Technical Correspondence

Channel Equalization Using Neural Networks: A Review

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***Abstract*—Equalization refers to any signal processing technique used at the receiver to combat intersymbol interference in dispersive channels. This paper reviews the applications of artificial neural networks (ANNs) in modeling nonlinear phenomenon of channel equalization. The literature associated with different feedforward neural network (NN) based equalizers like multilayer perceptron, functional-link ANN, radial basis function, and its variants are reviewed. Feedback-based NN architectures like recurrent NN equalizers are described. Training algorithms are compared in terms of convergence time and computational complexity for nonlinear channel models. Finally, some limitation of current research activities and further research direction is provided.**

***Index Terms*—Channel equalization, complex-valued neural networks (NNs), functional-link artificial NN (FLANN), multilayer perceptron (MLP), radial basis function (RBF).**

I. Introduction

Designing equalizers for complicated, fast-varying channels is an active area of academic research and development. In recent years, the art of using artificial neural network (ANN) for wireless communications has been gaining momentum. Linear equalizers generally employ linear filters with transversal or lattice structure and adaptation algorithm such as recursive least square (RLS), least mean square (LMS), fast RLS, square-root RLS, gradient RLS, etc. However, linear equalizers do not perform well on channels with deep spectral nulls. ANNs are capable of forming arbitrarily nonlinear decision boundaries to take up complex classification tasks [1]–[4]. This paper summarizes the selected applications of ANN in modeling nonlinear phenomenon of channel equalization. Equalization refers to any signal processing technique used at the receiver to combat intersymbol interference (ISI) in dispersive channels. Standard equalization techniques start by modeling a communication channel as an adaptive filter with a specific transfer function. The equalizer, which is part of the receiver, then estimates the parameters of this unknown transfer function, and attempts to undo the effects of this time-varying channel distortion [5]. The equalizer extracts the desired signal by applying adaptive algorithm using neural network (NN), which minimizes the error between the equalizer output and the delayed test signal, as depicted in Fig. 1. To extract the phase characteristics of the channel from the received data, it is necessary to use higher order statistics of the received signal. The nonlinear function of the output of the NN equalizer gives rise to higher order statistics of the received signal.

The channel, which may be linear or nonlinear, is modeled as a finite-impulse response (FIR) filter whose transfer function is given by

$$H(z) = \sum_{i=0}^{N} h(i) z^{-i} \tag{1}$$

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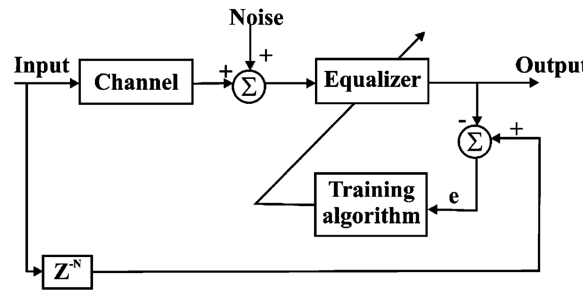

Fig. 1. Block diagram of an adaptive equalizer.

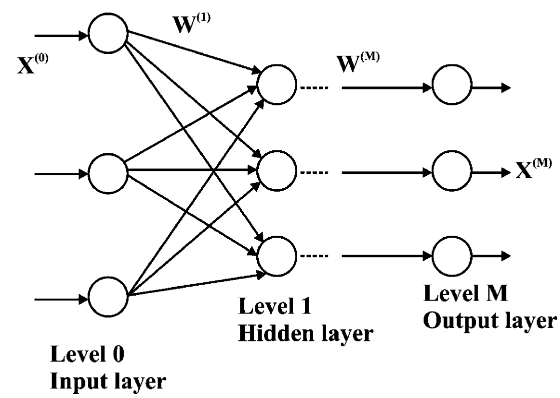

Fig. 2. MLP structure.

where $h(i)$ represent the channel tap values and N is the length of the FIR channel model. The signal $c(k)$ on passing through the channel is further subjected to additive white Gaussian noise $e(k)$ with zero mean and variance σ_n^2.

