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# Analysis of Deformation and Stress Distribution around Stopes in Mining using a Combination of Geostatistics and Numerical Analysis

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## Abstract

In the mining industry there are several methods to predict how the rock mass will behave. One of those methods is numerical analysis with modeling software, e.g. Finite Element Method (FEM). These numerical methods require good input parameters to yield good results. Today these models most likely consist of the mine geometry and a few different material property zones, like ore body and different zones between different lithologies. Since rock mass is not a homogeneous material this may not be sufficient to reproduce real conditions. A way to improve the model is to have more material property zones. A way to do that is to have more accurate input data. These data comes from testing drill cores. But it is time consuming and expensive to have extensive sampling campaigns. Solving that problem is geostatistics. With geostatistics a limited number of data points can be used to estimate material properties to areas that have little or none data points.

In this thesis geostatistical estimation is performed on mechanical properties from bore hole data. A data set with information from Rana Gruber AS provided by Steinar Ellefmo was used as input. The geostatistical estimation method ordinary kriging was performed on the data in the software Surpac. Results from the estimation were then put in the numerical analysis software Phase$^2$. In the numerical analysis, stress distribution and deformation was looked at.

It is concluded that the results clearly show that having numerous different material property zones have an effect on the stress distribution and deformation. This can mean that using this method will give a relative more accurate result, than not using geostatistics before numerical analysis.

## Sammendrag

I gruveindustrien finnes det flere metoder å vurdere hvordan berget vil oppføre seg. En av de metodene er numerisk analyse med endelig-element metode. Men denne metoden krever gode inndata for å produsere gode utdata. Modellene lagd etter denne metoden inneholder i dag vanligvis konturene til gruva og noen få soner med material egenskaper som f.eks malm, gråberg og omliggende berg. Siden bergmassen ikke er en homogen masse er dette muligens ikke tilstrekklig for å reprodusere faktiske forhold.
En måte å forbedre modellen er å ha flere soner med material egenskaper. Det kan gjøres ved å skaffe mer nøyaktige inndata. Disse data kommer fra testing av borkjerner. Men det er dyrt og tidkrevende å ha omfattende borehulls undersøkelser. For å løse dette problemet kan en benytte seg av geostatistikk. Med geostatistikk kan en ta en begreset mengde datapunkter og estimere seg frem til verdier i områder der det ikke finnes verdier.

I denne masteroppgaven utføres geostatistisk estimering på borkjerne data. Et data sett med informasjon fra Rana Gruber AS er satt opp av Steinar Ellefmo og er blitt brukt som inndata. Estimeringsmetoden som er brukt heter ordinær kriging, og dette ble utført i programmet Surpac. Resultatene fra estimeringen ble så brukt som inndata i numerisk analyse i et program som heter Phase$^2$. I denne analysen var det spenningsfordeling og deformasjon som ble sett på.

Det er konkludert med at resultatene tydelig viser at å ha flere soner med forskjellig materialegenskaper har en effekt på spenningforderling og deformasjon. Dette betyr at en slik metode kan gi relativt mer presise resultater i forhold til å ikke bruke geostatistikk før en numerisk analyse.

## Table of Contents

# List of Figures

## List of Tables

# 1. Introduction

This thesis gives an introduction to geostatistics and numerical modeling on a basic and generalized level. Numerical modeling is a method to simulate and evaluate the stability in mines and underground excavations. This is an important part of the mining process to predict how the rock mass will behave when excavated. A prerequisite for a good model and results are good input parameter(Nghia Quoc Trinh 2012). Usually the different geological units are given mechanical properties based on a mean derived from field and laboratory tests of drill cores. A numerical method could be improved if it incorporated geostatistics. Geostatistics incorporates the spatial variance in the rock mass, and the arithmetic mean that comes from basic statistics does not. Geostatistics is traditionally used in resource estimation, however it has been used on rock mechanic properties as well (Steinar L. Ellefmo 2008).

## 1.1 Objective and limitations

Geostatistical estimation is performed on a data set and then the results from this analysis are used as input in a numerical analysis. The analysis will assess the deformation and stress distributions around a selection of drifts. Focus will be on what prerequisites are needed to perform an analysis like this, and what assumptions that may be needed to make it work. Concepts that are looked at include stationarity and additivity. Pros, cons and limitations of using geostatistics for this are also discussed. The software used in this thesis is Surpac for the geostatistical analysis and Phase$^2$ for the numerical deformation and stress distribution analysis. There is a great deal of literature about geostatistics and numerical modeling and this paper will only cover the basic concepts. These methods will in this thesis be combined to illustrate how one can predict stability in tunnels underground constructions.

Most of the theory about geostatistics and numerical modeling is taken from the literature study done by me as part of a term project in the fall semester 2012,"Combining Geostatistics with Numerical Modeling**",** with some new additions**.** Geostatistics will be covered in more detail than numerical modeling. This is due to the fact that this is less commonly used in combination with mechanical properties of rock mass as this thesis covers, compared to the more traditional use with ore grade in mining operations.

A dataset containing real values and estimated values from bore holes are provided as the basis for the analysis. Geostatistical and numerical analysis are performed on this data and the results are presented and discussed.

It is important to note that the results produced in this thesis are not accurate enough to be used to predict the behavior and stability of rock mass in the mine. The results are to show how the process of an analysis like this can be performed with realistic data.

## 1.2 Introduction to Rana Gruber

### 1.2.1 The Company and mining operation

Rana Gruber is situated close to Mo i Rana in Nordland county. The company runs an iron mining operation(Ellefmo 2005).

The Kvannevann mine this thesis is looking at, started as an open pit mine, and then moved on to underground mining with Sublevel stoping mining method after 30 years of open pit mining.  In 2010 a new mining method was started, Sublevel caving, due to the higher capacity this method gives compared to Sublevel stoping. Sublevel caving is usually used on mineral deposits that are steep and continuous (RanaGruberAS 2012).

In short, Sublevel caving extracts the ore using sublevels that are developed in the ore body at regular vertical spacing (AtlasCopco 2002). Each of the sublevels features systematic layout with parallel drifts along or across the ore body. The sublevel drifts are driven from the footwall and across the ore body to reach the hanging wall. Figure 1 shows how this works in theory, and Figure 2 show how it looks at Rana Gruber, were the ore body is running parallel with the south west and north east line in the figure, as shown in Figure 3.

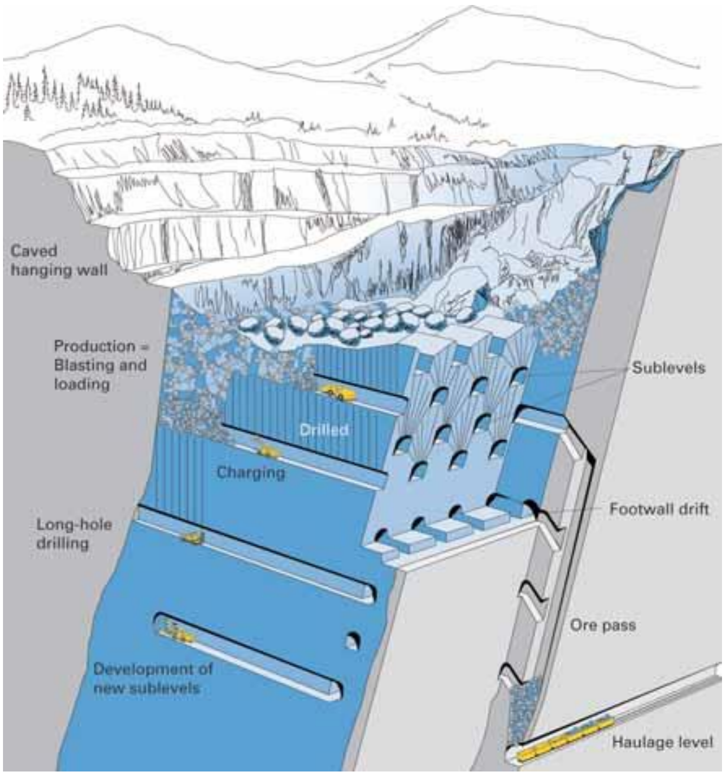


***Figure 1 Illustration of the basics of how Sublevel caving works(AtlasCopco 2002)***

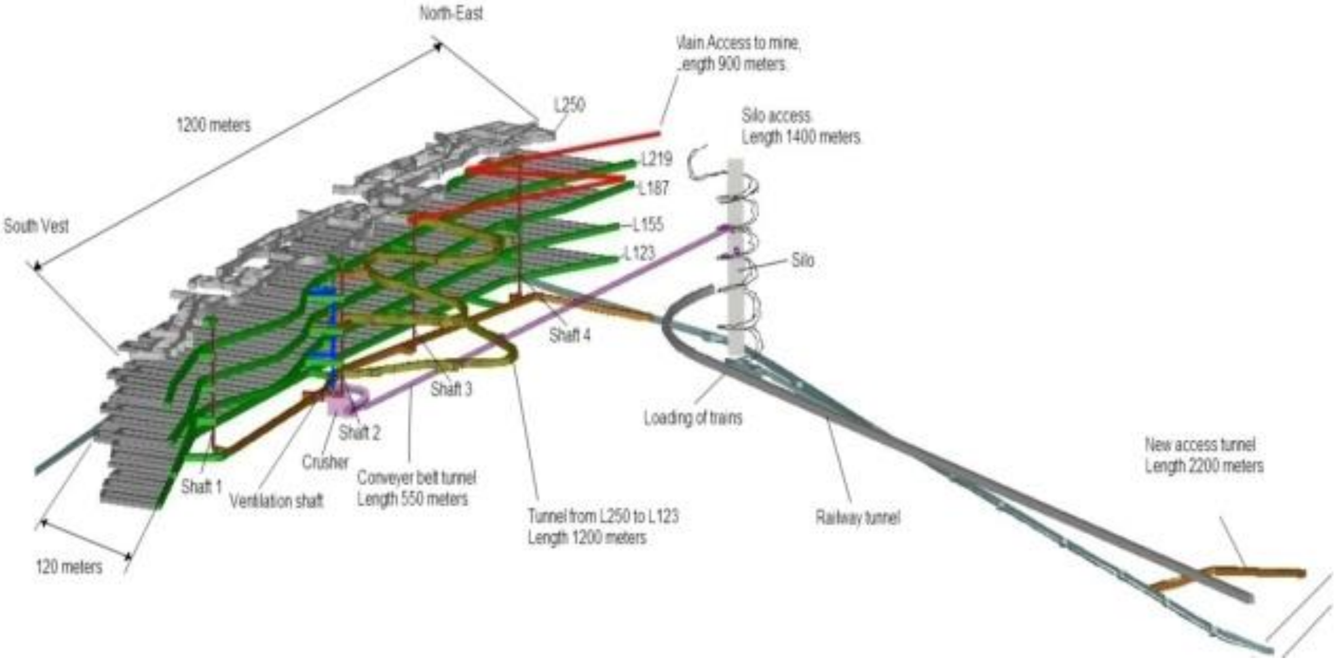


*Figure 2 Illustration of the drift layout at the Kvannevann mine (RanaGruberAS)*

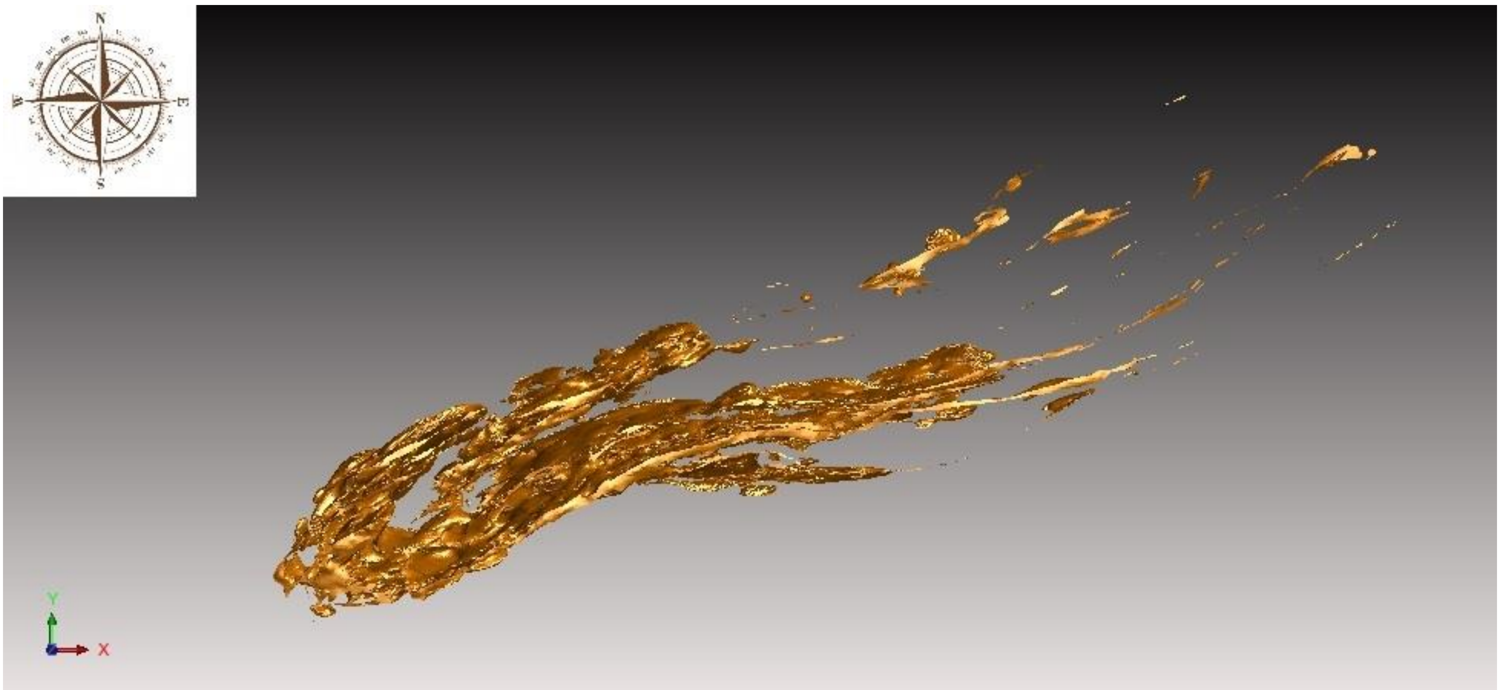


*Figure 3 Top down view of the iron ore outline in Surpac*

### 1.2.2 Geology

The geological region in this area is called the Dunderland formation. It is believed to be deposited in a submarine basin probably about 1000 Ma. Kvannevann mine is in this formation, and the dominating rocks are various types of mica schist, limestones/marbles, amphibolite and quartzite. The iron ore itself is made up of Hematite ad Magnetite. The Hematite is 97.5%-98% of the Kvannevann mine, while the Magnetite is 2.0%-2.5%. The various rocks were deposited horizontally, and then later folded so that it now is almost vertical with a dip of about 70-80$^{o}$ (Figure 4Figure 4 Vertical section of the Kvannevann mine).

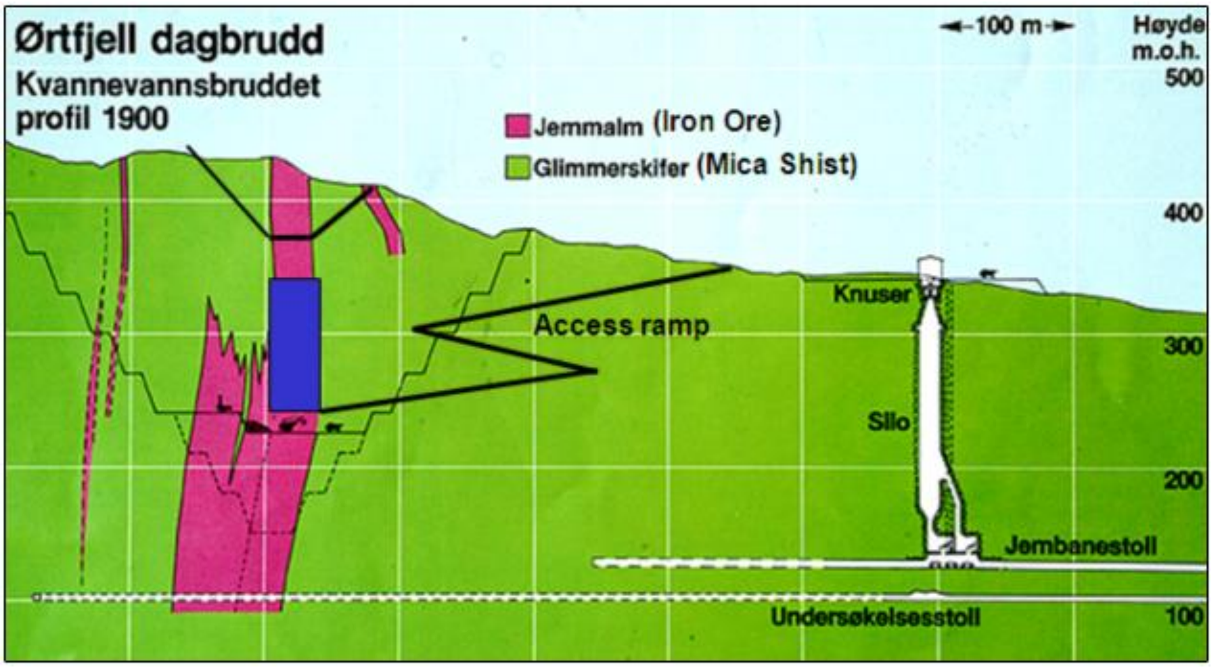


***Figure 4 Vertical section of the Kvannevann mine***

# 2. Geostatistics

## 2.1 Introduction

In the 1960's Matheron published "The Theory of Regionalized Variables and its Application" (Matheron 1971). Its application in geology and mining has given it the name geostatistics.

Geostatistics differs from normal statistics in that the data belong to some location in space(Edward H. Isaaks 1989). It simply looks at the spatial distribution of data. Often it is interesting to know the location of extreme values, a trend or the degree of continuity. Normal statistics do not incorporate the spatial features in its methods(Edward H. Isaaks 1989). In today's mining and tunneling industry the spatial variability of the rock mass is not taken into consideration(M. Stavropoulous 2007), except where there are big local variations that come from faults, contacts between different rock types and other variations. Experience and engineering judgment is the usual practice to assess the rock mass quality. Taking into account the inherent heterogeneity of the rock mass is not standard or does not get implemented in models. In fields such as hydrogeology, soil mechanics, contaminant transport and others, the use of spatial variability is well used and recognized.

## 2.2 Theory

If we assume that $h$ is the distance between two samples and we calculate the average value of a sample with respect to distance $m(h)$ (Clark 2001). This means that the distribution of difference only depends on h, and as a result the variance and the mean only depend on h. To calculate the mean the following equation can be used:

$$m*(h) = \frac{1}{n}\sum[y(x) - y(x+h)]$$

In this equation n is the number of samples and $y(x)$ is the value at point x. The * shows that the result is not theoretical and can be used when we calculate experimental values. This is the average difference in values between two samples. So in any direction over the sampled area this says that if you are $h$ distance between two points you expect the same mean values wherever you are.

### 2.2.1 Variogram

Now we look at the variance of the differences $2\gamma(h)$. This is the variogram and it varies with distance and direction. We assume that $h$ has the same difference in value where you sample it. To find the variogram use the equation below:

$$2\gamma^{*}(h) = \frac{1}{n}\sum[y(x) - y(x+h)]^{2}$$

In geostatistics we use the variogram which is $\gamma(h)$. Using the definition of the variogram, the experimental value can be calculated for many values of $h$ These results can be plotted as a graph as shown in Figure 5 where the x-axis is the distance between sample pairs and y-axis the variogram value.


***Figure 5 Variogram axis***

There is no variance when there is no distance between two points. As the distance of the samples grows large the variance will also grow larger until a distance where the samples can be assumed to be independent of each other is reached. At this point the value of the variogram will become constant and this is called the sill ($C$) of the variogram, whereas the distance is called the range ($a$). The shape described is an ideal model of a variogram and is called the spherical model. The spherical model can be calculated with the following equation:

$$\gamma(h) = C\left(\frac{3h}{2a} - \frac{1h^{3}}{2a^{3}}\right) \text{ where } h \le a$$

$$= C \qquad \text{where } h \ge a$$

This model and another model called the exponential model are commonly used. The exponential model rises slower than the spherical model and never quite reaches the sill, given as:

$$\gamma(h) = C\left[1 - e^{(-\frac{h}{a})}\right]$$

As you can see the described graph in Figure 6 shows how the two methods behave with the same range and sill

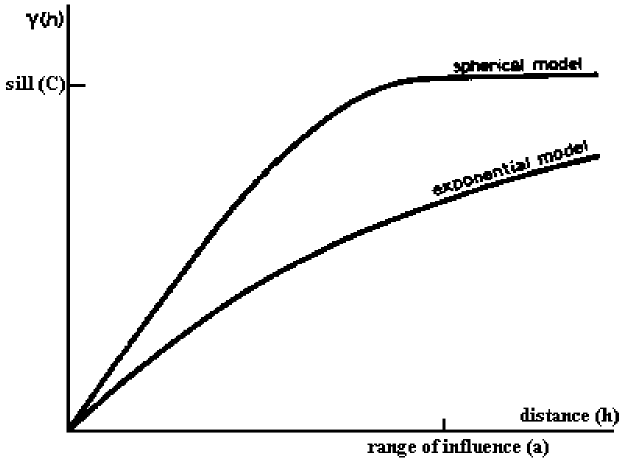


***Figure 6 Show how the two models differ***

Other variogram types include the de Wijsian model (logarithmic), Gaussian, and Linear. Another important model is the nugget effect, which shows if there is a discontinuity at the origin of the variogram.

### 2.2.2 Estimation

There are several ways to estimate values at unknown points. The most used methods are the kriging methods(Steinar L. Ellefmo 2012). Other ways to use distance as a way to estimate a point include, but are not limited to inverse distance and inverse distance squared. These methods assume that the relationship between the values only depend on the distance and nothing else. This does not incorporate high or low value areas, only the geometric placing of the samples. This is an easy and intuitive approach, however not sufficient enough.

In kriging, the method used depends on what kind of distribution the geological data has (Edward H. Isaaks 1989). Linear methods require normal distribution of the geological data. If it is not normally distributed a transformation may be applied to give it normal distribution. Or one can use non-linear kriging methods if there is lognormal distribution or no distribution at all.
The kriging estimation method is used to:

- Determine the weight of the values in known sample points using a variogram.

- Determine the value of a point that has an unknown value, based on the value of surrounding points.

One of the features of kriging is the better quality samples you have, the better the estimation will be. Another way to get better estimates is if you have evenly distributed data rather than clustered data. Kriging is based on the principle BLUE. This stands for "best linear unbiased estimator". The weights of the samples are linear combinations of the sample data.

$$y^* = \sum_i \lambda_i \cdot y_i$$

Here $\lambda$ is the weights that need to be determined. Because this is an unbiased estimator we want the mean error to equal to zero. The expected value between the estimator $y^*$ and the unknown $y$ is zero.

$$E\left[y^* - y\right] = 0$$

The focus here will be on Ordinary Kriging since it is the most commonly used kriging method. In this method the mean value is assumed unknown and stationary. Other common methods are Simple Kriging where the mean value is known and stationary. In Universal Kriging the mean value depends on location in space. In ordinary kriging we use a probability model where the mean error and error variance can be calculated. This gives the possibility of assigning weights to samples to get the average error to be zero and minimize the error variance. This gives:

$$E\left[y^* - y\right] = 0$$
$$E\left[y^*\right] = E\left[y\right] = m$$
$$E\left[y^*\right] = E\left[\sum_i \lambda_i \cdot y_i\right] = \sum_i \lambda_i \cdot E\left[y\right] = m$$

This leads to:

$$\sum_i \lambda_i \cdot m = m$$
$$\sum_i \lambda_i = 1$$

The last equation shows that the sum of the weights equals to 1.

We want the best estimator, which means that it looks to minimize the variance of the errors ( $\Phi$ ).

$$\Phi = Var\left[y^* - y\right] = 2\sum_i \lambda_i \cdot \bar{\gamma}(x_i, x_0) - \sum_i \sum_j \lambda_i \lambda_j \cdot \gamma(x_i, x_j) - \bar{\gamma}(y, y)$$

So $\Phi$ has to be minimized with regard to the weights and the requirement that the sum of the weights equals zero. To accomplish this one more parameter is introduced, the Lagrange parameter ( $\mu$ ). With the help of this parameter a system of equations can be created that will find the weights.

$$\sum_j^n \lambda_j \cdot \gamma(x_i, x_j) + \mu = \gamma(x_i, x_0)$$
$$i = 1, 2, ..., n$$

Then the minimized variance will be given as

$$\sigma_K^2 = \sum_i \lambda_i \cdot \gamma(x_i, x_0) + \mu$$

The weights can be found by writing up the equation as a matrix.

$$\begin{bmatrix} \gamma_{11} & \gamma_{12} & \bullet & \bullet & \gamma_{1n} & 1 \\ \gamma_{21} & \gamma_{22} & \bullet & \bullet & \gamma_{2n} & 1 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \gamma_{n1} & \gamma_{n1} & \bullet & \bullet & \gamma_{nn} & 1 \\ 1 & 1 & \bullet & \bullet & 1 & 0 \end{bmatrix} = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \bullet \\ \bullet \\ \lambda_n \\ \mu \end{bmatrix} = \begin{bmatrix} \gamma(x_1, x_0) \\ \gamma(x_2, x_0) \\ \bullet \\ \bullet \\ \gamma(x_n, x_0) \\ 1 \end{bmatrix}$$

When this is solved the weights for estimating the value of point $x_0$ is known.

