



NTNU – Trondheim
Norwegian University of
Science and Technology

Operation, Valuation and Electricity Sourcing for a Generic Aluminium Smelter

Eivind Fossan Aas
Sven Henrik Andresen

Industrial Economics and Technology Management

Submission date: June 2015

Supervisor: Stein-Erik Fleten, IØT

Norwegian University of Science and Technology
Department of Industrial Economics and Technology Management

Problem description

This thesis aims to analyse how electricity sourcing affects the operation and value of an aluminium smelter. The smelter can be shut down temporarily and permanently. Electricity procurement may vary with respect to electricity price, contract length and contract currency. There is uncertainty in exchange rates, metal prices and electricity prices, and these factors are modelled.

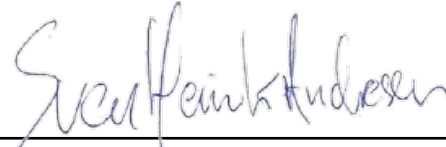
Preface

We have written this thesis as a fulfillment of our Master of Science degree in Industrial Economics and Technology Management at the Norwegian University of Science and Technology (NTNU). We would like to express a special appreciation and thanks to our supervisor, professor Stein-Erik Fleten at the Department of Industrial Economics and Technology Management (IØT), for the assistance offered. His participation, constructive criticism and knowledge in topics within option pricing, portfolio optimisation and commodity markets were of high importance in order to complete the thesis.


We would also like to express our gratitude towards Dr. Selvaprabu Nadarajah at the University of Illinois at Chicago and Denis Mazieres at Birkbeck, University of London and HSBC for acting as mentors and sharing their time, knowledge and expertise with us through constructive discussions during the process.

Finally, we are pleased to have been given the opportunity to present our paper at the 12th NESS conference in Trondheim, Norway, 9-11th of June, 2015 and that our supervisor has offered to present our paper at the ROW15: Real Option Workshop in Lappeenranta, Finland, 18-19th of August, 2015.

June 9, 2015



Sven Henrik Andresen



Eivind Fossan Aas

Abstract

Electricity prices vary across different geographic locations and affect the relative cost position of individual aluminium producers. Understanding the scope of electricity price risk is thus of high importance to industry players. We propose a sequential valuation and optimisation approach for investigating the relationship between operating policy, electricity sourcing and smelter value. The hybrid optimisation approach determines a heuristic operating policy with the least squares Monte Carlo (LSM) method and uses portfolio optimisation to find a corresponding electricity procurement scheme. We find that the resulting procurement scheme reduces the risk of shutdowns without compromising smelter value. In addition, the procurement scheme obtained when using demand derived from the heuristic operating policy outperforms the one found when treating demand as constant. Our findings show that there is substantial value in operational flexibility and suggest that decisions on electricity sourcing should be coupled with the operating policy. This could motivate the industry to adapt a valuation approach that captures the full value of operational flexibility and yields a corresponding operating policy.

Sammendrag

Prisen på elektrisitet kan variere i stor grad mellom geografiske områder, og påvirker konkurranseevnen til lokale aluminiumsprodusenter relativt til globale aktører. Det er derfor viktig at en aluminiumsprodusent har innsikt i omfanget av risikoen som er knyttet til elektrisitetsprisen. I denne oppgaven foreslår vi en sekvensiell verdsettelses- og optimeringsmetode for å undersøke forholdet mellom driftsstrategi, elektrisitetskjøp og verdi til et smelteverk. Den hybride optimeringstilnærmingen finner en heuristisk driftsstrategi for smelteren ved å benytte least squares Monte Carlo-metoden, for så å bruke porteføljeoptimering til å finne en tilhørende innkjøpsplan for elektrisitet. Resultatene tilsier at denne innkjøpsplanen reduserer risikoen for nedstengelser uten å gå på bekostning av verdien av smelteverket. Videre så gjør den resulterende innkjøpsplanen det bedre enn innkjøpsplanen generert når etterspørsel etter strøm antas å være konstant. Våre funn viser at det er betydelig verdi i operasjonell fleksibilitet hvilket impliserer at avgjørelser om elektrisitetskjøp burde tas basert på driftsstrategien. Dette kan motivere aluminiumsprodusenter til å ta i bruk en verdsettelsesmetode som fanger den fulle verdien av operasjonell fleksibilitet og som samtidig utarbeider en tilhørende driftsstrategi.

Contents

1	Introduction	1
2	Market and Institutional Context	3
2.1	Aluminium Production	3
2.2	The Aluminium Market	5
2.3	The Nordic Electricity Market	7
2.4	The Foreign Exchange Market	8
3	Dynamics of Risk Factors	11
3.1	Aluminium Price	11
3.2	Electricity Price	13
3.3	Exchange Rates	13
4	Methodology	15
4.1	Operating Policies and Smelter Valuation	15
4.2	Portfolio Optimisation	16
5	Summary and Contributions	19
6	Further Research	21
	Bibliography	23
	Appendix A Mathematical Elaborations	27
A.1	Calibrating Parameters of the Three-factor Extension	27
A.2	Correlated Random Draws	29
A.3	Calculating Expected Electricity Prices	29

Chapter 1

Introduction

Following the recent financial crisis, the aluminium industry has suffered from tight market conditions. Since it is a global industry, the relative cost positions of local producers are put under pressure by highly varying local electricity prices and fluctuating foreign exchange rates. In strive for competitive edge producers are thus focusing their efforts towards cost reductions and risk management. The main research question addressed in this thesis is:

How does electricity sourcing affect the operation and value of a generic smelter with operational flexibility? Three sub-questions follow:

1. What heuristic operating policy maximises the value of the smelter in an environment with uncertain aluminium prices, electricity prices and exchange rates?
2. Given an optimal policy, how should electricity be sourced to reduce cost and at the same time be aligned with management's appetite for risk?
3. How do different electricity procurement schemes perform when compared in terms of shutdown risk and smelter value?

The main contribution of this thesis is an article, "Operation, Valuation and Electricity Sourcing for a Generic Aluminium Smelter", addressing issues regarding operations and electricity sourcing for an aluminium smelter. The article [3] introduces a sequential valuation and optimisation approach for evaluating a smelter with operational flexibility and deriving a risk minimising electricity procurement scheme. The first step is to find a heuristic operating policy that maximises the expected payoff from the smelter. Due to the complexity of the problem we apply a numerical method to obtain a heuristic policy and an approximation of the smelter value. The approach used is the least squares Monte Carlo (LSM) method [18]. In the next step this policy is used as input to a two-stage stochastic program to find an optimal procurement scheme for electricity. The impact on shutdown risk and smelter value from the procurement scheme is finally evaluated by re-solving the first step of the solution approach, assuming that electricity is procured accordingly. At this final step the scheme is also compared to a range of benchmarks.

The thesis is organised as follows. Chapter 2 offers an introduction to the value chain of aluminium production and market insight for the aluminium, Nordic electricity and foreign exchange markets. Chapter 3 discusses the dynamics of the main risk factors in aluminium production. The LSM method and two-stage stochastic program are briefly presented in Chapter 4. Summary and contributions are provided in Chapter 5. In Chapter 6 we discuss limitations of our approach and make suggestions for further research. Finally, the article considered as the main contribution of this thesis is attached.

Chapter 2

Market and Institutional Context

The following sections provide a description of the value chain in aluminium production and offer a brief introduction to the aluminium, electricity and foreign exchange markets.

2.1 Aluminium Production

The production process of aluminium goes as follows. In a metal plant alumina is processed into aluminium using the Hall-Héroult process. Alumina is dissolved into molten cryolite and undergoes an electrolytic reduction to obtain aluminium. The process is extremely energy intensive, as a direct current of 150 to 250 kA is necessary to obtain the electrolytic reduction [14]. The process takes place in a bath of hot cryolite (around 960°C), hence access to reliable power sources is a necessity to ensure a high temperature in the bath at all times [10]. After the molten aluminium is extracted from the smelter, it is placed in large furnaces before being casted into other products. In the furnaces, the pure aluminium holds a temperature higher than 700°C while it is alloyed by adding other elements to further strengthen the material. The metal is then casted into different products specified by the end user. This final step is done in a casthouse. Producing aluminium is considered continuous, meaning that once the smelter is operating, it must continue to operate at all times in order to maintain a high temperature in the electrolytic baths. Short interruptions in the production process could potentially damage and reduce the lifetime of the pots due to cooling cracks in the cathode [41]. Aluminium producers do, however, have the optionality to shut down the smelter for longer time periods when facing unfavourable market conditions, but restarting the smelter entails high costs. The production process described above requires three main inputs; i) alumina, ii) electricity and iii) carbon.

i) Alumina is the direct base for aluminium and is refined from bauxite, a mineral that contains about 15-25% aluminium [25]. Bauxite is usually found several meters underground in a belt around the equator. After recovering bauxite the mineral is transported to crushing or washing plants before it is processed into aluminium oxide, commonly known as alumina. Mining of bauxite has become a multinational industry, and large aluminium producers tend towards complete vertical integration by acquiring their own mining facilities or engaging in joint ownerships with mining companies

[13][23]. Alumina refineries are typically constructed close to and dedicated to specific areas of bauxite mining, since bauxite is heterogeneous in terms of chemical characteristics based on its origin and is bulky in nature. The countries with the largest production of bauxite are Australia (30%), China (18%), Brazil (13%) and Indonesia (12%) [38]. In January 2014, Indonesia banned bauxite exports in order to motivate investments in domestic aluminium smelters. This could potentially have some impact on the global market for bauxite, as China is a net importer of bauxite, mainly from Indonesia. Thus, global prices of bauxite could strengthen somewhat due to lower supply [26]. Alumina prices tend to be closely linked to bauxite and aluminium prices, but industry practice is now moving in the direction of a separate price index for alumina [39].

ii) Carbon accounts for about 13% of the total production cost of primary aluminium [13], and is used for the cathodes and anodes in the electrolysis process of aluminium production. It is common for aluminium producers to own carbon electrode plants close to the smelter. The usage of carbon electrodes does lead to carbon emissions and certain countries have introduced a tax on carbon emissions, giving local producers a competitive disadvantage.

iii) Electrical power stands on average for 30% of the production cost of primary aluminium. Since electricity prices vary strongly across different geographic locations, electricity cost may be a source of competitive advantage for some producers. The average cost can vary from \$400 to \$1,000 per metric ton (mt) produced aluminium between industry players [13]. Aluminium producers have three options for electricity sourcing; they can purchase electricity through short-term or long-term commitments with electricity producers or invest in power plants. Since electricity is a dominating cost, several European aluminium producers own power generating assets despite it being considered a capital intensive strategy. In addition, to soften the effect of electricity price spikes, producers commonly trade in energy derivatives. Since 1980 there has been a nearly linear decrease in the average required MWh/mt produced aluminium, from 17 MWh/mt to 14.5 MWh/mt in 2012. Note that this is an average ratio, and that aluminium producers may lie above or below this ratio. A simple forecast based on a polynomial regression (see Figure 2.1) indicates that the ratio could converge towards 13.5 MWh/mt in the long-run if the same pattern continues. Despite increasing energy efficiency, increased energy costs have the last few years forced several aluminium producers to shut down their smelters.

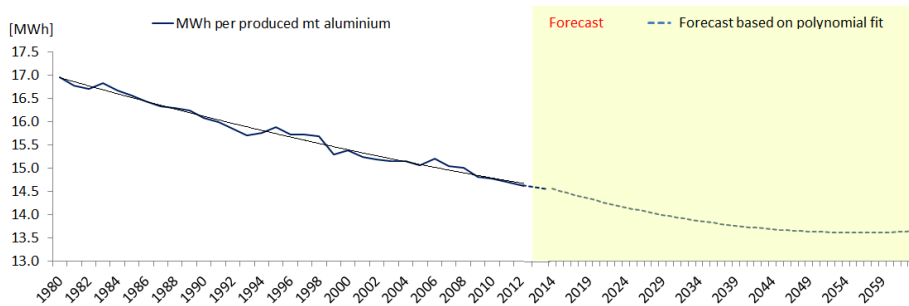


Fig. 2.1 Power consumption per produced mt primary aluminium [MWh/mt]. Source: The International Aluminium Institute.

2.2 The Aluminium Market

Aluminium was introduced on the London Metal Exchange (LME) in the end of 1978, whereas the contract that is traded today was introduced in 1987 [17]. In the period 1978-1996 the global production of aluminium experienced a compound annual growth rate (CAGR) of 2.2%. This more than doubled to 5.3% in the period 1996-2014, mainly driven by a strong increase in Chinese aluminium production (see Figure 2.2). Demand has, however, not always matched the same growth, especially the last few years. This has caused challenging market conditions for aluminium producers.

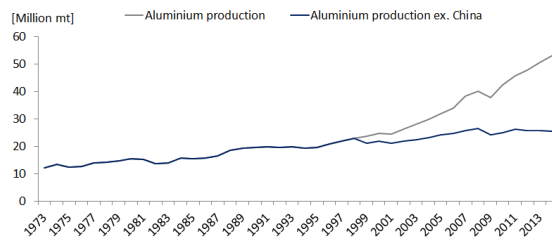


Fig. 2.2 Historical annual aluminium production 1973-2014. Source: The International Aluminium Institute.

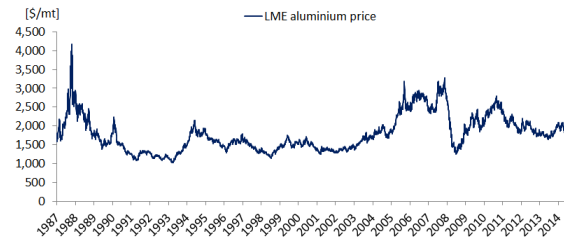


Fig. 2.3 Historical daily spot LME ring trade close aluminium price 1987-2015. Source: Reuters EcoWin Pro.

The LME price per mt aluminium in the time period 1987-2015 is plotted in Figure 2.3. In the period between 1987 and 1990 there was an extreme peak in the aluminium price, when it doubled to more than \$4,000 /mt before crashing. The rapid price increase was caused by low inventories and closed overcapacity combined with a strong increase in demand, which in turn resulted in a very tight supply and demand situation [30]. However, the dissolution of the Soviet Union caused large amounts of Russian aluminium to enter the market. Combined with heavy speculative trading in the futures market this caused the price to plunge. Following this volatile period the price fluctuated in the interval \$1,300-\$1,800 until the pre-financial crisis years. In this time period, increased period-to-period volatility and a weaker relationship between supply and demand and the price of the commodity, have been attributed to the financialization of commodities [6], a phenomenon caused by increased trading in futures. From 2005 to 2006 the price drastically increased from \$1,700 to

\$3,200 before varying between \$2,500 and \$3,200 until 2008, when the financial crisis struck. In a matter of few months the price dropped from \$3,200 to \$1,300 at its lowest. Since the aluminium price is quoted in U.S. dollars the price itself may be influenced by changes in the U.S. dollar trade weighted exchange rate, which is plotted in Figure 2.4. The movements in the aluminium price related to the financial crisis were strongly negatively correlated with this latter ratio. Within a short time after the price dropped a rebound wave materialized, which slowly died out and we currently see an aluminium price of around \$1,850-\$1,900. This decrease can partly be attributed to lower demand growth from China.

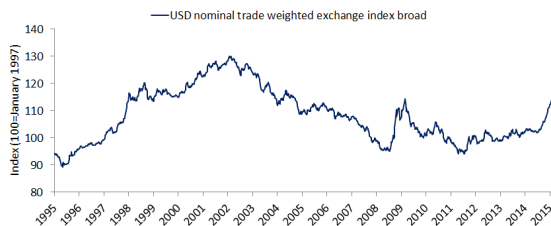


Fig. 2.4 USD nominal trade weighted exchange index broad, Federal Reserve. January 1997=100. Source: Reuters EcoWin Pro.

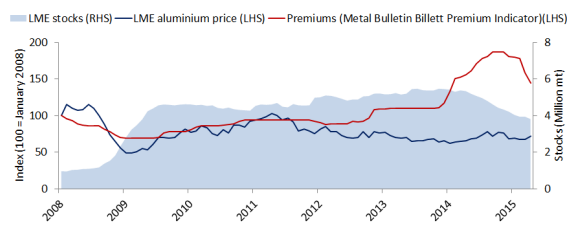


Fig. 2.5 LME stock levels, LME spot aluminium price and Metal Bulletin Billet Premium Indicator. January 2008=100. Source: Norsk Hydro Q1 Presentation 2015.

The buyer of aluminium often pays a premium on top of the aluminium price quoted at the LME. Cost of delivery and insurance were originally the factors determining the premium. However, in recent times analysts argue that the premium to a larger extent reflects market fundamentals [5]. Premiums have become a means for price negotiation and leverage for buyers to convince sellers to sell the metal instead of storing it. Figure 2.5 shows a positive correlation between storage levels and premiums the past couple of years. In addition, there seems to be a negative correlation between the aluminium price and premiums, which further strengthens the argument that premiums partly reflect market fundamentals. For the reasons mentioned above, premiums have fluctuated strongly the past few years increasing to high levels in late 2014. Premiums have however dropped drastically through April 2015, and are expected to drop even further by industry sources.

There is consolidation in the primary aluminium industry. As discussed earlier, the current market situation is tight, but the shutdown of capacity has brought some relaxation to a tight supply and demand situation. From the middle of 2014 demand has exceeded production (excl. China), and there is a physical market deficit. However, the global aluminium market including China is slightly oversupplied [27]. The strongest outlooks for demand growth are in the U.S. and in South-East Asia, while the eurozone is softening. Demand growth in 2014 excl. China was 3%. There have also been shifts in the end use applications of primary aluminium, which helps relieve the expected decrease in demand growth from China. New areas of application are e.g. within the automotive industry, transportation, consumer electronics, solar panning and wind farms. The end-use areas with the highest CAGR from 2004-2014 were electronics, construction and transport respectively [28], and the industry expects a CAGR of 20% in automotive demand the next eight years. Currently transport,

construction and electronics make up the largest share of demand.

2.3 The Nordic Electricity Market

Nord Pool Spot is Europe's leading power market, and offers both day-ahead and intraday trading for physical delivery of electricity. The power market has more than 380 members from 20 countries, and has a market share of 84% in the Nordic and Baltic region according to the 2013 annual report. Nord Pool Spot is licensed by the Norwegian Water Resource and Energy Directorate (NVE) to organise and operate the power market, and by the Norwegian Ministry of Petroleum and Energy to facilitate the power market with foreign countries [24].

In 2010, Nord Pool's marketplace for financial electricity contracts was acquired by NASDAQ OMX and is now known under the trade name NASDAQ OMX Commodities Europe. The most liquid financial contracts have a time horizon of up to three years and all traded financial contracts use the Nord Pool Spot system price as a reference price. They are cash settled, meaning that there is no physical delivery of electricity.

Since we value a generic smelter located in the Nordic region, we use the 1-year forward contract on the Nord Pool Spot system price as a proxy for the price of electricity. We set the prices in long-term bilateral contracts based on conditional expected prices of the 1-year forward price, a procedure that is thoroughly explained in Appendix A.3. Figure 2.6 shows a plot of the historical Nord Pool 1-year forward system price from 2001 to 2015.

