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TIME-VARIANT EQUALIZATION USING A NOVEL NON-LINEAR ADAPTIVE STRUCTURE

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SUMMARY

In much of the work which has been performed in the area of adaptive equalization over the last 30 years there has been a concentration of effort on systems which are *a priori* unknown but time invariant (or, at least only slowly variant with time). In certain key applications, most notably in digital mobile communications, this assumption of time invariance is not realistic. It is therefore important to focus on the particular requirements placed on adaptive equalizers in this sort of challenging environment.

This paper begins by examining the limitations of classical linear equalizers in the time-variant environment. A novel, non-linear, filter architecture which is designed to take advantage of the time-variant channel is then proposed. Two key points relating to this structure are that it has a complexity comparable to the standard linear forms and it only requires to be adapted using a basic stochastic gradient algorithm. Computer simulation studies are presented which demonstrate the ability of this structure to produce enhanced performance (in terms of bit error rate) across a range of fast-fading channel conditions when compared to standard linear equalizers. © 1998 John Wiley & Sons, Ltd.

Key words: adaptive equalization; time-varying channels

1. INTRODUCTION

Since the 1960s, a considerable effort has been devoted to research into adaptive equalizers in the area of digital data communications.[1] In the first instance this research has been dominated by the needs of either cable or fixed radio links (e.g. microwave line of sight). Such communication links have two key characteristics of relevance to the topic of this paper:

(i) The exact characteristic (transfer function) of the channel is *a priori* unknown.
(ii) The channel is essentially time invariant, or at least only slowly time variant.

The first of these characteristics results directly in the need for an adaptive solution to the equalization problem. However, the second characteristic partly solves the problem by permitting the use of adaptive solutions which invoke the pre-assumption of wide sense stationarity.

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In the more challenging environment of mobile communications, the previously common assumption of slow time variation is no longer tenable. Equalizer design must be considered within the constraints imposed by a fast-fading environment. The stationarity assumption referred to earlier has resulted in classical equalizers which are commonly based on recursive least-squares (RLS) or stochastic gradient algorithms.[2] It has been shown in earlier studies[3,4] that neither of these algorithms operate particularly well in a fast-fading environment when applied to linear-equalizer structures.

Given the problems outlined above, there are two potential routes for achieving a solution:

(i) Use of a modified approach to the design of the adaptation model which takes account of the dynamic nature of the adapting coefficients.
(ii) Adopting a filter structure which restricts the possible range of movement of the adapting coefficients, thus returning the problem to a less time-variant situation.

The first of these approaches has been studied[5,6] for the situation of direct-channel modelling by using predictor structures in the coefficient adaptation loop. This is a viable way to approach the equalization problem, provided one is willing to look at solutions such as maximum likelihood sequence estimation[7] or two-stage equalization[8] where the equalizer coefficients are calculated from the channel model. However, it should be noted that both these approaches are considerably more complex than single-stage equalization. The application of predictors in the equalizer case is not feasible, as the true equalizer coefficients exhibit dynamic characteristics which are even worse, in a time-varying sense, than the channel coefficients. The reason for this may be understood by noting that the equalizer is essentially the inverse of the channel (in the noise-free case) and this process of inversion greatly magnifies the degree of time variation.

This, then, leaves us with the second alternative, which is to design a new equalizer structure which restricts the possible movement of the individual adapted coefficients in the time-variant case. This, however, has to be done in the context of maintaining the full equalizer coefficient range (in order to maintain the optimality of the final solution). Given the contradictory nature of these two statements, it is clear that this can only be achieved by increasing the number of coefficients (but not the number of time samples). In this way, each sub-coefficient covers a much smaller range of values, but one must then devise a technique for switching between sub-coefficients. It is this technique[9] which is the subject of this paper.

The remainder of the paper is arranged as follows. Section 2 provides a brief description of the computer simulation structure, with particular emphasis on the time-varying channel model. Section 3 provides results obtained from classical linear equalizers using both LMS and RLS algorithms. Section 4 introduces the new 'amplitude banded' adaptive equalizer and Section 5 provides comparative performance results for a range of fading conditions. Section 6 gives the conclusion.

