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Analysis of camshaft gear transmission for a ship engine

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THE NORWEGIAN UNIVERSITY
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DEPARTMENT OF ENGINEERING DESIGN
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MASTER THESIS SPRING 2015
FOR
STUD.TECHN. KATRINE CLOSS

Analysis of camshaft gear transmission for a ship engine
Analyse av tannhjulsoverføringer for kamaksling på en skipsmotor.

An increase in power for an existing ship engine will cause higher loading on the camshaft. The project shall investigate the abilities of a current camshaft design to cope with the increased loading.
Calculations have shown that the gear transmission loading of the camshaft has significant dynamic contributions. A review of present static calculations is therefore desired with respect to the following effects

- Quality of present FE model and static computations
- Dynamics and backlash considerations
- Nonlinear contact effects
- Angular deflections of the transmission
- Safety factors for failure modes such as fatigue
- Vibrations and eigenfrequencies
- Tolerances
- Wear

An important factor for the dynamics of the camshaft system is the gear transmission between the crankshaft and camshaft. The complex mechanics of the gear transmission is the primary focus for this study. One seeks to investigate the transfer rate due to purely geometric (rigid) theory as well as the transfer rate when elasticity of the gear transmission itself is included. The model can also be used for a parameter study with respect to tolerances of the gear assembly.
Investigating and selecting suitable analysis software and methods will be an integrated part of the project.

Formal requirements:

Three weeks after start of the thesis work, an A3 sheet illustrating the work is to be handed in. A template for this presentation is available on the IPM's web site under the menu "Masteroppgave" (http://www.ntnu.no/ipm/masteroppgave). This sheet should be updated one week before the master's thesis is submitted.

Risk assessment of experimental activities shall always be performed. Experimental work defined in the problem description shall be planed and risk assessed up-front and within 3 weeks after receiving the problem text. Any specific experimental activities which are not properly covered by the general risk assessment shall be particularly assessed before performing the experimental work. Risk assessments should be signed by the supervisor and copies shall be included in the appendix of the thesis.

The thesis should include the signed problem text, and be written as a research report with summary both in English and Norwegian, conclusion, literature references, table of contents, etc. During preparation of the text, the candidate should make efforts to create a well arranged and well written report. To ease the evaluation of the thesis, it is important to cross-reference text, tables and figures. For evaluation of the work a thorough discussion of results is appreciated.

The thesis shall be submitted electronically via DAIM, NTNU's system for Digital Archiving and Submission of Master's theses.

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Institutt for produktutvikling og materialer

Preface

This thesis concludes my Master of Science education at the Norwegian University of Science and Technology in Trondheim. The thesis was written at the department of Engineering Design and Materials in cooperation with Bergen Engines Rolls-Royce AS on the subject of gear transmissions.

The work done for this thesis was primarily executed in the spring of 2015. The thesis was interrupted for some time and finished in early fall 2015 with a slightly change of focus to more parametric studies.

A study of the new B33:45 engine series from Bergen Engines AS (Rolls-Royce) is done with attention to the nonlinear dynamic effects of backlash in a spur gear pair.

I would like to express my deep gratitude to Professor Bjørn Haugen, for his excellent technical guidance and supervising. His willingness to give his time so generously has been very much appreciated. Without him this would not have been possible. A thanks to Bergen Engines for a very challenging and interesting topic.

Katrine Closs

Trondheim, October 1, 2015

Abstract

This thesis investigates the dynamic behavior of a spur gear pair. The gears studied are somewhat similar to the ones found in Bergen Engines medium-speed diesel engine with regards to design, but some assumptions have been made when it comes to stiffness and damping. Gear components are an important part in advanced machines and have been a subject of studies due to increasing demands for higher performance and reliability and noise control.

At first a dynamic model of a spur gear including time varying mesh stiffness and backlash is established in Fedem to be analyzed. The complicated phenomenon of tooth contact alternating between variable number of teeth pairs in contact and the presents of backlash, cause additional dynamic forces and is the main source of gear vibration. Backlash is unavoidable but also necessary in gear transmission preventing the teeth from jamming and allowing room for lubrication. The effect of backlash cause tooth separation and lead to contact loss during meshing.

Parametric studies were conducted to examine the effects of the following variables; backlash, torque and rotational speed. For a system with no backlash, the gear teeth remained in contact throughout the gear mesh and did not experience contact loss or tooth separation. For small values of backlash, the analysis showed zero values for the moment which is a result of tooth separation. The time of contact loss increased with increasing value of backlash.

For a systems with a given constant backlash, the variation in torque appeared as different values of time of contact loss. For decreasing values of torque the time of the tooth separation increased.

The analysis clearly shows that the model experience backside contact. Backside contact is a result of backlash and fluctuations caused by the time varying mesh stiffness due to the contact ratio alternating between number of teeth in contact. When these fluctuations coincide with the natural vibration, resonance occur and results in backside contact.

The results were in good agreement with the expectations and analytically showed the effects off backlash. The gears experience significant dynamic contributions as a cause of backlash and time variant mesh stiffness.

Sammendrag

Denne oppgaven undersøker de dynamiske effektene i et tannhjulspar. Tannhjulene som er analysert er basert på designet til tannhjulene som finnes i Bergen Engines sin motor, men avviker noe med tanke på stivhet og demping. Tannhjul som komponent er en viktig del i avanserte maskiner og har vært et emne som har blitt grundig analysert på grunn av økende krav om bedre utnyttelse og høyere driftssikkerhet og bedre støykontroll.

I første steg er en dynamisk modell av et tannhjulspar inkludert dødgang (backlash) og tidsvarierende inngrepsstivhet (time-varying mesh stiffness) modellert i Fedem for å ble analysert. Det kompliserte fenomenet som er når tannhjulene varierer mellom flere tenner i kontakt og tilstedeværelsen av dødgang, gir opphav til ytterligere dynamiske krefter som er hovedårsaken til vibrasjon i tannhjul. Dødgang mellom tannhjul er uunngåelig, men er også nødvendig i tannhjulsoverføringer for å hindre tennene fra å kjøre seg fast og for å tillater plass for smøring. Dødgang kan føre til separasjon mellom tannhjulstennene som resulterer i at tannhjulene mister kontakt under inngrep.

Parameter studier ble utført for å undersøke effekten av følgende variable; dødgang og moment. I et system uten dødgang, forblir tennene I kontakt under inngrepssyklusen og opplever ikke tap av kontakt mellom tannhjulene. For små verdier av dødgang, viser analysen nullverdier i momentet som er et resultat av tennene separeres. Lengden av tiden på disse nullverdiene øker med økende verdier av dødgang.

For et system med en gitt verdi på dødgangen, ble variasjonene av påført moment synlig i modellen som varierende tid hvor tennene ikke var i kontakt. Minkende verdier av påført moment, resulterer i lengere tid av seperasjon mellom tennene.

Analysen viser tydelig at modellen opplevder kontakt på baksiden av tennene. Dette er et resultat av dødgang og tidsvarierende inngrepsstivhet som kommer av varierende antall tenner i kontakt. Når disse svingningene sammenfaller med egenfrekvensen, oppstår det resonans og resulterer i kontakt på baksiden av tennene.

Resultatene stemmer godt med forventingene og viser analytisk effektene av dødgang i tannhjulsoverføringer. Tannhjulene opplever betydelig økning i dynamiske bidrag som et resultat av dødgang og tidsvarierende inngrepsstivhet.

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1 Introduction

Gears are the main component of mechanical transmission systems and are one of the most critical components in rotating machinery. Gearing systems are used to reduce the rotational speed, increase the available torque, change the direction of the power transmission, and distribute the available power between several machines [23]. The dynamics of a gear system is characterised by a periodically changing stiffness due to alternation between multiple teeth in contact over a mesh cycle.

Due to increased demands to high speed machinery and restrictions in the standards the topic has trough previous decades been widely researched. Researchers have found that a linear model is not sufficient enough to describe the real dynamic behaviours.

Vibration caused by transmission error is the primary concern in gear system due to increasing noise levels and dynamic forces which may lead to failure. The majority of the studies have researched the issues relating dynamic behaviour, but most of them have simplified the problem by neglecting some effects, specially the effects of backlash due to the serious difficulties in the analysis because the the strong nonlinear interaction in the dynamic equations [11].

However, some have included backlash in the analysis, Bonori and Pellicani who researched the nonlinear dynamics of spur gears with manufacturing errors using a one-degree-of-freedom system that included time varying stiffness, backlash and profile errors. They concluded that the presence of manufacturing errors magnifies the amplitude of vibration and leads to chaotic vibrations [2]. Along with Bonori and Pellicano, Wang also considered the effects of backlash.

Gear transmission has been widely researched over the past decades. Harris [5] introduced in 1958 that the behavior of spur gears at low speed could be described by a set of transmission error curves. He developed a diagram called the "Harris map" suggesting that the TE curves could be used to predict the dynamic behavior of a gear pair. Harris predicted that loss of contact is the principal source of non-linearity in the characteristics of gears. Gregory, Harris and Munro confirmed Harris's predictions experimentally [6].

Munro [13] later explained the fundamental mechanism behind profile relief explaining the effects of long and short relief.

Ozguven and Houser [24] reviewed numerous models concerning gear dynamics, describing in detail the main types of gear models that had been researched at that time. In their researched they classified the models into different categories.

Howard and Wang [7] presented a detailed analysis of various methods for modelling the torsional mesh stiffness of a involute gear pair in mesh. They found that the handover region between one and two teeth in mesh varies with the load.

Kahraman and Singh [9] analysed the effect of backlash associated with the time varying mesh stiffness using the harmonic balance method.

Parker et al. [15] compared the theoretical and experimental results of nonlinear behaviour of a spur gear pair including nonlinear mesh stiffness and backlash.

1.1 Scope

The main objective of this thesis is to (1) develop an analytical model for the spur gears system that incorporates the different nonlinear effects and (2) study the nonlinear oscillations of a spur gear pair with special attention to the effect of backlash. A parameter study will be executed in order to investigate to which extent a change in load case will have an impact on the system and what the effects are. A similar gear model as the gear pair model in the B33:45 series engine were used to investigate the influence of time varying mesh stiffness, damping and gear backlash.

1.2 Outline of thesis

In this thesis, Chapter 2 presents the theory used to understand and address the study. Chapter 3 deals with the complete modeling of the gears in Fedem. Chapter

4 presents the results from Fedem and an explanation of the findings. Chapter 5 provides discussion and conclusion of the study.

2 | Theory

2.1 Gears

Gears are used to transmit motion from one shaft to an other. The gear pair are spur gears with straight cut teeth and an involute gear profile. The involute gear profile comes from Leonhard Euler's spiral which is a spiral following a path traced by the end of a piece of string unwrapping from cylinder. The involute gear profile have teeth which are involutes of a circle. The benefits of having an involute profile are constant pressure angle, constant velocity ratio and less sensitive to change of center distance [8].

The terminology of a spur gear is described in Figure 2.1.

