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Improvement of response time and cutting capability of Blow Out Preventer (BOP)

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Master of Science in Product Design and Manufacturing
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Problem Description

Background
This master project is initiated by FMC Technologies and is about improvement of one of the company’s products for cutting drill pipes. The cutting ability is highly dependent of the pressure available for cutting. In his previous project, the student has contributed to increase the residual pressure by adding high pressure bottles and a constant pressure regulator between the high pressure gas bottles line and the low pressure hydraulic accumulator circuit. A test rig will be built and a test program with the purpose of evaluating the abovementioned approach will be conducted.

Aim
Verifications and further improvement of the accumulator system developed through the resent project.

The master thesis will contain the following tasks:

1. Evaluation of constant pressure regulators
2. Search in the marked for constant pressure regulators
3. Evaluation of refilling of high pressure accumulators concepts
4. Make necessary schematics and simulate the circuit in a suitable software (SimulationX)
5. Attend testing and calibrate numerical models an confidence (provided that the test project i granted within the period assigned for the master thesis)

Assignment given: 16. January 2008
Supervisor: Torbjørn Kristian Nielsen, EPT

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Summary

There is a general demand in the market for BlowOut Preventers (BOP), which are capable of cutting tougher materials and cutting them faster than what is available today. This will give better safety and a more efficient use of the equipment. Better cutting capabilities are directly related to the pressures available to the cutting actuator. Normal accumulators loose pressure while it is drained. To avoid this a gas tank with high pressure nitrogen is proposed to boost the pressures during the cutting operation.

A boost tank will however contain pressures which are much higher then the allowed pressure in the BOP circuit. It is therefore necessary with a governor which controls the pressure. Normal pressure regulating valves tend to have difficulties maintaining a constant pressure while the pressure in the boost tank drops. Therefore is a PI-governor proposed, which may increase the stability and reliability of the BOP.

This report focuses mainly on the design of the PI-governor and how to optimize its design and parameters. The results are promising and the solution shows a more stable pressure than both the old accumulator solution and normal pressure regulating valves.

There are however a few flaws with the design. First; may even very small leakages and changes in material properties severely reduce and/or destroy the efficiency of the governor. Small leakages may quickly increase the system pressures and empty the boost tank. Another problem is the overlap in normal proportional directional valves. This severely reduces the efficiency of the integrator in the governor.

A mechanism for refilling the boost tank using a high pressure supply line from the rig is found to work quite well in simulations. The nitrogen gas is pushed back into the boost tank and the system is reset.

Sammendrag

Det er i dag en etterspørsel i markedet etter BlowOutPreventere (BOP), som har mulighet til å kappe sterkere materialer og som kan kutte dem raskere enn hva som er tilgjengelig i dag. Kutteegenskapene til BOPer er direkte koblet med trykket tilgjengelig til aktuatorene i BOPen. Vanlige akkumulatorer taper trykk mens de blir tømt for olje. For å unngå dette er en høytrykks nitrogen tank foreslått som en 'boost' tank for å holde på trykke under kappingen.

Denne 'boost' tanken vil inneholde trykk som er mye høyere enn de maksimalt tillate trykkene i de vanlige hydraulikk kretsene. Derfor må det installeres en trykk regulator som kontrollerer trykket i kretsen. Vanlige trykk regulerende ventiler har ofte vanskeligheter med det varierende trykket fra gasstanken. Derfor blir en PI-regulator foreslått til å regulere trykket slik at BOPen fungerer bedre og mer stabilt.

Denne rapporten fokuserer først og fremst på designet av den nevnte PI-regulatoren og hvordan dens parametre bør velges. Resultatene virker lovende og viser trykk som er mer stabile enn både de gamle akkumulator løsningene og trykkregulerende ventiler.

Det er allikevel noen haker som ved designet. Lekkasjer og forrandringer i material egenskaper kan i stor grad påvirke trykket i kretsene. Selv små interne lekkasjer i ventilene kan helt eller delvis ødelegge hele regulatoren eller dens funksjoner. Et annet problem er ventilenes overlapping som hindrer en del av deres proporsjonale virkning. Spesielt hindrer slik overlapping den integrerende biten av regulatoren.

En mekanisme som kan etterforsyne systemet og gjøre det klart til en ny runde er helt nødvendig. Denne rapporten foreslår en løsning hvor en høytrykks slange med olje trykker nitrogenet tilbake på plass og på den måten nullstiller systemet.

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Nomenclature

$\dot{m}$	Mass flow
$\dot{Q}$	Heat flow
$\dot{W}$	Work flow
ψ	Phase margin
ρ	Density
$_0$	Subscript for start values
$_1$	Subscript for values ahead
$_2$	Subscript for values behind
$_{gas}$	Subscript for the gas/oil accumulator
$_n$	Subscript for normal values
$_{oil}$	Subscript describing the oil
$_{tank}$	Subscript for the gas tank
B	Compressibility oil
C	Pneumatic valve constant
c	Friction factor
c_p	Heat capasity constant pressure
C_v	Valve constant
c_v	Heat capasity constant volume
f	Constant describing the relation between flow and pressure
h	Entalpi
k	Spring constant
m	Mass
p	Pressure
Q	Volume flow

R	Gas constant
T	Temperature
t	Time
U	Internal energy
V	Volume
x	Displacement
Z	Compressibility gas
BWR	Benedict - Webb - Rubin equation
BOP	Blow-Out Preventer
SSR	Shear Seal Ram

1. Introduction

1.1. Blowout preventor

1.1.1. General

A Blow Out Preventor(BOP) is a commonly used device in subsea maintenance. It is mainly a steel pipe with two or more large ram-valves. Figure 1.1 shows a typical BOP with several fluid power operated ram-valves. This is not the actual BOP which is the subject of this report, but a general one.[1]

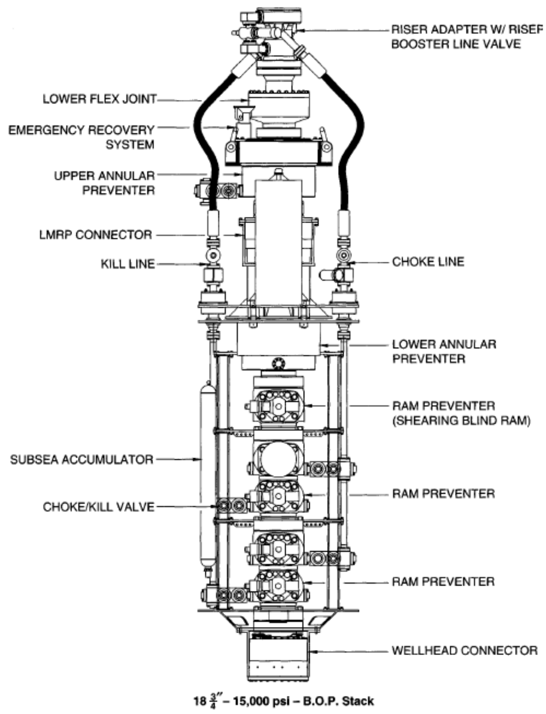


Figure 1.1.: Sketch of a BOP [Lyons and Plisga, 2005]

Blowout preventors have several functions, but the one which is of interest in this paper is its ability to prevent large blowouts of hydrocarbons from the well. There are several cases where such blowouts may occur. These include high pressures in the well and platform drift-off. More information on causes for subsea blowouts is provided by Holand [1997].

A blowout preventor is also used to close the well normally and stop the flow of hydrocarbons, but in this report it is the emergency close down which is of interest.

[1] Cover image:canadian wellsite

Figure 1.2.: The result of a blowout[Services, 2004]

During a normal operation, the BOP valves are operated with fluid power from the rig through the umbilical. In the event of an emergency may the umbilical be cut, and therefore must the BOP contain the necessary energy to operate the rams without a topside connection.

Figure 1.3 shows the location of the BOP on the seabed.

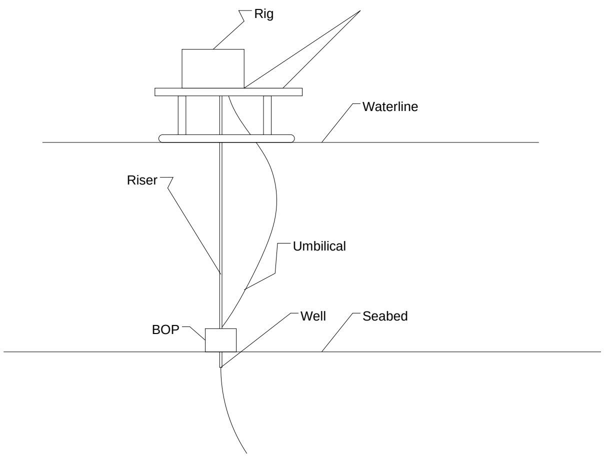


Figure 1.3.: Subsea system sketch

1.1.2. Ram valves

A Ridgid Arm Mooring (RAM) valve is a large valve which is designed to completely seal off the well and act as a barrier between the well and the sea. Two such barriers are required to always be in place. The barrier is achieved by inserting one blade from each side of the valve into the flow. These will then grip into each other and shutdown the flow.

There are several types of ram valves which have different tasks in addition to sealing the well. One may be designed to grip and hold equipment lowered into the well. Others are designed to cut equipment and pipes which are blocking for the ram. The latter is called a Shear Seal Ram.

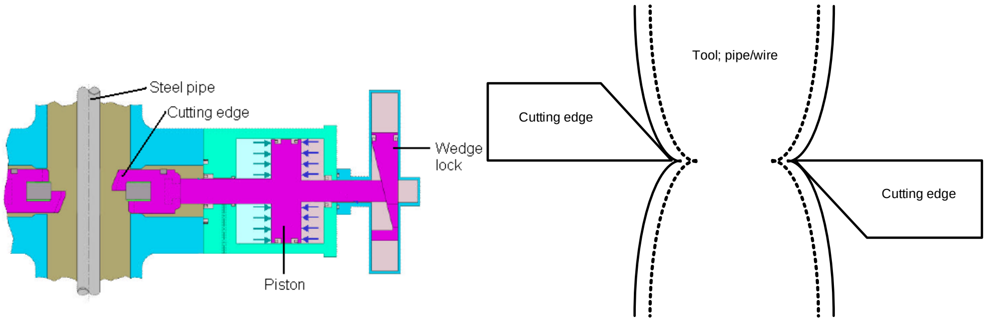


Figure 1.4.: Sketch of a SSR cutting a steel pipe [Left: Services, 2004]

During well maintenance, tools are lowered through the riser and into the well. Normally are such tools carried by either a wire-line or a steel pipe. While the tool is down, the ram are valves blocked and unable to close the well. The SSR is therefore a necessary tool for cutting during emergency shutdowns.

The maximum size and strength of the steel pipe is determined by the cutting capability of the SSR. Figure 1.5 shows the pressures in the hydraulic system powering the SSR during a test cut of a 3.5” steel pipe. The cutting occur from t=7 sec until t=12sec. The final pressure increase is the result of the wedge lock locking the ram into its final position.

The most important factor, limiting the cutting capability, is the supply pressure powering the ram. Increasing the supply pressure will increase both the speed and the force of the ram.

1.1.3. BOP accumulators

Normal bladder accumulators are the most common way of supplying a BOP with hydraulic pressure. These accumulators are located on the BOP and supply the valves with power. As shown in section 1.1.2, the pressure available from the accumulators is very important regarding the cutting capabilities and the closing speed. The cutting test shown in figure 1.5 shows a final accumulator pressure of 176 bar which is less then what is required by NORSOK D-002. NORSOK D-002 is the standard governing well intervention.

As shown, by Ostby [2007] in his project work, an increase in residue pressure in an accumulator will dramatically increase the required accumulator volume and weigth. Other solutions, other than normal bladder accumulators are therefore required to comply with the market demand to increase residue pressure.

1.2. High pressure accumulator with pressure governor

To solve the problems mentioned in section: 1.1.3, Ostby [2007] proposed to use a high pressure gas tank to pressurize the emergency oil circuits. Figure 1.6 shows a sketch with the main idea behind the high pressure accumulator solution, with a PI-governor.

When needed , a pilot operated valve (not shown in figure:1.6) in the control unit will connect the high pressure accumulator circuit with the required circuit. When the system is connected to the accumulator, the pressure will drop as a result of expanding oil and gas. The governor will open the valve to the high pressure gas tank, and thus ensuring that the pressure is maintained at a given level. To be able to maintain a constant pressure in the system, the parameters describing the valves, pistons, pipes and the rest of the system must be analyzed and optimized.

Much of the work in this thesis focus on the usage and the stability of the constant pressure regulator. In his project work proposes Ostby [2007] a hydraulic PI-regulator. A completely hydraulically regulator was proposed to remove any unnecessary electrical components which may, under rare circumstances, fail.

For such a solution to be effective, it is important that it has the possibility to be refilled from the surface. Even though it is very rare the accumulators are in use, it will be a loss of rig time, which is very expencive, if the accumulator have to be hoisted from the seabed and up to the rig when it has been used. This is an operation which may take several days.

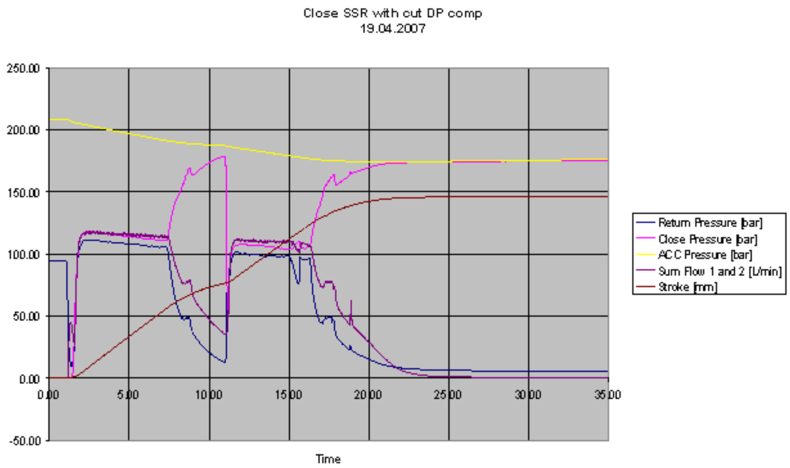
1.3. Goals

Because of the function of the BOPs, is it very important it closes quickly and always does it correctly. Therefore is its ability to close and the time it uses to do so the main focus of this paper.

No papers or patents describing a hydraulic PI-governor, which were usable for this kind of equipment, have been found, and therefore the main focus of this paper is to explore its possibilities with regards to a BOP. By comparing a normal pressure regulating valve to this PI-governor, it will be possible to evaluate its performance. Normal pressure regulating valves will not be examined in detail because its performance is well known and documented.[Nabi, Wacholder, and Dyan, 2000].

What affects the stability of the PI-governor and how can it be optimized are questions which will be discussed. Further will refill solutions and possible error modes be discussed. To verify all claims, simulation models be created, either on paper, in Simulink or SimulationX.

Each chapter in this report will address one of the tasks given in the project description.



(a) Pressures during the cut

(b) The pipe after the cut

Figure 1.5.: Results from cutting test of 3.5” S-135 Steel pipe[Int, 2007]

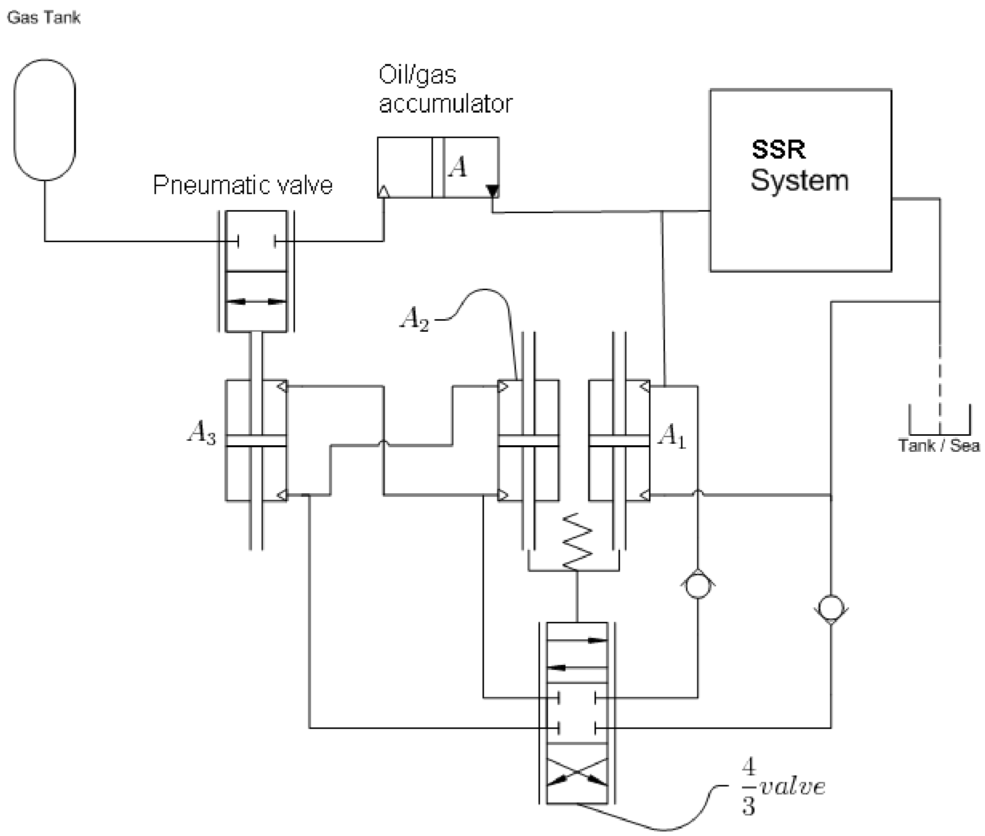


Figure 1.6.: Schematic of the PI governed pressure boost

2. Models

2.1. General

Both a linear and a non-linear model was created to evaluate the PI-governor. The linearization, which can be found in appendix A, was used to give an in depth evaluation of how the different parameters affect the overall system and to give some suggestions to how they should be chosen.

The non-linear model, which can be found in appendix B, was mainly used to verify the linear model. It was created in Simulink which has the possibility of linear analysis in non-linear models. Bode plots of the non-linear model have been made with the linear analysis software in Simulink.

The goals of the models are to maintain a constant pressure of $3000psi \approx 208bar$ plus the external pressure at $500m \approx 50bar$, which in total is $258bar$. The external pressure have to be included because during the emergency shutdown, the oil will be evacuated in to the sea to reduce the response time. The second goal is to reduce the time needed to close the SSR valves. This will however happen as a result of the first goal, and is therefore not used as an goal during the optimizations.

The models created in SimulationX (appendix D) have been used to simulate most of the cases where either changes in layout or errors have been implemented.

2.2. Requirements

To verify the linearized results was a non-linear model created in Simulink. For the models to be assumed to be correct, two requirements were created:

1. The Simulink model would be considered correct if the root mean square deviation of the pressure, was less then 2% of the reference value, and the pressure graphs had the same visual properties.

2. The linearized model would be considered correct if its logarithmic cross frequency in the bode plot was within ± 0.05 of the logarithmic cross frequency collected from the correct Simulink model, and the bode plot had the same visual properties.

If both these requirements are fulfilled, it will be assumed in this paper that both models are similar enough to the real system, and small changes in both will represent real changes.

The SimulationX model will be regarded as the most correct representation available until a real model is built.

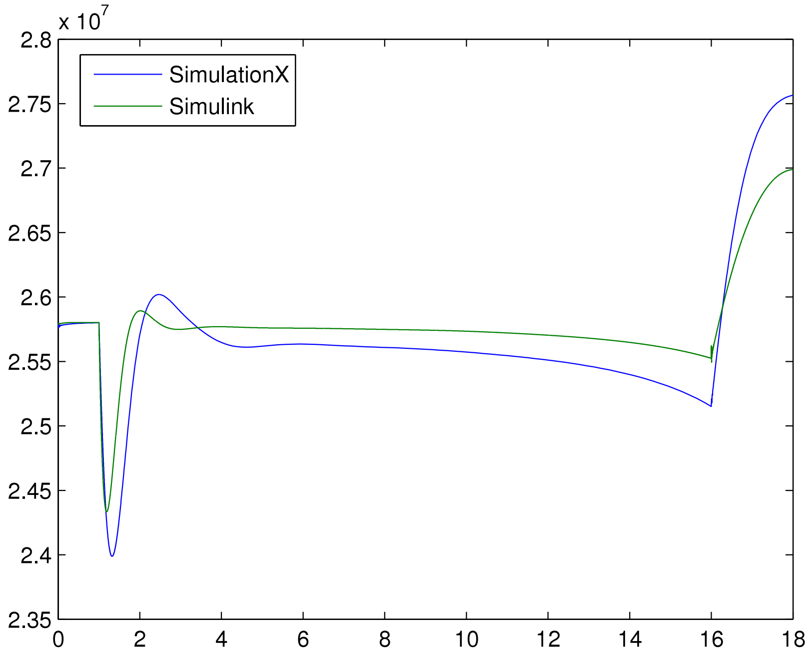


Figure 2.1.: Comparison of the oil pressure in SimulationX and Simulink

2.2.1. First

The Root Mean Square Deviation gives a good indication to how big the average difference between the two simulations are. The general formula is shown in (2.1).

$$\mathrm{RMSD}(\theta_1,\theta_2) = \sqrt{\mathrm{MSE}(\theta_1,\theta_2)} = \sqrt{\mathrm{E}((\theta_1-\theta_2)^2)} = \sqrt{\frac{\sum_{i=1}^{n}(x_{1,i}-x_{2,i})^2}{n}} \tag{2.1}$$

By applying (2.1) to the graphs in figure 2.1 with intervals of $0.05s$ the result becomes $\mathrm{RMSD} = 2.41bar$ which is less than 1% of the reference value of $258bar$. The two plots look quite alike, but there are some differences. These occurs because of simplifications in the non-linear model. But all inn all, the Simulink is model accepted with regards to the first requirement.

2.2.2. Second

The two bode plots of the linear model and the Simulink model is shown in figure 2.2
.

These two bode plots are very similar. The only real difference is found far to the left in the plot. This error is introduced by the linearization of the oil/gas accumulator, where some of the gas dynamics are simplified. There are several simplifications which may have caused this anomaly, but the linear model is all in all very similar to the one produced by Simulink. The cross frequency appears at $19.8 rad/s$ and $20 rad/s$, which is a deviation of $\log(20) - \log(19.8) = 4.3 \cdot 10^{-3}$. Thus is the non-linear model approved with regards to requirement two.