### 2.3 Method

The steps in a geostatistical analysis that uses Ordinary Kriging includes but not limited to (Edward H. Isaaks 1989):

- Basic statistics
- Create variogram model
- Estimation

First one should check for errors in the sampling procedure the following questions should be asked:

- Is the sampling done in clusters or a regular grid, are all the samples analyzed with the same method?
- Are the composites equal?
- Are there erratic values that seem unlikely?

These are some of the issues that need to be addressed before proceeding with the analysis of the data. When handling spatial data one effective tool is to visualize the data(Edward H. Isaaks 1989). The simplest way of doing this is by creating a data posting map, where each data point has a value and is plotted where it was sampled from. This can reveal errors in the data values or location. With an irregular sampling grid this can give clues to where the data were collected, and give away trends with high/low values. Irregular grids may come from topographical conditions like hills, swamps and dense growth.

The next step can be to create a contour map. In geology one is particularly interested in anomalies which a contour map (example Figure 7) can help show. This and the data posting map will give a good impression of how the data points and their values are distributed.

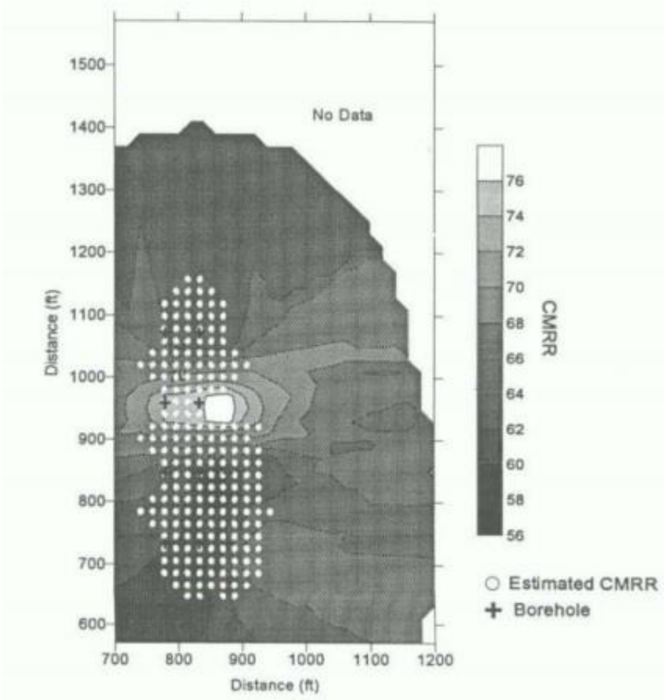


***Figure 7 Example taken from Riefenberg (1994)***

The next step would be to look for spatial continuity in the data. This is done in a variogram. The variogram describes the expected difference in value between pairs of samples with a given relative orientation. In other words, it shows continuity and the relationship of data with respect to its position. In practice many variograms are built up by more than one model, where the nugget effect is always used. A certain tolerance is used for the distance and direction. An example of this is shown in Figure 8. In this case all samples that are within the shaded region will be paired with the sample at (x,y).

First one creates an omnidirectional variogram where the directional tolerance is large. This will combine all the directions into one variogram. It can be interpreted of as an average of the various directional variograms. Often a variogram model is fitted with more than one model.

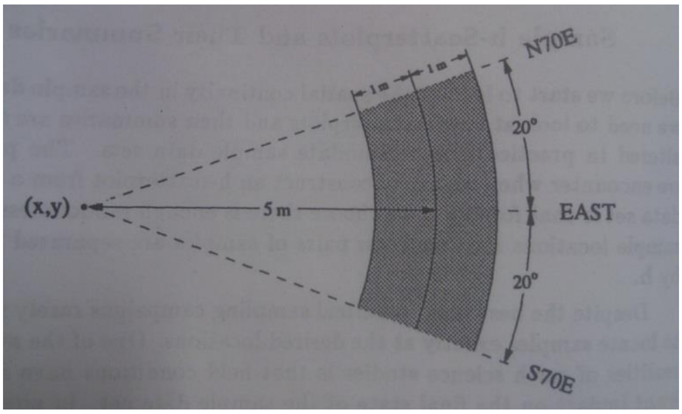


***Figure 8 Example of search strategy from (Edward H. Isaaks 1989)***

The omnidirectional variogram gives a useful starting point when establishing the model parameters. In this type of variogram the direction does not play a role, and one can therefore concentrate on finding the distance parameters that gives the clearest structure. This is usually done with a little trial and error. The next step after fitting an acceptable model to the semi-variogram is to find the anisotropy direction with the directional variograms. If the direction of anisotropy is not know from preliminary sampling and testing of the area one can look at the contour map to get an idea on how it can look. After the direction of anisotropy has been established the directional tolerance can be set so that it incorporate sufficient sample pairs to give a clear variogram.

When the variogram has been established, kriging estimation can be performed. The kriging method will assign weights to the samples already known which will then give an estimate at the unknown point.

## 2.4 Prerequisites

Stationarity and additivity are two important prerequisites so that a geostatistical analysis is a good estimation method.

For a variable to be additive it means that it has the following property: the linear average of smaller units have the same average as the linear average of a bigger unit that includes the smaller units (O. Bertoli 2003). This means that for the rock parameters, the average of the parameters in the sampled locations has to represent the average of the rock mass surrounding it.

The key aspect of stationarity is that the variance does not change with location (The importance of being stationary). There are several types of stationarity, where the most common are strictly, second order and intrinsic hypothesis.

<u>Strictly stationary</u>

Strictly stationary requires that independent of location the distribution is the same. This will say that mean, variance and all other distribution parameters have to be the same independent of location. Several non-linear kriging techniques require this type of stationarity, such as disjunctive, multigaussian, indicator and probability kriging.

<u>Second-order stationarity</u>

This level of stationarity requires that the expected value is the same everywhere and that the spatial covariance is the same too. This also means that the semi-variogram should look the same for each lag.

<u>The intrinsic hypothesis</u>

This form of stationarity does not require a constant expected value. It does, however, require the expected value of $[Z(x) - Z(x+h)]$ to be zero for all distances and directions of h. This is the form of stationarity that is used in ordinary kriging.

According to (Ellefmo 2005) there are three approaches to decide if a variable is stationary or not. If there is a trend, it may not be stationary. And if the variogram does not converge towards a sill it is not stationary. Lastly it is not stationary if the variance is plotted as a function of the mean, and if they increase together to create a proportional effect.

# 3. Numerical modeling

## 3.1 Introduction

Numerical modeling can be put into two main categories; continuous and discrete(Jing 2003). Continuous models are based on the fact that the rock mass is treated like a continuous media with a limited number of discontinuous models. The discrete models looks at the rock mass as discontinuous media that consists of blocks (block models).

The issue with getting accurate and reliable input parameters represents one of the greatest limitations for numerical analysis(Bjørn Nilsen 2009). Of the parameters that have proven to be generally difficult to assign a number are the elastic parameters Young's modulus and Poisson's ratio, and the virgin stress field. And during drill and blast the stress field will move, which makes it difficult to get good input parameters that get affected by it into the model. It is important to note that the accuracy of results from the numerical model is never more accurate than the input parameters.

## 3.2 Theory

The most common continuous method is the Finite Element Method (FEM)(L. Jing 2002). This method builds a geological model first, and then an element model around an excavation that is to be analyzed and by using numerical analysis it can simulate detailed stress distributions around the excavation. It is built up in a way so that the element density is largest around the opening to be analyzed, where it is of the greatest interest to get a detailed knowledge of the stress situation considering stability and support measures.

FEM is used to solve governing differential equations approximately. ODEs or PDEs are converted to a large system of algebraic equations and solved on computers. Quality of solution improves with increasing number of elements.(Subramnian 2009). Boundary value problems are also called field problems. The field is the domain of interest and most often represents a physical structure. The field variables are the dependent variables of interest governed by the differential equation. The boundary conditions are the specified values of the field variables (or related variables such as derivatives) on the boundaries of the field.

The body is modeled by dividing it into an equivalent system of many smaller bodies or units (finite elements) interconnected at points common to two or more elements (nodes or nodal points) and/or boundary lines and surfaces. A node is a specific point in the finite element at which the value of the field variable is to be calculated. The values of the field variable computed at the nodes are used to approximate the values at non-nodal points by interpolation of the nodal values. In the finite element analysis, the nodal values of the field variable are treated as unknown constants that are to be determined. The interpolation functions are most often polynomial forms of the independent variables, derived to satisfy certain required conditions at the nodes. The interpolation functions are predetermined, known functions of the independent variables; and these functions describe the variation of the field variable within the finite element.

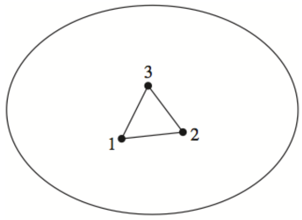


***Figure 9 Example of a triangular node (UniversityofVictoria)***

So the steps can be summarized into the following(Jan Blachowski 2012):

- Discretization of the continuum into a finite number of elements
- Analysis of particular elements of the discrete
- Formulation and solving set of equations describing the model
- Calculation of displacement, stress and strain values

## 3.3 General Method

To ensure that the numerical analysis is good the following steps should be followed (Nghia Quoc Trinh 2012):

<u>Define the problem.</u>

Here the modeler needs to define what the problem is correctly and get an understanding of it. If it is a stability issue in a mine with a certain mine method, or it is tunneling in poor or hard rock masses, and etc. This step can help selecting a suitable model and what the goal of the analysis is.

Selecting the most suitable method for the problem.

With the knowledge the modeler has from the previous step, he/she should be able to choose a suitable method for the analysis. Different methods have each its own limits, advantages and disadvantages that will affect the result of the analysis. Rock mass quality, static or dynamic, 2D or 3D are amongst the factors that go into choosing method. There might even be so complicated situations that a combination of methods would be the best choice.

Designing the geometry

Now that the method is chosen, the construction of the geometry can be started. The actual geometry in most cases can be too complicated to be put into the model. Thus it is necessary to simplify it. It is important to get a design as close as possible to the real situation, and even if it is a little off it will still give a useful analysis of the problem.

Getting the right input parameters for the model.

This is one of the most important steps in numerical analysis. The quality of the inputs directly relates to the quality of the results. To ensure this, the sampling and testing should be done needs to be done correctly. These parameters can found by sampling and site investigations, and laboratory and in-situ tests. Also considered in this article is how the quality of the input parameters can be better by using geostatistics.

Verify the model.

This step can be combined with evaluating the input parameters. The verification can be done by using known situations and run the model to see if the results are reasonable, or run the model with known stages and compare the results to observations and data from in-situ measurements. If the results are not reasonable, the model needs to be revised and improved by looking at the model and the input parameters. After the model has been improved, further analysis can be done.

Presenting and interpreting results.

There are numerous ways to present the results from a model, and the presentation of the results differs from what kind of problem that is to be analyzed. The most common results to be presented are contour plots of the stress distribution, displacement, yielded zone, distribution of pore pressure, and etc. The results also need to be interpreted if it is to give any meaning. Experience from practical work and numerical modeling is essential to interpret the results correctly.

Evaluation and follow up.

There are always uncertainties in numerical analysis and so the analyses should not be considered complete after getting the results, however a following up procedure should be done. The following up can be observations along the construction to see if the reality behaves as expected from the model. It may be necessary to monitor the behavior of the rock mass and compare with the results obtained from the model. The monitoring equipment can be extensometers to check displacements, loading cells to check loading condition, or stress sensors to monitor the stress change. The numerical analyses should be reviewed and improved any time along the following up process.

## 3.4 Input parameters

These are the parameters that are being used in numerical modeling. Input parameters for rock mass are in-situ stress, rock mass strength and displacements characteristics(Nghia Quoc Trinh 2012). The in-situ stress can be obtained through stress measurements. When there lack of stress information, it is normal to assume vertical stress caused by gravity. For the rock mass strength, the input parameters can be obtained from site investigations and mapping, laboratory or/and in-situ tests and measurements, and from reference projects. The most common parameters that comes from testing core samples are:

- E-Module (elastic modulus, stress-strain ratio)
- Poisson’s ratio ( ratio between axial and radial strain)
- Uniaxial Compressive Strength ( amount axial stress a sample can take before failure)

In Phase2, in addition to the “peak strength” there is an option to put the “residual strength”. The residual strength is the strength of the rock mass when it exceeds its peak strength. This happens when the material is plastic. When joints are included in the analysis, it is important to obtain the strength parameters for the joints or joint sets since these are a crucial part to analyze the stability(Pauli Syrjänen 2003).

## 4. Combining geostatistics with numerical modeling – small case study

The idea of combining geostatistics with numerical modeling has been studied earlier A method like this will be something that can link the data from geological mapping, core drilling investigation and laboratory tests with a numerical model. The use of geostatistics to estimate rock parameters that has an influence on stability has been done before (Pauli Syrjänen 2003, Steinar L. Ellefmo 2008). In these studies the common parameters to estimate are RQD or GSI, joint frequency, number of joints and joint roughness. All of this is determined from drill core logging, field testing or other applicable methods.

Another paper on the subject is also summarized here.

When having estimated these values, it can also be possible to use them as input in a numerical model. Stavropoulous 2007 did this with the Finite Difference Method (FDM). They used geological and borehole geotechnical data in a kriging interpolation scheme to see if they could effectively reproduce the spatial variability of rock mass quality (Rock Mass Rating, RMR). The Kriging estimation were done between borehole sampling locations, and put at the centroids of the elements of the numerical model.

The paper (M. Stavropoulous 2007) comes to the conclusion that a method like this could improve the design and help to better cope with large uncertainties and variations in rock properties. The highlights of the proposed method are:

1. Link the geological model made up of geological mapping, core drilling investigations and other measurements with a numerical method.
2. Consider the spatial heterogeneity of rock mass quality through geostatistical analysis (ordinary kriging).
3. Link the kriging model of the main geotechnical parameters (deformability and strength) with the geological- numerical model.

The extra time consumption this method has is approximately 6-20min extra effort.

A method like this can be good to predict the stress and deformation situation in a mine or underground excavation. In other studies where the stability of a mine is being modeled, they only look at the different lithology as a whole with the same strength parameter throughout (e.g. (T. Villegas 2008)). This comes from an arithmetic mean from all the borehole samples. If a geostatistical method is applied beforehand it is possible to have different domains within each lithology. This could increase the accuracy of the results for the modeled areas.

# 5. The use of Geostatistics in Surpac

Surpac is a geology and mine planning software(Surpac 2013). Its tools and applications include drill hole data management, geological modeling, block modeling, geostatistics, mine design, mine planning, resource estimation and more.

It was chosen in this thesis since it can handle drill hole data, block modeling and geostatistics which are critical for this thesis.

To begin with the bore hole data is imported to Surpac, which is done by setting it up as a database. Then this database is converted into a string file that contains all the information about the drill holes (composites, depth, location, mechanical properties of logged drill cores etc.). With this string created, the software can perform analysis of the data from this.

First off with the geostatistical analysis is basic statistics. Mean, std. dev., histograms etc. is calculated in the program. This can give a better understanding of the statistical properties of the data.

Now it is time to create experimental variograms and variogram maps. Omnidirectional variograms are created first to get a feeling of how the data is distributed, and to get a pointer of what the variogram modeling parameters will be. Then the directional experimental variograms are created. These are created in evenly spaced directions to cover a half circle. Usually the data have a structure or a shape where the data are more correlated to each other in certain directions. If the structure is in the shape of a plane with dip direction and dip, directional variograms are created down the plane and perpendicular to the plane to get a better estimate of the anisotropy and direction of continuity.

After the experimental variograms are created a variogram model is created based on the parameters the omnidirectional experimental variograms gave. Then they are adjusted to fit the experimental models. This is when the anisotropy with major and minor axis of continuity is established. These variogram models are used as the search parameters that are being used later in the kriging estimation.

Now a block model is created over the area that is to be estimated. The blocks have different size in different directions so that they are longer in the direction of continuity. Attributes that are getting estimated are given to the model, so it knows what attributes to receive. And it is constrained by various things like topography, ore body etc.

Next step now is estimation, with kriging, inverse distance or other estimation methods. With ordinary kriging the search parameters are the ones from the variogram models. Together with this information and the orientation of the search ellipsoid that gives the anisotropy.

After all that the block model can be constrained even further by removing all blocks that have no value. This will give an impression of high and low areas. At last the estimated block model can be sliced into sections where information can be extracted to be used for further analysis.

# 6. Results

## 6.1 Results from Geostatistical Analysis

### 6.1.1Basic Statistics

These are the results from the basic statistics performed in Surpac (Table 1)

***Table 1 Basic Statistics from Surpac***

| Variable | EMODULE | JPRM | POISSON | UCS |
|---|---|---|---|---|
| | | | | |
| Number of samples | 21750 | 12693 | 21750 | 21750 |
| Minimum value | 5,0 | 0,33 | 0,01 | 20,3 |
| Maximum value | 98,9 | 38,67 | 0,49 | 270,1 |
| | | | | |
| Mean | 21,9 | 4,72 | 0,24 | 84,5 |
| Median | 20,5 | 4,00 | 0,24 | 83,1 |
| Geometric Mean | 20,3 | 4,13 | 0,22 | 79,7 |
| Variance | 76,4 | 6,83 | 0,01 | 781,6 |
| Standard Deviation | 8,7 | 2,61 | 0,08 | 28,0 |
| Coefficient of variation | 0,4 | 0,55 | 0,34 | 0,3 |
| Skewness | 1,0 | 1,87 | 0,20 | 0,4 |
| | | | | |
| 10,0 Percentile | 12,0 | 2,00 | 0,12 | 49,6 |
| 20,0 Percentile | 14,4 | 2,67 | 0,18 | 59,3 |
| 30,0 Percentile | 16,5 | 3,00 | 0,20 | 66,7 |
| 40,0 Percentile | 18,5 | 3,67 | 0,22 | 74,8 |
| 50,0 Percentile (median) | 20,5 | 4,00 | 0,24 | 83,1 |
| 60,0 Percentile | 22,7 | 4,67 | 0,26 | 91,4 |
| 70,0 Percentile | 25,3 | 5,33 | 0,27 | 99,7 |
| 80,0 Percentile | 28,7 | 6,33 | 0,29 | 109,2 |
| 90,0 Percentile | 33,8 | 8,00 | 0,33 | 121,0 |
| 95,0 Percentile | 38,1 | 9,67 | 0,39 | 130,7 |
| 97,5 Percentile | 42,2 | 11,67 | 0,43 | 138,2 |

In Figure 10, 11, 12 and 13 histogram and cumulative percentage curve graphed together for each of the parameters.