2. SIMULATION STRUCTURE

The basic data transmission model used is shown in Figure 1. The transmitted data, $s(n)$, is a simple random data set $\{-1, +1\}$ with $+1$ and -1 being equiprobable. Symbol-spaced

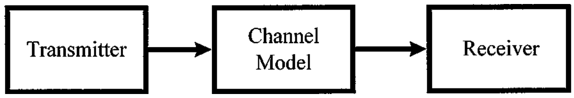

Figure 1. Basic transmission model

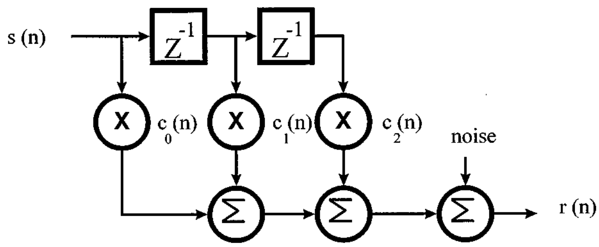

Figure 2. The simulation channel model

sampling with perfect phase synchronization is assumed. The channel model employed is a finite-impulse response filter shown in Figure 2 such that the received samples are given by

$$r(n) = \sum_{k=0}^{2} c_k(n)s(n-k) + \eta(n)$$

where $c_k(n)$ are a set of time-variant coefficients which describe the multipath environment for the baseband channel model. The term $\eta(n)$ is additive white Gaussian noise.

The time variation in the channel model coefficients is introduced by the application of a second-order Markov model where a white Gaussian noise source drives a second-order Butterworth low-pass filter. The bandwidth of the Butterworth filters determines the relative bandwidth (fade rate) of the channel time variation. The assumed sample rate used in this paper is 2400 bits s^{-1} and the fade rates used in the channel model vary between 0 and 2 Hz, i.e. varying between time invariant and rapid fading. Figure 3 shows an example of the channel coefficients generated by this model over 30 000 iterations for a fade rate of 0·1 Hz.

3. LINEAR EQUALIZERS

The linear equalizer structure employed in this study was a simple feed-forward linear filter trained using a delayed version of the actual input sequence. The output of the equalizer is, therefore,

