Deliverability of Wells in Shallow Confined Aquifers of Non-Integer Spatial Dimensions

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***Abstract*— We propose a generalized methodology to predict the deliverability of wells in fractional drainage areas. A fractional model is characterized by a single term power-law variation of the rock properties. Such a model may provide a more realistic alternative than the traditional homogeneous reservoir model. For example, the traditional models may not be applicable for sparse fracture networks. We assume the validity of the fractional model and find that the homogeneous solution is included in the generalized model as a special case. Simultaneous solution of the reservoir- and vertical lift equation leads to an algebraic equation of second degree. Hence, the prediction of the flow rate delivered into a pipe-line at any pressure is easily available. We investigate the sensitivity of the flow rate to variations in wellbore head, reservoir and flow string properties.**

Keywords; Sparse fracture network; Fractional reservoirs; Deliverability;

I. INTRODUCTION

The present study deals with an analytical model for the interaction of a confined aquifer and a well in a sparsely fractured rock. For such a network, the assumption of a homogeneous reservoir is problematic. A fractional model may be more attractive. We apply the method to predict the effect of centrifugal pumps and tubing size in fractional reservoirs.

The flow space of a fracture network obviously does not fill the bulk volume (Euclidian space). Barker [1] proposed a generalized radial flow (GRF) well test model to account for the non-filling property, which was characterized by a fractional space dimension. The non-integer space dimension, which may be thought of as the result of embedding, shows up as a real number exponent in the spatial variable.

Doe [2] pointed out that the fractional model includes non-fractured porous media. If the fracture network is self-similar and has some connectivity, then it is of fractal type (Chang and Yortsos [3]). Both descriptions lead to rock properties of single term power law expressions. The latter method is attractive since it provides a reasonably simple algorithm to construct random fracture networks based on reservoir statistics (Acuna and Yortsos [4]). In addition, the technique has a theoretical justification, (Acuna et al. [5]).

A definition of fractal dimension is given in [6] The most intuitive way to obtain a measure of the fractal dimensions is by the box-counting method which is explained in the same reference. An application of the technique to obtain a measure of the fractal density for a sparse fracture network is discussed in [7].

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The mass fracture density, d_f, which is akin to fractal density, is a key variable. The connectivity index, θ, which is related to the spectral density, [3], does not show up in the fractional model.

Due to the simple structure, the deliverability equations may be obtained by straight forward integration. Many network configurations may give rise to the same fractional dimension. Hence, the result of the calculation may be regarded as the average or expected deliverability with a given fractional space dimension and connectivity.

II. THEORY

We consider an analytical model that depends on one spatial variable. Eucledian dimensions $d = 1$, 2 and 3, corresponds to linear, sylindrical and spherical geometry, repectively.

The theory depends on the following assumptions:

i) Single phase flow of water in a sparsely fractured reservoir with rock properties of power law trends.

ii) Constant production rate from a confined aquifer.

iii) The fracture network is embedded in a Euclidian space of cylindrical shape, i.e. $d = 2$. The network may be characterized by a fractional dimension, d_f.

A. *The aquifer equations:*

We use fractal nomenclature. The same calculation methodology may be used for any conceptual model that leads to rock properties in the form of a single power law term.

Fractional mass density: $\rho_f \propto r^{d_f - d}$.

This study is restricted to Euclidian dimension $d = 2$ which corresponds to radial geometry. This choice leads to:

Porosity:

$$n = n_w r^{d_f - 2}. \quad (1)$$

Transmissibility:

$$T = T_w r^{d_f - 2 - \theta}. \quad (2)$$

Flow parameter:

$$\beta = d_f - \theta - 1. \quad (3)$$

Inflow performance relationship:

$$H_w = H_{res} - Q / PI \ . \quad (4)$$

The above equation, eq.(4), shows up as a straight line on piezometric head vs. flow rate plot. This will also be the case for an aquifer with homogeneous rock properties. The difference is that the existence of a fracture network is accounted for in the fractional model.

B. Flow Periods:

We consider steady and pseudo steady flow. These flow periods are dominated by the external boundary conditions, constant head, H_e , and a no flow boundary, $\left(\partial H / \partial r\right)_e = 0,$ respectively. An actual case will fall somewhere in between the two ideal cases. For pseudo steady flow the time derivative, $\partial H / \partial t$, is constant for all values of the spatial variable, r . Hence, the drawdown between two spatial positions becomes independent of time, which leads to a constant productivity index. The no flow boundaries may be due to physical boundaries or interference between production wells.