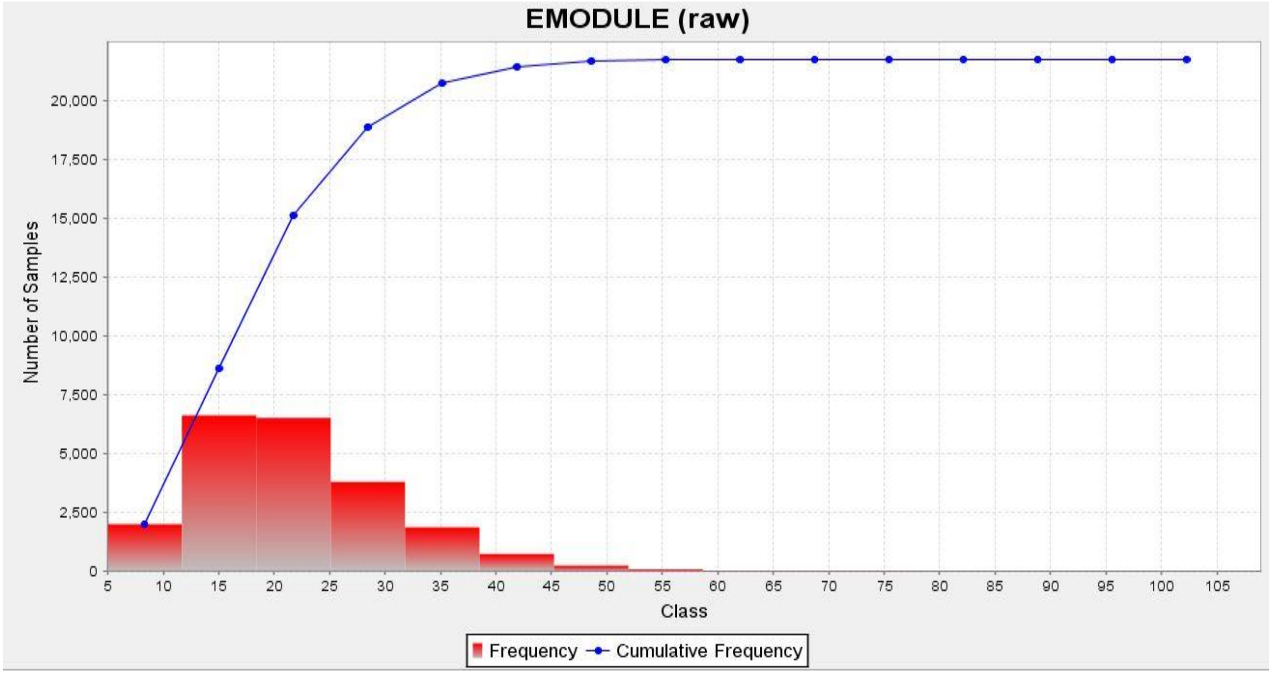


*Figure 10 Histogram and cumulative percent of Emodule*

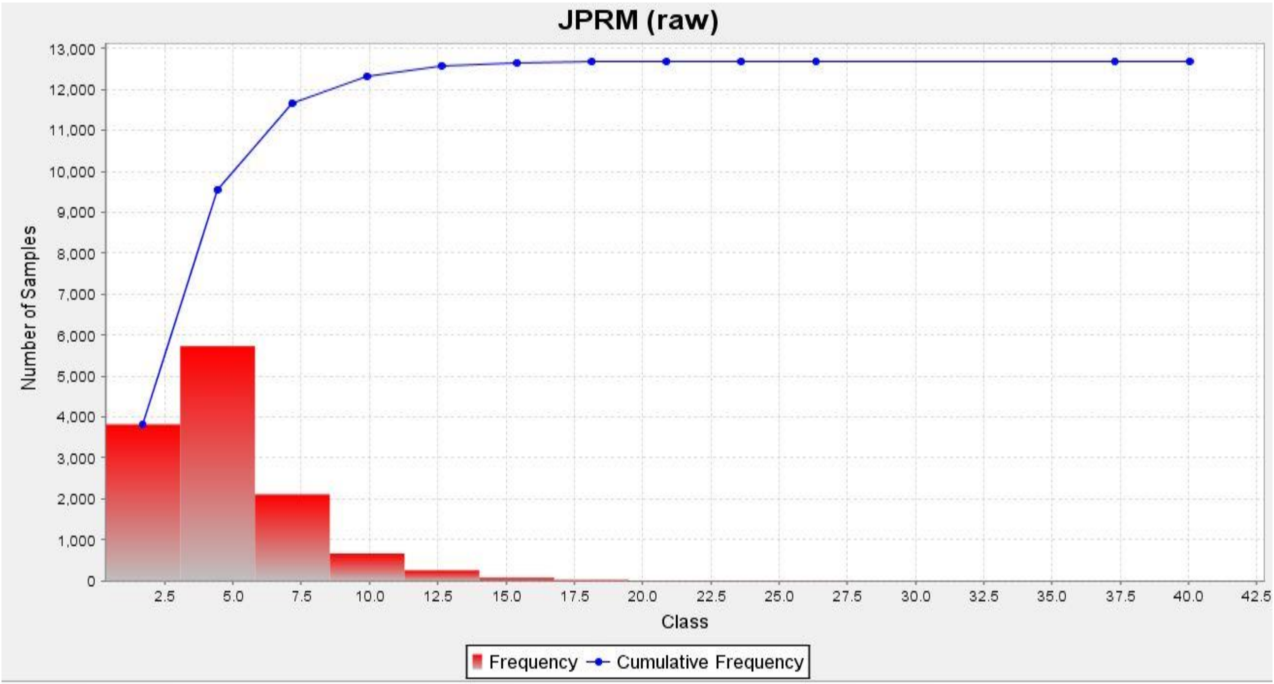


*Figure 11 Histogram and cumulative percent of JPRM*

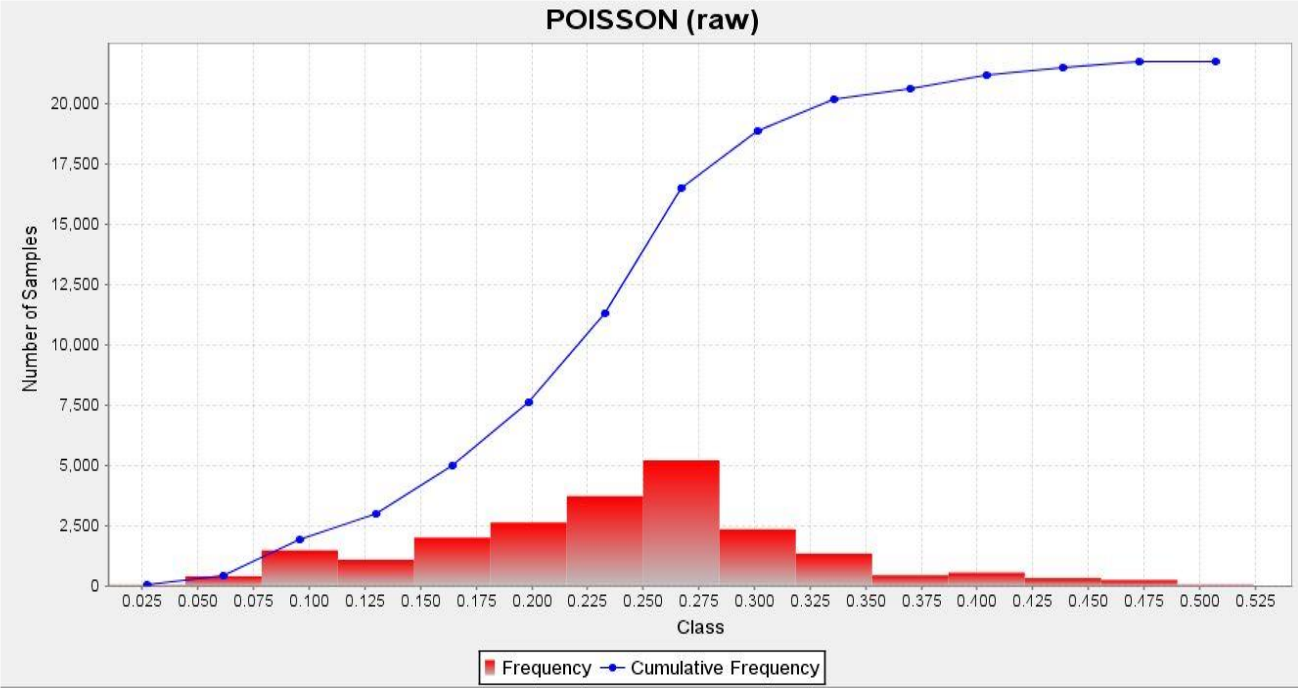


*Figure 12 Histogram and cumulative percent of Poisson's ratio*

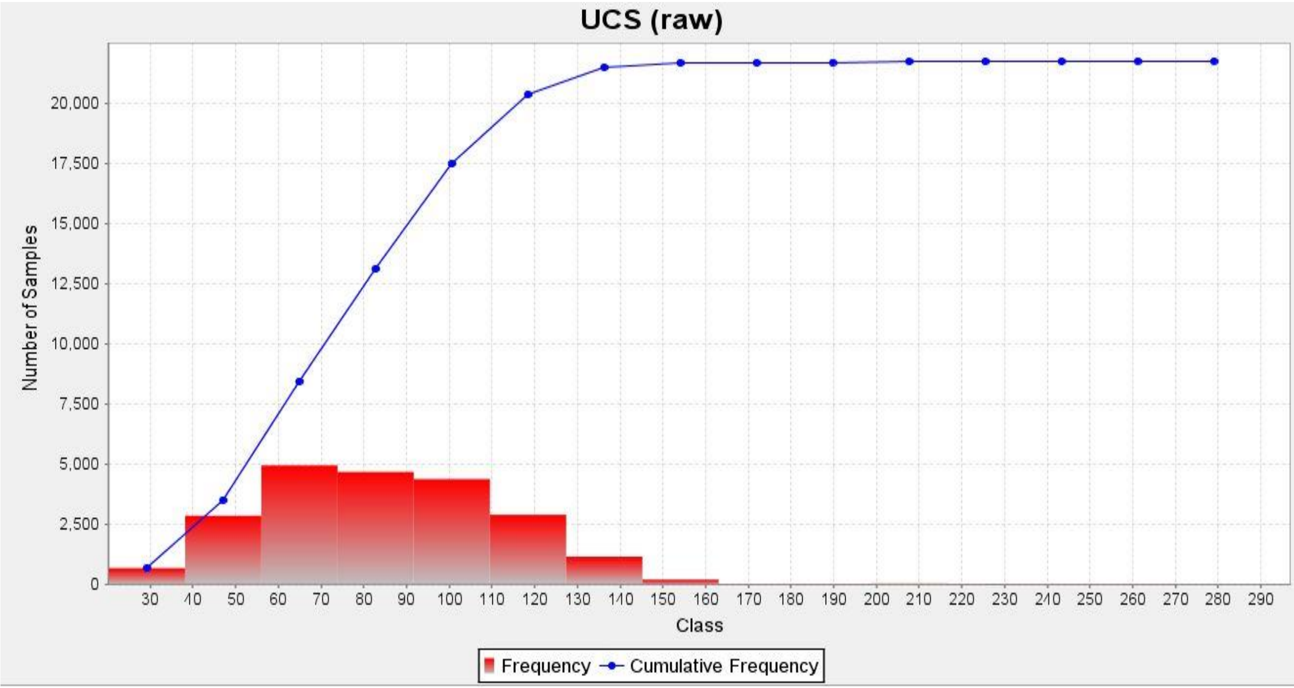


*Figure 13 Histogram and cumulative percent of UCS*

### 6.1.2 Variograms

The next sections are divided into the 4 different parameters that are estimated. Each parameter has experimental variograms for different directions and horizontal, parallel to the plane (plane means how the lithology is and the different layers of rock make up a plane) and perpendicular to the plane. E-modulus is shown in Figure 14, 15 and 16.

**<u>E-Module</u>**

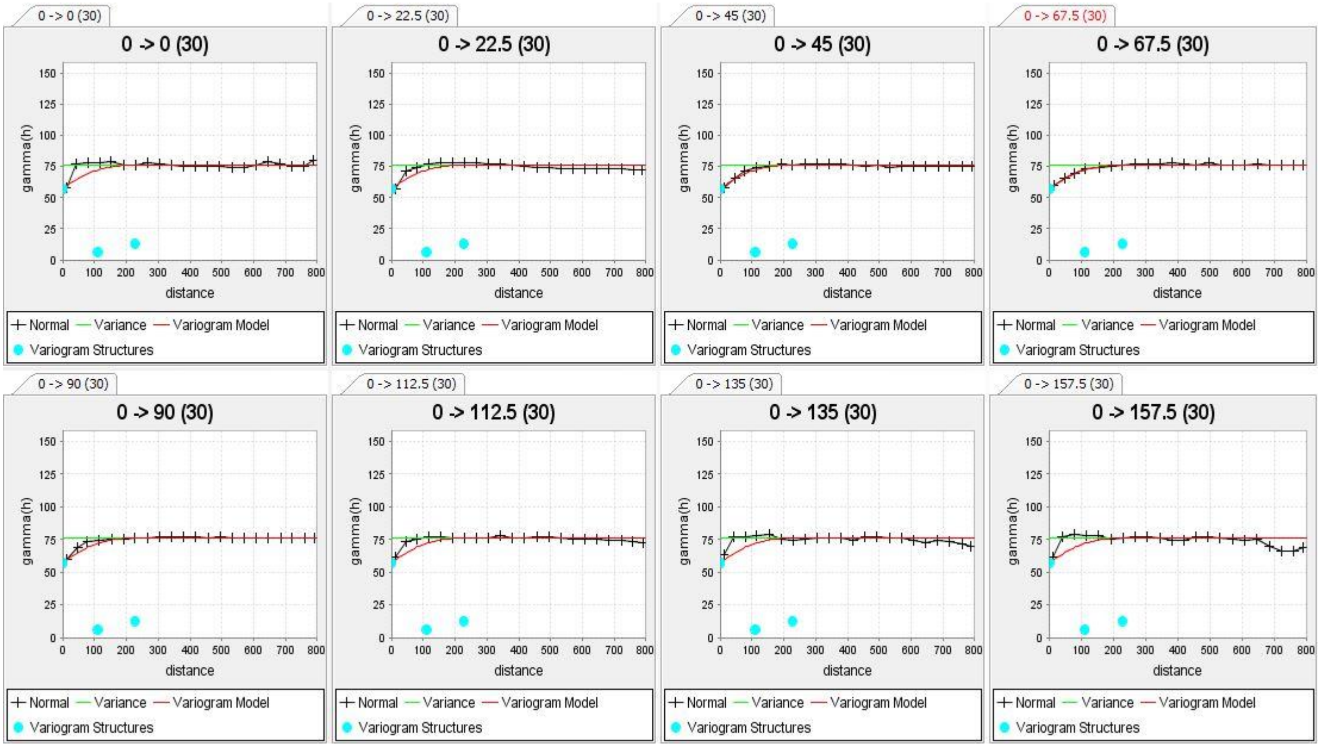


*Figure 14 Experimental Variogram Horizontal*

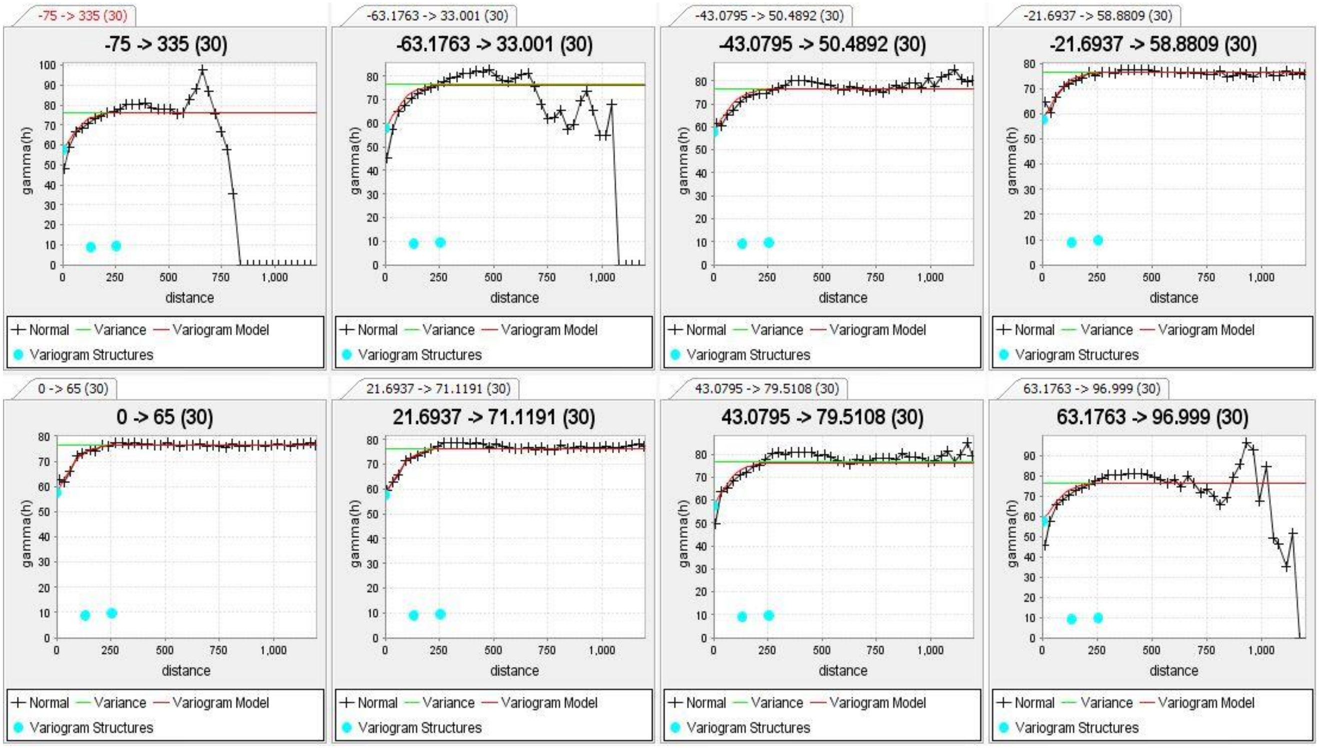


*Figure 15 Experimental Variogram parallel to plane*

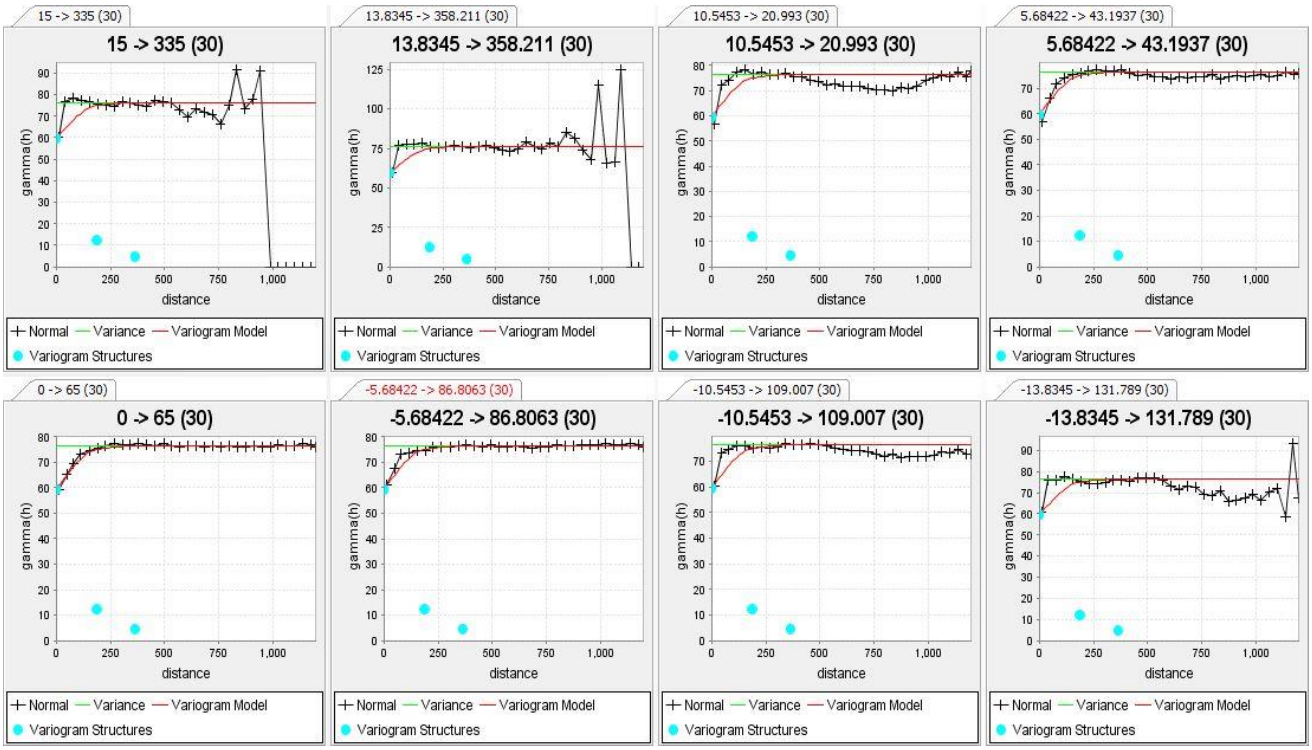


*Figure 16 Experimental Variogram perpendicular to plane*

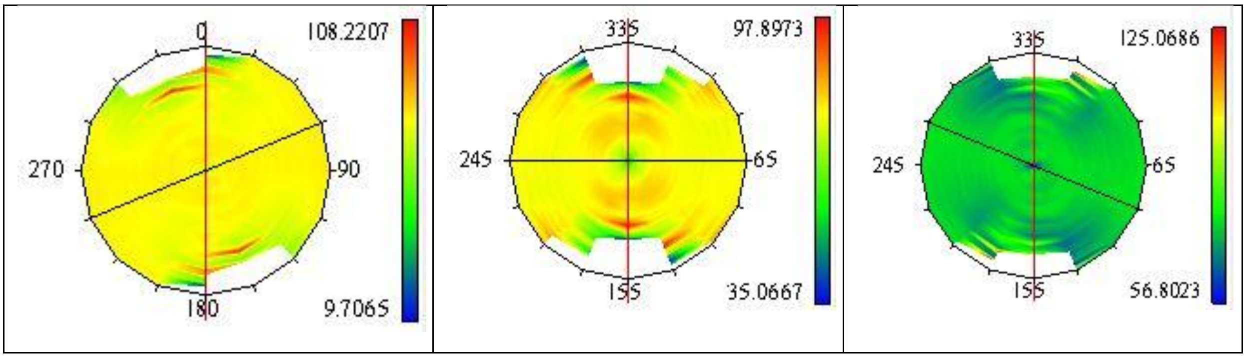


***Figure 17 Variogram maps horizontal, parallel and perpendicular to plane indicating direction of maximum continuity***

In the variograms maps in Figure 17Figure 19, 21 and 25 the direction of maximum continuity is the line that is not vertical. The maps that indicate parallel and perpendicular to the plane are turned 65$^{o}$ counter clockwise. JPRM is shown in Figure 18, 19 and 20.

## <u>Joints Per Meter</u>

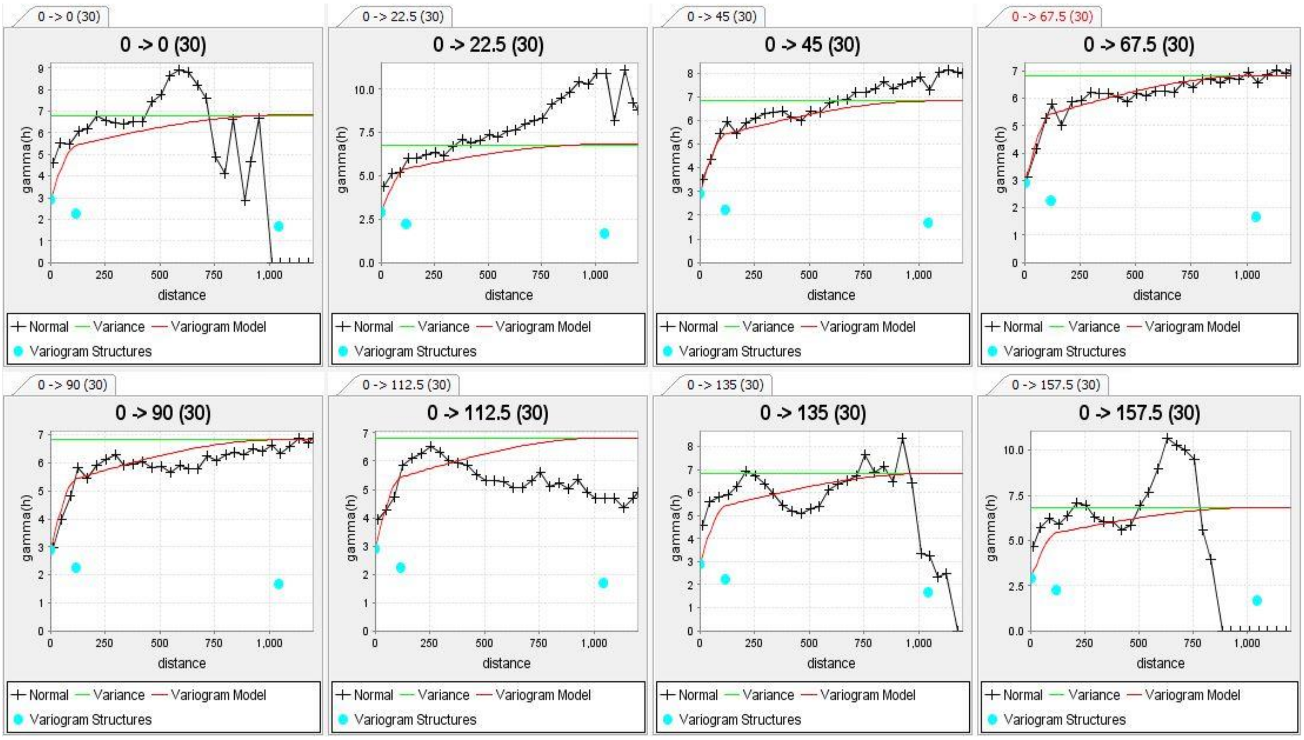


***Figure 18 Experimental Variogram horizontal***

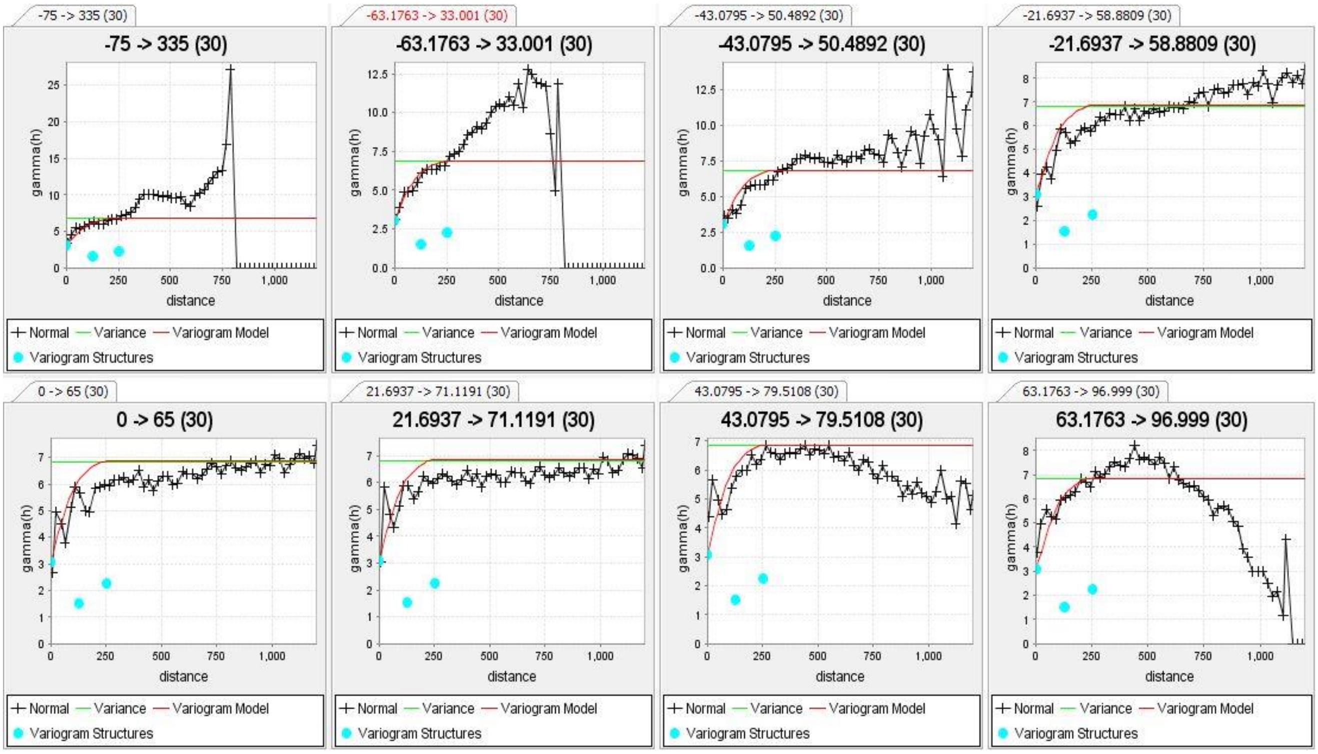


***Figure 19 Experimental Variogram parallel to plane***

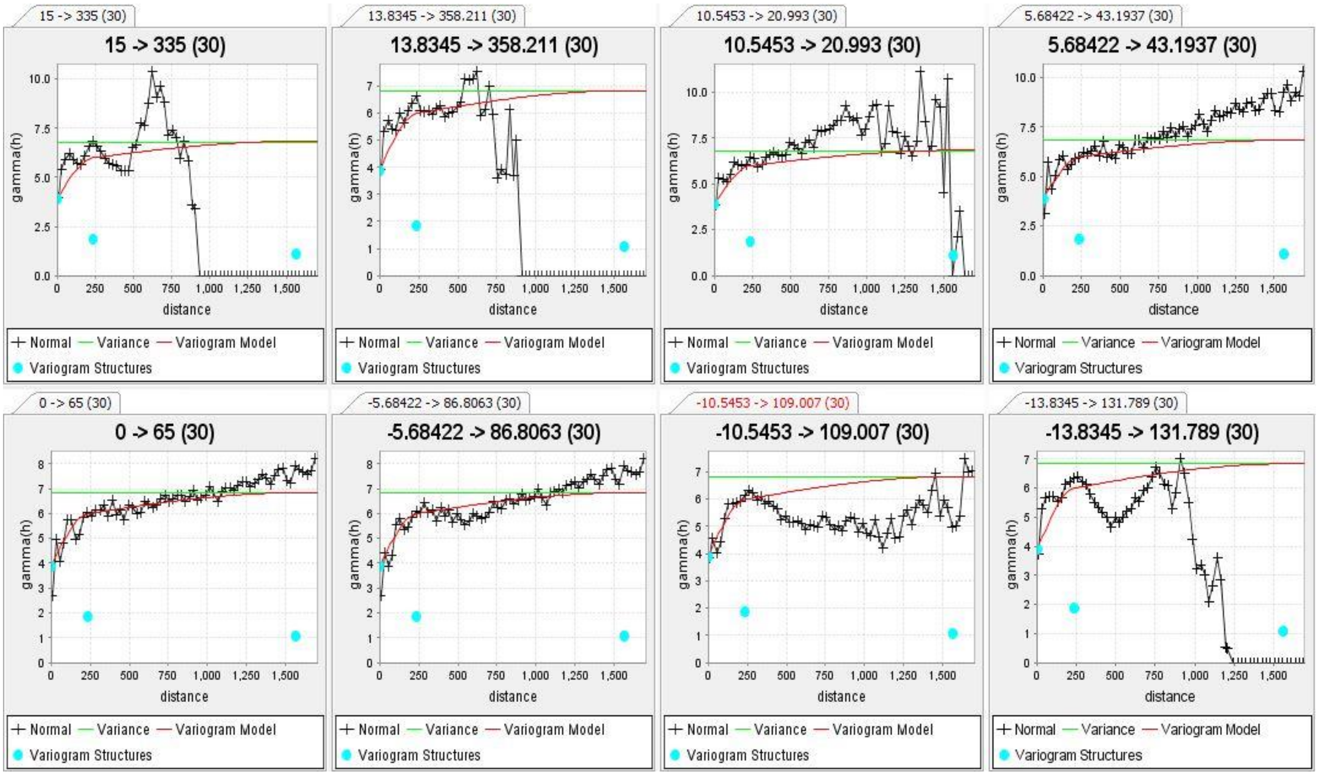


***Figure 20 Experimental Variogram perpendicular to plane***

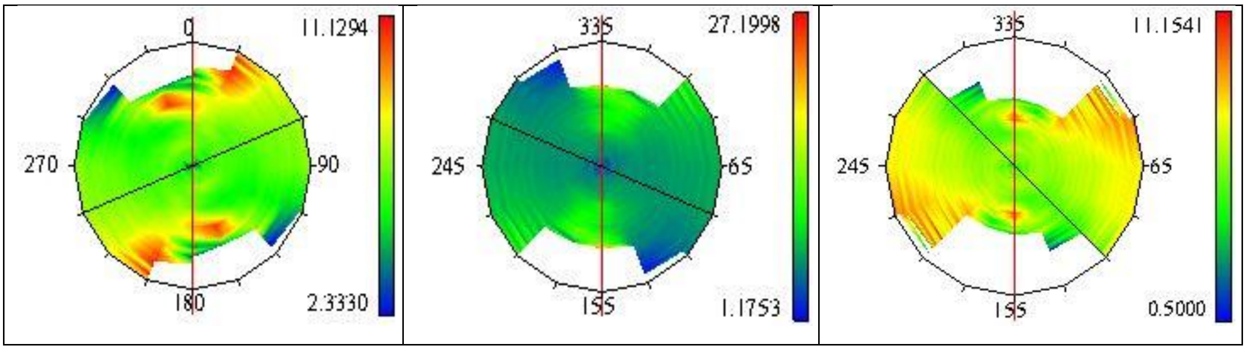


***Figure 21 Variogram maps horizontal, parallel and perpendicular to plane indicating direction of maximum continuity***

## **Poisson's Ratio**

Poisson's ratio is shown in Figure 22, 23 and 24.

