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Performance Analysis of ALOHA and CSMA in Spatially Distributed Wireless Networks

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Problem Description

The field of wireless communications is rapidly expanding in our technology-driven world today. In many applications, it is taking over most other forms of communication. However, despite its many advantages, wireless communication networks have many problems that still remain unsolved, where one of the major problems that is under extensive research is scarcity of resources such as bandwidth, time slots or channel. The problem of scarcity of resources is often addressed through Medium Access Control (MAC) layer design, where protocols such as ALOHA and CSMA may be used for communication between the nodes in the network.

In this project, we are interested in analyzing the performances of these MAC protocols, and understanding their behaviors in spatially distributed (ad hoc) wireless networks. The model we wish to use for the analysis is somewhat different than the traditional model, in the way that it allows transmissions to be carried out between many transmitter-receiver pairs simultaneously. The performance of the MAC protocols should be evaluated in terms of probability of outage. If possible, we are also interested in developing mathematical expressions for the probability of outage for ALOHA and CSMA in our modeling framework, and making comparisons between the performances of these MAC protocols.

Assignment given: 20. January 2007
Supervisor: Geir Egil Øien, IET

Preface

The following report is the result of my Master's thesis work for the Norwegian University of Science and Technology (NTNU). The project was carried out at the department of Electrical and Computer Engineering at the University of Minnesota (UMN) in Minneapolis, USA, and addresses the challenges present in the MAC layer design of wireless communication systems. In particular, this project considers "Performance Analysis of ALOHA and CSMA in Spatially Distributed Wireless Networks".

This research project has greatly increased my aspiration and motivation within the field of wireless communications, and familiarized me with the immense research potentials and interest in this field. I have throughout the project received great support from professors and faculty members at both NTNU and UMN. Having performed the project at UMN has been particularly rewarding and educational for me, as it has exposed me to broader research areas and introduced me to other distinguished professors within my field of interest.

Hereby, I would like to express my special thanks to my advisors, Professor Geir Øien at NTNU and Professor Nihar Jindal at UMN, for their guidance and support along the way. Also, I would like to thank Professor Georgios Giannakis at UMN for useful discussions and encouragement.

Mariam Kaynia
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Abstract

In this thesis the performance of the ALOHA and CSMA MAC protocols are analyzed in spatially distributed wireless networks. The main measurement metric used is *probability of outage*, a metric that is referred to in most of the related research done in this field, but has not been treated in detail thus far. Some of the research done on other performance metrics such as transmission capacity, throughput, bit error rate, spatial reuse and delay are also noted and described in this report.

In our system model, users/packets arrive randomly in space and time according to a Poisson point process, and are thereby transmitted to their intended destinations using a fully distributed MAC protocol (either ALOHA or CSMA). Our model allows simultaneous transmissions between many transmitter-receiver pairs in the network. An SINR-based model is considered, and a packet transmission is encountered as successful if the received SINR is above a predetermined threshold value for the entire duration of the packet.

Accurate bounds on the probability of outage, which is a function of the density of transmissions, are developed for both MAC protocols. The methods used to reach the obtained analytical results are presented in detail, and these results are shown to follow the simulation results tightly. We also present the methods used, which turned out to not be as successful as desired in terms of following the simulation results for all densities.

Furthermore, the derived bounds for the probability of outage are used to determine the performance advantage that CSMA provides over ALOHA and also to gain insight into the design of general MAC protocols for ad hoc networks. Our final comparison results show that CSMA performs better than unslotted ALOHA, and worse than slotted ALOHA in terms of probability of outage.

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Abbreviations

ACK	ACKnowledgement
BER	Bit Error Rate
BPSK	Binary Phase Shift Keying
CA	Collision Avoidance
CDF	Cumulative Distribution Function
CDMA	Code Division Multiple Access
CSMA	Carrier Sense Multiple Access
CSMA/CA	Carrier Sense Multiple Access with Collision Avoidance
CSMA/CD	Carrier Sense Multiple Access with Collision Detection
CTS	Clear To Send
dB	deciBel
DS	Direct Sequence
FH	Frequency Hopping
IEEE	Institute of Electrical and Electronics Engineers
INRIA	Institut National de Recherche en Informatique et en Automatique
IP	Internet Protocol
LAN	Local Area Network

LB	Lower Bound
MAC	Medium Access Control
MAN	Metropolitan Area Network
MANET	Mobile Ad Hoc Network
MIMO	Multiple Input Multiple Output
MP2MP	MultiPoint-to-MultiPoint
NTNU	Norwegian University of Science and Technology
OSI	Open Systems Interconnection
PDF	Probability Density Function
PPP	Poisson Point Process
PR-CSMA	Per-Route Carrier Sense Multiple Access
QoS	Quality of Service
RTS	Request To Send
RX	Receiver
SNR	Signal To Noise Ratio
SINR	Signal To Interference plus Noise Ratio
SR	Spatial Reuse
TCP	Transmission Control Protocol
TX	Transmitter
UMN	University of Minnesota
WLAN	Wireless Local Area Network

Chapter 1

Introduction

Spatially distributed, and equivalently *ad hoc*, wireless networks consist of sets of transmitting and receiving nodes distributed randomly in space, having the responsibility to transmit, relay, and receive data packets without a centralized control. These networks have the advantage of avoiding the cost, installation and maintenance of network infrastructure, being available to be rapidly deployed and reconfigured, and exhibiting great robustness owing to their distributed nature, node redundancy, and lack of single points of failure. Naturally, these desirable properties introduce many challenges in the design of such wireless networks, thus placing this field of study under extensive research for several decades.

The problem of scarcity of resources in communications systems is often addressed through *Medium Access Control* (MAC) layer design. The *quality-of-service* (QoS) of networks is critically dependent on the MAC protocol used for communication between the nodes. The challenge of designing an ad hoc network then becomes to decide what system performance metric to optimize, which MAC protocol to apply in order to get the best performance, and which system parameter values to choose in order to reach a desired goal of performance. In the work done thus far on MAC protocols, performance metrics such as transport capacity, throughput, bit error rate, spatial reuse, and delay are applied for analysis. In this report, the performance of two of the most widely used MAC protocols, namely ALOHA and CSMA, are investigated and compared with each other in terms of probability of outage.

1.1 Problem Statement

Despite the extensive interest in and research done on spatially distributed networks [4]-[8] [10] [28] [31] and the frequent application of various MAC protocols [9] [12]-[16] [23], many problems still remain unsolved. In most of the previous research done in this domain, the ad hoc wireless network models used is based on having many transmitters communicating with a single common receiver. Such models set a valid basis for performing comparisons between the MAC protocols, but they fail to address many of the issues that arise in networks that contain multiple simultaneous communication links, as is the case in many real ad hoc networks.

Hence, the model we consider in this project allows simultaneous communication between many transmitters and receivers in a random spatially distributed network. In particular, we analyze the performance of ALOHA and CSMA in terms of probability of outage in an ad hoc network under less restricted modeling assumptions. A packet is said to be in *outage* if the received SINR falls below the required threshold β at any time during the packet transmission. Literature review is performed on previous related work in order to investigate the performance of ALOHA and CSMA in different contexts. We perform our performance analysis based on a novel technique that involves geometric considerations. To the knowledge of the author, this report is the first to analyze the performances of the ALOHA and CSMA protocols in terms of probability of outage in a ad hoc network model that is stochastic and has so much variability and degrees of freedom as our model possesses.

The performance analysis carried out in other related work asserts that CSMA performs better than both slotted and unslotted ALOHA in terms of capacity, throughput, and bit error rate [9] [10] [14] [28]-[31]. This superiority in performance of CSMA over ALOHA is naturally followed by tradeoffs in other domains such as transmission rate and delay [10] [13] [29]. Except for some comparisons performed based on simulations, thus far CSMA does not appear to have been treated in detail analytically and in a network model that resembles real ad hoc networks. This is surprising particularly since CSMA is one of the most widely used MAC protocols today. Hence, in the following chapters we consider the performances of both ALOHA and CSMA in a network model that is based on stochastic SINR simulations, which allows for parallel communication between several transmitter-receiver links.

1.2 Structure and Goal of This Thesis

Motivated by the great need for understanding the exact performance of MAC protocols, this thesis contains several novel contributions. In particular, the model considered for the analysis allows for more of the factors that are desirable in many spatially distributed networks than most previously considered models. Naturally, the model still contains some simplifications in order to allow for the mathematical analysis to be carried out. However, these simplifications and assumptions will be justified and where needed, it will be shown through simulations that they do not affect the final results significantly, and that they are indeed reasonable.

This thesis is structured as follows. Firstly, we give a thorough overview of the background concepts, as well as familiarizing the reader with some of the related work that has been done on the performances of MAC protocols in different contexts. Secondly, the model in which the analysis is performed is introduced. This model is SINR-based, and is a close representation of a real ad hoc network. Thirdly, the performances of ALOHA and CSMA are analyzed. In the case of ALOHA we look at both the slotted and unslotted system, and for both ALOHA and CSMA, we present all the different approaches we use in order to carry out the performance analysis mathematically. Finally, the performances of these MAC protocols are compared through simulations and obtained formulas. The metric used for analysis is probability of outage, a performance metric that has not been treated in detail in previous related work.

The performance assessment is undertaken analytically and formulas are obtained for the probability of outage for the ALOHA and CSMA protocols. Simulations are then carried out to either disregard or confirm the obtained analytical results. Discussions and explanations are provided for all the results.

Chapter 2
Background and Related Work

In this chapter, we give a thorough background description on the concepts that are essential for understanding the analysis that is to follow in the remaining of this thesis. Furthermore, a literature review is performed and some of the past research and associated main results related to the topic of this document are presented.

2.1 Ad Hoc Wireless Networks

An *ad hoc wireless network* is a collection of mobile nodes that self-configure to form a network without the aid of any established infrastructure, and are connected to each other by wireless links [3]. Ad hoc wireless networks have the interesting characteristic that they can be tailored to specific applications and they can be formed from whatever network nodes are available, as described in section 2.1.2. Also these networks have the advantage of avoiding the cost, installation and maintenance of network infrastructure, they can be rapidly deployed and reconfigured, and they exhibit great robustness owing to their distributed nature, node redundancy, and lack of single points of failure. However, despite the advantages and advances in wireless communications over the last several decades, many performance, design, and fundamental capabilities of these networks remain suboptimal and are hence under further research and development.

2.1.1 Characteristics of Ad Hoc Networks

Ad hoc networks are classified by most researchers as a special subset of wireless networks. The design of ad hoc networks may be addressed from different perspectives, or through different *layers* referring to the well-known OSI model. These layers and the interrelation between them are illustrated in Figure 1. The work done in this project is primarily related to the MAC layer, often denoted as the *data link layer*. More on the characteristics of ad hoc networks is to be found in the rest of this section.

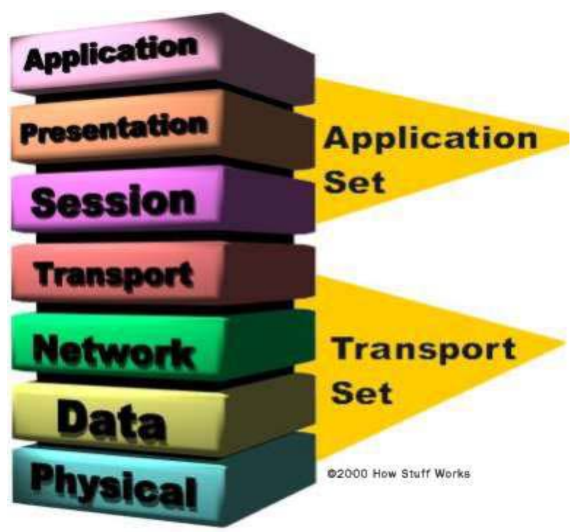


Figure 1 – The OSI model illustrating the relation between the seven layers of communications and computer network protocol design [42].

Mobility: Perhaps the main characteristic of ad hoc networks is the fact that nodes can be rapidly repositioned and moved. Rapid deployment of wireless communications in areas with no infrastructure often implies that the users must explore and sense their surroundings, communicate based on the sensed conditions, and if necessary form teams that in turn coordinate among themselves to create a taskforce or a mission [3]. The choice of the mobility model, such as individual random mobility, group mobility, motion along preplanned routes, etc, has a major impact on the selection of a routing scheme and can thus influence performance.

Self-organization: The ad hoc network must autonomously determine its own configuration parameters, including addressing, routing, clustering, position identification, power control, etc. [3] In some cases, e.g. in solar arrays, special nodes can coordinate their motion and dynamically distribute themselves in order to provide coverage over a larger area. The models considered in this project are all based on self-organization of the nodes.

Energy conservation: In most ad hoc networks, such as sensor networks, laptops, etc., the nodes have limited power supply, and an energy efficient design is thus critical for longevity of the mission.

Scalability: This is the ability of a network to maintain good performance under growing amount of load, by using additional resources [38]. In applications where the ad hoc network can grow to several thousand nodes, it is useful to allow for scalability. For wireless 'infrastructure' networks, scalability is handled by a hierarchical construction, where mobile IP or local handoff techniques may be used to overcome the problem of limited mobility. The main reason to design for scalability is reduced cost and effort. [40]

Security: Ad hoc networks are generally vulnerable to attacks from intruders that eavesdrop and jam the channel. There are two types of attacks, namely active and passive attacks [36]. An active attacker tends to disrupt operations, e.g. an impostor posing as a legitimate node intercepts control and data packets. Due to the complexity of ad hoc networks, these active attackers are very hard to detect, and thus much research has been done in this area. Whereas active attackers are eventually discovered and physically disabled, passive attackers are never discovered by the network. Passive attackers, which eavesdrop but do not modify the message stream in any way, are unique to ad hoc networks, and can be even more insidious than the active ones. Like a 'bug', it is placed in the network, monitoring the data and control traffic patterns, and relaying information back to the ‘enemy’ via special communication channels with low energy and low probability of detection [36].

Multihopping: A multihop network is a network where the path from source to destination traverses several other nodes [15]. Ad hoc networks often use multiple hops for obstacle negotiation, spectrum reuse, and energy conservation [2]. The communication between the two nodes in each hop uses the same concepts of wireless signal transmissions, and such a one-hop link is thus the basis of the communications considered in our research project.

2.1.2 Applications

Compared to other wireless technologies, such as cellular telephony and wireless internet, ad hoc networks are indeed the slowest to materialize in the commercial domain. This is quite surprising since the concept of ad hoc wireless networking was born in the early 70’s and was soon after discovered by the military to be a good potential for packet switching [3].

So why so slow development and deployment of commercial ad hoc applications? The main reason is that the original application scenarios were not targeted towards mass users, but rather towards unfriendly, remote infrastructure-less areas, such as the military and homeland security scenario.

Recently a new concept has emerged which may help extend ad hoc networking to commercial applications, namely *opportunistic ad hoc networking* [33]. This has been partially prompted from the recognition that the techniques used in wireless telephony and wireless LANs have their limits. Ad hoc networks can be used ‘opportunistically’ to connect smaller networks to areas not reached by the wireless telephony and wireless LANs [33].

Another important family of ad hoc networks is *sensor networks*, which can be viewed as a subset of ad hoc networks [27]. Sensor networks involve low energy operations, low form factors and low cost, while combining transport and processing. There are some major differences between ad hoc and sensor networks. At the Physical, MAC and Network layers, the major innovations and unique features of sensor networks are the miniaturization, the embedding in the application contexts, and the compliance with extreme energy constraints. At the application layer, the most unique and novel feature of sensor networks is the integration of transport and in-network processing of the data that are sensed by the nodes. The concepts and algorithms discussed for ad hoc wireless networks in this project report also apply to sensor networks.

2.1.3 Design Challenges

One of the main challenges in designing ad hoc networks is to create a network that performs efficiently across layers of communication. There are tradeoffs between optimizations in the different layers, and thus ad hoc networks are designed based on their application. *Cross-layer interaction* [3] refers to the fact that it is virtually impossible to design a 'universal' protocol (routing, MAC, multicast, transport, etc.) and expect that it will function efficiently in all conditions and applications. Since there often is an extreme range of variability in the system parameters of ad hoc networks, it is important to tune the network protocols to the radios, and the applications to the network protocols. Even more important is the concept that in many cases the MAC protocols, routing and applications must be jointly designed (i.e., cross-layer design), and due to the dynamical change of parameters the protocols must be adaptively tuned as well.

Other challenges in the design of ad hoc networks are mobility of the communication nodes and scalability of the network. As already discussed, mobility is one of the main characteristics of ad hoc wireless networks, and this property means that the network must in many applications be able to cope with the communicating nodes being in motion. Note that mobility, which is often viewed as the #1 enemy of wireless ad hoc networks, if properly characterized, modeled, predicted and taken into account, can indeed be of great help in the design of scaleable protocols. Also, large number of nodes and changes in the number of nodes in the network are common in ad hoc networks, and must thus be addressed in the design. The two aspects, mobility and scale, are in fact intertwined, and vary independent of each other. The scalability of ad hoc networks is taken into account and investigated in the network model considered in this project.

2.2 Random Access Protocols

In most data applications, data are generated at random time instances and the system typically has more users than can be accommodated simultaneously. This is where random access strategies come into picture, making sure that the channel and other resources in the network are effectively shared between the active users. Random access techniques are based on the premise of packetized data, and the resulting schemes are usually termed *packet radios*. Using a shared transmission medium in random access channels introduces many design challenges, which have been addressed from many different perspectives within all the seven 'layers' in the OSI model (Figure 1). One of these design considerations is the medium access control (MAC) protocol design in the data layer, which has been of great significance in channel access control schemes.

Random access techniques were pioneered by Norman Abrahamson with the ALOHA protocol in 1970 [25]. Modifications of the ALOHA scheme, and many other MAC protocols have also been proposed. In this report, focus is primarily set on ALOHA and CSMA[1] and their modifications, as these are among the most widely used protocols today, due to their simplicity and the fact that they were among the first MAC protocols to be implemented.

In this chapter, the concept and characteristics, design challenges, and the various types of ALOHA and CSMA are considered. Their concepts, related problems, and modified

[1] ALOHA, and Carrier Sense Multiple Access (CSMA) are to be described in sections 2.2.2 and 2.2.3 respectively.

versions of the protocols are discussed, and the performance differences between ALOHA and CSMA are further emphasized through analytical work and simulations in the next chapters of this report.

2.2.1 Medium Access Control

Medium Access Control (MAC) is an algorithm performed locally by communicating nodes, i.e. no scheduling or global control unit is applied, allowing an efficient access to and a fair share of a communication resource, such as a radio frequency, a bandwidth, or time slots. Due to the scarce channel bandwidth and other resources available in ad hoc networks, there is a need to regulate the nodes' access to their shared channel. This gives rise to the challenge of designing efficient MAC protocols.

Performance of MAC protocols may be evaluated using success probability, capacity, throughput, bit error rate, spatial reuse, and delay. While the *scaling behavior* of these metrics is considered in some research [9] [10] [29] [30] [31] relatively few *quantitative* results on are available.

2.2.2 Slotted and Unslotted ALOHA

In the ALOHA protocol, data is packetized at the transmitter, which instantly initiates its packet transmissions whenever there is data to be sent. This characteristic that the transmitter sends its packets immediately regardless of the channel condition is indeed the distinguishing feature of ALOHA. This property makes ALOHA extremely inefficient owing to the fact that the transmissions collide, resulting in a low throughput. In order to increase this throughput, *slotted* ALOHA is introduced [43]. Other modifications of the ALOHA protocol include carrier sensing[2] [1] [3] [34], collision avoidance [21] [22], and collision detection [36] [44]. These modifications and their inherent properties in terms of the above-mentioned metrics are introduced and discussed in the remaining of this chapter.

