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Dimensjonering av fiberarmerte betongelementer

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TITLE:

Design of Fibre Reinforced Concrete Elements

Dimensjonering av fiberarmerte betongelementer

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SUMMARY:

For the moment, have several countries developed a draft for a design guideline for fibre reinforced concrete. However, none of them is officially approved.

The scope of this thesis has been to gain knowledge about fibre reinforced concrete, by going through available guideline drafts and relevant material.

The knowledge has then been applied in design of fibre reinforced concrete beams. A goal of this thesis has been to check fibres ability to replace conventional reinforcement in the designed beams.

The beams have been designed and verified both ultimate limit state and serviceability limit state. All beams surpassed the design load with a reasonable safety margin.

This implies that much of the reinforcement can be removed, which could result in less labor-intensive work, improved working conditions and potential cost savings. However, more research is necessary to gain more knowledge on the subject, and to verify the design rules.

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Abstract

The concrete industry is constantly looking for improvements. A step in this progress is to increase the tensile capacity of concrete. Research has proved that this can be achieved by adding steel fibres into the concrete. The concept is on the rise, and can potentially revolutionize the concrete industry.

At the present time, several countries have developed design guideline drafts for fibre reinforced concrete. However, no guideline is officially approved. The intention of this thesis is to gain knowledge about fibre reinforced concrete by researching available guidelines and relevant material. The knowledge was applied in the design of fibre reinforced concrete beams. The thesis goal has been to check the fibres ability to replace conventional reinforcement.

Two different fibre reinforced concrete mixtures have been applied in the design of the beams, a B35 1 vol-% fibre mixture and a B 65 2 vol-% fibre mixture. Each type of concrete was used in designing two straight-end beams (slack-reinforced and prestressed) and one dapped-end beam(slack-reinforced). Resulting in a total of 3 beams for each concrete mixture. In addition, three beams were designed with conventional reinforcement to be used as reference beams.

The beams were designed and verified at both ultimate limit state and serviceability limit state. All beams surpassed the design load with a reasonable safety margin. This implies that much of the reinforcement can be removed, which could result in less labor intensive-work, improved working conditions and potential cost savings. However, more research is necessary to gain more knowledge on the subject, and to verify the design rules.

Sammendrag

Betongindustrien er på konstant utkikk etter forbedringer. Et skritt i denne retningen er å øke strekkapasiteten til betong. Forskning har vist at dette kan oppnås ved å tilsette stålfiber i betongen. Konseptet er under utvikling, og kan potensielt revolusjonere betongindustrien.

Til nå har flere land utviklet et utkast for retningslinjer for dimensjonering med fiberarmert betong, men det er midlertidig ingen av dem som har blitt offisielt godkjent. Intensjonen til denne masteroppgava er å tilegne seg kunnskap gjennom tilgjengelige retningslinjer og relevante materialer. Og dette ble benyttet dimensjonering av fiberarmerte bjelker. Målet i oppgaven var å sjekke muligheten for fiber til å erstatte den konvensjonelle armeringen.

To ulike fiberarmerte betongblandinger ble brukt i utforming av bjelkene, en B35 1 vol-% fiber og en B65 2 vol-%. For hver av de betongtypene ble det ble dimensjonert to bjelker med rette bjelkeender (slakkarmert og forspent) samt en bjelke med avtrappet bjelkeender (slakkarmert). Til sammen ble det tre bjelker for hver av betongtypene. I tillegg ble dimensjonert tre bjelker uten fiber som ble brukt som referanse.

Bjelkene ble utformet og dimensjonert i både buddtilstand og brukstilstand. Samtlige av bjelkene tålte den dimensjonerende lasten med en hvis sikkerhetsmargin. Dette medfører en mengde armering kan fjernes samt en reduksjon i arbeidskrevende arbeid, forbedret arbeidsvilkår samt potensiell reduksjon i kostnader. Allikevel må mer forskning til for å tilføye mer kunnskap til emnet og til verifisere dimensjoneringsreglene.

Acknowledgment

This Master‘s thesis is written for the Department of Structural Engineering at the Norwegian Institute of Science and Technology (NTNU).

This Master‘s thesis have been accomplished during 20 working weeks in the spring semester of 2015. Being two students working close together have made the work fun and exciting, even tough it has been though at times. Furthermore, the collaboration have contributed to deep conversations that have been of great value for the work.

The work of this thesis has offered us a great educational experience, and a chance to employ the knowledge we have acquired during our studies. Is has been an enjoyable task, but both frustrating and hectic at times.

We would like to thank our supervisor Professor Terje Kanstad for sharing his knowledge and guiding us whenever we needed help. He has supported and inspired us through our research.

We would also like to thank Elena Vidal Sarmiento, for helping us with the multi-layer procedure.

Contents

Chapter 1

Introduction

Concrete is by far the most used construction material in the world. In 2011, it was a global average consumption of 4 tonnes per person [12]. The material is easily accessible, cheap and castable in almost every desired form.

Concrete is known for its high compressive strength and poor tensile strength. Use of concrete in construction can be dated back to Roman Empire, with the construction of Pantheon. However, it was first in the end of the 19th century the benefits from introducing steel reinforcement into concrete were discovered. François Hennebique is credited as one on of the most important inventors of the modern reinforced method of construction. He was a French pioneer who patented a concrete construction system in 1892 [13, p.64-75]. From that year off, it became more and more usual to apply reinforcement in concrete to take handle tensile forces.

Even though the combination of these two materials work great together, there are some challenges. On one hand, concrete is a very brittle material. On the other hand, steel is a very ductile material. This results in crack development in the concrete in order to utilize the reinforcement‘s strength.

Crack development can result in reduced service and unaesthetic look. Prestressing the steel, so the concrete remains in compression during service is an effective way to reduce crack growth. An alternative way is to mix steel fibres into the concrete. The fibres will bind the concrete together, and limit crack spacing and crack widths.

In every construction processes does economy play a key role, and expenses related to man-hours is one of the greatest posts in the construction budgets. Placing of reinforcement is both time consuming and backbreaking to work with where it is a tough job to bend and tie the steel bars into place. Consequently is wear and tear injuries not rare, which leads to great costs for the society.

So, how can we bring the concrete technology a step further?

Fibre reinforced concrete (FRC) is about mixing fibre into the concrete mixture to get a concrete with great tensile strength. This could contribute to a construction process were much of the conventional reinforcement could be skipped, resulting in a cheaper and more effective construction process.

Brief About the Historical Development of Fibre Reinforced Concrete (FRC)

Reinforcing brittle materials by fibres to improve their mechanical properties is an age-old concept. For example, for approximately 3500 years ago, were clay sun backed bricks reinforced with horsehair and straw. Later, in the beginning of the 20th century, were cement-based paste reinforced with asbestos used for production of plates for roofing and pipes [11]. Due to asbestos harmful effects on humans, it has been completely abandoned.

First in the early 1960s, modern-day use of fibres in concrete started. The absence of asbestos created a need for new fibres that could be used as reinforcement.

According to Balaguru and Shah [8, p.1] fibres used in concrete can be broadly categorized in four categories:

1. *Metallic fibres* - Certainly the most important for structural concrete.
2. *Polymeric fibres* - Primarily used at a low volume fraction to control cracking of concrete at an early stage.
3. *Mineral fibres* - The predominant mineral fibre is glass, which is extensively used in production of thin-sheet concrete products [8]. A major problem with glass is lack of the durability in a alkaline environment.
4. *Naturally occurring fibers* - Naturally occurring fibres (hemp, animal hair, straw, etc.) can have a great potential as structural element in the developing parts of the world.

1.1 State of the Art

FRC is commonly used to prevent shrinkage and thermal cracking which occurs when the structure is restrained and exceeds the tensile strain capacity of the concrete. Common application areas are:

- Slabs cast directly on ground
- In slabs to prevent shrinkage
- Shotcrete used for rock support in tunnels and caverns.

Currently is FRC almost absent in load-carrying structures. Lack of commonly accepted design rules is one of the main reasons to the limited application of FRC. A general accepted guideline would definitely ease the use of FRC.

Many guidelines of FRC have been proposed. Among them, a German guideline [4] given by The German Committee for Reinforced Concrete. In Norway, is a draft under development by the Norwegian Concrete Association [9]. Similar has both Sweden and Denmark developed drafts for FRC.

Even though there exist numerous of drafts, none of them been generally accepted. It is still a lot of work to be done, as for instance to verify them.

A lot of resources have been spent the last years to get more knowledge about FRC. In 2007 presented COIN - The Concrete Innovation Centre - a budget of NOK 200 mill that should be used over 8 years. The money was financed by the Research Council of Norway, industrial partners and SINTEF Building and Infrastructure and NTNU.

Some of this money have been applied at master thesis the last few years in order to develop a fibre reinforced concrete mixtures with great performance. COIN‘s goal is to get a concrete with at least 15 MPa tensile strength after initiation of cracks. Additionally have different design approaches been tested to check their precision to predict failure.

Chapter 2

Concrete Technology

A short description of the composition and mechanical properties of concrete is presented in this chapter. The purpose is to provide essential knowledge before fibre reinforced concrete is presented later. The information in this chapter can be found in the book "Concrete technology 1" [17].

2.1 Mixing Concrete

Concrete is a composite material which consist mainly of water, cement and aggregate. The mixture of cement and water is named cement paste and become hard during hydration. The cement paste is approximately 30 % of the total mixture volume of normal concrete. The rest consist of a aggregate of stone material, which can be both coarse and fine aggregate. A wide scale of the stone sizes is in the aggregate is favourable to obtain a high compaction and strength of the material. A typical concrete mixture is shown in Table 2.1 [17].

Phase	Materials	Quantity (kg/m^3)	Desity (kg/m^3)	Volume (l/m^3)	Volume, phase (l/m^3)
Matrix	Cement	360	3120	115	296+17=313
	Silica	18	2200	8	
	Water	170	1000	170	
	Admixtures	3,0	1040	3,0	
	Sand 0-8 mm	936	2700	329+17	
Particles	Gravel 8-16 mm	455	2700	169	667
	Gravel 16-22 mm	455	2700	169	
	Air content	-	-	20	20
	Sum	2397	-	1000	1000

Table 2.1: Example of mix design (jacobsen et al. table 4.2)

Additives may improve the concrete‘s properties. Silica fume (SF) and fly ash (FA) are additives commonly used in concrete in Norway. SF contain reactive silica that reacts with the calcium hydroxide (a reaction product of the Portland cement hydration) to produce more of the C-S-H gel. FA, on the other hand, contain significant amounts of alumina and iron oxide in the glassy phase. The main glassy phase in FA is silicon dioxide (as in SF), amounting to around 50 % [17]. There is also a glassy aluminates phase in FA that is reactive to form products with binder properties. However, this is less important than the C-S-H formation. Addition of SF and FA are thereby used as replacement of some of the cement, or as an addition to improve the properties. FA may also be used to improve the workability of the concrete. Both products are industrial by-products, and their use is advantageous both from economic and environmental point of view.

Admixtures are chemical agents added at small dosages to improve certain properties of the concrete. The purposes of adding admixtures are many. The purpose of plasticizers (P) and superplasticizers (SP), which are the admixtures most sold in Norway, is to increase the workability of the concrete while keeping the strength and water content constant. The effect of this is a possible reduction of the water to cement ratio, resulting in increased strength of the concrete.

The air content in concrete is another important factor. In general, air entrained concrete have lower compressive strength compared to the same concrete without air entrained. As a rule of thumb, each volume percent of air in addition to air content of the reference concrete (which is about 1.5-2 %) gives 5 % reduction in the compressive strength. The primarily purpose of air entrainment is to increase the durability of hardened concrete subjected to freeze-thaw. The secondary purpose is to increase the workability of the concrete at plastic state.

The Particle-Matrix Model

Concrete typically contain 7-8 constituents, and it is hard to perform a prediction of how all these constituents each influence the properties of the concrete. Instead, it is easier to look at the concrete as a two-phase system and describe each phase at a time. Jacobsen et al. [17] describes the two phases as follows:

- *The matrix phase* consist of free water, admixtures and all solid materials with particle size less than 0,125 mm e.g. cement, silica fume and the filler of the aggregates. The phase can be regarded as a heavy viscous fluid, and it is the flow able component that fills the voids between the aggregate particles.
- *The particle phase* consist of the remaining of the aggregate, particles with a diameter size larger than 0,125mm. The particle phase is a friction material. Absorbed water in the aggregated is regarded as a part of the particle phase, i.e. increasing its density.

2.2 Workability

Workability is the properties of the concrete at the fresh state. The workability is defined after its stability, mobility, and ability to be compacted:

- *Stability* ability of the concrete to stay homogeneous throughout the fresh phase, which include every step from mixing to filling and compaction. Lack of stability may lead to separation, which occurs when the internal friction and cohesion between the particles is too low to counteract the effect of different densities between the concrete constituents.

- *Mobility* is the ability of the fresh concrete to move due to forces acting on it. It depend on resistance to internal flow, the friction and internal cohesion between the particles.
- *Ability to be compacted* is the ability of the concrete to fill out the formwork and let off encapsulated air pockets during reworking.

Ideally, it would be preferable to optimize all these characteristics, but it needs to be a trade-off. All of them needs to be examined together to design a concrete recipe for the project at hand. For instance can the mobility easily be increased by addition of water, but increased water content will at the same time decrease the internal friction and consequently increase separation.

2.3 Shrinkage and Creep

Concrete elements will shrink and creep during service, where both of the effects cause contraction of the concrete. However, there is a fundamental difference between shrinkage and creep.

Creep is additional deformation that develops over time due to loads pressing the concrete together during service, whereas instantaneous contraction is not defined as creep.

While creep is due to loads, is shrinkage independent of loads. The process is chemical and can be roughly divided into three categories:

- *Plastic shrinkage* -Caused by water evaporating from the surface during the fresh state. If the water evaporation from the surface is greater than the ability of the concrete to transport water to the surface (bleeding), the surface will dry out. The result is under-pressure of water at the surface due to the low w/c ratio and a bulk contraction of the concrete.
- *Autogenous shrinkage* -This is the self-produced shrinkage of the concrete. For instance, the chemical shrinkage is a part of the autogenous shrinkage. Chem-

ical shrinkage is because water and cement exhibit a loss of volume after hydration. The volume loss creates pores in the concrete, the loss can be estimated as 0.06 cm^3 per gram of reacted cement [17].

- *Drying shrinkage* -Caused by water evaporating from the hardened concrete surface when it is exposed to dry air. It differs from the plastic shrinkage which is in the initial phase. Eventually the concrete will dry out and tensile forces occur at the surface, resulting in cracks.

2.4 Self Compacting Concrete (SCC)

Self compacting concrete (SCC) is a type of concrete where the compaction is only taken care of by gravity. The idea is simple and the benefits are huge.

Mixture proportions for SCC differ from those of ordinary concrete. It has a higher volume fraction of the matrix. The high volume fraction of the matrix make sure that the that it is sufficient spacing between the aggregate. Too low spacing result in too high level of internal stress and too low flowability. SCC does also have a reduced maximum aggregate size compared to ordinary structural concrete.

Producing a highly flowable concrete is easy, making it stable is what makes it tricky. Segregation easily occurs, since the matrix phase is highly fluid. Large inner flow resistance reduce the risk of segregation. However, in order to ensure the highly viscous concrete to flow out by itself, it must have low resistance towards start of movement.

SCC has a large volume of matrix and low water to binder ratio. Superplasticizer is added to reduce the amount of needed water for hydration of the concrete, and special admixtures such as stabilizing agents are often used to increase the viscosity.

Due to relatively high matrix volume and low water to cement ratio, SCC is often associated with high strength concrete. SCC allows a new way of construction. New design of formworks can be constructed since no poker vibration is needed to consolidate the concrete, which will ease the construction of difficult geometrical constructions.

Chapter 3

Fibre Reinforced Concrete

Fibres are added to the concrete to improve the tensile strength. It works in the same order as traditional reinforcement, it bridge cracks. The effect primarily occurs after the brittle concrete has cracked. The post cracking behaviour is not as ductile as for ordinary reinforcement and is strongly dependent on the crack width. Important aspects, such as mixing procedure, fibre orientation and material properties of FRC, is presented in this chapter.

3.1 Mixing Fibre Reinforced Concrete

Addition of fibre in concrete will in general make the concrete less workable. The fibres will increase the porosity of the concrete. The porosity of fibre reinforced concrete is increasing with larger aggregates, as seen in Figure 3.1. Therefore, it is often necessary to increase the filler content (particles < 0.125 mm) or the fine-to-coarse aggregate ratio in the mix composition to obtain an optimum packing density. Then again, more water is needed. High level of additives and water reducing admixtures is also used in the mix composition.

In general will the workability of the concrete decrease as the fibre length increases. For SCC and conventional casting will it exist a upper limit of fibre volume. Crossing this

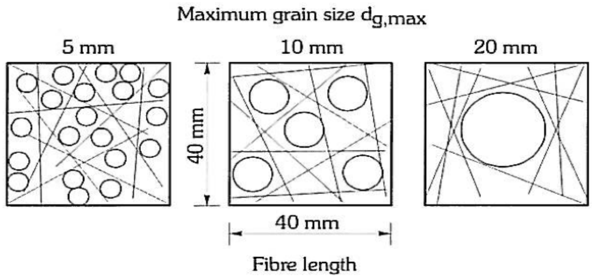

Figure 3.1: Increasing porosity with larger aggregates [9, Figure 5.1]

volume will result in balling of the fibres. The amount of fibre reinforcement is normally limited to a 2 vol-% of the concrete [8, p. 449].

3.2 Fibre Orientation

The orientation of fibres in FRC is important for the tensile strength. It is desired to have as many fibres as possible perpendicular to the the crack opening. The steel fibres transfer tensile forces across cracks similar as traditional reinforcement.

Evenly Distributed in Space?