The input to the equalizer is given by

$$x(k) = c(k) + e(k). \tag{2}$$

The remaining part of this paper is organized as follows. Section II introduces the multilayer perceptron (MLP) equalizer, and its merits and limitations are discussed. Section III reviews the literature associated with functional-link ANN (FLANN) based equalizers. In Section IV, the application of radial basis function (RBF) for channel equalization is presented. Section V summarizes the literature associated with recurrent NN (RNN) based equalizer and its applications. Section VI concludes this paper.

II. Channel Equalization Using MLP

The multilayer structure of an MLP network shown in Fig. 2 is composed of an input layer, an output layer, and one or more hidden layers. Through the multilayer structure one can attain nonlinear mapping from input to output signals for nonlinear equalization [6]. Generally, the back propagation (BP) algorithm is used to train the MLP networks [7]. One of the advantages of the BP algorithm is that its hardware circuit can be easily realized. MLP equalizers are superior to conventional transversal and decision feedback (DF) equalizers in terms of the equalizer performance and symbol error rate (SER), but they suffer from local minimum problem [8], [9].

TABLE I
APPROXIMATE COMPUTATIONAL LOAD [18]

Equalizer	Flops
Split complex BP	4 500 000
Fully complex BP	4 400 000
Complex RPROP	1 100 000
DFE	24 000

Zerguine [10] has proposed an MLP-based DF equalizer with lattice filter to overcome the local minimum problem that improves the performance of MLP, but at the cost of increased network complexity.

A. Complex MLP

For equalization of QAM signals, complex NN equalizers have been proposed [11]. Researchers have designed a complex MLP and extended the BP algorithm to the complex domain [12], [13]. Two approaches for the development of the complex NN is discussed in [14]. The first one looks for fully complex activation functions, which can satisfy a conflicting relationship between the boundedness and the differentiability of a complex function. The second approach employs so called "split" complex activation functions, where two conventional real-valued activation functions process the in-phase and quadrature component. Kim and Adali have presented a complex BP algorithm using elementary transcendental functions (ETFs), which further simplify the fully complex weight update formulas [15], [16]. The complex ETFs provide well-defined derivatives for optimization of the fully complex BP algorithm.

A new complex phase-invariant activation function, which satisfies all the essential properties for a complex-valued activation function has been proposed in [17] that uses a complex-valued exponential activation function, which has singularity at infinity. The minimization criterion uses a logarithmic error function that minimizes both the errors in magnitude and phase. A complex version of the resilient propagation (RPROP) has also been presented, which is used for realistic mobile systems [18]. RPROP is a local adaptive learning scheme where the basic principle is to eliminate the harmful influence of the size of the partial derivative on the weight step. The advantage of the RPROP algorithm over BP algorithm is that it converges faster, and thus, needs less training time. In complex RPROP (CRPROP), separate update values for the real and imaginary parts of the weights are computed. The adaptation effort is not blurred by unforeseeable gradient behavior, because only the sign of the partial derivative is used to perform both learning and adaptation. The equalizer is tested on global system for mobile communications (GSM) channel modeled as five-tap FIR filter. Performance comparisons made in terms of bit error rates (BERs) and computational complexity show that the MLP network trained with complex RPROP algorithm achieves approximately as good BERs as the MLP network trained with complex BP, but with smaller computational load, as shown in Table I.

B. Algorithms for Faster Convergence of MLP

The major limitation of the MLP network is its slow convergence to a local or global minimum of the error performance surface. This limitation is due to the reason that the BP algorithm operates on the basis of first-order information, i.e., the gradient of error performance with respect to its weight. The convergence can be accelerated by utilizing the second-order information like the Hessian matrix, which is defined as the second-order partial derivatives of the error performance surface with respect to weights or the extended Kalman filter (EKF), unscented Kalman filter (UKF), and the natural gradient (NG) descent algorithms. Ibnkahla and Yuan [19] have applied the NG descent algorithm for nonlinear satellite mobile channels and proved that its performance is superior to the conventional BP algorithm. The error computation for NG algorithm involves the calculation of inverse of the Fisher information matrix, which is computationally costly for a large number of neurons. Genetic algorithms [20] can also be used in solving the local minimum problem and feature extraction tools, like wavelet transforms can be incorporated in the receiver before applying NN.

