NTNU
Norwegian University of
Science and Technology

Cosine Modulated Filter Banks Systems in the Presence of Multipath Transmission

Natalia Pérez Tejada

Master of Science in Electronics
Submission date: March 2008
Supervisor: Tor Audun Ramstad, IET

Norwegian University of Science and Technology
Department of Electronics and Telecommunications

Problem Description

Cosine Modulated Filter Banks Systems in the Presende of Multipath Transmission

Assignment given: 03. May 2007
Supervisor: Tor Audun Ramstad, IET

Cosine Modulated Filter Banks Systems in the Presence of Multipath

Natalia Pérez Tejada

September 20, 2007

Acknowledgments

First of all I must thank my parents, Jose Luis and Carmen, for having trusted me, and to my older brother Jose Luis that has been along this years my referent.

I am grateful to Kari Enge for giving the oportunity to came here at NTNU. Also I would want to thank to my advisor Professor Tor A. Ramstad for the guiadance throughout the project and his contributons to the ideas and contents of this Thesis. I have learnt a lot from this Thesis.

I want to thank to my friends in Barcelona for their support althought being so much kilometres away from me, they have helped me to be connected to them. Thanks to my Norwegian friends I have shared my stay with in Trondheim, and they have given me their friendship and they support, thanks specially to Carolin, Maria and Alfonso, to make me feel as at home.

And finally I would like to say thanks to my flatmate and friend Mireia, for helping and advising me in so many ways during my stay in Trondheim.

Contents

1 Introduction **1**
1.1 Communication 1
1.1.1 Process of information transmission 1
1.1.2 Wireless Communication 2
1.2 Organisation of the Document 2

2 Multicarrier Modulation **3**
2.1 Introduction to Multicarrier Modulation 3
2.1.1 Frequency Division Multiplex (FDM) 4
2.1.2 Orthogonal Frequency Division Multiplex (OFDM) . . 5
2.1.3 Introduction FMT 5
2.1.4 Advantatges of FMT respect to classic OFDM 6
2.2 Multirate Digital Signal Processing 6
2.2.1 Interpolation 7
2.2.2 Decimation 8
2.2.3 The noble Identities 8
2.3 Filter Bank Structures 8
2.3.1 PR Filter Bank Conditions 9
2.3.2 Polyphase Filter Bank Structures 11
2.4 Introduction to Cosine Modulated Filter Bank 13
2.4.1 Quadrature Mirror Filter Bank 14
2.4.2 Cosine Modulated Filter Bank 14

3 Cosine Modulated Filter Banks **16**
3.1 Main Concepts 16
3.1.1 Cosine Transformation 16
3.1.2 Lattice Filters 18
3.1.3 Orthogonal Lattice Filters 19
3.2 Examples 20
3.2.1 4 channel system and 2 lattice structures 20
3.2.2 System for N=64, q=2 21

4 Channel Analysis in CMFB **25**
4.1 Inter-Symbol Interference (ISI) 25
4.2 Channel analysis 27
4.2.1 Characterization of Fading Multipath Channels 27
4.2.2 Multipath Channel 28
4.2.3 AWGN Fading Channel 30

5 Equalization Methods **34**
5.1 The ICI problem 34
5.1.1 ICI cancelling 34
5.1.2 Results 35
5.2 Types of Equalizers 36
5.2.1 Linear Equalizer 36
5.2.2 Decision-Feedback Equalizer (DFE) 38
5.3 Equalization in CMFB 39
5.3.1 Equalization Schemes 39
5.3.2 LMS 41
5.4 ISI cancellation 42
5.4.1 Decision Feedback equalizer in each subchannel 43
5.4.2 Decision Feedback equalizer after the Multipath channel 44
5.4.3 Error results 45
5.5 Other results 48
5.6 Two level signal analysis: PAM-2 (Pulse Amplitude Modulation) 50

6 Conclusions and Future Work **53**
6.1 Conclusions 53
6.2 Future Work 54

A Derivation of the Lattice Structure **56**
A.1 The Analysis Filter Bank: derivation of the Polyphase structure 57
A.2 Implementation of a CMFB with N=8 and q=1 60
A.2.1 Programs 60
A.2.2 Results 61

B Lattice Coefficients **63**
B.1 Lattice Coefficients for N=4 and q=2 64
B.2 Lattice Coefficients for N=64 and q=2 65

C Matrix A N=64 and q=2 **66**

List of Abbreviations

A/D Analog/Discrete

AWGN Additive White Gaussian Noise

BER Bit Error Rate

CMA Constant Modulus Algorithm

CMFB Cosine Modulated Filter Bank

DFE Decision Feedback Equalizer

DFME Decision Feedback Multichannel Equalizer

DFT Discrete Fourier Transform

FDM Frequency Division Multiplex

FFT Fast Fourier Transform

FIR Finite Impulse Response

FMT Filtered MultiTone

FSE Fractionally Spaced Symbols

ICI InterCarrier Interference

IFFT Inverse Fast Fourier Transform

ISI InterSymbol Interference

LMS Least Mean Square

MSE Mean Square Error

NTNU Norwegian University of Science and Technology

OFDM Orthogonal Frequency Division Multiplex

postDFME PostDFT simplified DFME

PR Perfect Reconstruction

preDFME PreDFT simplified DFME

RLS Recursive Least Squares

SNR Signal to Noise Ratio

List of Figures

2.1 Frequency response of the transmitted signal in a MCM system 4
2.2 General transmission diagram using MCM 4
2.3 Cyclic prefix in an OFDM symbol 5
2.4 Interpolator scheme 7
2.5 Example with N=3 7
2.6 Scheme of Decimation 8
2.7 Example with N=3 9
2.8 The noble identities 9
2.9 Uniform decimating filter bank 10
2.10 Polyphase implementation for a FIR filter 12
2.11 Polyphase structure of an analysis and synthesis filter bank . 13
2.12 Channel arrangement of a cosine-modulated filter bank 15

3.1 Cosine anti transformation 18
3.2 Cosine transformation 18
3.3 General lattice structure for the analysis filter bank 20
3.4 Unit sample and frequency response of the prototype filter h(k) 21
3.5 Magnitude responses of a four-channel cosine modulated system with q=2 22
3.6 Detailed Transmission System 23
3.7 Unit sample and frequency response for a CMFB with N=64 channels and q=2 24
3.8 Magnitude responses of a 64-channel cosine modulated system with q=2 24

4.1 Equivalent subchannel with additive noise 27
4.2 Real environment in a Multipath channel 28
4.3 Frequency response for a fixed channel with multipath transmission 28
4.4 Transmitted and received signal for sample no. 1 in a fading channel 29
4.5 Received signal in channel no. 1 in a fading channel 30

4.6 Transmitted and received signal for sample no. 57 in a fading channel . . . 30
4.7 Received signal in channel no. 57 in a fading channel . . . 31
4.8 Fading Multipath Channel with Additive Gaussian Noise . . . 31
4.9 Transmitted and received signal for sample no. 1 in a fading channel with noise . . . 31
4.10 Received signal in channel no. 1 in a fading channel with noise 32
4.11 Transmitted and received signal for sample no. 57 in a fading channel with noise . . . 32
4.12 Received signal in channel no. 57 in a fading channel with noise 33

5.1 In left figure is depicted the received signal in a Fading Channel with any compensation, in the right figure is depicted the received signal when the matrix **M** is applied . . . 36
5.2 Linear transversal filter . . . 37
5.3 Symbol-spaced Equalizer scheme . . . 38
5.4 Structure of decision-feedback equalizer . . . 38
5.5 Adaptive filter LMS . . . 41
5.6 General Subchannel structure with DFE equalizer . . . 42
5.7 Received Signal in Channel 57 in a fading channel, supposing no noise . . . 43
5.8 Received Signal in Channel 57 in a fading channel supposing additive Gaussian noise . . . 43
5.9 Received Signal in Channel 57 in a fading channel, supposing no noise . . . 44
5.10 Received Signal in Channel 57 in a fading channel supposing additive Gaussian noise . . . 45
5.11 Error in the received signal in a Multipath fading channel . . 45
5.12 Errors in channel 57 in a Multipath fading channel with equalizers . . . 46
5.13 Error in the received signal in a Multipath fading channel with AWGN . . . 46
5.14 Errors in channel 57 in an AWGN Multipath fading channel with equalizers . . . 47
5.15 In the left figure the theoretical signal is depicted and in the right the received signal is depicted in a multipath channel without noise . . . 48
5.16 Received signal for a multpath channel with Gaussian noise added . . . 48
5.17 ICI cancellation in a fading channel without AWGN noise . . 49
5.18 Received symbols for channels 16 and 57 in a multipath channel with no noise . . . 49
5.19 Received symbols for channels 16 and 57 in a multipath channel with AWGN noise . . . 50

5.20 Received symbols when 1 equalizer is used in a multipath channel with no noise 50
5.21 Received symbols when 64 equalizers are used in a multipath channel with no noise 51
5.22 Received symbols when 1 equalizer is used in a multipath channel with AWGN noise 51
5.23 Received symbols when 64 equalizers are used in a multipath channel with AWGN noise 52
5.24 Empirical Error Rates for channel 57 52

A.1 Unit Sample Response and Frequency Response of the Prototype Filter 62
A.2 Frequency Response for an 8 channel CMFB 62

List of Tables

3.1 Relation between lattice coefficients and the prototype filter unit sample response 21

5.1 MSE for the received signal in channel no. 57 47
5.2 SNR in dB for channel no. 57 47
5.3 SNR in dB for channel no. 16 and 57 52

A.1 Filter coefficients and relation with the Lattice Coefficients . . 62

Abstract

In some systems it is desirable to transmit the signal over several channels. This type of transmission is usually used in environments where one or more channels can be out of order during some time, that makes the channel unreachable for some periods of time. By transmitting the signal over several channels we are providing the system signal diversity. Another form of Multichannel communication is Multicarrier communications, where the frequency band is divided into subchannels, and the signal is transmitted on each of the the subchannels.

In this Thesis the main concepts of the parallel filter banks based on cosine-modulated filter banks are presented. Using uniform channel separation, an efficient implementation consisting of an FFT and subfilters and critical sampling of each channel, and FIR filters.

There are several problems of signal design for complex channels that have randomly time-variant impulse responses. The time-variant impulse responses of these channels are a consequence of the constantly changing physical characteristic of the media. First we will focus on an ideal CMFB, without any distortion in the channel, then we will see that the recovering of the transmitted signal is perfect.

A more realistic analysis of the environment in OFDM needs to be done. For this, the main problems are exposed in this type of channels, then we will give the simulation results for an specific channel with multipath transmission first without noise and after introducing AWGN noise.

After detected the distortions introduced to the received signal we are going to discuss the different ways to combat the Intersymbol Interference and the Intercarrier Interference. The channel distortion results in an Intersymbol Interference and an Inter carrier Interference, which if left uncompensated, may cause errors, as we will see. The solution to the ISI problem is called *equalizer* and consists on designing a receiver that will be capable of cancel or reduce the ISI in the received signal.

Chapter 1

Introduction

1.1 Communication

Communication is a process that allows exchange of information by several methods. It requires to use some symbols from a language to be exchanged. There are auditory means, such as speaking or singing, and nonverbal, such as body language, sign language, touch or even visual contact.

Communication happens in different ways, at many levels and many fields of study have been dedicated to communications. Communication is usually described along a few major dimensions, such as content (what type of things are communicated), source (by whom), form (in which form), channel (through which medium), destination/receiver (to whom), purpose/pragmatic aspect (with what kind of results). The form depends on the symbol systems used. Together, communication content and form make messages that are sent towards a destination over a channel. Communication can be unidirectional, bidirectional or broadcast.

Depending on the focus (who, what, in which form, to whom, to which effect), there exist various classifications. Some of those systematical questions are elaborated in Communication theory.

1.1.1 Process of information transmission

Communication can be seen as processes of information transmission governed by three levels of semiotic rules: Syntactic (formal properties of signs and symbols), pragmatic (concerned with the relations between signs/expressions and their users) and semantic (study of relationships between signs and symbols and what they represent). Therefore, communication is a kind of social interaction where at least two interacting agents share a common set of signs and a common set of semiotic rules. .

In a simplied model, information or content (e.g. a message in natural language) is sent in some form from an emitter to a receiver. In a slightly more complex form a sender and a receiver are linked reciprocally.

In the presence of "communication noise" on the transmission channel (air, in this case) received and decoded content can become faulty in the sense that it will contain errors and thus probably not cause the desired effect.

1.1.2 Wireless Communication

The wireless is referred to telecommunications systems such as radio transmitters and receiver, remote controls, computer networks, network terminals... which use some form of energy (e.g.,radio frequency (RF), infrared light, laser light, visible light, acoustic energy, etc.) to transfer information without the use of wires. Information is transferred in this manner over both short and long distances.

Wireless communication involves radio frequency communication, microwave communication or infrared (IR) short-range communication.

Its applications may involve point-to-point communication, point-to-multipoint communication, broadcasting , cellular networks and other wireless networks.

In the last 50 years, wireless communications industry experienced drastic changes driven by many technology innovations. And quite often there are start-up companies emerging and growing into multi-nationals.

The problem in this types of communications is reliably and efficiently transmitting information signals over imperfect channels. Trying to avoid these imperfections modulations such MCM(Multicarrier Modulation) are developed.

1.2 Organisation of the Document

In chapter 2 and 3 there is explained the theoretical background of the system that is going to be investigated. In Chapter 3 the system is analysed in front an ideal channel. After this ideal analysis in chapter 4 a non-ideal system is analyses, the signal is sent through a Multipath channel, that distorts the signal, and after it is added Gaussian noise to see its effects on the signal. In chapter 5 it is studied different ways to combat the distortios that are introduced by the non-ideal channel, and finally, in last chapter the conclusions are exposed.

Chapter 2

Multicarrier Modulation

This chapter gives an introduction to multicarrier systems. We will explain 3 different types such as FDM (Frequency Division Multiplex), OFDM (Orthogonal Frequency Division Multiplex), and FMT(Filtered Multi Tone), with a comparation between the last two. Also, we will explain the main concepts in a multirate system, and we will make an introduction to Cosine Modulated FIlter Bank. Finally the chapter will end comparing the Quadrature Mirror Filter Bank and the Cosine Modulated Filter Bank.

2.1 Introduction to Multicarrier Modulation

In some systems it is desirable to transmit the signal over several channels. This type of transmission is usually used in environments where one or more channels can be out of order during some time. For example, radio channels, such as ionospheric scatter channels and tropospheric scatter suffer from signal fading due to multipath. That makes the channel unreachable for some periods of time. By transmitting the signal over several channels we are providing the system signal diversity. Another form of Multichannel communication is Multicarrier communications, where the frequency band is divided into subchannels, and the signal is transmitted on each of the the subchannels. When we are transmitting over a fading channel the signal is distorted, and ISI (Inter symbol interference) may be produced in the received signal. In order to reduce it, the channel has to be made more efficient. One technique is the presented before, to subdivide the channel bandwidth into a number of subchannels.

In the next sections we are presenting some schemes for Multicarrier Modulation and presenting its characteristics.

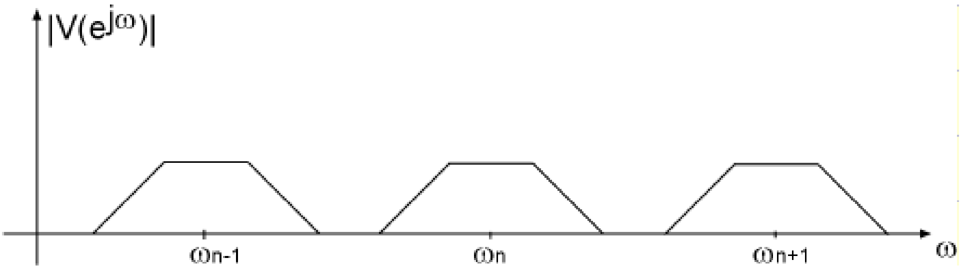

Figure 2.1: Frequency response of the transmitted signal in a MCM system

2.1.1 Frequency Division Multiplex (FDM)

This is the classical Multicarrier System, where the available bandwidth is divided into M frequency subchannels. All the subchannels are completely separated and do not interfere between them. Then, no aliasing is produced and the orthogonality is kept. Each subchannel is frequence multiplexed and can be modulated with separate symbols. We need to use filters to separate a single channel. As there are not ideal bandwidth filters we need to use a guard band to separate the individual subchannels. There has to be a compromise between the filter length and the guard interval length, because a long guard interval implies a waste of bandwidth.

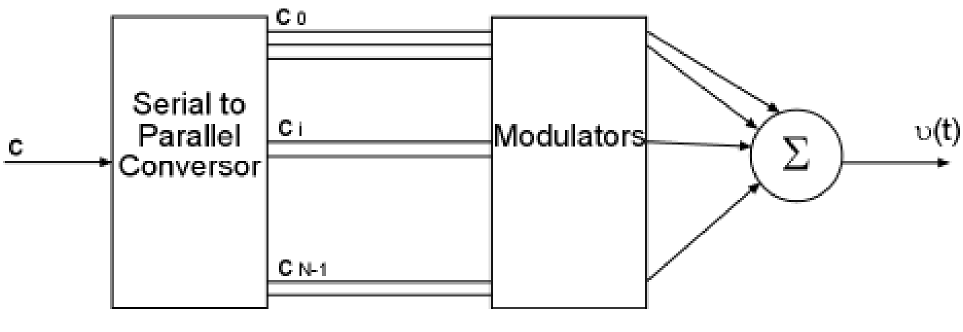

Figure 2.2: General transmission diagram using MCM

The structure of the transmission system of a MCM system is depicted in figure 2.2. The serial input c is split into M parallel streams in the Serial-to-Parallel converter. Then, the input bitrate R is reduced to R/M in each of the M parallel sequences. The discrete symbols c_i are modulated, so they are shifted to the desired frequency. The relationship between the center of the frequency ω_0 of the lowest carrier and the nth carrier is

$$\omega_n = \omega_0 + 2\pi\frac{n}{T} \quad n = 0, 1, 2, \ldots, M-1 \tag{2.1}$$

where M is the number of channels. Then the transmitted signal is

$$v(t) = \sum_{n=0}^{M-1} \sum_{l=-\infty}^{+\infty} c_n(l) g_n(t - lT) \exp\left(-\jmath\omega_n t\right) \qquad (2.2)$$

2.1.2 Orthogonal Frequency Division Multiplex (OFDM)

In OFDM the bandwith is subdivided in M subbands. Each subband is used by one sinusoidal subcarrier, so for each subband, for rectangular symbols and duration T. The baseband representation of the signal is

$$x(t) = \frac{1}{\sqrt{M}} \sum_{k=-\infty}^{\infty} \sum_{i=0}^{M-1} A^{(i)}(k) h(t - kT) \exp\left(\jmath i \frac{2\pi}{T} t\right) \qquad (2.3)$$

where $h(t)$ is a rectangular pulse of duration T and $A^{(i)}(k)$ are the QAM or QPSK symbols. The OFDM symbol duraton is T. Each of the N subcarriers are centerd in frequency $f_i = \frac{i}{T}$, with $i = 0, 1, 2, \ldots, M - 1$. The spectrum of these subcarriers will be the convolution of the Fourier Transform of a simple exponential at frequency $f_i = \frac{i}{T}$ with the Fourier Transform of a rectangular pulse of duration T. So the spectrum of each subcarrier will be a sinc function centered at frequency $f_i = \frac{i}{T}$.