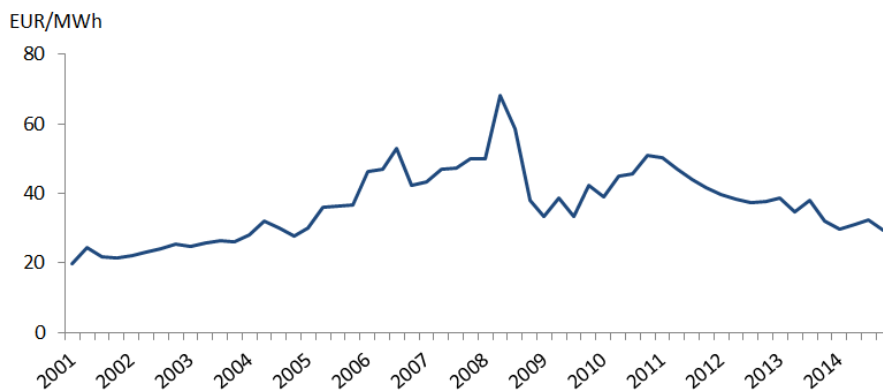


Fig. 2.6 Nord Pool 1-year forward system price, quarterly intervals. Source: Reuters EcoWin Pro.

In Figure 2.6 we see that the electricity price increased steadily from 2001 to 2008. From 2008 to 2009 the price fell significantly, but has remained stable thereafter. The system price peaked for a short period of time prior to the financial crisis in 2008. This was due to a new cap-and-trade quota system on CO₂ emissions launched by the European Union Emissions Trading System for the period 2008-2012. The quota system put an upward pressure on fossil fuel power plants, thereby affecting the system price of electricity in the Nordic region. The financial crisis led to a reduction in global production levels, and as a consequence there was an oversupply of emission quotas. Thus,

a correction in the system price took place during the first half of 2009, in which it returned to pre-quota levels [12].

Several factors may influence the Nord Pool Spot system price in the years to come. In an effort to integrate the transmission system in Europe, two new transmission cables are under construction from Norway to Germany and Great Britain, a market change that is expected to put an upward pressure on the system price. Other important factors are the price of CO₂ quotas, share of production capacity from renewable sources and hydro reservoir levels [35].

2.4 The Foreign Exchange Market

Currencies are traded in the the foreign exchange market. It is by far the largest market in the world in terms of value, and operates continuously during weekdays. Hence, it is characterised as one of the most efficient markets in the world.

Foreign exchange rates can be fixed or floating. With a fixed exchange rate, one currency is pegged to another. This implicates that monetary policies are undertaken in order to maintain a constant exchange rate. The contrary to fixed exchange rates are floating exchange rates. With this practice, other targets than only the foreign exchange rate determine the monetary policies undertaken. Hence, exchange rates may fluctuate.

Several factors affect floating exchange rates. They usually fall into three main categories; economic factors, political factors and market psychology. Economic factors are mainly fiscal policies from central banks, government spending, government surplus and deficits, balance of trades and economic growth. Political stability and anticipations make up the political factors. Political instability often has negative effects on a nation's economy, hence negatively impact foreign exchange rates. On the other hand, a responsible government may have stimulating effects in periods with financial difficulties. Psychological effects are e.g. speculations and rumors, expectations regarding long and short-term trends and fear of capital flight.

Since the price of aluminium is denominated in U.S. dollars and parts of the costs are incurred in local currencies, a local aluminium producer is exposed to fluctuating exchange rates. An appreciation or depreciation of the local currency against the U.S. dollar will only impact the relative cost position of the local producer and not competitors. In this thesis we consider a smelter located in the Nordics, hence we will focus on the U.S. dollar/Euro (USD/EUR) and U.S dollar/Norwegian krone (USD/NOK) floating exchange rates.

The euro currency was first introduced in January 1999, thus we use an approximated exchange rate from Reuters EcoWin Pro to extend the length of the time series. We observe in Figure 2.7 that the USD/EUR exchange seems to be mean reverting around a stationary level of approximately 1.2, however there are longer time periods where the exchange rate deviates from this level. During the recession in the early 1980s, when financial instability hit most industrial countries, we observe an appreciation of the U.S. dollar to the approximated euro. We also observe unusually high short-term

volatility during the currency crisis in 1992 and financial crisis in 2008.

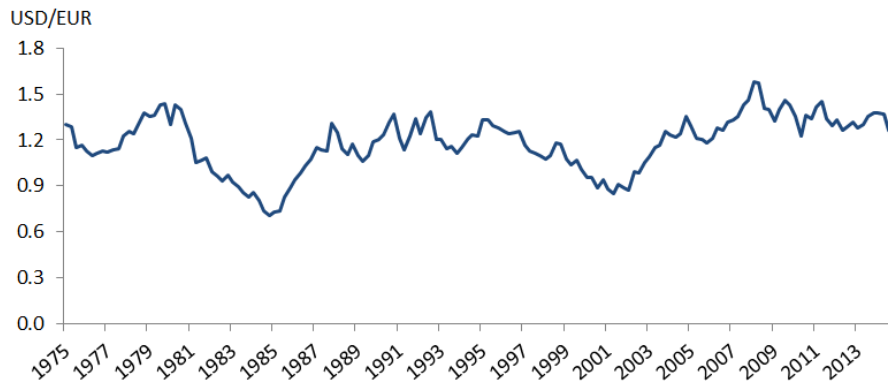


Fig. 2.7 USD/EUR exchange rate, quarterly intervals. Source: Reuters EcoWin Pro.

Figure 2.8 shows historical data for the USD/NOK exchange rate. The exchange rate seems to be mean reverting around a stationary level of approximately 0.15. Comparing historical data of the USD/EUR and USD/NOK exchange rates in Figure 2.7 and Figure 2.8 the exchange rates seem to follow a somewhat similar pattern, which indicates a possible positive correlation between these.

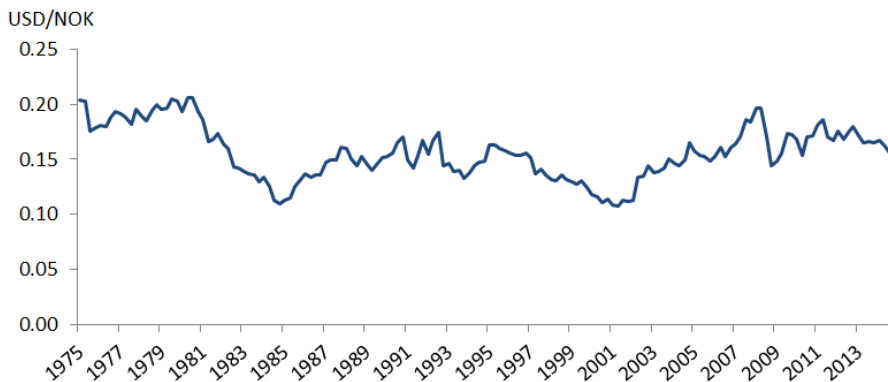


Fig. 2.8 USD/NOK exchange rate, quarterly intervals. Source: Reuters EcoWin Pro.

Chapter 3

Dynamics of Risk Factors

3.1 Aluminium Price

[9] suggest the use of a mean reverting process for modelling the stochastic behaviour of commodity prices. The intuition behind mean reversion in commodity prices comes from basic microeconomic theory. This states that when prices increase, high cost producers will enter the market, which in turn will increase the supply and push down the price. Conversely, when prices are low, high cost producers will leave the market, which will decrease the supply and increase prices. Purchasing of commodities often has a time aspect to it, since immediate delivery is usually not possible. The value of having the commodity now as opposed to in the future is captured by the convenience yield and can be positive or negative. Convenience yield is normally subtracted from the drift of the stochastic process used to describe the dynamics of the commodity. One basic single-factor mean reverting process is the Ornstein-Uhlenbeck process as described in e.g. [9]. Jumps have also been added to form a mean reverting process with jump diffusions, which has been popular for capturing the short-term dynamics of the electricity price, also seen in [9]. Despite a strong position in existing literature, reversion to a constant mean in commodity prices is an increasingly discussed topic. [36] summarises some of the literature that document the non-presence of mean reversion in commodity prices. There are arguments that the standard supply and demand relationship from microeconomics no longer holds in many commodity markets due to the financialization of commodities, which makes the pricing mechanisms in the markets much more complex. Alternative proposed processes are random walk or geometric Brownian motion (GBM), the latter much used in real options due to its mathematical convenience.

The processes discussed above are single-factor processes that mainly originate from pre-1990. The superiority of multi-factor models over a single-factor model for commodities, is discussed by among other [7],[15], [29] and [31]. Two such models are the two-factor model and three-factor extension presented in [32] and [33]. They claim that the spot price of oil is determined by two different factors, namely short-term deviations and a fluctuating equilibrium price. Short-term deviations are caused by factors such as changes in inventory levels and seasonality, while changes in

the long-term level are determined by macroeconomic factors such as technology and the geopolitical environment. This approach captures the mean reverting behaviour of commodity prices, but allows for uncertainty in the equilibrium level to which prices revert. Dynamics in simulations are thus enriched as opposed to with a single-factor process. The popularity of futures trading further emphasizes the interest in understanding the long-term dynamics. [33] and [32] have been devoted much attention (see [1], [8] and [36]), and their results have been applied in a range of real options problems. Calibration of the three-factor extension can be done by using the Kalman Filter. Refer to Appendix A.1 for details of the calibration procedure.

To evaluate the presence of mean reversion in the aluminium price it is relevant to conduct statistical hypothesis testing of stationarity. In order to test whether a time series has a stationary mean level to which it reverts one can among other conduct an augmented Dickey-Fuller (ADF) test, Kwiatkowski–Phillips–Schmidt–Shin (KPSS) test and variance ratio (VR) test. A high-level overview of how to interpret the results from these tests can be found in Table 3.1. Results from the tests applied on historical monthly, quarterly and yearly spot LME aluminium prices are shown in Table 3.2.

TABLE 3.1 INTERPRETATION OF RESULTS FROM STATISTICAL TESTS

Statistical test	Hypotheses	Conclusion
ADF	H_0 : Series contains a unit-root	False= Cannot reject hypothesis of unit-root. Not stationary
	H_1 : Series does not contain a unit-root	True= Can reject hypothesis of unit-root. Stationary.
KPSS	H_0 : Series is trend or level stationary	False= Cannot reject hypothesis of trend-stationarity.
	H_1 : Series is not trend or level stationary	True= Can reject hypothesis of trend-stationarity.
VR	H_0 : Series is a random walk.	False= Cannot reject hypothesis of a random walk. Not stationary.
	H_1 : Series is not a random walk	True= Can reject hypothesis of a random walk. Stationary.

TABLE 3.2 STATISTICAL HYPOTHESIS TESTING OF STATIONARITY

Period	ADF1	ADF2	ADF3	KPSS1	KPSS2	KPSS3	VR	Period	ADF1	ADF2	ADF3	KPSS1	KPSS2	KPSS3	VR
1987-2015	FALSE	FALSE	FALSE	TRUE	TRUE	TRUE	FALSE	1987-2015	FALSE	FALSE	FALSE	TRUE	TRUE	TRUE	FALSE
1987-2007	FALSE	FALSE	FALSE	TRUE	TRUE	TRUE	FALSE	1987-2007	FALSE	FALSE	FALSE	TRUE	TRUE	TRUE	FALSE
2009-2015	FALSE	FALSE	FALSE	TRUE	TRUE	TRUE	FALSE	2009-2015	FALSE	FALSE	FALSE	TRUE	TRUE	TRUE	FALSE

(a) Results from statistical hypothesis testing monthly time series

(b) Results from statistical hypothesis testing log of monthly time series

Period	ADF1	ADF2	ADF3	KPSS1	KPSS2	KPSS3	VR
1987-2015	FALSE	FALSE	TRUE	TRUE	TRUE	TRUE	FALSE
1987-2007	FALSE	FALSE	FALSE	TRUE	TRUE	TRUE	FALSE
2009-2015	FALSE	FALSE	FALSE	TRUE	TRUE	TRUE	FALSE

(c) Results from statistical hypothesis testing quarterly time series

(d) Results from statistical hypothesis testing log of quarterly time series

Period	ADF1	ADF2	ADF3	KPSS1	KPSS2	KPSS3	VR
1987-2015	FALSE	TRUE	FALSE	TRUE	FALSE	FALSE	FALSE

(e) Results from statistical hypothesis testing yearly time series

Period	ADF1	ADF2	ADF3	KPSS1	KPSS2	KPSS3	VR
1987-2015	FALSE	TRUE	FALSE	TRUE	FALSE	FALSE	FALSE

(f) Results from statistical hypothesis testing log of yearly time series

We see from Table 3.2 that there is some evidence of stationarity for low granularity time series. However, for high granularity levels and according to the VR test there is no evidence of stationarity. In addition, there are two potential issues posed by the small time frame of the historical data. First,

the Augmented Dickey-Fuller test often fails to reject a unit-root for short time series [2]. Secondly, it is difficult to distinguish between mean reversion and random walk for data within a small time frame. Finally, it can be argued that a sample set of only 27 observations is not sufficiently large to capture long-term dynamics with a single-factor process.

Due to the issues discussed above regarding proving a stationary mean level for the aluminium or log aluminium price, a single-factor mean reverting process for the aluminium price seems inapplicable. Recent literature suggests the use of a multi-factor model to capture the dynamics of commodity prices. One such process that is widely accepted in the literature is the three-factor extension in [32]. Therefore, we use this in the article [3] to capture the dynamics of the aluminium price.

3.2 Electricity Price

Certain characteristic properties of electricity affect the dynamics of the electricity price. Most importantly, electricity cannot be stored. Hence, it is subject to real-time consumption and relies on prices to balance supply and demand. Furthermore, electricity is dependent on a transmission system to be transported from the producer to the consumer. Therefore, constraints on the transmission capacity between regions contribute to differences in the electricity price across geographic locations. Electricity prices are highly cyclical due to rapid changes in supply and demand, and fluctuations are often daily, weekly and yearly. There exists extensive literature on how to capture the dynamics of hourly and weekly movements in the electricity price, and price processes often include combinations of autoregressive components, GARCH models and jump diffusion components, e.g. refer to [34].

The purpose of this thesis is to evaluate strategic decisions made on a yearly basis. It is thus of less relevance to adapt a process that mainly focuses on capturing the short-term dynamics of the electricity price. As opposed to for short-term prices, the literature on long-term electricity price trends is limited. [19] argue that the dynamics of electricity prices can be captured with a two-factor model that takes both short-term and long-term fluctuations into account. Long-term dynamics of the price are calibrated from historical forward curves. However, there is limited historical data on Nordic electricity forward curves. On the grounds of this and the fact that rapid short-term fluctuations in the electricity price are irrelevant from a lower granularity point of view, we argue that the long-term dynamics of the electricity price needed for our purpose can successfully be captured with a single-factor autoregressive process.

3.3 Exchange Rates

[22] argue that forecasting nominal exchange rates using empirical macroeconomic processes is close to impossible, since most time series processes fail to beat the random walk.

Recent research, such as [11] and [37], conclude that even though exchange rates seem to move

randomly in the short-run, the medium and long-term behaviour of real exchange rates should be forecasted based on the theory of purchasing power parity (PPP). The International Monetary Fund defines this as: "*The rate at which the currency of one country would have to be converted into that of another country to buy the same amount of goods and services in each country*". In other words, a basket of goods in two different countries should have the same price when expressed in the same currency. [40] use this theory to argue that in the long-run, exchange rates revert back to a stationary mean level. They compare processes based on PPP to the random walk, and conclude that the random walk performs just as well as processes based on PPP in the short-run, but to accurately forecast long-term effects of real exchange rates mean reversion is required.

In this thesis we are interested in the long-term dynamics of exchange rates, since the time frame considered for the aluminium smelter is 40 years. It follows from the arguments mentioned above that a random walk or other non-stationary processes for foreign exchange rates may yield unrealistic extreme values in long-term simulations. Therefore, we adapt a process for exchange rates that reverts to a stationary mean level in the long-run.

A fitted process can be evaluated with Q-Q plots of residuals and plots of historical volatility. A Q-Q plot illustrates to what extent the residuals from the fitted process resemble the residuals drawn from a normal distribution. Further on, extreme deviations are easily identified and can be investigated. These may in fact often be explained by extraordinary events. Volatility plots are used to check for constant volatility, and should show no signs of volatility clustering. Volatility should therefore be the same for both high and low values of the studied real exchange rate.

Chapter 4

Methodology

4.1 Operating Policies and Smelter Valuation

Finding a heuristic operating policy and an approximation of the smelter value make up the first step of the sequential solution approach to the combined problem. We formulate a stochastic dynamic program (SDP) that must be solved numerically due to its complexity. For this purpose we use the least squares Monte Carlo (LSM) method. [18] are the pioneers of LSM, which is a scenario-based approach for solving American type claims, and it has been widely applied within the fields of financial and real options. In short terms, it enables the use of Monte Carlo simulations for unbiased value approximations by avoiding perfect foresight. The decision to exercise American options are based on comparing the value of keeping the option alive for one more period (the continuation value) with exercising now. The main idea of the LSM method is to approximate these continuation values by regressing the next period continuation values on the current values of the explanatory variables, which is the equivalent of a conditional expectation function. In the specific problem studied in this thesis, the aluminium smelter receives a cash flow at each stage from the corresponding operating state. Cash flows are the basis of the valuation, and depend on the risk factors. The optimal operating state at a certain stage in a certain scenario is the one with the highest sum of cash flow and expected continuation value. The different operating states of the smelter are described in Table 4.1.

Choosing the functional form of the regression in the LSM method is challenging as the function should resemble the shape of the value function, but is simplified by the fact that the LSM algorithm only depends on the fitted value of the regression, and not on the correlation between the independent variables. Possible choices for a regression basis mentioned by [18] are Laguerre, Hermite and Jacobi polynomials as well as only simple powers of the state variables.

In this thesis, backwards dynamic programming and least squares multivariate regression are applied iteratively to estimate the value of the aluminium smelter at each time step from maturity to now. The discounted expected continuation values of keeping the smelter operating or mothballed at a certain stage and scenario are easily calculated by multiplying the regression basis for the values of the state variables with the derived regression coefficients for that time step. Actually, two re-

TABLE 4.1 DESCRIPTION OF OPERATING STATES

Operating state	Description
Operating	Smelter is operating and owners receive the net cash flow from production and sale of aluminium.
Mothballed	Smelter is temporarily shut down. Owners receive the net cash flow from sale of pre-ordered electricity in the spot market and may have to pay some operating expenses. The work force has been laid-off and production may restart if favourable market conditions occur. It is assumed that the smelter can be in this state for a maximum of three consecutive years ¹ .
Closed	Owners have shut down the smelter permanently and there is no optionality to restart operations. Smelter will not generate any future cash flows. Upon closure a closure cost is paid and remaining amount of pre-ordered electricity is sold. Value of the latter may be positive or negative and stems from differences between conditional expected prices at the contract order date and at the time of closure.

gressions are carried out at each time step. The first regression is performed in order to approximate the continuation value of an operating smelter, and the second regression is performed in order to approximate the continuation value of a mothballed smelter. To avoid a biased approximation of the continuation values, we generate two sets of scenarios. First, an in-sample set of scenarios for the state variables is generated to calculate regression coefficients for each time step. Secondly, an out-of-sample set of scenarios for the state variables is used to approximate continuation values. We use 10,000 correlated scenarios for both sets (refer to Appendix A.2 for details on correlated random draws). The conditional expected continuation values of keeping the smelter operating or mothballed are then calculated by multiplying the coefficients generated from the in-sample set with the regression basis for the values of the state variables from the out-of-sample set. By repeating the above procedure at every time step, an operating policy for the smelter is derived for each scenario. Finally, the approximated smelter value is found by applying the heuristic operating policy, calculating the net present value of the resulting cash flows and averaging over all scenarios. In this first step of the sequential solution approach, electricity is assumed to be procured only through 1-year forward contracts. MATLAB[®] [21] has been used to implement the LSM method in the attached article [3].