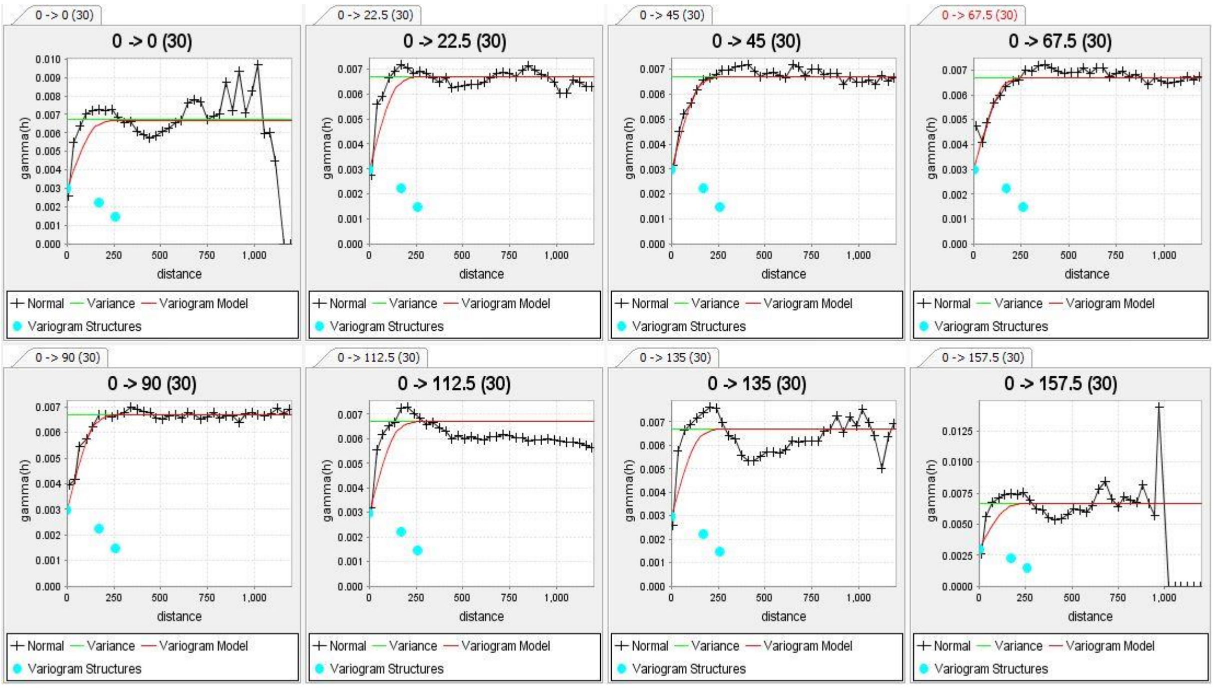


***Figure 22 Experimental Variogram horizontal***

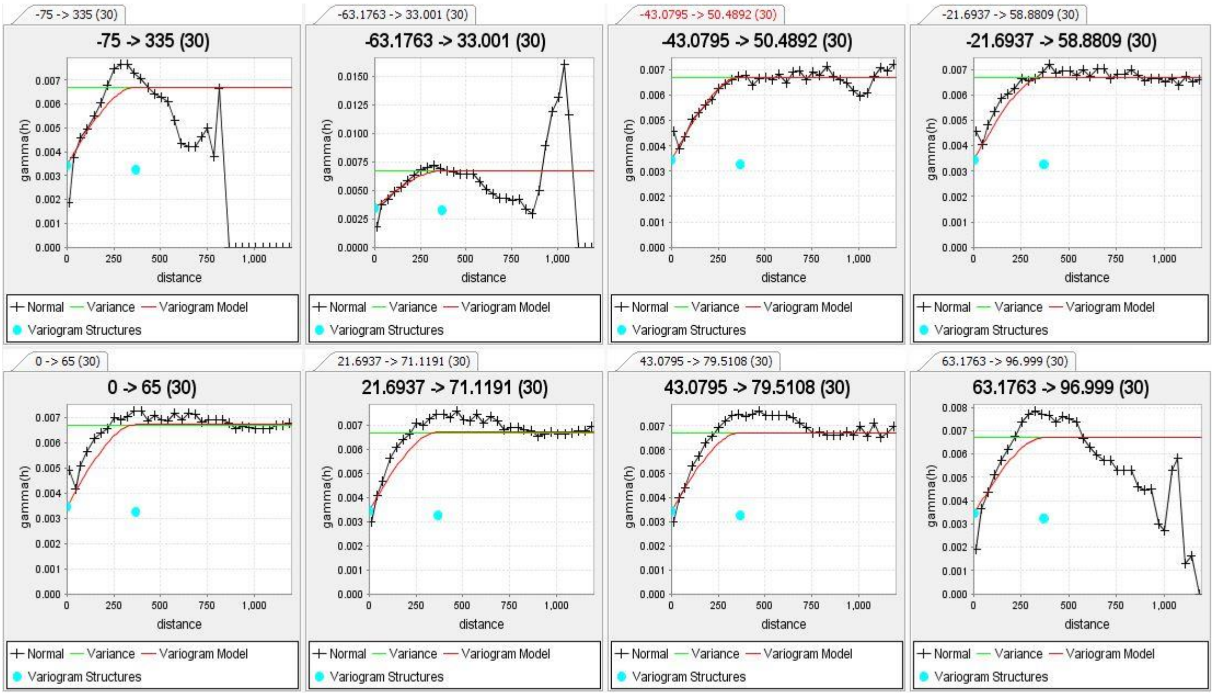


***Figure 23 Experimental Variogram parallel to plane***

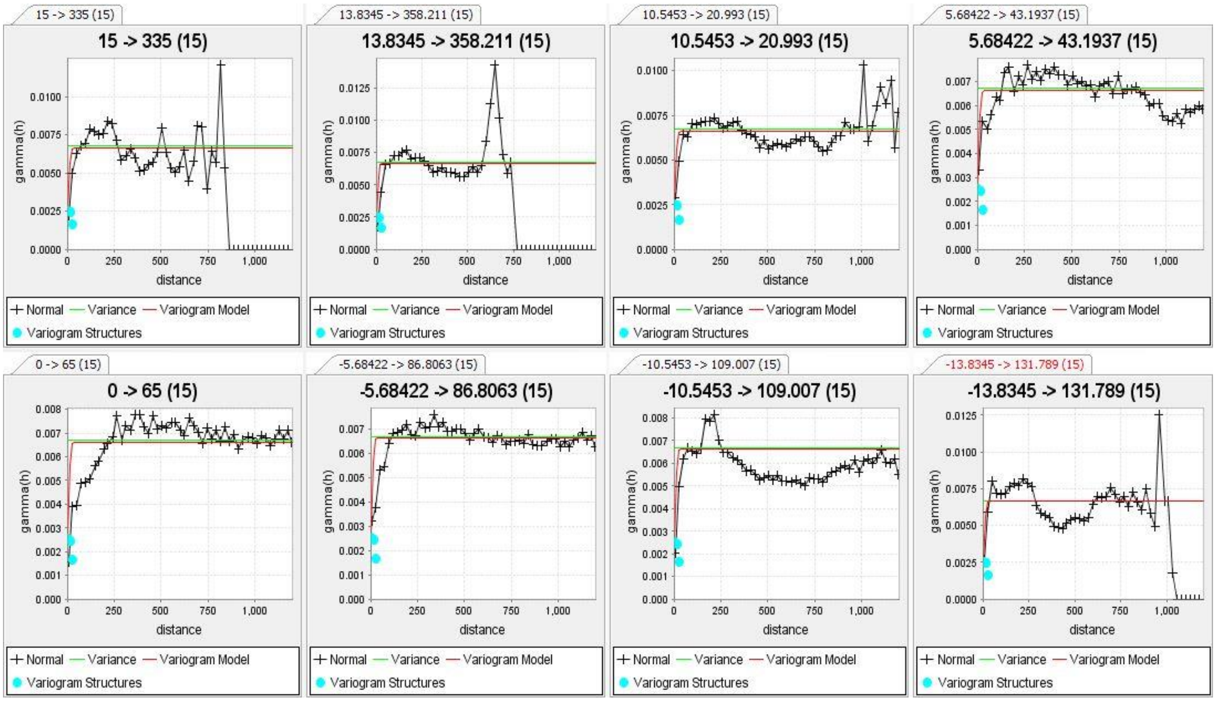


***Figure 24 Experimental Variogram perpendicular to plane***

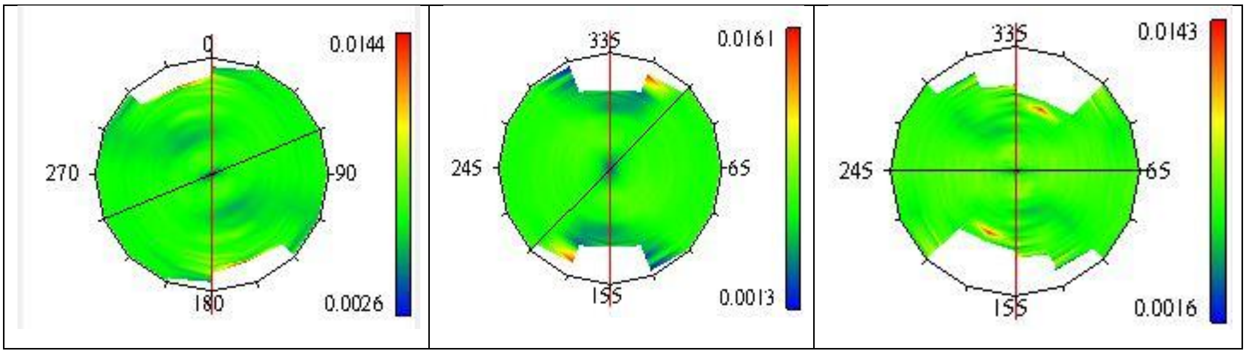


***Figure 25 Variogram maps horizontal, parallel and perpendicular to plane indicating direction of maximum continuity***

## <u>Uniaxial Compressive Strength</u>

UCS is shown in Figure 26, 27 and 28.

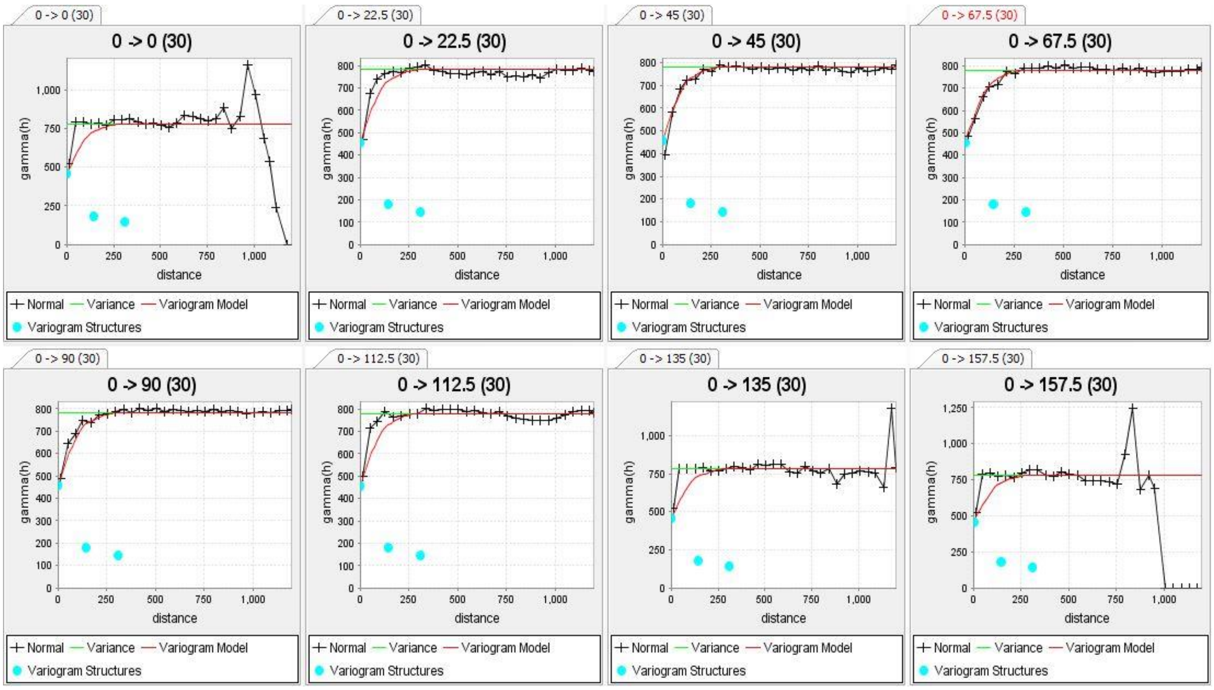


***Figure 26 Experimental Variogram horizontal***

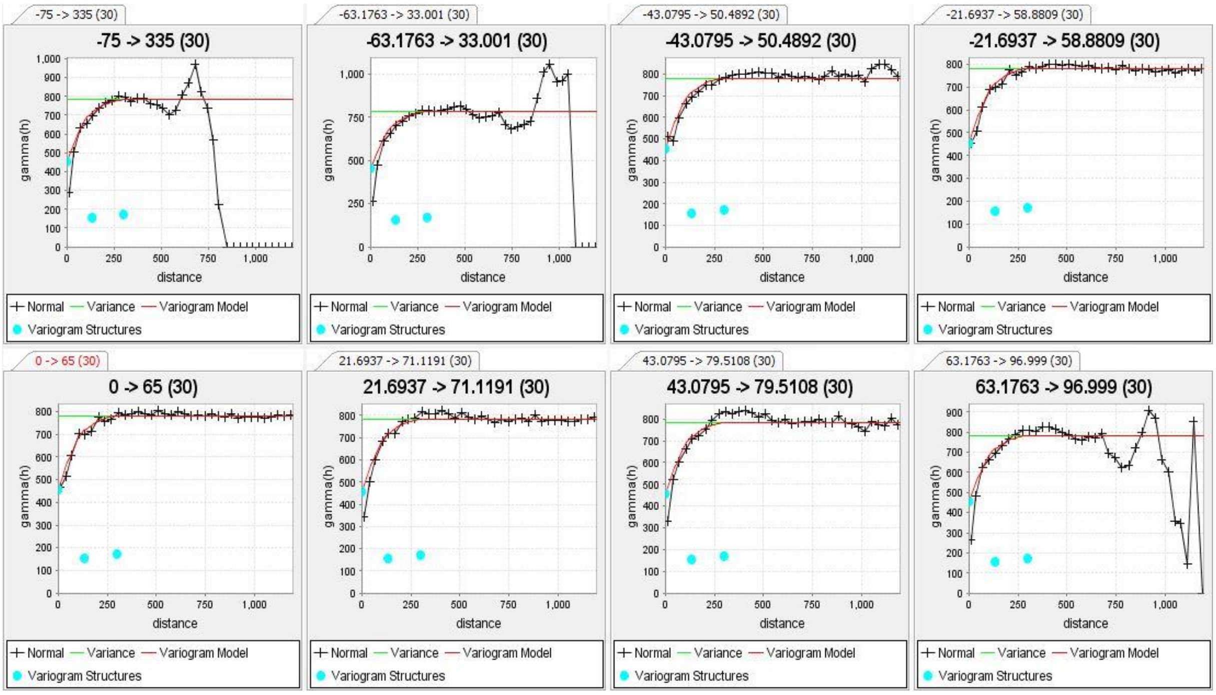


*Figure 27 Experimental Variogram parallel to plane*

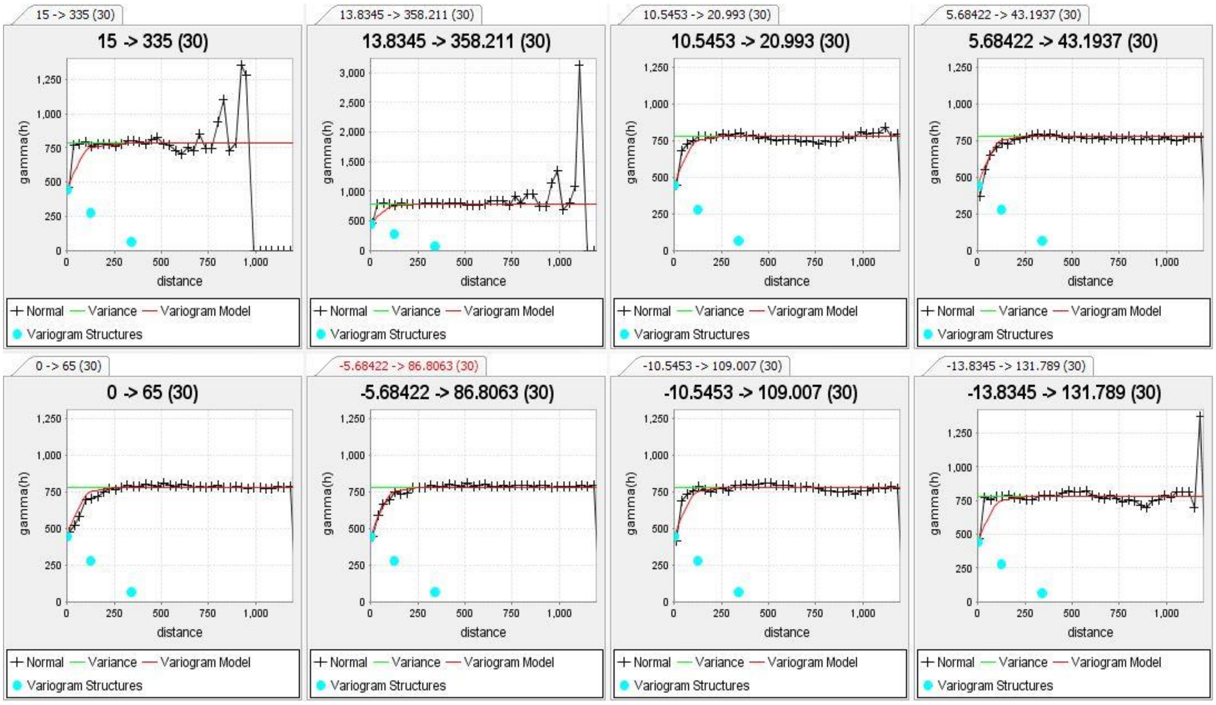


*Figure 28 Expermental Variogram perpendicuar to plane*

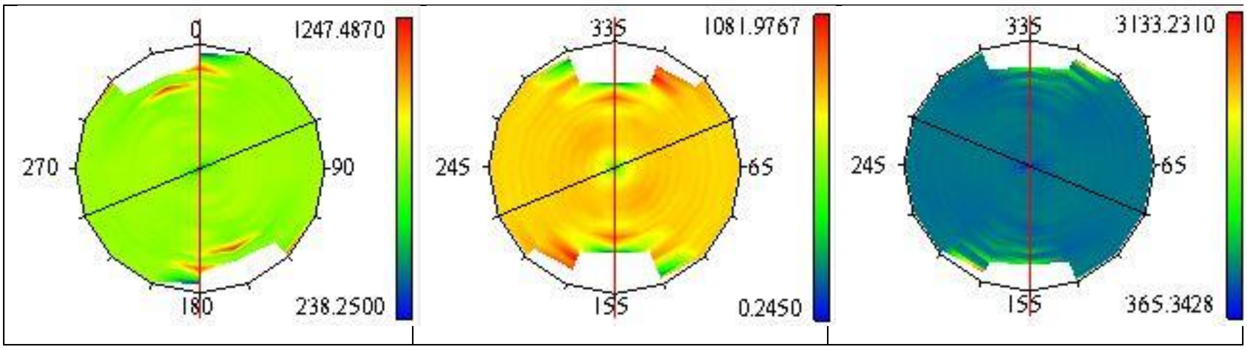

***Figure 29 Variogram maps horizontal, parallel and perpendicular to plane indicating direction of maximum continuity***

The anisotropy data used for the search ellipsoid is given in Table 2

***Table 2 Anisotropy data***

| | Emodule | JPRM | Poisson | UCS |
|---|---|---|---|---|
| Lag | 38 | 36 | 34 | 32 |
| Range | | | | |
| Horizontal | 248 | 880 | 232 | 303 |
| Parallel to plane | 223 | 318 | 216 | 246 |
| Perpendicular to plane | 34 | 550 | 62 | 30 |
| | | | | |
| Major/minor | 7,3 | 2,8 | 3,7 | 10,1 |
| Major/semimajor | 1,1 | 1,6 | 1,1 | 1,2 |

### 6.1.3 Block Model

These are top down view of the block model for the different parameters. Each block is 60 m high, 60 m long in easterly direction and 30 m wide in north direction. Each attribute have a constraint to not show blocks with no values. The blocks models are in Figure 31, 33, 35 and 37. There are three sections (Figure 30) named section 1, 2 and 3 from right to left. The sections are sliced for each of the indicated locations in the block model, and are viewed from southwest to northeast as vertical sections. In Figure 32, 34, 36 and 36 section 2 has been sliced to show how the model looks in a vertical profile.