In spite of the overlapping spectra of the sucarriers, the resulting sinc(f) spectrum has zero ISI as well as zero ICI because the adjacent carriers are at the nulls of the sinc(f) function. But this orthogonality is not maintained after a multipath channel.

To combat this, in OFDM is introduced an guard interval (cyclic prefix) in the transmission, so that it will not affect the spectral properties of the signal. This prefix is removed before the processing part in reception. When the symbols are longer that the maximum delay spread we can consider it as frequency flat fading for each subchannel, and this can be easily equalized by a single tap filter

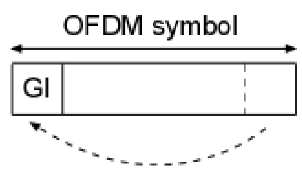

Figure 2.3: Cyclic prefix in an OFDM symbol

2.1.3 Introduction FMT

This filterbank technique has been widely investigated. It has been applied to the signal processing, image processing and antenna systems, especially in wireless mobile communications. The Orthogonal Frequency Division Multiplexing, introduced before, is a special form of filter bank multicarrier technique. The FMT is also a a filter bank multicarrier technique.

Filterd Multitone (FMT) is a multichannel modulation scheme where the subchannels are computed using a modulated filterbank structure. It achieves a high level of spectral containment and enables independent processing in the receiver of the subchannels. This makes it a potential candidate for broadband communication systems, where we need to reduce the impact of frequency selectivity, narrowband interference, crosstalk, etc.

In general, the block diagram of a FMT system consists in M channels and for each channel an upsampler and a downsampler by N in the transmitter and receiver respectively. The filter bank is composed from a single real finite impulse response prototype filter (FIR) with impulse response $h[n]$ and frequency response $H(\exp(\jmath\omega))$. For each subchannel, the transmission filter is a shifted version of the prototype filter. In FMT systems, in the subchannels output signal, ISI distortion, is usually present because of the characteristics of the channel, and this has to be incorporated to the receiver design.

2.1.4 Advantatges of FMT respect to classic OFDM

As we have seen before, to combat ISI the *Cyclic Prefix* is used in OFDM , it leads to a loss in transmission efficiency, as opposed to CMFB, that does not use a cyclic prefix to combat ISI. It rather uses equalizers as we will see in the next chapters.

In OFDM there is a significant power leaking into adjacent bands. In order to reduce interference in the adjacent bands, a number of subcarriers at the edges of the multiplex are not used. These virtual carriers are also introduced because the low pass filter following the Digital to Analog conversor (D/A) will distort the subcarriers close to the pass band edges. Due to the higher spectral containment in FMT, it needs fewer virtual carriers that will provide higher data througput.

2.2 Multirate Digital Signal Processing

In this section we are going to present the main characteristics and the main concepts of these schemes. In a multirate system different parts of the system can work at different clock rates. This is made with the upsamplers or downsamplers, that can manipulate the original signal samples rates. To avoid negative effects of the sampling rate alteration filters must be used. In this thesis critical samplig is considered , so that M (number of channels) is equal to N (downsampling/upsampling factor).

2.2.1 Interpolation

Interpolation and decimation have many applications. In this case they are used to change the sampling rate of the signal by a factor of N. Interpolation makes increase of the sample rate by this factor. If the sampling frequency is F_M, then the new sampling frequency is $F'_M = N \cdot F_M$. $N-1$ zeros are interlaced between two consecutive samples of the original sequence. The expansion of the signal in time domain corresponds to a compression in frequency domain. Then a low pass filter $h(m)$ is necessary to cancel the undesired images of the compressed spectrum that has been introduced. The scheme is as follows.

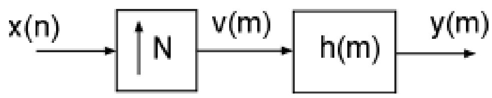

Figure 2.4: Interpolator scheme

Mathematically the process in time corresponds to

$$v(m) = \begin{cases} x(\frac{m}{N}), & m = 0, \pm N, \pm 2N, ... \\ 0, & other \end{cases} \tag{2.4}$$

$$y(m) = v(m) * h(m) \tag{2.5}$$

where the filter is a low pass filter with $\omega_c = \pi/N$. In figure 2.5 a graphical example is shown where $V(\exp(\jmath\omega)) = X(\exp(\jmath\omega N))$ is the Fourier transformation of equation 2.4 in the frequency domain.

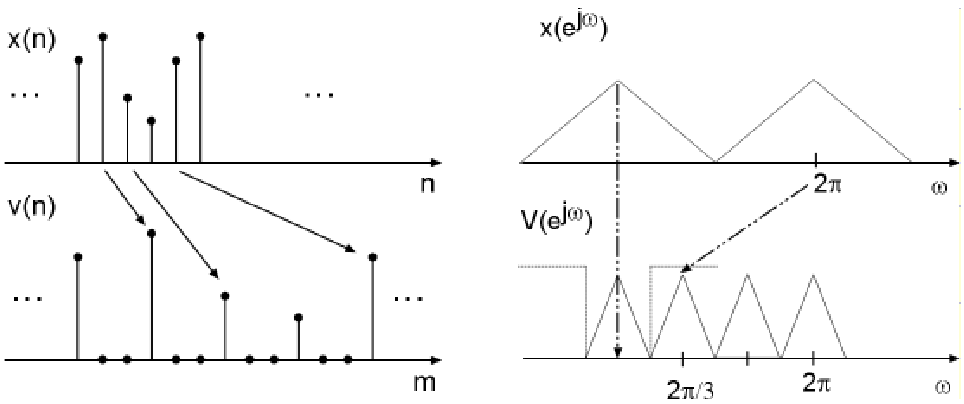

Figure 2.5: Example with N=3

2.2.2 Decimation

A decimation system ($N > 1$) is a lineal system. It is necessary to use a low pass filter for avoiding aliasing. In general, if the sequence bandwith $x(n)$ is bigger than π/N, it will produce aliasing. To avoid information loss caused by the spectral overlapping, a low pass filter $H(\exp(\jmath\omega))$ with a cut frequency of $\omega_c = \pi/N$ has to be inserted before the downsampler. The

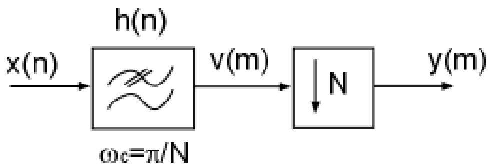

Figure 2.6: Scheme of Decimation

downsampling of the sequence reduces its sampling rate by an integer factor N. The downsampler takes only every Nth sample of the input signal $x(N)$. Mathematically this is described as

$$y(m) = v(Nn) \tag{2.6}$$

In frequency domain, the input-output relation is given by

$$V(\exp(\jmath\omega)) = \frac{1}{N}\sum_{k=0}^{N-1} \exp(\jmath(\omega - 2\frac{\pi}{N}k)) \tag{2.7}$$

and

$$Y(\exp(\jmath\omega)) = V(\exp(\jmath\frac{\omega}{N})) \tag{2.8}$$

In figure 2.7 an example for $N = 3$ can be seen for time domain and frequency domain

2.2.3 The noble Identities

In multirate systems we have some identities to obtain some simplifications for implementation. In filter bank theory the two identities of figure 2.8 are used, that are valid when the transfer function $G(z)$ is rational. In figure 2.8 can be seen that upsampling and downsampling modules can be interchanged by the filter, changing its z-dependency.

2.3 Filter Bank Structures

These systems provides effective ways to represent signals for processing, and understanding purposes. Filter banks have applications in lots of processing

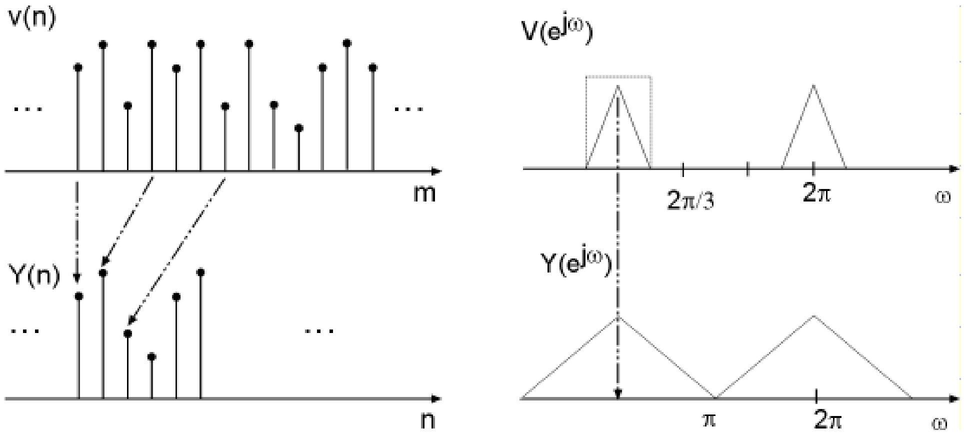

Figure 2.7: Example with N=3

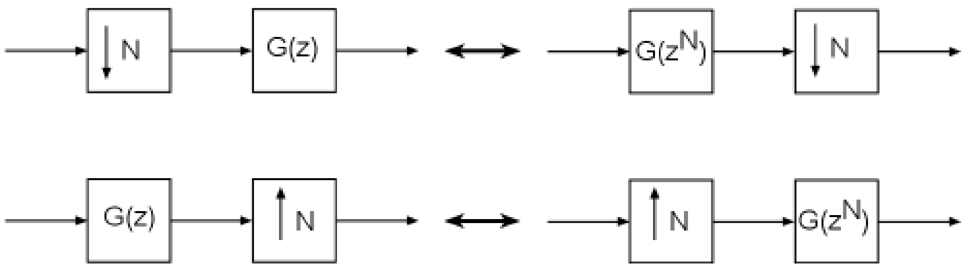

Figure 2.8: The noble identities

fields. The input signal $x_n(k)$ is passed through a bank of N analysis filters. The N filtered signals are then decimated by N to preserve the sampling rate. At the synthesis part, the signals are combined by a set of upsamplers and N synhesis filters, to get the resconstructed signal $y_n(k)$.

2.3.1 PR Filter Bank Conditions

In this section we will present the PR-requirement in the z-domain for a uniform filter bank where the output signals are critically sampled. First of all we have to make some assumptions. We are using critical sampling, this means that the number of channels N is equal to the downsampling factor. We will use the notation for signals used in the analysis filter bank shown in figure 2.9, and we will suppose ideal filters. Using the relations in section 2.2 for channel no. 0, we get the input and output relation

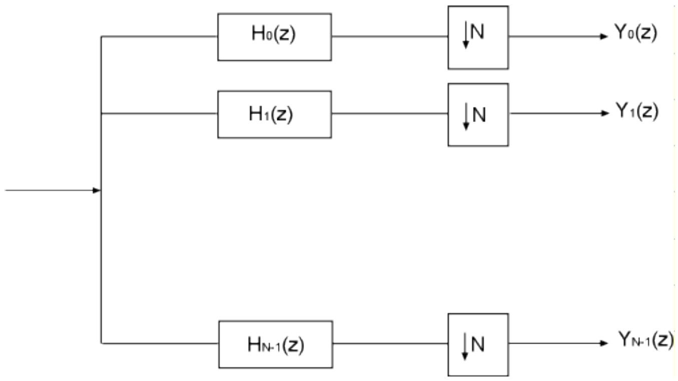

Figure 2.9: Uniform decimating filter bank

$$Y_n(z) = \frac{1}{N}\sum_{m=0}^{N-1} H_n(z^{1/N}\exp(-\jmath\frac{2\pi}{N}m))X(z^{1/N}\exp(-\jmath\frac{2\pi}{N}m)) \quad (2.9)$$

where $H_n(z)X(z)$ is the signal in channel no. n before the downsampler. This expression can also be expressed in terms of z^N

$$Y_n(z^N) = \frac{1}{N}\sum_{m=0}^{N-1} H_n(z\exp(-\jmath\frac{2\pi}{N}m))X(z\exp(-\jmath\frac{2\pi}{N}m)) \quad (2.10)$$

If we put these relations in matrix notation defining $\mathbf{x}(z)$ as the column vector ad $\mathbf{H}(z)$ as the *alias component matrix*

$$x_m(z) = X(z\exp(-\jmath\frac{2\pi}{N}m)) \quad (2.11)$$

$$H_{nm}(z) = H_n(z\exp(-\jmath\frac{2\pi}{N}m)) \quad (2.12)$$

Now, collecting the output signals in vector $\mathbf{y}(z)$, we can put equation 2.10 as

$$\mathbf{y}(z^N) = \frac{1}{N}\mathbf{H}(z)\mathbf{x}(z) \quad (2.13)$$

If $\mathbf{H}(z)$ is not singular, the equation can be inverted to get $\mathbf{x}(z)$ from $\mathbf{y}(z)$

$$\mathbf{x}(z) = N\mathbf{H}^{-1}(z)\mathbf{y}(z^N) = \mathbf{G}(z)\mathbf{y}(z^N) \quad (2.14)$$

where $\mathbf{G}(z)$ represents the synthesis filter.

Synthesis Filter Bank

The original signal is the first element of the vector $\mathbf{x}(z)$, that is the signal reconstructed from

$$\mathbf{X}(z) = x_0(z) = \sum_{i=0}^{N-1} G_{0i}(z)\mathbf{Y}_i(z^N) \tag{2.15}$$

where the matrix elements G_{0i} can be expressed as

$$G_{0i}(z) = \frac{N\hat{G}_i(z)}{\det \mathbf{H}(z)} \tag{2.16}$$

and $\hat{G}_i(z)$ is the *cofactor* of the matrix element H_{0i}, then the matrix 2.15 can be rewritten as

$$\mathbf{X}(z) = x_0(z) = \frac{N}{\det \mathbf{H}(z)} \sum_{i=0}^{N-1} \hat{G}_i(z)\mathbf{Y}_i(z^N) \tag{2.17}$$

With the analysis bank given, and the obtained synthesis filter bank, perfect reconstruction property is guaranteed in a two-stage structure. In general, the filter order of the synthesis filter bank is higher than in the analysis filter bank, and even having FIR analysis filter bank, the synthesis bank may have IIR filters.

2.3.2 Polyphase Filter Bank Structures

In this section we are going to explain the Polyphase structure for the analysis and synthesis filter bank. This structure is used in most parallel systems. In this case we are only going to explain the Polyphase decomposition of a FIR filter.

Polyphase decomposition of FIR Filters

The polyphase decomposition of $H(z)$ is

$$H(z) = \sum_{k=0}^{N-1} P_k(z^N)z^{-k} \tag{2.18}$$

where $P_k(z)$ is the k*th polyphase component* of the analysis filter, in figure 2.10 can be seen polyphase filter bank.

Assume that the unit sample response is given by $\{h(l), l = 0, 1, 2, \ldots, L-1\}$. Then the polyphase unit sample responses are given by

$$p_k(m) = h(k + mN), \quad \begin{array}{l} k = 0, 1, \ldots, N-1 \\ m = 1, 2, \ldots, [L/N] - 1 \end{array} \tag{2.19}$$

The polyphase filter coefficients are sampled versions of the original ones

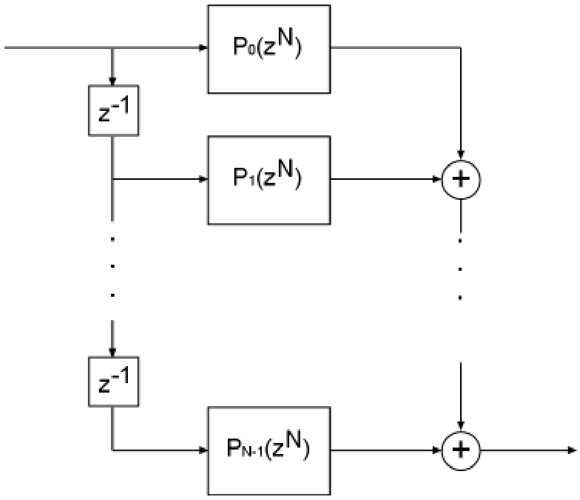

Figure 2.10: Polyphase implementation for a FIR filter

Polyphase Filter Banks

Now, we assume that we have N transfer functions that represent the analysis and synthesis filter bank channels.

$$H_n(z), \quad n = 0, 1, \ldots, N-1 \tag{2.20}$$

$$G_n(z), \quad n = 0, 1, \ldots, N-1 \tag{2.21}$$

If we split each filter in its polyphase components, then we get

$$H_n(z) = \sum_{k=0}^{N-1} P_{nk}(z^N) z^{-k} \tag{2.22}$$

$$G_n(z) = \sum_{k=0}^{N-1} Q_{nk}(z^N) z^{-k} \tag{2.23}$$

The column vector $\mathbf{h}(z)$ is composed of the transfer functions $\{H_0(z), H_1(z), \ldots, H_{N-1}(z)\}$, and the polyphase matrix $\mathbf{P}(z^N)$ is composed of the elements $P_{nk}(z^N)$ in the case of the analysis filter bank. In the synthesis filter bank the polyphase matrix $\mathbf{Q}(z^N)$ is composed by the elements $Q_{nk}(z^N)$. The analysis filter bank transfer functions can be expressed as

$$\mathbf{h}(z) = \mathbf{P}(z^N) \begin{bmatrix} 1 \\ z^{-1} \\ \vdots \\ z^{-(N-1)} \end{bmatrix} = \mathbf{P}(z^N)\mathbf{d}_N(z) \tag{2.24}$$

where $\mathbf{d}_N$ is the vector of the delay elements. The N outputs of $\mathbf{y}(z)$ are calculated from $\mathbf{Y}(z) = \mathbf{h}(z)X(z)$.
In the synthesis filter bank transfer functions are

$$\mathbf{g}(z) = \mathbf{Q}^T(z^N)\tilde{\mathbf{d}}_N(z) \tag{2.25}$$

where $\tilde{\mathbf{d}}_N(z) = [z^{-(N-1)}, z^{-(N-2)}, \ldots, z^{-1}, 1]^T$ is a reversed version of $\mathbf{d}_N(z)$

Perfect Reconstruction Filter Banks

The polyphase decomposition can be used for any filter particularlly to multirate systems because of the z^N dependency of the polyphase matrix. Using the noble identities explained in section 2.2.3 we can move the upsamplers and downsamplers across the polyphase matrices in order to change the argument z^N to z. I figure 2.11 can be seen the final strucuture after these changes.