4.2 Portfolio Optimisation

The second step of the solution approach to the combined problem is to use the results from solving the SDP with the LSM method, as input in an optimisation routine that finds an optimal electricity procurement scheme. The heuristic operating policy is used as basis to determine the demand for electricity at each time step in each scenario, whereas the cash flows are used to determine the risk of the procurement scheme.

We assume that the producer can reduce electricity price risk, by procuring electricity through long-term bilateral contracts. The contracts can have a duration of 5, 10 or 20 years, and can be

¹Cost of reactivating the smelter furnaces will with time increase to the point where reopening the smelter will no longer be an option [4]. Assumption of maximum three years based on input from industry sources.

denominated in USD, EUR or NOK. The prices used in long-term contracts are calculated based on conditional expected electricity prices and exchange rates at the time of contract entry. Refer to Appendix A.3 for pricing details. Since the smelter is valued on a USD per produced mt aluminium basis, entering into a long-term USD contract implicates hedging both electricity price and exchange rate risk. Management can also hedge electricity price risk by entering into long-term EUR and NOK contracts, but will then be exposed to exchange rate risks upon delivery.

In order to find an optimal portfolio of electricity contracts we construct a two-stage stochastic program that minimises a relationship between electricity cost and risk of low cash flows. Optimal portfolio strategies with risk measures were first introduced by [20]. The idea is to maximise the return of a portfolio with an upper bound on variance. Since variance is a quadratic risk measure, efforts have been made to find a linear risk measure. The two-stage stochastic program we have formulated in [3] uses Conditional Value-at-Risk (CVaR) as risk measure. This risk measure is linear and coherent, hence we are able to solve the portfolio optimisation problem using linear programming. Note that CVaR is a tail statistic and is therefore fragile towards estimation errors in risk factors. A large number of observations is necessary in order to accurately estimate the parameters of the risk factors. Otherwise, CVaR might be ineffective in capturing the underlying risk of the portfolio.

The optimisation routine works as follows. First, the heuristic operating policy from the previous step of the sequential solution approach is used to determine the demand for electricity at each time step in each scenario, hence treating demand as stochastic. In addition, the simulated cash flows, electricity prices and exchange rates from the previous step are taken as input. Next, the optimisation routine determines an electricity procurement scheme in the form of a portfolio with 1-year forwards and long-term bilateral contracts that matches the derived demand and risk preferences. Note that the amount of electricity procured through long-term contracts is the same across all scenarios. However, in some scenarios the smelter may be in a mothballed state at the time of delivery of pre-ordered electricity. In those cases, we allow for pre-ordered electricity to be sold in the spot market. Correspondingly, if the amount of pre-ordered electricity does not fulfill the demand at a certain point in time in a scenario, the remaining demand is procured through 1-year forwards.

A resulting procurement scheme is finally evaluated by repeating the first step of the solution approach, assuming electricity is procured accordingly. We use two benchmarks to evaluate the result of the optimal scheme. The first is a series of static procurement schemes. Secondly, we perform the optimisation without using the heuristic operating policy from the LSM method as input, hence treating demand as constant. The second benchmark gives us an estimate of the additional value gained from first determining an operating policy before finding a procurement scheme.

The resulting procurement scheme implicates lower downside cash flow risk than when electricity is procured only through 1-year forward contracts. Hence, assuming that electricity is procured according to the resulting procurement scheme when determining a heuristic operating policy for the smelter could yield a different heuristic operating policy than before, e.g. an operating policy

with fewer mothballs or closures. This implicates that the procurement scheme could potentially be further improved by repeating the steps of our solution approach in an iterative manner. In the article [3] we only consider one iteration.

In summary our sequential solution approach is as follows. We first use the LSM method to determine a heuristic operating policy and simulations of risk factors and cash flows, assuming electricity is only procured through 1-year forward contracts. This is used as input to the two-stage stochastic program to find an electricity procurement scheme that matches risk preferences. The procurement scheme is then fed back into the first step to evaluate its effect on mothballs and closure risk, as well as smelter value.

The two-stage stochastic program was solved using Mosel programming language and the software Xpress-Optimiser version 26.01.04.

Chapter 5

Summary and Contributions

Existing literature has only to some extent studied the operation of an aluminium smelter. Electricity procurement strategies for large consumers is a more studied field, but to our knowledge existing literature has not considered the combined problem of operation, valuation and electricity sourcing for an aluminium smelter. We contribute to existing literature by introducing a sequential solution approach to this combined problem. The analyses yield insight into how electricity sourcing decisions impact shutdown risk and smelter value, and could improve industry players' understanding of the scope of electricity price risk.

Research question 1 was addressed by formulating a stochastic dynamic program (SDP). This was solved numerically using the least squares Monte Carlo (LSM) method. We found the LSM method to be an attractive approach for determining a heuristic operating policy that maximises the approximated smelter value. The risk of mothballs and closures is then easily assessed by analysing the resulting heuristic operating policy. In addition, we found that introducing the flexibility of mothballs when already having closure flexibility is of noticeable value.

To answer research question 2, we used the demand for electricity derived from the heuristic operating policy as input to an optimisation routine that finds an electricity procurement scheme based on a trade-off between CVaR of cash flows and total electricity cost. Somewhat surprisingly, we found the optimal procurement scheme to be a mix of 1-year forwards and medium-term bilateral contracts.

Finally, research question 3 was addressed by determining operating strategies and value approximations of the smelter with different electricity procurement schemes. We found that optimising electricity procurement reduces the risk of mothballs, without compromising on closure risk and smelter value. The sequential approach is favoured by the observation that the electricity procurement scheme derived from using the heuristic operating policy as basis for demand, outperforms the scheme found when assuming constant demand. Approximated smelter value is higher and shutdown risks lower. Furthermore, the electricity procurement scheme also outperforms more generic procurement schemes as e.g. only very long-term contracts. Long-term contracts were found to increase closure risk and reduce the risk of mothballs, but yielded a substantial lower smelter value than the scheme found with the sequential solution approach.

Chapter 6

Further Research

The two-stage solution approach is an initial step in the direction of solving the combined problem of operation, valuation and electricity sourcing for an aluminium smelter. Our work shows that there are clear benefits of an approach that takes all the latter elements into account, as opposed to treating them as strictly independent. The analyses shed light on interesting findings that could be basis for further research.

Industry players such as Norsk Hydro ASA and Alcoa Inc. are concerned with electricity sourcing. They look at the opportunity to buy own power assets and usually enter into long-term bilateral agreements for electricity sourcing with large utility companies. However, we find that it would be more beneficial with a greater exposure towards short-term electricity prices. Our solution approach includes high uncertainty in several risk factors other than electricity, which could overshadow the isolated electricity price risk, and thus yield different electricity procurement schemes than if electricity price risk was to be treated in a more isolated way. Still, a question that arises is whether industry players overestimate the significance of electricity sourcing or if isolating the electricity price risk would yield findings more in-line with what is observed in practice? In addition, are there potentially other elements than only price risk, e.g. reliability, that motivate industry players to enter into long-term agreements with utility companies? Addressing these questions in further research would be an interesting extension of our work and could potentially have an impact on current industry practice.

Portfolio optimisation has some fallacies that are important to be aware of. For risk-minimising portfolio optimisation to yield fully correct results, a very large number of observations is needed. When using a risk measure such as CVaR, estimation errors may be significant, namely because CVaR measures tail risk. As extreme events occur with low probability, a large number of observations would be needed to accurately estimate the distribution of such events in the underlying population and to avoid overfitting. This is further emphasised by [16]. Having the required number of observations to make CVaR risk estimates fully accurate is rarely the case, hence it is important to be aware of this limitation when assessing the implications of the results.

The current sequential solution approach could potentially be approved in further research with

some alterations to underlying assumptions of electricity price dynamics. The approach is sensitive to the enrichment of the underlying electricity price and exchange rate processes, which currently are assumed to be single-factor processes. Electricity procurement schemes are tightly dependent on the process used to describe the dynamics of the electricity price. Introducing state of the art multi-factor and forward curve models would enrich the forecasted dynamics of the electricity price and introduce more risk factors. Intuitively, one would expect that introducing more risk factors in the electricity price would shift the electricity procurement scheme towards long-term contracts. It would be interesting to analyse whether this is the case.

Finally, the aluminium producer can potentially earn profits when selling pre-ordered electricity in the spot market. The aspect of whether to include potential speculative gains from electricity price trading in the operating decisions is easily changed in our solution approach. Analyses moving in the direction of comparing pure financial and social considerations would be an interesting extension of the current work and could be applicable for both aluminium producers and social planners.

Bibliography

- [1] Aiube, F. A. L. and Samanez, C. P. (2014). On the Comparison of Schwartz and Smith's Two- and Three-Factor Models on Commodity Prices. *Applied Economics*, 46(30):3736–3749.
- [2] Alexander, C. (2008). *Quantitative Methods in Finance*. The Wiley Finance Series. John Wiley & Sons, 2008, West Sussex UK.
- [3] Andresen, S. and Aas, E. (2015). Operation, Valuation and Electricity Sourcing for a Generic Aluminium Smelter. *Working Paper. Trondheim, Norway: Norwegian University of Science and Technology*.
- [4] Bastian-Pinto, C. L., Brandão, L. E. T., Raphael, R., and Ozorio, L. M. (2013). Financial Valuation of Operational Flexibilities in the Aluminum Industry using Real Option Theory. In *17th Annual International Conference Real Options: Theory Meets Practice*.
- [5] Cao, Y. (2014). Metal Bulletin Research Aluminium Weekly Market Tracker September 15 2014: What Factors Are Driving Aluminium Premiums? [Online] Available from: <http://www.metalbulletinresearch.com/Article/3381516/What-factors-are-driving-aluminium-premiums.html>. [2015-05-20].
- [6] Cheng, I. H. and Xiong, W. (2014). Financialization of Commodity Markets. *Annual Review of Financial Economics*, 6:419–441.
- [7] Cortazar, G., Gravet, M., and Urzua, J. (2008). The Valuation of Multidimensional American Real Options Using the LSM Simulation Method. *Computers & Operations Research*, 35(1):113–129.
- [8] Cortazar, G. and Schwartz, E. S. (2003). Implementing a Stochastic Model for Oil Futures Prices. *Energy Economics*, 25(3):215–238.
- [9] Dixit, A. K. and Pindyck, R. S. (1994). *Investment Under Uncertainty*. Princeton University press, New Jersey US.
- [10] Dubal Aluminium (2014). The primary aluminium production process. [Online] Available from: <https://www.dubal.ae/who-we-are/our-production-process/aluminium-smelting.aspx>. [2014-11-14].
- [11] Engel, C., Mark, N. C., and West, K. D. (2007). Exchange Rate Models are Not as Bad as You Think. Technical report, National Bureau of Economic Research.
- [12] European Commission (2009). Market Observatory for Energy. [Online] Available from: <http://www.cne.es/cgi-bin/BRSCGI.exe?CMD=VEROBJ&MLKOB=450764202525>. [2015-05-22].
- [13] Garen, J., Jepsen, C., and Scott, F. (2009). Economic Forces Shaping the Aluminum Industry. [Online] Available from: http://www.secat.net/sustainablealuminum.org/pdf/Economic_Forces.

- pdf. *University of Kentucky, Lexington. Report prepared for the Sloan Center for a Sustainable Aluminum Industry.*
- [14] Harton, T. W. (2010). Virtual Smelter Modelling for Metal Flow Management. In *Proceedings from the 45th Annual Conference of the ORSNZ.*
- [15] Jafarizadeh, B. and Bratvold, R. B. (2012). A Two-Factor Price Process for Modeling Uncertainty in Oil Prices. In *SPE Hydrocarbon Economics and Evaluation Symposium, Calgary, Alberta, Canada. Society of Petroleum Engineer.*
- [16] Lim, A. E. B., Shanthikumar, J. G. Y., and Vahn, G. Y. (2011). Conditional Value-at-Risk in Portfolio Optimization: Coherent but Fragile. *Operations Research Letters*, 39(3):163–171.
- [17] London Metal Exchange (2015). History. [Online] Available from: <http://www.lme.com/about-us/history/>. [2015-03-10].
- [18] Longstaff, F. A. and Schwartz, E. S. (2001). Valuing American Options by Simulation: A Simple Least-Squares Approach. *The Review of Financial Studies*, 14(1):113–147.
- [19] Lucia, J. J. and Schwartz, E. S. (2002). Electricity Prices and Power Derivatives: Evidence from the Nordic Power Exchange. *Review of Derivatives Research*, 5(1):5–50.
- [20] Markowitz, H. (1952). Portfolio Selection. *The Journal of Finance*, 7(1):77–91.
- [21] MATLAB (2014). *Version 8.3.0.532 (R2014a)*. The MathWorks Inc., Natick, Massachusetts.
- [22] Meese, R. A. and Rogoff, K. (1983). Empirical Exchange Rate Models of the Seventies: Do They Fit Out of Sample? *Journal of International Economics*, 14(1-2):3–24.
- [23] Nappi, C. (2013). The Global Aluminium Industry 40 years from 1972. [Online] Available from: http://www.world-aluminium.org/media/filer_public/2013/02/25/an_outlook_of_the_global_aluminium_industry_1972_-_present_day.pdf. [2015-03-10].
- [24] Nord Pool Spot AS (2014). Annual Report 2013. [Online] Available from: http://www.nordpoolspot.com/globalassets/download-center/annual-report/annual-report_nord-pool-spot_2013.pdf. [2015-03-20].
- [25] Norsk Hydro (2012). Aluminium, Environment and Society. [Online] Available from: http://www.hydro.com/upload/Aluminium/Download/Aluminium_environment-and-society.pdf. [2014-12-11].
- [26] Norsk Hydro (2014). Market Outlook: Capital Markets Day 2014. [Online] Available from: http://www.hydro.com/upload/Documents/Presentations/Capital%20Markets%20Day/2014/Market_CMD_2014.pdf. [2014-12-11].
- [27] Norsk Hydro (2015a). First Quarter Presentation. [Online] Available from: http://www.hydro.com/upload/Documents/Presentations/Quarterly/2015/presentation_q1_2015_en.pdf. [2015-05-20].
- [28] Norsk Hydro (2015b). Investor Presentation May 2015. [Online] Available from: http://www.hydro.com/upload/Documents/Presentations/Investor/inv_pres_apr_2015.pdf. [2015-05-22].
- [29] Ozorio, L. M., Shevchenko, P. V., and Bastian-Pinto, C. L. (2013). The Choice of Stochastic Process in Real Option Valuation II: Selecting Multiple Factor Models. In *Proceedings of the 17th Annual International Conference Real Options: Theory Meets Practice.*

- [30] Plunkert, P. A. (1998). Aluminium. [Online] Available from: <http://minerals.usgs.gov/minerals/pubs/commodity/aluminum/050798.pdf>. [2015-03-10].
- [31] Schwartz, E. S. (1997). The Stochastic Behavior of Commodity Prices: Implications for Valuation and Hedging. *The Journal of Finance*, 52(3):923–973.
- [32] Schwartz, E. S. and Smith, J. E. (1998). Short-Term Variations and Long-Term Dynamics in Commodity Prices: Incorporating A Stochastic Growth Rate. *Note from earlier version of the paper "Short-Term Variations and Long-Term Dynamics in Commodity Prices*, Available from: https://faculty.fuqua.duke.edu/~jes9/bio/Shortlong_stochastic_growth_rate_extension.pdf.
- [33] Schwartz, E. S. and Smith, J. E. (2000). Short-Term Variations and Long-Term Dynamics in Commodity Prices. *Management Science*, 46(7):893–911.
- [34] Skantze, P., Ilic, M., and Chapman, J. (2000). Stochastic modeling of electric power prices in a multi-market environment. In *Power Engineering Society Winter Meeting, IEEE*, volume 2, pages 1109–1114.
- [35] Statnett (2013). Kabler til Tyskland og Storbritannia – Analyse av Samf.øk. Nytte ved Spothandel. [Online] Available from: <http://www.statnett.no/PageFiles/7245/Dokumenter/Andre%20dokumenter/2013-05-13%20Analyserapport%20samfunns%C3%B8konomisk%20nytte%20ved%20spothandel.pdf>. [2015-05-24].
- [36] Tang, K. (2012). Time-Varying Long-Run Mean of Commodity Prices and the Modeling of Futures Term Structures. *Quantitative Finance*, 12(5):781–790.
- [37] Taylor, A. M. and Taylor, M. P. (2004). The Purchasing Power Parity Debate. *Journal of Economic Perspectives*, 18(4):135–158.
- [38] U.S. Geological Survey (2013). Annual Statistics Mineral Commodity Summaries: Bauxite and Alumina, February 2014. [Online] Available from: <http://minerals.usgs.gov/minerals/pubs/commodity/bauxite/mcs-2014-bauxi.pdf>. [2014-12-11].
- [39] Walker, S. (2015). Engineering & Mining Journal Tuesday, 10 March 2015 - Bauxite and Alumina: Growth Maintained. [Online] Available from: http://www.e-mj.com/features/5039-bauxite-and-alumina-growth-maintained.html#.VXXGes_tlBc. [2015-06-08].
- [40] Zorzi, M. C., Muck, J., and Rubaszek, M. (2015). Real Exchange Rate Forecasting and PPP: This Time the Random Walk Loses. *An earlier version was published as ECB Working Paper No. 1576, 2013*.
- [41] Øye, H. A. and Sørli, M. (2011). Power Failure, Restart and Repair. *Aluminum International Today*, 23(2):25–27.

Appendix A

Mathematical Elaborations

Note that the notations used in the following subsections are intended only for the definitions and derivations given, thus the same notation may be used with different meaning between the respective subsections.

