Production planning and sales allocation in the salmon farming industry

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Problem formulation

In this thesis, a multistage stochastic optimization model for planning of production and allocation of sales in the salmon farming industry will be developed. The model maximizes the expected profit. The objective of the thesis is to provide decision support regarding when to harvest fish, how to distribute harvested fish between different product groups, how to allocate sales between contracts and spot sales in different markets and how to transport products within the production network and to the market.

The thesis will include the following elements:

- Presentation and discussion of the problem, including a discussion of the relevant uncertainty parameters
- Formulation and implementation of a stochastic optimization model
- Discussion of the results from a test case

Summary

In this thesis a multistage stochastic optimization model has been developed for production planning and sales allocation within the salmon farming industry. To the authors knowledge, an optimization model for this problem has not been developed before.

The background for this thesis is a tactical planning problem spanning production and sales under uncertainty. The goal is to identify profitable solutions that are sufficiently flexible in accounting for this uncertainty.

The model is inspired by the work done by Hæreid (2011), but addresses more aspects of the value chain. The most important innovations in the model is that processed products and inventory management is included. Furthermore, the scope are extended to include global operations. Additionally, the model offers a more detailed modeling of contract and transport decisions. The model also incorporate both market and product price dependencies.

The model is implemented in two versions: one deterministic and one three-stage stochastic version. To illustrate how they can be used, the different models are tested on instances inspired of Marine Harvest's global production network. The input data used are however based on publicly available information. To model uncertainty in salmon price and sea water temperature, different seasonal ARIMA-models are used to model these uncertain time series. Based on these forecasts, residual scenarios are generated by using a copula-based scenario generation heuristic.

The developed model acts according to the authors initial assumptions and gives reasonable results compared to how the industry operates as of today. The value of including stochastic programming constitutes 5 % savings, and tests indicate that the model holds in-sample stability.

Sammendrag

I denne masteroppgaven har en multi-stegs stokastisk optimeringsmodell for produksjonsplanlegging og salgsallokering innen lakseoppdrett blitt utviklet. Såvidt forfatterene har kjennskap til, er det aldri tidligere blitt utviklet en optimeringsmodell for dette problemet.

Bakgrunnen for oppgaven er et taktisk planleggingsproblem som omhandler produksjon og salgsallokering under usikkerhet. Målet med denne modellen er å identifisere profitable beslutninger som er tilstrekkelig fleksible til å håndtere denne usikkerheten.

Modellen er inspirert av arbeidet til Hæreid (2011), men tar for seg en langt større del av verdikjeden. De viktigste nyvinningene i modellen er at prosesserte produkter og lagerstyring er inkludert. Videre er omfanget utvidet til å favne global virksomhet. I tillegg tilbyr modellen en mer detaljert modellering av kontrakter og transportbeslutninger, og lar priser avhenge av hvilket marked handelen gjennomføres i.

Modellen er implementert i to versjoner; en deterministisk og en tre-stegs stokastisk versjon. For å illustrere hvordan de kan brukes i praksis er modellene blitt testet på instanser inspirert av the globale produksjonsnettverket til Marine Harvest. Datagrunnlaget som er brukt er basert på offentlig tilgjengelig informasjon. For å modellere usikkerhet i pris og vanntemperatur er prognoser utviklet ved hjelp av ulike sesongbaserte ARIMA-modeller. Basert på prognosene er det generert et sett med scenarier ved hjelp av et copula-basert scenariogenereringsverktøy.

Modellen som er utviklet oppfører seg som tiltenkt og gir resultater som er fornuftige sett i sammenheng med bransjen. Verdien av å inkludere usikkerhet er vist å utgjøre 5%, og tester indikerer at modellen innehar in-sample stabilitet.

Preface

This master thesis is written within Applied Economics and Operations Management at the Department of Industrial Economics and Technology Management, Norwegian University of Science and Technology (NTNU). This thesis is motivated by the operations of the global salmon producer Marine Harvest and previous master theses written on the same subject.

We have received much appreciated help and guidance in completing this thesis, both from faculty at NTNU, co-students and industry contacts. We would especially like to thank our supervisor, professor at NTNU, Asgeir Tomasgard for his patience, guidance, interesting discussions and valuable feedback. We would also like to thank Tormod Johannessen and his colleagues at Marine Harvest for valuable help in answering questions about both Marine Harvest and the industry in general.

We would additionally like to express our gratitude to Stein Erik Fleten, professor at NTNU, Håkon Tjelmeland, professor at NTNU, Paul T. Aandahl, analyst the Norwegian Seafood Council and Michael Kaut, researcher at SINTEF. Our work has greatly benefited from the help and feedback received from all mentioned above.

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Glossary

Aquaculture Aquaculture, also known as fish farming, marine culture etc., defined as the human cultivation of organisms in water

Biomass Biomass is defined as the total weight of live fish.

FOB Free on board, meaning that the seller pays for transportation of the goods to the port of shipment

Production fish Fish with too low quality to be sold as fresh or frozen HOG salmon

Salmon Short for Atlantic salmon (Salmo salar)

Salmonids Collective name for all salmon fish species, including Atlantic salmon, Pacific salmon and trout

Secondary processing Any value adding processing beyond HOG

wellboats Boats with large wells of circulating seawater

Acronyms

ACF autocorrelation function

ADF Augmented Dickey Fuller's test for stationarity

AIC Akaike's Information Criterion

AR-model autoregressive model

ARCH autoregressive conditional heteroscedastic model

ARIMA autoregressive integrated moving average model

BIC Bayesian Information Criterion

CAGR compounded annual growth rate

CDF Cumulative Distribution Function

CWQ Carcass Weight Equivalent

DET deterministic model

EDEV expected result of using the dynamic solution

EEV expected value of expected solution

EPV Expected product value - Products in inventory in last time period

ESV Expected salmon value - Value of fish in farms in last time period

EU27 The 27 member nations in EU

EV expected value

EVPI expected value of perfect information

FTE Full-time equivalent (employee)

GARCH generalized autoregressive conditional heteroscedastic model

GWE gutted weight equivalent

HOG head-on gutted salmon

i.i.d. Independent and identically distributed random variables

MA-model moving average model

MAB maxium allowable biomass

MAP modified atmosphere packaging

MOOP multi-objective optimization problems

MT metric tonnes

PACF partial autocorrelation function

PW product weight

RP recourse problem

SP solution of stochastic problem

SST Sea surface temperature

VAP value added products

VSS value of stochastic solution

WFE whole fish equivalent

WS wait-and-see solution

Chapter 1

Introduction

Due to rapid growth in the world population, there will be an increased demand for protein. According to United Nations (2012) the world population is expected to reach 9,6 billion by 2050. Furthermore, with an estimated 800 million people already suffering from chronic malnourishment today (FAO, 2014), the need for sustainable protein sources will become increasingly important in the years to come. Although 70% of the Earth's surface is covered by water, only 6.5 % of the protein consumed by humans is produced in this element (Marine Harvest, 2014b). Consequently, there is a much greater potential for utilizing the aquaculture resources such as farmed Atlantic salmon (*Salmo salar*).

While agriculture is activities related to cultivating of soil and animals, aquaculture deals with production of farm bred marine species (National Geographic, 2015). In 2012, fish trade represented about 10% of total agricultural exports and 1% of world merchandise trade (FAO, 2014). For the next decade, total production from capture and aquaculture is expected to exceed the production of beef, pork and poultry combined (Mathiesen, 2012).

Commercial salmon aquaculture began in late 1960s. Since then, the industry has experienced extraordinary growth, and the farming of salmonids have dominated the worldwide market for salmon since 1999 (Marine Harvest, 2014b). Today, salmon aquaculture is the fastest growing food production system in the world (WWF, 2015), and the total amount of globally farmed salmon reached 2 040 300 metric tonnes (MT) in 2013 (Kontali Analyse, 2014). Moreover, the industry has grown to become one of Norway's most important industries, and as of today, seafood is Norway's third largest export article, only surpassed by Oil and Gas (Regjeringen, 2015).

Production planning in the salmon farming industry poses an interesting modeling challenge. Due to the rapid shift towards consolidation in the salmon farming industry,

where the number of salmon farming companies in Norway is reduced by nearly 70% since 1994 (Kontali Analyse, 2014), there is believed to be a significant potential with the use of operations research. As companies grow both in size and geographical presence, the planning process become increasingly more complex. Consequently, a need for better decision support tools and portfolio management has arisen, explaining the background for this thesis.

Key characteristics of the salmon farming industry are uncertainty regarding biomass development and future prices. Combined with long production cycles, these uncertainties represent a large challenge for the salmon producers. It is therefore essential to account for this uncertainty during the planning process. Thus, the problem is well suited for stochastic programming. The purpose of this thesis is to develop a decision support tool in the form of a multistage stochastic optimization model. The model seeks to maximize total profits for a global and vertically integrated salmon producer. Marine Harvest is chosen as the case studied. The scope is limited to the part of the value chain downstream of the fresh water salmon production.

The thesis is structured as follows. Chapter 2 gives an introduction to the salmon farming industry, and the value chain, operations and production network of Marine Harvest are outlined. In Chapter 3, the salmon market, global trade patterns and price behaviour are discussed in detail. A comprehensive problem description is given in Chapter 4 before relevant theory to this problem is presented in Chapter 5. In Chapter 6 relevant aspects to the model are discussed, before the mathematical model formulation is given in Chapter 7. The case used for testing the model is presented in Chapter 8, alongside with an explanation of the input data and the scenario generation. In Chapter 9, a discussion of the results and model application is presented. Finally, Chapter 10 summarize this thesis, while Chapter 11 points out possible directions for future work.

Chapter 2

Salmon aquaculture

In this chapter, the Salmon aquaculture industry is introduced. Section 2.1 gives a brief description of how salmon aquaculture is defined, and describes its main characteristics. In Section 2.2, each part of the value chain in the salmon farming industry is presented, while the most important regulations are presented in Section 2.3. Revenues and costs related to the salmon farming industry are briefly discussed in Section 2.4, whereas Marine Harvest's operations are described in Section 2.5. Uncertainty and risk related to the value chain operations will be discussed later in Chapter 4.

Throughout this thesis, the phrase salmon implies Atlantic salmon, and aquaculture is short for Atlantic salmon aquaculture. Companies operating in the salmon farming industry are referred to as salmon producers. Unless otherwise noted, weights are given in whole fish equivalents (WFE) and measured in MT. WFE is the weight of the fish after slaughtering but before gutting (Kontali Analyse, 2007).

2.1 Industry background

Due to biological and climate constraints including seawater temperature requirements, Atlantic salmon is almost exclusively farmed in Norway, Chile, Scotland and Canada, as illustrated by Figure 2.1. These countries jointly represent approximately 95 % of the world supply (Seafish, 2012). Norway is the largest producer in terms of volume, supplying 56 % of the global demand for farmed salmon in 2013 (Kontali Analyse, 2014).

The salmon industry is subject to government regulations, and licenses are needed in order to be granted the right to produce salmonids. The acquisition of a license often takes years, and in most countries, each license has an upper limit on allowable biomass in production. The licenses also specifies which region the license is valid for.

Figure 2.1: Main salmon producing regions worldwide.

As regulations differ from country to country, the reader is referred to Section 2.3 for more detailed descriptions of the regulatory environment of salmon farming.

Approximately 80 % of Atlantic salmon is marketed fresh. Either as fresh head-on gutted salmon (HOG), steaks, fillets or other elaborated fresh products. A small, but increasing percentage of the harvested salmon is processed into value-added products (Marine Harvest, 2014b; Seafish, 2012; Brækkan, 2014). Value added products (VAP), can be defined as all products that have been secondary processed (Marine Harvest, 2014b). Examples include, but are not limited to, fillets, steaks, smoked and cured salmon and ready meals.

The largest global market for farmed Atlantic salmon is the EU, followed by the US and Russia. The compounded annual growth rate (CAGR) for the EU has been approximately 9 % the last years. Large emerging markets such as Brazil and some countries in Asia are also starting to embrace salmon, showing a CAGR of approximately 20 % (Brækkan, 2014). In the same time period, both salmon prices and price volatility have increased (Øglend and Sikveland, 2008; Øglend, 2013).

Salmon experienced falling prices through most of the 1980s and 1990s caused by steady improvements in productivity (Øglend, 2013). The real price in 2008 was less than one-third of the price in the early 1980s. As a result, salmon has become a part of everyday diets. This have fueled product development as lower prices have made a

Year	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013E
Volume [MT]	1 205	1 251	1 272	1 396	1 495	1 474	1 456	1 633	1 996	2 040
Change	5%	4%	2%	10%	7%	1%	-1%	12%	22%	2%

Table 2.1: Historical worldwide harvest volumes in metric tonnes of Altantic salmon (Kontali Analyse, 2014).

new array of products affordable for consumers. There seems to be a trend towards an increased number of value added products which increase the total demand for salmon (Asche and Bjørndal, 2011, p87).

An overview of different weight conversion ratios for farmed salmon is given in Table A.1 in Appendix A.

2.2 The Salmon farming value chain

The value chain can be roughly divided into six steps, as presented in Figure 2.2. Note that not all fish go through step 5. The majority of harvested fish are marketed as fresh HOG salmon, and are sent directly from the slaughter house to the market. The total production cycle takes about three years to complete (Marine Harvest, 2014b).
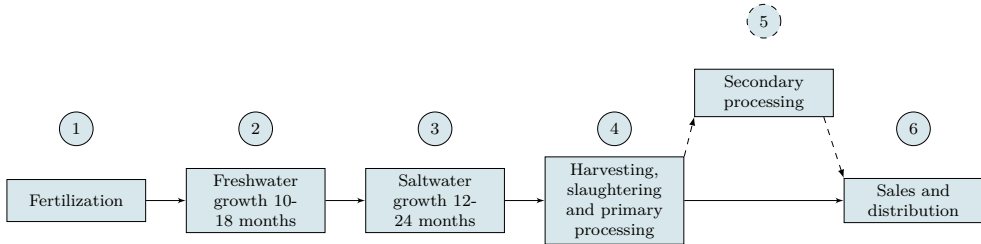
Figure 2.2: The salmon farming value chain.

2.2.1 Fresh water growth

The freshwater phase takes between 10-18 months, depending on growth, lighting conditions and the weight of the fish when it is deployed in saltwater. Towards the end of the fresh water phase, the fish goes through a physiological process called smoltification which makes them capable of surviving in seawater. This process can take place when the fish weigh from 40-120 grams (FAO, 2015), and the fish are then called *smolt*. The producer have the possibility to speed up the growth by light manipulation thus accelerating the smoltification process with up to 6 months. Light manipulated smolt are said to be generation S0, while smolt grown through normal conditions are called generation S1 smolt (FAO, 2015).

When a fish reaches the smolt stage it must be deployed into salt water within a short period of time, approximately two weeks, or it will de-smoltificate. This is undesired, as the desmoltification process is tiresome for the fish, leading to slower growth and a weakened immune system. Smolt transfer to seawater facilities is usually carried out in large tanks aboard wellboats, trucks or helicopters (FAO, 2015).

2.2.2   Salt water growth

At the beginning of the salt water phase, smolt are deployed in large sea water cages at the farming facilities. Each cage may contain from 30 000 fish and up to 200 000 fish (Cermaq, 2015). Seawater sites are chosen based on a number of different factors, such as suitability with regard to water temperature, salinity, flow and exchange rates, licensing regulations and proximity to other facilities (FAO, 2015). Two of the most important criterions is that the waterflow is sufficient to eliminate waste and to supply well oxygenated water, and that the site is sheltered to minimize escape during bad weather (Marine Harvest, 2014b).

In Norway, smolt is mainly released into seawater twice a year, in autumn and in spring (Marine Harvest, 2014b). It is normal that each cage only contains a single generation of fish (FAO, 2015). Due to biological and climatic reasons, smolt should only be transferred to sea during the warmer half of the year, namely March to October in Norway and September to March in Chile (Asche and Bjørndal, 2011).

Growth in the salt water process is dependent upon many factors, such as weight of the fish, light, feeding and sea water temperature (Asche and Bjørndal, 2011). Light and feeding may be controlled by the producer. In contrast, the sea water temperature can not be controlled. The growth rate increases with increasing temperature. The optimal temperature range for salmon is between $8 - 14\,^{\circ}\mathrm{C}$ (Marine Harvest, 2014b). At higher temperatures, growth rates decrease and the mortality rate increases, while growth rates become very low at temperatures substantially below $3\,^{\circ}\mathrm{C}$ (Brækkan, 2014). Mass mortality can occur at temperatures below $0\,^{\circ}\mathrm{C}$.

Production countries in the North experience a cyclic temperature, whereas Chile experience sea water temperatures consistently higher and more stable (Marine Harvest, 2014b). Note that Chile, due to being located on the southern hemisphere, experience opposite seasons of Norway, Scotland and Canada.

Most of the growth occurs during the warmer half of the year, both due to higher sea water temperatures and more daylight. This results in a S-shaped curved for

the biomass level in Europe having a S-curve, with a low point in May and a peak in October, while Chile experience more steady biomass levels throughout the year. Figure 2.3 show the Norwegian biomass development in 2013 and 2014. The availability of different weight classes for the market also have a seasonal pattern. This is due to the fact that salmon growth is not a linear function over time and that there is conducted smolt release (Asche and Bjørndal, 2011). During the summer supply is notably different than the rest of the year, as the producer go from harvesting fish belonging to S0s to harvesting fish belonging to S1s, and the large fish, S0s, and the small fish, S1s, dominate the supply (Marine Harvest, 2014b).
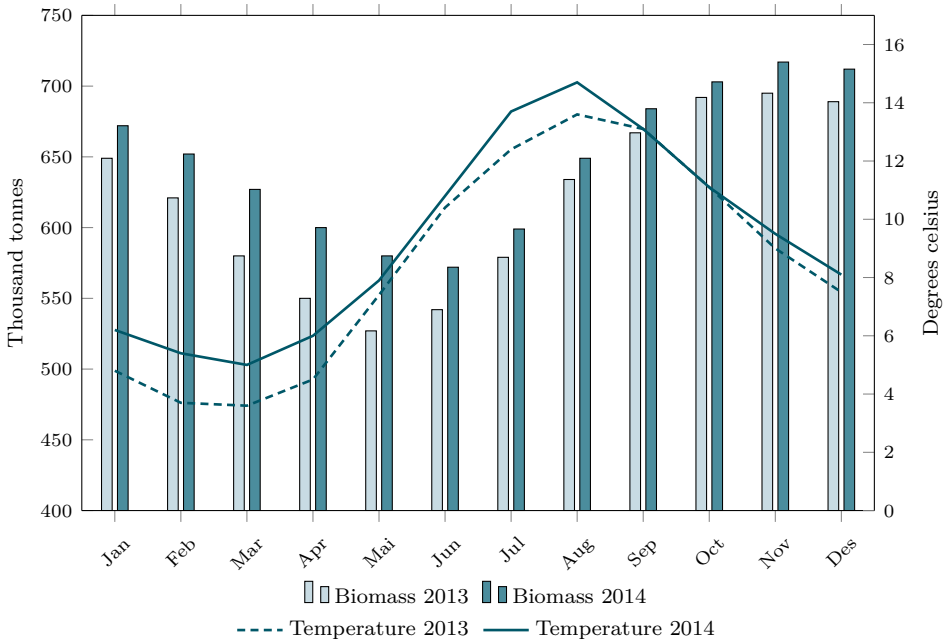
Figure 2.3: Norwegian biomass development (Marius Gaard, 2015).

2.2.3   Harvest

Harvest is performed continuously throughout the year. This contributes to a stable supply and predictable operations. Throughout the harvest, it is important to minimize the stress inflicted on the fish, and thereby maximizing product quality (FAO, 2015). The harvest is done by using sweep nets for sorting the fish into separate weight classes. This is necessary as fish of different weight classes are used for different purposes, and as variations in fish sizes slows down the slaughtering process due a need for constantly adjust the machinery.

After the sorting, the fish are pumped from the pens into wellboats for transport to the slaughtering facilities, which are boats with circulating seawater. Alternatively, the fish may be slaughtered immediately on the boat and then put on ice.

Most farmed salmon become sexually mature when they reach a weight of about 5-7 kg. As sexual maturity causes growth to stagnate, and the flesh of the fish to deteriorate the fish become unsuitable for human consumption (Asche and Guttormsen, 2001). Hence, sexual maturity puts a limit to how long a farmer can keep the fish in the pens. However, the timing of the sexual maturity can be manipulated to some degree by use of techniques such as artificial light (Hansen et al., 1992).

The economic optimal harvest weight is in the area of 4-5 kg (Marine Harvest, 2014b). However, fish are commonly marketed in the range between 3.5 and 7 kg. The reason for harvesting before the optimal harvest weight is reached might be poor growth due to for example cold water or disease, or a need to harvest fish ahead of plan to be able to fill contracts or take advantage of market opportunities. Reasons for harvesting fish after target weight is met might be hold ups in the slaughterhouses due to under capacity, need to balance harvest due to wellboat capacity considerations, or temporary high salmon prices.

After a site is harvested, the location is fallowed for 2 to 6 months before the next generation is deployed at the same location (Marine Harvest, 2014b). This is however done in practice by moving the farm to other locations by using designated tow boats (Evjemo, J.O., personal communication, 2015).

2.2.4 Slaughtering and primary processing

Slaughtering and primary processing are performed at the same facility. The fish arrives in large wellboats and are gutted before the fish are graded and packed in crates filled with ice (FAO, 2015; Cermaq, 2015). The fish are then ready for dispatch to the customers as a product called fresh HOG salmon, an abbreviation for fresh, head-on gutted superior salmon.

2.2.5 Secondary processing

Secondary processing includes all value adding beyond primary processing. Examples include filleting, fillet trimming, portioning, smoking, and the production of ready meals.

Globally, HOG salmon constitutes for 75 % of the total value of consumed farmed salmon (Statistisk Sentralbyrå (SSB), 2015b), which implies that most fish are packed at the slaughterhouses and sold directly to the market. The remaining 25 % of the farmed salmon are secondary processed. If the secondary processing consist of a relatively simple process, such as filleting, both the primary and secondary processing may be performed at the slaughter facility.

Secondary processing normally takes place at designated production facilities. Not all processing facilities can produce all types of value added products. Note that these facilities are more geographically dispersed than the fish farms and the slaughter facilities.

Fillets are the largest category of value added salmon products in terms of volume. Another important category of value added salmon products is smoked salmon (FAO, 2015). The market shares of different VAPs in the EU market are depicted in Figure 2.4. The figure shows that fillets have the largest market share of 47 %, followed by smoked salmon at 28 %.
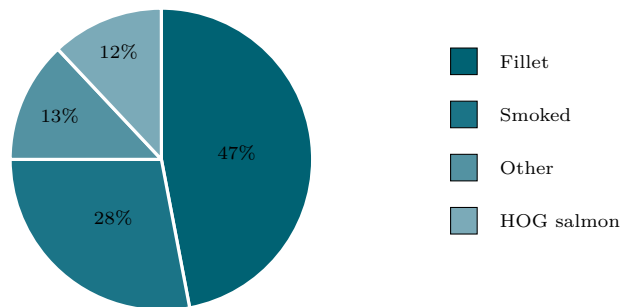
Figure 2.4: Product market share in the EU (Marine Harvest, 2014b).

2.2.6 Distribution

Smolt are distributed from fresh water facilities to salt water facilities, either by truck, helicopter or wellboat (Seafish, 2012). From the fish farm, the fish is transported by a wellboat to the slaughterhouse. From the slaughter house, the goods are transported directly to the market or via a processing facility, either by truck, boat or air freight (Cermaq, 2015). An illustration of the distribution network is given in Figure 2.5.

The products are transported in different states depending on durability of the product and distance to be covered. Among the different transportation states, supercooled is
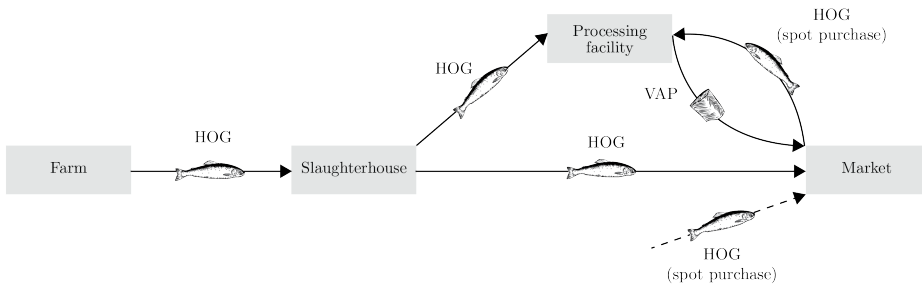
Figure 2.5: Illustration of the flow of products through the value chain.

the most common. The product is then cooled to $0\,°\mathrm{C}$, reaching a stage in between fresh and frozen. Supercooling is good for increasing the durability of the fish, while at the same time maintaining its quality. However, significant volumes are also transported in a frozen state due to long shipping distances in the global market (Einen et al., 2002).

Mode of transportation depends on both type of end product and distance to the markets. For fresh products, the risk of deterioration makes fast delivery imperative. Fresh products are therefore exclusively transported by air freight in a supercooled state over long distances. For shorter distances, it may also be possible to use trucks or boats. As transportation by plane is relatively expensive, frozen products are more often transported by trucks or ship.

2.3 Regulations

The industry is governed by numerous regulations setting the framework in which the salmon producers find themselves. The focus in this section is to describe the regulations most relevant to the problem studied in this thesis. Therefore, no exhaustive presentation of restrictions applying to the salmon farmers will be given. In particular, restrictions concerning feeding and slaughtering routines and control of fish welfare or restrictions that apply to the processes upstream of the salt water farming are omitted.

2.3.1 Norway

In Norway, the salmon farmers are subject to a relatively extensive set of regulations aiming at ensuring an ethical and sustainable development in the industry. Legislation is exercised by the Ministry of Fisheries and Coastal Affairs, the Directorate of Fisheries, and The Norwegian Food Safety Authority. The reader is referred to *Akvakulturdriftsforskriften* (2008) for a complete list of the Norwegian regulations concerning operation of aquaculture facilities in Norway.

Licensing has been practiced in Norway since 1973, the legal framework of which is described in *Laksetildelingsforskriften* (2004). A licence is valid within a specified region, and the licence states an upper limit for the amount of biomass, e.g. the maximum allowable biomass (MAB) of live fish. In most parts of Norway the biomass limit per license is 780 metric tonnes, while in the northernmost parts the limit is 900 metric tonnes.

To prevent companies from having disproportionate amounts of market power, Norwegian authorities have decided that no more than 25 % of the available licenses can be owned by the same industry player. Additionally, if one company were to control more than 15 % of the total allowable biomass in Norway, this would require a permission from the Ministry of Fisheries and Coastal Affairs.

The Regulation of Abatement of Sea Lice in Aquaculture Facilities of 2009 specifies that fallowing must be conducted simultaneously for all facilities within a set of lice zones (*Luseforskriften* 2009). The purpose of this regulation is to reduce the occurrence of sea lice by removing all potential hosts for the parasite.

2.3.2 Scotland

In Scotland, instead of a formal license, a permission from three institutions is required prior to development of a fish farming site. The producer needs a planning permission from the local regional council, a marine licence from Marine Scotland and a discharge license from Scottish Environment Protection Agency (Marine Harvest, 2014b). Licences can be traded and although there are no restriction on number of licences per company, there is a limit on production quantity assigned by the Competition Commission Authorities (Marine Harvest, 2014b).

Additionally, for each site there exists a MAB limit which is based on environmental concerns, meaning that the MAB limit for various sites is non-uniform, varying

between 100 MT to 2,500 MT depending on site characteristics and its geographic location (Marine Harvest, 2014b).

2.3.3 Canada

In Canada, both provincial and federal location specific licenses are required to operate a fish farm site (Marine Harvest, 2014b). Additionally, salmon producers must receive a License of Operation from the Federal Government. Here, the MAB limit and the allowed environmental impact of the site are specified. A typical site license will range in size from 2,000 MT to 5,000 MT of MAB (Marine Harvest, 2014b).

None of the licences can be traded between actors, but must be transferred to a different operator through a Government Assignment Process. This may be a time consuming procedure (Marine Harvest, 2014b).

2.3.4 Chile

The Chilean salmon farming industry is regulated by The Fishery and Aquaculture Law (Alvial et al., 2012). Chile practice licensing based on two authorizations. The first is an authorization to operate an aquaculture facility. These licences are valid for an infinite amount of time, and players can trade and rent licences from each other (Bjørndal, 2002; Marine Harvest, 2014b). The second authorization is for the physical area, and is restricted both to a specified species and a specified limit of production or stocking density (Marine Harvest, 2014b).

Except for the licensing practice, few regulations govern the salmon farming industry in Chile compared to the other salmon producing countries (Bjørndal, 2002).

2.4 Revenues and costs

The revenue stream is generated mainly from sales through bilateral contracts, futures contracts and sales on the spot market. Other operating revenues may be from sales of eggs, fish feed and redistribution of smolt and rentals (Norwegian Directorate of Fisheries, 2014a).

The Norwegian Directorate of Fisheries provide statistics regarding average costs of producing salmon and rainbow trout. In other producing countries however, official data is more scarce.

Table 2.2 presents estimated Norwegian operating costs for 2013. As the table show, feed is by far the most important cost component, representing 45.4 % of the total production costs. Miscellaneous operating costs constitute for the second largest part of the expenses.

Cost component	NOK/kg	Relative share
Smolt costs	2.19	8,64 %
Feeding cost	11.50	45,41 %
Insurance cost	0.11	0,44 %
Labour cost	1.80	7,10 %
Historic depreciation cost	1.23	4,87 %
Other operating cost	5.58	22,03 %
Net financial cost	0.28	1,10 %
Operational costs	**22.69**	**89,58 %**
Slaughtering costs (incl. transportation)	2.64	10,42 %
Total production costs	**25.33**	**100%**

Table 2.2: Estimated costs per kilogram fish (WFE) produced in Norway (Norwegian Directorate of Fisheries, 2014b).

Logistics costs come in addition to the costs listed. With a partial exception of Scotland, all producing countries are geared towards export markets. Transportation costs will therefore have a major impact on a country's competitiveness (Bjørndal, 2002). There are also costs related to mortality and disease.

Naturally, costs of production vary depending on geographical location (FAO, 2015). Bjørndal (2002) compares cost data for Norwegian and Chilean farms and concludes that Chilean cost levels are similar to or lower than Norway. However, the cost composition is different; Chilean feed, labour and processing costs are lower, but transportation costs are higher. Furthermore, industry sources normally indicate that average production costs in Scotland are 0.1–0.3 euro per kilogram higher than in Norway (Bjørndal, 2002). Note however that labour cost is a minor part of the total cost as much of the production is automatized.

2.5   Marine Harvest

Marine Harvest is the largest producer of farmed Atlantic salmon in the world, with a global market share of approximately 25 % (Marine Harvest, 2011a). Worldwide, Marine Harvest employ 11 715 people (FTE), and is represented in 23 countries while

supplying more than 70 markets. In 2014 the company had a turnover of NOK 25.5 billion (Marine Harvest, 2014a), and the corporate headquarter is located in Bergen, Norway.

In addition to Atlantic salmon, Marine Harvest also farm and cultivate other species. However, Atlantic salmon is by far the most important species, accounting for 91.7 % of the groups total revenues in 2014. Also, Marine Harvest almost exclusively produces salmon of superios quality (91 %) (Marine Harvest, 2014a).

The company controls the whole supply chain, where operations span from fish feed production through farming, processing, distribution, sales and marketing. Upstream and downstream integration is used actively as a strategy to stabilize costs, control product quality, improve efficiency and make the company less exposed to the economical cycles of salmon prices (Marine Harvest, 2013).

2.5.1 Salmon farming

Marine Harvest operate salmon farms in Norway, Scotland, Ireland, Chile, Canada and at the Faeroe Islands. The production sites along with the 2013 harvest volumes are depicted in Figure 2.6. Figure 2.7 illustrates the harvest volumes of Marine Harvest in reference to total supply of the respective countries. As seen, Norwegian production dominate the company's production, followed by Chile, Scotland and Canada. Production in Ireland and the Faroe Islands is notably smaller, representing only 4 % of Marine Harvest's total production.

One of the reasons for this geographical diversification is to hedge against location specific risk and thereby reduce total production costs. Historical numbers show that this has led to a more stable EBIT over time (Marine Harvest, 2014b, p. 43). Another factor influencing the choice of location strategy is that Marine Harvest is a global company, with markets all over the world. By producing in different regions, they also reduce transportation costs and have the opportunity to deliver fresh fish in a larger share of the world.

A consequence of Norway being the largest producing country, Norway is divided into four production regions; Region South, Region Mid, Region West and Region North. Each of the other farming countries are treated as one producing region (Marine Harvest, 2014b). Details of Marine Harvest's production network is displayed in Table 2.3. Notice how the seawater facilities outnumber the processing facilities, with an overall ratio of approximately 20:1.
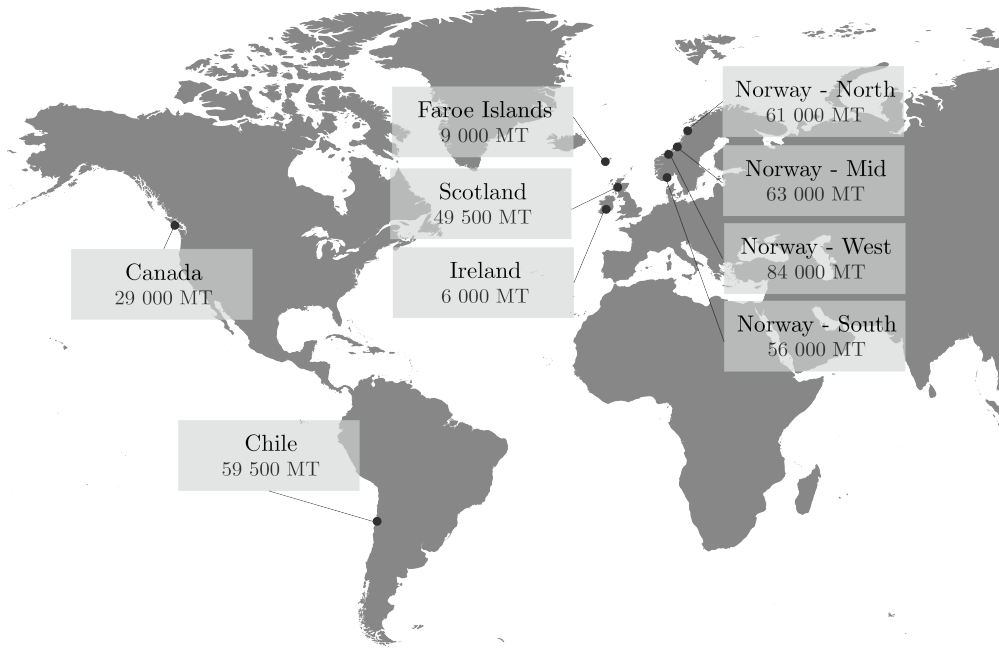
Figure 2.6: Map of Marine Harvest farming facilities with farmed volumes (Marine Harvest, 2014b).
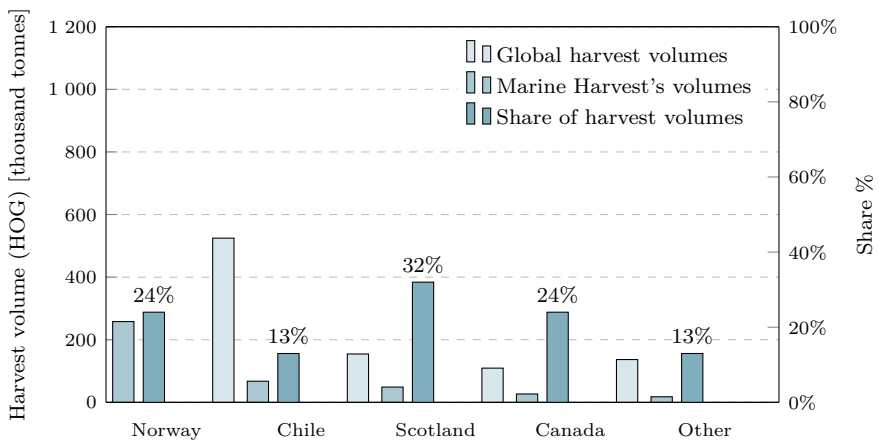
Figure 2.7: Marine Harvest's share of global harvest volumes in different producing countries (Marine Harvest, 2014a).

Marine Harvest production network in Norway		
Region	Sea water facilities	Processing facilities
Norway - South	27	1
Norway - West	29	1
Norway - Mid	35	1
Norway - North	34	1
Scotland	25	1
Ireland	10	1
Faroe Islands	4	0
Canada	37	2
Chile	53	4

Table 2.3: Marine Harvest's fish farming network (Marine Harvest, 2013; Norge, 2015).

Marine Harvest also operates their own fish feed production facility, located in Norway. Their feed production started in July 2014, and the company is planning to expand their fish feed production capacity to deliver up to 80 % of the feed required from their Norwegian farming operations in the near future (Marine Harvest, 2014b; Marine Harvest, 2014a).

The costs of producing fish is a frequently used benchmark in determining the efficiency of salmon farming. For Marine Harvest's different producing countries these costs are summarized in Table 2.4. The costs are given for one kilogram of HOG salmon delivered in box at the processing plant, and they do not include costs such as freight and marketing for sale. As the table illustrates, Norway is the country where feed has the highest relative costs, which may explain their investments in a feed producing facility. One would expect that the salaries in Norway would take a higher share, but these costs depends on the level of automation in the farming process, whereas Norway generally has a high degree of automation in their production (Marine Harvest, 2014b).

2.5.2   Secondary processing

Marine Harvest conduct secondary processing of seafood around the world. Their global network of VAP facilities is illustrated in Figure 2.8. Most of their plants are either defined as smokehouses, fresh pre-packed plants or frozen plants, although some facilities serve as hybrid plants (Marine Harvest, 2014b).

As value added products are generally associated with more stable consumer prices, Marine Harvest's VAP department is primarily used as a strategic tool to reduce

Production cost of HOG per country								
	Norway		Canada		Scotland		Chile	
	NOK/kg	*%*	*NOK/kg*	*%*	*NOK/kg*	*%*	*NOK/kg*	*%*
Feed	12.40	50.2 %	13.11	41.2 %	13.98	45.6 %	13.19	43.5 %
Primary	2.52	10.2 %	3.25	10.2 %	2.33	7.6 %	2.44	8.0 %
Smolt	2.31	9.4 %	3.02	9.5 %	2.89	9.4 %	3.09	10.2 %
Salary	1.51	6.1 %	3.02	9.5 %	1.68	5.5 %	1.07	3.5 %
Maintenance	0.82	3.3 %	1.33	4.2 %	0.75	2.4 %	1.31	4.3 %
Wellboat	1.02	4.1 %	1.16	3.6 %	2.05	6.7 %	1.72	5.7 %
Depreciation	0.77	3.1 %	1.22	3.8 %	1.03	3.3 %	0.89	2.9 %
Sales	0.56	2.3 %	0.00	0.0 %	0.00	0.0 %	0.00	0.0 %
Mortality	0.15	0.6 %	0.00	0.0 %	0.37	1.2 %	0.65	2.2 %
Other	2.64	10.7 %	5.74	18.0 %	5.59	18.2 %	5.94	19.6 %
Sum	**24.70**	100.0 %	**31.84**	100.0 %	**30.66**	100.0 %	**30.29**	100.0 %

Table 2.4: Cost of producing HOG salmon in different countries. Costs are given as NOK/kg and as a percentage share of total production costs (Marine Harvest, 2014b).
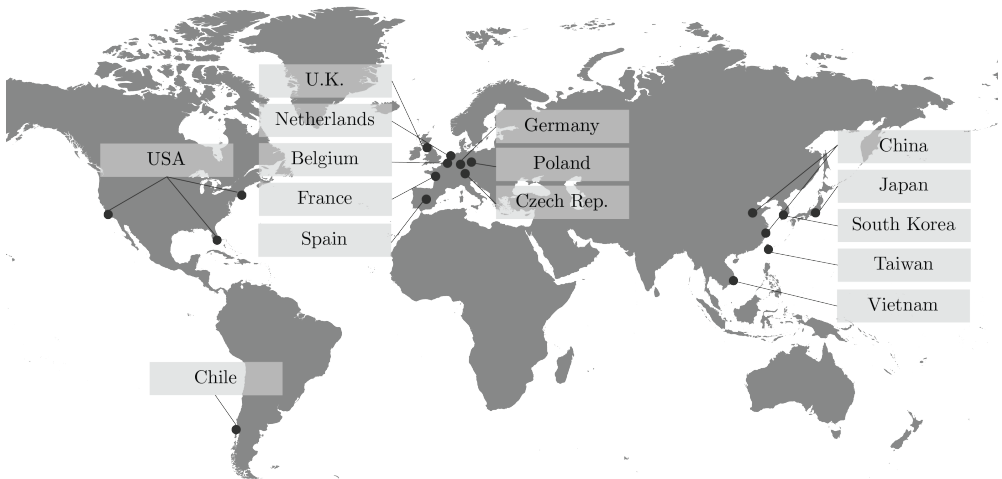
Figure 2.8: Map of Marine Harvest VAP facilities (Marine Harvest, 2014b).

exposure to spot price movements and net earning volatility (Marine Harvest, 2014a). In later years Marine Harvest have expressed a goal of moving in the direction of increased focus on value added products, and the company is working on increasing its capacity for producing more value added products (Marine Harvest, 2014a, p 38).

In terms of both volume and value, smoked salmon is the most important processed product supplied by Marine Harvest. Their main markets for smoked salmon are France, Italy, Germany and Belgium (Marine Harvest, 2014b). In accordance with the strategy of increased VAP focus, Marine Harvest acquired Morpol between 2012 and 2013. Morpol is a world leading secondary processor of salmon with processing facilities in Poland, United Kingdom and Vietnam. As a result of this acquisition Marine Harvest became the largest producer of smoked salmon worldwide (Marine Harvest, 2013). Marine Harvest are also in the midst of restructuring their existing VAP facilities. As a consequence of this strategy Marine Harvest reduced their European VAP entities from 13 to 8 during 2014 (Marine Harvest, 2014a).

2.5.3   Sales and distribution

Marine Harvest divide it's customers into retailers, industry and distributors and food service companies. Food service represent customers that prepare and serve the salmon, such as hotels, cafes and restaurants. The different producing regions tend to differ in their emphasis on the different costumer groups. Figure A.1 in Appendix A illustrate the relative share of income that these customer groups represent in each of their largest producing countries. An additional diagram is included for the VAP division as this division is treated separately within the organization. Due to the size of the figure, it is placed in the appendix.

The figure show large differences in customer segment focus. Norway and Scotland exhibit similar patterns, where the largest volumes are sold to the industry segment. Retail represent a modest share, and the food service share is almost neglectable.

Both Marine Harvest Canada and Marince Harvest Chile sell most of their production to other distributors, where Marine Harvest Chile has a production geared towards value added products. Products that go through secondary processing internally are mostly sold to retailers and the food service companies.

The sales policy of Marine Harvest is aimed at limiting the exposure to fluctuations in the salmon price by using contracts. Thus, 20-50 % of their harvest volumes are normally bound to contracts (Marine Harvest, 2013). The majority of these are long term

contracts, although futures contracts are used to a limited extent as a risk reduction tool. The contracts shares are generally higher for value added products. Additionally, the contract share also varies among the company's production countries, as evident from Figure 2.9. The variation in these numbers can be explained from the different customer segments which the production countries serve.
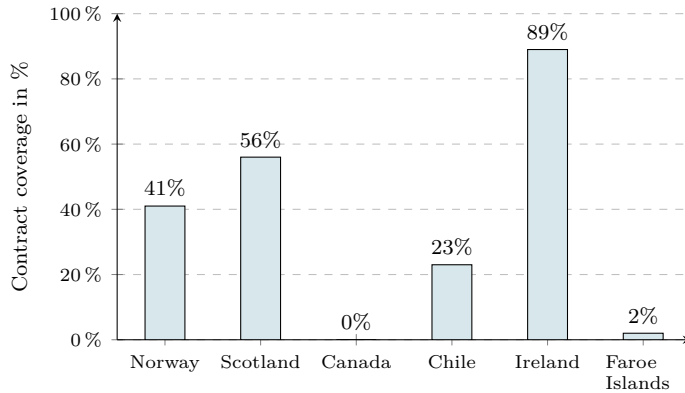
Figure 2.9: Marine Harvest's contract shares by production country (Marine Harvest, 2014a).

Figure 2.10 depict the geographic distribution of Marine Harvest's revenues. The European market clearly dominate, representing 70 % of total income. The second most important market to Marine Harvest in terms of revenue is the Americas, representing 16 % of total revenues, followed by Asia (9 %) and Russia (3 %). The latter decreased from 5 % in 2013 because of the imposed trade ban (Marine Harvest, 2014a).
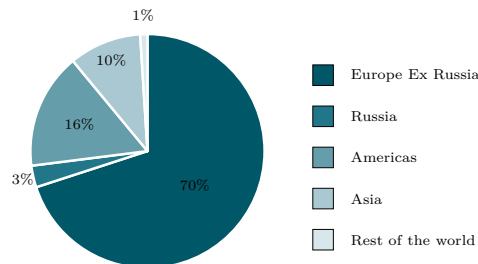
Figure 2.10: Marine Harvest's sales revenue distribution by geography 2014 (Marine Harvest, 2014a).

Marine Harvest's revenues by product mix are displayed in Figure 2.11. HOG salmon is clearly the most important product in terms of revenue representing 48 % of total income, while 22 % of total revenue is sold as fresh, elaborated products, which implies that fresh products represent 70 % of the total income.
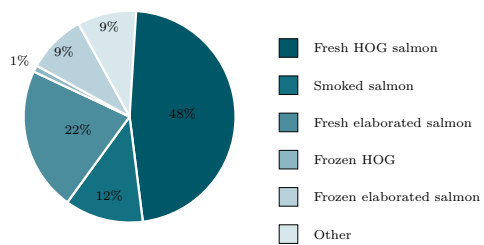
Figure 2.11: Marine Harvest's revenue distribution by product
group 2013 (Marine Harvest, 2013).

Chapter 3

The global salmon market

As Norway and Chile export salmon to almost 150 countries, salmon is considered global product (Asche and Bjørndal, 2011, p. 83). However, the supply and demand patterns varies substantially across different producing countries, both in terms of volume and product mix. As is common for perishable commodities, the salmon prices are volatile. Furthermore, the market experience inefficiencies because the industry is not yet considered mature.

In this chapter, both the supply and the demand side of the salmon market are explored in further detail. Section 3.1 gives a market overview, while Section 3.2 and Section 3.3 presents the supply and demand side of the market respectively. Next, the salmon price behavior is discussed in detail in Section 3.4, whereas relevant sales mechanisms are explained in Section 3.5. For a more thorough introduction to salmon price behaviour, the reader is referred to Øglend and Sikveland (2008) or Asche and Bjørndal (2011).

3.1   Overview of the global market for farmed salmon

Norway, Chile, UK and Canada dominate the supply of the industry, representing more than 90 % of the total global supply of farmed salmon. The main markets for farmed salmon have traditionally been the EU, the U.S. and Japan. The production volumes of each of these countries and their relative share of the world supply are given in Table 3.1. The table shows that Norway is the dominating actor with a supply share of approximately 56 %.

The size of domestic production and domestic consumption of a selected number of regions globally are illustrated in Figure 3.1, where the arrows indicate patterns that dominate the global trade of salmon. Note that the need for air freight to some markets give substantially different market prices (Marine Harvest, 2014b).

Country	Harvest quantity	Share of world supply
Norway	1 143 700 MT	56.06 %
Chile	468 200 MT	22.95 %
UK	157 800 MT	7.73 %
Canada	115 100 MT	5.64 %
Faeroe Islands	72 600 MT	3.56 %
USA	20 300 MT	0.99 %
Ireland	10 900 MT	0.53 %
Others	51 700 MT	2.53 %
Total	2 040 300 MT	100.00 %

Table 3.1: Harvest quantities of global farmed Atlantic Salmon 2013 (Kontali Analyse, 2014).
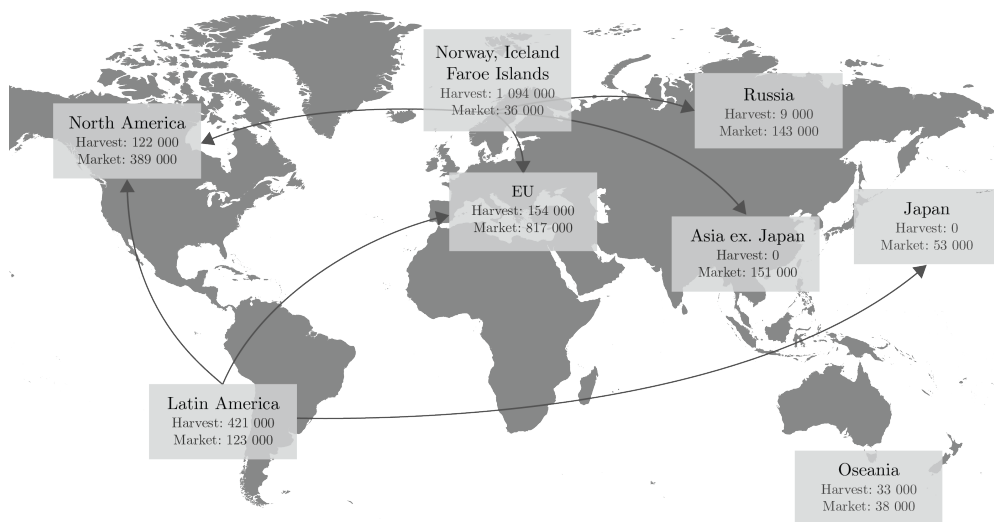
Figure 3.1: Global trade of farmed salmon (Marine Harvest, 2014b).

A complicating feature of the salmon market is that the salmon producers operate under different trade agreements. Canada and the USA belong to The North American Free Trade Agreement (NAFTA), Scotland and Ireland belong to the EU, while Norway has a trade agreement with the EU. This has led to numerous trade conflicts and periods where some producers have experienced regulated market access (Asche and Bjørndal, 2011, p136).