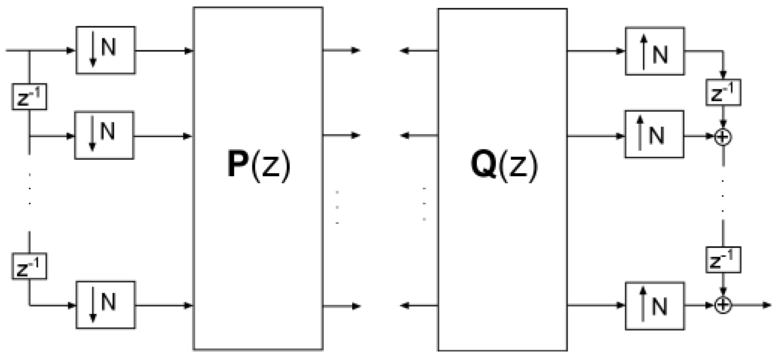

Figure 2.11: Polyphase structure of an analysis and synthesis filter bank

The *Perfect Reconstruction* condition is

$$\mathbf{P}(z)\mathbf{Q}(z) = z^{-k_0}\mathbf{I} \tag{2.26}$$

where $\mathbf{I}$ is de Identity matrix and k_0 is the system delay. It is said that we get perfect reconstruction when the recovered signal $y_n(k)$ is equal to the transmitted signal $x_n(k)$ with only a delay difference.

2.4 Introduction to Cosine Modulated Filter Bank

In general there are two types of Cosine Modulated Filter Bank:

- QMF-Banks (Quadrature Mirror Filter Banks)
- CMFB (Cosine Modulated Filter Banks)

2.4.1 Quadrature Mirror Filter Bank

They belong to the family of modulated filter banks. The analysis and synthesis filters are obtained by the cosine modulation of a linear-phase, low-pass prototype filter $H(z)$. M-channel QMF Bank can satisfy the PR condition, when the length of the linear-phase prototype filter N is constrained to be $N = 2M$. It can be found a detailed explanation in [16] and [17].

Vaydyanathan's CMFB uses the cosine modulation term given by $h_n(k)$

$$h_n(k) = 2h(k)\cos\left((\frac{\pi}{N})(n+\frac{1}{2})(k-\frac{L-1}{2}) + (-1)^n\frac{\pi}{4}\right) \tag{2.27}$$

where $h(k)$ is the prototype filter, L is the length and N the number of channels. This filter bank does not have as many advantatges as the next one, because as we are seeing in next section, it does not have the symmetric properties that the next one has. As can be seen in [2]

- It needs the double of superlattices.
- The implementation of the first lattice stages is different from the higher stages.
- The transform matrix $\mathbf{T}$ has a higher order and is more complex.

2.4.2 Cosine Modulated Filter Bank

This is a cosine-modulated version of a prototype lowpass FIR filter. With a uniform filter bank where each of the N channels occupies the same bandwith $(\frac{\pi}{N})$, and the channel filter band edges are ideally located at $n\frac{\pi}{N}$, for $n = 0, 1, \ldots, N$. $L = 2Nq$ is the lenght of the prototype filter, with $q \in \mathbb{Z}_+$, and it is defined by its unit sample response $h(k)$.

The unit sample response is obtained by modulating the prototype filter response as [1]

$$h_n(k) = h(k)\cos\left(\frac{\pi}{N}(n+\frac{1}{2})(k+\frac{1}{2})\right) = h(k)T_N(n,k) \tag{2.28}$$

The modulation term $T_N(n,k)$ represents the cosine transformation of type IV. The index N represents the number of channels.

In [2] this cosine modulated filter bank is studied for use in (OFDM) FMT systems. The following distortions have been identified:

- Additive white Gaussian Noise (AWGN)
- A relative motion between transmitter and receiver causes a Doppler shift in the transmitted signal

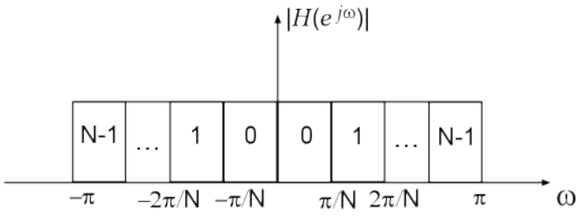

Figure 2.12: Channel arrangement of a cosine-modulated filter bank

- The D/A converter at the transmitter and the A/D converter ate the receiver are not perfectly synchronized

The coefficients derivated in Thesis [2] and used in this Thesis have been optimized for the AWGN distortion. With this coefficients we are going to study the behavior fo the CMFB when the signal is transmitted through a Multipath Channel.

Chapter 3

Cosine Modulated Filter Banks

In this chapter we present the main concepts of the parallel filter banks based on cosine-modulated filter banks. Using uniform channel separation, an efficient implementation consisting of an FFT and subfilters and critical sampling (upsampling and downsampling= N (no. of channels)) of each channel, and FIR filters.

3.1 Main Concepts

In this section we are going to extend the explanation of Cosine Modulated Filter Bank introduced in the previous chapter.

3.1.1 Cosine Transformation

In general, the transformation

$$h_n(k) = h(k)\cos\left(\frac{\pi}{N}(n+\frac{1}{2})(k+\frac{1}{2})\right) = h(k)T_N(n,k) \qquad (3.1)$$

has been implemented considering $T_N(n,k)$ as a matrix [1][2], but it has a high complexity to compute the elements on the matrix, overall if it is a complex system with a high number of channels or lattice structures (q).

It can be shown that the complexity can be reduced significantly by using the Fast Fourier Transform.

The equation 3.1 with $x(n)$ as its inputs can be rewritten as:

$$
\begin{aligned}
y_n(k) &= x(k)\cos\left(\frac{\pi}{N}(n+\frac{1}{2})(k+\frac{1}{2})\right) = \\
&= \Re\left(x(k)\exp\left(\jmath\frac{\pi}{N}(n+\frac{1}{2})(k+\frac{1}{2})\right)\right) \\
&= \Re\left(x(k)\exp\left(\jmath\frac{\pi}{N}(nk+\frac{n}{2}+\frac{k}{2}+\frac{1}{4})\right)\right) \\
&= \Re\left(x(k)\exp(\jmath\frac{\pi}{N}nk)\exp(\jmath\frac{\pi}{N}\frac{n}{2})\exp(\jmath\frac{\pi}{N}\frac{k}{2})\exp(\jmath\frac{\pi}{N}\frac{1}{4})\right)
\end{aligned}
\tag{3.2}
$$

We know that the Discrete Fourier Transform of a discrete signal $x(n)$ is

$$
X(k) = \sum_{n=0}^{N-1}\exp\left(\jmath 2\frac{\pi}{N}kn\right)x(n) \tag{3.3}
$$

Changing $N \rightarrow 2N$ in 3.3 we get the expression for the 2N points FFT

$$
X(k) = \sum_{n=0}^{2N-1}\exp\left(\jmath 2\frac{\pi}{2N}kn\right)x(n) = FFT(x(n), 2N) \tag{3.4}
$$

we can use this result in 3.2 and get the transformation of x(n)

$$
\begin{aligned}
y(k) &= \Re\left(\sum_{n=0}^{2N-1}\exp(\jmath\frac{2\pi}{2N}nk)\exp(\jmath\frac{\pi}{2N}n)\exp(\jmath\frac{\pi}{2N}k)\exp(\jmath\frac{\pi}{4N})\right) \\
&= \Re\left(\exp(\jmath\frac{\pi}{2N}k)\exp(\jmath\frac{\pi}{4N})\sum_{n=0}^{2N-1}x(n)\exp(\jmath\frac{\pi}{2N}n)\exp(\jmath\frac{2\pi}{2N}nk)\right) \\
&= \Re\left(\exp(\jmath\frac{\pi}{2N}k)\exp(\jmath\frac{\pi}{4N})FFT\left(x(n)\exp(\jmath\frac{\pi}{2N}n), 2N\right)\right)
\end{aligned}
\tag{3.5}
$$

We can see in figure 3.1 the transmission scheme. In this case it is part of the synthesis filter bank (transmitter), and the figure 3.2 is the cosine transformation in the analysis filter bank.

We have to say, that when we make the 2N point IFFT and the input signal length is N, the output of the IFFT will be 2N length, however in the complete scheme we will only need to use the first N samples, because the signal of duration 2N is antisymmetric.

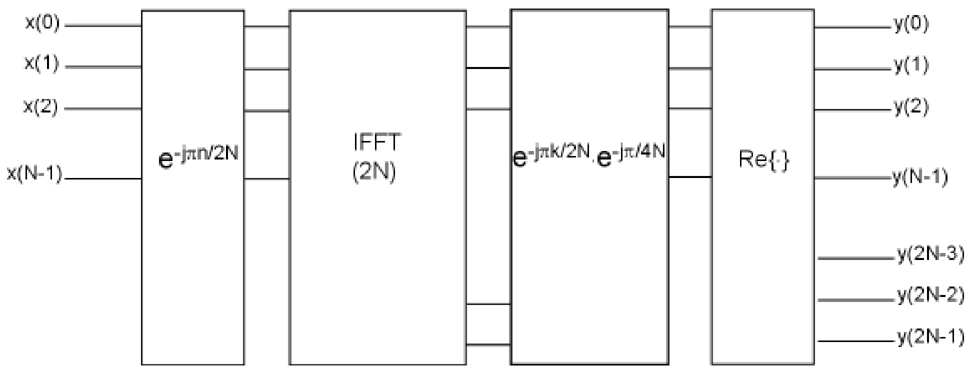

Figure 3.1: Cosine anti transformation

Then, when the receiver scheme is designed, it has to be noticed that it should be a signal of length 2N in the input of the cosine transformation, but although we will receive a N length signal, we know that it is antisymmetric and the input signal can be obtained from the original one.

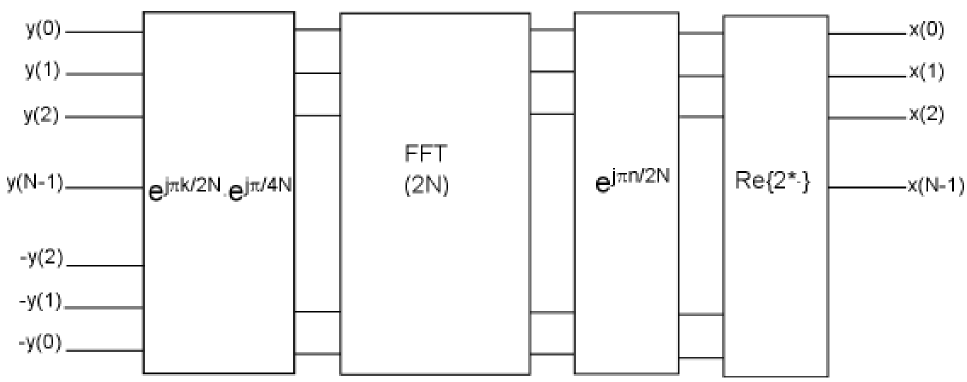

Figure 3.2: Cosine transformation

3.1.2 Lattice Filters

Based on the result in [1] we arrive at a lattice structure for the Analysis filter Bank using the polyphase structure of the prototype lowpass filter. Here we only are going to explain the main concepts of this useful structure.

Simple PR System

From the polyphase matrix (see Appendix A.1) we can extract simple submatrices $M_i(z)$, consisting of four nonzero elements located symmetrically around the center of the matrix:

$$M_i(z) = \begin{bmatrix} K_i(z^2) & -z^{-1}K_{2N-1-i}(z^2) \\ -z^{-1}K_{N+i}(z^2) & K_{N-1-i}(z^2) \end{bmatrix}, \quad i \in 0, 1, \ldots, N/2-1 \tag{3.6}$$

The submatrices represent two separate ports, we see that there is always interconnection between two and two branches only. The submatrices $M_i(z)$ are called *superlattices*.

In our particular case, the PR (Perfect Reconstruction) condition can easily be met such that the superlattice determinant is made equal to a pure delay. This is the case when each subfilter has only one nonzero coefficient.

The simplest applicable form of 3.6 is:

$$M_i(z) = \mu_i(z) = \begin{bmatrix} a_i z^{-2} & b_i z^{-1} \\ c_i z^{-1} & d_i \end{bmatrix}, \quad i \in 0, 1, \ldots, N/2-1 \tag{3.7}$$

The matrix inverse is given by

$$M_i^{-1}(z) = \mu_i^{-1}(z) = \frac{1}{(a_i d_i - b_i c_i)z^{-2}} \begin{bmatrix} d_i & -b_i z^{-1} \\ -c_i z^{-1} & a_i z^{-2} \end{bmatrix}, i \in 0, 1, \ldots, N/2-1 \tag{3.8}$$

Generalization of Lattice PR Systems

In our case we need a generalization of the previous results for higher order systems. In the previous section the superlattices were replaced by simple lattices, then, the superlattices may be composed as a cascade of $\mu_i(z)$, in the analysis filter bank, and by $\mu_i^{-1}(z)$ in the synthesis one.

$$M_i(z) = \prod_{k=1}^{q} \mu_{i,q-k+1}(z) \tag{3.9}$$

$$M_i^{-1}(z) = \prod_{k=1}^{q} \mu_{i,q-k+1}^{-1}(z) \tag{3.10}$$

3.1.3 Orthogonal Lattice Filters

For high orders systems it has a high complexity to design the filter coefficients, so one way to make it easier, is to use a type of filter, that only needs to design one parameter for each simple lattice structure. The relations are:

$$\begin{aligned} a_i &= d_i = \cos(\phi_i) \\ b_i &= -c_i = \sin(\phi_i) \end{aligned} \tag{3.11}$$

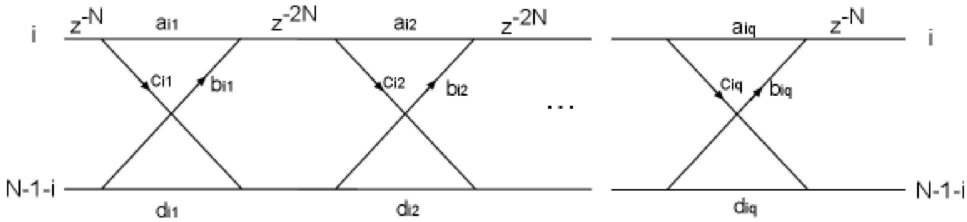

Figure 3.3: General lattice structure for the analysis filter bank

$$\mu_i(z) = \begin{bmatrix} \cos(\phi_i)z^{-2} & \sin(\phi_i)z^{-1} \\ -\sin(\phi_i)z^{-1} & \cos(\phi_i) \end{bmatrix} \tag{3.12}$$

Now the only coefficient that has to be designed for each lattice structure is ϕ_i. Also all the determinants of the synthesis lattices structures are equal to one with a delay of two samples, this makes the Analysis and Synthesis filters to be equal.

3.2 Examples

3.2.1 4 channel system and 2 lattice structures

We present here an example of the system developed with N=4 and 2 lattices stages, where we use the coefficients values in (B.1):

$$F(N,q) = F(4,2) = \begin{bmatrix} \phi_{01} & \phi_{02} \\ \phi_{11} & \phi_{12} \end{bmatrix} \tag{3.13}$$

where the analysis and filter banks are equal, then, the entire transmission system can be defined by the matrix in 3.13

For these coefficients the prototype filter is a low pass filter with cutoff frequency $wc = \pi/N$, and a length of L=16. The next figure depicts its unit sample response and its frequency response. The unit sample response is normalized so that the maximum amplitude is equal to 1, and the frequency responses are normalized to unit frequency by half the sampling frequency.

We have a relationship between the impulse response for channel no. 0, $h_0(k)$ according to 3.6 between the lattice coefficients and the prototype filter unit sample response. It is shown in table 3.1.

In the figure 3.5 can be seen the four impulse responses for all the subchannels and how they are modulated, the first channel is a low pass filter and the last channel is a high pass filter.

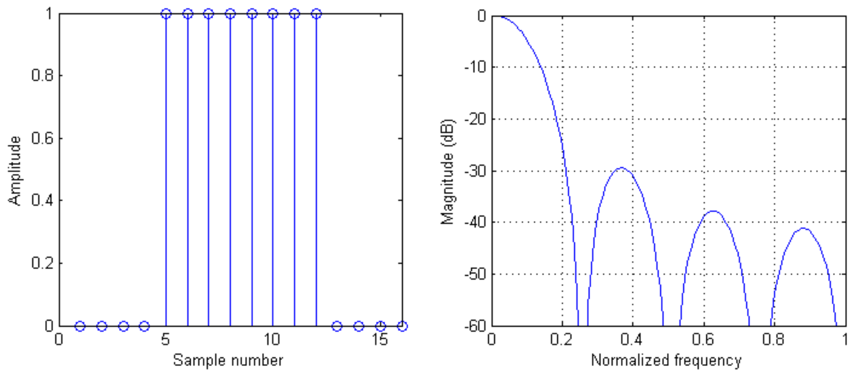

Figure 3.4: Unit sample and frequency response of the prototype filter h(k)

k	$h(k)$	k	$h(k)$
0	$+\cos(\phi_{21})\cos(\phi_{22})$	15	$+\cos(\phi_{21})\cos(\phi_{22})$
1	$+\cos(\phi_{11})\cos(\phi_{12})$	14	$+\cos(\phi_{11})\cos(\phi_{12})$
2	$+\sin(\phi_{11})\cos(\phi_{12})$	13	$+\sin(\phi_{11})\cos(\phi_{12})$
3	$+\sin(\phi_{21})\cos(\phi_{22})$	12	$+\sin(\phi_{21})\cos(\phi_{22})$
4	$-\cos(\phi_{21})\sin(\phi_{22})$	11	$-\cos(\phi_{21})\sin(\phi_{22})$
5	$-\cos(\phi_{11})\sin(\phi_{12})$	10	$-\cos(\phi_{11})\sin(\phi_{12})$
6	$+\sin(\phi_{11})\sin(\phi_{12})$	9	$+\sin(\phi_{11})\sin(\phi_{12})$
7	$+\sin(\phi_{21})\sin(\phi_{22})$	8	$+\sin(\phi_{21})\sin(\phi_{22})$

Table 3.1: Relation between lattice coefficients and the prototype filter unit sample response

3.2.2 System for N=64, q=2

In this section we are going to extend the four-channel system, with $q = 2$ to a more complex system with $N = 64$ and q=2. The coefficients of this system are taken from [2], and they have been optimized for a channel with additive white Gaussian noise.