$$\hat{s}(n) = \sum_{k=0}^{N-1} r(n-k)w_k(n)$$

and the output error is

$$e(n) = s(n-d) - \hat{s}(n)$$

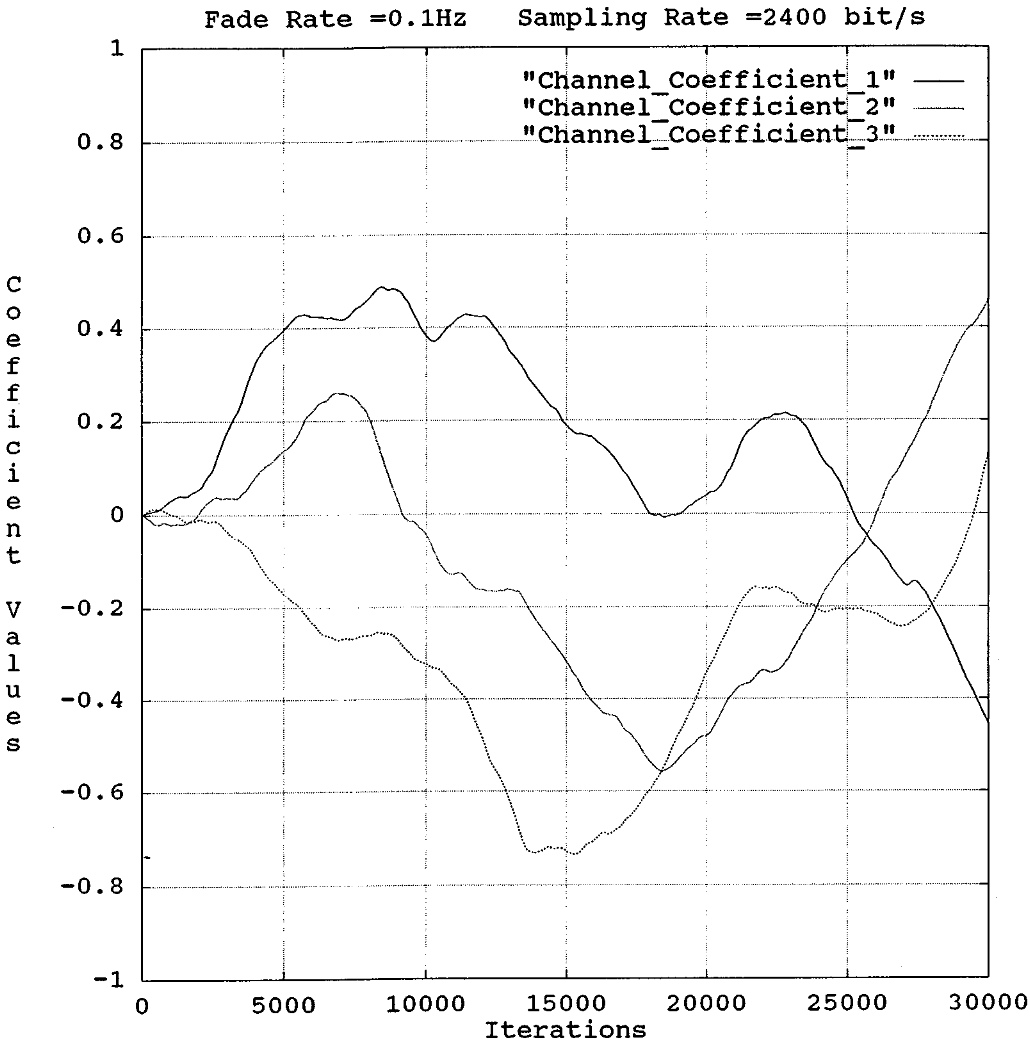

Figure 3. Channel coefficient trajectories for a fade rate of 0·1 Hz

Two forms of adaptation algorithm were employed in the simulations. The first was recursive least squares with a sliding exponential data window.[2] The other was the normalized least-mean-square algorithm (NLMS).[2]

The graph in Figure 4 shows the mean-square-error convergence for both of these algorithms when the simulated fade rate was 0·5 Hz. The result was generated by taking an - ensemble average over 25 independent experiments, the equalizer length was eight samples and the training delay was four samples. From this result it is clear that in even a moderately time-variant situation the RLS algorithm does not achieve a noticeably better result than NLMS.[3,4]

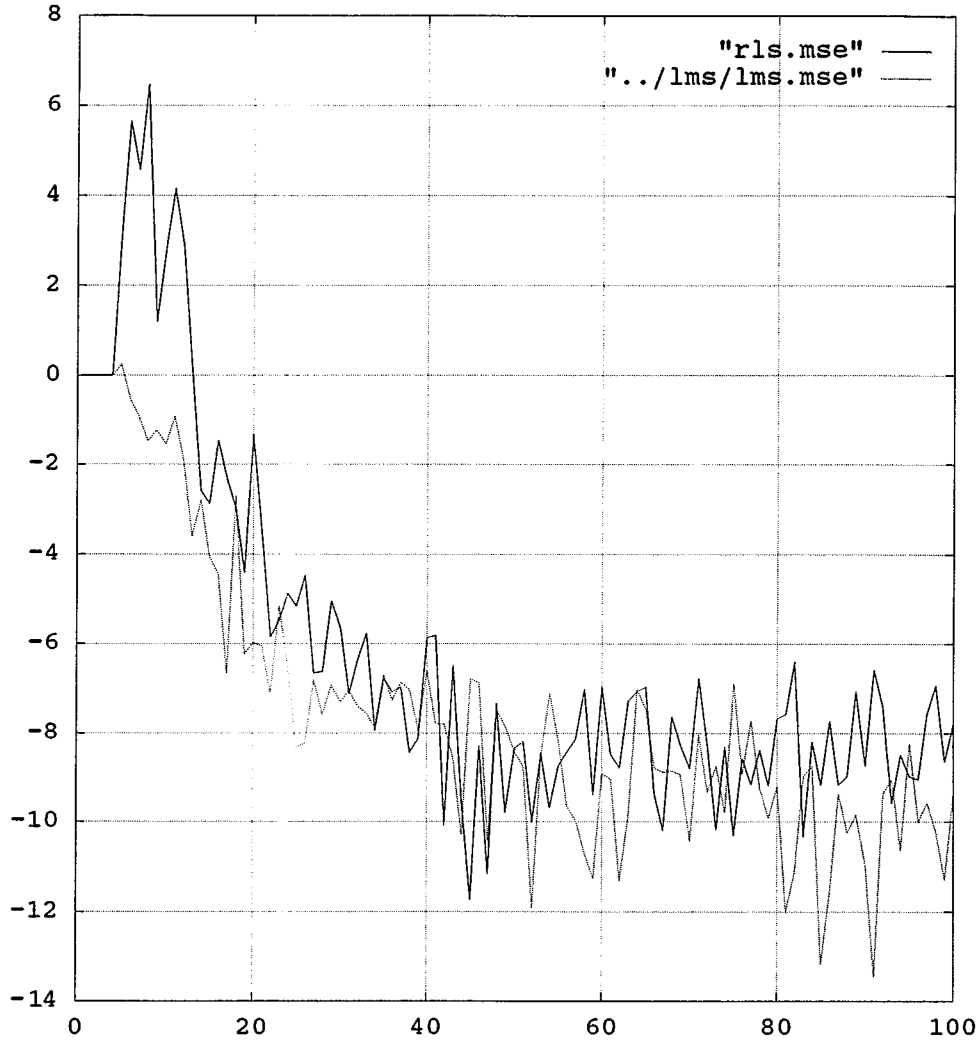

Figure 4. Comparative mean-square error convergence for LMS and RLS equalizers for a fade rate of 0·5 Hz