A.1 Calibrating Parameters of the Three-factor Extension

The expected values and covariance of the three factors in [32] are defined by (A.1)-(A.8).

$$\mathbb{E}^*[(\chi_t, \xi_t, \mu_t)] = [e^{-\kappa t} \chi_0 - (1 - e^{-\kappa t}) \frac{\lambda_\chi}{\kappa}, \xi_0 + (\bar{\mu}^* - \lambda_\xi)t + (\mu_0 - \bar{\mu}^*) \frac{(1 - e^{-\eta t})}{\eta}, \mu_0 - (\mu_0 - \bar{\mu}^*)(1 - e^{-\eta t})], \quad (\text{A.1})$$

$$\text{Cov}^*[(\chi_t, \xi_t, \mu_t)] = \begin{bmatrix} \sigma_{11}(t) & \sigma_{12}(t) & \sigma_{13}(t) \\ \sigma_{12}(t) & \sigma_{22}(t) & \sigma_{23}(t) \\ \sigma_{13}(t) & \sigma_{23}(t) & \sigma_{33}(t) \end{bmatrix} \quad (\text{A.2})$$

$$\sigma_{11}(t) = (1 - e^{-2\kappa t}) \frac{\sigma_\chi^2}{2\kappa} \quad (\text{A.3})$$

$$\sigma_{12}(t) = (1 - e^{-\kappa t}) \frac{\rho_{\chi\xi} \sigma_\chi \sigma_\xi}{\kappa} + \frac{\rho_{\chi\mu} \sigma_\chi \sigma_\mu}{\eta} \left(\frac{(1 - e^{-\kappa t})}{\kappa} - \frac{(1 - e^{-(\kappa+\eta)t})}{(\kappa + \eta)} \right) \quad (\text{A.4})$$

$$\sigma_{13}(t) = \rho_{\chi\mu} \sigma_\chi \sigma_\mu \frac{(1 - e^{-(\kappa+\eta)t})}{(\kappa + \eta)} \quad (\text{A.5})$$

$$\sigma_{22}(t) = \sigma_\xi^2 t + \frac{\rho_{\xi\mu} \sigma_\xi \sigma_\mu}{\eta} \left(t - \frac{(1 - e^{-\eta t})}{\eta} \right) + \frac{\sigma_\mu^2}{\eta^2} \left(t - 2 \frac{(1 - e^{-\eta t})}{\eta} + \frac{(1 - e^{-2\eta t})}{2\eta} \right) \quad (\text{A.6})$$

$$\sigma_{23}(t) = \rho_{\xi\mu} \sigma_\xi \sigma_\mu \frac{(1 - e^{-\eta t})}{\eta} + \frac{\sigma_\mu^2}{\eta} \left(\frac{(1 - e^{-\eta t})}{\eta} + \frac{(1 - e^{-2\eta t})}{2\eta} \right) \quad (\text{A.7})$$

$$\sigma_{33}(t) = \sigma_\mu^2 \frac{(1 - e^{-2\eta t})}{2\eta} \quad (\text{A.8})$$

Applying the Kalman Filter entails formulating a *transition equation* and *measurement equation* and calibrating these by the means of maximising a log-likelihood function. From (A.1)-(A.2) the two former may be formulated as (A.9) and (A.10) and the latter as (A.11).

$$\mathbf{x}_t = \mathbf{c} + \mathbf{Q}\mathbf{x}_{t-1} + \eta_t \quad (\text{A.9})$$

$$\mathbf{y}_t = \mathbf{d}_t + \mathbf{Z}_t\mathbf{x}_t + \varepsilon_t \quad (\text{A.10})$$

$$\Rightarrow \mathbf{x}_t = \underbrace{\mathbf{c} + \mathbf{Q}\mathbf{x}_{t-1}}_{\text{One-period ahead state variable estimates}} + \mathbf{K}_t \underbrace{(\mathbf{y}_t - \mathbf{d}_t - \mathbf{Z}_t(\mathbf{c} + \mathbf{Q}\mathbf{x}_{t-1}))}_{\text{Difference between predicted and observed price}}$$

where:

$\mathbf{x}_t \equiv [\chi_t, \xi_t, \mu_t]'$, a 3×1 vector of state variables

$\mathbf{c} \equiv [0, 0, \eta \bar{\mu} \Delta t]'$, a 3×1 vector

$\mathbf{d}_t \equiv [B(T_i)]'$, a $n \times 1$ vector

$$\mathbf{Q} \equiv \begin{bmatrix} e^{-\kappa \Delta t} & 0 & 0 \\ 0 & 1 & \Delta t \\ 0 & 0 & e^{-\eta \Delta t} \end{bmatrix}, \text{ a } 3 \times 3 \text{ matrix}$$

η_t is a 3×1 vector of disturbances

Δt =time step length

n_T =number of observations

$\mathbf{y}_t \equiv [\ln F_{T_i}]'$, a $n \times 1$ vector of observed futures prices

$\mathbf{Z}_t \equiv [e^{-\kappa T_i}, 1, e^{-\eta T_i}]$, a $n \times 3$ matrix

K_t is a correction factor

ε_t , a $n \times 1$ vector of disturbances

n =number of maturities for observed futures

$i = 1 \dots n$

$$B(T_i) = -(1 - e^{-\kappa T_i}) \frac{\lambda \chi}{\kappa} + (\bar{\mu}^* - \lambda \xi) T_i + \frac{1}{2} (\sigma_{11}(T_i) + \sigma_{22}(T_i) + 2\sigma_{12}(T_i))$$

$$\max_{\{\theta\}} \ln L = \sum_{t=1}^{n_T} \left(-\frac{n}{2} \ln(2\pi) - \frac{1}{2} \ln |\mathbf{Q}_{t|t-1}| - \frac{1}{2} \mathbf{v}_t' \mathbf{Q}_{t|t-1}^{-1} \mathbf{v}_t \right) \quad (\text{A.11})$$

θ is the set of parameters

\mathbf{v}_t are errors between observed and predicted prices

$\mathbf{Q}_{t|t-1}$ is the covariance matrix at time t

A.2 Correlated Random Draws

The main part of the scenario generation is to generate random draws, $\mathbf{L}(T', I) = \begin{pmatrix} \varepsilon_{1,1} & \cdots & \varepsilon_{1,I} \\ \vdots & \ddots & \vdots \\ \varepsilon_{T',1} & \cdots & \varepsilon_{T',I} \end{pmatrix}$

where $\varepsilon_{t,i} \sim N(0, \Sigma)$. Note that since we are generating scenarios for correlated processes, the epsilons should be correlated. To do this, we first generate random variables that are normally distributed with zero mean and variance 1. This yields for each t : $L_{t,i} \sim N(0, 1)$. Thus $c_1 L_{t,1} \dots c_i L_{t,i} \sim N(0, \sigma^2)$ where $\sigma^2 = c_1^2 + \dots + c_i^2$. Then $\mathbf{CL} \sim N(0, \mathbf{C}^T \mathbf{C})$, which reduces our problem to finding \mathbf{C} such that $\mathbf{C}^T \mathbf{C} = \Sigma$. The matrix \mathbf{C} is commonly referred to as the Cholesky-decomposition of Σ . From linear algebra we know that a symmetric positive-definite matrix \mathbf{K} can be expressed as $\mathbf{K} = \mathbf{U}^T \mathbf{D} \mathbf{U}$ where \mathbf{U} is an upper-triangular matrix and \mathbf{D} a diagonal matrix with non-negative elements. In our problem we have that $\Sigma = \mathbf{U}^T \mathbf{D} \mathbf{U}$, which yields the result $\mathbf{C} = \sqrt{\mathbf{D}} \mathbf{U}$.¹ Thus, the correlated random draws $\varepsilon_{t,i}$ are calculated by $\varepsilon(T', I) = \mathbf{CL}$. The matrix $\varepsilon(T', I)$ now represents correlated random price movements.

A.3 Calculating Expected Electricity Prices

Following is a description of how the conditional expected electricity prices used in the long-term contracts are calculated.

We assume that the log electricity price follows an AR(1) process, which is just a discretised version of the Ornstein-Uhlenbeck (OU) diffusion process. The OU-process is defined as:

$$dX_t = \kappa(\theta - X_t)dt + \sigma dW_t \quad (\text{A.12})$$

where x_t is the log electricity price, κ measures the speed of mean reversion, θ is the long-term mean level of the log electricity price and σ is the variance of the process. We have that $\kappa > 0$, $\theta > 0$ and $\sigma > 0$. To derive an expression for the conditional expectation of the process we must solve the stochastic differential equation (A.12). This requires a few steps of stochastic calculus. If we let $F_t = f(t, X_t)$, then Ito's lemma is given by:

$$dF_t = \frac{\partial f}{\partial t}(t, X_t)dt + \frac{\partial f}{\partial x}(t, X_t)dX_t + \frac{1}{2} \frac{\partial^2 f}{\partial x^2}(t, X_t)(dX_t)^2 \quad (\text{A.13})$$

¹If the covariance matrix is not positive definite it can be transformed through regularization, which works if the negative eigenvalue is close to zero.

We define $F_t = f(t, X_t) = e^{\kappa t} X_t$. Then we have that:

$$\begin{aligned}\frac{\partial f}{\partial t}(t, X_t) &= \kappa e^{\kappa t} X_t + e^{\kappa t} dX_t \\ \frac{\partial f}{\partial x}(t, X_t) &= e^{\kappa t} \\ \frac{\partial^2 f}{\partial x^2}(t, X_t) &= 0\end{aligned}$$

Substituting this into (A.13) yields:

$$dF_t = (\kappa e^{\kappa t} X_t + e^{\kappa t} dX_t)dt + e^{\kappa t} dX_t$$

By using the fact that $dX_t \times dt = 0$ and substituting in the expression of an OU-process (A.12) for dX_t we get:

$$\begin{aligned}dF_t &= \kappa e^{\kappa t} X_t dt + e^{\kappa t} (\kappa(\theta - X_t)dt + \sigma dW_t) \\ \Rightarrow dF_t &= d(e^{\kappa t} X_t) = \kappa \theta e^{\kappa t} dt + e^{\kappa t} \sigma dW_t\end{aligned}$$

Integrating both sides from 0 to t gives the following expression:

$$\begin{aligned}\int_0^t d(e^{\kappa s} X_s) &= \int_0^t \kappa \theta e^{\kappa s} ds + \int_0^t \sigma e^{\kappa s} dW_s \\ \Rightarrow \left[e^{\kappa s} X_s \right]_0^t &= \kappa \theta \left[\frac{1}{\kappa} e^{\kappa s} \right]_0^t + \sigma \int_0^t e^{\kappa s} dW_s \\ \Rightarrow e^{\kappa t} X_t - X_0 &= \theta e^{\kappa t} - \theta + \sigma \int_0^t e^{\kappa s} dW_s\end{aligned}$$

Multiplying both sides with $e^{-\kappa t}$ we get the expression for X_t :

$$X_t = X_0 e^{-\kappa t} + \theta - \theta e^{-\kappa t} + e^{-\kappa t} \sigma \int_0^t e^{\kappa s} dW_s$$

The conditional expectation of the log electricity price is:

$$\begin{aligned}E[X_t | X_0] &= E \left[X_0 e^{-\kappa t} + \theta - \theta e^{-\kappa t} + e^{-\kappa t} \sigma \int_0^t e^{\kappa s} dW_s \right] \\ &= X_0 e^{-\kappa t} + \theta - \theta e^{-\kappa t}, \quad \text{since } E \left[e^{-\kappa t} \sigma \int_0^t e^{\kappa s} dW_s \right] = 0\end{aligned} \tag{A.14}$$

The conditional variance of the log electricity price is:

$$\begin{aligned}
\text{Var}[X_t|X_0] &= \text{Var}\left(e^{-\kappa t} \sigma \int_0^t e^{\kappa s} dW_s\right) \\
&= \sigma^2 e^{-2\kappa t} E\left[\left(\int_0^t e^{\kappa s} dW_s\right)^2\right] \\
&= \sigma^2 e^{-2\kappa t} \int_0^t e^{2\kappa s} ds, \text{ (by Iso's isometry)} \\
&= \frac{\sigma^2}{2\kappa} (1 - e^{-2\kappa t})
\end{aligned} \tag{A.15}$$

Finally, let S_t denote the nominal electricity price. It is lognormally distributed. Using the formula for the expectation of a lognormally distributed variable, $e^{\mu + \frac{1}{2}\sigma^2}$, we get that the expected value of S_t conditional on the current price S_{t^*} is given by:

$$E[S_t | S_{t^*}] = e^{\left[X_{t^*} e^{\kappa(t-t^*)} + \theta - \theta e^{\kappa(t-t^*)} + \frac{1}{2} \frac{\sigma^2}{2\kappa} (1 - e^{-2\kappa(t-t^*)})\right]} \tag{A.16}$$

Equation (A.16) is used for calculating the expected prices used in long-term bilateral contracts. Note that the electricity price is modelled on a quarterly basis, which means that to calculate the expected electricity one year ahead we must use $t - t^* = 4$. To calculate expected electricity prices in terms of USD we must multiply with the conditional expected USD/EUR exchange rates. These are easily calculated with (A.14).

Operation, Valuation and Electricity Sourcing for a Generic Aluminium Smelter

Sven Henrik Andresen and Eivind Fossan Aas

Abstract—An aluminium producer is concerned with operating a smelter in a manner that maximises value and minimises shut-down risk. Operational flexibility is available through mothballs or closure, whereas procurement of electricity, a dominating input cost, may be conducted through 1-year forwards or long-term bilateral contracts. We present a sequential valuation and optimisation approach for evaluating a smelter and deriving a risk minimising electricity procurement scheme. Multiple risk factors are considered. An operating policy that maximises smelter value is found by solving a stochastic dynamic program with the least squares Monte Carlo (LSM) method. Electricity procurement is investigated using a two-stage stochastic program that minimises a trade-off between electricity cost and Conditional Value-at-Risk. The paper combines the two methods by using the heuristic operating policy found by the LSM method as input in the latter. We find that an aluminium producer can reduce the risk of mothballs, without compromising smelter value or closure risk, by procuring electricity according to the scheme obtained with our solution approach. The scheme derived from using the heuristic operating policy as basis for demand outperforms the one found when assuming constant demand.

Keywords—Least squares Monte Carlo, real options, portfolio optimisation, stochastic dynamic programming, electricity sourcing, Ornstein-Uhlenbeck, three-factor commodity process, Conditional Value-at-Risk (CVaR)

I. INTRODUCTION

Aluminium production is a classic industrial process, in which a smelter transforms alumina and carbon into aluminium through a power intensive electrolysis process. Electricity is a dominating production cost, and access to power is thus a critical aspect in deciding where to locate an aluminium smelter [1]. Smelters are typically constructed close to reliable and cheap power sources, such as next to dams in mountainous regions, in order to benefit from cheap hydroelectricity. Furthermore, they often take the role of being cornerstone businesses in their respective districts due to labour demands. A proper valuation of an aluminium smelter to be used as basis for decision-making is therefore of high importance in both business and social terms. Management of an aluminium smelter are concerned with operating the smelter in a way that maximises shareholder value, and has the flexibility to temporary shut down or abandon the smelter. The value of such flexibility can be captured through biased heuristics by the DCF capital budgeting approach, whereas it is rigorously captured by the real options approach (ROA). Through the

latter a heuristic operating policy can be determined together with the net present value of cash flows from operating the smelter. Finally, management may also choose to purchase electricity through a set of different contract types. Thus, there is a trade-off between total electricity cost and risk, as different contract portfolios yield different risk exposure.

Optimising the processes in aluminium production is a well-studied problem (see [2], [3], [4]), but existing literature has only to a limited extent studied aluminium smelters from a strategic management point of view.

[5] study the effects of operational flexibility for the specific case of an aluminium smelter. They find significant value in the flexibility to temporarily shut down the smelter. Electricity is assumed to be procured through long-term contracts at a fixed price, thus there is no uncertainty in electricity costs. The aluminium price is modelled with a single-factor geometric mean reverting process.

[6] study a related problem, however not for an aluminium smelter. They look at the extraction of a natural resource through an example of a copper mine with flexibility to temporary shut down or abandon operations. The problem is solved with a real options approach and their solution yields an optimal extraction policy for the mine. The output price is modelled stochastically as a geometric Brownian motion (GBM), whereas extraction costs are assumed constant. They apply stochastic control and continuous time arbitrage to derive an analytical solution when considering an infinite time horizon, whereas finite difference approximations of the valuation PDEs under no-arbitrage conditions are applied in their finite time horizon example.

[7] study the electricity procurement problem faced by a large consumer. They assume that three different sources of electricity are available to the consumer; limited self-production, spot market purchases and long-term contracts. The goal is to determine an optimal electricity procurement scheme with respect to Conditional Value-at-Risk (CVaR). Prices are treated as stochastic, whereas demand is assumed to be constant each period.

The combined problem of determining an operating policy and optimising electricity procurement has not been studied, and is a problem faced by a multinational smelting company that we have worked with.

Optimising the operating policy of a smelter leads to a real options problem that is typically formulated as an optimal control problem with multiple risk factors. This type of problem can be solved by PDE approaches [8] [9], approximate linear programming [10], stochastic programming [11] [12] and the least squares Monte Carlo (LSM) method [13] [14] [15]. LSM is the most popular approach for real options

S.H. Andresen and E.F. Aas are with the Department of Industrial Economics and Technology Management, Norwegian University of Science and Technology, Trondheim, 7491 Norway. E-mail: svenhea@stud.ntnu.no, eivinaas@stud.ntnu.no

problems due to its simplicity compared to other alternatives. It works as follows. [13], inspired by [14], suggest to approximate continuation values of American options by least squares regression based on Monte Carlo simulations of the state variables. Values of the state variables at the current time step are used as explanatory variables in the regression, and the continuation values of the different operating states are regressed on these. The described procedure is known as the regress-now variant. [16], [17] and [18] analyse the applicability of the LSM approach for general real options problems. Based on numerical results from comparisons with other methods, they all conclude that the LSM approach may be successfully used for multidimensional problems.

Our problem has multiple risk factors, which makes it hard to derive a closed-form expression for the valuation PDEs. Thus, the finite difference method used in [6] cannot be applied. The problem in [5] is the most similar to ours, but we assume stochastic electricity costs and that parts of the costs are incurred in local currency, thus adding exchange rate risk. We also argue that the LSM approach is a better alternative for approximating continuation values, rather than working directly with expectations of the stochastic variables. This is because the LSM approach assumes no knowledge about the expectations of underlying stochastic processes.

[18] have extended the work in [6]. They consider optimal control of a copper mine using the LSM approach and a three-factor model for copper prices. Their extension implicates that the real options approach suggested in the latter successfully can be used for multidimensional problems. They also argue, with references to [19], that the dynamics of commodity prices are better captured with multi-factor models. Their solution method is general and could easily be extended to include the relevant risk factors and be used to determine a heuristic operating policy for the aluminium smelter. However, it cannot be applied to the combined problem of determining an operating policy and optimising electricity procurement.

Choosing different portfolios of long-term electricity contracts may reduce the level of risk. [20] introduced the concept of portfolio optimisation with variance as risk measure. Subsequent portfolio optimisation problems focus on Value-at-Risk (VaR) as the risk measure (discussed by [21] and [22]). VaR measures the loss in market value over a time horizon that is exceeded by a given probability. Although its popularity, VaR has certain characteristics which are undesirable, such as lack of convexity and subadditivity. [23] therefore discuss the use of CVaR, the expected loss if VaR is exceeded. CVaR is a coherent risk measure, and suitable in this problem due to its linearity and consistency towards rational views on risk.

Since electricity sourcing is important for both electricity retailers and large consumers, efforts have been made in the past decade to approach this optimally. [24], [25] and [7] present stochastic simulation-based methods to optimally solve procurement problems. The two latter solve electricity procurement using CVaR as risk measure. [25] also include uncertainty in electricity demand from an electricity retailer's point of view, with demand treated as an independent stochastic process.

