An Analysis of the Swedish-Norwegian Electricity Certificate Market

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Problem statement

This thesis aims to model and analyze the Swedish-Norwegian Electricity Certificate Market. A stochastic model based on dynamic programming is implemented to assess the performance of this market and to forecast certificate prices. By looking into the implications of regulatory changes to the market structure, this work aims to contribute to the ongoing debate on the design of energy policies.
Preface

One of the greatest challenges of our time are the threats posed to earth’s ecosystems by human activity. Over the last 50 years the world population has nearly tripled, increasing from 2.5 to nearly 6.5 billion. During this period we have seen the greatest loss of biodiversity in human history while the atmospheric carbon dioxide concentration has increased to levels not seen on earth for 600,000 years. Scientists relate these levels to an increase in the global temperature levels and to more intense weather conditions. This change in atmospheric composition is caused mainly by the burning of fossil fuels.

In order to address these issues, a variety of environmental policies have been proposed. Traditional environmental policies have usually involved providing subsidies, or imposing obligations on market actors. However, over the last decades, several countries have introduced market-based support schemes favoring investments in renewable energy. Tradable Green Certificate (TGC) markets is an example of such a market-based energy policy which incentivize investments in renewable energy.

For this Master’s thesis, we implement a proposed a model for such a market, and discuss the effects of expectations and uncertainty on the performance of these support schemes. The purpose is to give a contribution to the ongoing debate on the design of environmental policies. Hopefully the thesis can lead to a better understanding of the mechanisms in these markets and ultimately to an increase in the deployment of renewable capacity.

We would like to acknowledge and extend our heartfelt gratitude to the following persons who have made the completion of this Master’s thesis possible: Our supervisor, Professor Stein Erik Fleten, for his vital encouragement and support throughout the project. We also wish to thank Ove Wolfgang for useful guidance and insightful questions during the process. Finally, we would like to thank our families, who have supported us every step of the way.

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Abstract

This thesis explores tradable green certificate markets, a support scheme for investments in renewable electricity production. A case-study on the Swedish-Norwegian Electricity Certificate market is conducted. In this market producers investing in new renewable capacity receive certificates based on their actual production. These are sold to retailers of electricity, which are required to buy certificates for a proportion of their total sales. The design and potential success of this multistate support mechanism is of great interest to policy makers and green investors. A stochastic model based on dynamic programming is implemented to assess the performance of this market. A discussion on market dynamics, uncertainty and expectations forms the basis for the model choices. The main findings from the model include i) Price expectations are not formed easily, and hence, investment decisions have to be made under a lot of uncertainty, ii) Under the current market structure, prices are expected to start at today’s level, while decreasing steadily towards zero in 2035 when the market is planned to end, iii) The prices are highly sensitive to changes in electricity consumption and generation of electricity, iv) Regulatory changes should be implemented carefully to avoid increased uncertainty and a consequent increase in price volatility and v) The price-based penalty further increases the volatility of the market. So far, the market has shown its ability to promote investments in renewable electricity production, however improvements of the market seem beneficial.

Summary in Norwegian

1 Introduction

Over the past years, an increased focus on emission reductions has led to the
development of a variety of energy policies penalizing emissive and promoting
clean industries. The tradable green certificate (TGC) market addressed in
this paper is an example of a market-based policy promoting electricity from
renewable energy sources. Lately, there has been considerable interest in such
markets, as several countries have launched these in attempts to implement
more cost-effective alternatives to the traditionally used Feed-In-Tariff regimes.

The main research question addressed in this thesis is:

How will the Swedish-Norwegian Electricity Certificate Market perform under
the current regulation, and how will it react to changes in regulation?

A series of sub-questions follow:

1. How are expectations of prices and penalties formed, and what are the
   rationales behind an investment decision?

2. What is the effects of uncertainty?

3. How can the regulations be changed to improve the performance of the
   market?

The main contribution of this Master’s thesis is the article, "A Dynamic
Model of the Swedish-Norwegian Electricity Market" [1]. It proposes a stochastic
dynamic programming approach to modelling the Swedish-Norwegian elec-
tricity certificate market by extending the work of Coulon et al. [2]. This type
of model has not been explored for this market before. A review of relevant
literature and a discussion on market dynamics, uncertainty and expectations
forms the basis for the model choices. Running the model for different scenarios,
it is shown that the model captures the market dynamics well. Policy changes
suggested by regulators’ for the upcoming market progress review are analyzed
and discussed. Additionally, several alternatives for the investment decision ra-
nionale, as well as different mechanisms for forming penalty fee expectations, is
explored. The article has been prepared for submission to a scientific journal.

The second article "Renewable portfolio standards, Tradable green certifi-
cates, Energy policy, Dynamic equilibrium, Market analysis" [3], were the au-
thors’ contribution to the EEM15 conference. The conference was held in Lis-
bon, where the article content was presented. The article provides a brief in-
troduction to the Swedish-Norwegian electricity certificate market along with a
case study adapting the model by Coulon et al. [2] to this market. The article
will be published in the IEEE Xplore database.

This thesis aims to contribute to a better understanding of the dynamics in
a TGC market. The model of Coulon et al. is extended by adding the penalty
fee as a state variable and including relationships between investments, the
electricity market and the Levelized Cost of Energy, \(LCOE\). Further, it in-
corporates all the important features in a TGC market, such as the investment
rate, the requirement level, the penalty level and a stochastic production of eligible electricity. By providing a better understanding of the formation of price expectations, the investment rationale and the role of uncertainty, the paper contributes to the debate on design of energy policies and to improvements in market efficiency. The proposed model can also be a supplemental tool for market actors and regulators.

Additional chapters, have been added to the thesis to provide the reader with background information not included in the main articles. The second chapter gives an introduction to the features of market-based energy policies, and discusses these in relation to more traditional subsidy and tax policies. Chapter 3 presents different methods for modeling such markets, while the final chapter discusses the modeling choices.

2 Market-Based Environmental Policies

Harnessing the power of market forces in the design of environmental policies is not a new idea. For a properly designed and implemented market-based policy, environmental-friendly behavior is encouraged through price signals rather than explicit taxes or subsidies. Tradable green certificate (TGC) markets and emission trading systems (ETS) are examples of such market-based policies used as alternatives to traditional renewable energy subsidies, often called Feed-In-Tariffs (FIT), and carbon taxes. TGC markets puts a lower bound on the quantity of electricity that must come from renewable sources. Emission trading systems puts an upper bound on the quantity of allowed emissions. The market then decides the price on renewable sources and emissions. These systems are often called cap and trade markets, as the government decides the quantity, while the price is determined in the market.

Though such market mechanisms have been in place for several years, their success has varied, and there is still ongoing debate on whether these policies yields the desired effects. Several papers have assessed the performance of TGC markets compared to FIT regimes. In theory, a TGC market would stimulate investments in renewable energy capacity at the lowest possible social cost, as the market prices the additional renewable electricity capacity correctly. Several European countries have however implemented successful FIT regimes at a modest social cost. Haas et al. [4] finds that TGC systems show to be less effective with respect to deployment of less mature renewable energy technologies. The intrinsic stability of FIT systems appears to be a key element in the success of these policies. According to Haas et al., a well-designed (dynamic) FIT system will provide a certain deployment of renewable electricity in a shorter time and at the lowest social cost. Midttun & Gautesen [5] argue that FIT regimes and TGC markets should be seen as complementary regulatory instruments targeting subsequent steps in the product cycle, on the way from early technology-conceptualisation and development towards competitive positioning in mature energy markets. Bergek and Jacobsson [6] find supporting arguments from an empirical study on the Swedish TGC market. They conclude that the TGC system reaches its target at a low cost for society, but at a higher cost for
consumers than necessary and with little incentive for technology development.
As of this moment it does not seem like any of the two policies have emerged as
the better alternative. In theory, a TGC policy has a lower social cost, while a
FIT regime provides less uncertainty to investors and provides stronger incen-
tives for technology change.

A TGC market can be implemented both for a single country, and as a
harmonized policy spanning a region of countries. In the report "European
RES-E Policy Analysis"[7] it is found that a EU-wide harmonized TGC market
will lower the total costs of meeting the EU renewable target. Morthorst [8]
argues that an international TGC market is not recommended if the TGC-
market is implemented with the goal of achieving the national CO2-reduction
targets. Even the most ambitious countries with regards to increasing their
TGC-quotas will only partly be gaining the CO2-reduction benefits themselves,
as the additional capacity may be built in other countries. Only when a green
certificate market is combined with a tradable permit scheme, the trade in
certificates is equivalent to the domestic development of renewables [9]. On the
other hand, del Rio [10] argues that if the policy priority is an increased share of
RES-E, the minimum costs harmonization should be favored by national energy
authorities. Then the total social costs are reduced, as the most profitable
investments are taken independent of borders.

3 Methods for Modeling an Electricity Certificate Market

Several organizations are currently working on models for the Swedish-Norwegian
market for electricity certificates. Best practices for modeling the market has
not yet been established, as can be seen from the several different approaches
that are being taken. This chapter aims to give an overview of the various
methods and a brief introduction to some of the existing models. Methods for
modeling a certificate market can be separated into two types, analytical and
numerical. The analytical models can provide a basic understanding of the mar-
et and some simple relations and results. An example of an analytical model is
described in the paper of Amundsen and Mortensen, on the effects from Green
Certificates and CO2-emission permits for the Danish market [11]. The model
is used to investigate the effects of several factors believed to impact the power
grid and its actors. They find that more rigorous CO2 constraints, as well as an
increased price for import wholesale electricity, may lead to a reduced capacity
for renewable energy production. Further, they are not able to conclude on
whether an increase in certificate requirements leads to a change in the capacity
of renewable energy, but they find that a higher share of consumption will stem
from renewables, as the total electricity consumption is reduced. Another exam-
ple is the analytical model described in the paper of Morthorst from 2003 where
he concludes that the long term certificate price is formed such that the sum
of the electricity spot price and the certificate price should equal the long run
marginal cost of investing in new renewable capacity [8]. Generally, analytical
models can be a good starting point before providing more advanced models.
From this point on, this text will focus on numerical models, enabling detailed
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Simulations of the market and its behavior. Numerical models can be divided into four main categories, discussed in the subsequent sections.

3.1 Fundamental Equilibrium

Fundamental equilibrium models formulate the market equilibrium models as large optimization problems, minimizing or maximizing an objective function subject to a set of constraints. Estimates for future prices are then obtained by solving the model for market equilibrium. Such models require perfect competition and rational behavior from all players in the market. Modeling supply and demand curves as step-functions yields LP-problems; this subsequently helps lowering complexity and hence lowers the runtime. Equilibrium models can both be static, considering only one time period, and dynamic, with relations between several time periods e.g. inventory equations for storage of certificates and energy over time.

A fundamental equilibrium model can be found in the paper of Unger and Ahlgren from 2005. They apply the Markal model-generator [12] for the energy systems of Denmark, Finland, Sweden and Norway. They maximize the objective function, chosen to be the sum of consumer and producer surplus. Among their findings is that the introduction of TGC quotas reduces wholesale electricity prices, while retail prices can be higher or lower depending on the TGC quota [13].

Another example of a fundamental partial equilibrium model is the recently developed model by Wolfgang & Jaehnert of the Swedish-Norwegian Electricity Certificate market [14]. The model is an extension of the EMPS model, a partial model for electricity markets, used by producers, regulators and system operators throughout Scandinavia. The extension introduces electricity certificates to this model. A price of certificates is calculated through stochastic dynamic programming with weekly time resolution for the remainder of the system lifetime and endogenously determined penalty rates. The strategy calculations are done for several fixed penalty rates. This is done to avoid introducing another state variable in this part of the solution algorithm, for the sake of computational tractability. Further, the expected penalty rate is calculated as an endogenous variable in the formal LP part of the model. The value obtained for the penalty rate is then used to choose the correct strategy, by interpolating between the fixed penalty rates used in the strategy calculation. This seems to be a well-functioning method. By using results from the EMPS model, the dependency between the electricity market and investments in additional capacity and production levels are directly included. They find the results to be consistent with other studies on similar TGC markets, i.e. in particular that “price-scenarios spread out such that the unconditional expected price of certificates is relatively stable in the planning period.” [14]

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1 An example of such optimization problems could be to minimize the total system costs subject to an inventory constraint and a requirement constraint.
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3.2 Agent Based Models
Agent based models aim to simulate the behavior and interaction between players in a system. Every individual agent is acting according to their observed situation based on a set of specified rules. Making these agents act realistically requires detailed information regarding behavioral pattern of each player type. This information is typically gathered by interviewing actual market actors. Once the agents are programmed, the market can be simulated. The simulation continues until it reaches a steady state. Such models allow for analysis of interaction between market actors. From the steady state situation, an equilibrium price can be obtained. Unlike the fundamental models, where rational behavior and perfect competition is assumed, this approach allows for the introduction of irrational behavior. Another positive feature is the possibility of implementing a form of learning process into the agents. Agents that performed well in the first period would want to act similar in the next, while agents that did poorly would try to adjust their behavior, hoping to improve their performance. The core strengths of these models include the ability to handle market power and imperfections more easily. A disadvantage of such an approach is that tracking the dependency between causes and effects could prove difficult.