[2] Carrier sensing is used in CSMA, which is discussed in section 2.2.3

- *Unslotted / Pure ALOHA:* In unslotted ALOHA, there are no time slots, and users transmit data packets as soon as they are formed. Note that since all the packets are of equal length *T*, any transmission initiated *T* seconds before will be an ongoing transmission at the start of a new packet arrival. A packet transmission as successful when no collisions occur. Hence, based on the probability that a transmission is successful and knowing that the number of packet arrivals in time is Poisson distributed, the throughput defined by equation (1) equals [1] [41]:

$$S = \frac{\lambda^t}{\varsigma} \cdot \Pr\,(\text{no arrivals during } [-T, T]) = \frac{\lambda^t}{\varsigma} e^{-2\cdot\frac{\lambda^t}{\zeta}} \tag{2}$$

This throughput is plotted versus the load, defined as λ^t / ζ, in Figure 2. This figure illustrates that the throughput increases with increasing load until the normalized load is 0.5, i.e. $\lambda^t = \zeta / 2$, for which the data rate is approximately 0.18. This means that as the number of users increases, the idle periods when no users are transmitting decrease, and hence the probability that there will be collisions rises. A load of $\lambda^t / \zeta = 0.5$ is the optimal balance between users generating enough packages to utilize the channel and the packets colliding

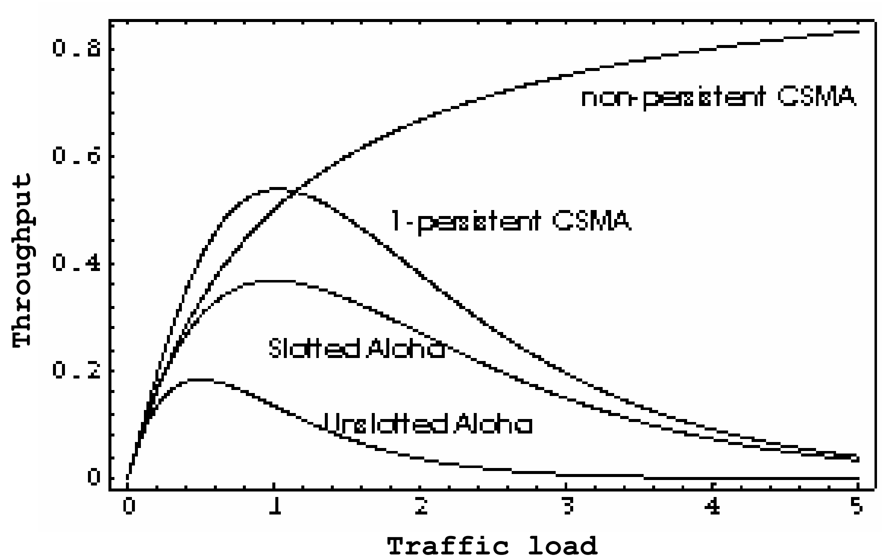


Figure 2 – The throughput *S* of ALOHA and CSMA versus the traffic load of the network [41].

infrequently.

Pure ALOHA is very inefficient, due to the fact that users can start their packet transmissions at any time, and any partial overlap of packets destroys the successful reception of many packets. This downside gives rise to *slotted* ALOHA, where users are synchronized and all packet transmissions are aligned in time.

- *Slotted ALOHA:* In slotted ALOHA, time is assumed to be slotted in time slots of length equal to one packet length *T*. Users can only start their transmissions in the beginning of each slot, resulting in no partial overlap, hence increasing the throughput. Using equation (1), gives the following throughput [1] [41]:

$$S = \frac{\lambda^t}{\varsigma} \cdot \Pr\,(\text{no arrivals during}\,[0, T]) = \frac{\lambda^t}{\varsigma} e^{-\frac{\lambda^t}{\varsigma}} \tag{3}$$

This throughput is also plotted in Figure 2, showing the enhancement in the performance compared to pure ALOHA. As can be seen from the graph, a maximum throughput of 0.37 is obtained for a normalized traffic load (λ^t / ζ) of 1. Thus by slotting the ALOHA protocol, the throughput may be doubled.

In order to increase the throughput even further, techniques such as *error correction coding* [35] [38] and *carrier sensing* [1] [3] [34] may be introduced. In the following section, the functionality and performance of *carrier sensing multiple access* are considered.

2.2.3 Carrier Sense Multiple Access (CSMA)

In the CSMA protocol, the transmitters sense the channel around them and delay their packet transmission if they sense that their transmission will not be successful, e.g. if the SINR at the receiver is expected to be less than the required β. The user can then either drop the package, which is rather impractical, or it can wait a random time before retransmitting the packet. This is called *random backoff*, and it precludes multiple users simultaneously transmitting when the channel is free. CSMA only works when each transmitter can sense the signal transmission of all other transmitters and when the propagation delay is small, both of which are assumed to be applicable in the model used in this project. These assumptions are reasonable in networks that are not too large, i.e. in LANs, MANs, and areas where the transmission time of each packet is relatively small compared to the length of the packet. Despite the requirement that each transmitter can potentially sense the transmission of all others, for the sake of simplicity of analysis and understanding the inherent problems of CSMA, the conventional model for CSMA assumes that each node can only hear its neighboring nodes.

- *Hidden and Exposed Terminal Problems:* There are a few problems inherent to the CSMA protocol, namely the *hidden terminal* and the *exposed terminal problem*. These problems are only present when each node can hear only its neighboring nodes, and not all active nodes in the network[3]. In order to understand these concepts, consider the network topology illustrated in Figure 3. In this setup each node can only hear its neighboring nodes, as is represented by the lines connecting the terminals.

To understand these problems, firstly note that in this setup a packet transmission is considered successful if there is no *collision* of transmissions. A collision is defined as the simultaneous reception of more than one packet at the receiving node. Now referring to the setup of Figure 3, for the hidden terminal problem, say both node 1 and 5 wish to send data to node 3. Since node 5 cannot sense node 1, and vice versa, both nodes will transmit their packets, and node 3 will thus get a collision of signals received. As for the exposed terminal problem, imagine that node 3 wishes to send a packet to node 5, at the same time as node 4 is sending a package to node 2. When node 3 senses its channel, it detects node 4's transmission and assumes the channel is busy. Node 3 will hence back off, although no collisions would have occurred. The hidden and exposed terminal problems both result in inefficiencies in the channel utilization.

[3] The model we use in our work assumes that all nodes overhear all others. Hence, we do not have the problem of hidden and exposed terminals.

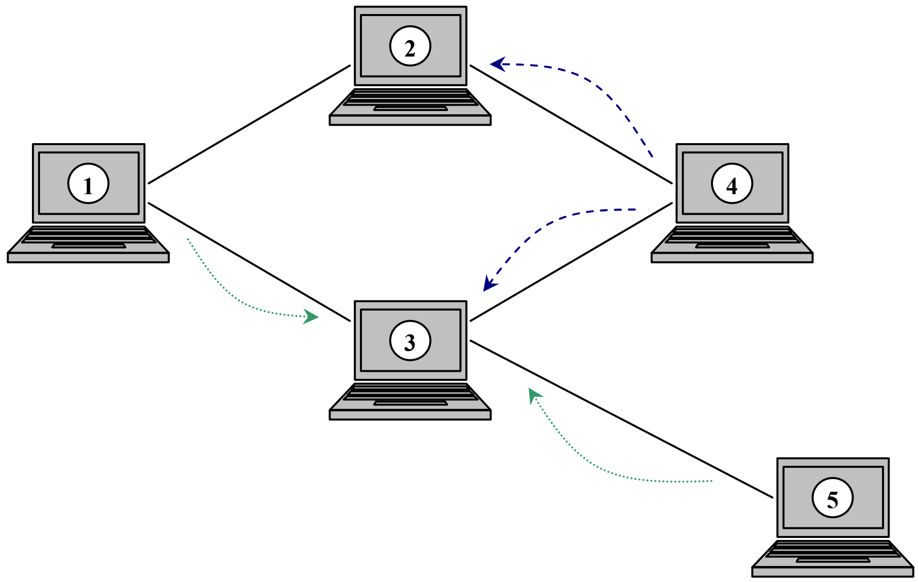


Figure 3 – Network topology to illustrate the hidden and exposed terminal problem.

Having considered these problems, let us return to the model of this project. In order to mitigate these effects for simplicity of the analysis, it is firstly assumed that each node can hear all others and that each transmitter is transmitting to its own dedicated receiver, meaning that the hidden and exposed terminal problems at the receiver are both avoided. However, also in our model simultaneous transmissions may cause problems; the signals of the packets that are sent at the same time are added together to cause what we refer to as *interference* at each receiving node, leading to degradation of the received signal and possibly the reception of erroneous packets. Using the topology of Figure 3 again and assuming that each of the terminals in the figure only transmits to its own receiver (not shown in the figure), it can be observed that when nodes 3 and 4 are transmitting to their receivers, their signals will result in interference on the signal received at both receiver 3 and 4. Thus by setting a condition on the value of the received SINR, denoted as β, each transmitting node can back off if the sensed SINR is less than this threshold β.

In order to overcome the hidden and exposed node problem in CSMA in the conventional model described previously, several modifications to the CSMA protocol are proposed, most of which are proposed and analyzed in detail by Kleinrock and Tobagi [23] more than 30 years ago. *Request-to-send* (RTS) and *clear-to-send* (CTS) packets are often used, resulting in what is known as collision avoidance CSMA [21], as described in the following.

- *Collision Avoidance CSMA* (*CSMA*/*CA*)*:* In order to overcome the hidden terminal problem, handshaking[4] is introduced. The handshaking comprises of an RTS packet and a CTS packet prior to each transmission. The node that wishes to send a packet first sends an RTS packet. If the potential receiver perceives an available channel, it will immediately respond with a CTS signal, which authorizes the initiating node to transmit its data packet. All other nodes that overhear the RTS and CTS packet, will thus know which transmitter and receiver pair are communicating, and will thus refrain from sending information if their transmission is expected to collide with the ongoing transmission and thus result in an SINR below β. This so-called *busy-signal* scheme best fits many-to-one networks, such as a single cellular base station and its user stations.

Despite the popularity of this modification, other methods have also been suggested. For instance, Ren and Guo propose in 2003 [12] a novel CSMA protocol for ad hoc networks in which the RTS/CTS packets are replaced by a so-called BROD packet, which is used at each node to keep track of the working status of the whole network. The main function of the BROD packet is to broadcast the source and the destination of the data packet, which is ready to transmit. This packet has a lifetime of only one hop, and is discarded at each node as soon as it informs it of the condition of the network. This modification of the CSMA protocol is shown to result in a significant improvement in the performance of CSMA in networks where the hidden node and exposed node problem is severe.

- *Nonpersistent, 1-persistent, and p-persistent CSMA:* These modifications of the CSMA protocol are described briefly in this section, for the sake of the subsequent discussions of

[4] Handshaking refers to some kind of agreement between the transmitter and receiver in order to avoid collisions.

Chapter 5. In the nonpersistent CSMA protocol, if the medium is idle, the transmitting node initiates its packet transmission immediately. If the medium is busy, the transmitter waits a random amount of time before transmitting again. This random backoff reduces the probability of collisions. If the backoff time is too long, nonpersistent CSMA results in idle time being wasted. For the sake of comparison with equation (2) for ALOHA, the throughput of nonpersistent CSMA for a given normalized load of $L = \frac{\lambda^t}{\varsigma}$ is given by [41]:

$$S = \frac{L}{1+L} \tag{4}$$

In 1-persistent CSMA, when the medium is busy, the transmitter continues to listen until the medium becomes idle, and then transmits immediately. This reduces the waste of idle time, but results in a collision if two nodes want to retransmit at the same time.

In the p-persistent CSMA protocol, when the medium is idle, data is transmitted with probability *p*, and delayed for the worst case propagation delay for one packet with probability (1 – *p*). If the medium is busy, the transmitter continues to listen until the medium becomes idle, and then transmits with probability *p*. If the transmission is delayed by one time slot, the transmitter senses the medium again and repeats the procedure just described. The p-persistent CSMA thus balances between the tradeoffs of the non-persistent and the 1-persistent CSMA. In short, the different degrees of persistence indicate how long the transmitter that is backing off should wait before retransmitting its packet.

2.3 Previous Related Work on Performance Analysis

There has been a notable amount of research done on the performance of ALOHA in ad hoc networks. A number of different researchers have analyzed slotted ALOHA using a Poisson model for transmitter locations [4] [6]. Perhaps the closest work is the analysis performed on success probability by using so-called *interference-free guard zones* [7], as described further in section 2.3.1. Moreover, some work has been done to compare the ALOHA and

CSMA protocols in terms of capacity [4] [10] [14] [28] [29], throughput [10] [13] [29] [30] [31], bit error rate [9] [24], spatial reuse [15] [27], and transmission delay [13]. All the above-mentioned performance metrics that are used in the related work will be defined in the corresponding sections, and they are all seen to be in some way based on signal-to-interference-plus-noise-ratio (SINR) measurements, where the SINR is defined as:

$$SINR = \frac{Signal\ \ Power}{Noise\ \ Power + Interference\ \ Power} \tag{5}$$

Despite all the research done on MAC protocols thus far, neither unslotted ALOHA nor CSMA appear to have been analyzed in a network model that closely represents a real ad hoc network, i.e. one that is stochastic, continuous in time, and allows for simultaneous communication between nodes. Since unslotted systems are of great interest in today's ad hoc networks and CSMA is one of today's most widely used MAC protocols, we will cover both these topics in our work that is described in Chapters 4 and 5. In the following sections, we give an overview over most of the related research performed on the performance of ALOHA and CSMA, the obtained results, and wherever possible, their relevance to our work.

2.3.1 Performance in Terms of Success Probability

Perhaps the closest work to our analysis is that of Hasan and Andrews [7], where success probability is analyzed for a similar spatial model. Success probability is defined as the probability that a transmission is received successfully at the receiver, i.e. that the measured SINR is above a certain threshold β. However, in their model, they assume the use of a scheduling mechanism that creates an *interference-free guard zone*, which is in effect a theoretical circle around the receiver, within which the amount of interference is below a required threshold. The model used in their paper [7] is similar to our network model, which is described in detail in Chapter 3, with the only difference that the system model of [7] is slotted. Naturally, the size of the guard zone affects throughput in an ad hoc network since nodes are inhibited within the guard zone, and the optimal guard zone size is thereby studied. By applying stochastic geometry, Hasan and Andrews derive an optimal guard zone expression that maximizes the density of successful transmissions under an outage constraint.

The understanding of the guard zone and how it is affected by the network parameters is useful as it helps researchers and designers improve the efficiency of multiple access and scheduling protocols for ad hoc networks. In our work, we will use the same concept of guard zones, and develop the model of [4] to include continuous-time transmissions. Note that the CSMA mechanism that we will consider in our work is able to suppress some nearby interferers, but is not able to create a perfect guard zone. This will be described in detail in Chapters 4 and 5.

Haenggi [6] considers the performance of slotted ALOHA in terms of success probability in a network that follows the Poisson point process. The model used allows simultaneous transmissions, and in this slotted system, every node transmits packets independently with probability p in each timeslot. No CSI is assumed anywhere in the channel, the noise in the channel is approximated to zero, and the measured signal-to-interference-ratio (SIR) is required to be above a certain threshold in order for the packet transmission to be encountered as successful. Let r_i denote the distance between the receiver under observation and the i-th interfering transmitter. By using the approximation [6]: $\log\left(1-\frac{p}{1+r_i}\right)\le\frac{-p}{1+r_i}$,

Haenggi finds the probability of success p_s to be:

$$p_s \approx e^{-p/\sigma} \qquad \text{and} \qquad \sigma = \frac{1}{\sum_i \frac{1}{1+r_i}} \tag{6}$$

where σ determines the degree of spatial reuse in the network, where spatial reuse is defined as the distance from the transmitter to the receiver divided by the mean distance between adjacent transmitters. If $\sigma = 0$, no simultaneous transmissions are possible, and for $\sigma \to \infty$, there are no collisions at all.

Similarly, Weber et al. in find the probability of outage[5] for slotted ALOHA to be $1-e^{-(\#\ of\ nodes)}$ [4], which resembles equation (6). The model used in [4] is also similar to ours with the exception that their system is slotted. In their paper, Weber et al. find the lower and upper bounds on the probability of outage for ALOHA as a function of spatial density. These results will be discussed further in our analysis of ALOHA in Chapter 4.

2.3.2 Performance in Terms of Capacity

As briefly mentioned in the previous section, Weber et al. [4] consider a model that allows for parallel communication between each transmitter and its receiver. The system is slotted, only path loss is considered in the channel model, the transmissions are single-hop, and a constraint is imposed on the probability of outage. Outage occurs when the received SINR is below a certain SINR threshold of β. Within this model the capacity of the network is analyzed through the concept of *transmission capacity*, C. This is defined to be the *area spectral efficiency* of the successful transmissions, i.e. the sum of the maximum average data rates per bandwidth per unit area [37] that is supported by the channel:

$$C = \lambda^{\varepsilon}\, b\, (1-\varepsilon) \tag{7}$$

where λ^{ε} is the optimal *contention density* of nodes in the network [1/m^2], corresponding to the maximum spatial density of nodes that can contend for the channel subject to a outage probability constraint ε, and b is the average rate that a successful user achieves [bits/s/Hz]. Consequently, C has unit [bits/s/Hz/m^2], indicating how many of the packets transmitted per unit area were received without being in outage.

In [10], Gupta and Kumar consider the *throughput capacity* of an ad hoc wireless network with spatial density of λ^{s}, and define this to be the maximum amount of information that can be sent successfully over the channel. In their paper they consider the SINR at the receiving nodes, and pose the requirement that the SINR must exceed a SINR threshold β in order for

[5] The probability of outage is equal to (1 – success probability).

each packet transmission to be considered successful. This specification is equivalent to the specifications of our model, as described in Chapter 3. However, what makes their model rather impractical is the fact that they employ a deterministic channel access scheme, making their model a deterministic SINR model, and hence precluding the occurrences of outages. In order to accurately model the behavior of a distributed ad hoc network at the physical and MAC layer, a stochastic SINR requirement must be used. Furthermore, the properties of random access channels and their stochastic behaviors must also be included. In this way the probability of outage is considered in the model, enabling a more realistic evaluation of real-life channels to be performed.

More recent work, such as [28] and [29], has considered the stochastic behavior of channels, and has thereby shown that the scaling of transport capacity depends of the amount of attenuation in the channel. In the low-attenuation regime with no channel absorption and small path loss, the transport capacity can be unbounded even under a fixed power constraint. In the high-attenuation regime with channel absorption and high path loss, the transport capacity is bounded by the total available power.

Also Ferrari and Tonguz have considered the transport capacities of the ALOHA and PR-CSMA[6] protocol [14]. The proposed transmission scheme is packetized, and it does not employ retransmissions. Through analytical work and simulations, Ferrari and Tonguz show that for low transmission densities the performance of ALOHA in terms of transport capacity is almost two orders of magnitude higher than that of CSMA. However, for increasing densities, while the effective transport capacity for ALOHA drops to zero, the effective transport capacity for PR-CSMA keeps on increasing, making PR-CSMA more beneficial at higher densities. This is indeed in correspondence with our obtained results as well as described in Chapter 6.

In their paper [14], Ferrari and Tonguz also consider the minimum data-rate necessary to maximize the effective transport capacity for a given traffic load. The minimum required

[6] PR-CSMA stands for “per route” CSMA. This term is applied in multihop networks, where each path (“route”) is considered separately.

data-rate in the case of ALOHA is significantly larger than that in PR-CSMA, due to the fact that in order to reduce the inter-node interference, the data rate needs to be increased. Furthermore, it is shown that in order to maximize the transport capacity, the data rate has to belong to a specific range, which is found to be narrower for the ALOHA protocol compared to CSMA. Hence, when using ALOHA, one has to be careful with the choice of the data rate based on a particular value of the traffic load.