We do not find existing solution approaches to be directly

applicable to our combined problem. Therefore, we propose a sequential solution approach that uses the regress-now LSM to determine a heuristic operating policy and stochastic programming to find a favourable electricity procurement scheme. Contrary to existing literature on procurement optimisation, we propose to use the demand derived from the operating policy found with the LSM method as input in the portfolio optimisation problem, as an alternative to treating demand as an independent stochastic process. We argue that this way of treating demand is better anchored in reality compared to treating it as an independent stochastic process. This yields a hybrid optimisation approach that combines the LSM method and stochastic programming.

We contribute to existing literature by combining the LSM method with risk-minimising electricity portfolio optimisation. Contributions are; (1) evaluation of an aluminium smelter using the LSM method and a three-factor model for the aluminium price and (2) use of demand from optimal control problem as input in a portfolio optimisation problem as opposed to treating demand as an independent stochastic process.

The paper is organised as follows. Section II gives a short description of the business problem faced by an aluminium producer. In Section III we conduct an empirical analysis of the risk factors, whereas section IV in detail describes how the LSM method and electricity portfolio optimisation are used in a sequential solution approach. Results from the previous sections are presented, discussed and interpreted in Section V. Finally, we draw our conclusions in Section VI.

II. BUSINESS PROBLEM

Management of an aluminium smelter faces a range of problems due to volatile input and output prices, as well as exchange rates. The most important risk factors on the input side are the electricity price and exchange rates, whereas the aluminium price is the main source of risk on the output side.

Electricity and alumina each represents approximately 30% of total costs. Management is mainly concerned with hedging the highly volatile electricity cost, as energy cost is a major determinant of international differences in aluminium production. Conversely, the market for alumina is globalised and changes in these prices will affect all market participants. In addition, aluminium producers are often vertically integrated with their own mineral extraction, and are thus less exposed to fluctuations in alumina prices. Changes in electricity prices, on the other hand, might affect only one producer and alter the relative cost position of the firm. The producer must therefore determine a trade-off between risk and total electricity cost, and can purchase electricity through 1-year forwards or long-term bilateral contracts. With short-term exposure the producer may benefit from periods with low prices, but at the same time faces greater risk of high prices compared to with long-term exposure. Finally, risk from fluctuating exchange rates impact the spread between local electricity costs and the aluminium price, denominated in U.S. dollars (USD), as well as processing costs incurred in local currencies.

The aluminium price quoted on the London Metal Exchange (LME) has a major impact on the revenues of an aluminium

producer. On top of this, the producer often receives a premium that takes into account elements such as cost of delivery and insurance. Short-term aluminium price risk is typically mitigated by hedging, but the producer stays exposed to long-term price risk. The rationale behind this can be understood by investigating other commodity markets. [26] study the hedging activities of oil and gas producers. They argue that hedging output prices does not increase market value, since a shareholder takes a position in a company precisely to increase risk exposure to the market in which the company operates. Therefore, hedging long-term aluminium prices should not be beneficial, since it implicates hedging the very market risk shareholders seek exposure to.

Management is concerned with operating the smelter in a value-maximising manner. The smelter can be in three different states; operating, mothballed or closed. When the smelter is operating, management receives the net cash flows from producing and selling aluminium. In a mothballed state, low or no operating costs are incurred and pre-ordered electricity is sold in the spot market each period the smelter stays mothballed. The cost of reactivating the smelter furnaces will with time increase to the point where reopening the smelter will no longer be an option [5]. Therefore, the smelter can only stay mothballed for a limited number of consecutive time periods. A closed smelter receives no cash flows, and upon closure all pre-ordered electricity is sold at once. Value of the latter may be positive or negative, and stems from differences between conditional expected prices at the contract order date and time of closure.

Once operating, a smelter is assumed to have a very long lifetime, thus a planning horizon of 20-40 years is typically considered by industry players. Furthermore, aluminium producers are concerned with relative cost positions. As output prices are denominated in USD and parts of the costs are incurred in local currencies, it is reasonable to value the smelter in USD to emphasise the relative cost position. Since we make no assumptions about economies of scale, we value the smelter on a USD per produced metric ton basis.

Motivated by the above, we seek to determine how to operate the smelter, then given the resulting electricity demand identify and evaluate a favourable procurement scheme. The solution approach should take the most important risk factors into account, namely electricity price, aluminium price and exchange rate risk. We consider a time horizon of 40 years.

III. DYNAMICS OF RISK FACTORS

A. Aluminium prices

[27] and [28] suggest the use of a mean reverting process to forecast the behaviour of commodity prices. The intuition behind mean reversion in commodity prices comes from basic microeconomic theory. This states that when prices increase, high cost producers will enter the market, which in turn will increase the supply and push down the price. Conversely, when prices are low, high cost producers will leave the market, which will decrease the supply and drive up the price. One basic mean reverting process is the Ornstein-Uhlenbeck process, as described in among other [19] and [27]. Statistical hypothesis

testing¹ of stationarity in the monthly, quarterly and yearly aluminium price time series and log of the aluminium price time series do however not provide strong evidence of a stationary mean level (refer to Table I and Table II for test results from monthly time series data).

TABLE I. RESULTS FROM STATISTICAL HYPOTHESIS TESTING OF STATIONARITY IN MONTHLY TIME SERIES. FALSE: DO NOT REJECT H_0 , TRUE: REJECT H_0

Period	ADF1	ADF2	ADF3	KPSS1	KPSS2	KPSS3	VR
1987-2015	False	False	False	True	True	True	False
1987-2007	False	False	False	True	True	True	False
2009-2015	False	False	False	True	True	True	False

TABLE II. RESULTS FROM STATISTICAL HYPOTHESIS TESTING OF STATIONARITY IN LOG OF MONTHLY TIME SERIES. FALSE: DO NOT REJECT H_0 , TRUE: REJECT H_0

Period	ADF1	ADF2	ADF3	KPSS1	KPSS2	KPSS3	VR
1987-2015	False	False	False	True	True	True	False
1987-2007	False	False	False	True	True	True	False
2009-2015	False	False	False	True	True	True	False

Economic intuition induces that aluminium prices are mean reverting, but lack of statistical evidence for a stationary mean level motivates the use of an alternative approach to capture the dynamics of the aluminium price, that still takes into account the mean reverting nature of commodity prices. The superiority of multi-factor models over a single-factor model for commodities is discussed by among other [18], [19], [29] and [30]. Two such models are the two-factor model and three-factor extension presented in [31] and [32]. The two-factor model allows for mean reversion in prices, but uncertainty in the equilibrium level to which prices revert. The three-factor extension allows for the growth rate of the equilibrium level to be modelled stochastically. On basis of the discussion above and wide acceptance of the latter model in the literature, we therefore choose to use the three-factor extension to capture the dynamics of the aluminium price. Following the notation in [32] it is formulated in its risk-neutral form as:

$$d\chi_t^* = (-\kappa\chi_t^* - \lambda_\chi)dt + \sigma_\chi dZ_\chi^* \quad (1)$$

$$d\xi_t^* = (\mu_t^* - \lambda_\xi)dt + \sigma_\xi dZ_\xi^* \quad (2)$$

$$d\mu_t^* = (-\eta(\mu_t^* - \bar{\mu}) - \lambda_\mu)dt + \sigma_\mu dZ_\mu^* \quad (3)$$

$$S_t^{A*} = \ln(\chi_t^* + \xi_t^*) \quad (4)$$

At time t , χ_t^* is the short-term deviation from the equilibrium price, ξ_t^* is the equilibrium price and μ_t^* is the growth rate of the equilibrium price. dZ_χ^* , dZ_ξ^* and dZ_μ^* are correlated Brownian motions, λ_χ , λ_ξ and λ_μ are risk premiums of the respective processes, κ and η are mean reversion parameters and $\bar{\mu}$ is the unadjusted mean level of the growth rate. The parameters of (1)-(3) are calibrated by applying the Kalman Filter [33] on the log of historical monthly spot and futures

¹ADFX= augmented Dickey-Fuller test on X lags. H_0 : Series contains a unit-root. KPSSX= Kwiatkowski-Phillips-Schmidt-Shin test on X lags. H_0 : Series is trend or level stationary. VR= variance ratio test. H_0 : Series is a random walk.

prices by maximising the log-likelihood function. Historical monthly data on LME futures prices is collected from Reuters to calibrate the processes. Calibration is done on monthly data as satisfactory forward data only is available from 2009, and lower granularity yields too few data points. Parameters are however easily transformed to be expressed in terms of years. [32] and [34] point out challenges in obtaining reliable estimates of risk premiums that are significantly different from zero, especially for small data sets. This implies that estimates of λ_χ , λ_ξ and λ_μ are not likely to be reliable and thus there will be difficulties when simulating futures prices, as risk premiums are key for risk-neutral pricing of futures contracts. However, the aluminium smelter in this paper does not rely on futures contracts for aluminium, thus only simulations of the spot price are needed. To avoid further issues with risk premiums, the risk premiums are set to zero in the simulations, effectively not using the risk-neutral form of the process. Equations (1)-(3) are discretised in order to conduct Monte Carlo simulations of quarterly prices for the time horizon considered.

B. Electricity prices

Historically, electricity prices have been characterised as highly volatile due to certain unique properties of electricity with significant impact on the price dynamics, e.g. the commodity is non-storable and highly demand-driven in the short-run. The short-term effects may be daily, weekly or yearly, and is modelled by [35]. In the long-run, the effects of the short-term spikes in electricity prices diminish. [36] argue that electricity prices follow a two-factor model, where the process depends on a stochastic long-term equilibrium component and a short-term mean reverting component. To avoid being affected by short-term price movements, we consider quarterly quoted electricity prices using historical data on the 1-year forward Nord Pool (NP) system price, later referred to as 1-year or 1-y forward price. Historical quarterly prices are collected from Reuters. Since long-term forward prices are only available from 2009 and onwards, we have few data points on which to calibrate the long-term equilibrium component in the two-factor model. Hence, it is less applicable compared to alternative stochastic models.

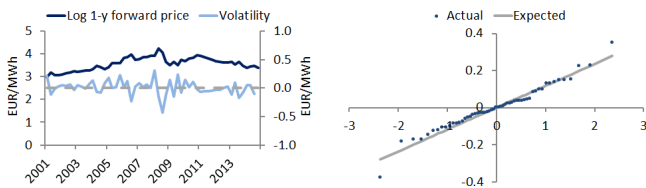


Fig. 1. Quarterly 1-y forward log price.

Fig. 2. Q-Q plot of residuals 1-y forward log electricity price.

Figure 1 shows a plot of historical log prices and volatility for the 1-year forward price. We can see from the figure that, disregarding the financial crisis, the volatility is constant during the selected time period. The high volatility observed during the financial crisis was due to the extraordinary event of new CO₂ quotas entering the market. Figure 2 shows a quantile-quantile (Q-Q) plot of an autoregressive process of order one

(AR(1)) fitted to the quarterly log electricity price. We choose to fit the log price in order to normalise the residuals from the process. Except for two extreme points observed during the entry of new CO₂ quotas, the figure shows that the residuals from the fitted process are normally distributed. On the basis of constant volatility and normally distributed residuals, we argue that the dynamics of the quarterly 1-year forward log price can be captured with an AR(1) process, described in (5).

$$\ln(S_t^e) = \alpha_{Se} + \beta_{Se} \ln(S_{t-1}^e) + \epsilon_{Se} \quad (5)$$

C. Exchange rates

According to [37], predicting foreign exchange rates using a random walk process performs as well as several other time series processes, e.g. autoregressive processes, when considering a one to twelve month horizon. However, simulating a random walk over 40 years could potentially yield scenarios with highly unrealistic exchange rates, which motivates the use of an alternative process.

Figures 3-6 show the same analysis as in Figures 1-2 for a fitted AR(1) process on both USD/EUR and USD/NOK exchange rates in quarterly intervals using data from Reuters. Figures 3 and 5 show that the two exchange rates have approximately constant volatility throughout the selected time period. As we can see from the Q-Q plots, the residuals from the fitted USD/EUR exchange rate process are normally distributed. The residuals from the fitted USD/NOK process have some deviations from a normal distribution due to financial crises, actions from central banks and other extraordinary events.

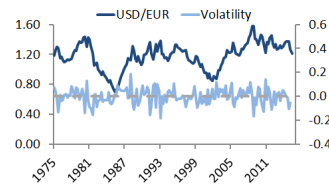


Fig. 3. Quarterly USD/EUR exchange rate.

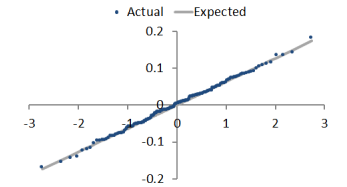


Fig. 4. Q-Q plot of USD/EUR residuals.

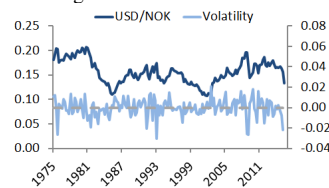


Fig. 5. Quarterly USD/NOK exchange rate.

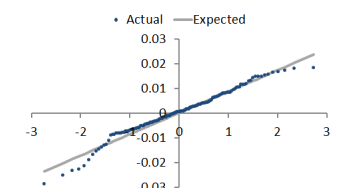


Fig. 6. Q-Q plot of USD/NOK residuals.

[38] argue that even though real exchange rates seem to move randomly in the short-run, there is empirical evidence that exchange rates move towards a long-term purchasing power parity. This implicates that a mean reverting process is necessary in order to capture long-term movements of real exchange rates. We adapt an AR(1) process fitted on real values as described by (6) and (7).

$$E_t^{kr} = \alpha_{E^{kr}} + \beta_{E^{kr}}(E_{t-1}^{kr}) + \epsilon_{E^{kr}} \quad (6)$$

$$E_t^e = \alpha_{E^e} + \beta_{E^e}(E_{t-1}^e) + \epsilon_{E^e} \quad (7)$$

IV. SOLUTION APPROACH

In order to evaluate operation and electricity sourcing for the aluminium smelter we use the LSM method and a two-stage stochastic program in a sequential approach. First, we evaluate the operation of the smelter with the LSM method assuming electricity is procured through 1-year forward contracts. The heuristic operating policy derived in the previous step is then used as input to the optimisation routine to investigate different electricity procurement schemes. We then select a favourable scheme and re-evaluate the smelter with the LSM method assuming electricity is procured accordingly, in order to investigate its effect on shutdown risk and smelter value. The information flow between the different steps of the sequence is illustrated in Figure 7.

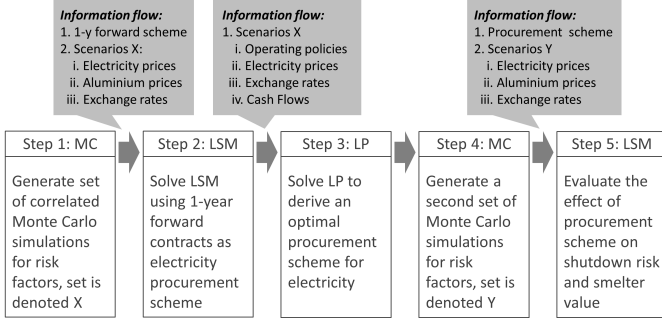


Fig. 7. Illustration of sequential solution approach.

A. Aluminium smelter operating policy

This section outlines the details of Step 2 in Figure 7. In order to find the net present value (NPV) of a smelter with operational flexibility one must also determine the optimal operating mode at any given time. As the complexity of the problem prevents us from deriving an analytical solution, a numerical method must be applied to obtain a heuristic operating policy and a lower bound on the smelter value. We use the regress-now variant of LSM for this purpose.

Every year the aluminium producer decides whether to operate the smelter for one more year, mothball or close. At the end of the planning horizon of T years, the smelter must either be closed or is assumed to operate in perpetuity. The value of the latter is approximated as the perpetuity of the cash flow in year T . The time required for state transitions is typically much shorter than a year. Therefore, we assume that transitions between states are immediate. We denote the aluminium and electricity price as S^A and S^e . The USD/NOK and USD/EUR exchange rates are denoted by E^{kr} and E^e . Let $\Pi^P(S^A, S^e, E^{kr}, E^e)$ and $\Pi^M(S^e, E^e)$ denote the respective cash flows of the producing and mothballs states. To streamline our presentation, we provide precise definitions of

these cash flows in Appendix A. Switching from producing to mothballed or closed incur costs K^{PM} and K^{PC} respectively. A transition from mothballs to closed has a cost of K^{MC} and a transition from mothballs to producing incurs cost K^{MP} . In all transitions to the closed state the producer receives the net value of pre-ordered electricity, denoted by $I_R(S^e, E^e)$ (see Appendix A for detailed definition).

The following is a formulation of the aluminium producer's problem formulated as a stochastic dynamic program (SDP). We let $\mathcal{T} = \{0, \dots, T\}$ denote the decision stages. The smelter's operating status is represented by an endogenous component, whereas the aluminium price, electricity price and exchange rates make up the exogenous component. We let P denote the producing state, M the mothballed state and C the closed state. Furthermore, since we limit the number of consecutive stages in which the smelter can stay mothballed to three, we let $M2$ and $M3$ denote a smelter in mothballs for two and three consecutive stages respectively. The endogenous set of states can then be defined as $\mathcal{X} = \{P, M, C, M2, M3\}$. We assume that the aluminium smelter is producing at stage $i = 0$. At stage i the exogenous set is denoted by the vector $\mathbf{z}_i = \{S_i^A, S_i^e, E^{kr}, E^e\}$. Thus, at stage i the state space is $\mathcal{X} \times \mathbb{R}^4$. We denote the decision to continue at the current state as d_N . Likewise, d_M , d_C and d_O represent the decisions to mothball, close and restart operations respectively. The set of decisions at endogenous state $x \in \mathcal{X}$ is defined by set

$$\mathcal{D}(x) = \begin{cases} \{d_N, d_M, d_C\}, & \text{if } x = P \\ \{d_N, d_C, d_O\}, & \text{if } x \in \{M, M2\} \\ \{d_C, d_O\}, & \text{if } x = M3 \\ \{d_N\}, & \text{if } x = C \end{cases}$$

The aluminium producer's decision results in an immediate cash flow. We define these cash flows using the function

$$r_i(x, \mathbf{z}_i, d) = \begin{cases} \Pi^P(\mathbf{z}_i), & \text{if } (x, d) = (P, d_N) \\ \Pi^M(\mathbf{z}_i) - K^{PM}, & \text{if } (x, d) = (P, d_M) \\ I_R(\mathbf{z}_i) - K^{PC}, & \text{if } (x, d) = (P, d_C) \\ \Pi^P(\mathbf{z}_i) - K^{MP}, & \text{if } x \in \{M, M2, M3\} \text{ and } d = d_O \\ I_R(\mathbf{z}_i) - K^{MC}, & \text{if } x \in \{M, M2, M3\} \text{ and } d = d_C \\ \Pi^M(\mathbf{z}_i), & \text{if } x \in \{M, M2\} \text{ and } d = d_N \\ 0, & \text{if } (x, d) = (C, d_N) \end{cases}$$

The function $f(x, d)$ defines the state transitions illustrated in Figure 8 (refer to Appendix A for a formal definition). The exogenous factors evolve according to the diffusion processes in Section III. Let ρ denote the discrete discount rate. In order to find an optimal operating policy the following stochastic dynamic program must be solved:

$$V_i(x, \mathbf{z}_i) = \max_{d \in \mathcal{D}(x)} r(x, \mathbf{z}_i, d) + \frac{1}{1 + \rho} \mathbb{E}[V_{i+1}(f(x, d), \mathbf{z}_{i+1}) | \mathbf{z}_i], \quad \forall (i, x, \mathbf{z}_i) \in \mathcal{T} \times \mathcal{X} \times \mathbb{R}^4 \quad (8)$$

where $V_i(x, \mathbf{z}_i)$ is the value function at stage i and state (x, \mathbf{z}_i) .