Pseudo steady state, Jelmert [8]:

$$PI = \frac{Q}{H_{\mathrm{Res}} - H_w} = \frac{2\pi T_w}{\frac{d_f}{(\theta+2)}\left(\frac{1}{1-\beta} - \frac{1}{\theta+2+d_f}\right) r_{eD}^{1-\beta} - \frac{1}{1-\beta} + S}. \quad (5)$$

Steady state, Jelmert [9]:

$$PI = \frac{Q}{H_{\mathrm{Res}} - H_w} = \frac{2\pi T_w (1-\beta)}{r_{eD}^{1-\beta} + (1-\beta) S} \ . \quad (6)$$

$H_{\mathrm{Res}} = H_e$ for steady state flow. $H_{\mathrm{Res}} = \overline{H}$ for pseudo-steady flow.

Observe that there is a discontinuity in the equation for $\beta = 1$. This case must be handled separately. The result is a logarithmic function rather than a power law function in the denominator. It may be shown that there is a continuous transition over the discontinuity, [10]. Hence, the homogeneous solution is included in the fractional one as a special case.

C. Tubing equations:

A shallow aquifer usually depends on pumps to bring water to the surface. We consider centrifugal pumps.

The tubing intake head is:

$$H_{in} = H_{whout} + H_{Frict} - \Delta H_{Pump}. \quad (7)$$

$$H_{in} = H_{whout} + f \frac{\rho L \frac{1}{2} v^2}{D_{it}} - \Delta H_{Pump}. \quad (8)$$

$$v = \frac{Q}{A_{it}} \ . \quad (9)$$

Tubing intake head:

$$H_{in} = H_{whout} + f \frac{\rho L \frac{1}{2} Q^2}{D_{it} A_{it}} - \Delta H_{Pump} \ . \quad (10)$$

D. Determination of deliverability:

Deliverability is the flowrate which can be fed into a pipeline at a specified pressure or head. The flowrate may be obtained from the condition that the tubing intake, H_{in}, and wellbore head, H_w, is the same. Then:

$$H_{\mathrm{Res}} - \frac{1}{PI} Q - H_{whout} + + f \frac{\rho L \frac{1}{2} Q^2}{D_{it} A_{it}} + \Delta H_{Pump} = 0. \quad (11)$$

This is an algebraic equation of second degree. The solution is:

$$Q = -\frac{\frac{1}{PI} - \sqrt{\left(\frac{1}{PI}\right)^2 + \frac{2 f \rho L \left(H_{res} - H_w + \Delta H_{Pump}\right)}{D_{it} A_{it}^2}}}{\frac{f \rho L}{D_{it} A_{it}^2}}. \quad (12)$$

The solution may also be obtained graphically as the intersect between the reservoir and tubing curves (broken line curves) as shown in Fig. 1 and Fig. 2.

III. Application of the Proposed Model

As an example application, we consider a hypothetical well with data as shown in Table 1.

TABLE I. Well and Reservoir Data

Tubing data	
Length	75 m
Inner diameter	0.2 m
Wellbore radius	0.1 m
Friction factor	0.001
Head at top	75 m
Reservoir data	
Thickness	50 m
ReservoirHead	74 m
Wellbore permeability	1.5 D
Wellbore porosity	0.2
Fluid data	
Density	1000 kg/m^3
Viscosity	0,001 Pas

Flow depends on artificial lift since the length of the tubing is 75 m while the reservoir head is 74 m only.

The flow rate depends on the interaction between the flow string (tubing and pump) and the reservoir. The effect of the flow string is shown as a broken line curves. Flow rates are given by cross-points of the flow string and reservoir curves. Then, the tubing intake, H_{in}, and wellbore head, H_w, are the same.

Observe that the flow rate increases with decreasing distance to the outer boundary. This is because resevoir drawdown is applied to a shorter distance. The flow rate increases with increasing lift capacity of the pumps.

