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Managerial Risk Profile in Hedge Funds with Multiple High-Water Marks

Numerical Modelling and Fund Structure Analysis

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Problem Description

- Discuss previous literature and numerical methods for the modelling of managerial risk-taking in hedge funds.
- Develop a numerical model with both multiple evaluation periods and multiple high-water mark considerations.
- Analyse and compare different hedge fund structures with regards to managerial risk-taking and manager-investor risk misalignment.

Preface

This thesis is submitted as a part of the M.Sc. degree in Industrial Economics and Technology Management, at The Norwegian University of Science and Technology (NTNU). The numerical calculations and plots are performed using MATLAB R2014a. The thesis is finalised in LaTeX.

We express our utmost gratitude to our supervisor, Associate Professor Einar Belsom at the Department of Industrial Economics and Technology Management, for excellent guidance throughout the process.

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Thomas A. Herud, Eirik F. Rime & Christian F. Scheel[1]

Abstract

We investigate a hedge fund manager's risk-taking profile and evaluate how fund composition and multiple evaluation periods affect risk-levels. The fund composition refers to the specific characteristics that result from investors entering the fund at different points in time, implying various maturities and strike levels for high-water mark incentive contracts. Multiple evaluation periods is the inclusion of long-term managerial compensation to the decision process. Using a numerical simulation framework we compute and analyse the optimal behaviour of a fund manager with constant relative risk aversion. In existing literature - where the fund is depicted as one, homogeneous pool of investments, evaluated over a single period - manager risk-taking fluctuates heavily depending on time to maturity and moneyness of the manager's incentive option. By introducing a diversified fund composition as well as accounting for multiple evaluation periods, the manager's behaviour is considerably less extreme, reducing manager-investor risk misalignment. Our results suggests that the fund composition is relevant to the individual investor, as it potentially affects the overall risk profile and thereby the value of the investment. Furthermore, our results prove an interesting study in examining the efficiency of option-like incentive contracts in relieving agency problems.

[1]Submitted for the degree in M.Sc. Industrial Economics and Technology Management, Norwegian University of Science and Technology (NTNU)
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Sammendrag

I denne oppgaven undersøker vi en hedgefondforvalters risikoprofil og vurderer hvordan fondets sammensetning og forvalters optimeringshorisont påvirker risikotaking. Fondets sammensetning refererer til når investorene entrer fondet, noe som vil påvirke forfallsdato og utgangspris på forvalterens insentivopsjoner. Optimeringshorisont refererer til antall evalueringsperioder, hvilket innbefatter at også langsiktig kompensasjon inkluderes i beslutningsprosessen av nåværende risikonivå. Ved hjelp av numerisk simulering beregner og analyserer vi den optimale risikotakningen til en fondsforvalter med konstant relativ risikoaversjon. I tidligere litteratur, hvor forvalters kompensasjon er modellert som en enkeltopsjon og vurdert over én periode, har forvaleren svært varierende risikotakning som er sterkt avhengig av forfallstid og verdi på opsjonene. Ved å innføre en mer diversifisert fondssammensetning, samt innføre flere evalueringsperioder, finner vi at forvalters beslutningstaking er langt mindre ekstrem. Dette bidrar til å redusere avvikene mellom forvalters risikotaking og investors optimale nivå. Våre resultater tyder dermed på at fondets investorsammensetning er relevant for den enkelte investor, da det påvirker fondets overordnede risikoprofil og dermed verdien av investeringen. Videre er resultatene en interessant studie av opsjonskontrakters effektivitet.

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1 Introduction

The hedge fund industry does not only manage a considerable amount of capital, it also has the privilege of bypassing both many regulations and the obligation of disclosing their investment strategies.[1] Understanding how managers react to incentives and aligning their interests with the investor's therefore is paramount from a capital management perspective. The efficiency of hedge fund contracts is also relevant to regulators as hedge funds are becoming increasingly accessible. Furthermore, it is an interesting study of general managerial behaviour, as parallels can be drawn to remuneration of executives in the corporate world.[2]

A typical hedge fund contract can be viewed as a principal-agent problem where the optimal contract aligns the manager's incentives with investor's interests. Generally speaking, hedge funds usually apply two basic mechanisms in an effort to mitigate principal-agent problems: incentive contracts and ownership structure.[3] Contracts are commonly comprised of an annual *management fee* equal to a given percentage of total assets under management, and an *incentive fee* entitling the manager to a share of the profits in a given evaluation period. The incentive fee works similarly to a European call option, where the fund value is the underlying asset, the evaluation period is the time-to-maturity and a high-water mark (HWM) is the strike price. This is hence a loss provision contract where the HWM defines a minimum required return in order to profit from the incentive fee. The overall issue concerning these contracts is that they potentially induce a hazardous level of risk-taking, largely offset from the investor's optimal level (Carpenter, 2000; Kouwenberg and Ziemba,

[1]As Brav et al. (2008, p. 1730) wrote in *The Journal of Finance*: "Hedge funds employ highly incentivised managers who manage large unregulated pools of capital."

[2]Agarwal et al. (2009, p. 2221) notes that corporate managerial incentives are "hard to interpret given significant endogenity", and that the hedge fund environment serves as an interesting substitute.

[3]Other solutions to agency problems such as market forces and government regulations are less predominant, due to the nature of hedge funds (Ackermann et al. (1999)).

2007; Hodder and Jackwerth, 2007; Asheim, 2014).

In this paper, we explore the dynamics of these incentive contracts, both its structural components and how these components affect risk-taking and risk misalignment between manager and investors. We do this by building a numerical model displaying a manager's optimal risk allocation depending on time to maturity and fund movement. The central contribution of our work is the development of a model which incorporates the situation where a manager manages a fund comprised of multiple investors, implying a number of overlapping incentive contracts. The incentive fee contract then resembles a portfolio of call options, where each option has its own strike price dictated by the fund value upon entrance. This is fundamentally different from existing research.

Previous work such as Hodder and Jackwerth (2007) and Asheim (2014) model the fund as if it consisted of only one single investor, modelling the incentive contract as a single option with one HWM and termination date. The benchmark is merely conceptualised as an aggregate of all investor benchmarks. Our model on the other hand, is extended with the purpose of both capturing the possibility and depicting the result of modelling a fund containing multiple incentive contracts, where investors have entered the fund at different times and fund values.[4] We believe this is a necessary addition as it provides greater realism and also provides new insights in hedge fund research. Our hypothesis is that a manager exposed to a diverse set of incentive contracts will be less prone to excessive risk-taking. Given that the contracts are sufficiently diverse, the optimal risk level for one contract will adversely affect the prospects of another, forcing the manager to balance out his appetite for risk to reach a combined optimum for all contracts.

Before going into further details on how we tackle the problem at hand, it is necessary to briefly assess similar works in order to fully grasp the complexity of the issue. A great deal of hedge fund literature has been concerned with remuneration structure, where especially the effects of the incentive fee has been under much academic scrutiny. What is clear from an overall perspective is how divided the results are. The obvious intuition behind the incentive contracts is that by directly linking managerial remuneration to fund performance, the fund should perform better. And since an investment in a hedge fund is essentially a bet on the manager's skills, the HWM loss-recovery contract can be seen as a manager's way to signal his abilities

[4] We refer to this characteristic as "fund composition".

towards investors.[5] However, the fact that the incentive fee is only directly affected by gains and not losses suggests a potential adverse risk-taking behaviour. After all, the value of a call-option on future gains is positively correlated with volatility; a decision variable left entirely to the manager's discretion. Two natural questions that are being asked therefore, is *i*) whether fee structure predicts fund performance, and *ii*) in what way different fund-characteristics affect risk misalignment between manager and investor. The first question has mainly been investigated through empirical studies and the latter through theoretical models, and results are at best mixed.

Addressing the first question, empirical studies show conflicting results regarding the relationship between incentive fee, fund performance and risk-taking, and the ambiguity persists across industries.[6] In the case of hedge funds, asymmetric remuneration structure is found to be positively related to performance by several papers. Ackermann et al. (1999) notes that funds charging higher fees are associated with better performance, and Liang (1999) that hedge funds provide superior Sharpe ratios with returns positively related to incentive fees with loss-recovery provisions. On the contrary, Brown and Goetzmann (2001) find that high-fee funds perform no better than lower-fee funds, and Kouwenberg and Ziemba (2007) concludes that hedge funds with incentive fees actually have significantly lower mean returns.

These conflicting results are addressed by Agarwal et al. (2009), who suggests that the fee-rate itself is a deficient proxy for the manager's exposure to fund returns. This is because pay-performance sensitivity also depends on other fund-characteristics such as the timing and magnitude of investor capital flows. In other words, Agarwal et al. (2009) empirically apply the very same notion of a diverse fund composition as we do in our model, where managerial compensation is best described as a series of diverse options-like contracts. Introducing a *delta* to account for the total managerial compensation to a 1% move in the fund, they conclude that if measured correctly, greater incentives yield superior performance. This discussion strongly supports our hypothesis on the relevance of fund composition, and that a single incentive option not sufficiently emulates reality.

The second question of how different fund-characteristics affect risk misalignment between manager and investor, has been explored by several papers applying both

[5]According to Aragon and Qian (2009) HWM provisions are more common amongst less-reputable managers in an effort do reduce asymmetric information on managerial quality.

[6]Analysing venture capital firms, Gompers and Lerner (1999) "observe no relation between incentive compensation and performance."

numerical and analytical approaches. What is evident from these studies is how sensitive the results are to the choice of framework, assumptions and simplifications. The issue is to successfully manage the trade-off between reducing complexity and oversimplifying the problem to a state where it loses touch with reality. The basic idea of solving the dynamic investment problem of an optimal allocation between a risky and risk-less asset, was pioneered by Merton (1969). In his case, the investor is the decision maker. Carpenter (2000) translates this problem to a setting where the choice of risk-allocation is outsourced to a manager exposed to incentive compensation. As a consequence, the risk-allocation becomes dependent on the moneyness and time to maturity of the option-like contract, and will therefore vary greatly to that of an investor. Kouwenberg and Ziemba (2007) provide further extensions to the field by factoring in management fee and managerial ownership, and crucially also alter the manager's preference to risk. While Carpenter (2000) describes the manager with hyperbolic absolute risk aversion (HARA), Kouwenberg and Ziemba (2007) applies Prospect Theory developed by Kahneman and Tversky (1979).

These additions, especially with respect to risk-preference, have significant implications on optimal risk-allocation sensitivity to changes in incentive fees. While Carpenter (2000) finds that for a fund above HWM, a manager's response to an increased incentive fee percentage is to reduce fund volatility, Kouwenberg and Ziemba (2007) concludes with the exact opposite. They also find that a significant ownership share (above 30%) considerably reduces risk-taking, which is unsurprising considering this also exposes the manager to a possible downside. However, both papers as well as Hodder and Jackwerth (2007) and Asheim (2014) agree on the notion that the incentive fee inflicts adverse behaviour, frequently resulting in risk-misalignment. This view is challenged by Panageas and Westerfield (2009) and Guasoni and Obłój (2013), who both provide analytical solutions in a continuous-time framework with infinite horizon. They find that managers place a constant portfolio risk, although Guasoni and Obłój (2013) concludes that this only applies to managers who are either risk-neutral or with low aversion to risk.

From the brief discussion above it is apparent that research results vary across the applied models, and that observed managerial behaviour is highly dependent on assumptions about reality and modelling methods. In investigating these issues further we develop a numerical framework analogous to that of Hodder and Jackwerth (2007). There are two reasons why we believe that these types of frameworks serves as a well-suited environment for exploring hedge fund contracts: Firstly, empirical data is difficult to obtain, making theoretical models a more attractive alternative.[7] Secondly, as noted by Asheim (2014), analytical solutions require severe simplifications due to the complexity of the problem, making numerical approaches more preferable. As an example, the continuous time framework of Panageas and Westerfield (2009) with perpetually renewed call options on the fund value, does not sufficiently emulate the reality of a hedge fund with yearly evaluation periods over a finite number of years.

In section 2 we develop the fundamental model upon which we subsequently include further additions. Section 3 establishes a base case scenario analogous to previous literature, and present different evaluation metrics in order to quantify our results. In accordance with Hodder and Jackwerth (2007) and Asheim (2014) we find that when the fund composition only consists of one incentive contract evaluated over a single optimisation period, the manager exerts extreme risk-taking behaviour often both far above and below that of an investor. We then extend our model in two directions. Section 4 introduces and presents the results for a situation with multiple evaluation periods where the manager is forced to consider the value of continuation. Due to the loss-recovery structure of the incentive contracts, this means that poor performance in the current evaluation period also inflicts future compensation. In section 5 we develop the central contribution of this paper, by investigating how a diverse composition of multiple incentive contracts further induce a risk aligning effect. Both of these extensions are motivated by the belief that they provide greater realism to the model. Section 6 then synthesises our findings and evaluates different contractual parameters such as managerial ownership and fee size. In section 7 we examine the cost of contractual inefficiencies, and in light of our quantitative results, we point towards an alternative remuneration structure that could benefit both manager and investor. Section 8 presents our concluding remarks.

[7]Brown and Goetzmann (2001): "absence of regulatory oversight, means that reliable data on hedge funds are hard to come by, and careful analysis of the conduct and performance of this sector of the market is difficult."

2 Fundamental Modelling

In the following section, the fundamental model setup is described. We begin by developing the process of the fund movement, before we elaborate on the mechanics of managerial remuneration and preference to risk. We first develop the model setup for the most basic case; a single evaluation period with one incentive contract. In subsequent sections we then develop the extensions of multi-period evaluation (section 4) and multiple incentive contracts with different HWMs and maturities (section 5).

The overall goal of the model is to depict the risk-profile of a hedge fund manager controlling the assets in a hedge fund. We apply a dynamic programming approach where the decision taking is discretised utilising a two-dimensional node network in time and fund values. The fund follows a stochastic process which determines the distribution of possible fund values. At each point the manager, displaying a constant relative risk aversion (CRRA) utility function, maximises his expected utility of remuneration by altering the allocated proportion of risky and riskless assets. The underlying model setup is based on the work done by Hodder and Jackwerth (2007), but where our extensions result in significant structural additions.

2.1 Modelling a Hedge Fund

An important attribute of hedge funds is the flexibility in investment options, as hedge funds are largely unregulated. Hedge funds are allowed to short sell, apply leverage and invest in derivatives. Our model is simplified with regards to the wide range of investment possibilities, and the manager's decision variable is merely a choice of aggregated leverage rather than specific securities. In order to model the amount of risk a hedge fund manager is willing to acquire in a given situation, we limit the hedge fund manager to invest in a risky asset S and risk-free government bond B. This is a common simplification similar to the original approach of Merton (1969).

The risk-free asset grows with an annual rate r, yielding a continuous return $dB_t = rB_t dt$. For the process of the risky asset we turn to the continuous stochastic process of a geometric Brownian motion (GBM),

$$dS_t = \mu S_t dt + \sigma S_t dW_t, \tag{2.1.1}$$

where S is the value of the risky asset, μ is the drift, σ is the standard deviation and W_t is a Wiener process. The *Markov property* of the GBM is particularly powerful as the probability distribution for future values depends only on the current state without any path dependency. With the assumption of GBM follows the assumption of returns being lognormally distributed. Considering the risk-seeking behaviour associated with hedge funds, the existence of fat tails and a significant downside is predominant. Applying GBM should thus be considered as a strong simplification.[1] However, given its mathematical tractability and its wide acceptance in academia, the use of GBM is considered suitable for the purpose of this paper. We leave it to further research to account for the special characteristics of hedge fund return distributions, such as negative skewness and excess kurtosis.

For the numerical values of μ and σ we use the *Credit Suisse Hedge Fund Index* database. This database reports an average annual return of 8.48% and an annual standard deviation of 7.11% for the overall hedge fund index.[2] The Sharpe ratio is reported to have a value of 0.81, calculated using a rolling 90 day T-bill rate. The implied T-bill rate is then 2.72%, which we set to be the risk-free annual return r. It should be noted that these numbers do not necessarily reflect a realistic risk measurement, since the Sharpe ratio could overstate fund performance considering the aforementioned skewness and excess kurtosis of hedge fund return distributions. There also exist several biases related to hedge fund data reporting that potentially could affect the parameters.[3] In addition, the database does not state the applied leverage, which directly amplifies return and volatility. Even if a hedge fund were

[1]"Many hedge fund index return distributions are not normal and exhibit negative skewness and positive excess kurtosis" (Kat and Brooks, 2001).

[2]The index is continuously updated with data beginning in January 1994. Stated numbers are obtained June 2015.

[3]Reporting biases - survival bias, instant history bias and selection bias - are thoroughly discussed by existing papers such as Fung and Hsieh (2000).

obligated to disclose financial statements such as a balance sheet, it would still be difficult to determine the realistic level of leverage. This is because leverage, in many cases, not only consists of outright borrowing but is also comprised of investments in derivatives and structured notes. However, the Sharpe ratio is still applicable since it, in accordance with the Capital Allocation Line, is not dependant on leverage.[4] Therefore, as long as the ratio between excess return and applied risk is realistic, our qualitative results will not be affected by the possible inaccuracies in parameter choice. We apply the reported data as the basis for unlevered parameters.[5]

The set of investment opportunities results in the total hedge fund portfolio following the process,

$$\begin{aligned} dX_t &= \kappa X_t dS_t + (1-\kappa) X_t dB_t \\ &= \kappa X_t(\mu dt + \sigma dW_t) + (1-\kappa) X_t r dt \\ &= [\kappa\mu + (1-\kappa) r] X_t dt + \kappa\sigma X_t dW_t \ , \end{aligned} \tag{2.1.2}$$

where X_t denotes the fund size at time t.[6] The weight allocation invested in the risky asset is assigned κ and will serve as the decision variable for the optimisation problem.

With the hedge fund being lognormally distributed it is convenient to express the fund process by the logreturn $d(logX_t)$. We set $F(X_t) = logX_t$ and apply Ito's Lemma to find the process of dF,

$$dF = \frac{\partial F}{\partial t} dt + \frac{\partial F}{\partial X_t} dX_t + \frac{1}{2}\frac{\partial^2 F}{\partial X_t^2}(dX_t)^2,$$

where the differentials are given by,

$$\frac{\partial F}{\partial t} = 0 \ , \quad \frac{\partial F}{\partial X_t} = \frac{1}{X_t} \ , \quad \frac{\partial^2 F}{\partial X_t^2} = \frac{-1}{X_t^2}.$$

As $(dt)^2 = 0$, $dWdt = 0$ and $(dW)^2 = dt$ the total differential reduces to,

[4] Assuming that borrowing at risk-free rate is possible.
[5] We refer to appendix A.2 for a list of our standard parameters.
[6] $\kappa X_t = S_t$ and $(1-\kappa)X_t = B_t$.

$$dF = \frac{\kappa\mu + (1-\kappa)rX_t dt + \kappa\sigma X_t dW_t}{X_t} - \frac{1}{2}\frac{\kappa^2\sigma^2 X_t^2 dt}{X_t^2}$$
$$= [\kappa\mu + (1-\kappa)r - \frac{1}{2}\kappa^2\sigma^2]dt + \kappa\sigma dW_t. \quad (2.1.3)$$

The Wiener process dW_t is normally distributed with mean 0 and standard deviation $\sqrt{dt}$. We can rewrite dW_t as $\sqrt{dt}dZ_t$, where dZ_t is standard normally distributed $N(0,1)$. For a given time step dt the logreturns are thus normally distributed with mean $[\kappa\mu + (1-\kappa)r - \frac{1}{2}\kappa^2\sigma^2]dt$ and standard deviation $\kappa\sigma\sqrt{dt}$. Due to the concavity of the log function and as a consequence of Jensens's Inequality, the mean is corrected with the term $-\frac{1}{2}\kappa^2\sigma^2$. Substituting dF for $logX_{t+dt} - logX_t$ we can derive an equation for the value of the hedge fund at time $t + dt$ given the value at time t,

$$logX_{t+dt} - logX_t = [\kappa\mu + (1-\kappa)r - \frac{1}{2}\kappa^2\sigma^2]dt + \kappa\sigma\sqrt{dt}dZ_t$$
$$X_{t+dt} = X_t e^{[\kappa\mu + (1-\kappa)r - \frac{1}{2}\kappa^2\sigma^2]dt + \kappa\sigma\sqrt{dt}dZ_t}. \quad (2.1.4)$$

The assumption of a lognormally distributed hedge fund and the corresponding properties will be used in section 2.4, where the probability of possible fund moves is derived.