An example is the Norwegian production in 1991 where Norwegian salmon farmers were accused of dumping salmon prices in the US market while receiving government subsidies. This resulted in an average tariff of 26 % being levied on all Norwegian salmon entering the US market. Consequently, Norwegian producers were effectively barred from this market and their market share were partly overtaken by Chilean producers. It falls outside the scope of this thesis to go into depth in describing and analyzing trade conflicts and economical politics in the salmon farming industry. However, the reader should be aware that such trade conflicts and free trade agreements do exist and thereby affect world trade patterns.

3.2 Salmon supply

Each of the main producing countries have characteristics in terms of export volumes, major markets and product mix patterns.

3.2.1 Norwegian production

As described in Section 2.1, Norway is the world's largest salmon producer in terms of volume. Availability of suitable farming sites, relatively stable seawater temperatures, heavy government investments and good infrastructure are all factors that have contributed to Norway's leading position despite high operating costs.

The EU is by far the most important market for Norwegian salmon representing approximately 66 % of the total Norwegian exports in 2013 (Kontali Analyse, 2014). Within the EU, France is the main market for Norwegian salmon, representing about 16 % of the total Norwegian export value, followed by Poland and Denmark with about 10 % and 8 % respectively (Asche and Bjørndal, 2011). In France, the imported salmon is used both for domestic consumption and as input in secondary processing, whereas Poland and Denmark are markets with limited consumption, and the majority of the imported volume are re-exported to other EU countries (Asche and Bjørndal, 2011).

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Russia has over the past few years grown to become the second most important market for Norwegian salmon, receiving about 8 % of the exports in 2008 (Asche and Bjørndal, 2011). However, after the introduction of the import embargo in August 2014, direct Norwegian exports to Russia have depleted.
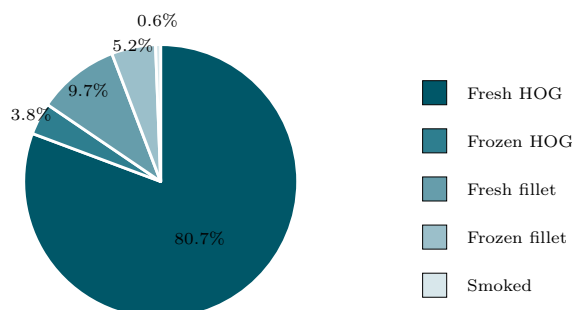
Figure 3.2: Product shares in Norwegian export in 2013 (Kontali Analyse, 2014).

Figure 3.2 presents the product mix of Norwegian exports. Fresh HOG is by far the most important salmon export article for Norway, representing about 81 % of the total export volume in 2013. As evident from the figure, secondary processed products represent only a small fraction of the total export value. Two of the main reasons for this product mix are considerably higher tariffs on processed products as compared to unprocessed products and high Norwegian labour costs (Asche and Bjørndal, 2011).

3.2.2 Chilean production

The Chilean salmon industry is concentrated around Puerto Montt and Chiloé Island, but some production also take place further south. Chile is the single large salmon producing country where salmon is not a native specie. Nevertheless, several biological and climatic factors contribute to making Chile well suited for salmon farming with stable sea water temperatures. The Chilean government has also actively promoted the development of salmon aquaculture, resulting in more liberal regulations than in other salmon producing countries (Asche and Bjørndal, 2011).

Chilean export volumes for 2013 are presented by product group in Figure 3.3. Compared to Norway, the share of value added products is substantially higher in Chile with about 66 % of total exports. This corresponds to the global trend, where value added products have become increasingly important during the last few years, especially in the US and the EU (Asche and Bjørndal, 2011).

Due to low domestic consumption approximately 90 % of the Chilean salmon produc-
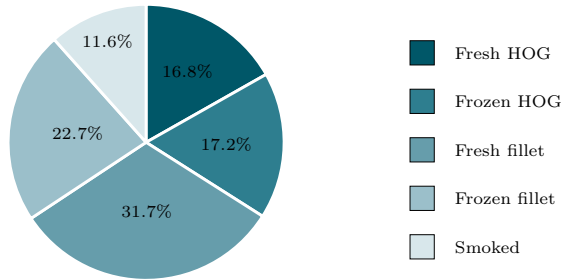
Figure 3.3: Product shares in Chilean export in 2013 (Kontali Analyse, 2014).

tion is exported to the US. Other markets such as South-America and Asia have in later years also emerged as significant (Bjørndal, 2002). As long distances require that fresh products are transported by air, the Chilean farming industry face challenges in terms of costs.

Furthermore, the Chilean salmon farming industry is still in the process of recovering from a serious outbreak of Infectious Salmon Anemia (ISA) in 2007. This disease outbreak resulted in a significant decline in the production of Atlantic salmon in 2009 and 2010 (Alvial et al., 2012).

3.2.3   Scottish production
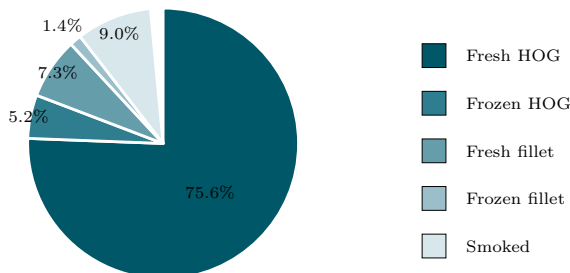
Figure 3.4: Product shares in Scottish export in 2013 (Kontali Analyse, 2014).

Scotland have in contrast to other producing countries a large domestic market resulting in exports less than 50 % of the total production. As evident in Figure 3.4 the export is dominated by fresh HOG salmon. The main export market for Scottish salmon producers is continental Europe, followed by the US. It has proved difficult for Scottish farmers to compete with Norwegian farmers on price. Consequently, the Scot-

tish farmers have focused on differentiation with production of higher quality salmon (Asche and Bjørndal, 2011).

3.2.4 Canadian production
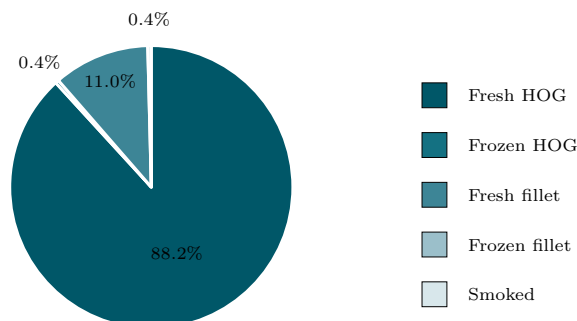
Figure 3.5: Product shares in Canadian export in 2013 (Kontali Analyse, 2014).

Salmon farming in Canada started in British Columbia and was later developed in eastern Canada. Today 65 % of Canadian production is located in British Columbia, while the remaining 35 % is produced in New Brunswick. The US is the main market for Canadian salmon, followed by the domestic market. Canadian exports per product group in 2014 is presented in Figure 3.5. As evident by the figure, fresh HOG salmon dominates the Canadian exports (Kontali Analyse, 2014).

3.3 Main markets for farmed salmon

An integrated market implies that changes in price in one market will affect the prices in other major markets. Price differences for markets in geographical proximity are corrected relatively quickly, whereas differences between continents takes longer time to disappear. In particular, the link between the North American and European markets is weak because of different main suppliers (Asche and Bjørndal, 2011).

The salmon markets differ by seasonal demand patterns due to different cultures, holiday traditions and culinary preferences, along with differences in price sensitivity and preferences regarding product mix. These relationships are important to understand, as they affect the global trade patterns.

Historically the EU, the US and Japan have been the main importers of salmon (Asche, 1997). In later years, Japanese imports have stagnated, whereas Russia have become a
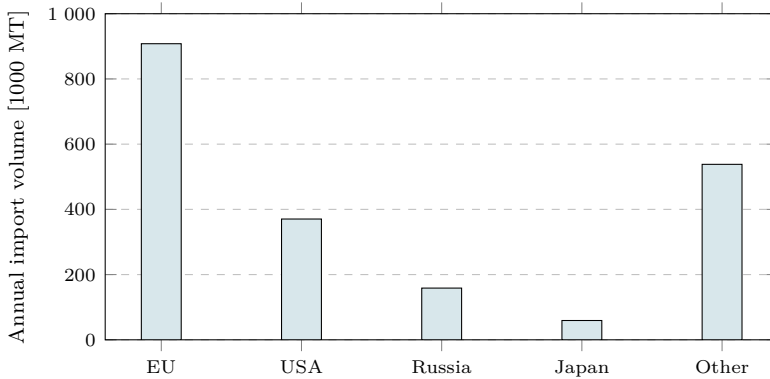
Figure 3.6: Main salmon importers by volume (Kontali Analyse, 2014).

new important market. This is illustrated in Figure 3.6 where yearly import volumes in 2014 are depicted. There is also significant development in several emerging markets such as Brazil, Russia, eastern Europe and Southeast Asia where salmon consumption has increased significantly since 2000 (Asche and Bjørndal, 2011).

The EU

The EU is the largest market for farmed salmon, where the largest importers within the EU are France, Germany and the UK. Other significant markets are Spain, Italy, Denmark and Poland (Asche and Bjørndal, 2011). The EU is not a homogeneous market as the individual countries differ in consumption patterns and preferences. Most of the volumes imported to the EU originate from Norway. However, a substantial share of the salmon originating in Norway is shipped via a third country, such as Denmark or Poland due to processing.

The processing industry is large within the EU. Consequently, several countries are both large importers and exporters with a limited domestic demand. Nine out of the ten main importers of smoked salmon are European countries, with the exception being USA (Tradedata.net, 2013). Poland is the world's leading exporter of smoked salmon, and exported close to 40 000 MT in 2013 (Tradedata.net, 2013).

The United States

The US is the third largest salmon market globally, and their consumption is increasing. Although the US is the world's largest producer of wild salmon, the domestic market is increasingly dominated by imported farmed salmon, with Chile and Canada as the main suppliers (Asche and Bjørndal, 2011). The import of fillets compared to

other products has increased rapidly in later years, reaching 57 % of the total imported volume in 2008 (Asche and Bjørndal, 2011). Interestingly, there is a distinct pattern that most of the imported fish originate from Canada, whereas the value added products mostly originate from Chile.

Russia

Russia have emerged as an important market for farmed salmon during recent years, experiencing major growth in the years after 2000. Based on numbers from 2008, 72 % of imports are fresh salmon, whereas 28 % are frozen products (Asche and Bjørndal, 2011). The share of fresh products have historically been even higher, but for the last few years, imports of frozen fillets have started to reach significant volumes.

Japan

Japan has historically been an important market for salmon. The country has the worlds largest per capita salmon consumption, and as domestic production is low, Japan has become the single largest fish importer in the world (Asche and Bjørndal, 2011). However, both imports and consumption have been stagnant in recent years.

The Japanese market is the most diversified salmon market in the world, with wild and farmed species competing in the same market. Findings in Asche, Guttormsen, Sebulonsen, et al. (2005) suggest that the different salmon species are close substitutes in the market. Consequently, the demand for Atlantic salmon can be affected by changes in supply of those substitutes. Historically, Japanese salmon consumption has been subject to strong seasonal patterns, partly because salmon is consumed in the celebration of different holidays and major social events.

In contrast to the EU and US markets, Japan has neither trade conflicts nor trade restrictions (Asche, Guttormsen, Sebulonsen, et al., 2005). Moreover, Japan is the only one of the major markets market in which fish from all the producing regions are present. In contrast to the US and most countries in Europe, Japan may be considered a mature market for salmon (Asche and Bjørndal, 2011).

An interesting feature of the Japanese market is that customers here are often willing to pay an extra large premium for larger fish, as a significant part of imported salmon is consumed as sashimi, and thus the structure and looks of the flesh is very important (Thorbeck, S., personal communication, 2015).

3.4 Salmon Prices

Through the 1980s and 1990s, the salmon prices experienced an steady decrease, before the salmon prices seem to have leveled out for the last fifteen years. This suggest that demand and production have reached equilibrium (Øglend, 2013).

Today's salmon farming industry is characterized by high price volatility (Øglend and Sikveland, 2008). This poses an important challenge for all actors involved in salmon production as it is impossible to predict future price development perfectly. However, having a thorough understanding of the underlying forces for the salmon price is vital for calculating good price forecasts.

3.4.1 Reference prices and price indexes

Reported HOG prices are normally reported in terms of specific reference prices. The three reference prices most widely used worldwide are: (1) the NOS/FHL FCA Oslo price, (2) the Urner Barry FOB Seattle price, (3) and the Urner Barry FCA Miami price. These prices are based on prices for superior quality fish, and are not reflecting freight. Furthermore, each reference price refers to one specific product. The FCA Oslo price refers to the price per kilogram HOG salmon between 4-5 kg packed fresh in a standard box and delivered in Oslo. The FCA Oslo price represents about two thirds of the global quantities for Atlantic salmon (Marine Harvest, 2014b; Urner Barry, 2012), and is thus the single most important reference price used.

Furthermore, NASDAQ (2015a) have their own reference price published for weekly spot prices for different sized HOG salmon. This index is calculated based on reports of achieved prices and volumes made by a representative panel of Norwegian salmon exporters and producers (NASDAQ, 2015a).

Fish Pool ASA (2015) also have a price index calculated on a monthly basis. This price is merely mentioned because the synthetic Fish Pool Index (FPI) is often used in financial settlement of all financial forward Fish Pool contracts.

As the different reference prices are calculated in different ways, they must be used with care. E.g. if the NASDAQ price should be comparable to an achieved selling price FOB from Oslo it must be adjusted by subtracting general sales and admission expenses and freight to Oslo and terminal cost (Marine Harvest, 2014b).

3.4.2 Salmon price volatility

As discussed later in Section 4.3, salmon prices are one of the most important sources of risk faced by a salmon producer, as salmon prices exhibit large week-to-week fluctuations. As a result, the industry has historically experienced substantial variations in margins, with periods of low salmon prices leading to frequent bankruptcies (Bergfjord, 2009). This is evident when examining historical data on yearly average operating margins in Norway, as depicted in Figure 3.7. The cycles in profitability also create trade tensions, as most of the salmon is produced in a different country from where it is consumed (Asche and Bjørndal, 2011).
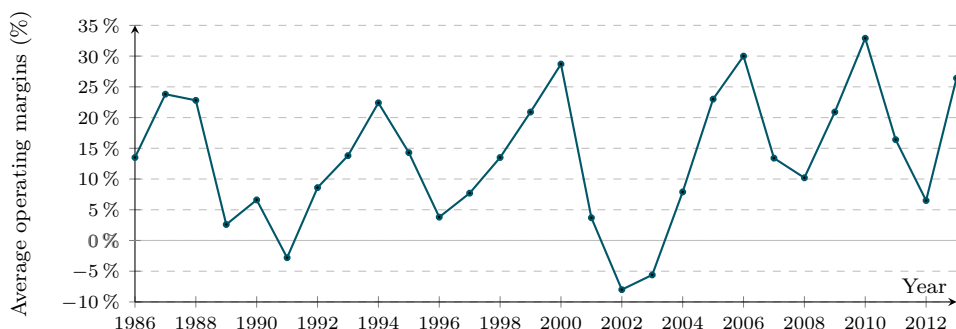
Figure 3.7: Average Norwegian operating margins 1986-2013 (Norwegian Directorate of Fisheries, 2014b).

There are several drivers of price volatility in the salmon farming industry, the most important of which being the short term in-elasticity of supply, market imperfections and the availability of substitutes.

Short term supply in-elasticity

Supply is governed by biomass development which varies with several uncertain factors, including sea water temperature and disease, as discussed in Section 2.2. As salmon reach sexual maturity around the same time they reach harvest-ready sizes, the producer has limited flexibility in the timing of the harvest. As such, short term supply of fresh salmon is both inelastic and uncertain, i.e. the salmon quantity supplied is to a very small degree affected by a change in the price of salmon (Andersen et al., 2008). This lack of supply price elasticity explains the periods consisting of both observed over and under-supply.

Market imperfections

Because of immature market places for salmon products as well as potentially high transportation cost between different markets for fresh products, the market prices are not perfectly correlated. As salmon is largely sold on a bilateral basis either through long term contracts or spot transactions, the price for each trade is not based on an index, but agreed upon through negotiations (Thorbeck, S., Helgesen, M., personal communication, 2015). The transaction costs from trading in one market and selling it in another may also contribute to imperfect market prices, as the difference in achieved prices in different markets are observed to be substantial when analyzing historic market prices (Norwegian Seafood Council, 2015). Fluctuations in exchange rates and regional differences in taxes, tariffs and transportation costs further complicate the matter.

Effect of substitute availability

Market integration studies indicate that farmed salmon competes closely with wild salmon (Asche and Bjørndal, 2011). Although many customers prefer farmed Atlantic salmon because of quality considerations and the stable supply, other customers such as the European low price retail chains Lidl and Aldo are somewhat flexible in terms of salmon species (Thorbeck, S., personal communication, 2015). Consequently, the prices for farmed Atlantic salmon is also affected by the supply of other salmonoids.

Other drivers of price volatility

For seasonally produced commodities, there is normally a greater price volatility in the period prior to the production or harvest period and in periods of high prices (Peterson and Tomek, 2005; Øglend, 2013). This assumption is also valid for salmon prices. In periods where demand exceeds supply, prices are allowed to persist above the long term price equilibrium. Emptying of inventories may also lead to producers being unable to fulfill the excess demand, causing price spikes and thus larger than average volatility (Øglend and Sikveland, 2008). Additionally, since salmon growth is highest in the summer and early autumn, the price volatility is also highest in this part of the year (Øglend, 2013).

3.4.3 Prices in different markets

There are shown to exist systematic differences in price levels between countries (Larsen and Asche, 2011). These differences can be caused by currency fluctuations

and differences in transportation costs and taxes from the major supplier regions to the different markets.

As an example, there are generally higher prices in Japan than in the EU due to higher transportation costs, as illustrated by Figure 3.8. The high price correlations can be explained by the fact that efficient currency markets cause producers to be indifferent as to where they sell their products as long as the prices are equal after a conversion to a common currency (Asche and Bjørndal, 2011).
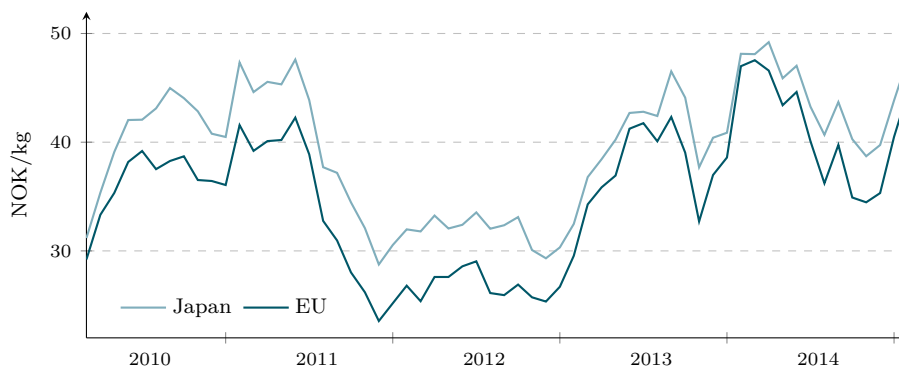
Figure 3.8: Monthly Norwegian export prices of HOG salmon to the EU and Japan (Norwegian Seafood council, 2014).

The small market differences observed in Figure 3.8, is caused by the producers' inability to react instantly to changes, and by variations in the quality compositions in the two markets (Asche and Bjørndal, 2011).

3.4.4 Price of different products

According to Asche and Bjørndal (2011), salmon is not a homogeneous product. Consumers are willing to pay a premium for quality attributes such as geographic origin, colour and size, where the most important factor is the latter.

The price of fresh HOG salmon

As discussed in Section 2.2, fish of different sizes are regarded as different products by the customers and thereby obtain different prices. The prices of fish in different weight classes are in general highly correlated, although Asche, Guttormsen, and Tveterås (2001) and Øglend (2010) conclude that the correlation decrease with increasing weight difference. This relationship is illustrated by Figure 3.9. As the figure indicates, there exist a premium on large fish relative to the price of smaller fish. However, seasonal

shortage in supply of small fish compared to large fish may give an opposite effect for short time periods. An interesting feature is that there seems to be a relatively stable pattern in the relative price development of different sized fish.
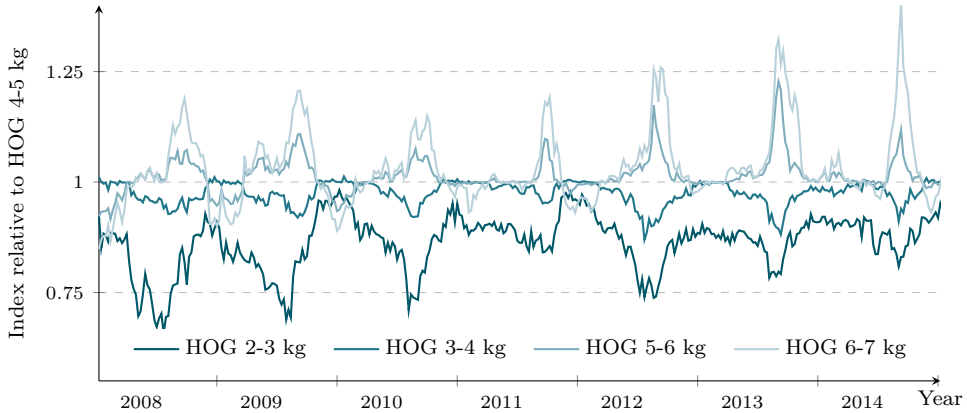
Figure 3.9: Prices for different weight classes relative to HOG salmon 4-5 kg, 2008-2014 (NASDAQ, 2014).

Asche and Guttormsen (2001) conclude that the main reason for these patterns is the biology of the salmon. As growth is uneven over the year and most salmon are deployed in salt water in spring or autumn, salmon will tend to grow in cohorts. This relationship causes the price of small fish to be relatively higher in the winter than in the summer months. Furthermore, to avoid large salmon becoming sexually mature during autumn, most of the larger fish are sold during summer. This behaviour result in a limited supply of large fish in autumn, driving the relative price of large fish up (Asche and Guttormsen, 2001).

The price of frozen HOG salmon

As evident from Figure 3.10, the fresh and frozen HOG salmon has a similar historic price development, but the price of frozen HOG salmon is less volatile. Furthermore, as frozen product sales are normally bound by long term contracts, it takes time to adjust for market volatility in price. This in turn gives a price lag compared to fluctuations in the fresh HOG salmon price (Marine Harvest, 2011b). According to Øglend (2010), freezing the fish will in general decrease its market value, as a premium exists on the willingness to pay for fresh fish, whereas the demand elasticity is higher for fresh products than for other product groups.

One main advantage of freezing the fish is the longer durability and the flexibility this may give the salmon producer. Øglend (2010) found that where price shocks for frozen
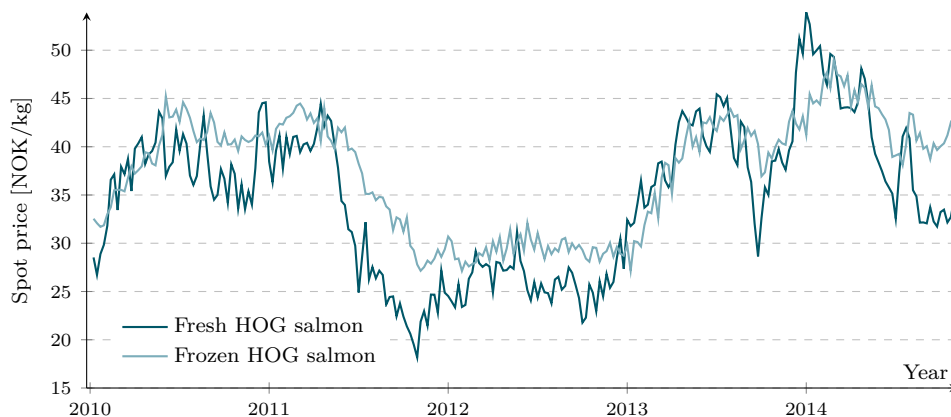
Figure 3.10: Weekly average Norwegian export prices for fresh and frozen salmon (Statistisk Sentralbyrå (SSB), 2015a).

products severely affected the price volatility of fresh product, the opposite had no significant effects on the frozen product price volatility. This supports the hypothesis that storage can be a buffer to price shocks.

The price of value added products

By producing value added products, the relative costs for the raw material decrease compared to total production costs. The relation between HOG salmon and the value added product thereby become weaker (Asche and Bjørndal, 2011). Still, the prices of fresh HOG salmon and value added products are linked. As most of the VAP types are more durable than fresh HOGs salmon, processing can be seen as a way of storing it. This offers flexibility for the producer and the opportunity to control market supply more independently of the salmon growth cycle. Stocks of the different product groups are also highly correlated, meaning that arbitrage opportunities potentially exist across different products for the producers (Øglend, 2010).

Even though value added products are sold at a premium to cover processing costs, the operating margins are often lower than on fresh fish. According to Marine Harvest (2013), the prices on value added products tends to be less volatile. This holds for most value added products except for smoked salmon, see Figure 3.11. As such, the production of valued added products may be attractive for risk averse salmon producers.
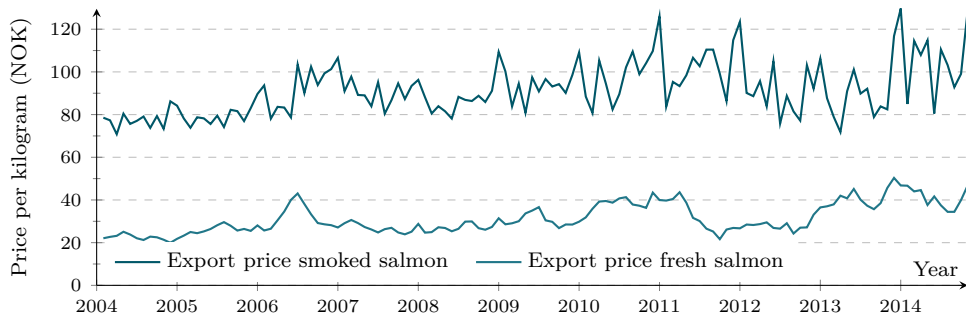
Figure 3.11: Monthly average Norwegian export price smoked salmon (Statistisk Sentral-byrå (SSB), 2014).

3.5 Sales mechanisms

Salmon products are sold both through the spot price contracts, futures contracts and long term contract. NASDAQ (2014) define spot contracts as contracts where prices and volumes are agreed between buyer and seller either in the week prior to invoicing or in the week of invoicing. In a futures contract, price is specified in advance, whereas in long term contracts, different pricing regimes exist. Note that as futures is a purely financial instrument primarily used for industry actors as an aid to reduce risk (Fish Pool ASA, 2015), this subject fall outside the scope of this thesis and are thus omitted from further discussions.

3.5.1 Spot price contracts

Spot transactions are conducted through negotiations or auctions (Aandahl, P.T., personal communication, 2015). Either the seller or the buyer contact a potential business partner to initiate a deal. As such, the spot market for salmon consists of a set of bilateral contracts with one delivery, and differs from other commodity markets, such as the markets for oil, electricity and steel. According to (Helgesen, M., personal communication, 2015), the spot price is only vaguely used in negotiations because of the structure of the market place and the low price resolution (Helgesen, M., personal communication, 2015).

Spot contracts are normally used for fresh HOG salmon and to some extent for other fresh products, such as fresh fillets. According to Helgesen (personal communication, 2015), spot contracts on fresh value added products often have a duration longer than the ones used for fresh HOG salmon to allow time for processing.

Salmon producers such as Marine Harvest also purchase HOG salmon in the spot

market to fulfill contract obligations or to be purchased as input to the production of value added products (Marine Harvest Canada, 2008)(Johannesen, T., personal communication, 2015). According to Johannesen (personal communication, 2015), only fresh HOG salmon is purchased by Marine Harvest in the spot market.

3.5.2 Long-term contracts

The time horizon of the long-term contracts used in the salmon farming industry is typically 3-12 months (Marine Harvest, 2014b; Guillotreau and Jiménez-Toribio, 2011). According to Larsen and Asche (2011), the expected revenues from using fixed price contracts are neither higher nor lower than the revenues generated through spot sales. Larsen and Asche (2011) argue that the use of fixed price contracts primarily change the profile of the revenue flows, and not the long term total revenues.

Contracts are mainly used to reduce risk and transaction costs (Brækkan, 2014). Using contracts, the producer is guaranteed a minimum order, and the buyer is guaranteed a minimum supply. Moreover, the use of contracts may reduce price volatility. The reduced uncertainty in demand and thereby quantity flow for the producer allows for better production planning and capacity utilization. Furthermore, the costs related to negotiation and administration, as well as costs of following up the customer ahead of, during and after delivery, can be reduced when using contracts.

The industry operates with a large number of different contract designs, with variations in both price and contracted volume. However, little is known about the use of contracts and the detailed contract designs as this it is considered to be sensitive information (Larsen and Asche, 2011). As such, empirical knowledge about the contract schemes used by the industry is limited.

Each contract can contain one or multiple deliveries, and the volume to be delivered each time can be fixed or flexible. Price can be incorporated through a number of different arrangement, where the price schemes included in this thesis are fixed price contracts, adjustable price contracts and partially adjustable price contracts.

One way to determine price in a contract is simply to set a price that will remain fixed throughout the contract period. Results based on a study conducted in 2006 indicate that about 25 % of the contracts in use where such fixed price contracts (Larsen and Asche, 2011).

Another commonly used practice is to specify a contract price which is later adjusted

according to a predefined agreement. This specific contract states an interval within the contract price is fixed, with the possibility for adjustment if the spot price is above or below the predefined interval. One can either agree on a full refund of the difference outside of this interval, or a relative share of the difference paid to the losing party. Such contracts are in this thesis referred to as partially adjustable contracts.

Of the different contract designs, fixed price contracts give more control over future revenue, thereby giving the producer better income predictability. Adjustable price contracts on the other hand, are more exposed against the spot market, but assures volume predictability. The last contract type, the partially adjustable contracts will give a trade off between the income and volume predictability.

Chapter 4

Problem description

The problem studied in this thesis is a portfolio optimization problem within the salmon industry. It considers the value chain for a global salmon producer with a high degree of vertical integration selling it's products all across the world.

Operations research papers have previously been written on behalf of the salmon farming value chain. However, to the authors' knowledge no work have yet been published on global portfolio optimization within the salmon industry while considering the complete value chain downstream of salt water growth. As the industry is still considered immature, there is most likely a significant potential in more optimal planning. Consequently, a need for solving the portfolio optimization problem in the salmon farming industry has been identified.

In this chapter, the problem scope is defined in Section 4.1. Thereafter, main decisions included in the problem are discussed in Section 4.2. Finally, Section 4.3 present the main sources of uncertainty and risk faced by a salmon producer.

4.1 Problem scope

The objective of this thesis is to develop a decision support tool in form of an optimization model to be used in management of the salmon farming value chain as an integrated unit. The optimal decisions must honor contract obligations and respect government regulations, where the goal is to maximise the profits throughout the planning horizon.

Coordination of the supply chain is essential because demands in different regions are not perfectly correlated. Safety inventories, buffer resources and raw material can be utilized from a global perspective rather than with focus on local capacities and local demand (Tomasgard and Høeg, 2005), resulting in improved overall performance.

To help manage the problem, a decision support tool is developed. The output of this tool is intended for management staff that are in charge of company-wide plans developed to streamline value chain operations, and should be used to develop plans and guidelines centrally. These plans should then be distributed downwards in the company hierarchy, where they will act as a framework for the more detailed operational plans developed by the local managers. The goal is to make robust, flexible decisions limiting the effects of any bottlenecks, while at the same time exploiting market opportunities.

The problem scope is to calculate the optimal harvest, production and inventory levels. Furthermore, decisions regarding material-flow between the different stages of the value chain, and sales allocation strategies, i.e. decisions regarding the spot market exposure and which contracts to enter, are included. According to Schmidt and Wilhelm (2000), the tactical decisions in a multi-national logistics network include production levels at all plants, assembly policy, inventory levels and dealing with material-flow between all stages in the value chain, from suppliers to customers. Given this definition, the problem studied in this thesis can thus be defined as a tactical problem.
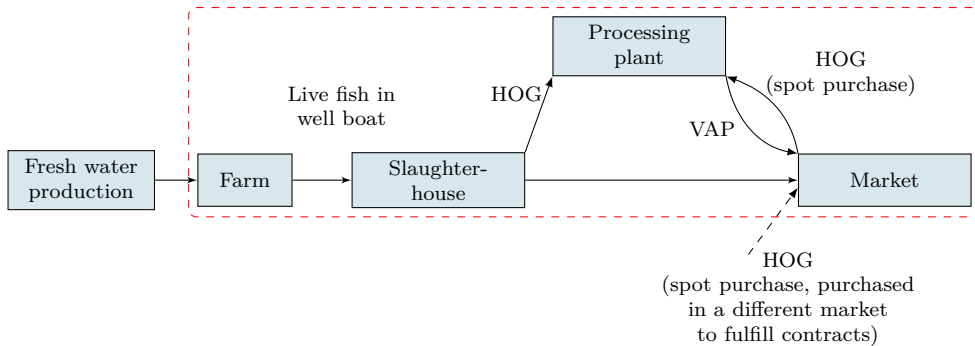
Figure 4.1: Illustration of the problem scope.

The focus of this thesis is the supply chain activities downstream of the salt water growth phase, i.e. harvesting, slaughtering, processing, sales and distribution. In Figure 4.1, the parts of the value chain included in this problem are highlighted by a red rectangle. As such, the freshwater production phase where salmon eggs are developed into smolt will not be addressed. Furthermore, decisions regarding smolt production and deployment fall outside the scope of this thesis. As discussed in Section 2.2, the production cycle of smolt has a lead time of approximately a year (Marine Harvest, 2014b). This implies that including decisions regarding smolt ordering aspires for a planning horizon longer than what is typically used in tactical problems. Moreover, smolt release is already described in existing literature, see Rynning-Tønnesen and

Øveraas (2012) and Frøystein and Kure (2013). To the authors' knowledge, these models can be used independently to create input to the decision support model developed with little loss in value.

Because the salmon farmer operate under uncertain conditions, it is important that both this uncertainty and the dynamics of the decision processes are captured in the planning process. As new information is revealed regarding uncertain parameters, the decision maker will adjust his decisions to adapt to the changing environment. To facilitate the development of robust, flexible solutions and thereby reducing the risk faced by the salmon producer this thesis takes a stochastic approach to uncertainty. Although many aspects in the value chain are stochastic in nature, this thesis only considers uncertainty in price and biomass development as depicted in Figure 4.2.
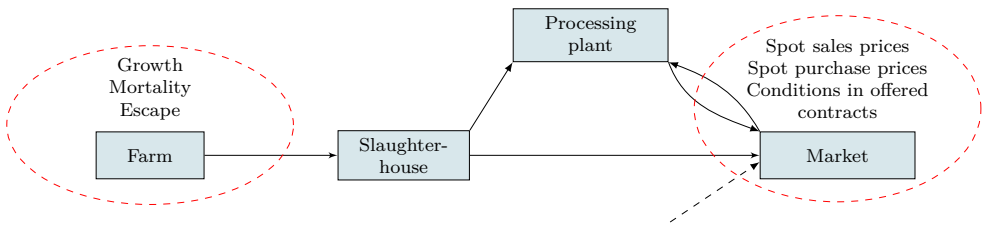
Figure 4.2: Illustration of the main sources of uncertainty in the value chain.

4.2 Decisions

In this thesis, decisions are categorized in two ways. Harvesting, production and transportation decisions, as well as how much to sell and buy in the spot market, are performed with a relatively high frequency and are therefore referred to as operational decisions. Decisions regarding which long term contracts to enter are performed less frequent. Contract decisions are therefore referred to as tactical decisions.

4.2.1 Harvesting and slaughtering

Understanding the drivers of biomass development can potentially have large implications on profitability. Having more biomass than foreseen due to higher than expected growth might necessitate unplanned harvest to satisfy maxium allowable biomass (MAB) restrictions, or to avoid fish reaching sexual maturity. Such unplanned harvest might result in a need to hire extra well boat capacity and make staff work overtime, which can prove to be costly. Additionally, unplanned harvest increase exposure to price risk, as the salmon must be marketed within a limited time window.

Moreover, higher growth than expected might lead to salmon reaching delivery weight sooner than planned. This can necessitate contract re-negotiations or the need to purchase products on the spot market in order to fulfill contracts obligations. Both outcomes can be costly as re-negotiation might result in a discount, and that buying products for fulfilling contract obligations may increase purchase costs.

Having a lower biomass than expected, either due to slower growth, escapes or disease, might have a similar outcome in terms of contract re-negotiations and the need to purchase products in the spot market. In addition, low biomass levels can lead to uneven production levels in slaughterhouses and processing facilities. Consequently, good biomass forecasting procedures are key to maintaining high profitability.

The harvest decision include both what number of fish to harvest from each fish farm, and what size distribution these fish should have. When making these decisions, both forecasted future biomass, wellboat capacities and slaughtering capacities must be taken into consideration.

As the economic optimal harvest weight is in the range of 4-5 kg (Marine Harvest, 2014b), most fish should be harvested in this weight interval. Yet, there are reasons for deviating from this target. Number of fish and the size harvested might be affected by events outside of the salmon producer's control, such as poor growth due to unfavorable weather conditions or diseases, premature sexual maturation, or hold ups in the value chain. Regulations must be honored, which can cause slaughter prior to reaching target weight to not surpass the MAB level and maximum allowed lice levels. Furthermore, there are incentives to have a steady slaughtering of fish in order to keep operational costs down by fully utilize the available capacity (Johannesen, P.T., personal communication, 2015), or to spread risk (Asche and Bjørndal, 2011). Additionally, there are large seasonal variations in demand, meaning that fish outside target weight might be harvested and slaughtered in order to fulfill peak demand.

As the price of salmon is highly volatile, these decisions are consequently made under a high degree of uncertainty. Although the time window for when a fish should be harvested is short, there is some flexibility in timing the harvest. In the vent of low prices, the producer might hold the salmon a short while and wait for price to change or sell it immediately (Øglend and Sikveland, 2008), giving the harvest decision a speculative dimension. This follows by continuation of cultivation being one of the only possibilities for storing goods with limited durability, such as fresh salmon (Øglend and Sikveland, 2008). However, delaying harvest comes at a price. The producer have to pay to keep the fish in the pen, a cost consisting both of extra feed expenses, and

the alternative cost of having the fish fill up the pen instead of deploying new smolt.

4.2.2 Production and inventory planning

Once harvested and slaughtered, the salmon producer must decide how to utilize the slaughtered fish. Fish could either be sold as fresh or frozen HOG salmon, or be processed further into value added products (VAP). As much of the processing is performed at separate facilities, the volume sent to processing and the volume sent directly to the market must be decided upon, meaning that the production decision and the logistical decisions are intertwined.

As discussed, there are limited options in the timing of harvest. When prices are low, another alternative is then to harvest and slaughter the fish, and then freezing it in hopes of better prices in the future. As the stock of frozen fish is less dependent on immediate effects of changes in the live stock, there is improved storage flexibility in keeping stocks of frozen fish. As such, frozen fish can provide a buffer for stochastic movements in the live stock. (Øglend, 2010, p 85)

Although the margins for VAP are generally lower than for fresh HOG salmon (Marine Harvest, 2014b), the prices for VAP are also generally somewhat less volatile, as discussed in Section 3.4. Combined with the lower degradation rate of such products, this makes production of VAP interesting, as it offers the producer more flexibility in terms of when to sell. After processing, the processed products can either be sent directly to customer, or be stored in inventory for a certain amount of time related to the product's durability. If prices are expected to rise in the future, it may be beneficial to postpone the sale as long as the expected price increase covers the inventory costs. Conversely, when the prices are high, the salmon producer may find it beneficial to empty or significantly reduce its inventories.

Øglend and Sikveland (2008) argue that the availability of inventories helps smooth prices, and that the utilization of inventories today comes at a trade-off of lower inventories tomorrow such that the option of smoothing prices in the future has decreased. As such, the product-mix can be used as a means to make the company less influenced by the spot price volatility. This flexibility may give more robust production plans, which are especially interesting for risk averse producers, as they tend to favor more stable profits over higher expected profits. Another argument for producing to stock is to do so in anticipation of the predictable seasonal patterns of demand discussed in Section 3.4. As a side note, it is worth mentioning that severe quality downgrades may

increase the allocation to processed products (Aandahl, P.T., personal communication, 2015).

Note that production decisions are restricted by transportation capacities and by the capacities at processing plants, as well as the inventory capacities.

4.2.3 Sales allocation

For a salmon producer there are several different ways to interact with customers in the global market. In the problem studied in this thesis, the transactions modelled are categorized as either spot transactions or contract transactions.

Spot transactions

As defined in Section 3.5, spot transactions are short term contracts agreed upon either the week prior to invoicing or in the week of invoicing. Although salmon producers can be considered price-takers in the short run, the salmon producer must decide what volumes to sell when. As prices differ in different market, another important aspect of the decision is in which markets to sell the products. Due to the imperfections in the market price correlations, efficient market allocation can potentially have an positive impact on the company's revenue, even though these decisions must be made under uncertainty. The ability to react fast to price changes in a market could give the salmon producer a significant advantage. Note that the market allocation is not a purely financial decision, but is restricted by transportation capacities.

Furthermore, the salmon producer must decide how much fresh HOG salmon to purchase in the spot market at any given time. Often, purchases are used as a mean to fulfill contract obligations when the internal supply is insufficient to cover the contracted volume. Spot purchases also occur in cases where the current spot price makes it more economical for the producer to purchase HOG salmon rather than using internal supply as raw material in the secondary processing. This may either be due to considerations regarding return on feed or to avoid high transportation costs from internal production sites to the processing facilities.

Long term contract transactions

Entering contracts is used mainly as a means to mitigate the risks associated with price, volume or both, as well as to achieve predictability in future sales (Larsen and

Asche, 2011). The predictability allows for better production planning and capacity utilization. Conversely, the downside of signing contracts is that because most contracts specifies a volume, the producer might have problems in keeping his contract obligations in case of poor growth. Furthermore, if large shares of the value chain capacity is tied up in contracts, this leaves less flexibility when problems or profitable opportunities arise or disruptions in the value chain occur. Situations where the producer finds it challenging to deliver can lead to contract re-negotiations, often resulting in less favorable pricing agreements. In using contracts where the price is fixed, there is also a risk of selling at a price lower than the spot price, thus missing out on extra revenue.

Which contracts to enter is arguably one of the most important decisions facing a salmon producer. The decision is two-faceted:

1. The producer must decide what proportion of total production should be tied up in contracts.
2. The producer must decide what types of contracts to aim for in negotiations.

As discussed in Section 3.5, the industry operates with several different types of contracts, differing both in how price and volume are established, the frequency of the deliveries and the time horizon of the agreement. Additionally, the producer should consider the network aspect of the contracts, e.g. how many customers he wants to engage contracts with, meaning if he wants to opt for fewer high-volume contracts, many low-volume contracts or something in-between.

The problem studied in this thesis includes three different contract types. They are all based on fixed volume agreements, but differ in how the price is established, and thereby in how they affect risk profile and predictability. Fixed price contracts give full predictability in terms of income, but you run the risk of missing out on income if spot price rises during the contract period. Adjustable price contracts expose the firm to the spot market volatility, thus giving less predictability. In using such contracts you will not be forced to sell at a price significantly lower than the spot price, but conversely the opportunity to sell at a price significantly higher than the spot price is lost as well. Partially adjustable price contracts, which is the last contract type included, give some predictability and lower price risk than the adjustable price contracts.

4.2.4 Logistics

The salmon producer has to continuously make a large number of decisions regarding logistics. All of the decisions previously discussed set the frame for, and are at the same time restricted by the logistics decisions. For example, harvest and slaughter decisions decide what volume of fish must be transported from a farm to a nearby slaughtering facility with the use of a wellboat. Furthermore, spot sales and contract volumes dictate which volume of a specified product that must arrive in each market at any given time.

The producer normally hire transport capacity from an external logistics provider. It is common for the provider to specify a minimum price the producer must pay for a specified time period regardless of how much of their services are used. Thus, to minimize costs the producer should seek to always use at least the amount of capacity corresponding to this minimum price.

The agreement could also specify an upper limit to the capacity that can be provided. If this capacity is surpassed, the producer would have to turn to the market and negotiate an agreement with an additional logistics provider. Due to increased transaction costs and a high probability of having to pay a premium price when conducting last minute negotiations, the producer have to weigh the benefit of the additional capacity against the added cost.

Deciding upon which transportation mode to use depends upon several factors:

1. The chosen transportation mode must be feasible across the distance. For example, ship freight is not possible between two inland locations.
2. The chosen transportation mode(s) must bring the product to the market in time before it is degraded. Thus, in most cases the number of eligible transportation modes is limited due to long transportation time.
3. Cost considerations. The cheapest choices are generally ship freight or transportation by trucks, whereas air freight is significantly more costly.

Consequently, there will be a trade-off between transportation costs and how fast the product must be delivered, which depends on how fast the product deteriorates. Therefore, air freight might be necessary to get the product to it's destination fast enough, even though it is the more expensive option.

4.3 Main sources of problem related risk

The main types of risk for the salmon producer are presented in Table 4.1, where *production risk* and *price risk* are the two types of risk a salmon producers primarily face (Øglend and Sikveland, 2008). In general, *production risk* influence the amount produced with a given input factor combination, and *price risk* influence the revenue one will obtain from the quantity produced (Just and Pope, 1978). Additionally, the salmon producer also face *institutional risk* primarily related to government regulations and risk related to changing customer behaviour.

Production risk	Financial risk	Institutional risk	Other
Growth rates	Price volatility	Changes in industry regulations	Changing customer preferences
Disease	Currency fluctuations	Availability of licenses	
Premature sexual maturity	Increased cost of input factors	Changes in tariffs and trade agreements	
Escape			

Table 4.1: Main risks faced by a salmon producer.

4.3.1 Production risk

The production risk salmon producers face is mainly due to uncertain output as a result of uncertain biomass development. Furthermore, the salmon producers also face production risk related to the primary and secondary processing of the fish, such as uncertain production costs and unstability in the production system. The main drivers of uncertainty in biomass development are growth uncertainty and uncertainty regarding loss of fish, see Table 4.2.

Drivers of production uncertainty	
Growth uncertainty	*Loss of fish*
Variation in growth rates	Escape
Water temperature	Young fish mortality
Fish health	Mortality caused by disease
Lightning conditions	
Feed	

Table 4.2: Drivers of production uncertainty.

Growth uncertainty

Growth uncertainty can be divided into the uncertainty caused by varying growth rates among fish of the same weight class, and uncertainty in overall growth due to uncertain growth conditions. An estimated proportion of 3-4% of the stock is classified as "loser fish", defined as fish that for unknown reasons do not keep up with the rest of the stock in terms of growth rates and quality (*Kyst og havbruk* 2009).

Overall growth is governed by water temperature, lighting conditions, type and amount of feed supplied and fish health. There are several important biological parameters affecting salmon health, including disease, water oxygen concentration, salinity, pH, ammonia and carbon dioxide content. As feed and lightning conditions can be controlled and the factors affecting health are usually acceptable, sea temperature and disease are the main source of growth uncertainty in salmon farming (Torgersen et al., 2008). Generally, the warmer the water, the higher the fish growth. However, upon reaching a temperature of approximately $17°C$, growth decreases, and higher temperatures can eventually lead to mortality. Diseases such as Sea Lice and Pancreas Disease affect growth negatively due to reduced fish appetite, and can at worst be fatal.

The water temperature at the farming facilities are exposed to both local and oceanic influence. At deep sea temperatures follow a relatively certain seasonal pattern, but near shore where marine farms are located, water temperature is much more unpredictable. Here, the temperature depends upon local weather conditions and on the exchange between coastal waters and the fjords. Figure 4.3 displays monthly mean temperatures for a Norwegian salmon farm for the years 1998 to 2006, showing considerable variation. Also worth noting is the fact that the variation is larger in periods with relatively higher water temperatures, e.g. in the period from June to September.