Kanstad and Døssland [18] tested a model based on the assumption that all fibres are evenly distributed if isotropic condition is assumed. Based on this assumption and some mathematical backbreaking calculation, equation 3.1 has been deduced [18]:

$$\rho = \frac{v_f}{2} \tag{3.1}$$

ρ is the unit area of fibres per unit concrete area, v_f is the fibre volume. This equation states that sate that 50 % of all fibres present in a unit volume, will cross a plane in any direction.

The beams casted in the research were cut at different lengths, with the purpose to count the fibres crossing the plane. The results did not show great accordance with the theoretical model. However, the test pointed out a need of better understanding of the casting process to understand the fibre distribution and orientation.

Flow

Use of steel fibre reinforced concrete require special attention related to casting. Tests done by Vandewalle, Heirmann and Rickstal [28] on the fibre orientation of fibre reinforced self-compacting beams (SCFRC), casted from one side, concluded that fibre tend to align themselves along the direction of the concrete flow. SCC have to be very stable to avoid an uneven distribution of fibres. If the concrete tends to separate, a higher level of steel fibres can be expected in the lower part of the structure.

While fibre orientation of SCC depends strongly on the concrete flow, the vibration is the main influence factor for vibrated concrete [14]. A immersion vibrator used for compaction will disperse the fibres, causing weak zones where no fibres are present. In full scale structures, an immersion vibrator might be the only possible solution to compact the concrete. If a vibration table is used to compact the concrete, it is likely that a that a planar-random orientation occur perpendicular to the direction of the flow [14].

Obstacles in the formwork, like reinforcement bars and cut-out may cause blocking of fibre. It is important to be sure there is sufficient spacing so the fibres do not get obstructed [14].

The Wall Effect

Casting of fibre reinforced concrete have shown that fibres tend to orientate themselves parallel to the form of the formwork. This tendency is named the wall effect.

Dupont [15, p.22-24] developed a method to estimate the fibre orientation for a fibre reinforced concrete beam. The beam is divided into three sections as seen in figure 3.2. In middle of the beam (zone 1), the fibres rotate freely in all direction, resulting in an orientation factor α_1 equal 0.5. Then one boundary condition is added for zone 2, giving an orientation factor α_2 equal 0.6. In the third zone where there are two boundary conditions, the orientation factor α_3 is set equal 0.84. Using these three orientation factors in equation 3.2 gives the overall orientation factor of the fibre reinforcement.

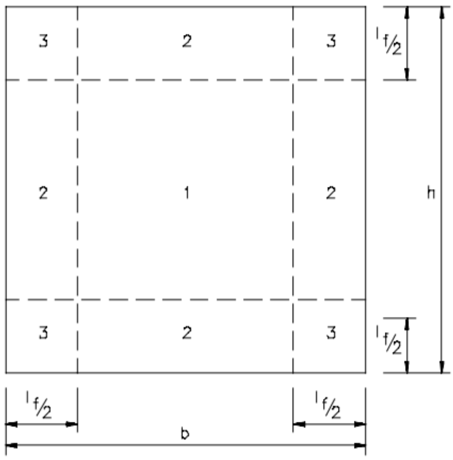

Figure 3.2: Wall effect, l_f is the length of the fibre, b is the width and h is the height of the beam. [15, Figure 3-1]

$$\alpha = \frac{\alpha_1 \times (b - l_f)(h - l_f) + \alpha_2 \times [(b - l_f)l_f + (h - l_f)l_f] + \alpha_3 \times l_f^2}{b \times h} \tag{3.2}$$

3.3 Tensile Capacity of FRC

Thorenfeldt has worked out equation 3.3 to estimate the tensile capacity of concrete based on the fibre orientation factor. The work is presented in Døsslands Ph.D. thesis [14]. This equation is the same as the one given in Norwegian Concrete Association‘s guideline draft for utilization of FRC in structural elements [9].

$$f_{\text{t.res}} = \eta \times v_f \times \sigma_{\text{average}} \tag{3.3}$$

η	is the capacity factor, indicating how much of the fibre force that are effective normal to the crack plane
v_f	is the fibre volume
σ_{average}	is the average stress in the fibres crossing the crack

The capacity factor η is set to $1/3$ for isotropic conditions, $1/2$ when the fibres are plane orientated, or 1 if the fibres are directed normal to the crack plane. Alternatively can the capacity factor be found [14] simplified as:

$$\eta = \begin{cases} \frac{2}{3} & When \quad 0.3 < \alpha < 0.5 \\ \eta = \frac{4}{3} & When \quad 0.5 < \alpha < 0.8 \end{cases}$$

The lower and upper limit of alpha is set equal to 0.3 and 0.8, because it would be highly unlikely that these levels would be exceeded in practical applications[14].

3.4 Residual Flexural Tensile Strength (f_R) and Residual Tensile Strength ($f_{\text{tk.res}}$)

To determine the tensile strength of concrete is a complicated process. It is very much dependent on the distribution and orientation of fibres, which is again very much implicated by the concrete mixture and casting process. Knowledge abut all these parameters is important in order to make a good workable concrete with high mechanical properties.

The standard NS-EN 15651:2005+A1 [2] describes a test procedure to determine the residual flexural tensile strength of FRC. The Norwegian Concrete Association [9] have proposed usage of this test procedure to determine the residual flexural tensile strength of FRC. In ultimate limit state the parameters ($f_{\text{tk.res2.5}}$ and $f_{\text{R.3}}$) is determined at 2.5 mm crack width. In serviceability limit state should the parameters ($f_{\text{tk.res0.5}}$ and $f_{\text{R.1}}$) at 0.5 mm crack width be used [9]. The residual tensile strength can be calculated as in equation 3.4 according the Norwegian Concrete Association [19].

$$f_{\text{ftk.res2.5}} = 0.37 \times f_{\text{Rk.3}} \tag{3.4}$$

Quality class	R0.5	R0.75	R1.0	R.125	R1.5	R1.75	R2.0	R2.5	R3.0
Residual tensile strength $f_{\text{tk.res2.5}}$	0,5	0,75	1,0	1,25	1,5	1,75	2,0	2,5	3,0
Residual flexural tensile strength $f_{\text{R.3}}$	1,3	2,0	2,7	3,4	4,0	4,7	5,4	6,7	8,1

Table 3.1: Quality class for tensile and flexural tensile strength [19, table 4]

The table 3.1 show how fibre reinforced concretes are standardized in order to describe the mechanical properties in a simple way. B30-R-1,5 for instance, is a concrete with a compressive strength of 30 MPa, at least 1.5 MPa characteristic residual tensile strength and residual flexural tensile strength of 4.0 MPa.

3.4.1 NS-EN 14651:2005+A1:2007

The NS- EN 14651 [2] beam test is a specialized customized test to determine the flexural tensile strength of FRC. The most important features of the test are given in this section in more or less the same words as written in the NS-EN 14651 Standard.

The determination of residual tensile strength and limit proportionality (LOP) is done by performing a deflection test on a specimen, casted at the same time and with the same concrete as the studied structure. It‘s a three point bending test with a 25 mm deep notch. The advantage of the notch is that the crack forms in a predefined position and not in the weakest section [14]. The LOP is described [2] as the stress at the tip of the notch, with the assumptions of uncracked mid-span section, linear stress distribution and subjected to a centre- point load. When applying the centre-point load on a simply supported notched beam, a load- crack mouth opening displacement ($CMOD$) or a load-deflection relationship can be measured.

The test specimens shall be prisms confirming to EN 12390-1 with normal size (width and depth) of 150mm and a length L so that 550mm < L < 700mm. The specified shape and size of test specimens are suitable for concrete with maximum size of aggregate no larger than 32mm and/or metallic fibres no longer than 60mm.

The test can be carried out in two different ways:

1. When the crack mouth opening displacement ($CMOD$) is measured, a displacement transducer shall be mounted along the longitudinal axis at the mid-width of the test specimen. The distance between the the bottom of the specimen and the line of measurement can not be more than 5 mm.

2. When the deflection is measured instead of the $CMOD$, a typical arrangement is as followed. A displacement transducer shall be mounted on a rigid frame that is fixed to the test specimen at mid-height over the supports. One end of the frame should be fixed to the specimen with a sliding fixture and the other end with rotating fixture. Since the transducer should measure the deflection, a thin plate fixed at one end can be placed at mid-width across the notch mouth at the point of measurement.

In order to find the deflection or $CMOD$ (vice versa), the approximated equation 3.5 can be used:

$$\delta = 0.85 \times CMOD + 0.4 \tag{3.5}$$

$CMOD$	is the $CMOD$ value, in millimetres, measured as the distance of notch
δ	is the deflection, in millimetres

Limit of proportionality (LOP) is the stress at the tip of the notch, which is assumed to act in an uncracked mid-span section, with linear stress distribution as shown in Figure 3.3, of a prism subjected to the centre-point load F_L. The load F_L shall be determined by drawing a line at a distance of 0,05 mm and parallel to the load axis of the load-$CMOD$ or load-deflection diagram and taking as F_L the highest load value in the interval of 0,05

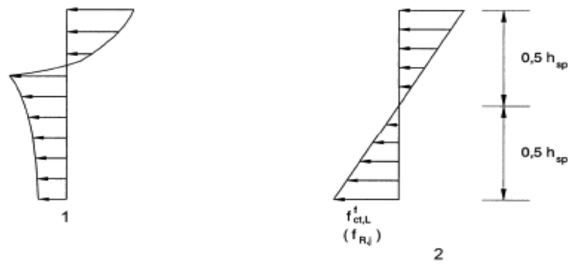

Figure 3.3: Stress distribution [2, Figure A.1]

mm. The values can then be plotted in a diagram, like shown in Figure 3.4.

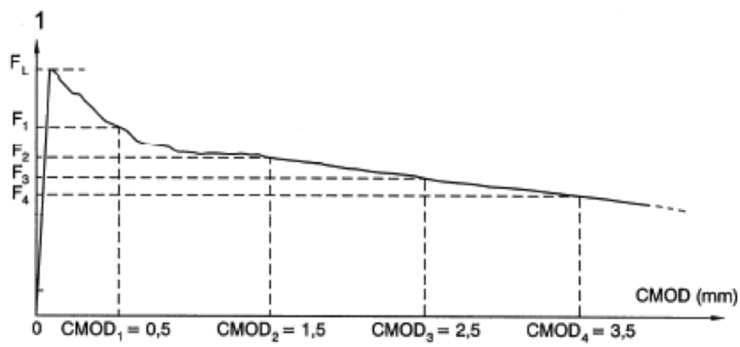

Figure 3.4: Typical load- CMOD graph measured from testing [2, Figure 7]

According to NS-EN 14651 [2], the LOP is given by the expression

$$f^f_{\text{ct,L}} = \frac{3F_L l}{2bh^2_{\text{sp}}} \tag{3.6}$$

where,

$f^f_{\text{ct,L}}$	is the LOP, given in MPa
F_L	is the load corresponding to the LOP, given in Newton
l	is the span length, in millimetres
b	is the width of the specimen, in millimetres
h_{sp}	is the distance between the tip of the notch and the top of the specimen, in millimetres

Residual flexural tensile strength, is the fictitious stress at the tip of the notch. It is assumed to act in an uncracked mid-span section, with a linear stress distribution as shown in Figure 3.3. The test prism is subjected to the centre- point load F_j which is of interest and is measured. The residual flexural strength $f_{R,j}$ is given Expression 3.7

$$f_{R,j} = \frac{3F_j l}{2bh_{sp}^2} \tag{3.7}$$

where

f_{Rj}	is residual flexural tensile strength, in Newton per square millimetre
F_j	the load corresponding to $CMOD = CMOD_j$ or $\delta = \delta_j$ (j = 1,2,3,4), in Newton
l	is the span length, in millimetres
b	is the width of the specimen, in millimetres
h_{sp}	is the distance between the tip of the notch and the top the specimen, in millimetres

3.5 Mechanical Properties of Fibre Reinforced Concrete

Concrete usually exhibit a large number of microcraks. Even before loading, due to for instance thermal expansion and shrinkage. A large number of those cracks would be expected at the interface between the coarse aggregate and the mortar, which is usually the weakest link in the composite system.

When concrete is applied a load, the matrix will transfer some of the load on the high-strength and high-modulus fibre. Hence, before any microcracks are initiated, the load will be carried by both the matrix and fibres. Ergo, it should be possible to increase the strength of concrete by addition of fibre with high strength and E-module. However, experimental studies has shown that fibres incorporated in concrete do not offer a substantial improvement in strength over corresponding mixtures without fibres [24]. This is mainly due to the low tensile strain capacity of cementitious matrixes and the fact that fibres might lead to higher porosity [24]. It is common to assume that the fibres will bind together the concrete, after cracking have occurred. Thereby, the stress-strain relations of concrete is unchanged in uncracked condition.

Post-Cracking Mechanisms

Fibres is added to improve the concretes post-cracking behavior and toughness. - "The capacity of transferring stresses after matrix cracking and the tensile strain at rupture - rather than the tensile strength" as Löfgren wroth in his PhD.thesis [24].

FRC does not break after initiation of cracks in the same brittle manner as plain concrete. This reflect the improved property of toughness (increased fracture energy). Toughness is the materials ability to absorb energy and deform plastically without fracturing. The fracture energy can for instance be calculated as the area under the curve in Figure 3.7.

3.5.1 Crack Bridging

The tensile fracture of concrete is a complex phenomenon. The resulting tensile bridging stress and dissipation of energy are a result of number of mechanisms. Both the aggregate and the fibres have the property to bridge cracks.

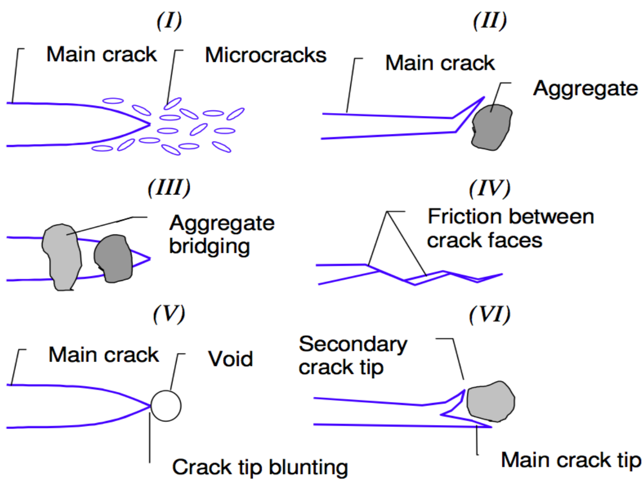

Figure 3.5: Toughening mechanisms in plain concrete [24, Figure 27]

In plain concrete, multiple crack mechanisms will be involved in the bridging process. The mechanisms showed in Figure 3.5 are: (1) crack shielding, (2) crack deflection, (3) aggregate bridging, (4) crack surface roughness-induced closure, (5) crack tip blunted by void and (6) crack branching [24]. The effect of those bridging mechanisms vary. However, the major toughening mechanisms are those of the crack wake (e.g. aggregate bridging). Aggregate bridging depends strongly on the aggregate and its bond to the matrix.

Fibres will increase the tensile bridging of concrete at a large scale. Addition of fibre in concrete will increase the fracture energy by a factor larger than 10 [24]. When the matrix crack and the crack approaches an isolated fibre, the mechanisms that may be expected to to take place and dissipate energy will be (as seen Figure 3.6): (1) matrix fracture and matrix spalling (or fragmentation), (2) fibre-matrix interface debonding, (3) post-debonding friction between fibre and matrix (fibre-pull out), (4) fibre fracture and (5) fibre abrasion and plastic deformation (or yielding) of the fibre.

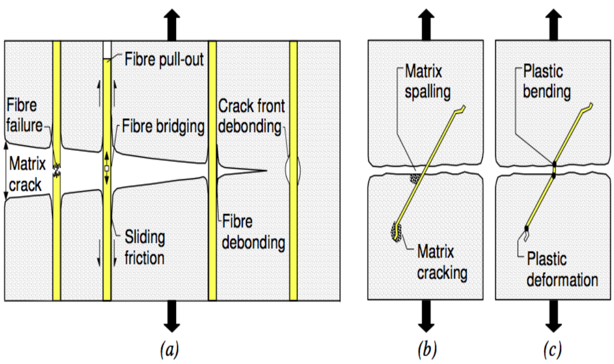

Figure 3.6: (a) Illustration of some of the toughening effects at the crack wake. (b) Matrix spalling and cracking. (c) Plastic deformation of inclined fibre during pull-out. [24, Figure 29]

The mechanical behavior of FRC is strongly dependent on the pull-out versus load behavior of the individual fibres. The pull-put behavior is dependant on: (1) the type of fibres and its mechanical and geometrical properties, (2) mechanical properties at the interface between the matrix and the fibres, (3) the angle of inclination of the fibre with respect to the the direction of loading, and the mechanical properties of the matrix [24]. The pull-out is considered to be a result of the gradual debonding of the interface surrounding the fibre, followed by frictional slip and pull-out [24]. In some cases the adhesion (chemical bond between fibres and matrix) will be negligible, and the friction between the fibre interface and the surroundings will be the governing mechanism.

Addition of fibres with hooked ends instead of straight fibres, will increase the pull-out resistance. This is due to the fact that energy will be dissipated to straighten out the physically deformed fibres. Figure 3.7 show a typical pull-out curve for both straight ended and hooked-ended fibres [24]. The ascending part (OA) is associated with elastic or adhesive bond. At the second part of the curve (AB), the debonding is initiated and progress until full debonding occurs (B). The straight fibres are now pulled out (BF) and only resistance by friction is offered.

Hooked-end fibres will resist a further increase in load (BC) due to the mechanical anchoring of the hooked end. After the slip of the hooked-end, the end will be deformed progressively (CE). A considerable energy dissipation take place to straighten and plastically deform the fibres. Then it is only the frictional force (EF) left to resist pull-out. The energy of the pull-out reaction could be calculated as the area under the curve in Figure 3.7.