2.3.3 Performance in Terms of Throughput

One of the most common metrics used to compare the performances of the various MAC protocols is *throughput*. This is defined as the ratio of the number of packets arriving successfully at the receiver over the total number of packet transmissions during a set time interval [10]. The throughput S can thus be expressed as:

$$S = \frac{\lambda^t}{\varsigma} \cdot p \tag{1}$$

where ζ is the rate of packet transmission [packets/s], λ^t is the packet arrival rate in time [packets/s], and p is the probability of successful packet reception. This probability is a function of both the random access protocol in use and the channel characteristics. As shown by Gupta and Kumar in 2000 [10] the global output of ad hoc networks, i.e. the throughput S, is inherently limited under a vast class of assumptions, i.e., the throughput of the network is dependent on many system parameters and on the restrictions set on the network model that is being used.

Also noteworthy is the work done in [13], where Yang and Yum consider slotted ALOHA and nonpersistent CSMA. The model assumes that packet arrivals, in combination with retransmission of packets, follow a Poisson point process with parameter λ. This parameter is referred to as the offered traffic to the slotted channel, and has unit [packets / T], where T

is the length of each packet. Based on this and the probability of a successful transmission denoted as p_s, the throughput S is found to be:

$$S = p_s \cdot \lambda \tag{8}$$

The simulation results in [13] show that nonpersistent CSMA in the slotted system has throughput not much different than slotted ALOHA if finite delay variance is to be guaranteed. The 1-persistent CSMA is superior to nonpersistent CSMA, a result that confirms the correct choice of using 1-persistent CSMA/CA for the IEEE 802.3 standard.

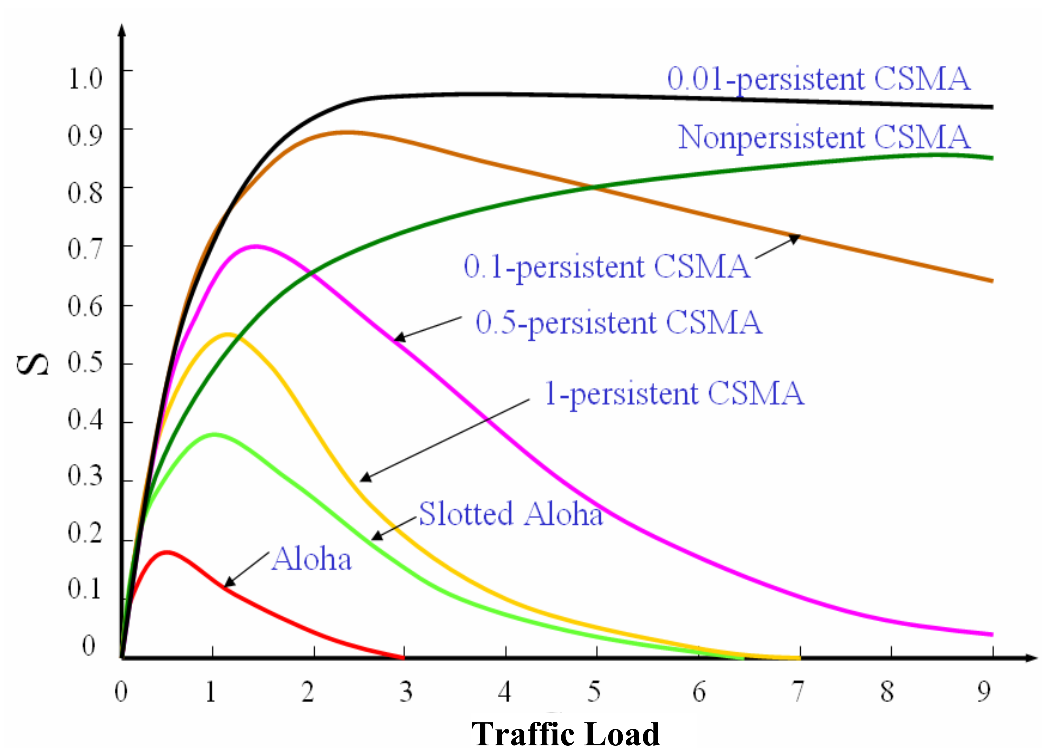


Figure 4 – Throughput of ALOHA and CSMA and the various modifications as a function of the offered traffic [23].

Figure 4 gives an overall comparison between the performances of ALOHA, CSMA, and the different modifications of these protocols in terms of throughput, clearly showing that CSMA provides higher throughput compared to ALOHA. In particular, we can see that the p-persistent CSMA protocols compete on offering the best performance based on the network load λ.

Others have also considered the performance of the ALOHA and CSMA protocols in terms of throughput. In [17] the impact of error-correction coding on multiple-cell and peer-to-peer ad hoc networks is studied. The results show that by applying modest coding, i.e. having a coding rate[7] of more than 2/3, increases the channel throughput significantly. While CSMA avoids many of the transmission collisions for which coding would be beneficial, because of the hidden terminal problem, some still remain. Compared to ALOHA, CSMA is shown to be beneficial when our goal is to maximize the throughput. Clearly, having coding in addition to the MAC protocol provides useful improvement.

2.3.4 Performance in Terms of Bit Error Rate

In order to compare the performance of ALOHA to that of CSMA, many papers use bit error rate (BER) as the measurement metric. In the following, the results obtained in these papers are restated and discussed.

In [9], Ferrari and Tonguz consider the BER performance of ALOHA and “per-route’ CSMA (PR-CSMA), where “per-route” means that we consider one single transmission path at a time in a multihop channel. The proposed transmission scheme is packetized, no retransmissions are employed, and collisions between packets are analyzed in terms of interference. Their network model constitutes reserved routes between the communicating nodes, i.e. there is a private path between transmitter and receiver, a property that resembles circuit switching. The spatial distribution of nodes is assumed to be uniform, with an intensity of λ^s, on a circular area A. The interference analysis is based on the transmission over a minimum length link and is carried out on a per-bit basis (as opposed to per-packet basis). The transmitted signal power ρ is constant for all transmitters, the transmitted packet

[7] Coding rate m/n means that each m-bit information symbol is transformed into an n-bit symbol.

has a fixed length of T (measured in bits), and the packet transmission process at each node follows a Poisson distribution with average packet transmission rate λ^t.

Based on this model [9], it is found through simulations that a BER floor appears for increasing packet size, and the performance becomes unacceptable. This is expected, because if the transmission rate λ^t remains fixed, larger packet sizes increase the probability of interference in the case of ALOHA where no channel sensing is present. In the case of CSMA, however, where channel sensing is present, Ferrari and Tonguz attempt to eliminate the interference completely by activating only one route at a time. The details of such analysis are performed in [24], and it is shown that the BER is now dependent not only on the geometry of the node distribution, but also on the spatial and temporal density of the network. The results indicate that the BER does not depend on the packet length T. Also, it should be noted that no error floor is observed in the BER range.

Finally, it is concluded in [9] that the performance of the PR-CSMA protocol in terms of BER is basically insensitive to data-rate, packet-generation rate, and packet length, while ALOHA significantly depends on these parameters. Also the PR-CSMA protocol suffers little performance loss, in terms of BER, with respect to the ideal case where there is no interference. Furthermore, these properties show that MAC and physical layers are strictly interrelated, and designing one without considering the other may lead to inefficiencies in ad hoc wireless network designs.

2.3.5 Performance in Terms of Spatial Reuse

Another metric that has been used to compare the performances of ALOHA and CSMA is *spatial reuse*, which is considered by Baccelli et al. in [15]. *Spatial reuse* (SR) is defined as the distance from the transmitter to the receiver, R, divided by the mean distance between adjacent emitters. For this last quantity, the mean distance between neighboring points in a

Poisson-Voronoi tessellation[8] is used. The SR as such gives an indication to how many simultaneous transmissions can proceed simultaneously without causing erroneous packet receptions. Naturally, one of the main goals in the design of networks is to maximize this entity. The model used in [15] applies the SR-ALOHA and CSMA schemes, and the performances of these protocols are analyzed and compared with each other in terms of SR.

Remarkably, the success probability for Rayleigh fading can be expressed as the product of the Laplace transforms of the noise and interference powers [15]. Indeed, this elegant equivalence of the Laplace transform was evaluated at the SIR threshold and the success probability was also pointed out in [27]. The simulation results of [15] show that the performance of SR-ALOHA is very close to that of the CSMA scheme for almost all values of the path loss exponent α. Nevertheless, the difference between the transmission densities of ALOHA and CSMA increases for larger values of α. What is most noteworthy is that for values of α close to 2, the optimized ALOHA scheme actually outperforms the CSMA scheme.

2.3.6 Performance in Terms of Delay Distribution

Performance of ALOHA and CSMA can also be compared in terms of end-to-end delay distributions. In many applications, such as applications when synchronization is required, it is of great significance to keep the delay of the signal transmissions as low as possible. In such cases, the choice of the MAC protocol becomes significant.

The delay distributions of ALOHA and CSMA are considered by Yang and Yum in 2003 [13]. In their paper, the closed form delay distributions of slotted ALOHA and nonpersistent CSMA/CA protocols under steady state are derived. The model used for the delay distribution analysis considers packet generations according to a Poisson process with the

[8] The Poisson-Voronoi tessellation is a special kind of decomposition of a metric space determined by the distances to a specified discrete set of points in space [46].

same packet length T for all packets. The maximum end-to-end propagation delay is denoted by τ, with the constraint: $2\tau < T$.

Simulation results show that the relative delay distributions of slotted ALOHA and nonpersistent CSMA/CA are quite similar [13]. Moreover, for the CSMA protocol, when the channel is sensed busy, the packet can attempt at the next slot. Thus, the access delay is continuous starting from the point $\tau = \tau_0$ compared to $\tau = 0$ in the case of ALOHA. Considering this delay offset, CSMA/CA can be said to have larger delays in average considering all packet transmissions, compared to ALOHA. However, the smaller delays in ALOHA are obtained at the expense of more transmission collisions and thereby a lower throughput, as was also mentioned in section 2.3.3.

Chapter 3

System Model

We wish to analyze the performances of ALOHA and CSMA in a network with randomly located users and random transmission times. We apply a model that is a close representation of a real ad hoc network. Some assumptions and simplifications are made to allow for mathematical analysis to be carried out. Justifications for these assumptions will also be made in this chapter.

3.1 Model Specifications and Assumptions

One possible model that could be used is as follows: transmitters are located on an infinite 2-D plane according to a homogeneous Poisson point process (PPP) with spatial density λ^s [# nodes / m^2]. The Poisson distribution with parameter λ^s is given as:

$$f(k;\lambda^s) = \frac{e^{-\lambda^s} \cdot (\lambda^s)^k}{k!} \tag{9}$$

which gives the direct probability that k occurs given that the expectation value is λ^s.

A typical transmitter is considered to be placed at the origin of a coordinate system, resulting in what is known as the Palm distribution for transmitters [26]. For our model, where the number of nodes on the plane follows a Poisson point process, and where the different point processes are independent, we can apply *Slivnyak's theorem* [46]. Within this framework, Slivnyak's theorem states that the Palm distribution indeed coincides with the Poisson point process, only with an additional point at the origin. Now shifting this entire point process so that the receiver associated with the typical transmitter lies at the origin, results in the conclusion that the conditional Palm distribution of potential interferers is a homogeneous Poisson point process with the same intensity.

Each transmitter receives packets in time according to an independent 1-D Poisson process with parameter λ^t [# packets / s], which represents the density of packet arrivals at each node. Each packet is then transmitted to its own dedicated receiver, meaning that each receiver gets its packets from a single transmitter, and the distance between each transmitter-receiver pair, denoted R, is fixed and equal for all pairs in the network. Recall that for a system with a Poisson arrival rate of λ^t, the interarrival times are exponentially distributed. The exponential distribution with parameter λ^t is defined as:

$$f(k;\lambda^t) = \lambda^t\, e^{-\lambda^t \cdot k} \qquad (10)$$

which gives the direct probability that the interarrival time between two packets is $1/k$, given that the expectation value for the interarrival time is $1/\lambda^t$.

If packets are assumed to have fixed length T [s], at each point in time the density of transmitters who have received a packet in the last T seconds is: $\lambda = T \cdot \lambda^s \cdot \lambda^t$. In order to analyze such a network, it would be necessary to average over the spatial (to fix locations) and temporal statistics (packet arrivals), which seems rather difficult.

An alternative is to assume that packets/users arrive at a random point in space and time, and then disappear after their packet is served (successfully or not). In the above model user locations are first fixed and then traffic is generated, while in this model user/packet locations are also random. As a result, there is a single process that describes both the spatial and temporal variations, which greatly simplifies analysis. We consider a finite area *A*, and model packet arrivals at each transmitter according to a 1-D Poisson point process with arrival rate $\lambda^{temporal} = (A/T)\lambda$. Upon arrival each packet is assigned to a random transmitter location (uniformly distributed in area *A*) and a receiver is randomly located a fixed distance *R* away, as shown in Figure 5.

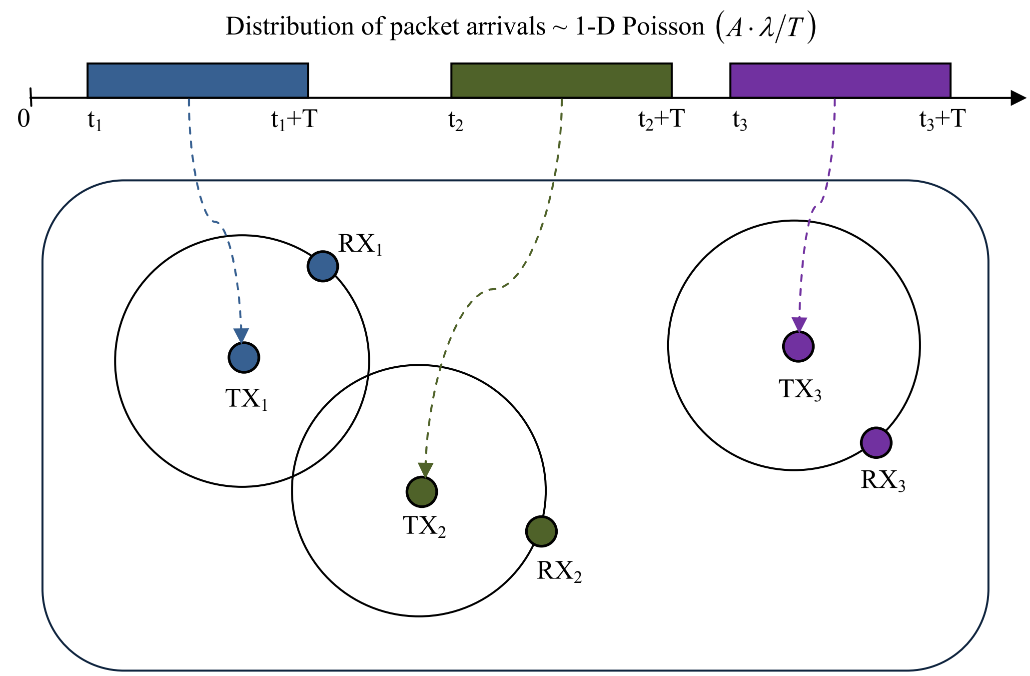


Figure 5 – Each new packet arrival is assigned to a transmitter-receiver pair, which is positioned randomly on the plane.

Note that the number of packet arrivals during a time interval of *T* seconds is Poisson($A \cdot \lambda$). When *A* is made large, this translates to a spatial density of λ, which is the same as in the model with fixed position of nodes, which was initially discussed. Therefore, results generated with our model can be fairly compared to the first network model with density λ. Furthermore, note that if slotted ALOHA is used (and the area *A* is taken to infinity) the two models are the same, because the set of transmitters during each time slot is a homogenous 2-D process with density λ. As mentioned earlier, the parameter for our Poisson distribution is $\lambda^{temporal} = A \cdot \lambda / T = A \cdot \lambda^s \cdot \lambda^t$, which indicates the density of packet arrivals for *all* nodes on the plane. That is, if we place all the possible packet transmissions between all the transmitter-receiver pairs on the place in a queue, then the rate of these packet transmissions is $\lambda^{temporal}$. Moreover, the density of nodes on the plane at a fixed time instant in our model is λ, whereas in the initial model with the fixed position of nodes, this spatial density of nodes would be λ^s. For the sake of intuition, we note that if we consider a unit area on the plane, the number of active nodes we will detect in this area is: $\lambda^t \cdot T$. On the other hand, if we look at our network at a fixed point in time, the number of active nodes on the entire plane is: $\lambda \cdot A$.

All transmitters are assumed to use the same transmission power ρ. This assumption has been made to simplify the analysis, and may in future work be extended to include variable transmit powers. Furthermore, it is assumed that there is sufficient bandwidth available in the channel, all of which is shared by all transmitters at the same time. No CSI is assumed anywhere in the network, and the propagation delay is assumed to be negligible. For future work, retransmissions and RTS/CTS packets can be added to the communication system, which will involve setting up packet queues and thereby result in delays.

For the channel model, only path loss attenuation effects (with exponent $\alpha > 2$) are considered, i.e. additional channel effects such as shadowing and fast fading are ignored, and the channel is considered to be constant for the duration of a transmission. Note that it is feasible to extend the work to include fading using the techniques developed in [8]. Each receiver sees interference from all the transmitters, and these interference powers are added to the channel noise η to result in a certain SINR at each receiver. If this SINR falls below the required threshold β at any time during the packet transmission, the packet is received in *outage*. With an outage constraint of ε, this is given as:

$$\Pr\left(\frac{\rho R^{-\alpha}}{\eta + \sum_i \rho |r_i|^{-\alpha}} \le \beta \right) \le \varepsilon \tag{11}$$

In the case of ALOHA, a transmitter starts its transmission as soon as the nodes are placed on the plane, regardless of the channel condition. Slotted ALOHA improves performance by removing partial outages, but this system requires synchronization. In the CSMA protocol the incoming receiver listens to the channel in the *beginning* of the packet, and if the measured SINR is below β, it informs its transmitter to cancel its transmission. No retransmissions are applied in our model. The properties of ALOHA and CSMA in our context are described in further depth in the corresponding chapters where the performance analysis is performed.

3.2 Justifications for Assumptions

Although our model involves some simplifications to allow for tractability, it contains many of the critical elements of a real ad hoc network. Firstly, the spatial Poisson distribution means that nodes are located randomly and independently in space; this is reasonable particularly in a network with substantial mobility or indiscriminate node placement, such as a very dense sensor network. The fixed distance between transmitter and receiver is clearly not a natural assumption; however, it has been rigorously shown in [6] and [7] that variable transmit distances do not result in fundamentally different capacity results. Hence, the fixed transmission distance was chosen to allow for simpler analytical considerations and crisper insights. Also, a fixed transmission power was chosen, in order to allow for simpler analysis, and also since power control is usually not used in actual ad hoc networks.

Furthermore, in order to simplify the analysis, any forms of coding or limitations on the number of bits that may be sent over the channel at a time are excluded. Focus is mainly set on what effects the choice of MAC algorithms have on the SINR and probability of outage at the receiver. Finally, scheduling introduces many other discussion issues, and is therefore left for future work.

3.3 Simulation Model

Our simulation model follows the descriptions of our system model, as was described above, closely. In particular, we consider a queue of packet arrivals, which follows a Poisson point process in time. Each packet transmission is assigned to a specific transmitter-receiver pair, which is then placed randomly on the plane. The number of transmitters on the 2-dimensional plane of area A follows a Poisson point process, i.e.:

$$\#\ of\ \ nodes = \lambda^s \cdot \lambda^t \cdot T \cdot A \tag{12}$$

where λ^s is the spatial density of nodes, λ^t is the temporal density of packet arrivals at each transmitter and T is the packet length in time.