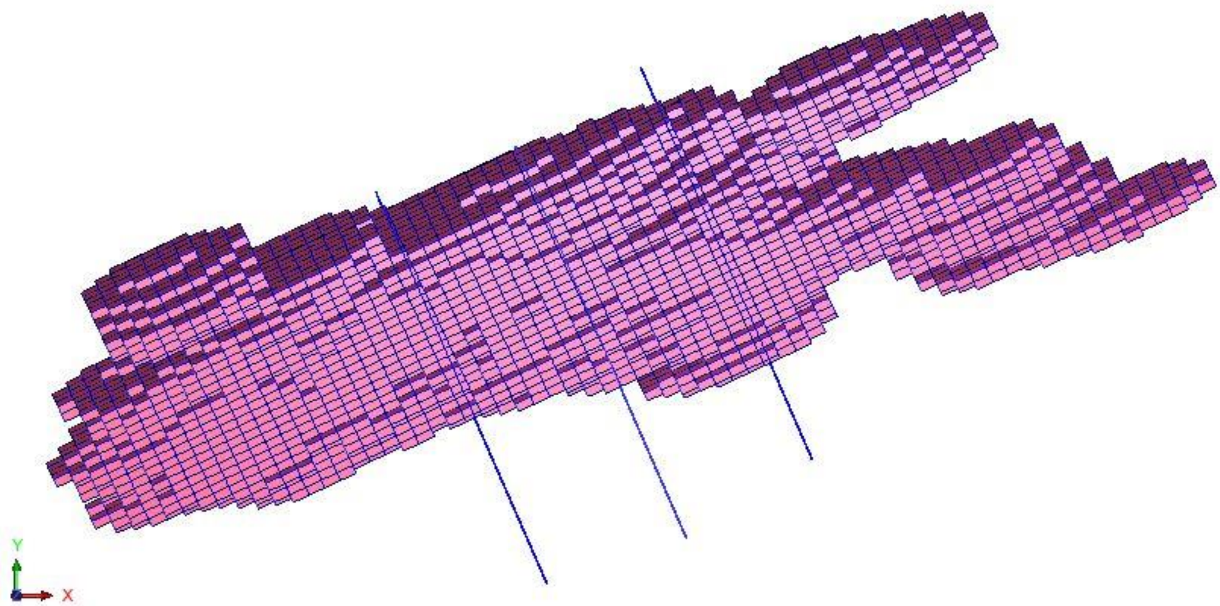


***Figure 30 Block model showing three section planes***

## E-Modulus

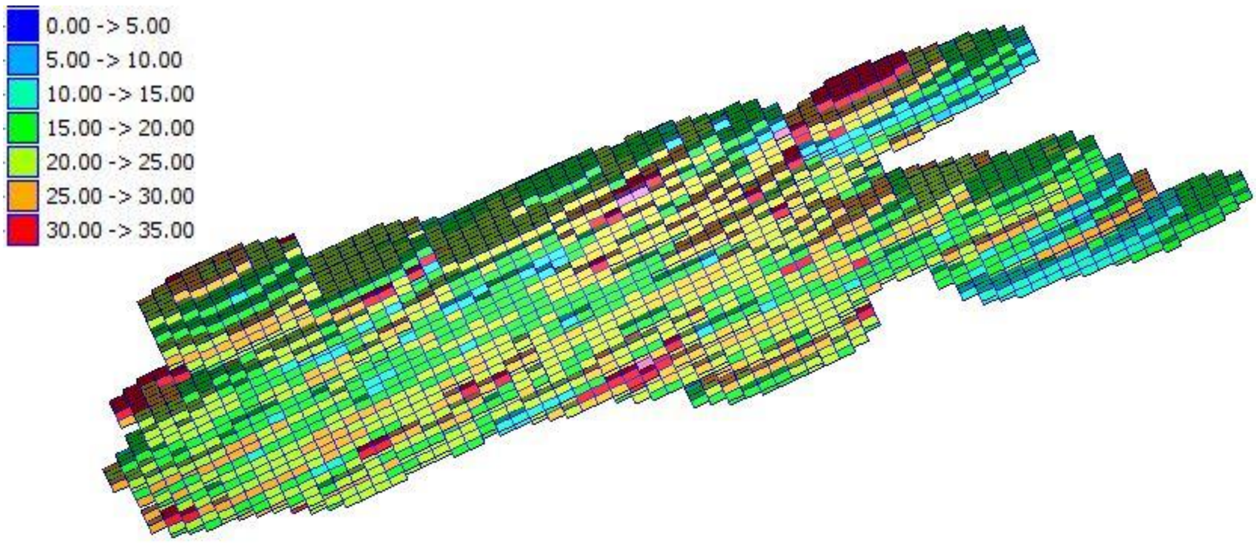


*Figure 31 Overview of E-Modulus Block Model*

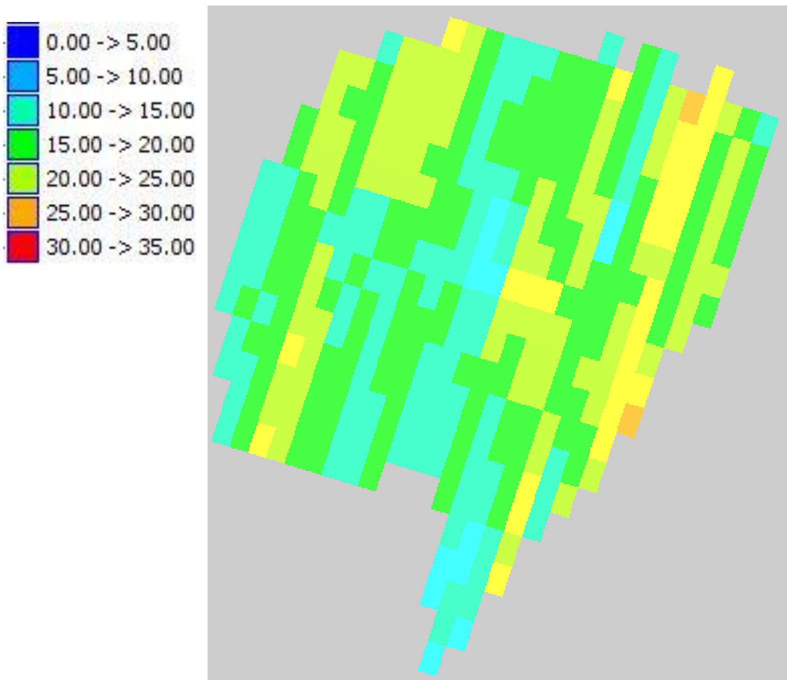


*Figure 32 Section view of block model*

**Joints Per Meter**

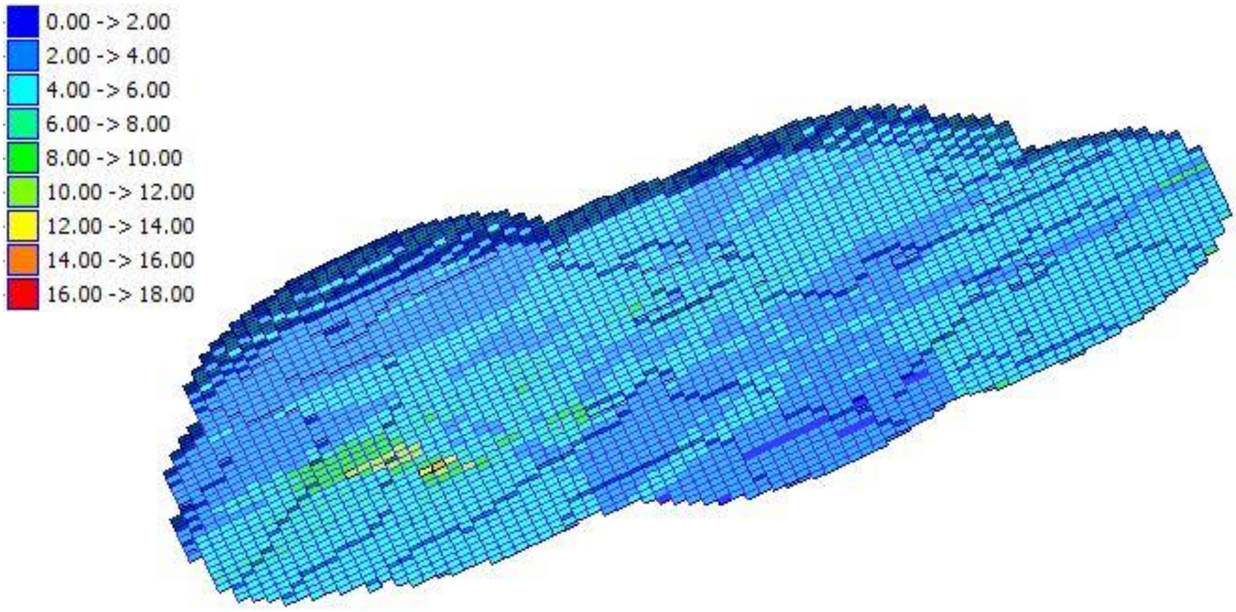


*Figure 33 Overview of JPRM block model*

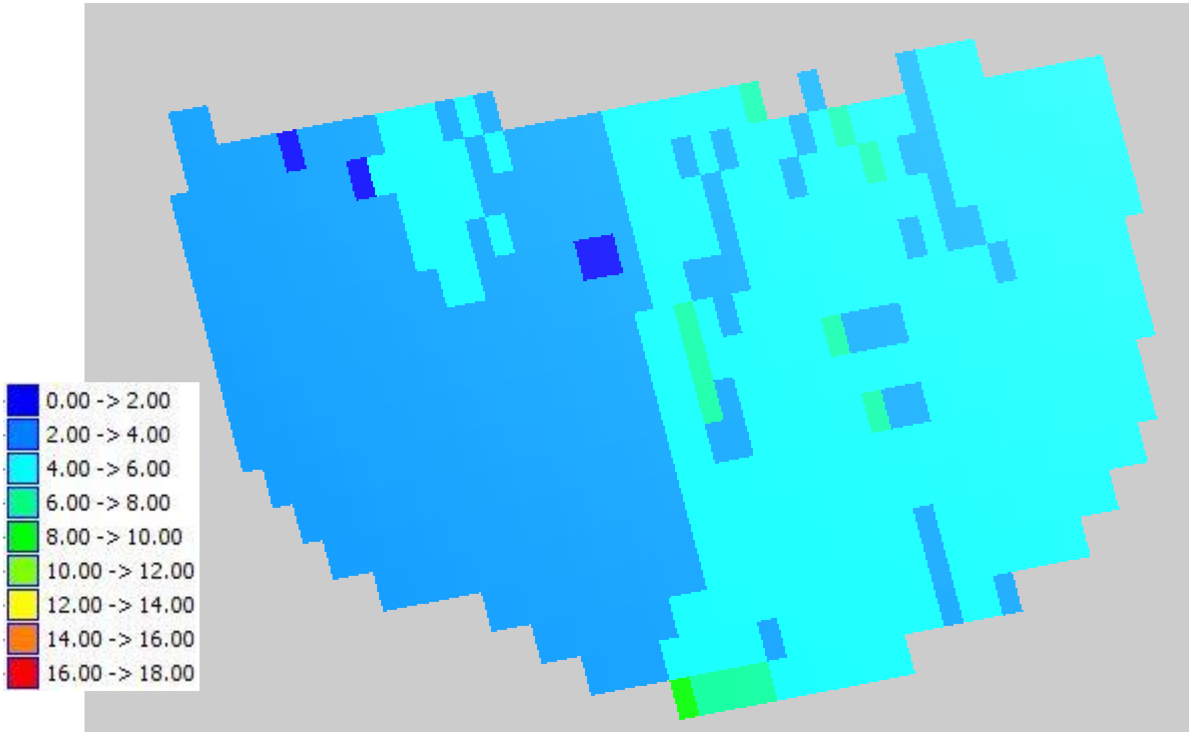


*Figure 34 Section view of block model*

**Poisson's Ratio**

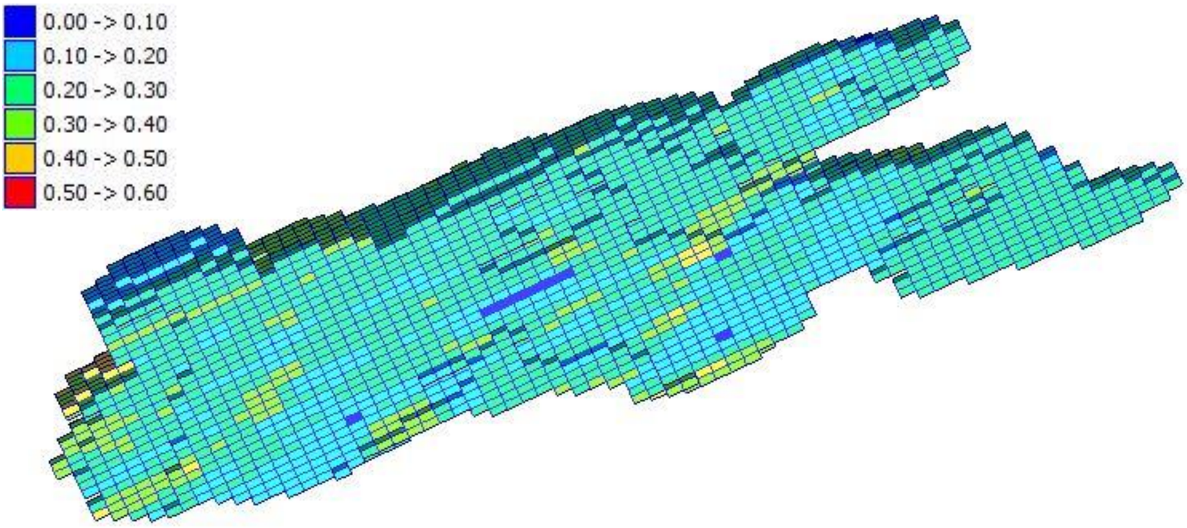


***Figure 35 Overview of Poissons ratio block model***

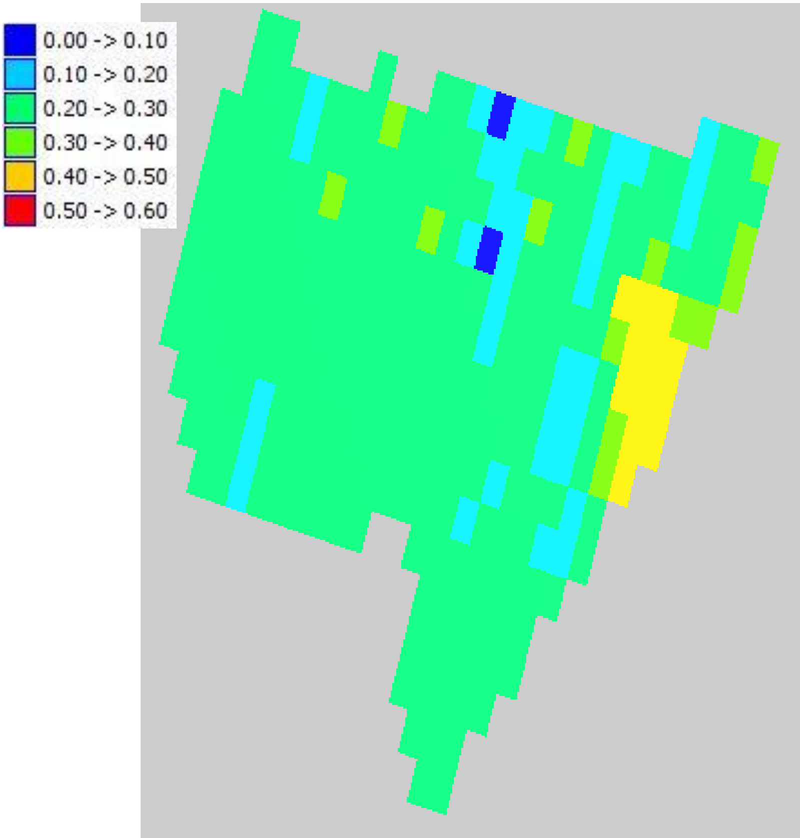


***Figure 36 Section view of block model***

## Uniaxial Compressive Strength

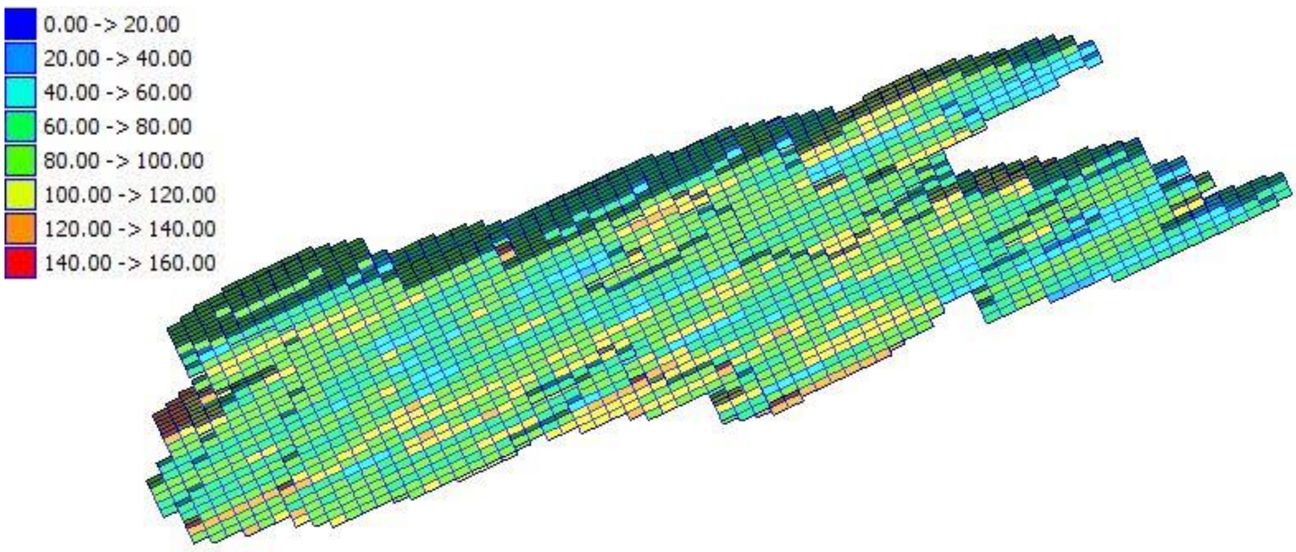


*Figure 37 Overview of UCS block model*

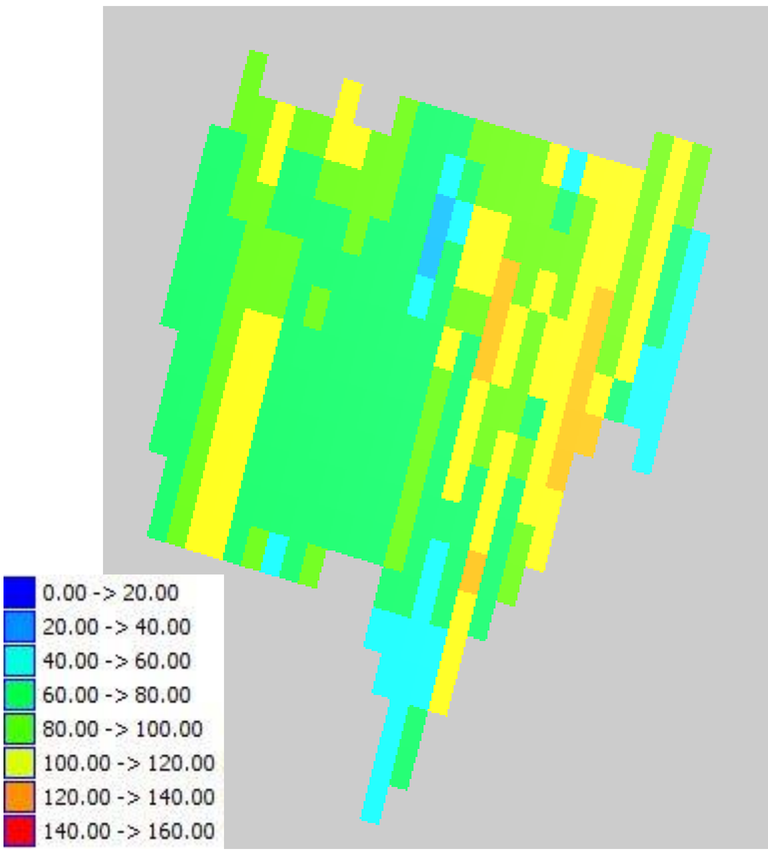


*Figure 38 Section view of block model*

Figure 39 show the three section planes close up and how they are positioned along the drifts.

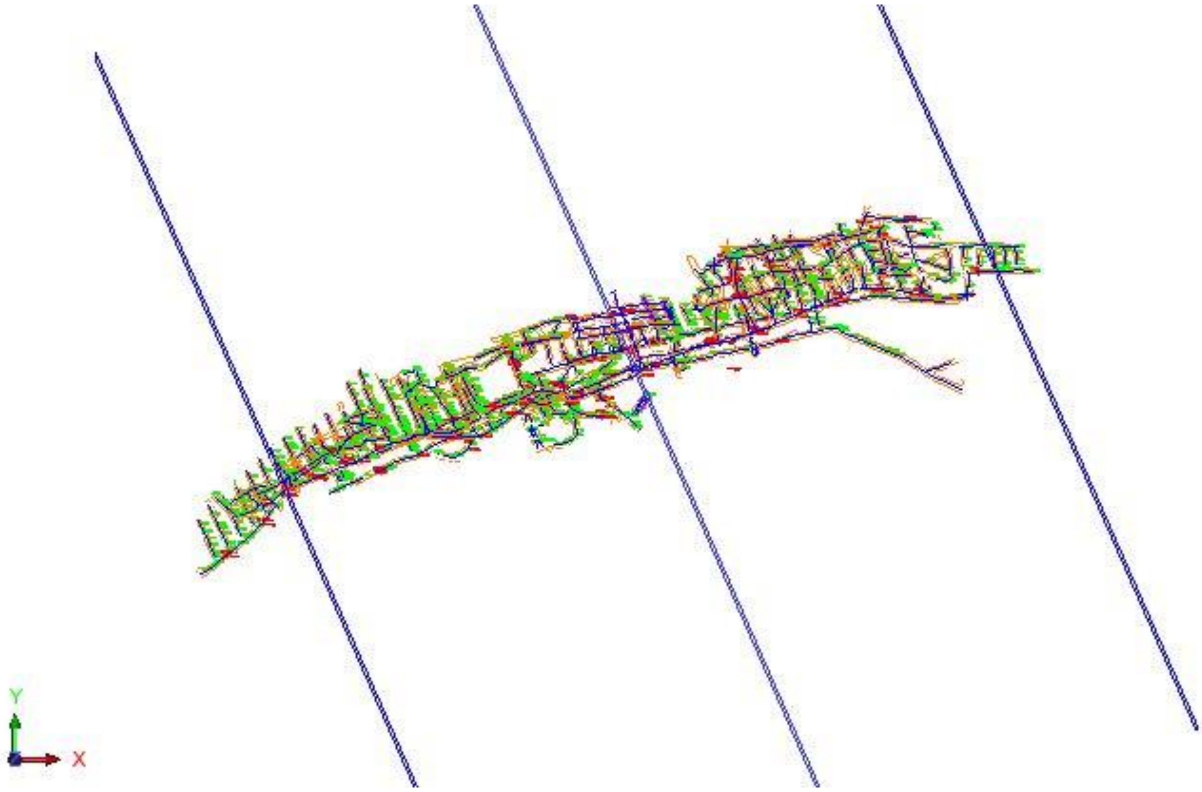


***Figure 39 The planes with the drifts***

Block attributes are extracted from the blocks that are numbered. These are around drifts that are being analyzed in Phase[2]. The drift analyzed is numbered 250 in Surpac.

**Section 1**

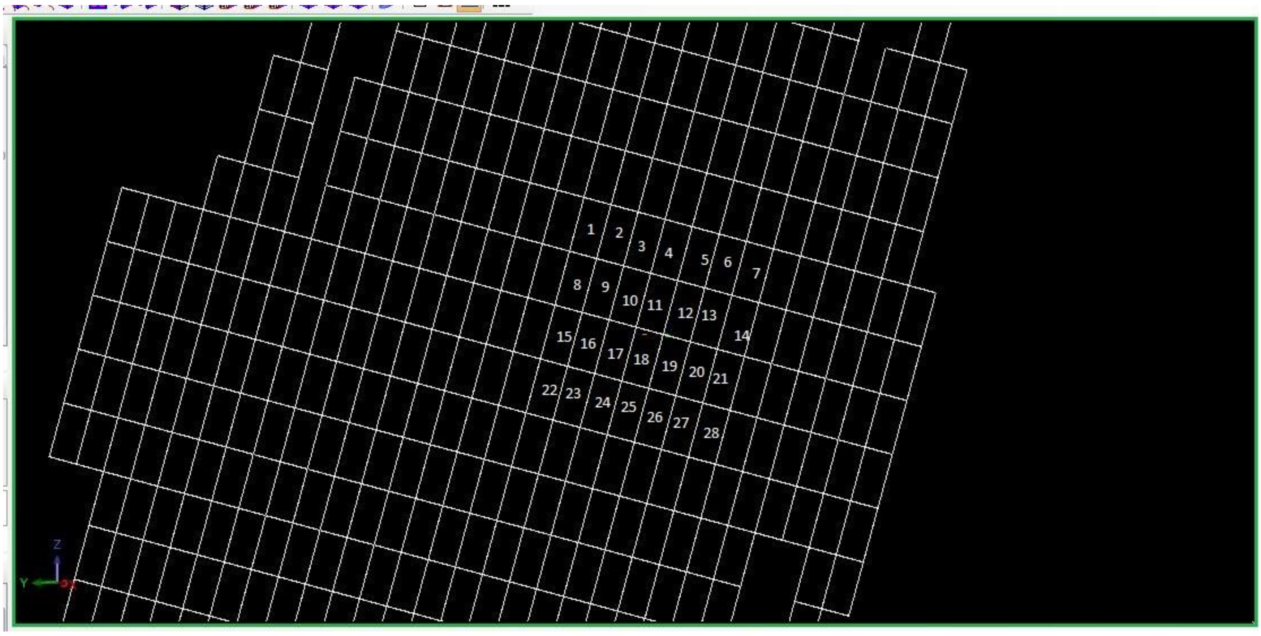


***Figure 40 Section 1 with numbered blocks***

The drift used later in Phase[2] is located in Figure 40 at the top of block 18. The values in each of the numbered blocks are given in Table 4.

