets);
meas_interf_csma_tx = zeros(1,packets);
true_interf_csma_tx = zeros(1,packets);
interf_aloha = zeros(1,packets);
interf_saloha = zeros(1,packets);
SINR_csma_rx = zeros(1,packets);
meas_SINR_csma_tx = zeros(1,packets);
true_SINR_csma_tx = zeros(1,packets);
SINR_aloha = zeros(1,packets);
SINR_saloha = zeros(1,packets);

outage_csma_occured_rx = zeros(1,packets);
outage_csma_occured_tx = zeros(1,packets);
outage_aloha_occured = zeros(1,packets);
outage_saloha_occured = zeros(1,packets);

arrival_time(1) = 5*rand + exprnd( 1/...
    (lambdas_vec(l)*L^2*lambda_t) );

for current = 1:packets

 arrival_slot(current)=roundup2(...
```

```
    arrival_time(current),T(q));
%***Positioning TXs and RXs uniformly*********
 TX_X(current) = L*rand;
 TX_Y(current) = L*rand;
 theta = 2*pi*rand;
 RX_X(current) = TX_X(current) + R*cos( theta );
 RX_Y(current) = TX_Y(current) + R*sin( theta );

 if( current > 1 )
  [x,departs] = find( arrival_time+T(q) > ...
  arrival_time(current-1) & arrival_time+T(q)...
  < arrival_time(current) );
 else
  departs = [];
 end

%**Find interference for the current packet arrival**
for pck = 1:(current-1)
  if( arrival_time(pck)+T(q) > ...
          arrival_time(current) )
   r = sqrt( (RX_X(current)-TX_X(pck))^2 ...
     + (RX_Y(current)-TX_Y(pck))^2 );
   interf_aloha(current)=interf_aloha(current)...
       + P_TX*r^(-alpha);

    if( transmit_rx(pck) == 1 )
    interf_csma_rx(current) = ...
        interf_csma_rx(current)...
     + P_TX * r^(-alpha);
    end
```

```
        if( transmit_tx(pck) == 1 )
        true_interf_csma_tx(current)= ...
            true_interf_csma_tx(current)...
         + P_TX * r^(-alpha);
        r = sqrt( (TX_X(current)-TX_X(pck))^2 ...
         + (TX_Y(current)-TX_Y(pck))^2 );
        meas_interf_csma_tx(current) = ...
            meas_interf_csma_tx(current)...
         + P_TX * r^(-alpha);
        end

      end

    if arrival_slot(pck)==arrival_slot(current)
      r = sqrt( (RX_X(current)-TX_X(pck))^2 + ...
          (RX_Y(current)-TX_Y(pck))^2 );
      interf_saloha(current)=interf_saloha(current)...
          + P_TX*r^(-alpha);
    end
  end

SINR_aloha(current)= P_TX*R^(-alpha) ...
    /(noise+interf_aloha(current));
SINR_saloha(current)= P_TX*R^(-alpha)...
    /(noise+interf_saloha(current));
SINR_csma_rx(current)= P_TX*R^(-alpha)...
    /(noise+interf_csma_rx(current));
meas_SINR_csma_tx(current)=P_TX*R^(-alpha) ...
    /(noise+meas_interf_csma_tx(current) );
true_SINR_csma_tx(current) = P_TX * R^(-alpha)...
```

```
 / ( noise + true_interf_csma_tx(current) );

if( SINR_aloha(current) < SINRo(q) )
 outage_aloha_occured(current) = 1;
end

if( SINR_saloha(current) < SINRo(q) )
 outage_saloha_occured(current) = 1;
end

if( SINR_csma_rx(current) < SINRo(q) )
 transmit_rx(current) = 0;
 outage_csma_occured_rx(current) = 1;
end



if( meas_SINR_csma_tx(current) < SINRo(q) )
  transmit_tx(current) = 0;
  outage_csma_occured_tx(current) = 1;
end

if( true_SINR_csma_tx(current) < SINRo(q) )
  outage_csma_occured_tx(current) = 1;
end