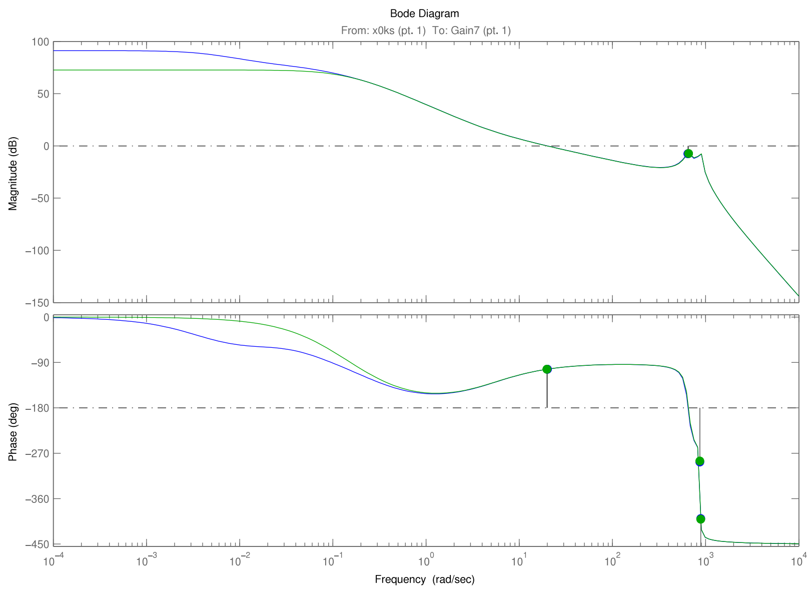


Figure 2.2.: Linear and Simulink bode plots

Parameter	Value	Unit
K_p	1	–
T_i	$\varepsilon[0.1, 1]$	s
A_1	≈ 0.1	m^2

Table 2.1.: Linear parameters

2.3. Results

Because both models are presented in the appendix, neither will be here. Only the results are presented.

2.3.1. Linear model

The linear model provided these parameters.

$$T_i = \frac{A_2}{C_v\sqrt{\frac{\Delta p_0}{2\rho}}} \tag{2.2}$$

$$K_p = \frac{A_2}{A_3} \tag{2.3}$$

T_i is the time constant of the integrator and the linear analysis show that it mainly contains parameters from the 4/3 proportional valve. K_p is described by piston parameters.

The resulting bode plot can be found in figure A.15. It shows a stable system with both good phase margin and gain margin. There are several other parameters which have been given values in the linearization chapter, but these are not included here because they are neither as critical nor as controllable as the ones shown in table 2.1.

Parameter	Value	Unit
K_p	5	–
T_i	0.2	s
A_1	0.1	m^2

Table 2.2.: Optimized parameters

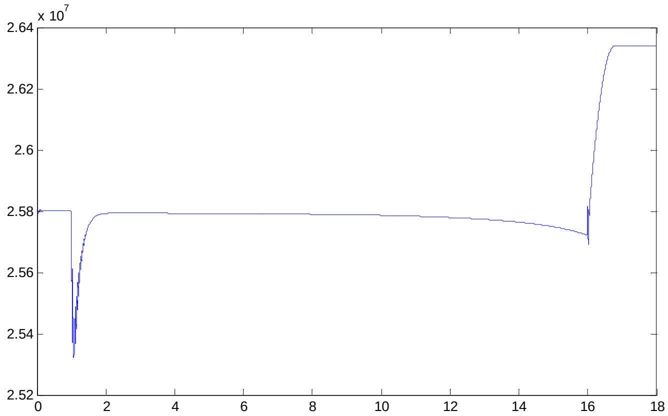


Figure 2.3.: Oil pressure with new parameters

2.3.2. non-linear model

The linear model was optimized by using the non-linear Simulink model. The results are shown in table 2.2.

More on the selection of parameters can be found in appendix: B. These parameters result in a stable system with satisfactory pressure variation. Resulting pressure plots can be found in figure 2.3.

2.3.3. Other important parameters

During the simulations with both the linear, non-linear and the SimulationX model several other parameters did stand out as important.

2.3.3.1. Accumulator gas volume

The volume of the gas in the oil/gas accumulator was expected to have some effect on the oil pressure. But as figure 2.4 shows, where $V_{gas} = 0.15l(green)\ 1.5l(blue)\ 15l(red)$, the volume does only affect the pressure when it is big enough to be some of a energy reservoir of its own. It should therefore be made as small as possible.

2.3.3.2. Pneumatic valve constant

As equation (A.24) shows, the size of the pneumatic valve constant will directly affect the gain of the system. In these simulations it is set to $3^{l}/_{bar\ s}$, but this may not be

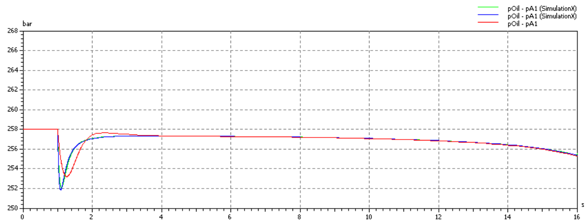


Figure 2.4.: Oil pressure when changing V_{gas}

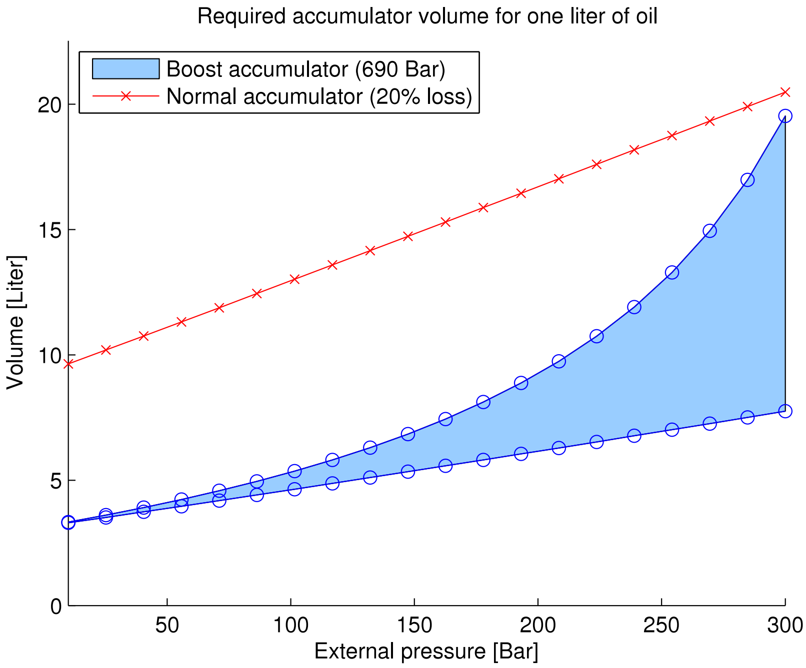


Figure 2.5.: Required gas volume

possible to acquire. Therefore must the regulator gain be changed to compensate for any changes in the valve.

2.3.3.3. Gas tank volume

The gas tank will be the biggest component in the new system as the accumulator were in the old one. Its size is dependent of the pressures required and the volume of oil required. Figure 2.5 shows how big the gas tank have to be compared to a conventional accumulator solution. Codes for creating this figure is found in appendix C.

The blue area represents the high pressure solution from this thesis. The huge variation in the required volume is a result of different refill solutions. The largest gas tank volume is required when the refill system cannot pressurize the tank higher than the topside pressure of 690*bar*. The other extremity is the lowest line which can be used if the refill solution manages to pressurize the gas tank to 690*bar* plus the external sea pressure.

The red line shows the normal accumulator solution given a refill of max pressure

($208bar = 3000psi$) plus the external. Which means that the best comparisons are the red line and the lower most blue line. The increase in volume required at larger depths is a result of both non ideal gas properties and the pressure loss not being relative to the total pressure but the relative pressure compared to the external pressure.

The result shows that at $500m$ is the required gas volume $3l$ gas per $1l$ oil with the boost solution, without pressure loss. While the required volume for a normal accumulator, with 20% pressure loss, is $11l$ gas per $1l$ oil.

2.4. SimulationX

2.4.1. General

During all the simulations, except the ones in this section, a very simplified model of the SSR unit was used. See equation (A.39). While this could not give completely realistic results, it did reduce the necessary time for each simulation while giving fairly good results. To be able to test the PI-governor and the high pressure system it was important that a more realistic model of the SSR system was used.

FMC usually simulates every new hydraulic circuit to verify and approve of its behavior. Such a model was also made for the SSR circuit where the high pressure boost system was intended. To test the final design possibilities were they simulated with the 'real' SSR circuit. A printout of these systems can be found in appendix D.

2.4.2. Simulation

2.4.2.1. Original

The original systems pressures and displacements are shown in figure 2.6. It shows the SSR closing at $t = 17s$ and the wedge lock closes at $t = 42s$. A wedge lock is a wedge which is driven in behind the SSR piston to lock it in place if the hydraulic pressure is lost. It can be seen in figure 1.4 on page 3. The most important values which can be extracted from this plot is the closing time of the SSR and the oil pressure at the end of the cut. Here is the pressure $220bar$.

2.4.2.2. Boost with PI governor and ideal 4/3 valve

Figure 2.7 shows how the improved system responds to the SSR circuit. The SSR finish cutting at $t = 13s$, the wedge lock closes at $t = 32s$ and the pressure during the final parts of the cutting is $256bar$. There are some pressure variations which are quite large. These are the result of changes in force needed to cut the pipe, integrator wind-up [Balchen et al., 2003] and time delay in the system.

2.4.2.3. Boost with PI governor and real 4/3 valve with displacement boost

This is the solution proposed in section 4.5 to minimize the effect of a real 4/3 valve with a 10% overlap. Figure 2.8 shows that this solution almost is identical as the one with the ideal 4/3 valve.

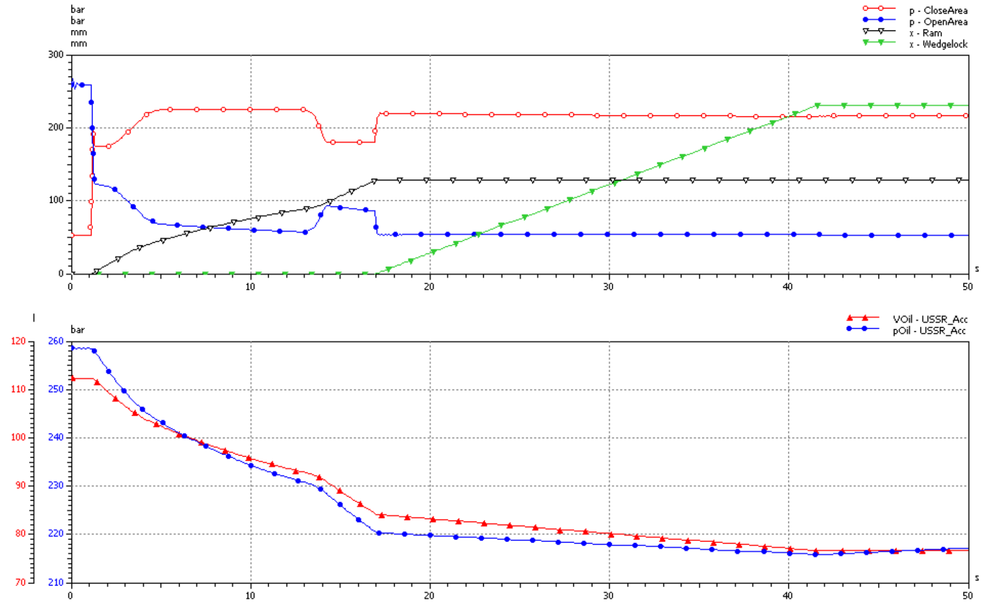


Figure 2.6.: Original plot

2.4.3. Result

As this section shows, a change from a normal accumulator to a PI governed pressure boost system will reduce the time needed to close the SSR unit, and also increase the pressure available for the cutting process.

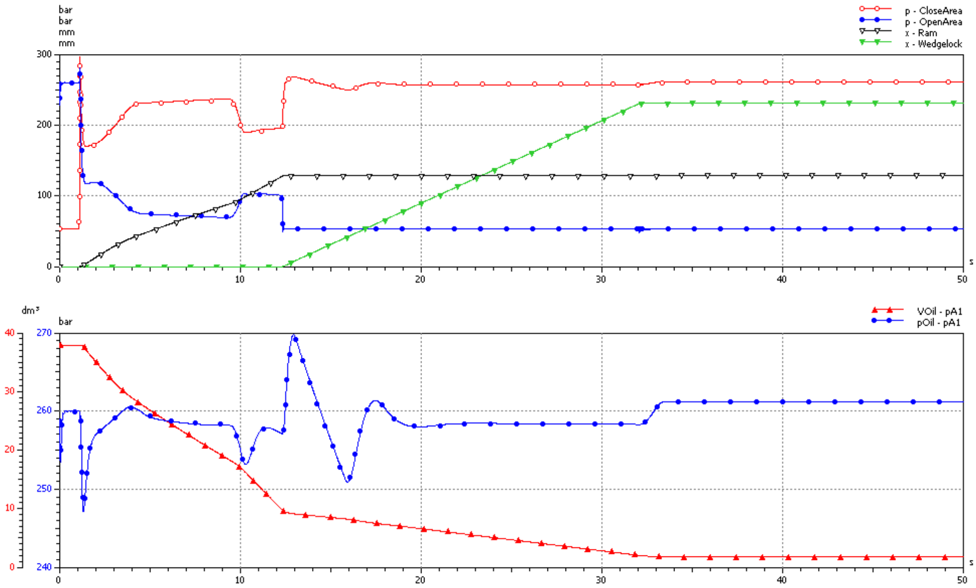


Figure 2.7.: Boost and ideal 4/3 valve

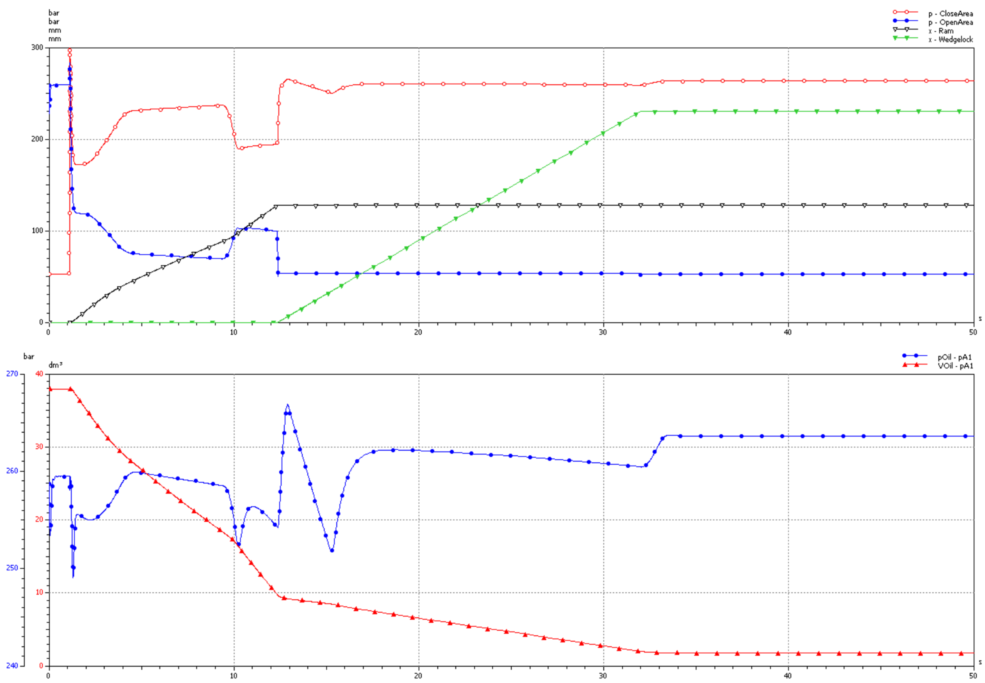


Figure 2.8.: Boost and real 4/3 valve

3. Refill

3.1. General

One important aspect of the high pressure gas tank solution is refill. Without some sort of mechanics to replenish the energy, the BOP must be hoisted to the surface and refilled there. Such an hoisting operation may take several days and is thus expensive and unnecessary.

3.2. Solutions

Note: In the sketches in this section, most of the governor and the SSR system are removed and only the parts vital for the refill process included. These simulation was conducted with SimulationX using the final recommended (which can be found in table 2.2 on page 10) values for K_p, T_i and the rest of the parameters. The SSR valves are closing between $t = \{[0,15],[600,615],[1200,1215]\}$, and the refill happens at $t = \{[250,450],[950,1150]\}$. Because the heat transfer is very important to the way the refill happens, these simulations are done with heat transfer and not adiabatically as the rest of the simulations in this paper.

3.2.1. Compressor

This solution uses a hydraulic motor and a compressor. Figure 3.1 shows a sketch of the idea. When the gas tank has reached its minimum pressure and the system is ready to refill, the hydraulic oil will flow down through the high pressure (HP) piping and through the motor. The motor runs a compressor which pumps the nitrogen gas back into the gas tank. While the gas in oil/gas accumulator is pumped out, the low pressure oil coming from the motor will run into the oil/gas accumulator refilling it. Any residue oil will be returned to the topside rig.

The pump solution have not been simulated in SimulationX, and the conclusions made in this chapter may therefore be incorrect. There were not made any simulations because SimulationX were missing tools for compressor simulation. An attempt was made to simulate this with the combination of simple elements such as pistons, drive shafts and rodes. The results varied a lot, even with small changes, and were therefore considered to unreliable.

Advantages:

- Can create the highest pressures in the gas tank
  - The compressor/motor can create higher pressures then the HP oil pressure.

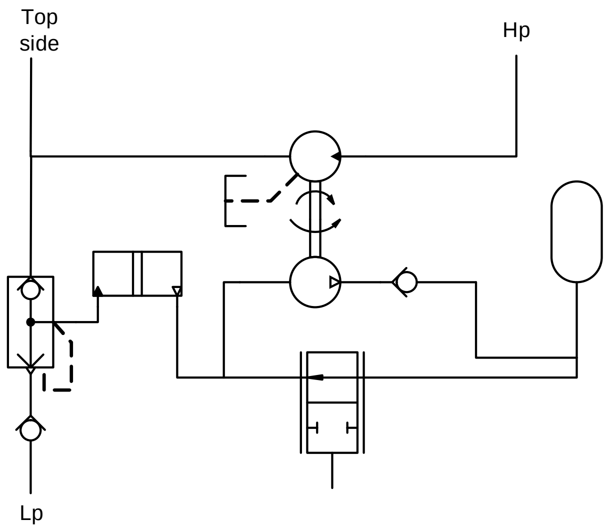


Figure 3.1.: Refill by pump

- Closed nitrogen system which may help avoid unwanted particles and moisture in the nitrogen
- Easy start of the refill process:
  - Pressurize the HP line to the motor and the rest is automatic.

Disadvantages:

- Large system with may components which may fail.
- Possible leakage problems with the compressor and motor.

3.2.2. Gas supply

3.2.2.1. General

This solution uses an extra hose, going through the umbilical, with high pressure nitrogen. When the gas tank is 'empty' and ready for refill, the gas in the oil/gas accumulator will be ejected into the sea. Fresh oil and nitrogen comes through the umbilical and replenish the system. This solution is the solution which has the smallest potential for high pressures in the gas tank.

$$p = p_{topside} + \int \rho_{gas}(z) \cdot g\, dz \tag{3.1}$$

Because of the non ideal behavior of nitrogen at high pressures, this equation was solved numerically with the BWR equation. See appendix C for the code. The result was an increase of the pressure compared to the topside pressure of 20bar at 500m of depth.

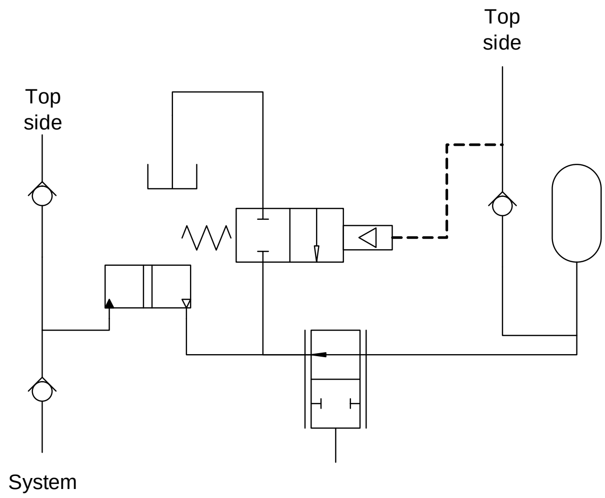
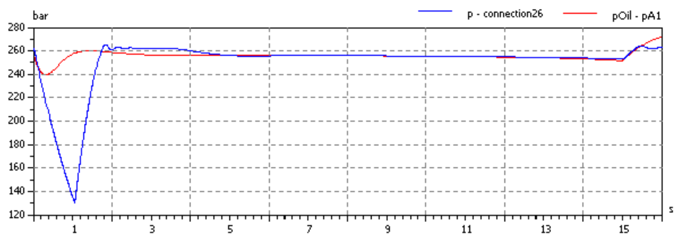
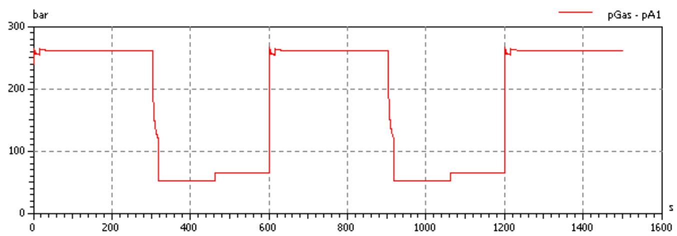


Figure 3.2.: Gas supply



(a) Oil pressure



(b) Pressure in the gas part of the oil/gas accumulator

Figure 3.3.: Simulation: Gas supply

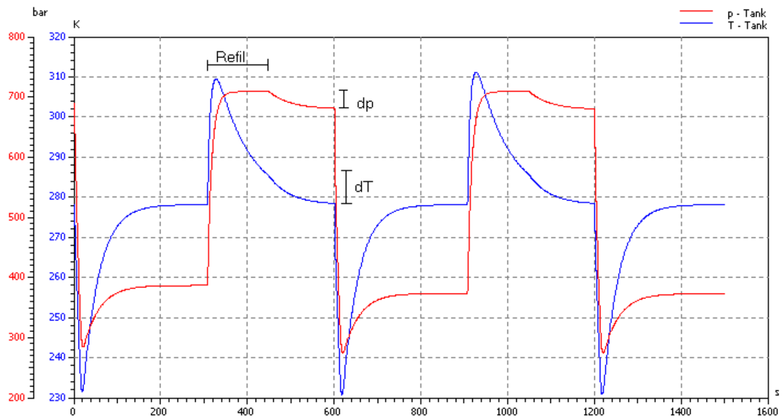


Figure 3.4.: Simulation: Gas supply. Tank pressure

3.2.2.2. Simulation

As figure 3.3 shows,there is a large difference between the no-refill solution (red line) and the refill-gas solution (blue line) as to how the oil pressure changes when the SSR starts to move. The final pressure is however almost the same. This difference occurs because the pressure in the accumulator (figure 3.3(b)) is severely reduced after the remaining gas has been ventilated into the sea. The pressure is not regained until the governor detects the 'missing' pressure, when the SSR starts to move, and the pneumatic valve is opened.