## Section 2

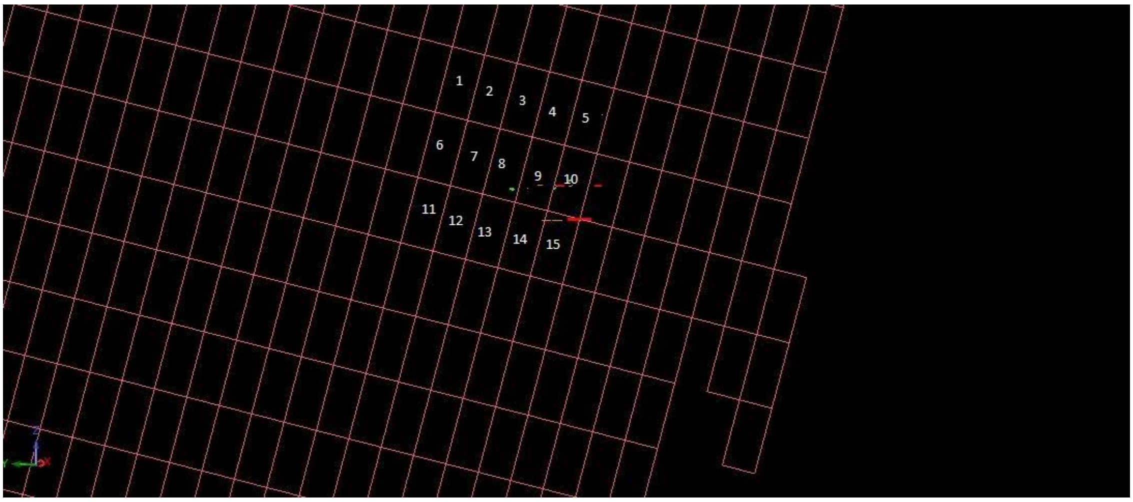


***Figure 41 Section 2 with numbered blocks***

The drift used later in Phase[2] is located in Figure 41 in block 8, bottom right. The block attributes are in Table 5

## Section 3

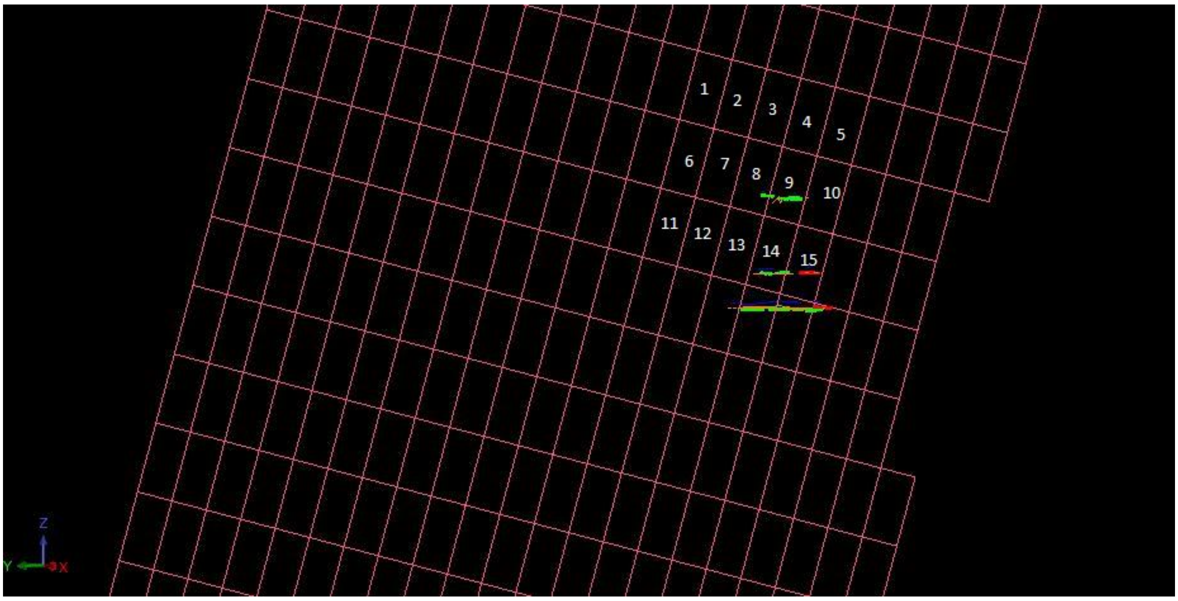


***Figure 42 Section 3 with numbered blocks***

The drift used later in Phase[2] is located in Figure 42 in block 8, bottom right. Block attributes are in Table 6

## 6.2 Results from Numerical Analysis

The models drawn in Phase$^2$ are based on the block model sections that have numbered blocks gathered from it. The input parameters used are the data gathered from the block model. RockLab was used to find failure criteria for each block, and also to calculate rock mass E-modulus. A constant stress field is used where the vertical stress is 10 MPa and horizontal is 20 MPa (Nghia Quoc Trinh 2011). The drift is 6,7 m wide and 7,1 m high.

### 6.2.1 Main principal stress

These figures show the stress distribution of the main principal stress in the sections. Figure 43, 44 and 45 give an overview of the stress situation in the surrounding area. There is a clear difference in the stress distribution in the different blocks, showing patterns of lower and higher stress zones. This is also shown in the different sections too.

**Overview**

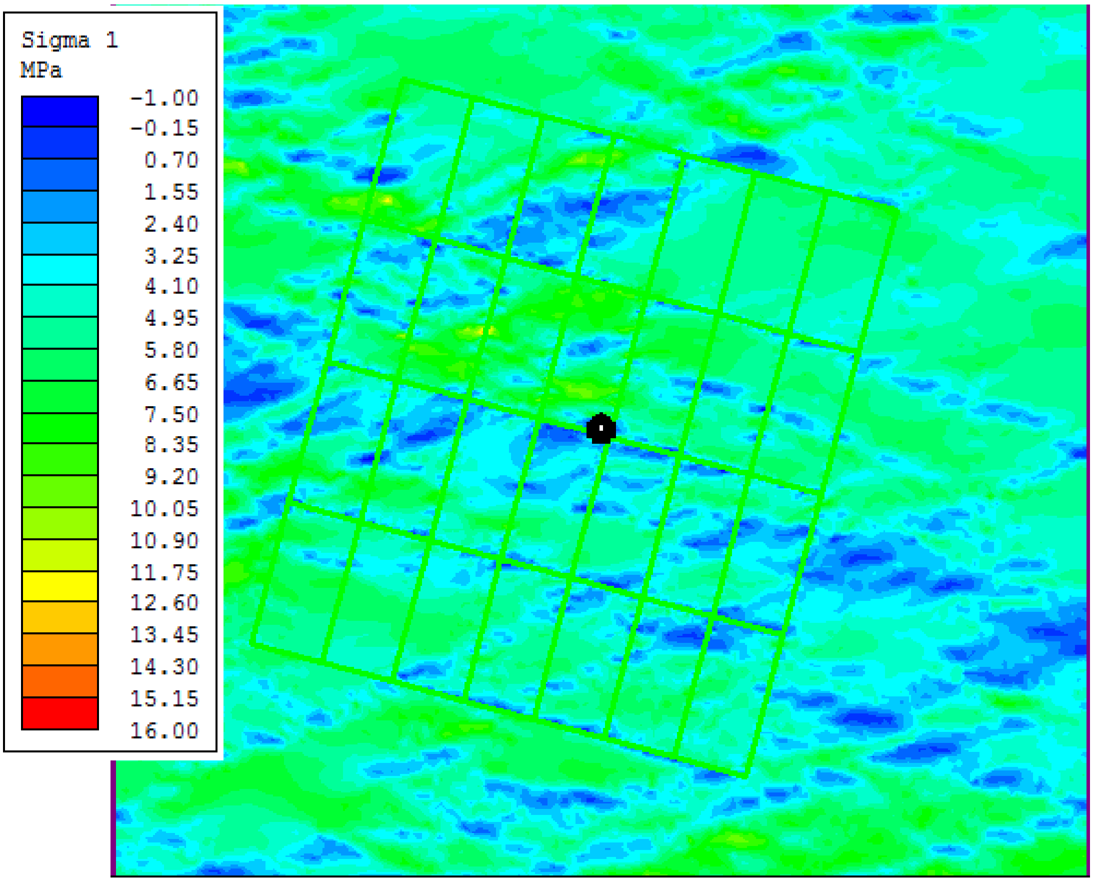


*Figure 43 Stress situation in section 1*

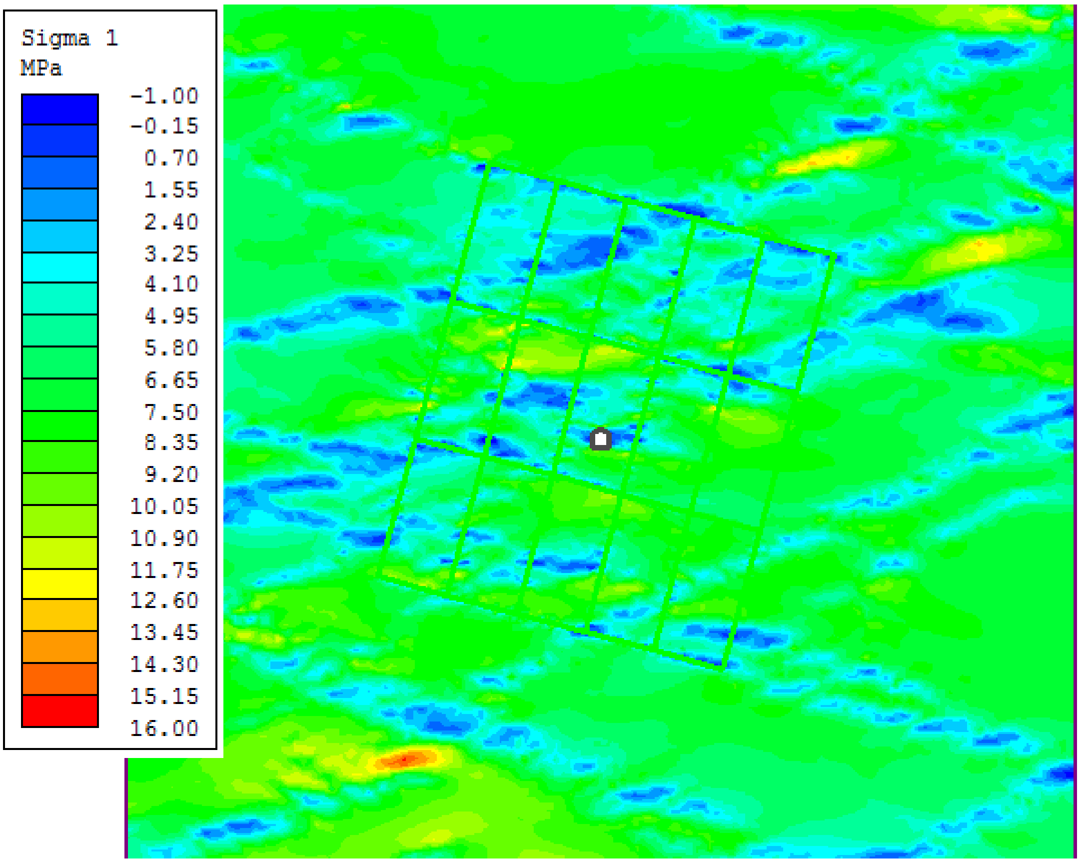


*Figure 44 Stress situation in section 2*

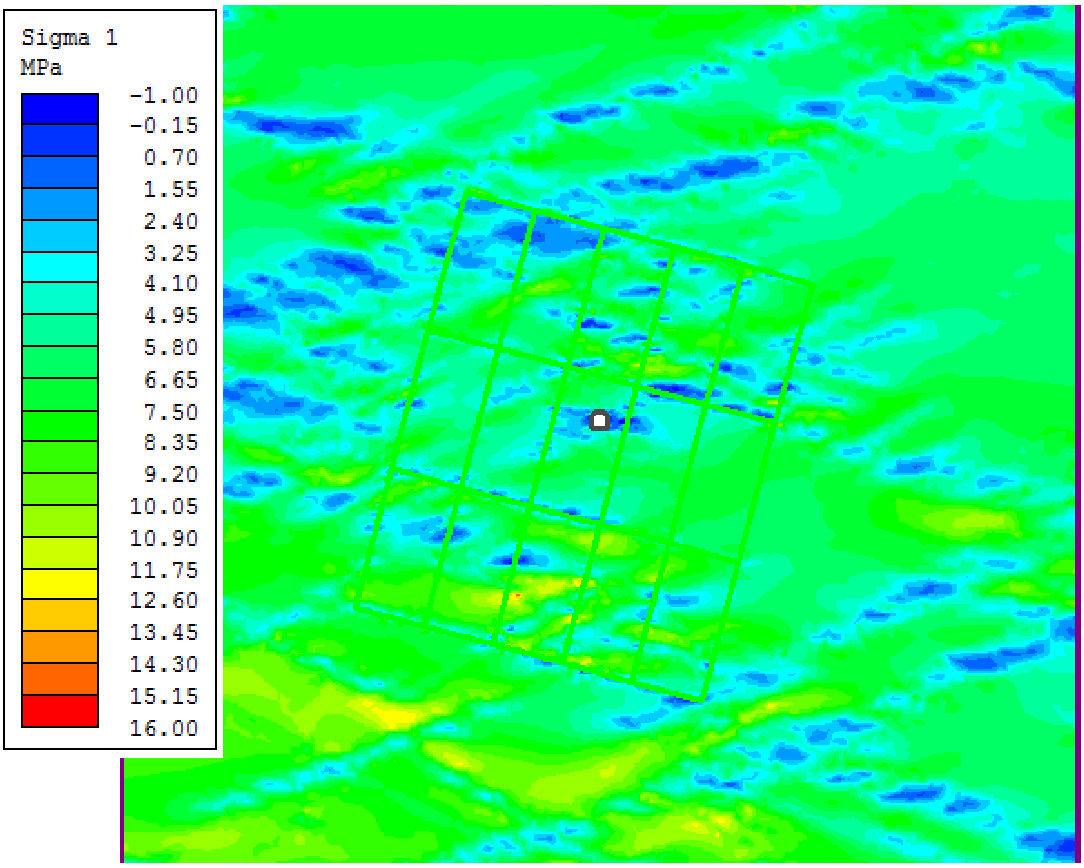


*Figure 45 Stress situation in section 3*

**Close up**

Here in Figure 46, 47 and 48 the sections are zoomed in, giving a better look at the stress distribution around the drifts. The figures show a distinct difference in the three sections with different blocks and parameters.

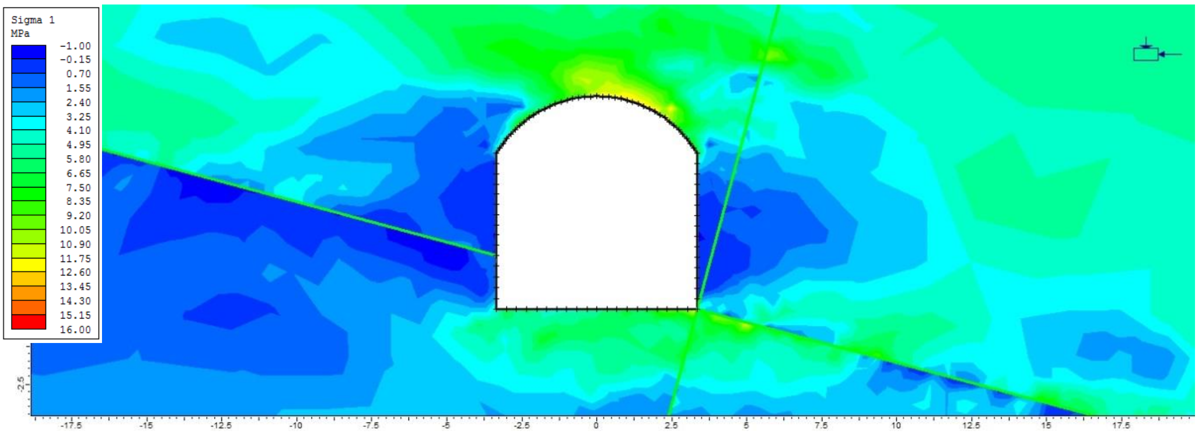


*Figure 46 Section 1 stress distribution around the drift*

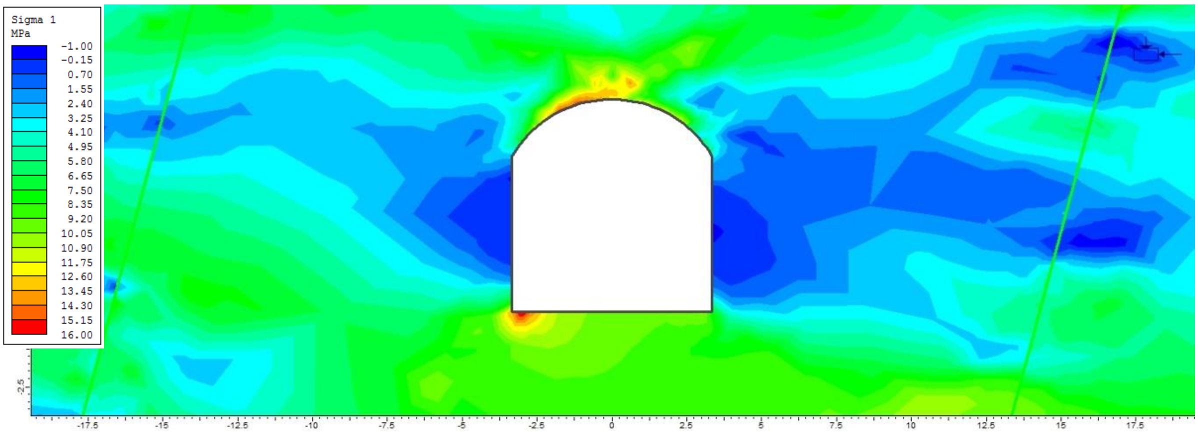


*Figure 47 Section 2 stress distribution around the drift*

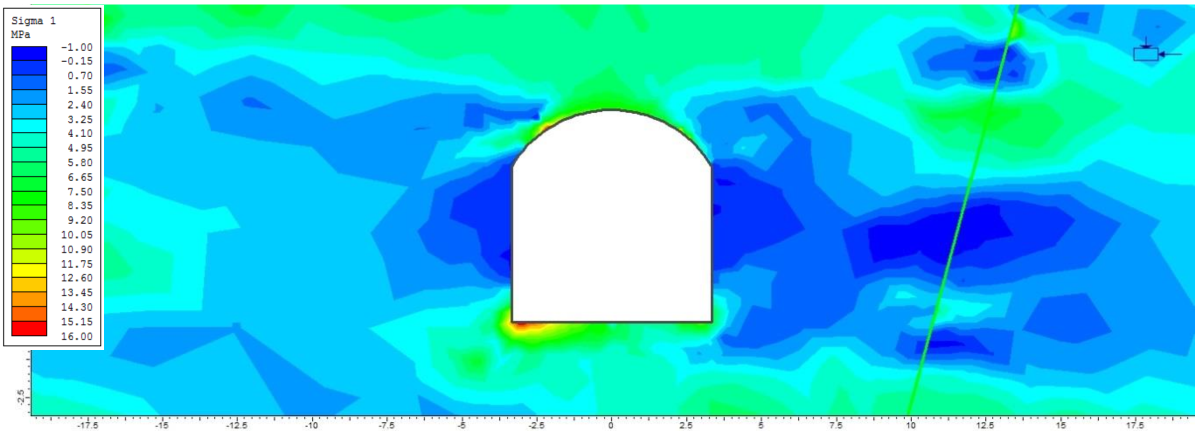


*Figure 48 Section 3 stress distribution around the drift*

Figure 49 show the stress of $\sigma_1$ along the boundary of the three drifts, going from the bottom left corner and counter clockwise.

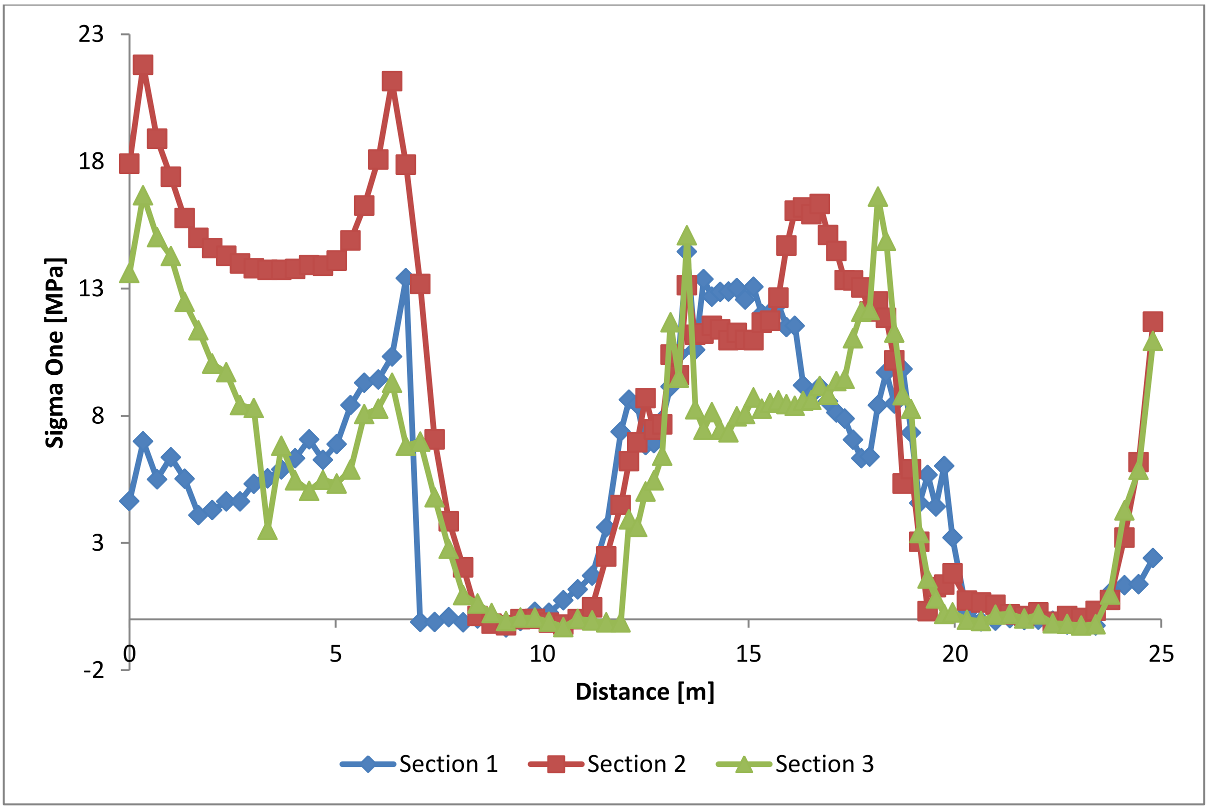


***Figure 49 Stress along the boundary of the drift***

### 6.2.2 Deformations

Here the deformations in the sections are shown over the whole block model in Figure 50, 51 and 52.