An example of an agent based model can be found in the paper of Aune et al. from 2012 [15]. They analyze the potential for cost reductions by allowing for trade in green certificates across member states. Their findings indicate that EU-wide trade in green certificates may cut the EU’s total cost of fulfilling the renewable target by as much as 70 percent compared to a situation with no trade.\(^2\)

3.3 Statistical Approaches
Statistical models are purely based on historical price data. They assume that what will happen in the future is reflected in history. From these historical data, parameters can be obtained to calibrate a parametric model. Exogenous variables can still be added to explain expected future changes. These systems has to be static, i.e. there are no dynamic relations between current and future model parameters.

An example of a statistical model can be found in the paper of Fagiani and Hakvoort from 2014 [16]. They use a GARCH model to analyze the causes of volatility and the effects of volatility on certificate prices. From their results it can be seen that regulatory changes strongly affect certificate markets, resulting in periods of higher volatility. During such periods of higher volatility, investors will require a higher rate of return. More specifically, they analyze whether certificate price volatility has changed after the creation of a joint Swedish-Norwegian electricity certificate market. Results indicate that the news about this extended market led to a period of increased price volatility between 2010

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\(^2\) An agent based model for the Swedish-Norwegian Electricity Certificate market has recently been developed. The project was led by the Norwegian companies Optimeering and Thema Consulting and was financed by the Norwegian Research Council. They have interviewed several market players, and are modeling the different types of players’ behavior to be able to predict prices and identify potential market improvements.
and 2011. They also note that this effect is not fully reversed, and that the market is still more volatile than before the change. This effect may be reversed as the market matures and quotas increase. However, a question raised by Fagiani and Hakvoort, is whether a larger market provides more stable prices.

3.4 System Dynamics

System dynamics modeling is an approach to understanding the behavior of complex systems over time. Such models are comprised of a coupled set of nonlinear differential equations, describing relations between actors, feedback loops and time delays. Further, the set of differential equations is used to simulate the system behavior.

An example of a system dynamics model can be found in the paper by A. Ford et al. from 2007 [17]. They simulate the price dynamics of a TGC market designed to support an aggressive mandate for wind generation in the northwestern USA. Their results indicate that the certificate prices climb rapidly to the penalty level in the early years of the market. This is met with increased investments in wind generation capacity, forcing certificate prices back down. Prices then stabilize (though smaller fluctuations still occur). The same results are obtained for simulations with different values for the uncertain parameters, such as price elasticity, penalty level and lead-time on investments. The results are consistent with actual price data from the early years of the Swedish electricity certificate market, which started in 2003.

3.5 Choice of Method

The model described in the first article is a fundamental model, with some aspects from system dynamics. It is an extended version of the work by Coulon et al. [2] for the SREC market in New Jersey. Our extended version is based on dynamic programming, with balance of certificates, annual eligible electricity production and penalty fee as state variables and with stochastic terms on the monthly electricity price and monthly production of eligible electricity. The model is solved for the certificate price for every combination of the state variables with a monthly time resolution by applying a backward recursion algorithm. A detailed explanation of the model is found in chapter four of the article. The main reasons for the choice of model are listed below:

1. The model by Coulon et al. showed promising results for the New Jersey SREC market.
2. An approach based on dynamic programming has not been implemented for Swedish-Norwegian Electricity Certificate Market before.
3. A fundamental model allows for tracking of dependencies between causes and effects.
4. The dynamic programming approach allows the modeling of complex relations between time-steps.
5. The model allows for the testing of multiple scenarios.

The main differences between the model described here and and the model of Coulon et al. are the inclusion of the penalty fee as an additional state variable, the integration of a dependency between the electricity price and the lifetime costs on investment and the possibility of infinite banking.

4 Evaluation of choices

Model building can be considered a simplification of reality. While addressing complex problems, identifying which are the important relationships that must be maintained and which are the ones that can be ignored or simplified is of the utmost importance. This discussion outlines which relationships were included in the electricity certificate model, how they were included and which relationships were left out. It serves as a supplement to the information presented in chapter 2 of the first article.

4.1 Included Relations

The price relationship is modeled such that the certificate price should equal the discounted value of the expected penalty times the probability of having to pay this penalty. This relation is chosen, as it is expected to best capture the market dynamics throughout the market lifetime. After an investment has been made, the short-term marginal cost of certificates is approximately zero. Hence, suppliers of certificates could some times be willing to sell at substantially lower prices than the long-term price required by investors to make an investment decision. This captures the effects from shifts in the certificate supply and demand curve. On the other hand, effects caused by a changed probability for having to pay the penalty may be under-estimated if the actors’ view on the probability for penalty does not change accordingly, following an unexpected shift in the supply or demand for certificates. It is noted that having one or a few of the actors in the market acting according to the assumed relation would suffice for the relation to hold. Further, in a scenario where the price suggested by the relation is lower than the level required for the marginal plant to be profitable, a possible outcome could be that no producers are willing to sell their certificates at this lower level. This would however be an inefficient market solution, which is not expected for liquid spot and futures markets.

The issuance relation describes the monthly eligible electricity production, i.e. issuance of certificates, and is modeled so that there is both seasonal and annual variance. This captures the fluctuating nature of generation from renewable energy sources, especially from wind energy, resulting from changing weather conditions. Since the annual issuance of certificates is a function of annual certificate-eligible generation, and since the requirement imposed on retailers is fulfilled once every year, long-term fluctuations are of special interest. Seasonal variance is not expected to affect the certificate price to any great extent. The magnitude of the annual variance is uncertain and will depend largely on the technology composition among certificate-eligible power plants. The variance
assumption is based on the regression of the historical issuance of certificates, and thus, represents the historical technology composition. This is assessed to be adequately precise, as the technology composition is only expected to experience small changes. To obtain more precise estimates of the annual variance, the technology composition of new investments could have been modeled endogenously. This is left out in order to maintain computational tractability and since the regression is believed to yield an adequate estimate.

The investment relation is modeled as a combination of a time-dependent term and a term depending on the basic profit equation for a power plant, i.e. the sum of the expected certificate and electricity price less the Levelized Cost of Energy of the investment opportunity. The time dependent investments are included as several power plants have already been planned and found profitable within a reasonable price range. At the same time, some investors are still considering new investments. These are expected to delay their decision until the certificate and electricity price is at a level that make their investment opportunity profitable. This will also take the lowered price expectations resulting from an increase in supply into account, when considering new investments. From the regression on historical prices and issuance it is found that there is very little or no correlation between investments and the profit equation. It is difficult to conclude on whether this mechanism is significant. There are substantial lead-times on new investments, which makes it hard to track the cause and effect for the investment-decision. In addition, investors may apply varying methods of forecasting the certificate and electricity price. In spite of this, it is natural to assume that there exists some degree of correlation, as a rational investor would require a positive estimation of future cash flows. A potential shortcoming of the model, could be, the implementation of wrong assumptions on the investment rationale. Further research would be needed to investigate this correlation.

The penalty equation is modeled so that the expected penalty is a function of all known information for the given month, i.e. the expected penalty equals 1.5 times the weighted average of the current price and the monthly prices observed since the last compliance date. An exception has been made for the first month following a compliance date, where the expected penalty is the average of 1.5 times the current price and the actual observed penalty from the previous year. Hence, stable price expectations are assumed in the formation of penalty fee expectations. An alternative form, where the prices over the remainder of the year is assumed equal to the price in the current time-step is also tested. This does not cause noticeable changes in the resulting forecasts.

The inclusion of the penalty fee as a state variable was made feasible due to the substantial improvements in run-time, described in Appendix A. Utilizing methods from approximate dynamic programming, or simulation of the penalty fee outside the model were among the alternatives considered. The first is difficult for the given approach, since this does not include an objective function. The latter was conducted inspired by the successful procedure described by Wolfgang and Jaehner [14]. The procedure showed promising results. The inclusion of an additional state variable is however considered to be the best alternative, as long as the computational tractability of the model is preserved.
4 EVALUATION OF CHOICES

4.2 Omitted Relations

This section discusses relations and dynamics that have been considered implemented but have been left out or modeled differently. Complexity and computational run-time have been essential factors in deciding whether to include, or leave out a relation from the model. The curse of dimensionality is one of the main challenges in dynamic programming [18]. Thus it is important to limit the number of state variables in order to preserve computational tractability.

The magnitude of the expected annual variation in electricity production is different for different renewable technologies. Hence, investors may have different rationales for investment in hydro, bio, wind or solar PV. The model does however, not distinguish between these different technologies. Renewables like bio and reservoir hydro always face an option of whether to produce or not and could have a stabilizing effect on the market. Hence, differentiating between technologies could have further refined the model. This is however not expected to have a large impact on the model output as these sources accounts for a small share of the total market. Furthermore, the inclusion of this feature would require an extensive qualitative study among potential investors and would also increase run-time.

The timing of investment decides when the power plant will be phased out from the certificate market. However, for this model, the phase-out of eligible capacity is not updated endogenously. This is done exogenously prior to each model run. Since the investment decision is modeled endogenously, this causes a potential mismatch between the time an investment’s capacity is phased in and the time it is phased out. Given the backwards recursive nature of the solution algorithm, this issue is not easily addressed, since the events are separated in time, and since the model is solved for the time step of the phase-out before it is solved for the time step when the investment decision is made. However, the effects of this mismatch should not be severe, as the endogenous timing of investments does not deviate substantially from the estimates made exogenously.

All new investments cause a positive shift in the supply of electricity, which consequently causes lower electricity prices. This effect could have been modeled through the inclusion of electricity price as an endogenous variable. The electricity price is however modeled exogenously. Investors in generation capacity are believed to care most about the long term trends of the electricity market. Further, changes in electricity prices caused by new investments are assumed incorporated in the futures prices of electricity, which is the input to the Schwartz-Smith model. Hence, investors’ expectations for the future of the electricity market is modeled through the long-term equilibrium level of a Schwartz-Smith 2-factor model.

Further, an immediate response from certificate price changes on investments is assumed. Implicitly, this translates into an assumption of no lead-time for investment opportunities. Including a lead-time would require the inclusion of another additional state variable. This would increase run-time substantially. The assumption could be justified by the Coulon et al.,[2] who find the lead-time to have modest impact on price forecasts.
An assumption is also made about Swedish investments made after 2020. While Norwegian investments must be made before 2020 in order to receive certificates, Swedish investments may be considered certificate-eligible after this time. These opportunity has been left out of the market model. This assumption is justified by the belief that investors will not make investments causing over-supply, and collapse in certificate prices. Hence, new investments are not believed to be made after 2020, given that the target of the market is met.
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Tradable Green Certificates for renewable support:  
The role of expectations and uncertainty

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Abstract

This paper explores tradable green certificate markets, a support scheme for investments in renewable electricity production. A case-study on the Swedish-Norwegian Electricity Certificate market is conducted. In this market producers investing in new renewable capacity receive certificates based on their actual production. These are sold to retailers of electricity, which are required to buy certificates for a proportion of their total sales. The design and potential success of this multistate support mechanism is of great interest to policy makers and green investors. A stochastic model based on dynamic programming is implemented to assess the performance of this market. A discussion on market dynamics, uncertainty and expectations forms the basis for the model choices. The main findings from the model include i) Price expectations are not formed easily, and hence, investment decisions have to be made under a lot of uncertainty, ii) Under the current market structure, prices are expected to start at today’s level, while decreasing steadily towards zero in 2035 when the market is planned to end, iii) The prices are highly sensitive to changes in electricity consumption and generation of electricity, iv) Regulatory changes should be implemented carefully to avoid increased uncertainty and a consequent increase in price volatility and v) The price-based penalty further increases the volatility of the market. So far, the market has shown its ability to promote investments in renewable electricity production, however improvements of the market seem beneficial.

Keywords: Renewable portfolio standards, Tradable green certificates, Energy policy, Dynamic equilibrium, Expectations, Uncertainty

1. Introduction

Over the last decades, several energy policies have been developed. The tradable green certificate (TGC) market in Sweden and Norway is an example of a market-based energy policy, used by governments to promote the development of increased share of electricity from renewable energy sources (RES-E) in the grid. In contrast to a pure subsidy policy, such as the Feed-In-Tariffs (FIT) used in several European countries, a TGC market is a quantity-based system relying on market forces to determine the certificate price, whereas the feed-in system is a price-based system with greater involvement from the government. The TGC market can be mimicked through a subsidy on green energy production and a tax on energy consumption under the restriction of budget neutrality [1]. Producers of renewable electricity receive subsidies through certificates for their production, while consumers of electricity are taxed through the addition of the certificate cost on the bill from their electricity retailer. However, the introduction of TGC quotas reduces wholesale electricity prices because of an increased supply of electricity with a low short term marginal cost. Thus, the tax is fully or partly redirected to the producers of conventional electricity [2] [3] [4].