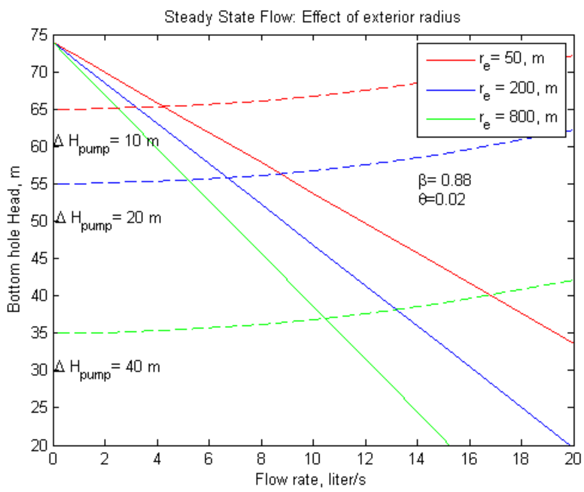

Figure 1. Effect of exterior radius, Steady state flow

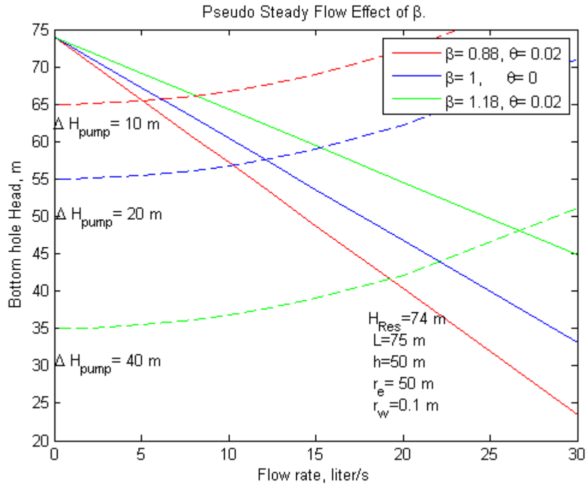

Figure 2. Effect of the flow parameter, β

Figure 2 shows the effect of the low parameter, β. The blue line, $\beta = 1$ and $\theta = 0$, corresponds to radial flow in a homogeneous reservoir. From eq.(3) we obtain $d_f = 2$ which is the same as the Eucledian dimension, $d = 2$. Hence, the fracture network fills the Eucledian space. From eq.(1) and eq.(2), this condition leads to homogeneous porosity and transmissibilty.

The green line, $\beta > 1$, is rare but possible. From eq.(3) obtain fractal dimension: $d_f = 2.2$. This case falls in between Eucledian dimensions $d = 2$ and 3, which corresponds to radial and spherical flow, respectively.

In all cases we have assumed: Atmospheric pressure at the top of the well. Length of tubing: 75 m. Inner diameter 0.2 m. Friction factor: 0.001. Head at top: 75 m. Reservoir head: 74 m. Reservoir thickness: 50 m. Wellbore permeability: 1.5 D. Wellbore porosity: 0.2.

The sensitivity of the fractal model to various reservoir variables has been discussed in [10].

IV. Discussion

The fractional model is an attractive alternative to the homogeneous radial one since the effect of sharp heterogeneities is accounted for in an approximate way. The result is an average value or expected value. Deterministic computations require detailed information which may be impossible to achieve. If a natural fracture network is self-similar and has some connectivity, the fractal model becomes plausible. The method is attractive since it provides a reasonably simple algorithm to construct random fracture networks based on reservoir statistics. In addition the technique has a theoretical justification (Acuna et al. [5]). A realization may be used for detailed local simulations by the finite element method. Many realizations may be used for analysis by geostatistics. This technique is time consuming and may involve more work than one normally is willing to spend. An analytical model will quickly predict the expected behavior in an approximate way. We recommend use of the approximate solution, especially for initial investigations.

A computer generated realization may be used for visualization of a possible fracture network. The opposite is also possible. One may observe physical realizations directly if the formation has an outcrop. Observations from outcrop studies and well testing may be integrated into the model or used to improve its credibility.

V. CONCLUSIONS

The methodology may be of interest in ground water hydrology, petroleum and geothermal engineering.

The deliverability of this class of sparsely fractured networks may be described by simple analytical solutions.

They are easy to program on a spread sheet. Generalized inflow equations based on the assumption of power law trends has been proposed.

The productivity may be improved by stimulation and densification of wells (decrease in external radius).

The generalized model may highlight problems that could go undetected by the homogeneous model.

We believe use of the generalized model may improve production forecasting and design of wells in sparsely fractured reservoirs, fractal or not.

The homogeneous model is included in the fractional model as a special case.

VI. NOMENCLATURE

A_{it} : Inner diameter of tubing, m
d_f : Euclidian dimension, Dimensionless
f : Friction factor, Dimensionless
H : Piezometric head, m
H_{Res} : Piezometric head of reservoir, m
$\overline{H}$: Volumetric average reservoir head, m
h : Reservoir thickness, m
L : Length of tubing, m
n : Porosity, Dimensionless
n_w : Porosity in vicinity of wellbore, Dimensionless
Q : Flowrate, m^3/s
r : Radial distance, m
r_D : Dimensionless distance, Dimensionless
r_e : External radius, m
S : Skin, Dimensionless
t : Time, s
T : Transmisciblity,
v : Velocity, m/s

Greek letters
Δ : Change
β : Flow parameter, Dimensionless
ρ : Density, kg/m^3
ρ_f : Fractal mass density
θ : Conductivity index, Dimensionless

Subscripts
in : Tubing intake
it : Inner tubing
out : Outflow
w : Well

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