An important issue is to continue the refill operation even after the pressure in the tank has reached the final value. This is because the filling process increases the temperature in the tank, and when the refill is complete and the temperature drops (dT) , the pressure will also drop (dp). This can be seen in figure: 3.4 where the refill only lasts for about half the time it is supposed to do. How long this refill operation have to go on depends on the rate of heat which is transferred from the gas tank and into the sea. This value have to be calculated when the properties of the actual gas tank is known.

Advantages

- Easy to control the final pressure in the gas tank
- Requires no high pressure oil supply

Disadvantages

- Not a closed gas system
  - May allow particles and moist into the system
  - Both a problem with the topside connection and the release valve into the sea
- Can not refill to as high pressures as the other solutions

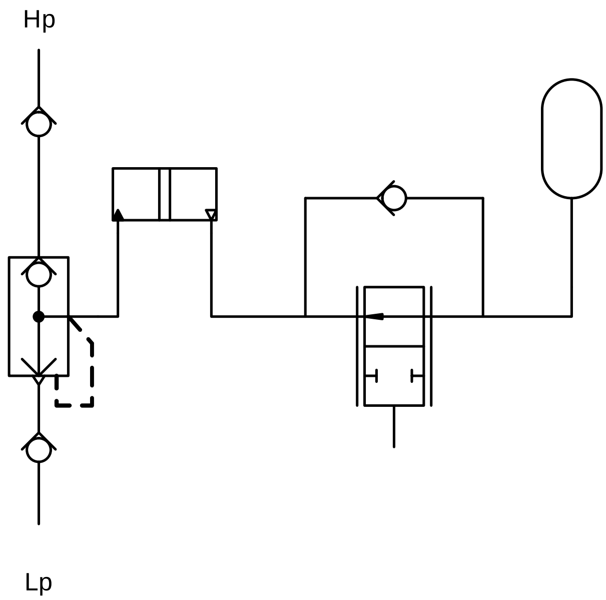


Figure 3.5.: Oil Refill

- Require a high pressure nitrogen hose
  - This is not normal in the existing umbillicals

3.2.3. Oil supply

3.2.3.1. General

By connecting the oil/gas accumulator to a high pressure supply hose from the umbilical, the gas can be forced back, through a bypass circuit, into the gas tank. This is a very simple refill method which require few extra components and it provides decent pressures.

$$p = p_{topside} + \int \rho_{oil}(z) \cdot g \; dz \approx 690bar + 42bar \; at \; 500m \tag{3.2}$$

To refill the tanks, all that is needed is to pressurize the high pressure line and the rest goes automatically. When the oil tank is full, the gas will be back in the tank and the system reset.

3.2.3.2. Simulation

Figure 3.6 shows the pressure when using the refill (red) and without (blue). Like the gas-refill, there is a large downwards spike at the beginning of the operation, but it is not as big as with the gas refill and it settles quickly. The final pressure is the same.

As with all the other refill solutions, it is important to continue refilling the tank even after it has reached its goal pressure. Figure: 3.7 shows what happens if the refill is stopped prematurely. When the excess heat is transferred into the sea and the temperature stabilizes, the pressure in the tank has dropped with close to 100bar (dp).

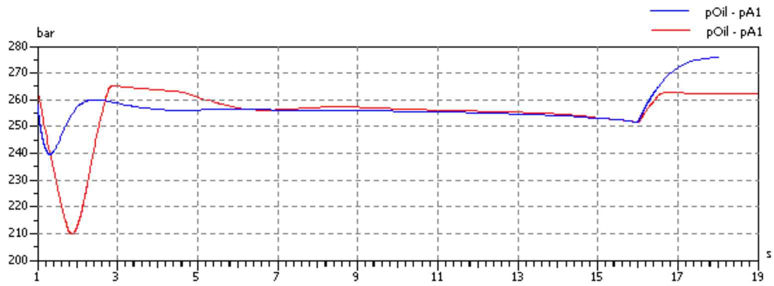


Figure 3.6.: Simulation: Oil Refill: Oil pressure

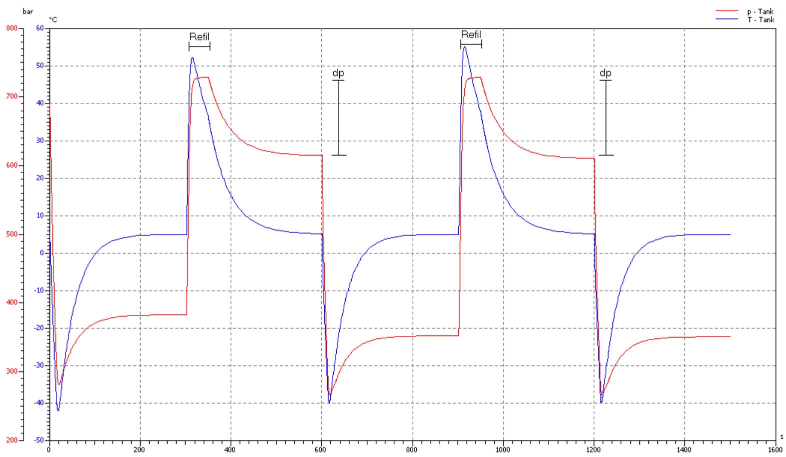


Figure 3.7.: Simulation: Oil Refill: Tank pressure

Advantages

- Very simple
- Require very few extra components
- Closed nitrogen system
- Does not create troubles for the governor
- Creates decent pressures which scale with the depth

Disadvantages

- Requires the oil/gas accumulator to be high pressure certified
  - This may result in bigger seals and thus more friction
- Require an extra safety valve stopping the high pressure oil from reaching the rest of the system

3.3. Conclusion

The best way for refilling the oil/gas accumulator is the 'Oil supply' solution. With this solution, the operator has the easiest job, and it requires very few new components. There is no extra parts needed other then an upgrade of the pressure class of the oil/gas accumulator and an extra valve for separating the high and low pressure circuits. As for supply line, it does need a high pressure line from the surface. This line is normally available through the umbilical and requires thus no extra installation.

These simulations also show that the governor is able to reset automatically after each run without any external action needed.

4. Error modes

4.1. General

In the whole constant pressure boost system, there are several items which may fail and create problems. In this chapter, some of the most problematic failures are simulated and evaluated. Some of the simulations are not real failures or errors, but physical properties which have been neglected in earlier simulations. This includes friction in the pistons and some leakages.

4.2. Wobbly connection to the pneumatic valve

The connections in the governor may for some reason become loose and the connection to the pneumatic valve may loosen a bit. Figure 4.1 shows a simulation of two cases. The relative play in this simulation is $1\%(red)$ and $5\%(green)$. The reference is shown with the black line.

As can be seen, such a loose connection will create some oscillations in the oil pressure but it will not become unstable.

4.3. Wrong preset and change in spring constant

A change in either the preset or the spring constant will result in the same error; a change in reference pressure. Figure 4.2 shows a simulation of changes in the preset of the spring. The changes are $\pm 1\%$ and the black line is the reference.

For small changes in both spring preset and the spring constant, the pressure will change linearly. Normally will this not affect stability only the absolute pressure. If the changes in the spring constant becomes large, it will affect the eigen frequency and therefore the stability. Because even small changes will largely affect the resulting oil pressure, it is important that the spring is kept in a homogeneous atmosphere. Perhaps in an oil chamber where the sea water cannot affect the spring properties. This have to be further analyzed by specialists on material mechanics or corrosion to ensure the correct precautions are made.

4.4. Friction in the pistons

In neither the linear approximation nor in the regular simulations have the stick friction in the pistons been included. Such friction is common and must be included in this analysis. The quantity of the stick force is mostly dependent on the seals used in the pistons, and the absolute value have to be collected from the manufacturer. Normally the will the stick friction vary from $1bar$ and upwards. In figure 4.3 are the

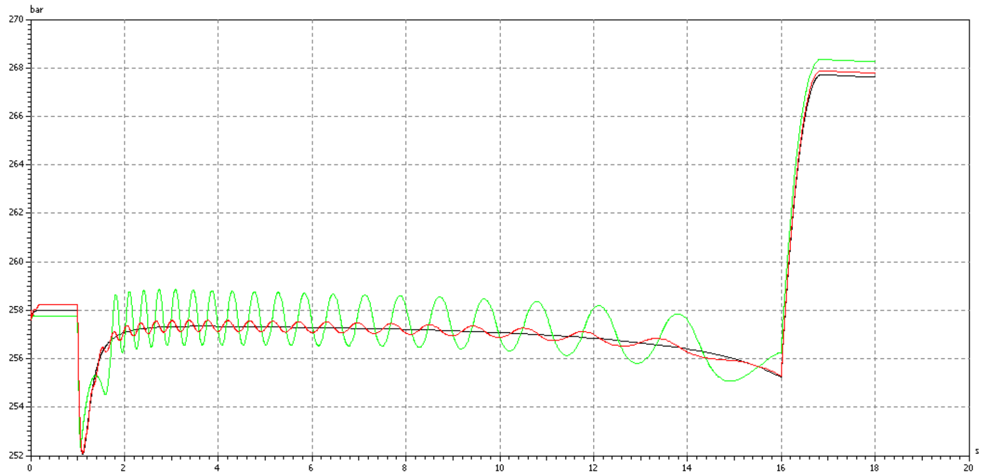


Figure 4.1.: Simulation of loose connection

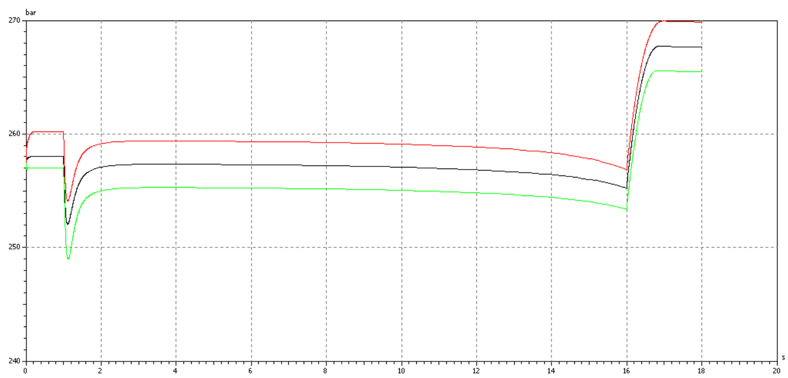


Figure 4.2.: Simulation of wrong spring preset

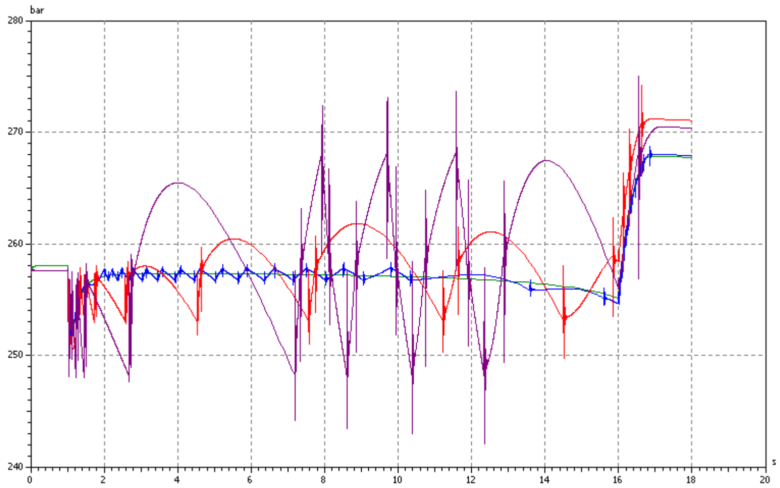


Figure 4.3.: Simulation of stick force in the pistons

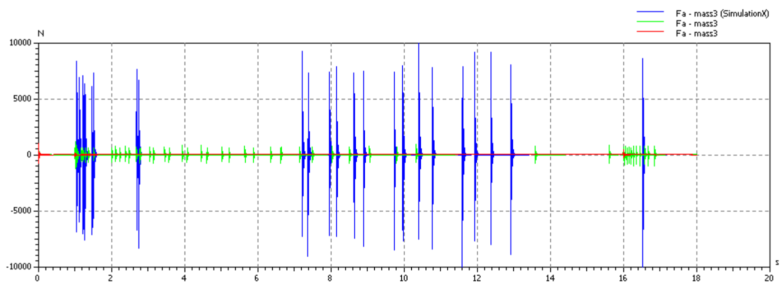


Figure 4.4.: Simulation of the inertial forces because of friction

stick frictions $1bar(blue)$, $5bar(red)$ and $10bar(purple)$ while the green line is the reference.

Such stick friction will create small instabilities and oscillations, but the system will all in all stay stable, and the pressure stay close to the expected value. If the stick values start to exceed $5bar$, the oil pressure may fluctuate so fast and with such force that it may become a problem with the safety and integrity of the entire structure. This can be seen in the $10bar$ line , where there are huge pressure changes and oscillations. Figure 4.4 shows the change in inertial forces of the moving piston in the oil/gas accumulator: $no\ friction(red)$ $1bar(green)$ and $10bar(blue)$.

4.5. Real 4/3 valve

As mentioned in section A.2.5.3, the 4/3 proportional valve used in most of the simulations is idealized. Figure A.10 shows how the real valve has a overlap where the flow remains zero without regard to slide valve position, where the ideal valve has no overlap. While the valve is in the overlap position,the effect of the integrator in the governor is removed. It is therefore very important to minimize the area where

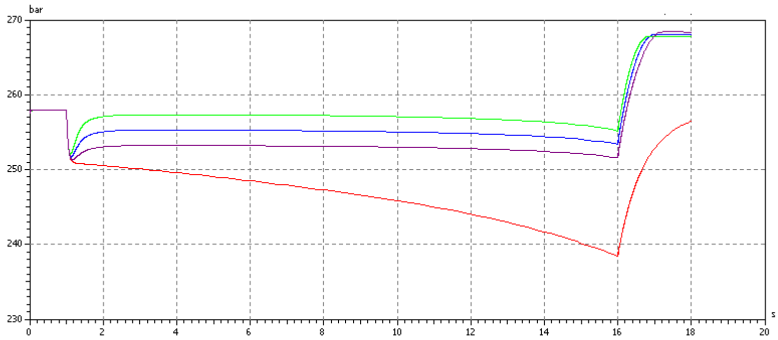


Figure 4.5.: Simulation of different overlaps

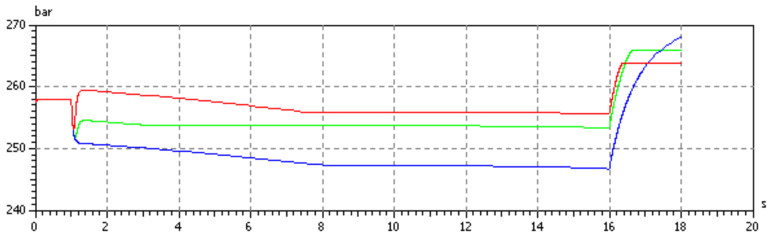


Figure 4.6.: Simulation of valve displacement boost

the valve is in this position. Figure 4.5 shows the results of different overlap sizes. Overlap: $0\%(green)$, $1\%(blue)$, $2\%(purple)$,$10\%(red)$

This figure shows the importance of the overlaps sizing. With 10% overlap, the integrator will not start integrating until the pressure drops with 10% which does not happen during this simulation.

The effect of the overlap can be significantly reduced by adding a hydraulic gear which increases the displacement of the valve. By increasing the displacement, the value T_i will be reduced by the same factor. Normally would this significantly reduce stability, but because of the overlap, the instability effect will be reduced / removed. In figure 4.6 is there a 10%overlap and the displacement have been boosted by: $2x(blue)$, $5x(green)$ and $10x(red)$.

With a boost of $5x$, the pressure will settle at $^{10\%}/_{5} = 2\% \approx 5bar$ of the desired value which is very good.

4.6. Leakages

There may occur some leakages in the governor. This section looks closer at leakages in the 4/3 valve and the pneumatic valve.

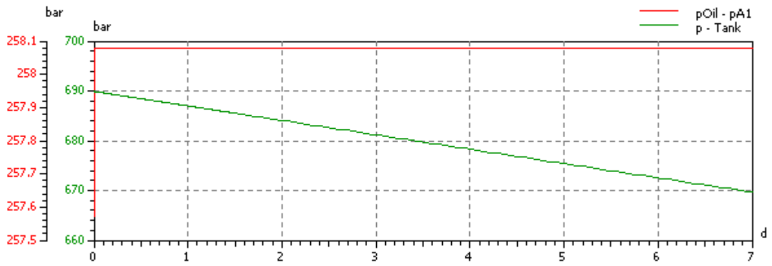


Figure 4.7.: Simulation of leakage in the 4/3 valve

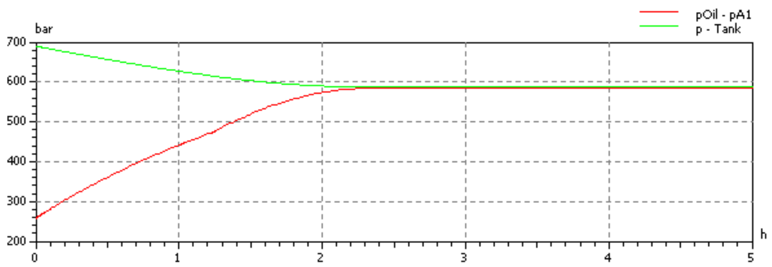


Figure 4.8.: Simulation of leakage in the pneumatic valve

4.6.1. In the proportional 4/3 valve

A leakage in the 4/3 valve will result in a change in the oil pressure. This change will then be handled by the governor and corrected. It is only the pressure in the gas tank which will be affected by this leakage. Dependent of the magnitude of the leak, the gas tank pressure will drop at different rates. A scenario with a leak of 1% of max flow over a 7 day period is shown in figure 4.7. The pressure drop is quite small and can easily be replenished with one of the refill mechanisms from chapter 3.

4.6.2. In the pneumatic valve

A leakage in the pneumatic valve will have much more dramatic consequences than the leakage in the proportional valve. When the gas leaks through the pneumatic valve, the oil pressure will increase, and the Governor try to close the valve but it can't because its already closed. Therefore, there is no way to stop the oil pressure increase. Figure shows a simulation where the leakage is $^1/_{10000}$ of the maximum flow through the valve. Already after a few hours, the gas tank will have emptied and the pressures have balanced out. The oil pressure is now much higher then the maximum allowed in the system.

It is therefore very important that the pneumatic valve is completely leakage free when in the closed position. If no such valve is available is this concept more or less useless.

4.7. Cold

When the nitrogen in the tank suddenly is allowed to expand, the temperature will drop. The worst case simulations (adiabatic) show a temperature drop of as much as $55K$. From $5°C$ to $-50°C$. This may provide problems with both the pneumatic valve and the oil. Any moist in the nitrogen will freeze and this may interfere with the pneumatic valve. The frozen moist may for instance clog the valve or freeze it in one position.

Another problem may occur if the piston in the oil/gas accumulator is a bad insulator and the oil is cooled, resulting in a higher viscosity and may create other problems in the system.

4.8. Conclusion

There are several areas where the real system may differ from the simulated one. Some of them are more important than the others.

Changes in either the spring constant or the spring preset position will change the preset value of the oil pressure relatively. Small changes can be tolerated, but the spring should be contained in a homogeneous environment where it is not affected by external factors. Such factors include pressure and temperature changes, corrosion and cracks.

Friction may also severely change the behavior of the governor. Even though it is impossible to completely remove friction, it should be reduced to a minimum. Heavy stick friction will cause large vibrations and osculations in both the oil pressure and the rig as a whole.

Leakage in the valves may completely destroy the low pressure circuits leading to the SSR. While a leakage in the proportional 4/3 valve easily can be replenished by starting the refill process, will a leakage in the pneumatic valve result in a much more dramatic problem. When the gas leaks through, the oil pressure will increase to levels much higher then the design pressure. It is thus very important the valve stays closed when it is supposed to be closed.

It is also important the nitrogen gas is as pure as possible, and the connection between the gas and the oil is somewhat insulated.

5. Other pressure governing solutions

While the general idea of a boost tank is kept are there other solutions, other than the PI-governor idea, to maintaining a constant pressure in the system. This chapter will present a few with their positive and negative aspects.

5.1. Pressure reducing valve

An ideal pressure reducing valve will in theory be able to maintain a constant pressure downstream of the valve. This however is only true when the internal dynamics of the valve are ignored. When introducing a valve with mass and friction, the result becomes somewhat different.

Pressure reducing valves are really simple proportional valves which may have problems regulating this system with its non-linearities. On the other hand are such valves much smaller and their mass and friction is smaller then the massive PI-governor. Therefore can the proportional gain be much higher which helps counter the non-linearities by lowering the response time of the governor.

There are mainly two different ways of governing the pressure by a pressure reducing valve.

5.1.1. Valve in the gas flow

The first placement of the pressure reduction valve is in the gas flow, replacing the pneumatic valve in the PI-governor system. This should ensure a correct pressure in the gas/oil accumulator and thus a correct pressure in the rest of the system.

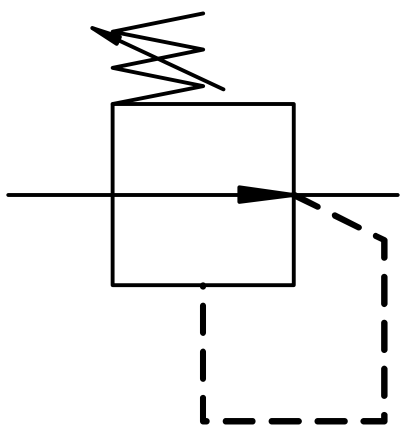


Figure 5.1.: Pressure reducing valve

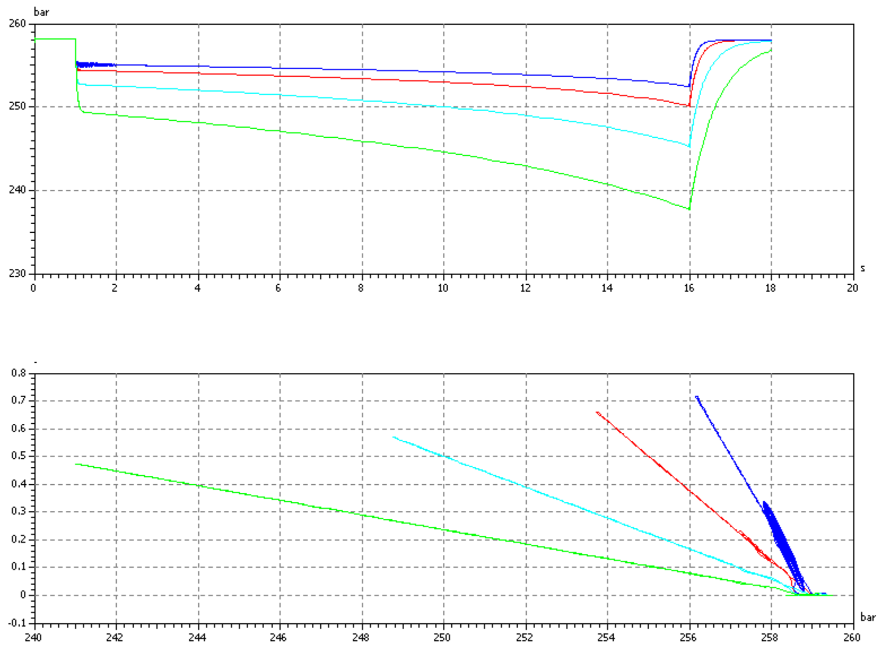


Figure 5.2.: Pressure reducing valve in the pneumatic system

Figure 5.2 shows the resulting oil pressure for different gain values, simulated in SimulationX. In the second figure are the gain values represented as valve opening related to the gas pressure. A steeper line represents a higher gain value.