%*Interference correction because of packet departures*
 for pck = 1:(current-1)
  for i = 1:length(departs)
     if( arrival_time(departs(i))+T(q) > ...
             arrival_time(pck) ...
```

```
            && arrival_time(departs(i))+T(q)<...
            arrival_time(pck) +...
            T(q) && departs(i)<pck )
        r = sqrt( (RX_X(pck)-TX_X(departs(i)))^2 +...
            (RX_Y(pck)-TX_Y(departs(i)))^2 );
        interf_aloha(pck) = interf_aloha(pck) - ...
            P_TX * r^(-alpha);

      if( transmit_rx(departs(i)) == 1 )
       interf_csma_rx(pck) = interf_csma_rx(pck) -...
           P_TX * r^(-alpha);
      end
      if( transmit_tx(departs(i)) == 1 )
      true_interf_csma_tx(pck) = ...
          true_interf_csma_tx(pck) - P_TX *...
          r^(-alpha);
      end
    end
  end

%***Interference because of new packet arrival****
 if( arrival_time(pck)+T(q) > arrival_time(current) )
  r = sqrt( (RX_X(pck)-TX_X(current))^2 + ...
   (RX_Y(pck)-TX_Y(current))^2 );
  interf_aloha(pck) = interf_aloha(pck) + ...
      P_TX * r^(-alpha);

    if( transmit_rx(current) == 1 )
    interf_csma_rx(pck)=interf_csma_rx(pck) +...
        P_TX * r^(-alpha);
```

```
 end
 if( transmit_tx(current) == 1 )
 true_interf_csma_tx(pck)=true_interf_csma_tx(pck)...
     + P_TX * r^(-alpha);
 end
end

if arrival_slot(pck) == arrival_slot(current)
r = sqrt( (RX_X(pck)-TX_X(current))^2 +...
    (RX_Y(pck)-TX_Y(current))^2);
interf_saloha(current)=interf_saloha(current) +...
    P_TX*r^(-alpha);
end

SINR_aloha(pck) = P_TX * R^(-alpha)...
    / ( noise + interf_aloha(pck));

if( SINR_aloha(pck) < SINRo(q) )
outage_aloha_occured(pck) = 1;
end

SINR_saloha(pck) = P_TX * R^(-alpha)...
    / ( noise + interf_saloha(pck) );
if( SINR_saloha(pck) < SINRo(q) )
outage_saloha_occured(pck) = 1;
end

SINR_csma_rx(pck) = P_TX * R^(-alpha)...
    / ( noise + interf_csma_rx(pck) );
if( SINR_csma_rx(pck) < SINRo(q) )
```

```
    outage_csma_occured_rx(pck) = 1;
    end

    true_SINR_csma_tx(pck) = P_TX * R^(-alpha)...
        / ( noise + true_interf_csma_tx(pck) );
    if( true_SINR_csma_tx(pck) < SINRo(q) )
    outage_csma_occured_tx(pck) = 1;
    end

   end
    arrival_time(current+1) = arrival_time(current)...
        + exprnd( 1/(lambdas_vec(l)*L^2*lambda_t) );
 end

 outage_csma_inst_rx(inst) = sum(...
     outage_csma_occured_rx)/packets;
 outage_csma_inst_tx(inst) = sum(...
     outage_csma_occured_tx)/packets;
 outage_aloha_inst(inst) = sum(...
     outage_aloha_occured)/packets;
 outage_saloha_inst(inst) = sum(...
     outage_saloha_occured)/packets;

end

%***********Calculating outage*******

   outage_csma_rx(l) = sum(...
       outage_csma_inst_rx)/max_inst;
   outage_csma_tx(l) = sum(...
```