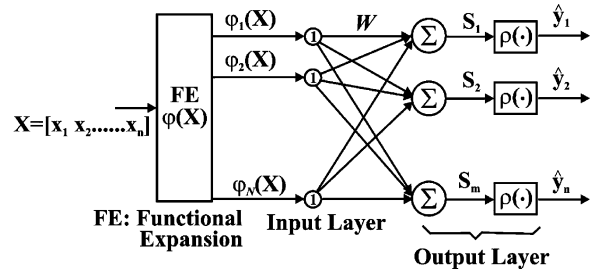

Fig. 3. FLANN structure.

TABLE II
COMPUTATIONAL COMPLEXITY EXAMPLE [26]

Number of operations	MLP	DF-CFLANN	RDF-CFLANN
Addition	136	106	74
Multiplication	290	181	125
tanh (.)	10	2	2
Order of Chebyshev polynomials	0	4	2

III. FLANN-BASED EQUALIZER

A FLANN given in Fig. 3 can be used to build a nonlinear channel equalizer. This network has a simple structure in which the nonlinearity is introduced by functional expansion of the input pattern by trigonometric polynomials and other basis function such as Gaussian or orthogonal polynomials such as Legendre and Chebyshev. The major difference between the hardware structures of MLP and FLANN is that FLANN only has input and output layers, and the hidden layers are completely replaced by the nonlinear mappings [21], [22].

The advantage of Chebyshev NN (ChNN) over FLANN is that the Chebyshev polynomials are computationally more efficient than using trigonometric polynomials to expand the input space for static function approximation, as well as nonlinear dynamic system identification. Patra and Kot [23] and Patra *et al.* [24] have used Chebyshev-polynomial-based FLANN structure for channel equalization of four quadratic-amplitude modulation (QAM) signals [23], [24].

A reduced DF-FLANN (RDF-FLANN) structure to lower the hardware cost without sacrificing system performance is proposed in [25] and the RDF-CFLANN-based equalizer is proposed in [26]. In the RDF-CFLANN structure, the output signals are directly fed to the input layer of the NN, instead of being taken as the input signals of the network. Performance comparisons made in terms of computational complexity for a particular case of four QAM signals shows that computational complexity of RDF-CFLANN is 30% less than CFLANN [26]. This is shown in Table II.

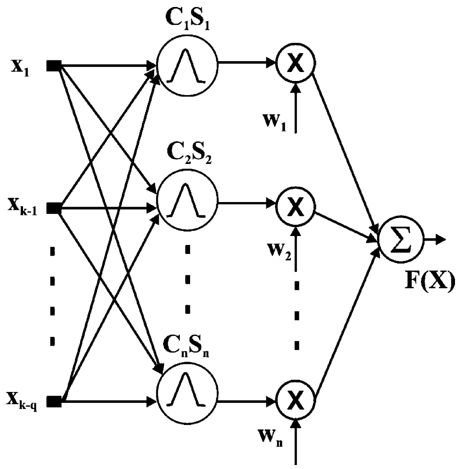

Fig. 4. RBF network.

Although FLANN-based equalizers exhibit better performance than MLP, its potential drawback is that as the input space dimensionality of the network is enlarged to reduce the BER, the complexity increases.

IV. RBF-Based Equalizer

RBFs have been applied in the area of NNs where they are used as a replacement for the sigmoidal transfer function [27]–[29]. Such networks have three layers: the input layer, the hidden layer with the RBF nonlinearity, and a linear output layer, as shown in Fig. 4. Because of the obvious reasons, the most popular choice for the nonlinearity is the Gaussian function. The RBF equalizer classifies the received signal according to the class of the center closest to the received vector. The output of the RBF NNs provides an attractive alternative to MLP and FLANN for channel equalization problems because the structure of the RBF network has a close relationship to Bayesian methods for channel equalization and interference rejection problems. Simulations performed on time-varying channels using a Rayleigh fading channel model to compare the performance of RBF with an adaptive maximum-likelihood sequence estimator (MLSE) show that the RBF equalizer produces superior performance with less computational complexity [30]–[32].