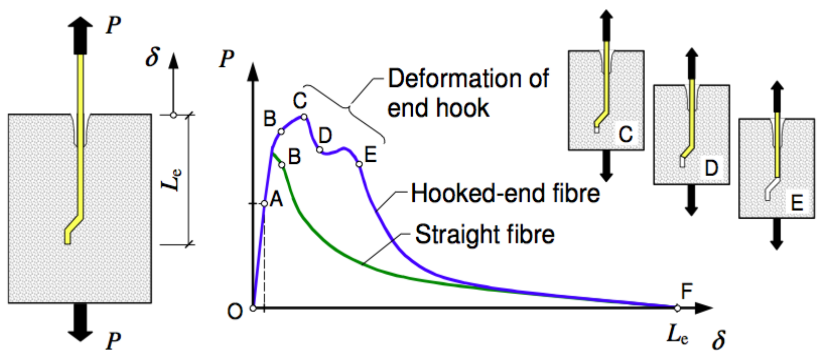

Figure 3.7: Fibre pull-out relationship between the end-slip and load for straight and hooked-ended fibres. [24, Figure 32]

The pull-out behavior also depend on the angle of inclination of the fibre. Especially the strength of the matrix plays an important role, as a weak matrix is prone to spalling (seen in Figure 3.9) and local damage. For stiff, but ductile fibres the pull-out load is almost as high and the work required to completely pull out the fibres are higher than that of fibres parallel with the load-direction [24].

The effect when a fibre is not orientated perpendicular to the crack and is pulled out of of the concrete, is named the subbing effect (Figure 3.8) [15].

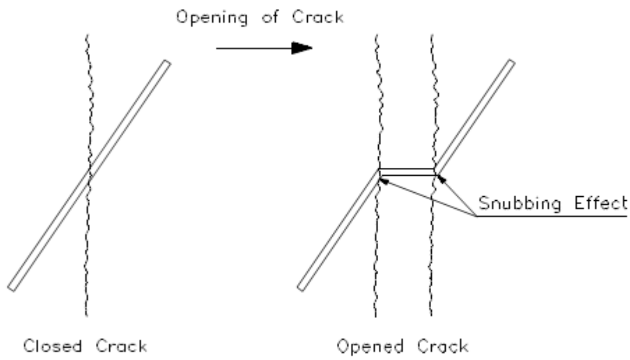

Figure 3.8: Snubbing effect [15, Figure 2-4]

A concentration of stresses is generated at the place where the fibre is forced to bend. The concrete between the fibre and the crack is crushed or pulled off. If concrete spalling occurs, the fibre may easily bend as shown in Figure 3.9 and the stress carried by the fibre will be reduced [15].

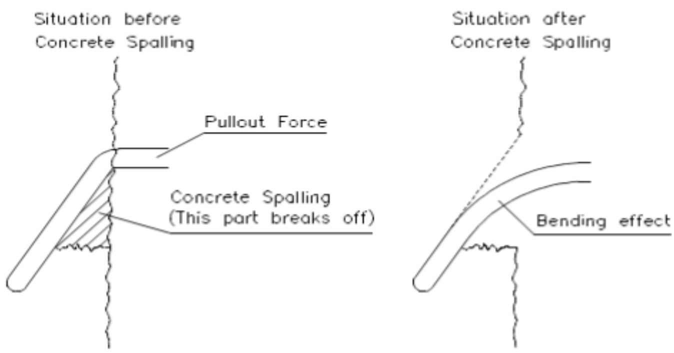

Figure 3.9: Concrete spalling [15, Figure 2-5]

For steel fibre, it can be assumed that the concrete spalling and snubbing occurs together [15]. The greater the concrete part that breaks of is, the smaller is the snubbing effect.

In Figure 3.10 is the pullout force plotted against the embedment angle. In the ascending part of the diagram, the pull out force will increase as the inclination angle increases.

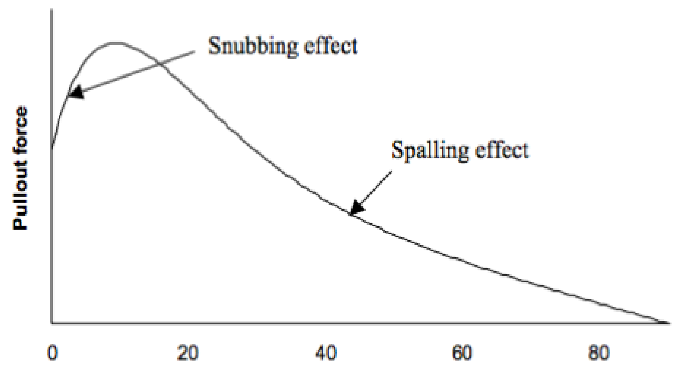

Figure 3.10: Snobbing effect versus spalling effect [15, Figure 2-6]

However, when a certain level of inclination is exceeded, spalling will occur and the snubbing effect will decrease.

If many fibres are close to each other, the efficiency of the fibres may be reduced. Multiple fibres with embedment zones close to each other, might lead to concrete failure instead of fibres being pulled out. Especially hooked-end fibres are postponed for this phenomena. To take care of this possibility of concrete failure in the the capacity calculations, one should multiply the tensile strength with a factor (<1). The higher the fibre content, the more risk to have this phenomena and lower factor will be required [15].

Length of Steel Fibres

The Norwegian design draft for FRC [9, 2.3.1] state that the length of the fibres shall be at least two times the maximum diameter of the aggregate. This is to ensure that the tension forces will pass the aggregate.

The properties of the concrete is influenced by the length of the fibres. Mohammed Alias Yusuf et. al. studied the mechanical properties of hybrid steel fibre (fibres with different lengths) reinforced concrete, some of his major conclusions are presented in a research paper [16] written by Gul, Bashir and Naqash. The results reveled that longer fibre performed better in flexural bending and in tension, while short fibres performed better in compression compared to concrete with longer steel fibre.

3.6 Flexural Design

The ductility of concrete is significantly increased by addition of steel fibres. Hence, the concrete strains before failure will be larger. A laboratory test done on the strain at failure of FRC-beams, showed values of 0.0066 at failure [8]. This is more than twice the value of plain concrete (value of 0.003). The American Concrete Institute Code recommends a value of 0.003 as the maximum limit for usable strain at compression [8] for plain concrete. In addition, is the tensile strength contribution of the plain concrete neglected.

The traditional way to calculate the moment capacity of a beam, is to use some assumptions to set up two equations involving force and moment. Then the equations are solved to get the moment capacity of the beam. The amount of tension reinforcement is limited by most codes in order to ensure a ductile failure by yielding of the reinforcing steel rather than crushing of concrete. However, the tensile strength of fibre concrete could be substantial, an higher levels of strain could be accepted [8].

To take into account the new properties, the existing design equations needs to be modified. For instance, the area of steel could be adjusted to balance the compression force created by the concrete. This would permit simultaneous failure of concrete and steel, which is not allowed by most codes in fear of a brittle failure. With addition of steel fibres, brittle failure could be avoided. Hence, a balanced failure condition could be used for design [8].

3.7 Shear Strength

The addition of steel fibres to reinforced concrete is in general known to increase its shear strength. If sufficient fibres are added, a brittle shear failure can be avoided in favour for a more ductile behavior [23]. Laboratory tests have confirmed that fibres can enhance the shear resistance. Common test variables of FRC is the shear-to span depth ratio (a/d), volume fractions, fibre type, and the compressive strength of concrete. The shear span a is defined as the distance from the load point to the nearest support point,

d is defined as the depth of the beam measured from the the extreme-compression fibres to the central gravity of flexural steel reinforcement.

Kwak, Eberhard and Kim [23] conducted a laboratory test on the shear strength of FRC beams without stirrups. The three variables mentioned above were the parameters of their investigation. The beams in their test with the lowest a/d-ratio performed best, with a increase in shear strength in a range of 69 to 80 %, compared to similar beams without fibres. The fibres also reduced the crack spacing and sizes, and made the failure mode more ductile.

It is known that the the amount of fibres crossing the shear plane influences the shear capacity due to the dowel effect, similar as tradition reinforcement. In plain concrete it is the aggregate interlock and friction at the crack face that transfer the shear stress across the crack. The cracking strength of FRC is not affected by fibres before the matrix cracks. When is has cracked, the fibres will be activated and start to be pulled out, resulting in a significant toughening mechanism [24].

Determining the shear capacity of fibre reinforced concrete is not easy due to large numbers of parameters. Many design equations have been developed, but no one has in general been approved.

Modulus of Elasticity of Steel Fibre Reinforced Concrete

The elastic modulus of concrete is a key parameter reflecting the ability of concrete to deform elastically. Gul, Alsana and Naqash [16] conducted a trial on the E-modulus of of FRC with different fibre fractions and aspects ratios (length of fibre to diameter of the cross-section). Their investigation concluded that the modulus of elasticity is significantly increased by the addition of fibres. The E-modulus increased with an increase in fibre volume fraction.

Chapter 4

Design of Beam Ends

Design of beam ends with traditional rebars is a well covered topic. Practice has shown that a great amount of rebar is needed to resist the large shear forces. The application is cumbersome and time-consuming. Hence, it would be of great interest to utilize the strength contributions from fibres. This chapter focus on the design of beam ends. The design approach for regular concrete is presented first, followed-up by some proposed design calculations for fibre reinforced concrete.

Then further on design models for fibre reinforced concrete.

4.1 Strut-and-Tie Design Model

Design in accordance with "*Betongelementboken bind C*" [30] is common practise in Norway. The design is based on the Strut-and-Tie Method, which idealize that concrete and reinforcement build up together an assembly of axially loaded members. These members are connected at nodes to form a truss.

To get a better understanding of how the method i applicated in the design of beam-ends, will the some of the design checks from "*Betongelementboken bind C, chapter* 8.2" [30] be presented in this chapter.

4.1.1 Design of Straight Beams

The anchoring capacity of the rebar in tension must to be checked. Limited support length will often result in need of additional rebar in the ends. For beams with small support lengths, this control is the most crucial for the design of anchor capacity [30]. For stirrup-reinforced concrete, we assume that the first crack develop from the edge of the shim (Figure 4.1), up in the compression zone of the concrete. Exactly how the crack develop is unknown [30].

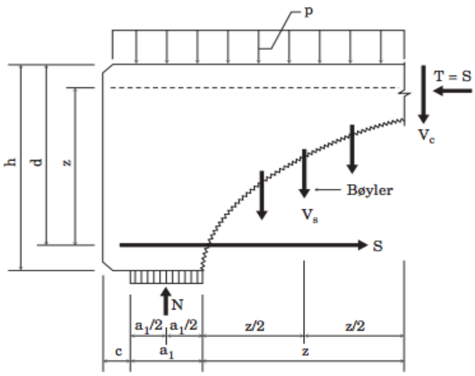

Figure 4.1: Force model to check anchor capacity of tension tension reinforcement [30, Figure C 8.3]

Vertical equilibrium of forces of Figure 4.1 gives Equation 4.1. The external load p is neglected in the following equations for simplification.

$$N = P + V_s + V_c \tag{4.1}$$

N	is the support force
P	is the outer force
V_s	is the total shearforce carried by the stirrups
V_c	is shearforce carried by the concrete compression zone

Equilibrium of moment give Equation 4.3. It is seen that the force S depend on the the amount of stirrups. This force must be properly anchored in both ends of the crack [30].

$$S \times z = V_s\left(\frac{z}{2} + \frac{a_1}{2}\right) + V_c\left(z + \frac{a_1}{2}\right) \tag{4.2}$$

$$S = 0,5V_s\left(1 + \frac{a_1}{z}\right) + V_c\left(1 + \frac{a_1}{2 * z}\right) \tag{4.3}$$

4.1.2 Design of Dapped-End Beams

The concept of dapped-end beams are extensively used in many areas in civil engineering. The supporting corbels can be recessed into the depth of the beam, which result in reduced floor height. The use of dapped en beams facilitates the erection of a precast concrete structure, due to greater lateral stability of an isolated dapped-end beam than that of an isolated beam supported at its bottom face [26].

It is recommended to keep the ratio between a_0/d in magnitude of 0,4 to 0,6. This ratio will ensure a practical rebar layout. The height of the nib should be at least half the beam height and the length of the nib should be less than 0,7 of the nib height. To use the design rules, a_0 must be less or equal to d and H_{Ed} less or equal to N_{Ed} [30]. Figure 4.2 show recommended geometrical design of a dapped-end beam.

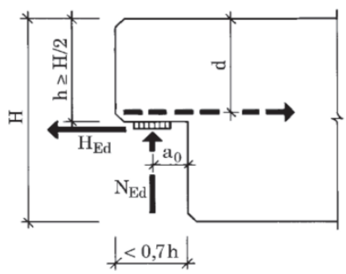

Figure 4.2: Design of dapped-end [30, Figure C 8.17]

Figure 4.3 shows how the forces travel through a dapped-end beam. If the cut out part of the beam is big, it will be developed large tensile forces perpendicular on the direction of the main reinforcement. If that is the case, additional reinforcement must be installed to handle these forces [30].

Figure 4.4 show a simplified force model for dapped-end beam. The vertical load N is uptaken by the vertical reinforcement F_v and the vertical component of the inclined

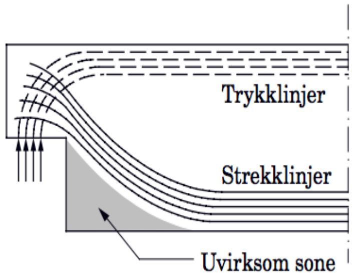

Figure 4.3: Show how forces travel through a dapped-end beam [30, Figure C 8.18]

reinforcement F_α. The vertical reinforcement F_v will also take some of the splitting tension forces from the anchoring of the horizontal reinforcement F_s in beam-nib. To reduce the splitting tension forces, it is favourable to anchor the main horizontal reinforcement in the nib over a great length.

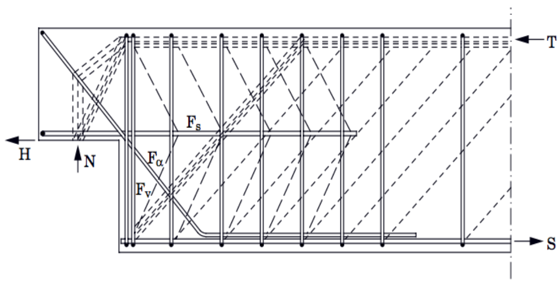

Figure 4.4: Simplified force model [30, Figure C 8.19]

It is limited how much inclined reinforcement is possible to install. Therefore, does usually the vertical reinforcement carry most of the load, which result in heavy concentration of vertical reinforcement in the beam-end [30].

Experiments have shown that nibs work as beam on its own. It is desired to to get the tension forces down from the nib to the flexural reinforcement in the beam. Inclined reinforcement is much more effective than vertical reinforcement doing this job [30].

"$BetongelementbokenbindC$" [30] splits the design calculation of the support load N of a dapped-end beam in two parts:

1. Contribution from the horizontal reinforcement, N_α.

2. Contribution from the vertical reinforcement, N_v.

Inclined Reinforcement A_{sa}

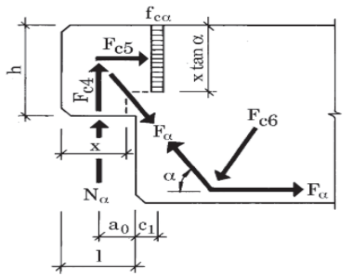

Figure 4.5: Inclined reinforcement force model [30, Figure C 8.22]

First choose a dimension and number of the inclined reinforcement. Then define the angle of the inclined reinforcement, a angle according to Equation 4.4 is recommended [30]. h and l is the height and length of the nib. c_1 is the the distance from the end of the beam to the center of the vertical stirrups (seen in Figure 4.5).

$$tan(\alpha) = \left(\frac{h}{l + c_1}\right) \tag{4.4}$$

The inclined reinforcements contribution to the the load capacity N is given by Equation 4.5, the contribution of the inclined reinforcement is the vertical component N_α.

$$N_\alpha = F_\alpha \times \sin(\alpha) \qquad F_\alpha = f_{yd'} \times A_{s\alpha} \tag{4.5}$$

The tension in the inclined reinforcement will give a total compression force F_{c5} in the upper part of the beam which is calculated as shown in Equation 4.6:

$$F_{c5} = \frac{N_\alpha}{\tan(\alpha)} \tag{4.6}$$

The compression per area $f_{c\alpha}$ in the upper part of the beam is calculated by Equation 4.7. The equation divide simply the force F_{c5} over the compression area. $x \times tan(\alpha)$ is the height of the compression zone.

$$f_{c\alpha} = \frac{N_\alpha}{b \times \tan(\alpha) \times x \times \tan(\alpha)} = \frac{N_\alpha}{b \times x \times \tan^2(\alpha)} \tag{4.7}$$

Horizontal Rebars A_s and Horizontal Stirrups A_{sb}

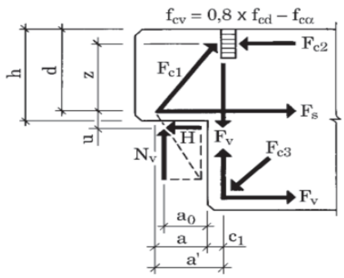

Figure 4.6: Vertical reinforcement force model [30, Figure C 8.23]

The contribution from F_α (N_α) in order to increase the shear capacity was found by Equation 4.5. The rest of the shear load must be taken care of by the horizontal reinforcement and vertical reinforcement. To find how much load resistance that will be required from the horizontal and vertical reinforcement, is N_α subtracted from the needed load resistance. This is done according to Equation 4.8:

$$N_v = N - N_\alpha \tag{4.8}$$

The length a‘ which is shown on Figure 4.6 is needed. It is the length from the end of the main nib reinforcement (F_s) to the center of the vertical reinforcement (F_v). The Expression 4.9 for the calculation of the length a‘, also include a variable $u \times \frac{H}{N}$, which is a factor depending on the difference between the horizontal load H and vertical load N.