An important parameter on the gear teeth is the pitch circle, which is an imaginary circle, defined by the tangential circles that occur between the mated gear pairs. A lot of theoretical calculations are based on this pitch circle. The diameter of the pitch circle is called the pitch diameter, d or D. The module, m, is the ratio of the pitch diameter to the number of teeth. The circular pitch, p, is the distance between two similar points on adjacent teeth. It can be calculated from the number of teeth, N, and the pitch diameter. The base pitch, P_b, is the circular pitch in plane of rotation at the base circle, expressed

$$P_b = \frac{\cos\phi\pi}{P} \tag{2.1}$$

where,

P	=	Diametrial pitch
ϕ	=	Pressure angle

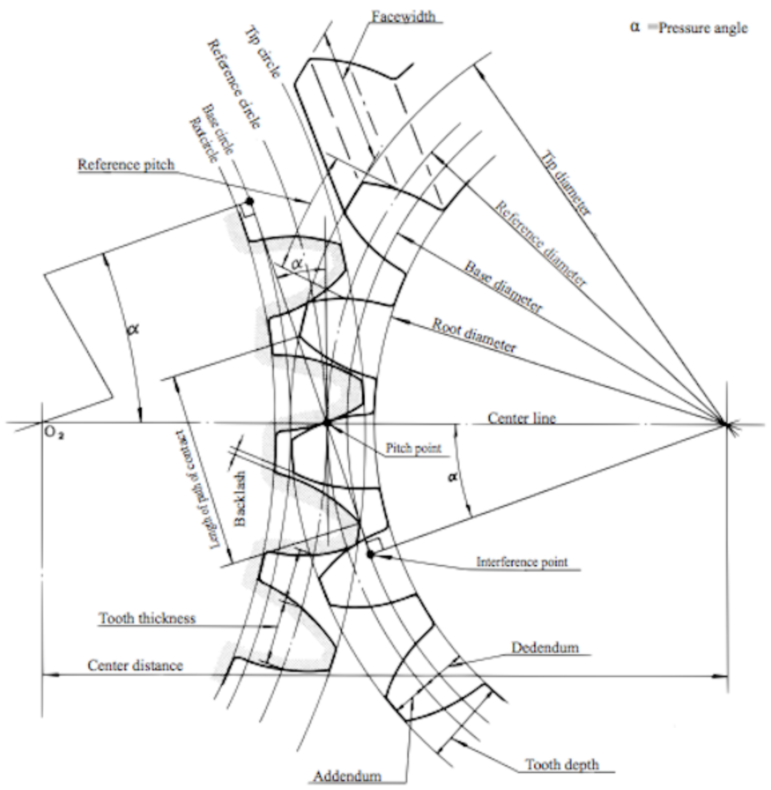


Figure 2.1: Nomenclature of spur gear teeth

The addendum is the radial distance between the pitch circle and the tip of the tooth. The dedendum is the radial distance between the pitch circle and the root of the tooth. It is important to be aware of that in a gear pair the addendum in the *gear* should not be larger than the dedendum of the *pinion*, this would lead to the mating teeth to interfere inappropriately and may cause gear jamming. There must be a clearance, and the clearance is defined as the difference between the dedendum in the gear and the addendum in the pinion.

The length of action, g_α is the length on the line of action which the point of contact moves during the engagement of the teeth.

The diametral pitch, P, is the ratio between number of teeth and the pitch diameter. For a standard gear tooth, the addendum is (in terms of the module m) $1m$ and for the dedendum $1.25m$. The thickness of the tooth, t is measured on the pitch circle.

$$P = \frac{N}{d} \tag{2.2}$$

$$m = \frac{d}{N} \tag{2.3}$$

$$p = \frac{\pi d}{N} = \pi m \tag{2.4}$$

$$t = \frac{P}{2} \tag{2.5}$$

,where

d	=	Pitch diameter
m	=	Module
N	=	Number of teeth
p	=	Circular pitch
P	=	Diametral pitch
t	=	thickness

2.1.1 Gear tooth action

When two gears mesh the surface of the gear teeth meet and motion is transmitted. In order to obtain correct meshing certain parameters for the pinion and gear must be equal, most importantly the module and pressure angle.

It is also necessary to have the right shape of the teeth to get the velocity ratio to remain constant during meshing. When the velocity ratio between a gear pair is constant at all times, it is known as *conjugate gear tooth action*. In order for a gear pair to transmit constant angular velocity ratio, the tooth profiles must be shaped in such a way that the line of action passes through a fixed point, called the pitch point, P, intersecting the line of centers. This is known as the fundamental law of gearing.

Therefore, the pitch line velocity can be defines as

$$V = r_p \omega_p = r_g \omega_g \tag{2.6}$$

Where, r_p and r_g are the pitch radii of the pinion and gear respectively and ω_p and ω_g the angular velocity of the pinion and gear.

For gears to mesh properly the pressure angle must be equal for the pair. The pressure angle is the angle between the line of action and a normal to the line connecting the centers. Knowing the pressure angle and the pitch radius it is possible to determine the base circle radius r_b.

$$r_b = r \times \cos\phi \tag{2.7}$$

2.1.2 Contact ratio

The contact ratio, CR is defined as the average number of teeth in contact during mating. Figuratively, as the first tooth ceases contact, the next tooth must already have come into engagement in order not to loose contact.

Due to the risk of deformation, contact ratios should be greater than 1.2 in order not to loose contact [17]. It is important to be aware of that the theoretical values of the contact ratio are greater then the actual values.

Contact ratio can be calculated as,

$$CR = \frac{\sqrt{r_{ap}^2 - r_{bp}^2} + \sqrt{r_{ag}^2 - r_{bg}^2} - c\sin\phi}{p_b} \tag{2.8}$$

where,

r_{ap}, r_{ag} = addendum radii of pinion and gear
r_{bp}, r_{bg} = base circle radii of pinion and gear
c = center distance
p_b = base circle pitch

High contact ratio (HCR) gears have a contact ratio greater than 2 i.e. that a minimum of two teeth always share the load.

High contact ratio can be achieved by increasing the number of teeth, lowering the pressure angle or increasing the addendum factor. By only increasing the addendum factor the overall configuration of the gear system will stay intact.

2.1.3 Mesh stiffness

The mesh stiffness of gears can be described as the ability to resist deformation during a gear mesh.

The general definition of stiffness is

$$K = \frac{F}{\delta} \tag{2.9}$$

where F is the applied force
δ is the displacement caused by the force.

Gear noise and vibration is considered to be difficult to control. One of the main contributors is transmission error (TE), defined as the difference between the theoretical and actual angular position . When a gear pair transmit torque the combined torsional mesh stiffness varies throughout the mesh cycle causing variation in angular position (transmission error). If the TE occur at the same frequency as the shaft, noise is enhanced. It is desirable to minimize the amount of TE in order to ensure proper operation[20].

The equation for transmission error has been defined by Welbourn, expressed as;

$$TE = \theta_g - Z\theta_p \tag{2.10}$$

where Z is the gear ratio and $\theta_{p,g}$ is the angular rotation for pinion and gear in radians.

2.1.4 The combined torsional mesh stiffness

The combined torsional mesh stiffness is defined by Sirichai as the ratio between the torsional load and the total elastic angular rotation of the gear body. The angular rotation is defined as the angle the wheel turns as a result of bending, shearing and contact of the gear when in loaded mesh with the fixed mating gear.

The mesh stiffness varies with time during meshing and depends on several parameters, the contact ratio and the point of contact being the most relevant. The variation of mesh stiffness is a major cause of vibration, noise and instability in a geared system.

Each tooth can be thought of as a spring, so the torsional mesh stiffness alternates between the stiffness of a single spring and a double spring in parallel. The torsional mesh stiffness will increase and decrease significantly throughout the mesh cycle. This can be used to determine the transmission error [20].

The single and combined torsional mesh stiffness of a single tooth pair contact

For a single tooth pair in contact the the single tooth torsional mesh stiffness is defined as the ratio between the torsional load, T and the elastic angular rotation, θ of the gear body.

$$K = \frac{T}{\theta} \tag{2.11}$$

During meshing of a single tooth pair, the single tooth torsional mesh stiffness of the pinion, K_p is decreasing while the increasing for the gear. The combined single pair torsional mesh stiffness can be found by combining the two stiffness as springs connected in series for a contact point B;

$$K_m = \frac{K_p \times K_G}{K_p + K_G} \tag{2.12}$$

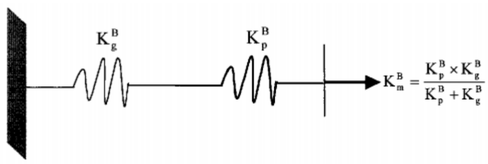


Figure 2.2: Spring connected in series

The combined double pair torsional mesh stiffness, $K_m^{A,D}$ can be found by combining the combined single pair torsional mesh stiffness's K_m^A and K_m^D as springs connected in parallel as shown in Figure 2.3: Spring connected in parallel. along the line of contact A to D.

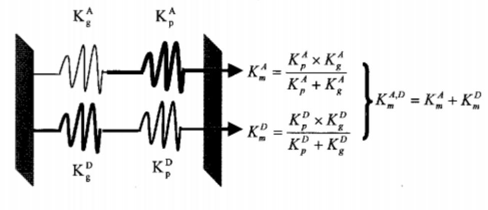


Figure 2.3: Spring connected in parallel.

2.2 Backlash

Backlash, b, is described as the difference between the circular thickness and the width of space between teeth i.e. the clearance between the two mating gear teeth. Backlash can also be described as the amount of lost motion when reversing the movement.

A gear pair is bound to have some backlash to prevent jamming and make room for lubrication for the gears to operate properly. Also manufacturing errors, thermal expansion and wear needs to be taken into account. Backlash is designed by reducing the thickness of both gear and pinion or just the gear, leaving the pinion to its full size. Another possibility is to move the gears further apart.

However, backlash can result in additional dynamic forces and is the main cause of vibration and gear noise.

Vibrations caused by backlash can cause tooth separation and loss in contact between the teeth. Due to this the gear has piecewise linear stiffness characteristics [22]. Backside contact, where the the gears alternates between frontside and backside may occur because of the time varying torques caused by the injection pumps and valve springs. This will cause complex behavior and intense vibration. The presence of backlash makes the system extremely nonlinear.

The contact loss can be modeled as a discontinuation of a piecewise linear function, where the discontinuation represents the backlash, see Figure 2.4.

The displacement function $g_i[\delta_i(t), b_i]$ can be expressed as

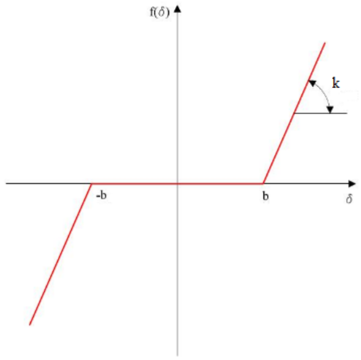


Figure 2.4: Nonlinear function of backlash

$$g_i[\delta_i(t)] = \begin{cases} \delta_i(t) - b_i & \text{if } \delta_i(t) > b_i \\ 0 & \text{if } -b_i \leq \delta_i(t) \leq b_i \\ \delta_i(t) + b_i & \text{if } \delta_i(t) < -b_i \end{cases} \tag{2.13}$$

where $2b_i$ represents the width of the backlash for mesh i=1:2, $\delta_i(t)$ teeth deflection [22].