As has been observed historically, both TGC markets and emission trading markets are intrinsically susceptible to unstable prices, which can potentially swing rapidly from nearly zero to the penalty level, despite relatively small changes in the underlying supply and demand forces [5]. Understanding the price dynamics is of major importance for market actors. Considering an investment, investors must form certificate price expectations. Retailers would need to form price expectations when considering the timing of the certificate purchases needed to meet their obligations. Further, the regulators will need to understand the dynamics of this price formation, in order to design markets that function well and yield the desired effects. Earlier works on equilibrium price formation in similar markets for emission allowances include the paper of Montgomery [6] showing how cost minimization can form an equilibrium price, and Rubin [7] who provides an analysis of the inter-temporal effects of banking and borrowing between time-steps. Further, Daskalakis et al. [8] find that this has significant implications in terms of futures pricing. Carmona et al.

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[9] suggest that the conditional probability for a deficit of allowance credits times the penalty characterizes the equilibrium price. This view is supported in e.g., [10], [11] and [12]. In an early work on TGC markets, Morthorst [13] suggest that the certificate price should equal the cost of renewable generation less the electricity price. Coulon et al. [5] and Wolgang & Jaehnert [14] describe stochastic models for TGC markets, both making major contributions to an improved knowledge of the market dynamics. While both assume a price equilibrium in accordance with the condition suggested in [9], [14] argues that this should not be seen as contradictory to the price equilibrium suggested in [13]. Other works analyzing TGC markets include fundamental equilibrium models [3], system dynamic approaches [15], agent-based approaches [16] and econometric studies [17].

Addressing the uncertainty in TGC markets should be of the utmost importance to market regulators. The value of flexibility increases with uncertainty, thus technologies with high variable and low fixed costs become more attractive as uncertainty rises [18]. Kildegaard [19] finds evidence for this shift towards the low fixed-cost alternative in case-studies for TGC markets in Britain, Sweden, and Texas. Further he argues that there is an asymmetric risk of over-investment, resulting in collapsing certificate prices and thus capital losses. Either, this will prevent the investor from building new capacity or the investor would require a significant risk premium as compensation. Haas et al. [20] find that the intrinsic stability of FIT systems appears to be a key element in the success of these systems. Further, they find that TGC systems show to be less effective with respect to deployment of less mature RES-E. This view is agreed upon by Bergek & Jacobsson [21] and Midttun & Gautesen [22]. They also suggest that FIT regimes and TGC markets should be seen as complementary regulatory instruments targeting subsequent steps in the product cycle. Agnolucci [23] argues that long-term contracts offered to producers will yield more certainty for green investors, and thus lower the price of certificates. Van der Linden also discusses cases where retailers are obliged to offer such long-term contracts. In these cases the TGC market is gradually converging towards a FIT regime [24]. Building on a classical theory of commodity storage presented by Wright & Williams [25] and Deaton & Laroque [26], Amundsen et al. [27] find that banking reduces price volatility significantly and also lowers average prices. Based on the model presented in [5], Khazaei et al. [28] suggest a new compliance policy that they show will reduce the price volatility of certificates.

In this TGC market, called the Swedish-Norwegian Electricity Certificate Market, the target is to achieve an additional 26.4 TWh of annual renewable generation. Supply is established by letting the regulator decide which projects fulfill the requirements for being certificate-eligible. Once qualified, these producers are awarded a number of certificates based on their actual monthly eligible production. New power plants have to be in operation by the end of 2020 to receive certificates. Certificates are then received monthly for a period of 15 years from the time operations start. Producers are allowed to bank their certificates and may time their certificate sales to maximize profits, i.e. either by selling immediately or by waiting for higher expected prices. Demand is established by imposing a requirement on each individual retailer as a proportion of the actual amount of electricity he has sold to his customers. If a retailer does not fulfill this requirement by holding enough certificates at the compliance dates, he is fined a penalty fee. The penalty fee is calculated as 1.5 times the average price over the 12 months prior to the compliance date. At the compliance date, March 31 every year, the required amount of certificates is handed in and cancelled. Certificates are traded in the spot and futures market, both over-the-counter and on the Nasdaq OMX.

Despite the clear need for better understanding of the price dynamics in a TGC market, there is a limited amount of academic research addressing the price volatility and stabilization mechanisms of such markets. This paper aims to contribute to a better understanding of the dynamics in such markets, through a case-study on the Swedish-Norwegian Electricity Certificate Market. A flexible way of modeling this TGC market, based on stochastic dynamic programming, is proposed. The proposed model extends the work of Coulon et al. [5] on the New Jersey SREC market, by adding the penalty fee as a state variable and including relationships between investments, the electricity market and the Levelized Cost of Energy (LCoE). Further, it incorporates all the important features in a TGC market, such as the investment rate, the requirement level, the penalty level and a stochastic production of eligible electricity. By providing a better understanding of the formation of price expectations, the investment rationale and the role of uncertainty, the paper contributes to the ongoing debate on the design of energy policies. The proposed model can also be a supplemental tool for market actors and regulators.

Chapter 2 aims to give an overview of the dynamics in the market, including how price expectations are formed, the effects of a price dependent penalty and how investment decisions are made. Chapter 3 proposes a method for assessing the investors’ expectations of electricity prices utilizing the two-factor model of Schwartz & Smith [29]. The approach of Benth [30] is implemented to extract smooth forward curves, which are further used as input to a Kalman-filter, calibrating the two-factor model. Chapter 4 introduces the electricity certificate model. A mathematical formulation of the model is provided, and the modeling of the different market dy-
dynamics is explained. Further, the model implementation is presented. Chapter 5 provides the calibration of the model along with the results from the base-case. Chapter 6 provides an analysis of the market for different supply and demand scenarios, different investment rationales and for suggested policy changes. Chapter 7 gives some concluding remarks about this certificate market and lists some conditions for stable prices.

2. Market Dynamics

This chapter discusses market dynamics, including how price expectations are formed, the effects of a price dependent penalty and how investment decisions are made. The reasoning behind the modeling choices made in chapter 4 are also provided.

2.1. Price Expectations

The formation of certificate price expectations in the market is of major importance. Demand for certificates is based on the consumers’ total consumption of electricity. This consumption is typically found to be fairly price inelastic, however it is still exposed to annual variation, as changes in consumer patterns cause changing demand. Thus, the annual demand for certificates is not easily predictable. Supply of certificates depends on actual generation of electricity, which for most renewable sources are affected by weather conditions. It is found to vary between years in addition to a seasonal variance. For Denmark, it is estimated that the maximum variation in the annual wind power generation is approximately ±20%, with a standard deviation of approximately 10% [27]. As the short-term marginal cost of wind energy is close to zero, volatility in the supply of certificates should be expected, as the production decision is based on weather conditions rather than prices seen in the market.

The following two conditions for explaining rational formation of price expectations are suggested.

1. The certificate price \( p^C \) should be such that the marginal plant is profitable, i.e. the price should equal the difference between the levelized cost of energy (LCoE) of the marginal plant, \( L^m \), and the electricity price, \( p^E \).

\[
 p^C = L^m - p^E \tag{1}
\]

2. The certificate price should equal the discounted value of the expected penalty, \( \pi \), times the probability of having to pay this penalty, that is, the probability of a shortage in the certificate balance, \( b \).

\[
 p^C = e^{-r}E[\pi]E[1_{\{b=0\}}] \tag{2}
\]

The first condition follows from the expected profitability of an investment in a TGC market. Normally, an investment is profitable if the expected electricity price is higher than the investment’s LCoE. In a TGC market, decisions to invest in renewable electricity capacity will be made on the basis of both expected electricity- and certificate prices. Investments are assumed made when the price of certificates is higher than the LCoE of the plant less the price of electricity. Consequently, the renewable plants with the lowest LCoE will be built first, since these are profitable at a lower certificate price. If the expected prices increase, so does the number of profitable investments. However, expected prices should at all times be such that the expected marginal plant is profitable. Higher prices than this level will indicate that the market is not perfectly competitive, as the price is higher than the long-run marginal cost. One should note that investors might hold a real option mindset, putting a high value on the flexibility achieved by delaying an investment, hence the required certificate price for making an investment might be slightly higher than the price suggested by (1).

The requirement imposed on the retailers is set such that the investments will add up to a total of 26.4 TWh of additional annual capacity. Thus, if the market works as planned, combined with a perfect foresight of the LCoE of the different investment opportunities, the price level at the end of the investment period is predictable. It is worth mentioning that new investments may cause decreasing electricity prices, thus a higher certificate price is required for the marginal plant to be profitable. Realistically the LCoE curve is not known in detail by all players in the market, thus creating slightly different price expectations, which again intuitively will cause more volatile prices. On the other hand, if new investments are less (more) than 26.4 TWh in the end of 2020 there will be a deficit (surplus) of certificates in the market, and the price of certificates will start rising (falling). Then the regulators are likely to implement changes in market design, e.g. by extending the investment period or reducing the requirement on retailers. Such regulatory changes may cause increased volatility and thus an increased required rate of return [17]. Increased required rate of return results in an increased LCoE, thus a higher certificate price will be needed for the marginal plant to be profitable.

The second condition follows from the expected payoff of a certificate. When new certificates are issued, electricity producers are faced with the decision of whether to sell their certificates immediately or bank their certificates to wait for higher prices. Producers will sell their certificates at the current price, unless the present value of an expected future price exceeds the current price. When the present value of a future price is higher, producers will bank their certificates until
the willingness to pay among retailers increases to a level at which producers are willing to sell. Hence, a player acting to maximize profit is likely to sell/bank his certificates such that the market reaches an equilibrium where the players are indifferent between selling today and selling tomorrow. Along the lines of Carmona et al. [9], the retailers’ willingness to pay is assumed equal to the net present value of the penalty they expect imposed if obligations are not met, times the probability of not being able to meet these obligations, i.e. the probability of a shortage of certificates in the market. Predicting these penalty fees is difficult, as they are based on prices observed over the preceding year of a given compliance date. Following from this, in a period immediately after a compliance date, no information is available for calculating the penalty fee of the next compliance date. As this compliance date approaches, more information becomes available. The probability of a shortage of certificates is based on the current balance of certificates, issuance of new certificates and the requirement imposed on retailers. Lower balance increases expected prices, while a lower requirement level decreases expected prices.

As stated in the introduction, both the requirement and the issuance are subject to unpredictable variations on an annual basis. Combined with the unpredictable penalty level this may, as for the first price condition, cause different price expectations, and thus more volatile prices.

To achieve market equilibrium, both of the above conditions should be met at the same time. For modeling purposes, an interesting question is whether fulfilling one of the conditions automatically leads to the fulfillment of the other. For a market in equilibrium, one would expect this to be the case. This has not been shown empirically, it is however natural to assume that the suppliers of certificates ensure the fulfillment of the first condition, while the demand side ensures the fulfillment of the second condition\(^1\).

2.2. Market Instability

The price dependent penalty causes some interesting effects on price expectations and thus prices, and may potentially cause large price increases or price drops. This can be described as a mathematical instability. An increase in the certificate price will lead to an increase in the expected penalty, which subsequently may cause the prices to rise even further. Given no intervention from market regulators, this may result in a spiral that could potentially lead to prices climbing without bounds or collapsing towards zero, depending on the initial price movement. As will be discussed below, there seems to be some stabilizing factors in the market, which ensures that spiraling prices do not occur.

For a TGC market in equilibrium, expected increases in demand should be offset by a corresponding increase in expected investments, hence stabilizing the market price. However, in the event of unexpected price increases due to new unexpected information, the lead time of new generation capacity will cause the expected increase in capacity to lag the observed increase in price, hence causing a mismatch between supply and demand which in turns causes prices to remain at a higher level for some time period. The lead-time of new capacity will decide the duration of the time period needed for the system to stabilize at a new equilibrium level. In the long run, the market will meet an unexpected increase in certificate prices with an increased rate of investment in renewable electricity capacity. This will cause the supply of certificates to increase, which in turns causes falling price expectations and finally lowered prices. Bio power and dammed hydro could to some extent benefit from these periods of higher prices by increasing production. The percentage share of such “stabilizing renewables” in the mix of certificate-eligible electricity, will decide the magnitude of the described instability.

The percentage share of stabilizing renewable energy sources is low in this market and investments made after 2020 are not certificate-eligible. Hence, other mechanisms are required to avoid the potential market instability. Stabilizing price expectations will work as such a mechanism. If an initial price movement is not met with a correspondingly large change in penalty expectations and a subsequent change in price expectations, the market will not be facing upward or downward spiraling prices. Such stabilizing price expectations may stem from a belief among actors that the regulators will act in extreme cases to stabilize the system. This gives an interesting finding about price expectations, particularly that the price expectations are of major importance for actual observed prices. If prices are expected to remain in a stable area, they most likely will. If prices are expected to change, they may start spiraling upwards or downwards. Thus, a stable market requires stable price expectations.