Average monthly seawater temperatures in each of the main production countries are displayed in Figure 4.4. Naturally, sea water temperatures in different countries follow different patterns. Due to the geographical closeness Norway and Scotland follow a similar pattern, with temperatures around $7\,°C$ in winter and $13\ °C$ in summer. The Canadian temperatures follow the same seasonal patten, but are more stable. Chilean temperatures follow an opposite pattern due to opposite seasons on the southern hemisphere, with temperatures around 14 in winter and 10 in summer.
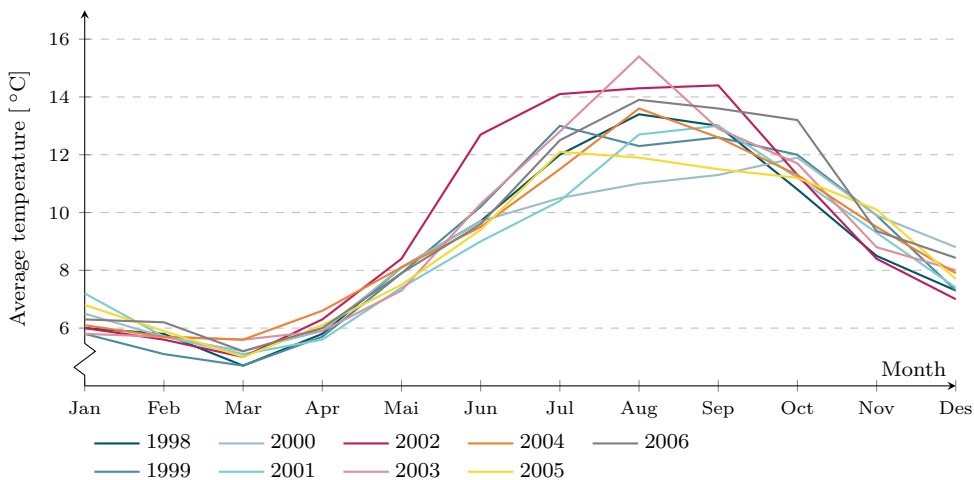
Figure 4.3: Monthly average sea temperatures for a Norwegian salmon farm (National Oceanic and Atmospheric Administration, 2015).
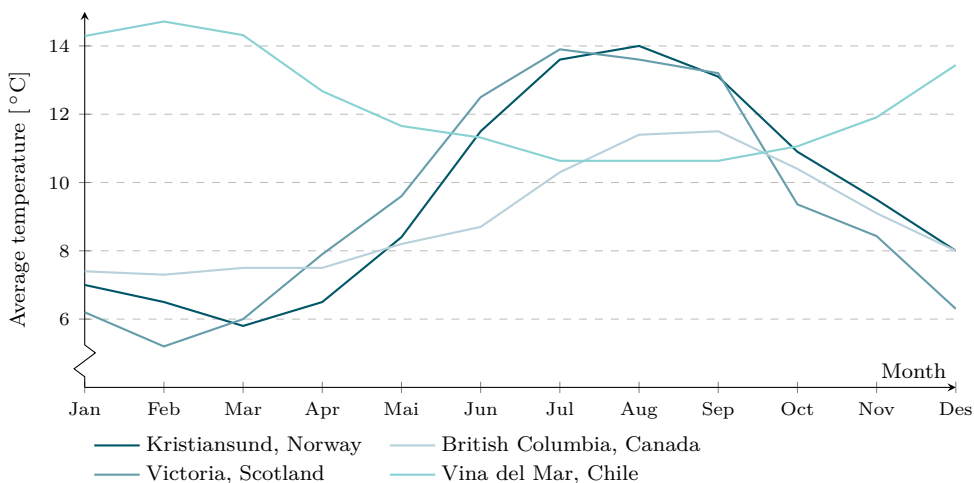
Figure 4.4: Monthly average sea temperatures for different producing regions around the world (National Oceanic and Atmospheric Administration, 2015).

Loss of fish

Loss of fish can be divided into escape and mortality. Reports by fish farming companies to the Norwegian Directorate of Fisheries indicate that escapes are heavily dominated by structural failures of equipment due to e.g. storms, while operational related-failure, escapes due to external factors and escapes from land-based facilities make up lesser proportions (Jensen, Dempster, et al., 2010). Typically, the structural failures are caused by tears in equipment due to storms or propeller damage, or escapes of small fish through nets. Large escapes, defined as incidents where more than 10 000 fish escape represent only 19% of all escapes, but amount to 91% of all escaped fish in terms of volume (Jensen, Lader, et al., 2012). Figure 4.5 displays the recorded number of fish escaped in Norway, showing a great variation in the number of escaped fish indicating that escape is a source of uncertainty to the salmon farmer. Note that the numbers for 2013 and 2014 are preliminary. However, as escapes represent less than 1% of the total production loss, other factors will be the main drivers of production uncertainty. The total loss of fish in the Norwegian salmon industry is estimated to be 16.3% in 2014 (Mattilsynet, 2014).
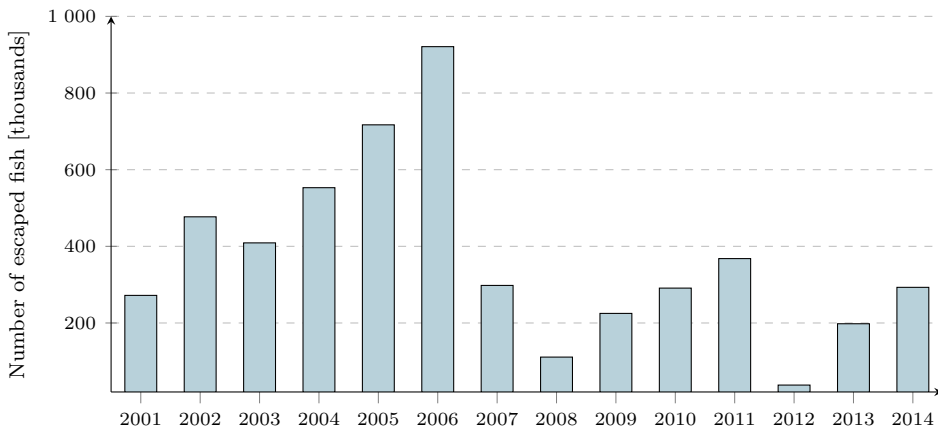
Figure 4.5: Reported number of escaped fish in Norway
(Norwegian Directorate of Fisheries, 2014c).

Mortality can be divided into mortality caused by disease, and young fish mortality due to deformities, injuries from transportation and handling, or difficulties in regards to transitioning to salt water. The latter is the most important cause of mortality - 80% of deaths occur before the fish reach 0.5 kg (Rynning-Tønnesen and Øveraas, 2012). Additionally, there is the natural mortality rate, meaning mortality during the growth phase not caused by disease. This is relatively steady, and as such its impact on the biomass development is regarded less important.

Mass mortality, such as in ISA crisis in Chile in 2008, might however have a severe effect. Mass mortality is caused by infectious diseases such as Pancreatic Disease (PD), Infectious Pancreatic Necrosis (IPN), Infectious Salmon Anemia (ISA) and Gill Disease (GD), and the outbreaks of such diseases is therefore an important source of uncertainty for the salmon farmer.

Additionally, diseases affect biomass development by inducing premature slaughtering to avoid the disease to spread. For instance, lice outbreaks may force the producer to fallow certain sites. Less severe diseases can cause loss of fish appetite and thereby slower growth. The main causes of salmon mortality as experienced by the Irish salmon farming industry in 1988 to 1992 are depicted in Table 4.3, showing PD to be the dominant cause.

Cause of mortality	% of total mortality
Pancreas Disease	51.3
Transfer	15.8
Unknown	12.9
Vibriosis	10.8
Algal blooms	6.6
Furunculosis	2.6

Table 4.3: Causes of mortality in Irish salmon farming 1988-1992 (Wheatley et al., 1995, p200).

4.3.2   Financial risk

As the salmon market is a highly volatile market, price uncertainty will greatly affect the optimal production and sales plans, and thereby the expected profit. To maximize profits, it is optimal for the producer to produce and sell more in periods with high prices and less in periods with low prices. This leads to low capacity utilization and low efficiency.

The costs of price volatility are transferred to the entire value chain. Volatile demand in the different stages in the value chain can create huge Bullwhip effects, causing overall supply chain efficiency to decrease further. Moreover, as the first-hand prices fluctuate and the retail prices are relative stable, many intermediary agents in the value chain, such as fish processors, can experience substantial variability of capacity utilization and profits (Øglend and Sikveland, 2008).

Changes in the price of input factors, such as fish feed, egg, smolt and electricity increases the risk related to salmon production. If factor cost rises quickly and large

parts of future production is locked to fixed price contracts, margins will diminish.

With financial risk being a global problem, currency fluctuations can significantly change cost structure and revenue streams of the producer, and the net effect of such fluctuations can potentially become quite significant. However, it is possible to mitigate much of this risk by currency hedging, and long term raw material and electricity contracts. The use of such instruments differs between the salmon producers, and may also introduce other risk factors such as hold-ups in the contractual relationship.

4.3.3   Institutional and other risk

In addition to the risks already discussed, the salmon producers face the vague, but potential risk from changes in regulations and other institutional risks. For instance, availability of licenses or new governmental impositions regarding lice treatment may transform the operating conditions. Such uncertainty may also adversely affect the willingness to future investments, although this topic is outside of the scope of this thesis.

Furthermore, changes in tariffs and trade agreements, including anti-dumping legislation poses potential market allocation challenges, such as the recent trade imports of some or all of the Norwegian fish to Russia (Dahl et al., 2014) and China (Mathisen, 2015). However, as the salmon market is a global market, different market channels or substitute markets may arise, as evident after the Russian trade restrictions imposed in 2014 (Roaldseth and Flatset, 2015). Furthermore, the effect of removal of one market channel will be less severe for large global industry actors such as Marine Harvest.

A long term risk is the effects of changing consumer preferences and market trends, potentially changing both the size and configuration of the total demand for salmon products. Public perception of the sustainability and environmental impact of the salmon farming industry can affect demand in a positive or negative direction. However, how to treat such qualitative risks is regarded as outside the scope of this thesis.

Chapter 5

Overview of relevant theory

This chapter contains a brief overview over relevant theory for this thesis, and is meant to serve as a framework for later discussions. The chapter provides the reader with an overview of important terms and concepts that may contribute to understanding the model presented in later chapters. However, it is assumed that the reader is familiar with basic principles from operations research and mathematical programming. A short introduction to stochastic programming and stochastic modeling is given in Section 5.1. Section 5.2 gives an overview of relevant theory on forecasting and how it may be used in conjunction with scenario generation.

5.1 Representing uncertainty

Uncertainty is present in the vast majority of real life decisions. Whether it is supply or demand, revenues or costs, technology or production, uncertainty is a factor that must be accounted for. In operations research there are two dominating modeling approaches to deal with uncertainty, namely the *deterministic approach* and the *stochastic approach*, while stochastic programming was first introduced by George Dantzig in the 1950s. In general, deterministic models are often chosen due to their simplicity, even when these models poorly describes the reality. However, when uncertainty plays an important role in the problems, deterministic models may fail to give an adequate description of the reality, and a model that takes the stochasticity into account may be necessary. While deterministic programming is the most common method used in early operations research, stochastic programming has increased in popularity during recent years due to technology development and increased computing power (Kaut and Wallace, 2007). For a more thorough introduction to stochastic programming in general, the reader is referred to Kall and Wallace (1994) or Birge and Louveaux (2011).

5.1.1   Introduction to stochastic programming

The simplest approach to model uncertainty is by using a deterministic problem formulation. Then, it is assumed that everything possible outcome of the future is known with certainty. Uncertainty are then not included explicitly, but rather represented implicit by replacing all uncertain variables by their expected values. When all uncertain variables in the problem are replaced by their expected values, one obtain what is called the *expected value problem* or *mean value problem*. Then the solution is known as the expected value (EV) solution.

A sensitivity analysis can be used to test the model robustness by varying the uncertain parameters and inspecting the resulting changes in the optimal solution. Alternatively, a scenario analysis can be conducted. In a scenario analysis, a set of scenarios are created and solved individually by using the expected values of each scenario. The decision maker can then generate different scenarios based on his or her's knowledge in order to find the best solution.

An alternative to the implicit representation of uncertainty from an deterministic approach is to explicitly model uncertainty. In stochastic programming, this is done by structuring the model in a way that emphasize when decisions are made and account for what information is currently available to the decision maker. In Figure 5.1, the uncertainty is represented by ω, which influence the decision on y. All decision variables x are not subject to uncertainty and are thereby treated deterministically.
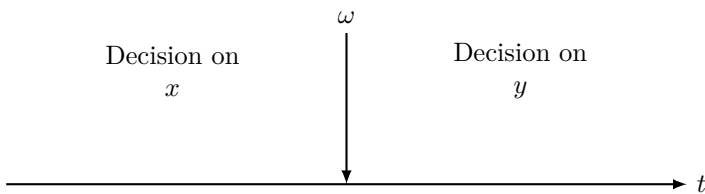
Figure 5.1: Realization of the uncertain variable ω which influence the decision on y.

As the uncertain variables in stochastic programming represent all possible outcomes, one fundamental assumption in stochastic programming is that the probability distribution of the uncertain variables is either known or can be estimated. Together, the uncertain variables and their probability distributions form the basis of how uncertainty in possible future realizations are constructed. Therefore, the stochastic method is most efficient when the future can be described with a discrete distribution. If the distributions are continuous, one have to discretize the set of possible outcomes. In most cases, this will be a simplification of reality, making the model less accurate.

When working with a stochastic problem, it is important to know the difference between three important terms, namely *time period*, *stage*, and *scenario*. A *time period* relates to the time passing in the decision process, e.g. a day, a week or a month. A *stage* can consist of one or more time periods, and is a point in time where new, useful information is revealed (Kall and Wallace, 1994). As it is only meaningful to make new decisions when new, relevant information become evident, stages are also the points in time when decisions are made. A *scenario* is a possible outcome of the problem during all time periods in the planning horizon. With T time periods and ξ_t is a vector describing what happens in time period t (i.e. a realization of ξ_t), Kall and Wallace (1994) defines a *scenario* as $s = \{\xi_0^s, \xi_1^s, \ldots, \xi_T^s\}$ which represent one possible future. As scenarios are meant to describe the possible outcomes of the uncertain variables a set of scenarios are often represented by a *scenario tree*. A scenario tree illustrate in which time period or state the uncertain variable is realized. Higle (2005) defines a scenario tree as a structured representation of the stochastic elements in a problem and how they may evolve over time during the problem's planning horizon. See Figure 5.2 for a illustration of how the three mentioned terms are connected.

By introducing stages this gives an opportunity to adapt a solution to a specific outcome observed, by either exploiting other opportunities or mitigating negative effects of the realization of observed uncertainty. Problems presented in a such way are often called recourse problems. In recourse models, the main idea is to utilize all information revealed prior to when decisions have to be taken. Recourse models seeks to postpone decisions as long as possible, thereby adding value by making more informed decisions. The opportunity to adapt a solution to the specific outcome observed are therefore often referred to by the term *recourse*. Recourse models are therefore always presented as models in which there are two or more stages.

Scenario trees

Before recourse formulations are presented, it may be useful to have a clear understanding of why the three terms presented are of importance. This becomes clear when introducing the concept of scenario trees. The main advantage with the scenario tree is that it may contribute to a better understanding of the structure of a recourse problem. The probability of the occurrence of each outcome is also often included in the tree (Pflug and Pichler, 2014b).

Figure 5.2 represent a simple scenario tree with 6 time periods, 3 stages and 4 scenarios. The circles represent nodes. Each node represents a possible state of the problem where

a decision can be made, and thus, in every node values for the uncertain variables are given. The nodes within a time period represent all the possible states the problem can be in at that time. In the figure, nodes belonging to the same time period are located vertically above one another. The single node in the first time period is referred to as the *root node*, and represents the initial state. The nodes in the last time period are called *leaf nodes*. A complete path from the root node to any leaf node represent a scenario.
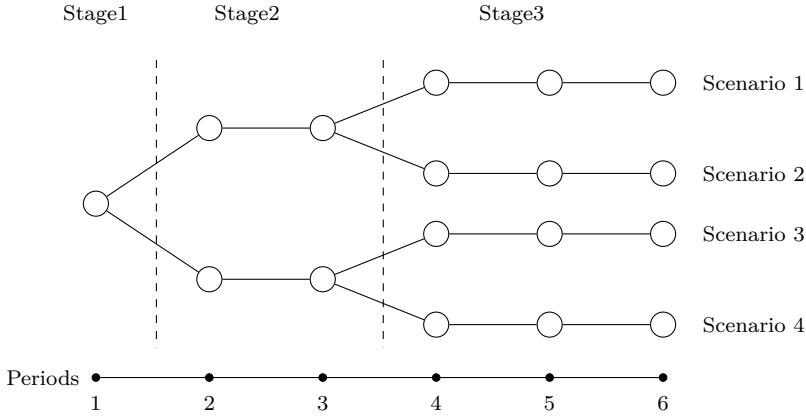
Figure 5.2: Scenario tree.

Uncertainty is resolved twice, represented by the stippled vertical lines. Each time there are two possible realizations of the random variable, resulting in branching the tree structure. Therefore, the problem illustrated has three stages. The first stage consists of one period, the second consists of two, whereas the last consist of three periods. In each stage, all uncertainty in the stage is resolved in the first time period. The remaining time periods in that stage can therefore be considered deterministic.

In practice, decisions are only made when new relevant information becomes available, or in other words, in nodes that are the result of a branching (Midthun, 2007). This implies that decisions regarding time period 3 are made in time period 2, and decisions regarding time period 5 and 6 are made in time period 4.

Two-stage recourse programs

The most widely applied and studied stochastic programming model are the two-stage linear recourse program. The recourse model traces its roots back to Dantzig (1955) and Beale (1955). In such programs, the decision process is divided into two stages. In the first stage, the decision maker has to commit to a decision before any information

relevant to the uncertainties becomes available. After making the first stage decision, a random event revealing information relevant to the uncertainties occur. This event most likely affect the outcome of the first-stage decision. A recourse decision can then be taken in the second stage, adjusting the first stage decisions according to the new information.

The solution to a two-stage linear program with recourse consist of a single first stage decision, often denoted x, followed by a set of recourse decisions defining the optimal strategy given every possible outcome of the random variables, often denoted y (Philpott, 2011). While x is referred to as the here-and-now-decision, y is referred to as the recourse decision. Because the optimal solution only gives one value for the first stage decision, x must be chosen so that it allows flexibility to handle all possible scenarios, using each scenario's probability as weighting. This is why a stochastic model is willing to pay for flexibility, while a deterministic model is not.Note that the stochastic valuation methods in the following sections are based on a maximization problem.

The first stage problem can be formulated as equations (5.1) - (5.3) and the second stage problem can be formulated as in equations (5.4) - (5.6). The second stage problem may also be referred to as the subproblem or recourse problem (Higle, 2005).

$$\max Z = cx + E[h(x, \tilde{\omega})] \tag{5.1}$$

$$Ax = b \tag{5.2}$$

$$x \geq 0 \tag{5.3}$$

$$h(x, \omega) = \max g_\omega y_\omega \tag{5.4}$$

$$W_\omega y_\omega \leq r_\omega - T_\omega x, \qquad\qquad \omega \in \Omega \tag{5.5}$$

$$y_\omega \geq 0, \qquad\qquad \omega \in \Omega \tag{5.6}$$

The uncertainty is represented by $\tilde{\omega}$ in equation (5.1). In the second stage, equation (5.4), the uncertainty $\tilde{\omega}$ in interpreted as a deterministic scenario. The variable $\tilde{\omega}$ is a discrete random variable with probability distribution independent of the decision vector x, and has probability $p_\omega = P(\tilde{\omega} = \omega)$ for each scenario ω in the set of scenarios Ω. The objective function of the first stage decisions takes uncertainty into account by the term $E[h(x, \tilde{\omega})]$. This term represent the expected value of the recourse decision, and is referred to as the value function or recourse function (Birge and Louveaux,

2011).

The formulation in equations (5.1) - (5.4) is generally referred to as an *implicit* representation of the stochastic problem (Birge and Louveaux, 2011). The formulation is also known as *node* formulation or *compact* form.

An alternative formulation widely used is the *explicit* formulation, also called an *extensive* formulation or *scenario* formulation (Higle, 2005). This formulation is given in equations (5.7) - (5.10).

$$\max Z = \sum_{\omega \in \Omega} p_\omega (cx_\omega + g_\omega y_\omega) \tag{5.7}$$

$$T_\omega x + W_\omega y_\omega \le r_\omega, \qquad\qquad \omega \in \Omega \tag{5.8}$$

$$x_\omega - x = 0, \qquad\qquad \omega \in \Omega \tag{5.9}$$

$$x_\omega, y_\omega \ge 0, \qquad\qquad \omega \in \Omega \tag{5.10}$$

In this formulation the here-and-now-decision from equation (5.1) is made scenario specific in x_ω, while constraints (5.9), which are known as non-anticipativity constraints, ensure that decisions honor the information structure of the problem (Higle, 2005). The objective function is also reformulated, such that it now maximizes the expected value of the second stage decision directly with respect to the probability, $p_\omega = P\{\tilde{\omega} = \omega\}$, of each scenario $\omega \in \Omega$. There is also extra constraints added, see constraints (5.9). The non-anticipativity constraints force the decisions taken in different scenarios to be consistent with the information available in each stage by forcing the here-and-now-decision (first stage decision) to take the same value in all scenarios. Thus it is not necessary to separate the first stage and second stage (recourse) decisions as in the implicit formulation. Note that constraints (5.9) are just one of numerous ways in which non-anticipativity constraints can be formulated. See Figure 5.3 for a illustration of how the non-anticipativity constraints acts compared to the structure in Figure 5.2.

Multi-stage recourse programs

The multi-stage recourse problem represents a planning situation where new information is revealed at several points during the planning horizon and decisions have to be made continuously based on the available information (Higle, 2005). The explicit formulation of the two-stage recourse problem previously presented can easily
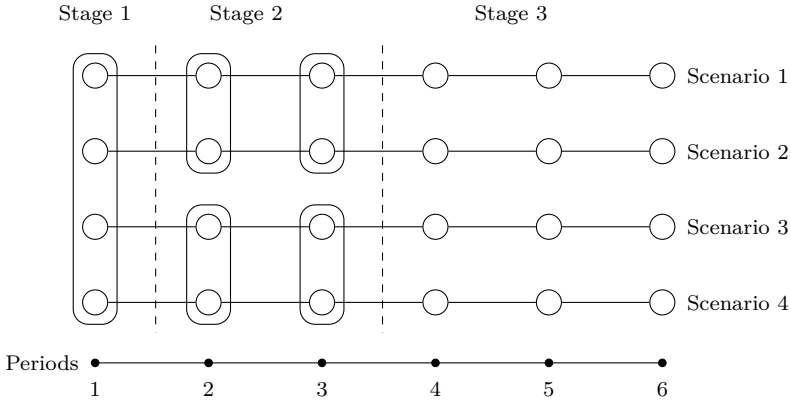
Figure 5.3: Scenario tree with non-anticipativity constraints.

be generalized to a multi-stage model:

$$\max Z = \sum_{\omega \in \Omega} p_\omega \sum_{t \in T} c x_\omega^t \tag{5.11}$$

$$\sum_{j=1}^{t} A_\omega^{tj} x_\omega^j \le b_\omega^t, \qquad t \in T, \omega \in \Omega \tag{5.12}$$

$$x_\omega^t - x_n^t = 0, \qquad t \in \mathcal{T}(n), \omega \in \Omega(n), n \in \mathcal{N} \tag{5.13}$$

$$x_\omega^t \ge 0, \qquad t \in T, \omega \in \Omega \tag{5.14}$$

In this formulation, the first-stage and recourse decisions are no longer separated by different variable names. Instead, x_n^t represent the decision made in period t. N denotes the set of non-anticipative decisions. As for the two stage extensive formulation, the non-anticipativity constraints (5.13) are used to enforce the right relationship between decisions and information structure. Constraints (5.13) ensure that each decision x_ω^t in time period $t \in T(n)$ are equal in all scenarios $\omega \in \Omega(n)$.

5.1.2 Evaluation and valuation of recourse models

Even though most real decision problems are subject to uncertainty, it is not guaranteed that an inclusion of uncertainty in the models is always beneficial. The deterministic models have the important advantage of being easier to understand and use compared to a stochastic model. The deterministic models also require fewer variables, which makes the implementation easier and reduces the solution times.

Therefore, it is useful to have tools for evaluating whether using a stochastic model is necessary or if it is sufficient to use a deterministic model. Evaluation of stochastic models may be especially valuable when the models are used continuously while the computational burden is high (Wallace, 2003). Two methods for valuating the stochastic models are presented, that is the *expected value of perfect information (EVPI)* and the *value of stochastic solution (VSS)*. In order to explain how they are calculated, some notation must be introduced.

To explain these measures, some terminology must be introduced. When the all random variables in the problem are replaced by their expected values, one obtains the expected value problem or mean value problem. The solution of such a problem is known as the expected value (EV) solution. The expected value of using the EV solution when uncertainty is included is known as EEV, and will be explained in more detail below.

The *recourse problem (RP)*, also known as the here-and-now solution, is the optimal solution to the recourse problem. Figure 5.5 illustrates the relationship between the different terms used in this section. Finally, the expected value of the set of optimal solutions for all scenarios is referred to as the *wait-and-see solution (WS)*. WS represents the expected solution if all uncertainty is resolved, and is thus a theoretic measure only, giving an upper bound for the objective value. If a explicit formulation is used, the WS-model can be constructed by removing the non-anticipativity requirements from the RP model. The solution of the WS model is what would be the optimal strategy given the occurrence of each scenario. In other words, the WS-model gives a set of optimal solutions given a specific scenario, and not one single solution that should be implemented as the RP gives.

For all recourse problems where the objective is maximized, the following property holds true:

$$EEV \leq RP \leq WS \tag{5.15}$$

Expected value of perfect information (EVPI)

The EVPI is defined by Birge and Louveaux (2011) as a measure of the maximum amount a decision maker would be willing to pay in return for complete and accurate information about the future, thereby removing all uncertainty.

In real-world business decisions there is rarely an option to remove all uncertainty.

However, the EVPI provide a benchmark about the value of taking actions to reduce uncertainty. Thus, the EVPI gives an indication of whether it is worth making an effort to reduce the uncertainty present in the problem, for instance by investing in more advanced forecasting and decision support tools. The higher the EVPI, the more the decision maker stands to gain by reducing uncertainty. Conversely, if the expenses by collecting information exceed the EVPI, it is unnecessary to make an effort to reduce the uncertainty (Forsberg and Guttormsen, 2006). EVPI can thus be thought of as an expected improvement in the solution when perfect information is available, compared to being uncertain about the future (Kall and Wallace, 1994).

Mathematically, EVPI is calculated as the difference between the objective function value of the wait-and-see solution and the solution of the recourse problem, shown in equation (5.16) with the notation used by Birge and Louveaux (2011).

$$EVPI = WS - RP \tag{5.16}$$

Value of stochastic solution (VSS)

The difference in objective value from solving the same problem by using both the stochastic and the deterministic approach is called the VSS. The VSS is calculated by comparing the RP solution with the EEV solution, as shown in equation (5.17).

$$VSS = RP - EEV \tag{5.17}$$

The solution of a recourse problem will always have a objective value better than or equal to the one from a deterministic model of the same problem, meaning VSS is always non-negative (Birge and Louveaux, 2011). Because a deterministic model does not take advantage of information about the probability distribution of the random variables, the VSS is often largest for problems where the probability distributions are non-symmetric (Birge and Louveaux, 2011).

Value of the stochastic solution in a multi-stage model

The EEV is calculated in different ways depending on the problem structure. EEV for a two-stage problem is found by first solving the deterministic expected value problem and obtaining the EV solution. Then, the first stage decisions are fixed to the values from the EV solution. The EEV is the expected value of all WS solutions of the model with fixed first stage decisions.

When calculating the EEV for multi-stage problems, the calculations become more intricate as one encounter an issue regarding which variables to fixate. Since the WS model is not restricted by non-anticipativity constraints and each scenario is solved as if they are independent, fixing the first stage solutions may result in a negative VSS since the first stage solution in the EV problem performs better than the solution of the RP (Escudero et al., 2007). Therefore, the EEV has to be redefined for a multi-stage problem in order give a meaningful VSS. Escudero et al. (2007) suggest two approaches to how this can be done.

First approach: The value of the stochastic solution in stage and time period t

As already mentioned, the problem of using VSS as for the two-stage stochastic problem in a multi-stage stochastic model is the lack of non-anticipativity constraints. Escudero et al. (2007) propose a solution to this problem by introducing the EEV_s as an alternative to the EEV used in two-stage stochastic models. Instead of inserting the EV solution into the WS problem, the EEV_s is calculated by inserting the EV solution into the recourse problem, RP, as given by equations (5.11)-(5.14). In this way the non-anticipativity constraints are handled by fixing the decision variables up to stage s to the solution of the expected value problem (the EV solution). The subscript s the EEV has adopted denotes the current stage. Note that Escudero et al. (2007) uses t for both stages and time periods due to that all examples and scenario trees in the paper have an equal number of time periods and stages. However, in this thesis the notation and equations are modified to fit a multi-stage RP with more time periods than stages which is common in a stochastic model.

$$EEV_s = \begin{cases} \text{RP model,} & \text{equations (5.11)-(5.14)} \\ \text{s.t. } x_w^t = \bar{x}^t, & t \in \mathcal{T}(s-1), \omega \in \Omega. \end{cases}$$

Here, $\bar{x}^t$ are the optimal values obtained by solving the expected value problem. The set of scenarios, $\omega \in \Omega$, is defined as before. The EEV_s are only defined for $t \in \mathcal{T}(s-1)$, because the decision variables are only locked for all time periods in all stages prior to stage s. By also defining EEV_s for $s = 1$, the EEV_1 equals the RP model and solution.

As for the EEV_s, the measure VSS also adopts the same subscript s. The value of the stochastic solution, VVS_s, is then defined as

$$VSS_s = RP - EEV_s \tag{5.18}$$

A relation that may make the difference between EEV, EEV_s and RP easier to understand is the fact that $EEV_{s+1} \leq EEV_s, \forall t = 1, \ldots, T-1$. Meaning that VSS_s is a measure of the cost of ignoring uncertainty until stage s in the decision making process. It can be seen as the performance of the deterministic model by using average values for the stochastic parameters for all stages up to stage s.

Second approach: The dynamic value of the stochastic solution

In the second approach Escudero et al. (2007) focus on finding a more realistic value for the EV solution. This is done by redefining the concept of the expected value problem. Instead of using the scenario tree which is introduced in preceding sections, Escudero et al. (2007) divides the scenario tree into sub-trees and are thereby able to solve a set of expected value problem for each sub-tree, hence the term *dynamic*. The scenario tree are also redefined to consist of scenario groups indexed by g, where time periods within each stage are grouped together into one node. Consequently, scenarios that has the same realizations of the uncertain parameters up to that stage are grouped together, and the corresponding expected value problem is indexed by the scenario group index, EV_g. $\mathcal{G}$ denotes the set of scenario groups g, and $\mathcal{G}_s$ denotes the set of scenario groups in stage s. The set of scenarios that belong to scenario group g are also indexed by the same subscript, Ω_g. The notation Escudero et al. (2007) use are illustrated in Figure 5.4. Note that π defines the immediate predecessor of each node, and that the scenario groups are descendant scenarios of the specific node g.

The approach can be described as an iterative procedure where the expected value problem are solved for each scenario group consecutively, starting with the scenario group representing the root node. The optimal solution is denoted by Z_{EV}^g, and the solution obtained by solving the expected value problems for the set of scenario groups in a given stage is referred to as the *dynamic solution of the expected value problem*. After solving the fist stage EV_g, the optimal first-stage decisions are fixated for all succeeding sub-trees before the procedure is repeated successively, with continuous locking the decisions from the ancestor scenario groups.

This leads to the definition of the $EDEV_s$, which is the expected result in stage s of using the dynamic solution of the expected value problem, as the expected value of the optimal values of the EV_g problems, with $g \in \mathcal{G}_s$. This is expressed as

$$EDEV_s = \sum_{g \in \mathcal{G}_s} p_g Z_{EV}^g, \quad t \in \mathcal{T} \tag{5.19}$$

where Z_{EV}^g is the optimal value for the model EV_g and p_g represents the likelihood of

$$\Omega = \Omega_1 = \{10, 11, \ldots, 17\}; \ \Omega_2 = \{10, 11, 12\}$$

$$\mathcal{G} = \{1, 2, 3, \ldots, 17\}$$

$$\mathcal{G}_2 = \{2, 3, 4\}; t(5) = 3$$

$$\mathcal{S} = \{1, 2, 3\}, S = 3, \mathcal{T} = \{1, 2, 3, 4\}, T = 4$$

$$\pi(9) = 4, \pi(14) = 7$$
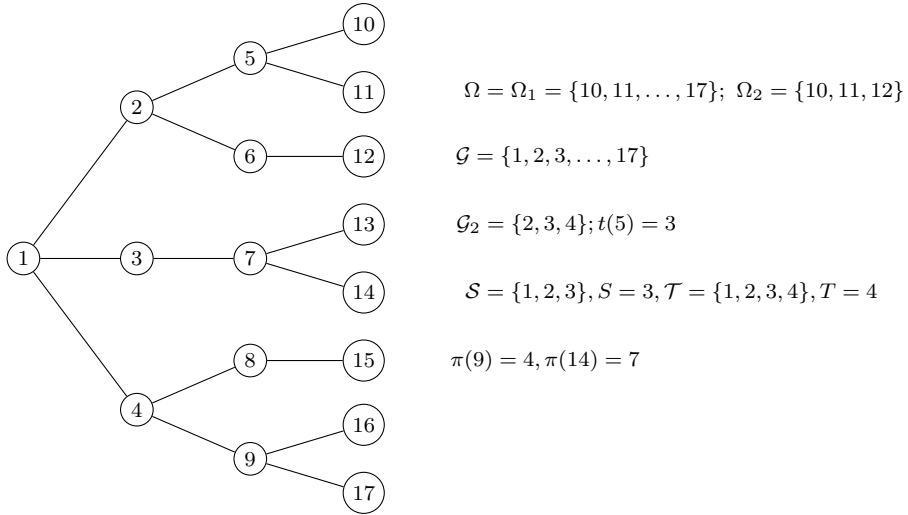
Figure 5.4: Illustration of how Escudero defines his notation in valuating a multi-stage model (Escudero et al., 2007).

the scenario group g, which is obtained as $p_g = \sum_{\omega \in \Omega_g} p_\omega$. This is the same probability for each scenario as in equation (5.11).

The dynamic value of the stochastic solution, VSS^D, is then defined as the value of ignoring uncertainty throughout the planning period

$$VSS^D = RP - EDEV_s \tag{5.20}$$

The relationship between the EVPI and the VSS is illustrated in Figure 5.5.
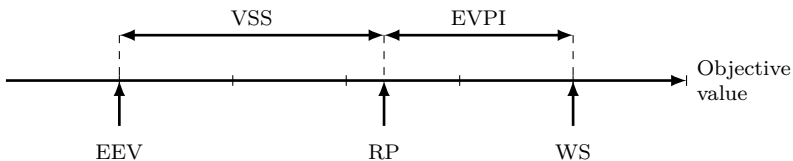
Figure 5.5: Illustration of the relationship between VSS and EVPI

5.2 Forecasting and scenario generation

If uncertainty is present in the reality one wants to model, good forecasts and scenarios are directly linked to the quality of the results the model gives (Kaut and Wallace, 2007), and bad scenarios can ruin an otherwise flawless model. It is not straightforward to describe all the possible outcomes of nature, but by approximating the probability distributions of the stochastic parameters as discrete distributions, i.e. lists of realizations (scenarios), and their probabilities one may get good estimates (Kaut, 2014). The aforementioned example by Higle and Wallace (2003) highlights the importance of forecasting to be able to give an as good representation of the future as possible, and forecasts are frequently used as a basis for scenario generation in stochastic programming, where the scenarios intend to describe the possible deviations from the forecasts (Midthun et al., 2009).

Dependent of problem and solution methods there are different methods for generating good scenarios. Ideally one would use a high number of scenarios to increase the quality of the result of the model, but to be able to solve the model within reasonable time there is often a trade off between the number of scenarios and solution time. While this section focus on the most relevant methods for the problem studied in this thesis, the reader is referred to Kaut and Wallace (2007) and Dupačová et al. (2000) for an overview of different scenario generation methods from the literature.

5.2.1 Introduction to scenario generation

Dependent of problem and solution methods there are different methods for generating good scenarios. Kaut and Wallace (2003a) divide the most important pure scenario generation methods in five groups; (1) conditional sampling, either by direct sampling from the distribution or indirectly sampling the process by calculating the samples using an explicit formula, (2) sampling from specified marginals and correlations, where the user specifies the marginal distributions and correlation matrix, (3) moment matching, and construction of the unknown distribution and marginals by using known moments, (4) path-based methods, where complete paths are generated to form a fan which is then clustered together in all-but-the-last period to transform the fan to a scenario tree, and (5) optimal discretization, where one tries to find an approximation of the stochastic process that minimizes an error in the objective function. There are also other scenario generation methods described in the literature, see e.g. Kaut (2014) and Dupačová et al. (2000). Kaut (2014) presents a method which generates scenarios by using copulas instead of correlations.

Sampling

The most common method for scenario generation is conditional sampling (Kaut and Wallace, 2003a). This method is based on sampling a number of values either directly from the distribution of the stochastic process at every node or by calculating the samples by the use of an explicit formula and thereby indirectly sampling the process. The method has the advantage of being easy to use, but the disadvantage of being restricted by sampling of only one univariate random variable. By combining separately sampled marginals one may sample a random vector, resulting in a tree that grows exponentially with the random vector's dimension.

Moment matching

When working with stochastic programming the scenarios are often drawn from a distribution with known statistical properties. However, if one do not know the distribution functions of the marginals, their moments may be good descriptors for the distributions. The degree of moments to a distribution are infinite, but often the four first are used to describe the distribution, the mean, variance, skewness and kurtosis. Together with the correlation matrix and other statistical properties, these may be used to construct a discrete distribution of scenarios which combined satisfy the same properties.

The procedures for generating scenarios are rather different, and the main reason for choosing between the different methods depends on the available information about the distribution the scenario tree should be made from. If the distribution functions of the marginals are unknown, moment matching is the preferred method. In moment matching the marginals are described by their different moments, that is mean, variance, skewness, kurtosis and higher order moments. According to Kaut and Wallace (2003a) it is often sufficient to match the mean, variance and correlations in mean-variance models, while most optimization models require more moments since matching of the right properties is an important issue in moment-matching methods. However, Kaut and Wallace (2003a) have experienced that the first four moments often gives a good enough description of the marginals, and to the authors' knowledge this is the most common approach.

Høyland, Kaut, et al. (2003) have developed an algorithm that uses moment-matching to produce a discrete joint distribution consistent with specified values for the first four marginal moments and correlations. The main reason for developing the algorithm was because scenario generation may be one of the most challenging and time consuming

parts of stochastic programming. In large problems, where there is a need for many scenarios in terms of many asset classes, early developed scenario generation methods are often the bottleneck in portfolio optimization, e.g. the method used in Høyland and Wallace (2001) (Høyland, Kaut, et al., 2003). This algorithm also forms the basis for the scenario generation method used in this thesis.

However, this algorithm itself do not produce complete scenarios, but residual term scenarios from the moments that describe the distribution of the time series one want to generate future scenarios from.

Copula based scenario generation

To generate scenarios from e.g. elliptical distributions such as the Normal distributions or the Student's t-distribution, correlations may be sufficient. However, to model asymmetric dependence the method might fail. This is also investigated by Kaut and Wallace (2011), and their work confirms that correlations can lead to sub-optimal solutions to a stochastic model (Kaut, 2014). If the reader is unfamiliar with the term copula, Kaut (2014) describes a copula as a joint cumulative distribution function for a multivariate random vector in which the marginal probability distribution of each variable is uniform. Kaut (2014) also shows that a multivariate CDF is fully determined by the marginal CDFs and the copula, and that a copula is a full description of the dependence between the margins.

Kaut and Wallace (2007) suggest a two-step procedure for generating scenarios by using copulas; (1) generate scenarios for the desired copula, (2) transform the margins using the inverse CDFs to get scenarios that have both the correct copula and marginal distributions. The second step is trivial if the marginal CDFs are continuous functions. Since a copula is a multivariate distribution with standard uniform margins, the margins of the scenario-distribution will constitute a sample from the same uniform distribution. Therefore, transformation of the margins by using the inverse CDFs result in scenarios that have both the correct copula and marginal distributions (Kaut, 2014). As the only available method for the first step have been sampling, there is difficult to represent the future with acceptable quality taking into account the high requirement on number of scenarios from sampling methods. As a consequence, Kaut (2014) has developed an heuristic that achieve a comparable quality in the solutions with fewer scenarios. For a bivariate case there is actually a MIP-model which is able to find exact residuals for the scenarios. However, to handle multivariate scenario generation methods, and faster scenario generation for bivariate distribution,

the heuristic proves to give scenarios of high quality in very short time, read > 1000 in approximately 6 seconds when 45 bivariate copulas are matched.

5.2.2 Forecasting

The salmon industry is characterized by large seasonal variations, e.g. with regards to water temperatures and price variations due to variations in the supply. Because of this, the authors will further explain models that handles seasonality in another way than e.g. exponential smoothing. Even if one can adjust models such as exponential smoothing to incorporate seasonality in the data by using seasonal factors for each sample, this is considered a weaker approach to get the best prognosis. Autoregressive integrated moving average models (ARIMA) on the other side aims to describe autocorrelations in the data (Hyndman and Athanasopoulos, 2015).

Autoregressive (AR) models

Regression analysis is a statistical process used to estimate the relationship between variables. In multiple regression models, a linear combination of predictors are used to forecast the variable of interest. In an AR-model model the forecast of the variable of interest is expressed as a linear combination of past variables, and this is where the difference between regular regression models and autoregression models lies. Namely, the term *auto*regression indicates that it is a regression of the variable itself (Hyndman and Athanasopoulos, 2015). Consequently, an AR-model is a representation of some random time varying process which is linearly dependent of its own previous values. This is one useful way to represent time varying processes found in e.g. nature or economics. The AR-model represent the process Z_t with regress to the value of Z in past values plus a random shock (Wei, 2006). The random shock is often considered as white noise drawn from a normal distribution with zero mean, and finite variance, $\mathcal{N}(0, \sigma^2)$. Equation (5.21) shows how the AR-model looks like.

$$Z_t = \mu + \sum_{i=1}^{p} \phi_i Z_{t-i} + \varepsilon_t, \quad t \in \mathcal{T} \tag{5.21}$$

where μ is a constant, $\varphi_1, \ldots, \varphi_p$ are the parameters of the model and ε_t is the white noise process, $\varepsilon_t \sim \mathcal{N}(0, \sigma^2)$.

Moving average (MA) models

The MA-model model is another useful representation of the time varying random process Z_t. Rather than using past variable values to forecast the variable of interest, the model use past forecast errors (Hyndman and Athanasopoulos, 2015), ε_t. The MA-model expresses Z_t as a linear combination of a sequence of uncorrelated random variables (Wei, 2006). The moving average representation can be seen in equation (5.22).

$$Z_t = \mu + \varepsilon_t + \sum_{i=1}^{q} \psi_i \varepsilon_{t-i} \tag{5.22}$$

where μ is a constant, $\psi_t, \ldots, \psi_{t-q}$ are the parameters of the model, and $\varepsilon_t, \ldots, \varepsilon_{t-q}$ is the white noise error terms, $\varepsilon_t \sim \mathcal{N}(0, \sigma^2)$.

As one can see, in the autoregressive model the shock affects the value of the evolving variable infinitely into the future. In the moving average model the shock only affects the value of the evolving variable q periods into the future. The shock is also propagated to the future values of the variables directly, while in the autoregressive model the shock affects future values of the evolving variable indirectly.

Autoregressive integrated moving average (ARIMA) models

As shown in preceding sections, a stationary process can be represented either in an autoregressive form or in a moving average form. Both of these formulations contain a high number of variables that needs to be estimated, which is a problem that arise in either representation. Even for a finite-order autoregressive model or finite-order moving average model a high-order model is needed to get a good approximation. The high number of parameters will in general reduce the efficiency of the estimation. This can be improved by combining the AR-model and MA-model into a useful mixed autoregressive moving average process (ARMA).

The process one wants to forecast is often non-stationary, and to account for non-stationarity one often uses what is called the generalization of the ARMA-model. The generalization of the ARMA-model is expressed as an ARIMA. The ARIMA-model reduces the non-stationary time series to a stationary time series by taking a proper degree of differencing (Wei, 2006). This relates to Step 1 in the scenario generation procedure already presented.

The ARIMA-model, equation (5.23), consist of the autoregressive representation, a

differencing part, and the moving average representation.

$$Z'_t = c + \phi_1 Z'_{t-1} + \cdots + \phi_p Z'_{t-p} + \psi_1 \varepsilon_{t-1} + \cdots + \psi_q \varepsilon_{t-q} + \varepsilon_t \qquad (5.23)$$

where Z'_t is the differenced series, $Z'_t = Z_t - Z_{t-1}$. This is called an ARIMA(p,d,q) model, where **p** is the order of the autoregressive part, **d** is the degree of first differencing involved and **q** is the order of the moving average part.

Seasonal autoregressive integrated moving average models

So far only the non-seasonal ARIMA-models are presented. However, ARIMA-models are also able to model a wide range of seasonal data by adding seasonal terms to the non-seasonal ARIMA-model just presented. The seasonal terms are very similar to the non-seasonal components of the ARIMA-model, but as for non-stationarity the seasonal periods needs to be differenced. The removal of seasonality is exemplified for an ARIMA$(1, 1, 1) \times (1, 1, 1)_{12}$ model data in equation (5.24). Note that the subscript 12 refers to the lenght of the seasonal period and that equation (5.24) uses what is called a backshift operator, B. The reason for using this backshift operator is its ease of use when combining differences, as the operator can be treated using ordinary algebraic rules. This is not to be confused with an operator in the strict mathematical sense. As easily illustrated by Hyndman and Athanasopoulos (2015), it is much easier to work with the backshift operator when one starts combining components to form more complicated models, such as ARIMA(p,d,q) models. This is also exemplified in the project thesis by Denstad and Ulsund (2015). The backshift operator B is often called a lag operator in the literature, commonly denoted L.

$$\underbrace{(1 - \phi_1 B)}_{\substack{\text{Non-Seasonal} \\ \text{AR(1)}}} \overbrace{(1 - \Phi_1 B^{12})}^{\substack{\text{Seasonal} \\ \text{AR(1)}}} \underbrace{(1 - B)}_{\substack{\text{Non-Seasonal} \\ \text{difference}}} \overbrace{(1 - B^{12})}^{\substack{\text{Seasonal} \\ \text{difference}}} Z_t = \underbrace{(1 + \psi_1 B)}_{\substack{\text{Non-Seasonal} \\ \text{MA(1)}}} \overbrace{(1 + \Psi_1 B^{12})}^{\substack{\text{Seasonal} \\ \text{MA(1)}}} \varepsilon_t \qquad (5.24)$$

As one can see the seasonal terms are just simply multiplied with the non-seasonal terms.

Model identification and forecasting method

In forecasting, one aims to find the forecasting model that in the best way possible approximates the underlying process one wishes to forecast, such as demand or price. One must therefore analyze the data foundation on which the forecast is based, and through testing find the model best suited.

Note that often there is a need for transformations on the forecasted time series in

order to e.g. stabilize variance in the time series. One method for variance stabilizing is the power transformation introduced by Box and Cox (1964), another commonly used variance stabilizing method is the logarithmic transformation (Hyndman and Athanasopoulos, 2015).

The Box-Jenkins method, named after George E.P. Box and Gwilym M. Jenkins (Box, Jenkins, et al., 2008), is one method widely used for model identification when forecasting is necessary. The Box-Jenkins method finds the best fit of a time-series model based on past values of the time series one wants to forecast. The original Box-Jenkins methodology is a three-stage iterative process consisting of the following steps:

1. **Identification and selection of an appropriate model**
   - Detecting stationarity and seasonality
   - Identifying p and q
2. **Estimation of parameters**
   - Non-linear estimation problem
   - Almost exclusively done by software
3. **Model diagnostics**
   - Model validation
   - Repeat the process if results are not satisfactory

The model identifies candidate forecasting models based on the values of the time series one wants to forecast, and can thereby be used to select the most suitable forecasting model by comparison of results attained from the different models suggested. This relates to Step 2 and 3 in the scenario generation procedure where the forecasting of the time series is the last part of Step 3 where the model found by the Box-Jenkins method is used. Note that such forecasting is almost exclusively performed using statistical software suitable for forecasting. For a more detailed description of the Box-Jenkins methodology, the reader is referred to the project thesis by (Denstad and Ulsund, 2015). Denstad and Ulsund (2015) also describe more in detail how the suggested models from the Box-Jenkins methodology can be validated.

5.2.3 Scenario generation by combining a time series model with the residual term scenarios

The methodology for scenario generation which is used in this thesis are a modified version of the methodology used by Schütz and Tomasgard (2011), Schütz et al. (2009a), and Rynning-Tønnesen and Øveraas (2012) which is based on the currently

unpublished work of Nowak and Tomasgard (2007).

1. Ensure stationarity and remove seasonality in the time series.
2. Identify candidate time series models.
3. Select the best model and forecast the time series with historical data as input.
4. Create residual term scenarios from preferred scenarios generation method.
5. Combining the time series model with the residual term scenarios.

Steps 1 through 3 are related to the forecasting of the time series, described in Section 5.2.2. The scenario generation in Step 4 depends on the time series one wants to forecast, its properties and to some extent the decision makers knowledge and preference on scenario generation procedure. After a scenario generation method is chosen, the residual term scenarios are added to the forecasting model, step 5, in the time period the decision maker faces uncertainty. For an illustration of the procedure from step 3 to 5, see Figure 5.6. Note that the notation is the same as in Section 5.2.2. It should also be noted that Figure 5.6 uses an stationary, nonseasonal AR-model for the time series forecast.