```
        outage_csma_inst_tx)/max_inst;

    outage_aloha(l) = sum(...
        outage_aloha_inst)/max_inst;
    outage_saloha(l) = sum(...
        outage_saloha_inst)/max_inst;

end
    outage_csma_varT_rx(q)=outage_csma_rx(l);
    outage_csma_varT_tx(q)=outage_csma_tx(l);

    outage_aloha_varT(q) =outage_aloha(l);
    outage_saloha_varT(q) =outage_saloha(l);
end

figure(1);
semilogy(T,outage_aloha_varT,'b:*');
hold on; grid on;
semilogy(T,outage_saloha_varT,'r:s')
xlabel('Packet duration, T')
ylabel('Probability of Outage')

figure(2)
semilogy(T,outage_csma_varT_rx,'r:d');
hold on; grid on;
semilogy(T,outage_csma_varT_tx,'b:o');

xlabel('Packet duration, T')
ylabel('Probability of Outage')
```

```
%********LB : ALOHA***************************'
stepz=10;
Tt=0:stepz:250;
alpha=3;
SINRo = 2.^(PL./(Tt*W))-1;

for m=1:length(alpha)

complex=find(SINRo > (P_TX*R^(-alpha))/...
    noise);
s = ( ((R^-alpha(m))./SINRo) - (noise/P_TX) ).^...
    (-1/alpha(m));
lambdas = lambdas_vec*lambda_t.*Tt;

%Pure ALOHA
Pout_aloha = 1 - exp(-2.*lambdas.*pi.*s.^2);
Pout_aloha(complex) = 1;
%Slotted ALOHA
Pout_saloha = 1 - exp(-lambdas.*pi.*s.^2);
Pout_saloha(complex) = 1;

%decimate for markers
dec=floor(linspace(1,length(s),10));
for u=1:length(dec)
Pout_aloha_plot(u)=Pout_aloha(dec(u));
Pout_saloha_plot(u)=Pout_saloha(dec(u));
end
dec=(dec.*stepz)-stepz;

figure(1)
```

```
semilogy(Tt,Pout_aloha,'b-');
semilogy(Tt,Pout_saloha,'r-');
legend('ALOHA simulated', ...
    'Slotted ALOHA simulated',...
    'ALOHA analytic',...
    'Slotted ALOHA analytic')

%****LB : CSMA TX- and RX-sensing*********
steps=10;
t=[0:steps:250];
SINRo2 = 2.^(PL./(t*W))-1;
complex=find(SINRo > (P_TX*R^(-alpha))...
    /noise);
s2 = ( ((R^-alpha(m))./SINRo2) - ...
    (noise/P_TX) ).^(-1/alpha(m));
s2(complex)=10^10; %workaround:...
%set s large instead of complex valued
prob_backoff=zeros(1,length(s2));
for q=1:length(s2)
fh = @(x) quadl(@(phi) ( 1/(2*pi)*(1 - 1/pi*acos( ...
    (x+2*R^2-s2(q)^2-2*R*sqrt(x).*cos(phi))...
    ./(2*R*sqrt(x+R^2-2*R*sqrt(x).*cos(phi))))) )...
    .*(pi*lambdas_vec*lambda_t*t(q)*...
    exp(-pi*lambdas_vec*...
    lambda_t*t(q)*x)), acos( ...
    (x-s2(q)^2+2*R*s2(q))./(2*R*sqrt(x)) ),...
    (2*pi - acos( (x-s2(q)^2+2*R*s2(q))./...
    (2*R*sqrt(x)) )));
x = 0.001:s2(q)^2/1000:s2(q)^2;
func = arrayfun(fh, x);
```