In order to accommodate our numerical framework, the continuous fund movement needs to be evaluated at discrete points. As previously mentioned, we base our computations upon a two-dimensional grid of nodes where each node (n, t) represents a fund value at a given time-step. Each time-step is defined as $\Delta t = \frac{1}{\tau}$ with τ being the total number of portfolio revisions in one evaluation period. In our model, one evaluation period corresponds to one year. τ is set to 12, implying monthly portfolio revisions. The size of Δt thereby defines the granularity of the network along the time axis. For the fund size axis we introduce the parameter C denoting the constant logarithmic change in fund value between two nodes at a given point in time. This is a convenient approach since it is the log-return that is normally distributed and not the fund size itself. We also let the fund values be incremented by the risk-free return as we move forward in time, guaranteeing a risk-free return when being purely invested in the riskless asset. The logarithmic return achieved from a given node to a node in the next time step is then,

$$log\left(\frac{X_{t+\Delta t,\,n+j}}{X_{t,n}}\right) = r\Delta t + jC, \qquad (2.1.5)$$

where j determines the number of node-moves up or down. The total value of the fund can thereby be expressed as

$$X_{t+\Delta t,\,n+\Delta n} = X_{t,n}e^{r\Delta t + jC}. \qquad (2.1.6)$$

2.2 Remuneration of Hedge Fund Managers

The remuneration of hedge fund managers is usually divided into two parts; one fixed percentage of the total value of the fund, referred to as the management fee, and a share of the profit above a certain required return, denoted the incentive fee. A common structure is 2% management fee and 20% incentive fee, labelled as a "two-twenty structure".[7] The required return is usually structured as a loss provision high-water mark (HWM), indicating a threshold based on previous fund value plus a "hurdle rate". The hurdle rate should at least amount to the risk-free rate which the investor would expect when investing in government bonds, but may also be based on other benchmarks. While mutual funds seek to produce positive relative returns, hedge fund managers seek positive absolute returns and have to make up all previous losses to derive their compensation from the incentive fee. Thus they are called absolute return managers (Fung and Hsieh, 1997). Figure 2.1 shows how the HWM increases after every evaluation period unless the fund decreases in value and naturally remains the same until the loss is recovered. In our model the HWM is by default set equal to the initial fund value augmented by the risk-free rate.

The remuneration W received from managing the fund for one period can be expressed as,

$$W = bX_T + c\max(0, X_T - \text{HWM}), \qquad (2.2.7)$$

where X_T is the value of the fund at maturity T, b is the management fee and c is

[7]Kouwenberg and Ziemba (2007) notes that a 20% incentive fee is the industry standard, relying on the Zurich Hedge Fund Universe database (year 1995-2000).

the incentive fee. The two-twenty structure is applied unless otherwise is stated.

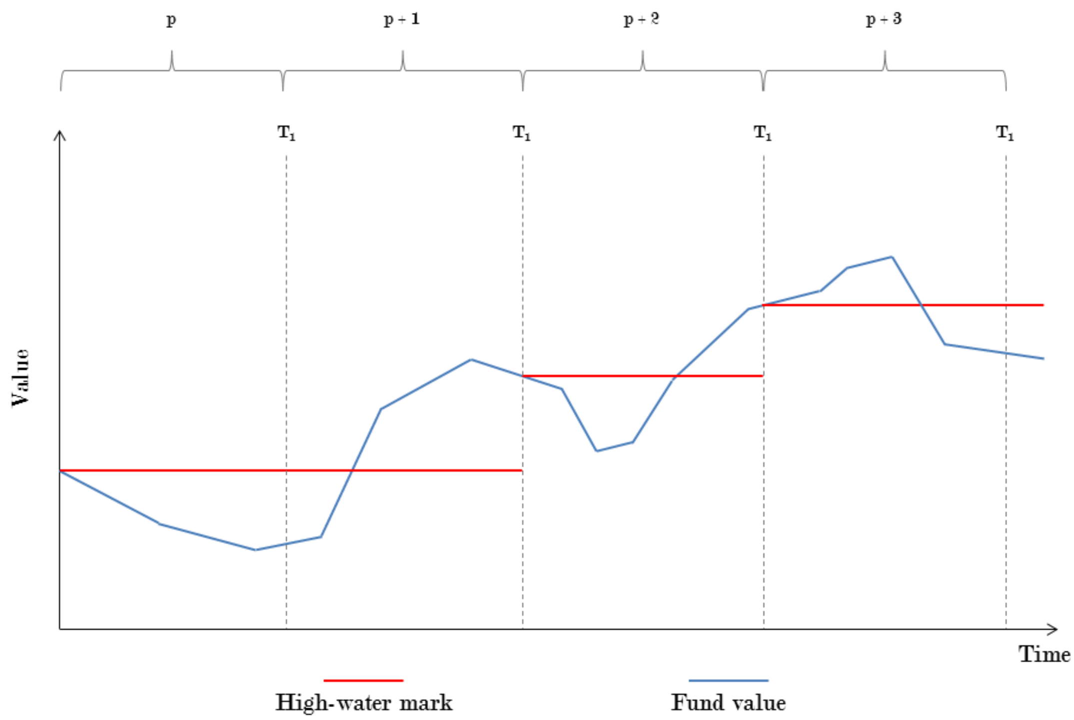

Figure 2.1: The dynamics of a high-water mark

In addition to remuneration it is common for a manager to hold a substantial personal investment in the fund (Fung and Hsieh, 1999), in this case assigned a share a. Managerial ownership has in previous literature been highlighted as an effective tool in order to mitigate risk misalignment.[8] The total wealth obtained by the manager at maturity thereby becomes,

$$W = aX_T + b(1-a)X_T + c(1-a)\max(0, X_T - \text{HWM}). \tag{2.2.8}$$

[8]Kouwenberg and Ziemba (2007) notes that "with a stake of 10% or less, the manager behaves extremely risk-seeking as a result of the incentive fee."

2.3 Managerial Preference to Risk and Utility

In order to depict a realistic picture of managerial behaviour, it is essential to successfully incorporate a sensible representation of risk preference. After all, the manager does not merely seek to maximise his wealth, but rather his expected utility. Relying on previous work, we see that the choice of utility-model has the potential to significantly impact the results. As mentioned in the introduction, Kouwenberg and Ziemba (2007) for instance show how a loss averse manager's reaction to increased incentive fees is completely opposite to that of a manager with HARA-utility. Asheim (2014) conducts a thorough discussion on this topic. Extending the work of Hodder and Jackwerth (2007) he includes a more refined utility model based on Prospect Theory (PT), and demonstrates how this addition impacts the managers decisions.

Although PT is recognised as a more avante-garde descriptive model for depicting actual behaviour, we adhere to using CRRA in our model. We admit to the shortcomings this choice implies, however it comes with two significant benefits. Firstly, it greatly simplifies our computations. This is due to the path dependency that occurs in PT with the use of reference points, where future utility depends on status quo. In our dynamic programming approach, status quo depends on future utility, making PT a "chicken or the egg problem". The second benefit of applying CRRA is that it makes our results more comparable to the majority of previous papers, which also apply CRRA. We therefore make this compromise for tractability purposes, and although we aim to conduct a realistic analysis of managerial behaviour, we do not suggest that the absolute values of risk-taking reflect reality completely. Rather we aspire to depict and discuss the *differences* between the original approach and our novel extension of the model.

Equation 2.3.9 shows the applied CRRA utility where γ is the risk aversion coefficient, determining the concavity of the utility curve.

$$U = \frac{W^{1-\gamma} - 1}{1 - \gamma} \tag{2.3.9}$$

As a general assumption we set both the manager and investor risk aversion to the value of 4, which is in line with both Asheim (2014) and Hodder and Jackwerth (2007). However, it is plausible that the risk aversion constant of the manager could be different to that of the investor. Examining the effects this may have on risk-misalignment is therefore a relevant future extension.

it to further research to examine the effects this may have on risk-misalignment.

2.4 Utility Optimisation Procedure

The aim of the model is to determine the risk allocation that maximises the hedge fund manager's expected utility for every given node in our two-dimensional node network. With manager wealth being calculated at maturity, we incrementally build a matrix of expected utilities and corresponding optimal risk allocation $\kappa_{t,n}$ based on subsequent nodes. First, the wealth obtained in the terminal nodes is calculated using equation 2.2.8,

$$W_n = aX_{T,n} + b(1-a)X_{T,n} + c(1-a)max(0, X_{T,n} - \text{HWM}). \tag{2.4.10}$$

The corresponding terminal utility is calculated using equation 2.3.9,

$$U_n = \frac{(W_n)^{1-\gamma} - 1}{1-\gamma}. \tag{2.4.11}$$

We then calculate the expected utility for each node in the preceding time step $T - \Delta t$ using the terminal utilities,

$$E[U_{t,n}] \begin{cases} \sum_{j=-J}^{J} p_{j,\kappa} U_{n+j} & \text{if } t = T - \Delta t \\ \sum_{j=-J}^{J} p_{j,\kappa} E[U_{t+\Delta t, n+j}] & \text{if } t < T - \Delta t \end{cases} \tag{2.4.12}$$

where J determines the search span and $p_{j,\kappa}$ is the probability of reaching j nodes up or down from n with a given kappa-choice. By altering κ the manager changes the probability distribution of the fund move and consequently the expected utility in each node, thus enabling him to obtain the optimal risk allocation. As we step backwards in time we repeat the procedure of calculating expected utilities and optimal risk-allocations until the model reaches $t = 0$. We assume that the portfolio can be revised without incurred transaction costs. Figure 2.2 depicts the recursive expected utility calculation.

With regards to our numerical approach we apply a binary search algorithm in order to approximate a continuous κ-choice. This allows us to increase the number of kappas significantly compared to previous papers without compromising computational run-time. Instead of computing the expected utility for each κ, the algorithm iteratively cuts the set of kappas in half by eliminating the part towards which the utility is decreasing.[9] We model with kappas from 0 to 20 with 0.01 as step size.

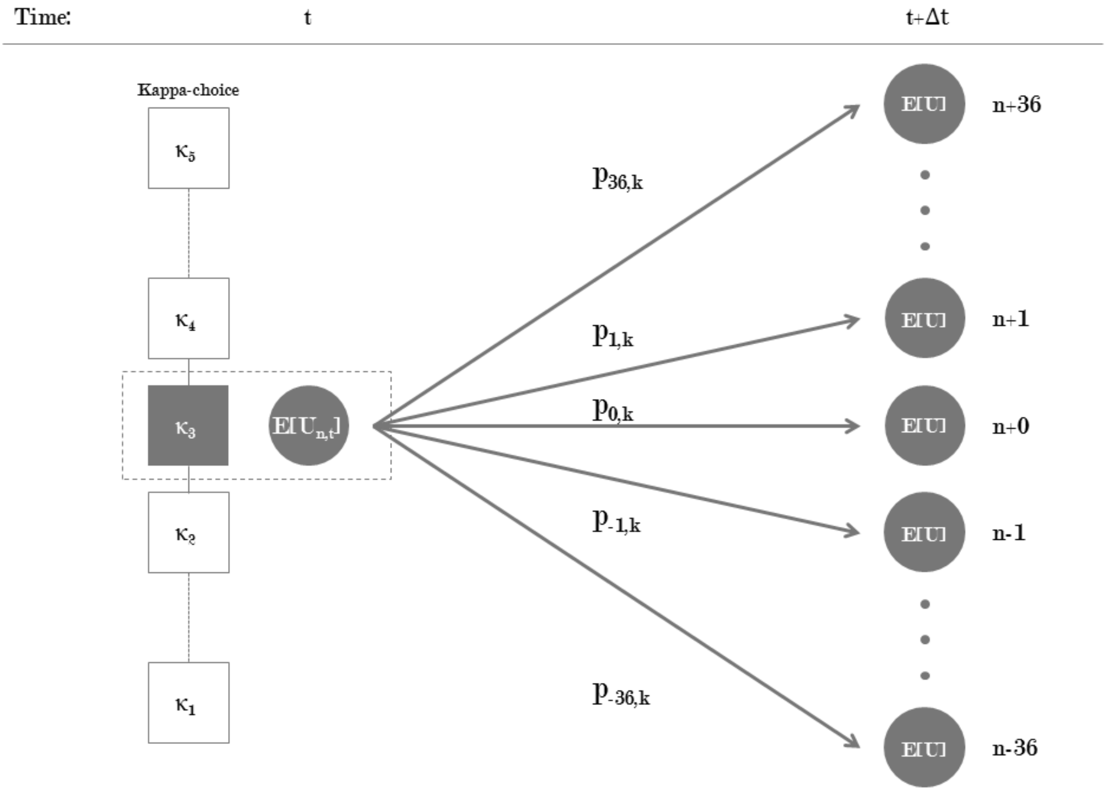

Figure 2.2: Determining optimal κ and utility in a node

The choice of span-width and step size C is made by considering the probability distribution of the returns within one time-step, compromising between computational run-time, granularity and coverage of the total sample space. We find that with 36 up and down moves and a C value of 0.0064 the model accounts for 98% or more of the sample space for kappas lower than 6.[10] In order to obtain the probabilities $p_{j,\kappa}$ of the possible up and down moves in our discrete framework, we apply the discretised process of the log-returns derived in equation 2.1.3. For a given risk

[9]This works because the relation between utility and κ is strictly concave with no local maxima.
[10]See Appendix A.2 for the calculation of the value of C.

proportion κ and a time-step of Δt, the log-return of the hedge fund is drawn from the normal distribution,

$$N(\mu_{\kappa,\Delta t}, \sigma_{\kappa,\Delta t}) = N((\kappa\mu + r(1-\kappa) - \frac{1}{2}\kappa^2\sigma^2)\Delta t,\ \kappa\sigma\sqrt{\Delta t}). \tag{2.4.13}$$

We observe that the mean and standard deviation are dependent on Δt, but independent of the current state X_t and time t. This simplifies our computations, allowing us to arrange the probability distribution as a look-up table which can be applied throughout the network. The probability of a given fund-move is approximated by computing the normal density of the move and multiplying it by a normalising constant, ensuring that the probabilities sum to one.

$$p_{j,\kappa} = \frac{\frac{1}{\sqrt{2\pi}\sigma_{\kappa,\Delta_t}} \mathrm{e}^{-\frac{1}{2}\left(\frac{r\Delta t + jC - \mu_{\kappa,\Delta t}}{\sigma_{\kappa,\Delta t}}\right)^2}}{\sum_{j=-36}^{36} \frac{1}{\sqrt{2\pi}\sigma_{\kappa,\Delta t}} e^{-\frac{1}{2}\left(\frac{r\Delta t + jC - \mu_{\kappa,\Delta t}}{\sigma_{\kappa,\Delta t}}\right)^2}} \tag{2.4.14}$$

Any discretisation of a continuous distribution will be inaccurate to some degree. This is due to the limitation of the span of possible outcomes and the granularity of fund size increments. In our model this inaccuracy only becomes evident for small and large values of κ. In the range $0 < \kappa \leq 0.2$ the volatility of the fund is so small that the likelihood of moving up or down becomes insignificant. In other words, the effect of increased leverage is nonexistent. For very large κ-choices (10 and upwards) the probability distribution becomes unrealistically flat, as the probabilities will be spread evenly over the 73 possible moves.

2.5 Boundary Conditions

Because the expected utilities are calculated based on nodes above and below the current position, we need to apply certain conditions at the boundaries of the grid. In nodes close to the upper and lower boundaries, the utility calculation is based on nodes located outside the grid, thus requiring some alternative method of calculation.

Lower boundary
As the incentive fee is a very important part of the remuneration for the manager, a poorly managed fund with its incentive fees considerably out of the money will face the threat of liquidation, allowing the manager to start afresh.[11] Some existing work opens for both exogenous and endogenous liquidation of the fund, such as Lan et al. (2012). The exogenous liquidation represents automatic termination of the fund due to poor performance. The investors will simply retract their money if the fund value crosses a specified liquidation boundary. Hodder and Jackwerth (2007) shows how this exogenous boundary condition can be combined with an endogenous American-style shutdown option, giving the manager the possibility of liquidating the fund whenever this is more profitable than continuation.

We restrain to only using an exogenous liquidation boundary where the fund is terminated at values lower than 50% of the initial fund value. This is in line with the initial exogenous boundary set by Hodder and Jackwerth (2007), however we allow the fund to move below the boundary in the final step. In the case of fund liquidation, the manager receives a management fee corresponding to the time period preceding liquidation, including his own share of the fund:

$$W_{t,n}^{\text{lower}} = aX_{t,n} + b\frac{t}{T}(1-a)X_{t,n}. \tag{2.5.15}$$

Another approach is that the manager neither collects incentive nor management fee after stochastic liquidation, as stated by Lan et al. (2012). The expected utilities are calculated as before, with each utility being multiplied by the probability of achieving it.

Upper boundary
While there is an intuitive rationale behind the lower boundary liquidation, the upper boundary is simply needed to limit the number of calculations. In our model, the number of nodes to the upper boundary is, in a trade-off between run-time and accuracy, set to 108 nodes above the HWM, which represents a 106% increase from the initial fund value. The wealth calculated in these nodes are set to be equal to the wealth received had the fund been terminated in the given node, rewarding full incentive and management fee,

[11] Although it may impair the manager's ability to attract new investors, fund liquidation is a real possibility in the world of hedge funds (Baquero et al., 2005) with performance as "a significant driver of liquidation" (Getmansky et al., 2004).

$$W_{t,n}^{\text{upper}} = aX_{t,n} + b(1-a)X_{t,n} + c(1-a)(X_{t,n} - \text{HWM}). \qquad (2.5.16)$$

3 Observing and Measuring Risk-misalignment

In order to effectively analyse how fund structure and contractual parameters affect risk misalignment in our model, we first need to establish a reasonable method of comparison. We develop the following three metrics: First is the expected kappa-difference between manager and optimal investor choice, providing information about the expected misalignment of an evaluation period. Second is the certainty equivalent for the investor, indicating the gain from a given setup. The third measure is the certainty equivalent for the manager, solely incurring from managerial remuneration.

Before elaborating on these metrics, it is useful to develop some motivation to why they are needed. This is best done through displaying how deviating managerial risk taking potentially could be from the level preferred by an investor. Attaining the optimal solution for an investor is fairly straight-forward: Given that the investor has the same preference to risk as the manager, his optimal risk allocation would be identical to that of a manager who owns the entire fund himself. Or expressed differently, we find how the manager would behave had he managed his own wealth rather than being incentivised by a contractual agreement. In our model setup this simply entails setting the managerial ownership share a to 1 in equation 2.4.10 and removing the liquidation boundary. This is analogous to the asset allocation problem of Merton (1969) with the closed form solution,

$$\kappa^* = \frac{\mu - r}{\gamma\sigma^2}, \tag{3.0.1}$$

resulting in a constant level of risk depending on expected return, volatility and risk aversion. Applying our parameters in the formula, we find $\kappa^* = 2.84$. Our model yields a constant value of 2.83, depicted in figure 3.1. The reason why our model does not display the exact value of 2.84 is because of the inherent inaccuracies in the model, such as the lower and upper boundary and the discretisation of probabilities.