What can be interpreted of this figure is, that the placement of a pressure reducing valve in the pneumatic pipeline may be very usefull. It reaches almost the same end pressure values as the PI-governor ($253bar$ and $255bar$, ref:$258bar$), but it requires the pressure reducing valve to almost touch its stability limits. As can be seen with the blue line which oscillates quite rapidly at the start.

The main advantage of this solution is the reduced need of special components and space. A small and simple valve can replace the entire PI-governor.

5.1.2. Valve in the oil flow

By placing the pressure reducing valve after the oil/gas accumulator, the system can be simplified even more. The gas tank and the accumulator can be combined into one single unit. This is by far the least space and component demanding solution. It however does not produce any particularly good results with regards to neither stability nor residue pressure.

Figure 5.3 shows the resulting oil pressures for different gain values. And as in figure 5.2 is the gain represented by a opening vs pressure plot. The bad performance may be the result of the much lower compressibility of the oil which results in a much steeper connection between density and pressure. This effect could be reduced by introducing a small accumulator to absorb some of the oscillations. An extra accumulator would however remove the benefits of the pressure reducing valve in the oil

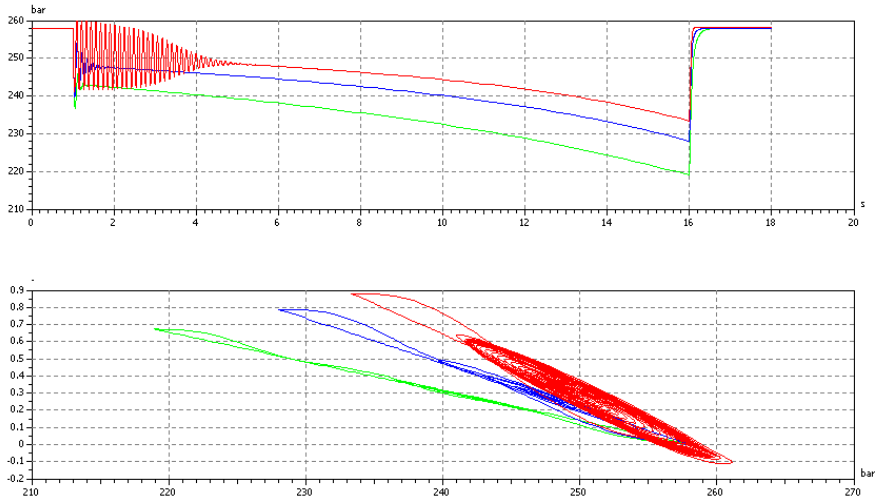


Figure 5.3.: Pressure reducing valve after the accumulator

stream.

5.2. Hydraulic PI-governor

Before the electric governors were invented, hydraulic-mechanical governors were used. Kværner developed such a PI-governor to govern hydro power plants. See figure 5.4. The governor can however not be used in the pressure boost system. It is based on a constant oil flow through the governor, both to measure the input signal and in the integrator. It solves the problem with a overlapping integrator by allowing a constant flow, which runs through it and not into the actuator collecting the integrated flow. When the drawback moves, the openings will change and allow the oil to move in and out of the actuator. More on this can be found in Kverner-Brug.

Other hydraulic-mechanical PI-governors have not been discovered. Searches in publication databases and patent databases have given no results.

5.3. Conclusion

The usage of a pressure reducing valve will reduce the size and complexity of the governor. It can also reach almost the same pressures as the PI-governor. However does such a solution not seem quite as stable as the PI governor. To conclude on this matter is a further study of the effect of the pressure reducing valve required. One possible valve which may prove to be usable is the Dome-Loaded Pressure Regulator, which dynamics are described by Nabi et al. [2000]. They conclude with this kind of valve is a very fast responding valve which is suitable for high flows, but it requires quite some optimization for it to become good for the specific problem. No simulation of this valve was conducted because of the advanced layout of the valve and the lack of good pneumatic elements, usable for modeling this valve, in SimulationX.

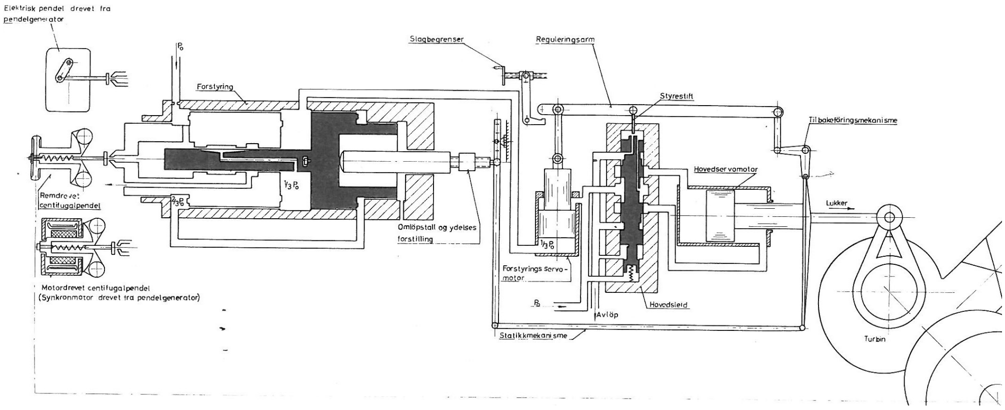


Figure 5.4.: Kværners hydraulic mechanical PI governor[Kverner-Brug]

6. Real life testing

In the project description, there was an optional point which involved building of either a small scale model or a real size model of the high pressure PI-governor. It became clear during the project work that a full scale model could not be built because of a lack of a collaborating partner. Statoil was first proposed as such a partner, but a series of events prevented this from working out.

Then the work on a scale model was started, but soon it became clear that there were no supplier which could deliver the necessary parts within the period of the project. This was mainly because of the special parts needed for this. A mechanically operated valve with the specifications needed was not stock line. Neither was the mechanically operated high pressure pneumatic valve stock. To manufacturer such parts would require way to much time. Such setbacka were crucial and the scale model was canceled.

Because the model tests was impossible to carry through, does it not imply that building a full scale or a small scale model is impossible. It just requires more time and resources then what was available during the project.

7. Final reflection

Even though the main idea shows promising results, there are some drawbacks and kinks which should be further examined. First should the physical layout of the system and what specific parts which should be used investigated. Most of the components used in this report are not real components, but components which have properties which give the wanted result. This is especially important for the valves which size is selected only to fit the model. A result of this was detected when the physical model was in planning. None of the suppliers had anything that could match the wanted specification in stock, and almost every part had to be custom made.

Another problem is unknown variables. Variables which should be included, but are not because of simplifications or just plain ignorance. Therefore should a prototype be built to look for errors and irregularities which may cause failure.

A third aspect to consider before building this is other solutions. Some have been proposed in this report, but none of them give very satisfactory results. However, there may be better designs available which may give better results. Or there may even be completely other solutions which the author never discovered.

On the topic of refill, there are several possible ways to do this, all which somewhat fulfills the task. The choice should be based on what is the most important to achieve. For instance; if either high pressures or simple design is the most important.

Finally should it be noted that all the conclusions made in this report is based on simulations. None of the results are exact simulations of the real world and, therefore, as mentioned earlier, should a test model be built.

8. Conclusion

A boost system to enhance the cutting capabilities of the SSR on a BOP is very favorable. Compared to normal accumulator solutions does the gastank only require $3/11$(at $500m$) of the volume. In addition to a reduced volume does it also reduce the response time and increasing the pressures. To maintain a constant pressure, a pressure regulator is used. There are several available in the market, but all rely on a proportional governor. In this case, where there are large variations in the pressures and lots of non-linearities, such a governor may be inadequate. As shown in this report, the normal pressure reducing valves tend to become unstable when they reach the level of accuracy a Proportional-Integrational governor have.

When carefully selecting parameters, a PI-governor will be able to maintain a pressure which stay very close to the preferred pressure. This solution has shown good results in simulations both with simple step responses and more complex behavior of a 'real' SSR. The 4/3 proportional valve have to be carefully selected. As shown earlier will an overlap reduce / remove the effect of the integrator. A hydraulic gear, enhancing the valve movement, will reduce the effect of such an overlap.

There is a need for a refill possibility, which can be solved in several ways. Using a high pressure oil supply line from rig through the umbilical to push the gas back into the gas tank and thus resetting the system, seems to be the best solution. One important issue to keep in mind is the time spent refilling. Simulations show that the refill process should continue long after the goal pressure is reached. This is because the temperature should have enough time to stabilize at the ambient temperature, and thus prevent pressure loss due to temperature change.

All in all is the schematic, which was collected from the earlier project work, the same as the one now used for the final design. No changes, except in sizings, have been made to the original schematic.

Finally was there supposed to be a model test of the final design to verify the simulation results. This was hindered by a series of events. The model build was canceled, early in the process, because it appeared to be impossible to complete it within the bounderies of the project.

9. Further work

The next step in the process of creating this boost system, should be building either a scale model or a full scale model. By doing so, the work in this project can be verified and, til now, unknown variables can be added to the simulations.

Further more should the possibility of better designed pressure reduction valves be examined. There are some designs which are not simulated or evaluated carefully in this report because of either lack of data or their complexity. These may provide better results then the ones used in the simulations. Therefore should someone with in depth knowledge of pressure reduction valves be consulted.

If both the test gives good results and no simpler solution is found, should the work with certifying the new components for sub sea use, be initiated. Each new component have to be certified, for sub sea use, according to code, regulations and legislation.

When all this is done, there is no big obstacles left and the PI-governed boost system should be ready.

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A. Linearization

A.1. General

This chapter explains how each of the parts included in the pressure governor were analyzed with regards to stability. The linearization process will be explained in detail for each object.

Common values

Where there have been inserted values are these collected from SimulationX. They are shown in table A.1. The subscript $_{gas}$ is used for parameters in the gas/oil accumulator and the subscript $_{tank}$ is used for parameters in the high pressure gas tank.

A.1.1. What is linearization

By linearization of an equation it is possible to analyze it by control engineering methods. But such an linearization also removes some of the dynamics and properties of the equation. The results can therefore not be 'trusted', but should only be used as guidelines. The linearized equations can often provide important information on what parameters in the equation which is the most important with regards to stability and response.

Each equation were linearized using this method:

$$f = f(x_1, x_2, ..., x_n) \tag{A.1}$$

$$\frac{df}{d\vec{x}} \mid_{\vec{x}=\vec{x_0}} = [\frac{\delta f}{\delta x_1}, \frac{\delta f}{\delta x_2}, ..., \frac{\delta f}{\delta x_n}] \tag{A.2}$$

$$df \mid_{\vec{x}=\vec{x_0}} = [\frac{\delta f}{\delta x_1}, \frac{\delta f}{\delta x_2}, ..., \frac{\delta f}{\delta x_n}] \cdot d\vec{x} \tag{A.3}$$

$$df \mid_{\vec{x}=\vec{x_0}} = \frac{\delta f}{\delta x_1} \mid_{x_1=x_{10}} \cdot dx_1 + \frac{\delta f}{\delta x_2} \mid_{x_2=x_{20}} \cdot dx_2 + ... + \frac{\delta f}{\delta x_n} \mid_{x_n=x_{n0}} \cdot dx_n \tag{A.4}$$

Equation (A.4) shows the result of the general linearization process around the operating point $\vec{x} = \vec{x_0}$. Here are each variable in the vector $\vec{x}$, a parameter describing one property in the system. At one operating point, $\vec{x} = \vec{x_0}$, it can be assumed that small changes in each of the parameters will result in small and linear changes in the other parameters.

Parameter	Value	Unit
c_p	1438	$\frac{J}{kg\ K}$
c_v	835	$\frac{J}{kg\ K}$
C_0	$3 \cdot 0.18$	$\frac{l}{bar\ s}$
f_0	$2.277\ 10^{12}$	$\frac{N\ s^2}{m^6}$
$h_u = h_i$	250000	$\frac{J}{kg}$
m_{gas}	0.9585	kg
m_{tank}	34.4	kg
$\dot{m}$	0.4431	$^{kg}/_{s}$
p_{tank}	656	bar
p_{oil}	258	bar
p_{gas}	258	bar
Q_{oil}	1.6	$^{l}/_{s}$
R	297	$^{J}/_{kg\ K}$
T_{gas}	275	K
T_{tank}	275	K
V_{gas}	3.3	l
Z_{tank}	1.6381	$-$
Z_{gas}	1.0677	$-$
V_{oil}	30	l
B	10^9	$^{m^2}/_{N}$

Table A.1.: Common values used

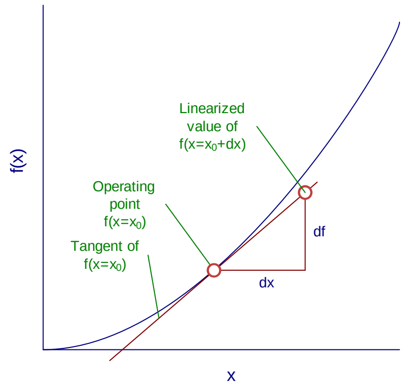


Figure A.1.: Linearization example

Figure A.1 shows an example of how the linearization is done and the changes are calculated. The values of the operating point $\vec{x} = \vec{x}_0$ are extracted from SimulationX simulations. Because of the nature of the pressure governor will the the operating point $\vec{x} = \vec{x}_0$ change during the closing sequence. Therefore each linearization have to be done at several operating points to see how the stability changes.

Note: All the simulations and liberalizations are done adiabatically. This is done because it is the worst-case and will result in the lowest pressures and lowest temperatures in the gas tank. The parameter values used are extracted from the SimulationX simulation 1 second after the simulation. This is done because the system is most unstable early in the run. The values before $t = 1s$ are varying quite a lot because of the step start and are therefor not very representative for the rest of the simulation. The initial values can neither be used since some of them are zero, i.e mass flow through the pneumatic valve. Using such a value would result in the mass flow staying at zero no matter what changes in other parameters are made. (See equation A.17)

A.1.2. Laplace transformation

When doing a Laplace transformation the main parameter shifts from being a variable in time into a variable in frequency. This allows for frequency response analysis where s is the frequency.

The general definition of the Laplace transformation is:

$$F(s) = \mathcal{L}\{f(t)\} = \int_{0^-}^{\infty} e^{-st} f(t)\, dt. \tag{A.5}$$

In this thesis there are two results of these equation which is often used:

$$\mathcal{L}\{\frac{df(t)}{dt}\} = sF(s) + f(0^-) \tag{A.6}$$

$$\mathcal{L}\{\int_0^t f(\tau)\, d\tau\} = \frac{1}{s}F(s) \tag{A.7}$$

While in the frequency domain do the normal mathematical functions apply for the s variable. This allows for easy transformation and combination of the different equations. More on this subject can be found in:Balchen et al. [2003]

A.2. Each part

A.2.1. Gas tank

The gas tank is the power source of the system and its main parameters change rapidly as it is emptying. Both the pressure and the temperature will decrease a lot. To describe the tank both the ideal gas equation (A.8) with compressibility and the first law of thermodynamics (A.9) are used[Moran and Shapiro, 2004].

$$pV = ZmRT \tag{A.8}$$

$$\frac{dU}{dt} = \dot{Q} - \dot{W} + \dot{m}_i h_i - \dot{m}_u h_u \tag{A.9}$$

Assuming constant volume, compressibility and gas constant, the change in pressure can be written:

$$\frac{dp}{p_0} = \frac{dm}{m_0} + \frac{dT}{T_0} \tag{A.10}$$

By assuming adiabatic properties and no work done ($\dot{Q} = 0, \dot{W} = 0$), little change in mass ($m = m_0$) and all the change in internal energy results in a change in temperature ($\frac{dU}{dt} = c_v m_0 \frac{dT}{dt}$) (A.9) can be written:

$$c_v m_0 \frac{dT}{dt} = -\dot{m}_u h_u \tag{A.11}$$

$$dT = -\frac{\dot{m}_u h_u}{c_v m_0 s} \tag{A.12}$$

When combining (A.10) and (A.11) and writing $dm = -\frac{\dot{m}}{s}$ the result becomes:

$$\frac{dp}{p_0} = -\frac{\dot{m}}{m_0 s} - \frac{\dot{m}}{m_0 s}\frac{h_u}{c_v T_0} \tag{A.13}$$

$$\frac{\frac{dp}{p_0}}{\frac{\dot{m}}{\dot{m}_0}} = -\frac{1}{s}\left(1 + \frac{h_u}{c_v T_0}\right)\frac{\dot{m}_0}{m_0} \tag{A.14}$$

Using values extracted for different operating points $t = \{2, 4, 6, 8, 10, 12\}$ from SimulationX we get the following bode plot of the gas tank:

The plot does not show much except that the cross frequency changes as the mass is left out of the tank. The changes are however rather small.

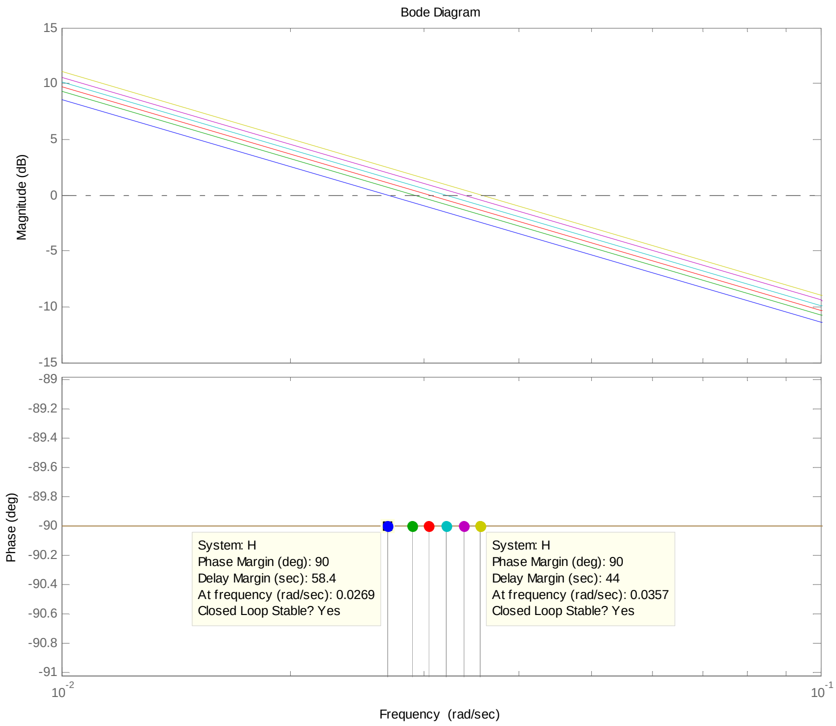


Figure A.2.: Bode plot of the gas tank

A.2.2. Pneumatic valve

The pneumatic vale between the gas tank and the oil-gas accumulator can be described with equations: (A.15). Subscript $_1$ refers to before the valve and $_2$ after. $_n$ refers to the normal condition at a reference pressure. $b = 0.53 = (\frac{2}{1+\kappa})^{(\frac{\kappa}{\kappa-1})}$. C is the valve parameter which is a function of the opening.

$$\frac{p_2}{p_1} > b || \dot{m} = C\rho_n \sqrt{\frac{T_n}{R_n}} p_1 \sqrt{\frac{R_1}{T_1}} \sqrt{1 - \left(\frac{p_2}{p_1} - b\right)^2 (1-b)^{-2}} \quad \text{(A.15)}$$

$$\frac{p_2}{p_1} < b || \dot{m} = C\rho_n \sqrt{\frac{T_n}{R_n}} p_1 \sqrt{\frac{R_1}{T_1}} \quad \text{(A.16)}$$

The two equations [Sim] are a result of a shock building in the valve. If the pressure ratio becomes to large, will the mass flow through the valve no longer be a function of the pressure behind, only the pressure in front. More on this subject can be found in: White [2003]

These equations are not linear with regards to the interesting parameters and must be linearized before they can be evaluated. To easen the job is both p_2 and R_1considered constant. The job of finding the constants K_1, K_2, K_3 is done with Maple 11.

$$d\dot{m} = K_1 dC + K_2 dp_1 + K_3 dT_1 \quad \text{(A.17)}$$

$$K_1 = \rho_n \sqrt{\frac{T_n}{R_n}} p_{10} \sqrt{\frac{R_1}{T_1}} \sqrt{1 - \left(\frac{p_{20}}{p_{10}} - b\right)^2 (1-b)^{-2}} \quad \text{(A.18)}$$

$$K_2 = -2 C_0 \rho_n \sqrt{\frac{T_n}{R_n}} \sqrt{\frac{R_1}{T_1}} \frac{((p_{10} - 1/2\, p_{20})\, b - 1/2\, p_{10})}{p_{10}(b-1)^2 \sqrt{1 - \left(\frac{p_{20}}{p_{10}} - b\right)^2 (1-b)^{-2}}} \quad \text{(A.19)}$$

$$K_3 = -1/2\, C_0 \rho_n \sqrt{\frac{T_n}{R_n}} p_{10} \sqrt{\frac{T_{10}}{R_1}} \frac{\sqrt{1 - \left(\frac{p_{20}}{p_{10}} - b\right)^2 (1-b)^{-2}}}{T_{10}^2} \quad \text{(A.20)}$$

If $\frac{p_2}{p_1} < b$ then the constants will be as follows.

$$K_1 = \rho_n \sqrt{\frac{T_n}{R_n}} p_{10} \sqrt{\frac{R_1}{T_{10}}} \quad \text{(A.21)}$$

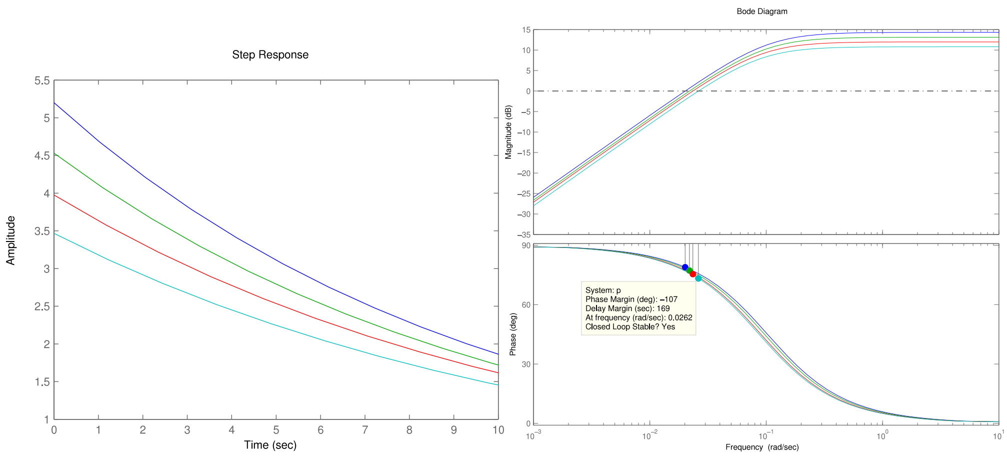
$$K_2 = C_0 \rho_n \sqrt{\frac{T_n}{R_n}} \sqrt{\frac{R_1}{T_{10}}} \quad \text{(A.22)}$$

$$K_3 = -1/2\, C_0 \rho_n \sqrt{\frac{T_n}{R_n}} p_{10} \sqrt{\frac{T_{10}}{R_1}} \frac{1}{T_{10}^2} \quad \text{(A.23)}$$

Using operating points extracted from simulations and measurements the transfer function of the pneumatic valve becomes rather simple.