```
analytical_csma_rxsens_nobackoff(q) =...
    real(sum(func*s2(q)^2/1000));
analytical_csma_txsens_nobackoff(q) =...
    quadl( @(x) (pi*lambdas_vec*...
    lambda_t*t(q)*exp(-pi*lambdas_vec*...
    lambda_t*t(q)*x)) * ...
    (1 - (1/pi)*acos( (x+R^2-s2(q)^2)/...
    (2*R*sqrt(x)) )), ...
    (s2(q)-R)^2, s2(q)^2 );
prob_backoff(q)=1-(lambertw(pi*lambdas_vec*...
    lambda_t*t(q)*s2(q)^2)...
    /(pi*lambdas_vec*lambda_t*t(q)*s2(q)^2));
analytical_csma_rxsens_total(q) = ...
    analytical_csma_rxsens_nobackoff(q)...
    *(1-prob_backoff(q)) + prob_backoff(q);
analytical_csma_txsens_total(q) =...
    prob_backoff(q) + (1-prob_backoff(q))...
    *analytical_csma_txsens_nobackoff(q) + ...
    (1-analytical_csma_txsens_nobackoff(q))*...
    prob_backoff(q)*...
    (1-2/(pi*s2(q)^2)*(s2(q)^2*acos(R/(2*s2(q)))...
    -R*s2(q)/2*sqrt(1-R^2/4/s2(q)^2)) );
test(q)=(1-analytical_csma_txsens_nobackoff(q))*...
    prob_backoff(q)*(1-2/(pi*s2(q)^2)*(s2(q)^2*...
    acos(R/(2*s2(q)))-R*s2(q)/2*...
    sqrt(1-R^2/4/s2(q)^2)) );
end

Pout_csma_rxsens(m,:)= ...
    analytical_csma_rxsens_total;
```

```
Pout_csma_rxsens(m,complex)=1;
Pout_csma_txsens(m,:)=...
    analytical_csma_txsens_total;
Pout_csma_txsens(m,complex)=1;

% decimate for markers
dec2=floor(linspace(1,length(s2),10));
for u=1:length(dec2)
Pout_csma_rxsens_plot(m,u)=...
    Pout_csma_rxsens(m,dec2(u))  ;
Pout_csma_txsens_plot(m,u)=...
    Pout_csma_txsens(m,dec2(u))  ;
end
dec2=(dec2.*steps)-steps;

figure(2); hold on;
semilogy(t,Pout_csma_rxsens(m,:),'r-');
semilogy(t,Pout_csma_txsens(m,:),'b-');
xlabel('Packet duration, T');
ylabel('Probability of Outage');
legend('CSMA-RX simulated','CSMA-TX simulated',...
    'CSMA-RX analytic','CSMA-TX analytic')
xlim([0 t(end)])

end

figure(3);
h1=semilogy(NaN,NaN,'b-*',Tt,Pout_aloha,...
    'b-',dec,Pout_aloha_plot,'b*');
hold on; grid on;
```

```
h2=semilogy(NaN,NaN,'r-d',Tt,Pout_saloha,...
    'r-',dec,Pout_saloha_plot,'rd');
h3=semilogy(NaN,NaN,'m->',...
    t,Pout_csma_rxsens(m,:),'m-',...
    dec2,Pout_csma_rxsens_plot(m,:),'m>');
h4=semilogy(NaN,NaN,'k-<',t,...
    Pout_csma_txsens(m,:),'k-',...
    dec2,Pout_csma_txsens_plot(m,:),'k<');
legend([h1(1),h2(1),h3(1),h4(1)],'ALOHA',...
    'Slotted ALOHA','CSMA-RX','CSMA-TX')
```

A.2 MATLAB code for μ_{opt}

```
%***Initializing system parameters*****%
T=20:0.1:200;
P_TX=1;
PL=100;
W=1;
R=1;
noise=0;
lambda_t = 0.1;
lambda_s = 10^-3;
alpha=[2.5:0.5:6];
SINRo = 2.^(PL./(T*W))-1;

for m=1:length(alpha)

%-Analytic Pure and Slotted ALOHA----%
```

```
s(m,:) = ( ((R^-alpha(m))./SINRo) - ...
    (noise/P_TX) ).^(-1/alpha(m));
lambdas = lambda_s*lambda_t.*T;

Pout_aloha(m,:) = 1 - exp(-2.*lambdas...
    .*pi.*s(m,:).^2); %Pure ALOHA
opt_RoW(m)=PL/T(find(Pout_aloha(m,:)==...
    min(Pout_aloha(m,:))))/W;