The nodes that are encountered as active, based on either the ALOHA or the CSMA scheme, start transmitting their packets as soon as they are placed on the plane. For the ALOHA protocol, all packets are encountered as active upon arrival, while in the CSMA protocol, only those nodes that sense their received SINR to be above a SINR threshold β are encountered as active. The active transmitter-receiver pairs remain on the plane for the duration of the packet, T, before they disappear.

We assume AWGN channel between all nodes, and the noise power is chosen to be relatively small. The transmitted power ρ and the distance between each transmitter and receiver R are chosen as to give a received power of 1 at a distance of 1 unit away from each transmitter. Furthermore, the SINR threshold β is set to 0 dB, which results in the outage probability having insignificant dependence on the path loss exponent α. This is explained in further depth in section 4.2.2.

The values of the system parameters used in the simulations are tabulated below:

Parameter	Value	Unit	Description
α	3	-	Path loss exponent
L	40	*Meters*	Length of each side of an LxL plane
R	1	*Meters*	Distance between TX and designated RX
ρ	1	*Watts*	Transmission power for each transmitter
T	[1, 100]	*Seconds*	Length of each data packet transmitted
β	0	*dB*	Required threshold for received SINR
λ^s	[10^{-5}, 10]	*Nodes/m*^{2}	Spatial density of nodes
λ^t	0.1	*Packets/sec*	Temporal density of transmission packets

In chapters 4, 5 and 6, simulations are used to either confirm or reject the derived analytical results. Hence, in the following section we make sure our simulation algorithm and model is in fact valid.

3.3.1 Confirming Simulation Model

Since the simulation results were used to prove that our derived analytical results were correct, we have to make sure our simulation results are in fact valid. Hence, we perform a CDF simulation of the cumulative distribution function (CDF) of the SINR in the ALOHA protocol, and compare it with existing results [4]. This CDF represents the ratio of the number of successful transmissions over the total number of transmissions.

The model used for this CDF simulation is as just described, with the only difference that the system used here is slotted. We choose this because the existing results all use slotted systems. Figure 6 shows the simulated CDF plotted versus SINR, for a fixed spatial density of $\lambda = 0.01$.

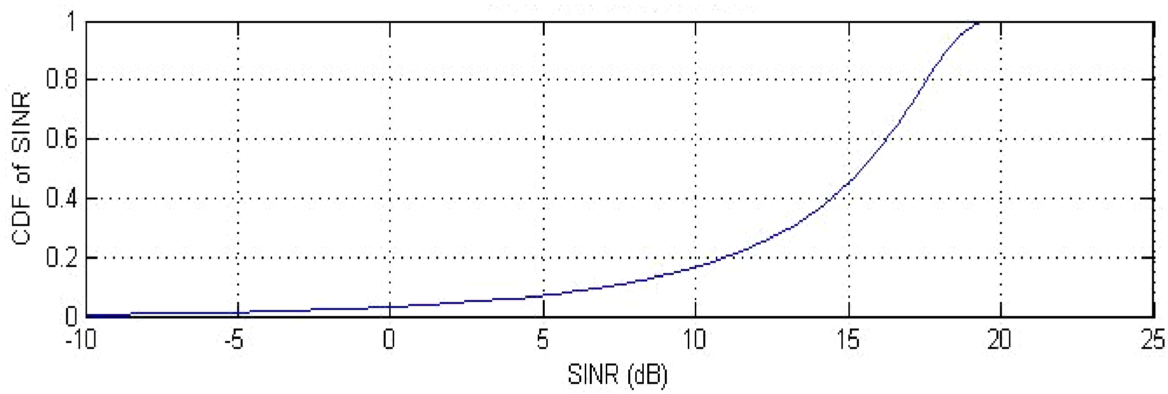


Figure 6 – CDF of SINR for the ALOHA scheme.

As indicated by the graph, as the SINR threshold increases, the probability that the received SINR will be below this value will increase. When SINR reaches its maximum, which occurs when there is no interference in the system (SINR = SNR = 20 dB), all receivers will have a SINR below this value, resulting in a CDF of 1. These results are equivalent to what was found in [4], indicating that the ALOHA model is implemented correctly. Since the only difference between the ALOHA and CSMA protocols is that transmissions in CSMA are only initiated if the sensed SINR in the channel is above a threshold β, and our CSMA simulations are based on the ALOHA simulation model, we can thereby conclude that our CSMA simulations are also correct.

Chapter 4

Performance Analysis of ALOHA

This section involves the performance analysis of slotted and unslotted ALOHA in the system model described in the previous section. The performances of these two systems are compared both analytically and through simulations.

4.1 Slotted ALOHA

Due to the nature of our model, in order to find the probability of outage, we look at the lower bound for the probability of outage, which is found by considering an area around the receiver that is measuring the SINR of the channel.

Weber et al. consider the ALOHA protocol in a slotted version of our network model [4], i.e. transmitters can only start their packet transmissions at the beginning of the next time slot after the packet has been formed. Thus there is no partial overlap of transmitted packets, something that is intuitively expected to decrease the probability of outage. Moreover, as discussed in section 2.3.1, Hasan and Andrews consider the role and significance of *guard zones* [7], which we will apply for our analysis. Define s to be the

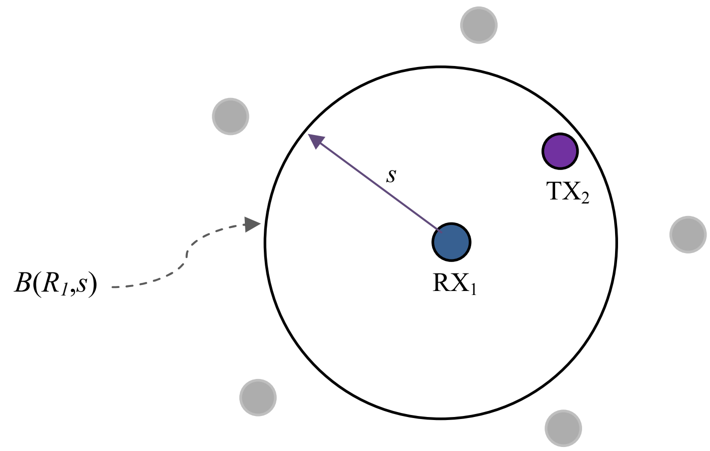


Figure 7 – Illustrating the guard zone of RX_1. When at least one interferer TX_2 falls within a distance *s* away from RX_1, i.e. within $B(R_1, s)$, it causes outage for RX_1.

distance between the receiver under observation and its closest interfering transmitter that causes the SINR to fall just below an SINR threshold of β. Setting the equation for the SINR equal to β and solving for *s*, we obtain:

$$s = \left(\frac{R^{-\alpha}}{\beta} - \frac{\eta}{\rho} \right)^{-1/\alpha} \tag{13}$$

Consider the area $B(R_1, s)$, which is a circle of radius *s* around the receiver under observation, say RX_1, as illustrated in Figure 7. To evaluate the interference that a typical receiver sees, define the function $p(r)$ as the signal power at a distance *r* from the transmitter, when the transmitter's intended receiver is a distance *R* away. Thus $p(r) = \rho \cdot r^{-\alpha}$, while the signal power received at the intended receiver is: $p(R) = \rho \cdot R^{-\alpha}$. With *s* as specified in equation (13), we can now find an analytical lower bound for the probability of outage P_{outage} of ALOHA. The lower bound (LB) on the outage is the case when there is at least one transmitter inside the region $B(R_1, s)$. This is a *lower* bound because it does not take into account the probability that outage can occur from of the aggregation of the interference powers of many transmitters outside of $B(R_1, s)$, even though there are no transmitters within $B(R_1, s)$.

Theorem 1: The lower bound for the probability of outage for slotted ALOHA is:

$$P_{out}^{LB}(Slotted\ \ ALOHA) = 1 - e^{-\lambda\pi s^2} \tag{14}$$

Proof of Theorem 1: Outage can be thought of as simply determined by the positions of the interfering nodes [4]. Hence, one situation that would result receiver RX_1 to go into outage is if at least one active transmitter is closer to the receiver than *s*, i.e. if one or more transmitters fall within the area $B(R_1, s)$, RX_1 will receive its packets in outage. Hence, the lower bound on the outage probability for a slotted ALOHA scheme is found by taking 1 minus the probability that there are no transmitters in $B(R_1, s)$, which is simply the void probability for $B(R_1, s)$ [26]. The reason only the lower bound is considered here, is because this can be found analytically, and also, as it is shown in [4], the upper and lower bounds on the probability of outage are both very close to the actual simulation results of the slotted ALOHA protocol. In fact, the lower bound is seen to follow the simulation results asymptotically, and hence focusing on this alone is sufficient for deriving an approximate expression for the actual probability of outage of ALOHA.

Based on the above arguments and the discussions on probability of success in section 2.3.1, we obtain:

$$\begin{aligned} &P_{out}^{LB}(Slotted\ \ ALOHA) \\ &\qquad = P(\geq 1\, TX\ \ inside\ \ B(R_i, s)\ \ during\ \ [0,T]) \\ &\qquad = 1 - e^{-\lambda\pi s^2} \end{aligned}$$

where *s* is given by equation (13). For small values of the density *λ*, this equation for the outage probability may be approximated by:

$$P_{out}^{LB}(Slotted\ ALOHA) \approx \lambda\pi s^2 \tag{15}$$

This is a linear function of the density, and through simulations this is shown to be relatively exact for outage probabilities up to 0.5. We will refer to the approximation of equation (15) later for the sake of comparison and intuition. In the following we will use and develop equation (14) further to cover the domain of continuous-time transmissions for both the ALOHA and CSMA protocols.

4.2 Unslotted ALOHA

If no synchronization is possible and delay is insignificant in the system that uses ALOHA, the nodes will have to use *unslotted* ALOHA for communication. Two methods were used to analyze the performance of unslotted ALOHA. The first approach was shown to not be successful for higher densities, and is in the line of the 'RX-RX Approach' that is one of the methods used for deriving the probability of outage for CSMA, which will be considered in Chapter 5. The second approach, which gives promising results, is in the line of the 'RX-TX Approach' and 'TX-TX Approach' used for the CSMA analysis.

4.2.1 RX-RX Approach

In this approach, we consider the guard zone of each receiver, which is a circle of radius s centered on the receiver as illustrated in Figure 7, where the expression for s is given in equation (13). Based on the same argument that was used for slotted ALOHA, we consider the lower bound for the outage to be the probability that at least one active transmitter is closer to the receiver RX_1 than s, i.e. if more than one transmitter falls within the area $B(R_1, s)$, as shown in Figure 8.

As before, we assume that receivers are placed on the plane according to a Poisson point process, and denote the random distance between the receiver under observation, RX_1, and the receiver of its closest interfering node, RX_2, as d. Conditioning on this distance, we find the probability that a new packet arrival will cause outage for an ongoing transmission between TX_1 and RX_1, if it is placed a distance d away from RX_1. Based on the adherent property of ALOHA, the incoming transmitter-receiver pair TX_2–RX_2 is placed on the plane and starts its transmission regardless of whether it is in outage itself or if it causes outage

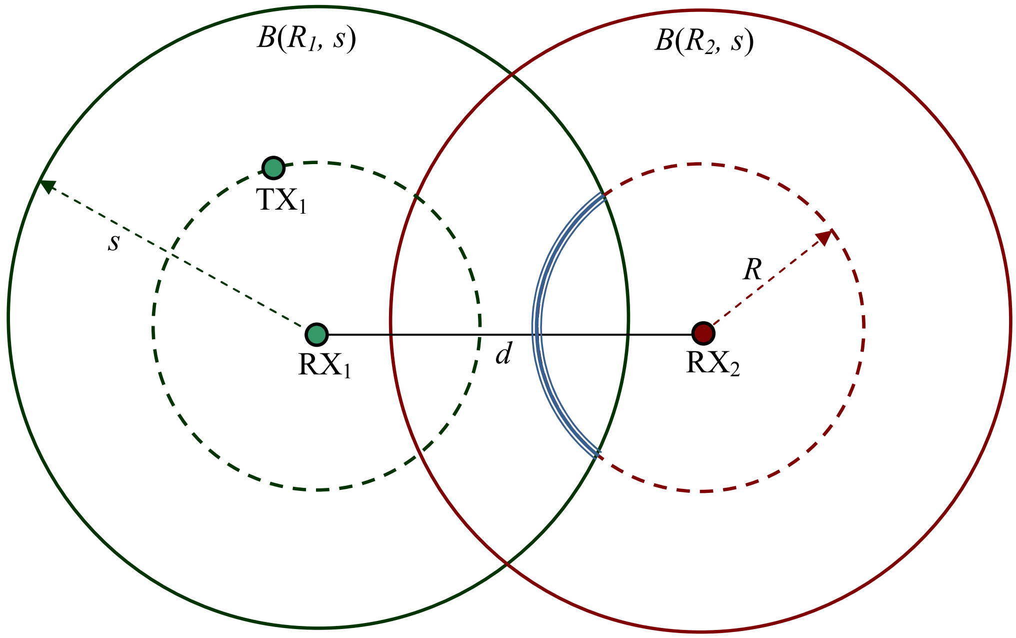
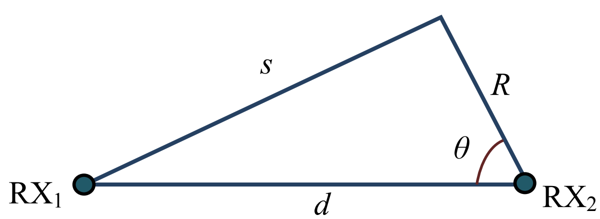


Figure 8 – This figure illustrates how we get outage in an ongoing transmission TX_1, with the arrival of a new packet through TX_2 when ALOHA is applied. If TX_2 falls on the marked section of the circle of radius R around RX_2, the packet of TX_1 is received in outage.



$$s^2 = d^2 + R^2 - 2Rd\cos(\theta)$$

$$\theta = \cos^{-1}\left(\frac{d^2 + R^2 - s^2}{2Rd}\right)$$

Figure 9 – Setup for calculating the probability of outage given d.

for another transmission. Hence, the arrival of RX_2 will only cause outage for RX_1 if the transmitter TX_2 is placed anywhere inside $B(R_1, s)$. Conditioning on d and using the setup of Figure 9, we derive this probability to be:

$$P_{out}^{LB}(Unslotted\ \ ALOHA \mid d) = \frac{1}{\pi}\cos^{-1}\left(\frac{d^2 + R^2 - s^2}{2R\,d}\right) \tag{16}$$

To find the total probability of outage, we integrate d over its distribution. Due to the assumption that d follows a Poisson point process, and based on the results in [4], we know that the pdf for d^2 is given as:

$$p(d^2) = \pi\lambda \cdot e^{-\pi\lambda d^2} \tag{17}$$

Since the position of TX_2 determines the probability of outage, we choose the integration limits such that TX_2 falls within $B(R_1, s)$, i.e. d^2 is integrated from 0 to $(s + R)^2$. This results in the following expression for the probability of outage of unslotted ALOHA:

$$P_{out}^{LB}(Unslotted \;\; ALOHA) = \int_0^{(s+R)^2} \frac{1}{\pi} \cos^{-1}\left(\frac{d^2 + R^2 - s^2}{2R\,d} \right) \left(\pi\lambda \cdot e^{-\pi\lambda d^2} \right) d(d^2) \tag{18}$$

As already mentioned, note that this is a lower bound on the outage probability because outage can also occur even though there are no transmitters inside $B(R_1, s)$, which is the case when there exist many interfering nodes outside of $B(R_1, s)$.

The simulation results, as shown in Figure 10 confirm these obtained results for lower densities, but fail to comply for higher values of λ. We suspect that the reason for this is our assumption that the distance between the *receivers* is Poisson distributed. This is not entirely true, because it is indeed the transmitters that are placed on the plane based on a PPP, meaning that if we are taking a particular receiver under consideration, and outage is caused by the entrance of a transmitter, then it is in fact the distance between the receiver and incoming transmitter that is Poisson distributed. This deviation becomes more evident for higher densities, because as the density increases, d decreases, and thus the integral in equation (18) exhibits greater dependence on d.

As this method resulted in too much deviation from the desired result, we turn to another approach, which considers the distance between the receiver on the plane and the incoming transmitter to follow a Poisson point process.

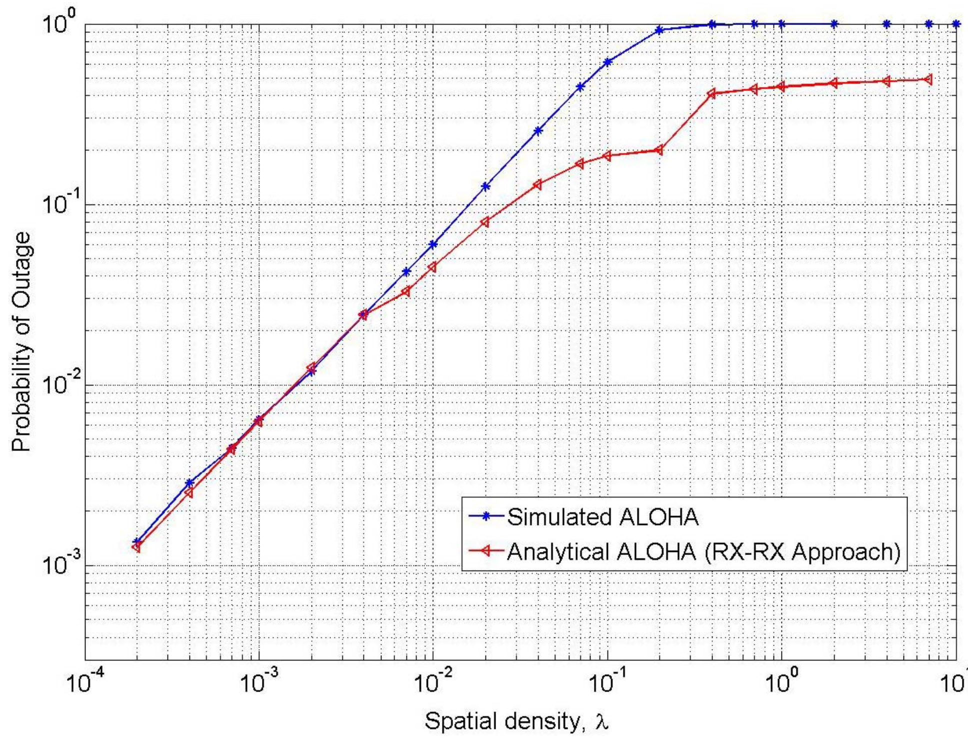


Figure 10 – Simulation and analytical results for the probability of outage of unslotted ALOHA. For higher densities, the deviation between analytical and simulation results increases.

4.2.2 RX-TX and TX-TX Approach

As described above, we look at the guard zone $B(R_i, s)$ around each receiver, and claim that we have outage if one or more transmitters fall inside this area. However, now we consider the distance between RX_1 and TX_2 to follow a Poisson distribution[9], and apply the results obtained in the case of slotted ALOHA (Theorem 1) to obtain the outage probability for unslotted ALOHA. As we will see shortly, these results follow the simulation results tightly, thus confirming our method and obtained equations, and we can hence obtain the following theorem.