2.3. The Investment Decision and Expected Investments

Understanding the rationale behind the investment decision is of major importance when modeling the electricity certificate market. Currently, almost six years remain of the investment period, leading to an expected annual addition of 2.7 TWh’s of new renewable generation if the 26.4 TWh target is to be reached. One essential question is whether the target will be met or whether the total investments will be either lower or higher than the target. Failing to reach the target would again cause prices to rise or fall as described in chapter

\(^1\)Thanks to Ove Wolfgang for posing this question.
2.1. Predicting the timing and the level of investments is essential to understanding the rationale behind the investment decision. Below, three such rationales are suggested.

1. A given number of investments will be made annually regardless of the prices and costs. This rationale stems from an assumption that when prices are situated within an expected price range, a sufficient amount of profitable investment opportunities exist to reach the 26.4 TWh target.\textsuperscript{2} Thus, all investors that believe their investment opportunities to be among the cheapest will make their investments independent of price movements. Specifically in Norway, there are investment opportunities in hydro power that are profitable even if certificates become worthless. These investments may then be made even if the investor knows that this will cause over-investments, and thus a collapse in certificate prices. In the opposite case, when the profitability of all investment opportunities depend on the certificate price, it is assumed that the investment will not be made if it will cause the total capacity to exceed the target. Additionally, considering the significant lead times of the different technologies, investment decisions must soon be made if the new plants are to be considered certificate-eligible. As a result of this, a potential theory could be that all the power plants needed to reach the 26.4 TWh target of new capacity have already been planned. If this is the case, movements in certificate price will be of little to no importance for new investments.

2. The investment decision is based solely on the certificate price, and a positive feedback from increased prices on the investment level is expected. Independent of the electricity prices and the cost structures, the rationale suggests that there exists a number of profitable investment opportunities, and that the number is increasing as certificate prices increases.

3. The investment decision is based on the profit equation for a certificate-eligible power plant operating in a TGC market, i.e. investments will be made if the sum of the electricity price and the certificate price less the LCoE of the current investment opportunity is greater than zero. If the certificate price is formed according to the first price formation condition presented in 2.1, a plant is profitable in the long run if its LCoE is lower than the LCoE of the marginal plant. Further, stemming from this investment rationale, the expected rate of investments will be changing directly with a changed expectation of a power plant’s marginal profit.

2.4. Assumptions and Model Choices

This subsection aims to explain the choices of modeling presented in chapter 4 based on the previous discussion. The background for the price equation, the investment equation, the issuance equation and the penalty equation are presented.

The price equation is modeled in accordance with the second price condition presented in (2), i.e. the certificate price should equal the discounted value of the expected penalty times the probability of having to pay this penalty. This relationship is chosen, since it is expected to best capture the dynamics in the market throughout the period. The short-term marginal cost of a certificate after the investment decision is made is approximately zero. The suppliers of certificates will thus be willing to sell certificates at significantly lower prices than suggested by (1). Fluctuations are thus assumed to be captured better by (2). Particularly in scenarios where supply exceeds (falls behind) demand, certificate prices are expected to decline (incline). However, for scenarios with stable expectations for supply and demand, one would assume the price expectations also to be in accordance with the condition presented in (1). It is also noted that having one or a few of the actors in the market acting according to the assumed relations would suffice for the relations to hold. Amundsen & Bergman [31] support these rational price expectations. They conclude that the problem of market power will be eliminated for a common Swedish-Norwegian certificate market.

The issuance equation describes the monthly issuance of certificates, and is chosen to be modeled such that it is subject to both seasonal and annual variance. This captures the fluctuations in generation stemming from renewable energy sources, and especially wind power, due to changing weather conditions. For valuation purposes, the annual fluctuations in issuance of certificates are of greatest importance, since the requirement imposed on retailers is only fulfilled annually. Thus, seasonal variances should not affect the certificate price.

The investment equation is modeled as a combination of the first and third investment rationales presented in section 2.3, i.e. some investments are occurring as a
function of time only, while some investments occur if the sum of the expected certificate and electricity price exceeds the LCoE of the investment opportunity. Time dependent investments are included, since several power plants have already been planned and found profitable within a reasonable price range. Other investors are expected to delay their investment decision until their project becomes profitable. These are expected to base their decision on their expectations of certificate and electricity price. Modeling investments in this way is also expected to capture the downward pressure on prices caused by new investments. The result should be an adequate number of investments, such that the target of 26.4 TWh of new capacity is met, and prices neither climb nor collapse.

The penalty equation is modeled such that the expected penalty is a function of all information known in a given month, i.e. the expected penalty equals 1.5 times the average of the current price and the prices over the months preceding the last compliance date. An exception has been made for the first month following a compliance date, where the expected penalty is calculated as the average of 1.5 times the current price and the actual observed penalty the previous year. The reasoning behind this modeling choice stems from an assumption that agents would believe the prices to remain in a stable area throughout the period and thus only use the currently available prices to form their expectations of the penalty.

3. Electricity Price Simulation

Including a dependency between investments and electricity prices entails a method of assessing and forecasting the investors’ long-term expectations of future electricity prices. For this, a Schwartz-Smith 2-factor model [29] has been used:

\[ X_t = \chi_t + \xi_t \]  \hspace{1cm} (3)

Here \( \chi_t \) and \( \xi_t \) are unobserved state variables representing time \( t \) short term deviation in log prices and time \( t \) equilibrium levels for log prices respectively. Short run deviations \( (\chi_t) \) are assumed to revert towards zero following an Ornstein-Uhlenbeck process

\[ d\chi_t = -\kappa\chi_t dt + \sigma_{\chi} dz_{\chi}, \]  \hspace{1cm} (4)

while equilibrium level \( (\xi_t) \) is assumed to follow a Brownian motion process

\[ d\xi_t = \mu_{\xi} dt + \sigma_{\xi} dz_{\xi}. \]  \hspace{1cm} (5)

The unobserved state variables have been estimated using a Kalman filter process. In typical implementations of the Kalman filter procedure, missing data problems are severe [32]. These known shortcomings have been overcome utilizing Benth et al.’s method [30] for extracting smooth forward curves from average-based commodity contracts with seasonal variation. For any given time, there exists futures contracts with settlement over the next month, quarter and year. This means that for the same date, several futures prices are observed. The contracts can be interpreted as swaps, as they are swapping a fixed price commodity against a floating price commodity in the settlement period. Benth et al.’s method takes these observed contracts and create smooth forward curves, fitting the contracts to a seasonality function under a maximum smoothness criteria. For any given time to expiry, the resulting forward curves can be interpreted as the future price on a contract with daily settlement. Further, these are used as input to the Kalman filter. The construction of forward curves will cause some information loss, but is necessary given the nature of delivery in the electricity market.

Furthermore, following from the output of the Schwartz-Smith 2-factor model, the long-term equilibrium level has been used as input for the electricity certificate model. This is motivated by the assumption that the long-term prices are the ones that are relevant to investors making investment decisions. The expected long-term equilibrium level of the electricity prices is given by (6).

\[
\ln(E[S_t]) = \ln(E[X_t]) + \frac{1}{2}\text{Var}[X_t] \\
= e^{-\gamma t}\left[\chi_0 + \xi_0 + \mu_{\xi} t + \frac{1}{2}\left(1 - e^{-2\gamma t}\right)\frac{\sigma_{\chi}^2}{2\kappa} + \frac{2(1 - e^{-\gamma t})\sigma_{\xi}^2}{\kappa} + 2\ln(1 + \frac{\rho_{\chi\xi}\sigma_{\chi}\sigma_{\xi}}{\kappa})\right] \\ + \frac{1}{2}\left(1 - e^{-2\gamma t}\right)\frac{\sigma_{\chi}^2}{2\kappa} + \sigma_{\xi}^2 t + 2(1 - e^{-\gamma t})\frac{\rho_{\chi\xi}\sigma_{\chi}\sigma_{\xi}}{\kappa}\right) \\
(6)
\]

4. Electricity Certificate Model

The Electricity Certificate model extends Coulou et al.’s modeling of the New Jersey Solar Renewable Electricity Certificate (SREC) and the equations described in their paper [5]. A noticeable difference between the New Jersey and the Swedish-Norwegian market is that while the New Jersey market has a fixed penalty, the Swedish-Norwegian market has a penalty that is dependent on the average of the prices observed over the course of the preceding year. The inclusion of this feature has required the introduction of another state variable, penalty \( \pi \). Additionally, the model has been extended to include a dependency between investments, LCoE and electricity prices. The possibility of infinite certificate banking, present in the Swedish-Norwegian market has also been included. Further, parameters have been estimated to reflect historical values from the Swedish market over the period 2004-2011.
4.1. Mathematical Formulation

\[ p_t^C = \max_{\psi \in \{N_t|t=1,...,T\}} e^{-r(t-i)}E[\rho_t]E_{i}[1_{\{b_i=0\}}] \]  
\[ p_t^C = e^{-r\Delta t}E(p_{t+1}^C) \text{ when } t \notin N \]  

As discussed in chapter 2.4, the modeling of the price is based on (1). At a compliance date, the holder of a certificate will avoid the penalty imposed on those who do not comply as he hands in his certificate. Further, if the balance of certificates in the market directly following a compliance date is 0, one can assume that investors would have been willing to pay the amount of the penalty fee for one certificate. Eq. (7a) states that at any time \( t \), the value of the certificate \( p_t \) is the maximum of the discounted expected future penalty fees it can be used to avoid, discounted at the rate \( r \) times the probability of having to pay this penalty. Eq. (7b), i.e. the Martingale condition, follows implicitly from (7a) and states that, except at compliance dates, the current price is the discounted expected future price.

\[ b_t = \begin{cases} \max(0, b_{t-1} + \int_{t-1}^{t} g_u du - R_t) & t \in N \\ b_{t-1} + \int_{t-1}^{t} g_u du & t \notin N \end{cases} \]  

Eq. (8) is a standard inventory equation, and keeps track of the accumulated number of certificates, banking, in the market at any given time. At any time step \( t \), the currently banked balance \( b_t \) is a function of the previous balance \( b_{t-1} \) and the accumulated issuance since the previous time step, \( \int_{t-1}^{t} g_u du \). If the current time step is part of the set of compliance dates \( N \), (8) accounts for a reduction in the number of certificates in the market, equal to the requirement \( R_t \) at the given date. The balance can never be negative, hence the max statement.

\[ g_t = g_t(p^C, p^F) \exp(a_1 \sin(4\pi t) + a_2 \cos(4\pi t) + a_3 \sin(2\pi t) + a_4 \cos(2\pi t) + e_t^N) \]  

The seasonality and annual variation in eligible electricity generation, and hence certificate issuance, discussed in 2.4, is accounted for by a seasonality function and a stochastic process on the form shown in (9). The state variable \( g_t \) represents the certificate-eligible annual capacity and is a function of the certificate price, electricity price and LCoE. This is motivated by the assumption that investors are likely to invest more while prices are high. Seasonal changes are modeled by the sine and cosine functions, while a noise term is added to reflect the uncertainty in annual generation from wind and hydro.

\[ \frac{\ln(g_{t+\Delta t}) - \ln(g_t)}{\Delta t} = a_5 + a_6(\max(0, (p_t^F + \sigma_d^2 dz_t) + p_t^C - L_t)) - C_t, \text{ for } a_5 \in \mathbb{R}, a_6 > 0 \]  

As discussed in chapter 2.4, rationale 1 and rationale 3 will be used as background for modeling new investments. Eq. (10) accounts for the monthly change in certificate-eligible generation capacity. The parameter \( a_6 \) accounts for the logical effect that producers are likely to invest more as the marginal profits associated with the investment rises. \( a_5 \) represents the growth of generation not related to marginal profit. It is an independent term describing the drift in investments over time. A noise term \( \sigma_d dz_t \) representing the uncertainty in the equilibrium level is added to the forecasted electricity price. \( C_t \) represents the monthly phase-out of certificates, occurring when a power plant has received certificates for 15 years.

\[ \text{E}[\pi_t] = \left\{ \begin{array}{ll} 1.5 \cdot \sum_{i=1}^{l} t_{-\max(1N_t|t=\infty)} \frac{p_i}{t_{-\max(1N_t|t=\infty)}} & t-1 \notin N \\ 1.5 \cdot \left( \frac{p_i}{t_{-\max(1N_t|t=\infty)}} \right) & t-1 \in N \end{array} \right. \]  

As discussed in chapter 2.4 the expected penalty will be modeled with an assumption that agents only use currently available price information in their formation of penalty expectations. Eq. (11) states that at all times, the penalty level should be 1.5 times the average of prices observed since the last compliance date, except in the first month after the compliance date, where the expected penalty is 1.5 times the average of the current price and the actual observed penalty the previous year.

\[ p_t^C = p_t^C \]  

It is here assumed, as stated in (12), that the investment decision is made based on the current price. This follows since the model captures all future price expectations in the current price. Following from this relation, is an immediate price feedback on certificate-eligible generation capacity. This is done to avoid the inclusion of another state variable, hence reducing dimensionality and preserving the computational tractability. The assumption could be justified further by Coulon et al. [5], who find the lead-time to have modest impact on price forecasts.