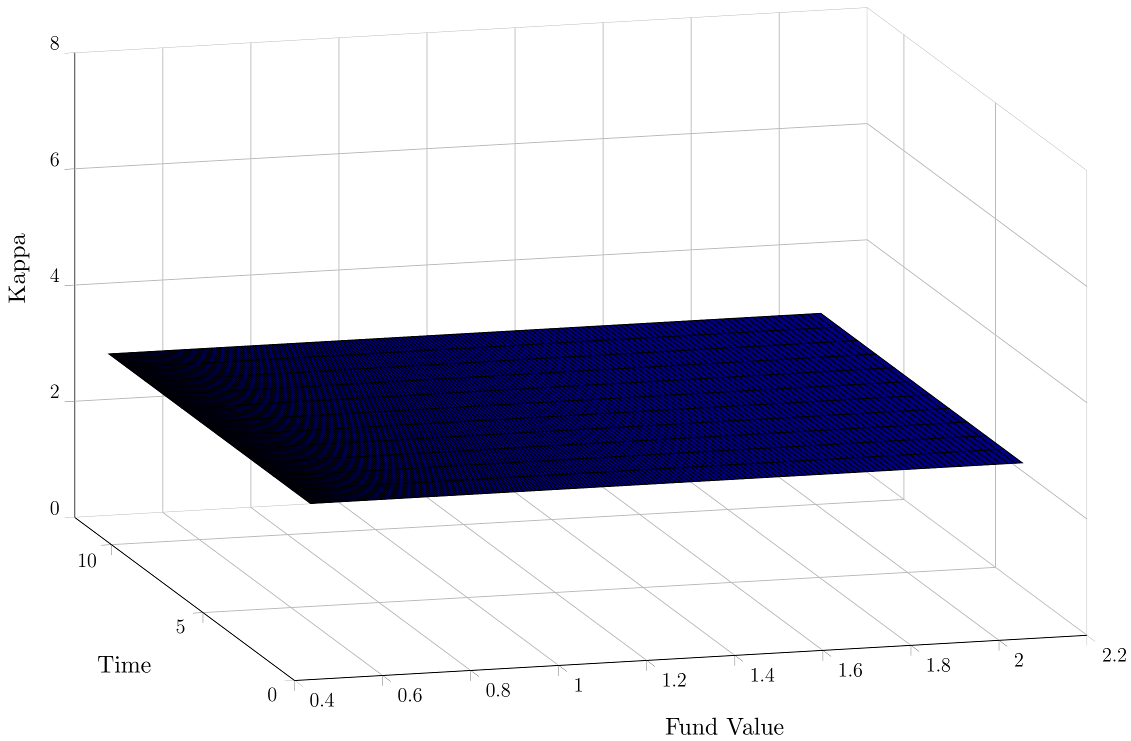

Figure 3.1: Merton flat, 100% managerial ownership

Turning back to the case where the fund is managed *on behalf* of the investor, the picture is radically changed. When applying our model in its most basic form, including a single 2/20 incentive contract, zero managerial ownership share and discontinuing the fund at the end of the period, we observe severe risk-misalignment. This provides a suitable illustration of the potential agency problems facing the investors. We refer to this scenario as the "base case"; an extreme situation which we can use as a reference point in evaluating the further extensions introduced in the model.

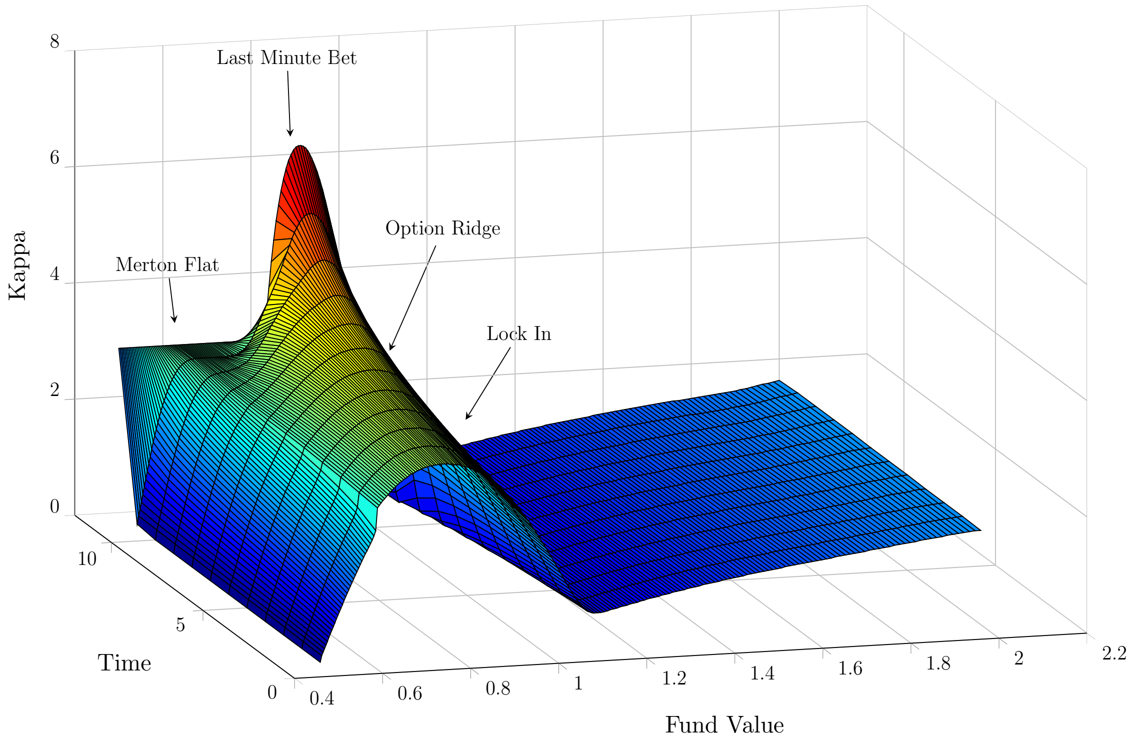

Figure 3.2: Base case, single period, single contract

As is evident in figure 3.2, the risk-taking is highly sensitive to the state of the option-like incentive contract, depending on both option moneyness and remaining time to remuneration. Located directly below the HWM is the *option ridge* where the manager seizes the opportunity to increase the probability of the incentive fee finishing in the money. As the end of the period closes in, we observe the increasing convexity along the ridge, culminating in a *last minute bet* right before termination. Farther below the HWM the effect of the incentive option diminishes as the prospect of moving into the money deteriorates, and close to maturity the risk levels resembles that of a Merton constant. We observe that the risk quickly reduces to zero as the liquidation boundary approaches in order to avoid fund closure. This is in line with the single period results of Hodder and Jackwerth (2007).

Papers such as Carpenter (2000), modelling without liquidation boundary and managerial ownership, observe rapid increase in risk-taking as the fund value decreases towards zero. This as the manager has no incentive to reduce the risk. Basak et al. (2007) also model without liquidation boundary but with an implicit managerial ownership, and observe risk-taking moving towards the Merton constant. This is because the impact of the incentive fee diminishes compared to the loss of his own share as the fund value moves substantially below the HWM.

Above the HWM, we observe that the manager quickly reduces the risk to a level significantly below the Merton constant in order to lock in his accrued earnings. Too low levels of risk is potentially just as undesirable as excessive risk-taking, since this means missing out on profits for the investor. The lock-in effect could in fact affect the expected utility of the investor more gravely than the risk peak. This as the probability of being located just above the HWM is higher than being located just below. In our default model setup the HWM is, as explained, set equal to the initial fund value augmented by the risk-free rate. With the hedge fund having a drift equal to the risk-free rate or higher, depending on the kappa-choice, the probability of being in a state located in the lock-in area is high. Changing the hurdle rate of the HWM will thus affect the results to some degree, which we investigate further in section 6.3. The reader should note that the lock-in effect will be an important determinant of the risk-misalignment in the results presented.

Comparing figure 3.1 to 3.2, it is easy to make a compelling case on the severity of risk misalignment in hedge funds. However, the realism of the scenario is questionable, and an inclusion of a more diversified fund composition evaluated over multiple periods would suggest a reduction in the misalignment. Even though it is possible to conclude simply by observation how different setups and additions to the model affect the risk-taking, it is useful to develop some metrics to quantify the actual differences and implications. In the following we run through the three metrics we apply for measuring the deviations from the optimal case.

3.1 Expected Kappa-difference

To begin with we want to find a metric that displays the overall misalignment in risk-taking. Asheim (2014) develops a metric of an average difference in κ-choice from the optimal. It can be argued that this average value is overly naive since the difference in each node (t, n) is not weighted by the probability of ever reaching that state. We therefore develop a metric that determines the expected kappa-difference experienced in the initial node. As we have 12 portfolio revisions during one evaluation period, we have to find the expected kappa-difference for each revision t. We determine the expected kappa-differences in a similar manner to how we calculate the expected utility of the manager. The absolute kappa-difference $|\kappa_{t,n} - \kappa^*|$ is first calculated for every node in a revision step. Then we recursively determine the expected kappa-difference $E[\Delta\kappa_t]$ for time step t, using the probabilities derived from the actual kappa-choices of a given setup, and moving backwards to the initial node. When this procedure is done for all revision steps, we are able to determine the total expected kappa-difference,

$$E[\Delta\kappa] = \frac{1}{\tau}\sum_{t=1}^{\tau} E[\Delta\kappa_t]. \tag{3.1.2}$$

Even though the expected kappa-difference gives a relevant measure on risk-misalignment it is difficult to relate to in absolute terms. However, it is a simple method for observing the changes in misalignment across different scenarios.

3.2 Certainty Equivalents for Manager and Investor

With the kappa-difference we measure the total risk misalignment for a given scenario, however we do not know what this really implies for the two parties. We therefore include a metric of certainty equivalents in order to measure actual payoff from a set of κ-choices. To determine the certainty equivalent we begin by determining the expected utility for the investor and the manager. We recursively run through the node network calculating expected utilities based on the terminal values and the specific κ-choice pattern, using equation 2.4.12. The expected utility is then found in the node located at the initial fund value, $E[U_0]$. The approach is similar for both investor and manager except the terminal utilities for the investor is based on investor wealth instead of managerial remuneration.[1] The optimal utility for the investor is calculated by using the Merton constant κ^*, while all other values are calculated applying the specific κ-pattern determined by the manager while optimising utility of his own total remuneration.

The concept of utility is not immediately comprehensible, as the calculated number does not provide much information other than observing the change in utility. We therefore monetise the expected utility to its certainty equivalent. This is simply done by inverting the CRRA-utility function and obtaining the according wealth. We then express the certainty equivalents as a return on the initial investor fund share $X_0(1-a)$,

[1]Investor wealth is simply the investor's share of the terminal fund value X_T subtracted managerial remuneration.

$$\Pi_{inv} = \frac{(E[U]_{inv}(1-\gamma)+1)^{-(1-\gamma)} - X_0(1-a)}{X_0(1-a)} \tag{3.2.3}$$

$$\Pi_{mng} = \frac{(E[U]_{mng}(1-\gamma)+1)^{-(1-\gamma)}}{X_0(1-a)}. \tag{3.2.4}$$

Table 3.1 displays the three misalignment metrics presented above for the optimal Merton flat and the "base case". We observe that there is a fairly large difference in certainty equivalents for the investor, indicating the adverse effects of a misaligned incentive contract. For the manager, we observe that the base case is clearly preferable to the the Merton case.

Scenario	$E[\Delta\kappa]$	Π_{inv}	Π_{mng}
Merton flat	0	7.40%	2.72%
Base case	1.613	4.57%	3.09%

Table 3.1: Misalignment in "base case"

4 Multi-period Evaluation

From the results in section 3 we observe the convex relationship between risk and time to maturity, where risk seeking behaviour increases to extreme levels as the evaluation period closes in.[1] With a fund value slightly below HWM, the manager will seize the opportunity to increase volatility and thereby his chances of ending his option in the money. From a realistic point of view, modelling risk-taking with a single evaluation period is clearly a crude simplification as it is very unlikely that the manager does not suffer any downside from poor results other than missing out on short-term remuneration. First of all, hedge funds usually endure for more than one evaluation period, forcing the manager to also consider the value of continuation. The loss recovery structure of the HWM effectively inflicts remuneration in periods following subpar performance. Secondly, an underperforming manager would suffer social and professional costs, affecting his track record and chances of raising capital for future funds. In the following section we extend our model with regards to the first remark mentioned above - including the value of continuation to the optimisation process - and display the dampening effects this implies. We also perform a brief discussion of managerial myopia with the purpose of establishing a realistic perceived value of future payouts.

4.1 Model Extensions

In order to incorporate multi-period evaluation, structural additions to the model are required. We introduce a multi-period model based on the one introduced by

[1]Seeing the incentive option as an European call, this convexity translates to the option 'gamma'.

Hodder and Jackwerth (2007).[2] The greatest challenge is the path dependency that occurs due to the HWM mechanism, with the expected value of continuation depending on preceding fund performance. If the hedge fund finishes *below* the HWM, the manager will begin the next period out of the money. The expected utility will thus be negatively affected as the loss has to be recovered before earning the incentive fee (see figure 2.1). When finishing *above* the HWM, the subsequent period is assigned a new HWM corresponding to the increased fund value. We will shortly return to how this problem is solved.

The model incorporates P number of evaluation periods. We begin the multi-period in the same way as before, starting at the terminal nodes in the last period P and recursively moving backwards. Running period P is then similar to a single period case, as there are no subsequent periods to account for. In principle, the expected utility found at the beginning of period P, $E[U_{0,n}]$, will represent the value of continuing from the preceding period $P-1$, $E[U_n^{Con}]$. In order for the manager to account for both periods in the decision process of $P-1$, the terminal utility in $P-1$ is augmented by the value of continuation. Due to the concave property of the utility function, the utilities cannot simply be added together. Thus we convert the continuation value into wealth W_n^{Con}, and then add it to the terminal wealth of period $P-1$ before the sum is converted back to utility,

$$U_n^{P-1} = \frac{(W_n + W_n^{Con})^{1-\gamma} - 1}{1-\gamma}. \tag{4.1.1}$$

U_n^{P-1} then represents the augmented terminal utility of period $P-1$ for a given fund value n. The process of augmenting the value of continuation to the preceding period is repeated until reaching $P=0$. There are, however, two complicating factors which need to be taken into consideration. First, we have to factor in whether period $P-1$ finishes above or below HWM, as this will clearly affect the value of continuing for another period. Second, the value of continuation needs to be adjusted by a realistic discount rate.

Finishing below HWM

When finishing period $P-1$ at or below the HWM, the HWM remains at the initial level. For the terminal nodes in $P-1$ below HWM, the value of continuation is conversely calculated using the expected utility of starting period P at the

[2]We refer to appendix A.5 for an explanation of why this approach is applied in favor of a traditional dynamic programming approach

corresponding fund value. This concept is illustrated in figure 4.1.

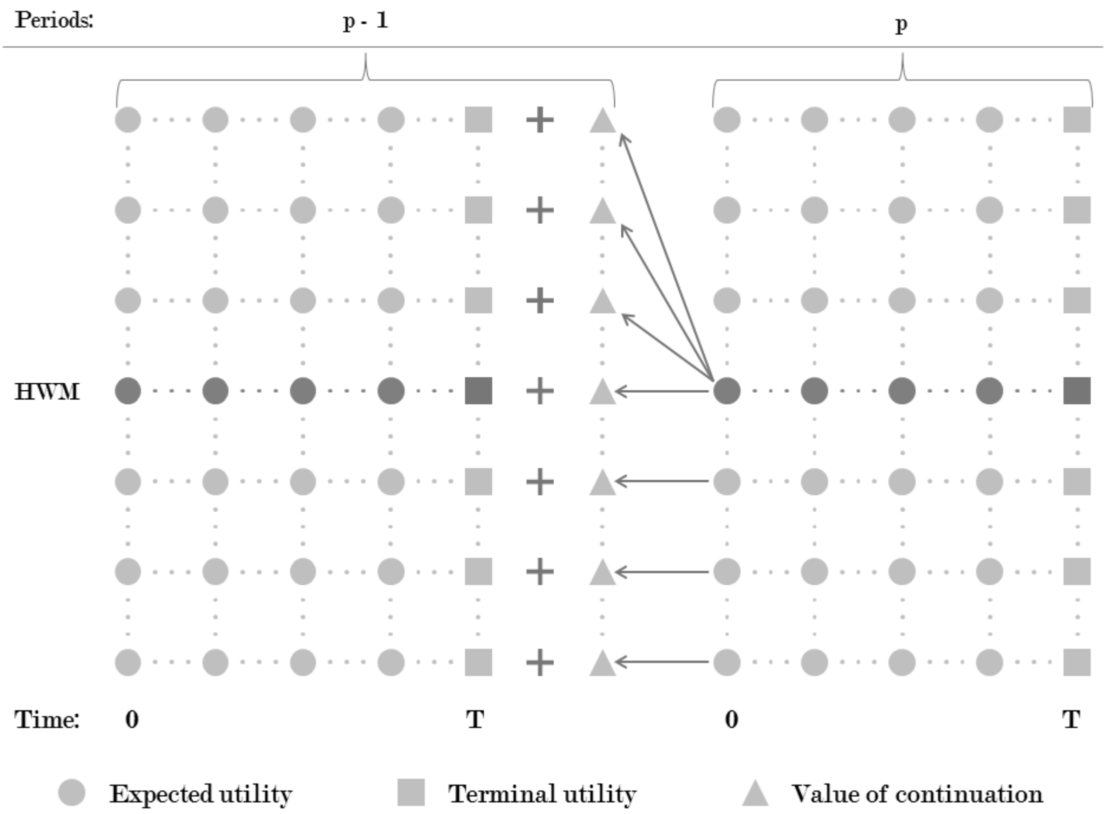

Figure 4.1: Continuation utilities

Finishing above HWM

When finishing period $P-1$ above the HWM, the calculations are somewhat more complex. Since the HWM is reset when finishing above benchmark, a naive procedure would be to simply apply the expected utility at HWM in period P as the value of continuation, since this corresponds to the value of managing the fund for another period (also illustrated in figure 4.1). However, this is an inaccurate approximation since finishing above HWM implies that the fund has been growing, and the value of managing a larger fund is obviously more profitable. Instead, we use the expected utility of starting period P at the HWM as a basis on which we adjust for the increase in fund value.

In order to find the marginal utility of managing a larger fund, we estimate the delta of the option-like contract, namely the percentage increase in the manager's certainty equivalent given an increase in the assets under management.[3] We do this by iteratively running period P for initial fund values, and corresponding HWMs, equal to the possible fund values above the original HWM. For each run the certainty equivalent in the node located at the new HWM is reported as a multiple of the original.

Figure 4.2 shows the resulting compounded change in the certainty equivalent in the initial node given the different initial fund sizes.