Theorem 2: The lower bound for the probability of outage for continuous-time ALOHA is:

[9] This gives the same result as considering the distance between TX_1 and TX_2 to be Poisson distributed.

$$P_{out}^{LB}(Unslotted\ \ ALOHA) = 1 - e^{-2\lambda\pi s^2} \tag{19}$$

Proof of Theorem 2: Consider the equation for the lower bound for the probability of outage of ALOHA in the slotted system, and note that this indicates that there are no active transmissions inside $B(R_1, s)$ during the time period [0, T]. Extending this concept to a continuous-time system means that any transmission that started time T before will still be an ongoing transmission when the new packet transmission is about to start, thus contributing to the outage probability of RX_1. Thus, we now require that there are no transmissions inside $B(R_1, s)$ during the period [-T, T]. That is:

$$\begin{aligned}
&P_{out}^{LB}(Unslotted\ ALOHA)\\
&\quad = P(\geq 1\ TX\ inside\ B(R_i, s)\ during\ [-T, T])\\
&\quad = P(outage\ in\ [-T,0] \cup outage\ in\ [0,T])\\
&\quad = P(outage\ in\ [-T,0]) + P(outage\ in\ [0,T]) - P(outage\ in\ [-T,0]) \cdot P(outage\ in\ [0,T])\\
&\quad = 2 \cdot (1 - e^{-\lambda\pi s^2}) - (1 - e^{-\lambda\pi s^2}) \cdot (1 - e^{-\lambda\pi s^2})\\
&\quad = 1 - e^{-2\lambda\pi s^2}
\end{aligned}$$

Note that this derivation is valid because the probability of outage in [-T, 0] is independent of the probability of outage in [0, T], since all packets are of equal length T. That is, the set of active transmissions at time 0 is independent of those at time T. For small values of the density λ the probability of outage for unslotted ALOHA may be approximated with:

$$P_{out}^{LB}\left(Unslotted\ \ ALOHA\right) \approx 2\lambda\pi s^2 \tag{20}$$

Comparing this with equation (15) we see that slotted ALOHA performs better than unslotted ALOHA by a factor of 2 in terms of probability of outage. This is expected and consistent with the results obtained from the conventional model for the slotted and unslotted ALOHA protocols.

Figure 11 shows the probability of outage versus density for both the slotted and unslotted ALOHA. The lower bounds given in (14) and (19) are plotted along with the simulation results. The plot illustrates that the bound is indeed tight, particularly at lower densities. As expected, slotted ALOHA outperforms unslotted ALOHA in terms of probability of outage, by approximately a factor of 2.

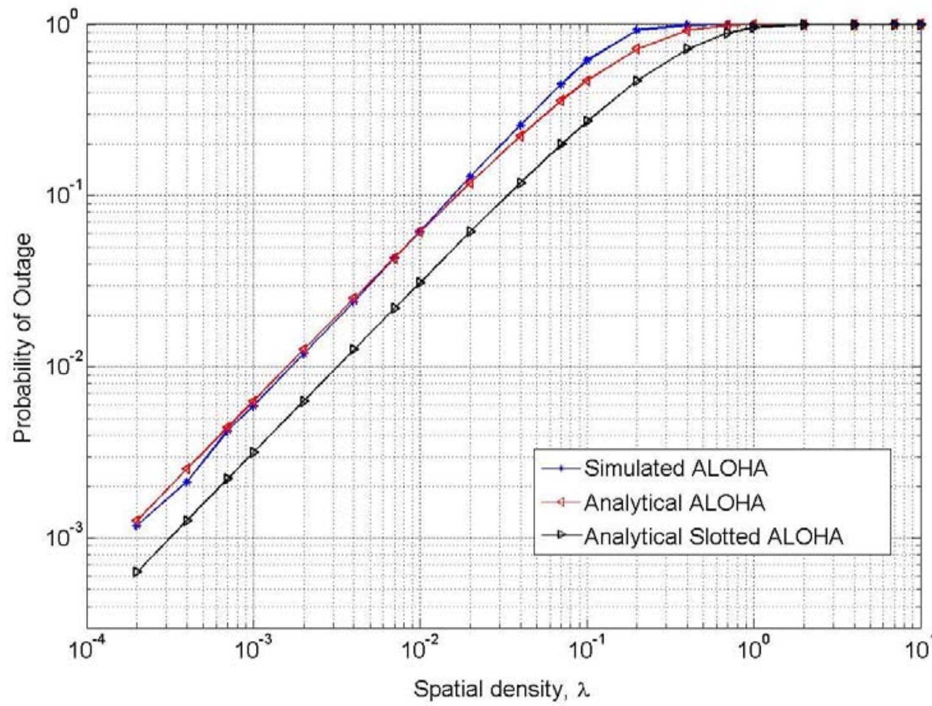


Figure 11 – The probability of outage for slotted and unslotted ALOHA along with the simulation results. The analytical results follow the simulation results tightly, and slotted ALOHA is shown to outperform unslotted ALOHA by about 100% in terms of probability of outage.

As noted in Chapter 3, we set β equal to 1, resulting in the outage probability having approximately no dependence on the path loss exponent α. This can be seen by inserting $\beta = 1$ in equation (13), and by noting that $\eta/\rho \ll 1$, giving us $s \approx R$. Hence, there is approximately no more dependence on α in the equations for the outage probabilities. This argument is consistent with the simulation results, as shown in Figure 12. Moreover, through the simulations performed for other SINR thresholds β, it was observed that the probability of outage decreases as the path loss exponent α increases. This is due to the fact that an increase in α results in a greater decrease in the interference power at the receiver as compared to the signal power, thereby resulting in a higher received SINR.

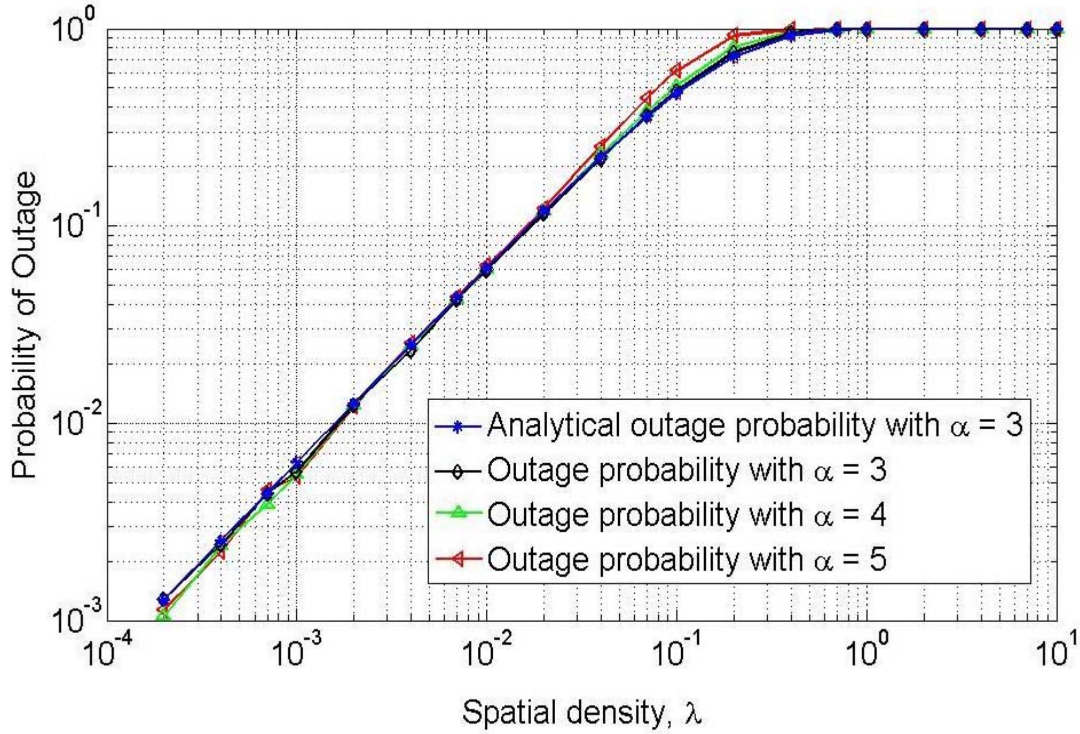


Figure 12 – The probability of outage for unslotted ALOHA is approximately independent on the path loss exponent α when $\beta = 1$.

Chapter 5

Performance Analysis of CSMA

This chapter involves the performance analysis of CSMA in the system model described in Chapter 3. The probability that an incoming transmitter backs off, the probability that outage occurs during a packet transmission, and the total probability of outage are all derived analytically, and presented along with corresponding simulation results.

5.1 Definition and Approach

In the CSMA protocol, the transmitter backs off or drops its packet if the measured SINR at the receiver at the *beginning* of the packet is sensed to be below the predetermined SINR threshold β. The probability that this happens, i.e. the probability that the packet is in outage at the start of the packet, is called the *backoff probability* for CSMA. If the measured SINR is below the threshold any time along the packet duration, the packet that is transmitted is received in outage at the receiver. Note that the receiver measures the channel, and informs its transmitter over a control channel whether it should start its transmission or not. This is equivalent to the popular RTS–CTS handshaking; when the transmitter has a packet to transmit, it sends a request-to-send (RTS) signal to the receiver. The receiver then senses the channel around it and if the measured SINR is above β, it sends a clear-to-send (CTS) signal back to allow the transmitter to initiate its packet transmission. If the SINR is below β, the transmitter backs off or drops its packet. The RTS–CTS signal transmissions are assumed to not introduce any delays, and the signals are transmitted over a control channel as to not affect the SINR level in the channel.

To derive a lower bound on the outage probability, we only consider the effect that the nearest interferer (corresponding to TX_2 in Figure 7) has on the receiver under observation (RX_1). Denoting the distance between RX_1 and TX_2 as d, d^2 follows a Poisson distribution. Furthermore, note that the total probability of outage for CSMA is given as the probability of the union of the probability that outage occurs in the beginning of the packet, $P_{backoff}$, and the probability that the transmitted packet goes into outage during the packet duration T. Now, we use the addition rule $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, where A and B are two events that are not independent of each other. Letting A be the probability that outage occurs in the beginning of the packet, and B the probability that the packet goes into outage in the duration of the packet , we obtain:

$$
\begin{aligned}
P_{out}^{LB}(CSMA) &= P_{backoff} + P_{out}^{LB}(CSMA \mid no\ backoff) - P_{backoff} \cdot P_{out}^{LB}(CSMA \mid no\ backoff) \\
&= P_{backoff} + (1 - P_{backoff}) P_{out}^{LB}(CSMA \mid no\ backoff) \qquad (21)
\end{aligned}
$$

Mathematical expressions for the probabilities $P_{backoff}$ and $P_{out}^{LB}(CSMA \mid no\ backoff)$ are found in the following sections.

5.2 Probability of Backoff

In the CSMA protocol, an incoming node backs off if the accumulation of the interference from all other active transmitters results in a received SINR that is below β. As briefly mentioned above, a lower bound for the outage probability may be obtained by only considering the nearest transmitter. That is, if an incoming receiver falls within a distance s away from an already active transmitter on the plane, this incoming node backs off. Because of the backoff property of CSMA, the number of transmitters on the plane no longer follows a Poisson point process. Nevertheless, as an approximation, we assume that the transmitters still follow a Poisson distribution, and as we will see from the simulation results, this assumption is reasonable.

Theorem 3: The approximate probability that an incoming transmitter using CSMA backs off is given by:

$$P_{backoff} = 1 - \exp\left(-\pi \lambda (1 - P_{backoff}) s^2\right) \tag{22}$$

The solution to this can be given in closed form in terms of the Lambert function $W_0(\cdot)$ as [45]:

$$\begin{aligned} P_{backoff} &= 1 - \frac{1}{\pi \lambda s^2} W_0\left(\pi \lambda s^2\right) \\ &= 1 - \frac{1}{\pi \lambda s^2} \sum_{n=1}^{\infty} \frac{(-n)^{n-1}}{n!} \left(\pi \lambda s^2\right)^n \\ &= \pi \lambda s^2 - \frac{3}{2}\left(\pi \lambda s^2\right)^2 + \frac{8}{3}\left(\pi \lambda s^2\right)^3 - \frac{125}{24}\left(\pi \lambda s^2\right)^4 + \cdots \end{aligned} \tag{23}$$

Proof of Theorem 3: Consider at a fixed point in time a new packet arrival that is assigned to a transmitter-receiver pair. In order for this new transmitter to start its transmission, we require that the closest transmission that is already active on the plane is at least a distance s away from the incoming receiver. Only the time period [-T, 0] is of interest for the probability of backoff, because the decision on whether to back off or not is made at the beginning of each packet. Also, due to the backoff property of CSMA the density of nodes on the plane is now: $\lambda(1 - P_{backoff})$. Hence, using equation (14), the probability of backoff for a new packet arrival is found to be (22). Note that for small values of λ, the probability of backoff increases as a linear function of λ.

Similar results have also been obtained in other related work [16], but within other models, and without the use of guard zones. For the sake of understanding and comparison, we restate these results and relate them to our model and our obtained results. As already discussed, a continuous-time CSMA system has the property that a transmitted packet (considering a specific transmitter-receiver link) will back off to any defer-causing arrival any time within the previous packet transmission time. Hence, by the Poisson assumptions, the probability that a packet will back off is:

$$P_{backoff}(CSMA) = 1 - e^{-(Backoff\ \ rate)} \tag{24}$$

The backoff rate can be found by considering a 'test packet' being sent from a transmitter to its receiver, and an interference power detected at a location of distance *r* away from the test packet transmitter. Based on a given density, a backoff probability, and an area *D* over which the backoff probability is considered, the backoff rate is found to be [16]:

$$Backoff\ \ rate\ = (1 - P_{backoff})\lambda \cdot D \tag{25}$$

D can be interpreted as the area surrounding the test packet transmitter in which other transmissions would cause deferrals in the test packet. Relating this to our model, we may set *D* to be the area of the guard zone around each receiver, i.e., $D = \pi{\cdot}s^2$, where *s* is the distance to the closest interferer on the plane that would cause the receiver to go into outage. Combining equations (24) and (25) gives:

$$(1 - P_{backoff}) = e^{-(1-P_{backoff})\lambda D} \tag{26}$$

which is the same as our derived equation (22). This expression must be solved for $(1 - P_{backoff})$ either numerically or by using the Lambert function as given in equation (23). Note that this value is in fact the factor by which the probability of outage of CSMA will be smaller than that of unslotted ALOHA. In other words, by applying the CSMA algorithm over ALOHA, the transmission density of nodes can be increased by a factor of $(1 - P_{backoff})$ in order to get the same probability of outage.

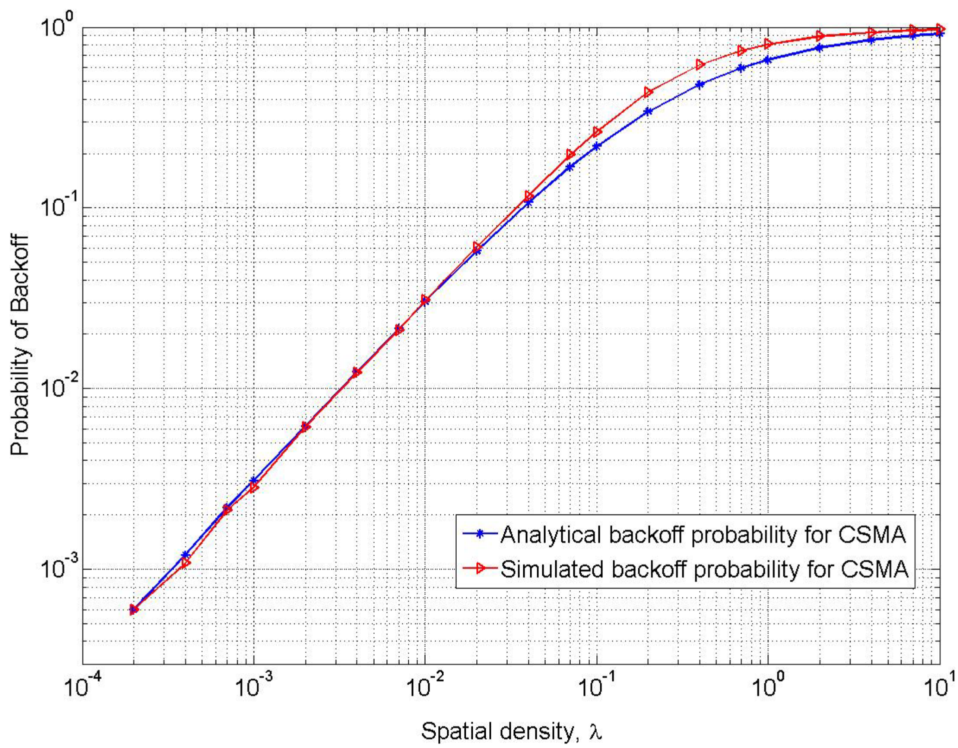


Figure 13 – Analytical and simulation results for the probability of backoff of the CSMA protocol.

The analytical backoff probability for CSMA is plotted versus λ in Figure 13, and is shown to follow the simulation results tightly. As expected, for higher spatial density of nodes, there is a greater probability that an incoming node backs off its transmission due to the higher level of interference. This indicates that the use of CSMA is more effective for higher transmission density, as we will also see when comparing ALOHA and CSMA in Chapter 6.

Note from the figure that for some lower densities, the simulation results are indeed lower than the analytical results, which is based on our discussions thus far supposed to be the *lower* bound for the outage. This deviation is due to the assumption that the number of active transmitters on the plane still follows a Poisson point process for the CSMA protocol. This approximation was made to allow for the mathematical expressions to be derived, but it is not exactly the case due of the backoff property of CSMA. The reason we still called this a lower bound is because it is based on the lower bound expression for the outage probability of ALOHA, as given by equation (14). Also, the outage probability for CSMA was based solely on the situation when there are one or more transmitters within a distance s away from the receiver under observation, and does not take into account the outage that may be caused from the aggregation of interference powers of many transmitters that lie outside the guard zone of the receiver. Despite this discussion, as can be seen from Figure 13, the deviation between the analytical and the actual results is very small, meaning that the approximations made for calculating the analytical expressions for the outage probability are indeed reasonable.

5.3 Probability of Outage

In this section we find the probability that a packet transmission goes into outage during its packet duration, i.e., the probability that the packet is received in outage, given that it was not in outage at the start of its transmission. This was obtained by considering three different approaches, which we refer to as the *RX-RX Approach*, the *TX-TX Approach*, and the *RX-TX Approach*, depending on which distance we consider to follow a Poisson distribution. The two first methods are found to have flaws, while the last method, the *RX-TX Approach*, is shown to follow the simulation results tightly.