4. NON-LINEAR EQUALIZER STRUCTURES

It is clear from the result in Section 3 that the basic linear equalizers do not provide good performance in the time-variant environment. In this section, we present a number of non-linear filter structures which may be employed in this context. The generic equalizer structure employed is shown in Figure 5. The output of the channel is input to a tapped delay line, the outputs of which are subjected to a non-linear transformation. This has the effect of greatly increasing the number of data points available, which are then applied to a weighting matrix, so that the output

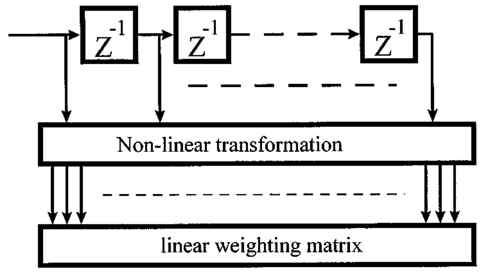

Figure 5. Generic LITP non-linear equalizer structure

Table I

$s(n)$	$s(n-1)$	$r(n)$
-1	-1	$-1-a$
-1	1	$-1+a$
1	-1	$1-a$
1	1	$1+a$

of the equalizer is a linearly weighted sum of the non-linear transform output, i.e. the architecture is linear in the parameters (LITP).

A number of such non-linear transform structures have been suggested in the past as possible solutions to this problem. These include both order statistic (OS)[10] and microstatistic (MF)[11] structures. We include comparative results for these structures in the following section.

The filter architectures mentioned above do fulfil the objective of providing a much richer range of coefficients, as described in Section 1, but they suffer from not really having any intuitive link to explain why they should perform better in a non-stationary environment. We now go on to describe a new structure which does match to the time-variant problem.

If we first consider a very simple channel having the transfer function

$$H(z) = 1 + az^{-1}$$

This channel has a single multipath element of relative amplitude a to the direct path. Given the binary nature of the input we can explicitly state all possible outputs from this channel (in the noise-free case), and these are given in Table I.

From Table I it can be seen that, provided the magnitude of a is less than 1, then it is possible to uniquely determine the value of a from the magnitude of $r(n)$. If we increase the number of variable coefficients in the channel model, then the range of values which cause possible ambiguities in the determination of the coefficients from the output magnitude increases. However, it can be stated that there remains a strong link between the instantaneous amplitude of the channel output and the channel coefficients themselves. Therefore, it follows that a link also exists between the amplitude and the equalizer coefficients. This reason leads us to the so-called 'amplitude banded' structure described below.

The amplitude banded architecture depends on the application of the idea that continuous adaptation may be successful in the time-variant environment, provided it is restricted to small ranges of the coefficient space. This is achieved by splitting the coefficient associated with each sample into a number (L) of separate coefficients. Only one of these sub-coefficients is used at any given sample instant and the particular coefficient used is chosen according to the following rule:

$$\hat{s}(n) = \mathbf{r}'(n)\boldsymbol{\theta}(n)$$

where

$$\mathbf{r}'(n) = [r(n)r(n-1) \cdots r(n-N+1)]$$

$$\boldsymbol{\theta}^t(n) = [\theta_1(n) \cdots \theta_N(n)]$$

$$\theta_p(n) = \sum_{q=1}^{L} x_{pq}(n) w_{qp}(n)$$

$$\mathbf{W}(n) = \begin{bmatrix} w_{11}(n) & w_{12}(n) & . & w_{1N}(n) \\ w_{21}(n) & w_{22}(n) & . & . \\ . & . & . & . \\ w_{L1}(n) & . & . & w_{LN}(n) \end{bmatrix}$$

$$\mathbf{X}(n) = \begin{bmatrix} x_{11}(n) & x_{12}(n) & . & x_{1L}(n) \\ x_{21}(n) & x_{22}(n) & . & . \\ . & . & . & . \\ x_{N1}(n) & x_{N2}(n) & . & x_{NL}(n) \end{bmatrix}$$

$$x_{ij} = \begin{cases} 1 & \text{if} \quad T_{L-1} < |r(n-i+1)| \\ 1 & \text{if} \quad T_j - 1 < |r(n-i+1)| \leqslant T_j \\ 0 & \text{otherwise} \end{cases}$$

$$j = 1, 2, \ldots L-1 \quad \text{and} \quad i = 0, 1, \ldots N-1$$

where L is the number of threshold levels and T_j is the predetermined amplitude range. The bracketed matrix product above has the effect of selecting one of the sub-coefficients from each column of $\mathbf{W}(n)$ dependent on the instantaneous amplitude of the input sample applied to that weight. During the adaptive phase only the N selected weights are updated by simply using the NLMS algorithm.