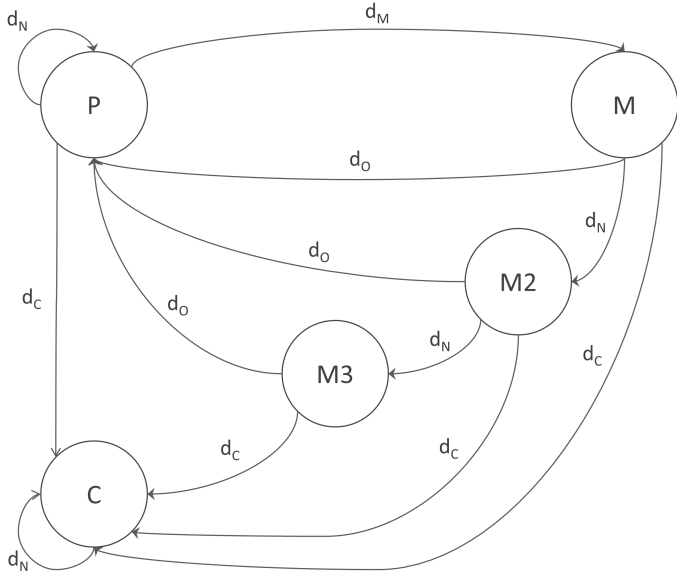


Fig. 8. State transitions.

LSM approximates the continuation function $W_i(x, \mathbf{z}_i) := \frac{1}{1+\rho} \mathbb{E}[V_{i+1}(x, \mathbf{z}_{i+1}) | \mathbf{z}_i]$ using basis functions in set $\Phi_{i,x} = \{\phi_{i,x,b}, b = 1, \dots, B_i\}$ where B_i is the number of basis functions at stage i and endogenous state x . Each $\phi_{i,x,b}$ is a function of \mathbf{z}_i . The continuation function approximation at (x, \mathbf{z}_i) is defined as $(\Phi_i, \beta_i)(x, \mathbf{z}_i) := \sum_{b=1}^{B_i} \phi_{i,x,b}(\mathbf{z}_i) \beta_{i,x,b}$ where $\beta_{i,x,b}$ is the coefficient in front of basis function b at time stage i and endogenous state x . The steps of the LSM procedure are as follows.

First we conduct Monte Carlo simulations of the exogenous information. We let $\hat{\mathbf{z}}_i(\omega)$ represent the stage i exogenous factor on sample path ω . The terminal values $\hat{v}_T(x, \hat{\mathbf{z}}_T(\omega))$ are calculated as in (9)-(11).

$$\hat{v}_T(P, \hat{\mathbf{z}}_T(\omega)) = \max\{r(P, \hat{\mathbf{z}}_T(\omega), d_N) + \frac{\Pi^P(\hat{\mathbf{z}}_T(\omega))}{\rho}, r(P, \hat{\mathbf{z}}_T(\omega), d_C)\}, \quad (9)$$

$$\hat{v}_T(M, \hat{\mathbf{z}}_T(\omega)) = \max\{r(M, \hat{\mathbf{z}}_T(\omega), d_O) + \frac{\Pi^P(\hat{\mathbf{z}}_T(\omega))}{\rho}, r(M, \hat{\mathbf{z}}_T(\omega), d_C)\}, \quad (10)$$

$$\hat{v}_T(C, \hat{\mathbf{z}}_T(\omega)) = 0 \quad (11)$$

Moving backwards from stage $T-1$ to stage 0, at each stage $i \in \{T-1, \dots, 0\}$ we (i) compute the value estimates along each sample path ω using the stage $i+1$ continuation function approximation:

$$v_i(x, \hat{\mathbf{z}}_i(\omega)) = \frac{1}{1+\rho} \max_{d_{i+1} \in \mathcal{D}(x)} r_{i+1}(x, \hat{\mathbf{z}}_{i+1}(\omega), d_{i+1}) + (\Phi_{i+1}, \beta_{i+1})(f(x, d_{i+1}), \hat{\mathbf{z}}_{i+1}(\omega))$$

and (ii) for each $x \in \mathcal{X}$, we compute the coefficients $\beta_{i,x,b}, \forall b$ by performing a least squares regression on the estimates $v_i(x, \hat{\mathbf{z}}_i(\omega)), \forall \omega$.

Having found the regression coefficients we can compute a feasible decision $d_i(x, \mathbf{z}_i)$ at a given stage i and state (x, \mathbf{z}_i) by solving the following optimisation problem:

$$d_i(x, \mathbf{z}_i) \in \arg \max_{d_i} r_i(x, \mathbf{z}_i, d_i) + (\Phi_i, \beta_i)(f(x, d_i), \mathbf{z}_i)$$

Therefore, LSM implicitly defines a heuristic operating policy. We simulate a separate set of sample paths of the exogenous information and simulate this operating policy to obtain a lower bound estimate on the smelter value.

A functional form for the regression basis $\phi_{i,x,b} \in \Phi_{i,x}$ must be chosen. [13] use weighted Laguerre polynomials in their original paper, but [39], [40], [41] and [42] argue that regressing on simple powers of the explanatory variables and cross products also provide fairly accurate numerical results compared to other forms of the explanatory variables. We choose to use the first four Laguerre polynomials as it is straightforward to implement and [40] show that this functional form provides fairly accurate results. These are defined as:

$$\begin{aligned} \Phi_{i,x,0}(\hat{\mathbf{z}}_i) &= 1 \\ \Phi_{i,x,1}(\hat{\mathbf{z}}_i) &= 1 - \hat{\mathbf{z}}_i \\ \Phi_{i,x,2}(\hat{\mathbf{z}}_i) &= \frac{1}{2} (\hat{\mathbf{z}}_i^2 - 4\hat{\mathbf{z}}_i + 2) \\ \Phi_{i,x,3}(\hat{\mathbf{z}}_i) &= \frac{1}{6} (-\hat{\mathbf{z}}_i^3 + 9\hat{\mathbf{z}}_i^2 - 18\hat{\mathbf{z}}_i + 6) \end{aligned}$$

Solving the problem described in this section yields a heuristic operating policy as well as an unbiased approximation of the smelter value.

B. Electricity sourcing

Optimising electricity procurement is Step 3 in Figure 7. Electricity can be procured through both long- and short-term contracts. Typically, short-term contracts have lower expected cost, but are more volatile. The challenge for a producer is to find a procurement scheme that minimises electricity costs, and at the same time limits the risk of incurring mothballs or closure in unfavourable market conditions. In this section we present a two-stage stochastic program that minimises a trade-off between the net present value of electricity costs and downside risk in cash flows from volatile electricity prices. We use CVaR as risk measure. Table III introduces the notation used, and the two-stage stochastic program is defined by (12)-(27).

TABLE III. DECLARATION OF TWO-STAGE STOCHASTIC PROGRAM TO SOLVE THE ELECTRICITY PROCUREMENT PROBLEM

Sets	
$N =$	Time stages $[1 \dots T_{max}]$
$L =$	Scenarios $[1 \dots L_{max}]$
$B =$	Contract lengths $[5, 10, 20]$
$C =$	Currencies $[USD, EUR, NOK]$
Indices	
$n \in N$	time
$m \in N$	time
$l \in L$	scenario
$b \in B$	contract length
$c \in C$	contract currency
Variables	
$q_{n,m,b,c}$	planned quantity of electricity delivered at time n originating from contract length b in currency c procured at time m
r_n^l	amount of electricity purchased in the 1-year forward market at time n in scenario l
y_n^l	total electricity cost in period n from previously ordered bilateral contracts in scenario l
x_n^l	loss over α -VaR at time n in scenario l
c_n^l	total electricity cost at time n in scenario l
ζ	α -VaR for every stage
δ_n	largest α -CVaR at time step n
Parameters	
T_{max}	number of stages (years) in the planning period
L_{max}	number of scenarios
D_{max}	maximum possible electricity demand in a period
B_{max}	number of contract lengths
C_{max}	number of contract currencies
ρ	per stage discount factor
α	probability level VaR
p^l	probability of scenario l
s_n^l	1-year forward electricity price at time n in scenario l denoted in EUR/MWh
$f_{n,m,c}^l$	expected forward price at time n conditional on electricity price at time m quoted in currency c in scenario l
D_n^l	electricity demand incurred from optimal operating policy at time n in scenario l
θ_n^l	USD/EUR exchange rate at time n in scenario l
λ	factor weight on CVaR versus minimising total electricity costs
CF_n^l	cash flow from aluminium smelter at time n in scenario l
ν	tax rate
u	factor to equalise the magnitude of CVaR and electricity cost in the objective function

$$\min_{\mathbf{q}, \mathbf{r}, \zeta, \delta} (1 - \lambda) \frac{1}{T_{max}} \sum_{n=2}^{T_{max}} \delta_n + \frac{\lambda}{u} \sum_{l=1}^{L_{max}} p^l \sum_{n=2}^{T_{max}} \frac{1}{(1 + \rho)^{n-1}} c_n^l \quad (12)$$

$$\text{s.t. } r_n^l + \sum_{c=1}^{C_{max}} \sum_{b=1}^{B_{max}} \sum_{m=1}^{T_{max}} q_{n,m,b,c} = D_n^l, \quad n \in N, l \in L \quad (13)$$

$$\sum_{c=1}^{C_{max}} \sum_{b=1}^{B_{max}} \sum_{m=1}^{T_{max}} q_{n,m,b,c} \leq D_{max}, \quad n \in N \quad (14)$$

$$\sum_{c=1}^{C_{max}} \sum_{b=1}^{B_{max}} \sum_{m=1}^{T_{max}} q_{n,m,b,c} f_{n,m,c}^l = y_n^l, \quad n \in N, l \in L \quad (15)$$

$$q_{n,m,b,c} = q_{m+1,m,b,c}, \quad m \in N, b \in B, c \in C, n \in [m + 2 \dots m + b] \quad (16)$$

$$q_{n,m,b,c} = 0, \quad m \in N, b \in B, c \in C, n \in [1 \dots m] \cup [m + b + 1 \dots N] \quad (17)$$

$$s_n^l \theta_n^l r_n^l + y_n^l = c_n^l, \quad n \in N, l \in L \quad (18)$$

$$\zeta + \frac{\sum_{l=1}^{L_{max}} p^l x_n^l}{1 - \alpha} \leq \delta_n, \quad n \in N \quad (19)$$

$$-CF_n^l - s_n^l \theta_n^l D_n^l (1 - \nu) + c_n^l (1 - \nu) - \zeta \leq x_n^l, \quad n \in N, l \in L \quad (20)$$

$$x_n^l \geq 0, \quad n \in N, l \in L \quad (21)$$

$$q_{n,m,b,c} \geq 0, \quad n \in N, m \in N, b \in B, c \in C \quad (22)$$

$$\delta_n \geq 0, \quad n \in N \quad (23)$$

$$y_n^l \geq 0, \quad n \in N, l \in L \quad (24)$$

$$\zeta \geq 0 \quad (25)$$

$$r_n^l \text{ is free}, \quad n \in N, l \in L \quad (26)$$

$$c_n^l \text{ is free}, \quad n \in N, l \in L \quad (27)$$

The portfolio optimisation takes the scenarios for exchange rates and electricity prices from Step 1 in Figure 7 as input. Demand, D_n^l , is derived from the heuristic operating policy found with the LSM method in Step 2 of the solution approach, e.g. when the smelter is mothballed there is no demand for electricity. Finally the cash flows, CF_n^l , calculated in Step 2 are used to calculate CVaR.

The objective function (12) is a trade-off between minimising CVaR and the NPV of electricity purchases, and is tractable by varying the parameter λ . Constraint (13) ensures that the demand at time n in scenario l is satisfied. If the smelter is closed and demand equals zero we allow for sale of pre-ordered electricity to the spot price, hence r_n^l is a free variable. Constraint (14) prevents pure speculation in long-term contracts by limiting the amount of electricity that can be purchased through these. Equation (15) calculates the realised electricity cost for delivery at time n from long-term contracts in scenario l . Note that (15) is not a constraint as y_n^l is a derived variable to ease the readability and hence does not increase the number of fundamental variables in the problem since it is substituted out by the solver. The prices of the long-term contracts are determined by $f_{n,m,c}^l$. Constraints (16)-(17) ensure consistency between the decision variables $q_{n,m,b,c}$. When entering a long-term contract at time m , $q_{n,m,b,c}$ stores the volume of this contract. Constraint (16) ensures that the volume of electricity is constant for each year of the contract's tenor. Constraint (17) ensures that $q_{n,m,b,c}$ cannot have value if it represents an infeasible contract, e.g. if n is smaller than m . Equation (18) is the total realised electricity cost at time n from purchasing electricity in the 1-year forward market and through long-term contracts. Note that c_n^l is also a derived variable in order to ease the readability. Constraint (19) corresponds to the CVaR constraint and (20) is the associated loss function. A loss at time n in scenario l is defined as a cash flow that is more negative than the α -VaR level. Finally, (21)-(27) are boundary constraints for the decision variables.

The two-stage stochastic program in (12)-(27) is solved with the software Xpress-Optimiser version 26.01.04². Using 10,000 scenarios, 40 time steps, three currencies and three possible long-term contract lengths we experienced a run time of 15-35 minutes for each value of λ .

The main output from the portfolio optimisation is the procurement scheme for long-term contracts, stored in $q_{n,m,b,c}$, along with 1-year forward purchases, stored in r_n^l .

C. Evaluation of procurement scheme

A favourable procurement scheme is chosen based on the relationship between total electricity cost and CVaR. Next, the scheme is evaluated by investigating its effect on shutdown risk and smelter value, and is compared to benchmark procurement schemes. This corresponds to Step 4 and Step 5 in Figure 7.

V. RESULTS

A. Calibrated parameters of risk factor processes

1) *Aluminium price:* The parameters of the three-factor extension for the aluminium price are calibrated from historical closing prices of monthly quoted futures³ at the London Metal Exchange (LME) in the period November 2009 to January 2015. This period has been carefully chosen in order to mitigate the effects of the financial crisis in 2007-2008. Calibrated parameter values are given in yearly terms in Table IV and the fit of the three-factor extension is illustrated in Figure 9.

TABLE IV. CALIBRATED PARAMETER VALUES THREE-FACTOR EXTENSION

Parameter	Estimate	Standard Error
κ	0.0415	0.0074
σ_χ	0.1966	0.0304
λ_χ	0.1295	0.0204
σ_ξ	0.1615	0.0158
η	2.8238	0.1710
$\bar{\mu}$	-0.0571	0.0079
$\bar{\mu}^*$	0.0500	1.0000
$(\bar{\mu}^* - \lambda_\xi)$	0.1165	0.0208
σ_μ	0.0530	0.0098
$\rho_{\chi\xi}$	-0.3827	0.1952
$\rho_{\chi\mu}$	0.2577	0.1444
$\rho_{\xi\mu}$	-0.5172	0.1543

Parameter estimates for $\bar{\mu}^*$, $\rho_{\chi\xi}$, $\rho_{\chi\mu}$ and $\rho_{\xi\mu}$ have undeniably high standard errors. Interpretations and conclusions based on these estimates must thus be done with special care. Table IV does not include estimates of λ_ξ or λ_μ . Point estimates of the latter parameters can be obtained by using $(\bar{\mu}^* - \lambda_\xi)$ together with differences in estimated long-term futures prices. Following the discussion in Section III-A we do not calculate these point estimates since λ_χ , λ_ξ and λ_μ are all set to zero in the simulations of (1)-(3). This means that we do not risk adjust the cash flows, but rather work under the real probability measure. Issues related to a high standard error for the estimate of $\bar{\mu}^*$ are thus also avoided.

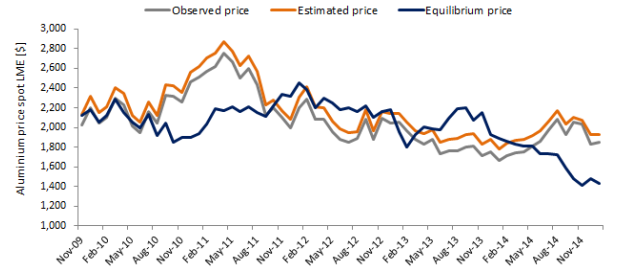


Fig. 9. Time series of the calibrated three-factor extension, with observed prices included.

The estimated value of κ corresponds to a half-life of 17 years for short-term deviations, which intuitively is unrealistically high. We argue that short-term deviations in the aluminium price are caused by events such as changes in storage levels, the market's perception of short-term scarcity and monetary disturbances such as a temporary increase in real interest rates [43], which cease within a year or two. A half-life of six months could therefore be realistic. Therefore, we set $\kappa = 1.17^4$. Simulating scenarios for quarterly prices over 40 years entails a few challenges when using the calibrated parameters from monthly time series data. A random walk for the equilibrium price yields unlikely values in some scenarios as the process may explode within the long time horizon that is simulated. This issue is pointed out by [44] who analyses simulation of the two-factor model. He argues that when used in long-term simulations, the equilibrium price should have a stationary mean. Therefore, we enforce a weak mean reversion to the equilibrium price when doing simulations by adjusting the discretised version of (2). We set the coefficient in front of ξ_{t-1} to 0.95 and add a constant term of 0.382386, the latter in order to match the average of the log spot price in the calibration period. In addition we assume that the mean growth rate $\bar{\mu}$ in (3) is zero.

2) *Electricity price:* The parameters of (7) are calibrated using Reuters data on 1-year forward NP system prices quoted in quarterly intervals. The time series starts in 2001 when the Nordic power market became fully integrated.

TABLE V. CALIBRATED PARAMETER VALUES FOR THE LOG ELECTRICITY PRICE DYNAMICS

Parameter	Estimate	Standard Error
α_{se}	0.471	0.203
β_{se}	0.870	0.057
σ_{se}	0.118	

The calibrated parameters of the log price process are given in quarterly terms in Table V, and correspond to a long-term

²Part of the FICO Xpress Optimisation Suite 7.7.

³Futures considered are 3, 6, 9, 12, 24, 48, 72, 96 and 120 month tenors.

⁴This is actually very similar to the κ estimate for crude oil obtained by [32].

mean level of 38.47⁵ EUR/MWh and a half-life of 1.25 years.

3) *Exchange rates:* The parameters of (6) and (7) are calibrated using historical data from Reuters EcoWin Pro quoted in quarterly intervals from 1974 to 2014. The EUR currency was first introduced in January 1999, but to increase the size of the data set we have used an extended time series approximated by Reuters to estimate parameters.