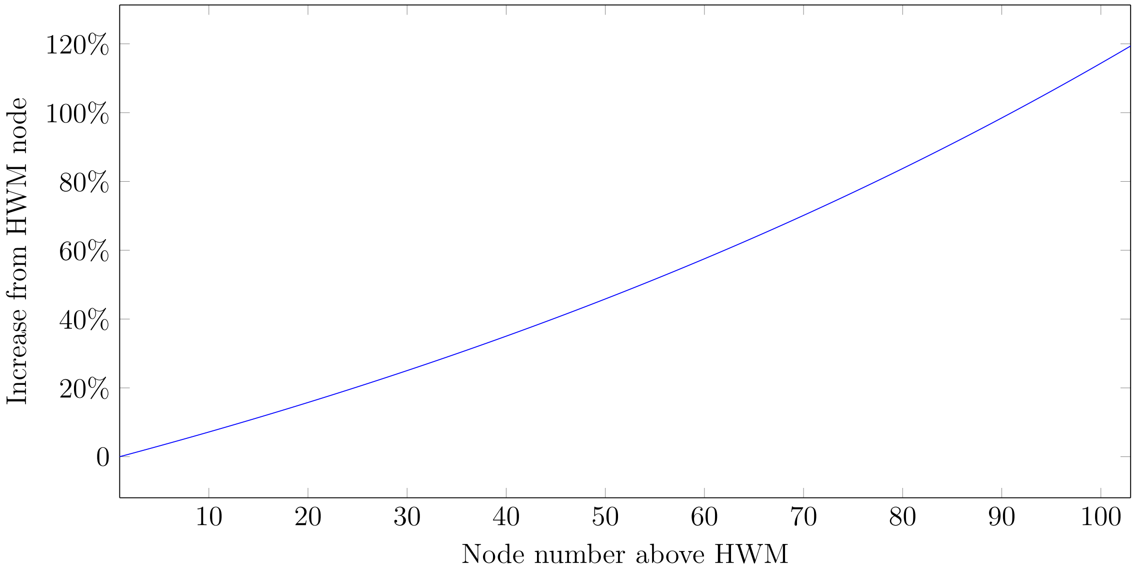

Figure 4.2: Percentage increase in certainty equivalent given fund size increase

We take use of these percentage changes to calculate the adjusted value of continuing into period P above the HWM node H,

$$W_n^{Con} = W_{\mathrm{H}}^{Con}(1 + K_n) \qquad if \quad n > H \tag{4.1.2}$$

where K is the percentage change between starting with the initial fund value and a value of n nodes above.

[3]This delta corresponds to that of Agarwal et al. (2009). As explained in their paper, the managerial option delta differs somewhat from the theoretical definition of an option delta in that it incorporates the delta of the manager's co-investment and managerial fee in addition to the option element of one or multiple contracts.

Discounting future remuneration

We have now found approximates to the certainty equivalent of continuing to period P for all possible ending states in period $P-1$. As the continuation value is based on wealth obtained one year ahead, we discount the certainty equivalent by a rate R to obtain the present value. The discount rate R should reflect the time-value of managerial remuneration and the uncertainty reflected in the fund. In addition, the illiquid attribute of hedge funds, both in terms of their investment strategies and lockup period on investor assets, suggests an additional illiquidity discount.[4] As discount values for hedge funds are difficult to obtain, we use the private equity industry as a proxy as is shares many characteristics to the hedge fund industry, such as compensation structure, lockup periods and risky investment strategies. Gompers and Lerner (1999) apply a 20% discount rate on the incentive compensation when evaluating venture capital firms. When surveying private equity firms, Gompers et al. (2014) find the typical target on rate of return to be between 20 and 25%. We apply a discount rate of 25% throughout the paper. This is considered as a relatively high value, implying conservative estimates for the value of continuation. A robustness analysis of discount rates is included in appendix A.4.

Auxiliary assumptions

The purpose of the multi-period model is to include the utility of future remuneration in the process of deciding the optimal risk-allocation. With regards to the fund process and managerial ownership, there is a slight distinction between whether the managerial remuneration received in each period is deducted from the fund value or reinvested. When reinvesting the remuneration, the managerial ownership share will increase slightly for each evaluation period. For simplicity, we assume that the manager remuneration is not reinvested, nor is the payment withdrawn from the investors' fund shares but rather conceptualised as an external payment. In this way the managerial ownership share remains constant through all evaluation periods, and we avoid having to adjust the total fund size for repeated payments.

4.2 Effect of Multiple Periods

When running our model with default parameters of 0% ownership and $R = 20\%$ we find results presented in figure 4.3, displaying how the risk peaks decline for each future period included. When analysing the results we are interested in the first

[4] Agarwal et al. (2009) notes a mean (median) lockup period of 0.8 (1.0) years.

period ($P = 1$), where the manager has the number of included periods in front of him. Table 4.1 synthesises these results by displaying the incurred reduction in misalignment from the base case in section 3. To assure the results of including different number of periods are comparable, we compute the manager and investor certainty equivalent for the first period only, and not as an aggregate of all future periods. The certainty equivalent can thus be interpreted as the expected periodical return.

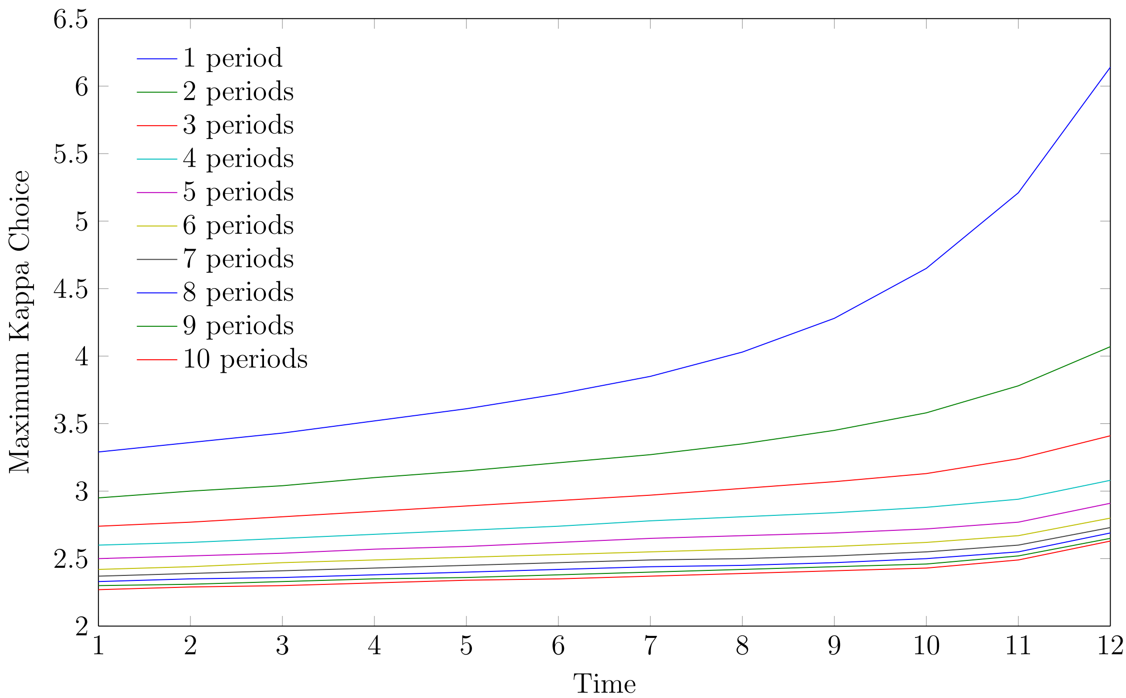

Figure 4.3: Option ridges, 1-10 periods

As can be seen from figure 4.3 and 4.4, accounting for the value of continuation clearly has a dampening effect on risk-taking. Comparing figure 4.4 to figure 3.2 we see that most of the excessive risk-taking is removed, and that the last minute bet is almost completely removed. Similarly, the lock-in effect is less apparent. However, since the HWM is reset for fund values above the current HWM and not for those below, multi-period evaluation predominantly dampens the last minute bet. This because potential losses would have to be recovered in subsequent periods.

With prolonged optimisation horizon the convexity of the incentive option decreases. This is also true for options that are either substantially in our out of the money. Consequently, as Hodder and Jackwerth (2007) notes, options maturing in future years will have current managerial behaviour effects roughly analogous to additional ownership share, dampening risk in previous periods. Figure 4.3 shows that for the first added periods, the risk-peaks are considerably lowered before the pattern

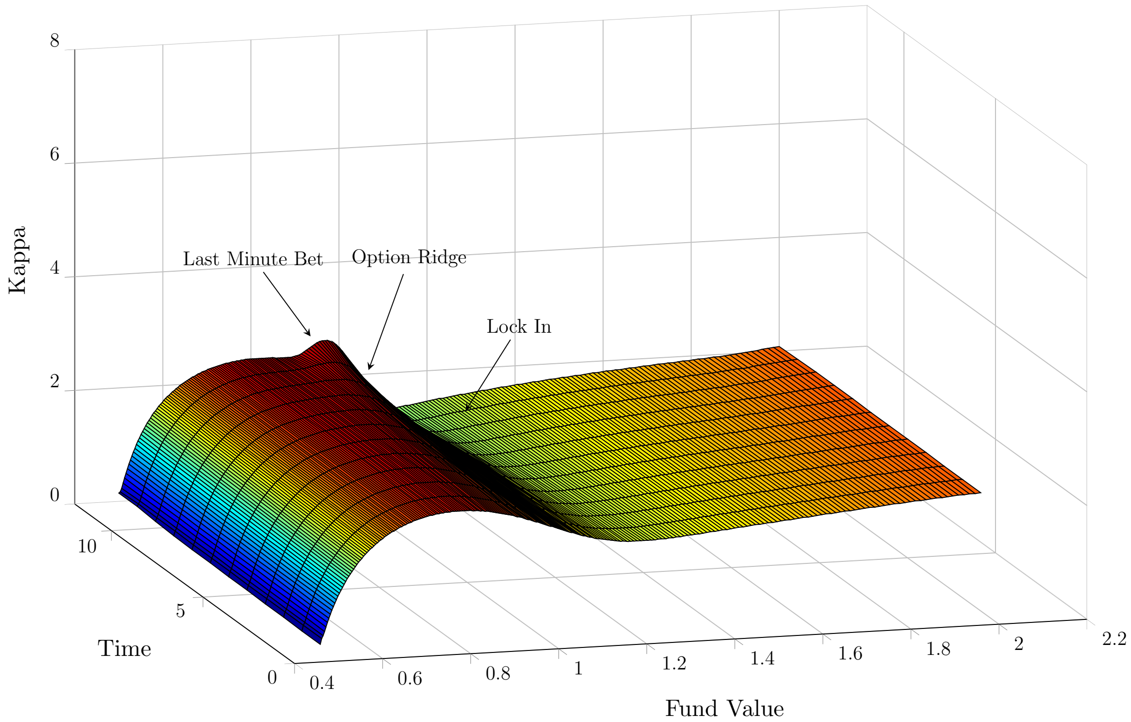

Figure 4.4: Multi period modelling, 10 periods

converges, in this case after including a number of 10 evaluation periods.[5] However, the convergence rate and risk-level in the first period depends on the assigned value of future periods, in part determined by R.

It is evident from table 4.1 that as risk-misalignment decreases, the investor experiences a gain in certainty equivalents at the expense of the manager.

Scenario	$E[\Delta\kappa]$	Π_{inv}	Π_{mng}
Merton flat	0	7.40%	2.72%
Base case	1.6132	4.57%	3.09%
2 periods	1.3826	5.40%	3.05%
5 periods	1.1086	6.19%	2.95%
10 periods	0.9747	6.51%	2.89%

Table 4.1: Misalignment in multi-period evaluation

[5] Adding further periods therefore has insignificant effect.

4.3 Considering Myopia

The realism of considering multiple evaluation periods is subject to managerial myopia. Myopically optimising managers will disregard the value of continuation, in our case analogous to the short-term single-period case presented in section 3. The opposite view is the perpetual approach applied by Panageas and Westerfield (2009) and Guasoni and Obłój (2013). With an infinite optimisation horizon the 'gamma' of the incentive option diminishes, resulting in constant risk-taking. These two opposing approaches each form the extremity of how a manager's view of future remuneration can be modelled. However, they both seem rather unrealistic, suggesting a middle ground is to be found. An example of this is Goetzmann et al.'s (2003) valuation of the incentive option who, for what they regard as reasonable parameters, find that the present value of future fees for the manager could be as high as 33% of the amount invested. Hodder and Jackwerth (2007) also suggest that the manager is likely to consider the value of continuation, but that it takes a horizon of decades for the risk-taking to approach the constant risky allocation, indicating the unlikeliness of a perpetual approach.

Turning to the academic research of myopic behaviour in general, the results are quite different from how short term behaviour is displayed in hedge fund literature. Benartzi and Thaler (1993); Thaler et al. (1997); Gneezy and Potters (1997) among others investigate how myopia is linked to loss-aversion, where frequent portfolio evaluation leads to *less* risk-taking due to a greater sensitivity to losses. In other words, longer time perspective actually makes risk seem more attractive. We acknowledge that there is a gap between the studies of behavioural economics and cognitive bias towards risk, and our more rational framework of utility maximisation. The ambition of this paper is not to bridge every aspect of risk-taking, but we conclude this brief discussion with the caveat that the multi-period framework not necessarily portrays a complete picture of human preference towards long-term wealth.

5 Multiple Incentive Contracts

We now turn to the main contribution of our paper - modelling a fund comprised by multiple incentive contracts. All previous research on incentive option modelling (Asheim, 2014; Hodder and Jackwerth, 2007; Goetzmann et al., 2003; Carpenter, 2000) model a fund as if it consisted of one investor, implying that the incentive fee is a single option with one HWM and termination date. This is a simplifying measure where the benchmark is conceptualised as an aggregate of all investor benchmarks. In terms of building a comprehensible and feasible model, this is a natural simplification to make. However, we believe it may be overly simplistic as potentially important aspects of the contractual scheme could be overlooked, as well as a single option could exaggerate pay-performance sensitivity and the level of induced risk-taking. Agarwal et al. (2009) present a similar argument in their empirical study of the incentive fee's ability to predict performance, stating that fee rate not fully captures managerial sensitivity to fund movements. This because two hedge fund managers with equal incentive fee rates may experience different incentives depending on the timing and magnitude of investors' capital flows and other contractual features. Agarwal et al. is the first to account for these properties when studying hedge funds empirically. To our knowledge, these innovations have not been incorporated in any previous numerical approach, hence the following results will provide new insight.

5.1 Model Extensions

We extend our model with the purpose of capturing the characteristics of a fund containing what we denote as multiple "pools" of investments. According to Lemke (2004), hedge funds often allow for additions or withdrawals by their investors on a monthly or quarterly basis. At each possible entry point, several investors may enter the fund, collectively making up an investment pool and a new incentive option maturing one evaluation period later. This implies that managerial remuneration now is subject to a more diversified composition of overlapping incentive contracts, resembling a portfolio of call options with individual strike prices. This increases the complexity of the utility optimisation procedure as each investment pool will have different HWMs depending on the fund value upon entrance, as well as different maturity dates. We model each setup as a steady-state situation with no inflow or outflow of net capital. Figure 5.1 depicts the mechanisms of multiple HWMs in greater detail.

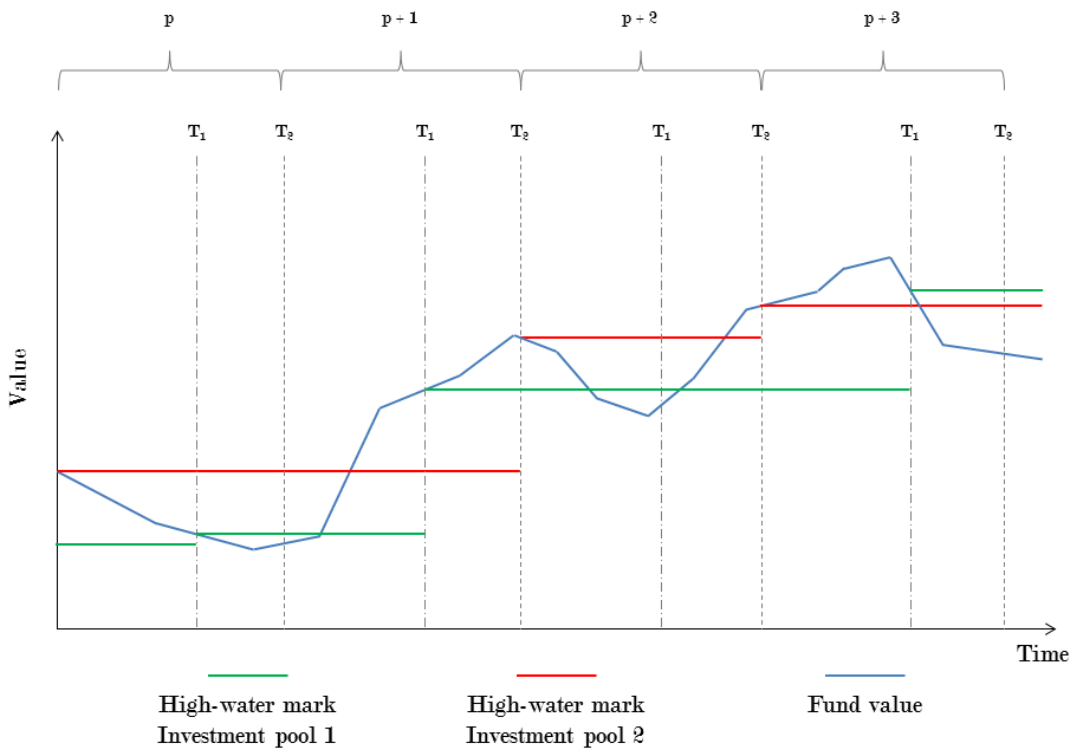

Figure 5.1: The dynamics of a high-water mark for two investment pools

The addition of multiple investment pools is incorporated in the model by running a number of parallel processes given by the number of investment pools I. Each process follows an equivalent procedure as with a single investment pool, beginning at the terminal nodes working backwards. The remuneration wealth obtained from managing the fund share of investment pool i is modelled redefining equation 2.2.7,

$$W_{i,n} = aw_iX_{T_i,n} + b(1-a)w_iX_{T_i,n} + c(1-a)max(0, w_iX_{T_i,n} - w_i\text{HWM}_i), \quad (5.1.1)$$

where w_i is the fund share of pool i and $X_{T_i,n}$ is the value of the fund at the respective remuneration date. The model combines the remuneration of the current period and the value of continuation in accordance with the multi-period perspective developed in the previous section,

$$U_{i,n} = \frac{(W_{i,n} + W_{i,n}^{Con})^{1-\gamma} - 1}{1-\gamma}. \tag{5.1.2}$$

With the notion that the manager chooses one overall risk level for the entire fund, the optimal κ is found considering the utility derived from managing all investment pools. For a given kappa, we first calculate the expected utility of each investment pool,

$$E\left[U_{i,\kappa,t,n}\right] = \sum_{j=-20}^{20} p_{\kappa,j} E(U_{i,t+\Delta t,n+j}), \tag{5.1.3}$$

but in order to sum the expected utilities they have to be converted to certainty equivalents,

$$W_{i,\kappa,t,n}^{Ceq} = (E[U_{i,\kappa,t,n}](1-\gamma)+1)^{\frac{1}{1-\gamma}}. \tag{5.1.4}$$

Finally, for a given time and fund state, the optimal kappa is the one which maximizes the utility of the summed certainty equivalents,

$$E\left[U_{\kappa,t,n}\right] = \frac{\left(\sum_{i\in I} W_{i,\kappa,t,n}^{Ceq}\right)^{1-\gamma} - 1}{1-\gamma}. \tag{5.1.5}$$

In the specific case of one single investment pool, we observe that the model reduces to the base case, where there is only one maturity and HWM to account for.