4.2. Implementation

The solution algorithm proceeds as follows:

1. A 3-D grid of values for \( b_t, g_t \) and \( \pi_t \) is chosen. For \( b_t \) and \( g_t \), the lower bounds are zero and the upper bounds are a little above the largest requirement. For \( \pi_t \), the lower bound is zero and the upper bound is a little above the highest expected penalty. Time is discretized in monthly steps, matching the frequency of historical generation data.
2. The dynamic program is initialized, evaluating the payoff of the certificate at the end of the market’s life \( t = T \) for every gridpoint \((b_t, \hat{g}_t, \pi_t)\). At this point, all information is known and hence, the program yields digital boundary price cubes:

(a) At grid points where there is a shortage of certificates (i.e. the balance is less than the requirement), investors are willing to pay the penalty, \( p_{C}^{T} = \pi_{T} \), for one certificate.

(b) At grid points where there is a surplus of certificates (i.e. the balance is higher than the requirement), investors are willing to pay \( p_{C}^{T} = 0 \) for one certificate.

3. From the boundary cube at \( t = T \), the dynamic program steps backward to \( t = (T - 1) \). Here it solves eqs. (7)-(12) and finds a price at every grid point using price information from the price cube at \( t = T \). The same procedure is then followed for every time step; information from price cube \( t + 1 \) is used to solve eqs. (7)-(12), finding a price at every grid point of price cube \( t \) with the FORTRAN subroutine \texttt{nag_rootsWithdrawSysFuncEasy} (c05nb) from the NAG Toolbox for Matlab [33], utilizing the Powell hybrid method.

The algorithm provides a price cube at every single time step. The price cubes show what the price would be at this time step, given a state \((b_t, \hat{g}_t, \pi_t)\). An example of the content of these cubes is shown in Figure 1, plotting the prices obtained for a chosen \( \pi \) over the grid \( b_t, \hat{g}_t \) for December 2020. For the given resolution level, the runtime of this procedure is approximately 1 hour on an Intel(R) Core(TM) i7-3770 CPU, multithreading at 3.4 GHz on all 4 available cores. Parallel processing and FORTRAN subroutines have been introduced to keep the model computationally tractable. This is discussed further in an appendix to the corresponding Master’s thesis introduction. [34].

From the algorithm presented above, the prices for every combination of state variables are found and stored in a 4-dimensional matrix. Furthermore, starting from the currently observed certificate-eligible capacity \( \hat{g}_{0} \), accumulated certificates in the market \( b_{0} \), penalty level \( \pi_{0} \) and time \( t_{0} \), the state transition equations are used to obtain the certificate price, \( p_{C}^{t} \), certificate-eligible capacity \( \hat{g}_{t} \), accumulated certificates in the market \( b_{t} \) and penalty level \( \pi_{t} \) for every time step throughout the period.

5. Calibration and Base-Case

This chapter describes the calibration of the model and provides results from the model for a base-case.

5.1. Model Input Estimation

The Schwartz-Smith procedure described briefly in chapter 3 yields the results shown in Figure 2. The figure shows estimated prices, observed prices and equilibrium price. As seen by the figure, the time series model replicates observed prices quite well. Eq. (6) and parameters obtained by the Schwartz-Smith procedure, shown in Table 2, are in turns used to calculate the long-term equilibrium level shown in Figure 3. This is used as a proxy for the investors expectations of the electricity price.

The input data for compliance requirements \( R_{t} \) and the LCoE curve are based on numbers received from the Norwegian Water Resources and Energy Directorate\(^3\) and the Swedish Energy Agency \([35] [36] [37]\).\(^4\) The annual requirement is calculated based on their forecasts of electricity consumption multiplied with the requirement quotas. Further, all known investments opportunities are sorted in ascending order based on their estimated LCoE. It is assumed that the less expensive investments are made first. Thus, in every month a certain number of investments is assumed to be made. The less expensive investment opportunity in the next month will then have a similar or higher LCoE than in the current month. Repeating this procedure for every month yields a time-

\(^3\)Courtesy of Leif Inge Husabø, The Norwegian Water Resources and Energy Directorate
\(^4\)The Norwegian Water Resources and Energy Directorate and the Swedish Energy Agency are the regulators of this certificate market.
Table 2: Parameters for the electricity price dynamics obtained from the Kalman filter

<table>
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<tr>
<th>$\kappa$</th>
<th>$\sigma_X$</th>
<th>$\lambda_X$</th>
<th>$\mu_C$</th>
<th>$\sigma_X$</th>
<th>$\rho_X$</th>
<th>$\chi_0$</th>
<th>$\xi_0$</th>
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<td>-1.21</td>
<td>-0.01</td>
<td>0.20</td>
<td>-0.17</td>
<td>3.19</td>
<td>-1.56</td>
</tr>
</tbody>
</table>

Figure 3: Schwartz-Smith long-term factor dependent LCoE curve.\(^5\) Cost reductions from technology development is omitted as the investment period only lasts until 2020. The timing of phase-out of power plants has been calculated based on data from all 4557 power plants currently accepted as certificate-eligible and expected annual investments. The discount rate used in the price equation has been set to 9.5%. This reflects the required rates of return at which the retail sellers in the Scandinavian electricity market operate.\(^6\)

The discount rate assumed in the calculation of LCoE for investments opportunities in Norway and Sweden has been set to respectively 4% and 8% by the Norwegian Water Resources and Energy Directorate and the Swedish Energy Directorate. Further, historical data on issuance of certificates and prices, provided by NECS [38] have been used to calibrate the stochastic generation function.

\[
\ln(\eta_t) = a_0 + a_1 \sin(4\pi t) + a_2 \cos(4\pi t) + a_3 \sin(2\pi t) + a_4 \cos(2\pi t) + a_5 t + a_6 (\max(0, p_t^{EI} + p_t^C - \text{LCOE}(t))) + \xi_t
\]  

In (13), (9), (10) and (12) have been combined to allow for parameter fitting, determining coefficients used in these equations from historical data. The regression, yields the regression coefficients shown in Table 3. The bootstrapped confidence intervals of these coefficients are shown in Table 4.

Table 3: Regressed parameters for the certificate issuance seasonality function (13)

<table>
<thead>
<tr>
<th>$a_0$</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
<th>$a_4$</th>
<th>$a_5$</th>
<th>$a_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.627</td>
<td>0.070</td>
<td>0.010</td>
<td>-0.309</td>
<td>0.221</td>
<td>0.087</td>
<td>0.0001</td>
</tr>
</tbody>
</table>

The Q-Q-plot of the residuals is shown in Figure 5, and indicates that the error terms are close to normally distributed [39], though there are some outliers in the lower left and the upper right corner. It is also noticed that the residuals do not show any patterns of autocorrelation. The residuals seem to be independent and normally distributed. The characteristics of the error term, shown in Table 5, are obtained. As seen, the residuals demonstrate some excess kurtosis, it is however close to zero and hence does not change the assumption that a normal distribution can be used to simulate noise. Neither does the slight negative skewness of the error terms [40]. Based on this discussion, it is decided to use a normal distribution with a mean of zero and a standard deviation of 0.080 for the renewable generation noise term $\xi_t$ in (9). For the electricity price stochastic term, $\sigma_x dz_x$, the standard deviation $\sigma_x$ obtained from the calibration of the Schwartz-Smith long term factor $\xi$ has been used.

Table 5: Error term characteristics for the renewable generation regression

<table>
<thead>
<tr>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Skew</th>
<th>Excess Kurt.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>0.080</td>
<td>-0.628</td>
<td>0.106</td>
</tr>
</tbody>
</table>

5.2. Model Output for the Base-Case Scenario

This section presents the base-case for the simulation of the certificate market. The model is run for the input regression shown in Figure 4.\(^7\) Testing and concluding on whether the relationship between marginal profit and generation, i.e. $a_6$, is significantly different from zero is difficult. It is however likely that the coefficient is positive, since some level of profit feedback is expected.\(^8\)

Since the regression is nonlinear, the goodness of fit is assessed based on the coefficient standard errors, the coefficient confidence intervals and a visualization of the

\[5\] The LCoE for the relevant time-period spans from €30/MWh to €46/MWh.

\[6\] Courtesy of Bjørn Erik Heiberg, Pareto (A Norwegian financial advisory firm)

\[7\] For nonlinear regressions, metrics like $R^2$ and the p-stat are not suitable for assessing the goodness of fit. These metrics are based on the assumption that the regression is linear.
described in section 5.1 from April 2015 until the last compliance date in March 2035. The requirement quota adjustment, suggested by the regulators to the 2015 market progress review⁸, has been included [37]. The resulting Figure 6 presents certificate prices, balance of certificates, certificate-eligible investments and the normal annual production of electricity from eligible power plants.

Figure 6a shows stable, but declining prices. The red line represents the average of 100 model realizations represented by the blue lines. Especially towards the end of the period a great variance between scenarios is observed. It is noticed that the modeled price for April 2015, €18.34/MWh, is slightly higher than the price actually observed in the market, €16.33/MWh. As the market approaches its planned end in 2035, the modeled prices decline rapidly towards zero. This is due to an expected surplus of certificates throughout the market’s lifetime. A time passes, the probability of scenarios resulting in a future deficit of certificates decreases. Similar results can be found in [14].

Figure 6b plots the accumulated investment in certificate-eligible capacity, as forecasted by the model, against time. A stable increase in investments is seen until the end of the investment period (2020). The parameter estimation from the regression in chapter 5.1 is used for the base-case. The time-dependent term was found to be substantially larger than the profit feedback term, thus the time-dependent term accounts for a large proportion of the expected investments. This results in a low level of uncertainty in the number of investments. Several different scenarios for the investment relation will be presented below. As seen from the figure, the model expects the target of 26.4 TWh’s of additional capacity from renewable energy sources to be met.

One can assume that the market is designed so that a small certificate surplus is expected if the market’s investment target is met. This is reflected in Figure 6c. The target is met and a stable balance surplus is seen throughout the period. The monthly incline in the certificate balance represents the expected monthly production of electricity, which in turns reflects the monthly issuance of certificates. The steep declines occurring annually represent the required quantities of certificates being cancelled at compliance dates.

Figure 6d shows the expected level of certificate-eligible annual capacity. Until the end of 2020 the eligible capacity is expected to increase due to investments seen in Figure 6b. In 2018 the first power plants declared certificate-eligible have received certificates for 15 years and the first phase-out of certificate-eligible capacity occurs. In Figure 6d this is best seen in the decline after 2020, when no new investments receive certificates. In 2035, when the market is planned to end, all the power plants have been phased out.

6. Analysis

This chapter aims to analyse and discuss how the market changes proposed by regulators for the 2015 progress review will impact the market. Additionally, changes in certificate supply and demand, different alternatives for the investment decision rationale and different mechanisms for forming penalty fee expectations, have been explored. Unless stated otherwise, input variables are equal to those presented in the base-case discussion.

6.1. Changes in Certificate Demand

Certificate demand is based on the actual observed consumption of electricity. Hence, a lower electricity consumption will yield a lower certificate demand. This section aims to investigate the effects of changes in electricity consumption. In figure 7, three scenarios with a varying level of electricity consumption have been compared to the base-case.

⁸Progress reviews are scheduled every fourth year to decide on regulatory changes to the market structure.
Figure 7a shows how a lower electricity consumption, and hence, certificate demand, yields lower prices throughout the period. This is natural as the probability of scenarios with a shortage of certificates decreases with an increased certificate balance. Correspondingly, a higher electricity consumption yields a lower certificate balance, and hence, higher prices throughout the period. The mechanism is illustrated well by Figure 7b which relates the minimum annual balance to the price level. The effect is observed to be noticeable even for small changes in demand. A realistic variance in eligible electricity production is found to cause similar variance in certificate prices. In both cases, extreme scenarios forcing prices to climb without bounds or collapsing towards zero can be seen. This illustrates the importance of stability in demand and supply for the formation of stable prices. Market actors report the estimation of consumption as a source of uncertainty [41]. This is supported by the large price changes seen from the model even for slight adjustments of input parameters. The updated forecast for the electricity consumption, provided by the Norwegian regulator, shows an annual 3-7% adjustment compared to the forecast provided at the launch of the market. Thus, such variation in the requirement can be considered realistic.