Forecasting method

Historical data | Predicted data

z_{t-2} z_{t-1} z_t $\hat{z}_{t+1}$ $\hat{z}_{t+2}$ $\hat{z}_{t+3}$

$$\hat{z}_{t+j} = \begin{cases} \mu + \displaystyle\sum_{i=1}^{N} \phi_i z_{t+j-i}\varepsilon_t, & j = 1 \\[2ex] \mu + \displaystyle\sum_{i=1}^{j-1} \phi_i \hat{z}_{t+j-i} + \sum_{i=1}^{N} \beta_i z_{t+j-i} + \varepsilon_t, & 1 < j \le N \\[2ex] \mu + \displaystyle\sum_{i=1}^{N} \phi_i \hat{z}_{t+j-i} + \varepsilon_t, & j > N \end{cases}$$

Scenario tree for the prediction error

ϵ_{t+1}^1

ϵ_{t+1}^2

ϵ_{t+1}^s

Resulting scenario tree

$\hat{z}_{t+1}^1$ $\hat{z}_{t+2}^1$ $\hat{z}_{t+3}^1$

z_{t-2} z_{t-1} z_t

$\hat{z}_{t+1}^s$ $\hat{z}_{t+2}^s$ $\hat{z}_{t+3}^s$

$$\hat{z}_{t+j}^s = \begin{cases} \mu + \displaystyle\sum_{i=1}^{N} \phi_i z_{t+j-i} + \varepsilon_t + \epsilon_{t+1}^s, & j = 1 \\[2ex] \mu + \displaystyle\sum_{i=1}^{j-1} \phi_i \hat{z}_{t+j-i}^s + \sum_{i=1}^{N} \beta_i z_{t+j-i} + \varepsilon_t, & 1 < j \le N \\[2ex] \mu + \displaystyle\sum_{i=1}^{N} \phi_i \hat{z}_{t+j-i}^s + \varepsilon_t, & j > N \end{cases}$$
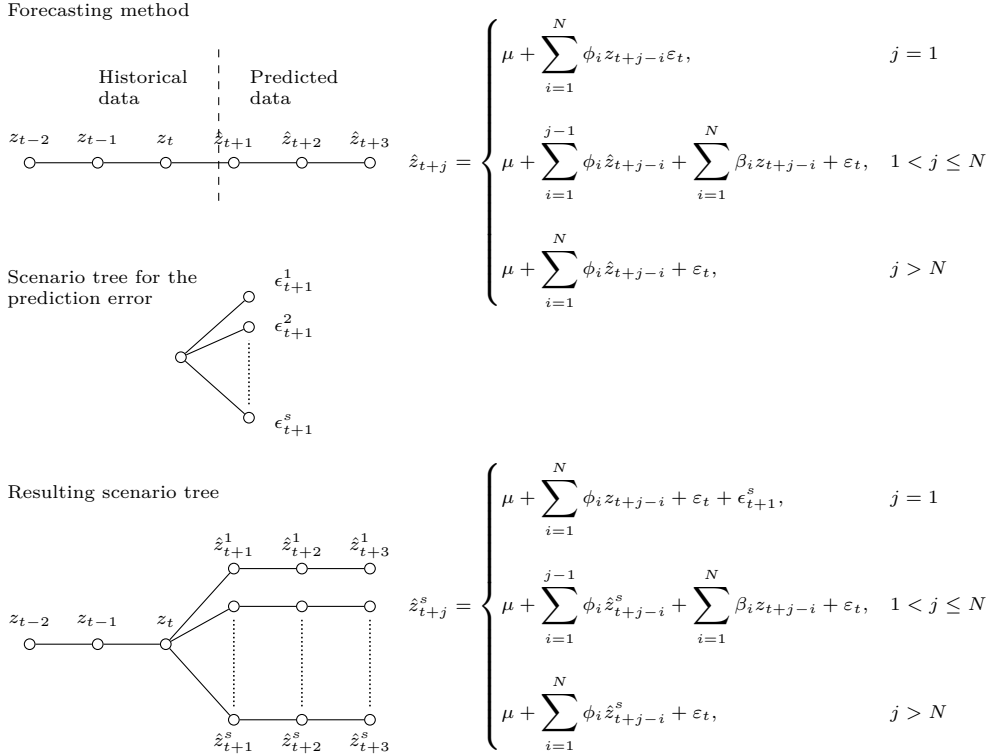
Figure 5.6: Procedure of scenario generation by combining the forecasted time series with the scenario tree obtained from the moment-matching procedure. The scenario tree for the prediction error forms the split in the resulting scenario tree used in the stochastic model.

Chapter 6

Model Introduction

Based on the problem described in Chapter 4, a stochastic optimization model has been developed. Before presenting the mathematical model in the following chapter, the model fundamentals will be explained. Section 6.1 presents a discussion about the planning horizon, before the chosen representation of time and the information structure is given in Section 6.2. Next, a conceptual representation of the model is presented in Section 6.3, while Section 6.4 give a description of the modeled network. Section 6.5 give an overview over the most relevant assumptions and simplifications, before modeling choices are explained in Section 6.6. Finally, possible model applications are outlined in Section 6.7.

6.1   Planning horizon and proposed time resolution

When deciding the length of the planning horizon, it is important that the length of the planning horizon is sufficiently long to avoid making sub-optimal short term decisions. On the other hand, models with long planning horizons quickly explode in size, especially when uncertainty is included. Furthermore, the forecasts used will also be increasingly less accurate, which will affect the added value of a longer planning horizon negatively. Based on this, planning horizons around a year seem reasonable as this will capture the seasonal cycles.

To be able to properly model the problem studied in this thesis, time is discretized over the length of the planning horizon, denoted $|\mathcal{T}|$. The model is developed with a time resolution of one week or less in mind. Note that using longer time periods, for instance a month, will make several of the simplifications and assumptions in this thesis invalid. The user should be aware how the time resolution affects the solution and must take this into account when analyzing the results.

The model is developed with the intent of being implemented in a rolling horizon

environment, implying that the model is run with updated information on a regular basis. A new run may be performed every time new information become available, when important decisions have to be made, or when a fixed number of time periods have passed (Swamidass, 2000). As Marine Harvest operate with planning cycles of 6 weeks (Johannesen, T., personal communication, 2015), this implies that running the model every 6th week should be sufficient. The model could also be run after irregular incidents, such as mass death due to disease or the introduction of new trade regulations.

6.2 Representation of time and information structure

The effects of all decisions are modeled as occurring immediately. For example when the decisions regarding harvest volumes are made, it is assumed that the harvest happen instantaneously and the time associated with sorting, harvesting, starving, slaughtering and processing is thus neglected. This can be partially remedied by adding average processing and administration times at each location into the transportation times. The scheduling problem that is present when planning the processing of the fish is also omitted, as this is outside the scope of this model. Note that the terms harvesting and slaughtering are used interchangeably throughout the rest of this thesis.
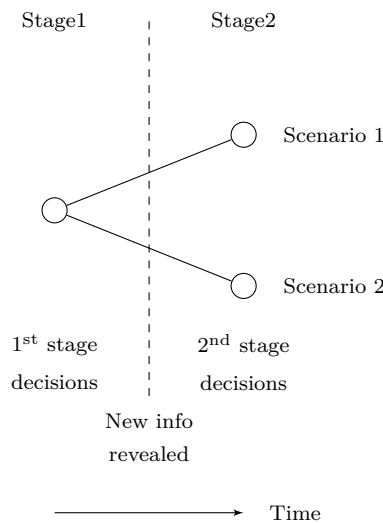
Figure 6.1: Illustration of the relation between decision-making and when is information revealed.

During the planning horizon, new information will become available. Specifically, information about weather conditions, disease outbreaks, prices and trends are revealed. When new information is obtained, the forecasts of future biomass and market development can be adjusted. In a stochastic programming model, this is modeled by including split nodes in the scenario tree. Information relevant to the future decisions is revealed immediately after the decisions represented by the split node have been made, as illustrated by Figure 6.1.

Because time is discretized and all decisions are taken in the beginning of each time period, production cannot begin and sales cannot be conducted in the middle of a time period. This is simplified into rounding the transportation time to the nearest integer. An alternative method could have been to distribute a share of the transport volume to the nearest floored time period and the remaining share to the nearest ceiled time period.

6.3   Conceptual model

To ease the understanding of the mathematical representation of the model, a conceptual model is presented in Table 6.1.

Maximize	Profits
Subject to	Biomass development constraints
	Harvesting constraints
	Mass balance constraints
	Inventory constraints
	Capacity constraints
	Contract fulfillment constraints

Table 6.1: Conceptual model

6.3.1   Model objective

The objective of the model is to maximize expected profits. As discussed by Hæreid (2011), using profit maximization as the objective bears great resemblance to maximizing biomass output. However, profit maximization has the advantage of also taking future price development into consideration.

6.3.2   Decisions

Due to varying frequency in the decision making, the decisions included in the model are are divided into two subsets, see Table 6.2. The tactical decisions will only be carried out at the beginning of the planning horizon, and are thus modeled as first stage decisions, whereas the operational decisions are carried out every time period

Tactical decisions	Operational decisions
Which contracts to enter	Harvest volumes
	Production volumes
	Inventory management
	Spot purchase and sales volumes
	Logistics

Table 6.2: Classification of decisions included in the model

The only tactical decision included in the model is the choice of which new contracts to enter. To make the model more oriented towards production planning, medium term slaughter and production plans could also have been included. According to Johannesen, Vinnes and Sværen (personal communication, 2015), Marine Harvest operate with 6-week slaughter and production plans. As such, the model could have been extended by including such operational plans as first stage decisions and punishing deviations from these plans. However, since this thesis focuses on the interaction with the market, the operational aspect of the problem is considered as outside the scope.

The tactical decisions give the frame within which the operational decisions are made. The common denominator for the operational decisions is that they are aimed toward using the harvested fish in the most profitable way possible while at the same time fulfilling signed contracts.

The operational decisions presented in Table 6.2 are mostly self-explanatory but are described to elucidate where and when these decisions are valid. The *harvest volume* decisions involves deciding the number of fish of each weight class that is to be harvested at each site, while *production volumes* are decisions regarding production volume of each processed product at each processing plant. The *inventory management* decisions are included to control the inventory balances, as well as how much that should be taken out of the inventories. *Spot purchase volumes* are decision about the spot purchase volume of each HOG weight class, whereas *spot sales volumes* means deciding what volume to sell of each product in each market. Finally, *logistics* means deciding about transportation volumes and by which transportation mode these volumes should be transported. All operational decisions are made every time period.

Furthermore, there are no explicit incentives for the model to keep a steady production level. This might lead to large and rapid changes in the production volumes, leading to solutions not suitable for real operational plans. This could have been enforced by including constraints for both minimum and maximum production or constraints for maximum allowed relative change from one time period to the next. Another option could have been to include a penalty for unused capacity in the objective function. This is however not included in the model presented in this thesis.

6.4   Network and mass flow description
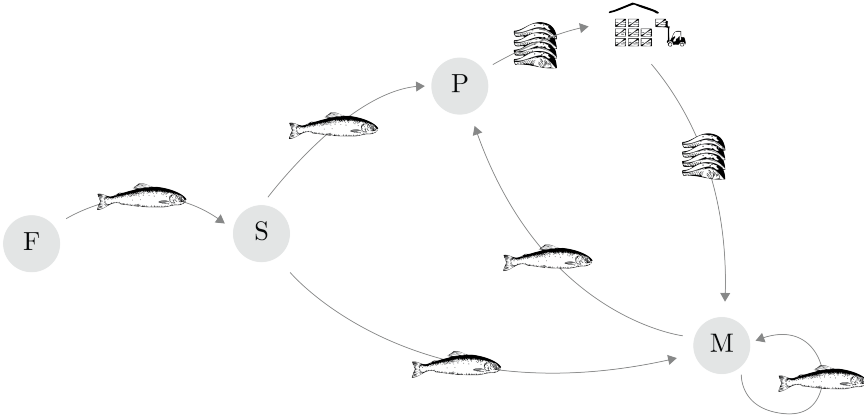
Figure 6.2: Conceptual illustration of the model network

The value chain is modeled as a set of arc flows within a node network. A set of arcs connects the nodes into a multi-commodity arc flow network, as there are multiple product that may flow between each node pair. The set of nodes, $\mathcal{N}$, are divided into subsets of farming nodes, $\mathcal{N}^F$, slaughterhouse nodes, $\mathcal{N}^S$, processing nodes, $\mathcal{N}^P$ and market nodes, $\mathcal{N}^M$. Additionally, the sets $\mathcal{D}_i$ and $\mathcal{O}_i$, which are the sets of origins and destinations for node i, are included to control the structure of the network. As illustrated in Figure 6.2, the fish farming nodes, F, are exclusively connected to slaughterhouse nodes, S, while the slaughter nodes are connected both to the processing nodes, P, and the market nodes, M. Note that each node in the schematic network presented below represents a set of nodes. Furthermore, it is important to stress the

distinction between the two different main uses of fresh HOG salmon:

1. HOG salmon as a singular product, marketed directly to a customer of Marine Harvest,
2. HOG salmon as raw material, used as input in secondary processing.

A separate arc is defined for each product that could flow between each node pair and for each transportation mode. While some slaughterhouses perform certain types of secondary processing, all products other than fresh HOG salmon are defined as secondary processed products to keep the model structure as simple as possible. As such, if salmon can be both slaughtered and processed at the same facility, this physical facility is represented as both a slaughter node and a processing node with both transportation cost and transportation time between them set to zero.

Fish marketed as fresh HOG salmon is sent directly from a slaughter node to a market node, whereas fish used as raw material in secondary processing are sent from the slaughter node to a processing node. As illustrated, processed products enter the inventory before being transported to market. A more thorough discussion on inventory modeling is found in Section 6.6.6.

Note that the market node, *M*, is connected to itself as well as to the processing node, *P*. This is because fresh HOG salmon purchased in the spot market can either be sold directly in the same market or in a different market or used as input into the processing of secondary processed products when it is economically beneficial.

The different products may be transported in different ways, depending on factors such as perishability and if the fish should be frozen or fresh. Consequently, each arc also defines the transportation mode that the product is transported by as illustrated in Figure 6.3 where as subset of all possible arcs between a node pair are shown. Note that only wellboats are used between farming and slaughter nodes.

6.5   Important assumptions and simplifications

The profit maximization problem modeled can be said to be a combination of a production planning problem, including production scheduling and inventory management, a transportation planning problem and a market allocation problem, implying a need for an overwhelming degree of details if to be modeled accurately. Therefore, several assumptions and simplifications have been when developing a mathematical model for
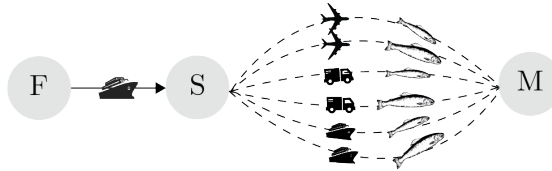
Figure 6.3: Illustration of how product flow and transportation modes are integrated.

the problem studied in this thesis. Assumptions and simplifications have also been made where necessary due to limited access to relevant information about industry specifics.

6.5.1   Uncertainty

In this model, biomass development and prices are included as the uncertain parameters. Biomass development is modeled by two separate sets of uncertain parameters, one representing growth and another representing the survival rate of the fish. As the development is highly dependent upon the fish size and on local conditions, both parameters are given per fish size and per farm. To be able to accurately model the price relations observed, they are modeled as separate for each product and each market.

6.5.2   Smolt release

As a result of excluding the decisions regarding the freshwater growth of the fish, a smolt release plan spanning the whole planning horizon is given as input. An alternative could have been to let smolt release volumes be decisions within the model. However, the long production cycle of smolt results in lead times of approximately a year, meaning smolt ordered within the planning horizon would not arrive until after the planning horizon has finished. Including such decisions would thus be of limited value.

6.5.3 Harvesting and slaughtering

As harvest is included as a decision in the model, it will find an efficient solution when considering the trade off between harvesting around the economical optimal weight in terms of return on feed and harvesting when market conditions are favourable.

To ensure that this trade off is modeled as realistically as possible, a caring cost is included in the objective function. This represent the cost of withholding fish from harvest and thus having to care for it for an additional time period. The caring cost consist mostly of feeding costs. As fish of different sizes need different amounts of feed, the cost is differentiated on fish weight.

To limit the model size, the set of weight classes are divided into two subsets. The smallest weight classes belong to the subset of weight classes deemed un-harvestable, and harvest of such fish is not allowed by the model. As fish in practice is never slaughtered before reaching a minimum weight of one kilogram, and sales of fish of 1-2 kg represent only a very minor portion of total sales, this mot severely affect the model.

All fixed costs related to farming, such as labour costs, electricity and depreciation, are excluded from the model as they do not affect the optimal decisions. Costs affiliated with harvesting and slaughtering, including costs of sorting and transportation of the fish, are also excluded based on the same argument. Furthermore, the handling of the fish at each location is not explicitly modeled. Consequently, the processing times normally associated with sorting, harvesting, transporting, starving and slaughtering are neglected in the model. Perfect sorting of fish is assumed possible, although the reader should also be aware of the practical limits in sorting the fish. The harvest is therefore conducted within a wide weight interval.

As discussed earlier, the salmon industry is subject to regulations regarding fallowing. However, according to (Evjemo, J.O., personal communication, 2015), fallowing is rarely performed by emptying the farms of fish. Rather, the farms are towed to other locations outside of the fallowing region. Because of this and the relatively short planning horizon and the global scope of the model, fallowing is excluded from the model.

6.5.4 Secondary processing

Maximum processing capacities are given for each product, and slaughtering capacities are given for each slaughtering facility. As such, it is implicitly assumed that the equipment is so specialized that no cross-use of machinery is allowed, i.e. excess capacity for producing one product cannot be utilised to produce another product. It would be possible to extend the model by introducing a set of machine types, and a set of products that can be produced at each machine type, with constraints controlling that only one product is produced at a time. Then, capacity could be given per machine type. However, as this is a higher level model developed to provide decision support for the senior management, such details are omitted. It is assumed that there are no minimum production volume requirements for any of the processing facilities.

6.5.5 Spot prices and spot transactions

Asheim et al. (2011) conclude that the price of farmed salmon has a limited effect on supplied quantity, giving a highly inelastic short-run supply, and that the biomass and seasonal factors are the main determinants of shifts in salmon supply in the short term. Based on this and the fact that Marine Harvest is only one of multiple suppliers, it is assumed that price is independent of Marine Harvest's supply. As Marine Harvest is the worlds largest salmon producer, this assumption cannot be regarded as fully realistic. However, as long as the model does not give extreme production levels, the levels supplied are assumed to have no effect on the current market situation. Further, it is assumed that Marine Harvest will never have a supply surpassing the total demand of the market. Consequently, the demand in the spot market is modeled as infinite.

As long as quality is maintained, weight is the most important factor in pricing. Consequently, weight is the only salmon characteristic included in the model, omitting other characteristics such as place of origin. Other factors, such as place of origin, are assumed to have no effect on the price obtained. As discussed in Chapter 3, salmon achieve different prices in different geographical markets. Each product is therefore modeled as having a separate price in each market in each time period.

The producer has the option of buying HOG from the spot market. In order to reduce arbitrage opportunities and to model the transaction costs affiliated with spot purchases, the price for selling and buying in the spot market are modeled as two different prices, where the price for buying exceeds the price obtained when selling.

6.5.6   Long-term contracts

It is usually the customer that approaches the producer with an offer, not the other way around (Thorbeck, S., personal communication, 2015). Consequently, all contracts are modeled as being initiated by the customer. As discussed by Hæreid (2011), contracts in the model can be interpreted in one of two ways. The set of contracts modeled can be seen as a representation of the contracts offered by potential customers. Another angle is to see the contracts included in the solution as a tool for guiding contract negotiations, in that they provide valuable insight in what contract characteristics that have the best potential for generating profits for the company.

The user will not have complete knowledge ahead of time about the contract characteristics. Nevertheless, all contract characteristics except for price are modeled deterministically to limit the model complexity.

Contracts are modeled as only being offered in the first stage. As the model should be implemented in a rolling horizon environment, the first stage contract decisions are thus the most important decisions in the model. Finally, including the contract decisions in the first stage gives the model more of a tactical approach.

If a contract is entered, the full amount must be delivered in the time period(s) specified in the contract. This is a simplification, as it is often possible to re-negotiate a contract during its proceeding. For instance, delaying the delivery time or deviating from the HOG weight class specified in the contract are often accepted by the counter-party if delivery problems arises (Helgesen, M., personal communication, 2015). As such, some of the contracts may not be entered because of infeasible terms in one or more scenarios, even though the contract price is beneficial. This flexibility could have been included in model, but is regarded as unnecessary when considering the little available information about contracts.

Another factor to consider is the aforementioned variety of creative contract designs that often occur in the salmon farming industry. In this model, the following contract schemes are considered:

- Fixed price contracts
- Adjustable price contracts
- Partly adjustable price contracts

In fixed price contracts, the price remains fixed throughout the contract duration, while the seller and buyer initially agree on a price that can later be adjusted according to the

spot price level. The price will then be adjusted by a relative share of the difference agreed upon in the contract terms. For instance, the buyer and seller could agree upon a price of 30 NOK/kg. If the realized spot price becomes 50 NOK/kg and the adjustment factor is 0.5, the buyer would end up paying 40 NOK/kg.

Partly adjustable contracts are similar to adjustable price contracts in terms of the price being adjusted according to the spot price. The only difference is that no adjustment is made if the differential is within a predefined interval given. The buyer and seller could for example agree that the price would only should be adjusted for the difference exceeding 5 NOK. If the same prices and the same adjustment factor are used, the buyer would now pay $30 + ((50 - 30) - 5) \cdot 0.5 = 37.5$ NOK/kg.

This comparison and the following adjustment is modeled as taking place simultaneous to contract delivery. In contracts involving multiple deliveries, the price will adjusted for each delivery. Furthermore, there are assumed that there are only one product and one included in every contract. This may however limit the contract flexibility somewhat, as the buyer may accept substitutes if there is limited supply of the product included in the model.

As will be discussed in further detail in Section 6.6, the model assumes a risk neutral producer. Consequently, neither volume only contracts or futures are included in the model formulation. Furthermore, a set of initial contracts are included as initial conditions. As this is not a decision, but a requirement, the revenue from initial contracts are not included in the objective function.

6.6 Modeling choices

When developing the model presented in this thesis, several modeling choices have beeen made. These choices will be discussed in the following section.

6.6.1 Risk aversion

When the profits are maximized in the objective function, the producer is implicitly assumed risk neutral. However, Marine Harvest can be assumed risk averse, as they have a large focus on selling much of its production through contracts, and are also investing in the production of value added products, which are known to have less margins, but also less volatility than fresh HOG salmon (Marine Harvest, 2014a; Marine Harvest, 2013).

The risk avertion could then be implemented with the use of multi-objective programming. Multi-objective programming has proven useful when a problem is characterized by the presence of many conflicting objectives (Marler and Arora, 2004). Denstad and Ulsund (2015) presents a problem where such multi-objective solution methods shows to be useful. Their objective was to maximize short term profits while at the same time minimizing risk of losing customers through maximizing customer satisfaction. The same methods could have been implemented for the problem faced in this thesis by including an risk minimizing objective in a multi-objective objective function. The reader is referred to Marler and Arora (2004) and Denstad and Ulsund (2015) for a detailed description of different multi-objective solution methods.

Contracts can for example be a useful measure when considering different measures for risk aversion. However, there are the literature proposes better risk measures than such crude measures, whee value at risk is one of them. Value at risk (VaR), is a single statistical measure of possible portfolio losses (Linsmeier and Pearson, 2000). VaR is thus a measure of losses resulting from normal market movements, and is simply a way to describe the magnitude of likely losses in a portfolio by aggregating all of the risks into a single number (Linsmeier and Pearson, 2000). However, minimization of VaR is not straightforward (Rockafellar and Uryasev, 2000). An inclusion of VaR would therefore most likely incorporate a separate optimization model which would require a different approach than the one used in this thesis.

Any option including risk aversion would increase the complexity of the model. Furthermore, it is important to give correct values of target levels, premiums and punishments in order to obtain a realistic representation of the reality. If these are not scaled correctly, it will severely affect the solution quality. Thus, modeling risk aversion demand more from the user. Because risk aversion is not included in this thesis, the model solution should be used as one of several input in the decision process, where other factors should include qualitative considerations as well as risk considerations. Not including risk aversion will affect the model solution, as the model will not put any extra value on risk reduction measures.

6.6.2 Biomass development

In order to describe the biomass development, the growth model developed by Hæreid (2011) is included in the complete model. Below, the characteristics of this growth model are explained.

Classification of fish

The industry classifies fish using a set of characteristics such as quality, age, weight, vaccines, feed type etc. Of these characteristics, weight is the only characteristic included in the model that differentiate the fish.

In order to avoid modeling individual fish, weight is discretizised. As such, live fish are modeled as belonging to a farming weight class v in the set of farming weight classes $\mathcal{V}$. Note that each weight class v represents a weight interval, so that the fish belonging to v is all fish with a weights that fall within this interval. In the model however, each weight class is defined by one specific weight, denoted G_v, representing the average weight of all fish belonging to weight class v. Consequently, the weight interval represented by a weight class v is given by an upper bound calculated as $\frac{G_{v+1}+G_v}{2}$ and a lower bound calculated as $\frac{G_v+G_{v-1}}{2}$. By classifying each individual fish as part of a farming weight class v with a defined weight, the modeling of the biomass and its growth is greatly simplified.

Hæreid (2011) shows that the discretization into weight classes must be sufficiently fine to get acceptable accuracy for the model. If the weight classes consist of a too large weight interval, fish can get stuck in one weight class without ever growing out of it. Consequently, it is important to choose sufficiently narrow intervals wen implementing the model. For a more detailed discussion of considerations regarding effects of different weight interval sizes, the reader is referred to Hæreid (2011).

When the fish is slaughtered and later marketed, such a fine discretization of weight is no longer necessary. To establish a connection between the way weight is treated in the farms and in the market, fish is classified in two ways. As soon as the fish is slaughtered, the fish is considered as part of a HOG weight class p that belongs to the set of HOG weight classes, $\mathcal{P}^{HOG}$. All farming weight classes included in HOG weight class p, are defined by the set $\mathcal{V}_p^H$.

Salt water growth

When modeling the biomass development, it is important to capture the uncertainty in future biomass that comes from the factors presented in Section 2.2. Consequently, growth is modeled as stochastic. The formulation is heavily inspired by a growth model developed by Hæreid (2011).

In short, growth is modeled by moving the fish between weight classes, so that a fish

advance from one class to a larger weight class as time passes. The growth in kilograms of a fish in a specified weight class is given by a stochastic parameter. The rate at which a fish grow depend upon the size, i.e. the weight class of the fish. Growth is also dependent upon location specific factors such as sea water temperature and light conditions, and these conditions vary with time. Therefore, the variable representing growth is defined for all combinations of weight classes v, farms f, time periods t and scenarios s, and is denoted σ_{vits}.

The number of fish in weight class v at the beginning of time period t in farm f in scenario s is given by the variable q_{vits}. At the beginning of each period, the number of fish belonging to each fish class is updated based on growth, mortality and escape during the previous time period. Mortality and escape are modeled jointly by the random variable ϵ_{vits}, representing the survival rate for a fish in fish class v in farm i in period t in scenario s. To get the number of fish currently in weight class v, the number of smolt of weight class v released is added and the number harvested is subtracted.

Calculating the weight of a fish at the beginning of time period t is done by adding the growth during period $t-1$, given by σ_{vits}, to the initial weight of the fish in time period $t-1$, given by G_v. The new weight of the fish will fall between the characteristic weight of two weight classes, as represented below. Therefore, additional calculations must be made in order to rightly distribute the fish into new fish classes.

$$G_{\underline{v}} \le (G_v + \sigma_{vits}) \le G_{\bar{v}}$$

If the fish of weight class v at the end of a time period t has a weight falling between the weight characterising weight class $\underline{v}$ and $\bar{v}$, this is handled by distributing a share of the fish into $\bar{v}$, and another share into $\underline{v}$. Given the growth, the distribution shares for fish starting out as fish class v in period t are calculated in the following way:

$$\alpha_{v\underline{v}its} = \frac{G_{\bar{v}} - (G_v + \sigma_{vis})}{G_{\bar{v}} - G_{\underline{v}}}$$

$$\alpha_{v\bar{v}its} = \frac{(G_v + \sigma_{vits}) - G_{\underline{v}}}{G_{\bar{v}} - G_{\underline{v}}}$$

The calculations of the distribution shares are part of the pre-processing. They are therefore not included as restrictions in the model, but are presented here to facilitate

understanding of the basis on which the model operate.

Upon working on this thesis, an error in how the these shares were calculated was discovered in the work of Hæreid (2011), where the shares were flipped in the original formulation. It is worth noting that this may have been a significant contributor to the detailed weight discretization advocated by (Hæreid, 2011). This matter is however not investigated further, as it is regarded as outside of the scope of this thesis.
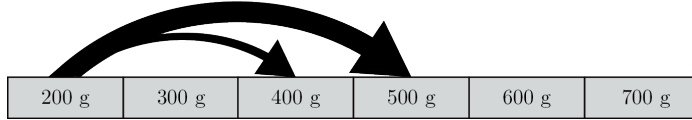
Figure 6.4: Illustration of the growth model

In reality, fish growth rates vary both across and within weight classes. In the model, the variation in growth within a weight class is ignored. All fish in a weight class v are assumed to grow at the same rate, making the weight of each fish in v identical and equal to G_v.

As q_{vits} represent the number of individual fish, updating the value of q_{vits} is in reality an integer-programming problem. To avoid the complexity involved with integer constraints, the variables q_{vits} are allowed to take on real values. Due to the magnitude of the number of the fish, this is a reasonable simplification.

Finally, the number of fish distributed into weight class v after belonging to weight class $\underline{v}$ the in $t-1$ is calculated by first multiplying the number of fish in $\underline{v}$ at the beginning of $t-1$ by the survival rate ζ_{vits}, representing the share of fish left after adjusting for mortality and escape. Then, this number is multiplied by the distribution share $\alpha_{v\bar{v}fts}$ to finally obtain the number of fish distributed into weight class v as a result of growth in time period $t-1$.

Biomass level

The total biomass available is tightly related to the salt water growth described above. However, additional factors affect the biomass level. Summnarized, the number of fish belonging to weight class v at farm i at the beginning of any given time period t, denoted q_{vits}, is determined by three factors:

1. The number of fish distributed into weight class v as a result of growth in time period $t-1$ adjusted for the survival rate

2. The number of fish of weight class v harvested in period t
3. The number of smolt of weight class v released in period t

6.6.3 Logistics

As this is a model which purpose is to generate guidelines for an entire corporation, there is no need for modeling individual vehicles. As such there is assumed that there will be available vehicles upon request. Transportation capacities and times are given for each transportation mode, where their values represent the average capacities and times for each node. Note that these transport capacities and times will be measures of low quality if the implemented network is too crude by aggressively combining a large set of individual facilities into one region. This should be taken into account when analyzing the solution, as using average values can give unrealistic solutions.

The salmon producer normally pay for transportation to the customer. Transportation costs are therefore included in the model objective. The producer do not normally own the transportation vessels, but rather hires capacity from a specialised logistics company (Helgesen, M., Aandahl, P.T., personal communication, 2015). As such, the producer often have to commit to use a certain minimum amount of transportation capacity to get a fair deal with a logistics operator. To model this relation, a punishing term for using too little capacity is included in the objective function. This punishment is meant to represent the economic sanctions presented in the contract as well as the loss of goodwill from the logistics company. Due to lack of information of the signed logistic contracts, it is assumed that the company will have one contract per mode so that the penalty term will be mode specific.

Additionally, it is possible to hire extra capacity in peak periods. Consequently, a surplus variable is introduced representing the amount of extra capacity hired in one time period. Hiring extra capacity at short notice will induce extra costs, both in terms of transaction costs and because external suppliers are expected to act opportunistically by demanding prices surpassing the marginal costs incurred by using capacity hired through long term contracts. Thus, transportation costs are set higher for the extra capacity than when using internal capacity for the same mode and distance.

Transportation capacities are defined per arc per transport mode. This means it is implicitly assumed that all vessels are able to transport all types of products. Furthermore, there is known to be a relatively predictable seasonal patterns in volumes traded in the salmon industry. This indicates that the salmon producer may have

long-term contracts with varying available capacity throughout the year, and as such, the available capacity is modeled as time dependent.

To simplify the model formulation and limit the amount of input data needed, no shipments underway in the beginning of the planning horizon are included in the model formulation. This may give some unrealistic starting effects, as the model would need some time periods to adjust to normal production. However, this is not included because of the lack of data.

6.6.4 Product age and durability

As salmon products are perishable, it is important to include the age of the products in the model formulation, especially since different products deteriorate at different rates. However, accurately keeping track of the age of different batches implies introducing a very large set of age variables, which implies a much more complex model. Instead, age is treated implicitly by exploiting the structure of the node flow network and through the introduction of artificial inventories.

As all fresh HOG salmon is sent directly from the slaughter node to the market node, ensuring that no expired products are sent to the market can be easily done by only including feasible arcs in terms of transportation time. Processed products on the other hand, cannot be treated in a similar fashion as they flow across two separate arcs. This is because it is the sum of the time to transport the raw material to the processing facility, the time spent at the processing facility and the time to transport the finished product to the market that must be below a certain limit. Consequently, the product can travel along two arcs that themselves are feasible, while the combination is infeasible. This concept is illustrated below in Figure 6.5:

In Figure 6.5, the F represents a farming node, P represents a processing node and M represents a market node and all arcs included in the figure are individually feasible in terms of transportation time. However, since the time it takes to travel each arc can vary from a matter of hours and up to over a week, the combination of two slow transportation modes from F to M might result in a total time in transit exceeding the durability of the product for fresh processed products such as fresh fillets. As an example, it will only be feasible to choose boat (arc c) between F and P if airfreight (arc d) is chosen for the latter part.

To ensure feasibility in such a formulation, one option would be to include precedence constraints. However, since this will introduce a relatively large extra set of constraints,
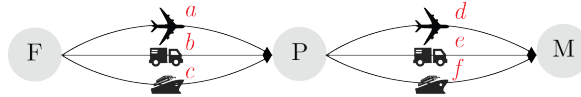
Figure 6.5: Illustration used to highlight the challenge of modeling age of processed products.

a reformulation into a path flow formulation for this part of the model is chosen instead.

Furthermore, as the durability varies greatly between the different processed products, from days for perishable products such as fresh fillets, to months for the more durable processed products, such as smoked salmon and frozen salmon products, the importance of controlling the age varies between the different products. Thus, processed products are divided into the two subset *perishable processed products* and *durable processed products*, where the path flow reformulation is only performed on the former product group.

Both these modeling choices are explained in further detail below.

1. Perishable and durable products

Perishable products are fresh processed products, such as fresh fillets, that have about the same durability as fresh HOG salmon. Durable processed products on the other hand are all non-fresh processed products, such as frozen, cured or smoked products. The rationale behind dividing processed products further into perishable and durable is that these products groups differ in two significant ways.

Firstly, upon beginning the production process of perishable products it is already decided where and when this product should be sold (Helgesen, M., personal communication, 2015). Because of the short durability it is therefore assumed that there exist no stock of such products beyond a temporary stock of products sent out the production day or shortly thereafter. Durable products on the other hand are often produced to stock, and the inventory are sometimes used as a mean to delay sales when prices are low in hope of better prices in the future (Aandahl, P.T., personal
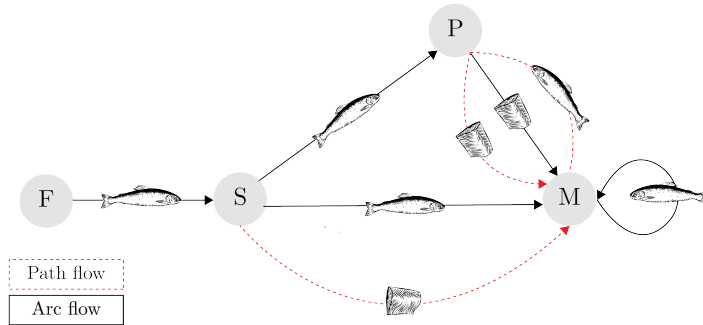
Figure 6.6: Mass flow with paths for perishable processed products and spot purchase. communication, 2015).

Secondly, durability is dependent upon different stages in the production cycle for the two different sets of products. For durable processed products it is assumed that production date determines the durability, whereas the slaughter date determines the durability for perishable processed products. Hence, the durability of durable processed products are controlled by the inclusion of artificial inventories, which will be explained in further detail in Section 6.6.6, whereas the transportation time to the processing facility is disregarded.

2. Path flow formulation for perishable products

For perishable processed products, a path flow formulation is used to control the total time that passes from time of slaughter to delivery. By only including paths that have a feasible total transport time, durability of these products is ensured without adding any age constraints. While it may be unconventional to combine an arc flow and a path flow formulation in one model, this is regarded as the best way to control the age of perishable processed products without increasing the number of variables and constraints excessively.

Each path specify a path from acquiring the raw material and to selling the finished product in the market place. Here, two sets of paths are defined, one from slaughter to market and one form market to market. Both go through a processing node and both include two individual transportation modes for each of the two arcs in the path.

These two sets of paths are illustrated in Figure 6.6, where the first set of paths are

illustrated by the lower red arc. The latter set of paths are included so that also HOG purchased in the market can be utilized to produce both perishable and durable processed products. This path flow formulation exploits the strict structure of the network, where all paths consists of exactly three nodes and two modes. As such, the number of possible paths is $SP * PM + 2MP$, which is significantly less than the number of possible paths in the complete network. This structure can also be exploited when generating feasible paths, as the algorithm only need to combine two and two arcs.

6.6.5 Modeling production

In this problem, the production of processed products can be modeled as a set of splitting processes based on the idea presented by Schütz et al. (2009b), where they represent the value chain as a sequence of production processes rather than a network of production facilities. Here, a distinction is made between combining and splitting processes, where the latter is the relevant process for the problem studied in this thesis.

Generally, a HOG weight class can be used as input for more than one product, and a product can be produced using one of several HOG weight classes. This can be illustrated by an example involving 3 raw material products and 3 finished products. Let raw material 1 be HOG of 2-3 kg, raw material 2 be HOG of 3-4 kg, and raw material 3 be HOG of 4-5 kg. Then, let product 1 be 50 g fillets, product 2 be 100 g fillets and product 3 be 200 g fillets. Let's assume that a 50 g fillet can be produced using either 2-3 kg or 3-4 kg HOG, 100 g fillets can be produced using any HOG size, and that the 200 g fillets can only be produced using the largest HOG size.

A challenge that arises in formulating the mass balance in the traditional is that several of the variables in the example above would have been counted in more than one constraint. To resolve this challenge, a set of production processes $\mathcal{B}$ is introduced. Each production process defines an input product, p, and an output product, q, where p is a HOG weight class, and q is a processed product.

A production process is defined by both the input and the output of the process. For example, producing 50 g smoked salmon portions using 3-4 kg HOG as raw material would be defined as one production process, whereas producing the same 50 g portions of smoked salmon using a 4-5 kg HOG as raw material would be defined as a separate production process.

For illustration, the set of processes $\mathcal{B}$ in the example above would consist of the

following processes:

$$\mathcal{B}_1 : 2 - 3 \text{ kg} \rightarrow 50 \text{ g fillet} \qquad \mathcal{B}_2 : 2 - 3 \text{ kg} \rightarrow 100 \text{ g fillet}$$

$$\mathcal{B}_3 : 3 - 4 \text{ kg} \rightarrow 50 \text{ g fillet} \qquad \mathcal{B}_4 : 3 - 4 \text{ kg} \rightarrow 100 \text{ g fillet}$$

$$\mathcal{B}_5 : 4 - 5 \text{ kg} \rightarrow 100 \text{ g fillet} \qquad \mathcal{B}_6 : 4 - 5 \text{ kg} \rightarrow 200 \text{ g fillet}$$

Furthermore, the sets $\mathcal{B}_{pq}$ and $\mathcal{U}_p$ is introduced. The former is a subset of $\mathcal{B}$ representing all production processes where p is input and q is output. The latter represents a reversed Bill of Materials of product p, meaning all products q that can be made using p as input. Then, the following set can be calculated:

$$\mathcal{B}_{\text{HOG 2-3}} : \{\text{process 1, process 2}\}, \quad \mathcal{U}_{\text{HOG 2-3}} : \{50 \text{ g fillet, } 100 \text{ g fillet}\}$$

$$\mathcal{B}_{\text{HOG 3-4}} : \{\text{process 3, process 4}\}, \quad \mathcal{U}_{\text{HOG 3-4}} : \{50 \text{ g fillet, } 100 \text{ g fillet}\}$$

$$\mathcal{B}_{\text{HOG 4-5}} : \{\text{process 5, process 6}\}, \quad \mathcal{U}_{\text{HOG 4-5}} : \{100 \text{ g fillet, } 200 \text{ g fillet}\}$$

By combining these sets, the challenge described is handled. Let y_{qb} be a decision variable representing the production volume of a product q produced by use of process b. Then, the sum

$$\sum_{q \in \mathcal{U}_p} \sum_{b \in \mathcal{B}_{pq}} y_{qb} \qquad p \in \mathcal{P}^{HOG} \tag{6.1}$$

will represent the total volume of a raw material p used in the production.

6.6.6 Inventory

As previously discussed, a clear distinction between perishable and durable processed products is made in this thesis because of the different importance of controlling the durability for the two product groups. However, it is still important to control how long the product spend in the inventory. The proposed solution to this problem is to divide the physical inventory into a set of artificial inventories where each artificial inventory represents a time interval, for example products between one and two months old. The artificial inventories do not need to be of equal length. A finer time discretization can therefore be used for time intervals of extra importance, for instance close to the product's expiry date.

For each time period, the oldest product batches in each artificial inventory are moved
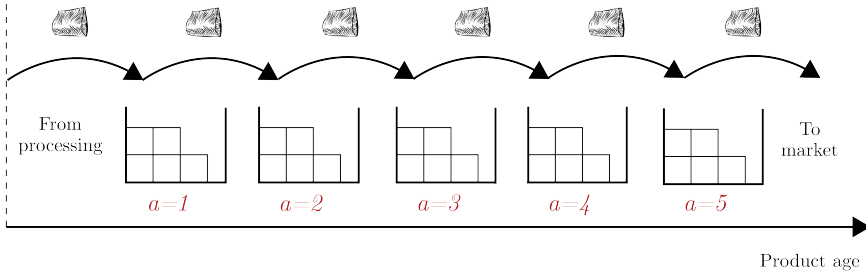
Figure 6.7: Illustration of the artificial inventories.

to the next artificial inventory, as illustrated in Figure 6.7. Here, i_a represent the ath artificial inventory, where i_1 consists of the products with the lowest age. The products can either remain in the same artificial inventory, be sent to the next artificial inventory or sent to the market. As such, the product volume in each artificial inventory can be calculated as the sum of the remaining share from the previous time period and the share sent from previous artificial inventory, minus the volume sent to the market. Note that the produced volume of each product is defined to always enter the first artificial inventory and that products can be sent to the market from all artificial inventories, although a market arc is only included for the last artificial inventory in the figure.

To update the inventory levels, an inventory distribution share $B_{\underline{a}a}$ is introduced, which defines the share of artificial inventory $\underline{a}$ that is moved to artificial inventory a from one time period to the next. As artificial inventory lengths shorter than the length of one time period will not improve the accuracy of the model, the complete matrix of inventory distributions, **B**, can be explicitly defined as follows:

$$\mathbf{B} = \begin{bmatrix} 1 - \frac{|t|}{T_1} & \frac{|t|}{T_1} & 0 & \dots & 0 \\ 0 & 1 - \frac{|t|}{T_2} & \frac{|t|}{T_2} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 - \frac{|t|}{T_n} \end{bmatrix}$$

Here, $|t|$ represents the length of the time period and T_a represents the length of the age interval for artificial inventory a. Thus, the diagonal represents the inventory share that remains in each artificial inventory from one time period and to the next, whereas the off-diagonal represents the inventory share that is moved to the next artificial

inventory. With an artificial inventory length of 4 weeks, the remaining share and moved share becomes 0.75 and 0.25 respectively. Note that this formulation implicitly disposes expired products because $\frac{|t|}{T_n}$ is not included anywhere in the matrix.

An important advantage of this formulation is that the durability requirements can be implicitly controlled by only defining a feasible set of artificial inventories for each product and only allowing feasible combinations of market flows and artificial inventories. As such, arcs with a high travel time cannot be defined for the last artificial inventories. Further, because of different storing requirements, where some products are stored chilled whereas others are stored frozen, there is an individual set of feasible artificial inventories for each product.

In this formulation, explicit durability constraints are traded in exchange for a larger set of inventory constraints and a larger set of flow variables from the processing locations. How large the penalty becomes depends upon the number of artificial inventories included and the number of feasible combinations of artificial inventories and flow variables to the market.

Note that since this model does not differentiate on quality, the model is indifferent to when the products reaches the markets as long as it is within the time requirements set by the decision-maker. Furthermore, as of the authors' knowledge, there is no empirical evidence that suggest this to be an important factor in setting the price, given that the product reaches the market within a reasonable time frame. However, the model will implicitly try to avoid to use the last artificial inventory as this will automatically dispose a share of the products. This might be somewhat unrealistic if the last artificial inventory spans a large time interval, but can be remedied with the use of a shorter time interval for the last artificial inventory. Furthermore, by including individual end values for each artificial and each product, the effect of stocking up a lot of old products at the end of the planning horizon can be avoided.

6.6.7 End of horizon boundary conditions

Measures should be taken to control the end of horizon behaviour of the model. If no such incentives or restrictions were included, the optimal solution would always be to harvest all biomass and sell all products in inventory in the final time period.

There are in principle three ways to handle this. One option is to require a certain biomass and inventory level after in the last period, another is to set a desired end level and include punishment terms in the objective function for deviations. A last

option is to price the biomass and inventory that is left at the end of the last period at a given value.

Alternative 1: Setting minimum levels

By requiring a minimum volume, one get the benefit of full control. The downside of this approach is that there will be no incentives to maintain higher levels of biomass and inventory than the ones specified, so that all optimal solutions will have these exact levels. Furthermore, in the specific case of modeling the salmon farming value chain minimum restrictions on biomass could lead to infeasibilities, for instance in the event of mass death due to a disease outbreak.

Alternative 2: Setting target levels

The benefit of setting a desired end level and including a punishment term for deviating from this in the objective is that one maintain some control while restricting only the model's flexibility through a punishment in the objective function. The degree of aggregation will also affect flexibility. As such, control is maintained. Here, long term plans could be used as end values. Finding the optimal weight on the punishment might however prove a challenge. If the punishment is set too high, the model flexibility diminishes, up to the point where the solution always include the exact desired level. If the punishment is set too low, the end of horizon behaviour is too loosely controlled and one might experience extreme solutions in the last time period. Another downside of this approach is that you lose the realistic representation of the economic trade off made by the producer at the end of the planning horizon.

Alternative 3: Including expected salmon and inventory value in the objective function

By altering the objective function by including the expected future value of salmon and inventory withheld in the last period, one explicitly includes the opportunity cost from leaving the fish and products. This method has the advantage of not restricting the flexibility of the model, and have been widely applied in stochastic models for water power planning, see for instance (Hveding, 1968; Steinsbø, 2008; Stage and Larsson, 1961). The approach models the economic trade off made by the producer in a more realistic way than the previous two. The producer wishes to withhold biomass if the expected salmon value at some point in the future exceeds the differential between the salmon value today and the cost of keeping the fish alive until that time. This trade

off is illustrated in Figure 6.8. For inventory, the rationale is the same, except the inventory cost then trades places with the cost of keeping fish alive.

The drawback of the model is that calculating the pricing of the withheld products is not straightforward, and might require considerable computational efforts. In comparison, when stating a final level using the required volume approach, it is often easier and more intuitive than to find the correct value. It might be natural to require the final reservoir level to equal either some historical mean for the date in question, or some level forecasted by a separate model, such as a strategic salmon production model.
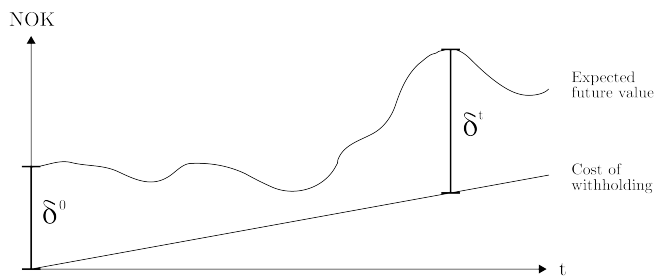
Figure 6.8: Illustration of the economic rationale of withholding biomass

Selection of alternative

Based on the discussion presented, the expected salmon value approach was chosen, as this approach gives a good representation of the economic trade-off facing the producer while not limiting model flexibility.

For the objective value to be a more realistic representation of the financial performance of the organization, there should be an adjustment for the expected values of the initial biomass and inventory levels. However, the goal is not for the objective value to be an accurate measure of financial performance, and as are constant values such an adjustment would not affect the solution. Consequently, no such adjustment is made.

6.7 Model applications

By running the model regularly, the producer should be able to achieve multiple benefits:

- Identify the most beneficial contract structures, and thereby uncover insight intowhich contracts to enter
- Identify parts of the value chain with under- or over-capacity
- Indicate to what degree the transportation policy is beneficial in terms of what transportation modes to use by comparing transportation mode mix in the model solution and in practice
- Disruption management, for example in situations similar to the Chile ISA crisis of 2008 or the Russian trade ban of 2014

Although the model is developed with a global, vertically integrated company in mind, less integrated companies controlling only part of the value chain could also benefit from the model. As discussed in Section 4.1, the model solution is meant to be used as decision support in making centralized production and sales plans at a senior level.

The model can also be adapted to perform various tests. An example would be to relax a set of restrictions such as capacity restrictions to perform cost/benefit analyses regarding future capacity investments. Another example is to run the model when relaxing all constraints regarding government regulations. Then, numeric values representing the potential profit gains if these restrictions were changed or lifted completely could be obtained.

The stochastic approach is demanding in terms of data, but as enterprise resource planning (ERP) systems and other software collecting and storing data are becoming more and more commonplace, much of the data needed are already available. It is required that the company has data on biomass volumes and inventory levels available. As producers need to keep track of the biomass development and volumes in inventory for other purposes, this should not pose a challenge. In addition, data on internal production and transportation times and costs and capacities are also required.

Chapter 7

Model formulation

In this chapter, the general mathematical model is presented. All sets, indices, constants and variables used in to describe the mathematical model are defined in Section 7.1. A presentation of the objective function is given in Section 7.2, whereas all constraints are described in Section 7.3. As all constraint apply to all scenarios s, the reader should be aware that the index s is omitted from the objective function and the constraints due to a relative high number of indices in most variables and to ease reading.

7.1 Definitions

Sets are denoted by capital, calligraphic letters with corresponding indexes and deterministic data are capitalized. Decision variables are denoted in lower case Latin letters, while Greek letters are used to denote stochastic data. All volumes refer to weight given in kilograms, while prices and costs are per kilogram.