$$a‘ = a_0 + u \times \frac{H}{N} + c_1 \tag{4.9}$$

Next step is to calculate the lever arm z of the internal forces. This is not a tricky step, but it needs some explanation. d is the effective height from the top of the beam to the main flexural reinforcement. c_2 is the distance from the top surface of the beam to the center of compression. z is defined according to Equation 4.10:

$$z = d - c_2 \tag{4.10}$$

Now look at the point where F_{c1} and F_s intersect. Note the angle between the two forces as β, and check the vertical equilibrium at the point. Then solve the equilibrium with respect to F_{c1} to get Equation 4.11:

$$F_{c1} \times \sin(\beta) = N_v \qquad \rightarrow \qquad F_{c1} = \frac{N_v}{\sin(\beta)} \tag{4.11}$$

Then examine the point where F_{c1} and F_{c2} intersect. Control the horizontal equilibrium at the point, and solve it with respect to F_{c2} to get Equation 4.12:

$$F_{c2} = F_{c1} \times \cos(\beta) = \frac{N_v}{\sin(\beta)} \times \cos(\beta) = N_v \times \frac{1}{tan(\beta)} = N_v \times \frac{a`}{z} \tag{4.12}$$

It is assumed that the compressive zone is fully utilized with height $2 \times c_2$. The reduction factor 0.8 is from Eurocode 2 [1], chapter 6.5.4(3). It has to do with the strut-and tie modeling and anchorage of the ties in the compression nodes. Now express the horizontal force F_{c2} by a compression block as seen in Figure 4.6, to get Equation 4.13:

$$F_{c2} = (0.8 f_{cd} - f_{c\alpha}) \times b \times (2c_2) \tag{4.13}$$

Use Equation 4.13, and solve it with respect of c_2 to get Equation 4.14:

$$c_2 = \frac{N_v \times a`}{(0.8 f_{fcd} - f_{c\alpha}) \times 2 \times b \times z} \tag{4.14}$$

Implant into Equation 4.14 that the internal lever arm is considered to be 0.8 $\times d$. This gives Equation 4.15:

$$c_2 = \frac{N_v \times a`}{1.6 \times b \times d(0.8 f_{fcd} - f_{c\alpha})} \tag{4.15}$$

Finally, use the results from Equation 4.10 and Equation 4.15 to write Equation 4.16 for internal lever arm without unknowns:

$$z = d - \frac{N_v \times a`}{1.6 \times b \times d(0.8 f_{cd} - f_{c\alpha})} \tag{4.16}$$

Now it is in interest to develop an expression for the horizontal force F_s. Take the horizontal equilibrium at the point where F_{c1} and F_s intersect to get Equation 4.17. It should also be mentioned that [30] also set an upper limit for the distance z which is $2 \times a`$.

$$F_s = H + F_{c1} \times \cos(\beta) = N_v \times \frac{a`}{z} + H \tag{4.17}$$

When the force is known, the required reinforcement area for the main nib reinforcement is calculated as shown in Equation 4.18.

$$A_s = \frac{F_s}{f_{yd}} \tag{4.18}$$

Reinforcement stirrups A_{sb}, as seen in Figure 4.7 is required in order to avoid cracking in the nib [30]. This is half of required main nib reinforcement area, Equation 4.19:

$$A_{sb} = \frac{0.5 F_s}{f_{yd}} \tag{4.19}$$

Vertical Reinforcement A_{sv}

The load N must be carried by the vertical and inclined reinforcement. As the inclined reinforcement rebar layout is chosen, the vertical component (N_α) of F_α is subtracted from the the load N. This is done as shown in Equation 4.20:

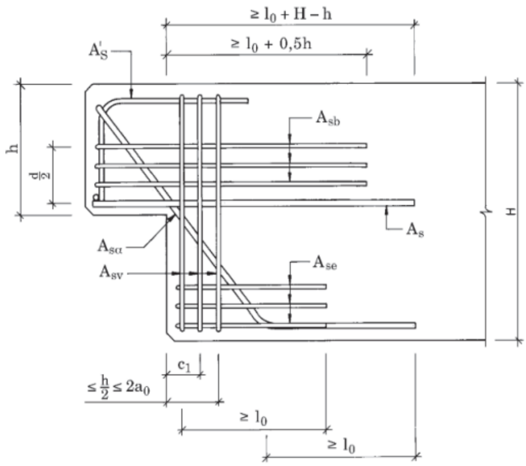

Figure 4.7: Reinforcement model [30, Figure C 8.21]

$$F_v = N_v = N - N_\alpha \tag{4.20}$$

The reinforcement area for the vertical reinforcement is given in Equation 4.21:

$$A_{sv} = \frac{N_v}{f_{yd}} \tag{4.21}$$

It is recommended that the vertical reinforcement should carry at least $\frac{2}{3}$ of the load N [30]. In some cases, the vertical reinforcement must be designed to take some of the splitting tensile forces due to the anchoring of the main reinforcement in the nib. However, practice has shown that beams that are heavy reinforced with stirrups A_{sv} in the end, have a reasonable amount of inclined reinforcement $A_{s\alpha}$ and anchorage length to take the force F_s, it is no trouble designing the vertical reinforcement without addition from the splitting tensile forces during normal load conditions.

End Anchorage Reinforcement A_{se}

It is recommended to use the same amount of horizontal stirrups A_{se} as the vertical stirrups A_{sv} (Equation 4.22. This is due to uncertainty of how the forces is exactly distributed.

$$A_{se} = A_{sv} \tag{4.22}$$

4.2 PCI Design Method

Yang and Lee [31] did a research on earlier conducted capacity tests of dapped-end beams. Among other goals of their research, was to check how the PCI design method could predict the capacity of those beams. The PCI design method is based on tests carried out by Mattock and Chan [26]. The method is quite easy to use, no strut-and tie modelling is required. The method predict 4 failure modes, that should be evaluated separately. Figure 4.8 show the relevant failure modes.

In order to apply the design method for design, the span to depth ratio a_1/d_d must be less than 1. a_1 is the effective shear span of the nib measured from the center of support to hanger reinforcement and d_d is the effective height of the nib. In "*Betongelement boken C* this ratio should be in magnitude of 0.4 to 0.6.

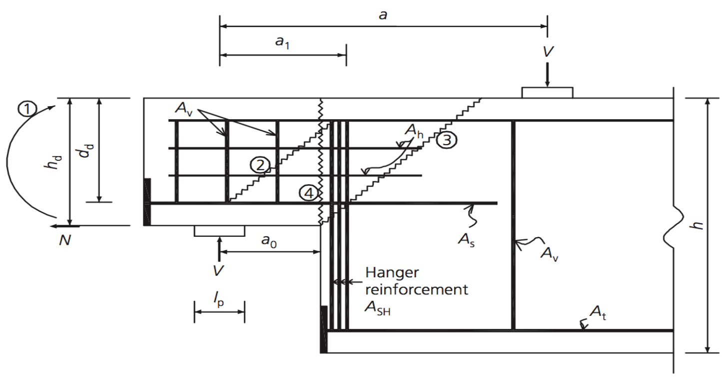

Figure 4.8: Potential failure planes according the PCI design method. [31, Figure 1]

1. Shear failure of the dapped-end beam, due to yielding of the the nib longitudinal reinforcement. The yielding is caused by moment in the nib.
2. Shear failure in the nib, caused by exceedance of the yield strength of the the horizontal shear reinforcement A_h and vertical shear reinforcement A_v.
3. Shear failure due to yielding of the hanger reinforcement A_{SH}.
4. Nib fails due to shear cut-off.

4.3 Softened Strut-and Tie Model (SST)

Lu et al. [25] conducted a capacity test on 12 dapped-end beams. The purpose of their research was to check out the accuracy of the softened strut-and tie models for design of dapped-end beams. The softened strut-and tie model was developed by Lu et. al. and described in their paper [25]. In addition of the 12 beams, were the beams tested by Mattock and Chan reviewed in order to estimate the capacity more accurate than what the PCI design method did [26].

The SST performed much better in predicting the ultimate failure load. In the most extreme case, the ratio ($\frac{V_{test}}{V_{calc}}$) of ultimate failure load and PCI method was 4.01. For the same beam, the ratio was 1.08 [25] for the SST method.

Figure 4.9 show the proposed SST model. The method is composed of diagonal and horizontal mechanism. Without going to deep into the method, is should be mentioned that three possible failure modes should be checked:

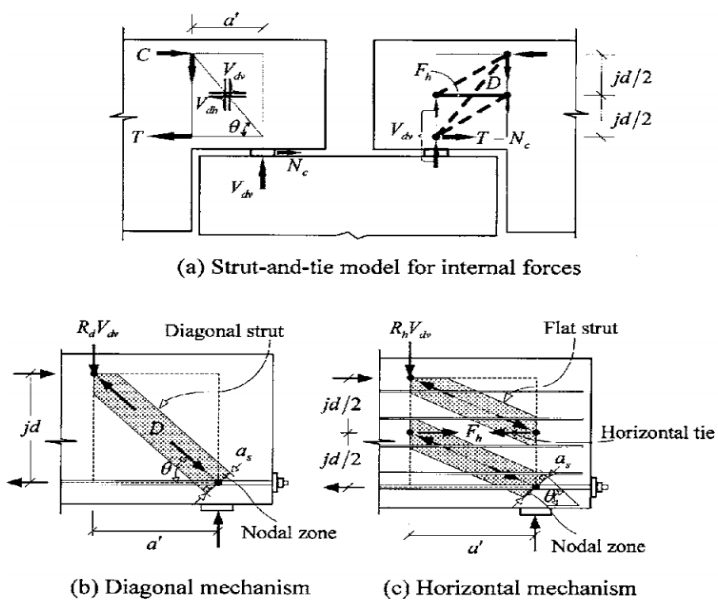

Figure 4.9: Strut-and-tie model for dapped ends [25, Figure 6]

- Failure due exceedance of the of the tensile capacity of the hanger bars.

- Shear failure due to moment in the nib.
- Compression failure of the diagonal (shown in figure 4.9 (b))

The method is more cumbersome than the PCI method, but show greater accordance with the test results. However, due to the complexity of the method, the method has not been applied further in the calculations of this thesis.

4.4 Simplified Strut-and Tie Model

The book "Ultimate Limit-State Design of Concrete Structures: A new approach" [22], written by Kotsovos and Pavlovic presents a simplified methodology to calculate the capacity of beams. The methodology behind simplified strut- and tie model is based on " thecompressive- force path (CFP) concept". The concept describe the behavior of failure to the related load capacity, which is likely to occur in compressive force path.

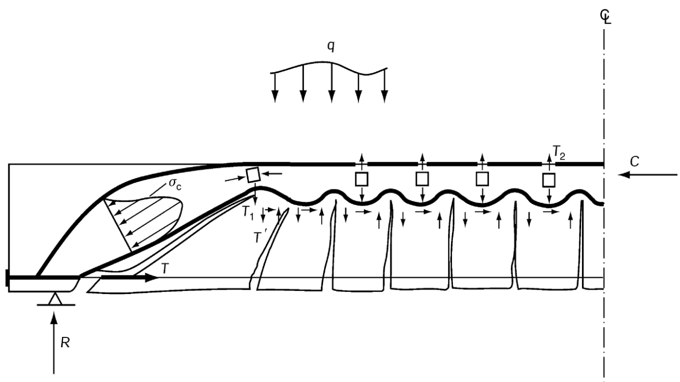

Figure 4.10: Indication of locations where tensile stresses are likely to develop within the uncracked portion [22, Figure 3.6.]

The Figure 4.10 provides a schematic representation of crack pattern in a simply supported beam under transverse loading. The corresponding internal forces, as depict, separates the uncracked portion from the remainder beam part, just before failure. Furthermore, figure 4.10 gives an indication where tensile stresses are likely to develop within the uncracked portion.

According to Kotsovos and Pavlovic [22], transverse tensile forces are likely to develop in four different regions:

a) Where the compressive stress path changes its direction, in intention to bring the force down to the support. By decomposing, a vertical force will occur and will be able to split the end block in the longitudinal direction.

b) In the interface between the uncracked and cracked concrete, a "comb-like" action is likely to occur and will create tensile stress in the fixed "cantilever beam" end.

c) In the adjacent regions to the deep flexural- or inclined cracks are caused due volume dilation corresponded to change of pressure intensity.

d) In the region where bending moment intend to be large or in section near the support, bond failure may happen. In aim to preserve moment equilibrium caused by the extension of the flexural crack, a force redistribution will occur and produce tensile stress in the section.

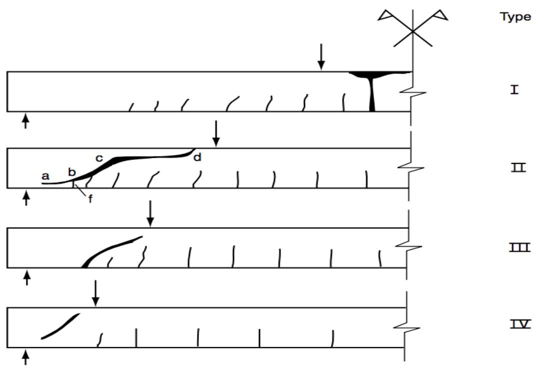

Figure 4.11: Mode of failure for different load situation[22, Figure 3.9.]

Four types of regimes apparent to occur by consider simply supported beam, without stirrups, under two point loading and let the span-to-depth (a_v/d) vary:

Type 1 behaviour corresponds to relatively large values of a_v/d (larger then 5) and is characterised by a flexural mode of failure. Item "c" gives a qualitative description of beam behaviour.

Type 2 behaviour corresponds to values of a_v/d between 2 and 5. The failure is characterised as brittle and are often associated with a deep inclined crack within the shear span of the beam or a nearly horizontal splitting of the compressive zone in the beam. Item "a" and "b" describe the failure mode. Although, bond failure in according to item “d” is likely to occur for type 2.

Type 3 behaviour corresponds to a_v/d value between 1-2 and is characterised with brittle failure. Such failure is associated with the development of an inclined crack within the shear span of the bream, but is in contrast to “type 2” independently from any pre-existing flexural or inclined crack. Furthermore, the inclined crack does not initiate instantaneous failure, but the load has to be increased. And in addition, beam failure intend to occurs outside of the shear span, where failure is caused by compressive volume dilation and can be qualitative described by item “c”.

Type 4 behaviour corresponds to values of a_v/d smaller then 1 and is characterised by two possible modes of failure. The first one is associated with a ductile mode of failure witch is caused within the middle narrow strip of the uncracked portion of the beam. The second one is a brittle mode of failure which is caused in the end block of uncracked portion of the beam in the region of the support. The mode of failure is usually dictated by the size of beam width, where larger size will attend to occur a more ductile behaviour. Item “c” describes the failure mechanism.

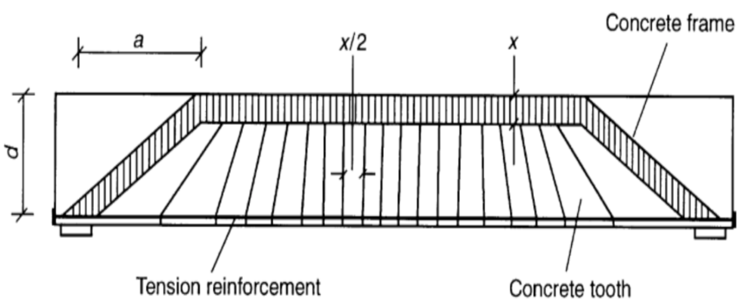

Figure 4.12: Physical model of simply supported beam under transverse loading[22, Figure 4.1.]

As Figure 4.12 illustrate, simplified strut and tie model is based on “comb-like” model with “teeth” fixed on to the horizontal element of a “frame” with “inclined legs”. The “frame” and the “teeth” also interact through a horizontal “tie” which is fully bonded to the “teeth” and anchored at the bottom ends of the “frame” legs.

By knowing beam depth and width, Kotsovos and Pavlovic[22] have posted a simple design method that is reliable to predict both loading capacity and mode of failure for situation with type 4 behaviour.

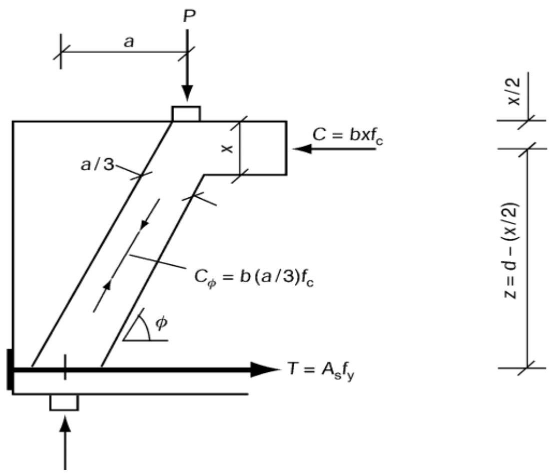

Figure 4.13: Assessment of load-carrying capacity of beam with type 4 behaviour[22, Figure 4.4.]

The depth of the horizontal element of the "frame" is assessed by satisfying the moment-equilibrium condition of the free body diagram, Figure 4.13, with respect to the intersection between the point loading (P) and compressive force (C). The moment equilibrium is shown in Equation 4.23 where the depth inclined leg is taken as equal to a/3 (where a is the shear span).

$$Tz = Sa \qquad \rightarrow \qquad T = \frac{Sa}{z} \tag{4.23}$$

By consider equilibrium in the horizontal direction, the compression height can be found as deduced in equation 4.24

$$T = C \qquad \rightarrow \qquad T = bxf_c \qquad \rightarrow \qquad x = \frac{T}{bf_c} \tag{4.24}$$

Equation 4.25 gives the diagonal compressive force.

$$C_{\varphi} = b\frac{a}{3}f_c \tag{4.25}$$

Brittle failure is prevented when the vertical component of the compressive force carried by the inclined leg to the "frame" is greater then, or equal to, the external load. If the condition is not satisfied, the beam width b should be increased to accomplish a ductile behaviour.