6.2. Regulators’ Suggested Market Changes for the 2015 Progress Review

The regulators’ suggestions for the 2015 Progress Review include four main elements. These are:

1. A slight adjustment of the quota path, to balance the market.
2. Extending the investment period to the end of 2021 while shifting the quota path upwards to compensate for the increasing supply.
3. Increasing the target of additional renewable energy capacity to 28.4 TWh by shifting the quota path upwards.
4. Increasing the number of power plants in the Norwegian transition system.9

The suggested adjustment of the quota path has already been included in the base-case, the other suggested changes are discussed in the following section.

Figure 8 shows the forecasted price and balance if new investments were to be considered certificate-eligible until the end of 2021. An extension of the investment period lowers the probability of good investments missing out on the system benefits due to lead-time issues. However,
as seen some from Figure 8b some of the investments are not expected to be made until 2021. This leads to a lower issuance of certificates in the preceding years, causing long-term effect on both certificate balance and certificate prices. The lowered issuance lowers balance, and thus higher and more volatile certificate prices are forecasted throughout the period. Approaching 2035, large variations between the generated market realizations are observed, following from uncertainty among actors of whether enough certificates will be available to fulfill the certificate obligations. If a deficit of certificates is expected, the prices will start climbing years in advance, consequently the penalty fee could start spiraling upwards. This scenario illustrates the importance of timing new investments. Even for this modest system change, both the expected prices and the volatility increases significantly. The regulators could avoid such effects by making appropriate changes to the requirement quotas.

Figure 9 shows forecasted prices and certificate balance given the inclusion of power plants in the Norwegian transition system to the market. The change accounts for a total of 2 TWh's of additional annual certificate-eligible generation capacity being included in the market over a period of 11 years. Figure 9 illustrates that forecasts are almost unchanged compared to the base-case. For this scenario the requirement quota levels have been updated accordingly. This further strengthens the importance of the requirement level even for small changes to the market structure.

Figure 10 presents the expected prices and balance following an increase of the target to 28.4 TWh. The expected reduction in certificate price caused by this change is estimated to €0.91/MWh in 2020. A linear change in requirement quotas is assumed to account for the additional issuance of certificates. For the first few years, the prices are slightly higher than in the base-case, they do however follow a similar path throughout the period.

Common to all the suggested changes in regulations, is the risk of investors losing their trust in a frequently revised market. This may increase volatility, and thus investors will require a higher rate of return on investments. Furthermore, considering a change in market regulations, its effects on supply and demand should be thoroughly assessed, as these are shown to impact prices substantially. Frequent adjustments of requirement quotas seem necessary in order to balance the system as the regulators’ electricity consumption forecasts are updated. As mentioned, estimating this consumption has been reported by actors as a considerable source of market uncertainty [41]. An alternative that might ease the formation of market expectations could be to change the requirement from a quota times the actual observed consumption of electricity to an absolute number of certificate per year. Thus, the only uncertainty would be on the supply side, with its stochastic issuance of certificates.
6.3. Penalty Expectations

The expectations of the penalty fee is an important factor in the forming of price expectations. An alternative to (11) is introduced in this section. While (11) assumes that an actor in the market expects the yearly average certificate price to equal the average of the certificate prices seen since the last compliance date, the proposed alternative assumes naive price expectations, and proposes that an actor in the market expects the certificates price over the remainder of the year to equal the certificate price observed today. The expected penalty fee can then be modeled as the weighted average of prices observed since the previous compliance date and prices expected over the remaining months of the year, i.e. the current price. Thus the current price is more heavily weighted. Running the model with the updated penalty expectation mechanism yielded the same results as from the base-case. One could expect more volatile price forecasts, as penalty expectations change more rapidly in response to current price changes. Realizing that the model output does not change suggests that the model is able to capture future expected prices in the price forecast of a given time step.

6.4. Investment Decision Rationale

Section 6.1 discusses the investment rationale. As seen from the regressed parameters, weak signs of feedback from expected profits or certificate prices on investments are exhibited. This section will look at how a stronger feedback mechanism would influence the market. Figure 11 illustrates market forecast realizations given the investment rationale in (10), for three different levels of profit feedback, $a_6$, on investments, and a fixed time dependent drift term. Figure 12 illustrates a scenario given the investment rationale in (14), where the certificate price is assumed to drive investments alone.

\[
\frac{\ln(\hat{g}_{t+\Delta t}) - \ln(\hat{g}_t)}{\Delta t} = a_7 \hat{p}_f^C, \quad \text{for } a_7 > 0
\]  

Figure 11 illustrates that for a medium and high profit feedback on investments, only small differences in forecasts are observed. This is due to the downward-pressure on certificate prices resulting from new investments. Even for very high feedback, the accumulated investments does not increase much as this is expected by the model to cause a collapse in certificate prices. Thus the two scenarios turn out nearly identical. The scenario with a lower feedback yields higher prices. This stems from total investments being slightly below the 26.4 TWh target, thus causing a lower certificate issuance and balance throughout the period. Compared to the base-case, the resulting prices are lower throughout the period and inclining in the first few years. This stems from more investments in the beginning of the period, decreasing certificate prices, followed by a slightly higher balance throughout the period. Concluding from this, a higher profit feedback on investments yields higher investments, and hence lower prices, until a certain...
level where further investments are unprofitable due to the low certificate price. Figure 12 presents lower prices throughout the period. This is due to accumulated investments being higher than the target, thus increasing the supply of certificates. This shows that when investments are solely driven by the certificate price scenarios, over-investment is more likely to occur. Realistically, such scenarios may happen if less expensive hydro power investments are made even if the market target is exceeded, as these power plants are profitable even for a low certificate price.

7. Conclusions

This paper looks into the performance of the Swedish-Norwegian Electricity Certificate Market. The purpose is to contribute to a better understanding of the dynamics of the Swedish-Norwegian Electricity Certificate market, to investigate the consequences of changes in the dynamics and to investigate consequences of the changes in market structure suggested by regulators for the 2015 progress review. The certificate prices are expected to remain at the current level over the next few years, they are however expected to decline steadily towards zero as the market approaches its planned end. It is found that small changes to the market structure or small changes in the levels of electricity generation or consumption may cause large shifts in certificate price.

The current certificate price is already in the lower price range. Recent news have shown Norwegian wind projects being abandoned due to the low certificate and electricity prices. The results of this analysis show no evident signs of an imminent increase in certificate prices. The investment target is however expected to be met as profitable investment opportunities are believed to exist, especially in Sweden where a beneficial tax regime will benefit wind power investments until July 2016. If the investment target is not met, significantly higher prices are expected due to certificate shortage. It is worth noticing that the forecast does not seem to meet this increase with increased investments. This is due to the foreseen fall in expected certificate prices as new investments contribute to meeting the target. Thus, investments requiring a higher certificate price are not necessarily made. On the other hand, if the target is exceeded the certificate price will drop towards zero. It is however assumed, and also shown in the analysis, that this is not expected, since an investor is not believed to invest in a project causing the certificate price to remain at a significantly lower level. Thus, to meet the target, a sufficient number of investments must be profitable at the current price level and investors have to adjust their decision making to the actions of others to avoid over-investing. Concluding from this, it is found that technologies which only require a small subsidy to become profitable will be promoted through the certificate market, while new and more expensive technologies will not. This is also in accordance with theory of certificate markets, stating that at any time the least expensive investment will be made.

Periodic adjustments of the requirement quotas are found to be necessary to achieve stable prices throughout the lifetime of the market. While the stochastic nature of the issuance cannot be easily addressed, the uncertainty in the requirement is addressable. An alternative design that may ease the formation of price expectations could be to change the requirement of certificates to an absolute quantity, rather than the quota level multiplied with the electricity consumption. The required quantity of certificates will then have to be split between the retailers of electricity. This gives the market actors perfect information about the total annual certificate demand and the uncertainty left will be limited to the issuance of certificates, i.e. the generation of eligible electricity and the magnitude of the penalty fee. For such scenarios, more stable prices would be expected, declining within a range representing the uncertainty in electricity generation, as shown in the base-case in chapter 6.

The main challenge for this market is found to be the difficulties in forming rational price expectations. Slight changes in generation of eligible electricity or consumption of electricity are shown to cause considerable price movements. The actors in the market report that they
want more predictable supply and demand allowing for better forecasts of the market development. It is also found that the price depends largely on the behavior of the market participants. In particular, the possibility of over-investment constitutes a threat to the market stability. Further, it is found that the price-based penalty accelerates price change to a new equilibrium level, and causes more volatile prices. Recently, several changes to the market have been suggested by the regulators. Our analysis shows that these may not influence the price if the requirement quotas are changed accordingly. However, it is not obvious how these changes in the quota level should be made. Therefore, it is difficult for actors in the market to estimate certificate price changes following adjustments in market design. Concluding from this, it is shown that investors will make their investment decisions under a lot of uncertainty, thus low fixed-cost technologies may be prioritized before high fixed-cost technologies. Improvements made to stabilize the market could be beneficial.

References


[37] NVE, Energimyndigheten, Summary of the review of the electric certificate markets system.

URL http://seec.statnett.no/


Abstract—This paper explores the Swedish-Norwegian market for electricity certificates, which is a support scheme for investments in renewable electricity production. Producers investing in new renewable capacity receive certificates based on their actual production. Retailers of electricity are required to buy certificates for a proportion of their total sales. If a retailer’s obligation is not met, a penalty fee is imposed. The certificates are traded both bilaterally and as a financial instrument on the Nasdaq Commodity Exchange. The design and potential success of this multistate support mechanism will be of great interest to policy makers and green investors. The dynamic equilibrium model of Coulon, Khazaee, Powell (2014) is adapted to the Swedish market. It is found to replicate historical long-term trends and price levels well. Sensitivity analyses show that the key drivers of certificate prices are the penalty levels and the discount rate. Further it is shown that a higher rate of certificate price feedback on the investment rate dampens the price fluctuations around the trend line. The rate of feedback is uncertain, but is assessed to be larger than zero.

Index Terms—Renewable portfolio standards, Tradable Green Certificates, Energy policy, Dynamic equilibrium, Market analysis

I. THE SWEDISH-NORWEGIAN ELECTRICITY CERTIFICATE MARKET

Over the last decades, market-based energy policies have grown more popular. The Tradable Green Certificate (TGC) market in Sweden and Norway is an example of such a market-based energy policy, used by governments to promote the development of increased renewable capacity in the electricity market.

In this TGC market, supply is established by letting the regulator decide which projects fulfill the requirements for receiving certificates. Once qualified, producers of eligible electricity are awarded a number of certificates based on their actual monthly production. Producers are allowed to bank their certificates and may trade them to maximize profits, i.e. either by selling immediately or by waiting for higher expected prices. Demand is then established by requiring retailers to buy some of these certificates. The requirement imposed on each individual retailer is based on the amount of electricity he has sold to his customers. If a retailer does not fulfill this requirement by holding enough certificates at some specified compliance date, he is fined a penalty fee. Since the required number of certificates is calculated as a proportion of sold quantity, the total demand for electricity certificates is based on end users’ consumption of electricity. This demand for electricity is typically found to be fairly inelastic, and the total consumption is predictable within seasonal variance.

One might argue who actually pays for these systems. The legislation states that the costs should be charged consumers over their electricity bill [1]. In his paper from 2000, Morthorst, finds that the costs of such a system is in fact carried by the consumers [2]. Since electricity demand is inelastic, an increased electricity price will not cause significant reductions in the quantity of electricity consumed. Bye [3] disagrees with the view of Morthorst. He argues that the costs of TGC markets are actually paid by the producers. As more projects become profitable, production volumes, and hence supply, rise. According to Bye, this will cause lowered retail prices, even though the certificate costs have been added. Thus, the existing producers carry the costs of the system, and consumers benefit from the lowered prices1.

The Swedish electricity certificate market was opened in 2003, and is currently planned to last until 2035. From January 2012, the market was extended to include Norway [4]. By legislating the system, the lawmakers seek to add an additional 26.4 TWh of annual renewable capacity to the Swedish-Norwegian electricity market. Electricity producers receive one certificate per MWh of generation for their eligible power plants. A power plant declared eligible, will generate certificates during its 15 first years of production. All sources of renewable electricity production are equally entitled to certificates and there are no requirements regarding how this new capacity should be located geographically. This ensures that more profitable projects are realized first. However Swedish taxation rules make investing there favorable, compared to investing in Norway. In a report from 2012, Theme Consulting Group concludes that “up to 5.6 TWh of new renewable electricity in Norway, mainly wind, but also hydro, will be crowded out by more expensive Swedish wind power. The costs of meeting the certificate target of 26.4 TWh will therefore be higher than necessary” [5].

1Bye notes that the increased consumer surplus is exceeded by the decrease in producer surplus, and hence, the total social surplus experience a net decrease.
II. Price Dynamics

A. Price Equilibrium

The formation of certificate price expectations in the market is of major importance. Two conditions explaining the rational formation of expectations are presented.

1) The certificate price should equal the difference between the levelized cost of energy (LCoE) of the marginal plant and the electricity price.

2) The certificate price should equal the discounted value of the penalty times the probability of having to pay this penalty.