By combining equations (A.17) and (A.14) the most interesting equation for the pneumatic system emerges:

$$\frac{\dot{m}}{\dot{m}_0} = \frac{(\frac{K_1 C\, dx}{\dot{m}_0} + 1)s}{s + K_2(1 + \frac{h_i}{c_v T_o})\frac{p_{10}}{m_0} + K_3 \frac{h_i}{c_v m_o}} \quad \text{(A.24)}$$



(a) Step (b) Bode

Figure A.3.: Plot of the pneumatic transfer function

In (A.24) the value dC is replaced by $C \cdot dx$ where dx is the relative opening of the vale. This transfer function has few parameters which easily can be changed. Only C the valve parameter can be chosen by the user, but it is an important parameter because it scales linearly with the proportional gain of the opening. Figure (A.3) shows the step and bode plots of the transfer function for different operating points.($t = \{2s, 4s, 6s, 8s\}$\{blue,green,red,cyan\})

The step plot shows how important it is with an integrator in the governor. As the gas tank empties will the mass flow drop and a larger opening of the pneumatic valve is required. A normal proportional governor would maintain a constant opening which would result in a stationary deviation. This can also be seen directly from the transfer function, which turns into an derivator at low frequencies. To avoid this effect must the governor integrate the same frequencies.

A.2.3. Gas-oil tank

A.2.3.1. Gas part

As in section A.2.1 are the two main equations Ideal gas equation (A.8) and the first law of thermodynamics (A.9). The main differences between the main gas tank and the gas in the gas-oil tank is the change in volume and a flow into the tank. Differentiating (A.8) will now result in:

$$\frac{dp}{p_0} = \frac{dm}{m_0} + \frac{dT}{T_0} - \frac{dV}{V_0} \tag{A.25}$$

By assuming adiabatic properties ($\dot{Q} = 0$), little change in mass ($m = m_0$) and all the change in internal energy results in a change in temperature ($\frac{dU}{dt} = c_v m_0 \frac{dT}{dt}$) (A.9) can be written:

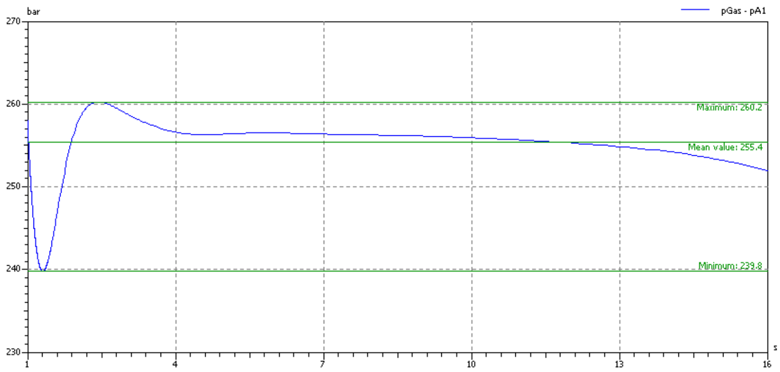


Figure A.4.: Gas pressure

$$
\begin{aligned}
c_p m_0 \frac{dT}{dt} &= -\dot{W} + \dot{m}_i h_i && \text{(A.26)} \\
W &= \int P dV = \int P \cdot A dx && \text{(A.27)} \\
P &\approx P_0 && \text{(A.28)} \\
\int \dot{W} dt &= W = P_0 \cdot A \int dx && \text{(A.29)} \\
dT &= -\frac{P_0 \cdot A \cdot dx}{c_p m_0} + \frac{\dot{m}_i h_i}{c_p m_o s} && \text{(A.30)}
\end{aligned}
$$

In (A.30) the integrator has been Laplace transformed into $^1/_s$. The assumption $P \approx P_0 = constant$ is valid if the governor is a good one and it maintains a constant pressure. Simulations with SimulationX shows that this assumption is valid, and the variation in gas pressure is small. The maximum deviation is 6.3% of the mean value. Figure: A.4 shows a plot of the gas pressure from SimulationX.

$$
\begin{aligned}
dV &= dx \cdot A && \text{(A.31)} \\
dm &= \frac{\dot{m}}{s} && \text{(A.32)}
\end{aligned}
$$

Combining (A.25) and (A.30) results in:

$$
\frac{dP}{P_0} = \left(1 + \frac{h_i}{c_p T_0}\right) \frac{\dot{m}}{m_0 s} - \left(\frac{P_0 V_0}{m_0 c_p T_0} + 1\right) \frac{A \cdot dx}{V_0} \qquad \text{(A.33)}
$$

A.2.3.2. Piston

The piston in the oil-gas tank is described as in figure A.5.

$$
\begin{aligned}
\sum F &= ma && \text{(A.34)} \\
(P_{gass} - P_{oil}) \cdot A - c \cdot \dot{x} &= m_{piston} \ddot{x} && \text{(A.35)}
\end{aligned}
$$

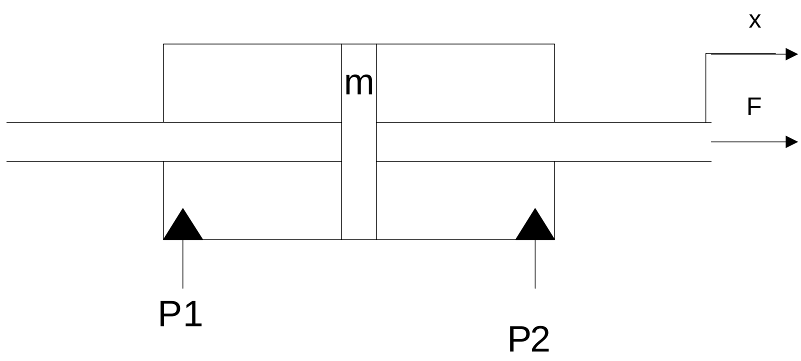


Figure A.5.: Sketch of the piston

Laplace transformed:

$$
\begin{aligned}
(P_{gass} - P_{oil}) \cdot A - c \cdot dx \cdot s &= m_{piston} dx \cdot s^2 && \text{(A.36)}\\
dx &= \frac{(P_{gass} - P_{oil}) \cdot A}{c \cdot s + m_{piston} \cdot s^2} && \text{(A.37)}
\end{aligned}
$$

A.2.3.3. SSR-system

The complete SSR-system is modeled by two simple equations [Goodwin, 1976]:

$$
\begin{aligned}
p_{oil} &= (Q_{in} - Q_{out}) \frac{B}{V_{oil}} \frac{1}{s} && \text{(A.38)}\\
p_{oil} &= f Q_{out}^2 && \text{(A.39)}
\end{aligned}
$$

The first equation calculates the oil pressure with regards to how much oil flows into/out of the oil volume after the tank. The second assumes oil flow out of the system as a function of pressure and a parameter f. Differentiating (A.39):

$$
\begin{aligned}
\frac{dp_{oil}}{p_{oil0}} &= \frac{df}{f_0} + 2\frac{dQ}{Q_0} && \text{(A.40)}\\
\frac{dp_{oil}}{p_{oil0}} &= \frac{p_{oil}}{p_{oil0}} - 1 && \text{(A.41)}\\
\frac{p_{oil}}{p_{oil0}} - 1 &= \frac{f}{f_0} - 1 + 2\frac{Q}{Q_0} - 2 && \text{(A.42)}\\
Q_{out} &= \frac{Q_0}{2}\left(\frac{p_{oil}}{p_{oil0}} + 2 - \frac{f}{f_0}\right) && \text{(A.43)}
\end{aligned}
$$

Combining (A.38) and (A.43), and writing $Q_{in} = A\frac{dx}{dt} = A \cdot dx \cdot s$ the transfer function becomes:

$$
p_{oil} = \frac{B}{V_{oil} s}(A \cdot dx \cdot s - \frac{Q_0}{2}(\frac{p_{oil}}{p_{oil0}} + 2 - \frac{f}{f_0})) \qquad \text{(A.44)}
$$

A.2.3.4. Combining SSR and piston

By combining (**??**) and (A.37) it is possible to find a connection between the oil and gas pressures.

$$
\begin{aligned}
p_{oil} &= \frac{2A^2B \cdot p_{oil0} \cdot (p_{gas} - p_{oil})}{(p_{oil0} \cdot 2V_{oil} \cdot s + Q_oB)(c + m_{piston} \cdot s)} && \text{(A.45)}\\
p_{gas} &= p_{oil}\left(1 + \frac{(p_{oil0} \cdot 2V \cdot s + Q_oB)(c + m_{piston} \cdot s)}{2A^2B \cdot p_{oil0}}\right) && \text{(A.46)}\\
\frac{p_{oil}}{p_{gas}} &= \frac{1}{(1 + \frac{V_{oil}}{B}\frac{2p_{olje}}{Q_0}s)(1 + \frac{m}{c}s)\frac{c \cdot Q_0}{A^2 \cdot 2p_{oil0}} + 1} && \text{(A.47)}
\end{aligned}
$$

This transfer function shows the connection between the oil and gas pressure. It is important that the oil pressure is a stable as possible and has as small oscillations as possible. The transfer function is on the same form as a normal mass-spring system:

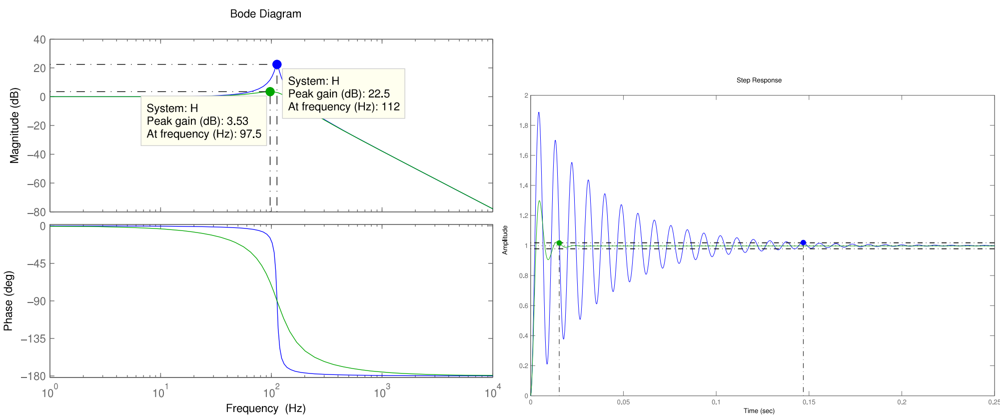
$$
\begin{aligned}
M\ddot{x} + C\dot{x} + Kx &= 0 && \text{(A.48)}\\
Mxs^2 + Cxs + Kx &= x_{ref} && \text{(A.49)}\\
\frac{x}{x_{ref}} &= \frac{1}{Ms^2 + Cs + K} && \text{(A.50)}
\end{aligned}
$$

Comparing (A.50) and (A.47) shows how the mass-spring system can be related to the transfer function.

$$
\begin{aligned}
K &= \frac{c \cdot Q_0}{A^2 \cdot 2p_{oil0}} + 1 && \text{(A.51)}\\
C &= \frac{V_{oil}}{B}\frac{c}{A^2} + \frac{mQ_0}{A^2 \cdot 2p_{oil0}} && \text{(A.52)}\\
M &= \frac{V_{oil}}{B}\frac{m}{A^2} && \text{(A.53)}
\end{aligned}
$$

Inserting common values into the parameters which can not easily be altered and thus are more or less locked by external parameters.

$$
\begin{aligned}
K &= \frac{c}{A^2} \cdot 3.07 \cdot 10^{-11} + 1 \approx 1 && \text{(A.54)}\\
C &= (c + 3.07m)\frac{10^{-11}}{A^2} && \text{(A.55)}\\
M &= \frac{m}{A^2}10^{-11} && \text{(A.56)}\\
\omega_0 &= \sqrt{\frac{K}{M}} = \sqrt{\frac{A^2}{m \cdot 10^{-11}}} && \text{(A.57)}\\
\zeta &= \frac{C}{2\sqrt{KM}} = \frac{(c + 3.07 \cdot m)}{2\sqrt{1 \cdot mA^2 10^{11}}} && \text{(A.58)}
\end{aligned}
$$



(a) Bodeplot (b) Step response

Figure A.6.: Response plots

By examining the main parameters ζ and ω_0, it is possible to decide on what each physical parameters should be. The piston mass also includes the mass of the oil which needs to be accelerated and with $0.01m^3 \cdot 850\frac{kg}{m^3} = 8.5kg$, and a piston weighted at max 10kg[Hydac, 2005] the $m \leq 20kg$. Using the highest m possible is the worst case scenario. The area (A) of the cylinder can be modified to almost any value by choosing several small or one big.

The solution of (A.48) given an under dampened system ($\zeta < 1$) is[Irgens, 1999]:

$$x = e^{-\zeta\omega_0 t}(A\cos(\omega_0 t) + B\sin(\omega_0 t)) \tag{A.59}$$

When the amplitude is less than 5% of the desired value it can be assumed to have stabilized. To ensure a quick as possible reaction of the system, the time until stability should be as small as possible. This results in:

$$0.05 = e^{-\zeta\omega_0 t} \tag{A.60}$$

$$ln(0.05) = -\zeta\omega_0 t \tag{A.61}$$

$$\frac{ln(0.05)}{-\frac{(c+3.07m)}{2m}} = t \tag{A.62}$$

To reduce the time until steady state, (A.62) shows that c should be as large as possible without $\zeta \geq 1$. This can be used to develop a relation between A and c, which should be considered when selecting equipment. But as long as there are some friction, which there will be, the result will be a quite low settling time: $c = 100\,{}^{Ns}/_{m} : t = 0.74s$ $c = 1000\,{}^{Ns}/_{m} : t = 0.11s$. The most important aspect is to ensure that the system stays under damped $\zeta < 1$, because a over dampened system will be considerably slower to respond. Figure A.6 (Blue: c=1000 Green: c=10000).

A.2.3.5. Combining the equations

Because of the complexity and number of parameters, Maple 11 is used to combine the different equations.

The transfer function between the oil pressure and the mass flow into the gas oil tank becomes:

$$
\begin{aligned}
\frac{\frac{p_{oil}}{p_{oil0}}}{\frac{\dot{m}}{m_0}} &= \frac{m_{piston}\, p_{oil0}\, V_{oil} s^3}{BA^2 p_{gas0}\left(1+\frac{h_i}{c_p T_0}\right)} \\
&+ \left(1/2\frac{m_{piston}\, Q_0}{A^2 p_{gas0}} + \frac{c \cdot p_{oil0} V_{oile}}{BA^2 p_{gas0}}\right)\frac{s^2}{\left(1+\frac{h_i}{c_p T_0}\right)} \\
&+ \left(\frac{p_{gass0}\, p_{oil0} V_{oil} c_p T_0}{B m_{gas0}} + \frac{cQ_0}{2A^2 p_{gas0}} + \frac{p_{oil0} V_{oil}}{B V_{gas0}} + \frac{p_{oil0}}{p_{gas0}}\right)\frac{s}{\left(1+\frac{h_i}{c_p T_0}\right)} \\
&+ \left(\frac{Q_0}{2V_{gas0}} + \frac{p_{gas0}\, Q_0\, c_p T_0}{2m_{gas0}}\right)\frac{1}{\left(1+\frac{h_i}{c_p T_0}\right)} \qquad \text{(A.63)}
\end{aligned}
$$

This is a quite ugly equation which is hard to analyze with good results. To easen the task the values from A.1 have been inserted.

$$
\frac{\frac{p_{oil}}{p_{oil0}}}{\frac{\dot{m}}{m_0}} = \frac{1}{(1.86\, m_{piston}\, s^3 + (1.94\, m_{piston} + 1.86c)\, s^2)\, \frac{10^{-11}}{A^2} + 0.80\, s + 0.185} \qquad \text{(A.64)}
$$

Finding the exact poles have proved to become hard and even using Maple the solution was unusable. An approximation may be used. Because of the great difference in the factors in the numerator it is safe to assume:

$$
\begin{aligned}
0.80\, s + 0.185 &\gg s^2(1.86\, m_{piston}\, s + (1.94\, m_{piston} + 1.86c))\frac{10^{-11}}{A^2} \,||s^2 \ll 10^4 |A \qquad \text{(A.65)} \\
s \ll 10^2 ||\, s &\approx \frac{0.185}{0.80} \approx 0.23 = T_1^{-1} \qquad \text{(A.66)} \\
s \gg 10^2 || \quad s^2 &= \frac{(0.80s + 0.185) A^2 10^{11}}{m_{piston} \cdot (s \cdot 1.86 + 1.94) + 1.86c} \approx \frac{0.80 A^2 10^{11}}{1.8 m_{piston}} \qquad \text{(A.67)} \\
s &= \pm\sqrt{\frac{8.0}{1.86 m_{piston}}} A\, 10^5 \approx \pm 950 = T_2^{-1} \qquad \text{(A.68)} \\
\frac{\frac{p_{oil}}{p_{oil0}}}{\frac{\dot{m}}{m_0}} &= \frac{\frac{1}{0.185}}{(T_1\, s + 1)(T_2\, s + 1)^2} \approx \frac{5.4}{(s/0.23 + 1)(s^2\, 8.89\, 10^{-11} \frac{m}{A^2} + 1)} \qquad \text{(A.69)}
\end{aligned}
$$

Equation (A.69) can be used as an approximation of the real transfer function to show where the phase crosses $\angle - 180°$. In the approximation is the friction ignored. This results in an unphysical behavior which can be seen in figure:A.7 (Green = real, Blue= approximated). This figure also shows how close the original and the approximated transfer functions are. Values chosen for this test are: $A = 0.01 m^2$ $c = 1000 Ns/m$ $m_{piston} = 5kg$.

The approximation shows how the frequency where the phase crosses $\angle - 180°$easily can be adjusted by changing either the mass or the area of the piston. Figure A.7 also shows how the friction doesn't affect the frequency given it is small enough not to interfere with the approximation ($c \gtrapprox 10000 Ns/m$).

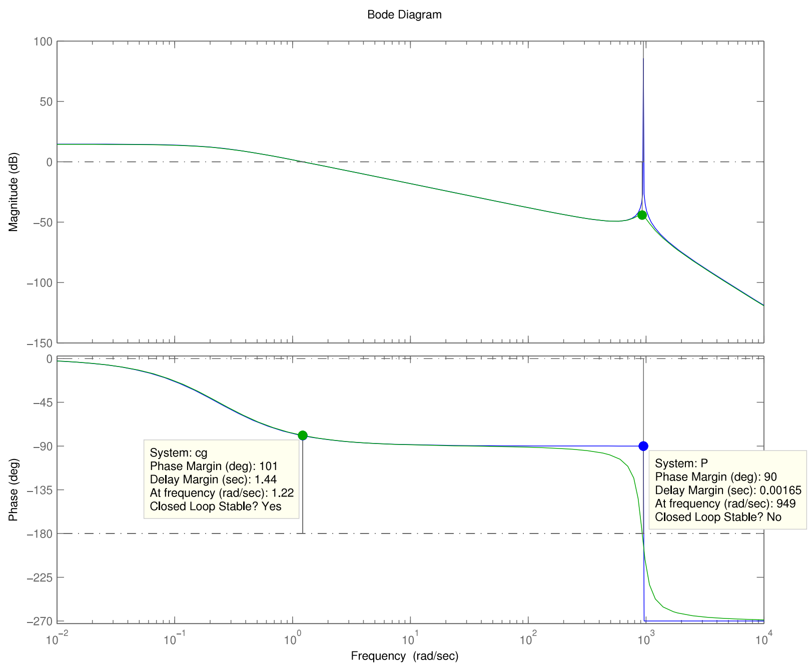


Figure A.7.: Bode plot of $\frac{\frac{p_{oil}}{p_{oil0}}}{\frac{\dot{m}}{m_0}}$.

A.2.4. System without governor

To create the best possible governor must the system as a whole, without a governor, be evaluated. Because of the linearity of each of the transfer functions they can be added in a log-log diagram. Figure A.8 shows the two equations for the pneumatic part (A.24) (blue) and the oil-gas tank (A.63) (green) separate and combined.

The result is a quite stable system with a large phase margin of 90° and a gain margin of ~ 21dB. The gain margin appears at the point where the peak of the mass of the piston appears. The height of this peak is very dependent upon the friction in the system.