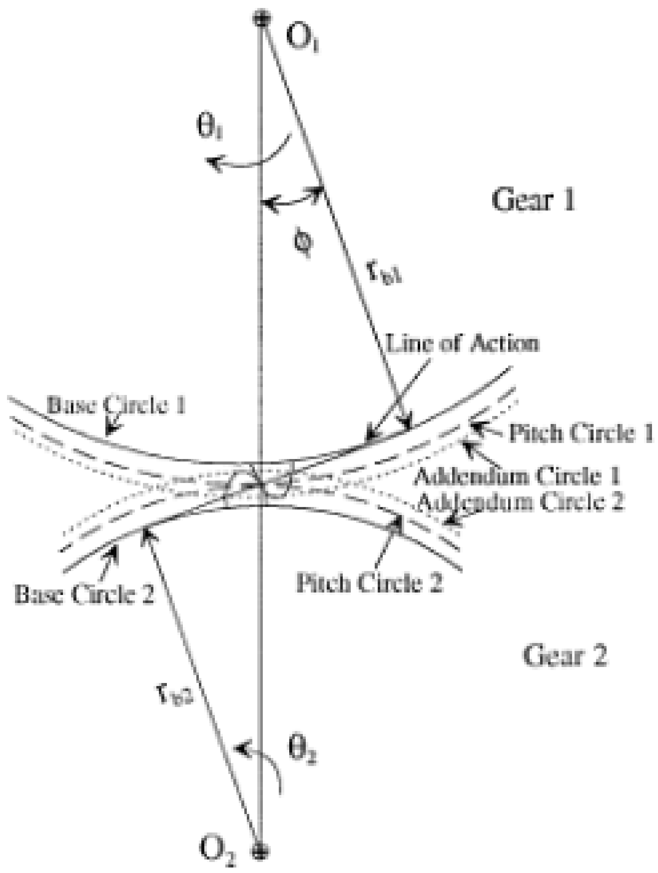


Figure 2.5: A gear pair

Considering a single degree of freedom gear pair model as shown in Figure 2.5. Angular rotation, θ_1 and applied torque, T_1 is defined as positive in clockwise direction, and θ_1 and T_2 positive in counterclockwise direction.

Because of backlash, three scenarios may occur. Frontside contact, contact loss i.e tooth separation and backside contact.

Mathematically these cases is expressed as following, [19].

Case 1- Fronside contact

$$r_{b1}\theta_1 - r_{b2}\theta_2 > b \tag{2.14}$$

Which occurs when the driving edge of gear 1 is in contact with the following edge of gear 2.

Case 2- Tooth separation

$$b > r_{b1}\theta_1 - r_{b2}\theta_2 > -b \tag{2.15}$$

Contact loss occurs when there is no force of interaction between the two gears.

Case 3- Backside contact

$$r_{b1}\theta_1 - r_{b2}\theta_2 < b \tag{2.16}$$

Which occurs when the trailing edge of gear 1 is in contact with the leading edge of gear 2.

2.3 Dynamic Simulations

Dynamic simulations is the study of how a physical system develops over time and cause behind those changes. Different from a static analysis, which only need one computation in order find a solution, dynamic analysis needs multiple repeated computations and is there for a more expensive analysis [1]. Vibrations is a big part of dynamics and is described as a repetitive, periodically or oscillating response to a mechanical system [16]. The rate at which the vibration cycles around its stable equilibrium with is called a frequency. If the frequency coincides with the natural motion of the system, it will respond more vigorously and the amplitude will increase. Operating at this resonance frequency is undesirable and can cause destruction.

2.3.1 Mechanical vibrations

Considering a system with a mass on the end of a spring, Hooke's law describes the system in equilibrium, while the equation of motion is described as

$$m\ddot{u} + c\dot{u} + ku = F(t) \tag{2.17}$$

Where m is the mass, c is the damping and k is the stiffness. u is the displacement of the mass with respect to time, $\dot{u}$ and $\ddot{u}$ is the velocity and the acceleration of the mass respectively. F(t) is the applied force. The force may be periodic and depend sinusoidally on time, expressed

$$F = a \sin \omega t \tag{2.18}$$

Where ω is the angular frequency.

Combining these describes the equation of motion of a harmonically driven linear damped oscillator.

$$m\ddot{u} + c\dot{u} + ku = a \sin \omega t \tag{2.19}$$

Mechanical vibrations may be classified in accordance to the load case; free vibration and forced vibrations. The simplest mechanical vibration occurs when the damping constant and the external applied forces are zero. This is known as an undamped free vibration.

$$m\ddot{u} + ku = 0 \tag{2.20}$$

The general solution to this is

$$\begin{aligned} u(t) &= C_1 \cos \omega t + C_2 \sin \omega t \\ &= Ae^{i\omega t} + Be^{-i\omega t} \end{aligned} \tag{2.21}$$

where $\omega = \sqrt{\frac{k}{m}}$ and describes the frequency at which the system oscillates without any damping. This motion is called a simple harmonic motion.

The period of the oscillation can be found by

$$T = \frac{2\pi}{\omega_0} \tag{2.22}$$

and the frequency, f is found $f = {}^1/_T$ with the unit Hz or s^{-1}.

Natural frequencies and mode shapes of a structure determine the dynamic behavior of a structure and how the structure will response to dynamic loading. The deformation shape of a structure at a specific natural frequency is determined by the structural properties and boundary conditions. Each natural frequency is associated to a mode shape.

Free vibrations occur when a system is excited by an initial condition and is then allowed to vibrate freely without further influence. A mechanical system in free vibration will oscillate with its natural frequency until further interactions or gradually reduce to zero due to damping.

The equation of motion for an undamped free vibration associated with Equation (2.20)

$$u(t) = A_0 \sin(\omega t + \phi) \tag{2.23}$$

where the amplitude A_0 is a constant that determines the maximum displacement from equilibrium, the phase constant ϕ determines the initial position of the oscillator when t=0. ω is the driving frequency.

2.3.2 Steady state motion

Equation (2.20) represents the motion of the oscillator. When the oscillator is disturbed it tends to oscillate with its natural frequency, ω_0, but it also being driven by the outside force at a different angular frequency,ω. The driving force is dominating because of damping to the system. The motion during this interference is called transient motion.

The motion of the driven oscillator is likely to have the same angular frequency as the driving force. However, it does not have to be in-phase with the driving force. It is normal for the displacement to lag behind the force.

The steady state motion can therefor be expressed

$$x(t) = A \sin{(\omega t - \delta)} \tag{2.24}$$

Where A is the amplitude, δ is the phase lag between the steady state motion and the driving force.

Three scenarios may occur, (1) the frequency of the driving force is much smaller than the natural frequency, $\omega << \omega_0$. (2) when the driving frequency is much bigger than the natural frequency, $\omega >> \omega_0$. (3) when the two become equal, $\omega = \omega_0$.

Considering Equation (2.20) again, for low frequency limit ($\omega << \omega_0$), the first and second term (for small damping and except when x=0) is small compared to the third term and can therefor be neglected and the equation becomes

$$x(t) = (\frac{a}{\omega_0^2}) \sin{(\omega t - \delta)} \tag{2.25}$$

This implies that the driving frequency is dominating, the amplitude is independent of the driving frequency and that the displacement are in phase with the driving force. This results in small amplitude oscillations in phase with the driving force.

For high frequency limit ($\omega >> \omega_0$) the first term in Equation (2.20) is dominating and the equation for high frequency limit is

$$x(t) = A \sin(\omega t - \pi) \tag{2.26}$$

The amplitude A depends on ω and the motion is in anti-phase with the force.

For resonance limit($\omega = \omega_0$) for ligthly damped oscillators the equation becomes

$$x(t) = \frac{a}{c\omega_0} \sin(\omega t - \frac{\pi}{2}) \tag{2.27}$$

The amplitude is $A = \frac{a}{c\omega_0}$, c is the damping constant. The amplitude will at resonance will exceed the amplitude in the low frequency by a factor of $\frac{\omega_0}{c}$ also called the Q-factor.

2.3.3 Damping

Damping reduces/prevents oscillation in a system by slowing down the motion of a system. The damping ratio, ζ

$$\zeta = \frac{c}{2\sqrt{mk}} \tag{2.28}$$

The value of the damping ratio determines the behavior of the system. A damped harmonic oscillator can be described as overdamped, critically damped or underdamped.

In cases where the system is overdamped, the damping ratio $\zeta > 0$, and it returns to equilibrium without oscillating. Larger values of ζ will cause the system to return to equilibrium slower. The displacement is described as

$$u(t) = C_1 e^{r_1 t} + C_2 e^{r_2 t} \tag{2.29}$$

In cases where the system is underdamped, the damping ratio $\zeta < 1$, it will oscillate with an amplitude gradually decreasing to zero. The displacement is described as

$$u(t) = C_1 e^{\lambda t} \cos \mu t + C_2 e^{\lambda t} \sin \mu t \quad (2.30)$$

For critically damped cases the system returns to equilibrium as fast as possible without oscillating, $\zeta = 1$.

The displacement is described as

$$u(t) = C_1 e^{rt} + C_2 t e^{rt} \quad (2.31)$$

The damping is expressed as

$$c = 2\sqrt{mk} \quad (2.32)$$

In cases where a system is critically damped, the damping coefficient, c is just large enough to prevent oscillation.

Damping ratios can be calculated using Rayleigh damping, which is commonly used in nonlinear dynamic analysis [4]. It is also known as proportional damping and expressed

$$C = \alpha \mathbf{M} + \beta \mathbf{K} \quad (2.33)$$

Where α and β are Rayleigh coefficients, α for mass proportional damping and β for stiffness proportional damping. The damping matrix is calculated as a sum of the mass and stiffness matrix multiplied by the damping constants. The damping constants are calculated from the damping ratio ζ. For cases with natural frequency for a given mode i of vibration, the damping ratio is expressed

$$\zeta_i = \frac{\alpha}{2\omega_i} + \frac{\beta \omega_i}{2} \quad (2.34)$$

Based on equation (2.33), the first term, containing α becomes dominant for higher natural frequencies, while the second term, containing β becomes dominant for lower frequencies [18].

2.4 The Finite Element Method

The finite element method (FEM) is a numerical technique for solving field problems like stress distribution, fluid flow, thermal fields etc. By dividing the subject of interest into a finite number of elements, the problem is discretized to a boundary value problem with a set of equations to be solved for each element. These equations are then systematically put together and solved numerically. Because of this discretization, finite element calculations will only provide an approximate solution, but the solution is generally increasingly accurate with increasing number of elements - at the cost of more equations to solve.

In structural analysis, the *direct stiffness method* is the most common technique for solving structural problems with the finite element method. The body of interest is separated into smaller idealized elements with interconnecting and shared nodes. Stiffness is found for each element and gathered in a stiffness matrix. This matrix is solved for the structure's unknown displacements at the nodes by matrix operations. This numerical method for solving a system of equations may be an extremely time consuming process, but is well suited for computer processors [10].