In this analysis, the channel will be considered ideal, so that the reconstruction is perfect and we will not have any loss. The received signal in the receiver will be the same as the one that is transmitted.

The coefficients used (B.2) results in a prototype filter that is considered a low pass filter with $L = 2Nq = 256$.

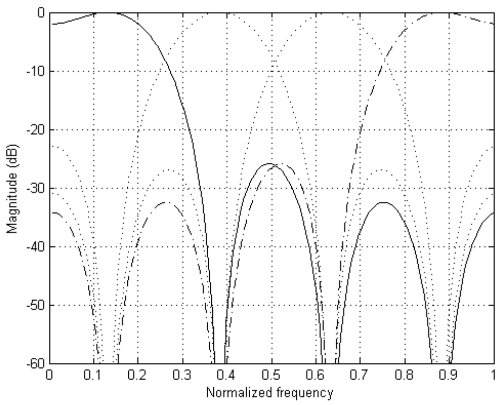

Figure 3.5: Magnitude responses of a four-channel cosine modulated system with q=2

The next figure represents the impulse response for this system in an ideal channel

This system with an ideal channel is not as much interesting as using a noisy channel or a fading channel. It does not represents a real situation, with delays and attenuation. These type of channels will be studied in the next chapter.

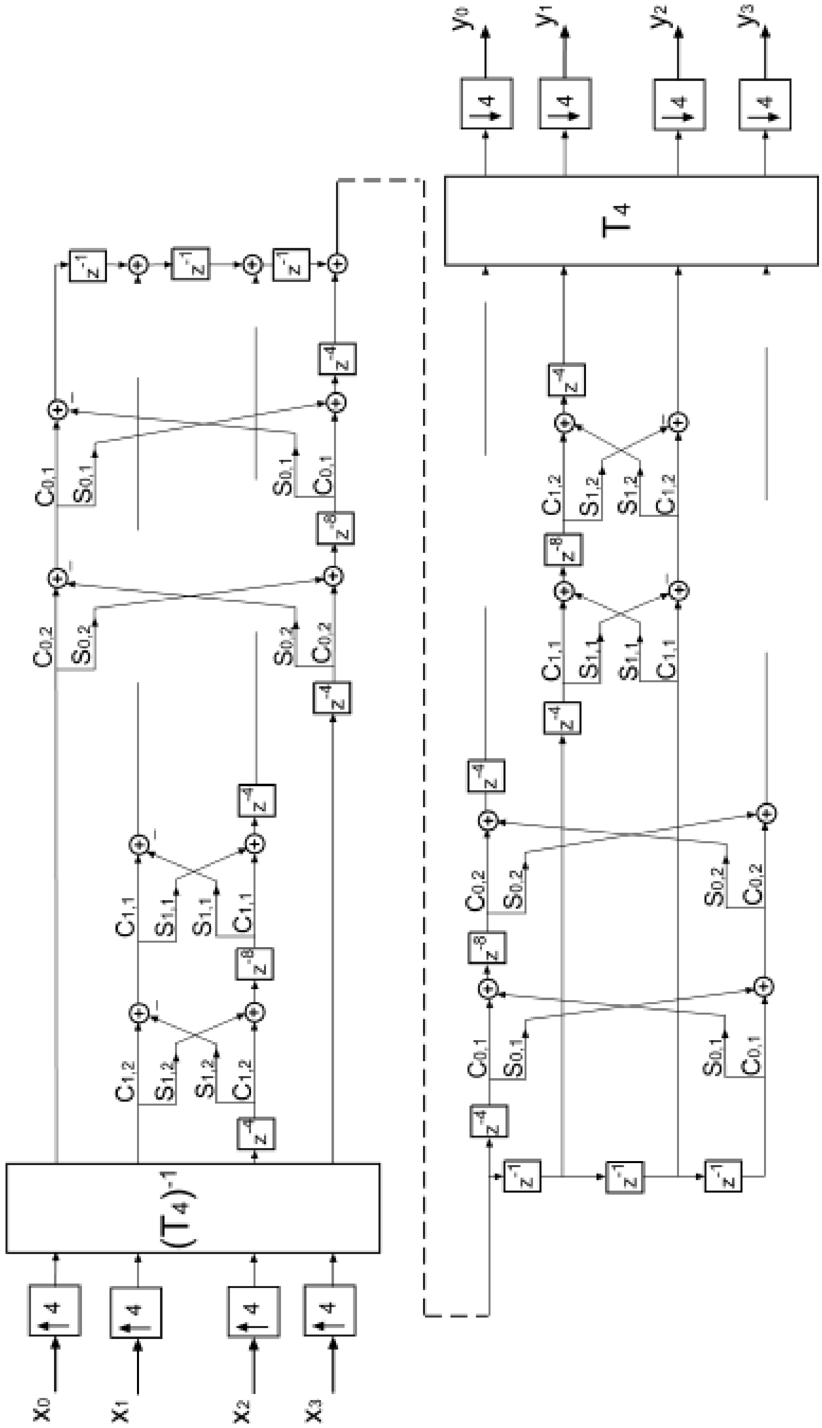

Figure 3.6: Detailed Transmission System

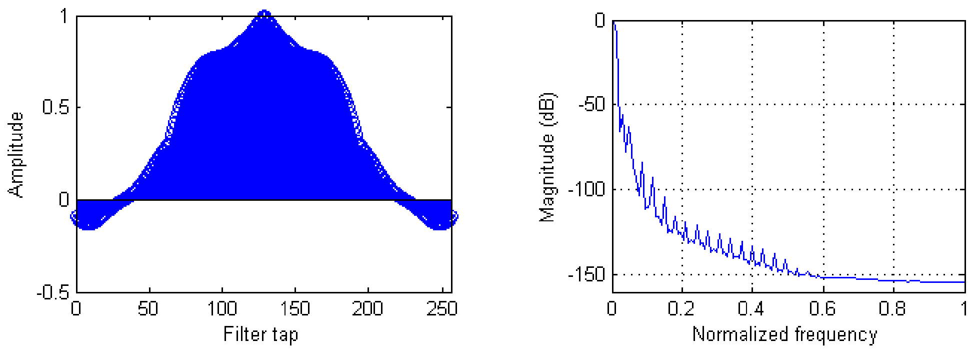

Figure 3.7: Unit sample and frequency response for a CMFB with N=64 channels and q=2

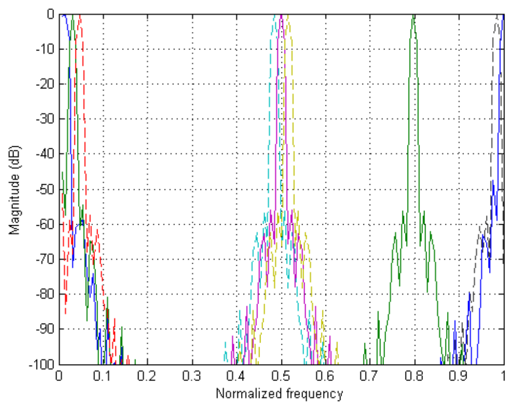

Figure 3.8: Magnitude responses of a 64-channel cosine modulated system with q=2

Chapter 4

Channel Analysis in CMFB

In this chapter we consider the problems of signal design for complex channels that have randomly time-variant impulse responses. The time-variant impulse responses of these channels are a consequence of the constantly changing physical characteristic of the media.

We will make an analysis more realistic of the environment in OFDM. In the previous chapter, we focused on an ideal CMFB, without any distortion in the channel, so that the transmitted signal was the same as that the received signal in the receiver, and then the recovering of the transmitted signal was perfect. First we will explain some concepts, then we will analyse a fading channel (without noise), finally we will see what results we get if we add white noise to the multipath channel. All the simulations have been done with the system seen in chapter 3 with $N = 64$ channels and $q = 2$ lattice structures.

4.1 Inter-Symbol Interference (ISI)

The equivalent low-pass transmitted signal for several different types of signal modulation techniques has the form

$$\sum_{n=0}^{\infty} I_n g(t - nT) \tag{4.1}$$

Where $\{I_n\}$ represents the discrete information sequence of symbols and $g(t)$ a pulse, which in this thesis is assumed to have a band limited frequency response characteristic $G(f)$ to $|f| \leq W$. This signal is transmitted over a channel with $C(f)$ as frequency response, also limited to $|f| \leq W$. So the received signal can be represented as

$$r(t) = \sum_{n=0}^{\infty} I_n h(t - nT) + \omega(t) \tag{4.2}$$

where

$$h(t) = \int_{-\infty}^{+\infty} g(t)c(t-\tau)d\tau \tag{4.3}$$

and $\omega(t)$ represents the additive Gaussian noise. Then the received signal is passed through a filter and is sampled at a rate of $1/T$ samples. In this example we use the optimum filter from the point of view of signal detection. That is a filter matched to the received pulse. The frequency response of that filter is $H^*(f)$. The output of the receiver signal is then

$$y(t) = \sum_{n=0}^{\infty} I_n x(t-nT) + n(t) \tag{4.4}$$

where x(t) is the pulse representing the response of the receiving filter to the input pulse $h(t)$ and $n(t)$ is the response of the receiving filter to the noise $\omega(t)$.

Now if we sampled $y(t)$ at times $t = kT + \tau_0$, $k = 0, 1, ...$, we have

$$y(kT+\tau_0) \equiv y_k \equiv \sum_{n=0}^{\infty} I_n x(kT - nT + \tau_0) + n(kT+\tau_0) \tag{4.5}$$

$$y_k = \sum_{n=0}^{\infty} I_n x_{k-n} + n_k \quad k = 0, 1, ... \tag{4.6}$$

where τ_0 is the transmission delay through the channel. The sample values can be expressed as

$$y_k = x_0 \left(I_k + \frac{1}{x_0} \sum_{\substack{n=0 \\ n \neq k}}^{\infty} I_n x_{k-n} \right) + n_k \quad k = 0, 1, ... \tag{4.7}$$

We regard x_0 as an arbitrary scale factor which we set to unity for convenience. Then

$$y_k = I_k + \sum_{\substack{n=0 \\ n \neq k}}^{\infty} I_n x_{k-n} + n_k \tag{4.8}$$

the term I_k represents the desired information symbol at the kth sampling instant. The term

$$\sum_{\substack{n=0 \\ n \neq k}}^{\infty} I_n x_{k-n}$$

represents the inter-symbol interference, and n_k is the additive Gaussian noise variable at the kth sampling instant.

4.2 Channel analysis

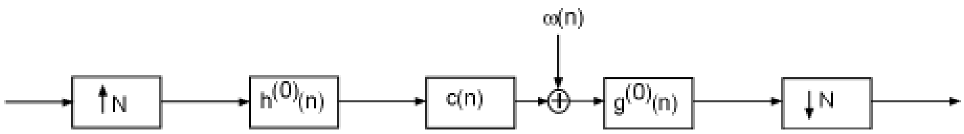

Figure 4.1: Equivalent subchannel with additive noise

4.2.1 Characterization of Fading Multipath Channels

If we transmit an extremely short pulse, ideally an impulse, over a time-variant multipath channel, the received signal might appear as a train of pulses. Then, one characteristic of a multipath medium is the time spread introduced in the signal which is transmitted through the channel. A second characteristic is due to the time variations in the medium. As a result of such time variations the nature of the multipath varies with time. The time variations appear to be unpredictable to the user of the channel, so it is reasonable to characterize the time-variant multipath channel statistically. Toward this end, we will examine the effects to the channel on a transmitted signal represented in general as

$$s(t) = \Re[u(t) \exp(\jmath 2\pi f_c t)] \tag{4.9}$$

We assume that there are multiple propagation paths. Associated with each path there is a propagation delay and an attenuation factor (τ and α respectively). Then, the received bandpass signal may be expressed in the form

$$x(t) = \sum_n \alpha_n(t) s[t - \tau_n(t)] \tag{4.10}$$

where $\alpha_n(t)$ is the attenuation factor for the signal received on the nth path and $\tau_n(t)$ is the propagation delay for the nth path. If we substitute $s(t)$ from 4.10 in 4.9, then the result is

$$x(t) = \Re\left(\left\{\sum_n \alpha_n(t) \exp(-\jmath 2\pi f_c \tau_n(t)) u[t - \tau_n(t)]\right\} \exp(\jmath 2\pi f_c t)]\right) \tag{4.11}$$

Then, is easy to see that the equivalent low-pass received signal is

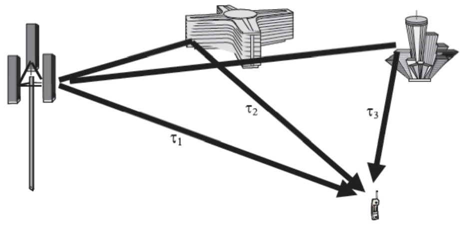

Figure 4.2: Real environment in a Multipath channel

$$r(t) = \sum_n \alpha_n(t) \exp(-\jmath 2\pi f_c \tau_n(t)) u[t - \tau_n(t)] \tag{4.12}$$

Since $r(t)$ is the response of an equivalent low-pass channel to the equivalent low-pass signal $u(t)$, it follows that the equivalent low-pass channel is described by the time-variant impulse response

$$c(\tau; t) = \sum_n \alpha_n(t) \exp(-\jmath 2\pi f_c \tau_n(t)) \delta[t - \tau_n(t)] \tag{4.13}$$

4.2.2 Multipath Channel

The channel that is going to be studied has only two paths, the one with the original signal, that does not have any attenuation and delay, and another path where the signal suffers a one sample delay ($\tau = 1$) and an attenuation of $\alpha = 0.4$

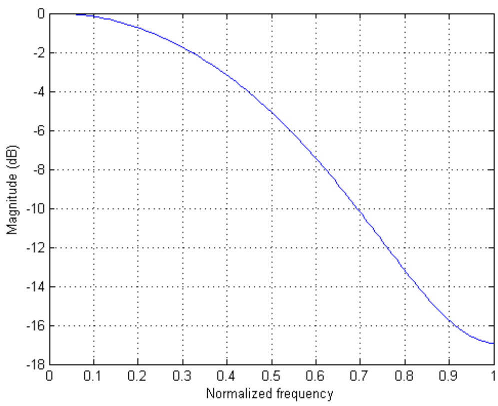

Figure 4.3: Frequency response for a fixed channel with multipath transmission

This fading channel makes that the users that are sending the information at high frequencies (the highest channels) suffer much more degradation that the ones that transmit in low frequencies (lowest channels). This implies that in the receiver you cannot recover the original signal transmitted by the user if the noise level is too high.

In particular we receive the signal with some ISI (Inter-symbol Interference) and some ICI (Inter-symbol Carrier). The ISI interference will be more strong that the ICI, and in the next chapter we will see how it can be removed.

First of all, we will see some examples of the signal sent and the signal received.

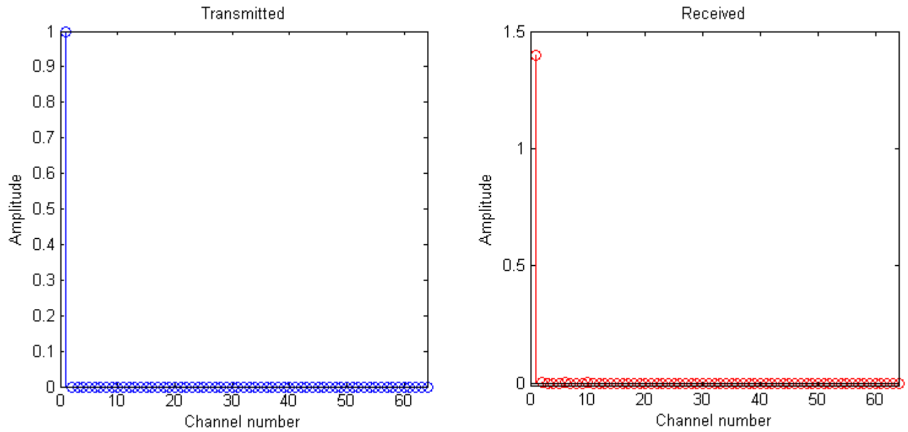

Figure 4.4: Transmitted and received signal for sample no. 1 in a fading channel

As we can see from figure 4.4, when we send a sample in the through the channel no. 0, the received signal is the same as the one sent, with only a difference of amplitude, that can be easily compensated, and with negligible ICI.

As we can see from c, we do not have ISI, that is because the channel no. 0 sends the information through the lowest frequency, and the fading channel that is under investigation has almost no distortion for these frequencies.

In the next examples we will see what happens when we use the higher channels that use the highest frequencies to transmit an impulse.

In figures 4.5 and 4.6 we can see that when we send information in higher channels the signal received is quite different from the one transmitted, this is because of the fading channel, that distorts the signal sent, then the signal is corrupted by ISI and ICI, as well as attenuated. The Inter-channel Interference is not as important as the Inter-symbol Interference.