4.5 Conducted Calculation Method

After having consulted over the presented design method, model 1 and -2 proposed in Backe-Hansen and Hamstad‘s master thesis [7] were decided to be utilized when taking fibre contribution into account.

Defining the Effective Fibre Length

A large amount of numerical analyzes have been conducted in previous master thesis in aims to identify tensions in simply supported dapped-end beams. In both thesis Backe-Hansen, Hamstad[7] and Kittelsen, Kristoffersen, Østberg[20] a two-dimensional Abaqus model with isotropic material features is modulated, added a point load close to the support. In the latter, Figure 4.14 depicts the computed tensile stress distribution where the green colour gives the higher level of tensile stress intensity. It may be noted the tensile stress in the bottom of the beam and in the inclined part close to the support.

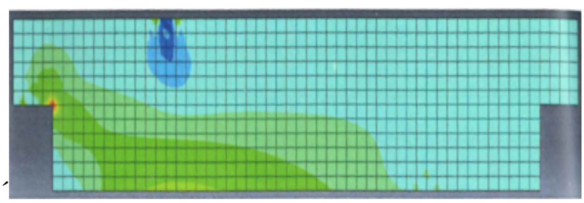

Figure 4.14: Numerical model - Tensile stress[20, Figure 7-13]

It´s assumed that cracks are likely to occur orthogonal relative to main tension force. Resulting force contribution from fibres depend on the residual tensile strength, Section 3.4, and effective cross section in accordance with fibres active zone. In both previous thesis, the effective width is set to the whole beam width, while so called effective fibre length, $L_{eff,fibre}$, appears to be more comprehensive to determine. Therefore, based on the numerical analysis, Kittelsen, Kristoffersen and Østberg[20] have posted a reliable $L_{eff,fibre}$- proposal, equation 4.26

$$L_{eff,fibre} = \frac{h}{2} \tag{4.26}$$

It seemed to be quite challenging to give an exact value for effective fibre length. Parameters like nib-height, load situation and placement of the support on nib will all together and individually affect $l_{eff,fibre}$. Thus, equation 4.26 will give a conservative value and can be optimized and improved.

Model 1

Model 1 is a modified version of dapped-end design in according to Betongelementboka [30]. The essence in the method gives the contribution in vertical load bearing from both hanger reinforcement and fibre. The contribution from hanger reinforcement is depict in strut- and tie model Figure 4.15 while the supplement from fibre is considered as an inclined tension rod, depict in Figure 4.16.

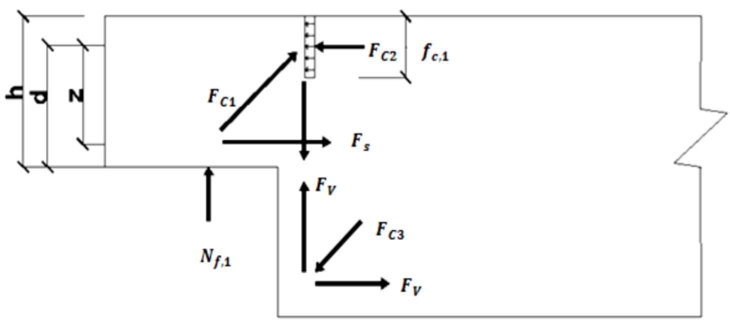

Figure 4.15: Strut- and tie model, hanger reinforcement[7, Figure 12.6]

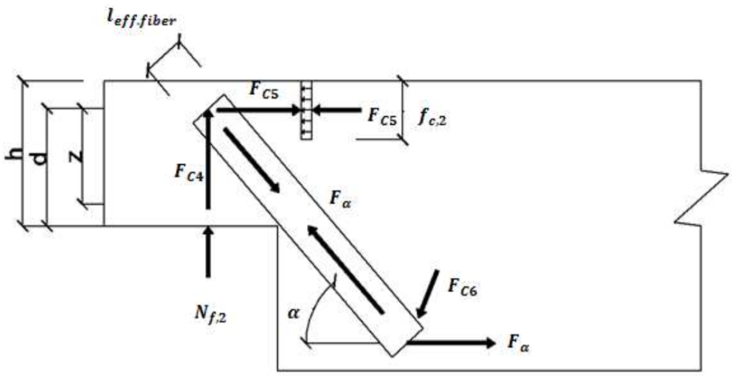

Figure 4.16: Strut- and tie model, fibre contribution[7, Figure 12.6]

Equation 4.27 gives the total force value in the support.

$$N_f = N_{f,1} + N_{f,2} \tag{4.27}$$

Equation 4.28 refers to the load at the support, which is carried by the hanger reinforcement.

$$N_{f,1} = F_v = A_{sv} f_{yk} \tag{4.28}$$

Equation 4.29 express the the total force of the stress fibre path.

$$F_\alpha = f_{td,res2,5} b l_{eff,fibre} \tag{4.29}$$

Where the vertical load contribution from F_α is given in the Equation 4.29

$$N_{f,2} = F_\alpha sin(\alpha) \tag{4.30}$$

According to Backe-Hansen and Hamstad[7] it should only consider contribution from fibre when vertical hanger reinforcement is left out. Although, the compression zone in the beam has be checked out for tension capacity in the concrete cf.EC2[6.5.2][1].

Model 2

In the second model, which is also based on Backe-Hansen and Hamstad, is unlike to model 1 by counting a another orientation to the fibre tension line. In this model the contribution from fibre is believed to act in an area parallel to the vertical stirrups, as illustrated in Figure 4.17

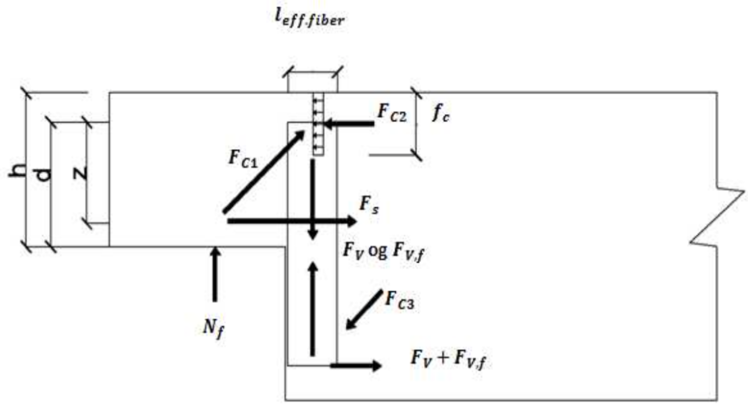

Figure 4.17: Strut- and tie model with combined contribution from both hanger reinforcement and fibres [7, Figure 12.6]

Equation 4.31 gives the total force value in the support.

$$N_f = F_v + F_{v,f} \tag{4.31}$$

Equation 4.32 refers to the load at the support, which is carried by the hanger reinforcement.

$$F_v = A_{sv} f_{yk} \tag{4.32}$$

Where the vertical load contribution from F_α is given in the Equation 4.28

$$F_{v,f} = f_{ftd,res2,5} b l_{eff,fibre} \tag{4.33}$$

Chapter 5

Design Methods

Norwegian Design Guidelines

On the initiative of the Concrete Innovation Centre (COIN), a guideline proposal for fibre reinforced concrete has been draft and released. The vision of COIN is creation of more attractive concrete buildings and constructions. Their primary goal is to fulfill this vision by bringing the development a major leap forward by more fundamental understanding of more advanced materials, including fibre reinforced concrete.

The Norwegian Concrete Association, known as the Norwegian society of concrete knowledge and technology, has developed a draft of a guideline[9] for fibre reinforced concrete. The design given in their guideline shall satisfy the Eurocode‘s requirements at Ultimate Limit State (ULS) and Service Limit State (SLS).

The two different guidelines, have derived their design calculations on the same base at ULS. Thereby, is just the the design checks at of Norwegian Concrete Association presented below.

However, the two different guidelines similarity is not that great at SLS. The reason is that COIN‘s guideline is designed on the base of Eurocode 2, while the Norwegian Concrete Association‘s guideline is based on the design of the next generation of Eurocode.

Thereby will both the design of COIN and the Norwegian Concrete Association be presented at SLS.

5.1 Ultimate Limit State

To figure out the design value to the residual tensile strength the partial safety factor γ_{cf} shall be applied and will be set equal 1.5. The design residual tensile strength can be determined according to Equation 5.1 [9, 4.2.1]

$$f_{ftd.res.2,5} = \frac{f_{ftk.2.5}}{\gamma_{cf}} \tag{5.1}$$

5.1.1 Bending Moment and Axial Forces

On the contrary of conventional concrete, fibre reinforced concrete can be designed to carry some of the tensile forces after initiation of cracking [9]. Thereby, could a tensile stress block as seen in Figure 5.1 be assumed in the elevation of the cross-section. As a simplification it is assumed that tensile zone have a uniformed tension distribution with a tension correspond to the residual tensile strength, $f_{\mathrm{ftd.2.5}}$. However, more precise calculations are given in the German SFRC guideline[10]. The German guideline assumes for instance a linear stress distribution in the tensile zone [9, 4.2.2].

Capacity for bending moment and axial force can be determined by assuming plane cross section remains plane after deformation. The stress-strain properties of fibre concrete in the compression zone, is similar as given the in the chapter [3.1.7] of the Eurocode 2 [1]. The conventional reinforcements stress-strain properties is as given in chapter [3.2.7] of the Eurocode 2.

In case of cross-sections exposed to pure tensile stress, shall strain in fibre concrete the be limited to less then 3/h ‰according to the Norwegian Concrete Association [9, 4.2.2]. It should be noted, that it is written in guideline that the strain limitation should be discussed. The German Design Code [4, 6.1] of fibre reinforced concrete, limit the strain to 25 ‰of the tension zone.

Construction that can cause major socially or economical damage or loss of life, shall prove bearing capacity by only take conventional reinforcement into consideration, with no contribution from the fibres. In that case, the partial safety factor should be be sett to 1,0 [9, 4.2.2].

5.1.2 Moment Capacity

It shall be proved that the load will be carried by both conventional reinforcement and the fibres assembled. [9, 4.2.3]. If the given residual tensile strength $f_{ftk.res2.5}$ is less than $2.5N/mm^2$, can a simplified design method be used. The principle of the method is shown in Figure 5.2. The residual tensile strength zone act over a height of $0.8h$ of the cross-section, and the internal lever arm is $0.5h$. Equation 5.2 express the bending moment capacity based on the simplified method.

$$M_{Rd} = 0.4 f_{ftd.res2.5} bh^2 \tag{5.2}$$

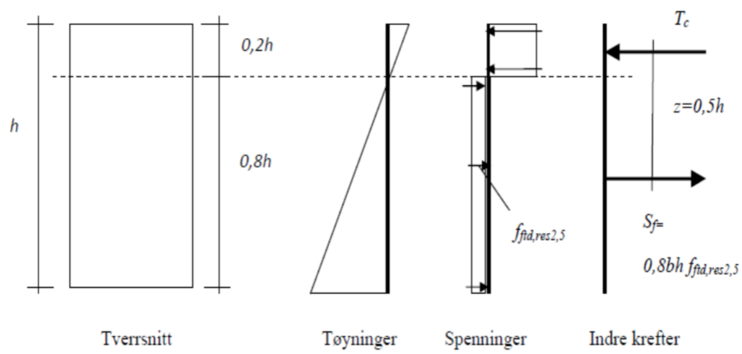

Figure 5.1: Stress and strain distribution of a rectangular cross-section exposed to pure bending [9, Figure 4.2]

If case of higher residual tensile strength than 2.5 MPa, a more advanced moment equilibrium will be the foundation of the equation for the moment capacity. Figure 7.1 show the strain and stress distribution of the cross-section of the a beam exposed to pure bending. The compression zone height shall be determined by axial equilibrium of the concrete compression force (T_c), the residual tensile force (S_f) and the reinforcement tensile force (S_a). The moment capacity is given by Equation 5.3:

$$M_{Rd} = S_f 0.5h + 0.1x + S_a(d - 0.4x) \tag{5.3}$$

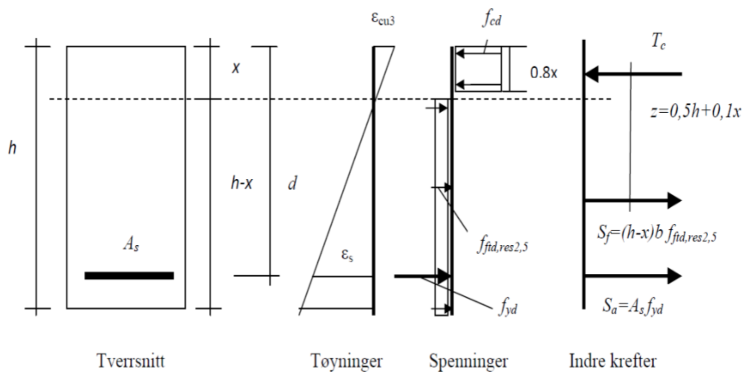

Figure 5.2: Stress and strain distribution of a rectangular cross-section exposed to pure bending [9, Figure 4.1]

5.1.3 Shear capacity

It is well documented with experimental studies that steel fibres increase the resistance against shear fracture [19], while it is not proved that synthetic fibres give the same promising results. Therefore, are the design rules given in the Norwegian Concrete Association‘s guideline [9] only valid for concrete supplied with steel fibres. Furthermore, must the cross-section to span ratio be less than three to apply the code.

A significant amount of promising methods and models have been developed to determine the shear capacity of fibre reinforced concrete. However, the Norwegian Concrete Association‘s guideline [9] only provides a guide for cross-section with interaction between longitudinal and fibre reinforcement without regular shear stirrups.

The design is based on section [6.2.2] of Eurocode 2 [1]. The section is modified to account for the additional strength offered by fibres. Equation 5.4 [9, 4.2.5] show that the shear strength $V_{Rd.c}$ is a result of the strength offered by both the fibres $V_{Rd.cf}$ and tensile rebar $V_{Rd.ct}$:

$$V_{Rd.c} = V_{Rd.ct} + V_{Rd_c f} \tag{5.4}$$

- $V_{Rd.ct}$ is found in EC2 [1], section [6.2.2].
- $V_{Rd.ct}$ is found by Equation 5.5. b the effective width of the section, d is the effective height.

$$V_{Rd,ct} = 0.6 \times f_{ftd.res2.5} \times b_w \times h \tag{5.5}$$

5.2 Serviceability Limit State

Serviceability limit state (SLS) defines critical states of a structure, regarding to specific requirements related to its use and purpose for its service life. Furthermore, these requirements ensure the structure‘s durability and ability to resist daily exposure.

Both the Norwegian Concrete Association‘s and the COIN‘s guideline [9] cover the design at SLS. However, it is a big difference between the two different proposed guidelines as discussed in the introduction of chapter 5. Thereby are the design checks of both guidelines given a proper walk-through.

5.2.1 Minimum Reinforcement

Minimum reinforcement area is derived in Eurocode 2 [1] by making sure that the element has a ductile behaviour at failure. Furthermore, it is also required due crack width limitation in expected tension areas. The determination is based upon the assumption that the tension zone in a cross-section should have the same capacity after cracking as immediately before[19]. For a fibre reinforced cross-sectional area this principle is expressed by Equation 5.6:

$$A_s\sigma_s + A_{ct2} f_{ftk.res2.5} \geq A_{ct} f_{cteff} \qquad or \qquad A_s \geq (A_{ct} f_{cteff} - A_{ct2} f_{ftk.res2.5})/\sigma_s \tag{5.6}$$

Where,

A_s	cross-sectional area of the main tensile reinforcement
σ_s	steel stress, usual the yield stress
A_{ct}	cross-sectional area of the tension zone before cracking
A_{ct2}	cross-sectional area of the tension zone after cracking
f_{cteff}	mean tensile strength when cracking occurs
$f_{ftk2.5}$	characteristic residual tensile strength

Minimum Reinforcement Rules of Beams According COIN

Tensile reinforcement

Equation 5.6 could be tough to use. The different cross-sectional areas after cracking are hard to estimate. Instead of doing this, have COIN [19] developed Equation 5.7 for the minimum tensile reinforcement. The equation is been based on the same principle as given in chapter 5.2.1, and applies for rectangular beams.

$$A_{s.min} = 0.26 b_t \times d \times \frac{f_{ctm} - 2.1 f_{ftk.res2.5}}{f_{yk}} \geq 0.0013 b_t \times d \times \left(1 - 2.1 \frac{f_{ftk.res2.5}}{f_{ctm}}\right) \tag{5.7}$$

f_{ctm}	average tensile strength of the concrete, EC2 Table 3.1[1]
$f_{ftk.res2,5}$	characteristic residual tensile strength
b_t	middle value of the width of the beam, which is subjected to tension
d	effective cross-sectional height

Shear Reinforcement

The minimum shear reinforcement is based on the Chapter [9.2.2] of Eurocode 2 [1]. Equation 5.8 is the same as given in the Eurocode. However, the shear reinforcement radio ($\rho_{w.min}$) is modified to utilize the strength contribution offered by the fibres. The new ratio is expressed by Equation 5.9.

$$\rho_w = A_{sw} / (s \times b_w \times sin(\alpha) \tag{5.8}$$

$$\rho_{w.min} = (0.1\sqrt{f_{ck}} - 0.3 f_{ftk.res2.5})/f_{yk} \tag{5.9}$$

s centre distance between shear reinforcement

A_{sw} cross-sectional area of shear reinforcement within s

b_w width of the web of the beam

α angle between the shear reinforcement and the longitudinal axis

Minimum Reinforcement Rules of Beams According Norwegian Concrete Association

Tensile Reinforcement

Minimum reinforcement is given by Equation 5.10. Like COIN‘s approach, is the equation very similar to the one found in Chapter [9.2.1.1] of Eurocode 2 [1]. COIN‘s have based their design on the the residual tensile strength at 2.5 mm crack width. The Norwegian Concrete Association have instead based their design on the characteristic flexural tensile strength at CMOD equal 0.5 mm (f_{Rk1}).

$$A_{s.min} = 0.26 \times \frac{f_{ctm} - f_{Ftsm}}{f_{yk}} \times b_t d \geq 0.0013 \times \left(1 - \frac{f_{Ftsm}}{f_{ctm}}\right) \times b_t d \tag{5.10}$$

f_{ctm} average tensile strength of the concrete, EC2 Table 3.1[1]

F_{Ftsm} Contribution from fibre reinforcement, Given in Equation 5.11

b_t middle value of the width of the beam, which is subjected to tension

d effective cross-sectional height

$$F_{Ftsm} = f_{Ftsk}/0.7 \qquad f_{Ftsk} = 0.45 \times f_{Rk1} \tag{5.11}$$

Shear Reinforcement

The expression of the shear reinforcement, is similar to COIN (Equation 5.8). However, the Norwegian Concrete Association have an other expression for the ($\rho_{w.min}$), which is given in Equation 5.12:

$$\rho_{w.min} = (0.1\sqrt{f_{ck}} - 0.2 f_{Fstm})/f_{yk} \tag{5.12}$$

5.2.2 Anchoring

Fibre in combination with conventional reinforcement, is considered by COIN to have the same anchorage capacity as conventional concrete. COIN refer to the rules of Eurocode 2 [1], in order to calculate the anchorage capacity.