As the model moves backward in the node network, a matrix containing the optimal κ-choices is incrementally being built. The expected utility derived from each individual investment pool is assigned to *separate* matrices, to be used in the preceding step $(t - \Delta t)$. Figure 5.2 depicts the modelling of the multiple investors and incorporation of continuation utilities.

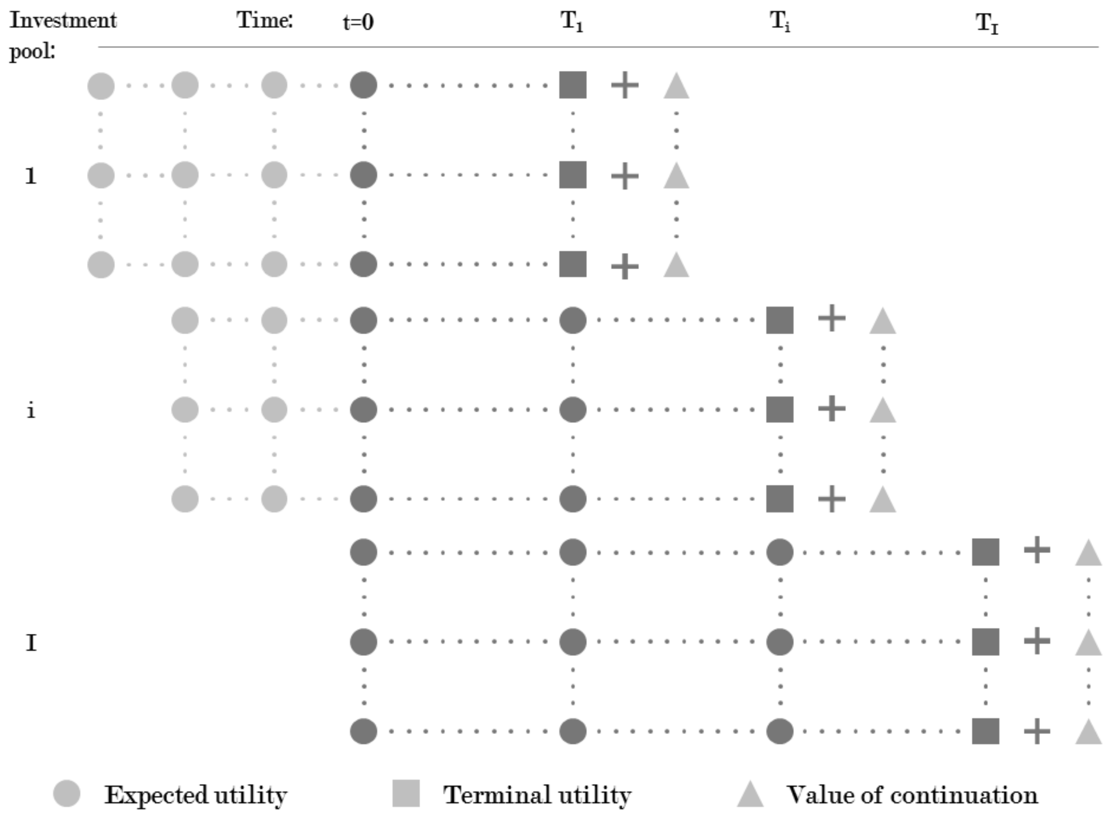

Figure 5.2: Multiple investors with overlapping evaluation periods

As for Π_{inv} and Π_{mng} we calculate these values based on the investment pool that matures at the last time-step of the period, with HWM corresponding to initial fund value. Since the certainty equivalents are expressed relative to the given investor's fund share, the results are comparable across different setups.

5.2 Effect of Different Maturities

In the following subsections we investigate the effects of different fund compositions. We begin by introducing a fund where the investment pools have entered the fund at different points in time but with equal HWMs and fund shares, resulting in a strip of overlapping incentive contracts along the time-axis. Figure 5.3 implements these characteristics, displaying the results of including two investment pools in a single period perspective.

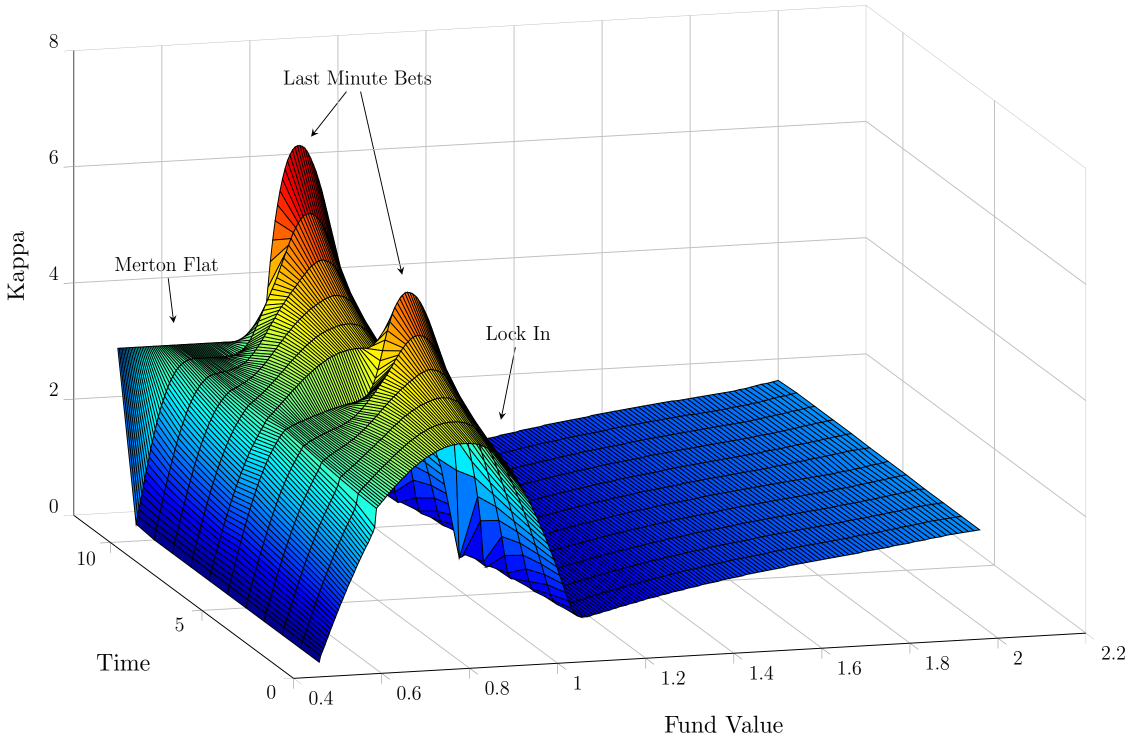

Figure 5.3: Two contracts, different maturities, single period

Adding an investment pool clearly has a dampening effect on the risk-taking, as the first peak is significantly reduced compared to the last. This shows the manager's tendency to even out the risk when investment pools of different termination overlap. The extreme risk-seeking behaviour close to maturity of one contract is offset by contracts further from maturity. This reluctance to compromise the prospects of future earnings is analogous to the effect of multiple evaluation periods seen in section 4. Table 5.1 shows the improvements in our misalignment measures when including different number of investment pools. The results from the base case is also included as a reference. As opposed to figure 5.3, where we for illustration purposes display the effects in a single period perspective, table 5.1 utilises the converged setup of 10 evaluation periods. The numbers for the one-pool scenario thus corresponds to the ten-period scenario in table 4.1. As found in table 5.1 the risk misalignment decreases as investment pools are added. We do however observe that reductions are slight due to the dampening effects already inherent from the multi-period extension. Unless otherwise is stated, further results from setups with multiple investment pools include 10 evaluation periods.

Scenario	$E[\Delta\kappa]$	Π_{inv}	Π_{mng}
Merton flat	0	7.40%	2.72%
Base case	1.6132	4.57%	3.09%
1 pool	0.9747	6.51%	2.89%
2 pools	0.9666	6.53%	2.87%
4 pools	0.9598	6.55%	2.87%
6 pools	0.9595	6.55%	2.87%

Table 5.1: Misalignment, multiple contracts, 10 periods

5.3 Effect of Different Investment Pool Sizes

In the case above, the pools were of equal size and evenly distributed, but the sizes of the investment pools will most likely differ. For the manager this will naturally mean that large options are assigned greater weight when deciding the risk-allocation. The interesting question is therefore how investors, and predominantly the smaller ones, could be adversely affected by the presence of larger investors.

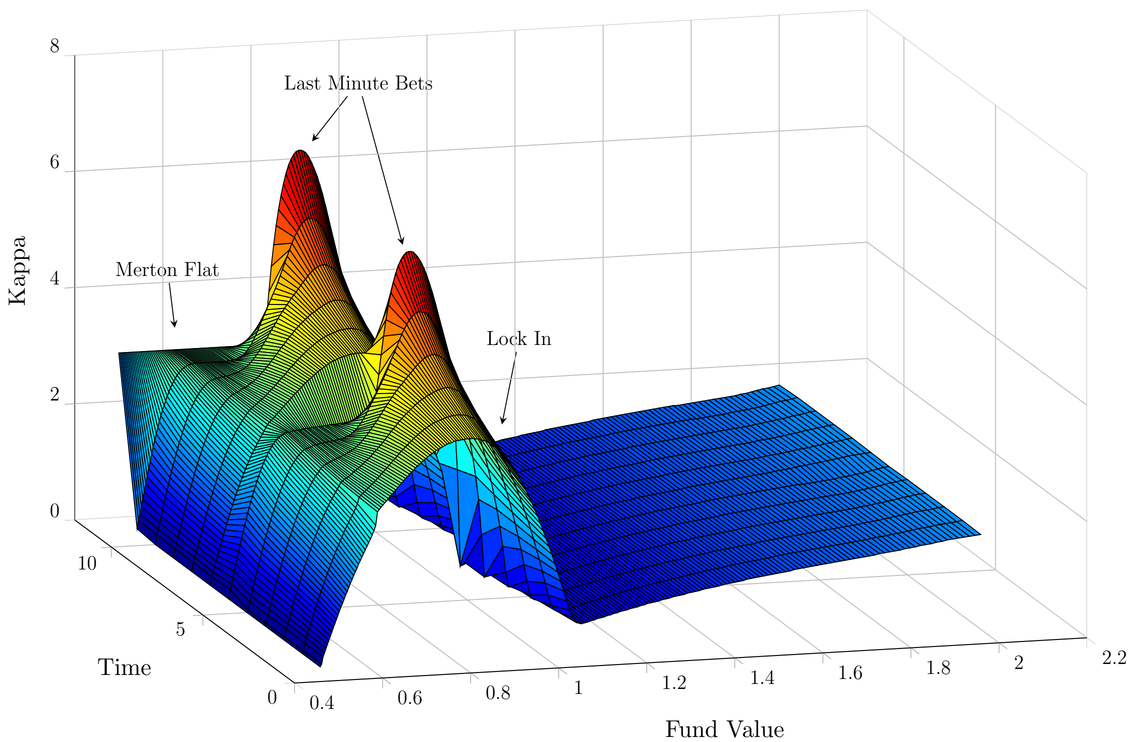

Figure 5.4: Two contracts, different maturities, different pool sizes, single period

Creating a scenario where an investor with 10% share enters the fund half a period after the remaining 90%, then illustrates the potential increase in misalignment for smaller investors. Figure 5.4 shows the first peak has increased from that of figure 5.3 since 90% of the fund matures at this date. Calculating the loss in certainty equivalents for the small investor, we find that in a single period case the 10% investor loses 0.15 percentage points as opposed to a 50/50 case. In other words, although the overall risk-peaks are lowered with multiple pools, the misalignment is unevenly distributed amongst the investors depending on fluctuations in size. Our 10/90% scenario is admittedly an unlikely and artificial construct, but it still shows that the overall dampening effect of multiple contracts involves some degree of ambiguity.

5.4 Effect of Different HWMs

So far, we have only looked at contractual variations along the time-axis, keeping the HWMs equal in order to isolate the effects. We now let the investment pools enter at different points in time in a fund that either increases or decreases in fund value, implying a strip of incentive contracts with different HWMs.

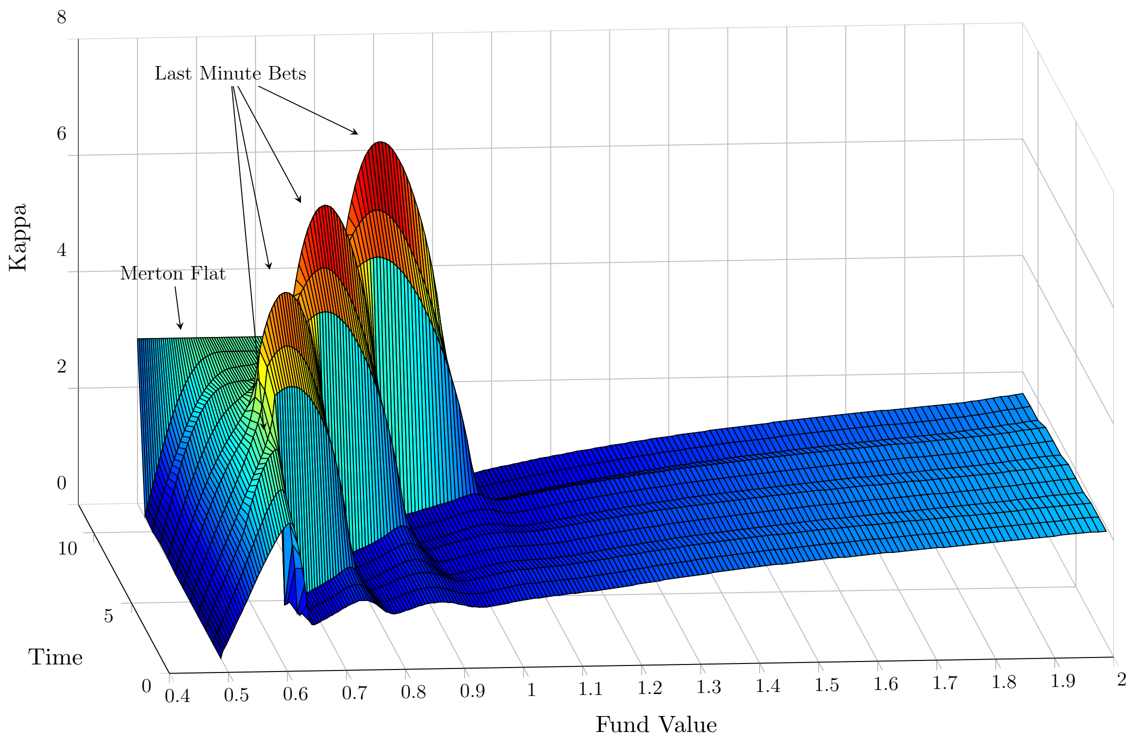

Figure 5.5: Four contracts, different maturities, increasing HWMs, single period

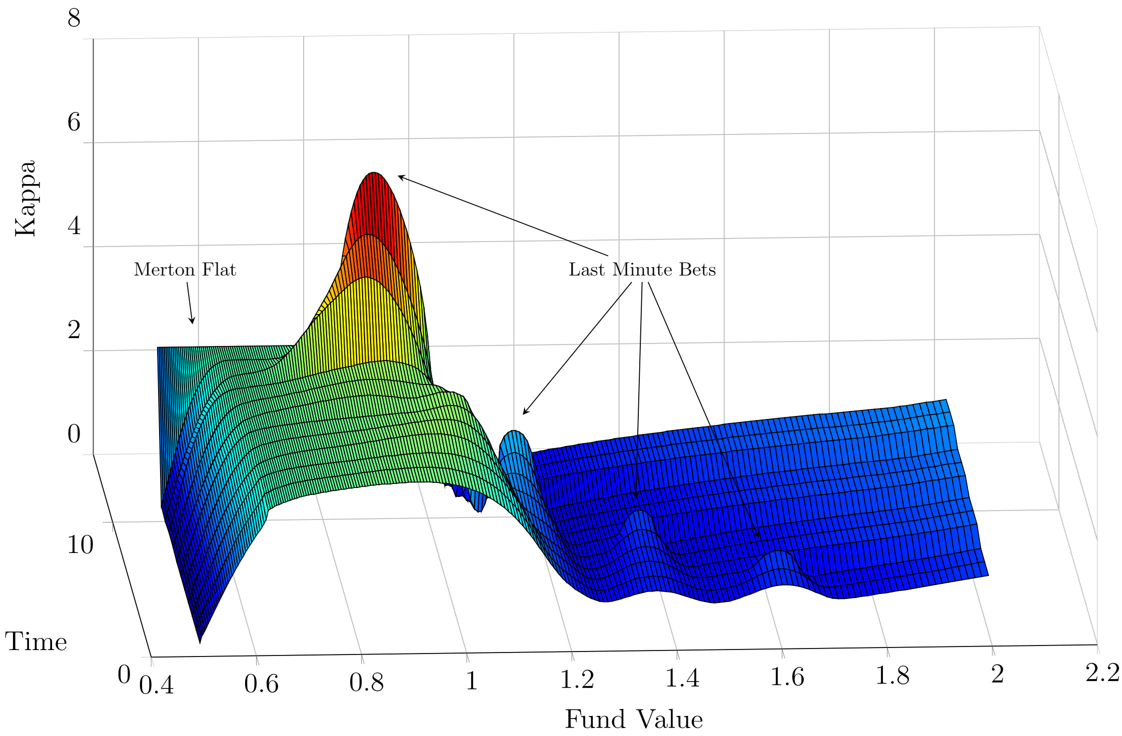

Figure 5.6: Four contracts, different maturities, decreasing HWMs, single period

We see from figure 5.5 and 5.6 that spread out HWMs will decrease the risk peaks and distribute risk more evenly across the fund value-axis. The diversity in moneyness for the different options counteracts the convexity of the option ridge for each single option, making the overall risk-taking less sensitive to fund size. This is true for both consecutive increasing and decreasing HWMs. For illustration purposes the graphs display the inclusion of four investment pools. As six investment pools yields slightly better results, these are presented in table 5.2. Table 5.3 contains information on the specific level of each HWM, indicating fund size upon entry.

An increasing HWM structure is a potential consequence of a fund in a persisting positive trend, a situation that is not unlikely. From table 5.2 we see that the resulting risk-misalignment for this situation is worse than for the constant and decreasing HWM scenarios. The manager is deep in the money for most of the option contracts and therefore prefers to lock in the profit, rather than risk losing the profit or in worst case suffer fund liquidation. Even though the fund has been on a positive trend, the investment pools will still retract their money if the fund drops 50% from the initial fund value of the current period. As earlier mentioned the lock-in area may affect the risk-misalignment to a great extent, as the risk is reduced to a level much lower than the optimal Merton level. With increasing HWMs the presence of the lock-in area is increased.

In our decreasing HWM scenario the manager is deep out of the money for most of the option contracts, which results in a relatively low expected remuneration from the incentive fees. In our model, the risk of fund liquidation is not increased compared to the two other situations as the liquidation boundary is absolute and all HWMs are above the initial fund value of the current period. When the effect of the option element diminishes and the probability of liquidation is low, the manager tends to adjust the risk to a level close to the Merton constant. These considerations results in the relatively low expected kappa-difference and high investor certainty equivalent for the decreasing HWMs scenario.