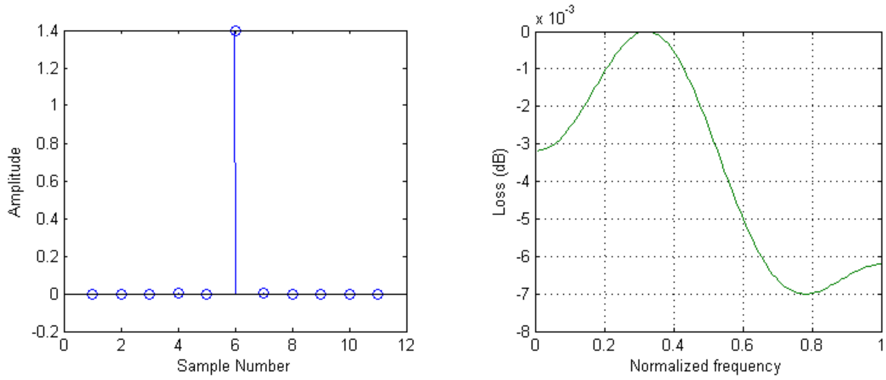

Figure 4.5: Received signal in channel no. 1 in a fading channel

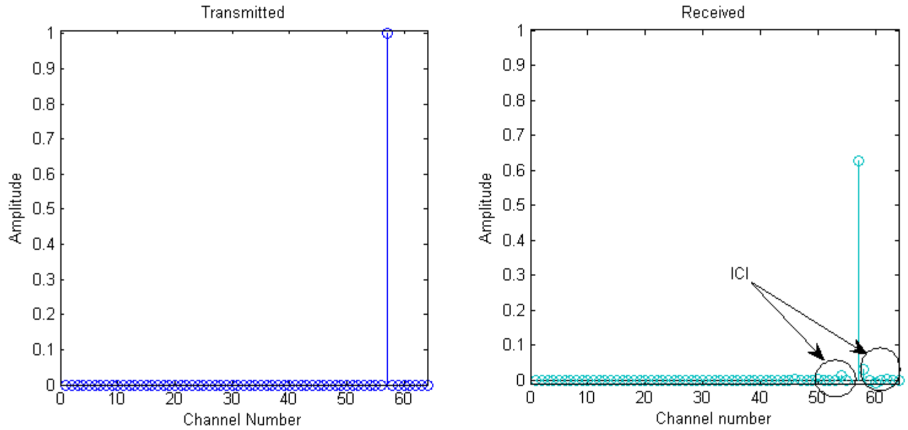

Figure 4.6: Transmitted and received signal for sample no. 57 in a fading channel

4.2.3 AWGN Fading Channel

But the situation in the previous section does not correspond to a realistic channel. In all transmissions the signal is distorted by noise. So in this section we have added after the fading channel White Gaussian Noise, to see what are the effects on the transmitted and received signal. With the noise that we have added, the channel has an SNR of 8dB.

Adding noise and analysing the same samples as in the previous section, we can see some differences with respect to ISI and ICI. In figure 4.9, for sample no. 1, we can see, that as we did not have ICI earlier, now it appears in all the channels. That is because of the noise added, and it is a problem that should be removed. Also, appears a little bit of ISI in the two forward

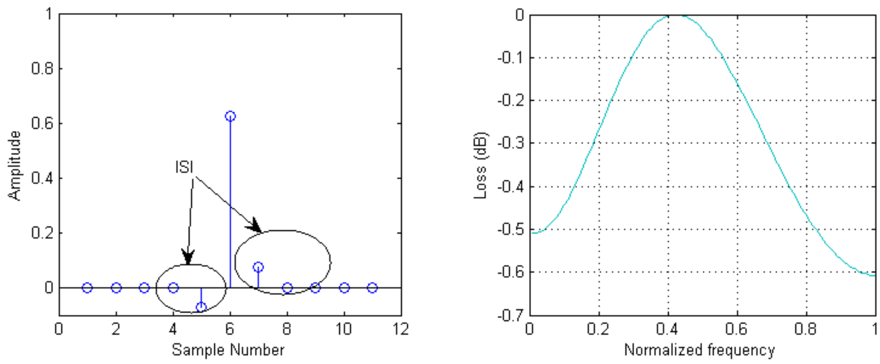

Figure 4.7: Received signal in channel no. 57 in a fading channel

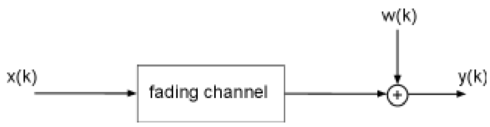

Figure 4.8: Fading Multipath Channel with Additive Gaussian Noise

and backward symbols in the channel one, where before we did not have.

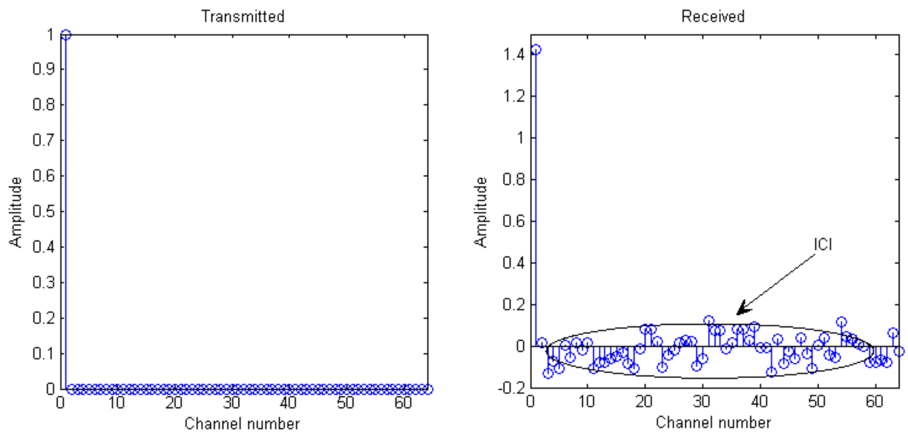

Figure 4.9: Transmitted and received signal for sample no. 1 in a fading channel with noise

In the previous section we saw that when receiving sample no. 57 already had ISI and ICI, that was due to the high distortion in the fading channel at high frequencies, but the ISI terms were only located next to the desired

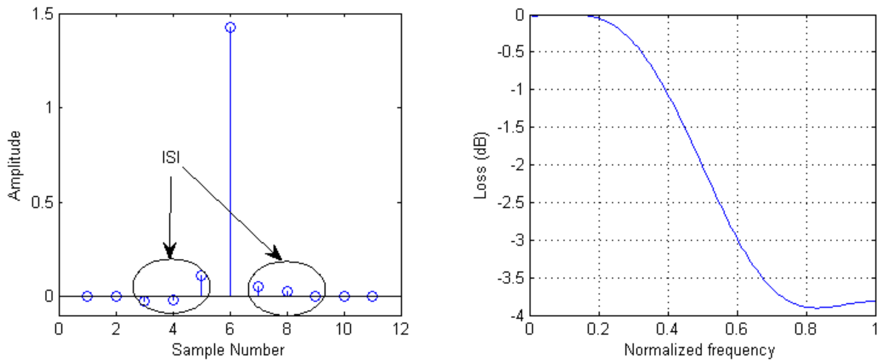

Figure 4.10: Received signal in channel no. 1 in a fading channel with noise

signal. In this case, due to the presence of noise in the channel, the ICI terms, as in the sample no. 1 are present in all the channels, and the ISI has increased. In the case of no noise, we only had ISI in the terms next to the desired signal, but now, it has appeared new ISI terms next to the ones that we already had.

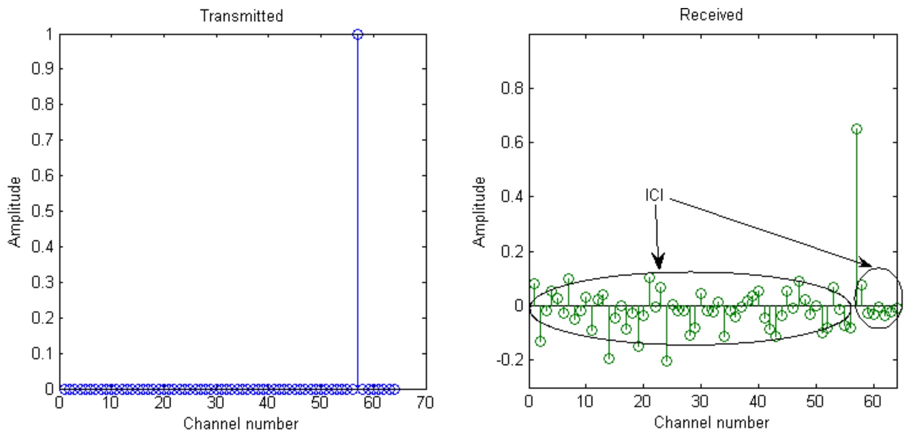

Figure 4.11: Transmitted and received signal for sample no. 57 in a fading channel with noise

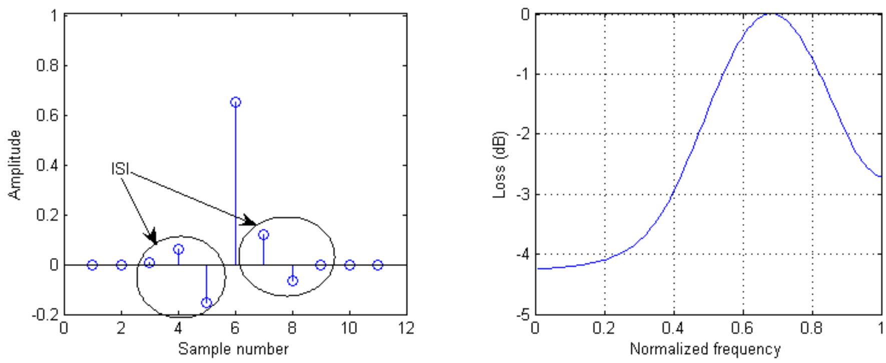

Figure 4.12: Received signal in channel no. 57 in a fading channel with noise

Chapter 5

Equalization Methods

In this chapter we are going to discuss the different ways to combat the Intersymbol Interference and the Intercarrier Interference. The channel distortion results in an Inter-symbol Interference and an Inter carrier Interference, which if left uncompensated, may cause errors, as we have seen in the previous chapter. The solution to the ISI problem is called *equalizer* and consists on designing a receiver that will be capable of cancel or reduce the ISI in the received signal.

All the simulations have been done with the system explained in Chapter 2 and 3, with $N = 64$ and $q = 2$. First we will explain a system for cancelling ICI distortion and then we will explain the possible ISI problem solutions.

As we have used MATLAB for the implementation of the system, we will explain the two possible equalizer that can be used:

- Linear Equalizer, based on the use of a linear filter with adjustable coefficients

- Decision-Feedback Equalizer, based on the use of previous detected symbols to suppress the ISI in the present symbol being detected.

5.1 The ICI problem

5.1.1 ICI cancelling

In this case it is supposed zero noise. The ICI distortion is not as important as the ISI one, as we have seen in previous chapter. But we have devised an simple system to cancel it. First of all we have to send a pilot for analysing the channel and to get the values of the signal received. One easy system to cancel it is by computing the gains and losses that the received signal suffers. We need to evaluate all the subchannels, for that we have to send a pilot signal over each one, to get all the necessary parameters.

Once the gains for all the subchannels have been calculated, we are able to build the parameter matrix with the calculated coefficients. This matrix will have the main diagonal with the amplitude of the signal received from the original pilot for each subchannel, so the index ik, if $i = k$ (main diagonal), the a_{ii} are the coefficients for the channel i over which the pilot has been sent, and if $i \neq k$, a_{ik} are the coefficients that correspond to the ICI received for the channel i:

$$A = \begin{bmatrix} a_{00} & a_{01} & a_{02} & 0 & & \dots & & 0 & 0 \\ a_{10} & a_{11} & a_{12} & 0 & & \dots & & 0 & 0 \\ & & a_{ik} & & & & & & \\ \dots & \dots & \dots & a_{i(i-1)} & a_{ii} & a_{i(i+1)} & \dots & \dots & \dots \\ & & & & & & & & \\ 0 & 0 & 0 & & \dots & 0 & a_{(N-2)(N-3)} & a_{(N-2)(N-2)} & a_{(N-2)(N-1)} \\ 0 & 0 & 0 & & \dots & 0 & a_{(N-1)(N-3)} & a_{(N-1)(N-2)} & a_{(N-1)(N-1)} \end{bmatrix} \tag{5.1}$$

Once we have built the matrix with the coefficients, then we have to find the inverse matrix:

$$\mathbf{M} = \mathbf{A^{-1}} = \frac{1}{\det(\mathbf{A})} \operatorname{adj}(\mathbf{A})^T \tag{5.2}$$

Now, with this matrix **M** we have the different gains to compensate ICI distortion in the transmitter. So before sending the signal over the CMFB there will be a block, where the signal will be multiplied by the matrix M to combat the ICI distortion.

$$\mathbf{y} = \mathbf{A}\mathbf{x} \qquad \text{and} \qquad \mathbf{x}' = \mathbf{A}^{-1}\mathbf{x} = \mathbf{M}\mathbf{x} \tag{5.3}$$

In equation 5.3, $\mathbf{x}' = \{x'(0), x'(1), \dots, x'(N-1)\}$ is the new input signal in the system, which is obtained from multiplying the vector $\mathbf{x}$ by the matrix of gains $\mathbf{M}$.

5.1.2 Results

In the figure 5.1, we can see the results obtained for the sample no. 57. In the previous chapter we saw that some ICI appeared when we sent a pilot over a multipath channel. This distortion was not as serious as the ISI one, but anyway, it should be reduced in order to receive a less corrupted signal. Introducing the concept explained in the previous section, we see that the received signal has reduced its distortion and the ICI has almost been cancelled.

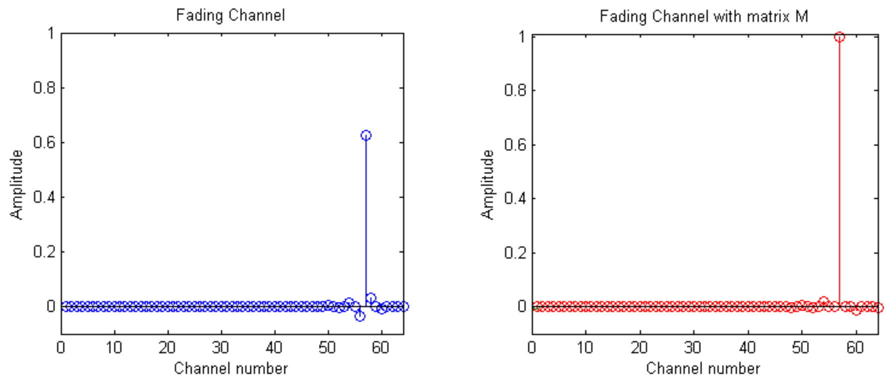

Figure 5.1: In left figure is depicted the received signal in a Fading Channel with any compensation, in the right figure is depicted the received signal when the matrix **M** is applied

5.2 Types of Equalizers

In this section we will explain in more detail about equalizers. We are going to explain two suboptimal channel equalization approaches to compensate the ISI. One approach employs a linear tansversal filter. These filter structures have a computational complexity that is a linear function of the channel dispersion length L.

5.2.1 Linear Equalizer

The linear filter most often used for equalization is the transversal filter shown in fig 5.2. The received signal is $\{\upsilon_k\}$

$$\upsilon_k = \sum_{n=0}^{L} f_n I_{k-n} + \eta_k \tag{5.4}$$

where $\{\eta_k\}$ is a white Gaussian noise sequence and $\{f_k\}$ is a set of tap coefficients of an equivalent discrete-time transversal filter having a transfer function $F(z)$.

Its output is the estimate of the information sequence $\{I_k\}$ and the estimate of the k symbol may be expressed as

$$\hat{I}_k = \sum_{j=-K}^{K} c_j \upsilon_{k-j} \tag{5.5}$$

where $\{c_j\}$ are the $2K+1$ complex valued tap weight coefficients of the filter. The estimated $\hat{I}_k$ is quantized to the nearest (in distance) information symbol to form the decision $\hat{I}_k$. If $\hat{I}_k$ is not identical to the transmitted information symbol I_k then we have error in the signal.

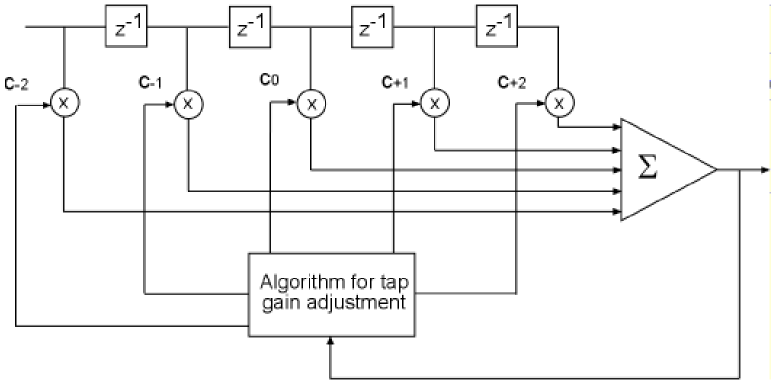

Figure 5.2: Linear transversal filter

These types of equalizer are divided in two groups:

- Symbol-spaced Equalizers
- Fractionally spaced equalizers (FSEs)

Symbol-spaced Equalizers

A symbol-spaced linear equalizer consists of a tapped delay line that stores samples from the input signal. Once per symbol period, the equalizer outputs a weighted sum of the values in the delay line and updates the weights to prepare for the next symbol period. This class of equalizer is called symbol-spaced because the sample rates of the input and output are equal.

In figure 5.3 we can see a schematic of a symbol-spaced linear equalizer with N weights, where the symbol period is T.

Fractionally spaced equalizers (FSEs)

In the previous linear equalizer structure, the equalized taps are spaced at the reciprocal of the symbol rate. This tap spacing is optimum if the equalizer is preceded by a filter matched to the channel distorted transmitted pulse. When the channel characteristics are unknown, the receiver filter is usually matched to the transmitted signal pulse and the sampling time is optimized for this suboptimal filter.

A fractionally spaced equalizer is a linear equalizer that is similar to a symbol-spaced linear equalizer, as described in the previous section. By contrast, however, a fractionally spaced equalizer receives K input samples before it produces one output sample and updates the weights, where K is an

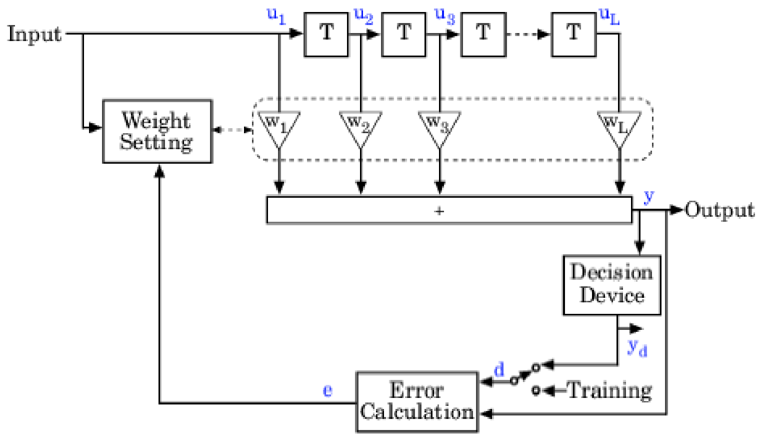

Figure 5.3: Symbol-spaced Equalizer scheme

integer. In many applications, K is 2. The output sample rate is 1/T, while the input sample rate is K/T. The weight-updating occurs at the output rate, which is the slower rate.