7.1.1 Sets and indices

Time and scenarios

$\mathcal{T}$	-	Set of time periods t
$\mathcal{S}$	-	Set of scenarios s
$\mathcal{K}$	-	Set of split nodes k in the scenario tree
$\mathcal{S}_k$	-	Set of scenarios passing through split node k

Nodes and transportation

$\mathcal{R}$	-	Set of farming regions r

$\mathcal{N}$	-	Set of all network nodes i, j
$\mathcal{N}^F$	-	Set of farming nodes i, $\mathcal{N}^F \subset \mathcal{N}$
$\mathcal{N}_r^F$	-	Set of farming nodes in farming region r, $\mathcal{N}_r^F \subset \mathcal{N}^F$
$\mathcal{N}^P$	-	Set of processing nodes i, $\mathcal{N}^P \subset \mathcal{N}$
$\mathcal{N}_p^P$	-	Set of processing nodes that can produce product p, $\mathcal{N}_p^P \subset \mathcal{N}^P$
$\mathcal{N}^S$	-	Set of slaughtering nodes i, $\mathcal{N}^S \subset \mathcal{N}$
$\mathcal{N}^M$	-	Set of market nodes i, $\mathcal{N}^M \subset \mathcal{N}$
$\mathcal{M}$	-	Set of transportation modes m
$\mathcal{F}$	-	Set of paths f used for modelling transportation of perishable fresh products
$\mathcal{F}_i$	-	Set of paths that include node i, $\mathcal{F}_i \subset \mathcal{F}$
$\mathcal{F}_{ijm}$	-	Set of paths that travel along arc (i, j) by transportation mode m, $\mathcal{F}_{ijm} \subset \mathcal{F}$
$\mathcal{O}_i$	-	Set of origins for node i, $\mathcal{O}_i \subset \mathcal{N}$
$\mathcal{D}_i$	-	Set of destinations for node i, $\mathcal{D}_i \subset \mathcal{N}$
$\mathcal{O}_i^F$	-	Set of paths originating in node i
$\mathcal{D}_i^F$	-	Set of paths arriving in node i

Products and inventory

$\mathcal{V}$	-	Set of fish weight classes v
$\mathcal{V}^H$	-	Set of fish weight classes v that are allowed to be harvested, $\mathcal{V}^H \subset \mathcal{V}$
$\mathcal{V}_p^H$	-	Set of fish weight classes included in HOG weight class p, $\mathcal{V}_p^H \subset \mathcal{V}^H$
$\mathcal{P}$	-	Set of all products p, q
$\mathcal{P}^{HOG}$	-	Set of all fresh HOG weight classes p, $\mathcal{P}^{HOG} \subset \mathcal{P}$
$\mathcal{P}^P$	-	Set of all perishable processed products p, $\mathcal{P}^P \subset \mathcal{P}$
$\mathcal{P}^{DUR}$	-	Set of all durable processed products p, $\mathcal{P}^{DUR} \subset \mathcal{P}$
$\mathcal{L}$	-	Set of inventory types l
$\mathcal{P}_l^L$	-	Set of products that can be stored in inventory type l, $\mathcal{P}_l^L \subset \mathcal{P}^{DUR}$
$\mathcal{A}$	-	Set of all artificial inventories a, differentiated by different product age intervals
$\mathcal{A}_p^P$	-	Set of artificial inventories related to product p, $\mathcal{A}_p^P \subset \mathcal{A}$
$\mathcal{B}$	-	Set of all production processes b
$\mathcal{B}_{pq}$	-	Set of all production processes having HOG class p as input and processed product q as output, $\mathcal{B}_{pq} \subset \mathcal{B}$
$\mathcal{U}_p$	-	Set of all processed products q that can be produced using HOG weight class p as raw material

Contracts

$\mathcal{C}$	-	Set of contracts offered during the planning horizon
$\mathcal{C}^F$	-	Set of fixed price contracts, $\mathcal{C}^F \subset \mathcal{C}$
$\mathcal{C}^A$	-	Set of adjustable contracts, $\mathcal{C}^A \subset \mathcal{C}$
$\mathcal{C}^{PA}$	-	Set of partly adjustable contracts, $\mathcal{C}^{PA} \subset \mathcal{C}$
$\mathcal{C}_c^T$	-	Set of time periods in which a delivery to contract c is made
$\mathcal{C}_{pit}$	-	Set of contracts involving product p and market i, having a delivery in time perid t, $\mathcal{C}_{pit} \subset \mathcal{C}$, $t \subset \mathcal{C}_c^T$

7.1.2 Deterministic data

Initial conditions

B_{vi}^0	-	Initial biomass given by number of fish of farming weight class v at farm i
I_{pia}^0	-	Initial inventory of product p in artificial inventory a at processing facility i
E_{pit}	-	Total volume of product p to be delivered in market i in period t according to current contract obligations
R_{vit}	-	Number of smolt of farming weight class v to be released at farm i at the beginning of time period t

Capacities, times and costs

N_i^F	-	Maximum allowable biomass at farming node i
N_r^R	-	Maximum allowable biomass in farming region r
S_i^{CAP}	-	Slaughtering capacity of slaughterhouse i, measured in number of fish per time period
C_v^C	-	Caring cost of keeping one kg of fish of farm weight class v one more time period
C_p^P	-	Cost of producing one kilogram of processed product group p
C_{pij}^{TAR}	-	Tariff for product p transported from node i to node j
C_{li}^I	-	Cost of keeping one kilogram of processed product in inventory type l for one time period at processing node i
C_{ijm}^T	-	Transportation cost from node i to node j using transportation mode m

C_{pf}^{F}	-	Combined transportation and tariff costs when transporting product p across path f
I_{il}^{CAP}	-	Inventory capacity of inventory type l at processing facility i
P_{pi}^{CAP}	-	Processing capacity; maximum number of kilograms of processed product p that can be produced at processing facility i per time period
T_{ijm}^{T}	-	Transportation time from node i to node j using transportation mode m
T_{f}^{F}	-	Transportation time from stating point to processing node in path f
T_{ijmt}^{CAP}	-	Maximum transportation capacity from node i to node j using transportation mode m in time period t
T_{m}^{LIM}	-	Minimum transportation capacity using transportation mode m per time period the producer is committed to

Contracts

Q_c	-	Volume to be delivered to fulfill contract c at each delivery
U_c	-	Price of contract c
A_c^C	-	Adjustment factor: The factor with which the relevant price difference is multiplied by when adjusting the price of contract c
F_c	-	Adjustment limit: The value in NOK the spot price and contract price for contract c must differ for the contract price to be adjusted, $c \in \mathcal{C}^A$
M_c^C	-	Market in contract c
P_c^C	-	Product in contract c

Punishment parameters

| K_m^{TMAX} | - | Punishment term per kilogram transported using transportation mode m after surpassing the maximum capacity m |
| K_m^{TMIN} | - | Punishment term per kilogram the transported volume using transportation mode m falls short of the minimum use of m the producer is bound by |

Parameters used to control end of horizon effects

| V_v^B | - | Expected future value of one fish of farming weight class v |

V_v^P -   Expected future value of one kilogram of processed product p

Other parameters

π_s -   Probability for scenario s

Q_k -   The time period corresponding to split node k

G_v -   Weight of fish belonging to weight class v

L^S -   Slaughtering yield, defined as the weight of a slaughtered fish in percentage of a live fish

L_b^P -   Process yield, defined as the volume of processed product q produced using process b given as a percentage of the volume of raw material p going into the process

$B_{\underline{a}a}$ -   Share of artificial inventory $\underline{a}$ that is moved to artificial inventory a from one time period to the next

7.1.3 Stochastic data

σ_{vits} -   Growth in kilograms for fish of farming weight class v at farm i in time period t and scenario s

ζ_{vits} -   Survival rate for fish in weight class v at farm i in time period t and scenario s

$\alpha_{\underline{v}vits}$ -   Share of fish in weight class $\underline{v}$ that has grown to become part of class v due to their growth at farm i in time period t and scenario s ($\underline{v} \leq v$)

ρ_{pits}^S -   Spot sales price for product p in market i in time period t and scenario s

ρ_{pits}^B -   Spot purchase price for HOG salmon of class p in market i in time period t in scenario s

ρ_c^C -   Contract price for contract c when the contract is made available

7.1.4 Variables

Helping variables

q_{vits} -   Number of fish belonging to fish weight class v at farm i in time period t in scenario s

i_{pitas} -   Volume of product p stocked in artificial inventory a at processing node i at the end of time period t in scenario s

o_{cs}	-	Revenue generated form contract c in scenario s
a_{ijmts}^{MAX}	-	Number of kilograms the volume of products transported by transportation mode m in time period t in scenario s that falls short of the minimum commitment to using m
a_{mts}^{MIN}	-	Number of kilograms of products transported by transportation mode m in time period t in scenario s surpassing the maximum capacity of m
z_{cpit}^{C}	-	Volume of product p sold trough contract c in market i in period t

Decision variables

h_{vits}	-	Number of fish harvested in HOG weight class v and harvested at farm i in time period t in scenario s
$x_{pijt\bar{t}s}^{LIVE}$	-	Volume of live fish belonging to HOG weight class p sent from farming node i to slaughter node j in time period t, with arrival in time period $\bar{t}$ $(t \le \bar{t})$
$x_{pijmt\bar{t}s}^{HOG}$	-	Volume of fresh HOG of weight class $p \in \mathcal{P}^{HOG}$ sent from node i to node j by transportation mode m in time period t in scenario s, with arrival in time period $\bar{t}$ $(t \le \bar{t})$
$x_{pijmt\bar{t}s}^{RAW}$	-	Volume of fresh HOG dedicated to be used as raw material for durable products, sent from slaughter node i to processing node j in time period t using transport mode m, with arrival in time period $\bar{t}$ $(t \le \bar{t})$
$x_{pijmt\bar{t}as}^{DUR}$	-	Volume of durable processed product p from artificial inventory a that is transported from processing facility i to market j by mode m in time period t in scenario s, with arrival in time period $\bar{t}$ $(t \le \bar{t})$
$r_{pbft\bar{t}s}^{P}$	-	Volume of perishable product p produced by using process b that is transported through path f in time period t in scenario s, with arrival in time period $\bar{t}$ $(t \le \bar{t})$
y_{pbits}^{DUR}	-	Volume of durable product $p \in \mathcal{P}^{D}$ produced using process b at processing facility i in scenario s
z_{pits}^{S}	-	Volume of product p sold in spot market i in time period t in scenario s
z_{pits}^{B}	-	Volume of product p bought in spot market i in timer period t in scenario s
δ_{c}^{C}	-	Binary variable, 1 if a contract c is entered, 0 otherwise

7.1.5 Variable description
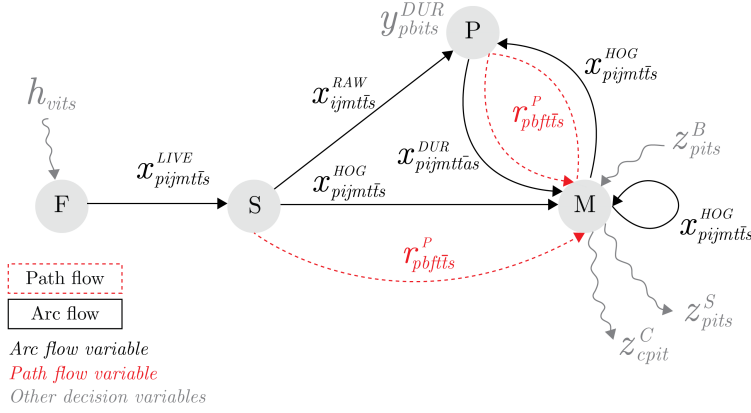
Figure 7.1: Variable illustration.

For a better understanding of the relationship between the variables, the reader is referred to Figure 7.1. Here, all flow variables are denoted x, except for the path flows of perishable processed products that are denoted $r^P_{pbft\bar{t}}$. Note that a processing facility in reality is included in path drawn from S to M. As the fish is transported in several different states, the flow variables are given a superscript according to the state in which the fish is transported. As such, the flow of live fish to the slaughter node is denoted $x^{LIVE}_{pijt\bar{t}}$, whereas flow of fresh HOG salmon to and from the market and flow of fish to secondary processing are referred to as $x^{HOG}_{pijmt\bar{t}}$ and $x^{RAW}_{pijmt\bar{t}}$, respectively. The flow variables do mostly follow the same structure. Note however that

- the raw material flow variable $x^{LIVE}_{ijt\bar{t}s}$ do not have a transportation mode index, as all transport between farm nodes and slaughter nodes is conducted by well boat
- the durable product flow variable $x^{DUR}_{pijmt\bar{t}as}$ do have an additional index a representing the artificial inventory the product is drawn from
- the flow variable $r^P_{pft\bar{t}s}$ does not have neither node nor transportation mode indexes, as both are defined by the path itself
- the production volume variable y^{DUR}_{pits} is only defined for the durable products. The production volume of perishable products is implicitly defined by the path flow volumes.

Furthermore, other decision variables are included in Figure 7.1 at their respective nodes to illustrate their relationship to the flow variables. Note however that z^C_{cpit} are included instead of δ_c, as z^C_{cpit} represents the volume sold through contracts, even

though this volume is explicitly defined by the value of δ_c. Also note that these two variables do not have any s index, as all contract decisions are defined as first stage decisions. The alternative would have been to use a set of constraints to force δ_c to be equal over all scenarios. However, initial tests indicated that such a formulation would give a small penalty in solution time, so the explicit formulation were chosen.

As a consequence of modeling the decisions as instantaneous, the biomass variable q_{vits} represent the biomass at farm i after the smolt release and harvest of period t is conducted. This relationship is illustrated by Figure 7.2.
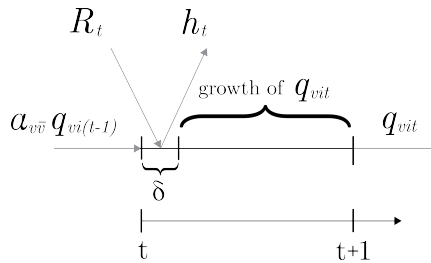
Figure 7.2: Time line for decisions and variables regarding biomass.

In contrast, the inventory variable i_{pitas} represent the outgoing inventory of time period t, as illustrated in Figure 7.3.
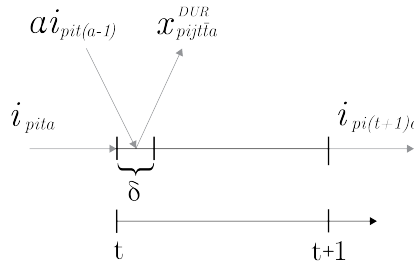
Figure 7.3: Time line for decisions and variables regarding inventory.

7.2   Objective function

The model objective is to maximize expected total profits from the value chain over the planning horizon. A conceptual representation of the model is presented below. Here, the first three terms represent all market transactions, both from spot sale, spot purchase and sale through long-term contracts. The next four terms represent

operational costs included in the model, whereas end of horizon effects and penalties are included in the last terms.

$$
\begin{aligned}
Max\ Z = \quad & Contract\ revenues + Spot\ revenues - Spot\ purchase\ costs \\
& - Transportation\ costs - Caring\ costs - Processing\ costs - Inventory\ costs \\
& + End\ of\ horizon\ effects + Penalty\ term
\end{aligned}
$$

Before the full objective term is presented, each term will be briefly addressed to give the reader a clear understanding of the models objective function. A short discussion on modeling choices made when formulating the objective and their potential consequences follows. Note that as all objective terms are summed over all scenarios s, this sum and the scenario specific probability are omitted in the explanations below.

7.2.1 Contract revenues

$$
Contract\ revenues = \sum_{c \in \mathcal{C}} o_c
$$

The total revenues generated through long term contracts are calculated by adding the contract generated from a contract c, o_c, for all contracts.

7.2.2 Spot revenues

$$
Spot\ revenues = \sum_{p \in \mathcal{P}} \sum_{i \in \mathcal{N}^M} \sum_{t \in \mathcal{T}} \rho_{pit}^S z_{pit}^S
$$

The spot revenues term calculates the total revenue generated from selling different products in the spot marked by multiplying the spot sales price, ρ_{pit}^S, and the volume sold in each market i, z_{pit}^S.

7.2.3 Spot purchase costs

$$
Spot\ purchase\ costs = \sum_{p \in \mathcal{P}^{HOG}} \sum_{i \in \mathcal{N}^M} \sum_{t \in \mathcal{T}} \rho_{pit}^B z_{pit}^B
$$

The spot purchases term calculates the costs of spot purchases, where z_{pit}^{B} is the purchase volume and ρ_{pits}^{P} is the spot purchase price. The spot prices differ by both HOG salmon weight class and market region, as explained in Section 3.4.

7.2.4 Transportation costs

$$Transportation\ costs =$$

$$\sum_{p\in\mathcal{P}\backslash\mathcal{P}^{P}} \sum_{i\in\mathcal{N}} \sum_{j\in\mathcal{D}_{i}} \sum_{m\in\mathcal{M}} \sum_{t\in\mathcal{T}} \sum_{\bar{t}\geq t} (C_{ijm}^{T} + C_{pij}^{TAR}) \left(x_{pijmt\bar{t}}^{HOG} + x_{pijmt\bar{t}}^{RAW} + \sum_{a\in\mathcal{A}_{p}} x_{pijmt\bar{t}a}^{DUR} \right)$$

$$+ \sum_{p\in\mathcal{P}^{P}} \sum_{f\in\mathcal{F}} \sum_{t\in\mathcal{T}} \sum_{\bar{t}\geq t} C_{pf}^{F} r_{pft\bar{t}}^{P}$$

The transportation costs are mainly included in order to differentiate which mode to use. Otherwise, the fastest transportation mode would always be preferred. The taxation costs are included in order to make the problem more realistic, as tolls and tariffs in some cases heavily influence trade patterns in the industry as discussed in Section 3.2 and Section 3.4.

The transportation costs term calculates the combined costs of transportation and tariffs between each node pair (i, j) by multiplying the total volume transported and the sum of the transportation cost and tariff factor per kilogram. As tariff rates differ in regards to what type of product is transported, the rates are defined for each product p. The transportation cost is also differentiated by transportation modes m, as faster shipping alternatives are generally more expensive than slower ones. The shipping rates are assumed to be constant. Note that T_{pf}^{C} if the total cost for using path f.

7.2.5 Caring costs

$$Caring\ costs = \sum_{v\in\mathcal{V}^{H}} \sum_{i\in\mathcal{N}^{P}} \sum_{t\in\mathcal{T}} G_{v} C_{v}^{C} q_{vit}$$

The caring costs term represent the costs associated with caring for the fish one additional time period, consisting mainly of feeding costs. Here, the total caring costs are

calculated by multiplying the cost of caring C_v^C, and the number of live fish q_{vit}. Note that caring costs are only included for harvestable fish classes. The reason for this is that the fish must reach a harvestable weight before it is included in any decision-making, i.e. whether it should be harvested now or not.

7.2.6 Processing costs

$$Processing\ costs = \sum_{p \in \mathcal{P}^D} \sum_{i \in \mathcal{N}^P} \sum_{t \in \mathcal{T}} C_p^P y_{pit}^{DUR} + \sum_{p \in \mathcal{P}^P} \sum_{b \in \mathcal{B}} \sum_{i \in \mathcal{N}^P} \sum_{f \in \mathcal{F}_i} \sum_{t \in \mathcal{T}} \sum_{\bar{t} \geq t} C_p^P r_{pbft\bar{t}}^P$$

The processing cost term represent the marginal costs of transforming HOG into secondary processed products. Here, C_p^P is the marginal cost of producing one more kilo of product p, whereas $x_{pbft\bar{t}}^P$ and y_{pit}^{DUR} represent the volume of perishable and non-perishable processed products produced at each processing location, respectively. As for caring costs, only variable costs are included, assuming that all fixed costs will incur irrespective of the product-mix. The marginal cost is also assumed to be constant.

7.2.7 Inventory costs

$$Inventory\ costs = \sum_{l \in \mathcal{L}} \sum_{p \in \mathcal{P}_l^L} \sum_{i \in \mathcal{N}^P} \sum_{t \in \mathcal{T}} \sum_{a \in \mathcal{A}_p} C_l^I i_{pita}$$

The term representing the inventory costs are calculated by multiplying the cost of storing one kilo of product in inventory type l for one time period, C_l^I, and the volume of each product p stored at each processing location, i_{pita}. The primary function of including inventory costs is to emphasize that storing ties up working capital and also incur discernible costs (Michalski, 2008).

7.2.8 End of horizon effects

$$End\ of\ horizon\ effects = \sum_{v \in \mathcal{V}} \sum_{i \in \mathcal{N}^F} V_v^B G_v q_{vi|\mathcal{T}|} + \sum_{p \in \mathcal{P}^D} \sum_{i \in \mathcal{N}^P} \sum_{a \in \mathcal{A}_p} V_p^P i_{pi|\mathcal{T}|a}$$

A term that controls the end of horizon effects are included in the objective function to avoid strange model behavior, as discussed in Section 6.6.7. The expected future

value of the biomass at the end of the planning horizon (ESV) and the expected value of the products in outgoing inventory (EPV). Note that $|\mathcal{T}|$ represent the last time period in the planning horizon.

7.2.9 Penalty terms

$$Penalty\ terms = \sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}}K_m^{TMAX}a_{mt}^{TMAX} + \sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}}K_m^{TMIN}a_{mt}^{TMIN}$$

The penalty terms are included to give incentives for a leveled use of transportation capacity. The first term gives a punishment for surpassing the maximum transport capacity by multiplying the cost of additional capacity K^{TMAX} by the additional capacity used a_{mt}^{TMAX}. The second term represent a punishment for using too little capacity by multiplying the punishment term K_m^{TMIN} by a_{mt}^{TMIN}.

7.2.10 Full objective function

$$\max Z = \sum_{s\in\mathcal{S}}\pi_s\left\{\sum_{c\in\mathcal{C}}o_{cs} + \sum_{t\in\mathcal{T}}\left[\sum_{p\in\mathcal{P}}\sum_{i\in\mathcal{N}^M}\rho_{pits}^S z_{pits}^S - \sum_{p\in\mathcal{P}^{HOG}}\sum_{i\in\mathcal{N}^M}\rho_{pits}^B z_{pits}^B\right.\right.$$

$$-\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{D}_i}\sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}}\sum_{\bar{t}\geq t}(C_{ijm}^T + C_{pij}^{TAX})\left(\sum_{p\in\mathcal{P}^{HOG}}(x_{pijmt\bar{t}s}^{HOG} + x_{pijmt\bar{t}s}^{RAW}) + \sum_{p\in\mathcal{P}^D}\sum_{a\in\mathcal{A}_p}x_{pijmt\bar{t}as}^{DUR}\right)$$

$$-\sum_{p\in\mathcal{P}^P}\sum_{b\in\mathcal{B}}\sum_{f\in\mathcal{F}}\sum_{t\in\mathcal{T}}\sum_{\bar{t}\geq t}(T_f^C + C_p^P)r_{pbft\bar{t}s}^P - \sum_{p\in\mathcal{P}^D}\sum_{b\in\mathcal{B}}\sum_{i\in\mathcal{N}^P}\sum_{t\in\mathcal{T}}C_p^P y_{pbits}^{DUR}$$

$$-\sum_{l\in\mathcal{L}}\sum_{p\in\mathcal{P}_l^L}\sum_{i\in\mathcal{N}^P}\sum_{t\in\mathcal{T}}\sum_{a\in\mathcal{A}_p}C_l^I i_{pitas} - \sum_{v\in\mathcal{V}}\sum_{i\in\mathcal{N}^P}\sum_{t\in\mathcal{T}}\sum_{s\in\mathcal{S}}C_v^C q_{vits}$$

$$-\sum_{m\in\mathcal{M}}\left(K_m^{TMAX}a_{mts}^{TMAX} + K_m^{TMIN}a_{mts}^{TMIN}\right)\bigg]$$

$$\left.+\sum_{v\in\mathcal{V}}\sum_{i\in\mathcal{N}^F}V_v^B G_v q_{vi|\mathcal{T}|s} + \sum_{p\in\mathcal{P}^D}\sum_{i\in\mathcal{N}^P}\sum_{a\in\mathcal{A}_p}V_p^P i_{pi|\mathcal{T}|as}\right\}$$

Although the objective function aims to maximize the profits it is important to note that the expression is in fact not an accurate expression of expected future profits,

as the objective function only includes cost elements relevant to the decision making process. Consequently, no fixed costs are included in the model objective as fixed costs would not affect the solution in any way. Furthermore, all costs up until the point where the fish is ready to leave the slaughter node are omitted, except for the caring cost. As all fish have to be slaughtered prior to selling, these costs would also not affect the solution.

As a simplification, the time value of money is ignored in the objective. This will result in solutions that are indifferent to when in the planning horizon income is generated, as opposed to a model where net present value or a similar measure was maximized which would favour income today over income at the end of the horizon. However, with a planning horizon of one year, this difference is virtually negligible.

7.3 Constraints

The production and sale of HOG salmon and salmon based products are limited by the biomass development, production, inventory and transportation capacities, durability requirements and market properties. Below, a mathematical representation of these relations are presented.

7.3.1 Biomass development constraints

Initial biomass

$$q_{vi1} + h_{vi1} = B_{vi}^0 \qquad v \in \mathcal{V}, \ i \in \mathcal{N}^F. \tag{7.1}$$

The biomass in the first time period, q_{vi1}, is set equal to initial biomass, B_{vi}^0, as there is no intial harvest. This is ensured by Constraints (7.1).

Growth

$$q_{vit} - \sum_{\underline{v} \leq v} \alpha_{\underline{v}vi(t-1)} \zeta_{vi(t-1)} q_{\underline{v}i(t-1)} + h_{vit} = R_{vit} \qquad v \in \mathcal{V}, \ i \in \mathcal{N}^F, \ t \in \mathcal{T} \setminus \{1\}. \tag{7.2}$$

Furthermore, Constraints (7.2) keep track of the development in biomass from one period to the next using the methodology described in Section 6.6.2. The number of

fish at farming location i in farming weight class v in the beginning of time period t, q_{vit}, is calculated as the sum of all fish growing into farming weight class v during previous time period added with the smolt release into farming weight class v and subtracted with the number of fish harvested from farming weight class v at that location. The number of fish that have grown into farming weight class v, is calculated as the sum of all fish growing into farming weight class v from farming weight classes smaller than or equal to v, denoted $\underline{v}$, adjusted by the survival rate from the previous time period, $\zeta_{vi(t-1)}$. This also includes the number of fish that remain in farming weight class v from the previous time period. Note that Constraints (7.2) are not defined for the first time period, as the initial biomass is defined by (7.1).

7.3.2 Harvesting and slaughtering constraints

Maximum allowable biomass

$$\sum_{v \in \mathcal{V}} G_v q_{vit} \leq Q_i^F \qquad\qquad i \in \mathcal{N}^F,\ t \in \mathcal{T}, \qquad (7.3)$$

$$\sum_{v \in \mathcal{V}} \sum_{i \in \mathcal{N}_r^R} G_v q_{vit} \leq Q_r^R \qquad\qquad r \in \mathcal{R},\ t \in \mathcal{T}. \qquad (7.4)$$

Because of the regulations described in Section 2.3, the salmon producer must obtain licenses in the regions it wants to operate, each with a defined upper limit for biomass. As such, the maximum allowable biomass MAB at each farming location and within each farming region are limited by Constraints (7.3) and Constraints (7.4) respectively. Here, the current licences held by the salmon producer and region regulations within each country determines the scalar values on the parameters Q_i^F and Q_r^R. Note that not all producing countries have both farm and regional regulations, so Constraints (7.3) and Constraints (7.4) will not be defined for all farms and regions when implemented on a real network.

Slaughter capacity

$$\sum_{v \in \mathcal{V}^H} \sum_{j \in \mathcal{O}_i} G_v h_{vjit\bar{t}} \leq S_i^{CAP} \qquad i \in \mathcal{N}^S,\ t \in \mathcal{T}. \qquad (7.5)$$

Constraints (7.5) ensure that the volume sent to each slaughtering location i cannot exceed the slaughtering capacity per time period, S_i^{CAP}, measured in kilograms. As

one farming location may send its products to several slaughtering locations, Constraints (7.5) must be summed over all farming locations allowed to deliver fish to that slaughtering location.

Note that the available biomass gives an upper limit to the harvest volume h_{vit}. However, due to the relation between the biomass variable q_{vit} and the harvest volume h_{vit} given by Constraints (7.2) combined with the non-negativity constraints ensure that the harvest volume can never exceed the biomass available.

7.3.3 Mass balance constraints

The mass balance constraints presented in this section ensures correct mass flow behaviour in the network. As these depends on which node they are valid for, the constraints are divided into groups based on which part of the network they are valid for.

Mass balances at farming nodes

$$\sum_{v \in \mathcal{V}_p^H} \sum_{i \in \mathcal{N}^F} G_v h_{vit} - \sum_{j \in \mathcal{D}_i} \sum_{\bar{t} \geq t} x_{pijt\bar{t}}^{LIVE} = 0 \qquad p \in \mathcal{P}^{HOG}, \ i \in \mathcal{N}^F, \ t \in \mathcal{T}. \qquad (7.6)$$

Constraints (7.6) ensure that the mass balances for each farming node i and sales weight class p are satisfied in each time period t, i.e. the total volume of fish of farming weight class v belonging to sales weight class p, $\mathcal{V}_p^H$, harvested at farm i equals the total volume of product p sent from farm i to all slaughter nodes j using all transportation modes m arriving in any time period $\bar{t} \geq t$.

In the growth model, the fish is treated as numbers of fish in each weight class. In all other parts of the model only volumes of fish are considered, so Constraints (7.6) are also used to translate number of fish into a volume of fish in each sales weight class p by multiplying the number of fish harvested and the average weight of each farming weight class v, G_v.

Mass balances at slaughtering nodes

$$\sum_{j \in \mathcal{O}_i} \sum_{\underline{t} \leq t} L^S x^{LIVE}_{pji\underline{t}t} - \sum_{j \in \{\mathcal{D}_i \cap \mathcal{N}^P\}} \sum_{m \in \mathcal{M}} \sum_{\bar{t} \geq t} x^{RAW}_{pijmt\bar{t}} - \sum_{j \in \{\mathcal{D}_i \cap \mathcal{N}^M\}} \sum_{m \in \mathcal{M}} \sum_{\bar{t} \geq t} x^{HOG}_{pijmt\bar{t}}$$

$$- \sum_{q \in \mathcal{U}_p} \sum_{b \in \mathcal{B}_{pq}} \sum_{f \in \mathcal{O}^F_i} \sum_{\bar{t} \geq t} \frac{1}{L_b} r_{qbft\bar{t}} = 0 \qquad p \in \mathcal{P}^{HOG}, \; i \in \mathcal{N}^S, \; t \in \mathcal{T} \qquad (7.7)$$

Constraints (7.7) ensure mass balance in the slaughter nodes. The volume flowing into the node equals the sum of HOG sent from all farms belonging to the set of origin nodes $\mathcal{O}_i$ to the slaughter node i arriving in time period t. Due to the weight loss related to the slaughter process, the incoming volume is divided by the slaughter yield factor L^S. This sum should equal the volume flowing out of the node, namely the flow of HOG to the markets, the flow of HOG used as raw material for durable products $x^{RAW}_{pijmt\bar{t}}$, and the flow of HOG used as raw material for perishable products $r_{qbft\bar{t}}$. Note that the latter flow is adjusted for the weight loss connected to the transformation from HOG to perishable products when using production process b by multiplying with the production yield factor L^P_b.

Mass balances at processing nodes

$$\sum_{j \in \mathcal{O}_i} \sum_{m \in \mathcal{M}} \sum_{\underline{t} \leq t} x^{RAW}_{pjimtt} - \sum_{q \in \mathcal{U}_p} \sum_{b \in \mathcal{B}_{pq}} \frac{1}{L^P_b} y^{DUR}_{qbit} = 0 \qquad p \in \mathcal{P}^{HOG}, \; i \in \mathcal{N}^P, \; t \in \mathcal{T}.$$

$$(7.8)$$

Constraints (7.8) ensure mass balance in the processing nodes. The volume of a HOG weight class p arriving in a processing node at time t must equal the sum of all durable products produced using p as raw material. Here, the flow of raw material, $x^{RAW}_{pjimt\underline{t}}$, is the sum of the flows from both the slaughtering nodes and HOG salmon purchased in the market to be used as input into the production of VAP. That volume is calculated by adding the production volume of all durable products q produced by a process $\mathcal{B}_{pq}$, where $\mathcal{B}_{pq}$ is defined as the set of processes where HOG weight class p is used as raw material for production of durable product q.

In order to adjust for the weight loss of the production process, the production volume is divided by the production yield factor L_b, thereby adjusting for weight loss due to for example removal of the skin, bones, head and tail. Here, it is assumed that all raw material arriving in the beginning of a time period is processed in the same time

period. Note that as perishable processed products are sent to the markets through separate paths, production constraints for such products are be omitted.

Mass balance in market nodes - sales

$$\sum_{j \in \mathcal{O}_i} \sum_{m \in \mathcal{M}} \sum_{\underline{t} \leq t} x^{HOG}_{pjim\underline{t}t} - \sum_{c \in \mathcal{C}_{pit}} z^C_{cpit} - z^S_{pit} - E_{pit} = 0 \quad p \in \mathcal{P}^{HOG} \ i \in \mathcal{N}^M, \ t \in \mathcal{T},$$

(7.9)

$$\sum_{f \in \mathcal{D}^F_i} \sum_{bin\mathcal{B}} \sum_{\underline{t} \leq t} r^P_{pbf\underline{t}t} - \sum_{c \in \mathcal{C}_{pit}} z^C_{cpit} - z^S_{pit} - E_{pit} = 0 \quad p \in \mathcal{P}^P, \ i \in \mathcal{N}^M, \ t \in \mathcal{T}.$$

(7.10)

$$\sum_{j \in \mathcal{O}_i} \sum_{m \in \mathcal{M}} \sum_{\underline{t} \leq t} \sum_{a \in \mathcal{A}_p} x^{DUR}_{pjim\underline{t}ta} - \sum_{c \in \mathcal{C}_{pit}} z^C_{cpit} - z^S_{pit} - E_{pit} = 0 \quad p \in \mathcal{P}^D, \ i \in \mathcal{N}^M, \ t \in \mathcal{T}$$

(7.11)

The total volume of a product p arriving in market i in time period t equals the sum of the volume of product p sold in market i in time period t through spot transactions, z^S_{pit}, and contracts, z^C_{cpit}. The sales constraints are specified for each of the different product categories in Constraints (7.9) - (7.11) representing the sales of fresh HOG salmon, perishable products and durable products respectively.

Mass balance market nodes - spot purchases

$$z^B_{pit} - \sum_{j \in \{\mathcal{D}_i \cup \mathcal{N}^M\}} \sum_{m \in \mathcal{M}} \sum_{\bar{t} \geq t} x^{HOG}_{pijmt\bar{t}} + \sum_{j \in \{\mathcal{D}_i \cup \mathcal{N}^P\}} \sum_{m \in \mathcal{M}} \sum_{\bar{t} \geq t} x^{RAW}_{pijmt\bar{t}}$$

$$+ \sum_{f \in \mathcal{O}^F_i} \sum_{q \in \mathcal{U}_p} \sum_{b \in \mathcal{B}_{pq}} \sum_{\bar{t} \geq t} \frac{1}{L^P_q} r_{qbft\bar{t}} = 0 \qquad p \in \mathcal{P}^{HOG}, \ i \in \mathcal{N}^M, \ t \in \mathcal{T}. \qquad (7.12)$$

Further, Constraints (7.12) control the spot purchases and ensure that the bought volumes are distributed correctly. The volume bought in one market node can be sent to another, or the same, market node to be sold on the spot market, it can be sent to a processing node where it is used as raw material in the production of durable products, or it can be sent via a processing node to a market node using a path, being transformed to a perishable product on the way.

7.3.4 Inventory constraints

As inventory model based on the discussion in Section 6.6 is presented below.

Initial inventory

$$i_{pi1a} = I_{pia}^0 - \sum_{\bar{t} \geq 1} x_{pijm1\bar{t}a}^{DUR} \qquad p \in \mathcal{P}^D,\ i \in \mathcal{N}^P,\ a \in \mathcal{A}_p^P \qquad (7.13)$$

Constraints (7.13) ensure correct initial inventory levels at each processing location i for the artificial inventories of each durable processed product p in the beginning of the first time period.

Inventory balance

$$i_{pit1} - B_{11}i_{pi(t-1)1} - y_{pi(t-1)}^{DUR} + \sum_{j \in \mathcal{D}_i} \sum_{m \in \mathcal{M}} \sum_{\bar{t} \geq (t-1)} x_{pijm(t-1)\bar{t}1}^{DUR} = 0$$

$$p \in \mathcal{P}^D,\ i \in \mathcal{N}^P,\ t \in \mathcal{T} \setminus \{1\}, \qquad (7.14)$$

$$i_{pita} - \sum_{\underline{a} \leq a} B_{\underline{a}a}i_{pi(t-1)\underline{a}} + \sum_{j \in \mathcal{D}_i} \sum_{m \in \mathcal{M}} \sum_{\bar{t} \geq (t-1)} x_{pijm(t-1)\bar{t}a}^{DUR} = 0$$

$$p \in \mathcal{P}^D,\ i \in \mathcal{N}^P,\ t \in \mathcal{T} \setminus \{1\},\ a \in \mathcal{A}_p^P \setminus \{1\}. \qquad (7.15)$$

Constraints (7.14) and Constraints (7.15) ensure that the inventory levels at the end of each time period are updated throughout the planning horizon. The inventory level of product p in artificial inventory a going leaving time period t, i_{pita}, is calculated as the sum of the inventory volumes from the previous time period that are moved to artificial inventory a, including the share that remains, subtracted by the volume sent to the market. For the first artificial inventory of product p, the production volume from the previous time period is also added.

Inventory capacity

$$\sum_{p \in \mathcal{P}_l^L} \sum_{a \in \mathcal{A}_p^P} i_{pita} \leq I_{il}^{CAP} \qquad i \in \mathcal{N}^P,\ l \in \mathcal{L},\ t \in \mathcal{T}. \qquad (7.16)$$

For each processing facility i, the inventory cannot exceed the inventory capacities. This is ensured by Constraints (7.16). As the different products require different

inventory types, for example refrigerated storage and freezer, the capacities are per inventory type l. Here, it is assumed that each product cannot be stored in more than exactly one storage type, otherwise one variable will be present in two or more inventory capacity constraints simultaneously. Note that the inventory capacity is per kg, whereas in reality the space occupied per kilogram product might differ significantly amongst products. As such, the inventory capacity will also be dependent upon other factors, such as number of products and shape and volume of the product. An easy modification to remedy this problem is to also include factors that represent the relative space occupied for each product.

7.3.5 Production and distribution constraints

Processing capacity

$$\sum_{b \in \mathcal{B}} \sum_{f \in \mathcal{F}_i} \sum_{\bar{t} \geq t} r_{pbf\underline{t}\bar{t}} \leq P_{pi}^{CAP} \quad p \in \mathcal{P}^P, \ i \in \mathcal{N}^P, \ t \in \mathcal{T} \mid \underline{t} = t - T_f^F, \ t \geq T_f^F \quad (7.17)$$

$$y_{pit}^{DUR} \leq P_{pi}^{CAP} \quad p \in \mathcal{P}^D, \ i \in \mathcal{N}^P, \ t \in \mathcal{T}. \quad (7.18)$$

Similarly to each slaughtering location, all processing locations will have an upper limit of production per time period. This is ensured by Constraints (7.17) and Constraints (7.18) for perishable and durable processed products respectively. Note that because perishable processed products are sent through separate paths that starts in a slaughtering node and ends in a market node, the time period in which the production occurs are defined as $\underline{t} + T_f^F$.

This formulation assumes that the production capacity is given per product p. Because of little available information about how the processing facilities used in the salmon industry are organized, these constraints are kept as simple as possible, assuming no relation between the production of different products. As the production capacity of one product is assumed to be independent of the production capacities of all other products, it is assumed that all products have separate production lines and no common machine usage. If machines are shared between products, it is possible to include shared capacities or constraints that ensure the correct relationships, depending upon the configuration of each individual facility.

Transportation capacities

$$\sum_{p\in\mathcal{P}}\sum_{\underline{t}\leq t}\sum_{\overline{t}\geq t}\left(x^{LIVE}_{pijmt\underline{t}}+x^{HOG}_{pijmt\underline{t}}+x^{RAW}_{pijmt\underline{t}}+\sum_{a\in\mathcal{A}^P_p}x^{DUR}_{pijmt\overline{t}a}+\sum_{b\in\mathcal{B}}\sum_{f\in\mathcal{F}_{ijm}}r_{pbft\underline{t}}\right)$$

$$-a^{MAX}_{ijmt}\leq T^{CAP}_{ijmt}\qquad i\in\mathcal{N},\ j\in\mathcal{D}_i,\ m\in\mathcal{M},\ t\in\mathcal{T}. \tag{7.19}$$

The transportation capacities within the network are controlled by Constraints (7.19), which control both the wellboat capacities and shipping capacities.

As discussed in Section 6.6, an option to surpass the maximum transport capacity by paying a penalty K^T_m is included. This penalty represents the transaction costs involved in re-negotiation and the added costs of short term agreements on small volumes.

$$\sum_{p\in\mathcal{P}}\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{D}_i}\sum_{\underline{t}\leq t}\sum_{\overline{t}\geq t}\left(x^{HOG}_{pijmt\underline{t}}+x^{RAW}_{pijmt\underline{t}}+\sum_{a\in\mathcal{A}^P_p}x^{DUR}_{pijmt\overline{t}a}+\sum_{b\in\mathcal{B}}\sum_{f\in\mathcal{F}}r_{pbft\underline{t}}\right)$$

$$+a^{MIN}_{mt}\geq T^{LIM}_m\qquad m\in\mathcal{M},\ t\in\mathcal{T}. \tag{7.20}$$

The commitment of using a minimum capacity demanded by the logistics firms are controlled by Constraints (7.20). The penalty is per time periods in order to give incentives for leveling the transport volumes throughout the planning horizon. Note the difference between Constraints (7.19) and (7.20), where the former are given locally, whereas the latter are given globally.

7.3.6 Contract fulfillment

$$z^c_{cpit}-Q_c\delta_c=0\qquad c\in\mathcal{C}\mid p=P^C_c,\ i=M^C_c\ t\in\mathcal{C}^T_c \tag{7.21}$$

All signed contracts must be fulfilled, which is ensured by Constraints (7.21) Note that as Q_c represent the volume to be delivered at each delivery in contract c, it is assumed that the total volume is uniformly distributed for multiple delivery contracts. Also note that all offered contracts are assumed to be fulfilled within the plnaning horizon. As δ_c is a first stage decision, no scenario index is included above.

7.3.7 Contract revenues

As described in Section 3.5, the industry operates with several different types of contracts, each of which must be modeled differently. The set of all contracts, $\mathcal{C}$, is therefore divided into subsets, where $\mathcal{C}^F$ represents all fixed price contracts. Furthermore, the set $\mathcal{C}^{PA}$ includes all contracts where the price is adjusted in relation to the realised spot price at delivery and $\mathcal{C}^A$ includes all contracts where the price is adjusted in relation to the realised spot price only if the price difference exceeds a pre defined limit. Common for all contract types is that the contract price is stochastic, while other contract characteristics are deterministic. Furthermore, all contracts only include one product, specified by P_c^C, and all deliveries to fulfill that contract is made in one market, specified by M_c^C. However, contracts can include either one or several deliveries, where the set $\mathcal{C}_c^T$ define the set of time periods in which delivery to contract c must be made.

Revenue of fixed price contracts

In a simple fixed price contract c, the buyer and the seller agree upon trading a fixed volume, Q_c, of a specified product, P_c^C, for a fixed price per kilogram, ρ_c^C. The product is to be delivered at the beginning of a specified time period t in a specified market, m.

$$Q_c \rho_c \delta_c^C = o_c \qquad c \in \mathcal{C}^F. \tag{7.22}$$

The binary variable δ_c^C is equal to 1 if the contract is entered, and equal to 0 if it is not. Thus, the contract revenue is calculated by multiplying δ_c^C by the contract price U_c with the contract volume Q_c. Note that this also holds for fixed price contracts with multiple deliveries as the price is flat for the entire contract period and Q_c is the total volume agreed to be delivered in contract c.

Revenue of adjustable price contracts

The second type of contracts modeled is a contract in which the product p, volume Q_c and delivery time t and place j is still fixed, but where the price can change in the

time between signing and fulfilling the contract.

$$\sum_{t\in\mathcal{C}_c^T} Q_c \left(\rho_c^C + A_c^C(\rho_{pit}^S - \rho_c^C)\right)\delta_c^C = o_c \qquad c \in \mathcal{C}^A \tag{7.23}$$

$$\sum_{t\in\mathcal{C}_c^T} Q_c \left(\rho_c^C - A_c^C(\rho_c^C - \rho_{pit}^S - F_c)^+ + A_c^C(\rho_{pit}^S - \rho_c^C - F_c)^+\right) = o_c$$

$$c \in \mathcal{C}^{PA} \mid i = M_c^C \tag{7.24}$$

Two methods of adjusting the price of a contract is included in the model. Both methods seek to limit the gap between achieved contract price and the comparable spot price. The revenue generated from adjustable price contracts for each of these two methods are calculated by (7.23) and (7.24).

The first method is the simplest. Here, a price per kilogram U_c is agreed upon when entering the contract. Come delivery in a later time period t, this price is adjusted by adding the difference between the contract price U_c and the realised spot price for the relevant product p in the relevant market i, ρ_{pit}^S, times a contract-specific adjustment constant A_c.

In the second method of adjusting the contract price the adjustment is made in the same manner, but the price is only adjusted if the difference between the initial contract price and the realised spot price is greater than an agreed upon value measured in NOK.

7.3.8 Non-anticipativity

The non-anticipativity Constraints (7.25) enforce the relationship between stages, periods and scenarios in the scenario tree. This is done by forcing decisions in a period t in a node k to be equal for all scenarios s passing through that node. The set $\mathcal{S}_k$ denotes the set of scenarios passing through the split node k, while Q_k is the time period corresponding to node k. $|\mathcal{S}_k|$ is the size of $\mathcal{S}_k$.

$$\frac{1}{|\mathcal{S}_k|} \sum_{s \in \mathcal{S}_k} (q_{vits'}, h_{vits'}, x^{LIVE}_{pijt\bar{t}s'}, x^{HOG}_{pijmt\bar{t}s'}, x^{RAW}_{pijmt\bar{t}s'}, r_{pbft\bar{t}s'}, x^{DUR}_{pijmt\bar{t}as'},$$

$$y^{DUR}_{pbits'}, i_{pitas'}, z^{B}_{pits'}, z^{S}_{pits'}, z^{C}_{cpits'}, \delta^{C}_{cs'}) =$$

$$(q_{vits}, h_{vits}, x^{LIVE}_{pijt\bar{t}s}, x^{HOG}_{pijmt\bar{t}s}, x^{RAW}_{pijmt\bar{t}s}, r_{pbft\bar{t}s}, x^{DUR}_{pijmt\bar{t}as},$$

$$y^{DUR}_{pbits}, i_{pitas}, z^{B}_{pits}, z^{S}_{pits}, z^{C}_{cpits},) \tag{7.25}$$

$$v \in \mathcal{V}, \ p \in \mathcal{P}, \ i \in \mathcal{N}, \ j \in \mathcal{D}_i, \ f \in \mathcal{F}, \ t \in \mathcal{T}, \bar{t} \in \mathcal{T}, \ k \in \mathcal{K} \ s \in \mathcal{S}$$

7.3.9 Variable constraints

$$q_{vits} \geq 0 \quad v \in \mathcal{V}, \ i \in \mathcal{N}^F, \ t \in \mathcal{T}, \ s \in \mathcal{S} \tag{7.26}$$

$$i_{pitas} \geq 0 \quad p \in \mathcal{P}, \ i \in \mathcal{N}^P, \ t \in \mathcal{T}, \ a \in \mathcal{A}, \ s \in \mathcal{S} \tag{7.27}$$

$$o_{cs} \geq 0 \quad c \in \mathcal{C}, \ s \in \mathcal{S} \tag{7.28}$$

$$a^{TMAX}_{mts} \geq 0 \quad m \in \mathcal{M}, \ t \in \mathcal{T}, \ s \in \mathcal{S} \tag{7.29}$$

$$a^{TMIN}_{mts} \geq 0 \quad m \in \mathcal{M}, \ t \in \mathcal{T}, \ s \in \mathcal{S} \tag{7.30}$$

$$h_{vits} \geq 0 \quad v \in \mathcal{V}, \ i \in \mathcal{N}^F, \ t \in \mathcal{T}, \ s \in \mathcal{S} \tag{7.31}$$

$$x^{LIVE}_{ijt\bar{t}s} \geq 0 \quad i \in \mathcal{N}, \ j \in \mathcal{D}_i, \ t \in \mathcal{T}, \ \bar{t} \in \mathcal{T}, \ s \in \mathcal{S} \mid t \leq \bar{t} \tag{7.32}$$

$$x^{HOG}_{pijmt\bar{t}s} \geq 0 \quad p \in \mathcal{P}^{HOG}, \ i \in \mathcal{N}, \ j \in \mathcal{D}_i, \ m \in \mathcal{M}, \ t \in \mathcal{T}, \ \bar{t} \in \mathcal{T}, \ s \in \mathcal{S} \mid t \leq \bar{t} \tag{7.33}$$

$$x^{RAW}_{pijmt\bar{t}s} \geq 0 \quad p \in \mathcal{P}^{HOG}, \ i \in \mathcal{N}, \ j \in \mathcal{D}_i, \ m \in \mathcal{M}, \ t \in \mathcal{T}, \ \bar{t} \in \mathcal{T}, \ s \in \mathcal{S} \mid t \leq \bar{t} \tag{7.34}$$

$$r_{pbft\bar{t}s} \geq 0 \quad p \in \mathcal{P}^P, \ b \in \mathcal{B}, \ f \in \mathcal{F}, \ t \in \mathcal{T}, \ \bar{t} \in \mathcal{T}, \ s \in \mathcal{S} \mid t \leq \bar{t} \tag{7.35}$$

$$x^{DUR}_{pbijmt\bar{t}as} \geq 0 \qquad p \in \mathcal{P}^D, \ b \in \mathcal{B}, \ i \in \mathcal{N}, \ j \in \mathcal{N}, \ m \in \mathcal{M}, \ t \in \mathcal{T}, \ \bar{t} \in \mathcal{T},$$

$$a \in \mathcal{A}_p, \ s \in \mathcal{S} \mid t \leq \bar{t} \tag{7.36}$$

$$y^{DUR}_{pbits} \geq 0 \qquad p \in \mathcal{P}^P, \ i \in \mathcal{N}^P, \ t \in \mathcal{T}, \ s \in \mathcal{S} \tag{7.37}$$

$$z^{B}_{pits} \geq 0 \qquad p \in \mathcal{P}^{HOG}, \ i \in \mathcal{N}^M, \ t \in \mathcal{T}, \ s \in \mathcal{S} \tag{7.38}$$

$$z^{S}_{pits} \geq 0 \qquad p \in \mathcal{P}, \ i \in \mathcal{N}^M, \ t \in \mathcal{T}, \ s \in \mathcal{S} \tag{7.39}$$

$$z^{C}_{cpits} \geq 0 \qquad c \in \mathcal{C}, \ p \in \mathcal{P}, \ i \in \mathcal{N}^M, \ t \in \mathcal{C}^T_c, \ s \in \mathcal{S} \tag{7.40}$$

$$\delta^{C}_{c} \in \{0,1\} \qquad c \in \mathcal{C}. \tag{7.41}$$

Chapter 8

Model implementation

The mathematical model presented in Chapter 7 was implemented in two versions, one deterministic and one stochastic. Both models are written in `Mosel` and implemented in FICO® Xpress v.7.8.0. The tests were performed at a computational cluster owned by the Department of Industrial Economics and Technology Management at NTNU. Most tests were performed on nodes from rack 2 or 3, with 2xAMD Opteron 2431 2,4 GHz CPU and 24.0 GB of installed RAM memory. The implemented model with 100 scenarios was run at rack 5, with HP BL686 G7, 4 x AMD Opteron 6274 2,2 GHz CPU and 128 Gb of installed RAM memory.

