A SIMPLIFIED STOCHASTIC MODEL FOR THE AERONAUTICAL MOBILE RADIO CHANNEL

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ABSTRACT

The aeronautical mobile radio channel is analyzed.A stochastic model for the channel is proposed in terms of the transmission coefficient. The power spectrum and the correlation functions are derived for cases of practical interest. The model is useful for predicting the error rate performance of digital modulation techniques. The results from a flight test showed the validity of the proposed model.

I. Introduction

The aeronautical mobile radio channel appears in many applications. Air/ground radio communication between aircraft and ground radio sites is one of the most important applications. Most of the published literature, e.g. [4], [5] focused on the satellite aircraft channel. In this paper the air/ground channel is studied and a stochastic model is proposed. Voice and data transmission between an aircraft and a ground radio site has several applications outlined in this introduction. Voice communications by radio is used for operating Air Traffic Control (ATC) functions and for providing flight services [10]. Data communications by radio is used for commercial purposes.The very high frequency (VHF) band of 118-137 MHz was assigned for civilian air/ground communications. This band is shared by ATC, commercial data communications and other flight services. The VHF band is divided into 760 channels with 25KHz as the channel separation. The current voice radio system used by the FAA controllers is based on amplitude modulation (AM). The bandwidth for each baseband voice channel is approximately 3 KHz. Future air/ground systems are expected to use digital modulation instead of AM to increase the capacity and improve the performance of the system by utilizing the new digital signal processing technology. Evaluation of different digital modulation techniques for future VHF air/ground communications systems requires a good understanding of the nature of the channel to model the signals accurately. A stochastic model for the air/ground channel is proposed in this paper. The statistical properties of the channel such as the correlation functions and power spectrum are derived in section II. These functions are useful in predicting the error rate performance of digital modulation techniques [6] using Pawula's theorem [7].

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The results from a flight test to characterize the air/ground channel are presented in section III. It is concluded that there is a close agreement between the experimental results and the proposed stochastic model.

II. Stochastic model of the channel

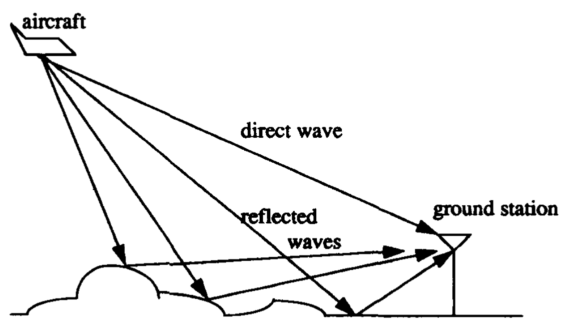

Fig.(1) Multipath reflections of the signal from rough terrain

The propagation path between aircraft and ground terminals is not only by a direct line-of-sight route, but via multipath reflections as shown in Fig.1. Thus, the signal received by the aircraft consists of a number of plane waves whose amplitudes, phases and angles of arrivals relative to the direction of the aircraft motion are random. These waves interfere and produce a varying field strength at the aircraft antenna. With the short wavelengths in the VHF and UHF bands and high aircraft speeds, the received signal is expected to fade rapidly during the aircraft motion. This multipath fading causes error in the transmission of digital signals. Since different modulation schemes have different error rates in a fading environment, it is important to study the fading characteristics of the air/ground channel to select a digital modulation that is robust to this fading.

A convenient way of characterizing the air/ground radio communication channel is in terms of its transmission coefficient [2]. The transmission coefficient represents the amplitude and phase of the received signal when a unit amplitude continuous phase (CW) is transmitted. If the transmitted wave is given by

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$\cos 2\pi ft = Re\{exp(j2\pi ft)\}$

where Re denotes taking the real part, then the received signal will be

$Re\{T(f,t)\, exp(j2\pi ft)\}$

where T(f,t) is the complex transmission coefficient which is a function of the time t and the frequency f of the transmitted wave. Fading is represented by the variations in the magnitude of T(f,t) as time is varied. Variation in the phase of T(f,t) as time is varied is often termed random FM. Variations in amplitude and phase of T(f,t) as frequency is varied are called frequency selective fading and phase distortion of the channel respectively.

For a given transmitted frequency f, the received signal consisting of many plane waves with different Doppler shifts will have a power spectrum spread around the transmitted frequency. The power spectrum of $T(f, t)$ can be expressed in terms of the strength of the signal received with each Doppler shift. The statistical properties of T(f,t) that are useful in computing the error rate performance of digital modulation schemes in the air/ground channel are investigated in this section.