**<u>Overview Total Displacement</u>**

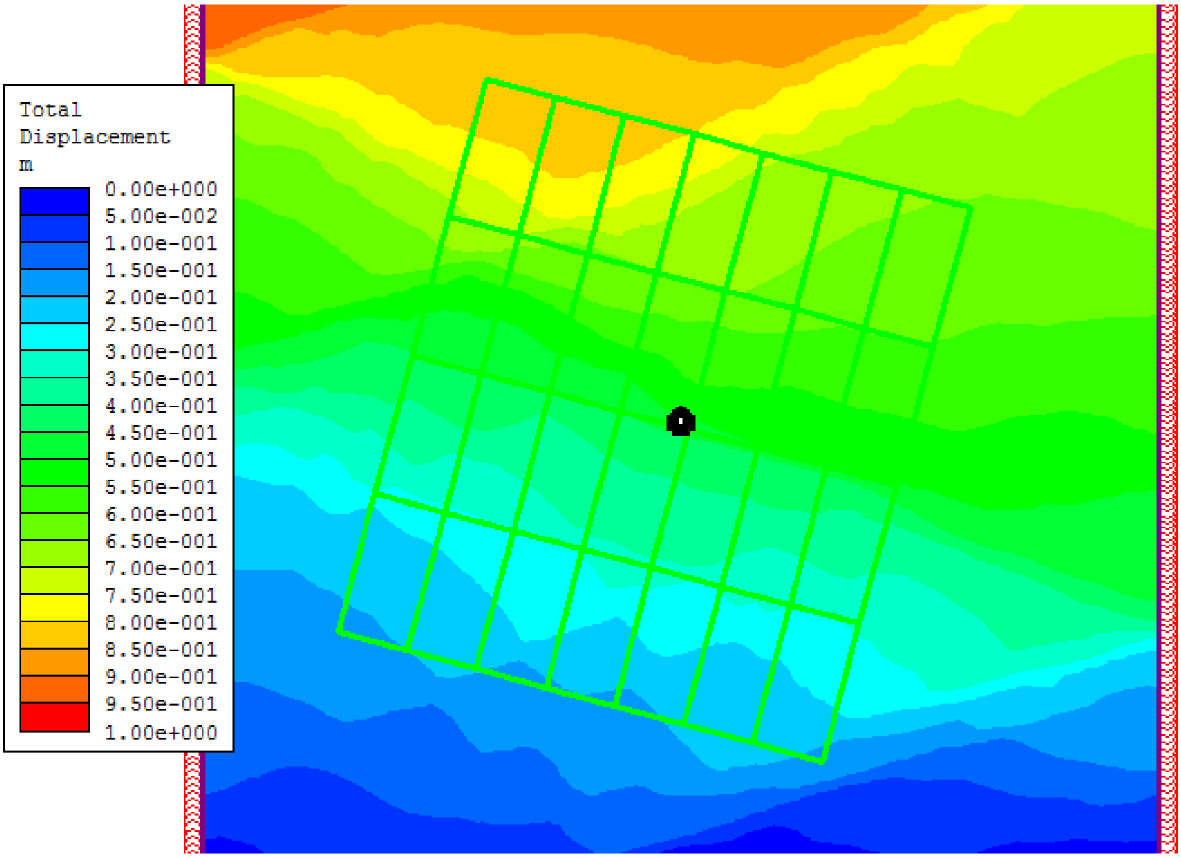


***Figure 50 Section 1 total displacement***

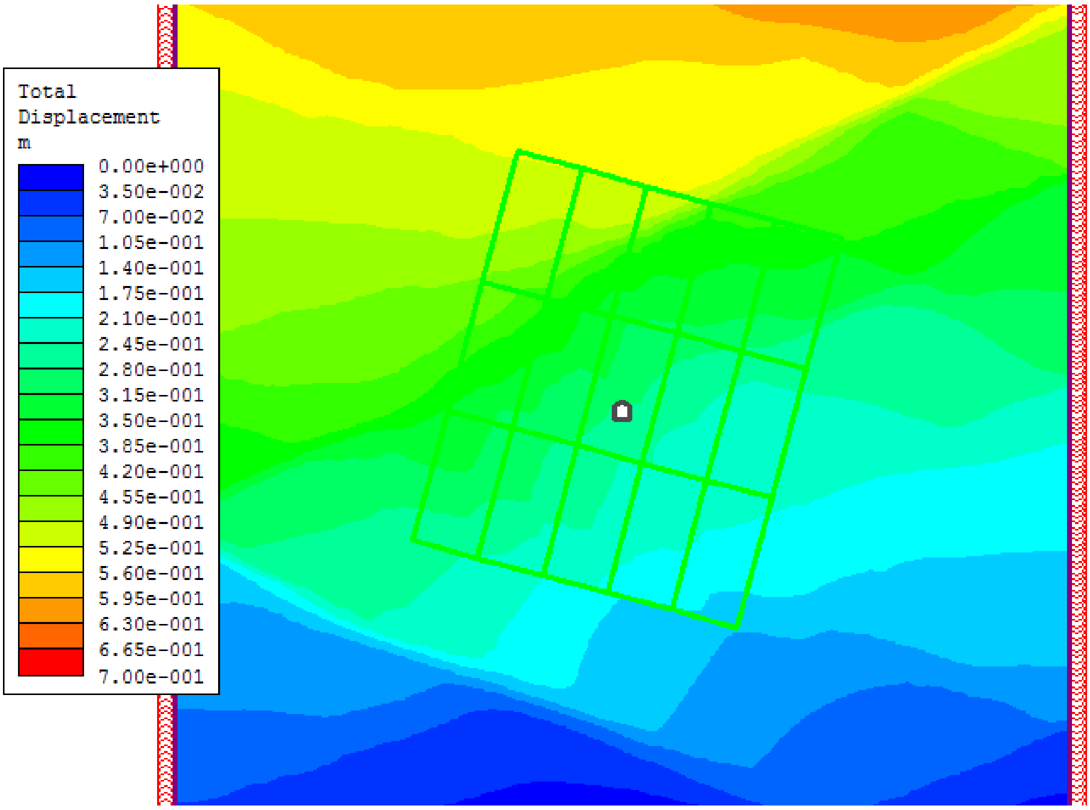


***Figure 51 Section 2 total displacement***

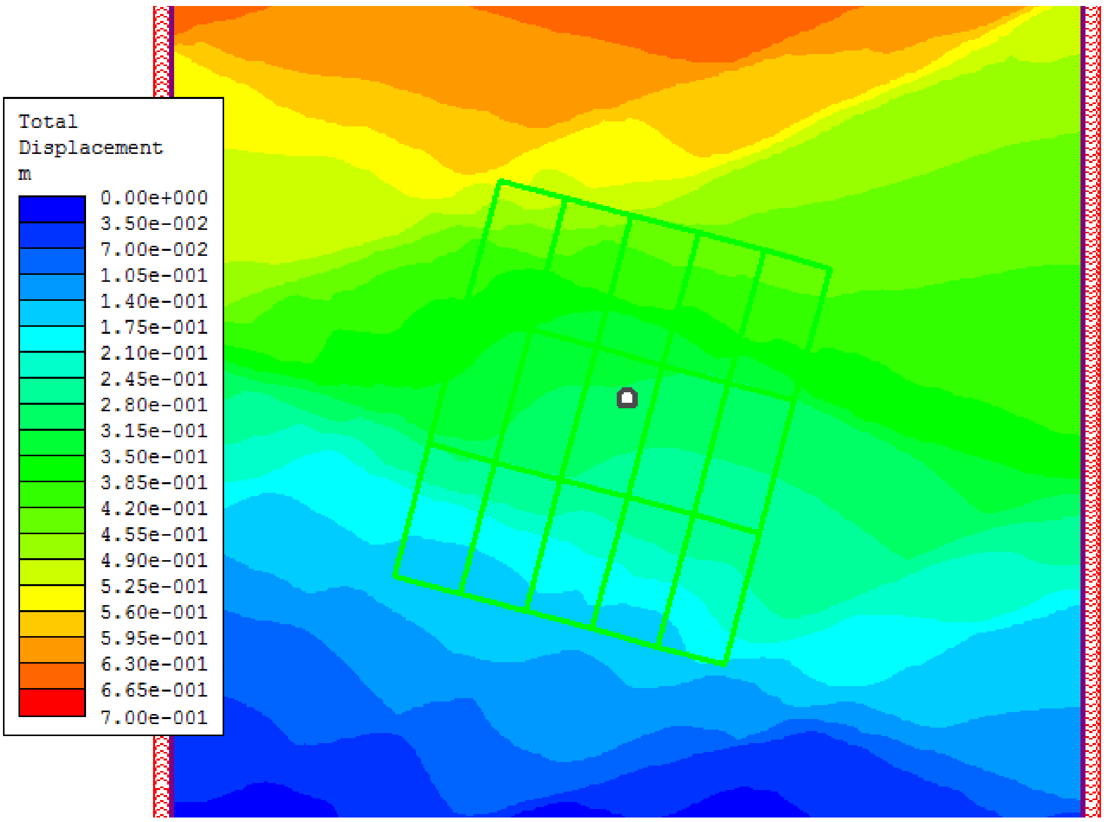


***Figure 52 Section 3 total displacement***

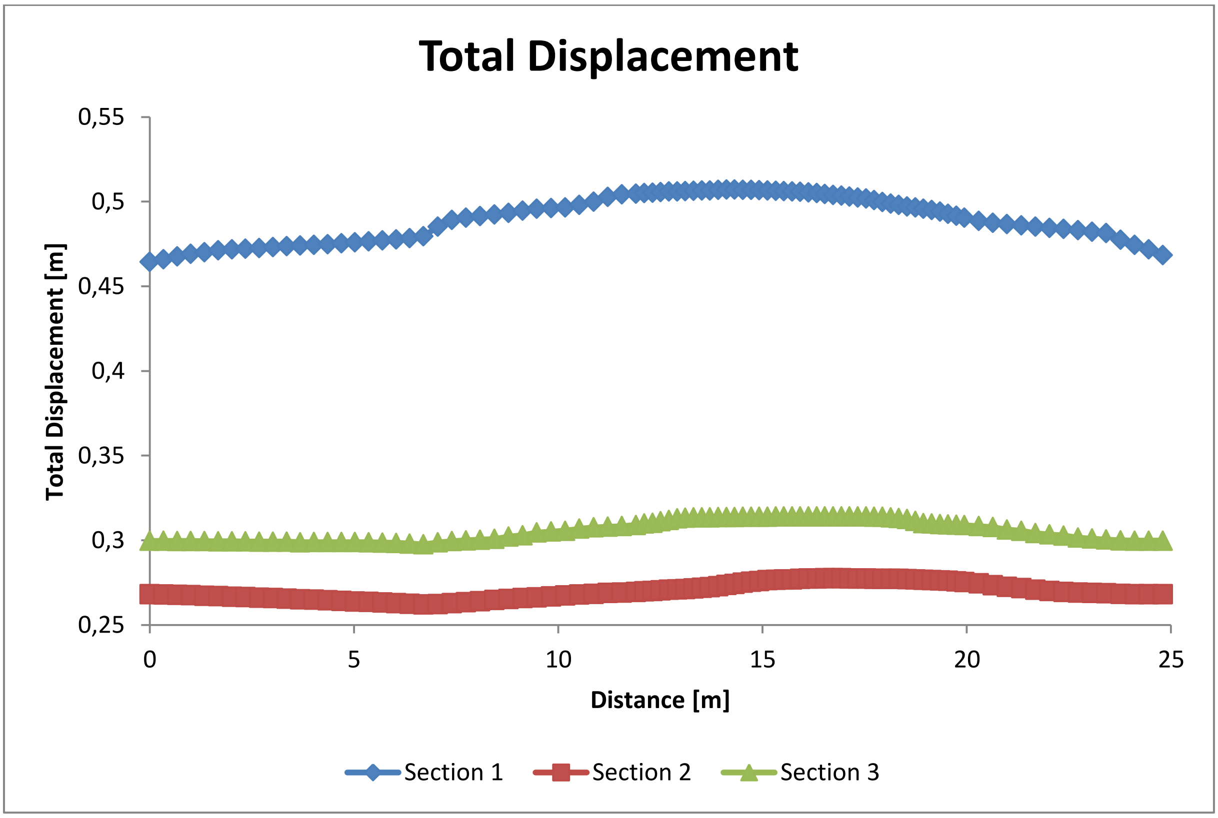


Figure 53 show the total displacement on the boundary of the three drifts, going from the bottom left corner and counter clockwise.

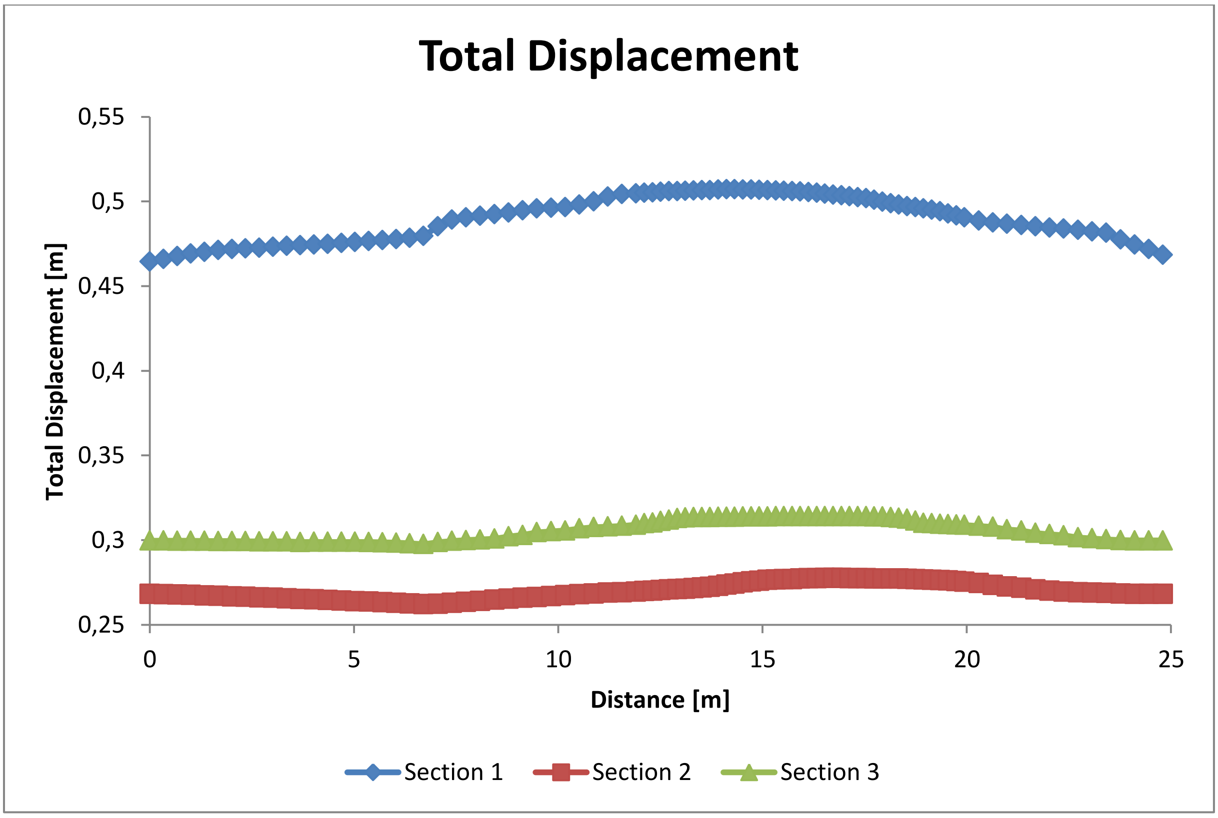


***Figure 53 Total displacement along the boundary of the drift***

## Overview of Horizontal Displacement

The horizontal distribution does, like the principal stress figures, show a clear structure with different zones relatively of higher and lower displacement (Figure 54, 55 and 56).

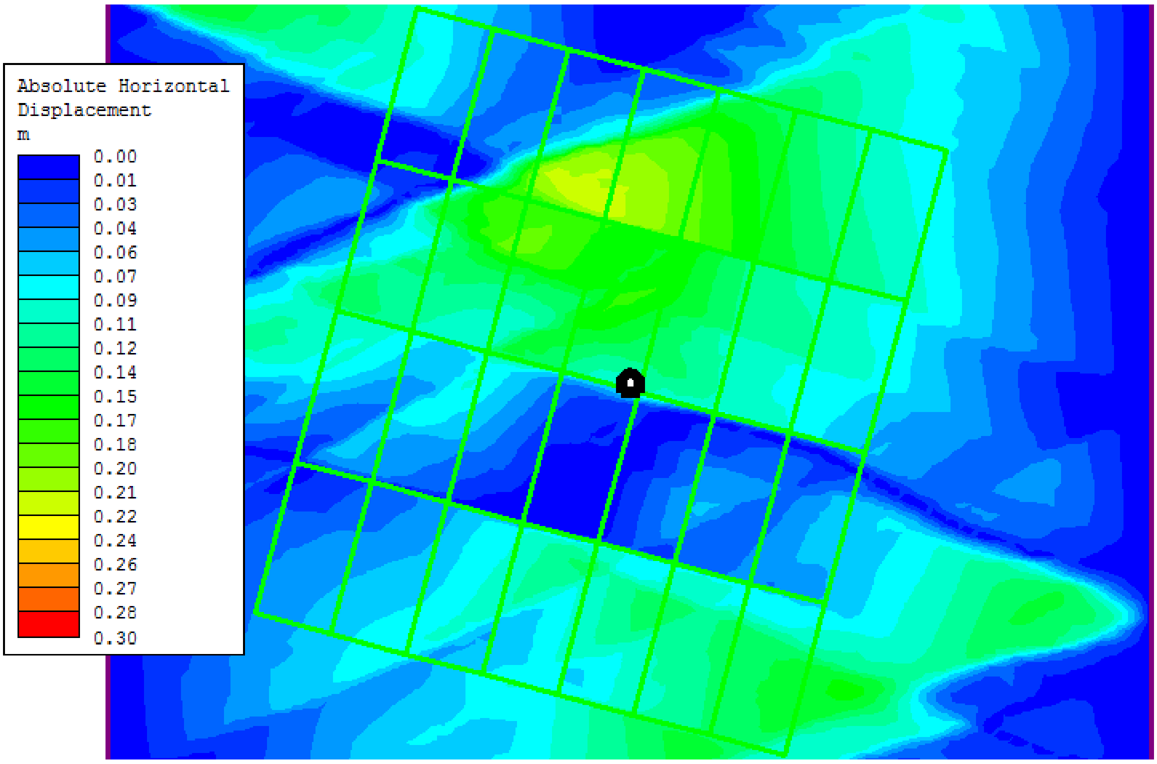


***Figure 54 Section 1 horizontal displacement***

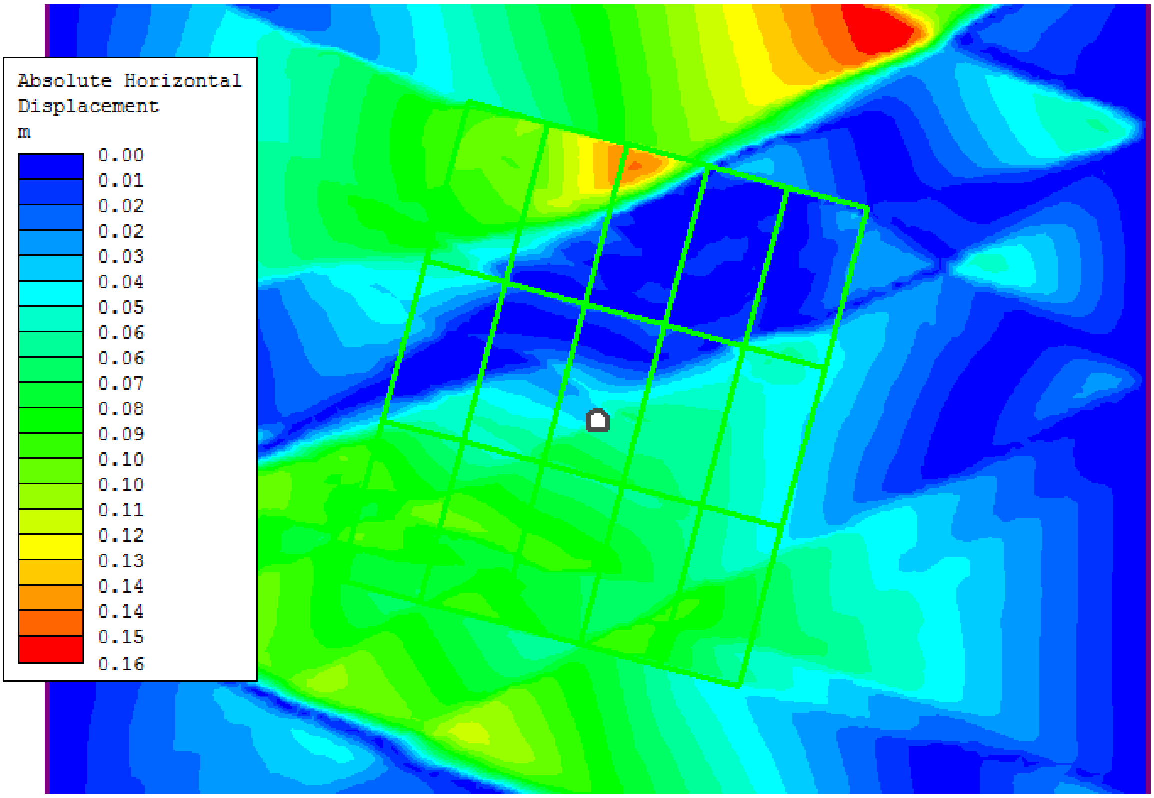


***Figure 55 Section 2 horizontal displacement***

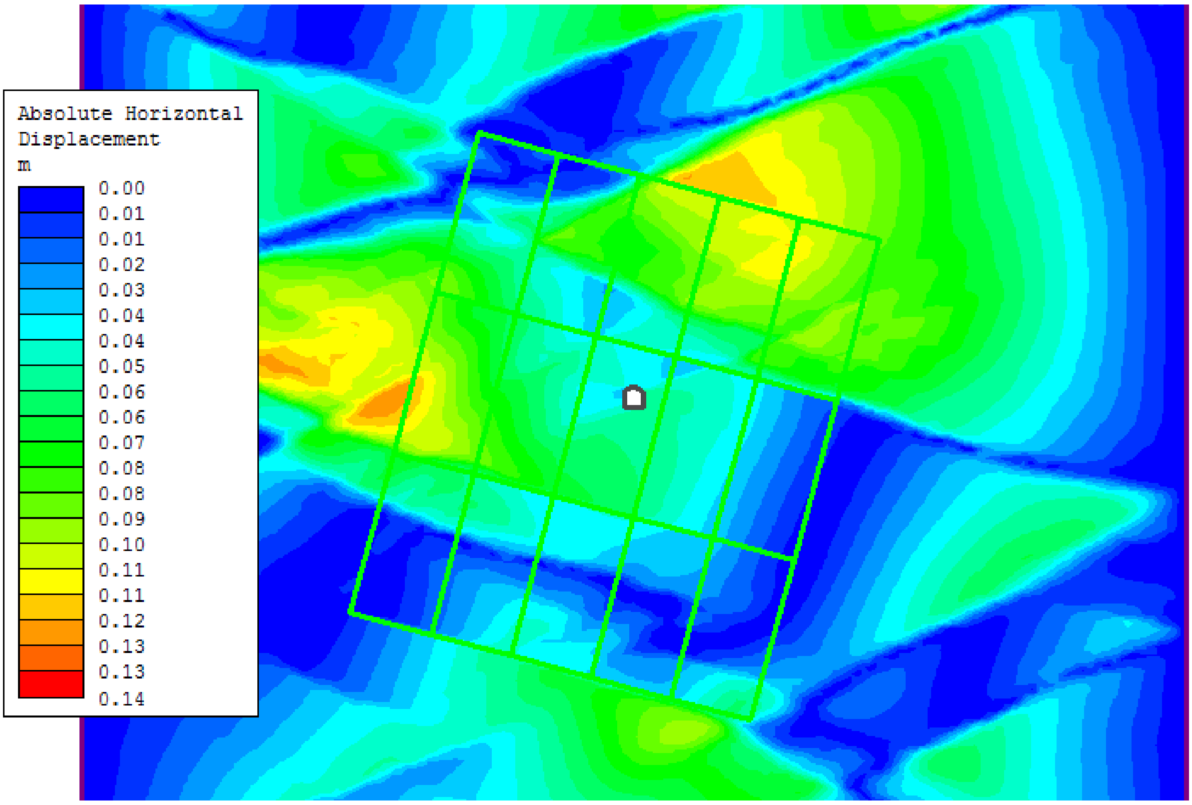


***Figure 56 Section 3 horizontal displacement***

## Close up Horizontal Displacement

Figure 57, 58 and 59 indicate three completely different deformation situations when given blocks with different attributes.