In order to model the problem as a stochastic linear program that is computationally tractable with acceptable solution times, some assumptions and simplifications have been made. Assumptions and simplifications are also made where necessary due to limited access to relevant information about industry specifics such as production costs and capacities. Based on the discussion in Section 6.1, planning horizon chosen is one year with a time resolution of one week.

This chapter starts by a short overview over the most important problem specific assumptions and simplifications presented in Section 8.1. In Section 8.2, the case study is presented. Case specific problem reduction efforts are described in Section 8.3, whereas Section 8.4 elaborates how uncertainty is represented in terms of scenarios. Finally, Section 8.5 explains how the input data is generated.

8.1 Problem specific assumptions and simplifications

The most important assumptions and simplifications made when the model is implemented are summarized below.

Location aggregation

For the problem to be computationally solvable, sets of nearby locations are aggregated into regions which acts as single facility. This is described in further detail in Section 8.2.1.

Farm similarity

Due to geographical closeness, it is reasonable to assume that the farms belonging to the same slaughter region have similar growth and mortality rates. Consequently, these farms share growth and mortality scenarios.

Survival rates

As no detailed data have been available regarding historical mortality or escape in Marine Harvest's operations, the survival rate at all farms have been set deterministic equal to 1.

Maximum Allowable Biomass and fallowing

As the implemented network consists of large farming regions and not individual farms, MAB and fallowing constraints are not implemented, as they would not be meaningful when running the model on a user case with such a high level of farm aggregation.

Transportation capacities

As no data is available on the details of Marine Harvest's agreements with their logistics provider, maximum capacities implemented are based on qualified guesses. In consequence, no punishments for using too little capacity is implemented. Furthermore, no option to purchase extra capacity is included in the implementation.

Transportation times

Exact transport times are gathered in the pre-processing. However, in the implementation, these transportation times are rounded, so that goods transported by a mode taking less than 0.5 time periods are modeled as arriving instantly, and goods transported by a mode taking 0.5 time periods or more are modeled as arriving the following time period.

Production processes

As the product groups implemented are generalized, all harvested fish sizes are allowed as input into all value added products.

Spot purchase prices

The spot purchase price is modelled by a fixed mark up on the corresponding spot sales price.

Tariffes

As it has not been possible to attain a complete set of tariffs for all products between all countries included in the model, the tariff factor have been omitted in the implementation.

8.2 Case study

The implemented model is tested on a case study representing Marine Harvest's global value chain. For testing purposes, details in the case study are adjusted to construct a set of instances. However, the most important features, such as the structure of the network, remain unchanged throughout.

8.2.1 Network

As the number of facilities Marine Harvest operate is massive, there is a need for aggregation. Consequently, each node in the implemented network represent all facilities within the specified region. Biomass, capacities and inventory levels are aggregated for the physical facilities belonging to each node, and the nodes included are listed in Table 8.1. The implemented network is presented in Table D.1 in Appendix D.

Slaughterhouses	Processing regions	Markets
Norway North	Poland	EU
Norway Mid	France	Russia
Norway West	Italy	North America
Norway South	Belgium	South America
Scotland	Germany	Japan
Ireland/Faroe Islands	US	Asia excl. Japan
Canada	Chile	
Chile	China	
	Hong Kong	

Table 8.1: Nodes in the implemented network.

As a general rule, each country in which Marine Harvest conducts salmon farming is represented by two farming regions and one slaughter region. However, some exceptions are made. Marine Harvest's production in Norway vastly exceeds the numbers of other countries. Marine Harvest itself divides the country in four, and Norway is therefore implemented as four separate regions. Each Norwegian region also has a slaughter node and two farming nodes. Note that because of the geographical proximity, Norway North and Mid, as well as Norway West and South, are allowed to use each others slaughtering locations depending upon where capacity is available. This correspond with Marine Harvest's practice of today (Johannessen, T., Vinnes, K. and Sværen, U., personal communication, 2015). Furthermore, the production in Faroe
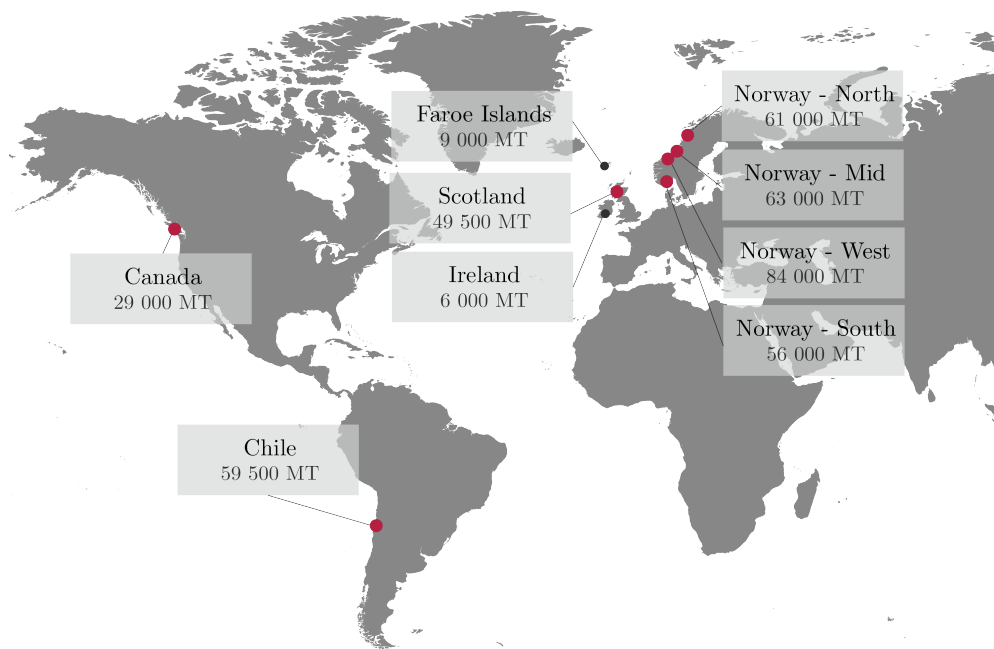
Figure 8.1: Comparison of Marine Harvest's farming network and implemented farming regions.

Islands and Ireland is significantly smaller compared to the other countries, and no farming or slaughter nodes are included for these countries.
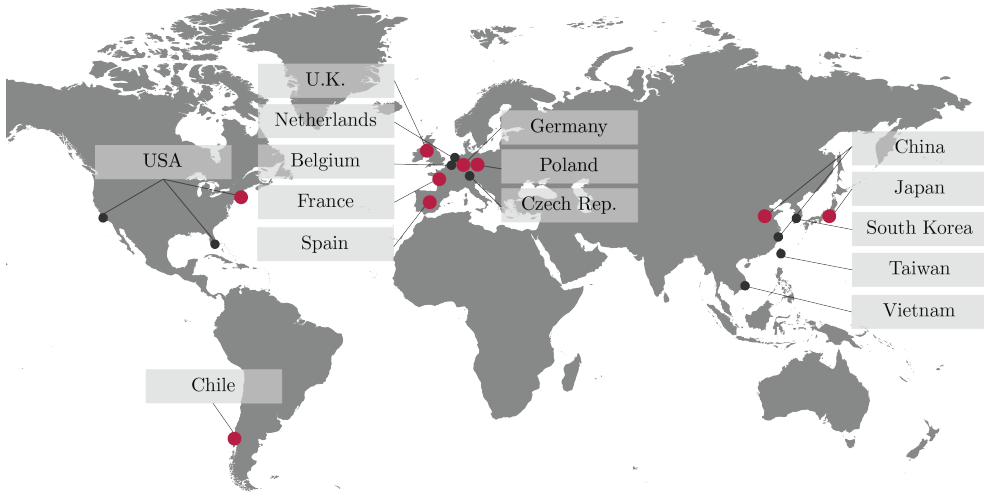
Figure 8.2: Comparison of Marine Harvest's processing network and implemented processing regions.

Based on available information about Marine Harvest's production network, 10 processing facilities are included Marine Harvest (2014b), Marine Harvest (2013), Marine Harvest (2012), and Marine Harvest (2011b)(Johannessen, T., personal communication, 2015). In practice there are significantly fewer processing facilities than there are slaughter houses. However, slaughtering is a standardized process, whereas processing facilities tend to be specialized and hence only able to produce a subset of products. It is therefore included a higher number of processing nodes in the network to give the model enough flexibility in producing all types of processed products.

The selection of which markets to use is based on the following considerations: (1) First and foremost, they must represent a demand high enough to say something meaningful about the price differences between the markets. Preferably, the market sizes should also be relatively even. (2) Enough information must be available in order to generate a data set that includes that market. (3) The trade-off between making a computationally tractable model and the desired degree of detail are considered.

With this in mind, the six markets presented in Section 8.3.1 were chosen. This corresponds well with the divisions Marine Harvest use themselves. An exception is made for America, where Marine Harvest refer to Americas as one trading region, whereas the America is treated as two regions in the model, North and South America.
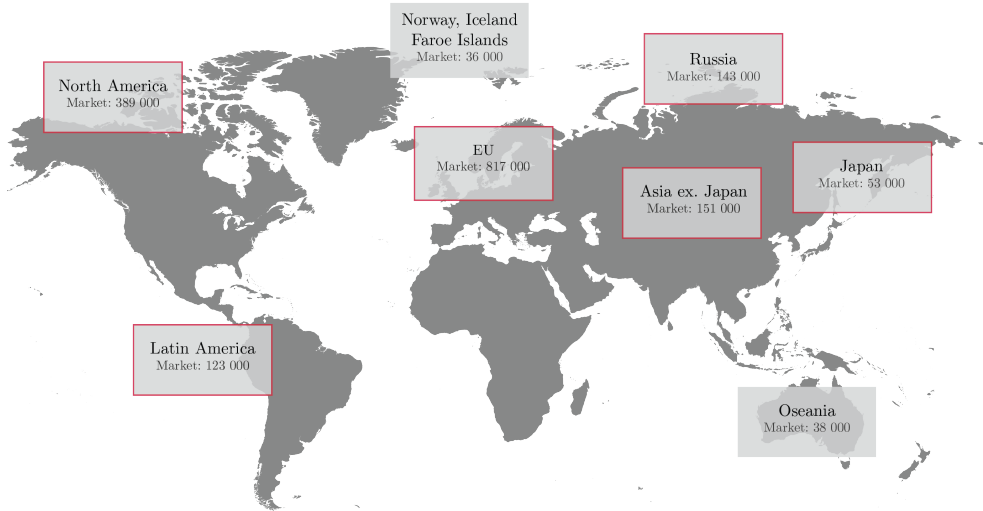
Figure 8.3: Comparison of main markets for salmon and implemented markets, including market sizes.

This is done to be able to model traveling times and costs more accurately.

A comparison of Marine Harvest's farming and processing operations and the implemented network is illustrated by Figure 8.1 and Figure 8.2. Figure 8.3 visualize the main global markets and their respective sizes and compare it to the markets included in the model. In all of the mentioned figures, nodes included are highlighted in red, while regions that are part of the real network but excluded from the model are depicted in grey.

8.2.2 Product groups and transportation modes

82 farming weight classes are used to keep track of the weight of the fish in the salt water growth phase. As smaller weight intervals are necessary for modeling growth of smaller fish, the sizes of the farming weight classes are not uniform. The relation between the farming weight classes and the HOG sales weight classes implemented is illustrated in Figure 8.4. Note that sales weight class 0 represent the farming weight classes that are defined as unharvestable in the model, representing fish in the weight interval 0.01-1.99 kg.

In total, nine product groups are included in the user case, five of which are different sized fresh HOG and four of which are processed products, see Table 8.2. These product groups are chosen to best represent the total product mix based on historical
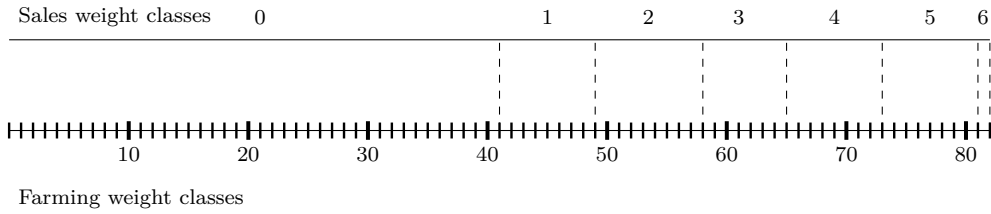
Figure 8.4: Illustration of the relation between the farming weight classes
v and HOG sales weight classes p.

values of traded volumes of different products.

HOG	VAP
2-3 kg	Frozen HOG salmon
3-4 kg	Fresh fillets
4-5 kg	Frozen fillets
5-6 kg	Smoked salmon
6-7 kg	

Table 8.2: Product groups included in the user case.

As discussed in Chapter 3, fresh HOG salmon of different weight classes are used
for different purposes, forming the rationale behind dividing fresh HOG salmon into
several weight classes. The included sales weight classes have a resolution of one kilo-
gram. This is because providers of statistical data on salmon spot prices, such as
NASDAQ and SSB, record their prices according to these intervals. The processed
product groups are chosen based on available data on the Marine Harvest trade vol-
umes of different products, see Figure 2.11. Here, all frozen HOG salmon weight
classes are aggregated into one single product group. Fillets include all fillet sizes,
with and without additional differentiation such as seasoning and marinades. The
smoked salmon-category includes all types of smoked salmon, such as whole, smoked
portions, cuts and toppings. Consequently, other elaborated products such as cured
salmon and ready meals are not included. As this is a very heterogeneous group of
products, it would be hard to calculate meaningful production costs and sales prices.
Additionally, the existing data for these products is insufficient.

For transportation it is chosen three different modes; truck, boat and airplane. The
transportation modes are not further differentiated within each mode, e.g. in terms
of speed, size or ability to transport chilled versus frozen goods, as this would add
complexity without contribute to better problem insight.

8.3 Problem reduction

Solving a stochastic model is time consuming, and for the model to be of practical use it must have an acceptable solution time. It is therefore of great importance to take measures to reduce the problem size without too much loss of details.

8.3.1 Network reduction

Because of the problem structure, the network size can be considerably reduced. As there is a clear structure on how the products flows through the network, a set of directed arcs should be used. Furthermore, as many arcs are never used in a practical setting because of travel time considerations, while others are only used for small volumes, it is possible reduce the network size down to a size that is practically solvable. As an example, all transport between the European production sites and American markets are excluded because the small volumes that are actually sent.
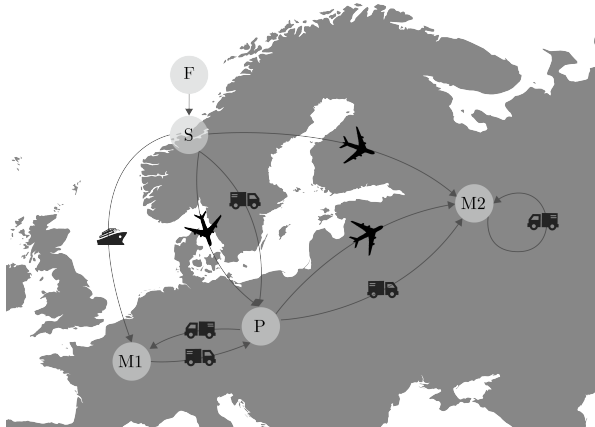
Figure 8.5: Example of a partial transportation network within the value chain for two markets in Europe.

Furthermore, not all remaining arcs are defined for all transportation modes because of the same reasons, especially travel time considerations. In addition, many modes cannot be used because of the physical design of the network. Boat can for example not be used to transport products from the processing facility in Poland and to the Russian market, as depicted in Figure 8.5. Here, a set of possible transportation modes m in one part of the network is shown as an illustrative example.

Paths are included only if the total path time is below a one time period, i.e. a week, where 6 hours was added to represent handling, production and inbound and outbound logistics. The limit were set so that the products can reach the market within

a reasonable time frame when considering the durability of fresh fillet. Furthermore, the maximum cost path cost was set to 20 NOK/kg.

8.3.2 Variable reduction

As each flow-variable $x_{pijmt\bar{t}s}$ defines both which product that is transported across the distance ij and the transportation mode used, some combinations of indexes are impossible. Variables with such combinations of indexes are thus not included in the model. As these variables would never be included in a feasible solution, not including them reduces the computational effort without affecting the solution.

8.3.3 Simplifications of the model formulation
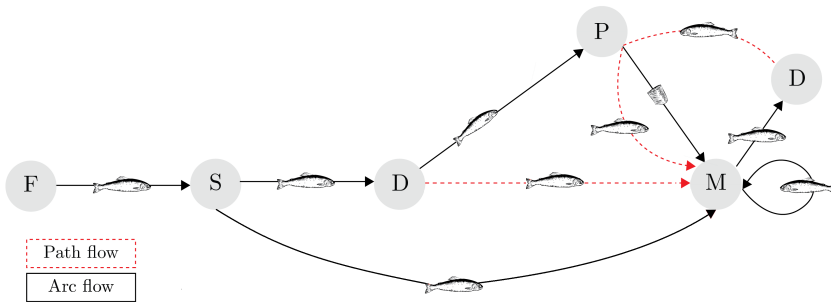
Figure 8.6: Illustration of the implemented network.

As all HOG weight classes p are allowed to be used as input for all processed products q, there is no longer a need to differentiate the flow from the distribution node to the processing node on HOG weight class. Keeping track of the total volume is sufficient, which makes it possible to reduce the problem size further by the inclusion of two dummy nodes. The new network with the dummy nodes depicted as D are shown in Figure 8.6. With this reformulation of the network, a total of more than 92 000 variables per scenario, or 80 % of the relevant variables, can be excluded from the problem in exchange for 676 new constraints and variables per scenario.

The inclusion of one dummy node for each slaughter node and each market node gives the model the flexibility to differentiate on HOG weight classes in the mass balances, while at the same time avoids to send five separate volumes of raw material to the processing nodes, one for each HOG sales weight class. Let $\mathcal{N}^{SD}$ be the set of dummy nodes assigned to the slaughter nodes. Then, (7.7) and (7.8) can be modified into

Constraints (8.1) - (8.3):

$$\sum_{j\in\mathcal{O}_i}\sum_{\underline{t}\leq t}L^S x^{LIVE}_{pji\underline{t}t} - \sum_{j\in\mathcal{D}_i}\sum_{m\in\mathcal{M}}\sum_{\bar{t}\geq t}x^{HOG}_{pijmt\bar{t}} = 0 \qquad p\in\mathcal{P}^{HOG},\ i\in\mathcal{N}^S,\ t\in\mathcal{T},\quad (8.1)$$

$$\sum_{p\in\mathcal{P}^{HOG}}\sum_{j\in\{\mathcal{O}_i\cup\mathcal{N}^S\}}x^{HOG}_{pjit} - \sum_{q\in\mathcal{P}^P}\sum_{f\in\mathcal{F}_i}\sum_{\bar{t}\geq t}\frac{1}{L^P_q}r_{qft\bar{t}} - \sum_{j\in\{\mathcal{D}_i\cup\mathcal{N}^P\}}\sum_{m\in\mathcal{M}}\sum_{\bar{t}\geq t}x^{RAW}_{ijmt\bar{t}} = 0$$

$$i\in\mathcal{N}^D,\ t\in\mathcal{T}, \qquad\qquad\qquad (8.2)$$

$$\sum_{j\in\mathcal{O}_i}\sum_{m\in\mathcal{M}}\sum_{\underline{t}\leq t}x^{RAW}_{jimt\underline{t}} - \sum_{p\in\mathcal{P}^{DUR}}\frac{1}{L^P_b}y^{DUR}_{pit} = 0 \qquad i\in\mathcal{N}^P,\ t\in\mathcal{T}. \qquad (8.3)$$

Now, let $\mathcal{N}^{MD}$ be the set of dummy nodes assigned to the market nodes. Constraints (7.12) can then be modified into Constraints (8.4) - (8.5):

$$z^B_{pit} - \sum_{j\in\mathcal{D}_i}\sum_{m\in\mathcal{M}}\sum_{\bar{t}\geq t}x^{HOG}_{pijmt\bar{t}} = 0 \qquad p\in\mathcal{P}^{HOG},\ i\in\mathcal{N}^M,\ t\in\mathcal{T}, \qquad (8.4)$$

$$\sum_{p\in\mathcal{P}^{HOG}}\sum_{j\in\{\mathcal{O}_i\cup\mathcal{N}^M\}}x^{HOG}_{pjit} - \sum_{j\in\{\mathcal{D}_i\cup\mathcal{N}^P\}}\sum_{m\in\mathcal{M}}\sum_{\bar{t}\geq t}x^{RAW}_{pijmt\bar{t}} - \sum_{f\in\mathcal{O}^F_i}\sum_{q\in\mathcal{P}^P}\sum_{\bar{t}\geq t}\frac{1}{L^P_q}r_{qft\bar{t}} = 0$$

$$i\in\mathcal{N}^{DM},\ t\in\mathcal{T}. \qquad\qquad\qquad (8.5)$$

Note that in addition the constraint adjustments above, $x^{HOG}_{pijmt\bar{t}}$ is defined for two new sets of arc, i.e. from the slaughter nodes and market nodes and to their respective dummy nodes. Also note that since no transportation mode is physically used to the dummy nodes and the travel time will always be zero, the mode index is and one time index are omitted for x^{HOG}_{pijt} Constraints (8.2) and (8.5).

8.4 Representing uncertainty

In stochastic programming, it is important to choose a scenario generation method that is able to quickly find scenarios that constitute a good fit of the uncertainty modeled. Because of the sparse available historical data, there is a need for data manipulation in order to get data in the right resolution while at the same time keeping the dependence between the stochastic variables. Based on this, the copula-based heuristic developed by Kaut (2014) is chosen as scenario generation method. Sampling and other methods discussed in Section 5.2 are rejected mostly due to the need for known probability distributions and complementary historical data sets.

To generate the scenarios, the authors have used a modified version of the already mentioned procedure used by Schütz and Tomasgard (2011), Schütz et al. (2009a), and Rynning-Tønnesen and Øveraas (2012). All parameter estimation for the time series models are done in the statistical software R. The historical data for the temperatures used as input to the scenario generation procedure are weekly averages based on daily sea water temperatures gathered from National Oceanic and Atmospheric Administration (2015). Product price scenarios are generated from weekly historical spot prices provided by Norwegian Seafood Council (2015) and NASDAQ (2015b) from the years 2010-2014.

Another option for generating forecasts for price and temperature could have been to use historical data directly. The main advantage of using historic data to represent the future is that this is an easy and fast method which require minimal pre-processing of the data. However, the main drawback is that this is considered to give a bad representation of the future. The method also restricts the possibility to make as many scenarios that are needed to give a good representation of future uncertainty while accounting for market correlations and non-seasonal trends.

8.4.1 Forecasting

The first step of scenario generation is to build a good forecasting model which is used as a basis. Initially one have to ensure stationarity and remove seasonality in the time series. However, before one starts with the removal of trend and seasonality, it is advisable to inspect the time series to acquire as much knowledge as possible of the time series a priori. By plotting the time series one usually get a good idea about whether the series actually contains a trend or seasonality. Non-constant variances, and other non-normal and non-stationary phenomena may also be discovered by inspection.

Inspection also plays an important part in postulating a possible data transformation of the time series. Differencing is, together with variance-stabilizing transformation, one of the most commonly used transformations in time series analysis. Note that variance stabilizing transformations such as power transformations require non-negative values, and as differencing may create negative values, the variance stabilizing transformations should always be applied before taking differences (Wei, 2006).

8.4.2 Variance stabilizing transformation of the time series

Øglend and Sikveland (2008) have shown that salmon prices often are *heteroskedastic*, where heteroscedasticity means uneven variance in the time series. Several statistical tests are developed to test for the null hypotesis of homoscedasticity, which tests if all random variables in a time series have finite variance (Wei, 2006). One of these

tests, the *F-test*, are used to investigate if the variance in the time series are equal
to the variance of a Normal distribution with the same mean and standard deviation
as the time series (Härdle and Simar, 2003). These tests indicated that the variance
is equal to the variance of the corresponding Normal distribution. However, because
this is a rather weak approach to test for heteroscedasticity (Tjelmeland, H., personal
communication, 2015), a variance stabilizing technique are applied to the time series.

Two of the most used variance-stabilizing transformations is the logarithmic trans-
form and the Box-Cox. In this thesis the Box-Cox transformation is used to stabilize
variance, and the log-likelihood plot for finding optimal transformation parameter for
HOG salmon in weight class 4-5 kg is presented in Figure 8.7a. The Box-Cox trans-
formation combines a logarithmic transformation with a power transformation based
on maximum likelihood estimation and is described in equation (8.6).

$$BC(Z_t, \lambda) = \begin{cases} \frac{Z_t^\lambda - 1}{\lambda} & \text{if } \lambda \neq 0 \\ \ln Z_t & \text{if } \lambda = 0 \end{cases} \tag{8.6}$$

As a time series analysis is unchanged by linear transformations, equation (8.6) and
equation (8.7) are equivalent.

$$BC(Z_t, \lambda) = \begin{cases} Z_t^\lambda & \text{if } \lambda \neq 0 \\ \ln Z_t & \text{if } \lambda = 0 \end{cases} \tag{8.7}$$

For validating the variance stabilizing transformation it is common to apply the Box-
Cox test once more on the transformed series. As Figure 8.7b shows, the trans-
formation parameter, λ, for transform HOG salmon prices then equals to one and no
further transformation is needed. Note that the spot price time series for HOG salmon
in weight class 4-5 kg, together with the temperatures for Norway Mid, are used as
examples throughout the rest of this section.

8.4.3 Removal of non-stationarity

To detect non-stationarity there are several methods, of which the easiest method is
to inspect the ACF and PACF plot for the time series. If the ACF decays very slowly
and the sample PACF cuts off after the first lag, this is an indication of the need
for differencing (Wei, 2006). The ACF and PACF for HOG salmon 4-5 kg historical
prices and Norway Mid temperatures are presented in Figure 8.8. As a result of Wei
(2006)'s deduction, Figure 8.8a and Figure 8.8b which shows the ACF and PACF plots
for HOG salmon 4-5 kg indicate that the price series should be differenced. For the

(a) Before Box-Cox transf.　　　　　**(b)** After Box-Cox transf.
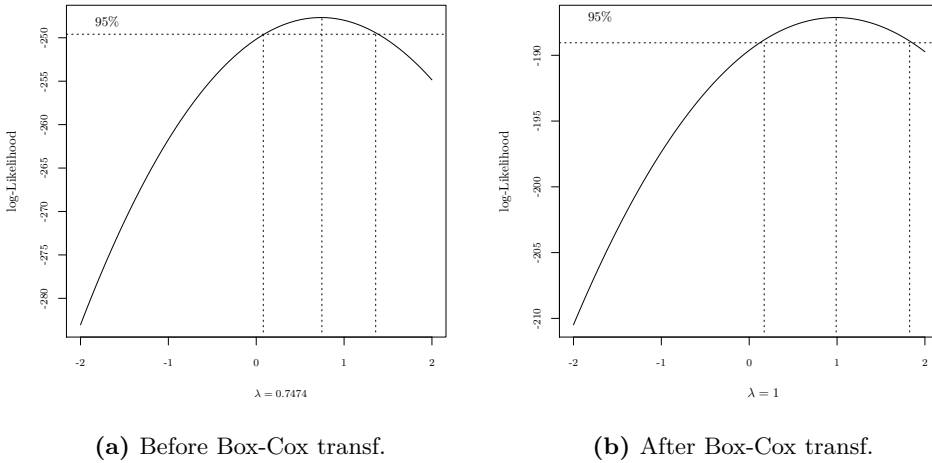
Figure 8.7: The figure shows the log-Likelihood function for HOG salmon in sales weight class 4-5 kg.

temperatures series, Figure 8.8c indicates a decaying ACF. However, as there is no clear cut off for the PACF in Figure 8.8d it can not be said that the temperature series should be differenced.

The Augmented Dickey Fuller (ADF) test for stationarity proposed by Dickey and Fuller (1979) is another, complimentary method for identifying non-stationarity. The test indicates whether the null hypothesis, H_0, of non-stationarity should be rejected in favor of the alternative hypothesis, H_1, of stationarity in the mean. The ADF-tests of the two time series used as examples in this Section are presented in Table 8.3. These tests confirms the assumption that non-stationary differencing are only needed for the HOG salmon 4-5 kg price series, and that the temperatures of Norway Mid are stationary in the mean. On further inspection, the same holds true for all price and temperature time series.

Series	Statistic	5% critical value	p-value	Reject H_0 at 5%?
HOG 4-5 kg	-0.7224	-2.89	0.9671	**No**
Norway Mid	-4.5417	-2.89	≤ 0.01	**Yes**

Table 8.3: Augmented Dickey-Fuller test for HOG salmon 4-5 kg spot price and Norway Mid temperatures.

8.4.4　Identification of time series model

After the time series are properly differenced, the ACF and PACF may again be useful in determining candidate time series models. This manual interpretation of such mathematical tools may in many cases be very useful, and sometimes enough.

137
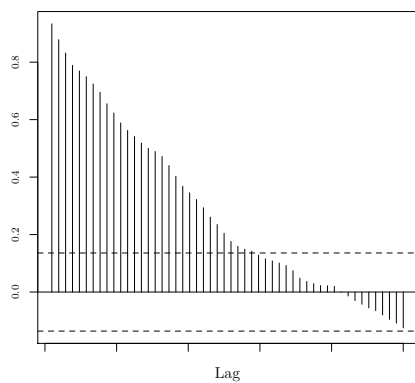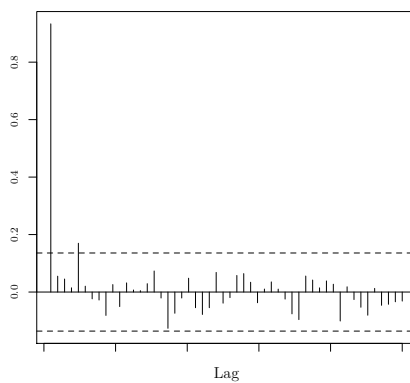
(a) ACF HOG 4-5 kg

(b) PACF HOG 4-5 kg
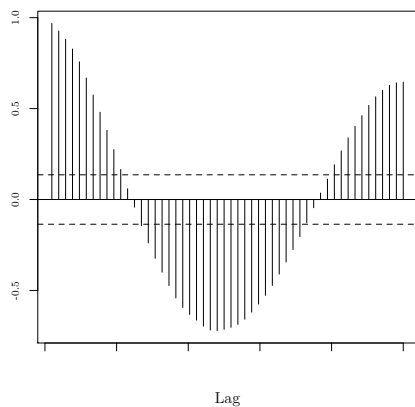
(c) ACF Norway Mid
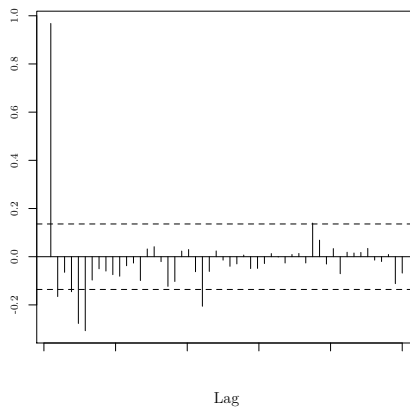
(d) PACF Norway Mid

Figure 8.8: ACF and PACF for HOG salmon 4-5 kg and Norway Mid temperature time series.

However, there are two well known criterions which is often used in determining the order for the right time series model; the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC). The AIC, named after Hirotugu Akaike, was introduced by Akaike (1973) and Akaike (1974), and is a measure of the relative quality of a statistical model for a given set of data. The AIC offers a relative estimate of the information lost in the process that generates a set of data, and by doing so it deals with the trade off between the goodness of fit of the model and the model's complexity. The BIC is closely related to the AIC, but were developed by Akaike after Shibata (1976) showed that the AIC tends to overestimate the order of the autoregression (Wei, 2006; Akaike, 1978; Akaike, 1979). In identifying candidate time series model, both of these criterions have been used, in addition to inspection of the ACFs and PACFs.

The ACF and PACF plots can be used to determine the order of an $ARMA(p, q)$, $ARIMA(p, d, q)$ or $ARIMA(p, d, q) \times (P, D, Q)_m$, (seasonal ARIMA). In doing so, there is a set of characteristics that may help. These are summarized in Table 8.4. The subscript m in the last model represent the seasonal period, as will it throughout the rest of this chapter.

Process	ACF	PACF
$AR(p)$	Tails off as exponential decay or damped sine wave	Cuts off after lag p
$MA(q)$	Cuts off after lag q	Tails off as exponential decay or damped sine wave
$ARMA(p, q)$	Tails off after lag $(q - p)$	Tails off after lag $(p - q)$

Table 8.4: Characteristics of theoretical ACF and PACF for stationary processes (Wei, 2006).

If the ACF is positive at the seasonal period, the analyst should consider adding an seasonal autoregressive term in the model. If the ACF is negative at the seasonal period, one should consider adding a seasonal moving average term (Nau, 2015). The seasonal term may be identified as necessary by looking for a significant partial auto correlation when the lag equals the length of the seasonal data in the PACF plot.

The ACF and PACF plots for the differenced Norway Mid temperatures are presented in Figure 8.9a and 8.9b respectively. Figure 8.9b clearly shows that the PACF cuts off after lag 1. Furthermore, the ACF in Figure 8.9a is negative for the seasonal period of 52, which indicates that the temperature should be forecasted with an $ARIMA(1, 0, 0) \times (0, 1, 1)_{52}$ model. By same analogous the ACF and PACF for HOG salmon 4-5 kg indicates that an $ARIMA(0, 1, 1) \times (0, 1, 1)_{52}$ model is suited for forecasting the spot price for HOG salmon in weight class 4-5 kg. Both the AIC and

BIC have been used to confirm the validity of these models, and the same procedure is used for the six other temperature forecasts.

(a) ACF

(b) PACF
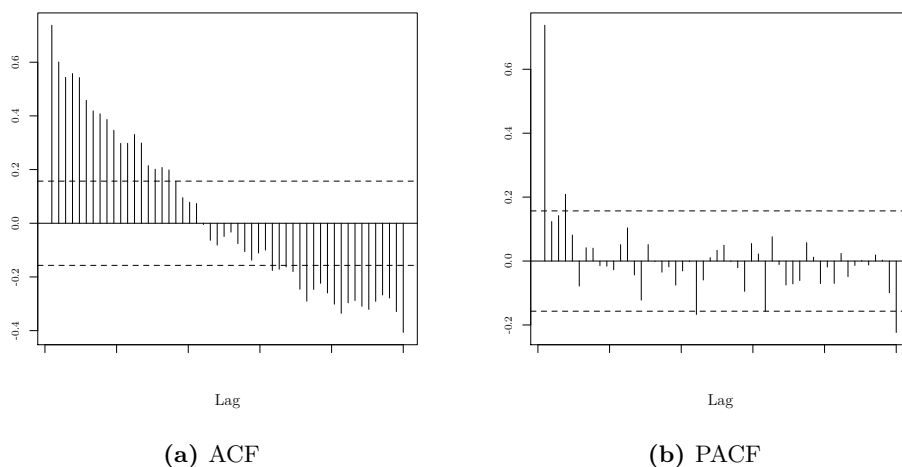
Figure 8.9: ACF and PACF for the seasonally differenced Norway South temperature time series.

The seasonal moving average model which is used for HOG salmon 4-5 kg was first introduced by Box and Jenkins to represent international air travel data, and has since been found to be very useful for representing a variety of seasonal time series (Wei, 2006). Note that this model also may be referred to as a seasonal exponential smoothing model.

Based on the findings of Øglend and Sikveland (2008) of *heteroscedasticity* in the time series for salmon prices, one could argue to use time series models that incorporate this heteroscedasticity, such as e.g autoregressive conditional heteroscedasticity (ARCH) or generalized autoregressive conditional heteroscedasticity (GARCH) models. However, because of the applied variance stabilizing techniques such as the Box-Cox transformation of the time series, and that it is somewhat uncertain if the time series actually used in this thesis holds heteroscedasticity, ARIMA-models are chosen to forecast the time series.

For the eight additional spot price forecasts, the point estimate from HOG salmon 4-5 kg is used as a co-variate in each of the other spot price forecasting models. HOG salmon 4-5 kg is chosen due to being the product that are most frequent traded in terms of volume, in addition to being used in the production of other VAPs. HOG salmon 4-5 kg prices has also high correlations with the other prices, and are therefore experienced as a good reference price time series.

A co-variate is alternatively explained as an *independent variable*. More specifically, a

co-variate is used as a secondary variable that may affect the relationship between the dependent variable and other independent variables of primary interest. Equation (8.8) illustrates how a co-variate is included in an ARIMA$(p, d, q) \times (P, D, Q)_m$-model. As the reader may recall from Equation (5.24) in Section 5.2, the only difference from the ARIMA$(p, d, q) \times (P, D, Q)_m$-model is that the ARIMAX$(p, d, q) \times (P, D, Q)_m$-model in Equation (8.8) have adopted the autoregressive term βX_t where β is the parameter for the co-variate regression, and X_t is the co-variate. Not that the compact notation in Equation (8.8) is the same as the most frequent notation used in literature for the well-known Box-Jenkins multiplicative seasonal ARIMA-model.

$$\phi_p(B)\Phi_P(B^s)(1 - B)^d(1 - B^s)^D(Z_t - \beta X_t) = \psi_q(B)\Psi_Q(B^s)\varepsilon_t \qquad (8.8)$$

Here, d is the order of ordinary differencing, D is the order of seasonal differencing and β is the exogenous variable's X_t coefficient. The other terms, including the model parameters, are merely a more compact notation replacing the following equations:

$$\phi_p(B) = 1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p, \qquad \text{(AR)}$$

$$\psi_q(B) = 1 - \psi_1 B - \psi_2 B^2 - \cdots - \psi_q B^q, \qquad \text{(MA)}$$

$$\Phi_P(B^s) = 1 - \Phi_1 B^s - \Phi_2 B^{2s} - \cdots - \Phi_P B^{Ps}, \qquad \text{(SAR)}$$

$$\Psi_Q(B^s) = 1 - \Psi_1 B^s - \Psi_2 B^{2s} - \cdots - \Psi_Q B^{Qs}, \qquad \text{(SMA)}.$$

For an ARIMAX$(0, 1, 1) \times (0, 1, 1)_{52}$ model, which is used in the spot price forecasts in this thesis, the equation becomes:

$$\begin{aligned}
Z_t =& Z_{t-1} + Z_{t-52} - Z_{t-53} \\
& + \beta X_t - \beta X_{t-1} - \beta X_{t-52} + \beta X_{t-53} \\
& + \varepsilon_t + \psi_1 \varepsilon_{t-1} + \Psi_1 \varepsilon_{t-52} + \psi_1 \Psi_1 \varepsilon_{t-53}
\end{aligned} \qquad (8.9)$$

Despite high correlations in the temperatures for regions on the Northern hemisphere, the authors have chosen not to include an exogenous variable as a co-variate in the forecasting-models for the temperatures for each region. As discussed in Section 4.3.1, the sea temperature at the fish farms depends heavily on local weather conditions and on the exchange between coastal waters and fjords. Thus, local influence dominate the short-term temperature-fluctuatuions.

8.4.5   Scenario generation method

To generate the scenario residuals, the authors have used have used the copula based scenario generation heuristic developed by Kaut (2014), where the residuals are generated from historical values for each time series to be forecasted. The binaries and the source code for the scenario generation procedure is public available on Michal Kaut's webpage (Michal Kaut, 2015), as well as the coherent paper by Kaut (2014).

To generate the scenarios, the residuals generated by the copula-based heuristic are added to the forecasting model's equation in the time period of the split, see Figure 5.6 in Section 5.2. For an general ARIMA$(p, d, q) \times (P, D, Q)_m$ this is illustrated in Equation (8.10).

$$\phi_p(B)\Phi_P(B^s)(1 - B)^d(1 - B^s)^D(Z_t - \beta X_t) = \psi_q(B)\Psi_Q(B^s)\varepsilon_t + \epsilon_{ts} \qquad (8.10)$$

The residual term, ϵ_{ts}, will affect the resulting future forecasts in different ways depending on the time series historical values and the estimated parameters for the forecasting model. If the time series is stationary, the scenarios will converge. How fast one may expect the scenarios to converge depends on the values of the AR and MA parameters, and the number of AR and MA terms in the model. If the series are non-stationary it is difficult to give a qualified answer to how the behaviour of the time series prediction. However, if the non-stationary time series are forecasted with an AR-model, the confidence interval for the possible scenarios will converge. For a MA-model, the scenario residuals are expected to affect the forecasted scenarios q periods after the split is done, regardless of the time series are stationary or non-stationary. Furthermore, if the forecasting model involves a seasonal MA term and the forecast is longer than the seasonal period, m, of the time series, the first predicted values may affect the time series forecast Q periods into the next season as well.

For the Norway Mid temperatures, which are forecasted with an ARIMA$(1, 0, 0) \times (0, 1, 1)_{52}$-model, the temperature scenarios converge with time, see Figure 8.10. This coincide with the series expected behaviour, as the time series are found stationary in the mean and as an autoregressive term is used in the forecasting model. As already mentioned, how fast the series converge depend on both the order of the autoregressive term and the value of the parameters in the forecasting model. The different parameters that are fitted over historical temperature data are given in Table 8.5. For the spot prices, which are forecasted with an seasonal moving average model, the scenarios do not converge. However, the residual term added in the split affects the solution for one time period, making the time series either converge or diverge to or from the deterministic forecast respectively. How large this step is depends on

Temperature coefficients

Region	λ	AR(1)	SAR(1)
Norway North	0.2222	0.7509	-0.6452
Norway Mid	0	0.7043	-0.8106
Norway West	0.6667	0.7519	-0.9997
Norway South	0.7071	0.6536	-0.6434
Scotland	0.4242	0.7678	-0.9902
Canada	-0.4242	0.8135	-0.9977
Chile	-0.8282	0.6484	-0.9991

Table 8.5: Coefficients for the Box-Cox transformation parameter, AR-term and seasonal AR-term for each region's temperature model.

the moving average parameter. As Table 8.6 show, the moving average parameter for smoked salmon is rather large, meaning the spot prices for smoked salmon have only a small spread between the different scenarios as soon as one time period after the first split.

Price coefficients

Product	λ	MA(1)	SMA(1)	X_{reg}
HOG 2-3 kg	0.5858	-0.2213	-0.7069	0.9018
HOG 3-4 kg	0.7878	-0.18227	-0.9996	0.9859
HOG 4-5 kg	0.7474	-0.23782	-0.6569	0
HOG 5-6 kg	0.5454	0.310378	-0.3315	0.9816
HOG 6-7 kg	0.5454	0.111327	-0.6212	0.9581
Fresh fillet	1.5556	-0.2993	-0.7485	0.3085
Frozen HOG	0.6667	-0.5044	-0.4312	-0.0147
Frozen fillet	1.7171	-0.6054	-0.9999	-0.0302
Smoked	0	-0.8698	-0.6365	0.2118

Table 8.6: Coefficients for the Box-Cox transformation parameter, MA-term, seasonal MA-term and the exogenous variable parameter for each product price model.

Figure 8.11 illustrates the scenario tree for HOG salmon in weight class 4-5 kg, where the same splits as in Figure 8.10 are applied. The splits that are used in the illustrative scenarios trees are only chosen to give a clear picture of how the scenario tree evolves through time. As later discussed in Chapter 9, the model was run with larger combinations of splits, and consequently the scenario trees used in the computational studies are more detailed for most of the instances. For illustrative purposes, it is chosen to present the scenarios with a 5-split in the first period, and a 3-split in the sixth.
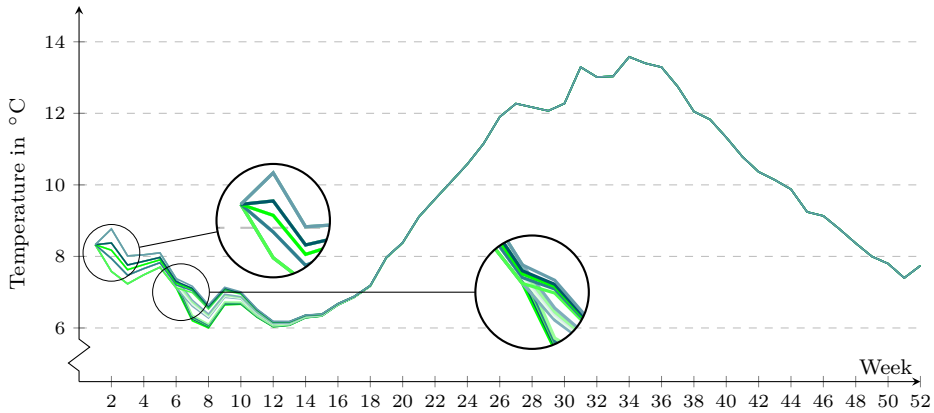
Figure 8.10: Temperature scenarios generated for Mid Norway with split in time period 1 and 6. Note that the temperatures are forecasted with an autoregressive model which differs from the moving average model used for spot price forecasts in Figure 8.11. The most conspicuous difference is that the temperature scenarios converge back to the forecast, while the price scenarios do not converge.
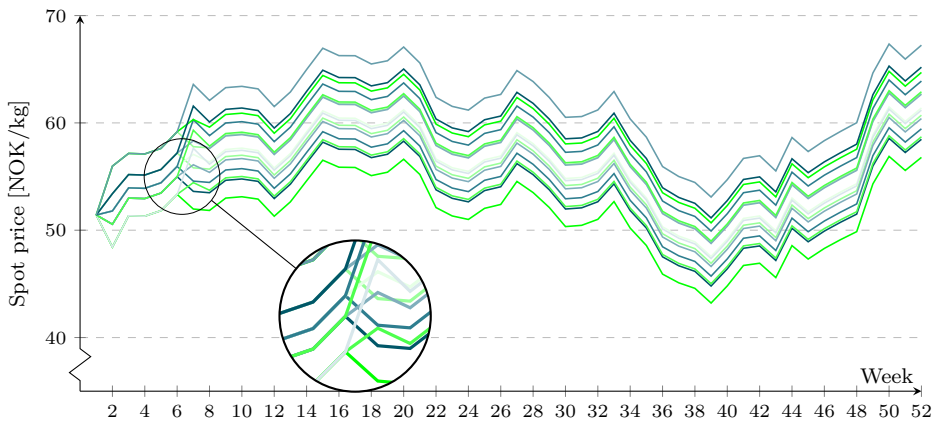
Figure 8.11: Spot price scenarios generated for HOG salmon in weight class 4-5 kg with split in time period 1 and 6. The scenarios have the same splits as in Figure 8.10. The difference is however is that the spot prices are forecasted with an seasonal moving average model. HOG salmon 4-5 kg is also the exogenous variable used as a covariate in the other forecasts of salmon spot prices.

8.4.6   Market prices

Due to lack of price data differentiated both on product and market, it was not possible to create market specific price scenarios. Consequently, the market prices were implemented by use of mark ups and mark downs for each market compared to the Norwegian market. The Norwegian market was used as a reference market, and mark ups and mark downs were based on historical data for Norwegian HOG export prices to different countries. Market prices were generated using weighted averages, and extrapolated to all product groups.

To induce the less than perfect correlation between the market prices seen in empirical studies, an error term was added to this marked up price. The error terms were generated in the same way as the scenario residuals by using Kaut (2014)'s copula scenario heuristic. In addition, by generating a total of 52 residual mark ups and downs for each price series, this method ensured that dependencies between the different product's price series were kept throughout the time horizon.

8.4.7   Scenario tree structure

A scenario tree is characterized by the total number of scenarios, the number of stages, placement of the split nodes and the bushiness of the tree, meaning the number of branches at each split (Pflug and Pichler, 2014a). The number of scenarios in a stochastic model will have an considerable impact both for the solution time of the model and for the solution itself. The structure of a scenario tree can also affect the solution. For instance, a comparatively longer deterministic period prior to the first split may give the model more time to adjust for uncertainty in later time periods. Therefore, the scenario tree structure must be chosen with care.

Having a large number of scenarios will also in most cases give a more realistic representation of the uncertainties. The drawback is that a large number of scenarios lead to an rapid increase in problem size, and thereby increases the solution time of the model. The number of scenarios should therefore be chosen as a trade-off between solution time and solution quality obtained.