For three dimensional structures, solving the governing differential equations analytically is difficult if not impossible. But by using weighted residual methods, the solutions is replaced with approximate algebraic equations. The governing system equations are partial differential equations of strong form which is difficult to solve for practical engineering problems because of continuity requirements. However, by applying a finite sum of test functions, the solution is well approximated and the continuity requirement is lowered.

The weak form of the equilibrium forces can be used to establish the matrix relationship between the nodal forces and nodal displacements. The nodal displacements are interpolated locally over each element using shape functions. As an illustration, consider the two dimensional quadrilateral element and its shape functions:

$$
\begin{aligned}
N_1(\xi,\eta) &= \frac{1}{4}(1-\xi-\eta+\xi\eta) \\
N_2(\xi,\eta) &= \frac{1}{4}(1+\xi-\eta-\xi\eta) \\
N_3(\xi,\eta) &= \frac{1}{4}(1+\xi+\eta+\xi\eta) \\
N_1(\xi,\eta) &= \frac{1}{4}(1-\xi+\eta-\xi\eta)
\end{aligned}
\tag{2.35}
$$

3 | Gear Modeling

3.1 Introduction

The camshaft gear drive consists of two gear pairs with individual gear mesh. The crankshaft gear wheel transmits power to a big idler gear, which is connected coaxial with a smaller idler gear that transmits power to the camshaft gear wheel, see Figure 3.1. The shaft connecting the two idler gear is assumed rigid.

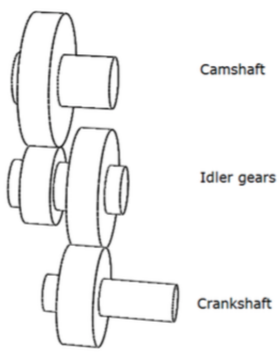


Figure 3.1: Gear drive system [21]

Diesel engines often experience dynamic behavior followed by a rattling motion of teeth within the backlash, often referred to as gear hammering. It originates in that the transmitted torque is small relative to the dynamic loads. Dynamic effects may not be neglected for precise simulations. Gear hammering in gears occur because of camshaft torque variation, time variant mesh stiffness and the presence of backlash.

The gear hammering can be separated into two actions: separation and impact. Ideally the angular speed of the two gears are equal when the teeth enters the mesh, but because of periodically combustion processes the torque varies and become negative. The angular speed on the cam gear will be faster relative to the driving gear which results in teeth separation. There is also a possibility that the cam gear teeth will pass through the backlash area and collide on the non working side of the driving gear teeth. During the next injection the cam gear will pass through the backlash and collide on the working side of the gear teeth again [3].

The dynamic model is characterized by two degrees of freedom that is associated with teeth deflection at every stage. The deflection is defined as the relative displacement of the teeth along the line of action for gear i, expressed in equation (3.1)

$$\delta_1(t) = r_1\theta_1(t) + r_2\theta_2(t) \tag{3.1a}$$

$$\delta_2(t) = r_3\theta_3(t) + r_4\theta_4(t) \tag{3.1b}$$

3.2 Material

Gear materials are selected to provide the optimal combinations of properties. An important psychical property to consider is the wear resistance. The mechanical properties of the material selected for the gears are described in Table 3.1.

Material Properties		
Modulus of elasticity	[MPa]	210 000
Poisson's ratio	[mm/mm]	0.3
Mass density	[$10^3 kg/m^3$]	7.85

Table 3.1: Material Properties

3.3 Gear dimensions

The gear data is provided by Rolls-Royce and the gears are modelled in accordance with the production drawings.

Parameter		Pinion 1	Gear 2	Pinion 3	Gear 4
		Crancshaft	Big Idler	Small Idler	Camshaft
Module	m_n	8	8	9	9
Number of teeth	z	59	68	34	59
Pressure angle	α [deg]	20	20	20	20
Profile shift coeff.	x [mm]	-0.626	-0.626	-0.082	-0.082
Backlash	b_{min} [mm]	0.15	0.15	0.15	0.15
	b_{max} [mm]	0.43	0.43	0.43	0.43
Base diameter	d_b [mm]	443.535	511.193	287.546	498.977
Face width	w [mm]	90	88	135	130
Inertia	I [Kgm^2]	4.034	3.06	0.998	4.503
Center distance	a [mm]	497	497	417	417
Mass	m [kg]	104	59	75	78.1

Table 3.2: Gear specifics

3.4 The model in Fedem

The physical model of the complete system is presented in Figure 3.2a. The dynamic model associated with the physical model is illustrated in Figure 3.2b. The system analysed however, consists of two spur gears represented by rigid disks connected by a translational, viscous damped spring along the pressure line of the gears, as seen i Figure 3.4. The shafts and bearings are considered rigid. Each gear is modeled with a moment of inertia.

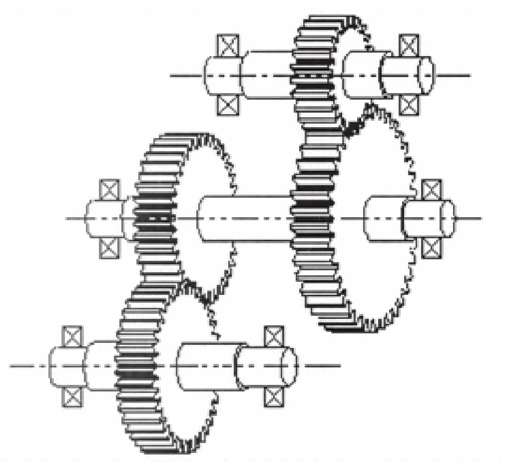
The equation of motion of the non-linear model can be described in the general form as

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{f} \tag{3.2}$$

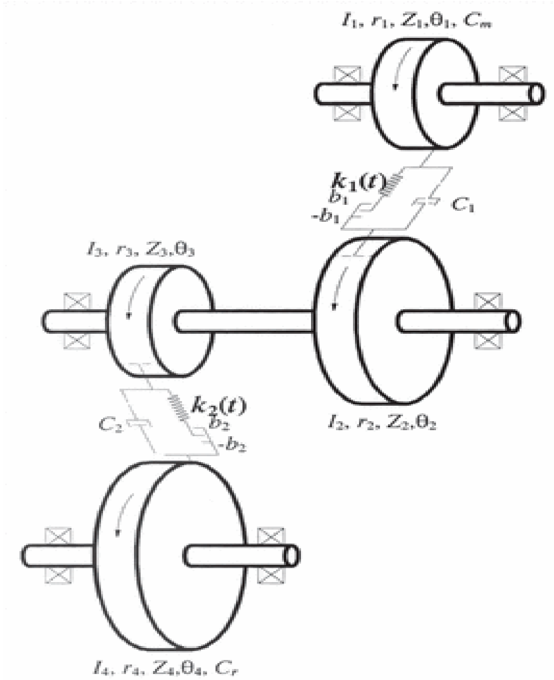
where **M**, **C** and **K** represents respectively inertia, damping and stiffness matrices.

$\mathbf{M}$= diag$(m_1, m_2, ..., m_n)$ is the inertia matrix where m_i=I_i/r_1^2. $\mathbf{u}$=$[x_1, x_2, ..., x_n]^T$.

The stiffness matrix **K** expressed as



(a) Physical model of the two-stage gear system



(b) Dynamic model of the two-stage gear system

Figure 3.2: Gear model [22]

$$\mathbf{K} = \begin{bmatrix} k_1 & -k_1 & & \cdots & \\ -k_1 & (k_1+k_2) & -k_2 & & \\ & -k_2 & (k_2+k_3) & -k_3 & \\ & & \ddots & & \\ & & & & -k_{n-1} \\ & & & -k_{n-1} & k_{n-1} \end{bmatrix} \tag{3.3}$$

The damping matrix $\mathbf{C}$ expressed as

$$\mathbf{C} = \begin{bmatrix} c_1 & -c_1 & & \cdots & \\ -c_1 & (c_1+c_2) & -c_2 & & \\ & -c_2 & (c_2+c_3) & -c_3 & \\ & & \ddots & & \\ & & & & -c_{n-1} \\ & & & -c_{n-1} & c_{n-1} \end{bmatrix} \tag{3.4}$$

The equation of motion for the two stage gear pair system can be expressed as

$$I_1\ddot{\theta}_1(t) + r_1c_1[r_1\dot{\theta}_1(t) - r_2\dot{\theta}_2(t)] + r_1k_1(t)g_1[\delta_1(t)] = T_1(t) \tag{3.5a}$$

$$\begin{aligned} I_{23}\ddot{\theta}_2(t) + r_2c_1[r_1\dot{\theta}_1(t) - r_2\dot{\theta}_2(t)] + r_2k_1(t)g_1[\delta_1(t)]+ \\ r_3c_2[r_3\dot{\theta}_2(t) - r_4\dot{\theta}_4(t)] + r_3k_2(t)g_1[\delta_1(t)] = T_{23}(t) \end{aligned} \tag{3.5b}$$

$$I_4\ddot{\theta}_4(t) + r_4c_2[r_3\dot{\theta}_2(t) - r_4\dot{\theta}_4(t)] + r_4k_2(t)g_2[\delta_1(t)] = T_4(t) \tag{3.5c}$$

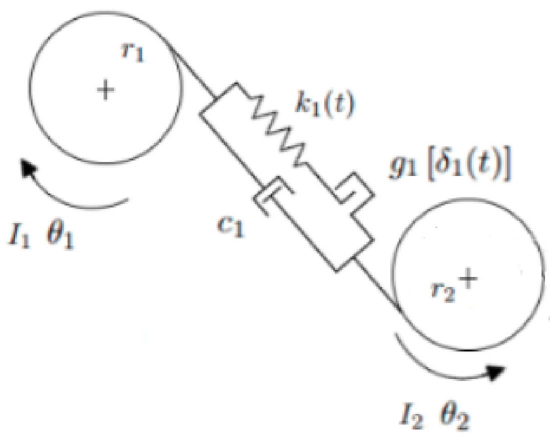


Figure 3.4: Single gear pair

For a single gear pair, shown in Figure 3.4 the equations of motions can be expressed as

$$I_1\ddot{\theta}_1(t) + r_1c_1[r_1\dot{\theta}_1(t) - r_2\dot{\theta}_2(t)] + r_1k_1(t)g_1[\delta_1(t)] = T_1(t) \tag{3.6a}$$

$$I_2\ddot{\theta}_2(t) - r_2c_2[r_1\dot{\theta}_1(t) - r_2\dot{\theta}_2(t)] - r_2k_2(t)g_2[\delta_1(t)] = -T_2(t) \tag{3.6b}$$

Where T_i are the torques applied for gear i=1,2,3,4. k_i are the time varying mesh stiffness for mesh i=1,2. $g_i[\delta_i(t)]$ is a function describing the backlash for mesh i, see section 2.2 for further explanation. c_i is the constant damping acting along the line of action for mesh i.

The gear mesh stiffness is represented as a time varying mesh stiffness and a non-linear displacement function which includes gear backlash. The time varying mesh stiffness is modeled as a periodic rectangular wave function, illustrated in Figure 3.5.

Where K_{max} and K_{min} is the maximum and minimum value of mesh stiffness respectively.