A.2.5. Governor

A.2.5.1. Piston

There are three pistons, with through shaft, in the governor. All of these are governed by the same set of equations:

$$\sum F = ma \tag{A.70}$$

$$F = -(p_1 - p_2)A + c\frac{dx}{dt} + m\frac{d^2x}{dt^2} \tag{A.71}$$

$$F = (p_2 - p_1)A + cxs + mxs^2 \tag{A.72}$$

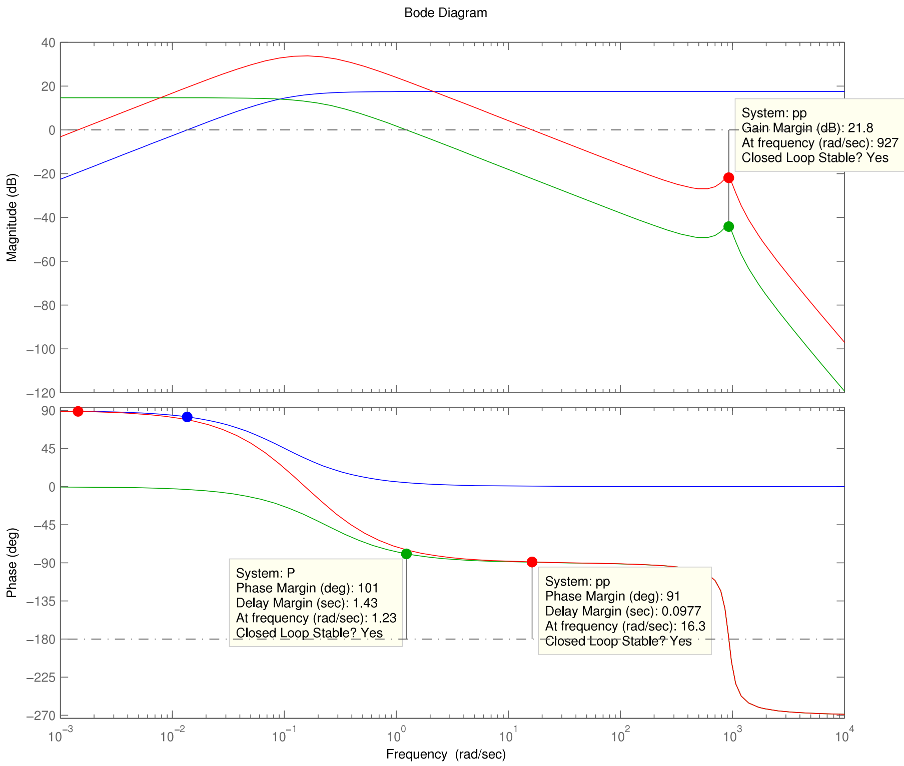


Figure A.8.: System bode plot without governor

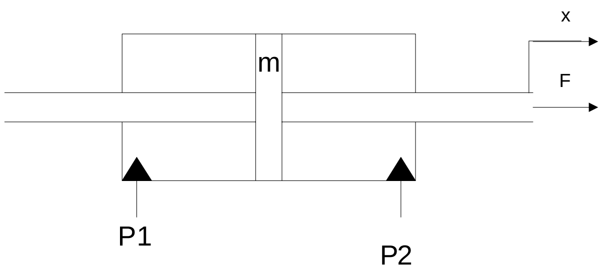


Figure A.9.: Piston sketch

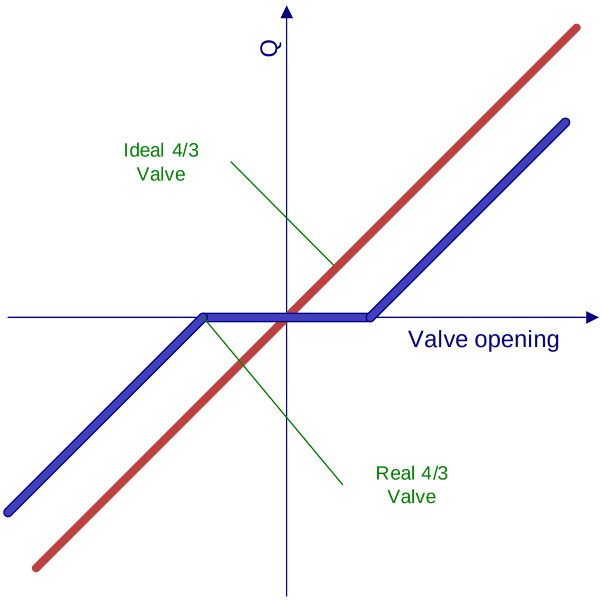


Figure A.10.: 4/3 Valve comparison

A.2.5.2. Spring

The spring is a normal linear spring of any sort. It is very important that the spring preforms as expected. Its job is to set the reference force for balancing the pressures. $x_0$is the preset value of the spring.

$$
\begin{aligned}
F &= k(x-x_0) && \text{(A.73)}\\
\Delta p \cdot A &= kx_0 && \text{(A.74)}\\
x_0 &= \frac{\Delta p \cdot A}{k} && \text{(A.75)}
\end{aligned}
$$

(A.75) show how to set the preset value of the spring. Every derogation for either k or x_0, will result in a wrong pressure in the system.

A.2.5.3. 4/3 - valve

In this simulation is the 4/3 valve regarded rather ideally. Figure A.10 shows the difference between a real and an ideal 4/3 valve. The real curve shows how no flow goes through the valve at small openings. The problems with this are further discussed in section 4.5.

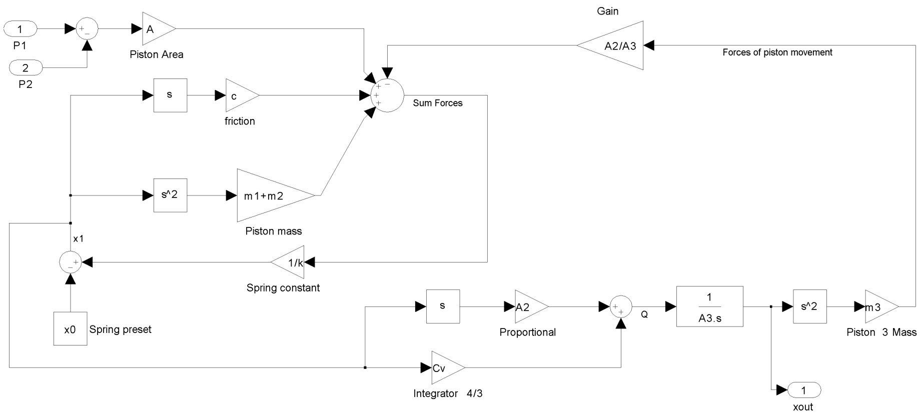


Figure A.11.: Block diagram: governor

$$Q = C_v x \sqrt{\frac{\Delta p}{2\rho}} \tag{A.76}$$

$$dQ = \frac{c_v}{\sqrt{2\rho}}(\sqrt{\Delta p_0}dx + {}^1\!/\!{}_2 \frac{x_0}{\sqrt{\Delta p_0}} d\Delta p) \tag{A.77}$$

$$x_0 \approx 0 \tag{A.78}$$

$$Q = C_v x \sqrt{\frac{\Delta p_0}{2\rho}} \tag{A.79}$$

The assumption of $x_0 \approx 0$ is valid because the valve starts at $x = 0$ where it normally also operates. Assuming a good governor will the value $d\Delta p \approx 0$. C_v is the valve constant.

A.2.5.4. Block diagram

Combining these equations into a block diagram results in figure: A.11

Figure A.12 is a result of simplifying A.11. This reveals the proportional and integrational elements of the governor. As expected are the valve parameters related to the integrator and the piston sizes related to the proportional part.

The values in figure A.12 are as follows:

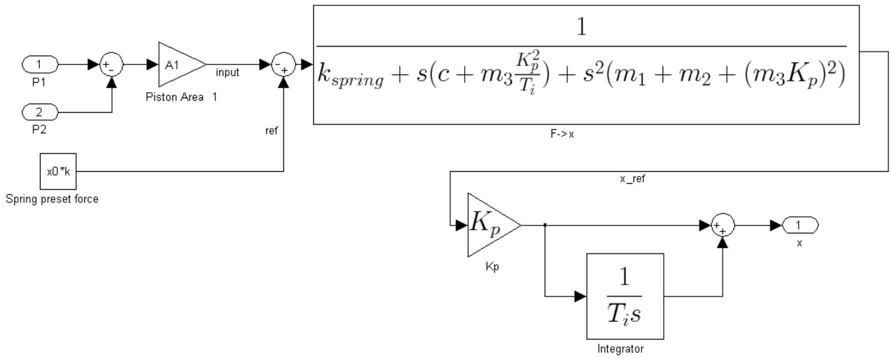


Figure A.12.: Simplified block diagram: governor

$$T_i = \frac{A_2}{C_v\sqrt{\frac{\Delta p_0}{2\rho}}} \tag{A.80}$$

$$K_p = \frac{A_2}{A_3} \tag{A.81}$$

As can be seen in figure A.12 is the governor a combination of a normal PI governor with a mass-dampener-spring system attached. Because the elements in the mass-dampener-spring system are affected by the properties of the PI governor can the parameters of either be chosen freely.

Many of the same considerations as made in section A.2.3.4 with the oil/gas tank have to be made here. The general solution for the mass-dampener-spring system is the same:

$$
\begin{aligned}
x &= e^{-\zeta\omega_0 t}(A\cos(\omega_0 t)+B\sin(\omega_0 t)) && \text{(A.82)}\\
K &= k_{spring} && \text{(A.83)}\\
C &= c+m_3\frac{K_p^2}{T_i} && \text{(A.84)}\\
M &= m_1+m_2+m_3K_p^2 && \text{(A.85)}\\
\omega_0 &= \sqrt{\frac{k_{spring}}{m_1+m_2+m_3K_p^2}} && \text{(A.86)}\\
\zeta &= \frac{c+m_3\frac{K_p^2}{T_i}}{2\sqrt{k_{spring}(m_1+m_2+m_3K_p^2)}} && \text{(A.87)}\\
t_{steady} &= ln(0.05)\frac{2(m_1+m_2+m_3K_p^2)}{c+m_3\frac{K_p^2}{T_i}} && \text{(A.88)}
\end{aligned}
$$

Equation (A.88) shows that the stiffness of the spring does not affect the settling time of the system. This allows the spring stiffness to be chosen freely in a way resulting in $\zeta \approx 1$. And therefore can the response time of the governor become quite fast. In addition can one see that the mass of piston 1 and 2 affect the settling much more then the mass of the third piston. This should be considered when choosing the final piston parts for the project.

The resulting output is normalized on the same scale as the opening of the 4/3-valve and is thus: $x_{out} = [-1,1]$. If the real displacement needed for opening the pneumatic valve is different from the real opening (which it most probably will be), must this difference be countered by adjusting the value of $K_p = K_{p0}\frac{x_{pneumatic}}{x_{4/3}}$.

A.2.5.5. Selecting governor parameters

From figure A.8 the values of K_p and T_i can be approximated to $K_p = 15dB$ and $T_i = 10$. These values have to be corrected because of the dynamics of the governor. Figure A.13 shows the bode plot of the governor with $K_p = 1, T_i = \{15, 1.5, 0.15\}, A_1 = 0.01m^2$

When using a PI-governor will the phase, as can be seen in figure (A.13), start at $\angle -90°$. In this particular governor will the phase start to rise as the frequency passes the inverted time constant of the integrator. The final drop to $\angle -180°$ happens when the frequency reaches the eigen frequency of the moving masses in the governor.

Because the phase of the PI-governor naturally starts at $\angle -90°$, is the frequency when the system reaches a phase of $\angle -90°$very interesting. Figure A.8 shows this happening between $\omega = 1 rad/s$ and $\omega = 10 rad/s$. It is therefor important to lift the governor phase higher then $\angle -90°$at these frequencies to improve the stability. The value of T_i should therefore be somewhere between 0.1 and 1. This should be sufficient to remove the stationary deviation which occurs because of the derivation in (A.24): $T_i < T_d \approx 10$

An very important parameter for this governor is the spring constant k_s. The larger the spring constant is the less effected will the system be of the friction and the mov-

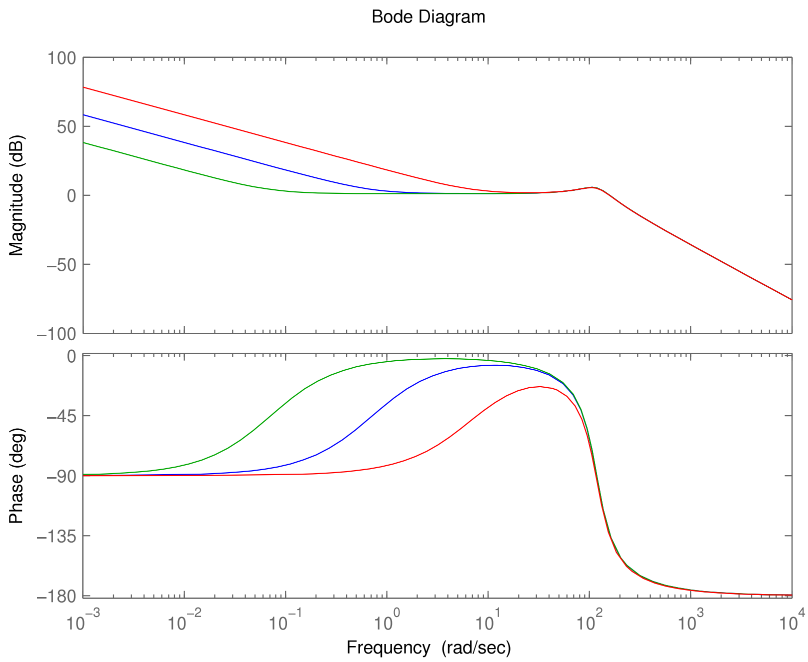


Figure A.13.: Bode plot governor

ing masses. The choice of spring constant is largely dependent on the area of the first piston, A_1, as can be seen in equation (A.75). Changing the spring constant also affects the eigen frequency of the system which can be written: $\omega_0 = \sqrt{\frac{K}{M}}$, and thus also when the governor $\angle - 180°$. An increase in A_1 will normally also increase the mass of piston 1 proportionally, but this increase will affect the eigen frequency less then the increase of the spring constant (A.86). Figure A.14 ($A_1 = \{0.1m^2, 0.01m^2, 0.001m^2\}\{blue, green, red\}$) shows how the bode plot is affected by different values of A_1.

A.3. Conclusion

By linearising the system and the governor, simple analysis may be conducted to reveal important parameters and determine their values. Such a linearization will never be completely accurate but it may give a hint in which direction things in the real world may turn.

The previous sections show how some of the parameters in the system should be chosen.

- $K_p = 1$
- $T_i \approx 0.2$
- $A_1 \approx 0.1m^2$

With these values selected and values from SimulationX a bode plot of the open linearized system is shown in figure A.15 (green:governor, blue:system, red:combined).

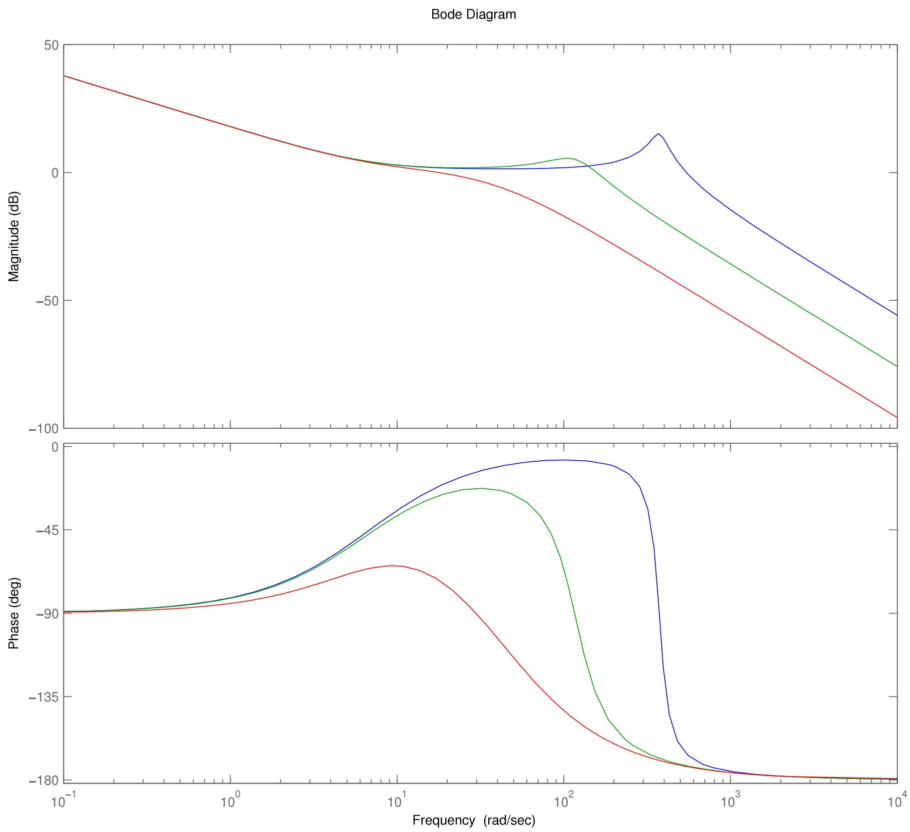


Figure A.14.: Bode plot for the governor changing

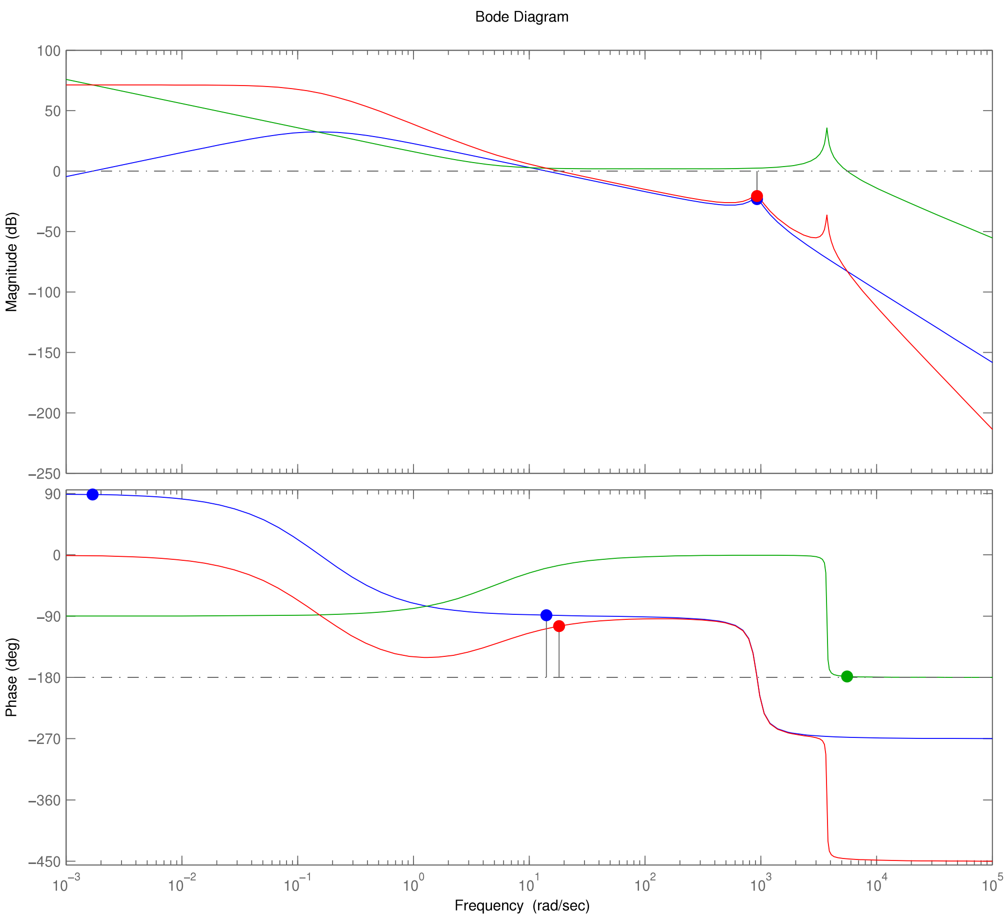


Figure A.15.: Bode plot open-loop

The resulting bode plot shows a gain margin of 23dB and a phase margin of $\angle 75°$. This is sufficient with regards to the basic rules of thumb, with a gain margin of 6dB and a phase margin of $\angle 30°$. The minimum phase margin is however $\angle 30°$ at a frequency of $1^{rad}/_{s}$, still this is sufficient. Normally would such a large gain margin allow an increase of $K_p$by $16-17$dB. As can be seen in (A.84) and (A.85) does K_p affect both the 'mass' and the 'dampening' of the mass-dampener-spring system. An increase in K_p will increase both M and C, which reduces the response and thus the stability. Therefore can not $K_p$be increased as much as could be expected. The final choice of K_p is made with a non linear model in appendix B.

A.3.1. Derivator?

The choice of a PI-governor and not a PID-governor was based on the fact that it became to hard to build. A partial derivator could have been very useful, helping with an increase of the phase angle near $\omega = 1^{rad}/_{s}$, where the phase is a bit low. The idea was to use a design which resembled a normal shock absorber. Several attempts were made to create a simple derivator, but simulations showed they caused to much disturbance in the system.

A.3.2. Conclusion

The linearization of the system reveals several important parameters which should be chosen carefully to create a quick and stable system. Especially are the areas and masses of the pistons together with the size of the valves very important parameters for stability. The values extracted from the linearized model match somewhat good with the non-linear model, but they may only be regarded as pointers for what values should be chosen for the nonlinear system.

B. Simulink model

B.1. General

This model is built in Simulink using the same equations which are the basis of the linearization in chapter A. Figures B.1 and B.2 show the models as they are displayed in Simulink. Further study of these models are possible through the models which are included on the appendix CD. The analysis is not as thorough as with the linear model for two reasons. First are the two models more or less the same, so what goes in the linear model goes here. Secondly is it harder/impossible to analytically analyze each parameter as it is possible with the linear model.

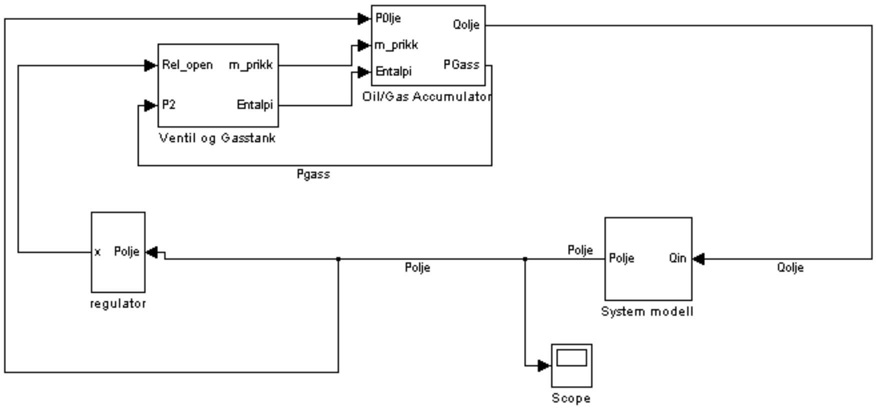
The main reason for this model is to verify the linear model and to optimize the parameters found there.