TABLE VI. CALIBRATED PARAMETER VALUES FOR THE USD/EUR EXCHANGE RATE

Parameter	Estimate	Standard Error
α_{E^e}	0.077	0.034
β_{E^e}	0.935	0.028
σ_{E^e}	0.040	

TABLE VII. CALIBRATED PARAMETER VALUES FOR THE USD/NOK EXCHANGE RATE

Parameter	Estimate	Standard Error
$\alpha_{E^{kr}}$	0.010	0.004
$\beta_{E^{kr}}$	0.933	0.028
$\sigma_{E^{kr}}$	0.001	

The calibrated parameters are given in quarterly terms in Tables VI and VII. For the USD/EUR exchange rate the parameters correspond to a long-term mean of 1.183 USD/EUR and a half-life of 2.57 years. The USD/NOK exchange rate has an estimated long-term mean of 0.152 USD/NOK and a half-life of 2.51 years.

4) *Simulations:* We perform Monte Carlo simulations of the discretised price processes to generate sample paths. Correlations between the processes are captured through Cholesky decomposition of the covariance matrix.

B. Evaluating smelter operation when sourcing electricity from 1-year forward contracts

The parameters used for deriving the aluminium smelter cash flows are listed in Table VIII. Note that we use a real discount rate, hence all inputs are in real values. The problem was solved under the assumption that all electricity is purchased through 1-year forward contracts.

The smelter value approximations for different multiples of the actual volatilities of the aluminium and electricity price processes are shown in Table IX. It is evident that an increase in electricity price volatility has a negative impact on the

⁵We have used the following. Log electricity price, denoted by \tilde{X} , follows an Ornstein-Uhlenbeck (OU) process. Then $d\tilde{X}_t = \kappa^*(\theta^* - \tilde{X}_t)dt + \sigma_{S^e}dW_t$. Discretisation yields $\Delta t = 1$, $\kappa^* = \frac{1 - \beta_{S^e}}{\Delta t}$ and $\theta^* = \frac{\alpha_{S^e}}{\kappa^* \Delta t}$. θ^* is the long-term expected value of an OU process. The conditional variance when t goes towards infinity is by Ito's isometry given as $\lim_{t \rightarrow \infty} \text{Var}[\tilde{X}_t | \tilde{X}_0] = \frac{\sigma_{S^e}^2}{2\kappa^*}$. Using this variance together with θ^* in the formula for the expected value of a lognormally distributed variable, the expected long-term real electricity price when substituting for κ^* and θ^* is given by $\tilde{S}^e = e^{\left[\frac{\alpha_{S^e}}{(1 - \beta_{S^e})} + \frac{1}{2} \left(\frac{\sigma_{S^e}^2}{2(1 - \beta_{S^e})} \right) \right]}$.

⁶Parts of the costs in the electrolysis step are incurred in local currency, there is thus some exchange rate risk related to this part of total electrolysis cost.

TABLE VIII. PARAMETER VALUES

Parameter	Parameter description	Parameter value
C_a	Carbon cost per mt aluminium	400 USD/mt
A	Alumina cost per mt aluminium	758 USD/mt
ρ	WACC (real)	5%
ν	Company tax rate	27%
B^L	Electrolysis cost local currency ⁶	3,173 NOK/mt
B^U	Electrolysis cost USD	163 USD/mt
C^L	Casthouse cost local currency	1,692 NOK/mt
C^U	Casthouse income USD	425 USD/mt
M^*	El. consumption rate per mt aluminium	14 MWh/mt
O^M	Annual operating cost for a mothballed smelter	0 USD/mt
K^{PC}	After-tax switching cost operating to closed	2,000 USD/mt
K^{PM}	After-tax switching cost operating to mothballed	1,000 USD/mt
K^{MC}	After-tax switching cost mothballed to closed	1,000 USD/mt
K^{MP}	After-tax switching cost mothballed to operating	1,000 USD/mt
T^{temp}	Max number of consecutive years mothballed	3 years
S^{AP}	Premium in percentage of aluminium price	10%

TABLE IX. SENSITIVITY TABLE OF SMELTER VALUE WITH FULL OPERATIONAL FLEXIBILITY

		σ_{el}				
		0.5x	0.75x	1.0x	1.25x	1.5x
σ_{alu}	0.5x	854	807	748	649	566
	0.75x	1,213	1,151	1,140	1,020	986
	1.0x	1,794	1,698	1,602	1,575	1,517
	1.25x	2,384	2,353	2,334	2,235	2,112
	1.5x	3,158	3,091	2,979	2,944	2,813

TABLE X. VALUE INCREASE OF ADDING FULL OPERATIONAL FLEXIBILITY

		σ_{el}				
		0.5x	0.75x	1.0x	1.25x	1.5x
σ_{alu}	0.5x	100	129	159	180	224
	0.75x	194	199	218	269	321
	1.0x	289	312	321	366	426
	1.25x	403	432	436	477	510
	1.5x	498	504	535	571	607

TABLE XI. VALUE INCREASE OF ADDING MOTHBALLS ON TOP OF CLOSURE OPTIONALITY

		σ_{el}				
		0.5x	0.75x	1.0x	1.25x	1.5x
σ_{alu}	0.5x	2	9	19	25	40
	0.75x	15	27	38	55	82
	1.0x	43	55	57	94	113
	1.25x	77	83	96	124	143
	1.5x	142	143	149	178	185

smelter value, a result that may be counter intuitive. Increased volatility yields larger random terms. Since we simulate log prices the simulated state space of real prices is lognormally distributed. Hence, positive random terms have a greater effect on the real price than negative random terms. As electricity is a cost, higher volatility will thus have a net negative effect on the smelter value. Increased volatility in the electricity price will also trigger more mothballs and closures, thus accumulated shutdown costs increase. Conversely, higher volatility in the aluminium price has a positive effect on the smelter value, as a higher aluminium price has a positive cash flow effect.

Table X shows the value increase from adding both closure and mothballs optionality compared to having no closure optionality, while Table XI shows the additional value gained by adding mothballs optionality when already having closure

optionality. In the base case the value increase of having full operational flexibility compared to no flexibility is \$321/mt, which represents an increase of 25%. Somewhat surprisingly there is a noticeable increase in value attributed to introducing the option to mothball when already having the option to close. The increase in value is \$57/mt, or 4% in relative terms, but when electricity price volatility is high, the value increases with up to 8%. Increased volatility in the aluminium price and/or the electricity price increase(s) the value of both closure and mothballs. This is an expected result since operational flexibility is of higher value in more uncertain times. For instance, when having operational flexibility, periods with high electricity prices may be hedged by the means of mothballs or closure, which increases the smelter value.

This observation is also emphasised by Figure 10, which plots the percentage of scenarios with closures, mothballs and no shutdowns respectively as function of electricity price volatility. It is evident that higher electricity price volatility yields an increased probability of closures and/or mothballs. The same pattern materialises for the aluminium price and is illustrated in Figure 11. Hedging aluminium price risk is however beyond the scope of this paper, but we consider hedging the electricity price risk. Optimising electricity procurement with respect to some risk measure is, based on the above rationale, a potential means for reducing closure and mothballs risk, an issue that is further discussed in Section V-D.

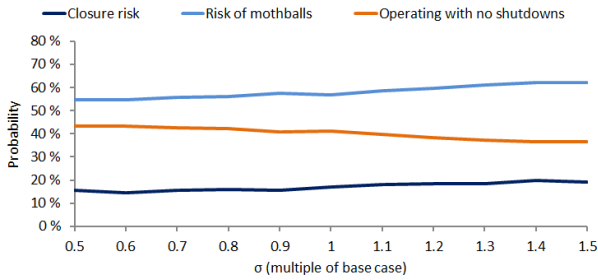


Fig. 10. Shutdown risk at different volatility levels for the electricity price.

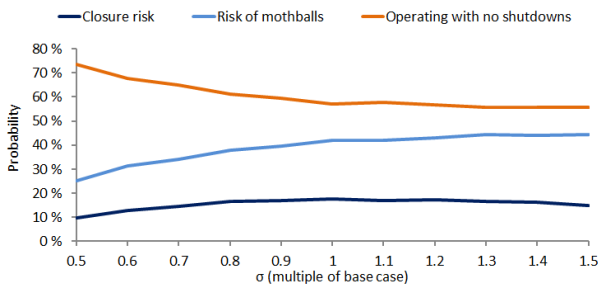


Fig. 11. Shutdown risk at different volatility levels for the aluminium price.

A final remark is that the smelter value estimates, when repeating the LSM procedure on new simulation sets for risk factors, converge with an increasing number of scenarios being used in the simulations. Figure 12 illustrates this feature. We

use 10,000 simulated scenarios when determining an operating policy and approximating the smelter value.

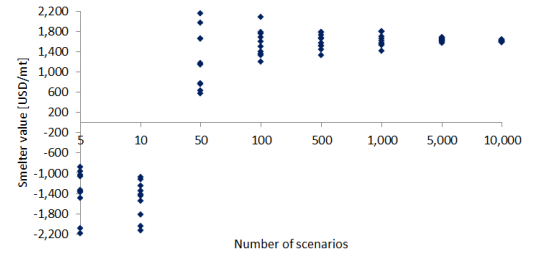


Fig. 12. Convergence of results, as function of the number of in- and out-of sample scenarios. For illustrative reasons, only 10 example runs for each number of scenarios are plotted.

C. Results from optimising electricity sourcing

Using the operating policy, cash flows and underlying risk factor simulations as input to the problem in (12)-(27), we are able to investigate different electricity procurement schemes. Figure 13 illustrates the relationship between total electricity cost and average CVaR over all operating years for different values of λ . For low values of λ we have risk aversion, whereas higher values of λ yield higher risk tolerance. We observe a high total electricity cost and low CVaR when λ is small. As λ increases the producer becomes more risk tolerant with respect to electricity price volatility and more emphasis is put on minimising the total electricity cost. Hence, riskier contracts are selected, electricity cost is reduced and average CVaR increases. An interesting observation is that real changes to the CVaR in Figure 13 are relatively small. This is due to the fact that the loss function in (20) includes not only risk from high electricity prices, but also risk from low aluminium prices. The implication of this is that when management is risk averse, CVaR will still be relatively large due to aluminium price risk. The latter is dominating and reducing electricity price risk has only a limited impact on total system risk compared to reducing aluminium price risk, which can be observed when comparing Figure 10 and Figure 11. This argument is also supported by Figure 14, which displays the same analysis as in Figure 13, but where the volatility of the aluminium price process is halved. We can observe that CVaR is reduced as a result of lower total risk, and the difference between the highest and the lowest CVaR is larger in magnitude. This is because a greater part of the system risk now originates from electricity prices.

In Figure 15 we can see how the average electricity portfolio mix over all simulated years varies with different values of λ . As expected, when management is risk averse the portfolio consists of a combination of 1-year forwards and long-term contracts. When λ increases and thus risk tolerance increases, a larger share of the portfolio is comprised of 1-year forward contracts. An interesting observation from Figure 15(a) is the large share of 1-year forward contracts when minimising CVaR alone. As the 1-year forward contracts are the most volatile contracts, this result might be counterintuitive. It appears

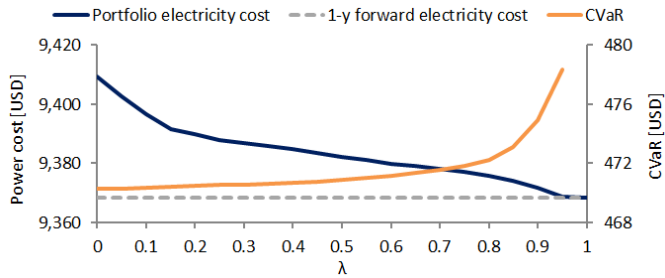


Fig. 13. CVaR and electricity cost against λ , $\sigma_{alu} = 1.0x$ base case.

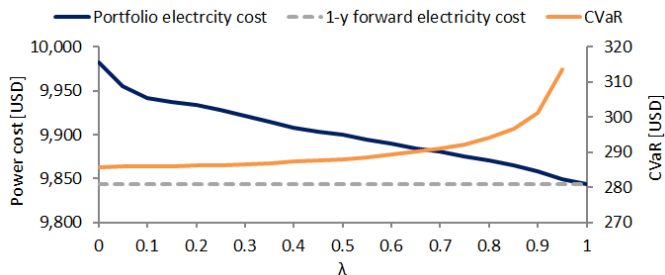


Fig. 14. CVaR and electricity cost against λ , $\sigma_{alu} = 0.5x$ base case.

because the CVaR is calculated using smelter cash flows, hence incorporates aluminium price risk. At a certain CVaR level, all risk originates from low aluminium prices, hence procuring a larger share of forward contracts will not reduce the risk of losses. For the same reason the longest contracts with duration of 20 years are never purchased. Since aluminium price risk becomes more dominating for low values of λ , 20-year contracts will not contribute to reducing CVaR. We confirmed this by reducing the volatility of the aluminium price process by 50%. We found that 20-year contracts now to a much larger extent were procured in order to reduce risk. For λ equal to zero, that is maximum risk aversion, we observed that the procurement scheme only included long-term contracts.

Another observation when studying the derived procurement scheme is that, even though it is possible to enter into contracts denominated in three different currencies, the portfolio optimisation always finds the USD currency to be most favourable. This result is unexpected, and may occur due to the risk measure used in the two-stage stochastic program. Since the smelter is valued on a USD per produced mt aluminium basis, entering into a long-term USD contract implicates hedging both electricity price and exchange rate risk. Since CVaR is a tail statistic, it will penalise the EUR and NOK contracts for being exposed to fluctuations in foreign exchange rates. Hence, when minimising risk the two-stage stochastic program will favour USD contracts over other currencies.

To evaluate the stability of the solution, Step 1 through 3 in Figure 7 were repeated ten times and the different resulting procurement schemes were compared. The results are analysed in Figure 16, which shows the average volume of each contract together with the maximum and minimum volume observed for

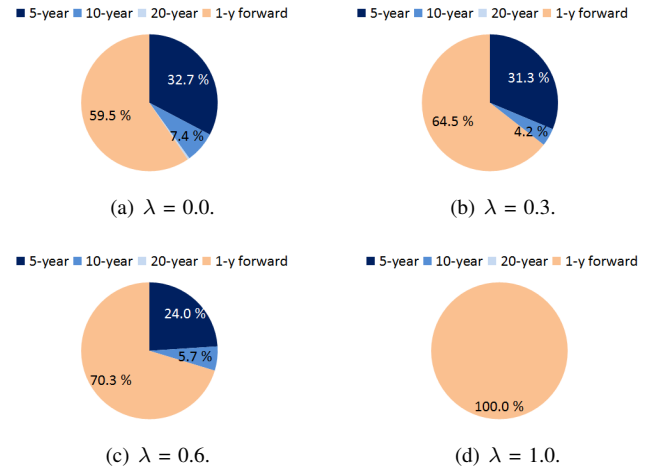


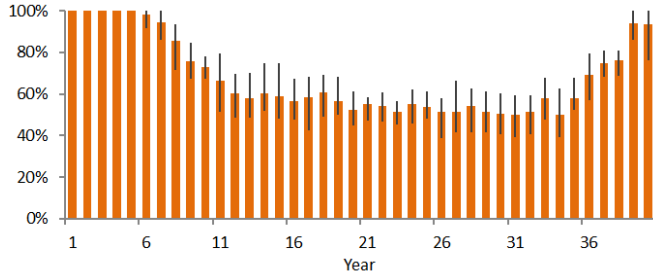
Fig. 15. Average contract mix for different values of λ .

each year in the ten schemes when λ equals 0.6. We observe that the portfolio optimisation yields fairly stable solutions. Figure 16(a) shows that the volume of 1-year forwards is relatively stable compared to the long-term contracts in 16(b) and 16(c). An explanation for this can be found in Figure 17, where we see evidence that the volume of the 10-year contract ordered in year 8 substitutes the volume of 5-year contracts. Exposure to the 1-year forward contract thus remains stable. Note that we assume that the smelter does not hold any contracts in the beginning of the period. Therefore, we argue that the procurement schemes in years 11-34 represent a steady-state, with minimal start-of and end-of-period effects.

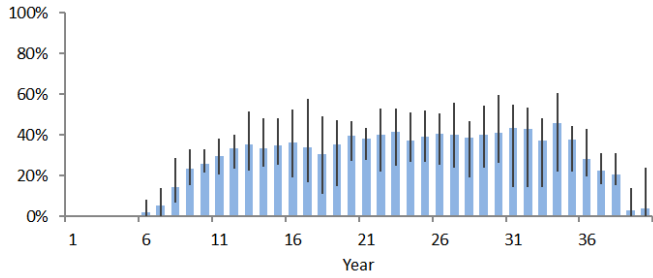
Utilising insights from the portfolio optimisation solution we are able to find electricity sourcing schemes that reduce the risk of losses from volatile electricity prices. Figure 13 illustrates that for $\lambda \in [0.3, 0.6]$, cost has decreased noticeably without compromising CVaR too much. We therefore consider the procurement schemes in this interval as favourable. In the following section we therefore use the scheme derived when $\lambda = 0.6$ as input in Step 4 and Step 5 (refer to Figure 7) in the sequential solution approach to investigate possible benefits.

D. Results from the integrated problem

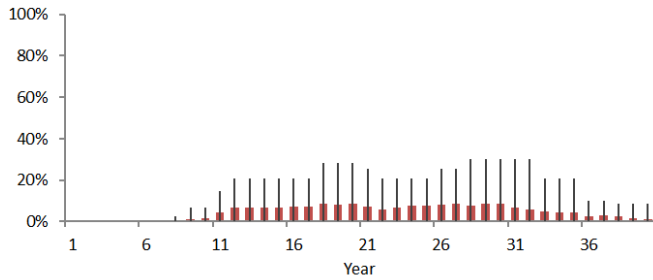
The procurement scheme in Figure 17 was evaluated by using it as input and re-solving the problem in Section IV-A. Table XII summarises key findings where the selected procurement scheme, called portfolio, is compared against static long-term schemes, a 1-year forward scheme and a two-stage benchmark portfolio, named constant demand. A static long-term procurement scheme assumes that demand is to be fulfilled only from one type of contract. E.g. with a 20-USD scheme electricity is purchased through a 20-year contract ordered the first year and another 20-year contract ordered in year 21, both denominated in USD. The two-stage benchmark is derived by solving the portfolio optimisation without the operating policy derived with the LSM method as input, thus treating demand as constant. The impact on smelter value and shutdown risk is shown in Figure 18 and Table XII.



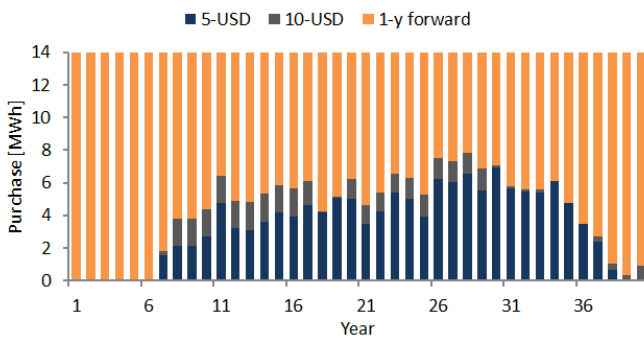
(a) Share of electricity from 1-year forwards.



(b) Share of electricity from 5-year contracts.



(c) Share of electricity from 10-year contracts.

Fig. 16. Average result and high-low lines of ten electricity procurement schemes for $\lambda = 0.6$.Fig. 17. Electricity procurement scheme for $\lambda = 0.6$.