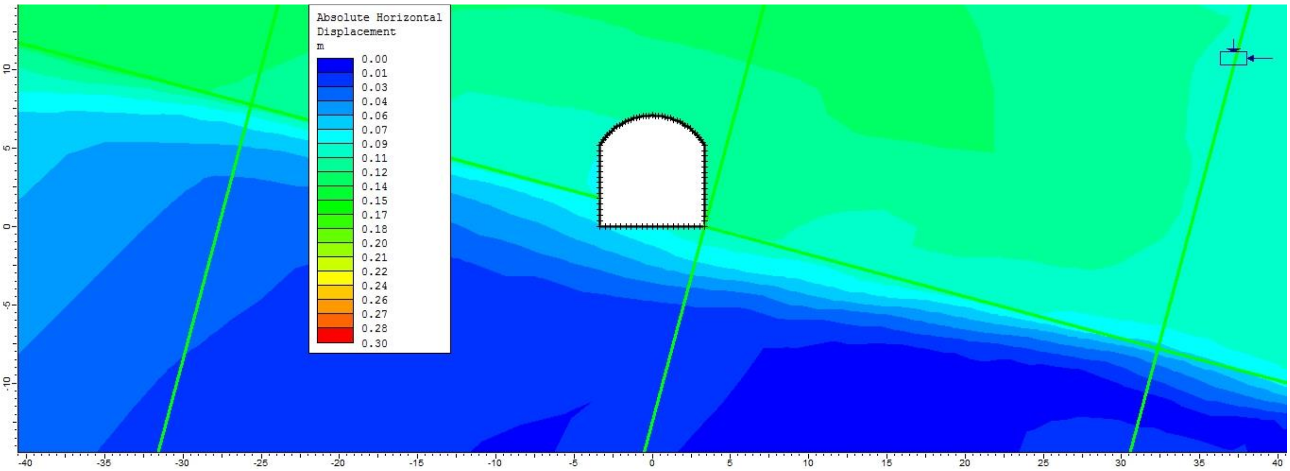


***Figure 57 Section 1 horizontal displacement around the drift***

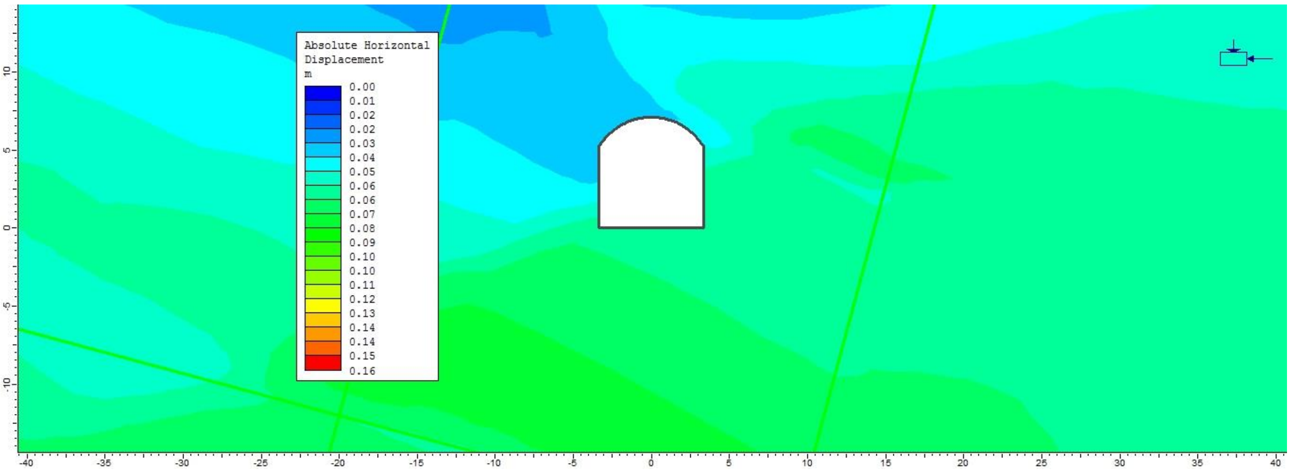


***Figure 58 Section 2 horizontal displacement around the drift***

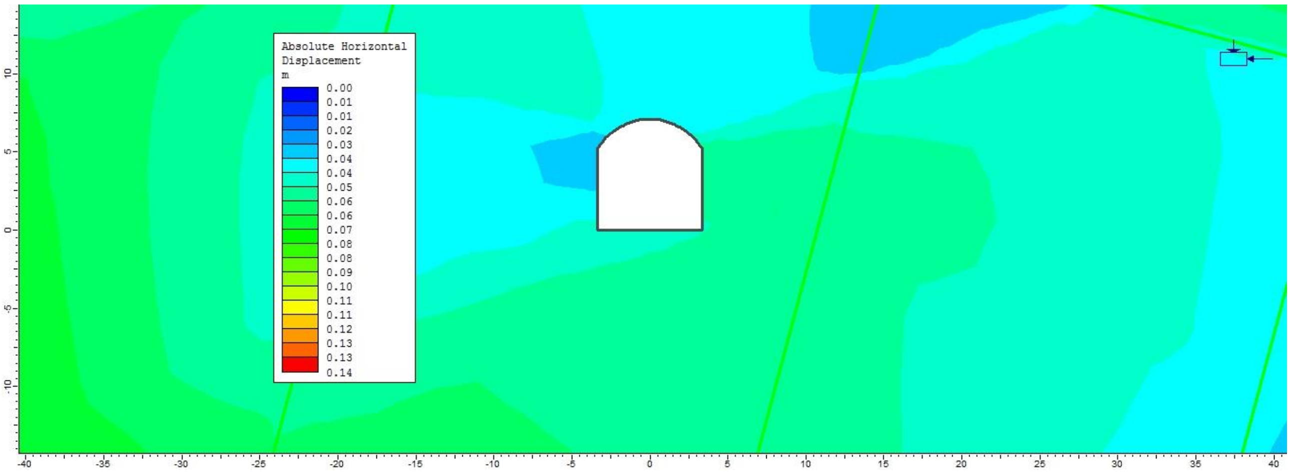


*Figure 59 Section 3 horizontal displacement around the drift*

Figure 60 show absolute horizontal displacement along the boundary of the three drifts, going from the bottom left corner and counter clockwise.

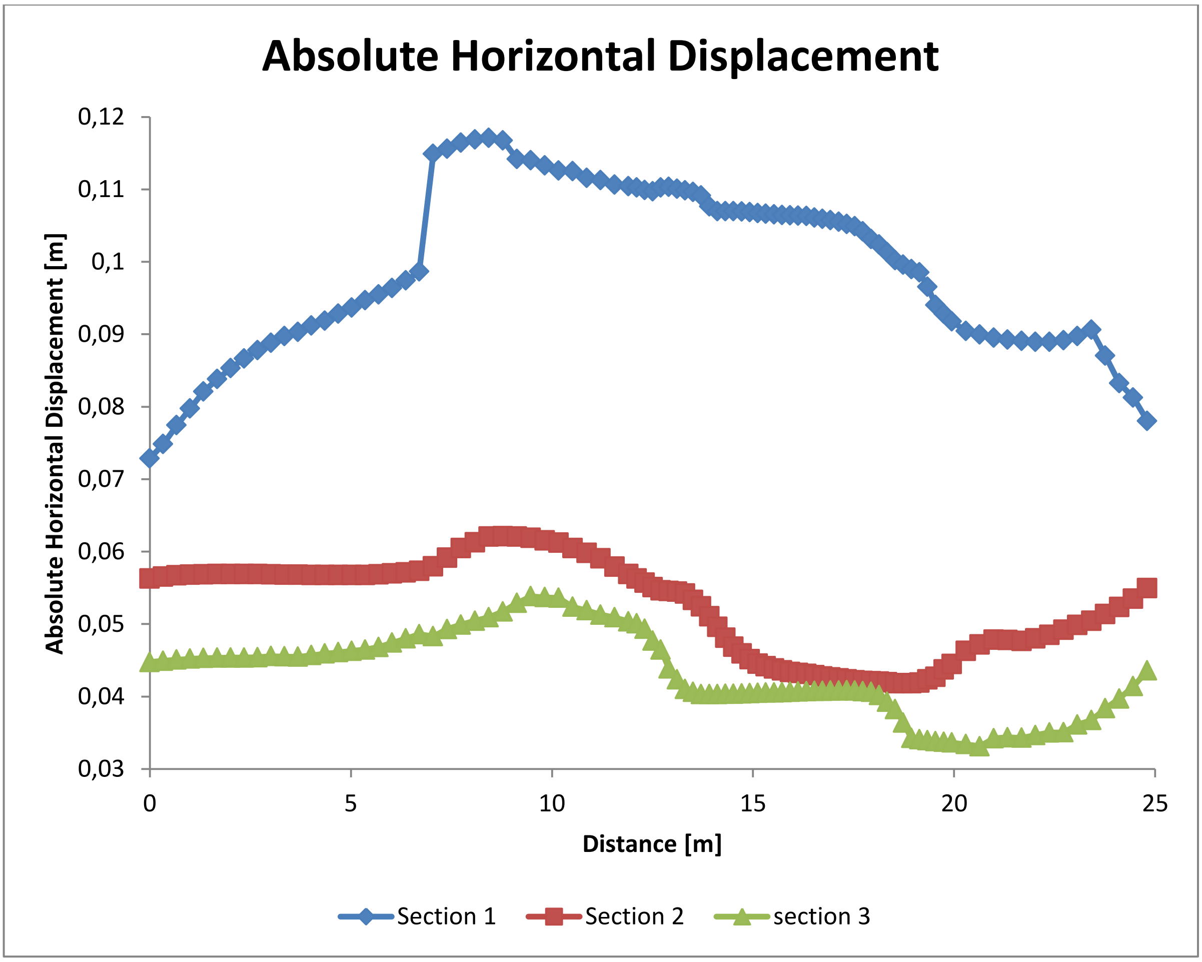


*Figure 60 Absolute horizontal displacement along the drift*

## 6.3 Relative comparison of attributes

The attributes in Figure 61, 55 and 56 are plotted on graphs and then piled on top of each other to illustrate how they move relative to each other. This makes it possible to see if the values are high and low compared to each other. Figure 62 and Figure 63 have a noticeable better coherence than Figure 61

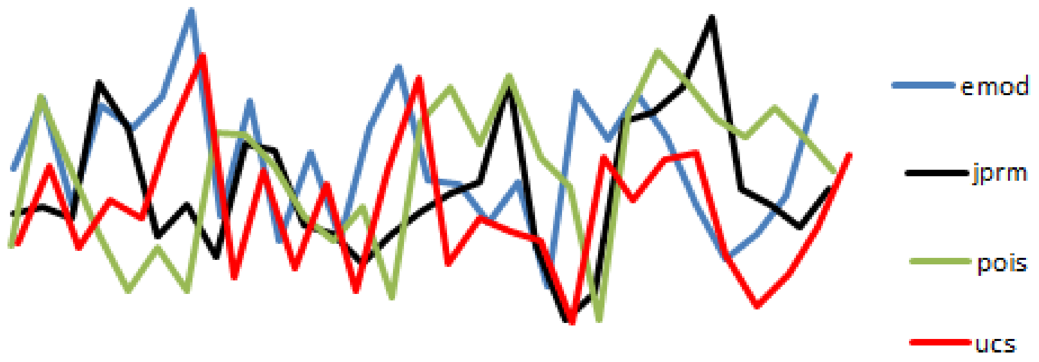


*Figure 61 Attributes section 1*

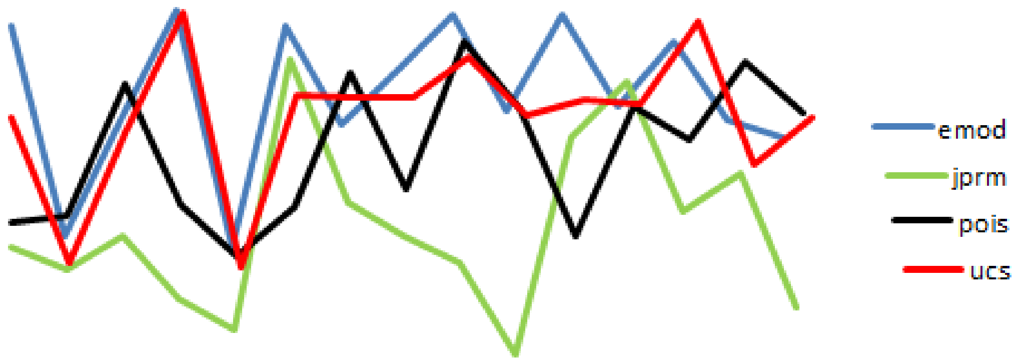


*Figure 62 Attributes section 2*

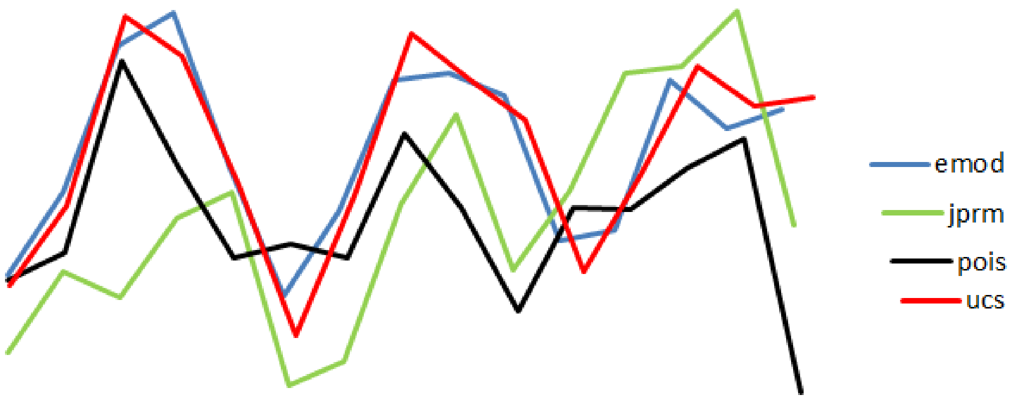


*Figure 63 Attributes section 3*

# 7. Discussion

The goals of all the results in this thesis are produced to show how geostatistics can improve the input parameters and design of a numerical model. The main purpose of this thesis is not to get the most accurate results. The emphasis is more towards the possibility of using the method and procedure proposed in this thesis. The results are good as long as they serve the purpose of illustrating the use of the proposed method and the difference it can make compared to not using it.

## 7.1 Geostatistics

The results from the basic statistical analysis showed the general trend of the data set. The histogram plots in Figure 10 through Figure 13 showed that the data set is close to normal distribution.

The different experimental variogram models that are drawn in Surpac are quite different depending on which parameter was plotted. E-module variograms showed the smoothest models of all the parameters. It also had the highest nugget effect, which suggests that there might be few samples close to each other. It also means that the estimation procedure becomes more like an averaging of the data (Edward H. Isaaks 1989). This means that inverse distance estimation might be as good as ordinary kriging. In this thesis ordinary kriging is used for all the estimation. This was due to lack of knowledge with geostatistical estimation methods and wanting continuity in the estimation, so all results come from the same estimation method.
The other parameters had rougher variograms, but they all exhibit the same trend with longer continuity in the direction of the ore body. This compares well with the actual situation, where the different lithologies follow the ore body and is in layers parallel to it. So as expected the variograms looks best in the direction parallel to the ore body and lithology layers, and have lower continuity perpendicular to the ore body and layers. This is also seen in the variogram maps were the lines of maximum continuity are about 65$^{o}$ north east, the same as the strike of the ore body.

One more issue arises with the variograms. This data set is not necessarily stationary with the intrinsic hypothesis fulfilled. This means that the entire data set is not well modeled by a stationary random function (Edward H. Isaaks 1989), which can be normal with earth data sets. This data set looks to be what can be called quasi-stationary (Ellefmo 2013). The variogram model smooth out at the sill in most cases, but then after a certain range it increases or decreases rapidly. So the data set is stationary within the range it is level at the sill, before it goes up or down. But just because the data set is inappropriate it doesn't mean that ordinary kriging should be abandoned in favor of example inverse distance method. Instead the data set can be subdivided into smaller separate populations, where stationarity might be appropriate.

Each model had spherical variogram models fitted to them, since this was the model with the best fit. The lag had to be adjusted for each parameter to get the smoothest model.

The block model was decided to be 60x30x60m (length x width x height). This was so that the blocks are big enough to contain sufficient samples when estimated. The block model might benefit with smaller blocks, but then enough samples might not be estimated giving a poor estimate. And with smaller blocks the work of getting the attributes into a numerical would be very tedious.

The estimation method used for all the parameters is ordinary kriging. Search parameters like maximum and minimum samples had to be determined, to ensure that there are enough samples to make a valid estimation, but not too many so the computer runs slow (Surpac 2012). The standard, which was between 3 and 15 samples, was chosen. Also maximum horizontal and vertical search distance had to be determined. This value was set a little further than the range, so that most of the data points have some correlation. The results of the kriging are shown as blocks colored by attribute (Figure 31, 33, 35 and 37). Here zones of higher and lower values are more easily visualized, giving a good picture of the where the rock quality is relatively high or low. In section views (Figure 32, 34, 36 and 38) layered zones that are the same dip as the ore body is easily visible. This agrees well with the actual situation where the different rock types are layered in that pattern. So the ordinary kriging results were as expected. If the results weren't like they are now, the estimation would be run over again with different search parameters. This is the good thing about knowing how the geology in the area is, one know approximately what the results should be. One can also see it by looking at the direction of maximum continuity in the variograms.

Three vertical sections from the block model were sliced to focus on smaller areas to make a numerical model from (Figure 30 and Figure 39). The drifts that are used in the model were chosen at random, the only criteria were that they were spread approximately equal throughout the ore body. The same drift was analyzed in all three sections, and was also chosen at random.

## 7.2 Numerical analysis

The numerical models are designed based on the block section in Figure 40, 41 and 42. When deciding on the number of blocks that needs to be used it was looked at two different numbers to see how far the influence might be. It was decided that about 70 m to each side was enough, but also have one with about 100 m to each side to see if it behaves any different. The model was set up with no restraint at the top, so that subsidence and deformation of the rock mass from above could be seen.

At first it was planned to use zones with different material properties. The idea was that parameters within an interval would be grouped together into one domain. One problem arises when doing it like this, and it is that the parameters don't always increase or decrease relative to each other. As it is seen in Figure 61 were a pattern is not present. Figure 62 have better correlation than Figure 61, but only for some areas. Figure 63 is the one that looks like it has the best fit. How good should the parameters fit with each other is a question that is raised. In the case of section 3 it could be that this is a fit that is more than good enough. But when the rest of the sections don't look to be like that, the safest and conservative approach is what is done in this thesis, give every block different properties.
One of the downsides with using geostatistics before numerical methods is time consumption and tedious work of getting attributes from each block, and then into the numerical software. With using the method in this thesis it can be very time consuming getting all the parameters into so many blocks (28 blocks in section 1 and 15 in section 2 and 3), and also using RockLab on all of them.

The parameter JPRM was not included in the numerical analysis. The author could not incorporate all the joints in an easy and practical way, and when consulting an expert it was decided by the author that it will be left out of the numerical analysis (Trinh 2013).

What is looked at in the numerical analysis is stress distribution from $\sigma_1$ (main stress) and total and horizontal displacement. These are important to assess stability (Rocscience 2013).

In Figure 43, 44 and 45 the stress situation is clearly different in each of the models. The only thing that is different between them before the analysis was performed are the material properties in the blocks. Since the legend is the same for all of them it is easy to see the difference, and all the different material properties clearly have an impact on the stress situation. Although the principal stress situation induced on this model does create a pattern by itself due to no restraints above, it is distinctly different in the different figures, showing zones of high and low stress in different areas.

When looking at the situation up close around the drift in Figure 46, 47 and 48 it is assumed that the different material properties have an effect on the stress situation around the drift. It has the same structure with the lowest stress in zones horizontal out from the walls of the drifts. But as seen from Figure 46, it has a block down to the left that have considerable less stress in it than the other blocks seen in Figure 46. So this has a considerable different stress situation close to the drift, which in turn can influence the stability of the drift.
These figures(43-48) and the graph in Figure 49 underlines the point that the stress situation is different at the same point in the drift when material properties are divided into smaller areas. And with that, may have greater accuracy with actual conditions.

The total (Figure 50, 51 and 52) and absolute horizontal displacements (Figure 54-59) give the same picture as the stress distribution, where there are zones with relatively high or low displacement. Total and absolute horizontal displacement was chosen to be shown because they give a good indication of how much the different blocks has an effect on the model. Absolute vertical displacement was almost identical and was deemed unnecessary to show.

The importance of using the right parameters to get good results from a numerical analysis is also underlined in a conference paper looking at the effect E-Modulus have in using different materials around an opening (R. E. Hammah 2006). It points out that the use of E-Modulus that is not representative can cause true behavior to be completely missed. This is what this thesis tries to shed light on. If f. ex only two zones are modeled (ore body and surrounding rock), true behavior of the rock mass may be overlooked due to the fact that the rock mass is a heterogeneous material. The introduction of geostatistics may help with that issue, making the models more accurate.

Why it that the method proposed in this thesis is not used to a greater extent? It seems like the benefit of using it outweighs the possible downsides (time consuming, core analysis, need knowledge of geostatistics and numerical modeling). In an conference paper these questions are also asked (R. E. Hammah 2006) in relation to using geostatistics with geotechnical engineering. There it was considered that there are a couple of reasons why the industry is not utilizing the possibility of better accuracy from numerical models. First the geostatistical software is hidden in huge mining-oriented programs, making it difficult to get into and learn. Also the results from geostatistics are not readily incorporated into subsequent geotechnical calculation and software. And also the expenses of purchasing software seem to be an issue.

# 8. Conclusions and recommendations

## 8.1 Conclusions

The modeling presented in this thesis is not intended to be a rigorous analysis of the deformation and stability of drifts in Rana Gruber, but help understand the general difference geostatistics makes in numerical analysis. The following conclusions can be drawn based on the study and analysis of using geostatistics with mechanical rock mass properties, and later the estimated results in a numerical analysis presented in this thesis:

- Rock mechanical parameters needs to be assessed for stationarity and additivity to determine a suitable geostatistical estimation method.
- Geostatistics can be a powerful tool to calculate the spatial variability of rock mass, given that a suitable method is used and that the data meets the prerequisites of the estimation method.
- Using a relatively detailed numerical model with numerous zones that have different material properties, the numerical analysis from that model will yield more detailed results, than e.g. only two zones.
- Using good quality input parameters in numerical analysis can make a big influence on the predicted behavior of the rock mass.

## 8.2 Recommendations for further study

- Using geostatistics in combination with numerical modeling on mines that have already had numerical analysis performed without geostatistics.
- Look more into the possibility of using different zones with material properties in numerical models, instead of dividing the model into block like in this thesis.

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# Appendix

***Table 3 Plane coordinates***

| Plane01 | Y | X | Z |
|---|---|---|---|
| 1 | 7366566,5 | 487720,7 | 973,134 |
| 2 | 7366557,9 | 487720,2 | -398,729 |
| 3 | 7367906,1 | 487129,8 | -406,414 |
| 4 | 7367918,2 | 487128,7 | 965,449 |
| | | | |
| **Plane02** | | | |
| 1 | 7366294,5 | 487284,3 | 973,134 |
| 2 | 7366285,9 | 487283,9 | -398,729 |
| 3 | 7367634,0 | 486693,4 | -406,414 |
| 4 | 7367646,2 | 486692,3 | 965,449 |
| | | | |
| **Plane03** | | | |
| 1 | 7366137,3 | 486800,9 | 970,397 |
| 2 | 7366128,7 | 486800,4 | -401,466 |
| 3 | 7367476,8 | 486210,0 | -409,151 |
| 4 | 7367488,9 | 486208,9 | 962,712 |

*Table 4 Parameters section 1*

| Block nr | Emodulus | JPRM | Poisson | UCS |
|---|---|---|---|---|
| 1 | 18750 | 3,3 | 0,17 | 70 |
| 2 | 22400 | 3,36 | 0,26 | 88 |
| 3 | 16880 | 3,25 | 0,22 | 69 |
| 4 | 21960 | 4,64 | 0,18 | 80 |
| 5 | 20810 | 4,16 | 0,15 | 76 |
| 6 | 22520 | 3,06 | 0,17 | 98 |
| 7 | 26900 | 3,38 | 0,15 | 115 |
| 8 | 16330 | 2,86 | 0,24 | 62 |
| 9 | 22290 | 4,02 | 0,24 | 87 |
| 10 | 15090 | 3,94 | 0,22 | 65 |
| 11 | 19540 | 3,18 | 0,19 | 85 |
| 12 | 15210 | 3,08 | 0,18 | 59 |
| 13 | 20800 | 2,78 | 0,20 | 88 |
| 14 | 24020 | 3,09 | 0,14 | 110 |
| 15 | 18180 | 3,31 | 0,25 | 65 |
| 16 | 17910 | 3,5 | 0,27 | 76 |
| 17 | 16000 | 3,62 | 0,24 | 73 |
| 18 | 18120 | 4,66 | 0,28 | 71 |
| 19 | 12700 | 2,96 | 0,23 | 51 |
| 20 | 22690 | 2,21 | 0,21 | 91 |
| 21 | 20260 | 2,47 | 0,13 | 80 |
| 22 | 22540 | 4,25 | 0,25 | 90 |
| 23 | 20400 | 4,33 | 0,29 | 92 |
| 24 | 16910 | 4,6 | 0,27 | 67 |
| 25 | 14110 | 5,3 | 0,25 | 55 |
| 26 | 15440 | 3,54 | 0,24 | 63 |
| 27 | 17320 | 3,38 | 0,26 | 74 |
| 28 | 22490 | 3,14 | 0,24 | 92 |
| Average | 19235 | 3,55 | 0,22 | 78 |

*Table 5 Parameters section 2*

| Block nr | Emodulus | JPRM | Poisson | UCS |
|---|---|---|---|---|
| 1 | 30760 | 3,47 | 0,19 | 101 |
| 2 | 17940 | 3,2 | 0,19 | 65 |
| 3 | 24980 | 3,59 | 0,27 | 97 |
| 4 | 31660 | 2,85 | 0,20 | 126 |
| 5 | 17290 | 2,45 | 0,17 | 64 |
| 6 | 30720 | 5,73 | 0,20 | 106 |
| 7 | 24680 | 3,99 | 0,28 | 105 |
| 8 | 27920 | 3,59 | 0,21 | 105 |
| 9 | 31430 | 3,27 | 0,30 | 115 |
| 10 | 25520 | 2,16 | 0,26 | 101 |
| 11 | 31360 | 4,78 | 0,18 | 105 |
| 12 | 25850 | 5,47 | 0,26 | 104 |
| 13 | 29790 | 3,88 | 0,24 | 124 |
| 14 | 25040 | 4,34 | 0,28 | 89 |
| 15 | 23970 | 2,73 | 0,25 | 101 |
| Average | 26594 | 3,70 | 0,23 | 101 |

*Table 6 Parameters section 3*

| Block nr | Emodulus | JPRM | Poisson | UCS |
|---|---|---|---|---|
| 1 | 16960 | 3,19 | 0,19 | 66 |
| 2 | 21940 | 4,45 | 0,21 | 85 |
| 3 | 30800 | 4,05 | 0,34 | 131 |
| 4 | 32790 | 5,27 | 0,27 | 121 |
| 5 | 23560 | 5,66 | 0,21 | 90 |
| 6 | 15750 | 2,67 | 0,22 | 54 |
| 7 | 20790 | 3,04 | 0,21 | 87 |
| 8 | 28670 | 5,48 | 0,29 | 127 |
| 9 | 29170 | 6,87 | 0,24 | 116 |
| 10 | 27730 | 4,47 | 0,17 | 106 |
| 11 | 18980 | 5,7 | 0,24 | 69 |
| 12 | 19660 | 7,52 | 0,24 | 92 |
| 13 | 28760 | 7,62 | 0,27 | 119 |
| 14 | 25760 | 8,47 | 0,29 | 109 |
| 15 | 26910 | 5,17 | 0,11 | 111 |
| Average | 24549 | 5,31 | 0,23 | 99 |