B.2. Simplifications

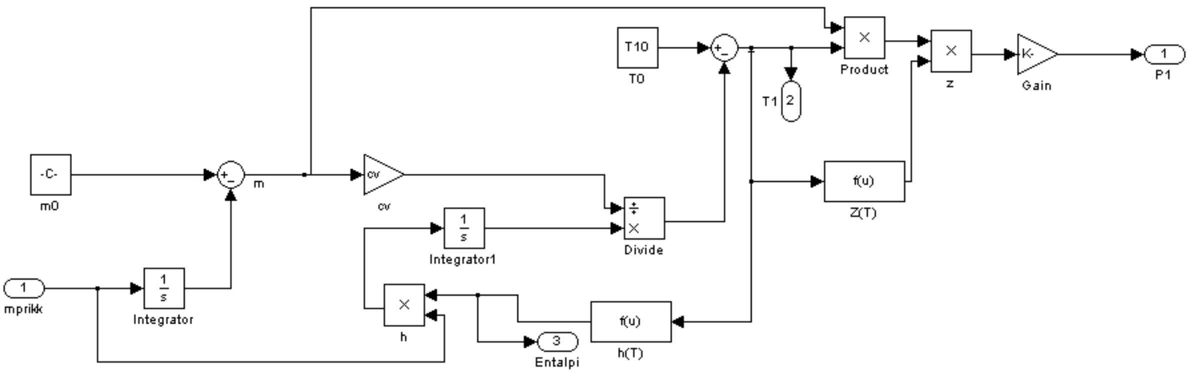
There have been some simplifications in these models with regards to both the equations and the real model:

- The gas in the accumulator is assumed to have a constant compressibility Z . And is calculated with the ideal gas equation: $pV = ZmRT$
- c_v and c_p are considered to be unaffected by temperature and constant.
- The enthalpy $h = h(T)$ is assumed to be a temperature function only
- The compressibility $Z = Z(T)$ in the gas tank is assumed to be a temperature function only
- The SSR-system is modeled by the equation $p = f\, Q^2$, where f is a constant
- The pressures in the pneumatic system is assumed to propagate instantly through the entire volumes

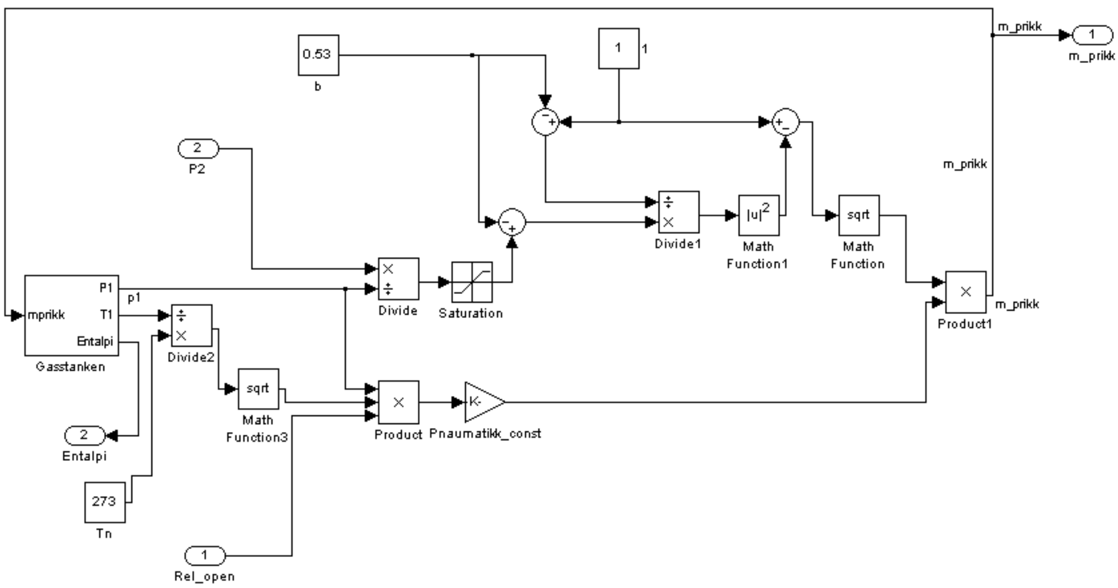
B.3. Simulink models



(a) Overall model

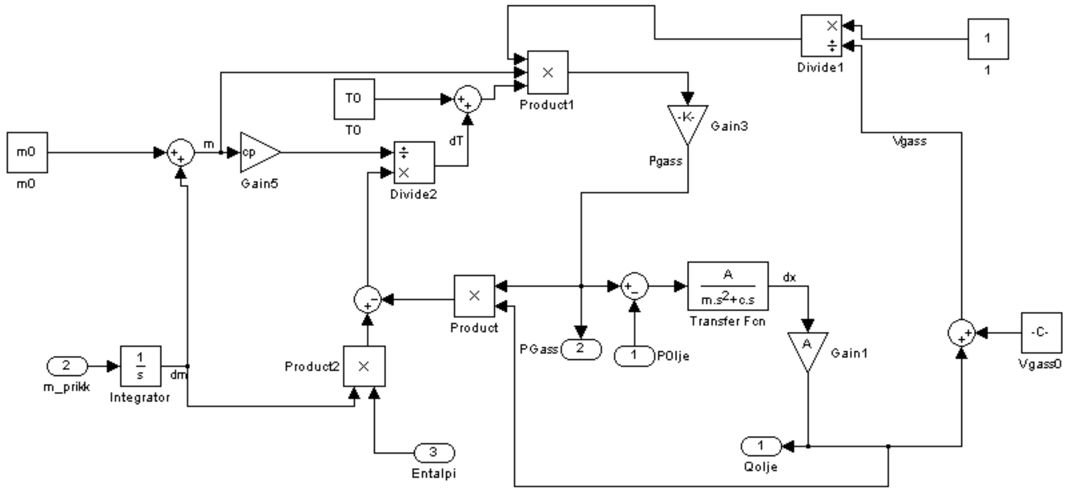


(b) Gas tank

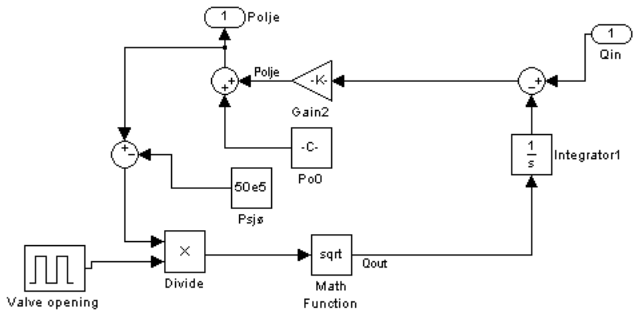


(c) Pneumatic valve

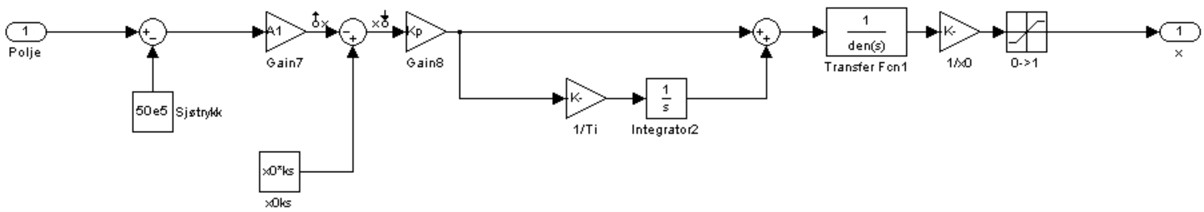
Figure B.1.: Simulink models



(a) Oil/gas accumulator



(b) SSR-system



(c) Governer

Figure B.2.: More Simulink models

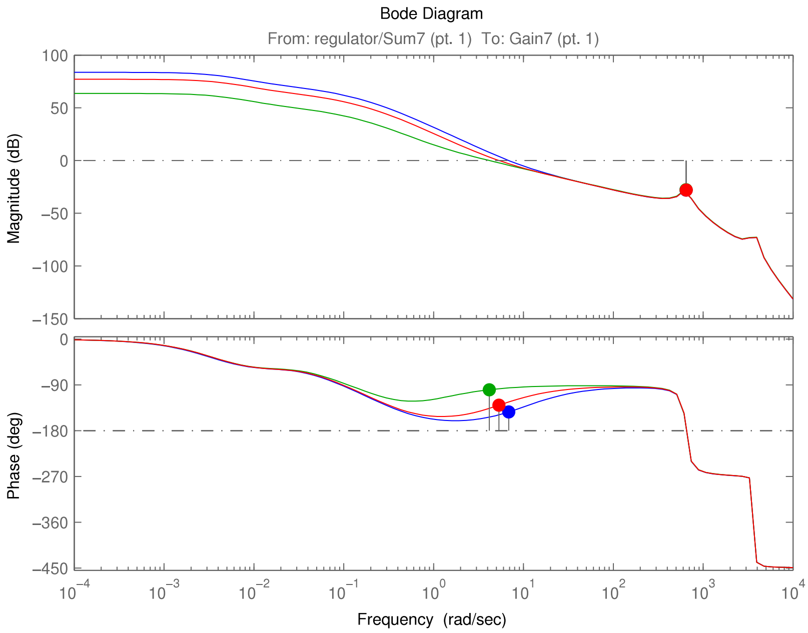


Figure B.3.: Choices for T_i

B.4. Results

B.4.1. T_i

As can be seen in figure 2.1 are the results very similar to the SimulationX and the linear model. Therefor can the Simulink model be used to optimize the design. First is $T_i = 0.1(blue)\ 0.2(red)\ 1(green)$ chosen.

Because a small T_i reduce the effect of the derivator in the pneumatic valve, must an value as small as possible be chosen. But the phase margin, ψ should never be smaller then $\angle 45°$[Balchen et al., 2003]. Figure B.3 shows $T_i = 0.1$ gives a phase margin below $\angle 45°$, while $T_i = 0.2$ gives $\psi = \angle 52°$. Therefore is T_i chosen to be 0.2.

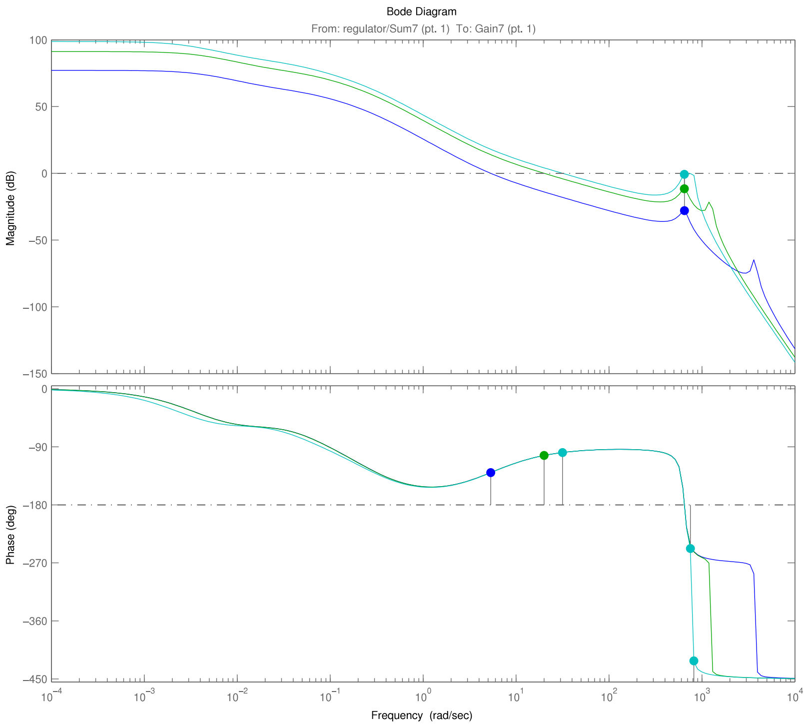


Figure B.4.: Choices for K_p

B.4.2. K_p

Next is the value $K_p = 1(blue)\ 5(green)\ 8(cyan)$ chosen.

Figure B.4 shows that $K_p = 1$ gives a gain margin $\Delta K = 22.5dB$, which would indicate that an increment of $10^{\frac{22.5}{20}} = 13.3$ should be possible. But as the figure shows does ΔK surpass the $0dB$ line already at $K_p = 8$. This happens because K_p affects more than just the gain, but also the stability of the governor. Therefor is $K_p = 5$ chosen where $\Delta K = 11.5dB$ which is more then the requirement of $\Delta K \geq 2 \approx 6dB$ [Balchen et al., 2003].

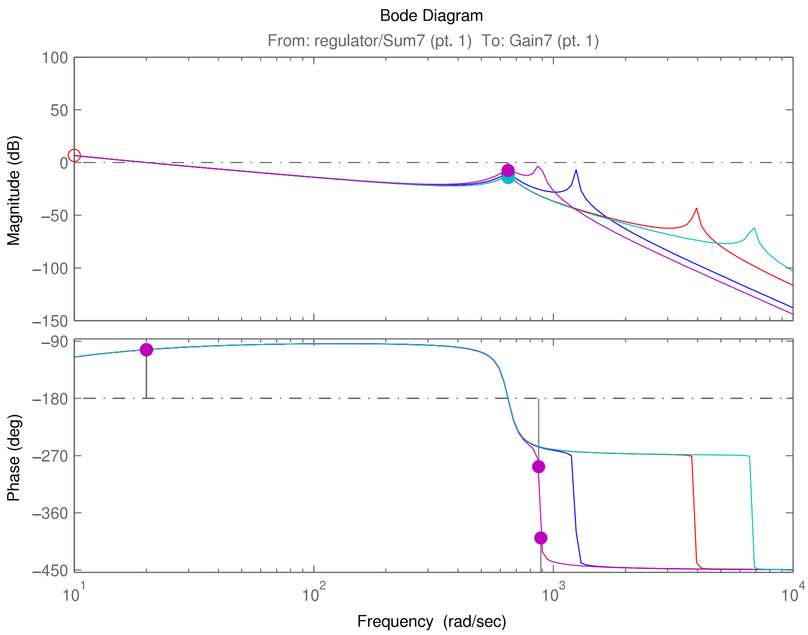


Figure B.5.: Choices for A_1

B.4.3. A_1

The suggested value for $A_1 = 0.1m^2$ is kept, because it should be as high as possible with in the physical limitations. As figure shows will a larger area A_1 not make any real difference, where $A_1 = 0.05m^2(purple)$ $0.1m^2(blue)$ $1m^2(red)$ $3m^2(cyan)$. But if the value is chosen to be less than $0.1m^2$, will the gain margin shrink.

B.5. Set-point and Disturbance response

The set point shows how good the governor manages to follow the reference input and should be as close to $1 = 0dB$ as possible. Disturbance response shows how well the system suppress disturbances. This parameter should be as low as possible ($< 0dB$).

Figure B.6 shows the two responses in a bode magnitude plot. It shows that the governor can handle frequencies up to $\approx 10\frac{rad}{s} = 1.6Hz$ which is a bit low, but good enough.

B.6. Changes during the simulation

As the gas tank empties and the piston in the accumulator moves will the system dynamics change. FigureB.7 shows how these changes affect the bode plot. The main change is in the gain. This occurs mainly as a result of the drop in gas tank pressure. Such a loss in gain reduces the systems ability to conquer changes in pressure, and the result is a lowered system pressure.

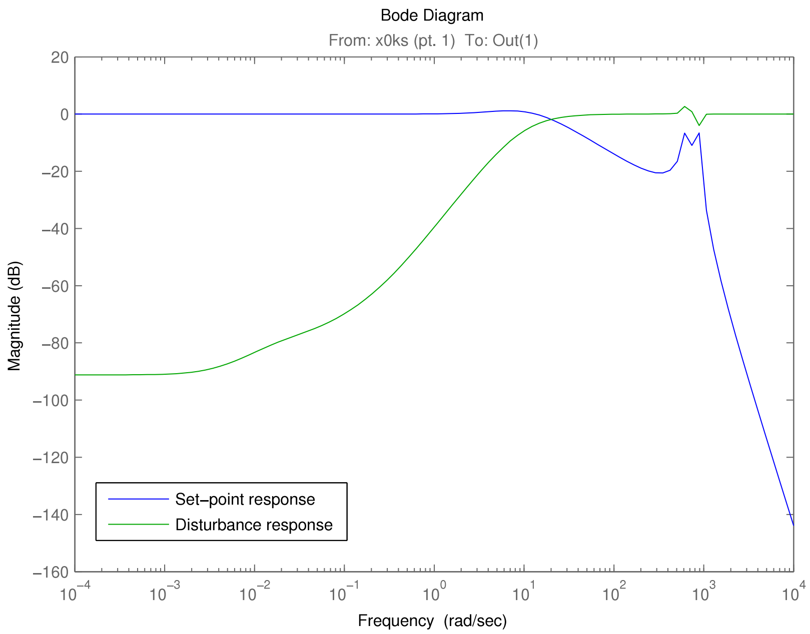


Figure B.6.: Set-point and disturbance response

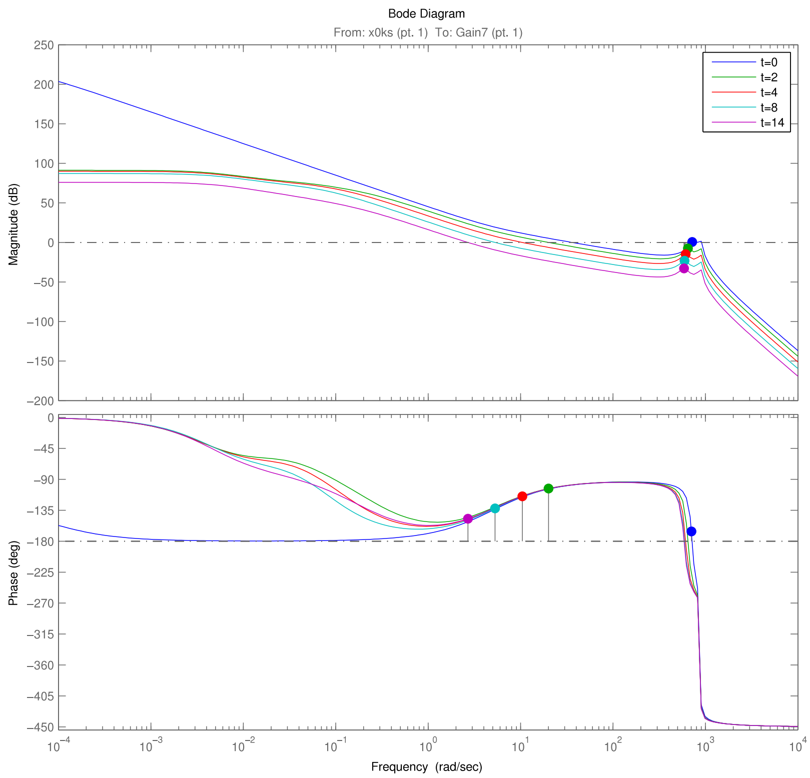


Figure B.7.: Bode plot at different steps in the simulation

C. Real gas simulations

Note: This chapter is directly copied from the project work of Ostby [2007] with a few modifications. Some of the code have been optimized to give more correct calculations. It was included because this explains the theory behind the real gas calculations which have been done in connection with this paper.

C.1. General

The normal way of describing a gas is to use the ideal gas equation:

$$pV = nRT \tag{C.1}$$

But this equations have several limitations, and it is normally only valid near 1 atm and 273 Kelvin. When either the temperature or the pressure deviate largely from the valid area there are effects in between molecules which change the behavior of the gas. Many scientists thus tried to create better and more correct equations describing gases.

C.2. Benedict-Webb-Rubin equation

The BWR equation was derived from earlier tries to create a better equation of state. This equation have become very popular and have been used in many applications. And there have been created numerous variations of it trying to further improve its quality.

$$p = \rho RT + \left(B_0RT - A_0 - \frac{C_0}{T^2}\right)\rho^2 + (bRT - a)\rho^3 + \alpha a\rho^6 + \frac{c\rho^3}{T^2}\left(1+\gamma\rho^2\right)e^{\left(-\gamma\rho^2\right)} \tag{C.2}$$

Moran and Shapiro [2004]

Equation: (C.2) is explicit with regards to p and to solve it for either ρ or T we have to use a numerical solver. The constants: $A_0, B_0, C_0, \alpha, \beta, \gamma$ are material constants which can be found in Moran and Shapiro [2004, Table A-24].

Note: This equation, even though much used, is not very accurate at the pressures and temperatures used in subsea. [Jacobsen et al., 2007] These equations and calculations following can only be used to give an estimate of pressures and temperatures. And they should thus not be regarded as correct values but only as guidelines.

C.3. Using matlab and BWR

C.3.1. Adiabatic simulations

In every simulation conduced in connection with this report the gas expansion in the tank is assumed to be adiabatic. This is mainly done because it gives the worst-case scenario where the temperature drop is the largest and thus the largest pressure drop and it will require the largest gas volume.

During an adiabatic expansion no heat is exchanged with the surroundings. This is of course a simplification which is only valid for very fast expansion. In the case of the boost accumulator, the effect of the convection with the gas tank wall and the convection with the sea water will reduce the temperature drop in the tank. As a result will the pressure drop become smaller and the controller unit will have an easier time controlling. Before this concept is realized must a more correct simulation which includes the convectional effects be conducted.

C.3.2. Just solving the equation

Algorithm 1 Solving with regards to pressure:

```
function P=bwr_P(rho,T)
% a=0.0254;
% A0=1.0676;
% b=0.002328;
% B0=0.04074;
% c=738.1;
% C0=8166;
% alfa=1.272*10^-4;
% gamma=0.0053;
a=2.54;
A0=106.76;
b=0.002328;
B0=0.04074;
c=73790;
C0=816400;
alfa=1.272*10^-4;
gamma=0.0053;
R=8.314472;
r=rho/28.02;
rRT=r*R*T;
B0RT=(B0*R*T-A0-C0/T^2)*r^2;
bRT=(b*R*T-a)*r^3;
alfaar=alfa*a*r^6;
crT=(c*r^3/T^2)*(1+gamma*r^2);
P=rRT+B0RT+bRT+alfaar+crT*exp(-gamma*r^2);
P=P*1000; % Konverterer til Pa
```

Algorithm 2 Solving with regards to density:

```
%Denne funksjonen løser Benedict-Webb-Rubin ligningen itterativt for rho.
%Innput er Trykket; P i bar. Og Temperaturen; T i Kelvin
function rho=bwr_rho(P,T)
P=P*100;
% a=0.0254;
% A0=1.0676;
% b=0.002328;
% B0=0.04074;
% c=738.1;
% C0=8166;
% alfa=1.272*1e-4;
% gamma=0.0053;
a=2.54;
A0=106.76;
b=0.002328;
B0=0.04074;
c=73790;
C0=816400;
alfa=1.272*10^-4;
gamma=0.0053;
R=8.314472;
rho=P/(R*T);
cor=0.9;
for i=1:50000
    r=rho(i);
    test=r*R*T;
    B0RT=(B0*R*T-A0-C0/T^2)*r^2;
    bRT=(b*R*T-a)*r^3;
    alfaar=alfa*a*r^6;
    crT=(c*r^3/T^2)*(1+gamma*r^2);
   % rho(i+1)=cor*r+(1-cor)*(P-B0RT-bRT-alfaar-crT*exp(-gamma*r^2))/(R*T);
   rho(i+1)=cor*r+(1-cor)*(P*r/(r*R*T+B0RT+bRT+alfaar+crT*exp(-gamma*r^2)));
     if(abs(1-abs(rho(i)/rho(i+1)))<0.000001)
     break
     end
end
rho=rho(i)*28.02;
```

C.3.3. Calculating necessary booster tank volume

This calculation first assumes a required mass needed to fill the required oil volume at the given pressure: p_{oil} and a guessed temperature: T_{init}. Then it starts to empty the gas tank with the mass calculated. This calculation is done repetatly with a new initial temperature wich gives a more correct required mass: m_{req}.

The evacuation of the gass tank is assumed to be adiabatic and the work done by the gas results in a temperature loss. This gives a result wich is conservative compared to the results given by SimulationX which uses a modified set of BWR equations.