5.3.1 RX-RX Approach

We consider the probability that a packet transmission goes into outage during its packet length (i.e., we assume that RX_1 has already sensed its own channel at the start of its transmission, and has decided to transmit). Consider an ongoing packet transmission between say TX_1 and RX_1, as shown in Figure 13. Then a new packet arrives and is assigned to TX_2. Instead of considering transmitters being placed on the plane according to a Poisson point process, with receivers being placed randomly a distance R away from its

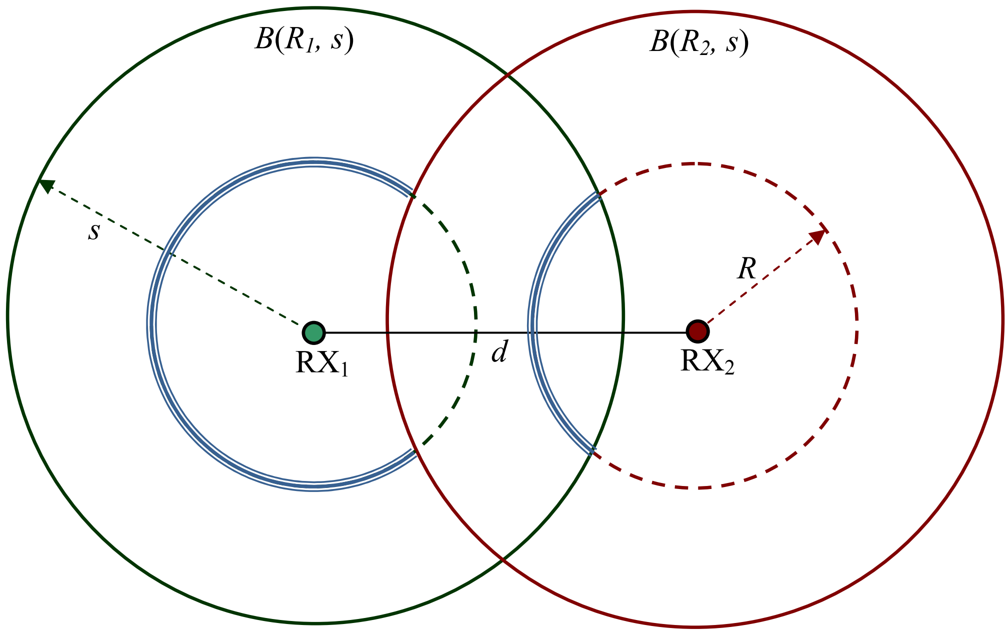


Figure 14 – This figure illustrates how we get outage in an ongoing transmission between TX_1 and RX_1, with the arrival of a new packet through TX_2 when CSMA is used. If TX_1 and TX_2 fall on the marked sections of the circles, the packet is received in outage at RX_1.

transmitter, in this approach we rather look at receivers as being placed randomly on the plane according to a PPP.

Consider Figure 14 where TX_1 and RX_1 are active nodes on the plane, when a new packet arrival occurs through the transmitter-receiver pair, TX_2-RX_2. As in the case of ALOHA, this new packet transmission may cause outage for the existing TX_1–RX_1 transmission if TX_2 falls within a distance s away from RX_1, i.e. within the guard zone $B(R_1, s)$. As discussed earlier, this probability is found by considering the setup of Figure 9, which gives us:

$$p\left(TX_2 \text{ } inside \text{ } B(R_1,s)\right)=\frac{1}{\pi}\cos^{-1}\left(\frac{d^2+R^2-s^2}{2Rd}\right) \tag{27}$$

Moreover, this outage occurs only if TX_2 does not back off. In order for this to happen, the closest transmitter to RX_2, which we assume to be TX_1, has to be outside of $B(R_2, s)$. Based on a similar derivation as above, the probability that TX_1 is positioned outside of $B(R_2, s)$ is found to be:

$$p\left(TX_1 \ outside\ of\ \ B(R_2,s)\,|\,d\right)=\frac{1}{2\pi}\left[2\pi-2\cos^{-1}\left(\frac{d^2+R^2-s^2}{2Rd}\right)\right] \quad (28)$$

These sections of the circles are marked in Figure 14. Based on this, we can now find the probability that an existing transmission goes into outage during its packet duration.

$$P_{out}^{LB}\left(CSMA|\,no\ backoff\right)=\int_{0}^{(s+R)^2}\frac{1}{\pi}\cos^{-1}\left(\frac{d^2+R^2-s^2}{2Rd}\right)\cdot\left[1-\frac{1}{\pi}\cos^{-1}\left(\frac{d^2+R^2-s^2}{2Rd}\right)\right]\cdot\pi\lambda e^{-\pi\lambda d^2}\,d(d^2) \quad (29)$$

The results of the simulations in Figure 15 show that this approach undercounts the situations that result in outage. For lower densities, there is a lower probability that the incoming transmitter backs off, and hence the results follow the simulation results closely.

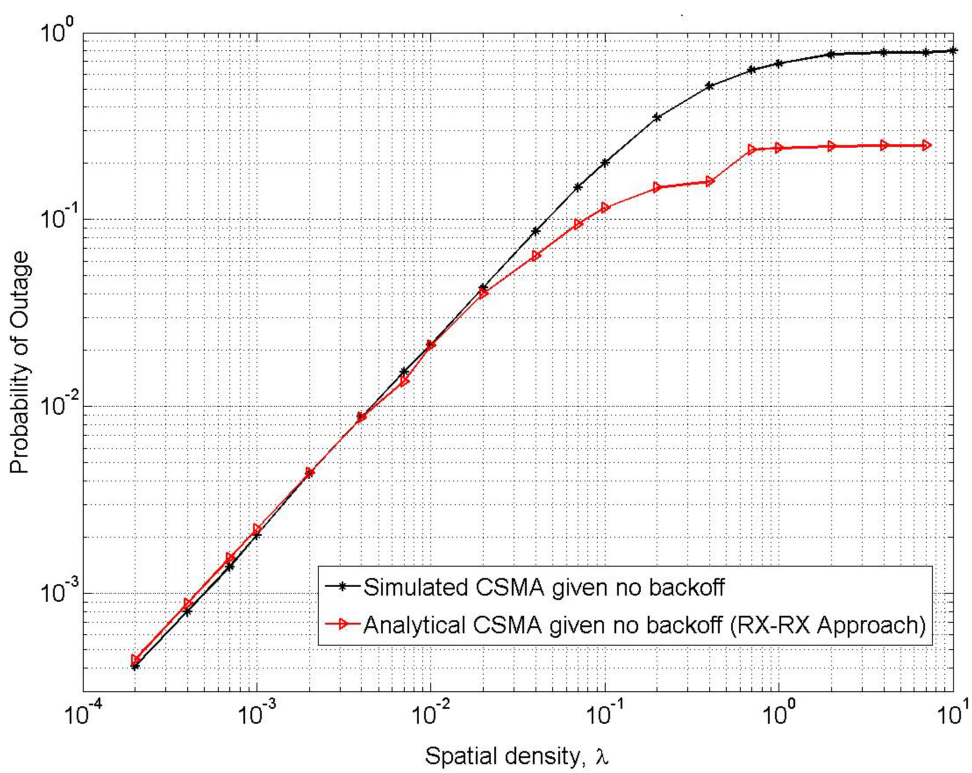


Figure 15 – Simulation and analytical results for the probability of outage of CSMA (using the RX–RX Approach). For higher densities, the deviation between analytical and simulation results increases.

For higher densities, however, this approach clearly fails. With the aim of mitigating the underestimation of the outage probability at higher densities we try another approach.

5.3.2 TX-TX Approach

In this approach we consider the distance between the transmitters, denoted x in Figure 16, to follow a Poisson distribution, and we look at areas and sections within which the transmitters and receivers must fall in order cause outage.

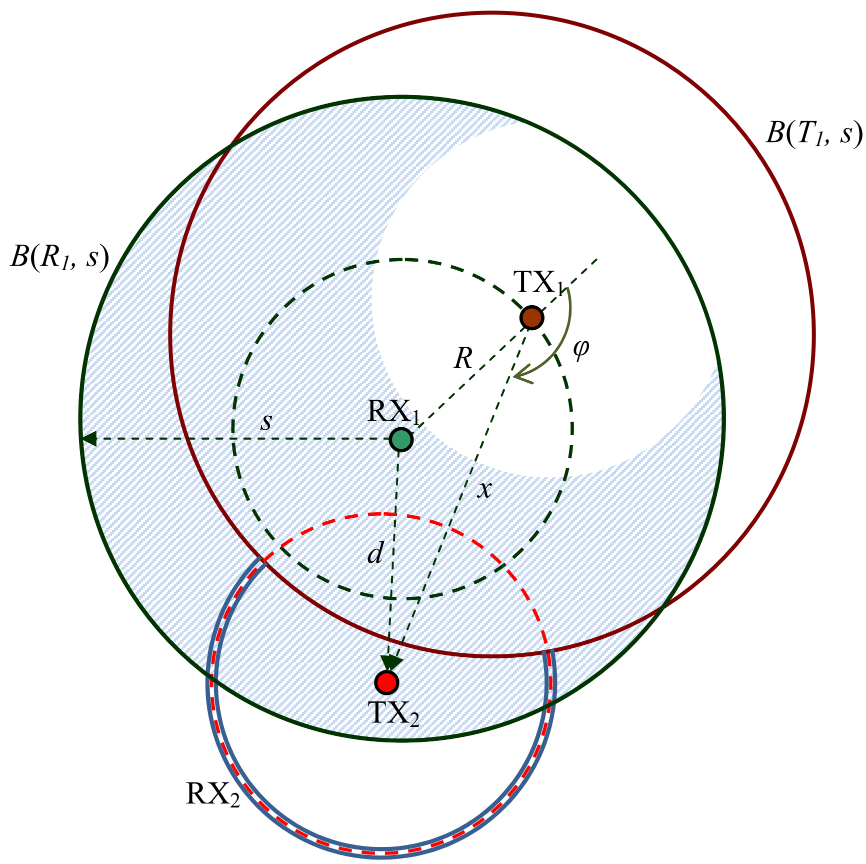


Figure 16 – The distance x between the transmitters follows a PPP. If an incoming transmitter TX_2 falls anywhere within the shaded area and RX_2 falls on the marked section of the circle of radius R centered around TX_2, then the new packet arrival will cause outage for the ongoing transmission between TX_1 and RX_1.

Based on the same arguments used thus far, outage is now caused if an incoming transmitter, TX_2, falls within the shaded area of $B(R_1, s)$ as shown in Figure 16, at the same time as the receive RX_2 is placed at least a distance s away from TX_1. Since the receiver is placed randomly on a circle around the transmitter, we know that the pdf of φ is $1/(2\pi)$. Now, in order to find the probability of outage, we integrate the pdf of the angle φ from α to β, where these limits are geometrically found to be:

$$\alpha = \pi - \cos^{-1}\left(\frac{R^2 + x^2 - s^2}{2Rx}\right) \qquad \wedge \qquad \beta = \pi + \cos^{-1}\left(\frac{R^2 + x^2 - s^2}{2Rx}\right) \tag{30}$$

In order for TX_2 to not back off, TX_1 must be placed outside of the guard zone of RX_2, i.e. $B(R_2, s)$. The probability that this happens is similar to equation (27) with d replaced by x. Since there is no dependence on φ in this equation, integration over the distribution of φ is equal to: $\beta - \alpha$, and we end up with the exact same equation as (28). Hence, this approach gives the same results as those obtained using the 'RX-RX Approach' described above. As we saw earlier, these results deviate from the simulation results at higher densities, and so we modify our approach once again, and consider the *RX-TX Approach*.

5.3.3 RX-TX Approach

Based on the same reasoning that was used for the TX-TX approach described in the previous section, outage is caused if an incoming transmitter TX_2 falls inside $B(R_1, s)$, as shown in Figure 17. The difference however is that it is now the distance d between the active receiver RX_1 and the incoming transmitter TX_2 that follows a Poisson distribution. The new packet arrival will only cause outage if the incoming transmitter TX_2 does not back off. This is determined by the probability that its receiver RX_2 is placed a distance s away from the active transmitter TX_1, i.e. outside of $B(T_1, s)$. Given the location of the new transmitter through the variables d and φ, we express x in terms of d, R, and φ, and thereby find the probability that the incoming transmitter decides to start its transmission after sensing the channel to be:

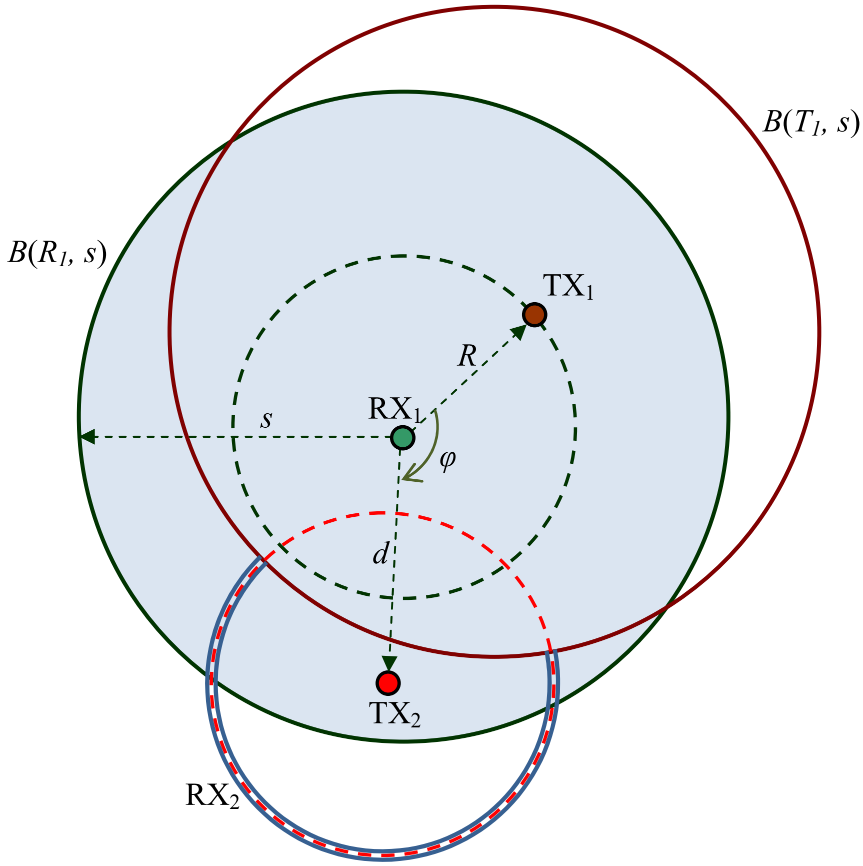


Figure 17 – Setup to find the probability that outage occurs during packet transmission between TX_1 and RX_1. Based on the 'RX-TX Approach', outage is caused if TX_2 falls inside $B(R_1, s)$, while RX_2 is located outside of $B(T_1, s)$ on a circle of radius R centered around TX_2.

$$P(transmit \mid d, \varphi) = 1 - \frac{1}{\pi} \cdot \cos\left(\frac{d^2 + 2R^2 - s^2 - 2Rd\cos\varphi}{2R\sqrt{d^2 + R^2 - 2Rd\cos\varphi}} \right) \tag{31}$$

Based on this expression we are now able to find the probability that a new packet arrival will cause outage for an ongoing transmission by integrating over the area in which the placement of a new transmitter would cause outage for an existing packet transmission. Note that equation (31) now has dependence on both φ and d.

Theorem 4: Considering an active transmitter-receiver pair, the probability that the packet is received in outage is:

$$P_{out}^{LB}(CSMA\,|\,no\ backoff) = \int_{0}^{s^2}\int_{\upsilon(d)}^{\omega(d)} \frac{1}{2\pi} \cdot P(transmit\ \,|\, d,\varphi) \cdot \pi\lambda e^{-\pi\lambda d^2}\, d\varphi\ d(d^2) \qquad (32)$$

where the integration limits for the angle φ are:

$$\upsilon(d) = \cos^{-1}\left(\frac{d^2 - s^2 + 2Rs}{2Rd}\right) \qquad \text{and} \qquad \omega(d) = 2\pi - \upsilon(d) \qquad (33)$$

Proof of Theorem 4: For a new packet arrival to cause outage for an existing transmission, firstly the transmitter TX_2 has to fall within $B(R_1, s)$, and secondly, its receiver RX_2 has to fall outside of $B(T_1, s)$, so that the new transmitter TX_2 will not back off its transmission. Since the distance R between the transmitter and receiver is constant, the new receiver must be positioned on the part of the circle centered on TX_2 that is at least a distance s away from TX_1. If all these conditions are satisfied, then the arrival of a new packet will result in outage for an ongoing packet transmission. Note that this outage probability only covers the transmitter-receiver pairs that are already active on the plane. Inserting equation (31) into the integral, while knowing that the pdf of d^2 is as given by equation (17), and the pdf of φ is $1/(2\pi)$, we double integrate over φ and d^2 to cover the area in which the existence of TX_2 may cause outage, and obtain equation (32).

The probability that a node will go into outage after it has started its transmission is plotted in Figure 18 as a function of the square of the distance d between an existing receiver RX_1 and an incoming transmitter TX_2. When d is small, the distance between TX_1 and RX_2 is also small, and hence there is a greater chance that RX_2 is in outage upon arrival. Thus there is a larger probability that TX_2 backs off[10], resulting in a lower probability of outage. As d increases, it indicates that the density of nodes in the network is decreasing, and therefore CSMA becomes less effective as fewer incoming nodes back off, resulting in a higher probability of outage. In other words, the CSMA protocol is most advantageous for higher density of nodes.

[10] Since we do not apply retransmissions in our model, the incoming transmitter "backs off" by dropping its packet.

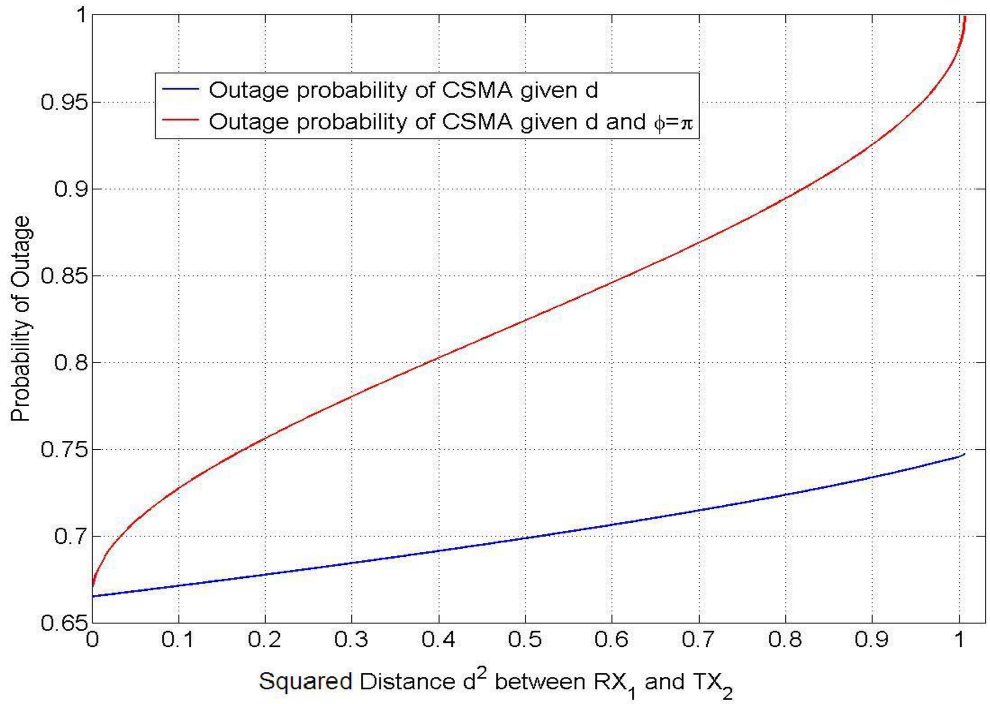


Figure 18 – Probability of outage for CSMA with respect to the square of the distance *d* between active receiver on the plane and incoming transmitter.