5. SIMULATION RESULTS

In this section, the data generator and channel model described in Section 2 were employed. All the equalizers were of length eight samples and the training delay was four samples. In the case of mean-square-error (MSE) plots an ensemble average of 50 independent runs was employed with a signal-to-noise ratio of 50 dB. In all cases the convergence gain was chosen to provide fastest

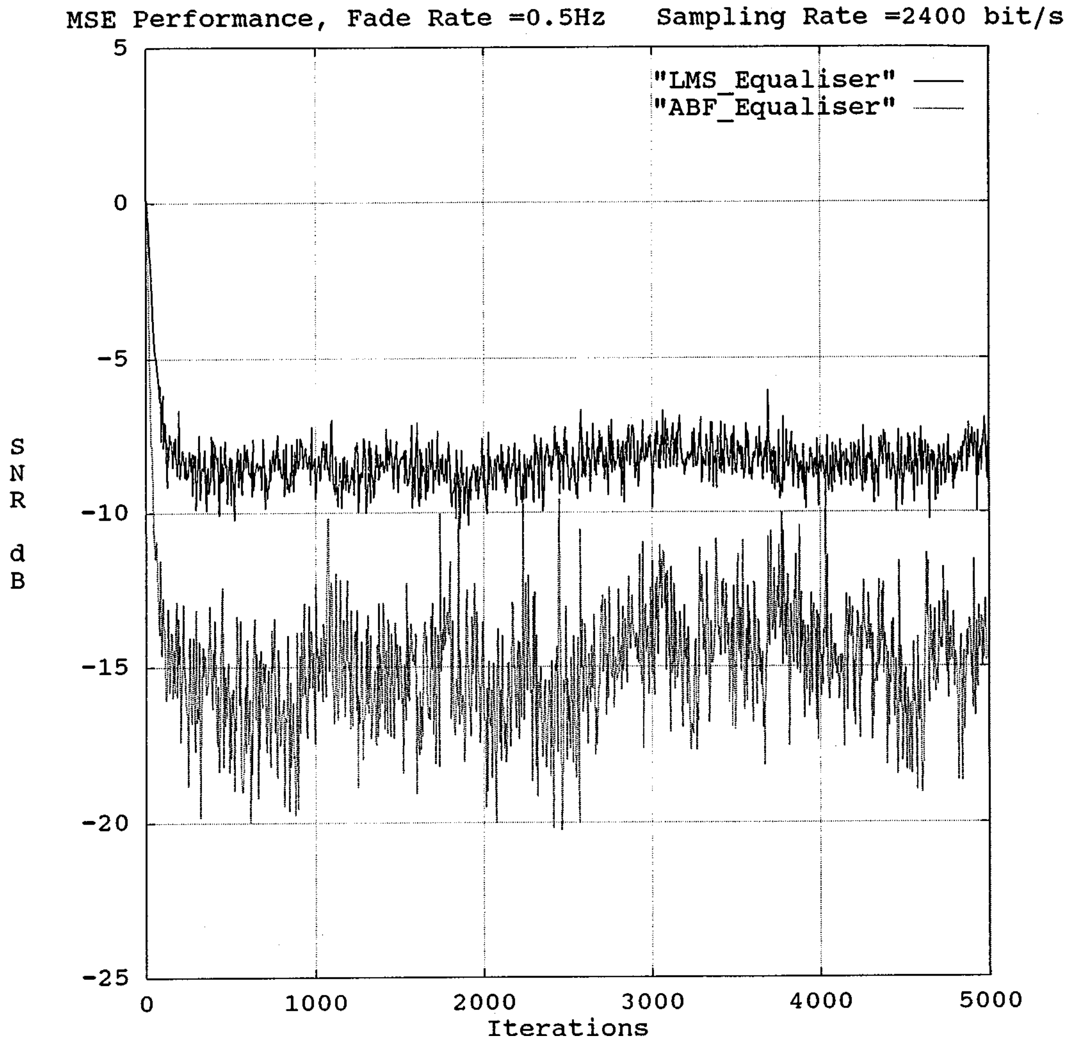

Figure 6. Convergence of a linear LMS and an amplitude banded equalizer for a fade rate of 0·5 Hz

convergence to the lowest attainable steady-state mean-squared error. All equalizers were trained using the correct training signal (i.e. no decision-directed training was attempted).