Pout_saloha(m,:) = 1 - exp(-lambdas.*pi...
    .*s(m,:).^2);   %Slotted ALOHA
opt_RoW_slotted(m)=PL/T(find(Pout_saloha(m,:)...
    ==min(Pout_saloha(m,:))))/W;

%------------Analytic CSMA TX- and RX-sensing------------%
    t=15:10:100;
    SINRo2 = 2.^(PL./(t*W))-1;
    s2 = ( ((R^-alpha(m))./SINRo2) - ...
        (noise/P_TX) ).^(-1/alpha(m));
    prob_backoff=zeros(1,length(s2));
    analytical_csma_rxsens_nobackoff=zeros(1,length(s2));
    analytical_csma_txsens_nobackoff=zeros(1,length(s2));
    analytical_csma_rxsens_total=zeros(1,length(s2));
    analytical_csma_txsens_total=zeros(1,length(s2));
    Pout_csma_rxsens=zeros(length(m),length(s2));
    Pout_csma_txsens=zeros(length(m),length(s2));

    for q=1:length(s2)
    fh = @(x) quadl(@(phi) ( 1/(2*pi)*(1 - ...
        1/pi*acos( (x+2*R^2-s2(q)^2-2*R*sqrt(x)...
```

```
    *cos(phi))./(2*R*sqrt(x+R^2-2*R*sqrt(x)...
    .*cos(phi)))) ) ).*(pi*lambda_s*...
    lambda_t*t(q)*exp(-pi*lambda_s*lambda_t...
    *t(q)*x)), acos( (x-s2(q)^2+2*R*s2(q))./...
    (2*R*sqrt(x)) ), (2*pi - acos( ...
    (x-s2(q)^2+2*R*s2(q))./(2*R*sqrt(x)) )));
x = 0.001:s2(q)^2/1000:s2(q)^2;
func = arrayfun(fh, x);
analytical_csma_rxsens_nobackoff(q) =...
    real(sum(func*s2(q)^2/1000));
analytical_csma_txsens_nobackoff(q) =...
    quadl( @(x) (pi*lambda_s*lambda_t*t(q)...
    *exp(-pi*lambda_s*lambda_t*t(q)*x)) *...
    (1 - 1/pi*acos( (x+R^2-s2(q)^2)/...
    (2*R*sqrt(x)) )), (s2(q)-R)^2, s2(q)^2 );
prob_backoff(q)=1-(lambertw(pi*lambda_s*...
    lambda_t*t(q)*s2(q)^2)/(pi*lambda_s*...
    lambda_t*t(q)*s2(q)^2));
analytical_csma_rxsens_total(q) = ...
    analytical_csma_rxsens_nobackoff(q)*...
    (1-prob_backoff(q)) + prob_backoff(q);
analytical_csma_txsens_total(q) = ...
    prob_backoff(q) + (1-prob_backoff(q))...
    *analytical_csma_txsens_nobackoff(q) +...
    (1-analytical_csma_txsens_nobackoff(q))...
    *prob_backoff(q)*(1-2/(pi*s2(q)^2)*...
    (s2(q)^2*acos(R/(2*s2(q)))-R*s2(q)/...
    2*sqrt(1-R^2/4/s2(q)^2)) );
end
Pout_csma_rxsens(m,:)=...
```

```
        analytical_csma_rxsens_total;
    Pout_csma_txsens(m,:)=...
        analytical_csma_txsens_total;

    opt_RoW_rxsens_csma(m)=PL/t(...
        find(Pout_csma_rxsens(m,:)==...
        min(Pout_csma_rxsens(m,:))))/W;
    opt_RoW_txsens_csma(m)=PL/t(...
        find(Pout_csma_txsens(m,:)==...
        min(Pout_csma_txsens(m,:))))/W;

end

figure(3)
plot(alpha,opt_RoW);hold on;
plot(alpha,opt_RoW_slotted);
plot(alpha,opt_RoW_rxsens_csma);
plot(alpha,opt_RoW_txsens_csma);
xlabel('Path loss exponent, \alpha')
ylabel('Optimal spectral efficiency')
```