The first condition follows from the way electricity producers make investment decisions. In a TGC market, decisions to invest in renewable electricity capacity will be made on the basis of expected electricity- and certificate prices. Assuming a fixed electricity price, investments are made when the certificate price is at a level that makes a new investment profitable. This happens when the price of certificates is higher than the LCoE of the plant less the price of electricity. Consequently, the renewable plants with the lowest LCoE will be built first, since these are profitable at a lower certificate price. As prices increase, so does the number of profitable investments. Subsequently, investments will be made until the price of certificates is equal to the LCoE of the marginal plant less the electricity price. If the adaptation to a price level of certificates yielding a 26.4 TWh of additional capacity is slower than planned, the regulators are likely to implement changes in market design. Such regulatory changes may cause increased volatility and thus an increased required rate of return on investments [6].

Furthermore, the second condition follows from the expected payoff of a certificate. When new certificates are issued, electricity producers are faced with the decision of whether to sell their certificates immediately or bank their certificates to wait for higher prices. Producers will sell their certificates at the current price, unless the present value of an expected future price exceeds the current price. When the present value of a future price is higher, producers will bank their certificates until the willingness to pay among retailers increase to a level at which producers are willing to sell. The retailers’ willingness to pay is assumed equal to the expected penalty faced if obligations are not met, times the probability of not being able to meet these obligations, i.e., a shortage of certificates in the market. Hence, a player acting to maximize profit is likely to sell/bank his certificates such that he reaches an equilibrium where he is indifferent between selling today and selling tomorrow.

To achieve market equilibrium, both of the above conditions should be met at the same time. For modeling purposes, an interesting question is whether fulfilling one of the conditions automatically leads to the fulfillment of the other. For a market in equilibrium, one would expect this to be the case, but this question remains to be answered.

B. Market Instability

Changes in certificate prices cause some interesting effects. The Swedish-Norwegian market operates with a penalty calculated as 1.5 times the average certificate price of the previous year. This leads to a short term mathematical instability. An increase in price will lead to an increase in the expected penalty, which subsequently may cause the certificate prices to rise even further. Given no intervention from market regulators, this spiral could potentially lead to prices climbing infinitely high or collapsing towards zero, depending on the initial price movement.

Bio power could to some extent, benefit from higher prices by increasing production. The percentage share of such “stabilizing renewables” in the mix of certificate eligible electricity, will decide the magnitude of the described instability. In the long run, the market will be able to meet an increase in certificate prices with an increased rate of investment in renewable electricity. This will cause the supply of certificates to increase, which in turn causes prices to decrease. The lead time of new capacity will decide the duration of the time period needed for the system to stabilize at a new equilibrium level.

C. Alternatives to Price Feedback on Investments

Both for the discussion in the previous section and for the example model in section III it is assumed that the level of investments in new renewable capacity is a positive function of the certificate price level. This assumption is logical, as an increase in prices will directly increase the gross profit of a renewable plant. Two extreme cases are considered. In the case of certificate prices moving towards infinity, all investment opportunities will be taken. In the case of prices collapsing towards zero, only investments profitable without TGC support will be taken. However, while prices fluctuate within an expected range, the dependency between prices and investment rate is considered uncertain and may vary for the different technologies.

Historically, both dammed and run-of-river hydro power have been profitable without subsidies. On the other hand, the profitability of wind and bio power have, with few exceptions, depended largely on subsidies. Assuming that projects waiting to be realized have similar characteristics, one can assess the dependency between investment levels and certificate

2Thanks to Ove Wolfgang for posing this question.
prices to be less prominent for hydro than for wind and bio. Information from a key producer in the Norwegian electricity market also indicates that their development of new hydro power plants does not depend on prices, but solely on access to capital. Wind and bio power projects will depend on positive certificate prices to be realized. Summing up, whether changes in certificate prices affects investment decisions is unclear. New renewable capacity must be in operation by the end of 2020 to benefit fully from the TGC support scheme. Considering the lead times of the different technologies, investment decisions must soon be made if the new facilities are to be considered certificate eligible. An interesting theory states that the power plants needed to reach 26.4 TWh of new capacity before 2020 have already been planned, since electricity producers have a long planning horizon for future investments. If this is the case, movements in certificate price will be of little to no importance before 2020. Movements in certificate price will thus only affect investments in Sweden after 2020, and prior to this, only expected electricity matter.

D. Expected Prices in the Market

As described in section II-A there are two alternative ways to describe the price equilibrium in a TGC market. However, the market price expected by market players is not necessarily equal to the equilibrium price. Among players, there might be a lack of information and ability to model and forecast prices\(^3\). Thus actual price expectations may not be consistent with theoretical prices. For players in the certificate market, price expectation may play a role both in investment decisions and for trading purposes. Today there is a futures market for certificates where contracts can be traded up to five years ahead. The futures market is in contango, with prices increasing with time to maturity. Thus, price expectations seem to be in accordance with the martingale condition\(^4\). Morthorst argues that a liquid futures market for certificates might increase long-term transparency in pricing while stabilizing expectations [2].

III. EXAMPLE MODEL

This example model is an adaption of Coulon et al.’s modeling of the New Jersey Solar Renewable Electricity Certificate (SREC) market based on the equations described in their paper [7]. The model was chosen due to the promising results it has shown for the New Jersey SREC market. Additionally, to the best of the authors’ knowledge, this is the first implementation of such an approach for the Swedish-Norwegian market.\(^5\)

Minor adjustments to the equations have been made to include the possibility of infinite banking of certificates. Parameters have also been updated to reflect historical values from the Swedish market during the period 2004-2011.

A. Mathematical Formulation

\[
b_t = \begin{cases} 
\max(0, b_{t-1} + \int_{t-1}^t g_{du} - R_t) & t \in \mathbb{N} \\
\hat{b}_{t-1} + \int_{t-1}^t g_{du} & t \notin \mathbb{N}
\end{cases}
\]  

(1)

Eq. (1) keeps track of the accumulated number of certificates, banking, in the market at any given time. At any time step \(t\), the currently banked balance \(b_t\) is a function of the previous balance \(b_{t-1}\) and the accumulated issuance since the previous time step \(\int_{t-1}^t g_{du}\). If the current time step is part of the set of compliance dates \(N\), (1) accounts for a reduction in the number of certificates in the market, equal to the requirement \(R_t\) at the given date. The balance can never be negative, hence the max statement.

\[
p_t = \max_{\tau \in \{[t], [t+1], \ldots, T\}} e^{-r(t-\tau)} \mathbb{E}_t[1\{b_t=0\}] 
\]  

(2a)

\[
p_t = e^{-rT} \mathbb{E}_t(p_{t+1}) \ 	ext{when} \ t \notin \mathbb{N} 
\]  

(2b)

At a compliance date, the holder of a certificate will avoid the penalty imposed on those who do not comply as he hands in his certificate. Further, if the balance of certificates in the market directly following a compliance date is 0, one can assume that investors would at least have been willing to pay the amount of the penalty fee for one certificate. Eq. (2a) states that at any time \(t\), the value of the certificate \(p_t\) is the maximum of the discounted expected future penalty fees it can be used to avoid, discounted at the rate \(r\). Eq. (2b), i.e. the Martingale condition, follows implicitly from (2a) and states that, except at compliance dates, the current price is the discounted expected future price.

\[
g_t = \hat{g}_t(p) \exp(a_1 \sin(4\pi t) + a_2 \cos(4\pi t) + a_3 \sin(2\pi t) + a_4 \cos(2\pi t) + \epsilon_t)
\]  

(3)

The seasonality of electricity consumption, and hence certificate generation, is accounted for by a stochastic process on the form shown in (3). \(\hat{g}_t\) represents the annualized issuance of certificates and is a function of price, \(p\). This is motivated by the assumption that investors are likely to invest more while certificate prices are high. Seasonal changes are modeled by the sine and cosine terms.\(^6\)

\(^3\)Some players in the market may utilize advanced forecasts for future prices.

\(^4\)The price in one time step is the discounted expected price at the next time step.

\(^5\)From 2003-2011, the electricity certificate market did only include Sweden. Norway did not enter the market until 2012. Hence, the model has been implemented for the Swedish market, to be able to replicate historical price data.
cosine functions, while a noise term is added to reflect the uncertainty of generation.

\[
\frac{\ln(g_{t+\Delta t}) - \ln(g_t)}{\Delta t} = a_5 + a_6 \bar{p}_t, \text{ for } a_5 \in \mathbb{R}, a_6 > 0 \quad (4)
\]

Eq. (4) accounts for increase in generation. The price feedback parameter \(a_6\) accounts for the logical effect that producers are likely to invest more as prices rise. \(a_5\) represents the growth of generation not related to price increases. It is an independent term describing the drift in investments over time.

\[
p_t = \bar{p}_t \quad (5)
\]

Bellman introduced the term “the curse of dimensionality”, referring to the exponentially increased execution time associated with the introduction of another state variable [8]. While Coulon’s generalized model uses a weighted price average to calculate price feedback, it is here assumed, as stated in eq. (5), that the current average price equals the spot price. This is done to reduce dimensionality, and lower runtime. The result of this adjustment is immediate price feedback on generation.

B. Implementation

It is assumed that the requirements and the penalties are known and fixed for each year. The assumption is done for the purpose of computational tractability. Further work will investigate whether it is possible to solve this price model within a reasonable timeframe, without making this assumption.

The solution algorithm proceeds as follows:
1) A grid of values for \(b_t\) and \(\tilde{g}_t\) is chosen with lower bounds zero and upper bounds a little above the largest requirement. Time is discretized in monthly steps, matching the frequency of historical generation data.
2) The dynamic program is initialized, evaluating the payoff of the certificate at the end of the market’s life \(t = T\) for every single gridpoint \((b_t, \tilde{g}_t)\). At this point, all information is known and hence, the program yields a digital boundary price surface:
   a) At grid points where there is a shortage of certificates (i.e. the balance is less than the requirement), investors are willing to pay the penalty, \(p_T = \pi^T\), for one certificate.
   b) At grid points where there is a surplus of certificates (i.e. the balance is higher than the requirement), investors are willing to pay \(p_T = 0\) for one certificate.
3) From the boundary surface at \(t = T\), the dynamic program steps backward to \(t = (T - 1)\). Here it solves 1-5 at every grid point using price information from the price surface at \(t = T\). The same procedure is then followed for every time step; information from price surface \(t + 1\) is being used to solve 1-5 for every grid point of price surface \(t\) with Matlab’s \texttt{fsolve} function [9].

The algorithm provides one price surface for every single time step. The price surfaces show what the price would be at this time step, given a state \((b_t, \tilde{g}_t)\). An example of the resulting price surfaces is shown in figure 1. In order to compare the modeled prices to historical data, one needs to extract the modeled price for the historical levels of \((b_t\) and \(\tilde{g}_t)\) for every time step. For the given resolution level, the the runtime is approximately 2.5 hours on an Intel(R) Core(TM) i7-3770 CPU at 3.4 GHz.

C. Results and Interpretation

Comparing model output to historical prices, it is seen that historical prices are replicated fairly well with modeled and historical prices fluctuating around the same trend line. While the modeled prices capture long term trends quite well, fluctuations are not captured. The model is requiring prices to equal discounted future prices. These future prices are dependent on penalties occurring once a year, thus short term fluctuations will not be captured. This also follows from the frequency of the input data which never exceeds monthly. Between compliance dates, graphs are smooth and increasing due to certificate prices satisfying the martingale condition given by (2a) at all time steps. At compliance dates price drops are sometimes observed. These drops stems from foregone possibilities of using certificates for compliance.

The level of price feedback is determined by regression parameter \(a_6\). A higher \(a_6\) reflects a greater degree of flexibility among producers of electricity. Producers respond more rapidly to price increases, investing in more capacity to overcome a shortage of certificates in the market. Prices are slightly lower for a greater level of price feedback. Higher feedback levels dampens fluctuations from the trend line.

As risk increases, so does the required rate of return. The higher the required rate of return, the steeper
the associated price curves. The sensitivity analysis indicates that a required rate of return of 15% seems to produce the best replication of historical prices. One can argue that this is high. However, not only are investors in the electricity certificate market exposed to price risk. They are also exposed to regulatory risk [6]. This is the risk that changes in regulations will materially impact the certificate price. One reason why such changes might occur is the mathematical instability of certificate prices, mentioned in section II-B.

Further, it is observed that higher (lower) penalty fees yield higher (lower) prices. This is as expected, as the price of a certificate is a positive function of future penalty fees.

Examining the results from the three sensitivity analyses, it is found that the best replication of historical data are produced using a discount rate = 15% with penalty fees at historical levels. For parameter $a_6$, the results are inconclusive.