Scenario	$E[\Delta\kappa]$	Π_{inv}	Π_{mng}
Merton flat	0	7.40%	2.72%
Base case	1.6132	4.57%	3.09%
6 pools, 10 periods, constant HWMs	0.9595	6.55%	2.87%
6 pools, 10 periods, decreasing HWMs	0.4135	7.21%	2.78%
6 pools, 10 periods increasing HWMs	1.1688	6.20%	2.78%

Table 5.2: Misalignment, multiple contracts across HWMs

Maturity		1st	2nd	3rd	4th	5th	6th
Decreasing	HWM	1.72	1.55	1.40	1.26	1.14	1.03
Increasing	HWM	0.59	0.66	0.73	0.82	0.92	1.03

Table 5.3: Decreasing and increasing HWMs

In these examples we have simply introduced new contracts at specific points in time at HWMs of our own choosing. This approach is somewhat unsophisticated in that we merely state arbitrary fund values for where the new contracts enter the fund, and evaluate the setup in a steady-state perspective. Calculating the overall value of a diverse fund composition proves to be very complex since the composition continuously changes depending on the flow of investors and a fluctuating fund value. However, we believe the results are still powerful in that we are able to display the direct implications of a contractual scheme where investors are assigned different HWMs and maturities.

5.5 Considerations and Common Practice

While we incorporate the above additions under the assumption that it provides valuable insight, we do not present any empirical evidence that hedge fund managers actually operate by issuing separate contracts. We merely view the situation from a technical perspective and base our assumptions upon how overlapping incentive contracts should be treated mathematically. Agarwal et al.'s (2009) work strongly supports our notion of capital flows and individually assigned incentive contracts, approaching the problem in a similar fashion. Brown and Goetzmann (2001) however, asks the same question but argues that investors entering midyear after a loss for the first half will apply previous year's high-water mark. They correctly acknowledge that this will benefit the midyear investor at the expense of current investors, and note that this imbalance can be reduced by the use of "equalising shares". Whether practitioners apply individual contracts or a rely on pragmatic simplifications, will probably vary between funds and depend on how often investors are allowed to enter the fund and redeem their investments – Arguably, the issue could be avoided by only raising and redeeming capital at predetermined dates, thereby issuing identical contracts on all capital.

Regardless of common practice, our results are still valid in that they illustrate the investor benefits of issuing separate incentive contracts. The main attribute of a diverse incentive option portfolio is its innate dampening effect on risk fluctuations. Secondly, individual contracts will increase the predictability of incurred fees, as opposed to the less sophisticated procedure described above where some investors could experience adverse treatment. Thirdly, it facilitates a more flexible capital flow, removing the contractual constraint of raising and redeeming capital at specific dates.[1] These considerations with the support of the above results, therefore suggests that practitioners as well as academics should not be indifferent to how individual incentive contracts are constructed. For investors specifically, the advantage of being in a fund comprised of diverse contracts is apparent.

[1]Investors could, however, still be prone to lock-up periods as a result of illiquid fund positions.

6 Evaluating Contractual Parameters

In the previous sections we have focused on how the implementation of diverse fund composition and multi-period evaluation affects risk-taking, keeping all other fund parameters constant.[1] In previous literature, however, much attention has been devoted to the adverse effect of fee size and the application of ownership share as the main tool for mitigating misalignment. It is therefore interesting to evaluate these parameters within the confines of our model. Specifically, we analyse the sensitivity of misalignment to these parameters for different model setups. We believe this exercise will provide further validation of our findings in that we will observe less sensitivity to changes in fee and ownership structure due to the risk aligning characteristics included in the model.

6.1 Sensitivity to Remuneration Structure

In the framework of our new extensions, we would like to find to what degree changes in the different rates affect the contractual misalignment. Carpenter (2000), Asheim (2014) and Kouwenberg and Ziemba (2007) preform similar analyses of how the overall risk profile evolves as a function of incentive fee size. As discussed in section 2.3 this is due to the application of different utility functions. For a manager with loss-aversion, Asheim and Kouwenberg and Ziemba finds that increased incentive fee increases the total volatility in the fund. Carpenter finds the opposite for a manager with HARA-utility. However, what direction the overall risk level moves is in itself not very informative, as it does not necessarily correlate with the change in misalignment. Rather, the move in risk should be considered relative to the

[1] Incentive fee = 20%, managerial fee = 2%, ownership share = 0%.

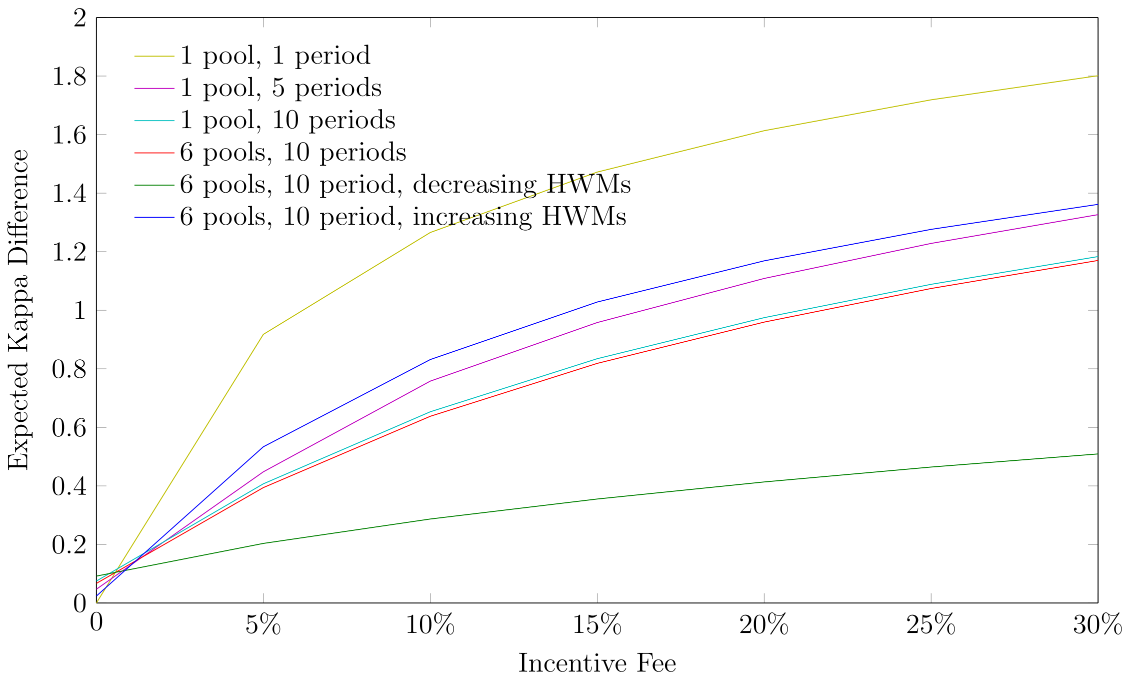

Figure 6.1: Sensitivity to incentive fee size for different setups

optimal Merton constant. Since the risk level for HARA and, in our case, CRRA-utility often lies beneath the optimal, an increase in risk could therefore imply a decrease in misalignment. We therefore rely on our previously developed metric of expected kappa-difference and find that although specific risk levels may move in different directions from that displayed in a loss-averse context, the misalignment is always reduced when lowering the incentive fee. This is consistent with Asheim and Kouwenberg and Ziemba.

In figure 6.1 we show how the expected kappa-difference is positively correlated with incentive fee size, and how the sensitivity changes depending on the model. We observe that for model scenarios that already are inherently less misaligned, the sensitivity towards movements in incentive fee is less. This is not surprising, being that the incentive fee is the chief cause of misalignment, however it shows that our previous results are robust across fee structure. This analysis also confirms the notion of Agarwal et al. (2009) that the mere size of the incentive fee is an incomplete measure of manager pay-performance sensitivity, and thereby his applied risk-taking.

The management fee is less controversial than the incentive fee, being that it does not have an option-like structure and therefore should not be a cause of misalignment. In fact, Asheim (2014) shows how the management fee has an innate dampening effect. This is because the payoff is linearly dependent on the total fund size and therefore also exposes the manager to losses. Figure 6.2 shows this very clearly.

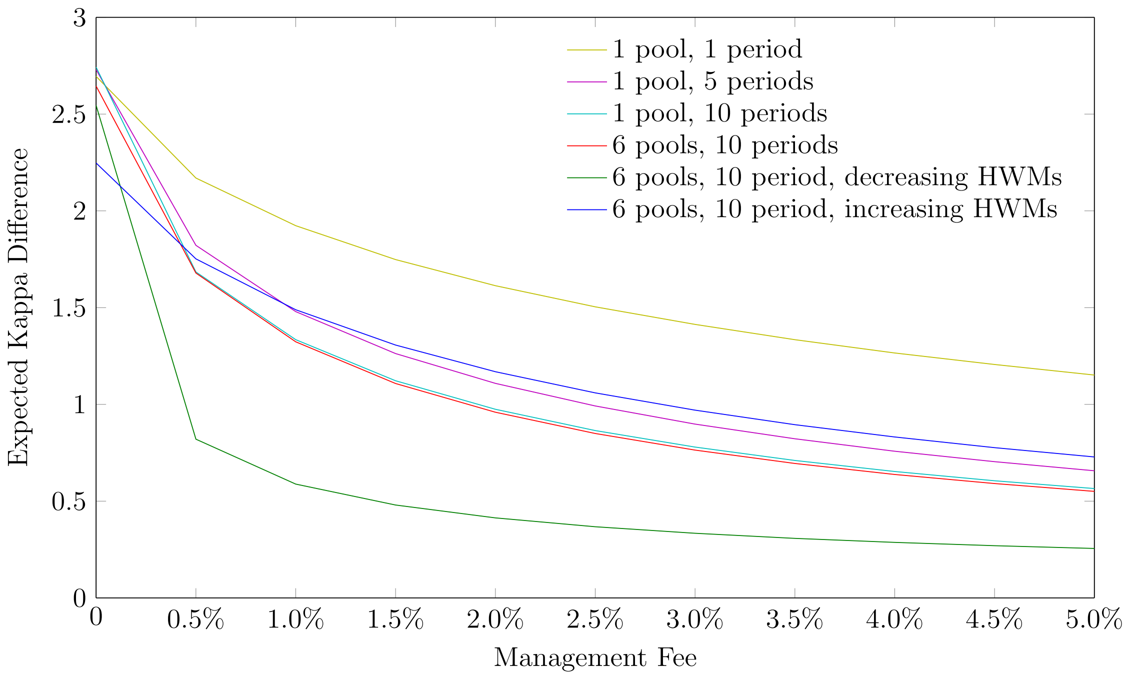

Figure 6.2: Sensitivity to management fee size for different setups

Not only does increased management fee reduce misalignment, the fee itself is an essential component in order to reap the benefits of multiple periods and incentive contracts. We observe that the sensitivity towards management fee is highest for values below 1%, and that multiple evaluation periods boosts the effect of the fee since it is also included in the value of continuation.

6.2 Sensitivity to Managerial Ownership

Until this point we have kept the managerial ownership a equal to zero in order to isolate other dampening effects. In this section we discuss the effects of managerial ownership in light of our different model extensions. The subject of managerial ownership in hedge funds is thoroughly discussed in existing literature and often considered a vital part of the contractual agreement.[2] Kouwenberg and Ziemba (2007) finds that with a an ownership share of 30% and above the manager acts very similar to the case of 100% ownership, and with 10% and less the adverse effects of the incentive fee becomes the dominant factor. Asheim (2014) confirms,

[2]Hodder and Jackwerth (2007) propose that a ownership of 10% or more is plausible for medium sized hedge funds. But for large hedge fund, holding assets of billion of dollars, the managerial ownership would likely be smaller, but still non-trivial.

in his single period model, managerial ownership to be a strong risk alignment tool, and that the effect is immediately significant at 3% ownership and above. However Asheim also notes that the significance of the managerial ownership is decreasing with longer horizons.

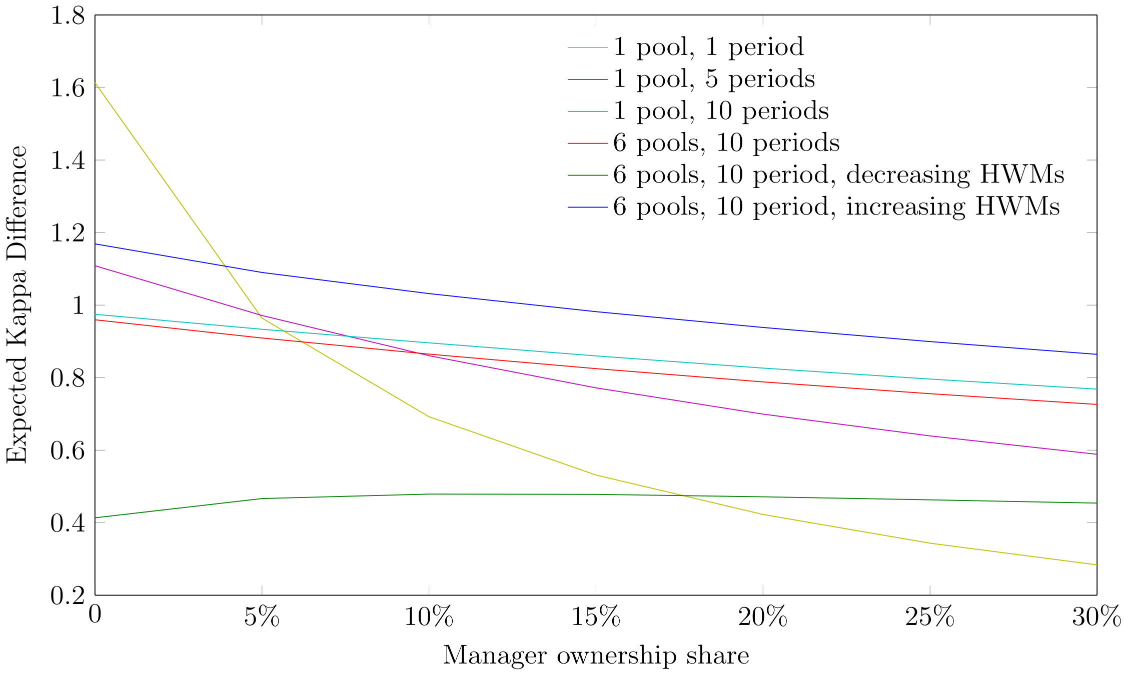

Figure 6.3: Sensitivity to ownership share for different setups

Figure 6.3 displays the expected kappa-difference sensitivity to ownership share for different scenarios. For the single period, single investor case the risk aligning impact of the managerial ownership is clearly evident. In line with Kouwenberg and Ziemba (2007), we find that with 30% ownership and above the manager acts closely to the case of 100% ownership. What is very interesting is how the impact of the managerial ownership decreases as other risk aligning characteristics are introduced, which is opposite from what we observed with the management fee. There are two reasons for this. First and foremost, as we include more periods the managerial remuneration represents a relatively larger part of his future expected wealth. Secondly, the wealth the manager derives from his own co-investment in the fund is in our multiple period model set as a payout in the very last period of evaluation. In our opinion this is a realistic assumption as the manager will have to lock up his own investment for the lifetime of the fund. As we discount the utility of continuation for each period with a discount rate of 25%, the impact of the managerial ownership diminishes as more periods are included to the evaluation. Appendix A.4 includes a sensitivity table for discount rates in a setup with 10% ownership share. This table shows that our results are fairly robust within the realistic span of 15% to 25% discount rate.

6.3 Sensitivity to Hurdle Rate

To our knowledge, the effect of increasing the hurdle rate as a means to relieve misalignment has not been thoroughly analysed in previous numerical research. Since the hurdle rate defines the strike level of the incentive fee relative to the initial fund value, an increase will directly reduce the value of the option-like contract. It may therefore seem redundant to investigate the misalignment sensitivity to a hurdle rate increase as it in essence is analogous to lowering the incentive fee. However, considering the prevalence of the incentive fee as industry standard, proposing a higher hurdle rate could prove more feasible in reality. This is also a compelling argument because it would favour superior managers, increasingly enabling them to signal their abilities.

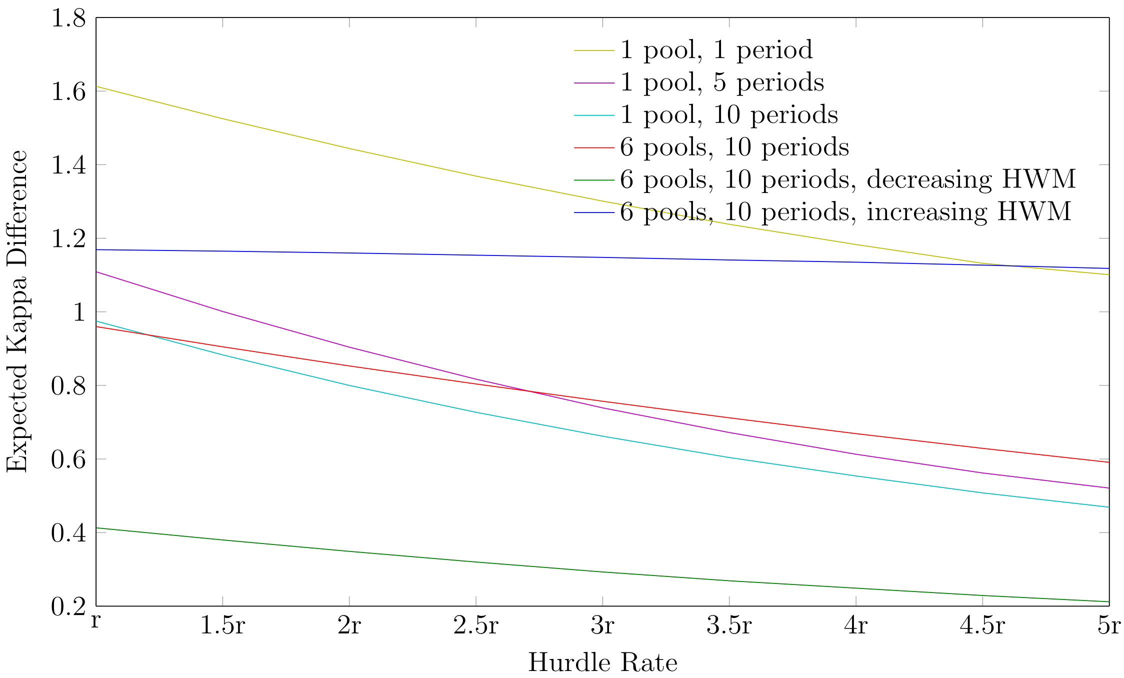

Figure 6.4: Sensitivity to hurdle rate for different setups

Figure 6.4 displays a brief analysis of adjusting the hurdle rate as a multiple of the risk-free rate. We observe a parallel shift in the sensitivity curves when increasing the number of periods, indicating that the benefit of increasing the hurdle rate is constant in absolute terms regardless of evaluation periods. On a general basis the increase in hurdle rate reduces the expected kappa-difference, as the effect of the incentive fee diminishes as options move out of the money. For the setup with overlapping incentive contracts, the sensitivity to the hurdle rate is generally lower. Since dividing the fund into multiple contracts reduces managerial sensitivity to the aggregated contractual scheme, changing the character of each contract has less effect than for a single-investor setup. For the setup with increasing HWMs, the hurdle rate increase has insignificant impact as the manager is deep in the money for most of the options. The risk aligning effect from the hurdle rate increase on the highest HWM is offset by the lower HWMs moving closer to the current fund value, leaving the manager less in the money for these options.

Tabular results, including changes in certainty equivalents, for all sensitivity assessments in section 6 are included in the appendix A.3.

7 Cost of Contractual Inefficiency

Throughout the paper we have shown how different fund structures affect the investor-manager risk misalignment and consequently their expected payoff. Clearly, there are ways to model both extreme and more moderate scenarios of adverse risk-allocations, but the solution to the problem is not yet apparent. So far, our results clearly indicate that the investor will to some extent suffer from contractual inefficiency unless the option-like structure of the contract is removed altogether. The question is either whether these inefficiencies can be justified by superior fund performance or if the contract can be modified in a way that benefits both investor and manager. We investigate the latter by synthesising our previous results and displaying how wealth is transferred between manager and investor.