Algorithm 3 Calculating nessecary booster tank volume

```
% -- Program for å beregne nødvendig boost volum for gitte parametre v2 --%
% Beregner nødvendig volum ved å se på massestrømmen ut av tanken og inn
% i olje/gass tanken.
% Antagelser: Adiabatisk, konstant temperatur i gass/olje tanken
% Gir bedre resultat enn v1
% Gir en alt for stor Delta T
%% -- Parametre -- %%
function TankVolum=boost_v2(P1,Poil,deltaV)
% P1 og Poil er i Bar, deltaV er i liter
P2 = Poil*1.1; % Bar
Tinit = 273; % Kelvin
Tres(1) = Tinit;
Tres(2) = Tinit*0.8;
Kappa = 1.6 ;
Ideal_Volume=deltaV/((P1/P2)^(1/Kappa)-1);
V0=ones(1,100)*Ideal_Volume/1000;

%% -- Beregning -- %%
n=2;
minit=zeros(50,1);
while abs(1-Tres(n)/Tres(n-1))>0.01; % Sørger for riktig valg av Tinit


    m_req=(deltaV)*bwr_rho(Poil,Tres(n))/1000;
    m_act=m_req*1.1;
    r=1;
    while abs(1-m_req/m_act)>0.001 && r<50;
        %% -- Kjøring av en ekspansjon --- %%
        i=1;
        minit(r)=V0(r)*bwr_rho(P1,Tinit);
        m=linspace(minit(r),minit(r)-m_req,10000); % Gassvolum
        T=ones(1,10000)*Tinit;
        P=ones(1,10000)*P1;

        while (P(i)>P2 && i<10000);
            i=i+1;
            rho2=m(i-1)/V0(r);
            P(i)=bwr_P(rho2,T(i-1))/100000;
            % Adiabat tilsier at alt arbeidet blir omgjort til varme:
```

```
            % W=int{-V}dp
            %dT=((P(i)-P(i-1))*100000*(-V0(r)))/(minit(r)*1040);
            % Cv = 1040 Cp =748
            h=(1.0416*T(i-1)-0.9171)*1e3;
            dT=(1-m(i-1)/m(i))*h/1040;
            T(i)=T(i-1)+dT;
        end
        m_act=minit(r)-m(i);
        r=r+1;
        % Velger nytt start volum ut i fra forskjellen mellom dv og
        % ønsket dv(deltaV)
        V0(r)=V0(r-1)+(1-m_act/m_req)*(V0(r-1));
    end
    n=n+1;
    Tres(n)=250;%T(i);
end
V0=V0(1,1:r)*1000;
subplot(2,1,1);plot(V0)
title(['V0: ', num2str(V0(r)),' --- Delta m: ' , num2str(m_act) ...
    , ' --- Delta P: ' , num2str(P(1,1)-P(1,i))...
    , ' --- Delta T: ' , num2str(T(1,1)-T(1,i))])
xlabel('Iteration')
ylabel('Volume')
P=P(1,1:i);
m=m(1,1:i);
T=T(1,1:i);
subplot(2,1,2);
[AX,H1,H2]=plotyy(m,P,m,T,'plot');
set(get(AX(1),'Ylabel'),'String','Pressure [Bar]')
set(get(AX(2),'Ylabel'),'String','Temperature [Kelvin]')
xlabel('max [kgs]')
TankVolum=V0(1,r);
```

D. SimulationX

Note: This chapter is directly copied from the project work of Ostby [2007] with a few modifications. It was included because this explains the basics behind SimulationX which the author does not believe is a commonly used program. A similar chapter explaining Simulink was not included for the opposite reason.

D.1. General

SimulationX is a numerical analysis software created by . It allows the user to draw the system he wants to simulate by inserting premade objects into the main window. There are a wast number of objects in several different categories. For instance hydraulic valves and lines, electric circuits, piston engines, evaporators and so on. Figure:D.1 shows an example of the workspace where the PI controller mentioned in section: is modeled.

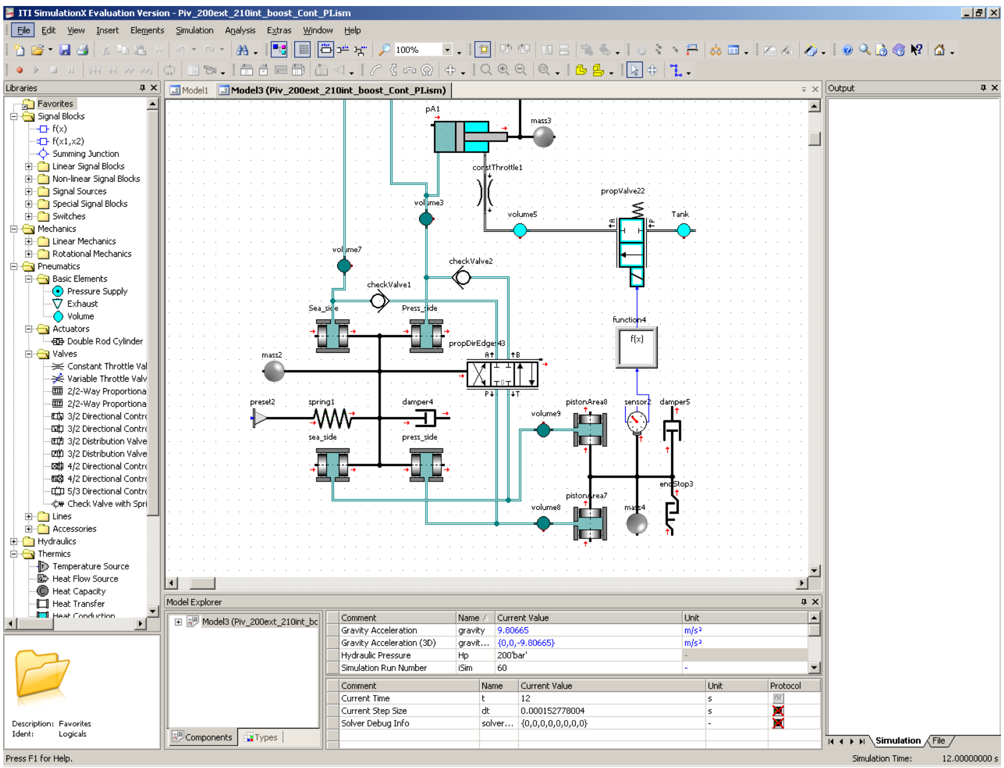


Figure D.1.: SimulationX workspace

To solve the systems created by the user SimulationX uses a set of 1st order non-linear differential equations. Each object have their set of equations. These equations are solved with a Backwards-Differential-Method solver.

D.2. Backwards Differential Method

BDM is an implicit method where the next time step is solved by the neighbor nodes in space and the last value in time for the node in question.

Example: The heat transfer function in one dimension:

$$\frac{\mathrm{d}u}{\mathrm{d}t} = \frac{\mathrm{d}^2u}{\mathrm{d}x^2} \tag{D.1}$$

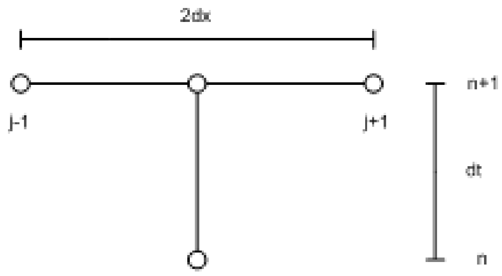


Figure D.2.: Backwards differential method

Using the scheme shown in figure: D.2 (D.1) can be written:

$$\frac{u_j^{n+1} - u_j^n}{\Delta t} = \frac{u_{j-1}^{n+1} - 2u_j^{n+1} + u_{j+1}^{n+1}}{\Delta x^2} \tag{D.2}$$

Here n is not the power of u, but a way to say at which time-step we are calculating. This equation is implicit for u_j^{n+1}, and it is solved with a matrix solver; e.g. TDMA Versteeg and Malalasekera [2007, Chapter 7].

D.3. The simulations

Only visual printouts have been included of the simulations. No parameters or start values are included. This because the amount of parameters for each simulation would fill 10-15 pages.

D.3.1. Simulation basis

Figure D.3 is a printout of the basic SimulationX model which is the most used model. It uses a simple valve to model the SSR actuators. The pneumatic valve, propValve22, have no mechanical connection and is therefore connected through a sensor and a function normalizing the signal. A valve, ConstThrottle1, is inserted just before the accumulator tank, pA1. It is very wide and does not affect the simulation. Its only function is to prevent two separate volumes, volume1 and pA1, to be connected without any losses. This would have caused simulation crashes.

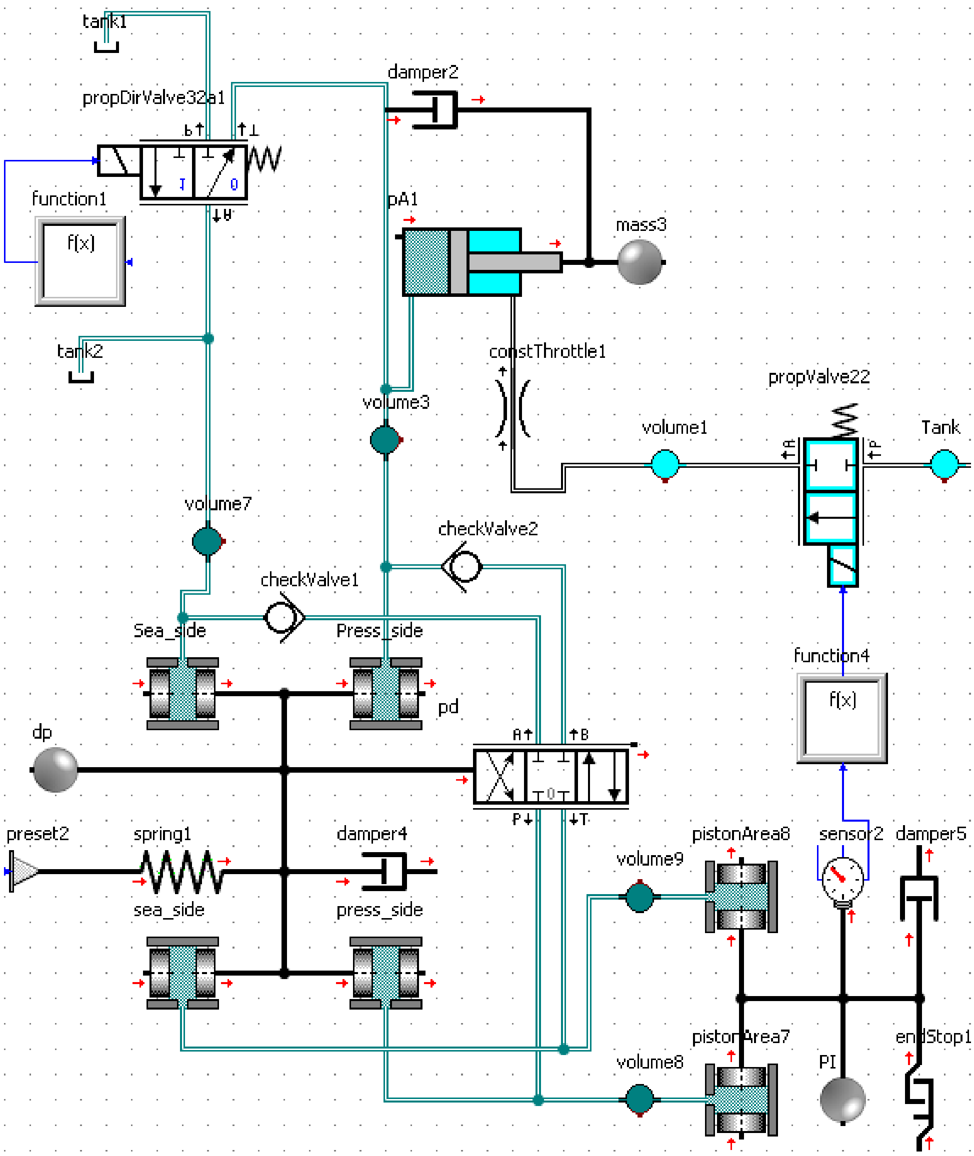


Figure D.3.: The most basic SimulationX model

D.3.2. Refill

D.3.2.1. Oil supply

Figure D.4

D.3.2.2. Gas supply

Figure D.5

D.3.3. FMCs correct SSR model

Figure D.6 shows FMCs SSR model. When used in this report have the accumulator unit, USSR_Acc, been replaced by the circuit shown in figure D.3. Because of its size is only half of the model shown in the figure. The right half is an exact mirror of the left.

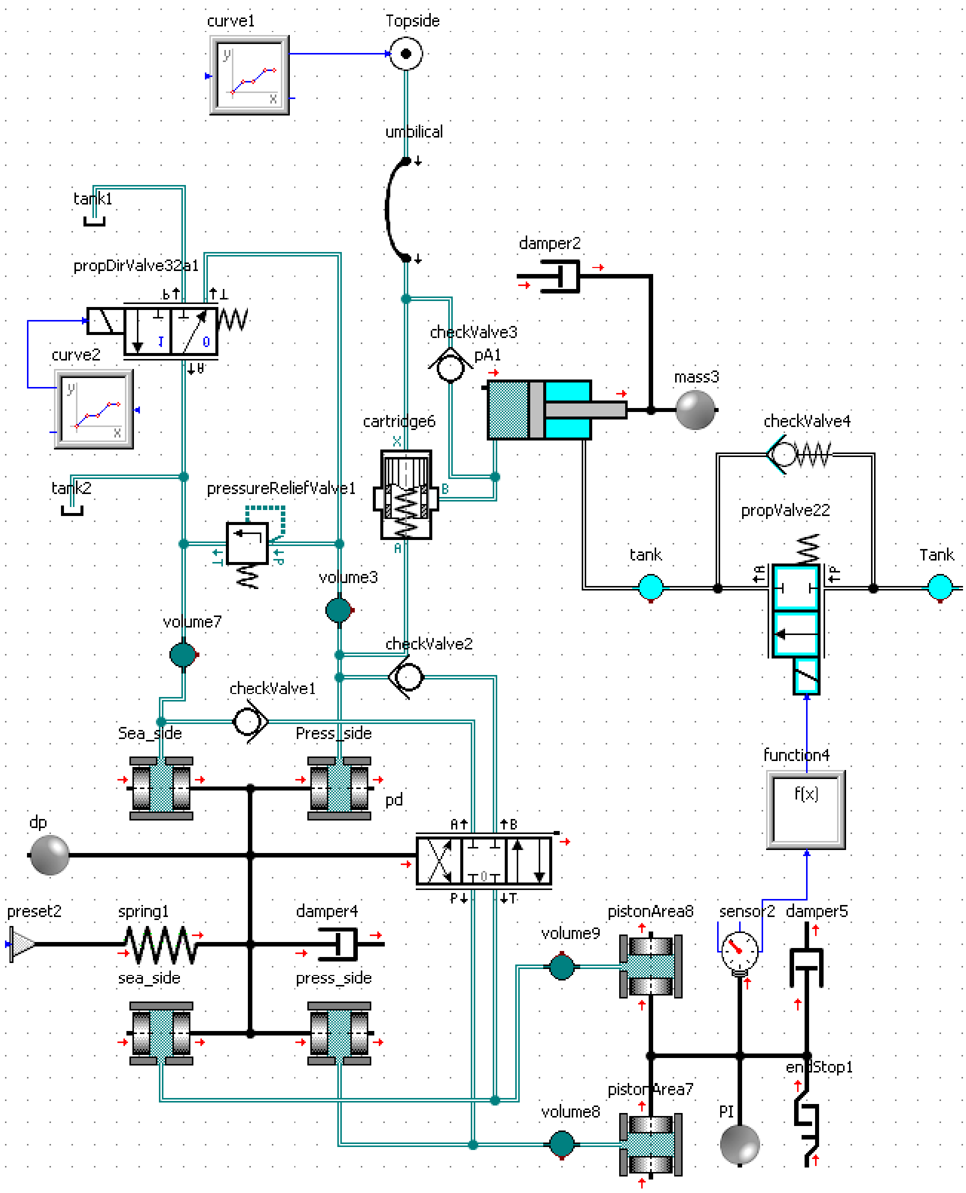


Figure D.4.: Refill with oil pressure modeled in SimulationX

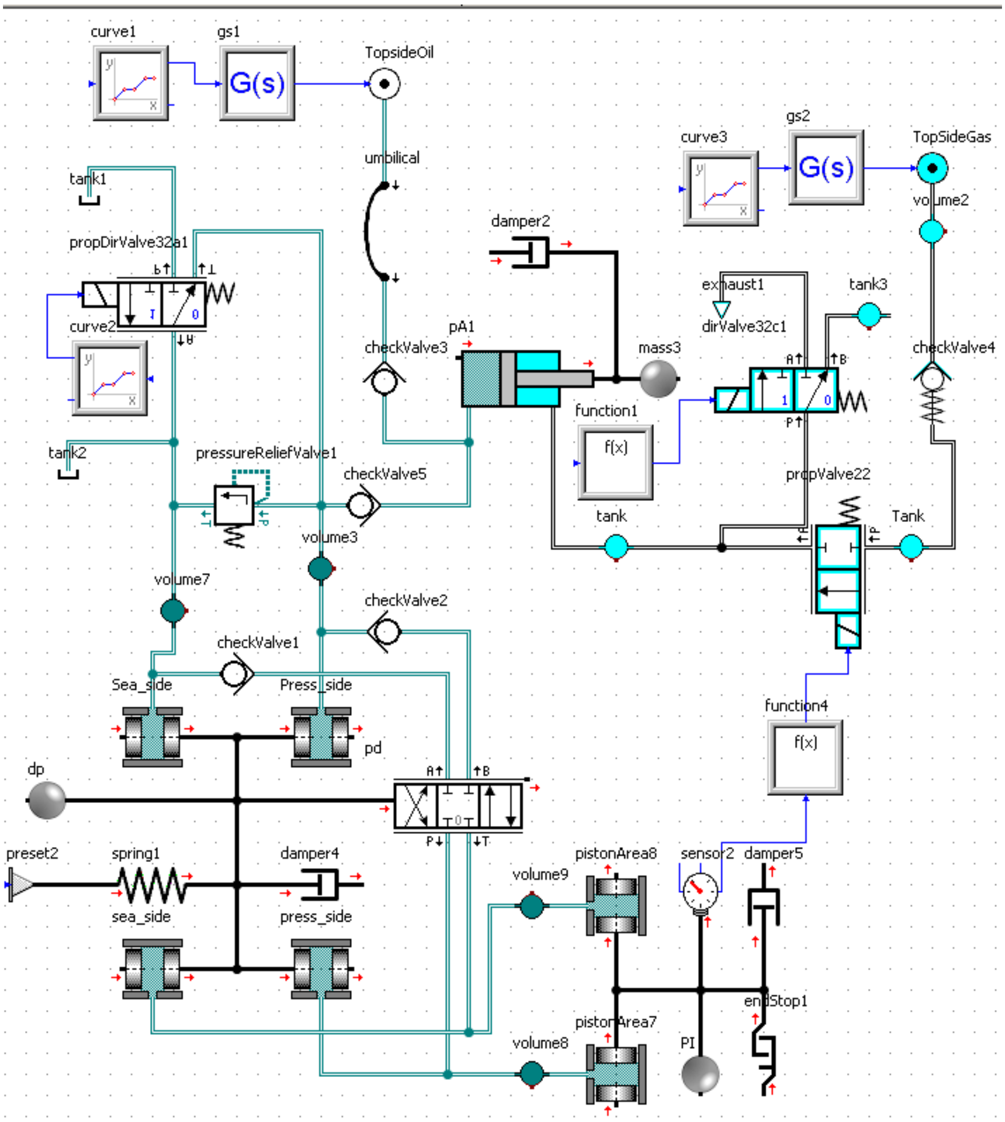


Figure D.5.: Refill with gas pressure modeled in SimulationX

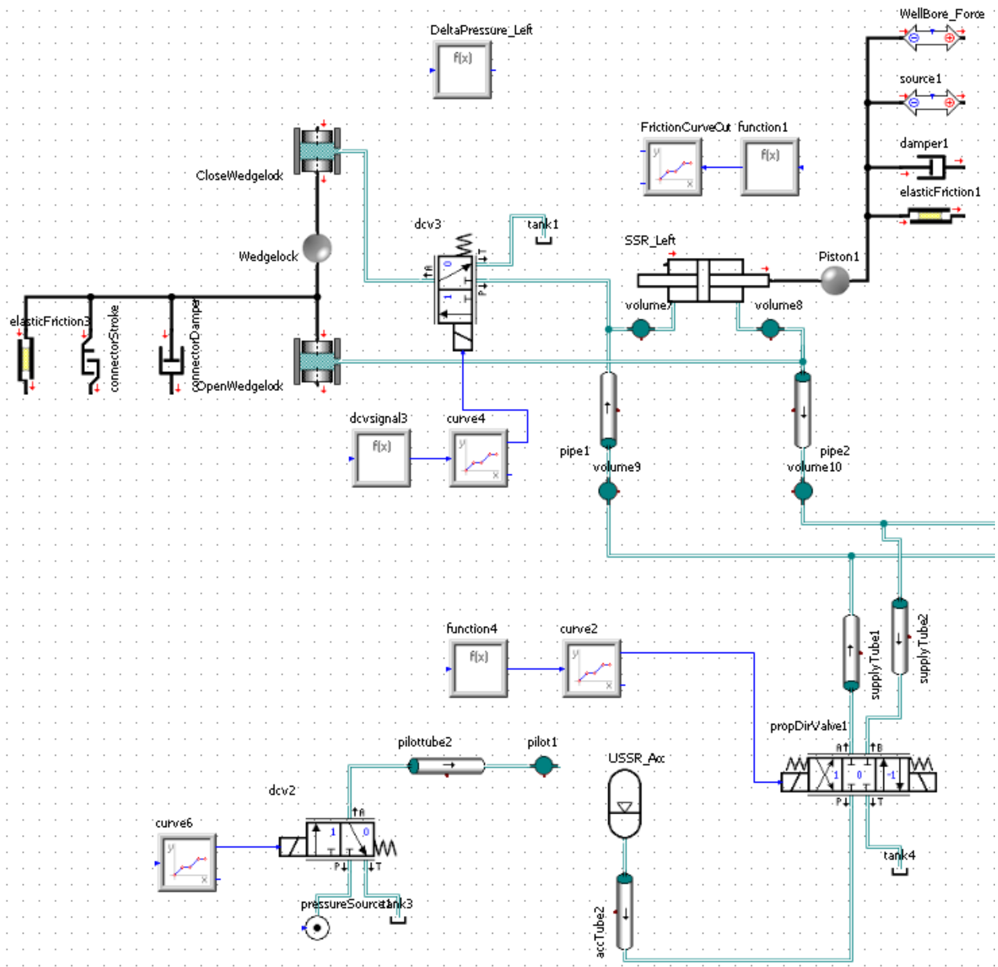


Figure D.6.: The most correct model of the SSR circuit created by FMC