5.2.2 Decision-Feedback Equalizer (DFE)

The *Decision-feedback equalizer* (DFE) consists of two filters, a feedforward filter and a feedback filter. Both have taps spaced at symbol interval T. The input to the feedforward filter is the sequence $\{\upsilon_k\}$. The feedforward filter is identical to the transversal equalizer. The feedback filter has as its input the sequence of decisions on previously detected symbols. The feedback filter is used to remove that part of the inter-symbol interference from the present estimate caused by previously detected symbols.

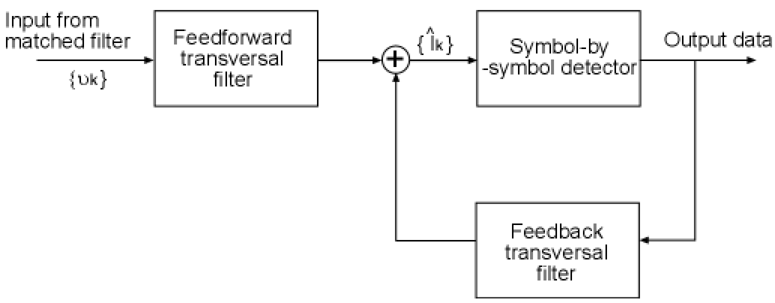

Figure 5.4: Structure of decision-feedback equalizer

5.3 Equalization in CMFB

The peculiarity of OFDM systems is that equalization of dispersive channels is performed simply by multiplying the signal at the output of each subchannel. The multiplication coefficient is related to the channel frequency response. However, this scheme only works if there is inserted a cyclic prefix in the transmitted signal. The insertion of this prefix reduces the spectral efficiency of the system.

As an alternative to OFDM, the Cosine Modulated Filter Bank, could be equalized by a multidimensional decision feedback equalizer (DFE). This results in an increase of the spectral efficiency for CMFB with respect to OFDM.

5.3.1 Equalization Schemes

In this section we will explain some simplified schemes and we will make some review of other known equalization procedures. We will begin introducing the M-point discrete Fourier transform (DFT) of the channel impulse response

$$\psi_m = \sum_{i=0}^{N_c-1} \exp\left(-\jmath 2\pi (\frac{m}{M})^i\right) c_e(i), \quad m = 0, 1, ...M-1 \tag{5.6}$$

OFDM

The usual equalization scheme for this modulation consists of multiplying the mth sample at the output of the FFT by the coefficient (zero-forcing criterion)

$$C_m = \frac{1}{\psi_m}, \quad m = 0, 1, ...M-1 \tag{5.7}$$

This technique uses a cyclic prefix at the transmitter to cope with the dispersive channel. The length of the effective channel may be reduced, and a shorter cyclic prefix may be used, by introducing a time-domain FIR filter immediately after the A/D converter.

Cosine Modulated Filter Bank

The CMFB is a particular case of Filtered Multitone (FMT), so we can apply the concepts of equalization in FMT to the CMFB. In this scheme ISI is always present in each subchannel. Then, two problems have to be solved, equalization of the transmit filters and equalization of the transmission channel.

- Decision Feedback Multichannel Equalizer (DFME): In this case we assume no ICI. The feedforward filter work on a subchannel base, a

DFE is inserted at the output of each subchannel. This scheme has a high computational complexity, moreover, the filter coefficients must be updated at least at time intervals to cope with the time-varying nature of the channel.

- PostDFT Simplified DFME (postDFME): This a simplified version of the previous one. We assume that over each subchannel the frequency response of the transmission channel is flat, i.e. it has a constant amplitude and a constant phase. This condition is related to the number of subchannels of the multicarrier system and to the coherence bandwidth of the channel. Under this assumption the transmission channel can be adaptively equalized by one tap per subchannel equalizer (as in OFDM with cyclic prefix), while the transmit filters can be equalized by a fixed DFE. Then, the cascade of transmit and receive filters turns out to be independent of the subchannel index. Hence, the feedforward and feedback filters are the same for all subchannels.

- PreDFT Simplified DFME (preDFME): In this case, the DFME is instead directly applied to the receiver signal. The receiver filters equalize the transmit filters, while the channel is equalized by the one tap per subchannel structure. In particular the feedforward and feedback subchannel filters are computed as a DFE of the corresponding polyphase components of the transmit prototype filter, the each branch of the receiver filter bank is different.

In order the apply these structures we have studied two simplified cases, using a DFE instead of using a Linear filter because of its robustness towards noise. We have computed a scheme with one equalizer in each subchannel (in this case N=64) in the output of the receiver, before the FFT. And another scheme equalizing the general channel before the receiver with one DFE equalizer. We have employed the Matlab Simulink Tool, and we have used a training sequence, obtained by sending a pilot through the channel.

Linear and decision-feedback equalizers are adaptive equalizers that use an adaptive algorithm when operating. The matlab toolbox supports these types of algorithms:

- Least mean square (LMS)
- Signed LMS, including these types: sign LMS, signed regressive LMS, and sign-sign LMS
- Normalized LMS
- Variable-step-size LMS
- Recursive least squares (RLS)
- Constant modulus algorithm (CMA)

5.3.2 LMS

In our case we have used Least Mean Square algorithm for the equalizer scheme, it is the most used adaptive filtering algorithm, in practice, this is attributed to its simplicity and robustness to signal statistics.

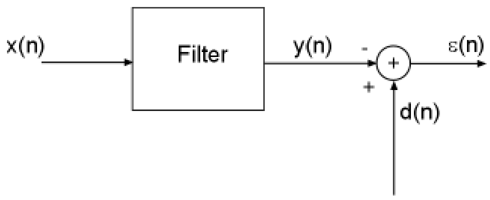

Figure 5.5: Adaptive filter LMS

In this section we are going to explain how it works and its main concepts.

The filter is an N-tap transversal adaptive filter, the input sequence is $x(n)$, the reference is $d(n)$ and the output sequence obtained is $y(n)$. We assume $y(n)$ to be real:

$$y(n) = \sum_{i=0}^{N-1} w_i(n)x(n-i) \tag{5.8}$$

The $w_i(n)$ coefficients are selected so that the difference error,

$$\varepsilon(n) = d(n) - y(n) \tag{5.9}$$

is minimized. The LMS algorithm adapts the filter coefficients so that $\varepsilon(n)$ are minimized in the mean square sense.

The LMS algorithm is a stochastic implementation of the steepest-descent algorithm. It replaces the cost function $\xi = E[\varepsilon^2(n)]$ by its instantaneous estimate $\hat{\xi}^2(n) = \varepsilon^2(n)$. Then for LMS we have the function

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \mu \nabla e^2(n) \tag{5.10}$$

where $\mathbf{w}(n) = [w_0(n) \quad w_1(n) \ldots w_{N-1}(n)]^T$, μ is step-size parameter and ∇ is the gradient operator defined as the column vector

$$\nabla = \left[\frac{\partial}{\partial w_0} \quad \frac{\partial}{\partial w_1} \cdots \frac{\partial}{\partial w_{N-1}} \right]^T \tag{5.11}$$

the ith element of the gradient vector $\nabla \varepsilon^2(n)$ is:

$$\frac{\partial \varepsilon^2(n)}{\partial w_i} = -2\varepsilon(n) \frac{\partial \varepsilon(n)}{\partial w_i} \tag{5.12}$$

Substituting 5.9 in 5.12 and noting that $d(n)$ is independent of $w_i(n)$ then

$$\frac{\partial \varepsilon^2(n)}{\partial w_i} = -2\varepsilon(n)\frac{\partial y(n)}{\partial w_i} \tag{5.13}$$

substituting 5.8 in this relation, we get

$$\frac{\partial \varepsilon^2(n)}{\partial w_i} = -2\varepsilon(n)x(n-i) \tag{5.14}$$

Using 5.12 in the last equation we obtain

$$\nabla \varepsilon^2(n) = -2\varepsilon(n)\mathbf{x}(n) \tag{5.15}$$

where $\mathbf{x}(n) = [x(n) \quad x(n-1)\ldots x(n-N+1)]^T$. So finally, substituting this into 5.10 we arrive to

$$\mathbf{w}(n+1) = \mathbf{w}(n) + 2\mu\varepsilon(n)\mathbf{x}(n) \tag{5.16}$$

We have seen how the LMS works with recursive adaptation of the filter coefficients after every new input sample, $x(n)$, arrives, and its correspondence with the desired sample $d(n)$. The equations 5.8, 5.9 and 5.16 are the three steps that follows the LMS algorithm to complete each iteration

5.4 ISI cancellation

In this section we are going to explain the results obtained after the simulation of two different schemes. For simplification first we have put a different equalizer for each subchannel, with a different training sequence for each one, then in the next section we have used one equalizer after the fading channel to equalize the signal. The equalizer used in both cases has been the DFE with a LMS algorithm with coefficients:

- 10 weights for the fordward filter
- 9 weights for the backward filter
- A reference tap index in the equalizer with value 1
- A step size of 0.03

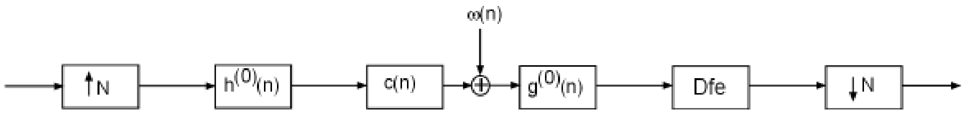

Figure 5.6: General Subchannel structure with DFE equalizer

5.4.1 Decision Feedback equalizer in each subchannel

Fading Channel

In the chapter before we present some results of the received signal, as we saw, in many of the received symbols ISI distortion appeared, in this chapter we have analysed the same channel as the one that before presented distortion in the reception (no.57), but with an equalizer to compensate the ISI. In this firsts results with a DFE in each subchannel and without noise in the fading channel, we see that the results are good, with a biggest difference of magnitude between the symbols of 1db.

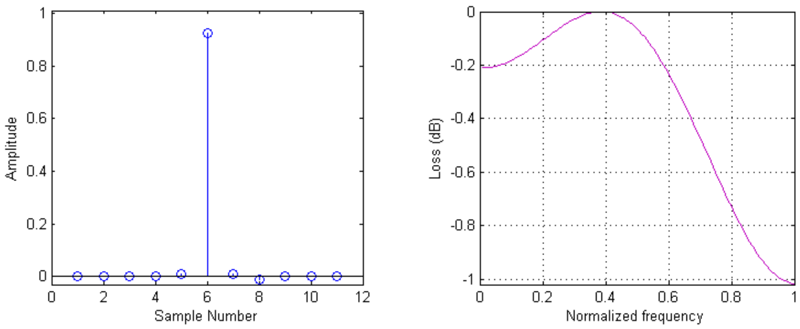

Figure 5.7: Received Signal in Channel 57 in a fading channel, supposing no noise

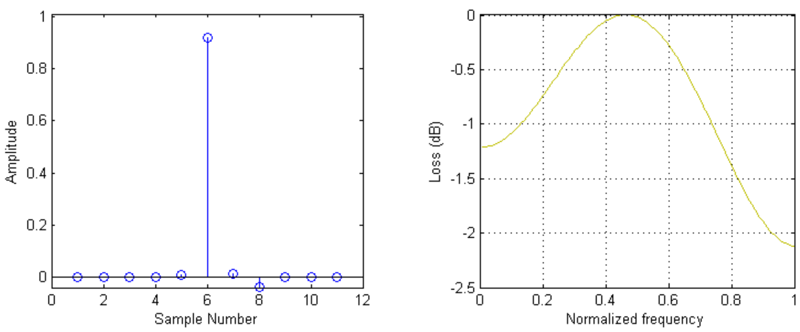

Figure 5.8: Received Signal in Channel 57 in a fading channel supposing additive Gaussian noise

AWGN Fading Channel

When we add a Gaussian noise to the channel, so that that we have an SNR of 8dB, the ISI is reduced considerably, and the original signal can be detected although it has some amplitude loss, as we can see in figure 5.8.

5.4.2 Decision Feedback equalizer after the Multipath channel

Now, in this case, we have introduced the equalizer after the fading channel (in both cases, with and without noise), this is a less complex scheme, because it does not need as much multiplications as when we have a different DFE for each subchannel. Here, we only need one, with its training sequence. As in the case before, the channel has been evaluated sending a pilot signal over it to know its characteristics.

Fading Channel

As we have seen before we have some ISI distortion with any equalizer, and now, without considering additive Gaussian noise we have got a big reduction of the ISI, but it cannot be appreciated if this is a better result than with an equalizer after each subchannel.

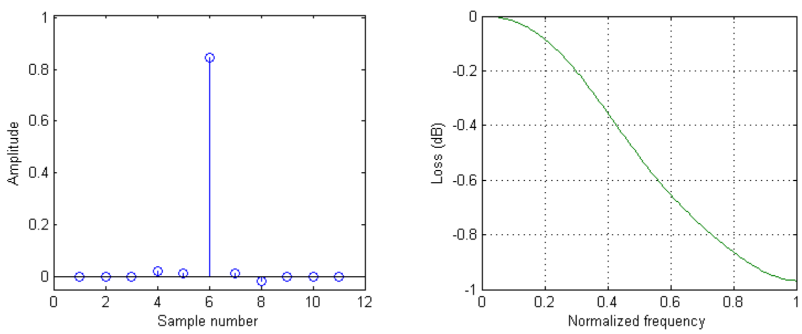

Figure 5.9: Received Signal in Channel 57 in a fading channel, supposing no noise

AWGN Fading Channel

In this case, the results have improved respect to the ones in the previous section. We received the signal with many ISI in all the symbols in channel no.57, and as we see in figure 5.10, there is high improvement of the results after introducing the equalizer.

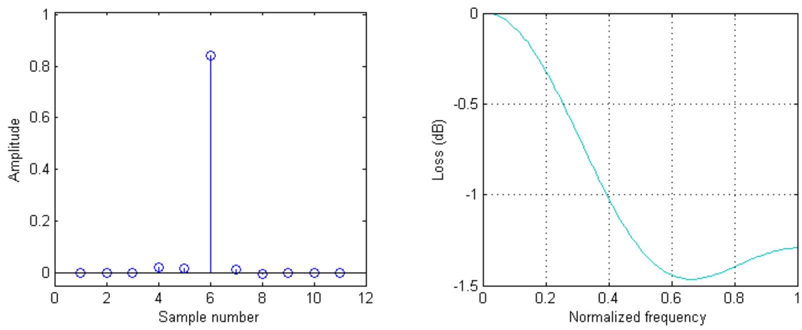

Figure 5.10: Received Signal in Channel 57 in a fading channel supposing additive Gaussian noise

5.4.3 Error results

In this section we are going to analyse the error obtained in the received signal for the different schemes explained. In the previous section we have the signals received, but it is hard too see which position of the equalizer in the scheme gives a better result. In the next figures we are able to see these results better.

Figure 5.11 depicts the error obtained without any equalization in the receiver, with a big error of the original symbol (sample no.6) with amplitude almost 0.4, and many errors in the previous and following symbols. In figure 5.12, in the first graph we can see, that the amplitude of the errors have been reduced with one DFE equalizer after the fading channel, but in the next figure, with 64 DFEs equalizers, the errors have been reduced more than in the previous case.

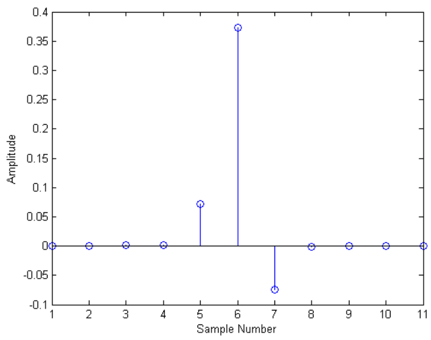

Figure 5.11: Error in the received signal in a Multipath fading channel

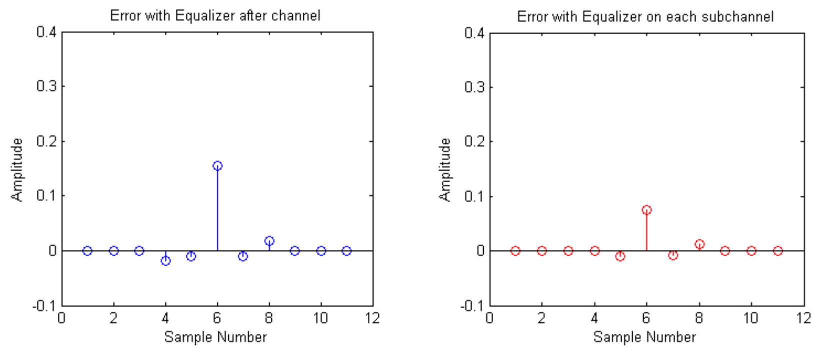

Figure 5.12: Errors in channel 57 in a Multipath fading channel with equalizers

In figures 5.13 and 5.14 we can see the results before but applied to a fading channel with additive white Gaussian noise. As before, we are able to reduce the amplitude of the ISI distortion as we see, that the amplitude of the errors in these symbols are much lower than 0.1 of amplitude. We see that with the equalizer after each channel, we have best results, as in the previous case, although one of the symbols have a bigger error than the other ones, it is not superior than 0.1.

Although that in this results, we get better results with the equalizer after each subchannel, it has a higher computational complexity than with just an equalizer after the fading channel.