7.1 Transferring Wealth from Manager to Investor

In section 4 we found that increasing the number of evaluation periods reduces risk-misalignment. As a consequence, increasing evaluation periods can also be used as a proxy for evaluating the effect of increased alignment on manager and investor expected payoff, without creating noise by altering contractual parameters.

Figure 7.1 has two key takeaways: First, when solely decreasing misalignment without changing anything else, the investor benefits at the manager's expense. Second, the marginal benefit for the investor is much greater than the marginal loss for the manager, implying a reduction in deadweight loss in the contract. We see both Π_{inv} and Π_{mng} converge towards their respective earnings Π^*_{inv} and Π^*_{mng} given that the manager chooses the Merton constant in every evaluation decision.

We define the difference between Π_{inv} and Π^*_{inv} as the contractual inefficiency,

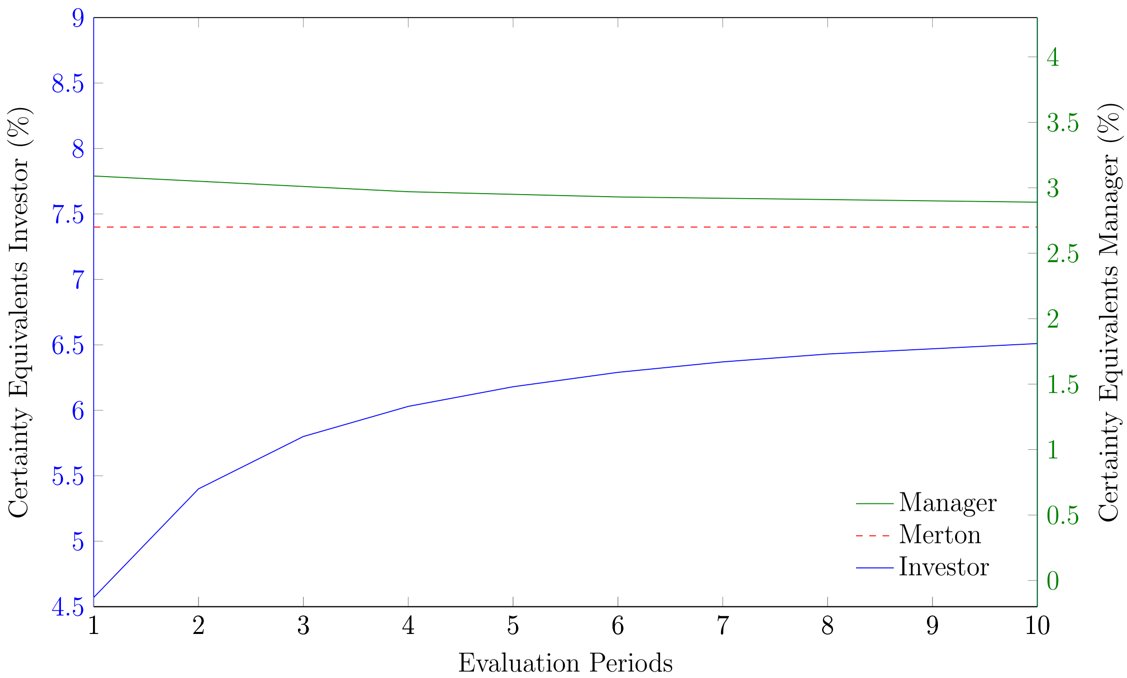

Figure 7.1: Risk misalignement effects on investor and manager certainty equivalents

$$\Delta\Pi_{inv} = \Pi_{inv} - \Pi^*_{inv}, \tag{7.1.1}$$

where $\Delta\Pi_{inv}$ then indicates what the contract costs and how much excess return the manager would be expected to generate in order to justify his fee structure. The investor then simply accepts that the contract is flawed and bets on superior manager skill. Table 7.1 summarises the contractual inefficiency for the different scenarios.[1] Naturally, the most costly scenarios are those that are most misaligned.

Scenario	$\Delta\Pi_{inv}$
1 pool, 1 period	-2.82 %
1 pool, 5 periods	-1.21 %
1 pool, 10 periods	-0.89 %
6 pools, 10 periods	-0.85 %
6 pools, 10 periods, decreasing HWM	-0.19 %
6 pools, 10 periods, increasing HWM	-1.19 %

Table 7.1: Cost of contractual inefficiencies

[1]With the standard setup of 20% incentive fee, 2% management fee and zero managerial ownership.

7.2 Searching for the Optimal Contract

As can be seen from previous results, a reduction in misalignment by itself solely benefits the investor. The reason for this is twofold: Firstly, reduced misalignment by default means that the solution moves from managerial optimum in the base case and towards investor optimum of Merton. Secondly, so far this reduction in misalignment has not been induced by anything that directly benefits the manager, such as for instance increase in management fee. This raises the question of whether there exists a setup where both manager and investor are better off.

As argued by Asheim (2014), the trivial solution to the misalignment problem is to remove the incentive fee option completely, however as he mentions, the fee has become an industry standard. Increasing management fee or managerial ownership in the fund is therefore perhaps a more realistic approach. In section 6.2 we show that by including more evaluation periods and investment pools, the risk aligning effect of the managerial ownership quickly diminishes, hence indicating that this measure is possibly not as effective as previous literature has suggested. This leaves us with the option of changing the managerial remuneration contract in order to counteract and optimise the investor-manager risk misalignment.

When looking at an increasing management fee as depicted in figure 7.2, we find that in the lowest regions of management fee, the certainty equivalents of the investor increase with increasing fees. The reason for this is the improved risk-alignment the fee produces. At a certain point, the increase turns net negative for the investor as the fee represents a larger marginal cost than the gain from increased alignment. For the incentive fee (figure 7.3), we find certainty equivalents to be monotonically decreasing for the investor as incentive fees increase - not only is the incentive fee expensive, it paradoxically also induces adverse performance.

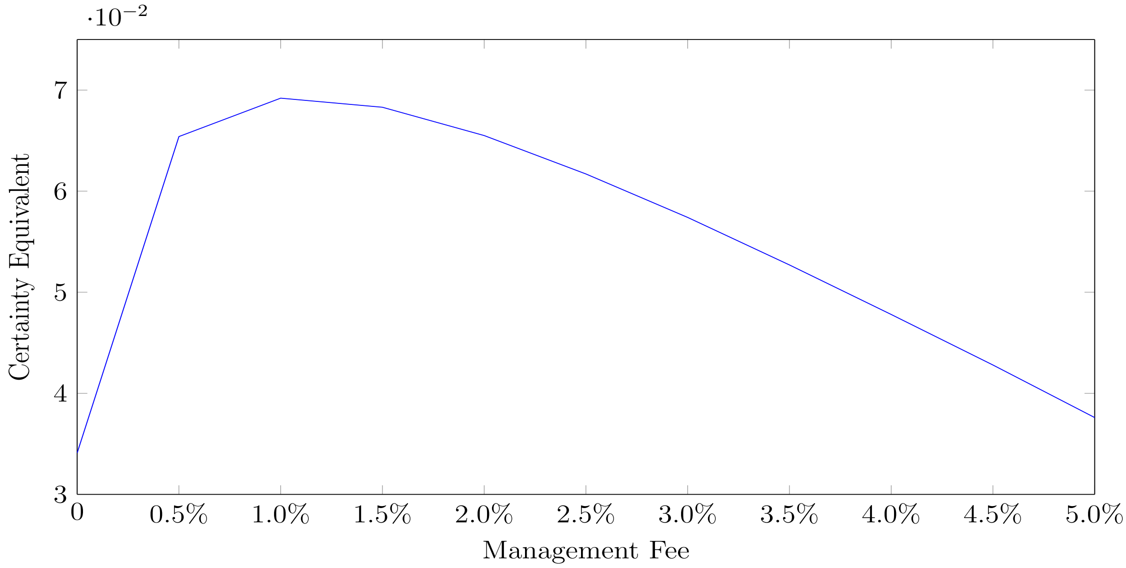

Figure 7.2: Certainty equivalents of investor by change in management fee

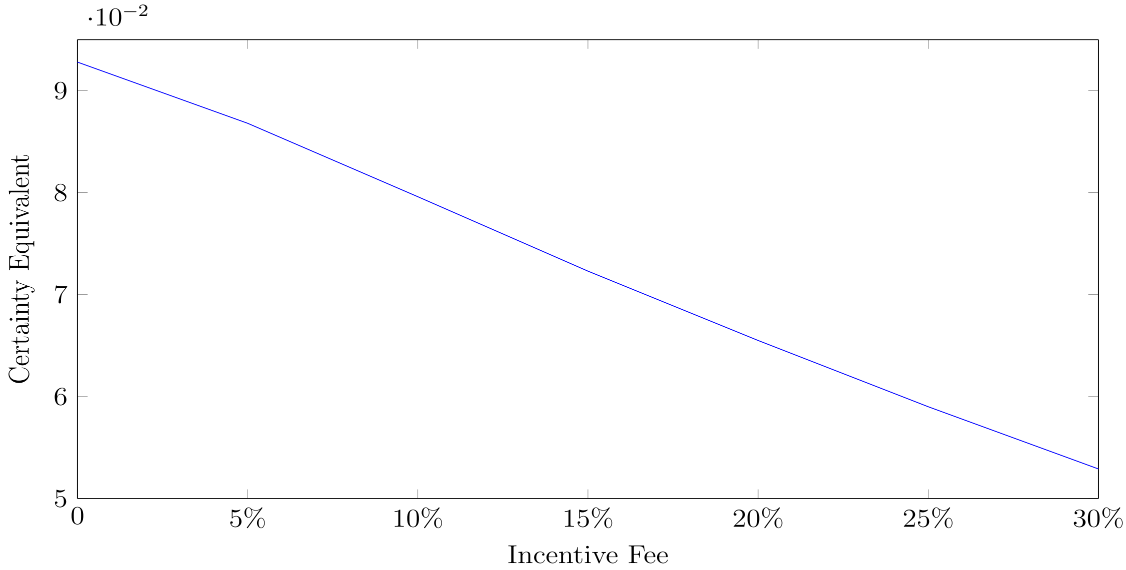

Figure 7.3: Certainty equivalents of investor by change in incentive fee

From observing 7.2 and 7.3, we see that the 2/20 structure cannot be optimal for the investor. Lowering both management fee and incentive fee could potentially increase investor's earnings, but would simultaneously produce a remuneration structure no manager would be willing to accept. Instead we search for a contract that keeps the managers certainty equivalents at the same level or higher, making the manager theoretically indifferent to the new structure while still benefiting the investor.

Management Fee		Incentive Fee 0 %	5 %	10 %	15 %	20 %	25 %	30 %
0.0 %	Π_{inv}	11.50 %	4.47 %	3.91 %	3.61 %	3.41 %	3.24 %	3.13 %
	Π_{mng}	0.00 %	0.02 %	0.02 %	0.02 %	0.02 %	0.02 %	0.02 %
0.5 %	Π_{inv}	10.95 %	9.54 %	8.26 %	7.30 %	6.54 %	5.91 %	5.38 %
	Π_{mng}	0.56 %	0.72 %	0.77 %	0.81 %	0.84 %	0.86 %	0.87 %
1.0 %	Π_{inv}	10.39 %	9.53 %	8.55 %	7.68 %	6.92 %	6.25 %	5.66 %
	Π_{mng}	1.12 %	1.34 %	1.44 %	1.50 %	1.55 %	1.59 %	1.62 %
1.5 %	Π_{inv}	9.84 %	9.15 %	8.33 %	7.55 %	6.83 %	6.16 %	5.56 %
	Π_{mng}	1.67 %	1.93 %	2.06 %	2.15 %	2.22 %	2.28 %	2.32 %
2.0 %	Π_{inv}	9.28 %	8.68 %	7.96 %	7.23 %	6.55 %	5.90 %	5.29 %
	Π_{mng}	2.23 %	2.52 %	2.67 %	2.78 %	2.87 %	2.94 %	3.00 %
2.5 %	Π_{inv}	8.72 %	8.17 %	7.50 %	6.83 %	6.17 %	5.53 %	4.93 %
	Π_{mng}	2.79 %	3.09 %	3.27 %	3.40 %	3.50 %	3.59 %	3.66 %
3.0 %	Π_{inv}	8.16 %	7.64 %	7.02 %	6.38 %	5.74 %	5.12 %	4.52 %
	Π_{mng}	3.35 %	3.66 %	3.86 %	4.01 %	4.13 %	4.22 %	4.31 %
3.5 %	Π_{inv}	7.61 %	7.10 %	6.51 %	5.89 %	5.27 %	4.66 %	4.07 %
	Π_{mng}	3.90 %	4.23 %	4.45 %	4.61 %	4.74 %	4.85 %	4.94 %
4.0 %	Π_{inv}	7.05 %	6.55 %	5.98 %	5.39 %	4.78 %	4.18 %	3.59 %
	Π_{mng}	4.46 %	4.80 %	5.03 %	5.20 %	5.35 %	5.47 %	5.57 %
4.5 %	Π_{inv}	6.49 %	6.00 %	5.45 %	4.87 %	4.28 %	3.69 %	3.10 %
	Π_{mng}	5.02 %	5.37 %	5.61 %	5.79 %	5.95 %	6.08 %	6.19 %
5.0 %	Π_{inv}	5.93 %	5.45 %	4.91 %	4.34 %	3.76 %	3.18 %	2.60 %
	Π_{mng}	5.58 %	5.93 %	6.18 %	6.38 %	6.54 %	6.68 %	6.80 %

Optimal contract (green: 2.5 % management fee with 0 % and 5 % incentive fee; 3.0 % management fee with 0 % incentive fee) — Better or indifferent for both investor and manager (yellow: 2.5 % with 10 % and 15 %; 3.0 % with 5 % and 10 %; 3.5 % and 4.0 % with 0 % and 5 %) — Standard contract (red: 2.0 % with 20 %)

Table 7.2: Certainty equivalents for investor and manager for different remuneration structure.

As can be found in table 7.2, we discover an optimal solution for the investor in the area between 2.5% to 3% management fee and 0% incentive fee, or in the area between 0% and 5% incentive fee and 2.5% management fee. Moving towards such a fee structure could in fact increase investor payoff with 2 percentage points in certainty equivalents. This solution surpasses that of the previous Merton optimum due to a significant lower fee payment. As mentioned, removing or dramatically reducing the incentive fee is perhaps an unrealistic solution. However from our results above, we find that even a moderate decrease to 15% incentive fee compensated by an increase to 2.5% management fee leaves both investor and manager with a larger return. Although the absolute values of these results are subject to numerous assumptions, we believe the pattern displayed from this analysis is powerful, suggesting that a contract solely based on management fee could remove contractual inefficiency and still benefit both parties.

It should however be emphasised that our model does not incorporate possible adverse side effects of reducing the incentive fee. Given that the fee somehow directly induced improved decision-making through increased manager motivation etc., this is not captured by our model. Furthermore, difficulties may arise for managers to sufficiently signal their skills if reducing incentive fees. Investors could perceive funds with lower incentive fees than the industry standard as a sign of inferiority. An alternative might be to simultaneously signal superior performance by increasing the hurdle rate, as mentioned in section 6.3.

8 Concluding Remarks

Our work should be viewed as an extension of the previous hedge fund literature that aspires to realistically model a manager's behaviour when compensated by a loss-provision incentive contract. The quantitative results are best understood as a measure of misalignment between the investor's optimal risk allocation, and the solution the manager chooses on investor's behalf. This misalignment could therefore be interpreted as the total cost of the contract, including cost of fees and the indirect alternative cost of adverse decision making. Furthermore, since this misalignment is obtained numerically, it is naturally dependent on a range of parameters and assumed contractual features. By constructing different scenarios, we illustrate and quantify how several realistic model extensions contribute in dampening the fluctuating risk-taking.

Although we do not suggest that previous research's concern of adverse risk-taking is unwarranted, our results imply that they may be somewhat exaggerated. We do admittedly confirm that there is a strong potential for adverse risk-misalignment inherent in the option-like compensation structure. However, by including a fairly uncontroversial number of five continuation periods at a relatively high discount rate of 25%, the misalignment in terms of certainty equivalents is more than halved. Consequently, there exists many ways to realistically reduce the displayed misalignment just by improving the model.

In this respect, we discover that the fund composition is highly relevant, and that diverse incentive contracts have an offsetting effect on each other, dampening the overall variations in risk. The implications of this is twofold: Firstly, viewing managerial pay-performance sensitivity to one single incentive option should be considered by academia as an oversimplification. Secondly, the benefits of assigning individual contracts, with maturities and benchmarks depending upon entry point, should be recognised by practitioners. Successfully incorporating a diverse portfolio of incentive options has the potential to relieve agency problems while still entitling the manager to a share of the profits. Investors should therefore not be indifferent to how they could benefit from each others presence, given that the fund issues separate contracts. We believe these results represent a solid foundation upon which future research should be conducted.

In a broader perspective it is clear that issuing options in order to increase performance comes at a price. Not only are they costly in that they directly redistributes wealth from investor to the manager, we also find potentially large incurred costs of the investor-manager risk misalignment. Put bluntly, the investor could end up paying extra for adverse performance. Previous literature highlights ownership share as an effective countermeasure, however we find its impact to diminish when accounting for long-term remuneration. Little suggests that the inefficiencies of the contract can be fully eradicated without completely removing the unsymmetrical incentive compensation. One would assume that doing so would seriously inflict managerial payoff, but in our model we find a potential for a Pareto improvement, where both manager and investor benefit from reduced misalignment. The reason for this is that when increasing contractual alignment, the marginal benefit for the investor is greater than the marginal loss of the manager, leaving a total surplus. By then increasing the management fee, some of the investor's gain is used to compensate for the manager's loss in incentive fee.

It should be noted that since our framework is very simplified with regards to managerial decision variables, our model may overlook positive implications of issuing greater incentives on performance. It could be argued that the incentive fee for instance induces motivation or attracts more skillful managers, resulting in higher expected returns - an effect not captured in our model. Therefore, although removing the fee has a profound positive effect in our model, there may exist adverse implications that is not covered by our framework. On the other hand, the conventional contract structure in the industry seems to rely more on industry habit and good faith rather than being solely based on scientific research. Goetzmann et al. (2003) even notes that the prevalence of the incentive contract might just be an accident of history, and that investing with a hedge fund manager would only appear to be rational given a large, positive risk-adjusted return. It therefore seems that a well performing manager is simply entitled to charge a large incentive fee, and that it is not the fee itself that makes him perform well.

Future research should in our opinion continue to challenge the status quo of hedge fund contracts, and in a broader perspective investigate whether option-like incentives really is an effective tool for relieving agency problems. More specifically, we suggest three extensions that could elevate the understanding of the field. First, we believe our investigation on fund composition shows great potential, and that future research should empirically study how contracts are issued and the implication on performance. This topic should also be further explored numerically, possibly with the purpose of thoroughly valuating the benefits of diverse incentive contracts. Second, the subject of utility maximisation for both investor and manager has only been superficially discussed in this paper. Preference to risk and possible differences between managers and investors are factors that could alter the results. Finally, further studies should consider whether there exists relevant decision variables or indirect parameters other than applied risk, that are sensitive to performance based incentives. This would provide further insight to whether option-like contracts also could portray aligning characteristics, or if they are in fact inherently unsuited in relieving agency problems.