Chen *et al.* [33] and Cha and Kassam [34] have independently proposed a complex RBF (CRBF) network, which is an extension of its real counterpart. The stochastic-gradient training algorithm (with Gaussian basis function) is used for training the aforementioned network. Many techniques have been developed in literature to tackle the problem of blind equalization using RBF [35]–[37]. A new fully complex learning algorithm for the feedforward NN (FNN) is the extreme learning machine (ELM) [38], [39], which can give better performance than traditional tuning-based learning methods for FNNs in terms of generalization and learning speed. ELM proposed by Huang *et al.* is a single hidden-layer FNN in which input weights and hidden-layer biases are randomly chosen based on some continuous distribution probability, and the output weights are then analytically calculated.

Unlike the BP algorithm that cannot be used to train the threshold networks, ELM reaches good solutions analytically. The learning speed of ELM is extremely fast compared to other traditional methods, as shown in Table III.

TABLE III
Time Comparison of Equalizers [40]

Algorithms	Neurons	Number of training data	Training time (seconds)
C-ELM	10	1000	0.032
CBP	10	1000	1.266
CMRAN	22	1000	25.481
CRBF	30	10000	46.331

TABLE IV
Comparison of Equalizer Complexity [43]

Algorithms	Neurons	Number of training data	CPU time for training in seconds
CGAP-RBF	18	1000	15.17
CMRAN	25	1400	65.70
CRBF	30	13500	98.78

A. Growing and Pruning RBF Networks

RBF network learning algorithm, called minimal resource allocation network (MRAN) was developed by Yingwei *et al.* [41]. Complex MRAN (CMRAN) algorithm is proposed in [42] in which the network begins with no hidden neurons. The algorithm adds new hidden neurons or adjusts the existing network parameters according to the training data received. The algorithm incorporates a pruning strategy that is used to remove the hidden neurons that do not contribute significantly to the output. The aforementioned algorithms are not fully complex, but use a split-complex approach, where the RBF activation function remains real, and the real and imaginary part of the input signal is processed separately.

A complex-valued growing and pruning (CGAP) RBF NN for communication channel equalization of four QAM and 16 QAM signal is proposed in [43]. By linking the significance of a neuron to the equalization accuracy, a growing and pruning strategy for a CRBF NN is derived. Further, for growing and pruning, the nearest neuron (based on the Euclidean distance to the latest input data) is tested for its significance, resulting in a more compact network. When there is no growing or pruning, a complex EKF is used to adjust the RBF network parameters. The performance of the CGAP-RBF equalizer is compared with several other equalizers such as CMRAN, CRBF, and with several nonlinear, complex channel equalization problems. The results presented in Table IV show that the CGAP-RBF equalizer is superior to other equalizers in terms of SER and network complexity.

The performance of RBF NN equalizers can be further enhanced by fuzzy logic methodology of approximate reasoning to develop neuro-fuzzy systems for solving real-world problems effectively [44], [45]. Batch processing algorithms can be replaced by sequential learning algorithms [46] in which the complete dataset is not required for training, but data for learning are either used one by one or in small blocks. Thus, sequential learning algorithms do not require retraining for every set of new data as compared to the batch learning algorithms.

V. RNN-Based Neural Equalizer

RNNs, often generalized as IIR filters with feedback, as shown in Fig. 5, are known to outperform FNNs such as MLP or RBF networks [47], [48]. Although the classical equalizers perform well over fixed channels, they may not be appropriate for fast-fading channels. The time-varying nature of fading channels can be interpreted as a dynamic system with uncertainties in its coefficients [49]. RNN with their ability to learn nonlinear mappings of arbitrary complexity may