The anchorage of the tensile reinforcement in not treated in the Norwegian Concrete Association‘s proposed guideline.

5.2.3 Crack Widths

Implementation of fibres into the concrete will give a significant increase in the material‘s ability to limit crack widths and distance. In fact, just a small amount of fibre will effectively limit the crack propagation, giving a virtually crack-free appearance of the the structure during service [19]. Cracks in Concrete is mainly caused by three different mechanisms: load, change of volume and chemical environment.

COIN‘s guideline provide design rules to determine the crack widths due to loads and volume change. The design concept of COIN is based recommendations of Löfgren. The Norwegian Concrete Association‘s guideline provide also formulas to calculate crack widths. Both approaches are presented below.

COIN‘s Approach

Crack width and crack distance calculation is based on principles described in chapter [7.3.4] of Eurocode 2 [1], with some modifications due to the fibres. The approach is based on the work of Døssland‘s thesis [14]. The approach is to calculate the stress in the reinforcement, by examination of the stress-strain relationship of a cross-section with uniform residual tensile strength ($f_{ftk.res2.5}$) in the tension zone. The tension in the reinforcement can be calculated according to the multi-layer method, which is presented a little later in this chapter.

The fibres will increase the pressure zone height in the cross-section, hence reduce the stress in the reinforcement. The calculations is based on stage II stiffness of concrete in compression and conventional reinforcement. While, the residual tensile strength is constant independent of load scenario.

Crack Caused by External Load

Equation 5.13 gives the general expression for crack widths caused by external loads:

$$w_k = s_{r.max}(\epsilon_{sm} - \epsilon_{cm}) \tag{5.13}$$

Where ϵ_{sm} and ϵ_{cm} represents the mean strain in the reinforcement and the concrete. The values shall be calculated accordance with chapter [7.3.4] of the Eurocode 2 [1]. However, the residual tensile strength ($f_{ftk.res2.5}$) is included in the calculation of the stress of the reinforcement, σ_s.

The Equation 5.14 is almost the same as given in the Eurocode. It is slightly modified with a factor k_5 to benefit the strength contribution of the fibres.

$$s_{r.max} = k_3 c + k_1 k_2 k_4 k_5 \frac{\phi}{\rho_{s.eff}} [mm] \tag{5.14}$$

where,

k_1	equal 0.8 if the reinforcement have sufficient adhesion with the concrete. If the reinforcement surface is almost smooth, shall it be sett equal 1.6
k_2	equal 0.5 in case of bending, or 1.0 in case of pure tension
k_3	= 3.4
k_4	= 0.425
k_5	$= (1 - \frac{f_{ftk.res2.5}}{f_{ctm}})$
ϕ	the diameter of the reinforcement
c	cover of the longitudinal reinforcement
$\rho_{s.eff}$	$= \frac{A_s + \xi_1^2 + A_p`}{A_{c.eff}}$ the values of $A_p`$, $A_{c.eff}$, and ξ_1 are found in Eurocode 2 [1], chapter [7.3.2](3)

Average crack width can be found by Equation 5.15:

$$s_r = \frac{s_{r.max}}{1.7} \tag{5.15}$$

Crack Caused by Shrinkage

To calculate crack widths caused by volume change is a bit challenging and seldom executed. However, the issue is quite relevant. For a waterproof construction for instance, could this be very important.

The method suggested in COIN's guideline [19] is based on a model where the cracks are modeled as springs. The number of cracks can be calculate according to Equation 5.16:

$$\frac{N(\sigma_s, f_{ftk.res2.5}) \times L}{E_c \times A_I}(1 + \phi_{ef}) + n \times w(\sigma_s) = R \times \epsilon_{cs} \times L \tag{5.16}$$

where,

$N(\sigma_s, f_{ftk,res2,5})$	axial loading in uncracked cross-section, expressed by Equation 5.18
L	length of the element
$A_I = A_c + A_s(\frac{E_s}{E_c} - 1)$	
ϕ_{ef}	creep number
n	number of cracks
R	degree of restraint (R=0 no restraint, R=1 full restraint)

$w(\sigma_s)$ is the crack width caused by volume changes of the concrete and can be determined by Equation 5.17:

$$w(\sigma_s) = 0,428[\frac{\phi\sigma_s^2}{0,22 f_{cm} E_s (1 + \frac{E_s A_s}{E_c A_{cf}})}]^{0,826} + \frac{\sigma_s}{E_s 4\phi} \tag{5.17}$$

$$N(\sigma_s, f_{ftk,res2,5}) = \sigma_s A_s + f_{ftk,res2,5}(A_c - A_s) \tag{5.18}$$

A crack is likely to be develop if axial load is greater then the initiate force ($N(\sigma_, f_{ftk,res2,5} \geq N_1$). The value of N_1 is given in Equation 5.19

$$N_1 = f_{fctm}[A_{ef} + (\frac{E_s}{E_c} - 1)A_s] \tag{5.19}$$

If $N(\sigma_s, f_{ftk,res2,5}) \leq N_1$ will the crack propagation stop, and the actual number of cracks can be calculated from equation 5.16. The method demands iterations and could be easily performed by a computational program. The number of cracks will decrease with increase of the residual tensile strength and reinforcement amount. Reduced reinforcement diameters will also limit the crack widths [19].

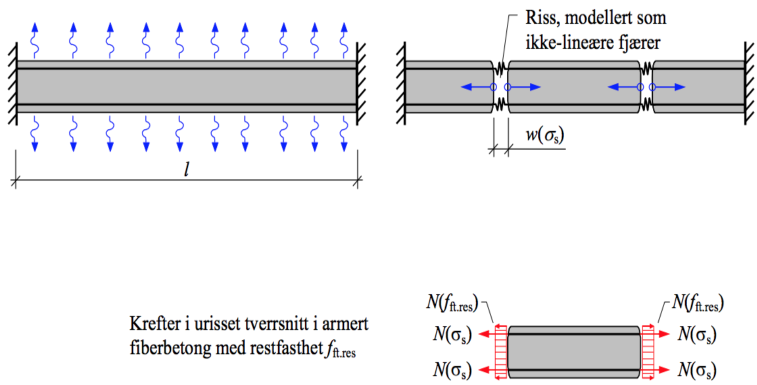

Figure 5.3: Model for analysing cracks caused by shrinkage[19, Figure6.4]

Norwegian Concrete Association's Approach

Equation 5.20 is given by the Norwegian Concrete Association [9] to calculate the size of the crack widths. The equation applies for fibre reinforced concrete in combination with conventional reinforcement.

The crack widths shall be checked with the requirements given in Eurocode 2 [1], table NA.7.1N.

$$w_d = 2\left(k \times c + \frac{1}{4}\frac{\phi_s}{\rho_{s.ef}} \times \frac{(f_{ctm} - f_{Ftsm})}{\tau_{bm}}\right) \times \frac{1}{E_S} \times (\sigma_s - \beta \times \sigma_{sr} + \eta_r \times \epsilon_{sh} \times E_S) \tag{5.20}$$

k	= 1.0
c	length of the concrete cover
ϕ_s	diameter of rebar
$\rho_{s.ef}$	$\frac{A_s}{A_{c.ef}}$ Defined in Eurocode 2 [1], Figure 7.1
f_{ctm}	average tensile strength of concrete
f_{Ftsm}	Contribution from fibre reinforcement, given in Equation 5.11
τ_{bm}	adhesion strength, see Table 5.1
E_S	modulus of elasticity of the reinforcement
σ_s	stress in the reinforcement located at the crack
β	coefficient related to the strain of the anchorage length of the reinforcement see Table 5.1
σ_{sr}	$= \frac{(f_{ctm} - f_{Ftsm}}{\rho_{c.ef}} \times (1 + \frac{E_S}{E_C}\rho_{c.ef})$
η_r	coefficient that account for the degree of restraint, see Table 5.1
ϵ_{sh}	strain caused by shrinkage

	Crack formation stage	Stabilized cracking stage
Short term, instantaneous loading	$\tau_{bm} = 1.8 \times f_{ctm}(t)$ $\beta = 0.6$ $\eta_r = 0$	$\tau_{bm} = 1.8 \times f_{ctm}(t)$ $\beta = 0.6$ $\eta_r = 0$
Long term, repeated loading	$\tau_{bm} = 1.35 \times f_{ctm}(t)$ $\beta = 0.6$ $\eta_r = 0$	$\tau_{bm} = 1.8 \times f_{ctm}(t)$ $\beta = 0.4$ $\eta_r = 1$

Table 5.1: Values for τ_{bm}, β and η_r for deformed reinforcing bars [9, Table 7.6-2]

The Norwegian Concrete Association's guideline do not provide any formulas that can be used to calculate crack widths, of structural members without reinforcement bars. However, the guideline state that if no reinforcement is needed at ULS, is it assumed that the member is uncracked as SLS.

5.3 Multi-Layer Simulation

The multi-layer method was developed by Hirdijk (1991) aimed to simulate the bending response for plain concrete. A research paper developed by Kooiman, van der Veen and Walraven [21] presents the procedure, and how it could be used to model the bending behavior of FRC.

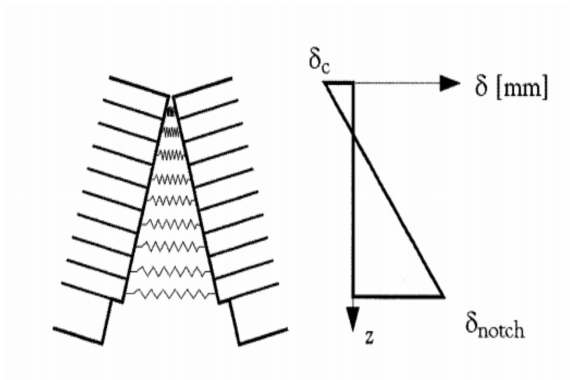

Figure 5.4: First principle of the multi-layer simulation procedure [21, Figure 26]

The multi-layer procedure is based on three principles [21]:

- *Finite number of layers* Figure 5.4 illustrate the first principle, that the beam is divided into two halves, which are connected by springs above the notch. Furthermore, it is assumed that the there will be a linear displacement distribution that describes the deformation.

- *Balance of forces to calculate the bending moment* The second principle of the procedure is demonstrated in Figure 5.5. The deformation of each spring is determined by calculating the average deformation of the corresponding layer. The force N required to deform each layer (spring) is calculated and added together. Equilibrium is found when N is is equal to zero, as shown in Equation 5.21. The internal bending moment is also calculated, according to Equation 5.22 The corresponding internal moment is equal to the external bending moment caused by the applied load. As a result, the external load can easily be determined from internal bending moment, as showed in Equation 5.23

$$N = \sum_{i=1}^{n} \sigma_i \times \times h_i \times b = 0 \tag{5.21}$$

$$M_{int} = \sum_{i=1}^{n} z_i \times \sigma_i \times h_i \times b \tag{5.22}$$

$$P = M/l \tag{5.23}$$

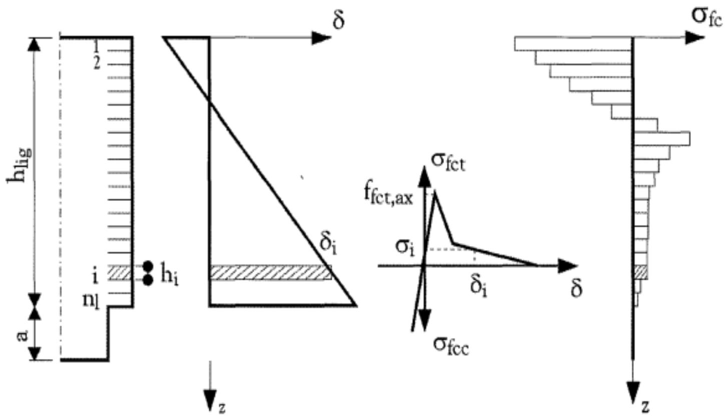

Figure 5.5: Second principle of the multi-layer simulation procedure [21, Figure 27]

- *Variation of strains* The third principle of the model is the increment procedure. In small steps the displacement at the notch is increased with a marginal displacement δ_{notch}. After each iteration is it necessary to calculate the axial force equilibrium of the cross-section, this is done by adjusting the displacement at the top of the beam d_c until N is equal to zero. At this state of equilibrium, the crack opening displacement at the notch δ_{notch} and the bending moment M, are plotted. By repeating the incremental steps, the load crack opening displacement diagram can be calculated, as illustrated in Figure 5.6

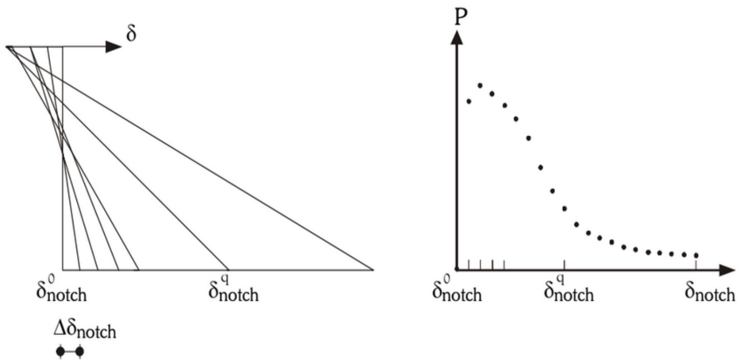

Figure 5.6: Third principle of the multi-layer simulation procedure [21, Figure 28]

The model ideally be programmed in a spreadsheet, were parameters easily can be changed investigate the behavior of a FRC exposed to bending moment.

5.4 Angle of Crack Development

Design with conventional concrete is based on the assumption that concrete can only carry compressive forces. The addition of fibres will give a significant increase in the tensile strength of the concrete. The traditional approach to estimate the inclinement angle of the cracks close to the beam support must be modified to account for the changed properties of the concrete.

Mohr‘s circle is an important tool that could be used to find the main directions of the forces in a material. A little concrete element is shown in Figure 5.7. The element is located close to the support of the beam, in the tensile zone. If a linear shear distribution is assumed in the tensile zone, the shear stress could be calculated according to Equation 5.24.

$$\tau = \frac{V}{z \times b} \tag{5.24}$$

The next step is to find the axial force σ_x and the vertical force σ_y for the element. σ_y is zero, and σ_x is small due to almost no bending moment at the beam-end. Figure 5.8

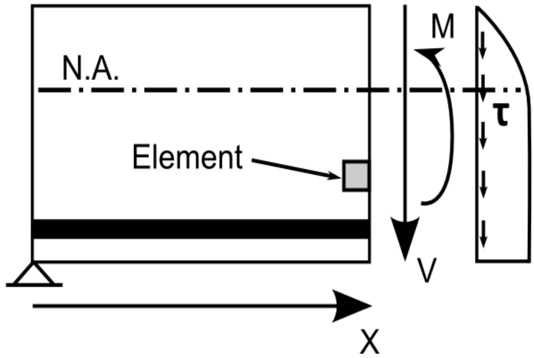

Figure 5.7: Element of interest

illustrate how the forces are plotted in Mohr‘s circle, to find the main forces and their directions. The crack develops perpendicular to σ_1 (the main direction of the tensile force of the element).

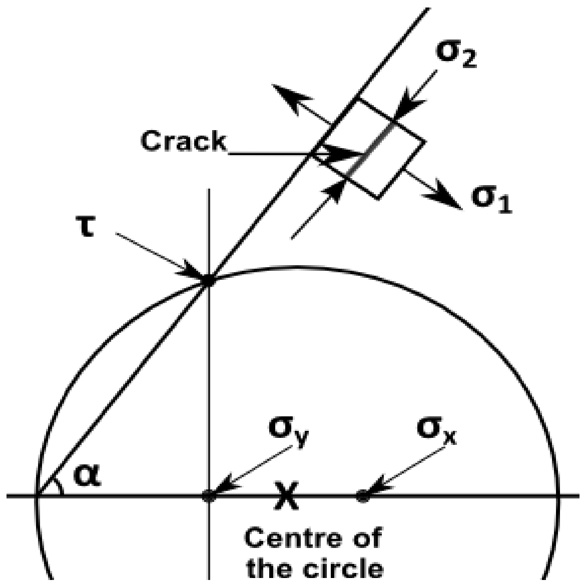

Figure 5.8: Mohr‘s circle

The crack angle will be a little higher than 45 °.

5.5 Anchor of Tensile Reinforcement in Fibre Reinforced Concrete

Eurocode 2 [1] covers the anchoring of tensile reinforcement. If cracking occurs, the fibres will tie together the cracked cross-section in a similar manner as stirrups. However, section 6.2.3 (7) needs to be modified slightly to calculate the attribution ΔF_{td} in the tensile reinforcement due to the shear force V_{Ed}.

The derivation of the expression ΔF_{td} could be divided into two steps. Both steps consist of examining the moment equilibrium close to the support-end of the beam. In step one, an uncracked cross-section is examined, while a cracked cross-section is considered in step two.