The result in Figure 6 shows a comparison in terms of MSE convergence of a linear LMS based equalizer and an amplitude banded equalizer which uses eight amplitude bands (64 sub-coefficients in total). The number of amplitude bands was determined empirically as there is a trade-off between a low number of bands being too near the linear structure and a large number of bands producing training problems, due to the need to access all the sub-coefficients during training. In fact, this resulted in choosing the same number of amplitude bands as the number of delays in the filter (i.e. eight). It is clear from Figure 6 that the amplitude-banded structure provides a

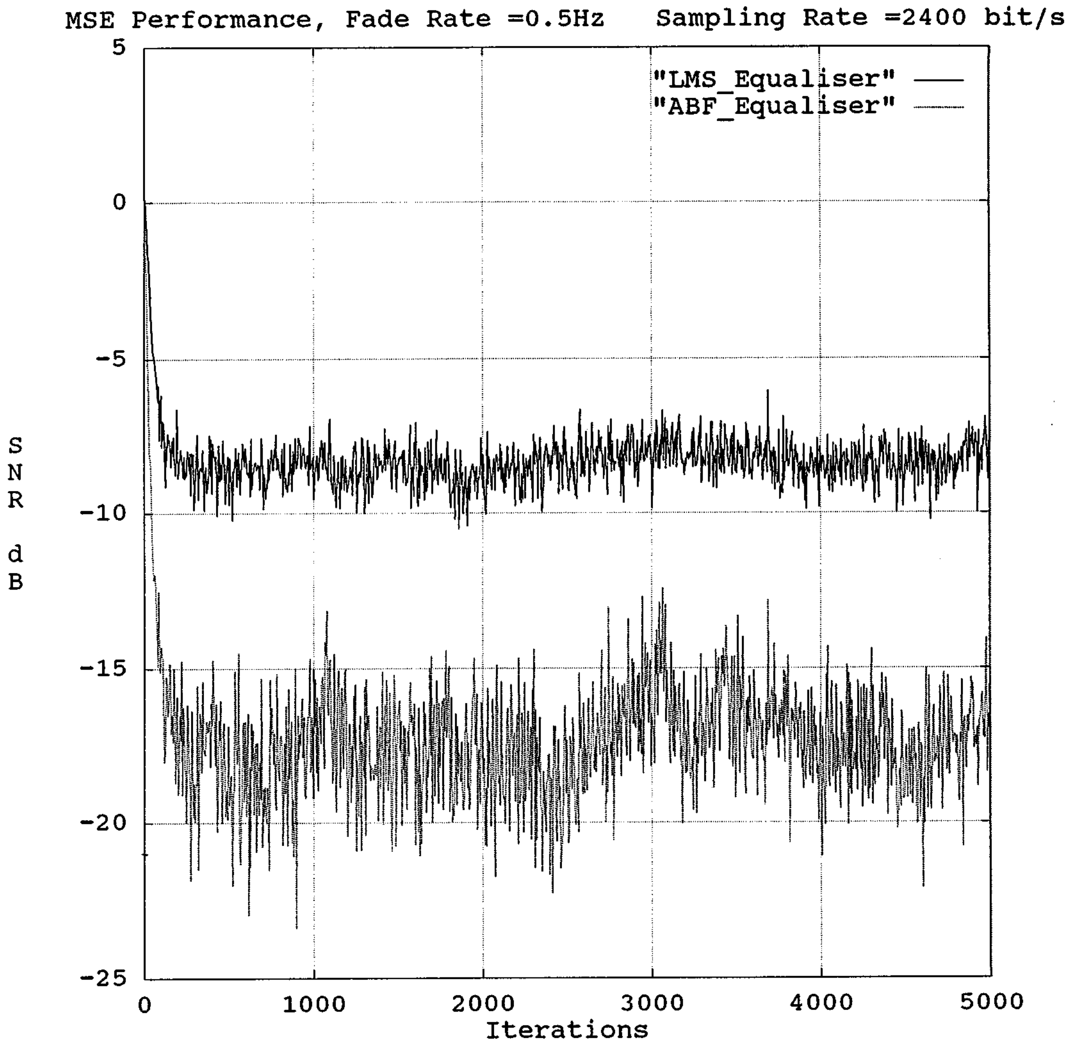

Figure 7. Result for the modified amplitude banded structure

substantial improvement in performance for this case, which is for a fade rate of 0·5 Hz. It should be noted, however, that periodic error 'spikes' do occur with the non-linear structure, possibly due to the amplitude ambiguities mentioned earlier. A solution to this is to train a linear equalizer in parallel with the amplitude-banded filter and, at each sample instant, choose the output providing the lowest instantaneous error. Figure 7 shows the MSE convergence for this modification and it is noted that the 'spikes' are now absent and the average performance enhancement is about 7 dB.