If a single tone unmodulated carrier with frequency f_c is transmitted, the received signal will in general consist of the following two components added to the white Gaussian noise:

1) A line-of-sight (direct) component of power P_s and carrier frequency $f_c + f_D$ where f_D is the Doppler shift of this line of sight component. This component is given by

$\sqrt{2P_s}\cos 2\pi(f_c+f_D)t$.

2) A diffuse component with power P_d due to the multipath reflected signals. This component is statistically analyzed in the following.

The diffuse component of the received signal consists of n waves and can be expressed as [8]

$$d(t) = \sum_n \alpha_n(t)\cos 2\pi(f_c+f_n)(t-\tau_n(t))$$

where $\alpha_n(t)$ is the attenuation factor for the signal received on the n th path, $\tau_n(t)$ is the propagation delay for the n th path relative to the specular component, and $f_n = (v/\lambda)\cos\theta_n$ is the Doppler shift of the n th path with θ_n the angle between the directions of the aircraft velocity v and the n th path.

The total power P_d of the diffuse component is the sum of the powers of the individual waves. d(t) can be written in the following form

$$d(t) = \sum_n \alpha_n(t)\cos[2\pi(f_c+f_n)t+\phi_n]$$

where ϕ_n is a random phase uniformly distributed between 0 and 2π.

Using trigonometric identities, d(t) can be expressed in terms of two quadrature components as

$$d(t) = x(t)\cos(2\pi f_c t) - y(t)\sin(2\pi f_c t)$$

where the quadrature components x(t) and y(t) are given by

$$x(t) = \sum_n \alpha_n(t)\cos(2\pi f_n t + \phi_n)$$

$$y(t) = \sum_n \alpha_n(t)\sin(2\pi f_n t + \phi_n)$$

At a given time both x(t) and y(t) are the sum of n independent zero mean random variables. Using the central limit theorem, x and y can be approximated as two zero mean Gaussian random variables with equal variance.

$$E(x) = E(y) = 0,\ E(x^2) = E(y^2) = P_d = \sigma_d^2$$

where E denotes the statistical expectation. x(t) and y(t) become zero mean Gaussian processes with equal variance σ_d^2. Therefore the probability density function of the amplitude (envelope) r of the received signal has a Rician distribution.

$$p(r) = \frac{r}{P_d}exp\left[-\frac{(r^2+2P_S)}{2P_d}\right]I_o\left(r\frac{\sqrt{2P_s}}{P_d}\right)$$

It is concluded that the fading is Rician with the ratio $K = P_s/P_d$ as a variable depending on the aircraft altitude and the ground reflection coefficient.

The power spectrum S(f) of the diffuse component d(t) can be derived as [2]

$$S(f) = \begin{cases} \dfrac{p(\theta_n)}{\sqrt{\left(\frac{v}{\lambda}\right)^2 - (f-f_c)^2}} \ldots\ldots\ldots\ldots |f-f_c| < \frac{v}{\lambda} \\ 0 \ldots\ldots\ldots\ldots elsewhere \end{cases}$$

where $p(\theta_n)$ is the power density with the angle and equals P_d times the probability density function of the angle. For θ_n uniformly distributed between θ_L and θ_H, the power spectral density is given by

$$S(f) = \begin{cases} \dfrac{P_d}{\theta_H-\theta_L}\dfrac{1}{\sqrt{\left(\frac{v}{\lambda}\right)^2-(f-f_c)^2}} \ldots (f_c+\frac{v}{\lambda}\cos\theta_H < f < f_c+\frac{v}{\lambda}\cos\theta_L) \\ 0 \ldots\ldots\ldots\ldots elsewhere \end{cases}$$

In general, $S(f)$ is not symmetric around f_c as shown in Fig(2).

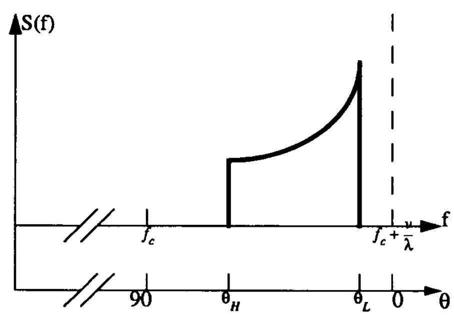

Fig.(2) Power spectrum of the diffuse signal component.