IV. CONCLUSIONS AND FURTHER WORK

The development over the market's first three years indicates that the goal of an additional 26.4 TWh of annual renewable capacity by 2020 is likely to be reached. Due to differences in the tax regimes of Norway and Sweden, some Swedish investment opportunities with higher LCoE are likely to be taken before some of their Norwegian counterparts with a lower LCoE, thus increasing the total social costs of the system. Whether the producers or the consumers are the ones carrying the cost of the system is debatable, however, the system is found to cause a reduction in the total social surplus. From this it is assessed that the system achieves the sought-after effects, but possibly at higher costs than necessary.

The effects of price changes have several important aspects. Due to the spiral dependence between previous certificate prices and penalty fees, it is pointed out that the system is mathematically unstable. However this instability has yet to result in abnormal changes in the certificate price. Further, the degree to which an increase in prices is met by an increased rate of investments in renewable capacity is unclear. However, from the regression done on historical data, this effect is assessed to exist.

The comparison of model output to historical Swedish certificate prices shows that though short-term fluctuations are not captured, long-term trends and price levels are replicated quite well. This indicates that the example model will be a suitable starting point for further work.

From sensitivity analyses done for the penalty fee, discount rate and the feedback effect, results are assessed to be in accordance with the expected behavior. Prices are observed to be positively dependent of the penalty fee, the slope of the price curves increase with the discount rate and an increased feedback effect dampens price fluctuations. Furthermore, the penalty fee and the discount rate seem to be the key drivers of the model.

Some important aspects of the market have yet to be implemented. In contrast to the fixed penalty used here, the penalties in the Swedish-Norwegian market depend on prices observed over the previous year. To include the penalty fee as an endogenous variable requires the introduction of another state variable, leading to a considerable increase in runtime. Generation of electricity, and thus the issuance of certificates depend on electricity prices and weather conditions. Including electricity price forecasts and weather forecasts into the model, allows for better estimates for the issuance of certificates, consequently increasing the quality of the certificate price forecasts. Further investigation in these points will result in a more sophisticated price model for the Swedish-Norwegian electricity market. This will be a useful tool for both investors and regulators.

REFERENCES

A A NOTE ON LOWERING MODEL RUN TIME FOR STOCHASTIC DYNAMIC PROGRAMS IN MATLAB

Appendices

A A note on lowering model run time for stochastic dynamic programs in Matlab

The introduction of a new state variable into a model leads to an exponential increase in model run time [18]. This was also true for the procedure presented in this paper, when the state variable $\pi_t$ was introduced. The model has however been kept computationally tractable by continuous improvement of the algorithm, which has lead to a dramatically decreased model run time.

A.1 Parallel processing

Due to the recursive nature of the algorithm described in this paper, the price cubes of every time step has to be calculated succeeding. However, for a given time step, solving the set of equations in grid points $(b_t, \hat{g}_t, \pi_t)$ are independent operations. This allows for parallel processing. The grid $b_t, \hat{g}_t$ is split into vectors, one vector for each $b_t$. Then for every point $b_t, \hat{g}_t$ of a given vector, the equations are solved. Splitting the grid into vectors, allows Matlab to exploit the power of multi-core processors. Iterating through the vectors with parfor loops from the Matlab Parallel Computing Toolbox, makes Matlab solve as many vectors as there are CPU-cores available, simultaneously. Solving the procedure locally on a quadcore processor, this procedure lowered the run time by 75 %, a substantial improvement. If one has access to a cluster, it is possible to scale this up, lowering the run time even further with Matlab cloudfor loops.

Figure 1: Parallel processing of a grid

![Parallel processing of a grid](image_url)
A.2 Introduction of Fortran subroutines

While Matlab is an intuitive programming language, it does not offer the fastest speed. It is an interpreted language, which means that most of its implementations execute instructions directly, without previous compiling of the program into machine-language instructions. Therefore, compiled languages like C, C++ and FORTRAN often offer substantial increases of speed. The *NAG Toolbox for Matlab*, is a set of subroutines written in FORTRAN. It offers FORTRAN implementations of many native Matlab functions. Swapping the native fixed point iteration procedure of Matlab, *fsolve*, with its FORTRAN subroutine counterpart, *nag_roots_withdraw_sys_func_easy (c05nb)*, more than doubled the speed of the electricity certificate model. The *c05nb* function finds a solution of a system of nonlinear equations by a modification of the Powell hybrid method [19].
B Extracting Smooth Forward Curves From Average-Based Electricity Contracts

As storage of electricity is extremely costly, electricity futures are traded on a constant delivery flow over the settlement period. Such contracts are classified as average-based forward contracts and are settled against the average spot price. In the Nordic market, such contracts are traded with different settlement periods. These settlement periods are often overlapping, which implies that for a given future date, several prices are observed in the market. For the contract with a yearly settlement period, one would pay the average yearly spot price, while for the contract with a monthly settlement period, one would pay the average spot price for that given month.

Benth et al. proposes the following model for extracting smooth forward curves from these contracts. The method takes the observed contracts and uses a Lagrangian method to create smooth forward curves, fitting the contracts to a seasonality function under a maximum smoothness criteria. For any given time to expiry, the resulting forward curves can be interpreted as the future price for a contract with daily settlement.

The forward price \( f(t) \) is decomposed into

\[
f(t) = s(t) + \varepsilon(t), \quad t \in [t_0, t_n]
\]

where \( s(t) \) represents a seasonality function and \( \varepsilon(t) \) represents an adjustment function that measures the forward curve’s deviation from seasonality.

For \( s(t) \) an estimate of the seasonality in the Nord Pool market, calibrated by Lucia and Schwartz [20] has been used.

\[
s(t) = 145.732 + 29.735 \times \cos \left( (t + 6.91) \frac{2\pi}{365} \right) \tag{1}
\]

The set of start and end dates of \( m \) contracts on the form \( F(T^S, T^E) \) is denoted

\[
\Phi = \{(T^S_1, T^E_1), (T^S_2, T^E_2), \ldots, (T^S_m, T^E_m)\}
\]

A new set of dates is constructed. Sorting the starting dates \( T^S \) and ending dates \( T^E \) in ascending order and removing duplicates yields a set on the form
B EXTRACTING SMOOTH FORWARD CURVES FROM AVERAGE-BASED ELECTRICITY CONTRACTS

\{ (t_1, t_2, ..., t_n) \}.

The adjustment function \( \varepsilon \) is implemented as a spline function on the form

\[
\varepsilon(t) = \begin{cases} 
\alpha_1 t^4 + \beta_1 t^3 + \gamma_1 t^2 + \delta_1 t + \epsilon_1 & t \in [t_0, t_1] \\
\alpha_2 t^4 + \beta_2 t^3 + \gamma_2 t^2 + \delta_2 t + \epsilon_2 & t \in [t_1, t_2] \\
\cdots & \\
\alpha_n t^4 + \beta_n t^3 + \gamma_n t^2 + \delta_n t + \epsilon_n & t \in [t_{n-1}, t_n]
\end{cases}
\]

To find the parameters

\[
x^T = [a_1 b_1 c_1 d_1 e_1 a_2 b_2 c_2 d_2 e_2, \ldots, a_n b_n c_n d_n e_n]
\]

of \( \varepsilon(t) \) the equality constrained convex quadratic programming problem

\[
\min_x \int_{t_0}^{t_n} [\varepsilon''(t; x)]^2 dt
\]

is solved subject to constraints in the connectivity and smoothness of derivatives at the knots, \( j = 1, \ldots, n - 1, \)

\[
(a_{j+1} - a_j) t_j^4 + (b_{j+1} - b_j) t_j^3 + (c_{j+1} - c_j) t_j^2 + (d_{j+1} - d_j) t_j + (e_{j+1} - e_j) = 0 \quad (3)
\]

\[
4(a_{j+1} - a_j) t_j^3 + 3(b_{j+1} - b_j) t_j^2 + 2(c_{j+1} - c_j) t_j + (d_{j+1} - d_j) = 0 \quad (4)
\]

\[
12(a_{j+1} - a_j) t_j^2 + 6(b_{j+1} - b_j) t_j + 2(c_{j+1} - c_j) = 0 \quad (5)
\]

\[
\varepsilon'(t_n; x) = 0 \quad (6)
\]

\[
P_s^C = \int_{T_t}^T w(r; t)(\varepsilon(t) + s(t)) dt
\]

where

\[
w(r; t_i) = \frac{e^{-r \tau_i}}{\sum_{j=1}^N e^{-r \tau_j}}
\]

Equations (2)-(7) is rewritten and solved with the method of Lagrange multipliers to obtain \( \hat{x} \) and \( \hat{\lambda} \).
C Predicting a long term factor for the electricity price as expected by investors

The logarithm of the electricity spot price is denoted by $X_t$. It is further decomposed into two factors:

$$X_t = \chi_t + \xi_t$$

(8)

$\chi_t$ represents the short-term deviation and $\xi_t$ represents the long-term equilibrium level. The short-term deviations are assumed to follow an Ornstein-Uhlenbeck process.

$$d\chi_t = -\kappa \chi_t dt + \sigma_\chi d\zeta_t,$$

(9)

The equilibrium level is assumed to follow a Brownian motion process.

$$d\xi_t = \mu_\xi dt + \sigma_\xi d\zeta_t.$$

(10)

$\chi_t$ and $\xi_t$ can be found to be jointly normally distributed with mean and covariance:

$$\mathbb{E}[(\chi_t, \xi_t)] = [e^{-\kappa t} \chi_0, \xi_0 + \mu_\xi t]$$

(11a)

$$\text{Cov}[(\chi_t, \xi_t)] = \begin{bmatrix}
(1 - e^{-2\kappa t}) \frac{\sigma_\chi^2}{2\kappa} & (1 - e^{-\kappa t}) \frac{\rho_{\chi \xi} \sigma_\chi \sigma_\xi}{\kappa} \\
(1 - e^{-\kappa t}) \frac{\rho_{\chi \xi} \sigma_\chi \sigma_\xi}{\kappa} & \sigma_\xi^2 t
\end{bmatrix}$$

(11b)

Thus, the logarithm of the spot price is normally distributed with:

$$\mathbb{E}[X_t] = e^{-\kappa t} \chi_0 + \xi_0 + \mu_\xi t$$

(12a)

$$\text{Var}[X_t] = (1 - e^{-2\kappa t}) \frac{\sigma_\chi^2}{2\kappa} + \sigma_\xi^2 t + 2(1 - e^{-\kappa t}) \frac{\rho_{\chi \xi} \sigma_\chi \sigma_\xi}{\kappa}$$

(12b)

Further, the expected spot is then log-normally distributed, and can be found from:

$$\ln(\mathbb{E}[S_t]) = \mathbb{E}[X_t] + \frac{1}{2} \text{Var}[X_t]$$

$$= e^{-\kappa t} \chi_0 + \xi_0 + \mu_\xi t + \frac{1}{2} \left( (1 - e^{-2\kappa t}) \frac{\sigma_\chi^2}{2\kappa} + \sigma_\xi^2 t + 2(1 - e^{-\kappa t}) \frac{\rho_{\chi \xi} \sigma_\chi \sigma_\xi}{\kappa} \right)$$

(13)

Thus, a forecast of future spot prices can be calculated given the state variables $\chi_t$ and $\xi_t$, and the required parameters.

A standard Kalman filter was applied for the calculation of the state variables, $\chi$ and $\xi$, and the required parameters for the electricity forecast, $\kappa$, $\sigma_\chi$, $\sigma_\xi$, $\rho_{\chi \xi}$, and $\mu_\xi$. The evolution of the state variables is described by the following transition equation.
\[ x_t = c + Qx_{t-1} + \eta_t, \ t = 1, \ldots, nT \]  

(14)

where:

- \( x_t = [\chi_t, \xi_t] \), a 2×1 vector of state variables
- \( c = [0, \mu \Delta t] \), a 2×1 vector
- \( Q = [\chi_t, \xi_t] \), a 2×2 matrix

\( \eta_t \) is a 2×1 vector of serially uncorrelated normally distributed disturbances with \( \mathbb{E}[\eta_t] = 0 \) and

\[
\text{Var}[\eta_t] = \begin{bmatrix}
(1 - e^{-2\kappa \Delta t}) \frac{\sigma^2}{2n} & (1 - e^{-\kappa \Delta t})(2\chi_t^2 \sigma_x^2 \sigma_{\xi t}^2) \\
(1 - e^{-\kappa \Delta t}) \frac{\sigma_{\xi t}^2 \sigma_x^2 \sigma^2}{n} & \sigma^2 \Delta t
\end{bmatrix}
\]

Based on these equations the Kalman filter is run recursively. In each time-step the observed forward prices and the previous time-step’s estimates of the state variables to calculate new estimates for the state variables. Further a correction is made in every time-step based on the difference between the actual observed prices and the prices following from the estimates variables. The estimation is done for an assumed set of parameters. The likelihood of the observations can be calculated for a given set of parameters. The Kalman filter can then be re-run, varying set of parameters to identify the set that maximizes the likelihood function. Finally, a solution for the state variables and the set of parameters is obtained.
D  Regression Error Term Characteristics

Figure 3: Characteristics of the coefficient regression error terms