The remaining results shown in Figures 8 and 9 are bit error rate (BER) curves for fade rates of 0·1 and 2·0 Hz, respectively. In addition to the filter structures mentioned earlier a multi-layer perceptron (MLP) equalizer[12] with 8 nodes in the input layer, 10 nodes in the second layer and

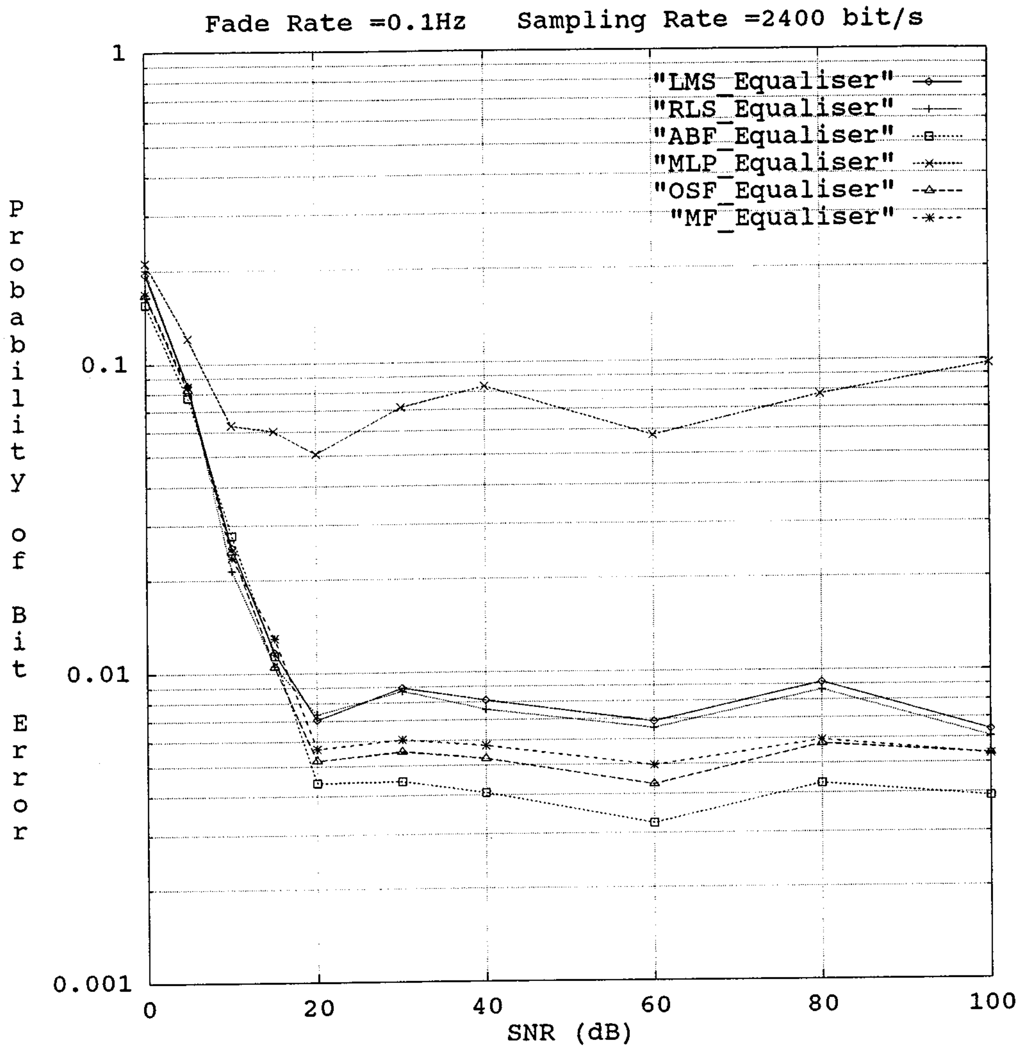

Figure 8. BER Performance of all equalizers for 0·1 Hz fade rate

a single output node has been included. This is not an LITP structure, but is included here as a comparison with a structure having additional weights which are not uniquely related to each delayed sample.

Figure 8 shows the BER performance at a fade rate of 0·1 Hz (slow fading). We note here that the LMS and RLS linear equalizers, produce very similar performance. The order statistic (OF) and microstatistic (MF) equalizers produce a BER about half that of the linear cases and the amplitude-banded structure (ABF) produces some further improvement. The MLP result is considerably worse than the linear case.