The autocorrelation function of x(t) and y(t) is
$E\{x(t)x(t+\tau)\} = E\{y(t)y(t+\tau)\}$

Expanding using trigonometric identities and after some manipulation we get

$$E\{x(t)x(t+\tau)\} = \sigma_d^2 E\{\cos(2\pi\tau\frac{v}{\lambda}\cos\theta_n)\}$$

The normalized autocorrelation function $g(\tau)$ is given by

$$g(\tau) = E\{\cos(2\pi\tau\frac{v}{\lambda}\cos\theta_n)\}$$

and depends on the distribution of the angles of arrival of the reflected waves θ_n.

Similarly, the crosscorrelation function of x(t) and y(t) can be derived as

$$E\{x(t)y(t+\tau)\} = -E\{y(t)x(t+\tau)\} = \sigma_d^2 E\{\sin(2\pi\tau\frac{v}{\lambda}\cos\theta_n)\}$$

and the normalized crosscorrelation function is given by

$$h(\tau) = E\{\sin(2\pi\tau\frac{v}{\lambda}\cos\theta_n)\}$$

During taxi and take off θ_n can be assumed uniformly distributed between 0 and 2π, and $g(\tau)$ is given by

$$g(\tau) = J_0(2\pi\tau\frac{v}{\lambda}) \text{ and } h(\tau) = 0$$

where J_0 is the zeroth order Bessel function. This correlation functions have been used extensively in the modeling of the land mobile radio channel [1], [2] and the mobile satellite channel [6], [9].

For other distributions of θ_n, it is not possible to obtain a closed form expression for the normalized correlation functions. However, we can obtain approximate expressions using Taylor's series of the trigonometric functions when the product of the time shift τ and the maximum Doppler frequency v/λ is small. Fortunately, this product is small enough in most practical situations of air/ground communications.

Let $z = 2\pi\tau\frac{v}{\lambda}$ represent this product in radians, then for $z \ll \pi$

$$g(\tau) = E\{\cos((z\cos\theta_n))\} \approx E\{1 - \frac{(z\cos\theta_n)^2}{2}\}$$

Or $g(\tau) = 1 - \frac{z^2}{4} - \frac{z^2}{4}E\{\cos 2\theta_n\}$ and $h(\tau) \approx zE\{\cos\theta_n\}$

In order to simplify the derivation of the correlation functions, we assume that θ_n is uniformly distributed between θ_L and θ_H. Therefore

$$E\{\cos 2\theta\} = \frac{\sin 2\theta_H - \sin 2\theta_L}{2(\theta_H - \theta_L)}$$

and the correlation functions become

$$g(\tau) = 1 - \frac{z^2}{4}\left(1 + \frac{\sin 2\theta_H - \sin 2\theta_L}{2(\theta_H - \theta_L)}\right) \text{ and}$$

$$h(\tau) = z\left(\frac{\sin\theta_H - \sin\theta_L}{\theta_H - \theta_L}\right)$$

Some interesting cases are considered below

1. If $\theta_L = 0$ and θ_H is very small, then

$$g(\tau) = 1 - \frac{z^2}{2} = g_{min}, h(\tau) = z = h_{max}$$

This case gives the minimum value of the autocorrelation function and the maximum value of the crosscorrelation function for a given z.

2. If $\theta_L = 0$ and $\theta_H \approx 3\frac{\pi}{4}$, the autocorrelation function is maximum. The maximum value of the autocorrelation function is given by

$$g(\tau) = 1 - \frac{z^2}{4}(1 - \frac{2}{3\pi}) = g_{max}$$

When the aircraft is at low altitude (or small elevation angle i.e. θ_L is close to zero), the assumption that θ has a uniform distribution between 0 and θ_H is reasonable and the autocorrelation function is bounded between g_{min} and g_{max} derived above.