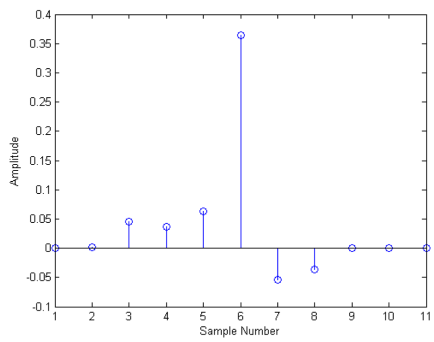

Figure 5.13: Error in the received signal in a Multipath fading channel with AWGN

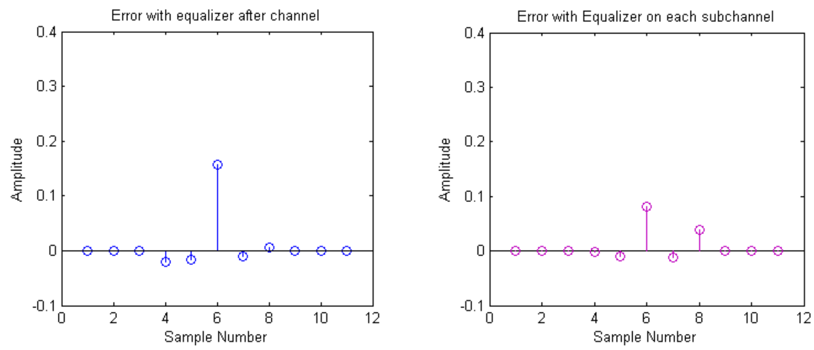

Figure 5.14: Errors in channel 57 in an AWGN Multipath fading channel with equalizers

	No equalizer	1 Equalizer	64 Equalizers
Fading Channel	0,013645	0,0022753	0,00054928
AWGN Fading Channel	0,013091	0,002876	0,00089163

Table 5.1: MSE for the received signal in channel no. 57

In the table 5.1 we can see the Mean Square Error, calculated for all the cases. We see, that it is reduced when we use one equalizer for each one of the subchannels. With any equalizer, we see that both for a fading channel without noise, and a fading AWGN channel we have more or less the same MSE, but when we introduce the equalizers, the result is better for a non AWGN channel, it is because of the noise introduces more distortion to the signal.

In the table 5.2 we are presenting the results obtained for SNR in the channel no. 57. As we can see we already have a good Signal to Noise ratio when using 1 equalizer, over 15 dB when no noise is added and almost 15 when AWGN has been added.

	No equalizer	1 Equalizer	64 Equalizers
Fading Channel	4.84	16.38	24.41
AWGN Fading Channel	5.51	14.76	21.78

Table 5.2: SNR in dB for channel no. 57

5.5 Other results

In this section, instead of sending only an impulse through one channel, we two different impulses have been sent through two different channels. In the figure 5.15 we can see that the channels chosen have been 16 and 57. When this samples are sent through a multipath channel (with or without AWGN noise) ICI appears in the received signal as it can be seen in figures 5.15 and 5.16, being ICI more important when a channel with additive Gaussian noise is used.

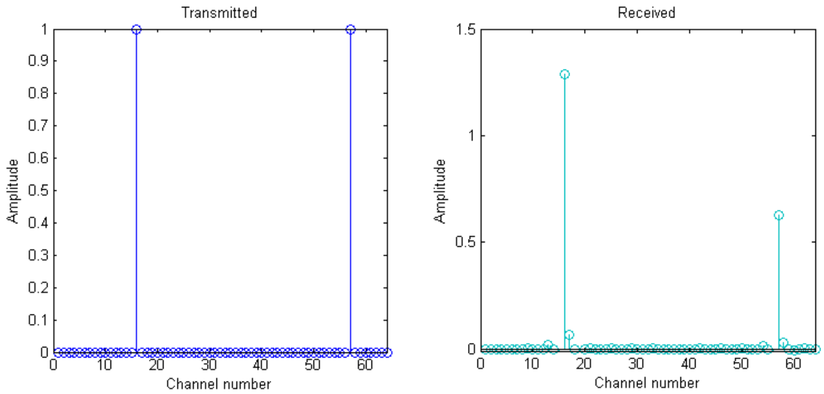

Figure 5.15: In the left figure the theoretical signal is depicted and in the right the received signal is depicted in a multipath channel without noise

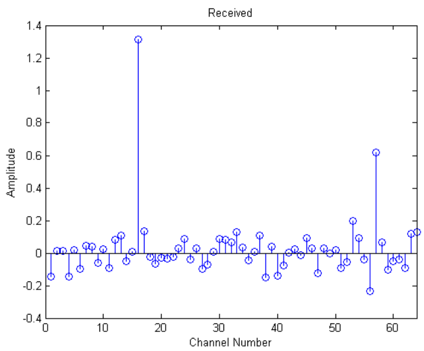

Figure 5.16: Received signal for a multpath channel with Gaussian noise added

With the matrix coefficients in Appendix C for a fading channel with no noise we have compensate the ICI error introduced when two samples are sent through two different channels.

Also in figures 5.18 and 5.19 we can see that the received symbols in both channels are corrupted by ISI distortion. When AWGN noise is introduced

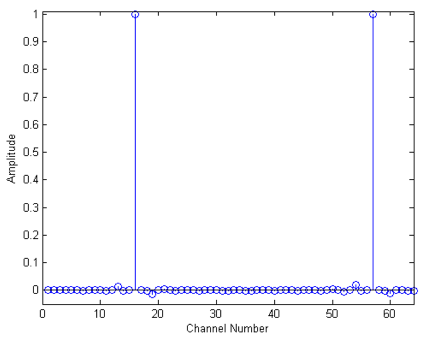

Figure 5.17: ICI cancellation in a fading channel without AWGN noise

to the channel ISI increases. In order to avoid this distortion, the same results as in the previous sections are applied to this case. First introducing one equalizer, and after introducing 64 equalizers after the receiver.

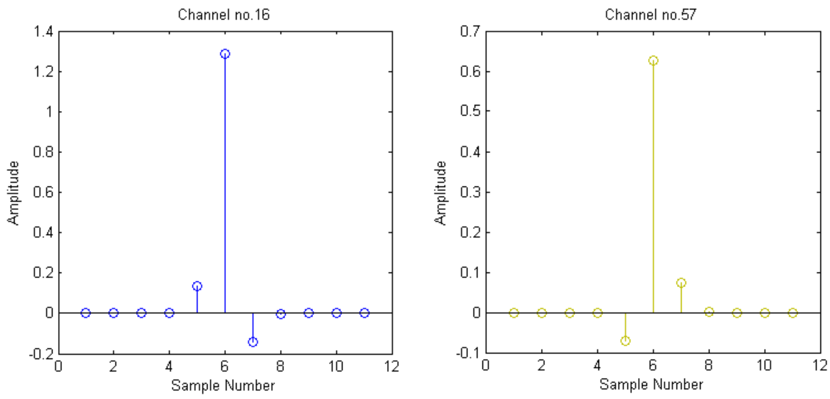

Figure 5.18: Received symbols for channels 16 and 57 in a multipath channel with no noise

As in the previous cases, the best results are given when using 64 equalizers, but with a higher computational complexity than in the case with one equalizer. In the Table 5.3 the SNR results for this system for channels no. 16 and 57.

When one equalizer is used the SNR is higher than 15 dB, that is a good relation Signal to Noise, when 64 equalizers are used the SNR increases until 30dB in channel no. 16, and 20 in channel no.57, however the high waste of resources when 64 equalizers are used and the SNR obtained with one equalizer makes the simpler scheme studied a good aproximation to cancel ISI.

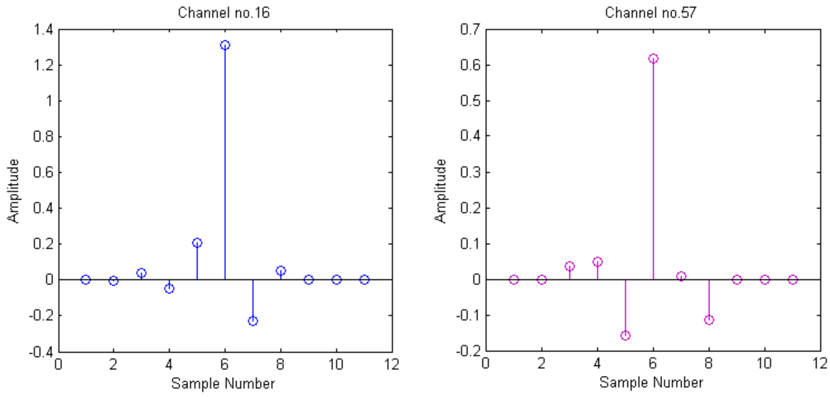

Figure 5.19: Received symbols for channels 16 and 57 in a multipath channel with AWGN noise

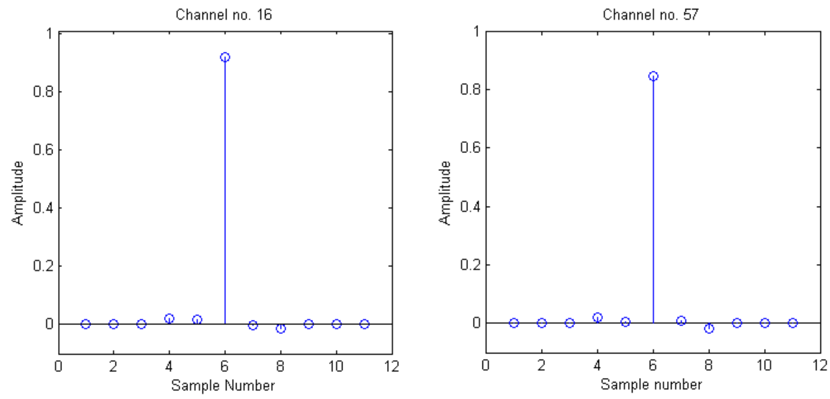

Figure 5.20: Received symbols when 1 equalizer is used in a multipath channel with no noise

5.6 Two level signal analysis: PAM-2 (Pulse Amplitude Modulation)

In this section we have evaluated the behaviour of a two level signal, for it, a PAM-2 modulation has been chosen. The main characteristics of this modulation is:

- Bits used in the modulaton: 2 bits (0,1)
- The symbols used are 0 and 1, which are modulated as -1 and +1 respectively
- In this case the the Bit Energy is equal to the Symbol Energy

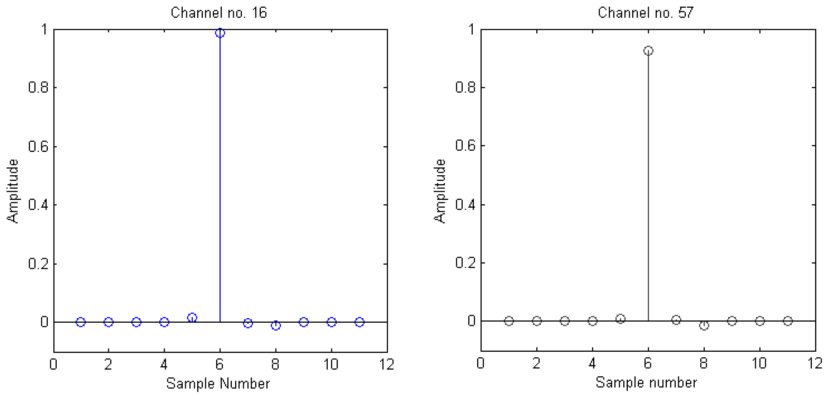

Figure 5.21: Received symbols when 64 equalizers are used in a multipath channel with no noise

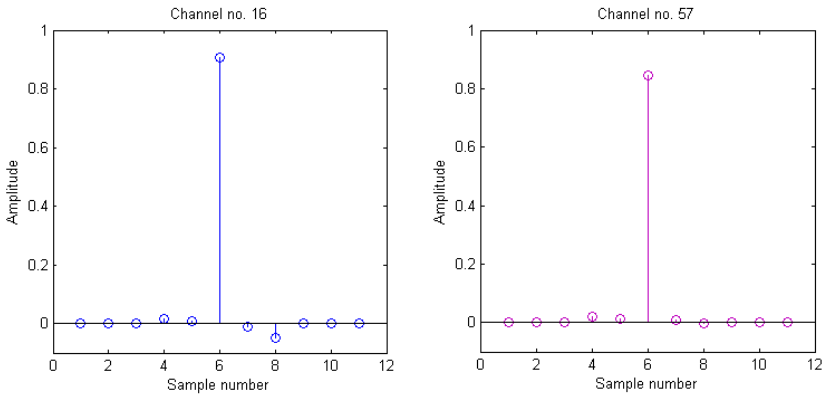

Figure 5.22: Received symbols when 1 equalizer is used in a multipath channel with AWGN noise

In this case we have made a Bit Error analysis depending on the SNR introduced. For that a PAM-2 modulated random signal has been sent through the channel. The results obtained are depicted in figure 5.24. We can see that the Error rates slightly decrease when equalizers are used and the Eb/N_0 is increased. In order to obtain better results a detailed analysis has to be done and an optimum equalizer has to be found to try to decrease the Bit Error Rate.

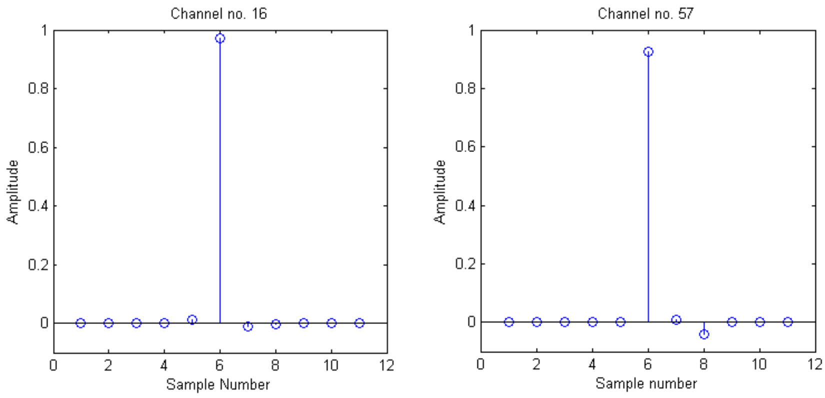

Figure 5.23: Received symbols when 64 equalizers are used in a multipath channel with AWGN noise

	Channel no.16		
	No equalizer	1 Equalizer	64 Equalizers
Fading Channel	12.69	23.47	36.76
AWGN Fading Channel	11.23	21.65	33.13
	Channel no.57		
Fading Channel	4.84	16.57	24.93
AWGN Fading Channel	1.03	16.31	23.92

Table 5.3: SNR in dB for channel no. 16 and 57

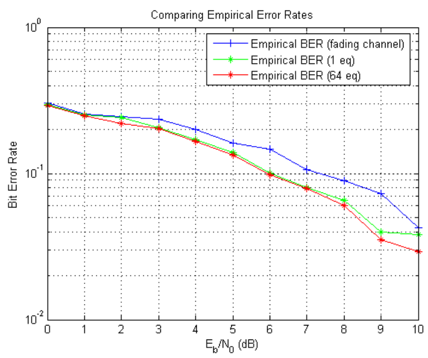

Figure 5.24: Empirical Error Rates for channel 57

Chapter 6

Conclusions and Future Work

A Cosine Modulated Filter Bank system has been studied. In [2] the coefficients of the filter were optimized for 3 types of distortion

- Additive White Gaussian Noise
- Doppler shift introduced by the relative motion between the transmitter and receiver
- Timing error between the converters in the transmitter and receiver

In this thesis we have used the coefficients given for a system of N=64 channels and q=2 lattice structures. In order to follow with the previous study, in this case we have analysed the behaviour of a transmitted signal over a non ideal channel, in this case affected by a Multipath fading channel.

6.1 Conclusions

We have seen, when a signal is sent through a fading channel, that the received signal is distorted, and the original signal can be recovered if the distortions are moderate. The signal received is corrupted by ICI and ISI, although ISI error is more important that ICI. First we have made an analysis in a channel without AWGN noise, and after we have added it to the system.

After the preliminary analyses we have seen that there were two problems for which we need a solution.

- For the ICI problem a simple system that consist on compensate the gains before the signal was sent over the channel has been developed. This needs of a previous analysis of the subchannels.
- For the ISI problem, we have introduced an *equalizer*. Several different equalizers have been explained to see which one fitted better to our system, depending on our necessities.

We have seen that the ICI distortion is not important, although it corrupts the received signal. It needs to be cancelled, and as it has been seen in chapter 5, it can be done in an easy way. At least, the ICI distortion is interchannel interference, this means that there is a loss of energy in the channels next to the subchannel used. As the channel is changing in time, it has to be analysed continuously. Then prior to the signal a pilot has to be sent to compute de gains. After this we have seen that applying this results the ICI is properly cancelled, and the signal received is more similar to the signal sent.

It is an easy method to be implemented to cancel the ICI distortion, although it produces a high resource waste to send a pilot to study the channel before and tansmission.

In previous chapters we saw that when you tansmit a signal through a fading channel, in the receiver the signal arrives distorted by ISI (and ICI as we haven seen previously). It is a serious problem and it has needed a harder study to solve the problem than in the ICI distortion.

In this case we have studied the usage of an equalizer. It has been tested and the results are given for two different schemes. The first scheme was with one equalizer after the fading channel, before the receptor, and the second scheme was developed introducing an equalizer in every subchannel, after the receiver subscheme. In the results we could see that the best results were given by the second scheme, however this scheme is more complex to compute due to use 64 equalizers instead of 1 equalizer. The number of multiplications and resources used in the 64-equalizers scheme is much higher than in the 1-equalizer scheme, however the results are not much better, so, the scheme with one equalizer gives quite good results and its complexity is low.

6.2 Future Work

Although OFDM systems have been largely studied, in this master thesis we have gone deeply into OFDM systems and developing it as a filter bank system and studied its behaviour in a multipath environment. However there are many characteristics that can be more exhaustively studied. ISI and ICI distortion have been separately studied in this project, and a more extended investigation of these concepts have to be done, for example in order to see how ICI distortion behaves when use an equalizer to cancel ISI.

Other methods apart of equalizers and gain matrix can be studied to cancel de distortions that a fading channel introduces in the signal, that will allow to make a further comparison between all possibilities, to see which one fits better in a multirate filter bank system.

A detailed study of the the hardware complexity cost in the different receiver schemes after using the equalizer block is needed too. And as we have seen in the previous chapter a further analysis has to be done in order to obtain a better BER (Bit Error Rate) when a M-level signal is transmitted through the system.