The value of the mesh stiffness used in the analysis are estimated values and chosen for the purpose to give distinct effect. $K_{max} = 3$ and $K_{min} = 2$ i.e that the difference in stiffness between three and two pair of teeth in contact are $^2/_3$.

The mesh stiffness is created in Fedem by a math expression describing one cycle of mesh stiffness variation. This cycle is then repeated in a loop by adding a function that repeats the mesh stiffness function in a 2π interval in a loop, illustrated in Figure 3.6. This function is added to a simple sensor connected to the revolute joint in Fedem.

Mesh stiffness variation is caused by the change in number of teeth in contact. This cause parametric instabilities and severe vibration in gear systems. As a result, gear resonance occurs, and by determining and identifying these conditions it is possible to minimize their effects.

The backlash is introduced to the system as a nonlinear function by discontinuity in the stiffness, presented in Figure 3.7, where the backlash value 2b= 0.043 mm.

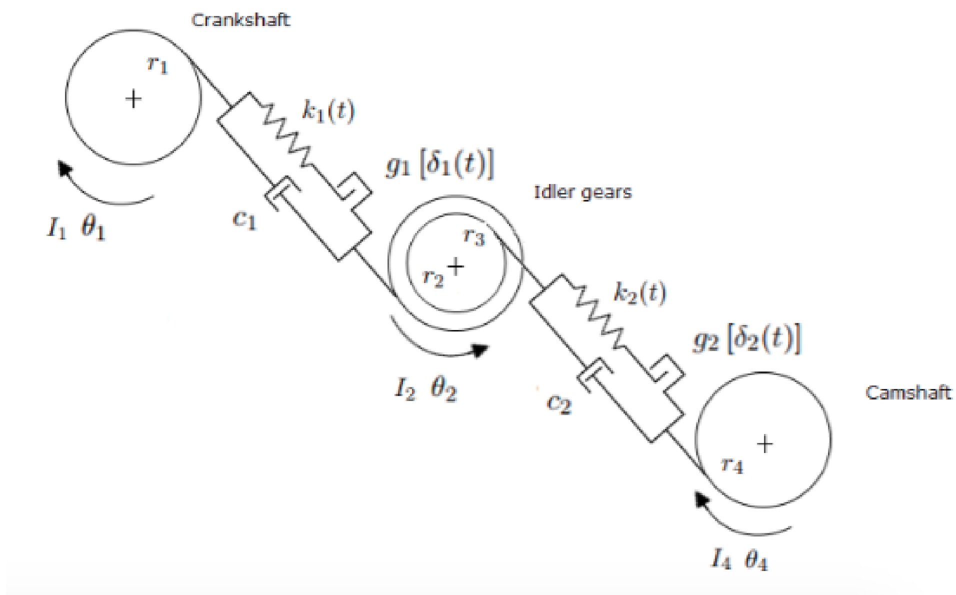


Figure 3.3: Nonlinear gear model

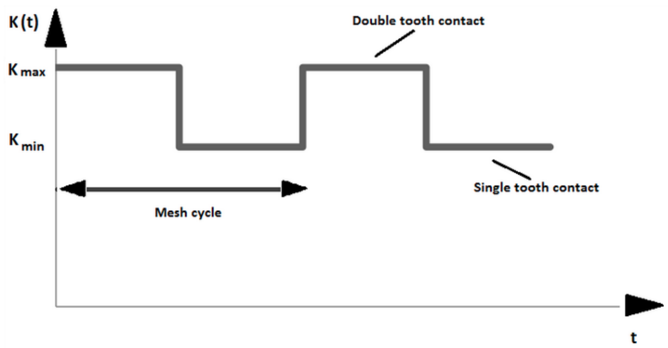


Figure 3.5: Time-varying mesh stiffness

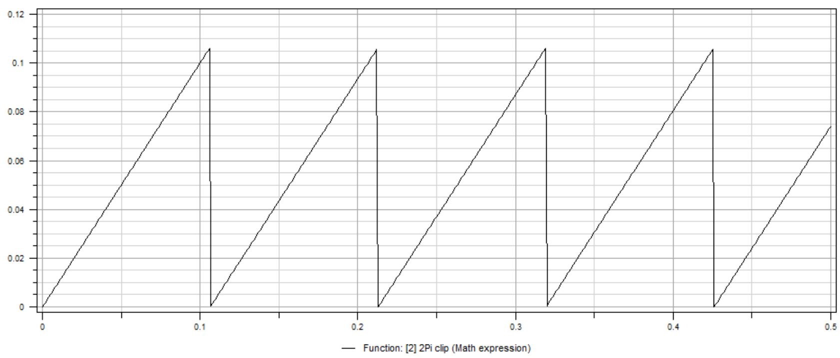


Figure 3.6: 2π clip off function

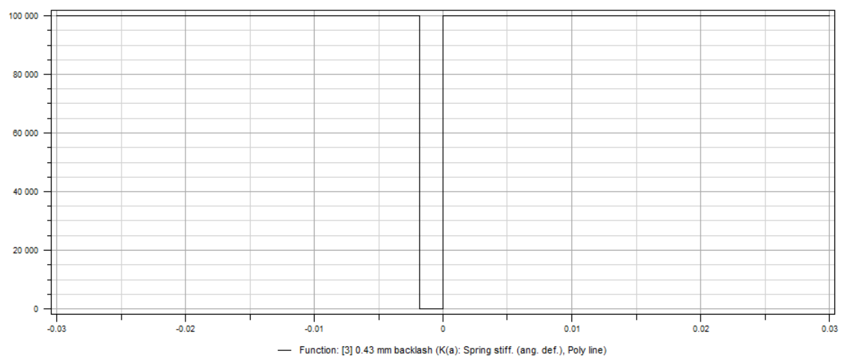


Figure 3.7: Backlash function

3.4.1 A single gear pair setup in Fedem

A single gear pair is modeled in Fedem to investigate the effects of backlash. Various parameters have been changed to analyse the different outcomes. Pinion 1 and Gear 2 are the gears used in this analysis, see Table 3.2.

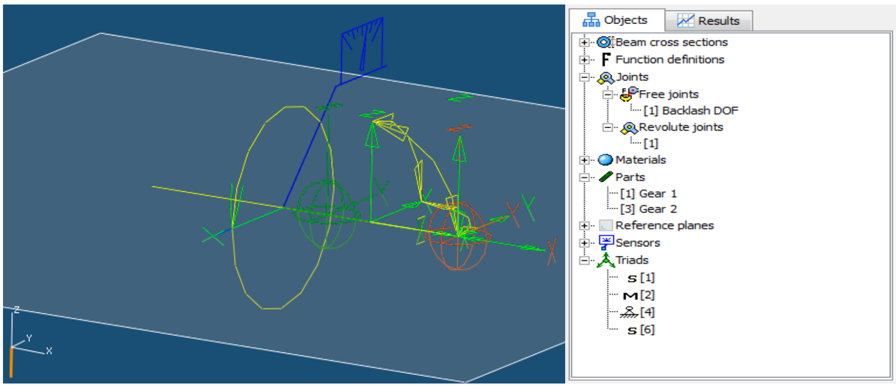


Figure 3.8: Model in Fedem

FE-nodes in Fedem are represented as triads and may be connected using a joint. The constraints between the two triads are assigned through the joint, applied on the joint DOFs. One of the joints triads is labeled master and one is labeled slave, where the slave following the master.

Triad 2 (master) and triad 1 (slave) is connected by a free joint. The free joint is assigned the time varying mesh stiffness, the backlash function and damping coefficient. The free joint it also fixed in translational direction and rotational direction around y and z, and free to rotate around one axis, x.

Triad 1 (slave) is connected to the ground, or earth (master triad 4), through a revolute joint. The revolute joint is assigned a angular velocity function trough a stress free angle change, and is moreover considered stiff. The stress free angle change varies as a function of time which is possible since the constraint type is set to spring-damper [1].

A generic part, representing gear 1, connects the two joints. This part is assigned the mass and the moment of inertia of gear 1. A second generic part, representing gear 2 is connected to triad 6 (slave).

4 | Results and Results

4.1 Backlash phenomenon

The phenomenon of backlash is visible in Figure 4.1

In this analysis the rotational speed is equal to 4π/sec, or 12.6 rad/sec corresponding to approximately 120 RPM. A ramp load is applied to triad 6 (slave) connected to gear 2. The ramp load, with an amplitude of 5 is applied from start and held for 0.1 second and then released to set the system into oscillatory motion. The damping coefficient is set to 0.1 Ns/m.

Three different backlash values are analyzed, 0 mm, the minimum value 0.15 mm and the maximum value 0.43 mm.

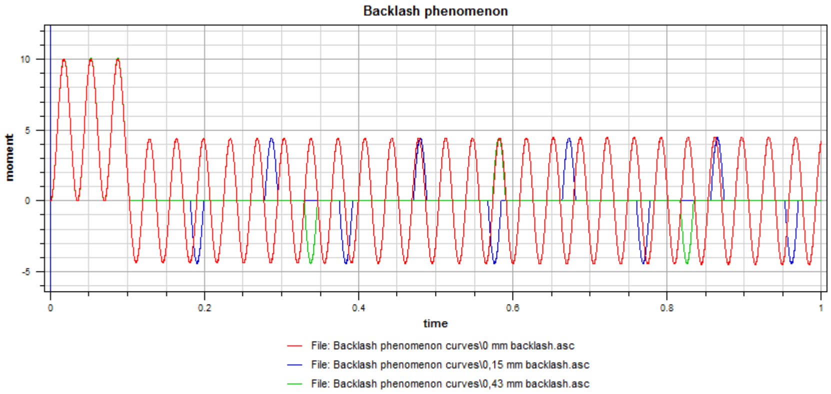


Figure 4.1: Backlash phenomenon

The area between 0 and 0.1 seconds displays the time when the system oscillates around the applied load. After 0.1 seconds the system has no external forces and will oscillate around equilibrium which is zero.

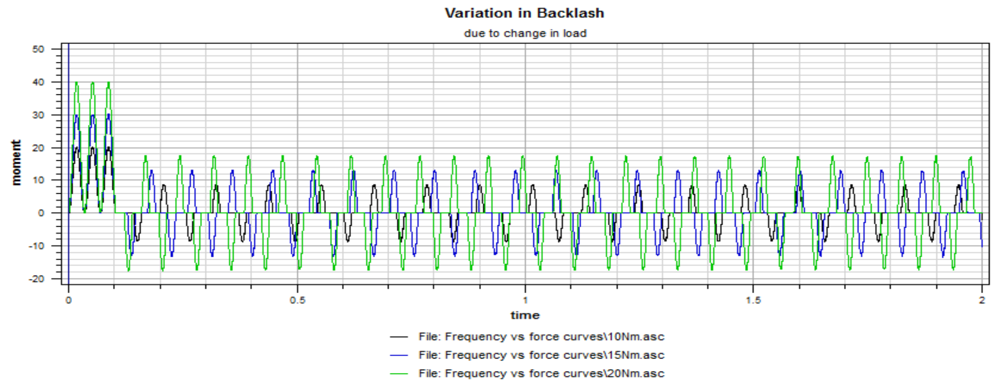
The red curve illustrated the moment vs time function where no backlash is present. It shows a continuous curve without any zero values i.e that the gears are continuously in contact with each other.