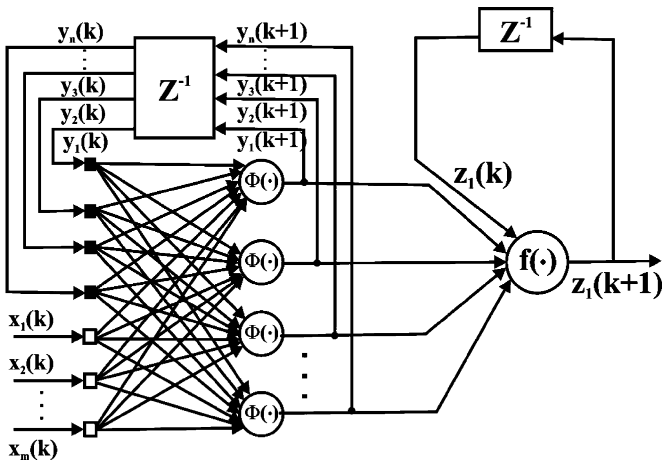

Fig. 5. Recurrent NN.

prove invaluable toward the solution of the challenging nonlinear blind equalization. Thus, RNN has been successfully applied to channel equalization of communication systems [50]–[52]. The RNN equalizer (RNE) with a small number of neurons outperforms the FNN equalizer for linear and nonlinear channels. Kechriotis *et al.* [47] have shown that nonlinear adaptive filters based on RNN can be used for both trained adaptation and blind equalization.

However, the RNE is very unstable due to its IIR structure. Ong *et al.* [53] proposed an adaptive DF recurrent neural equalizer, which not only models the IIR structure, but also overcomes the instability of the RNN-based equalizer. The BER performances for the nonlinear channel comprising high-density magnetic recording systems indicate that as the nonlinear distortion increases, the DFRNE outperforms the RNE and the DFE using MLP. Several algorithms have been proposed for training RNNs. The most widely used algorithm is the real-time recurrent learning (RTRL) algorithm and its complex version [54], [55]. Gradient-based learning approaches for training RNN are discussed in [56]. Major disadvantages of gradient-based methods are slow convergence rates and long training symbols required for satisfactory performance of channel equalization. Another disadvantage is the vanishing-gradients problem. A complex bilinear RNN (BLRNN) for equalization of a digital satellite channel is proposed in [57]. Since the BLRNN is based on bilinear polynomial, it can be used to model highly nonlinear systems, with time series more effectively. In [58], the focus is on learning algorithms for the RNE with suitably fast convergence and good tracking performance using relatively short training symbols. The EKF and UKF are used as training algorithms for the RNE [59], [60]. Results support the superiority of the UKF to the EKF in compensating the effect of non-Gaussian impulsive noise such as acoustic underwater channels and indoor wireless channels [61].

The EKF-based learning algorithms have drawbacks of high computational complexity and sensitivity to initial parameter selection. Gauss–Newton method training algorithm to achieve convergence rates close to second order with lower computation efforts is proposed in [62]. In this type of equalizer, a DFE is used with a soft-decision function (hyperbolic tangent function) at the forward filter output, during the training phase, which is replaced by hard limit or sign function (hard decision) during testing phase. Only two iterations are necessary to achieve a good estimative of the coefficients. A suitable and more generalized activation function can greatly improve the NNs' performance. In recent years, some researchers have devoted themselves to design new generalized activation functions to improve the capacity of NN [63]. Stability of the system and convergence rate are two main issues while designing RNNEs. Unsupervised nonlinear blind equalization based on a fuzzy structure [64], [65] and a prediction criterion field that can be implemented in a simple way in a hardware device could be a future research direction.

VI. Conclusion

In this paper, we have discussed various NN architectures and learning methods for solving the problem of channel equalization. The main drawback of the NN equalizers is the large computational complexity due to extensive training. The MLP network is simple to implement, but usually requires long training time. The main limitation of FLANN structure is that as the number of nodes in the input signal space is increased, the computational complexity increases. RBF-based NN equalizers are an attractive alternative and have successfully being applied for blind equalization. RNN-based equalizers, generalized as IIR filters, outperform feedforward NNs, including MLP, RBF, and FLANN. They are especially suitable for equalization of fading channels. The future research approach could be designing new generalized activation functions to improve the capacity of NN, implementations of neurofuzzy systems, and development of algorithms and NN structures to equalize time-varying channels with faster convergence and simpler architecture.

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