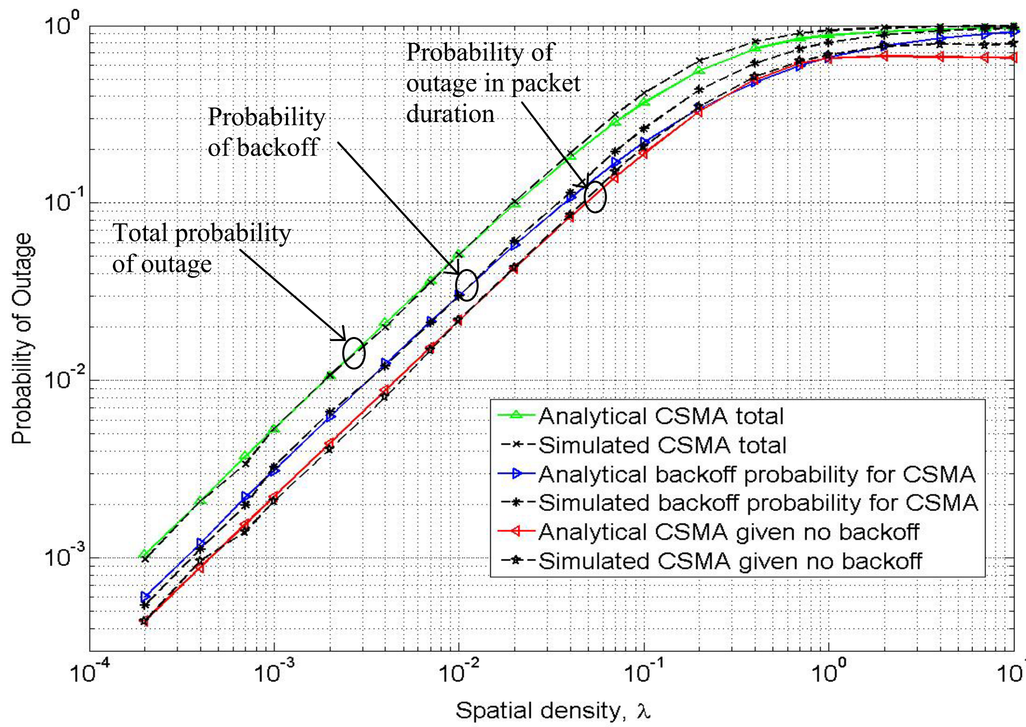


Figure 19 – The total probability of outage for CSMA, the probability of backoff, and the probability that the packet goes into outage during its transmission.

Figure 19 shows the total probability of outage for CSMA, as well as the backoff probability and the probability that an active transmission goes into outage during its packet length. The simulated results follow the analytical results tightly, hence validating our method and obtained formulas. The total probability of outage is found by using equations (21), (22) and (32).

Similar to the ALOHA protocol, the choice of $\beta = 1$ makes the outage probability approximately independent on the path loss exponent. As shown in Figure 20, the simulation results are insensitive to the path loss exponent α. As discussed earlier, this can be seen mathematically by inserting $\beta = 1$ in equation (13), obtaining $s \approx R$. This shows that s, as well as the expressions for the outage probabilities, are no longer dependent on α.

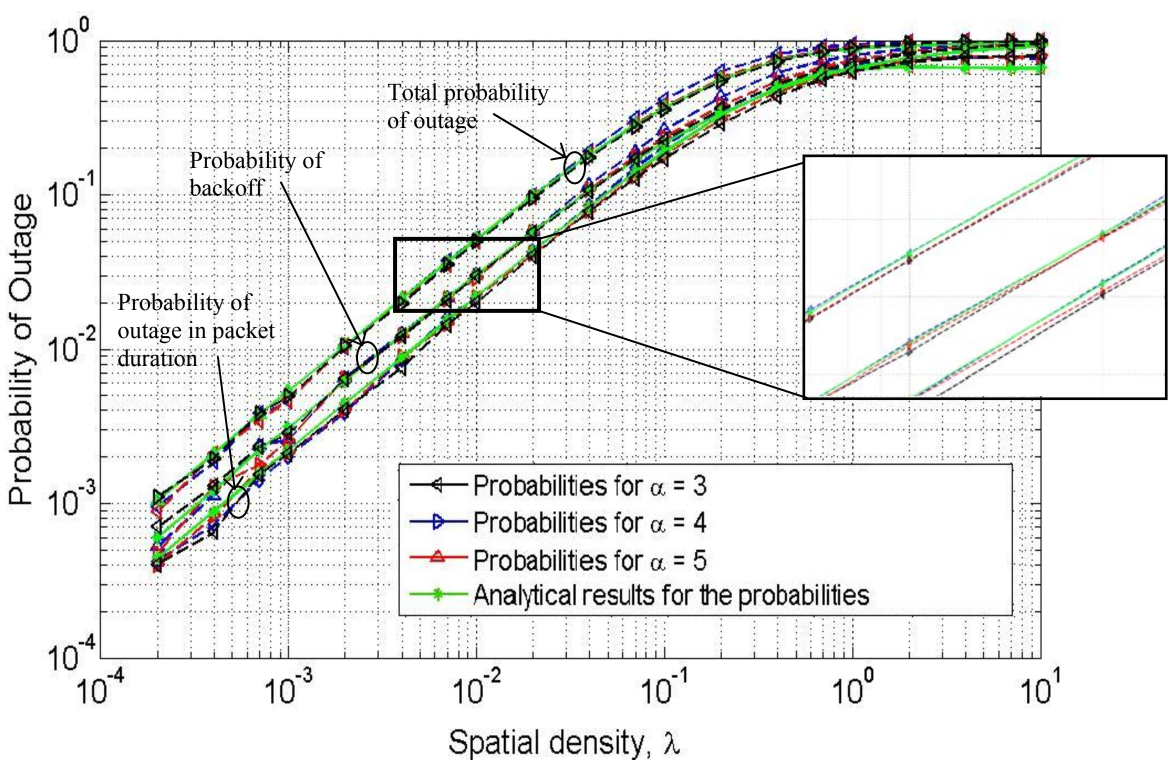


Figure 20 – Simulations of the total probability of outage, probability of backoff, and the probability that outage occurs during a packet duration, for $\beta = 1$ and path loss exponents α equal to 3, 4 and 5. The figure shows that the value of α has approximately no effect on the outage probabilities.

In Figure 21 we look at how much of the total probability of outage is due to the probability of backoff, i.e. the probability that outage occurs in the beginning of the packet, and how much is due to outage occurring during the packet duration. The lower densities are of greatest importance as they contain the least approximations. We see that approximately 42% of the total outage probability is due to backoff probability, and the remaining 58% of the outages occur in the middle of the packet transmissions. The probability of backoff clearly increases as the density of nodes increases. Note that at higher densities these two probabilities no longer add up to 1, because the total probability of outage can no longer be approximated as the summation of the probability of backoff and the probability of outage during the packet transmission; the exact equation for the total probability of outage is given in (21).

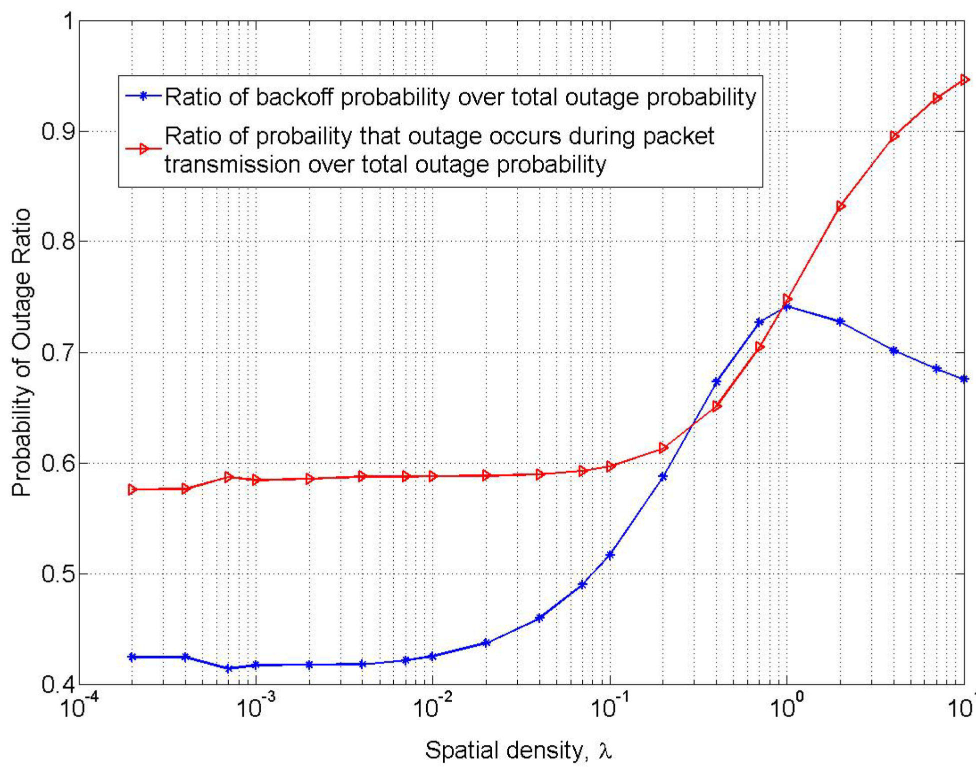


Figure 21 – The ratio of the backoff probability over the total outage probability of CSMA, and the ratio of the probability that a packet goes into outage during the packet duration over the total outage probability.

Chapter 6
Comparing Performance of ALOHA and CSMA

This chapter presents the simulations performed for the sake of comparison of the ALOHA and CSMA protocols. As seen in the previous sections the respective simulations follow the analytical results tightly. Hence, in this chapter we will only refer to the simulation results that are relevant for comparisons between the two protocols.

The primary difference between the unslotted ALOHA and the CSMA protocols is primarily due to the fact that in CSMA transmissions are only initiated when the measured SINR exceeds the SINR threshold β, while in ALOHA packets are transmitted as soon as they are received at the transmitter, regardless of the channel condition. Based on this difference intuition tells us that CSMA should perform better than ALOHA in terms of probability of outage. In order to see how much the improvement is, we refer to our simulation results. Figure 22 shows the simulation results of the total probability of outage for both ALOHA and CSMA (taken from Figures 11 and 19, respectively).

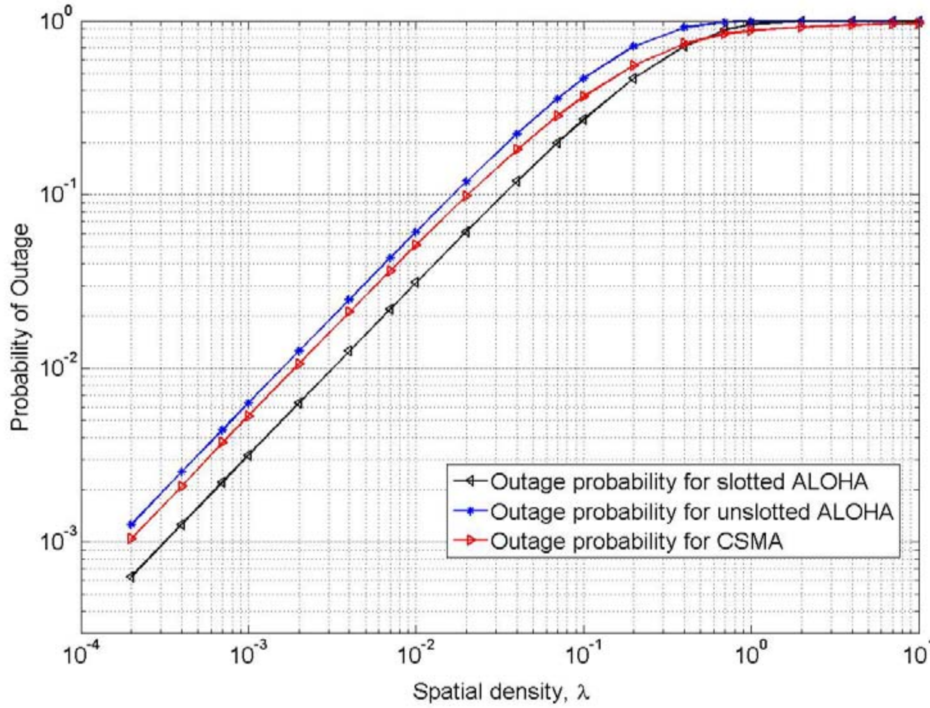


Figure 22 – Simulated probability of outage of slotted and unslotted ALOHA and CSMA, as a function of the spatial density.

As expected CSMA outperforms unslotted ALOHA, but it still exhibits more outage than slotted ALOHA. However, slotted ALOHA is not practical in many ad hoc networks, due to its inherent delay in transmissions and its need for synchronization. For a fixed probability of outage, the spatial density of nodes may be increased by 17% by using CSMA over unslotted ALOHA. If the system has synchronization abilities and transmission delays are insignificant, then for a fixed probability of outage the spatial density may be increased by another 70% by using unslotted ALOHA over CSMA. This means that for a fixed probability of outage unslotted ALOHA can have double the spatial density as opposed to the unslotted ALOHA. With a fixed spatial density, we see that ALOHA exhibits 20% more probability of outage compared to CSMA and 100% more compared to unslotted ALOHA.

The ratio of the outage probability of ALOHA over that of CSMA is plotted in Figure 23, showing that CSMA outperforms ALOHA by about 20% for the lower densities, where the probabilities of outage are most linear, and with a maximum of about 50% (based on

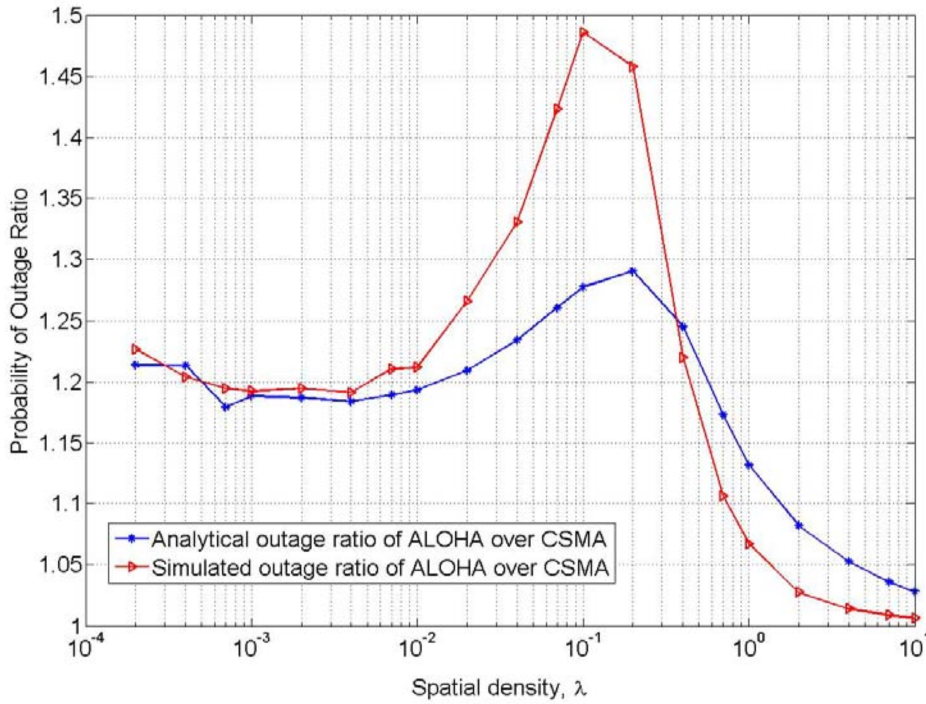


Figure 23 – Ratio of total outage probability of ALOHA over that of CSMA, using both analytical and simulation results.

simulation results). Moreover, the graph indicates that there exists an optimal density for which the use of CSMA is significantly beneficial over that of ALOHA. From the figure, we see that there is great deviation between the analytical results and the simulation results, which is primarily due to the fact that the analytical result for the probability of backoff for CSMA does not follow the simulation results tightly for higher densities, as was seen in Figure 13. This is because the assumption that the number of nodes on the plane follows a Poisson point process is only an approximation because of the backoff property of CSMA, as was discussed in section 5.2.

Minimizing the probability of outage is one of the main goals in the design of ad hoc networks. In our comparisons of the performance of unslotted ALOHA with that of CSMA, we see that due to carrier-sensing and the backoff strategies of CSMA, lower probability of outage may be obtained for a constant spatial density. Having said this, note however, that there are tradeoffs for this optimization of the probability of outage. For instance, when the transmitter backs off, packets are dropped and hence information is lost. If retransmissions were to be applied, so that packets that are not sent enter a queue and are retransmitted, then the data rate would be traded off. Also, delay is introduced and complexity of the transmitter and receiver is also increased. In other words, the improvement in the probability of outage has some tradeoffs for CSMA compared to unslotted ALOHA, such as decrease in data rate, increase in transmission delay, and increase in complexity. Hence, as an overall rule it can be said that designs should enable systems to adapt. Because performance depends critically on carrier sense threshold and its relationship to desired signal statistics, no single "factory-set" threshold will work well in all settings [16]. A robust MAC must include provisions for measurement and adaptation to the measured environment.

Chapter 7
Conclusions

In this project, the performances of the ALOHA and CSMA MAC protocols have been evaluated in terms of probability of outage. This is done in the context of a new modeling framework that allows for simultaneous communication between several transmitter-receiver links in a continuous-time system. Users/packets arrive randomly in space and time according to a Poisson point process, and are transmitted to their destinations according to the distributed MAC protocol at use. This model is SINR-based, and a packet transmission is encountered as successful if the received SINR is above a predetermined threshold value for the entire duration of the packet.

In this report, the methods used to reach the obtained analytical results are presented in detail. Accurate bounds to the probability of outage are derived, and then used to determine the performance advantage that CSMA provides over ALOHA and also to gain insight into the design of general MAC protocols for ad hoc networks. The analytical and simulation results show that CSMA outperforms unslotted ALOHA, but still exhibits more outage than slotted ALOHA. However, slotted ALOHA is not practical in many ad hoc networks, due to its inherent delay in transmissions and its need for synchronization. Note that the superiority in the performance of CSMA over ALOHA, and the advantages of using a slotted system over an unslotted system, are also naturally followed by tradeoffs in other domains such as data rate and delay.

More work may still be done to fully understand the behavior of MAC protocols in practical stochastic environments, some of which are mentioned in section 7.2. In the following section, the main findings and results obtained in this project are restated.

7.1 Main Findings and Results

Simulations have been performed on the ALOHA and CSMA protocols separately to fully understand their inherent properties and in order to confirm the obtained analytical results. Also, the two MAC protocols were compared and in terms of probability of outage and density of nodes.

Firstly, the performances of the MAC protocols were analyzed analytically, and simulations were used to either confirm or reject the obtained results. We found analytical lower bounds on the outage probability of slotted and unslotted ALOHA, given by (14) and (19), the probability that an incoming node using the CSMA protocol backs off (22), the probability that a packet goes into outage during its packet duration (32), and the total outage probability of CSMA by using equation (21). All the final analytical results were shown to follow the simulation results tightly. As expected, our results showed that slotted ALOHA performs better than unslotted ALOHA by a factor of 2 in terms of probability of outage. Moreover, we found that for lower densities approximately 42% of the total outage probability for CSMA is due to backoff probability, and the remaining 58% of the outages occur in the middle of the packet transmissions.

Next, the comparisons performed on the performances of ALOHA and CSMA showed that for a fixed probability of outage, the spatial density of nodes may be increased by 17% by using CSMA, and by 100% more by using slotted ALOHA, compared to unslotted

ALOHA. Furthermore, we showed that CSMA outperforms ALOHA in terms of probability of outage by about 20% for the lower node densities, where the probabilities of outage are approximately linear as a function of the density. A maximum advantage of 50% is obtained for higher density of nodes. We also concluded that the CSMA protocol is most advantageous when the distance between the nodes and packet arrivals is small, i.e. for a higher density of nodes.

7.2 Future Work

Due to the popularity of distributed MAC protocols in today's wireless networks, there is a great potential for future research and improvement within this area. We have applied the proposed approach of [4] and [7] to analyze the performances of ALOHA and CSMA, and as an immediate step, we wish to further utilize the results of [7] to study the optimal sensing zone for the flavor of CSMA considered here.