The blue and the green curve illustrates the moment function including backlash, respectively 0.15 mm and 0.43 mm backlash. They both contain an area where the moment is zero, meaning there is no contact between the gears. As the graph illustrates, with increasing the backlash, the time of contact loss also increases. This means that the model with 0.043 mm backlash experiences longer amount of time where the gears are no longer in contact than the model with 0.015 mm backlash.

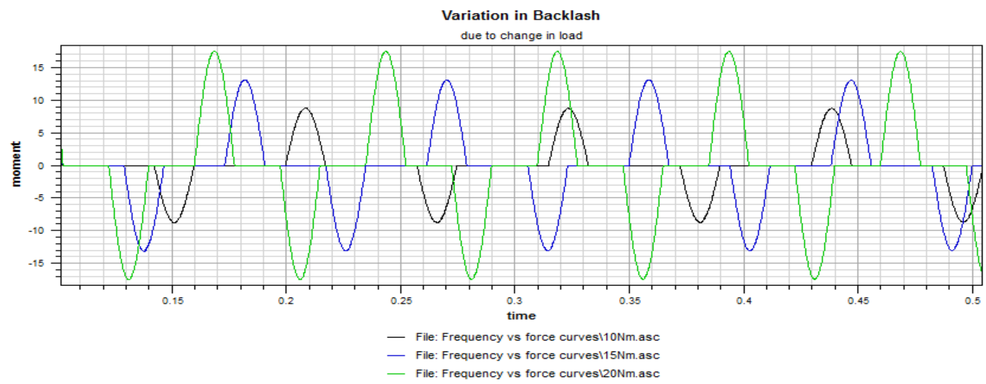
4.2 Moment response

In this analysis the backlash value is kept constant throughout the analysis at 0.15 mm. What varies is the amplitude of the load applied.

Figure 4.2 illustrates the moment function for 10Nm (black curve), 15 Nm (blue curve) and 20 Nm (green curve). The amount of time the moment equals zero, where the gears are not in contact with each other, depends on the magnitude of the applied load. For lower values of applied load the longer the separation time is, and vice versa, as can be seen from the figures below.



(a) Moment: 10Nm, 15Nm and 20Nm



(b) Closeup of Figure 4.2a

Figure 4.2: Variation of moment

This becomes even more clear when the applied load values are considerably higher, as Figure 4.3 and 4.4 shows. For higher loads the gears have almost non existing tooth separation. In Figure 4.5 the blue curve shows the moment reaction at at 1500 Nm, and it is almost continuous with only a small zero zone.

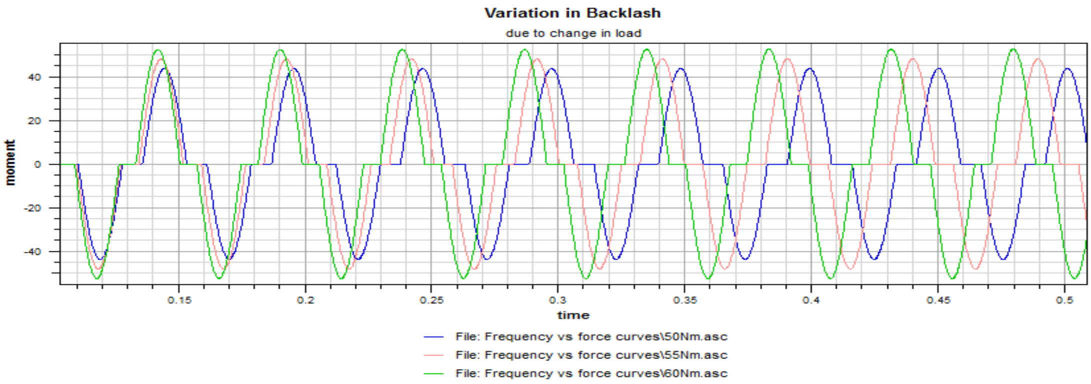


Figure 4.3: Moment: 50 Nm, 55 Nm and 60 Nm

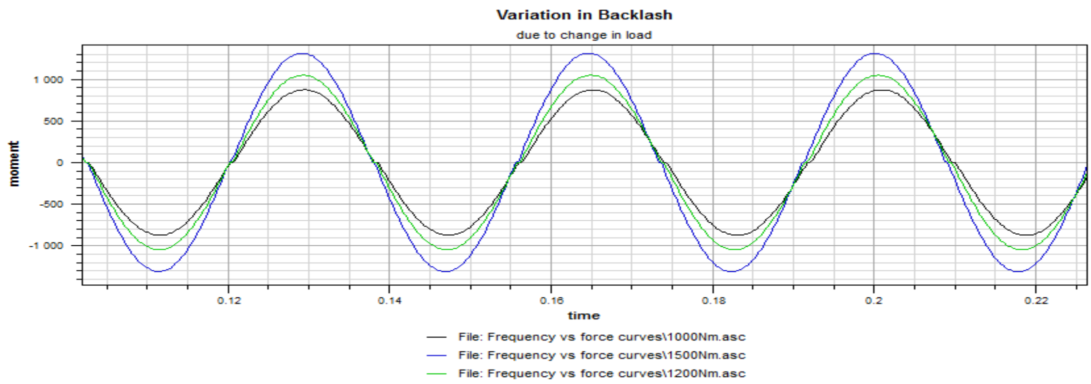


Figure 4.4: Moment: 1000 Nm, 1200 Nm and 1500 Nm

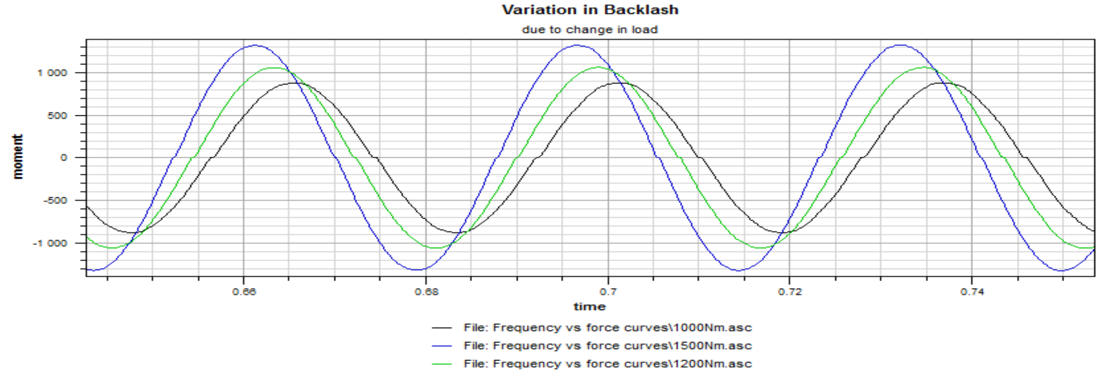


Figure 4.5: Closeup of Figure 4.4

Figure 4.6 shows that the frequency becomes independent of the force when the force reaches a certain level. For higher load applied, the backlash have less impact on the system.

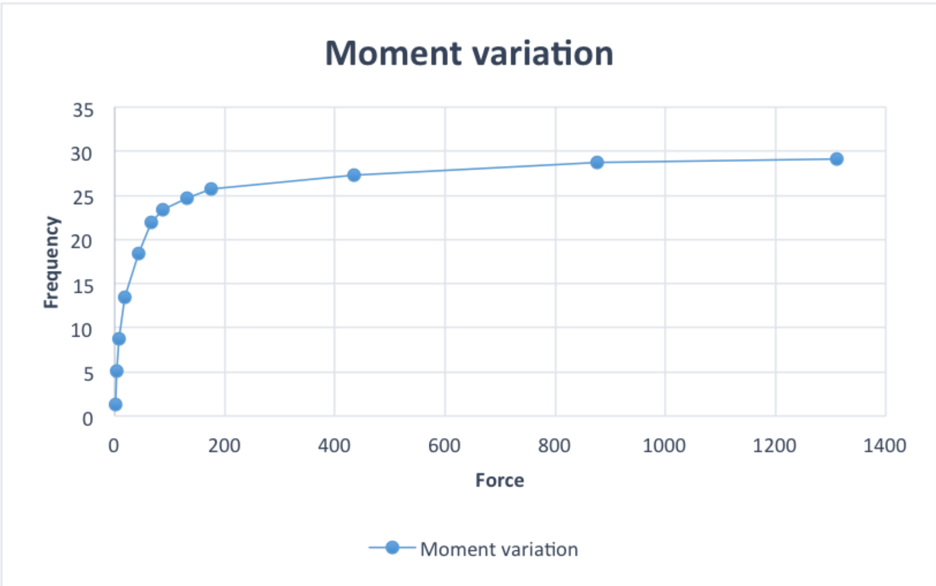


Figure 4.6: Moment variation

4.3 Dynamic Amplification Factor

Dynamic loads have significantly larger effects on a structure than static loads, even with the same magnitude, due to the inability a structure have to respond quickly. How the structure response to external forces is dependant on the frequency and the damping.

The dynamic amplification factor (DAF) expresses the relation between the dynamic and the static response.

$$DAF = \frac{\delta_{dynamic}}{\delta_{static}} \tag{4.1}$$

Where $\delta_{dynamic}$ and δ_{static} is the dynamic and static displacement respectively.

The effects of the static load is increased by the DAF to account for the dynamic contribute. It is determined by the ratio between the natural frequency and the force frequency, $\Omega = {}^{\omega}/_{\omega_n}$ and the damping ratio, ζ.

The maximum value of the DAF is given by the magnification factor, μ [12].

$$\mu = \frac{1}{\sqrt{(1-\Omega^2)^2 + (2\zeta\Omega)^2}} \tag{4.2}$$

Figure 4.7 shows that the response is at its greatest, Ω=1 when the load frequency is close to the same as the natural frequency for small damping ratios.

The figure clearly shows the effect of damping in a system. Theoretically, in undamped systems, $\zeta = 0$, the response tends towards infinity. For small and large values of Ω the damping ratio becomes negligible.

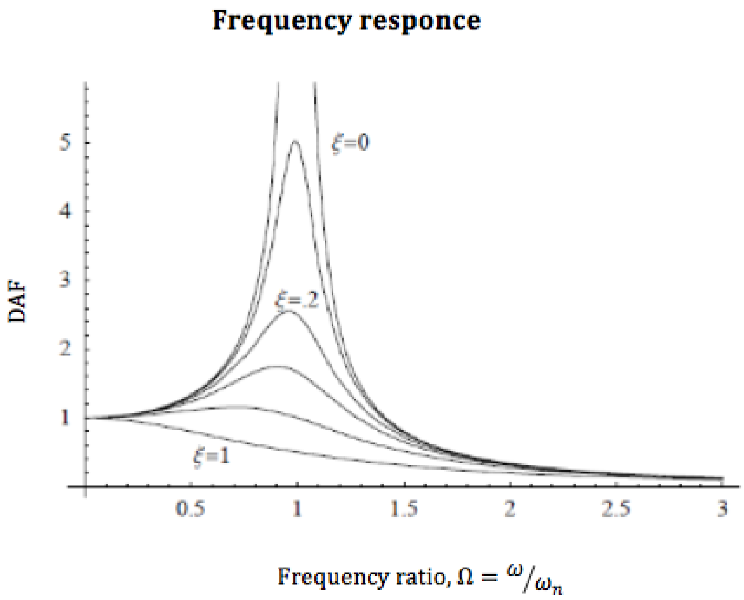


Figure 4.7: Frequancy responce function for a given structure

When the gear teeth enters the mesh the teeth collide and teeth deformation will occur [16]. This follows by a natural gear vibration frequency of f_n. The natural frequency of the gear mesh is expressed as

$$f_n = \frac{1}{2\pi}\sqrt{\frac{c}{m}} \tag{4.3}$$

where c is the damping and m is the mass.