The first step is to execute moment equilibrium about point A in Figure 5.9. This gives equation 5.25.

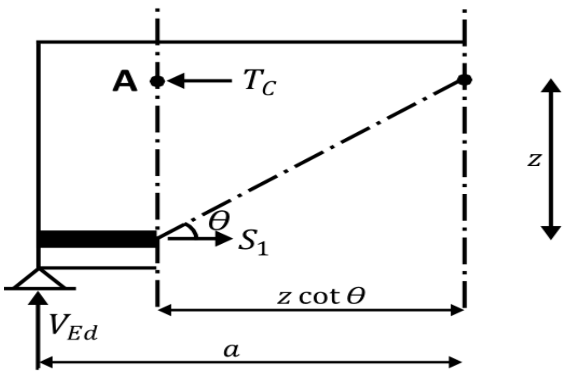

Figure 5.9: Beam in uncracked condition

$$S_1 \times z = M_1 = V_{Ed}(a - z \times cot\theta)$$

$$S_1 = \frac{M_1}{z} = \frac{V_{Ed}(a - z \times cot\theta)}{z} = \frac{V_{Ed} \times a}{z} - V_{Ed} \times cot\theta \tag{5.25}$$

Next step is to examine the moment equilibrium about point B in Figure 5.10.

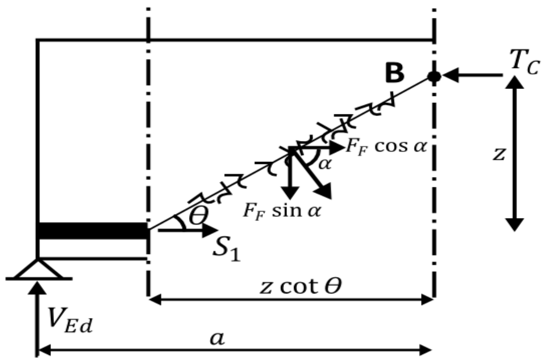

Figure 5.10: Beam in cracked condition

$$S \times z = V_{Ed} \times a - F_F(\frac{z \times cot\theta}{2} \times cos\theta + \frac{z}{2} \times sin\theta) \qquad \rightarrow F_f = f_{ftd.res.2,5}\frac{b \times z}{sin\theta}$$

$$= V_{Ed} \times a - F_F\frac{z}{2}(cot\theta \times cos\theta + sin\theta)$$

$$S = \frac{V_{Ed} \times a}{z} - 0.5F_F(cot\theta \times cos\theta + sin\theta) \tag{5.26}$$

Equation 5.26 expresses the force in the flexural reinforcement in a cracked cross-section.

Finally an expression of ΔF_{td} could be developed by subtracting the force from Equation 5.25 from Equation 5.26.

$$\Delta F_{td} = S - S_1$$

$$= \frac{V_{Ed} \times a}{z} - 0.5F_F(cot\theta \times cos\theta + sin\theta) - \frac{V_{Ed} \times a}{z} + V_{Ed} \times cot\theta$$

$$= V_{Ed} \times cot\theta - 0.5F_F(cot\theta \times cos\theta + sin\theta) \tag{5.27}$$

For simplification, θ is set equal to 45° in the expression developed for F_F. However, in Equatiion 5.27 the the addition force ΔF_{td} that must be anchored increase with increased angle.

5.6 Splitting Tensile Forces Caused by Concentrated Loads

An easy way to describe the occurence of splitting tensile forces, is to study a column which supports a point load centered at top. The situation is illustrated in Figure 5.11. The Figure shows the force distribution in an idealized material (not concrete). The stresses will try to to approach a constant value($\sigma_x = N/A$).

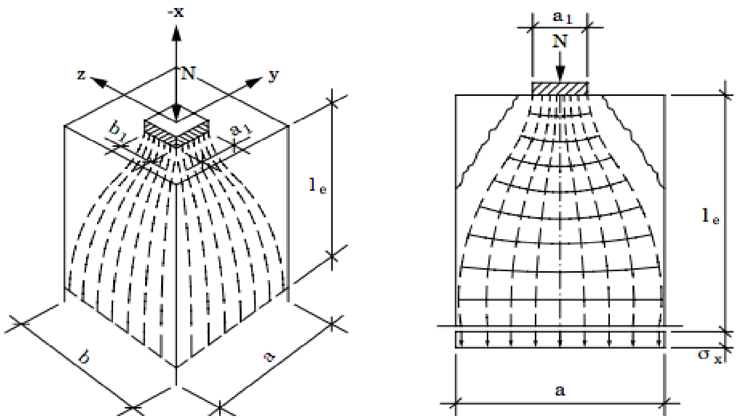

Figure 5.11: Distribution of main stresses [5, Figure B17.8]

The stress area will expand further down the column as shown in Figure 5.11. The length l_e describes the length of how far down the column, the force will continue to spread out before it approaches a constant value. The directions of the main forces will obviously be of great interest. Critical tensile forces will develop perpendicular to the compression forces The forces are named splitting tensile forces, and must be calculated and checked.

The friction in the concrete will provide some capacity to resist the splitting tensile forces. In most cases there must be installed additional reinforcement, to take care of the rest.

It is obvious that the fibre reinforcement increase the concretes capacity to withstand tensile forces. Betongelementboken Volume B [5, 17] provide some methods which could be applied to calculate the splitting tensile forces. The methods from Betongelementboken Volume B [5, Chapter 17] are presented here. The methods are followed by a discussion of how fibre reinforcement can contribute to increase the splitting tensile capacity.

First a centric point load is examined, the force-model is shown in Figure 5.12. An Equation for the splitting tensile forces is derived in Equation 5.28. The derivation is simply done by checking moment equilibrium about the support. This is same equation as given in Eurocode 2 [1, 9.8.4(2)].

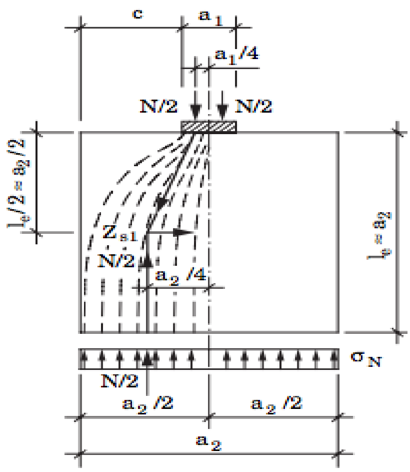

Figure 5.12: Basis to calculate the splitting tensile force [5, Figure 11.11]

$$Z_{s1} = \frac{\frac{N}{2}\left(\frac{a_2}{4} - \frac{a_1}{4}\right)}{\frac{a_2}{2}} = 0.25 \times N(1 - \frac{a_1}{a_2}) \tag{5.28}$$

In case of an eccentric point load, a simplified truss model illustrated in Figure 5.13 can be used. This method is considered to be conservative according to Betongelementboken [5]. Tensile forces at the surface will be higher for an eccentric point load than for a centric point load. Equation 5.28 can be used to calculate the tensile force Z_{s1}. Tthe tension force Z_{s2} at surface must also be calculated. Equation 5.29 is given Betongelementboken to calculate the tension force Z_{s2}.

$$Z_{s2} = \frac{0.015 \times N}{1 - \sqrt{2e/a}} > 0.02 \times N \tag{5.29}$$

In case of several loads, it will be safe to summarize all the loads $\left(Z_{s2} = \sum_{i=1}^{n} Z_{s2,n}\right)$.

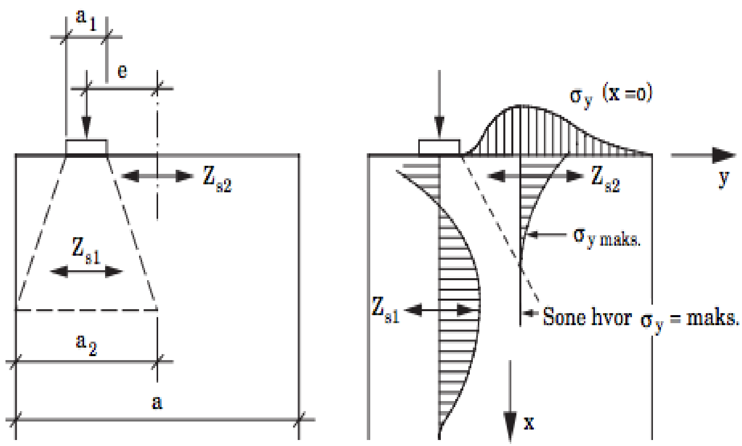

Figure 5.13: Splitting tensile force, in a case with eccentric load[5, Figure 17.12]

The horizontal rebar A_{sb} should according to Equation 4.19 be designed to have half the capacity of the main nib reinforcement. If the tensile strength of the horizontal rebar is utilized, could much of this reinforcement be excluded.

5.6.1 Increased Splitting Tensile Capacity due to Fibres

No design rules of fibres ability to increase the tensile capacity, was found in the research work for this thesis. An approach could be to multiply the residual tensile strength ($f_{td.res2.5}$) with an effective cross sectional area. This approach was chosen for the calculations in this thesis. The contribution from the fibres is calculated according to Expression 5.30:

$$SP_{std} = f_{td.res2.5} \times b_t \times d_t \tag{5.30}$$

b_t Effective width of the beam. The with of the beam is substituted 2× the concrete cover

d_t Effective height of cross-section. Distance from main tension reinforcement to the compression reinforcement.

Horizontal reinforcement is needed, can be calculated according Equation 5.31, if needed

$$A_{sb} = \frac{Z_{s1} - SP_{std}}{f_{yd}} \tag{5.31}$$

It should also be checked that the fibres provide enough capacity as required in Equation 4.19. If not, additional reinforcement must be installed.

Chapter 6

The Design

The scope of this thesis was to use our knowledge about fibre reinforced concrete for design purposes. This has been done by assuming a real design situation. Six different fibre reinforced beams were designed for the situation. Three similar concrete beams were designed without fibre reinforcement as reference beams as reference beams to check the fibres efficiency.

The design case and the different design solutions will be presented in this chapter.

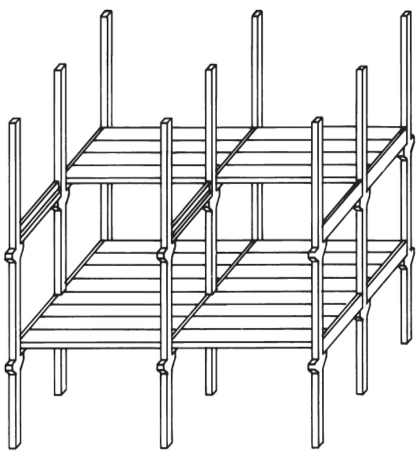

Figure 6.1: Reference structure [6, Figure A 3.1]

6.1 Case

A structure made of precast concrete elements is planned to be build. The structural system of the structure is illustrated in Figure 6.1. The structure will serve as an office building, Therefore the office load from $Eurocode$ 1 [3] is applied. The structural element that is going to be designed, is a beam with span of 6 m. The beam will support hollow cores that span over a length of 10 m.

6.2 The Choose of Concrete

The scope of this master thesis have been to apply our knowledge about fibre reinforced concrete in a best possible way for designing purposes. Much effort has been devoted to develop different fibre reinforced concrete mixtures with excellent performance. The mixture process is intricate and requires skilled labours. The attention of this thesis has not been to develop a fibre reinforced concrete. Therefore was is chosen to use the fibre reinforced concrete mixtures developed by Norbrøden and Weydahl's [27] and Kittelsen, Kristoffersen and Østbergs's thesis [20].

Norbrøden and Weydahl [27] had great success with their development of a fibre reinforced concrete. With B35 concrete and 1 vol-% of Dramix 65/60 fibres, they obtained an average residual tensile strength of 4.04 MPa. However, they based their residual tensile strength on Equation 3.3, which resulted in a value of 1.67 MPa
($f_{ftk.res2.5} = \eta v_f \sigma_{average} = \frac{1}{3} \times 0.01 \times 500$) [27]. This might seem a bit conservative.

Nr of test specimen	$F_{Rk,3}$ $[N/mm^2]$	$f_{tk.res2,5}$ $[N/mm^2]$
1	9.91	3.67
2	10.52	3.89
3	13.99	5.18
4	11.02	4.08
5	10.32	3.82
6	9.81	3.63

Table 6.1: Norbrøden and Weydahl results of the NS-EN 14651 test [27, Table 10.2]

Table 6.1 shows the test results of Norbrøden and Weydahl‘s conducted NS-EN 14651 test. With statistics, a characteristic strength from the the dataset in Table 6.1 can be calculated. Assuming normal distribution seems fair, but there is not enough data to use the standard distribution directly. The Student‘s t-distribution take into account the number of samples, and should be used when there are fewer than 30 samples. Equation 6.1 calculates the characteristic value from a limited data set according to the Student‘s t-distribution.

$$x_{k,pred} = \mu_x + t_{p_k,v} s_x \sqrt{1+\frac{1}{n}} \tag{6.1}$$

where,

m_x $= \frac{1}{n} \times \sum_{i=1}^{n} x_i$, the average value of the samples

n, is the number of samples

s_x $= \sqrt{\frac{1}{n-1} \times \sum_{i=1}^{n} (x_i - m_x)^2}$ the standard deviation of the samples

$t_{p_k,v}$ Inverse value from the t-distribution with degree of freedom $v = n-1$ and probability p. For instance: $t_{0.05,(6-1)}$=−2.015

The characteristic strength is calculated for the probability $p = 5$ % of failure. Usage of Student‘s t-distribution on Norbrøden and Weydahl‘s dataset resulted in a characteristic flexural (f_{Rk3}) and tensile strength ($f_{ftk.res2.5}$) of the 7.53 and 2.78 MPa. Those are the values which have been used in this thesis for design with the 1 vol-% fibre reinforced concrete.

Kittelsen, Kristoffersen and Østberg [20] developed in cooperation with Kjellmark a 2 vol-% 65/60 3D fibre reinforced concrete. They performed 15 flexural tensile strength tests that showed an average value of 17,4 MPa strength, which exeeds the COIN-target of 15 MPA [29].

However, the 0.05-percentile value of their trial showed a flexural bending strength of 13.2 MPa [20], less than 15 MPa. This was a bit of a setback, but Kittelsen et. al. concluded in their thesis that with a better understanding of the casting procedure. A more favourable and more controlled fibre-orientation could be obtained, thereby reducing the scatter of the test. This would obviously give a higher 0.05-percentile value.

However, this has not been done yet. The values from the Student's t-distribution were therefore chosen. A flexural-(f_{Rk3}) and tensile strength ($f_{ftk.res2.5}$) of 13,2 and 4,88 MPa has been applied in the beam design for this thesis.

The material properties of the two fibre reinforced concrete mixtures applied in the beam design in this thesis are given in Table 6.2:

Mix	B65, 2 vol-% 65/60 3D fibre reinforced concrete	1 vol-% Dramix fibre reinforced concrete
$f.ck$ [Mpa]	65	35
$f_{Rk,3}$	13.2	7.53
$f_{\mathrm{ftk.res2.5}}$ [Mpa]	4.88	2.78
f_{R1k} [Mpa]	11	6.11

Table 6.2: Concrete properties

6.3 Beams

Both conventional reinforcement and fibre reinforcement were used in the design of the beams. The reference beams were designed with B35 concrete. The purpose of the reference beams, was to compare them to the fibre reinforced beams. The target of the comparison was to examine the fibres effectiveness to replace conventional reinforcement. All the beams were designed with the same width and height($b \times h = 350$ $mm \times 600$ mm) to ease the design process. Furthermore, it would show the fibres ability to increase the design capacity.

Two fibre reinforced concrete mixtures were applied in the design of the beams in this thesis, a B35 1 vol-% fibre mixture and a B 65 2 vol-% fibre mixture. The properties of the fibre reinforced concrete mixtures are given in Table 6.2.

Each type of concrete was used for designing of two straight-end beams (slack-reinforced and prestressed) and one dapped-end beam(slack-reinforced). Resulting in a total of 3 beams for each concrete mixture. In addition, three beams with conventional reinforcement designed as reference beams. All the different beams are presented in Table

6.3:

Beam	A-1	A-2	A-3	B-1	B-2	B-3	C-1	C-2	C-3
Straight-end beam	X		X	X		X	X		X
Dapped-end beam		X			X			X	
Prestressed reinforced			X			X			X
Slack reinforced	X	X		X	X		X	X	
B35 concrete	X	X	X						
B35, 1 vol-% fibre reinforced				X	X	X			
B65, 2 vol-% fibre reinforced							X	X	X

Table 6.3: The different beams

All the reinforcement layouts are illustrated in the figures below (Figure 6.2- 6.10). Here is also the design calculations presented.

The loads for the load scenario is found in Appendix Load Scenario. The material factors for beam type A is found in Attachment A. Similar is the material factors for beam B found in Attachment B, while the material factors for beam C is found in Attachment C.

6.3.1 Beam A-1

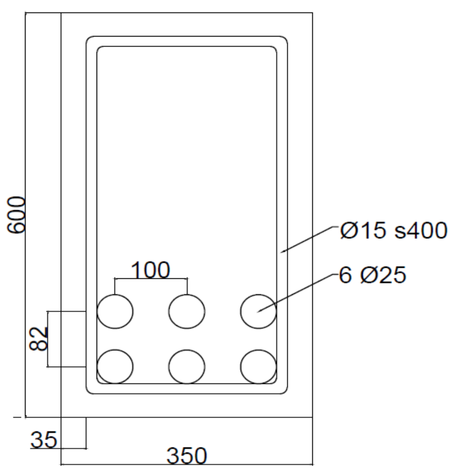

Figure 6.2: Reinforcement layout of beam A-1

Beam A-1 is a slack-reinforced beam casted with concrete quality B35.