Other possible extensions related to this research topic involve incorporating modifications to our network model and investigating new techniques in order to improve the performances of the MAC protocols. In the following, some potential research topics for further investigation are suggested.

i) Incorporate retransmissions in the model. This is expected to result in delays in the system, and a reduction in the data transmission rate.

ii) Add fading to the channel model, with Rayleigh fading as a worst-case scenario, and investigate the effects.

iii) Add adaptive power or rate control algorithms to the system to improve the performances of both MAC protocols.

iv) Consider applications that have access to enough bandwidth, divide the available bandwidth in smaller bands, and use the increase in the number of frequency bands to reduce the density of transmissions. For the bandwidth allocation, frequency hopping (FH) or direct sequence (DS) CDMA may be used, as has been suggested in [4].

v) Let the packet length T, or the transmission rate R, adapt to the quality of the channel, and investigate the results. That is, if the channel is busy and may result in too much interference, the transmission rate can be reduced in order to help improve the condition of the channel.

vi) In a power-constrained setting, adapt the transmission power P to the quality of the channel, i.e. transmit with lower power when the channel condition is good, and if too much interference is present, use more transmit power.

vii) Extend the model to include mobility of the nodes, and study and quantify the impact of this on the performance of the ad hoc wireless networks.

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Appendix

A.1 MATLAB Code for CDF of ALOHA

The following sections include the MATLAB codes for simulating the ALOHA and CSMA MAC protocols, resulting in the graphs presented in Chapter 5.

```
%--------------------System parameters-----------------------
lambda = 0.01;          % Number of nodes per m^2
rate = 1;               % Arrival rate
SINRo = 3;              % 4.77 dB
packet_length = 1;      % Packet length

%------------------Simulation variables--------------------
alpha = 3;              % Path loss exponent (alpha>2)
freq = 10^5;            % Frequency of operation
P_TX = 1;               % Transmitted power
L = 100;                % Defines area of nodes: LxL m2
R = 1;                  % Distance between RX and TX
noise = P_TX/100;       % Chosen to give SNR ~ 20-30 dB
```

```
N = poissrnd( lambda * L^2 );      % Number of TX/RX pairs

%------------Placing TX and RX and measuring SINR------------
max_inst = 100;
SINR = zeros(max_inst, N);
avg_SINR = zeros(1,max_inst);
corner_TX = zeros(1,max_inst);

for inst = 1:max_inst
   TX_X = L*rand(1,N);
   TX_Y = L*rand(1,N);
   RX_X = zeros(1,N);
   RX_Y = zeros(1,N);
   for n = 1:N
      theta = 2*pi*rand;
      RX_X(n) = TX_X(n) + R*cos( theta );
      RX_Y(n) = TX_Y(n) + R*sin( theta );
   end

   for RX = 1:N
      interference = 0;
      for TX = 1:N
         if( TX ~= RX )
            r = sqrt( (RX_X(RX)-TX_X(TX))^2 + (RX_Y(RX)-
                                 TX_Y(TX))^2 );
            interference = interference+P_TX*r^(-alpha);
         end
```

```
        end
        SINR(inst,RX) = P_TX*R^(-alpha)/(noise+interference);
    end

    avg_SINR(inst) = sum(SINR(inst,:)) / N;
    corner_TX(inst) = Find_TX( TX_X, TX_Y );
end

%--------------------------Results---------------------------
n = 1;
M = max(max(SINR));
cdf = zeros(1,M);
cdf_corner = zeros(1,M);
cdf_avg = zeros(1,M);

for sinr = 0:0.1:(M*1.5)
    cdf(n) = length( find(SINR <= sinr) ) / (N*max_inst);
    cdf_corner(n) = length( find(SINR(:,corner_TX) <= sinr) )
                                    / (length(corner_TX)*max_inst);
    cdf_avg(n) = length( find(avg_SINR<=sinr) ) / (max_inst);
    n = n + 1;
end

subplot(3,1,1); plot( 10*log10(0:0.1:(M*1.5)), cdf );
grid on; hold on;
xlabel('SINR (dB)'); ylabel('CDF of SINR');
title(['CDFs for \lambda=',num2str(lambda),', and  \alpha=',
```

```
            num2str(alpha)]);

subplot(3,1,2); plot( 10*log10(0:0.1:(M*1.5)), cdf_corner );
grid on; hold on;
xlabel('SINR (dB)'); ylabel('CDF of SINR for corner TX');

subplot(3,1,3); plot( 10*log10(0:0.1:(M*1.5)), cdf_avg );
grid on; hold on;
xlabel('SINR (dB)');
ylabel('CDF of SINR averaged over nodes');
```

A.2 MATLAB Code for ALOHA and CSMA

A.2.1 Analytical Simulations for the RX-RX and TX-TX Approaches

```
P_TX = 1;
noise = P_TX/100;
SINRo = 1;
alpha = 3;
lambda_t = 0.1;
T = 100;
R = 1;
s = ( (R^-alpha)/SINRo - noise/P_TX )^(-1/alpha);
lambdas_vec = [2*10^-5 4*10^-5 7*10^-5 10^-4 2*10^-4 4*10^-4 7*10^-4 10^-
        3 2*10^-3 4*10^-3 7*10^-3 10^-2 2*10^-2 4*10^-2 7*10^-2 10^-1 2*10^-1
        4*10^-1 7*10^-1];
```

```
K = length( lambdas_vec );
analytical_csma = zeros(1,K);

for l = 1:K
   ls = lambdas_vec(l)*lambda_t*T;
   fh = @(x) quadl(@(phi) ( 1/(2*pi)*(1 - 1/pi*acos( (x+2*R^2-s^2-
        2*R*sqrt(x).*cos(phi))./(2*R*sqrt(x+R^2-2*R*sqrt(x).*cos(phi)))) ) )
        .*(pi*ls*exp(-pi*ls*x)), acos( (x-s^2+2*R*s)./(2*R*sqrt(x)) ), (2*pi - acos(
        (x-s^2+2*R*s)./(2*R*sqrt(x)) )));

   x = 0.001:s^2/1000:s^2;
   func = arrayfun(fh, x);
   analytical_csma(l) = abs(sum(func(2:length(x))*s^2/1000));
end

figure(2); loglog(lambdas_vec*lambda_t*T, analytical_csma, 'r->'); grid on;
title('Analytical Outage Probability for CSMA'); axis equal
```

A.2.2 Analytical and Numerical Simulations for RX-TX Approach

```
%********************Initializing system parameters********************
P_TX = 1;                       % Transmitted power
R = 1;                          % Distance between RX and TX pair
noise = P_TX/100;               % Chosen to give SNR ~ 20-30 dB
SINRo = 1;                      % 0 dB
alpha = 3;                      % Path loss exponent (alpha>2)
L = 35;                         % Length of area LxL
```

```
T = 100;                          % Packet length in seconds
lambda_t = 0.1;
s = ( (R^-alpha)/SINRo - noise/P_TX )^(-1/alpha);
packets = 50000;
max_inst = 1;

lambdas_vec = [2*10^-5 4*10^-5 7*10^-5 10^-4 2*10^-4 4*10^-4 7*10^-4 10^-
        3 2*10^-3 4*10^-3 7*10^-3 10^-2 2*10^-2 4*10^-2 7*10^-2 10^-1 2*10^-1
        4*10^-1 7*10^-1 1];
K = length( lambdas_vec );
analytical_aloha = zeros(1,K);
analytical_csma_nobackoff = zeros(1,K);
analytical_csma_nobackoff2 = zeros(1,K);
analytical_csma_total = zeros(1,K);
outage_slotted = zeros(1,K);
outage_aloha = zeros(1,K);
outage_csma = zeros(1,K);
outage_csma_nobackoff = zeros(1,K);
prob_backoff_sim = zeros(1,K);
prob_backoff = zeros(1,K);
outage_percentage = zeros(1,K);
backoff_percentage = zeros(1,K);

for l = 1:K
   outage_aloha_inst = zeros(1,max_inst);
   outage_csma_inst = zeros(1,max_inst);
   outage_csma_nobackoff_inst = zeros(1,max_inst);
```

```
prob_backoff_sim_inst = zeros(1,max_inst);

for inst = 1:max_inst
    TX_X = zeros(1,packets);
    TX_Y = zeros(1,packets);
    RX_X = zeros(1,packets);
    RX_Y = zeros(1,packets);
    arrival_time = zeros(1,packets);
    transmit = ones(1,packets);
    interf_aloha = zeros(1,packets);
    interf_csma = zeros(1,packets);
    SINR_aloha = zeros(1,packets);
    SINR_csma = zeros(1,packets);
    outage_aloha_occured = zeros(1,packets);
    outage_csma_occured = zeros(1,packets);
    outage_csma_occured_nobackoff = zeros(1,packets);
    arrival_time(1) = 5*rand + exprnd( 1/(lambdas_vec(l)*L^2*lambda_t) );

    for current = 1:packets
        %******************Positioning TXs and RXs uniformly******************
        TX_X(current) = L*rand;
        TX_Y(current) = L*rand;
        theta = 2*pi*rand;
        RX_X(current) = TX_X(current) + R*cos( theta );
        RX_Y(current) = TX_Y(current) + R*sin( theta );

        if( current > 1 )
```

```
   [x,departs] = find( arrival_time+T > arrival_time(current-1) &
                              arrival_time+T < arrival_time(current) );
else
   departs = [];
end

%***********Find interference for the current packet arrival************
for pck = 1:(current-1)
   if( arrival_time(pck)+T > arrival_time(current) )
      r = sqrt( (RX_X(current)-TX_X(pck))^2 + (RX_Y(current)-
                                                  TX_Y(pck))^2
                                           );
      interf_aloha(current) = interf_aloha(current) + P_TX * r^(-alpha);
      if( transmit(pck) == 1 )
         interf_csma(current) = interf_csma(current) + P_TX * r^(-alpha);
      end
   end
end
SINR_aloha(current) = P_TX * R^(-alpha)/(noise+interf_aloha(current));
SINR_csma(current) = P_TX * R^(-alpha)/(noise+interf_csma(current));
if( SINR_aloha(current) < SINRo )
   outage_aloha_occured(current) = 1;
end
if( SINR_csma(current) < SINRo )
   transmit(current) = 0;
   outage_csma_occured(current) = 1;
end
```

```
for pck = 1:(current-1)
   %*****Interference correction because of packet departures*******
   for i = 1:length(departs)
      if( arrival_time(departs(i))+T > arrival_time(pck) &&
               arrival_time(departs(i))+T  < arrival_time(pck)+T &&
               departs(i)<pck )
         r = sqrt( (RX_X(pck)-TX_X(departs(i)))^2 + (RX_Y(pck)-
                                                   TX_Y(departs(i)))^2 );
         interf_aloha(pck) = interf_aloha(pck) - P_TX * r^(-alpha);
         if( transmit(departs(i)) == 1 )
            interf_csma(pck) = interf_csma(pck) - P_TX * r^(-alpha);
         end
      end
   end
   %************Interference because of new packet arrival************
   if( arrival_time(pck)+T > arrival_time(current) )
      r = sqrt( (RX_X(pck)-TX_X(current))^2 + (RX_Y(pck)-
                                                      TX_Y(current))^2 );
      interf_aloha(pck) = interf_aloha(pck) + P_TX * r^(-alpha);
      if( transmit(current) == 1 )
         interf_csma(pck) = interf_csma(pck) + P_TX * r^(-alpha);
      end
   end
   SINR_aloha(pck) = P_TX * R^(-alpha) / ( noise + interf_aloha(pck) );
   if( SINR_aloha(pck) < SINRo )
      outage_aloha_occured(pck) = 1;
```

```
            end
            SINR_csma(pck) = P_TX * R^(-alpha) / ( noise + interf_csma(pck) );
            if( SINR_csma(pck) < SINRo )
               outage_csma_occured(pck) = 1;
               if( transmit(pck)==1 )
                  outage_csma_occured_nobackoff(pck) = 1;
               end
            end
         end

         arrival_time(current+1) = arrival_time(current) +
                                         exprnd( 1/(lambdas_vec(l)*L^2*lambda_t) );
      end

      outage_aloha_inst(inst) = sum(outage_aloha_occured)/packets;
      outage_csma_inst(inst) = sum(outage_csma_occured)/packets;
      outage_csma_nobackoff_inst(inst) =
        sum(outage_csma_occured_nobackoff) / length(  find(transmit==1) );
      prob_backoff_sim_inst(inst) = length( find( transmit == 0 ) ) / packets;
   end

   %*************Calculating outage, averaged over instances***************
   ls = lambdas_vec(l)*lambda_t*T;
   outage_slotted(l) = 1 - exp(-ls*pi*s^2);
   analytical_aloha(l) = 1 - exp(-2*ls*pi*s^2);
   outage_aloha(l) = sum(outage_aloha_inst)/max_inst;
   outage_csma(l) = sum(outage_csma_inst)/max_inst;
```

```
outage_csma_nobackoff(l) =
      sum(outage_csma_nobackoff_inst)/max_inst;
prob_backoff_sim(l) = sum(prob_backoff_sim_inst)/max_inst;

fh = @(x) quadl(@(phi) ( 1/(2*pi)*(1 - 1/pi*acos( (x+2*R^2-s^2-
      2*R*sqrt(x).*cos(phi))./(2*R*sqrt(x+R^2-2*R*sqrt(x).*cos(phi)))) )
      ).*(pi*ls*exp(-pi*ls*x)), acos( (x-s^2+2*R*s)./(2*R*sqrt(x)) ), (2*pi - acos(
      (x-s^2+2*R*s)./(2*R*sqrt(x)) )));
x = 0.001:s^2/1000:s^2;
func = arrayfun(fh, x);
analytical_csma_nobackoff(l) = real(sum(func*s^2/1000));

fh2 = @(x) (1 - (quadl(@(phi) ( 1/(2*pi)*(1 - 1/pi*acos( (x+2*R^2-s^2-
      2*R*sqrt(x).*cos(phi))./(2*R*sqrt(x+R^2-2*R*sqrt(x).*cos(phi)))) ) ).*(pi*ls),
      acos( (x-s^2+2*R*s)./(2*R*sqrt(x)) ), (2*pi - acos( (x-
      s^2+2*R*s)./(2*R*sqrt(x)) ))) ));
func2 = arrayfun(fh2, x);
analytical_csma_nobackoff2(l) = 1 - real(sum(func2*s^2/1000));

while( abs(prob_backoff(l) - (1 - exp(-pi*ls*(1-prob_backoff(l))*s^2))) >
                                        0.0001 && prob_backoff(l) < 0.9999 )
   prob_backoff(l) = prob_backoff(l) + 0.0001;
end
analytical_csma_total(l) = analytical_csma_nobackoff(l)*(1-
                                        prob_backoff(l)) + prob_backoff(l);
outage_percentage(l) = analytical_csma_nobackoff(l) /
                                        analytical_csma_total(l);
```

```
    backoff_percentage(l) = prob_backoff(l) / analytical_csma_total(l);
end

figure(1);
loglog( lambdas_vec*lambda_t*T, outage_aloha, 'b-*' ); grid on; hold on;
loglog( lambdas_vec*lambda_t*T, analytical_aloha, 'r-<' );
loglog( lambdas_vec*lambda_t*T, outage_slotted, 'black->' );
xlabel( 'Spatial density, \lambda' ); ylabel('Probability of Outage');
legend( 'Simulated ALOHA','Analytical ALOHA','Analytical slotted ALOHA');
title(['Outage probability for ALOHA with SINRo=',num2str(SINRo),',
        \alpha=',num2str(alpha),', T=',num2str(T),', L=',num2str(L),', and
        \lambda_t=',num2str(lambda_t)]);

figure(2);
loglog( lambdas_vec*lambda_t*T, analytical_csma_total, 'g-^' );
grid on; hold on;
loglog( lambdas_vec*lambda_t*T, outage_csma, 'black--*' );
loglog( lambdas_vec*lambda_t*T, prob_backoff, 'b->' );
loglog( lambdas_vec*lambda_t*T, prob_backoff_sim, 'black--*' );
loglog( lambdas_vec*lambda_t*T, analytical_csma_nobackoff, 'r-<' );
loglog( lambdas_vec*lambda_t*T, outage_csma_nobackoff, 'black--*' );
xlabel( 'Spatial density, \lambda' ); ylabel('Probability of Outage');
legend( 'Analytical CSMA total','Simulated CSMA total','Analytical backoff
probability for CSMA','Simulated backoff probability for CSMA','Analytical
CSMA given no backoff','Simulated CSMA given no backoff');
title(['Outage probability for CSMA with SINRo=',num2str(SINRo),',
```

```
\alpha=',num2str(alpha),', T=',num2str(T),', L=',num2str(L),', and \
lambda_t=',num2str(lambda_t)]);

figure(3);
x = 0.001:s^2/1000:s^2;
f_method4 = arrayfun( @(x) quadl(@(phi) ( 1/(2*pi)*(1 - 1/pi*acos( (x+2*R^2-
s^2-2*R*sqrt(x).*cos(phi))./(2*R*sqrt(x+R^2-2*R*sqrt(x).*cos(phi)))) ) ), acos(
(x-s^2+2*R*s)./(2*R*sqrt(x)) ), (2*pi - acos( (x-s^2+2*R*s)./(2*R*sqrt(x)) ))), x);
f_method4_givenphi = 1 - 1/pi*acos( (x+2*R^2-s^2-
2*R*sqrt(x).*cos(pi))./(2*R*sqrt(x+R^2-2*R*sqrt(x).*cos(pi))));
plot(x,f_method4,'b-'); grid on; hold on;
plot(x,f_method4_givenphi,'r-'); legend('Outage probability for CSMA versus
d', 'Outage probability for CSMA versus and \phi=\pi');
xlabel( 'Distance d^2 between RX_1 and TX_2' ); ylabel('Probability of
Outage'); title('Outage Probability for CSMA');
analytical_zerolambda = real(sum(f_method4*s^2/1000));

figure(4);
semilogx( lambdas_vec*lambda_t*T, analytical_aloha ./
analytical_csma_total , 'b-*' ); grid  on; hold on;
semilogx( lambdas_vec*lambda_t*T, outage_aloha ./ outage_csma, 'r->' );
legend( 'Analytical outage ratio of ALOHA over CSMA ', 'Simulated outage
ratio of ALOHA over CSMA' );
xlabel( 'Spatial density, \lambda' ); ylabel('Probability of Outage Ratio');
title('Analytical Probability of Outage of ALOHA over CSMA');

figure(5);
```

```
loglog( lambdas_vec*lambda_t*T, prob_backoff, 'b-*' ); grid on; hold on;
loglog( lambdas_vec*lambda_t*T, prob_backoff_sim, 'r->' );
legend('Analytical backoff probability for CSMA','Simulated backoff
probability for CSMA');
xlabel( 'Spatial density, \lambda' ); ylabel('Probability of Backoff');
title('Probability of Backoff for CSMA');

figure(6);
loglog( lambdas_vec*lambda_t*T, outage_slotted, 'black-<' );
grid on; hold on;
loglog( lambdas_vec*lambda_t*T, outage_aloha, 'b-*' );
loglog( lambdas_vec*lambda_t*T, outage_csma, 'r->' );
legend('Outage probability for slotted ALOHA','Outage probability for
unslotted ALOHA','Outage probability for CSMA');
xlabel( 'Spatial density, \lambda' ); ylabel('Probability of Outage');
title('Probability of Outage for ALOHA and CSMA');

figure(7);
semilogx( lambdas_vec*lambda_t*T, outage_percentage, 'b-*' );
grid on; hold on;
semilogx( lambdas_vec*lambda_t*T, backoff_percentage, 'r->' );
legend('Ratio of backoff probability over total outage probability','Ratio of
probability that outage occurs duringpacket transmission) over total
P_outage');
xlabel( 'Spatial density, \lambda' ); ylabel('Probability of Outage Ratio');
title('Relation between P_backoff and P_outage (during packet
transmission)');
```