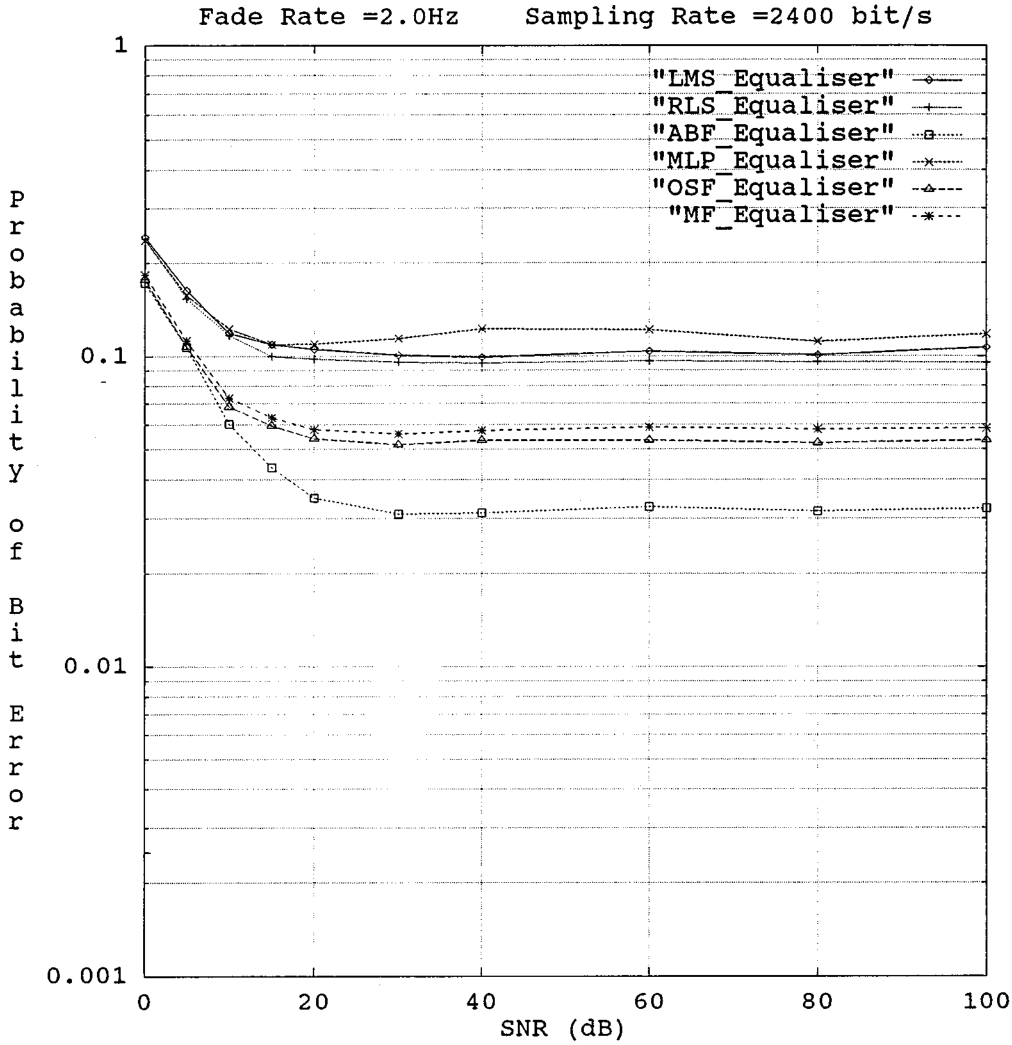

Figure 9. BER Performance of all equalizers for 2·0 Hz fade rate

Figure 9 is for the fast-fading case at a rate of 2·0 Hz. In this case the performance differences are clearer. Again the MLP produces the worst result, the LMS and RLS are marginally better. An improvement by about a factor of 4 is achieved by the OSF and MSF. The ABF again produces the best result by a considerable margin.

In general, the results presented here tend to verify the arguments presented in Section 4. Certainly, all three of the LITP structures do produce better performance in the time-variant case, thus lending weight to the argument that increasing the number of coefficients at each sample tap provides additional information relevant to the time variation. The clear performance advantage of the ABF also tends to verify the validity of the argument that there is an explicit linkage between instantaneous amplitude and the channel coeffiients. The failure of the MLP in this case is almost certainly due to the slow adaptation of the back-propagation algorithm which is unable to keep up with the time variations.

6. CONCLUSIONS

A new non-linear filter structure has been introduced which has been shown (by simulation studies) to provide considerable performance enhancement relative to linear equalizers in the time-variant environment. The structure proposed has the characteristic of requiring little additional complexity relative to the linear version. Although the amplitude-banded filter has N^2 coefficients only N of them are used at any sample instant.

Further work is concentrated on examining the exact nature of the amplitude bands with a view to optimizing the number of bands used and the relative widths of the bands.

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