The teeth mesh frequency f is expressed as

$$f = \frac{nZ}{60} = \frac{\omega Z}{2\pi} \tag{4.4}$$

where n is the rotational speed in rpm, ω is rotational speed in rad/sec and Z is the number of teeth. Gear mesh frequency increase with increasing rotational speed.

When the meshing frequency and the natural frequency become close or equal, resonance occurs and the system will oscillate with higher amplitude.

The gear pair is subjected to a constant moment of 500 Nm and a damping coefficient of 100 Ns/m. The backlash value is 0.043 mm and the mesh stiffness is 100000 N/m. The rotational speed is added to the model as a displacement function where the angle as a function of time is

$$\theta = 0.1t^2 \tag{4.5}$$

The velocity function is the deviated of the displacement function.

$$\dot{\theta} = \omega = 2 \times 0.1t \tag{4.6}$$

The dynamic amplification factor of the gear pair is shown in Figure 4.8

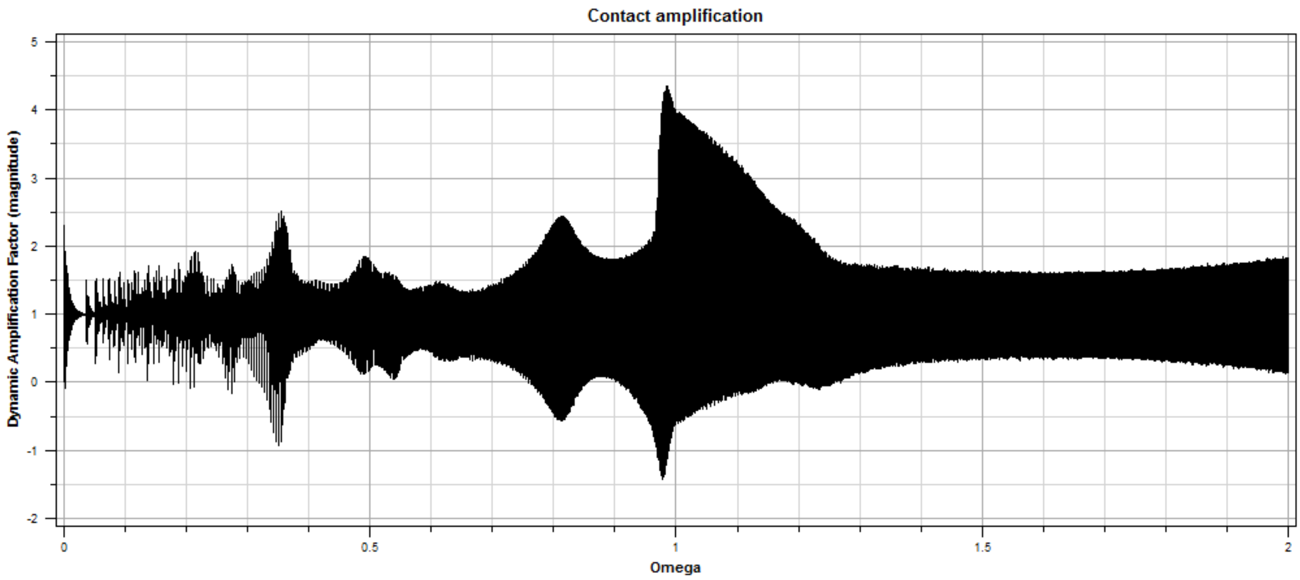


Figure 4.8: Dynamic Amplification Factor

The figure shows resonance when $\omega_0 = \omega$ i.e. $\Omega = 1$.

The natural frequency of the system is $f_n = 41.4$ Hz i.e a period of 0.0242 s. The velocity at which the mesh frequency and the natural frequency are equal, is at $\omega = \frac{2\pi}{59}/0.0242 = 4.4$ rad/sec. From Figure 4.9 the time when the velocity is 4.4 rad/sec is around 20 seconds. The most critical situation is when the mesh frequency coincides with the natural frequency. This will cause the system to oscillate with maximal amplitude and is the case at approximately 19.5 sec, see Figure 4.10.

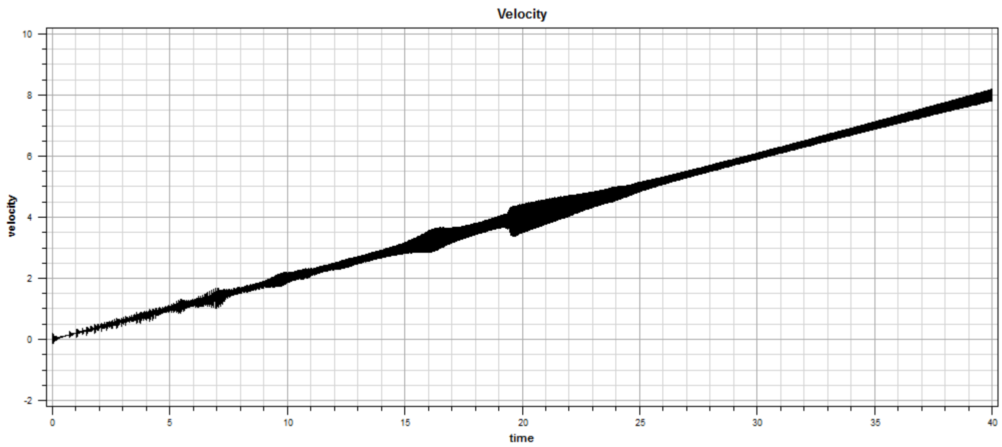


Figure 4.9: Velocity of the gear

4.4 Gear Contact Force

Figure 4.10 shows the gear contact force as a function of time. The system oscillates about the applied force before the motion is damped due to the applied damping coefficient. Because the gears alternate the number of teeth in contact, the mesh stiffness varies with time. This, along with the teeth impacts when they enter the mesh, excite dynamic forces and vibration. Because the gears operate over a wide speed range, it is likely that the natural frequency will be excited. The vibrations appear to be chaotic in the beginning. This is due to the fact that they coincide with a multiple of the natural frequency, see Table 4.1. In addition the the time of impact repetition in longer than the period of the natural frequency, causing the chaotic behavior.

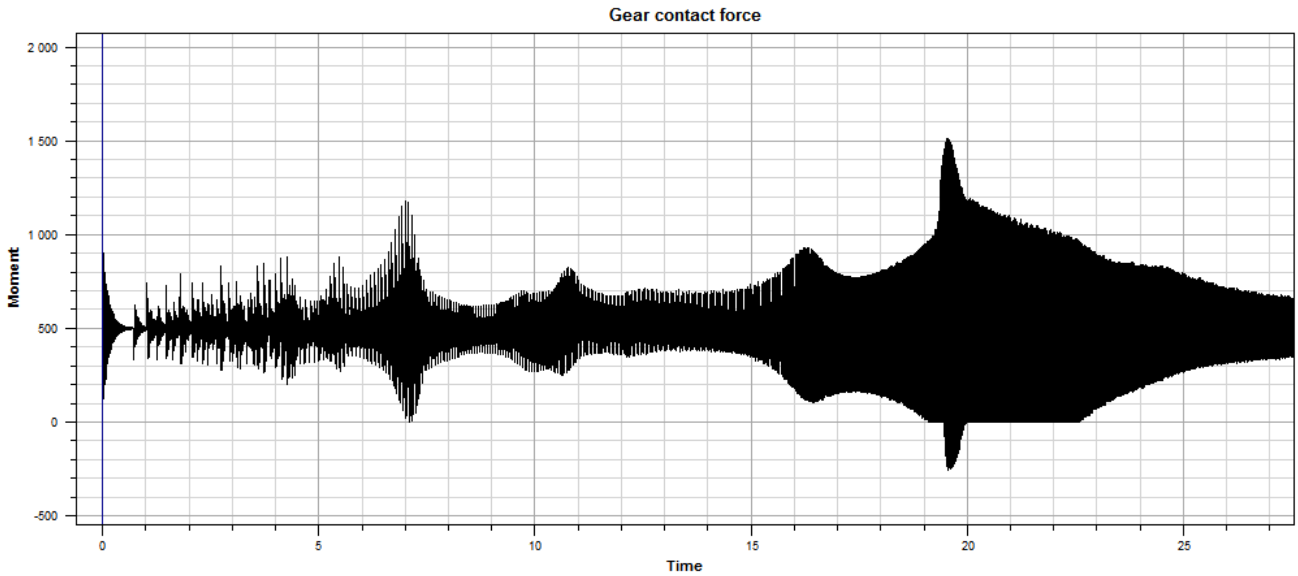


Figure 4.10: Gear Contact Force, 40 sec

From Figure 4.11 it is visible that the oscillating frequency and the load frequency coincides at several time step, and repeat it self at multiple of ω. Table 4.1 addresses the ratio between the oscillating frequency and the mesh frequency. Resonance is expected to occur when the ratio is i whole numbered multiple of the fundamental frequency.

Multiples of ω	
time [sec]	ω_v/ω_L
1	33
4,5	5
7	3
10	2
16	3
19.5	1
43	1.2
48	1

Table 4.1: Multiples of ω

At time step 19.5 and 48 seconds the ration between the two frequencies is one. However, at 19.5 seconds the amplitude is significantly higher than at 48 seconds. This can be explained by the phase shift. The frequencies are equal but the phase constant ϕ is different. It is natural to assume that the frequencies at 19.5 second experience more constructive interference than the ones at 48 seconds.