A Appendix

A.1 Notation

Table A.1: Notation

N	-	Set of nodes
I	-	Set of investment pools
T	-	Set of evaluations in evaluation period
$n \in N$	-	Node n
$i \in I$	-	Investment pool i
$t \in T$	-	Time t
$X_{t,n}$	-	Fund value at time t and node n
$W_{i,n}$	-	Terminal manager wealth in node n relating to investment pool i
$U_{i,n}$	-	Terminal manager utility in node n relating to investment pool i
$W_{i,n}^{Con}$	-	Continuation wealth in node n relating to investment pool i
$W_{i,n}^{Ceq}$	-	Certainty equivalent in node n relating to investment pool i
Π	-	Certainty equivalent as a share of the initial investment
HWM_i	-	High water mark for investment pool i
w_i	-	Fund share of investment pool i
κ	-	Share of risky asset in portfolio

A.2 Parameters

Table A.2: Standard parameters

Standard Parameters		
Model Setup		
Time to maturity	T	1 year
Revisions in one period	τ	12
Time step	Δt	$\frac{1}{\tau} = 0.0833$
Initial fund value	X_0	1
Liquidation boundary	ψ	0.5
Log X step	C	$\frac{log(\frac{1}{0.5})}{108} = 0.0064$
Risk allocation, kappa	κ	[0: 0.01 :20]
Return and standard devation		
Risky asset, annual return	μ	0.0848
Risky asset, annual volatility	σ	0.0711
Risk free, annual return	r	0.0272
Discount rate	R	25%
Manager remuneration		
Managerial ownership	a	0
Management fee, annual rate	b	2%
Incentive fee	c	20%
CRRA Utility function		
Risk aversion coefficient	γ	4

A.3 Results

	$E[\Delta\kappa]$	Π_{inv}	Π_{man}
Merton	0	7.40 %	2.72 %
Base case (1 pool 1 period)	1.613	4.57 %	3.09 %
1 pool 2 periods	1.383	5.40 %	3.05 %
1 pool 3 periods	1.249	5.80 %	3.01 %
1 pool 4 periods	1.164	6.03 %	2.97 %
1 pool 5 periods	1.109	6.18 %	2.95 %
1 pool 6 periods	1.069	6.29 %	2.93 %
1 pool 7 periods	1.036	6.37 %	2.92 %
1 pool 8 periods	1.012	6.43 %	2.91 %
1 pool 9 periods	0.991	6.47 %	2.90 %
1 pool 10 periods	0.975	6.51 %	2.89 %
2 pools 10 periods	0.967	6.53 %	2.87 %
3 pools 10 periods	0.962	6.54 %	2.87 %
4 pools 10 periods	0.960	6.55 %	2.87 %
6 pools 10 periods	0.960	6.55 %	2.87 %
2 pools 10 periods, 10/90%	0.963	6.54 %	2.87 %
6 pools 10 periods, decreasing HWMs	0.413	7.21 %	2.78 %
6 pools 10 periods, increasing HWMs	1.169	6.20 %	2.78 %

Table A.3: Table of results for standard parameters

A.3.1 Changing Management Fee

$E[\Delta\kappa]$	0 %	0.50 %	1.00 %	1.50 %	2.00 %	2.50 %	3.00 %	3.50 %	4.00 %	4.50 %	5.00 %
1 pool, 1 period	2.695	2.170	1.923	1.748	1.613	1.504	1.413	1.334	1.266	1.206	1.152
1 pool, 5 period	2.730	1.822	1.479	1.263	1.109	0.992	0.898	0.822	0.758	0.704	0.657
1 pool, 10 period	2.743	1.684	1.334	1.122	0.975	0.865	0.779	0.710	0.653	0.605	0.565
6 pool, 10 period	2.645	1.679	1.323	1.108	0.960	0.850	0.764	0.695	0.638	0.591	0.550
6 pool, 10 period, decreasing HWM	2.546	0.820	0.588	0.480	0.413	0.368	0.334	0.308	0.287	0.270	0.256
6 pools, 10 period, increasing HWM	2.247	1.752	1.489	1.307	1.169	1.059	0.970	0.895	0.832	0.776	0.728

Table A.4: Expected kappa-difference, 20% incentive fee, changing management fee

Π_{inv}	0 %	0.50 %	1.00 %	1.50 %	2.00 %	2.50 %	3.00 %	3.50 %	4.00 %	4.50 %	5.00 %
1 pool, 1 period	3.24 %	4.60 %	4.81 %	4.76 %	4.57 %	4.30 %	3.98 %	3.62 %	3.22 %	2.80 %	2.37 %
1 pool, 5 period	3.18 %	6.03 %	6.44 %	6.41 %	6.18 %	5.85 %	5.46 %	5.03 %	4.56 %	4.08 %	3.59 %
1 pool, 10 period	3.13 %	6.51 %	6.88 %	6.79 %	6.51 %	6.13 %	5.70 %	5.24 %	4.75 %	4.25 %	3.74 %
6 pools, 10 period	3.41 %	6.54 %	6.92 %	6.83 %	6.55 %	6.17 %	5.74 %	5.27 %	4.78 %	4.28 %	3.76 %
6 pools, 10 period, decreasing HWM	3.68 %	8.34 %	8.13 %	7.70 %	7.21 %	6.70 %	6.16 %	5.63 %	5.08 %	4.53 %	3.99 %
6 pools, 10 period, increasing HWM	5.16 %	6.34 %	6.54 %	6.45 %	6.20 %	5.87 %	5.48 %	5.05 %	4.59 %	4.11 %	3.62 %
Merton (w/ fees)	9.63 %	9.07 %	8.51 %	7.96 %	7.40 %	6.84 %	6.28 %	5.72 %	5.17 %	4.61 %	4.05 %

Table A.5: Investor certainty equivalent, 20% incentive fee, changing management fee

Π_{mng}	0 %	0.50 %	1.00 %	1.50 %	2.00 %	2.50 %	3.00 %	3.50 %	4.00 %	4.50 %	5.00 %
1 pool, 1 period	0.04 %	1.00 %	1.75 %	2.43 %	3.09 %	3.72 %	4.35 %	4.96 %	5.56 %	6.16 %	6.76 %
1 pool, 5 period	0.03 %	0.90 %	1.62 %	2.30 %	2.95 %	3.58 %	4.20 %	4.81 %	5.42 %	6.02 %	6.61 %
1 pool, 10 period	0.02 %	0.85 %	1.57 %	2.24 %	2.89 %	3.52 %	4.14 %	4.76 %	5.36 %	5.96 %	6.56 %
6 pools, 10 period	0.02 %	0.84 %	1.55 %	2.22 %	2.87 %	3.50 %	4.13 %	4.74 %	5.35 %	5.95 %	6.54 %
6 pools, 10 period, decreasing HWM	0.03 %	0.76 %	1.45 %	2.12 %	2.78 %	3.41 %	4.04 %	4.66 %	5.27 %	5.88 %	6.48 %
6 pools, 10 period, increasing HWM	0.02 %	0.75 %	1.45 %	2.12 %	2.78 %	3.41 %	4.04 %	4.66 %	5.27 %	5.87 %	6.48 %

Table A.6: Manager certainty equivalent, 20% incentive fee, changing management fee

A.3.2 Changing Incentive Fee

$E[\Delta\kappa]$	0 %	5.00 %	10.00 %	15.00 %	20.00 %	25.00 %	30.00 %
1 pool, 1 period	0.000	0.918	1.266	1.472	1.613	1.719	1.801
1 pool, 5 period	0.047	0.448	0.758	0.958	1.109	1.228	1.327
1 pool, 10 period	0.075	0.407	0.653	0.834	0.975	1.089	1.183
6 pools, 10 period	0.067	0.394	0.638	0.818	0.960	1.074	1.170
6 pools, 10 period, decreasing HWM	0.092	0.203	0.287	0.355	0.413	0.464	0.509
6 pools, 10 period, increasing HWM	0.024	0.534	0.832	1.028	1.169	1.276	1.362

Table A.7: Expected kappa-difference, 2% management fee, changing incentive fee

Π_{inv}	0 %	5.00 %	10.00 %	15.00 %	20.00 %	25.00 %	30.00 %
1 pool, 1 period	9.30 %	7.66 %	6.34 %	5.35 %	4.57 %	3.93 %	3.39 %
1 pool, 5 period	9.29 %	8.62 %	7.74 %	6.93 %	6.18 %	5.51 %	4.90 %
1 pool, 10 period	9.28 %	8.66 %	7.93 %	7.20 %	6.51 %	5.86 %	5.25 %
6 pools, 10 period	9.28 %	8.68 %	7.96 %	7.23 %	6.55 %	5.90 %	5.29 %
6 pools, 10 period, decreasing HWM	9.28 %	8.80 %	8.29 %	7.76 %	7.21 %	6.66 %	6.10 %
6 pools, 10 period, increasing HWM	9.29 %	8.53 %	7.67 %	6.89 %	6.20 %	5.59 %	5.03 %
Merton (w/ fees)	9.30 %	8.85 %	8.38 %	7.89 %	7.40 %	6.89 %	6.36 %

Table A.8: Investor certainty equivalent, 2% management fee, changing incentive fee

Π_{man}	0 %	5.00 %	10.00 %	15.00 %	20.00 %	25.00 %	30.00 %
1 pool, 1 period	2.23 %	2.56 %	2.78 %	2.95 %	3.09 %	3.21 %	3.31 %
1 pool, 5 period	2.23 %	2.53 %	2.71 %	2.84 %	2.95 %	3.04 %	3.12 %
1 pool, 10 period	2.23 %	2.52 %	2.68 %	2.80 %	2.89 %	2.97 %	3.03 %
6 pools, 10 period	2.23 %	2.52 %	2.67 %	2.78 %	2.87 %	2.94 %	3.00 %
6 pools, 10 period, decreasing HWM	2.23 %	2.50 %	2.64 %	2.72 %	2.78 %	2.82 %	2.85 %
6 pools, 10 period, increasing HWM	2.23 %	2.50 %	2.63 %	2.72 %	2.78 %	2.82 %	2.85 %

Table A.9: Manager certainty equivalent, 2% management fee, changing incentive fee

A.3.3 Changing Ownership Share

$E[\Delta\kappa]$	0 %	5.00 %	10.00 %	15.00 %	20.00 %	25.00 %	30.00 %
1 pool, 1 period	1.613	0.964	0.692	0.531	0.423	0.343	0.284
1 pool, 5 period	1.109	0.971	0.860	0.772	0.699	0.639	0.589
1 pool, 10 period	0.975	0.933	0.896	0.860	0.826	0.796	0.768
6 pools, 10 period	0.960	0.909	0.865	0.825	0.788	0.756	0.726
6 pools, 10 period, decreasing HWM	0.413	0.467	0.479	0.478	0.471	0.463	0.454
6 pools, 10 period, increasing HWM	1.169	1.090	1.032	0.982	0.938	0.899	0.864

Table A.10: Expected kappa-difference, 20% incentive fee, 2% management fee, changing ownership share

Π_{inv}	0 %	5.00 %	10.00 %	15.00 %	20.00 %	25.00 %	30.00 %
1 pool, 1 period	4.57 %	6.06 %	6.56 %	6.82 %	6.98 %	7.08 %	7.16 %
1 pool, 5 period	6.18 %	6.47 %	6.66 %	6.79 %	6.89 %	6.97 %	7.03 %
1 pool, 10 period	6.51 %	6.59 %	6.65 %	6.71 %	6.76 %	6.81 %	6.85 %
6 pools, 10 period	6.55 %	6.64 %	6.72 %	6.78 %	6.83 %	6.88 %	6.92 %
6 pools, 10 period, decreasing HWM	7.21 %	7.19 %	7.18 %	7.19 %	7.19 %	7.20 %	7.21 %
6 pools, 10 period, increasing HWM	6.20 %	6.36 %	6.47 %	6.56 %	6.63 %	6.70 %	6.75 %
Merton (w/ fees)	7.40 %	7.40 %	7.40 %	7.40 %	7.40 %	7.40 %	7.40 %

Table A.11: Investor certainty equivalent, 20% incentive fee, 2% management fee, changing ownership share

Π_{man}	0 %	5 %	10 %	15 %	20 %	25 %	30 %
1 pool, 1 period	3.09 %	2.96 %	2.89 %	2.84 %	2.82 %	2.80 %	2.78 %
1 pool, 5 period	2.95 %	2.91 %	2.89 %	2.87 %	2.86 %	2.84 %	2.83 %
1 pool, 10 period	2.89 %	2.88 %	2.87 %	2.86 %	2.85 %	2.84 %	2.84 %
6 pools, 10 period	2.87 %	2.86 %	2.85 %	2.84 %	2.83 %	2.82 %	2.82 %
6 pools, 10 period, decreasing HWM	2.78 %	2.78 %	2.78 %	2.78 %	2.78 %	2.77 %	2.77 %
6 pools, 10 period, increasing HWM	2.78 %	2.78 %	2.79 %	2.79 %	2.79 %	2.79 %	2.79 %

Table A.12: Manager certainty equivalent, 20% incentive fee, 2% management fee, changing ownership share

A.3.4 Changing Hurdle Rate

$E[\Delta\kappa]$	r	$1.5r$	$2r$	$2.5r$	$3r$	$3.5r$	$4r$	$4.5r$	$5r$
1 pool, 1 period	1.613	1.525	1.444	1.369	1.301	1.238	1.183	1.132	1.101
1 pool, 5 period	1.109	1.001	0.904	0.817	0.739	0.672	0.613	0.562	0.521
1 pool, 10 period	0.975	0.883	0.800	0.727	0.662	0.604	0.554	0.508	0.469
6 pools, 10 period	0.960	0.905	0.853	0.804	0.757	0.712	0.669	0.629	0.591
6 pools, 10 period, decreasing HWM	0.413	0.380	0.349	0.320	0.293	0.269	0.249	0.229	0.212
6 pools, 10 period, increasing HWM	1.169	1.165	1.160	1.154	1.148	1.141	1.135	1.127	1.118

Table A.13: Expected kappa-difference, 20% incentive fee, 2% management fee, changing hurdle rate

Π_{inv}	r	$1.5r$	$2r$	$2.5r$	$3r$	$3.5r$	$4r$	$4.5r$	$5r$
1 pool, 1 period	4.57 %	4.93 %	5.26 %	5.55 %	5.82 %	6.07 %	6.29 %	6.49 %	6.68 %
1 pool, 5 period	6.18 %	6.53 %	6.82 %	7.07 %	7.28 %	7.46 %	7.62 %	7.76 %	7.87 %
1 pool, 10 period	6.51 %	6.80 %	7.06 %	7.27 %	7.46 %	7.62 %	7.75 %	7.88 %	8.00 %
6 pools, 10 period	6.55 %	6.80 %	7.03 %	7.24 %	7.44 %	7.61 %	7.77 %	7.92 %	8.06 %
6 pools, 10 period, decreasing HWM	7.21 %	7.39 %	7.56 %	7.71 %	7.86 %	7.99 %	8.11 %	8.22 %	8.33 %
6 pools, 10 period, increasing HWM	6.20 %	6.39 %	6.57 %	6.73 %	6.89 %	7.03 %	7.17 %	7.29 %	7.40 %
Merton (w/ fees)	7.40 %	7.54 %	7.68 %	7.81 %	7.93 %	8.05 %	8.16 %	8.27 %	8.36 %

Table A.14: Investor certainty equivalent, 20% incentive fee, 2% management fee, changing hurdle rate

Π_{mng}	r	$1.5r$	$2r$	$2.5r$	$3r$	$3.5r$	$4r$	$4.5r$	$5r$
1 pool, 1 period	2.89 %	2.81 %	2.75 %	2.69 %	2.64 %	2.60 %	2.57 %	2.53 %	2.50 %
1 pool, 5 period	2.95 %	2.87 %	2.80 %	2.74 %	2.68 %	2.64 %	2.60 %	2.56 %	2.53 %
1 pool, 10 period	2.89 %	2.81 %	2.75 %	2.69 %	2.64 %	2.60 %	2.57 %	2.53 %	2.50 %
6 pools, 10 period	2.87 %	2.80 %	2.74 %	2.68 %	2.64 %	2.59 %	2.56 %	2.52 %	2.49 %
6 pools, 10 period, decreasing HWM	2.78 %	2.72 %	2.67 %	2.63 %	2.59 %	2.55 %	2.52 %	2.49 %	2.46 %
6 pools, 10 period, increasing HWM	2.78 %	2.70 %	2.64 %	2.58 %	2.53 %	2.49 %	2.45 %	2.41 %	2.38 %

Table A.15: Manager certainty equivalent, 20% incentive fee, 2% management fee, changing hurdle rate

A.4 Robustness of Discount Rate

R	5 %	10 %	15 %	20 %	25 %	30 %	35 %	40 %
$E[\Delta\kappa]$	0.794	0.845	0.892	0.935	0.975	1.010	1.042	1.070
Π_{inv}	6.85 %	6.77 %	6.68 %	6.59 %	6.51 %	6.42 %	6.34 %	6.27 %
Π_{mng}	2.80 %	2.83 %	2.85 %	2.87 %	2.89 %	2.91 %	2.92 %	2.94 %

Table A.16: How discount rate affect metrics (1 pool, 10 periods)

R	5 %	10 %	15 %	20 %	25 %	30 %	35 %	40 %
$E[\Delta\kappa]$	0.785	0.838	0.884	0.924	0.960	0.992	1.020	1.046
Π_{inv}	6.87 %	6.78 %	6.70 %	6.62 %	6.55 %	6.47 %	6.40 %	6.33 %
Π_{mng}	2.81 %	2.82 %	2.84 %	2.86 %	2.87 %	2.88 %	2.90 %	2.91 %

Table A.17: How discount rate affect metrics (6 pools, 10 periods)

R	5 %	10 %	15 %	20 %	25 %	30 %	35 %	40 %
$E[\Delta\kappa]$	0.639	0.700	0.759	0.814	0.865	0.911	0.954	0.991
Π_{inv}	7.03 %	6.95 %	6.85 %	6.75 %	6.65 %	6.55 %	6.45 %	6.36 %
Π_{mng}	2.77 %	2.80 %	2.82 %	2.84 %	2.87 %	2.89 %	2.91 %	2.92 %

Table A.18: How discount rate affect metrics (1 pool, 10 periods, 10% manager ownership)

R	5 %	10 %	15 %	20 %	25 %	30 %	35 %	40 %
$E[\Delta\kappa]$	0.639	0.700	0.759	0.814	0.865	0.911	0.954	0.991
Π_{inv}	7.05 %	6.97 %	6.89 %	6.81 %	6.72 %	6.63 %	6.54 %	6.45 %
Π_{mng}	2.77 %	2.79 %	2.81 %	2.83 %	2.85 %	2.86 %	2.88 %	2.89 %

Table A.19: How discount rate affect metrics (6 pools, 10 periods, 10% manager ownership)

A.5 Elaboration on Multi-period Modelling

Due to the path dependency that occurs with the high-water mark mechanism we introduce a multi-period model similar to that of Hodder and Jackwerth (2007). If we were to model multi-periods with a traditional recursive dynamic programming method, the model could easily become too complex to run as the number of periods included increase. Each period would have to be run for all possible HWMs and initial fund values. The number of possible states at entry of a period is given by,

$$M\left(N+\frac{1-M}{2}\right), \tag{A.5.1}$$

where N and M are the total number of possible fund values and HWMs respectively. This run-time multiple is linearly increasing with the number of periods, but as explained in section 5 we also include multiple investment pools to the model. The multiple is then raised to the power of investment pools added, which makes the traditional dynamic programming approach unsuitable for our purpose. With our approach this run-time multiple (A.5.1) is reduced to one. The trade-off is the inaccuracies that incur when approximating the continuation values.

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