In some applications, the structure of the scenario tree in terms of number of stages might be given by the problem structure itself. King and Wallace (2012) divide stochastic models into inherent two-stage models, also referred to as invest-and-use models, and inherently multistage models, also referred to as operational models. Models that are inherently two-stage are based on problems where there first is a major long-term decision, whereas the rest of the horizon represent use of the investment. In contrast, inherent multistage models incorporate only stages of the same type. In such models,

stages model the flow of information rather than a divsion between investment and use.

A two-stage scenario tree is easier to implement, both in terms of input data generation and coding of the model itself. Furthermore, two-stage models are more intuitive to use and analyze for a decision maker which is inexperienced in stochastic optimization. Conducting model evaluation by calculating the VSS is also more straight forward for a two-stage model, as the calculation of EEV become more complex in multistage problems. Furthermore, two-stage problems may either have the same number of realizations and higher bushiness or the same bushiness and a significantly smaller problem size than most multistage models. However, the two-stage model solution is generally too optimistic compared to a multistage model, as larger parts of the model is deterministic. On the other hand, the process of generating input data for multistage models are often much more complex. From the definitions above it is evident that the model presented in this thesis fall into the category of inherent multistage models, and should thus be modeled accordingly

After having decided upon the number of stages, the placement of the split nodes should be carefully chosen. Generally they should be placed where new information become available. In this problem, new information regarding both temperatures and prices become available on a daily basis. Splitting every time period would lead to an incredibly large tree resulting in the model only being able to solve very small problem instances. Consequently, one must choose a subset of time periods to split on. One could argue that the most rational placement would be before important decisions are made, or in periods with especially high uncertainty.

The decisions with the largest long-term impact in this problem are the contract decisions, and it should therefore be a split immediately after the contract decision on entering a contract have been made. As demand peak around Christmas, one could further argue that prices in December will impact total profits more than prices at other times of the year. By following this logic, a split should be placed a short time before the Christmas rush. However, this particular model is developed to be used as a rolling horizon model, and thus the early periods decisions are by far the most important ones. As a consequence, a detailed modeling of these decision takes precedence. Also recall that everything happening beyond time period six in the model is only included to model end effects. Marine Harvest also operate with a six week planning cycle (Johannessen, Vinnes and Sværen, peronal communication, 2015), splits are therefore placed in period 1 and 6. One could argue that the decision maker to some degree have insight into what the price and temperature development will be the coming six weeks, and that the splits should rather be placed in period 6 and

12. However, by placing the contact decisions in the first time period a split in the immediate period fall natural and, as discussed, thus giving the model a more tactical slant.

The final decision regarding the scenario tree structure is to chose a reasonable total number of scenarios. According to Michal Kaut (personal communication, 2015), one cannot expect a tree with good statistical properties if one have fewer branches than the number of stochastic parameters in each split. In this problem the number of stochastic parameters is 16 (9 prices, 7 temperatures), meaning a tree should preferably contain at least 256 scenarios. However, such a tree would have a solution time making the model unsuited for any practical applications (100 scenarios take approx. 3.5 days to solve for this problem). Based on this trade-off between solution time and statistical validity, the choice fell on a tree containing 40 scenarios.

8.5 Input data

As the input data highly affects the model solution, it is important to be aware of the sources of these data and how they have been manipulated.

8.5.1 Initial conditions

For each location, the initial biomass, i.e. the initial number of fish in each weight class v is given as input to the model. As no numbers of live stock for Marine Harvest is publicly available, this input is estimated based on other available numbers. In order to get harvest values comparable to the values for Marine Harvest, their annual harvest numbers per farming region (Marine Harvest, 2010; Marine Harvest, 2011b; Marine Harvest, 2012; Marine Harvest, 2013) were chosen as the basis for estimating the initial biomass levels. Then, the biomass were distributed to the different farming weight classes relative harvest volumes presented in (Marine Harvest, 2014b).

The smolt release plan in Norway is based on the numbers from The Directorate of Fisheries (2015) adjusted by the market share of Marine Harvest (0.25) (Marine Harvest, 2011a). These numbers are then extrapolated to the other regions based on the relative harvest volumes per farming region. As a simplification, only smolt of sizes 100 g, 150 g and 250 g are deployed, where a random smolt size is chosen for each smolt deployment. Additionally, as the smolt release values are per month, each smolt deployment is performed in a random week within that month.

Last, initial inventory levels are generated randomly based on historic production.

8.5.2   Contracts

As no data has been available regarding Marine Harvest's historical contract arrangements, the set of available contracts is generated by randomly choosing contract type, product, market and the set of delivery time periods for each contract. Note that no consumer preferences in different products are included. 20 initial contracts and 300 new contracts are used as input. The volumes per delivery are based on the capacity of a trailer, which is about 20 MT (Aandahl, P.T., personal communication, 2015). Here, most deliveries are set to the capacity of one trailer, while some are also set higher.

According to Larsen and Asche (2011), contract sales give on average the same revenues as spot sales. As such, the contract prices used as input are based on the average forecasted price for each product in each market and a random markup between 2 and 7 NOK/kg.

8.5.3   Growth

The growth model is implemented by use of a modified version of salmon food producer Skretting's growth matrix. This matrix gives the weekly growth in grams for a salmon belonging to a specified weight class given an average sea water temperature during that period. Note that the growth of the largest weight class is zero. This is done to ensure that fish does not grow out of the distribution.

8.5.4   End of horizon

The expected valuation of biomass and inventory is meant to represent the net future profits of the biomass and inventory, meaning the expected profits attained when selling the fish or processed products less the expected sales and administrative costs and other costs related to growing the fish and getting it to the customer. For biomass, the expected salmon value would additionally have to be adjusted for the expected caring, harvest and slaughter costs, whereas the expected product value would have to be adjusted for expected inventory costs.

The expected value before adjusting for the costs of withholding could either be given based on historical values obtained prior to the beginning of the planning horizon, or be based, fully or in part, on the prices realized during the planning horizon. On the other hand, Øglend and Sikveland (2008) argue that the high price volatility of salmon makes recent historical prices less influential on future prices. A final alternative is therefore to base the expected salmon value on the spot price immediately prior to running the model.

Based on the foregoing discussion, the use of historical averages is considered to be the best approach, and consequently this is the method implemented. To give the model incentives to reduce the volume of old products at the end of the planning horizon, the oldest artificial inventories are given an incrementally smaller value.

8.5.5 Constants

The values of the constants used in the model were estimated in the following way:

Caring costs

The caring costs are based on the numbers used in the work of Hæreid (2011).

Yield ratios

The slaughtering loss ratio used is 0.84, based on numbers found in Marine Harvest Industry Handbook 2014 and Salmon World (Kontali Analyse, 2014; Marine Harvest, 2014b). The processing yield of frozen whole fish is set to 1, as fresh and frozen HOG will weight the same. The processing yield for fillets is based on numbers found in Marine Harvest (2014b), and processing yield for smoked salmon is calculated based on volumes of input and output given in Marine Harvest (2013).

Production costs

Production costs are not available directly as they are regarded sensitive. As a result they are estimated from known profit margins and figures in the Marine Harvest annual report (Marine Harvest, 2014a).

Inventory costs

Inventory costs are based on numbers provided in Shaw and Muir (1987).

Production capacities

Production capacities are estimated based on available information on total yearly sales volumes that are assumed to be distributed evenly among the production regions modeled.

Transportation times and costs

Transportation times and transportation costs are calculated based on numbers provided by *World Freight Rates* (2015) and times found by the use of Google Maps' distance and direction API.

Spot purchase mark-up

Based on scaling of the model, the spot purchase mark-up is set to 6 NOK.

8.5.6   Input data generation

To easily generate different sets of scenarios dependent on the decision makers preference, an `Excel`-script is developed where the user may set the time period of the split and number of branches in each split. As such, the model can be easily be adopted to the level of available information and the desired degree of uncertainty. The `Excel`-script is found in Appendix E, which describe the electronic attachments to this thesis.

A large set of input data were also generated automatically by the use of `Python` scripts. This included most of the sets and matrices that were unchanged, such as the origin and destination sets, the contract sets and contract parameters and the path sets and path parameters

8.5.7   Testing of the input data

The models were initially tested on small instances for verification purposes, before moving on to larger, more realistic data sets. These tests are explained in further detail in Appendix C.

Chapter 9

Computational study

The model studied in this thesis was implemented as a deterministic and three-stage model. The three-stage version were tested on different scenario trees. In the following chapter, results and main findings from the tests performed on the different models are presented and discussed.

After verifying correct model behaviour, several tests were performed on larger instances meant to give a realistic representation of reality. The analysis performed can be grouped into four types:

1. Realism comparison
2. Stability testing
3. Sensitivity analysis
4. Case study

This chapter starts with an analysis of the sources of errors in Section 9.1, before the stochastic method is evaluated in Section 9.2. Next, the solutions from the base case are analyzed in Section 9.3, where the solution is compared to the actual numbers for Marine Harvest in 2014. Section 9.4 and 9.5 presents results of the stability testing and sensitivity analysis of key parameters, whereas a case study that analyses the effects of a trade ban to Russia is presented in Section 9.6. Considerations around the solution time and a discussion as how to improve the performance of the implemented model is evaluated in Section 9.7.

9.1 Sources of error

Before presenting the results and analysis, a general note of caution must be given regarding the model solution. As the data set is exclusively based on publicly available numbers, several parameters are unknown to the authors of this thesis. Accordingly, many key parameters are therefore approximated with the use of the publicly available figures as described in Chapter 8.

Most notably, these numbers includes the internal production costs, transportation

costs and all capacities, as well as financial figures such as the contract mark up, the spot purchase mark up and the values of ESV and EPV. Furthermore, all initial conditions, the smolt release plan and the set of available contracts are generated randomly with levels that are set to represent a real problem. It is important to be aware of these limitations when analyzing the solution, as deviations from the real numbers may change the results slightly in either direction. This fact will affect the validity of the results obtained, although the results still indicates that there are some areas that can be exploited by Marine Harvest to improve their profit margin. Consequently, most of the following analysis will focus on reality testing and why the model deviate from the reality.

No risk aversion

A consequence of disregarding the additional value of trading through contracts and selling VAP, and assuming risk neutrality, will most probably affect the solution. As the model acts risk neutral and no means for reducing the volatility in the income are included, the model is expected to act too opportunistic if used in the industry. For the results presented in this chapter, the share of processed products are thus expected to be lower than the real numbers. Furthermore, as the model is not run in a rolling horizon environment and contracts can only be entered in the first time period, the contract share cannot be compared to the real contract share. However, the analysis can give some indications as to a preferred contract design.

No mortality

Several of the simplifications made will to some extent have large implications for the solution and objective value. As no mortality or loss of fish is included, the solution will be somewhat optimistic. The VSS will thus be lower than it would if mortality scenarios were included. Both the sales allocation and product-mix will likely be affected by the simplification of unlimited demand. Factor costs such as caring costs, production costs and inventory costs are nor differentiated on location. Consequently, the model solution will not reflect effects such as the Chilean exports having a comparatively higher share of VAP due to cheaper labor.

Scenario generation

Another important factor is that the optimal solution is highly dependent on the price scenarios generated. Although considerable efforts have been made to generate a good ARIMA-model, there are some precautions that must be taken. Firstly, the lack of consistent price data may affect the results adversely. This is because two separate time series from Norwegian Seafood Council (2015) and NASDAQ (2015b) are used as

input, as they were the only available price data with the desired detailing level. This implies that the price relationships may not reflect the actual historic prices achieved by Marine Harvest.

The price forecasts used as input are also shown to be too optimistic when comparing with actual price data for 2014 NASDAQ (2015b). This is because of a significant rise in the historic prices towards the end of 2013, as depicted in Figure 9.1. This effect is especially evident for the frozen HOG price, as the price gap between fresh and frozen HOG was historically high at the end of 2013. This is highlighted in Figure 9.1
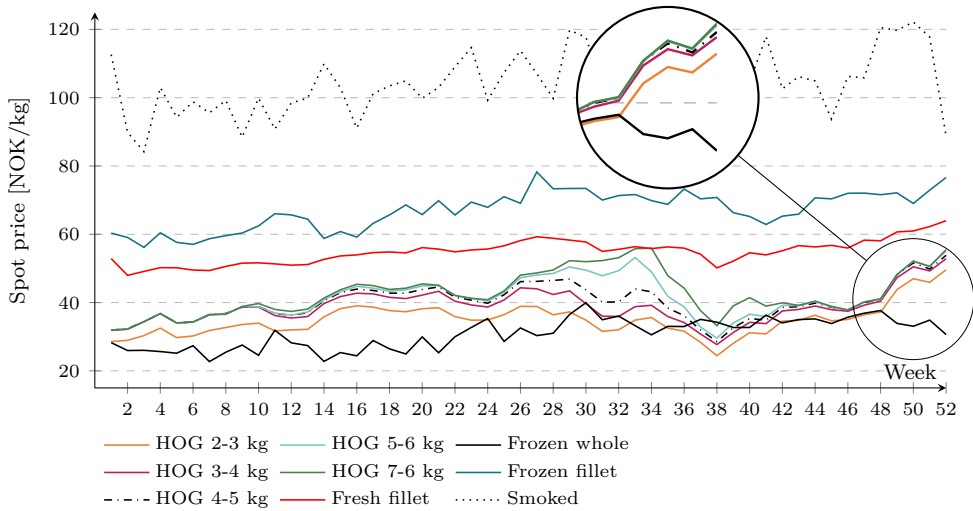
Figure 9.1: Historic product prices in 2013 for all fresh HOG salmon weight classes and all processed product categories included in the model.

In general, when using seasonal ARIMA-models the average level in the forecast will be mostly dependent upon the last historic value unless a trend is included or there is a strong seasonality in the data. As a trend was omitted because of the high price volatility, and the historic seasonal pattern is quite weak, a forecast hovering around the end values for 2013 was anticipated. When not implemented in a rolling horizon environment, the effect of the wrong forecast becomes quite large.

As all prices are based on Norwegian salmon prices with a markup adjustment, some markets may have unrealistically high prices. However, the use of a copula-based sampling method implies that the relationships between the markups and the distribution of the markups in each market should correspond well with the historical values.

No tariffs

As a consequence of excluding tariffs, the global trade patterns are expected to shift somewhat compared to actual numbers. As Canada and the USA belong to NAFTA,

and Scotland and Ireland belong to the EU, trade within these regions is expected to be lower in the model solution than in real life. Furthermore, as Russia has relatively high tariffs of 10 % (Eurasiatx - Eurasian market news, 2015), the trade with Russia is also expected to be to optimistic. This effect will be further amplified by the high rail tariffs in Russia making transport from the coast to the inland market costly (The Moscow Times, 2014). This cost is not intercepted in the model. The exclusion of tariffs can also affect product-mix, as primary and secondary processed products are taxed differently.

Temperature forecasting

The temperature scenarios are based on daily sea temperatures from National Oceanic and Atmospheric Administration (2015). National Oceanic and Atmospheric Administration (2015) uses 8-km resolution Global SST Observations to generate a composite gridded-image of the sea surface temperatures (SST) on a daily global scale. The temperatures are afterwards derived from this image, which is naturally subject to variable quality. The temperatures are also the skin sea surface temperatures, and may not represent the actual water temperature at the depths where the fish reside. As surface temperatures are probably higher, this could result in somewhat optimistic growth in the model. However, this time series was chosen as there are few alternative data sources of weekly temperatures.

One alternative could be to use Fiskeri- og Havbruksnæringens Landsforening (2015) for Norwegian sea temperatures. However, these temperatures are only given on a monthly basis, and would therefore give to few observations for fitting a forecasting model of high quality.

9.2 Evaluation of the stochastic method

As the problem is inherently multistage by nature, the performance of two three-stage implementations of the model is evaluated. The performance of the stochastic models can be described by using the concepts introduced in Section 5.1. For each model the RP, WS and expected value of expected solution (EEV) are calculated. The WS is calculated by relaxing all non-anticipativity constraints, and thus giving the model the freedom to solve each scenario deterministic. For the three-stage model, the EEV is calculated with the use of the second approach presented by Escudero et al. (2007) described in Section 5.1. This approach is chosen because of its higher degree of accuracy, albeit it is more time consuming to implement.

In Table 9.1, the EV, EEV, RP and WS solutions for two different instances are

listed, whereas EVPI and VSS are presented in Table 9.2. Both instances are three-stage models with splits in time period 1 and 6. The first instance has 15 scenarios with bushiness (5,3), while the second has 40 scenarios with bushiness (8,5). The latter is also used as the base case for the rest of the results.

Model	Nr of scenarios	EV	EEV	RP	WS
Three-stage	15	16 275.9	15 645.3	16 297.0	16 305.9
Three-stage	40	15 971.0	15 435.5	16 338.1	16 338.0

Table 9.1: Comparison of RP and EV solutions [mill. NOK].

In both instances, the WS solution is greater than the RP solution, which in turn is greater than the EDEV solution. This coheres with property (5.15), and is true for all recourse problems. Furthermore, both the EVPI and the VSS is lower for the 15 scenario instance than for the 40 scenario instance. This result is as expected, because more scenarios give a larger number of realizations of the future, which in turn give more value to the use of stochastic programming. The explanation for this, is that when including more scenarios, the fixation of decisions in the EDEV will have a more severe adverse effect on the objective value. Also note that the RP solution is higher when adding more scenarios, as will be explained in further detail below in Section 9.2.1.

Model	Nr of scenarios	VSS	VSS %	EVPI	EVPI %
Three-stage	15	651 700 000	3.9989 %	8 900 000	0.0546 %
Three-stage	40	892 580 000	5.4665 %	9 900 000	0.0606 %

Table 9.2: VSS and EVPI for three different model runs.

As Table 9.2 show, the EVPI is quite small for both instances. In these cases, the scenarios converge or partially converge and most decisions are performed each time period. As such, there seems to be less value of perfect information, since the scenario tree has a smaller span and the model has a lot of adjustability.

However, the VSS is significant for both instances, at approximately 4.0 % and 5.5 % respectively. This indicate that the effect of adding stochasticity in the planning process for Marine Harvest has a significant value, although the added complexity severely affects the solution time, as discussed in Section 9.7. The possible increase in profits must be considered against the added computational effort and the need for increased knowledge among the model users when deciding whether to use a deterministic or stochastic model.

9.2.1   The effects of different scenario tree structures

Model	Nr of scenarios	Profits [mill. NOK]	Diff
Four stage	45	16 399.7	71.6
Three stage	100	16 414.2	86.1
Three stage	40	16 328.1	31.1
Three stage	15	16 297.0	61.5
Two stage	40	16 235.5	264.5
Deterministic	1	15 971.0	

Table 9.3: Profits for different model instances, sorted highest to lowest.

As evident from Table 9.3, the solution to the RP problem seems to increase with both the number of stages and the number of scenarios included. Consequently, the inclusion of more scenarios seems to give the model more flexibility and thus more potential to adjust its production to earn relatively more in the high price scenarios compared to the relative loss in the low price scenarios. This may be a result of the bushiness chosen, as the increase in bushiness may give more probability for the scenario generator to also include some outliers. When analyzing the residuals used, this fact is especially prominent when looking at the matrix with three residuals for each price and temperature, where the spreads between the largest and smallest residuals are much lower as compared to the other branch matrices.

Furthermore, the share of spot sales relative to the contract share is increasing when adding more scenarios or adding more stages in the scenario tree. This seems to be a result of how the model postpones uncertain decisions when possible, as contract signings limits its flexibility in later time periods.

When considering the results above and the problem properties, where most decisions are taken every time period, the chosen scenario generation method may not been given the desired statistical properties. Usually, splits in the scenario tree are used as a mean to model specific stages in the planning period where new information is observed, or there are important events where the uncertainty in future outcomes should be taken into account. In this problem, the first six weeks were considered as the most important, and thus splits in week 1 and week 6 were implemented. However, when considering the results, there may have been better to model the uncertainty with a different scenario generation method. One alternative could be to simulate a large set of possible outcomes with the statistical properties of the historical data and then use a scenario reduction technique to get a realistic tree size (Tjelmeland, H., personal communication, 2015).

9.3 Base case analysis

The three-stage model described in Section 8.4 is used as a base case throughout this thesis. The base case solution is compared to actual numbers from the industry and Marine Harvest to test the realism of the model solution. When conducting this comparison, it is crucial to keep in mind the sources of error expected to cause deviations discussed in Section 9.1. Note also that only the base case is analyzed in this section, most results obtained here are similar in other runs.

9.3.1 Key figures

Table 9.4 summarize some key numbers and compare them to relevant values found on the Marine Harvest's annual report (Marine Harvest, 2014a). A percentage value giving the base case solution values relative to the real values is also included. Note that because of different calculation methods, the numbers for the base case solution cannot be directly compared to the numbers presented in the annual report.

Key Figures 2014 [MNOK]	Base case	MH	%
Volume harvested (GWE), tonnes	369 121	418 873	88%
Average price achieved (NOK/kg)	61.9	57.3	108%
Spot revenue	21 288	15 351	139%
Contract revenue	1 554	8 634	18%
Transportation costs	-459	-1 922	24%
Inventory costs	-0.55	-	-
Caring costs	-2 791	-4 751	59%
Spot purchase costs	-266	-	-
Processing costs	-1 450	-	-
Change in biomass value	-1 395	477	-292%
Change in inventory value	-125	650	-19%
Objective value/EBITDA	**16 328**	**5 220**	**313**%

Table 9.4: Key figures comparison of the base case solution and similar figures reported by Marine Harvest in 2014 (Marine Harvest, 2014a).

As Marine Harvest report their harvest volumes in gutted weight equivalent (GWE), the model solution is transformed from whole fish equivalent (WFE) to GWE for comparison purposes, whereas harvest volumes in the rest of the chapter are reported in WFE. Also note that the biomass and inventory values are presented as the change in value and not the absolute end value for easier interpretation.

The objective value is 16.3 billion NOK, compared to a reported EBITDA of 5.2 billion NOK. As discussed in Section 7.2, several large cost objects are excluded from

the objective function in the model, and these two values are therefore not comparable. The revenues is on the other hand easier to compare, and as evident from the average price achieved, the total revenues from the base case seems comparable to the actual revenues. However, as will be discussed in further detail below in Section 9.3.3, there are several differences between these two values.

Below, a visual representation of the distribution of the costs and revenues are given in Figure 9.2a and Figure 9.2b. A notable difference between model solution and real numbers is that the contract share in the model solution is only 7 %, compared to a reported 36 %. This is however not surprising, as contracts are modeled as a first stage decision. For a more realistic comparison of the contract share, the model should be run with a rolling horizon of six weeks, where contract decisions are included as first stage decisions in every run. Note that exclusion of the revenue generated from initial contracts also affect the result below.

(a) Revenues   **(b)** Costs
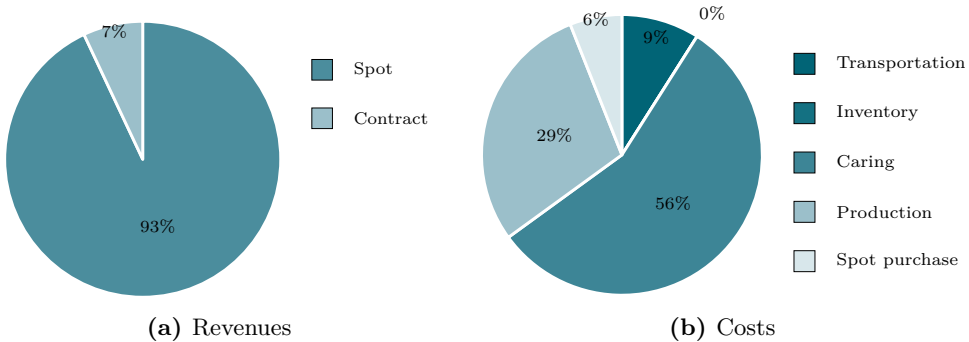
Figure 9.2: Revenues and costs generated in the base case solution

Furthermore, the real operating expenses and net changes in assets are also deviating significantly from the base case solution. When comparing the figures reported in Table 9.4, the absolute values of the transportation costs and caring costs both seems a bit low. Moreover, the share that goes to transportation seems unrealistically low, as well as the fact that where the model is getting rid of its assets, Marine Harvest is increasing both its live stock and product stock. Note that for some of the figures presented in Table 9.4, comparative values were not obtained.

In the following sections, a more detailed analysis of the values obtained from the base case will be given. The focus will be on explaining the deviations from the real values, as well as giving suggestions to how Marine Harvest may be able to improve their operational performance where relevant.

9.3.2 Production

Total volume harvested in the base case solution is 439 430 MT WFE, corresponding to 369 121 MT GWE. Compared to the reported volume from the annual report, this is 49 752 GWE tonnes lower, which represents a gap of about 12 %. This indicates that the estimated initial biomass may be a bit low, especially when considering that the model is using more biomass than what it is producing, whereas Marine Harvest is increasing its total biomass volume according to the annual report (Marine Harvest, 2014a).

Biomass development

In Figure 9.3, the biomass development is plotted. As evident from the figure, the biomass development is quite stable. When comparing the modeled biomass development to the figures presented in "Salmon Farming Industry Handbook 2014" (Marine Harvest, 2014b, p. 64), the S-shaped curve for biomass development presented there is not replicated by the model.
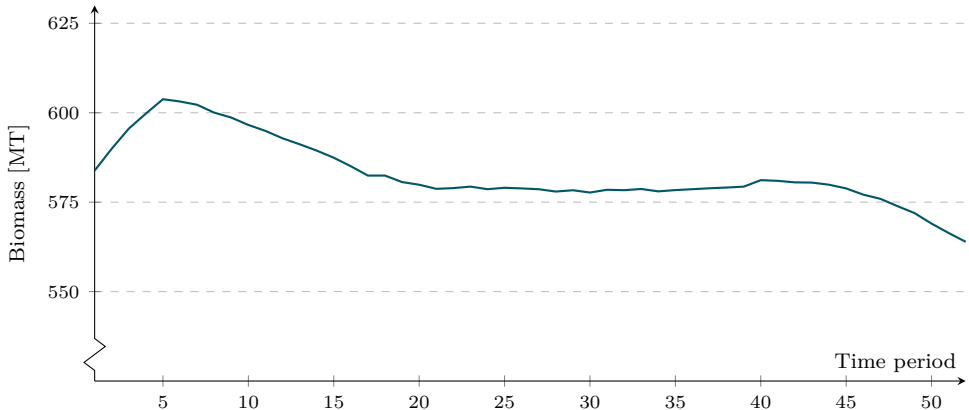
Figure 9.3: Total biomass in the base case solution

This may be an effect of several factors, one of the most probable of which being that the estimated smolt releases may deviate significantly from the actual smolt releases by Marine Harvest in 2014. The numbers used as input in the model are based on the smolt release numbers for Norway and extrapolated to the rest of the world, and do consequently not take country specific factors into account. In addition, the exclusion of fallowing may also give a more stable biomass development.

No harvest in the first time period may contribute to the small increase in the biomass at the beginning of the planning horizon. The decrease in the end of the year have a more realistic explanation, as it is caused by the price peak in December. Also, note that if implemented within a rolling horizon framework, the optimal biomass

development will most probably differ from the one in Figure 9.3.

Harvest per country

As illustrated by Figure 9.4, the harvest volume broken down per country seem to be quite realistic. The only exception being Canada, which has 61 % more harvest than what the reported value for 2014. There are two main reasons for this deviation. Firstly, only air freight is implemented as a viable transportation option for fresh HOG salmon from Chile and to the US market, whereas boat and truck can be used for transport fresh HOG salmon from Canada to the same market. Since the cost difference is very high, estimated to about 10-15 NOK/kg, and the quality of the fish is assumed equal, the model finds it more attractive to harvest relatively more in Canada. Since all costs are assumed equal and the forecasted prices in the US are based on prices for Norwegian salmon with a random markdown, the model have found the US market as a more attractive market than what it is in reality (Marine Harvest, 2014a).
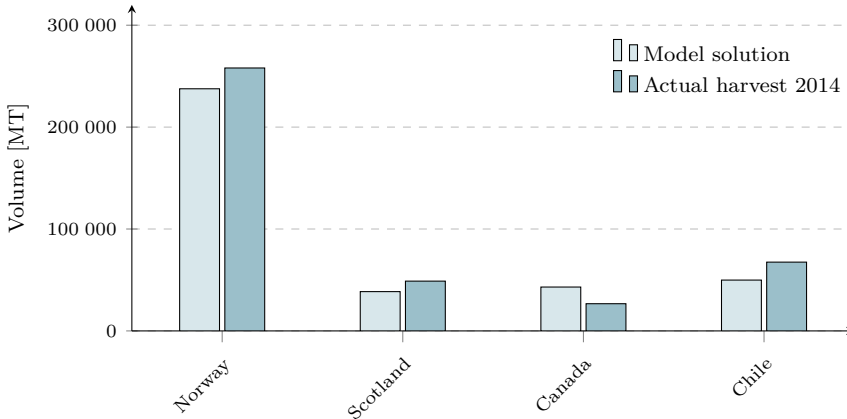
Figure 9.4: Comparison of harvest volumes pr slaughter region for Marine Harvest

9.3.3   Market transactions

The total revenues presented in Table 9.4 seems comparable to the revenues achieved by Marine Harvest for 2014, as the average price achieved were 61.9 NOK/kg compared to 57.3 NOK/kg for Marine Harvest. However, where Marine Harvest in reality has such a high price achievement because of a larger share of sales through processed products, the model obtains a higher price achievement because of the high average price for fresh HOG salmon. According to Marine Harvest, the average reference prices were 39.42 NOK/kg, 27,06 NOK/kg and 19.66 NOK/kg for FCA Oslo, Urner Barry Miami 2-3 pound and Urner Barry Seattle 10-12 pound respectively (Marine Harvest,

2014a). Furthermore, Marine Harvest had an average price achievement of 102 % in 2014. When comparing these reference prices to the average forecasted price for HOG of 57.4 NOK/kg, it is evident that the forecast was too optimistic. This is a result of the high price in the end of 2013, as discussed in Section 9.1.

Contract sales

In the base case, close to a third of the contracts are signed (94 out of 300). As the model is risk neutral, contracts entered must at least have an expected value equal to the expected value of selling the same product in the same market and in the same time periods at the spot market. Consequently, the contracts signed in the base case solution is increasing the profit, whereas contracts in reality are used as a risk reduction tool.

The number of contracts initially offered were distributed evenly between the three different contract types offered. In Figure 9.5, the relative distribution between the three different contract types are shown. As evident from the figure, fixed price contracts and partially adjustable price contracts are entered twice as often as adjustable price contracts. If assuming that the three different contract types have the same average markup on expectation, the results below indicate that partially adjustable contracts seem like the least preferable contract type. Which contract type that is the most preferable is however hard to quantify from the results presented in Figure 9.5.
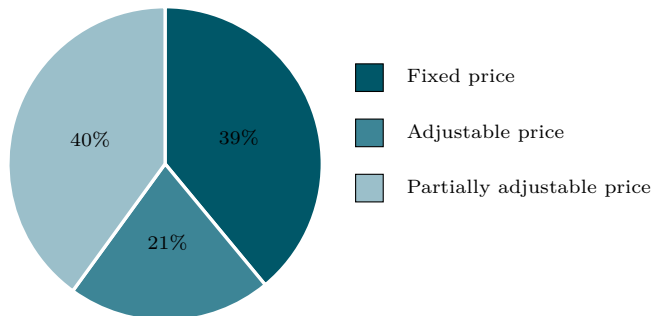
Figure 9.5: Signed contracts.

The reason for the clear preference for the two least flexible contracts seems to be the effects of included a markup for contract sales. The purpose of the markup is to assure that contracts are signed. However since this markup is always positive, the probability for losing some profits is higher than the probability for gaining extra profits if an adjustable price contract is entered. As such, a risk measure should be included instead of the markup if one wants to analyze which contract scheme that is

the most effective.

Spot sales

Figure 9.6b illustrates the total spot sales volume per time period. Contract sales volumes are excluded as they are not optional as soon as the contract is signed, and thus can obscure the pattern resulting from price changes. As evident from comparing the average spot prices for the different products to the forecasted prices, the sales volume closely follows the price development. However, the volatility in the sales volumes are much higher than the price volatility, as the model tries to exhaust even the slightest price differences. The characteristic sales volume peak immediately prior to Christmas discussed in Section 3.4 is clearly visible. Note that the low sales volumes in the first five time periods are a natural result of no volume in production or any volume flows in the transportation network in the beginning of the planning horizon.

Product mix

Figure 9.7a depict the product mix in the base case solution. As a reference, Figure 9.7b is included to illustrate the actual product mix reported in Marine Harvest's annual report (Marine Harvest, 2014a). The largest product volume is sold as fresh HOG salmon in both instances. However, the reported share of processed products are much higher in the annual report than in the model solution. This difference is largely a result of much lower fillet sales. Where fresh fillets are the second largest category with 24 % of the total sales according to the annual report, they only contribute to 1 % in the base case solution. There can be several explanation to this deviation where no risk aversion are the two most evident. A likely explanation is that a risk neutral model such as the model presented in this thesis will be indifferent to the price volatility of the products produced. Consequently, processed products with lower volatility at the expense of lower operating margin, such as salmon fillets (Marine Harvest, 2013), will be less preferred by the model. The model would rather try to maximize the profit by producing most of the product that have the highest expected profit, i.e. fresh HOG salmon.

As for the reasons why there is a much higher sale of frozen fillets, this seems to be a result of the relatively higher price volatility in the forecast and the added flexibility from allowing storage. Because the forecasted price of frozen fillets has more variation than the forecasted price for fresh fillets, frozen fillets have higher peaks that the exploits. Furthermore, as evident in Figure 9.12 which plot the inventory volumes, the model also utilizes the ability to stock up on products and sell them in time periods with price peaks.
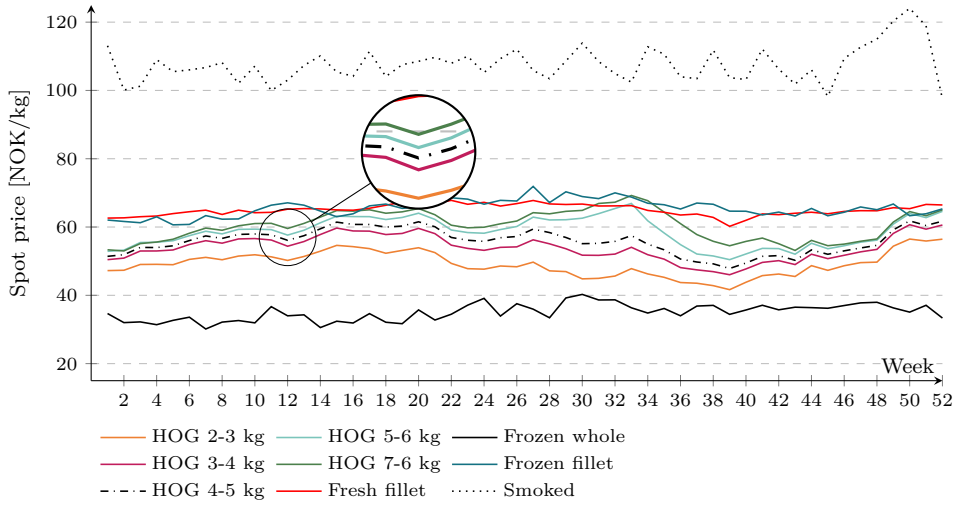
(a) Average spot prices for all scenarios in the 40 scenario base case.
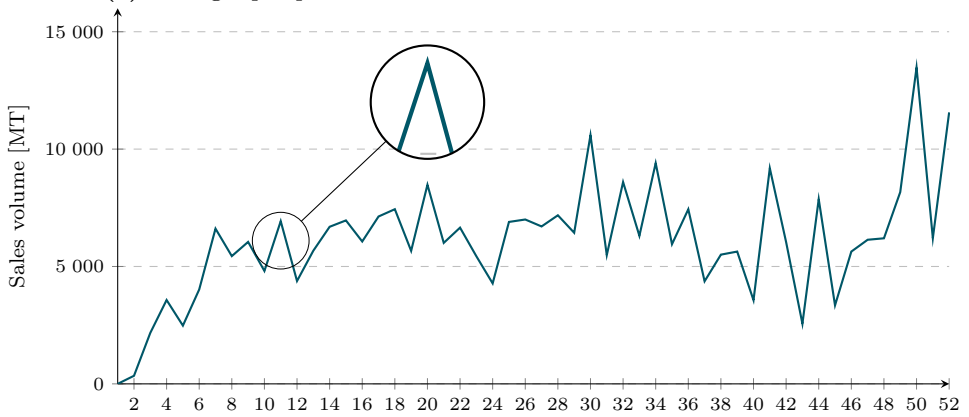
(b) Spot sale volumes per time period in base case solution.

Figure 9.6: Spot price and spot sale volumes in base case solution.

(a) Sales volume per product in model solution.

(b) Actual sales volume per product 2014.
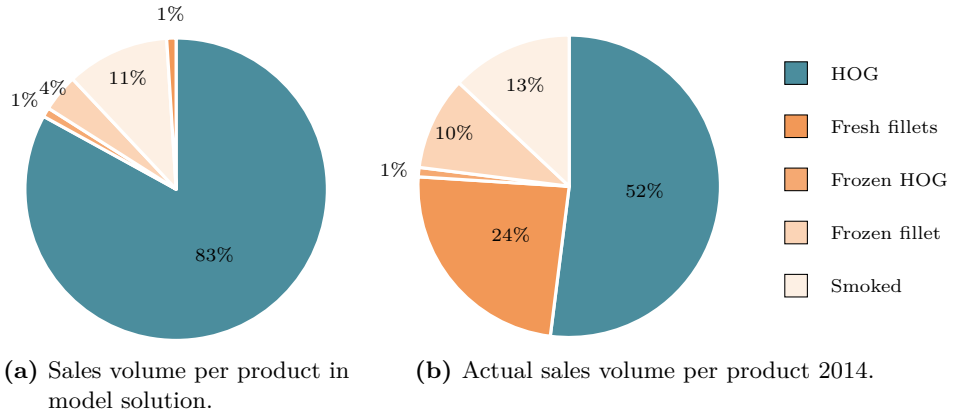
Figure 9.7: Comparison between sales volumes from model solution and actual sales volumes in 2014.

(a) Base case
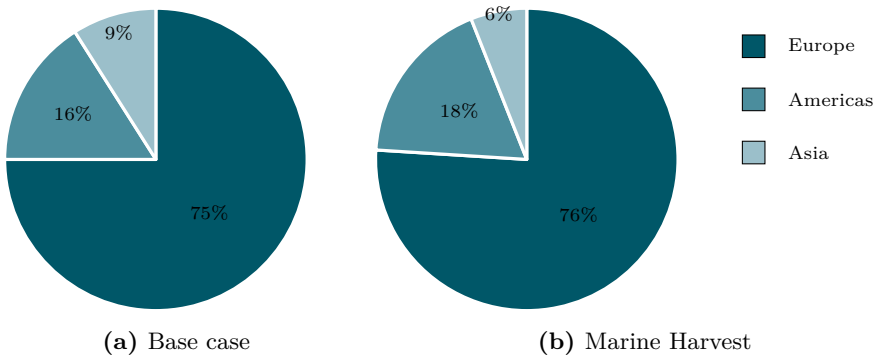
(b) Marine Harvest

Figure 9.8: Relative market sales from base case compared to the average values for Marine Harvest (Marine Harvest, 2014b).

Sales per market

How the sales volumes are distributed amongst the markets in the base case solution is illustrated by Figure 9.8 and Figure 9.9. From Figure 9.8, it seems like the distribution between the markets are on the base case solution is comparable to the average values obtained in "Industry Handbook 2014" (Marine Harvest, 2014b). However, the sales in Russia are significantly higher in the base case solution than the reported figures for Marine Harvest, where sales to Russia were only 3 % (5 % in 2013) (Marine Harvest, 2014a). This can be explained in part by the trade ban imposed by Russia blocking Norwegian exports. As such, a more probable explanation is that tariffs are not included in the model implementation. As the implemented transportation cost from Norway to Russia does not differ significantly to the cost to Europe, the higher prices in some periods implies that this will be an attractive market. If however Russia operate with higher import tariffs than Europe, this will make the market less profitable, so that more of the volume will probably be rerouted to Europe instead. Note that the numbers in "Industry Handbook 2014" are representative numbers based on historic averages, whereas the rest of the numbers used for comparison are based the performance of the base case.
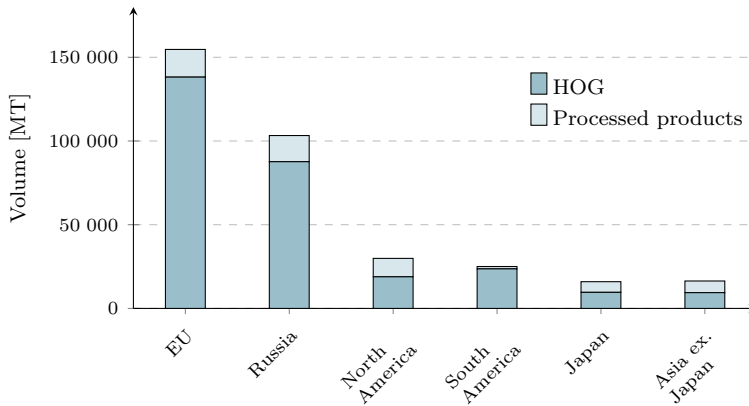
Figure 9.9: Base case sales volumes per market.

As for the share of processed product per market, this varies from 4 % the Asian markets and up to 36 % for North America. However, because of the aforementioned lack of suitable data for prices per product in each market, these differences are most probably random effects. The only exception is the high share of processed products sold in North America, which is a result of a nearly exclusive sale of Chilean salmon through processed products. If fresh HOG salmon is sold in the North American market, it must be transported by air freight, which in most instances give a much lower profit margin than to produce processed products and ship them to Norh America by

boat.

Sales distribution of fresh HOG salmon

Another interesting analysis is the weight distribution of the harvested fish. In Figure 9.10, the weight distribution in the model solution is compared to the general distribution of Norwegian harvest presented in Marine Harvest (2014b). The distribution shows a similar pattern as the actual distribution, with a relatively symmetric distribution around the 4-5 kg HOG salmon. However, the model solution suggest that there is more optimal to harvest a relatively larger harvest share of the smaller fish sizes from 2 to 4 kg, compared to the industry averages. This result is also supported by the solutions obtained in another master thesis written by Rynning-Tønnesen and Øveraas (2012), which showed a significant increase in the objective value when also smaller fish sizes were allowed to be harvested. This suggest that the premium paid on larger fish is not worth the extra cost of breeding the fish until it reaches the larger fish sizes. Furthermore, as higher caring cost will probably result in increased profitability for smaller fish sizes, and the reported caring cost from the solution of the base case is lower than the figure reported in the annual report (Marine Harvest, 2014a), the profitability for lower fish sizes may be even better.
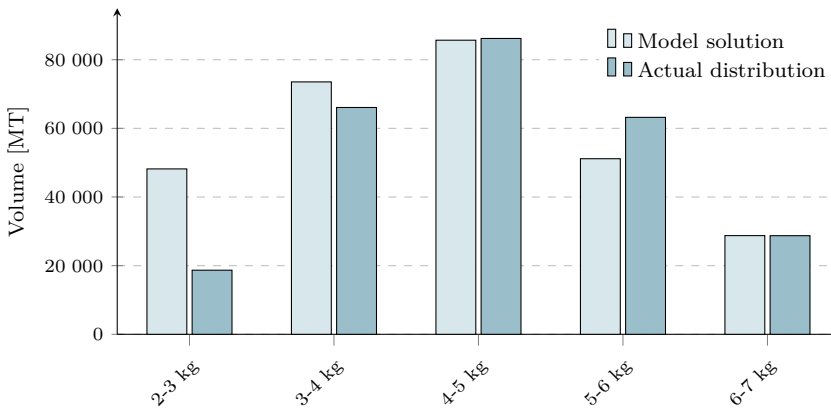
Figure 9.10: Comparison of harvest volumes pr HOG weight class.

Spot purchases

Looking at the spot purchases, most purchases amount to either 20 or 40 MT. This is seen as an indication of spot purchases being used as a means to fulfill contract obligations. A contract volume for each delivery is 20 or 40 MT for all contracts. Also, there are proven to exist arbitrage opportunities in the implemented network between Japan and Asia in a few time periods. As such, the transportation capacity

is binding in the corresponding time periods.

9.3.4   Logistics

Figure 9.11 represent the relative use of the different transportation modes included in the model. As expected, boat is the most widely used mode, followed by truck. Only a minor portion of the total volume transported, 4 %, is transported by air freight. Although no transportation data for Marine Harvest were available, the use of air freight seems a bit low, as this is the only way to transport fresh HOG salmon to Asia from Europe. However, this seems to be a result of the large price difference in using air freight compared to boat or truck. Since Canada is able to supply the Asian market with fresh products by boat, it is not surprising that it is mostly Canada that supplies the Asian markets, whereas only a small volume is transported from Norway to Asia. Because of this, there seems like a premium on Norwegian salmon should be implemented.
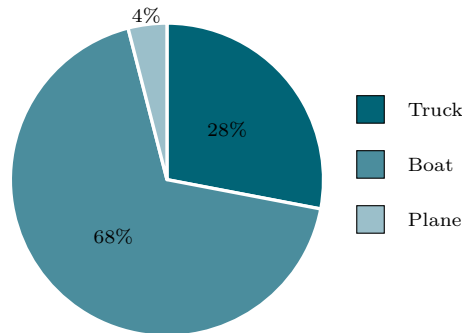
Figure 9.11: Volume transported by the different transportation modes included.

9.3.5   Inventory and end of horizon

The inventory profile of the different durable product groups included in the model are illustrated in Figure 9.12, where three interesting patterns are evident. The inventory of frozen HOG salmon steadily decreases throughout the planning horizon until it is empty and remains at zero for the remaining time horizon. This implies that the EPV values for frozen HOG salmon is too low as it is better to utilize the inventory in production instead of keeping stock at then end of the planning horizon. The production of frozen HOG is also very low because of the relatively low forecasted price.

Furthermore, the inventory profile of smoked salmon clearly show how the model uses
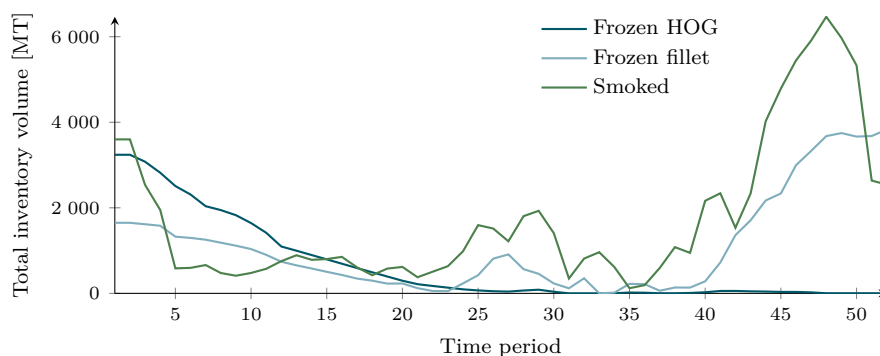
Figure 9.12: Inventory profiles per product group.

the inventory to maximize the utilization of the Christmas price peak. This seems reasonable as smoked salmon has the highest relative increase in its December price.

Another interesting fact is that the outgoing inventory of frozen fillets is much higher than the ingoing inventory, as well as it is emptied two times during the planning horizon. The reduction in the inventory in the middle of the planning horizon seems to align well with the forecasted price peak for frozen fillet. As frozen fillets are the only product that has a higher end value than ingoing value there are some indications that the EPV of frozen fillet is set comparably higher than the other values.

As for the ESV values they seems to be set at a reasonable level as the decrease in the biomass is 3.4 %. This is an indication that the end of horizon term in the objective function that controls biomass works as intended.

When comparing the effect of the values set for ESV and EPV it seems like where the ESV work as intended, the EPV have some contradicting effects. As both the inventory of smoked and of frozen HOG is reduced or even emptied at the end of the planning horizon these two values seem to be set too low. On the other hand, the model stock up on frozen fillet indicating that this value is set too high. In total the outgoing inventory is reduced by 27.4 % compared to the ingoing inventory. This model behavior indicates that controlling the end of horizon effects with the use of end values is tricky as they should somehow be set relative to the price forecast. On the other hand, if the user puts too much effort into deciding the "correct" outgoing volumes, the model could have just as well introduced a set of constraints to force the end biomass to the preferred values. However, if the user has a good method to decide "correct" end values and the user values more flexible end effects this should be the preferred method. In addition, as the model should be run in a rolling horizon environment the end values will never be used.

9.4   Stability testing

Stability testing may be used to evaluate the scenario generation method which is used. A model is said to be stable if several different scenario trees, generated using the same method and input, yields approximately the same optimal solution value in the optimization model (Kaut and Wallace, 2007). A stability test may thus be viewed as a robustness test of the model regarding how it is affected by the scenario generation method.

There are two main types of stability, in-sample stability and out-of-sample stability. A requirement for being a good scenario generation method is that different scenario trees should give approximately the same objective value from the stochastic model as in reality (Kaut and Wallace, 2003b). This is also the requirement for being considered a out of sample stable scenario generation method. The problem with this type of stability is that it must be possible to evaluate the objective function. Unfortunately, this may prove to be difficult, but the true objective function can be approximated by the value of the objective function obtained when using a very large scenario tree.

An in-sample stability test investigates how the discretization of the random variables in the scenario tree affects the value of the objective function. It can thus be said to be a test of the robustness of the discretization process, and must not be confused with being a measure of model quality (King and Wallace, 2012). A scenario generation method is said to be in-sample stable if several scenario trees generated from the same scenario generation procedure gives approximately the same optimal objective function values (Kaut and Wallace, 2003b).

King and Wallace (2012) describes two approaches to test for in-sample stability. Choosing between these two depends on whether the scenario generation procedure is random or deterministic. If a deterministic procedure is used several times with the exact same input, it will produce the same scenario tree each time. Conversely, a random procedure will not produce the same tree each time.