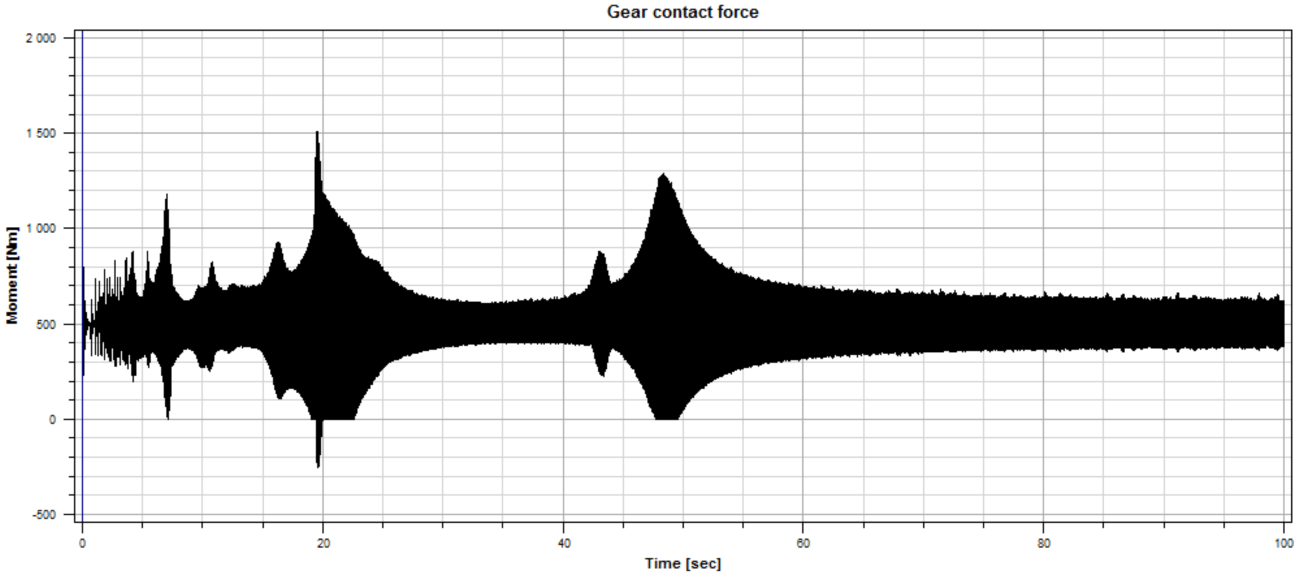


Figure 4.11: Gear Contact Force, 100 sec

At 19.5 seconds the lowest resonant frequency occur and is the so called fundamental frequency. The mesh frequency coincide with the natural frequency at a similar phase and cause vibrations to become stronger with higher amplitudes. This causes the gear to oscillate to the degree where backside contact occur. This phenomenon is visible in Figure 4.12 and 4.13 in that the moment is negative. The moment response correspond to the mechanism of the gear hammering and teeth collision explained in Section 3.1.

As stated earlier, when the teeth enters the mesh, tooth collision occur along with teeth deformations. Teeth impact is repeated with each entrance into mesh. The intensity of the impact is related to the magnitude of the backlash. The bigger the backlash the more intensive the impact between the gear teeth [14]. Since the mesh stiffness fluctuates from alternating between various number of teeth in contact, deformations will also fluctuate. Because of the gears inertia the response to these extremely small displacements which change in extremely short time are too slow, causing the force to increase, leading to high accelerations.

The mesh stiffness varies as a result of geometrical imperfections of the teeth, acceleration and deceleration of gears due to impact and changing contact ratios of the gears. The mesh stiffness directly affects the tooth deflection. The fluctuations of mesh stiffness is visible in the gear contact force figures. As the teeth changes from two pair of teeth in contact to three pair of teeth in contact, the stiffness increase, but the deformation happens so quickly the response is so slow due to inertia.

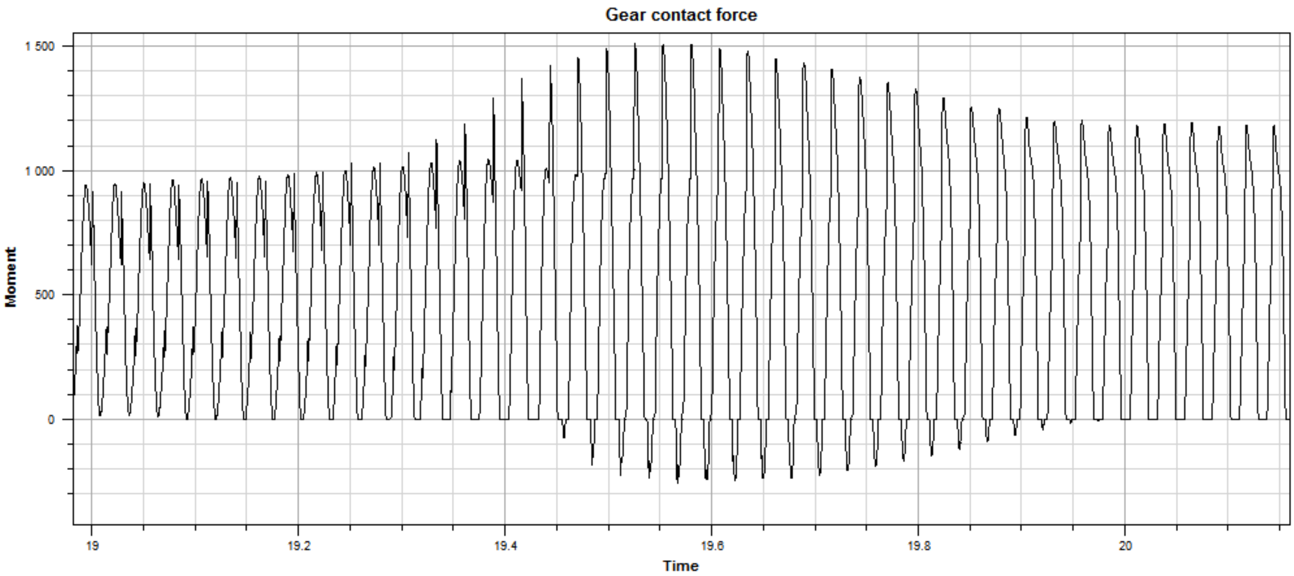


Figure 4.12: Gear Contact Force, closeup around resonance area

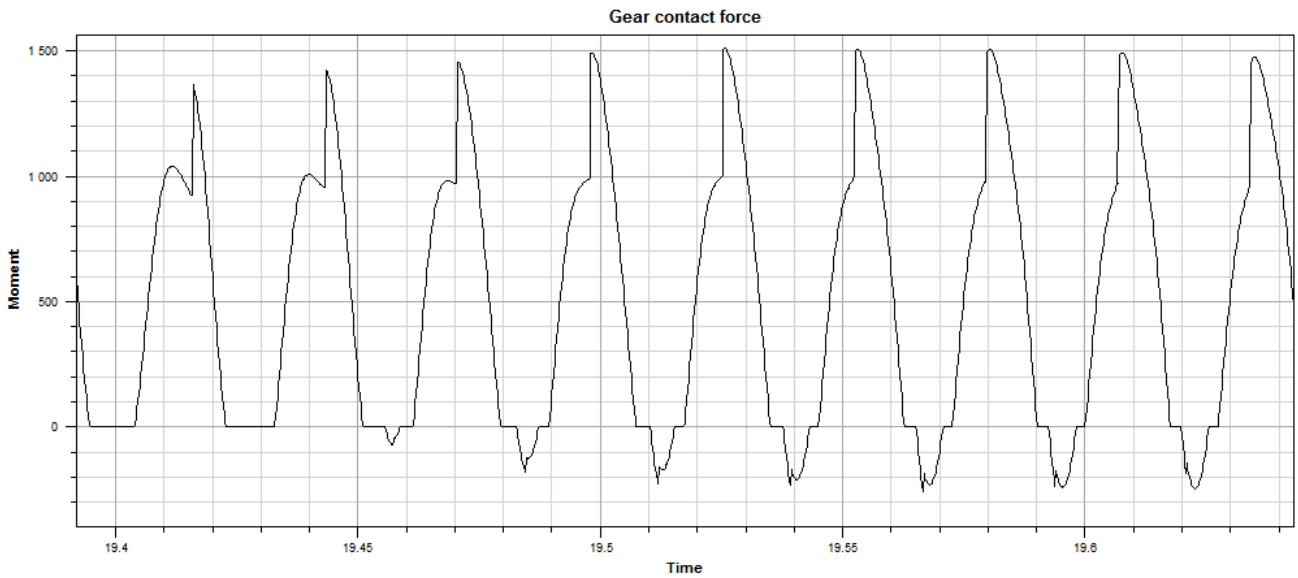


Figure 4.13: Gear Contact Force, backside contact

5 Conclusion

A gear pair system has been modeled and subjected to analysis. The focus has been on the dynamic behavior with the presence of backlash and time-varying mesh stiffness. The results obtained were used to study these effects.

The time of tooth separation increased as the value of backlash increased. For a gear pair with a constant backlash but different applied load, the time of tooth separation is considerably longer for low torques than for high. For higher force the frequency becomes independent of the force as a result of decreasing time of contact loss.

Vibration caused by transmission error is the primary concern in gear system due to increasing noise levels and dynamic behavior. Due to non-linearities such as time varying mesh stiffness and backlash gears experience dynamic forces, vibration and noise. Because the gears operate over a wide speed range, it is likely that the natural frequency will be excited. The most critical situation is when the mesh frequency coincides with the natural frequency. This will cause the system to oscillate with maximal amplitude. This is the case at 4.4 rad/sec, but also less severe cases at the multiple of ω, where the angular frequency is equal to the natural frequency, but the phase constant is off, causing destructive interference.

The analysis clearly shows that the model experience backside contact. Backside contact is a result of backlash and fluctuations caused by the time varying mesh stiffness due to the contact ratio alternating between number of teeth in contact. When these fluctuations coincide with the natural vibration, resonance occur, free vibrations becomes stronger and results in backside contact.

An action to reduce the effect of backlash is to increase the damping and thus the high impact on the gear teeth. This will reduce the gear hammering and thereby the gear noise.

5.1 Further Work

Some simplifications have been made in this analysis. The mesh stiffness is just an estimate and should be made an effort to numerically find the correct value. For this analysis the purpose has been to analyse the behavior and look at the response of the system. The same can be said for damping.

The shafts and bearings are considered rigid, but in reality they are not. Further work should include stiffness and damping in shafts and bearings.

The analysis does not look at the behavior of the gear for rotational speed more than 20 rad/sec or 190 rpm. The operational speed for the Bergen Engine diesel engine is at 750 rpm.

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6 | Appendix

NTNU		Utarbeidet av	Nummer	Dato
HMS	Kartlegging av risikofylt aktivitet	HMS-avd.	HMSRV2601	22.03.2011
		Godkjent av		Erstatter
		Rektor		01.12.2006

Enhet:

Dato: 23. april

Linjeleder: Torgeir Velo

Deltakere ved kartleggingen (m/ funksjon): Katrine Closs, Student   Bjørn Haugen, veileder
(Ansv. veileder, student, evt. medveiledere, evt. andre m. kompetanse)

Kort beskrivelse av hovedaktivitet/hovedprosess: Masteroppgave student xx. Tittel på oppgaven. Analysis of camshaft gear transmission for a ship engine

Er oppgaven rent teoretisk? (JA/NEI): JA   *«JA» betyr at veileder innestår for at oppgaven ikke inneholder noen aktiviteter som krever risikovurdering. Dersom «JA»: Beskriv kort aktiviteteten i kartleggingskjemaet under. Risikovurdering trenger ikke å fylles ut.*

Signaturer: *Ansvarlig veileder:* Bjørn Haugen   *Student:* Katrine Closs

ID nr.	Aktivitet/prosess	Ansvarlig	Eksisterende dokumentasjon	Eksisterende sikringstiltak	Lov, forskrift o.l.	Kommentar
	Kontorarbeid					