When the aircraft is flying over a lake or a smooth terrain at high altitude $\theta_L \neq 0$ but we can assume that the difference between θ_H and θ_L is very small. Therefore

$$E\{\cos 2\theta\} = \frac{2\sin(\theta_H - \theta_L)\cos(\theta_H + \theta_L)}{2(\theta_H - \theta_L)} \approx \cos 2\theta_H,$$

$$E\{\cos\theta\} \approx \cos\theta_H$$

and the correlation functions can be approximated by

$$g(\tau) = 1 - \frac{z^2}{4}[1 + \cos 2\theta_H] = 1 - \frac{(z\cos\theta_H)^2}{2}$$

$$h(\tau) = z\cos\theta_H$$

For example if θ_H is close to $\frac{\pi}{2}$, then $g(\tau) \approx 1$, $h(\tau) \approx 0$ and if $\theta_H = \frac{\pi}{4}$, then $g(\tau) = 1 - \frac{z^2}{4}$, $h(\tau) = \frac{z}{\sqrt{2}}$

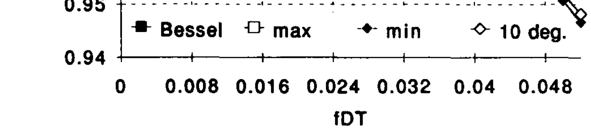

Fig.(3) Autocorrelation function

The autocorrelation function is shown in Fig.(3). It is clear that the autocorrelation function for the case of small grazing angle (10 degrees) and small beamwidth ($\theta_H \approx \theta_L$) is very close to the minimum function g_{min}.

III. Experimental data

MITRE participated in flight test analyses to characterize the air/ground channel fading. One test was conducted at Midway Airport in Chicago and another at St. Paul Airport in Minneapolis by an ad hoc group formed by the Airlines Electronic Engineering Committee (AEEC) data radio subcommittee. Each test included transmitting an unmodulated continuous wave (CW) from the aircraft radio and recording the received signal by ground radio after down conversion to a center frequency of 3 KHz. The transmitted frequency in both tests was in the 118-136 MHz band. The transmission took place during taxi, take-off, flying in the airport vicinity, landing and at the terminal. The recorded signals were sampled and analyzed to extract some channel statistics. The results showed Rician fading with strong line-of-sight component most of the time. However, severe fading was recorded during takeoff. Fig.(4) shows the spectrum measured at an altitude of 4000 feet, a range of 18 miles and at aircraft speed of 270 miles per hour. This speed yields a maximum Doppler shift $v/\lambda \cong 50$ Hz. The direct signal spectrum has its peak at 3204.5 Hz in Fig. (4) while the diffuse signal spectrum is contained in the 3192 Hz to 3195 Hz band. Comparing Fig. (2) and Fig. (4) shows a close agreement between the measured spectrum and the theoretical spectrum derived in the previous section and

$$f_c + \frac{v}{\lambda} = 3205 \, , f_c + \frac{v}{\lambda}\cos\theta_L = 3195 \, , f_c + \frac{v}{\lambda}\cos\theta_H = 3192$$

Solving the last three equations simultaneously yields

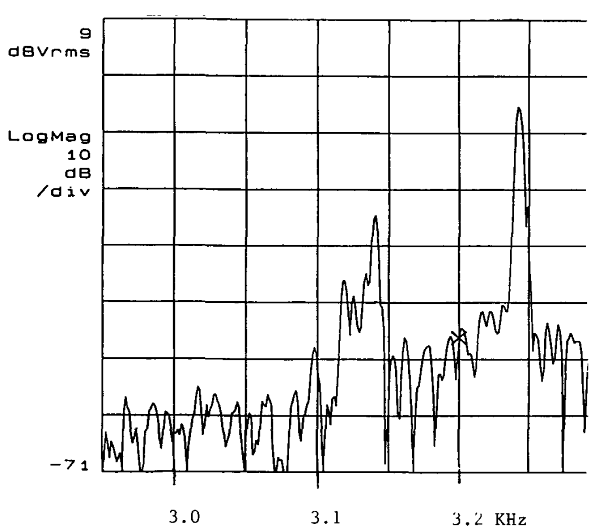

Fig. (4) Measured spectrum

$\theta_L = 75°$ and $\theta_H = 78.5°$. Thus, the diffuse signal component has a narrow beamwidth equal to $3.5°$.

IV. Conclusion

The VHF and UHF aeronautical mobile radio channel was studied in this paper. The important applications of this channel were described. The channel characteristics were investigated and a stochastic model was proposed. The correlation functions that are required for the analysis of the bit error rate (BER) performance of digital modulation techniques were derived. The spectrum of the transmission coefficient was derived and compared with experimental data. The comparison showed the validity of the stochastic model. A forthcoming paper will use this model in computing the BER performance of the digital modulation techniques that are candidates for future air/ground communication systems.

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