The copula based method by Kaut (2014) are actually an deterministic method as the residuals can be found by an MIP-model. However, the implemented heuristic by Kaut (2014) has random elements so that the procedure is considered to be random. The model is tested for in-sample stability by running the heuristic several times on the same input data which generated several different trees.

10 different trees were generated, and the optimal objective values are presented in Table 9.13. The numbers indicate only very small differences in the objective values, the largest one being 700 000 NOK, representing a mere 4 ‰of total objective value.

This indicate that the scenario generation method which is used in this thesis can be considered to hold in-sample stability.
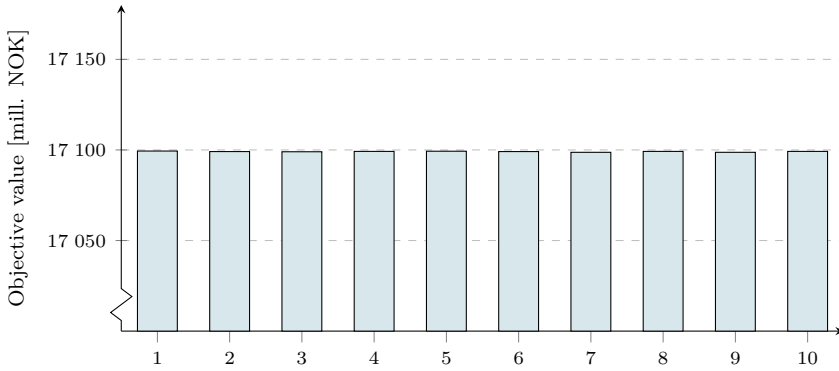
Figure 9.13: Comparison of optimal objective values from different runs.

Out-of-sample stability testing is omitted as the computational platform available, Solstorm, does not readily allow automation for solving a large number of second-stage problems. Furthermore, the solution time per second stage problem is relatively long (about 600 seconds), meaning testing for out-of sample stability would be a time consuming process. In any regard, an approximation of a more real objective value would imply using the scenarios generation heuristic to generate a much larger tree. This would then be a quite inaccurate referance point as the smaller trees would have been generated by the same heuristic.

9.5 Sensitivity analysis

Sensitivity analysis in mathematical programming is conducted to investigate how parameter changes impact the solution. The analysis conducted in this thesis focus on changes in capacities and markup values.

9.5.1 Markups

In the results presented so far, contract mark ups are fixed. To study the impact of this markup tests were conducted by running the model with both increased and reduced markups by 2 NOK. The objective function value from these runs are compared to the base case in Table 9.5. There were also conducted a similar sensitivity analysis for the spot purchase mark ups. These finding are presented in Table 9.6.

It is evident that the model is relatively stable regarding contract markups. The number of contracts signed is the same in all three runs. As expected, the objective function value increase when the mark up increase, and decrease when the mark up

	Change in mark-up	Objective function value [mill. NOK]	Contracts signed
+	2 NOK	16 328.7	94
	0 NOK	16 328.0	94
-	2 NOK	16 327.5	94

Table 9.5: Sensitivity to contract mark up changes.

decreases. However, as most of the revenue come from spot sales, the relative change in the objective function value is very small, making out less than 1 ‰of the total value in both instances.

	Change in markup	Objective function value [mill. NOK]	Spot purchase cost [mill. NOK]
+	2 NOK	16 322.9	88.5
	0 NOK	16 328.0	266.3
-	2 NOK	16 341.7	612.0

Table 9.6: Sensitivity to purchase mark up changes.

Some of the same patterns are evident when analyzing the spot purchase markup. The spot purchase cost represent only a minor part of the objective function, and the optimal objective function value is insignificantly affected. However, the spot purchase cost is highly affected. When the mark up increase, the total spot purchase cost decrease to less than one third of the base case costs. This indicate that the model find alternative sales allocations that do not invoke the need for spot purchases.

When the markup is decreased the spot purchase more than double. This will most likely be an effect of two reasons. The first is that cheaper spot purchases may induce spot purchases as more attractive to fulfill contract obligations. This is also supported by the fact that more contracts are entered when the spot purchase markup decrease. The other reason is because there exists arbitrage opportunities in the implemented network. As there are already proven to be arbitrage opportunities between Japan and Asia, this behavior is expected. As such, it is not surprising that the objective values increases significantly.

9.5.2   Capacities

To test the model sensitivity to capacity changes, several runs were conducted where the capacity of one type of resource was increased by 50 % in each instance. Figure 9.14 compares the objective function values of the different runs in which capacities were altered.
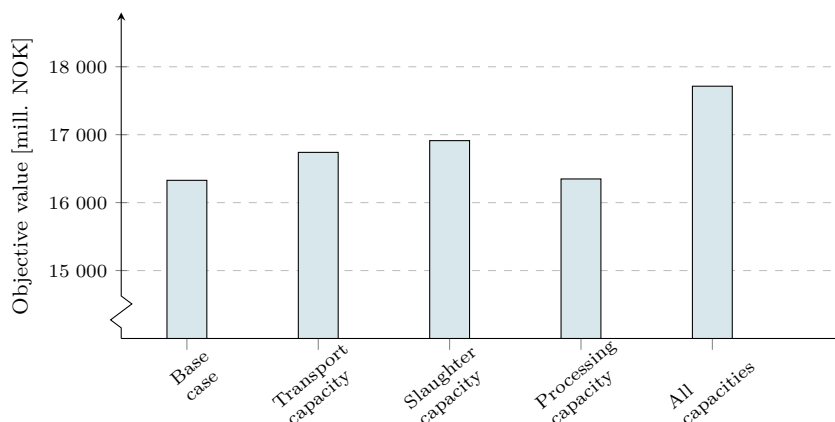
Figure 9.14: Comparison of objective function values after relaxing transport capacity constraints.

By increasing the transport capacities, the objective value increase by 2.52 %. However, an increase of slaughter capacity result in an even larger increase of 3.58 %. This indicate that the slaughter capacity are a bottleneck in the value chain to a larger degree than the transportation capacity. On the contrary, increasing processing capacity result in a mere 200 000 NOK increase in the objective function value which is considered to be insignificant. As the VAP share of the product mix in the model solution is smaller than in the actual production volumes from Marine Harvest's production is rarely conducted at maximum capacity. A 50 % increase of all capacities gives an objective value 8.48 % better than the base case, which is not surprisingly much better than the other increased capacities.

An interesting finding is that the relative share of truck as transportation increase with increased capacities, and the share of volume transported by airfreight decrease. This indicate that the use of the different modes are dictated from the available transportation rather that time considerations. This is most probably due to the fact that all transport which takes up to 3.5 days being rounded to zero in the model. As truck and boat are feasible for most distances, the model has higher freedom in terms of choosing the cheapest transportation alternative.

Another interesting finding is that an increase in the slaughter capacity resulted in a 20 % increase in harvest volumes. This may indicate that higher slaughter capacities would be reasonable if this also were true in reality. However, most likely is this just a result of the model having unlimited demand.

9.6   Case study - Trade ban

A special instance was designed to simulate the trade ban of Russia of 2014. To do so, all arcs connecting Norway and Russia were omitted from the network, whereas all other aspects of the model was kept equal to the base case. In analyzing the results, one sees that the share of VAPs of total sales increase. Smoked salmon increase from 11 % to 14 %, frozen HOG and frozen fillet increase from 5 % to 7 %, and the share of fresh fillets doubled from 1 % to 2 %. In other words, hindering trade between Norway and Russia result in a product mix more similar to the one observed in 2014 where the VAP share of sold products was larger than in the base case solution. As VAPs production increase, so does the transportation costs. This is a result of the need to transport the salmon through the VAPs processing nodes.

Sales in Russia are affected by an reduction of approximately 80 %. Most of the sales volume is re-distributed to Europe, and smaller portions go to Japan and the rest of Asia. The total harvest volume also decreased by 22 000 MT. Another interesting observations is that the production volumes at different processing locations shifts. Poland dominates in the base case, whereas Poland's relative share has diminished in this instance and the share of Chile and the US increased. In an economic sense this is also natural as cross Atlantic transportation is expensive.

9.7   Solution time considerations

The solution time of some test instances are presented in Table 9.7. The 15 scenario instance is solved in approximately 2 hours, which must be said to be acceptable. Especially when this solution time is compared to the solution time for the base case of about 11 hrs. Furthermore, the 100 scenarios model had a solution time of close to 3.5 days, while a model with aggregated time periods and 40 scenarios had a solution time of about 3 hrs.

Model	Nr of scenarios	Solution time [s]	Solution time [h]
Deterministic	1	100.95	0.03
Two stage	40	26779.4	7.44
Three stage	15	7309.15	2.03
Three stage	40	39806.9	11.06
Three stage	100	296201.0	82.28
Three stage - Agg.	40	10867	1.91

Table 9.7: Solution times of different model instances.

The table clearly shows the compromise between statistical tractability and solution

time when implementing a stochastic model. According to Kaut (personal communication, 2015), the number of residuals used in each split should at least be the same as the number of individual variables if one wants to retain most of the statistical properties between the different variables. Consequently, if one wants to have a scenario tree with a good representation of the reality, one should use the 100 scenario tree that has a bushiness of (10,10). However, because of the high solution time, such a large tree is not usable in practice. Consequently, a model with a smaller tree or model with aggregated time periods, such as the one explained below, would be used in practice.

9.7.1   Model with aggregated time periods

For rolling horizon models, a common mean to reduce solution time while still maintaining model detail is to aggregate the time periods in the less important parts of the planning horizon. By doing so the model size decreases dramatically while the more important periods of the planning horizon are still modeled in detail. However, there is a risk of getting solutions that deviate significantly from solutions attained without such aggregation as a result of the loss of details.

A test instance using aggregated time periods was implemented to analyze the gains and drawbacks by doing so on this particular problem. The time periods from time period 13 and onwards were aggregated, so that the first three months had a weekly discretization of time, whereas each time period represented one month for the rest of the horizon.

The solution time of the instance with aggregated time periods was reduced by 82.7 % compared to the base case. However, when comparing the solution of both runs for the first six weeks, these were found to deviate significantly from one another. Harvest and sales volumes were consistently larger in the aggregated version. Furthermore, the product mix deviated and the share of signed contracts decreased. This is most probably a result of the model's inability to give a correct representation of the dependencies between the time periods when they are too aggregated. This preliminary test thus indicate that aggregating time periods result in adverse effects that makes this approach unsuitable for this problem. The authors of this thesis regards that a better approach is to reduce the number of scenarios used.

Chapter 10

Concluding remarks

Understanding the uncertainty present in salmon farming is an important part of successfully planning operations. In this thesis, stochastic programming is used as a tool for including the uncertainty in a problem formulation. The scope of the model is a global production planning and sales allocations problem, spanning the whole value chain. To the authors' knowledge, this is the first model published within this area of research. The resulting multistage stochastic model considers both the uncertainty in biomass development and future salmon prices, and provides salmon producers with a tool that can aid them in making profitable decisions regarding harvest, production and sales.

A simplified version of the model is applied to the global network of Marine Harvest to illustrate how the model can be implemented and used. Discrete scenarios representing the uncertainty have been generated using seasonal ARIMA models and a copula-based scenario generation heuristic developed by Kaut (2014). Tests of the implemented model was run on a computational cluster due to its size. The results of these tests are analyzed to show how a quantitative assessment of the gains from implementing a stochastic solution can be performed.

The evaluation of the stochastic model indicate that the value of the stochastic solution is significant at 5 %. Tests also indicate in-sample stability of the scenario generation procedure, and that the model is relatively in-sensitive to changes in markup values. Furthermore, given the capacities implemented in this problem, calculations show that profits might be increased by close to 4 % by increasing the slaughtering capacity by 50 %. On a general note, the quality of the solution is highly dependent on the input data quality. As such, the results should be analyzed with care and the model should be implemented with a more realistic data set before making any general conclusions.

Chapter 11

Future research

Reduction of solution time and making the model and data more realistic have been identified as the main challenges for future work. As the actual solution time of the model is very high, improvements in solution should get more attention. The authors of this thesis regard that a reformulation of the problem, such as Bender's method should be investigated. Because of the large size of the problem, it may be beneficial to solve the problem as a set of paralell subproblems.

The growth model in the optimization model is based on a growth table from Skretting, which is only dependent on temperature and fish weight. As fish growth is dependent upon several factors, it would be interesting to improve this model so that other parameters were also taken into account, such as lighting conditions, as well as the inclusion of realistic mortality and escape scenarios.

As the model developed assumes risk neutrality, a degree of risk aversion should be included,for instance by implementing some form of value at risk model with the use of multi-objective optimization. For a discussion about different multi objective optimization techniques, see Denstad and Ulsund (2015).

Earlier research has shown that generalized autoregressive conditional heteroscedastic model (GARCH) models more accurately describes the stochastic volatility observed in the salmon price (Øglend, 2010), and these forecasting methods should therefore be investigated in further detail. In addition, other scenario generation methods could be applied to generate a more realistic representation of the future. As an example, Monte Carlo simulation could be used to generate a large set of scenarios before a scenario reduction technique is applied.

Bibliography

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Appendix A

Tables

Product	Weight conversion factor
Live fish	100 %
Loss of blood/starving	7 %
Harvest weight - Round bled fish (WFE)	93 %
Head on, gutted (HOG)	84 %
Head off, gutted	77 %
Fillet, skin on	56 - 64 %
Fillet, skin off	47 - 56 %

Table A.1: Weight conversion factors (Marine Harvest, 2014b).

(a) Norway

(b) Scotland

(c) Canada

(d) Chile

(e) VAP

Retail

Food service

Industry
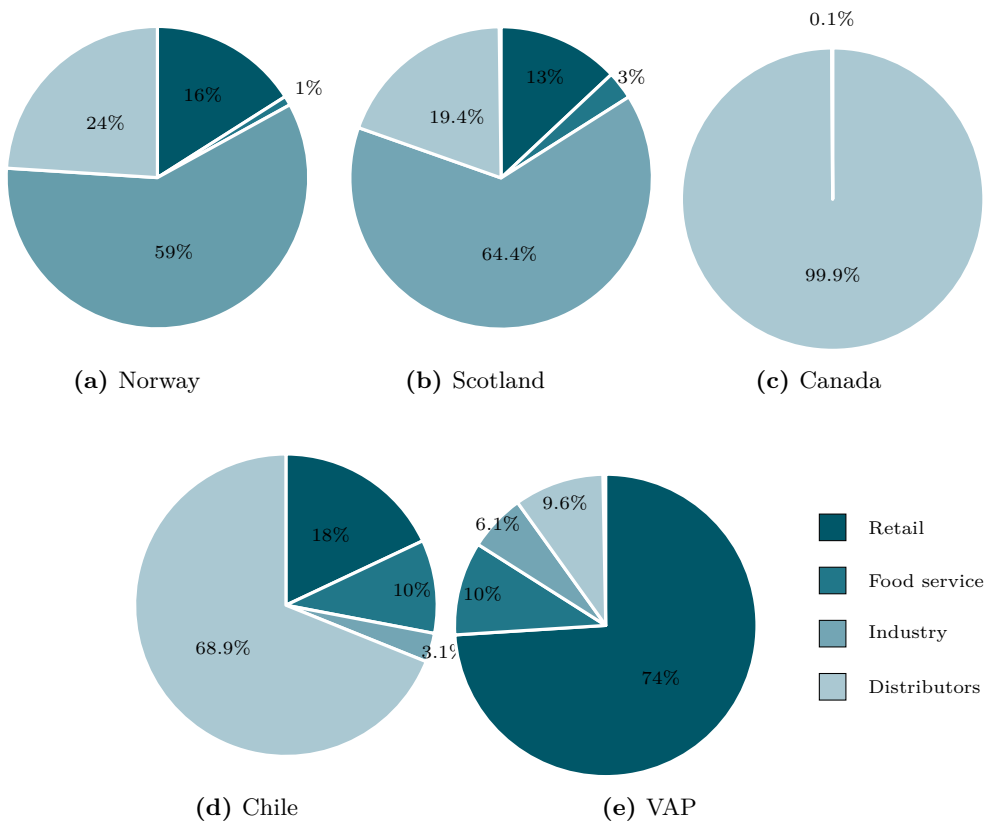
Distributors

Figure A.1: Distribution of Marine Harvest's products and VAP from different markets (Marine Harvest, 2014b).

Appendix B

Mathematical formulation

B.1 Sets and indices

B.1.1 Time and scenarios

$\mathcal{T}$ -   Set of time periods t

$\mathcal{S}$ -   Set of scenarios s

$\mathcal{K}$ -   Set of split nodes k in the scenario tree

$\mathcal{S}_k$ -   Set of scenarios passing through split node k

B.1.2 Nodes and transportation

$\mathcal{R}$ -   Set of farming regions r

$\mathcal{N}$ -   Set of all network nodes i, j

$\mathcal{N}^F$ -   Set of farming nodes i, $\mathcal{N}^F \subset \mathcal{N}$

$\mathcal{N}_r^F$ -   Set of farming nodes in farming region r, $\mathcal{N}_r^F \subset \mathcal{N}^F$

$\mathcal{N}^P$ -   Set of processing nodes i, $\mathcal{N}^P \subset \mathcal{N}$

$\mathcal{N}_p^P$ -   Set of processing nodes that can produce product p, $\mathcal{N}_p^P \subset \mathcal{N}^P$

$\mathcal{N}^S$ -   Set of slaughtering nodes i, $\mathcal{N}^S \subset \mathcal{N}$

$\mathcal{N}^M$ -   Set of market nodes i, $\mathcal{N}^M \subset \mathcal{N}$

$\mathcal{M}$ -   Set of transportation modes m

$\mathcal{F}$ -   Set of paths f used for modelling transportation of perishable fresh products

$\mathcal{F}_i$ -   Set of paths that include node i, $\mathcal{F}_i \subset \mathcal{F}$

$\mathcal{F}_{ijm}$ -   Set of paths that travel along arc (i, j) by transportation mode m, $\mathcal{F}_{ijm} \subset \mathcal{F}$

$\mathcal{O}_i$ -   Set of origins for node i, $\mathcal{O}_i \subset \mathcal{N}$

$\mathcal{D}_i$ -   Set of destinations for node i, $\mathcal{D}_i \subset \mathcal{N}$

$\mathcal{O}_i^F$ -   Set of paths originating in node i

$\mathcal{D}_i^F$ -   Set of paths arriving in node i

B.1.3 Products and inventory

$\mathcal{V}$	-	Set of fish weight classes v
$\mathcal{V}^H$	-	Set of fish weight classes v that are allowed to be harvested, $\mathcal{V}^H \subset \mathcal{V}$
$\mathcal{V}_p^H$	-	Set of fish weight classes included in HOG weight class p, $\mathcal{V}_p^H \subset \mathcal{V}^H$
$\mathcal{P}$	-	Set of all products p, q
$\mathcal{P}^{HOG}$	-	Set of all fresh HOG weight classes p, $\mathcal{P}^{HOG} \subset \mathcal{P}$
$\mathcal{P}^P$	-	Set of all perishable processed products p, $\mathcal{P}^P \subset \mathcal{P}$
$\mathcal{P}^{DUR}$	-	Set of all durable processed products p, $\mathcal{P}^{DUR} \subset \mathcal{P}$
$\mathcal{L}$	-	Set of inventory types l
$\mathcal{P}_l^L$	-	Set of products that can be stored in inventory type l, $\mathcal{P}_l^L \subset \mathcal{P}^{DUR}$
$\mathcal{A}$	-	Set of all artificial inventories a, differentiated by different product age intervals
$\mathcal{A}_p^P$	-	Set of artificial inventories related to product p, $\mathcal{A}_p^P \subset \mathcal{A}$
$\mathcal{B}$	-	Set of all production processes b
$\mathcal{B}_{pq}$	-	Set of all production processes having HOG class p as input and processed product q as output, $\mathcal{B}_{pq} \subset \mathcal{B}$
$\mathcal{U}_p$	-	Set of all processed products q that can be produced using HOG weight class p as raw material

B.1.4 Contracts

$\mathcal{C}$	-	Set of contracts offered during the planning horizon
$\mathcal{C}^F$	-	Set of fixed price contracts, $\mathcal{C}^F \subset \mathcal{C}$
$\mathcal{C}^A$	-	Set of adjustable contracts, $\mathcal{C}^A \subset \mathcal{C}$
$\mathcal{C}^{PA}$	-	Set of partly adjustable contracts, $\mathcal{C}^{PA} \subset \mathcal{C}$
$\mathcal{C}_c^T$	-	Set of time periods in which a delivery to contract c is made
$\mathcal{C}_{pit}$	-	Set of contracts involving product p and market i, having a delivery in time period t, $\mathcal{C}_{pit} \subset \mathcal{C}$, $t \subset \mathcal{C}_c^T$

B.2 Deterministic data

B.2.1 Initial conditions

B_{vi}^0	-	Initial biomass given by number of fish of farming weight class v at farm i
I_{pia}^0	-	Initial inventory of product p in artificial inventory a at processing facility i
E_{pit}	-	Total volume of product p to be delivered in market i in period t according to current contract obligations

R_{vit} -   Number of smolt of farming weight class v to be released at farm i at the beginning of time period t

B.2.2   Capacities, times and costs

N_i^F -   Maximum allowable biomass at farming node i

N_r^R -   Maximum allowable biomass in farming region r

S_i^{CAP} -   Slaughtering capacity of slaughterhouse i, measured in number of fish per time period

C_v^C -   Caring cost of keeping one kg of fish of farm weight class v one more time period

C_p^P -   Cost of producing one kilogram of processed product group p

C_{pij}^{TAR} -   Tariff for product p transported from node i to node j

C_{li}^I -   Cost of keeping one kilogram of processed product in inventory type l for one time period at processing node i

C_{ijm}^T -   Transportation cost from node i to node j using transportation mode m

C_{pf}^F -   Combined transportation and tariff costs when transporting product p across path f

I_{il}^{CAP} -   Inventory capacity of inventory type l at processing facility i

P_{pi}^{CAP} -   Processing capacity; maximum number of kilograms of processed product p that can be produced at processing facility i per time period

T_{ijm}^T -   Transportation time from node i to node j using transportation mode m

T_f^F -   Transportation time from stating point to processing node in path f

T_{ijmt}^{CAP} -   Maximum transportation capacity from node i to node j using transportation mode m in time period t

T_m^{LIM} -   Minimum transportation capacity using transportation mode m per time period the producer is committed to

B.2.3   Contracts

Q_c -   Volume to be delivered to fulfill contract c at each delivery

U_c -   Price of contract c

A_c^C -   Adjustment factor: The factor with which the relevant price difference is multiplied by when adjusting the price of contract c

F_c -   Adjustment limit: The value in NOK the spot price and contract price for contract c must differ for the contract price to be adjusted, $c \in \mathcal{C}^A$

M_c^C -   Market in contract c

P_c^C -   Product in contract c

B.2.4 Punishment parameters

K_m^{TMAX}	-	Punishment term per kilogram transported using transportation mode m after surpassing the maximum capacity m
K_m^{TMIN}	-	Punishment term per kilogram the transported volume using transportation mode m falls short of the minimum use of m the producer is bound by

B.2.5 Parameters used to control end of horizon effects

V_v^B	-	Expected future value of one fish of farming weight class v
V_v^P	-	Expected future value of one kilogram of processed product p

B.2.6 Other parameters

π_s	-	Probability for scenario s
Q_k	-	The time period corresponding to split node k
G_v	-	Weight of fish belonging to weight class v
L^S	-	Slaughtering yield, defined as the weight of a slaughtered fish in percentage of a live fish
L_b^P	-	Process yield, defined as the volume of processed product q produced using process b given as a percentage of the volume of raw material p going into the process
$B_{\underline{a}a}$	-	Share of artificial inventory $\underline{a}$ that is moved to artificial inventory a from one time period to the next

B.3 Stochastic data

σ_{vits}	-	Growth in kilograms for fish of farming weight class v at farm i in time period t and scenario s
ζ_{vits}	-	Survival rate for fish in weight class v at farm i in time period t and scenario s
$\alpha_{\underline{v}vits}$	-	Share of fish in weight class $\underline{v}$ that has grown to become part of class v due to their growth at farm i in time period t and scenario s ($\underline{v} \leq v$)
ρ_{pits}^S	-	Spot sales price for product p in market i in time period t and scenario s
ρ_{pits}^B	-	Spot purchase price for HOG salmon of class p in market i in time period t in scenario s
ρ_c^C	-	Contract price for contract c when the contract is made available

B.4 Variables

B.4.1 Helping variables

q_{vits}	-	Number of fish belonging to fish weight class v at farm i in time period t in scenario s
i_{pitas}	-	Volume of product p stocked in artificial inventory a at processing node i at the end of time period t in scenario s
o_{cs}	-	Revenue generated form contract c in scenario s
a_{ijmts}^{MAX}	-	Number of kilograms the volume of products transported by transportation mode m in time period t in scenario s that falls short of the minimum commitment to using m
a_{mts}^{MIN}	-	Number of kilograms of products transported by transportation mode m in time period t in scenario s surpassing the maximum capacity of m
z_{cpit}^{C}	-	Volume of product p sold trough contract c in market i in period t

B.4.2 Decision variables

h_{vits}	-	Number of fish harvested in HOG weight class v and harvested at farm i in time period t in scenario s
$x_{pijt\bar{t}s}^{LIVE}$	-	Volume of live fish belonging to HOG weight class p sent from farming node i to slaughter node j in time period t, with arrival in time period $\bar{t}$ ($t \leq \bar{t}$)
$x_{pijmt\bar{t}s}^{HOG}$	-	Volume of fresh HOG of weight class $p \in \mathcal{P}^{HOG}$ sent from node i to node j by transportation mode m in time period t in scenario s, with arrival in time period $\bar{t}$ ($t \leq \bar{t}$)
$x_{pijmt\bar{t}s}^{RAW}$	-	Volume of fresh HOG dedicated to be used as raw material for durable products, sent from slaughter node i to processing node j in time period t using transport mode m, with arrival in time period $\bar{t}$ ($t \leq \bar{t}$)
$x_{pijmt\bar{t}as}^{DUR}$	-	Volume of durable processed product p from artificial inventory a that is transported from processing facility i to market j by mode m in time period t in scenario s, with arrival in time period $\bar{t}$ ($t \leq \bar{t}$)
$r_{pbft\bar{t}s}^{P}$	-	Volume of perishable product p produced by using process b that is transported through path f in time period t in scenario s, with arrival in time period $\bar{t}$ ($t \leq \bar{t}$)
y_{pbits}^{DUR}	-	Volume of durable product $p \in \mathcal{P}^{D}$ produced using process b at processing facility i in scenario s

z_{pits}^S	-	Volume of product p sold in spot market i in time period t in scenario s
z_{pits}^B	-	Volume of product p bought in spot market i in timer period t in scenario s
δ_c^C	-	Binary variable, 1 if a contract c is entered, 0 otherwise

B.5 Full objective function

$$\max Z = \sum_{s \in \mathcal{S}} \pi_s \Bigg\{ \sum_{c \in \mathcal{C}} o_{cs} + \sum_{t \in \mathcal{T}} \Bigg[\sum_{p \in \mathcal{P}} \sum_{i \in \mathcal{N}^M} \rho_{pits}^S z_{pits}^S - \sum_{p \in \mathcal{P}^{HOG}} \sum_{i \in \mathcal{N}^M} \rho_{pits}^B z_{pits}^B$$

$$- \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{D}_i} \sum_{m \in \mathcal{M}} \sum_{t \in \mathcal{T}} \sum_{\bar{t} \geq t} (C_{ijm}^T + C_{pij}^{TAX}) \left(\sum_{p \in \mathcal{P}^{HOG}} (x_{pijmt\bar{t}s}^{HOG} + x_{pijmt\bar{t}s}^{RAW}) + \sum_{p \in \mathcal{P}^D} \sum_{a \in \mathcal{A}_p} x_{pijmt\bar{t}as}^{DUR} \right)$$

$$- \sum_{p \in \mathcal{P}^P} \sum_{b \in \mathcal{B}} \sum_{f \in \mathcal{F}} \sum_{t \in \mathcal{T}} \sum_{\bar{t} \geq t} (T_f^C + C_p^P) r_{pbft\bar{t}s}^P - \sum_{p \in \mathcal{P}^D} \sum_{b \in \mathcal{B}} \sum_{i \in \mathcal{N}^P} \sum_{t \in \mathcal{T}} C_p^P y_{pbits}^{DUR}$$

$$- \sum_{l \in \mathcal{L}} \sum_{p \in \mathcal{P}_l^L} \sum_{i \in \mathcal{N}^P} \sum_{t \in \mathcal{T}} \sum_{a \in \mathcal{A}_p} C_l^I i_{pitas} - \sum_{v \in \mathcal{V}} \sum_{i \in \mathcal{N}^P} \sum_{t \in \mathcal{T}} \sum_{s \in \mathcal{S}} C_v^C q_{vits}$$

$$- \sum_{m \in \mathcal{M}} \left(K_m^{TMAX} a_{mts}^{TMAX} + K_m^{TMIN} a_{mts}^{TMIN} \right) \Bigg]$$

$$+ \sum_{v \in \mathcal{V}} \sum_{i \in \mathcal{N}^F} V_v^B G_v q_{vi|\mathcal{T}|s} + \sum_{p \in \mathcal{P}^D} \sum_{i \in \mathcal{N}^P} \sum_{a \in \mathcal{A}_p} V_p^P i_{pi|\mathcal{T}|as} \Bigg\}$$

B.6 Constraints

$$q_{vi1} + h_{vi1} = B_{vi}^0 \qquad v \in \mathcal{V}, \ i \in \mathcal{N}^F. \tag{B.1}$$

$$q_{vit} - \sum_{\underline{v} \leq v} \alpha_{\underline{v}vi(t-1)} \zeta_{vi(t-1)} q_{\underline{v}i(t-1)} + h_{vit} = R_{vit} \qquad v \in \mathcal{V}, \ i \in \mathcal{N}^F, \ t \in \mathcal{T} \backslash \{1\}. \tag{B.2}$$

$$\sum_{v \in \mathcal{V}} G_v q_{vit} \leq Q_i^F \qquad i \in \mathcal{N}^F, \ t \in \mathcal{T}, \tag{B.3}$$

$$\sum_{v \in \mathcal{V}} \sum_{i \in \mathcal{N}_r^R} G_v q_{vit} \leq Q_r^R \qquad r \in \mathcal{R}, \ t \in \mathcal{T}. \tag{B.4}$$

$$\sum_{v \in \mathcal{V}^H} \sum_{j \in \mathcal{O}_i} G_v h_{vjit\bar{t}} \leq S_i^{CAP} \qquad i \in \mathcal{N}^S,\ t \in \mathcal{T}. \tag{B.5}$$

$$\sum_{v \in \mathcal{V}_p^H} \sum_{i \in \mathcal{N}^F} G_v h_{vit} - \sum_{j \in \mathcal{D}_i} \sum_{\bar{t} \geq t} x_{pijt\bar{t}}^{LIVE} = 0 \qquad p \in \mathcal{P}^{HOG},\ i \in \mathcal{N}^F,\ t \in \mathcal{T}. \tag{B.6}$$

$$\sum_{j \in \mathcal{O}_i} \sum_{\underline{t} \leq t} L^S x_{pji\underline{t}t}^{LIVE} - \sum_{j \in \{\mathcal{D}_i \cap \mathcal{N}^P\}} \sum_{m \in \mathcal{M}} \sum_{\bar{t} \geq t} x_{pijmt\bar{t}}^{RAW} - \sum_{j \in \{\mathcal{D}_i \cap \mathcal{N}^M\}} \sum_{m \in \mathcal{M}} \sum_{\bar{t} \geq t} x_{pijmt\bar{t}}^{HOG}$$

$$- \sum_{q \in \mathcal{U}_p} \sum_{b \in \mathcal{B}_{pq}} \sum_{f \in \mathcal{O}_i^F} \sum_{\bar{t} \geq t} \frac{1}{L_b} r_{qbft\bar{t}} = 0 \qquad p \in \mathcal{P}^{HOG},\ i \in \mathcal{N}^S,\ t \in \mathcal{T} \tag{B.7}$$

$$\sum_{j \in \mathcal{O}_i} \sum_{m \in \mathcal{M}} \sum_{\underline{t} \leq t} x_{pjimt}^{HOG} - \sum_{c \in \mathcal{C}_{pit}} z_{cpit}^C - z_{pit}^S - E_{pit} = 0 \qquad p \in \mathcal{P}^{HOG}\ i \in \mathcal{N}^M,\ t \in \mathcal{T},$$
$$\tag{B.8}$$

$$\sum_{f \in \mathcal{D}_i^F} \sum_{bin\mathcal{B}} \sum_{\underline{t} \leq t} r_{pbf\underline{t}t}^P - \sum_{c \in \mathcal{C}_{pit}} z_{cpit}^C - z_{pit}^S - E_{pit} = 0 \qquad p \in \mathcal{P}^P,\ i \in \mathcal{N}^M,\ t \in \mathcal{T}.$$
$$\tag{B.9}$$

$$\sum_{j \in \mathcal{O}_i} \sum_{m \in \mathcal{M}} \sum_{\underline{t} \leq t} \sum_{a \in \mathcal{A}_p} x_{pjimt\underline{t}a}^{DUR} - \sum_{c \in \mathcal{C}_{pit}} z_{cpit}^C - z_{pit}^S - E_{pit} = 0 \qquad p \in \mathcal{P}^D,\ i \in \mathcal{N}^M,\ t \in \mathcal{T}$$
$$\tag{B.10}$$

$$z_{pit}^B - \sum_{j \in \{\mathcal{D}_i \cup \mathcal{N}^F\}} \sum_{m \in \mathcal{M}} \sum_{\bar{t} \geq t} x_{pijmt\bar{t}}^{HOG} + \sum_{j \in \{\mathcal{D}_i \cup \mathcal{N}^P\}} \sum_{m \in \mathcal{M}} \sum_{\bar{t} \geq t} x_{pijmt\bar{t}}^{RAW}$$

$$+ \sum_{f \in \mathcal{O}_i^F} \sum_{q \in \mathcal{U}_p} \sum_{b \in \mathcal{B}_{pq}} \sum_{\bar{t} \geq t} \frac{1}{L_q^P} r_{qbft\bar{t}} = 0 \qquad p \in \mathcal{P}^{HOG},\ i \in \mathcal{N}^M,\ t \in \mathcal{T}. \tag{B.11}$$

$$i_{pi1a} = I_{pia}^0 - \sum_{\bar{t} \geq 1} x_{pijm1\bar{t}a}^{DUR} \qquad p \in \mathcal{P}^D,\ i \in \mathcal{N}^P,\ a \in \mathcal{A}_p^P \tag{B.12}$$

$$i_{pit1} - B_{11}i_{pi(t-1)1} - y_{pi(t-1)}^{DUR} + \sum_{j \in \mathcal{D}_i} \sum_{m \in \mathcal{M}} \sum_{\bar{t} \geq (t-1)} x_{pijm(t-1)\bar{t}1}^{DUR} = 0$$

$$p \in \mathcal{P}^D, \ i \in \mathcal{N}^P, \ t \in \mathcal{T} \setminus \{1\}, \tag{B.13}$$

$$i_{pita} - \sum_{\underline{a} \leq a} B_{\underline{a}a}i_{pi(t-1)\underline{a}} + \sum_{j \in \mathcal{D}_i} \sum_{m \in \mathcal{M}} \sum_{\bar{t} \geq (t-1)} x_{pijm(t-1)\bar{t}a}^{DUR} = 0$$

$$p \in \mathcal{P}^D, \ i \in \mathcal{N}^P, \ t \in \mathcal{T} \setminus \{1\}, \ a \in \mathcal{A}_p^P \setminus \{1\}. \tag{B.14}$$

Inventory capacity

$$\sum_{p \in \mathcal{P}_l^L} \sum_{a \in \mathcal{A}_p^P} i_{pita} \leq I_{il}^{CAP} \qquad i \in \mathcal{N}^P, \ l \in \mathcal{L}, \ t \in \mathcal{T}. \tag{B.15}$$

$$\sum_{b \in \mathcal{B}} \sum_{f \in \mathcal{F}_i} \sum_{\bar{t} \geq t} r_{pbf\underline{t}\bar{t}} \leq P_{pi}^{CAP} \quad p \in \mathcal{P}^P, \ i \in \mathcal{N}^P, \ t \in \mathcal{T} \mid \underline{t} = t - T_f^F, \ t \geq T_f^F \tag{B.16}$$

$$y_{pit}^{DUR} \leq P_{pi}^{CAP} \quad p \in \mathcal{P}^D, \ i \in \mathcal{N}^P, \ t \in \mathcal{T}. \tag{B.17}$$

$$\sum_{p \in \mathcal{P}} \sum_{\underline{t} \leq t} \sum_{\bar{t} \geq t} \left(x_{pijm\underline{t}\bar{t}}^{LIVE} + x_{pijm\underline{t}\bar{t}}^{HOG} + x_{pijm\underline{t}\bar{t}}^{RAW} + \sum_{a \in \mathcal{A}_p^P} x_{pijm\underline{t}\bar{t}a}^{DUR} + \sum_{b \in \mathcal{B}} \sum_{f \in \mathcal{F}_{ijm}} r_{pbf\underline{t}\bar{t}} \right)$$

$$-a_{ijmt}^{MAX} \leq T_{ijmt}^{CAP} \qquad i \in \mathcal{N}, \ j \in \mathcal{D}_i, \ m \in \mathcal{M}, \ t \in \mathcal{T}. \tag{B.18}$$

$$\sum_{p \in \mathcal{P}} \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{D}_i} \sum_{\underline{t} \leq t} \sum_{\bar{t} \geq t} \left(x_{pijm\underline{t}\bar{t}}^{HOG} + x_{pijm\underline{t}\bar{t}}^{RAW} + \sum_{a \in \mathcal{A}_p^P} x_{pijm\underline{t}\bar{t}a}^{DUR} + \sum_{b \in \mathcal{B}} \sum_{f \in \mathcal{F}} r_{pbf\underline{t}\bar{t}} \right)$$

$$+ a_{mt}^{MIN} \geq T_m^{LIM} \qquad m \in \mathcal{M}, \ t \in \mathcal{T}. \tag{B.19}$$

$$Q_c \rho_c \delta_c^C = o_c \qquad c \in \mathcal{C}^F. \tag{B.20}$$

Revenue of adjustable price contracts

$$\sum_{t \in \mathcal{C}_c^T} Q_c \left(\rho_c^C + A_c^C (\rho_{pit}^S - \rho_c^C) \right) \delta_c^C = o_c \qquad c \in \mathcal{C}^A \tag{B.21}$$

$$\sum_{t \in \mathcal{C}_c^T} Q_c \left(\rho_c^C - A_c^C (\rho_c^C - \rho_{pit}^S - F_c)^+ + A_c^C (\rho_{pit}^S - \rho_c^C - F_c)^+ \right) = o_c$$

$$c \in \mathcal{C}^{PA} \mid i = M_c^C \tag{B.22}$$

$$z_{cpit}^c - Q_c \delta_c = 0 \qquad c \in \mathcal{C} \mid p = P_c^C, \ i = M_c^C \ t \in \mathcal{C}_c^T \tag{B.23}$$

$$\frac{1}{|\mathcal{S}_k|} \sum_{s \in \mathcal{S}_k} (q_{vits'}, h_{vits'}, x_{pijt\bar{t}s'}^{LIVE}, x_{pijmt\bar{t}s'}^{HOG}, x_{pijmt\bar{t}s'}^{RAW}, r_{pbft\bar{t}s'}, x_{pijmt\bar{t}as'}^{DUR},$$

$$y_{pbits'}^{DUR}, i_{pitas'}, z_{pits'}^B, z_{pits'}^S, z_{cpits'}^C, \delta_{cs'}^C) =$$

$$(q_{vits}, h_{vits}, x_{pijt\bar{t}s}^{LIVE}, x_{pijmt\bar{t}s}^{HOG}, x_{pijmt\bar{t}s}^{RAW}, r_{pbft\bar{t}s}, x_{pijmt\bar{t}as}^{DUR},$$

$$y_{pbits}^{DUR}, i_{pitas}, z_{pits}^B, z_{pits}^S, z_{cpits}^C,) \tag{B.24}$$

$$v \in \mathcal{V}, \ p \in \mathcal{P}, \ i \in \mathcal{N}, \ j \in \mathcal{D}_i, \ f \in \mathcal{F}, \ t \in \mathcal{T}, \bar{t} \in \mathcal{T}, \ k \in \mathcal{K} \ s \in \mathcal{S}$$

$$q_{vits} \geq 0 \qquad v \in \mathcal{V}, \ i \in \mathcal{N}^F, \ t \in \mathcal{T}, \ s \in \mathcal{S} \tag{B.25}$$

$$i_{pitas} \geq 0 \qquad p \in \mathcal{P}, \ i \in \mathcal{N}^P, \ t \in \mathcal{T}, \ a \in \mathcal{A}, \ s \in \mathcal{S} \tag{B.26}$$

$$o_{cs} \geq 0 \qquad c \in \mathcal{C}, \ s \in \mathcal{S} \tag{B.27}$$

$$a_{mts}^{TMAX} \geq 0 \qquad m \in \mathcal{M}, \ t \in \mathcal{T}, \ s \in \mathcal{S} \tag{B.28}$$

$$a_{mts}^{TMIN} \geq 0 \qquad m \in \mathcal{M}, \ t \in \mathcal{T}, \ s \in \mathcal{S} \tag{B.29}$$

$$h_{vits} \geq 0 \qquad v \in \mathcal{V}, \ i \in \mathcal{N}^F, \ t \in \mathcal{T}, \ s \in \mathcal{S} \tag{B.30}$$

$$x_{ijt\bar{t}s}^{LIVE} \geq 0 \qquad i \in \mathcal{N}, \ j \in \mathcal{D}_i, \ t \in \mathcal{T}, \ \bar{t} \in \mathcal{T}, \ s \in \mathcal{S} \mid t \leq \bar{t} \tag{B.31}$$

$$\tag{B.32}$$

$$x^{HOG}_{pijmt\bar{t}s} \geq 0 \qquad p \in \mathcal{P}^{HOG},\ i \in \mathcal{N},\ j \in \mathcal{D}_i,\ m \in \mathcal{M},\ t \in \mathcal{T},\ \bar{t} \in \mathcal{T},\ s \in \mathcal{S} \mid t \leq \bar{t}$$
$$\text{(B.33)}$$

$$x^{RAW}_{pijmt\bar{t}s} \geq 0 \qquad p \in \mathcal{P}^{HOG},\ i \in \mathcal{N},\ j \in \mathcal{D}_i,\ m \in \mathcal{M},\ t \in \mathcal{T},\ \bar{t} \in \mathcal{T},\ s \in \mathcal{S} \mid t \leq \bar{t}$$
$$\text{(B.34)}$$

$$r_{pbft\bar{t}s} \geq 0 \qquad p \in \mathcal{P}^{P},\ b \in \mathcal{B},\ f \in \mathcal{F},\ t \in \mathcal{T},\ \bar{t} \in \mathcal{T},\ s \in \mathcal{S} \mid t \leq \bar{t} \qquad \text{(B.35)}$$

$$x^{DUR}_{pbijmt\bar{t}as} \geq 0 \qquad p \in \mathcal{P}^{D},\ b \in \mathcal{B},\ i \in \mathcal{N},\ j \in \mathcal{N},\ m \in \mathcal{M},\ t \in \mathcal{T},\ \bar{t} \in \mathcal{T},$$

$$a \in \mathcal{A}_p,\ s \in \mathcal{S} \mid t \leq \bar{t} \qquad \text{(B.36)}$$

$$y^{DUR}_{pbits} \geq 0 \qquad p \in \mathcal{P}^{P},\ i \in \mathcal{N}^{P},\ t \in \mathcal{T},\ s \in \mathcal{S} \qquad \text{(B.37)}$$

$$z^{B}_{pits} \geq 0 \qquad p \in \mathcal{P}^{HOG},\ i \in \mathcal{N}^{M},\ t \in \mathcal{T},\ s \in \mathcal{S} \qquad \text{(B.38)}$$

$$z^{S}_{pits} \geq 0 \qquad p \in \mathcal{P},\ i \in \mathcal{N}^{M},\ t \in \mathcal{T},\ s \in \mathcal{S} \qquad \text{(B.39)}$$

$$z^{C}_{cpits} \geq 0 \qquad c \in \mathcal{C},\ p \in \mathcal{P},\ i \in \mathcal{N}^{M},\ t \in \mathcal{C}^{T}_c,\ s \in \mathcal{S} \qquad \text{(B.40)}$$

$$\delta^{C}_{c} \in \{0,1\} \quad c \in \mathcal{C}. \qquad \text{(B.41)}$$

Appendix C

Tests

Initial tests conducted to validate the model and investigate certain parameters behaviour are presented in Table C.1.

#	Test	Test description	Expected result	Res.
1	Mass flow	Did three runs on a small test instance with two farming nodes, one slaughter node, one distribution node, one processing node and one market. The different tests had an increasing number of products included. Spot purchase was also included in test 3.	All mass balances should be equal.	OK.
2	Capacities	Included transportation, slaughter, production and inventory capacities. Tested with high temperatures and prices: (1) Very high production capacity and price of a durable product in the last time period. (2) Very high inventory capacity and price of a durable product in the last time period. (3) High initial smolt levels, equal marginal revenues.	In test 1 and 2, there should be an elevated sales of that product in the last time period. In test 3, transportation capacities should be restrictive.	OK.
3	Sales behaviour	Three runs where (1) The price of all products was set higher in the last time period (2) The price of one product surpassed the price of the other products in all periods (3) The price of all products were lower than total production costs	(1) The sales should be delayed to the last period as far as capacities allowed. (2) The sales should only be of the most profitable product. 3: Should not sell any products.	OK.
4	Initial contracts	One initial contract was included	The initial contract should be fulfilled, resulting in a lower total sales volume through spot sales and optional contracts compared to running without.	OK.
5	Optional contracts	Included two optional contracts. Did three different tests where (1) Price of both contracts below spot price (2) Prices of both contracts above spot price (3) Price of one contract below spot price and the other above	No contract were signed in the first test, while both were signed second test. Only the high price contract were signed in the third test.	OK.
6	Growth model	Let initial biomass consist of only fish size, while all prices set to zero.	Fish should not be harvested, rather grow into higher weight classes for following periods. Number of fish should be constant through the time horizon.	OK.
7	End of horizon effects	End of horizon terms included in the objective function. Performed several tests with changing values.	The model should keep an incrementally higher share of the biomass when the EPV and ESV values are increased.	OK.
8	Inventory	Included artificial inventories in the model. Tested with high prices for durable products in the last time period.	Products should or from production into artificial inventory 1, and thereafter be moved into higher order artificial inventories until sold.	OK.
9	Scenarios	A two-stage model where (1) One price scenario was used, (2) Two price scenarios equal to the scenario in test 1 were used, (3) Two price scenarios were used. One scenario equal to the scenario in test one, and the other scenario with a slightly higher price.	Test 1 and 2 should give the same solution. Test 3 should result in a slightly higher objective function value.	OK.

Table C.1: Initial tests

Appendix D

Full network

Table D.1 shows the implemented network for the problem studied in this thesis. Distribution nodes are excluded from the table as these are only in the mathematical formulation due to the need for both path and arc flow formulations. The table also reflect the most important parts of Marine Harvest's global network.

		Processing region									Market					
		Poland	Germany	UK	France	Spain	US	Chile	China	Japan	EU	Russia	North America	South America	Japan	Asia ex. Japan
Slaughter region	Norway - North										1				1	1
	Norway - Mid										1				1	1
	Norway - West										1				1	1
	Norway - South										1				1	1
	Scotland										1				1	1
	Canada												1		1	1
	Chile	1	1	1	1						1		1	1	1	1
Distribution	Norway - North	1	1	1	1	1										
	Norway - Mid		1	1	1	1										
	Norway - West		1	1	1	1										
	Norway - South	1	1	1	1	1										
	Scotland	1														
	Canada							1								
	Chile	1						1	1	1						
Processing region	Poland										1	1			1	1
	Germany										1	1			1	1
	UK										1	1			1	1
	France										1	1			1	1
	Spain										1	1			1	1
	US												1		1	1
	Chile												1	1	1	1
	China														1	1
	Japan														1	1
Market	EU										1	1			1	1
	Russia										1	1			1	1
	North America												1		1	1
	South America												1	1	1	1
	Japan														1	1
	Asia ex. Japan														1	1

Table D.1: Full implemented network

Appendix E

Electronic attachments

E.1 Excel-scripts

`Scenariogeneration.xlsm`

This is an `Excel` workbook which is based on numerous functions written in `Visual Basic`. The script generate all scenarios used in the stochastic model by calculating the used ARIMA-models

`ScenariogenerationAggregated.xlsm`

This script is equal to `Scenariogeneration.xlsm` except that this script generate the aggregated scenarios by using `Visual Basic` to forecast the fitted ARIMA-models

`MasterData.xlsm`

This is an `Excel` workbook which contains all constants and parameters used in the models implemented in `Mosel`

E.2 Text-files

`MasterConstants.txt`

All constant parameters needed for the problem studied in this thesis.

`InputMaster.txt`

All data needed for the stochastic programming. A `*.txt` file was needed in order to run the implemented model on the computational cluster

E.3 `Mosel-scripts`

`ConvertData.mos`

    Merely a `Mosel` script which read all data from `MasterData.xlsx` and `Scenariogeneration.xlsm` and writes the data to the `InputMaster.txt`-file to use as input to the `Mosel`-model which is evaluated on the computational cluster *Solstorm*

`MasterModelBaseCase.mos`

    This is the implementation of the `Mosel` script used to solve all base case instances on all scenario tree sizes. The script reads the `InputMaster.txt`-file which contain all input parameters

`MasterModelAggregated.mos`

    This is the aggregated implementation of the `Mosel` script used to solve the aggregated model instance

E.4 EDEV - Dynamic solution of stochastic solution

The same files are also attached for calculating the $EDEV_s$, (expected result in stage s of using the dynamic solution of the expected value problem) for uncertainty introduced in a three stage model. The first and second stage decisions are taking the optimal values of the deterministic solution from the previous stage.