Appendix A

Derivation of the Lattice Structure

A.1 The Analysis Filter Bank: derivation of the Polyphase structure

In this Appendix we are going to explain the derivation of the Polyphase matrix used in chapter 2. By modulating the prototype filter as

$$h_n(k) = h(k)\cos\left(\frac{\pi}{N}(n+\frac{1}{2})(k+\frac{1}{2})\right) = h(k)T_N(n,k) \qquad \text{(A.1)}$$

we can obtain the unit sample response for the channel no. n. $T_N(n,k)$ represents elements of a matrix $\mathbf{T}_N$, a cosine transformation of type IV. This transformation has several interesting symmetry properties. The coefficient $n+\frac{1}{2}$ represents the frequency shift of the prototype filter, where n is the channel number. The parameter $k+\frac{1}{2}$, is inserted to make the transform symmetryc. This transform has two useful properties:

- Periodic extension

$$T_N(n, m+2Nr) = (-1)^r T_N(n,m) \qquad \text{(A.2)}$$

- Symmetry

$$T_N(n, 2N-1-m) = -T_N(n,m) \qquad \text{(A.3)}$$

Then, we can calculate the output signals of channel no. n as

$$y_n(k) = \sum_{l=0}^{L-1} x(k-l)h(l)T_N(n,l), \quad n \in 0,1,\ldots,N-1 \qquad \text{(A.4)}$$

The summation can be split into a double sum by introducing two summation variables instead of l, as $l = m+2Nr$, where $m \in 0,1,\ldots,2N-1$, and $r \in 0,1,\ldots,q-1$. After some manipulations and downsampling the output signals by a factor N, to obtain critical sampling (this means that we conserve all the samples in the filtering process) we get:

$$y_n(iN) = \sum_{m=0}^{2N-1} T_N(n,m)\upsilon_m(i) \qquad \text{(A.5)}$$

where $\upsilon_m(i)$ is given by

$$\upsilon_m(i) = \sum_{r=0}^{q-1} \kappa_m(r)u_m(i-2r) \qquad \text{(A.6)}$$

The equation A.6 represents the convolution in the subfilters using the coefficients

$$\kappa_m(r) = (-1)^r h(m+2Nr) \qquad \text{(A.7)}$$

related to the prototype filter coefficients. We can see that the coefficients of the filter are downsampled by a factor $2N$. The inputs to the subfilters are selected samples of the input signal

$$u_m(i) = x(Ni - m) \tag{A.8}$$

In the Z-domain, the previous equations results in

$$V_m(z) = K_m(z^2)U_m(z) \tag{A.9}$$

where $V_m(z) = TZ\{v_m(i)\}$ and $U_m(z) = TZ\{U_m(i)\}$, and the subfilter transfer functions are expressed as

$$K_m(z^2) = \sum_{r=0}^{q-1} \kappa_m(r) z^{-2r} \tag{A.10}$$

The argument z^2 means that there are two delays between each non-zero coefficient in the filters. The filter systems looks like a polyphase structure, but in this case there are $2N$ subfilters, there are N independent subchannels and the subchannels are downsampled by N.

Observing some relations between the differents inputs to the subfilters and the symmetry of the transforms, it will help us to make the structure of the entire system, and its implementation more simple.

The first modification is based on

$$u_{m+N}(i) = u_m(i-1) \tag{A.11}$$

The second modification is based on the transform $\mathbf{T}_N$ property

$$y_n(i) = \sum_{m=0}^{2N-1} T_N(n,m)v_m(i) = \sum_{m=0}^{N-1} T_N(n,m)[v_m(i) - v_{2N-1-m}(i)] \tag{A.12}$$

The transform is square with N inputs and size $N \times N$. And the polyphase matrix of the CMFB can be expressed as the product of the cosine transform and the matrix representing the filter operations

$$\mathbf{P}(z) = \mathbf{T}_N \mathbf{K}(z). \tag{A.13}$$

The $\mathbf{K}(z)$ matrix structure is very important ($N \times N$ size). It consists of elements in each one of the diagonals:

$$\mathbf{K}(z) = \begin{bmatrix} K_0 & 0 & \cdots & 0 & -z^{-1}K_{2N-1} \\ 0 & K_1 & \cdots & -z^{-1}K_{2N-2} & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & -z^{-1}K_{N+1} & \cdots & K_{N-2} & 0 \\ -z^{-1}K_N & 0 & \cdots & 0 & K_{N-1} \end{bmatrix} \tag{A.14}$$

The argument z^2 is left out for all K's in the matrix elements. Observe that the polyphase matrix is a function of z, not only of z^2.

From the matrix A.14 simple submatrices $\mathbf{M}_i(z)$ can be extracted. These consist of four nonzero elements located symmetrically around the center of the matrix

$$M_i(z) = \begin{bmatrix} K_i(z^2) & -z^{-1}K_{2N-1-i}(z^2) \\ -z^{-1}K_{N+i}(z^2) & K_{N-1-i}(z^2) \end{bmatrix} \tag{A.15}$$

A.2 Implementation of a CMFB with N=8 and q=1

In this section we are going to present a simple system with $N = 8$ channels and 1 lattice structures

A.2.1 Programs

Cosine Transformation

```
%This program compute the cosine transformation of the input x_n(k)
%Y(k)=x(n)cos((pi/N)*(n+1/2)*(k+1/2))
function [g1,g2]=coseno(x)

%Variables
%N= number of channels
N=8;
n=0:1:N-1;
k=0:1:2*N-1;
g1=ones(1,N/2);
g2=ones(1,N/2);

y2=x.*exp(1i*pi*n/(2*N));
v2=y2';

% Second module
% Compute the DFT, the FFT algorithm works by columns
% and the input signal are by rows, so I has to be used
% the transposed vector in FFT
f1=fft(v2,2*N);
f2=f1';

%Third module
x5=f2.*exp(1i*pi/(4*N));
u=x5.*(exp(1i*pi*k/(2*N)));
z=real(u);

for m=1:1:N/2
g1(m)=z(m);
g2(m)=z(m+N/2);
end
```

Analysis Filter Lattice structure without delays

```
% This block computes a simple lattice structure
% without delays, they are applied before, in the simulink system
% q=1 lattice stage module
% Varibles:
% h(k) are the filter coefficients.
% P depends on the simulation duration
% g is the output signal
% w5,w7 are the input paramemter to the block

function [g,P] =trans(w5,w7,h);

N=8;
g=ones(P,N);
d=[h(4) h(3) h(2) h(1)];
c=[-h(5) -h(6) -h(7) -h(8)];
b=[-h(12) -h(11) -h(10) -h(9)];
a=[-h(13) -h(14) -h(15) -h(16)];

for n=1:1:P
for m=1:1:N/2
g(n,m)=[w5(1,m,n) w7(1,N+1-m,n)]*[d(m);-b(m)];
end
end

for n=1:1:P
for m=N/2+1:1:N
g(n,m)=[w5(1,m,n) w7(1,N+1-m,n)]*[a(N+1-m);-c(N+1-m)];
end
end
```

A.2.2 Results

In this case, as we can see in [2] the filter used is a High Pass Filter with coefficients of the lattice filter related to the prototype filter with the values shown in table A.1.

With an impulse respose and frequency response for the filter depicted in figure A.1:

This filter gives a frequency response for the channel no. 1 of a High pass Filter, and for channel no. 8 a Low pass impulse response

i	a_i	b_i	c_i	d_i
1	-h(12)=-0,075683	-h(11)=0,085839	-h(4)=-0,075683	h(3)=-0,063662
2	-h(13)=0,063662	-h(10)=-0,093549	-h(5)=0,085839	h(2)=0,050455
3	-h(14)=-0,050455	-h(9)=0,098363	-h(6)=-0,093549	h(1)=-0,036788
4	-h(15)=0,036788	-h(8)=-0,1	-h(7)=0,098363	h(0)=0,023387

Table A.1: Filter coefficients and relation with the Lattice Coefficients

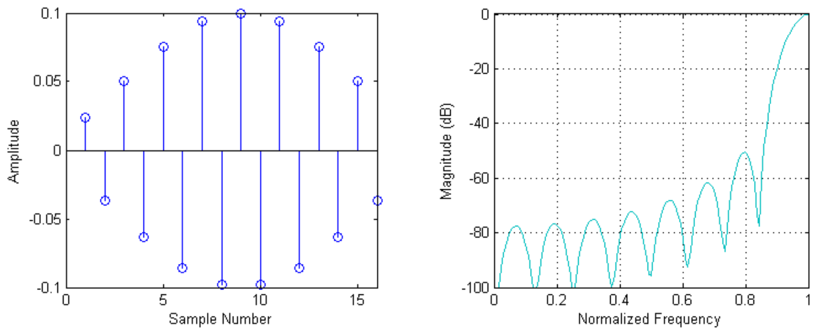

Figure A.1: Unit Sample Response and Frequency Response of the Prototype Filter

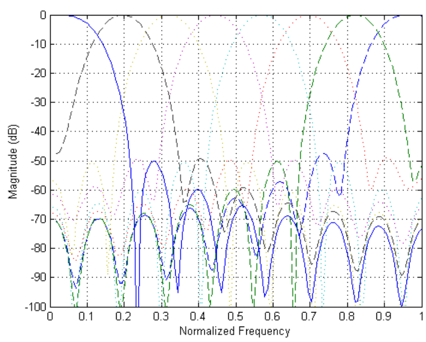

Figure A.2: Frequency Response for an 8 channel CMFB

Appendix B

Lattice Coefficients

B.1 Lattice Coefficients for N=4 and q=2

$$F(N,q) = F(4,2) = \begin{bmatrix} \phi_{01} & \phi_{02} \\ \phi_{11} & \phi_{12} \end{bmatrix} = \begin{bmatrix} 3\frac{\pi}{4} & \frac{\pi}{2} \\ 3\frac{\pi}{4} & \frac{\pi}{2} \end{bmatrix} \tag{B.1}$$

B.2 Lattice Coefficients for N=64 and q=2

$$F(64,2) = \begin{bmatrix} 2.35491 & 1.56802 \\ 2.35236 & 1.56225 \\ 2.34965 & 1.55612 \\ 2.34649 & 1.54960 \\ 2.34286 & 1.54242 \\ 2.33865 & 1.53445 \\ 2.33349 & 1.52587 \\ 2.32727 & 1.51668 \\ 2.32003 & 1.50679 \\ 2.31157 & 1.49640 \\ 2.30185 & 1.48564 \\ 2.29121 & 1.47430 \\ 2.27973 & 1.46244 \\ 2.26735 & 1.45034 \\ 2.25444 & 1.43778 \\ 2.24134 & 1.42464 \\ 2.22782 & 1.41126 \\ 2.21375 & 1.39790 \\ 2.19917 & 1.38458 \\ 2.18385 & 1.37155 \\ 2.16744 & 1.35914 \\ 2.14963 & 1.34758 \\ 2.12991 & 1.33730 \\ 2.10837 & 1.32812 \\ 2.08571 & 1.31922 \\ 2.06114 & 1.31133 \\ 2.03358 & 1.30549 \\ 2.00574 & 1.29899 \\ 1.98005 & 1.28947 \\ 1.95118 & 1.28238 \\ 1.91375 & 1.28327 \\ 1.87734 & 1.28227 \end{bmatrix} \quad \text{(B.2)}$$

Appendix C

Matrix A N=64 and q=2

$$M[0,31] = \begin{array}{ccc} a_{i(i-1)} & a_{ii} & a_{i+1} \end{array} \left[\begin{array}{ccc} 0 & 1,3993 & 0,0045799 \\ -0.0044017 & 1,3984 & 0,0089138 \\ -0.0090279 & 1.3964 & 0.013486 \\ 0.013372 & 1.3936 & 0.0017804 \\ -0,017907 & 1,3897 & 0,022286 \\ -0,02219 & 1,385 & 0,02652 \\ -0,026615 & 1,3793 & 0,030881 \\ -0,030786 & 1,3727 & 0,034977 \\ -0,035072 & 1,3652 & 0,039178 \\ -0,039084 & 1,3568 & 0,043098 \\ -0,04319 & 1,3475 & 0,047098 \\ -0,047007 & 1,3374 & 0,050803 \\ -0,050892 & 1,3265 & 0,054564 \\ -0,054477 & 1,3148 & 0,058019 \\ -0,058104 & 1,3024 & 0,061505 \\ -0,061422 & 1,2892 & 0,064676 \\ -0,064758 & 1,2753 & 0,067855 \\ -0,067773 & 1,2608 & 0,070707 \\ -0,070789 & 1,2456 & 0,073554 \\ -0,07347 & 1,2299 & 0,076056 \\ -0,076141 & 1,2136 & 0,078545 \\ -0,078459 & 1,1967 & 0,080673 \\ -0,08076 & 1,1794 & 0,08278 \\ -0,082692 & 1,1617 & 0,084512 \\ -0,0846 & 1,1436 & 0,086217 \\ -0,086129 & 1,1251 & 0,087538 \\ -0,087627 & 1,1063 & 0,088825 \\ -0,088736 & 1,0873 & 0,08972 \\ -0,089809 & 1,068 & 0,090578 \\ -0,090488 & 1,0486 & 0,091037 \\ -0,091128 & 1,0291 & 0,091458 \\ -0,091368 & 1,0095 & 0,091478 \end{array}\right] \quad (C.1)$$

$$M[32,63] = \begin{bmatrix} a_{i(i-1)} & a_{ii} & a_{i+1} \\ -0,091568 & 0,98989 & 0,091458 \\ -0,091368 & 0,97029 & 0,091037 \\ -0,091128 & 0,95077 & 0,090578 \\ -0,090488 & 0,93136 & 0,08972 \\ -0,089809 & 0,91211 & 0,088825 \\ -0,088736 & 0,89308 & 0,087538 \\ -0,087627 & 0,8743 & 0,086217 \\ -0,086129 & 0,85583 & 0,084512 \\ -0,0846 & 0,8377 & 0,08278 \\ -0,082692 & 0,81997 & 0,080673 \\ -0,08076 & 0,80266 & 0,078545 \\ -0,078459 & 0,78583 & 0,076056 \\ -0,076141 & 0,76952 & 0,073554 \\ -0,07347 & 0,75376 & 0,070707 \\ -0,070789 & 0,73859 & 0,067855 \\ -0,067773 & 0,72406 & 0,064676 \\ -0,064758 & 0,71018 & 0,061505 \\ -0,061422 & 0,69701 & 0,058019 \\ -0,058104 & 0,68456 & 0,054564 \\ -0,054477 & 0,67287 & 0,050803 \\ -0,050892 & 0,66197 & 0,047098 \\ -0,047007 & 0,65188 & 0,043098 \\ -0,04319 & 0,64263 & 0,039178 \\ -0,039084 & 0,63424 & 0,034977 \\ -0,035072 & 0,62674 & 0,030881 \\ -0,030786 & 0,62013 & 0,02652 \\ -0,026615 & 0,61443 & 0,022286 \\ -0,02219 & 0,60966 & 0,017804 \\ -0,017907 & 0,60584 & 0,013486 \\ -0,013372 & 0,60296 & 0,0089138 \\ -0,0090279 & 0,60103 & 0,0045799 \\ -0,0044017 & 0,60007 & 0 \end{bmatrix} \tag{C.2}$$

Bibliography

[1] Tor A. Ramstad, S.O Aase, J.H.Husøy *Subband Compression of Images: Principles and Examples*, Advance in Image Communication, Volume 6, ELSEVIER 1995

[2] Markus Knoch *Design of OFDM Systems Based on Cosine-Modulated Filter Banks*, Masther Thesis, NTNU 1997

[3] John G.Proakis *Digital Communications*, McGraw-Hill 1995

[4] Edward A. Lee, David G. Messerschmitt *Digital Communication*, Kluwer Academic Publishers, Netherlands 1990

[5] B. Farhang-Boroujeny *Adaptative Filters: Theory and Applications*, John Wiley & songs, 1998

[6] Iñaki Berenguer, Ian. J. Wassell *FMT Modulation: Receiver Filter Bank Definition for the derivation of an Efficient Implementation*, McGraw-Hill 1995

[7] Sun Qiaoyun, Gao Zhenming *The Research on Precoding in Filterd Multitone Modulation*, Networking and Mobile Computing on Wireless Communications. Proceedings, 2005 International Conference, vol. 1, pp. 661-664

[8] N Benvenuto, S. Tomasin, L.Tomba *Equalization Methods in OFDM and FMT Systems for Broadband Wireless Communications*, IEEE Transactions on Communications, sept 2002, vol. 50, pp 1413-1418

[9] Tor A.Ramstad, Joar P. Tanem *Cosine-Modulated Analysis-Synthesis Filter Bank with Critical Sampling and Perfect Reconstruction*, Proc. IEEE ICASSP 1991, pp. 1789-1792

[10] N Benvenuto, Stefano Tomasin, Luciano Tomba *Receiver Architectures for FMT Broadband Wireless Systems*, VTC 2001 Spring. IEEE VTS 53rd,vol 1, pp 643-647

[11] Radiocom notes ETSETB, 2006, http://www.gcr.tsc.upc.es/downloads.php?pid=5

[12] The Mathworks, 2007, http://www.mathworks.com/

[13] Miguel A. Lagunas, Josep Vidal, Ana I. Pérez-Neira *Procesado de Señal* CTTC Centre Tecnològic de Telecomunicaciones de Catalunya. Undergraduate Publications, 2004 http://www.cttc.es/publications/undergraduate/

[14] Ove Edfors, Magnus Sandell, Jan-Jaap van de Beek, Daniel Landström and Frank Sjöberg *An Introduction to Orthogonal Frequency Division Multiplexing*, Signal Processing Group at LuleåUniversity of Technology, September 1996

[15] Bahram Borna and Timothy N.Davidson, *Efficient Design of FMT Systems* ,IEEE Transactions on Communications, May 2006, Vol. 54, issue 5 pp. 794- 797

[16] P.P Vaidyanathan, *Multirate Systems and Filter Banks*, Englewood Cliffs: Prentice Hall, 1993

[17] R.D Koilpillai and P.P. Vaidyanathan, *Cosine-Modulated FIR Filter Banks Satisfying Perfect Reconstruction*,IEEE Transactions on Communications, May 2006, Vol. 54, issue 5, pp. 794- 797

[18] I. Berenguer, Ian J. Wassell *FMT Modulation: Receiver Filter Bank Definition for the derivation of an Efficiency Implementation*, Seventh International OFDM Workshop, Hamburg, Germany, September 2002.

[19] Pandharipande, A. *Principles of OFDM*, IEEE Potentials, Apr/May 2002, Volume 21, Issue 2, pp. 16-19