In Table XII, the two first columns show the share of scenarios where closures and mothballs have occurred respectively. Since multiple mothballs can occur in one scenario, the third column calculates the average number of mothballs per scenario. The fourth column shows the percentage of scenarios

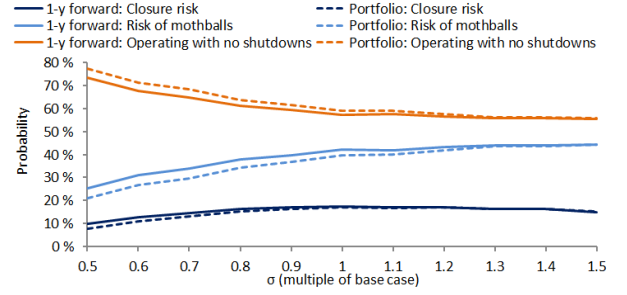


Fig. 18. Comparison of portfolio and 1-y forward shutdown risk at different volatility levels for the aluminium price.

TABLE XII. PERFORMANCE OF PORTFOLIO AGAINST STATIC CONTRACTS AND CONSTANT DEMAND OPTIMISATION, $\lambda = 0.6$

Contract type	Closures	Mothballs	# mothb. /scenario	No shutdowns	Value [USD/mt]
Portfolio	17 %	38 %	1.0	60 %	1,593
20-Nok	17 %	34 %	0.9	63 %	1,449
20-Eur	18 %	36 %	0.9	61 %	1,450
20-Usd	21 %	37 %	0.9	59 %	1,414
10-Nok	17 %	35 %	0.9	63 %	1,450
10-Eur	18 %	37 %	0.9	61 %	1,449
10-Usd	20 %	37 %	0.9	59 %	1,417
5-Nok	17 %	36 %	0.9	62 %	1,453
5-Eur	18 %	37 %	1.0	61 %	1,455
5-Usd	19 %	37 %	1.0	60 %	1,431
1-y forward	17 %	41 %	1.1	58 %	1,602
Const. demand	18 %	39 %	1.0	59 %	1,582

where no closures or mothballs are undertaken, whereas the last column gives us the approximated value of the smelter.

Relative to the 1-year forward case, the portfolio has fewer scenarios with mothballs, a lower average number of mothballs per scenario and an increase in the number of scenarios where the smelter operates with no shutdowns. Furthermore, there is a marginal decrease in smelter value when procuring electricity according to the portfolio compared to only 1-year forward contracts. In Table XII we also observe that the static 20-NOK scheme seems to be the most favourable procurement scheme in terms of shutdown risk. However, the smelter value is approximated to be 10% higher with the portfolio compared to the 20-NOK scheme. We argue that the considerable compromise in smelter value with the 20-NOK scheme is negative from a shareholder's point of view, which favours the portfolio. Finally, we observe that the portfolio outperforms the constant demand benchmark on most metrics. Hence, it is value adding to solve the optimisation problem with demand derived from the heuristic operating policy determined in Step 2. Overall, the results show that the portfolio reduces the risk of mothballs without compromising the value of the smelter significantly.

Since the portfolio is a mix of 1-year forward and long-term contracts, we would expect the closure risk to fall in the interval between what is observed for the 1-year and long-term electricity procurement schemes. A somewhat surprising observation is that there is an equal probability of shutting down the smelter permanently with the portfolio mix compared to the 1-year forward case, which favours the portfolio. We argue that closures have more negative impacts for stakeholders than

mothballs. Thus, it is desirable with an electricity procurement scheme that does not compromise on closure risk. Being able to reduce mothballs risk without increasing the closure risk is therefore an attractive feature with our findings.

In Figure 19 we further evaluate how the portfolio performs against static contracts for different values of λ . The line is a plot of electricity cost and CVaR from the optimal portfolio for different values of λ , and represents an efficient frontier. Portfolios that lie to the upper right of this frontier are sub-optimal, since they do not provide a lower electricity cost for the given level of risk. As expected, the figure illustrates that the static contracts perform worse in terms of CVaR and total electricity cost. We see that the difference in total electricity cost between the efficient frontier and static contracts is large for all contracts, and roughly similar to the difference in smelter value observed in Table XII.

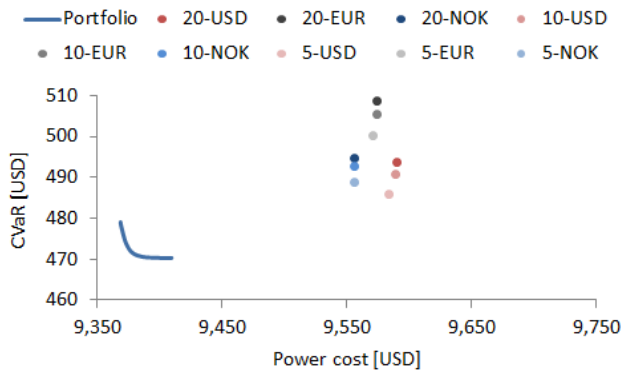


Fig. 19. Efficient frontier of portfolio compared to static contracts.

The insights and solution approach presented in the latter sections make up a valuable decision-making basis for management when planning long-term schemes for electricity procurement and for assessing the implications of the current market conditions on the robustness of smelters.

VI. CONCLUSION

We have shown how the least squares Monte Carlo (LSM) method can be used to produce unbiased value estimations of an aluminium smelter with mothballs and closure optionality. There is substantial value in operational flexibility, and adding the flexibility of mothballs when already having closure flexibility surprisingly yields a noticeable increase in smelter value. We argue that capturing the dynamics of the aluminium price, the most important risk factor, should be done according to recent literature which states that a single-factor mean reverting process is not well anchored in reality and that a more sophisticated approach should be used. The Schwartz-Smith three-factor extension in [32] is applied. We also developed a two-stage stochastic program that optimises the electricity procurement scheme with respect to a trade-off between Conditional Value-at-Risk (CVaR) and total electricity cost, using the heuristic operating policy derived with the LSM method and simulated state space of the risk factors as

input. The solution from the optimisation routine proved to be fairly stable. Re-evaluating the aluminium smelter using the optimal electricity procurement scheme yielded a reduction in the probability of mothballs, an increase in the probability of operating without shutdowns and no difference in closure risk. This without compromising smelter value. We also found that using the demand derived from the heuristic operating policy as input to the two-stage stochastic program as opposed to assuming constant demand resulted in a higher smelter value and lower risk of closures and mothballs. Using the sequential solution approach in an iterative manner could potentially yield a procurement scheme that further decreases shutdown risk. It may also be used for rolling simulations and rebalancing of bilateral electricity contract portfolios.

APPENDIX A SUPPLEMENTARY DETAILS STOCHASTIC DYNAMIC PROGRAM

Smelter cash flows depend on the elements listed in Table XIII, all expressed in per mt produced aluminium terms unless otherwise stated.

TABLE XIII. CASH FLOW ELEMENTS

Element	Description
S^A	Aluminium price
S^e	Electricity price
E^{kr}	USD/NOK exchange rate
E^e	USD/EUR exchange rate
C_a	Carbon cost
A	Alumina cost
ν	Company tax rate
B^L	Electrolysis cost local currency NOK per mt aluminium
B^U	Electrolysis cost USD
C^L	Casthouse cost local currency NOK per mt aluminium
C^U	Casthouse income USD. Casthouse is considered as an own factory with costs in NOK and revenues in USD
M^*	El. consumption rate MWh per mt aluminium
O^M	Annual operating cost for a mothballed smelter
S^e	Electricity price
S^A	Aluminium price
F^e	Average price of pre-ordered electricity contracts with delivery the current period, quoted in USD
E^{kr}	Exchange rate USD/NOK
E^e	Exchange rate USD/EUR
Q	Amount of pre-ordered electricity for the current period
S^{AP}	Premium in percentage of aluminium price

Π^P and Π^M are the cash flows of an operating and mothballed smelter respectively, and are defined as follows.

$$\begin{aligned} \Pi^P &= \left(S^A(1 + S^{AP}) - A - C_a - B^L E^{kr} - B^U \right. \\ &\quad \left. - C^L E^{kr} + C^U - S^e E^e (M^* - Q) - F^e Q \right) (1 - \nu) \\ \Pi^M &= \left(Q(S^e E^e - F^e) - O^M \right) (1 - \nu) \end{aligned}$$

I_R is the net present value of all pre-ordered electricity with delivery after the closure time. When procuring electricity through 1-year forwards this is just the difference between the price paid with the 1-year forward and the spot electricity price at closure time, all multiplied with the amount of purchased

electricity. The value of pre-ordered electricity with delivery at a date $t^* > t$, where t is the closure time, is the difference between F^e at t^* and the expected electricity price at time t^* conditional on the current electricity price. Then I_R is the discounted value of these differences adjusted for tax and multiplied with the respective amounts of pre-ordered electricity.

Figure 8 illustrates the transitions given (x, d_i) . The formal definition of the transition function $f(x, d)$ is:

$$f(x, d) = \begin{cases} x, & \text{if } x \in \{P, C\} \text{ and } d = d_N \\ M, & \text{if } x = P \text{ and } d = d_M \\ M2, & \text{if } x = M \text{ and } d = d_N \\ M3, & \text{if } x = M2 \text{ and } d = d_N \\ C, & \text{if } x \in \{P, M, M2, M3\} \text{ and } d = d_C \\ P, & \text{if } x \in \{M, M2, M3\} \text{ and } d = d_O \end{cases}$$

ACKNOWLEDGMENT

A special thanks is directed towards Dr. Selvaprabu Nadarajah and Denis Mazieres for acting as mentors and sharing their knowledge, expertise and brilliant ideas with us through constructive discussions during the process.

REFERENCES

- [1] H. A. Øye and M. Sørli, "Power Failure, Restart and Repair," *Aluminium International Today*, vol. 23, no. 2, pp. 25–27, 2011.
- [2] M. G. Nicholls, "Optimizing the Operations of an Aluminium Smelter Using Non-Linear Bi-Level Programming," in *Multilevel Optimization: Algorithms and Applications*, ser. Nonconvex Optimization and Its Applications, A. Migdalas, P. M. Pardalos, and P. Vörbrand, Eds. Springer US, 1998, vol. 20, pp. 117–148.
- [3] T. W. Harton, "Virtual Smelter Modelling for Metal Flow Management," in *Proceedings of the 45th Annual Conference of the ORSNZ*, 2010.
- [4] A. Molina-Garcia, M. Kessler, M. C. Bueso, J. A. Fuentes, E. Gómez-Lázaro, and F. Faura, "Modeling Aluminum Smelter Plants Using Sliced Inverse Regression With a View Towards Load Flexibility," *IEEE Transactions on Power Systems*, vol. 26, no. 1, pp. 282–293, 2011.
- [5] C. L. Bastian-Pinto, L. E. T. Brando, R. Raphael, and L. M. Ozorio, "Financial Valuation of Operational Flexibilities in the Aluminum Industry using Real Option Theory," in *Proceedings of the 17th Annual International Conference Real Options: Theory Meets Practice*, 2013.
- [6] M. J. Brennan and E. S. Schwartz, "Evaluating Natural Resource Investments," *The Journal of Business*, vol. 58, no. 2, pp. 135–157, 1985.
- [7] M. Carrión, A. B. Philpott, A. J. Conejo, and J. M. Arroyo, "A Stochastic Programming Approach to Electric Energy Procurement for Large Consumers," *IEEE Transactions on Power Systems*, vol. 22, no. 2, pp. 744–754, 2007.
- [8] M. Thompson, M. Davison, and H. Rasmussen, "Natural Gas Storage Valuation and Optimization: A Real Options Application," *Naval Research Logistics*, vol. 56, no. 3, pp. 226–238, 2009.
- [9] M. Thompson, "Optimal Economic Dispatch and Risk Management of Thermal Power Plants in Deregulated Markets," *Operations Research*, vol. 61, no. 4, pp. 791–809, 2013.
- [10] S. Nadarajah, F. Margot, and N. Secomandi, "Approximate Linear Programming Relaxations for the Real Options Management of Commodity Storage Assets," *Management Science*, 2015, Forthcoming.
- [11] S. E. Fleten, S. W. Wallace, and W. T. Ziemba, "Hedging Electricity Portfolios via Stochastic Programming," in *Decision making under uncertainty*. Springer, 2002, pp. 71–93.
- [12] S. Sen, L. Yu, and T. Genc, "A Stochastic Programming Approach to Power Portfolio Optimization," *Operations Research*, vol. 54, no. 1, pp. 55–72, 2006.
- [13] F. A. Longstaff and E. S. Schwartz, "Valuing American Options by Simulation: A Simple Least-Squares Approach," *The Review of Financial Studies*, vol. 14, no. 1, pp. 113–147, 2001.
- [14] J. F. Carriere, "Valuation of The Early-Exercise Price for Options Using Simulations and Nonparametric Regression," *Insurance: Mathematics and Economics*, vol. 19, no. 1, pp. 19–30, 1996.
- [15] J. N. Tsitsiklis, N. John, and B. V. Roy, "Regression Methods for Pricing Complex American-style Options," *IEEE Transactions on Neural Networks*, vol. 12, no. 4, pp. 694–703, 2001.
- [16] A. Gamba and M. Tesser, "Structural Estimation of Real Options Models," *Journal of Economic Dynamics and Control*, vol. 33, no. 4, pp. 798–816, 2009.
- [17] G. Alesii, "Assessing Least Squares Monte Carlo for the Kulatilaka Trigeorgis General Real Options Pricing Model," 2008, Working paper. Istituto per le Applicazioni del Calcolo "Mauro Picone" IAC Report Series n.147 6/2008. Available at: http://papers.ssrn.com/sol3/papers.cfm?abstract_id=153525.
- [18] G. Cortazar, M. Gravet, and J. Urzua, "The Valuation of Multidimensional American Real Options Using the LSM Simulation Method," *Computers & Operations Research*, vol. 35, no. 1, pp. 113–129, 2008.
- [19] E. S. Schwartz, "The Stochastic Behavior of Commodity Prices: Implications for Valuation and Hedging," *The Journal of Finance*, vol. 52, no. 3, pp. 923–973, 1997.
- [20] H. Markowitz, "Portfolio Selection," *The Journal of Finance*, vol. 7, no. 1, pp. 77–91, 1952.
- [21] D. Duffie and J. Pan, "An Overview of Value at Risk," *The Journal of Derivatives*, vol. 4, no. 3, pp. 7–49, 1997.
- [22] T. S. Beder, "VAR: Seductive but Dangerous," *Financial Analysts Journal*, vol. 51, no. 5, pp. 12–24, 1995.
- [23] R. Rockafellar and S. Uryasev, "Optimization of Conditional Value-at-Risk," *The Journal of Risk*, vol. 2, pp. 21–42, 2000.
- [24] C. Bao, G. Lee, and Z. Zhu, "A Simulation-Based Portfolio Optimization Approach with Least Squares Learning," in *Proceedings of the World Congress on Engineering 2014 Vol II*, July.
- [25] A. Hatami, H. Seifi, and M. Sheikh-El-Eslami, "A Stochastic-Based Decision-Making Framework for an Electricity Retailer: Time-of-Use Pricing and Electricity Portfolio Optimization," *IEEE Transactions on Power Systems*, vol. 26, no. 4, pp. 1808–1816, 2011.
- [26] Y. Jin and P. Jorion, "Firm Value and Hedging: Evidence from U.S. Oil and Gas Producers," *The Journal of Finance*, vol. 61, no. 2, pp. 893–919, 2006.
- [27] A. Dixit and R. Pindyck, *Investment Under Uncertainty*. New Jersey: Princeton University press, 1994.
- [28] H. Geman, *Commodities and Commodity Derivatives*. West Sussex, UK: Wiley, 2005.
- [29] B. Jafarizadeh and R. B. Bratvold, "A Two-Factor Price Process for Modeling Uncertainty in Oil Prices," in *Presented in SPE Hydrocarbon Economics and Evaluation Symposium, Calgary, Alberta, Canada. Society of Petroleum Engineer*, 2012.
- [30] L. M. Ozorio, P. V. Shevchenko, and C. L. Bastian-Pinto, "The Choice of Stochastic Process in Real Option Valuation II: Selecting Multiple Factor Models," in *Proceedings of the 17th Annual International Conference Real Options: Theory Meets Practice*, July 2013.
- [31] E. S. Schwartz and J. E. Smith, "Short-Term Variations and Long-Term Dynamics in Commodity Prices," *Management Science*, vol. 46, no. 7, pp. 893–911, 2000.
- [32] —, "Short-Term Variations and Long-Term Dynamics in Commodity Prices: Incorporating A Stochastic Growth Rate," 1998, Note from earlier version of the paper "Short-Term Variations and Long-Term Dynamics in Commodity Prices," Available from: https://faculty.fuqua.duke.edu/jes9/bio/Shortlong_stochastic_growth_rate_extension.pdf.

- [33] A. C. Harvey and N. Shephard, "Structural Time Series Models," *Handbook of Statistics*, vol. 11, 1993.
- [34] A. Andresen and J. M. Sollie, "Multi-factor Models and the Risk Premiums: A Simulation Study," *Energy Systems*, vol. 4, no. 3, pp. 301–314, 2013.
- [35] A. Escribano and J. I. Peña and P. Villaplana, "Modelling Electricity Prices: International Evidence*," *Oxford Bulletin of Economics and Statistics*, vol. 73, no. 5, pp. 622–650, 2011.
- [36] J. J. Lucia and E. S. Schwartz, "Electricity Prices and Power Derivatives: Evidence from the Nordic Power Exchange," *Review of Derivatives Research*, vol. 5, no. 1, pp. 5–50, 2002.
- [37] R. A. Meese and K. Rogoff, "Empirical Exchange Rate Models of the Seventies: Do They Fit Out of Sample?" *Journal of International Economics*, vol. 14, no. 2, pp. 3–24, 1983.
- [38] M. Ca'Zorzi, J. Muk, and M. Rubaszek, "Real Exchange Rate Forecasting and PPP: This Time the Random Walk Loses," 2015, *An earlier version was published as ECB Working Paper No. 1576, 2013.*
- [39] M. Moreno and J. F. Navas, "On the Robustness of Least-Squares Monte Carlo (LSM) for Pricing American Derivatives," *Review of Derivatives Research*, vol. 6, no. 2, 2003.
- [40] A. Rodrigues and M. J. R. Armada, "The Valuation of Real Options with the Least Squares Monte Carlo Simulation Method," 2006, *Braga, Portugal: Management Research Unit, University of Minho.*
- [41] N. Areal, A. Rodrigues, and M. J. R. Armada, "Improvements to the Least Squares Monte Carlo Option Valuation Method," 2008, *Braga, Portugal: Management Research Unit, University of Minho.*
- [42] R. Carmona and M. Ludkovski, "Valuation of Energy Storage: An Optimal Switching Approach," *Quantitative Finance*, vol. 10, no. 4, pp. 359–374, 2010.
- [43] S. K. Roache and N. Erbil, "How Commodity Price Curves and Inventories React to a Short-Run Scarcity Shock," Available from: http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1750710, 2010, international Monetary Fund (IMF), Working Papers.
- [44] G. A. Davis, "Technical Note: Simulating the Two-Factor Schwartz and Smith Model of Commodity Prices," *The Engineering Economist*, vol. 57, no. 2, pp. 130–140, 2012.