Beam A-1 -*Slack-reinforced beam with straight- ends*

Miscellaneous data

Covering, [EC2 4.4.1]	$c_{nom} := 35mm$
Vertical spacing	$a_v := 32mm$
Horizontal spazing	$a_h := 70mm$
Horizontal rebar	$Ø_s := 25mm$
Shear reinforcement	$Ø_v := 15mm$
Effective height	$d := h - c_{nom} - Ø_v - Ø_s - \frac{a_v}{2} = 509 \cdot mm$

Ultimate limit state

Moment capacity

Normal reinforced [Table 4.3], Sørensen	$K := 0.275$
Fully utilized preassure zone:	$d_f := \sqrt{\frac{M_{Ed}}{K \cdot f_{cd} \cdot b}} = 0.486\,m$
Strength grade < B50	$\alpha := 0.40$ $\quad \lambda := 0.8$
Internal lever arm	$z_f := (1 - 0.5\lambda \cdot \alpha) \cdot d_f = 0.408\,m$
Required tensile reinforcement	$A_s := \frac{M_{Ed}}{f_{yd} \cdot z_f} = 2540 \cdot mm^2$
Use 8Ø25	$A_s := 6 \cdot \frac{Ø_s^{\,2} \cdot \pi}{4} = 2945.2 \cdot mm^2$
Compressive height ratio	$\alpha := \frac{f_{yd} \cdot A_s}{\lambda \cdot f_{cd} \cdot b \cdot d} = 0.453$
Lever arm	$z := (1 - 0.5\lambda \cdot \alpha) \cdot d = 0.417\,m$
Moment capacity	$M_{Rd} := \lambda \cdot \alpha \cdot z \cdot f_{cd} \cdot b \cdot d = 533.683 \cdot kN \cdot m$

Min. reinforcement and ductility control

Minimum reinforcement requirement [EC2 NA.9.2.1.1.(1)]
$$A_{s.min} := 0.26 \cdot \frac{f_{ctm}}{f_{yk}} \cdot b \cdot d = 296.442 \cdot mm^2$$

Balanced reinforcement cross section
$$\alpha_b := \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{yd}} = 0.617$$

The cross section is under reinforced
$$A_{s.b} := \lambda \cdot \frac{f_{cd}}{f_{yd}} \cdot b \cdot d \cdot \alpha_b = 4.01 \times 10^3 \cdot mm^2$$

Check reinforcement strain
$$\varepsilon_s := \frac{1 - \alpha}{\alpha} \cdot \varepsilon_{cu} = 4.226 \times 10^{-3}$$

$$\varepsilon_s < \varepsilon_{ud}$$

Shear capacity

Without reinforcement

$$\rho_l := \frac{A_s}{b \cdot d} = 0.017$$

$$k := 1 + \sqrt{\frac{200mm}{d}} = 1.627$$

$$C_{Rd.c} := \frac{0.18}{1.5} = 0.12$$

EC2- [6.2(a)]
$$V_{Rd.c} := \left[C_{Rd.c} \cdot k \cdot \left(100 \rho_l \cdot 35\right)^{\frac{1}{3}} \right] \cdot b \cdot d \cdot \frac{N}{mm^2} = 134.518 \cdot kN$$

EC2- [6.10.aN]
$$v_{min} := 0.035 \cdot k^{\frac{2}{3}} \cdot 35^{0.5} = 0.286$$

EC2- [6.2.(b)]
$$V_{Rd.c.min} := v_{min} \cdot b \cdot d \cdot \frac{N}{mm^2} = 51.025 \cdot kN$$

Shear capacity, without reinforcement
$$V_{Rd.c} := \max\left(V_{Rd.c}, V_{Rd.c.min}\right) = 134.518 \cdot kN$$

With reinforcement

Shear reinforcment area in a vertical section
$$A_{sw} := 2 \cdot \frac{\pi \cdot Ø_v^2}{4} = 353.429 \cdot mm^2$$

Center distance, vertical hoops
$$s := 400mm$$

Cutting press capacity [EC2 6.2.3(3)]
$$V_{Rd.s} := \frac{A_{sw}}{s} \cdot f_{yk} \cdot 0.8 \cdot z \cdot \frac{1}{\tan(22grad)} = 364.572 \cdot kN$$

[NA.6.10.aN]- strength reduction factor
$$\nu_1 := 0.6$$

$$V_{Rd.max} := \frac{b \cdot z \cdot \nu_1 \cdot f_{cd}}{\frac{1}{\tan(22grad)} + \tan(22)} = 698.819 \cdot kN$$

Shear capacity, with reinforcement $V_{Rd} := \min(V_{Rd.s}, V_{Rd.max}) = 364.572 \cdot kN$

Min. reinforcement

Minimum shear reinforcement [EC2. 9.2.2(5)] $\rho_{w.min} := \frac{0.1 \cdot \sqrt{35}}{500} = 1.183 \times 10^{-3}$

Maximum distance according to EC2-9.2.2(5) $s_{max} := \frac{A_{sw}}{b \cdot \rho_{w.min}} = 853.435 \cdot mm$

Maximum center distance between the shear reinforcement [EC2. 9.2.2(6)] $S_{l.max} := 0.75 \cdot d \cdot \left(1 + \frac{1}{\tan(22grad)}\right) = 1.327 \cdot m$

Serviceability limit state

Uncracked cross- section (Stadium I)

Material stiffness ratio $\eta := \frac{E_s}{E_{cm}} = 5.882$

Transformed cross -section $A_t := A_c + (\eta - 1) \cdot A_s = 2.244 \times 10^5 \cdot mm^2$

The distance from the longitudinal reinforcement to concrete center of gravity $e_s := d - \frac{h}{2} = 209 \cdot mm$

Reduction distance to the center of gravity of reinforced cross-section. $y_t := \frac{(\eta - 1) \cdot A_s \cdot e_s}{A_t} = 13.394 \cdot mm$

Second moment of area, contribution from concrete $I_{c1} := \frac{b \cdot h^3}{12} + h \cdot b \cdot y_t^2 = 6.338 \times 10^9 \cdot mm^4$

Second moment of area, contribution from steel bars $I_{s1} := A_s \cdot (e_s - y_t)^2 = 1.127 \times 10^8 \cdot mm^4$

Cracking moment $M_{cr} := \frac{I_{c1} + \eta \cdot I_{s1}}{0.5 \cdot h - y_t} \cdot f_{ctm} = 78.162 \cdot kN \cdot m$

Cracking load $q_{cr} := \frac{M_{cr} \cdot 8}{l_b^2} = 17.369 \cdot \frac{kN}{m}$

Beam stiffness, stadium I $EI_I := E_{cm} \cdot I_{c1} + E_s \cdot I_{s1} = 2.38 \times 10^5 \cdot kN \cdot m^2$

Deflection, stadium I $\delta_I := \frac{5}{384} \cdot \frac{q_k \cdot l_b^4}{EI_I} = 5.385 \cdot mm$

Cracked corss- section, (Stadium II)

Reinforcement ratio $\rho := \frac{A_s}{b \cdot d} = 0.017$

Compression height ratio $\alpha_{II} := \sqrt{(\eta \cdot \rho)^2 + 2\eta \cdot \rho} - \eta \cdot \rho = 0.354$

Second moment of area, contribtution from the concrete $I_{c2} := \frac{b \cdot (\alpha_{II} \cdot d)^3}{12} + b \cdot \alpha_{II} \cdot d \cdot \left(\frac{\alpha_{II} \cdot d}{2}\right)^2 = 6.846 \times 10^8 \cdot mm^4$

Second moment of area, contribution from steel bars $I_{s2} := A_s \cdot \left[(1 - \alpha_{II})d\right]^2 = 3.181 \times 10^8 \cdot mm^4$

Beam stiffness, stadium II $EI_{II} := E_{cm} \cdot I_{c2} + E_s \cdot I_{s2} = 8.689 \times 10^4 \cdot kN \cdot m^2$

Deflection, stadium II $\delta_{II} := \frac{5.}{384} \cdot \frac{q_k \cdot l_b^4}{EI_{II}} = 14.752 \cdot mm$

Moment at SLS $M_k := \frac{q_k \cdot l_b^2}{8} = 341.82 \cdot kN \cdot m$

Load, long duration $\beta := 0.5$

Distrubution koeffisient $\zeta := 1 - \beta \cdot \left(\frac{M_{cr}}{M_k}\right)^2 = 0.974$

Deflection, stadium I and II, EC2 - 7.4.3 $\delta := \zeta \cdot \delta_{II} + (1 - \zeta) \cdot \delta_I = 14.507 \cdot mm$

Crack Controll

Cover factor $k_c := 1.3$

Max. accepted crack width, exposure class XC1 $w_{max} := 0.3 \cdot k_c \, mm = 0.39 \cdot mm$

Tension in reinforcement, stadium II $\sigma_s := E_s \cdot \frac{M_k \cdot (1 - \alpha_{II}) \cdot d}{EI_{II}} = 258.553 \cdot \frac{N}{mm^2}$

Will not satisfied requrements according to Tabel 7.2N where greatest rebar dimension is restrictet to less then ø16 for for tesile stresses less tten 280MPa. Requirements related to max rebar spacing is in according to Table 7.3N.

Crack width calculation

Factor based on duration to the load $k_t := 0.4$

Concrete mean tensile strength value at time by cracking $f_{ct.eff} := f_{ctm}$

Distance from neutral axis to pressure edge

$$x := \frac{A_c \cdot \frac{h}{2} + \eta \cdot A_s \cdot d}{A_c + \eta \cdot A_s} = 0.316\,m$$

Effective tensile height

$$h_{cef} := \min\left[2.5\cdot(h-d) + 1.5\cdot Ø_s, \frac{h}{2}, \frac{(h-x)}{3}\right] = 94.691\cdot mm$$

Effective tensile area $A_{c.eff} := h_{cef} \cdot b = 0.033\,m^2$

$$\rho_{p.eff} := \frac{A_s}{A_{c.eff}} = 0.089$$

Modulus of elasticity ratio

$$\alpha_e := \frac{E_s}{E_{cm}} = 5.882$$

$$\Delta := \frac{\sigma_s - k_t \cdot \frac{f_{ct.eff}}{\rho_{p.eff}} \cdot \left(1 + \alpha_e \cdot \rho_{p.eff}\right)}{E_s} = 1.183 \times 10^{-3}$$

Miscellaneous factors -NA.7.3.4 $k_3 := 3.4$ $k_4 := 0.425$ $k_1 := 0.8$ $k_2 := 0.5$

Cover of the longitudinal reabars $c_3 := c_{nom}$

Upper value for crack width

$$S_{r.max} := k_3 \cdot c_3 + k_1 \cdot k_2 \cdot k_4 \cdot \frac{Ø_s}{\rho_{p.eff}} = 166.824\cdot mm$$

Determined crack width $w_k := S_{r.max} \cdot \Delta = 0.197\cdot mm$

Creep impact on beam deflection

Dead load will be applied after 28 days, while live load will be applied after 50 days. 40% of the live load is considered as permanent load. The beam is located inside. indoor condition with 50% relativ humidity

Load condition:

Long term load 1: $g_k = 45.96 \cdot \frac{kN}{m}$

Long term load 2: $p_{lang} := 0.4 \cdot p_k = 12 \cdot \frac{kN}{m}$

Calculate creep number [EC2 annex B]

$$\alpha_1 := \left(\frac{35 \frac{N}{mm^2}}{f_{cm}} \right)^{0.7} = 0.866 \quad \alpha_2 := \left(\frac{35 \frac{N}{mm^2}}{f_{cm}} \right)^{0.2} = 0.96 \quad \alpha_3 := \left(\frac{35 \frac{N}{mm^2}}{f_{cm}} \right) = 0.814$$

Effective cross- section thickness $h_0 := \frac{2 \cdot A_c}{2 \cdot (b + h)} \cdot \frac{1}{mm} = 221.053$

Factor based on the relative humidity $\varphi_{RH} := \left(1 + \frac{1 - \frac{50}{100}}{0.1 \cdot h_0^{\frac{1}{3}}} \cdot \alpha_1 \right) \cdot \alpha_2 = 1.647$

Factor based on the contribution from the concrete strength, on normed creeping number $\beta_{f.cm} := \frac{16.8}{\sqrt{f_{cm} \cdot \frac{mm^2}{N}}} = 2.562$

Days for applied load 1 and 2, respectively $t_{0.28} := 28 \qquad t_{0.50} := 50$

Factor that takes the concrete age at loading into accounte for load 1 and 2, respectively

$$\beta_{t.0.28} := \frac{1}{\left(0.1 + t_{0.28}{}^{0.2}\right)} = 0.488$$

$$\beta_{t.0.50} := \frac{1}{\left(0.1 + t_{0.50}{}^{0.2}\right)} = 0.437$$

Creep number for load 1 $\varphi_{0.28} := \varphi_{RH} \cdot \beta_{f.cm} \cdot \beta_{t.0.28} = 2.061$

Creep number for load 2 $\varphi_{0.50} := \varphi_{RH} \cdot \beta_{f.cm} \cdot \beta_{t.0.50} = 1.845$

Stiffness calculation

Effective elastic modulus for load 1 $E_{c.1} := \frac{E_{cm}}{1 + \varphi_{0.28}} = 1.111 \times 10^4 \cdot \frac{N}{mm^2}$

Effective elastic modulus for load 2 $E_{c.2} := \frac{E_{cm}}{1 + \varphi_{0.50}} = 1.195 \times 10^4 \cdot \frac{N}{mm^2}$

Moment, load 1 $M_1 := \frac{g_k \cdot l_b^2}{8} = 206.82 \cdot kN \cdot m$

Moment, load 2 $M_2 := \frac{p_{lang} \cdot l_b^2}{8} = 54 \cdot kN \cdot m$

Mean elastic modulus $E_{c.middel} := \frac{M_1 + M_2}{\frac{M_1}{E_{c.1}} + \frac{M_2}{E_{c.2}}} = 1.127 \times 10^4 \cdot \frac{N}{mm^2}$

Elasticity ratio $\eta := \frac{E_s}{E_{c.middel}} = 17.742$

Compression heigth ratio $\alpha := \sqrt{(\eta \cdot \rho)^2 + 2 \cdot \eta \cdot \rho} - \eta \cdot \rho = 0.527$

Equivalent second moment of area $I_{cc} := 0.5 \cdot \alpha^2 \cdot \left(1 - \frac{\alpha}{3}\right) \cdot b \cdot d^3 = 5.281 \times 10^9 \cdot mm^4$

Beam stiffness, creep $EI := E_{c.middel} \cdot I_{cc} = 5.953 \times 10^{10} \cdot kN \cdot mm^2$

Deflection, creep $\delta_{creep} := \frac{5}{384} \cdot \frac{(g_k + p_{lang}) \cdot l_b^4}{EI} = 16.43 \cdot mm$

Shrinkage, influence on the beam deflection

Nominal skrinkage strain, Cement type N, EC2 -Tabel 3.2 $\varepsilon_{cd.0} := 10^{-3} \cdot \frac{0.46 + 0.38}{2} = 4.2 \times 10^{-4}$

Coefficient based on effective cross-section $k_h := 0.75 + (0.85 - 0.75) \cdot \frac{300 - h_0}{300 - 200} = 0.829$

Value will become 1 when t goes toward infinity $\beta_{ds} := 1$

Shrinkage stain after time	$\varepsilon_{cd} := \beta_{ds} \cdot k_h \cdot \varepsilon_{cd.0} = 3.482 \times 10^{-4}$
Autogeneic shrinkage stain	$\varepsilon_{ca} := 2.5 \cdot \left(f_{ck} \cdot \frac{mm^2}{N} - 10 \right) \cdot 10^{-6} = 6.25 \times 10^{-5}$
Free skrinkage strain	$\varepsilon_{cs} := \varepsilon_{ca} + \varepsilon_{cd} = 4.107 \times 10^{-4}$
Values from creep determination	$\rho = 0.017 \qquad \eta = 17.742 \qquad \alpha \cdot d = 0.268\,m$
Distance from neutral axis to pressure edge	$x := \frac{A_c \cdot \frac{h}{2} + \eta \cdot A_s \cdot d}{A_c + \eta \cdot A_s} = 0.342\,m$
Distance from center of gravity axis to tensile reinforcement	$e_0 := d - x = 167.357 \cdot mm$
Curvature caused by shrinkage	$\kappa_s := \frac{\varepsilon_{cs} \cdot E_s \cdot A_s \cdot e_0}{EI} = 6.801 \times 10^{-4} \frac{1}{m}$
Deflection, shrinkage	$\delta_{svinn} := \frac{l_b^2 \cdot \kappa_s}{8} = 3.06 \cdot mm$
Total deflection	$\delta_{total} := \delta_{svinn} + \delta_{creep} = 19.491 \cdot mm$

Stress limitation

Compressive ratio for stadium II behaviour.	$\alpha_{II} = 0.354$
Compressive stress	$\sigma_c := \frac{2M_k}{\alpha_{II} \cdot d^2 \cdot b \cdot \left(1 - \frac{\alpha_{II}}{3}\right)} = 24.125 \cdot MPa$
Max. compressive stress, EC2 -7.2.(2)	$0.6 \cdot f_{ck} = 21 \cdot MPa$
Tensile stress in the rebar	$\sigma_s = 258.553 \cdot \frac{N}{mm^2}$
Max. tensile stress in rebar, EC2- 7.2.(5):	$0.8 \cdot f_{yk} = 400 \cdot \frac{N}{mm^2}$

See that the compressive stress is slightly larger than accepted stress.

Anchoring

Anchoring force, EC2 - 6.2.3(7)

$$\Delta F_{td} := 0.5 \cdot V_{Ed} \cdot \frac{1}{\tan(22grad)} = 371.827 \cdot kN$$

Factors witch depend on adhesion condition and size to the bars.

$$\eta_1 := 1$$

$$\eta_2 := 1$$

Adhesion strength, EC2 -8.4.2(2)

$$f_{bd} := 2.25 \cdot \eta_1 \cdot \eta_2 \cdot f_{ctd} = 2.805 \cdot \frac{N}{mm^2}$$

Tension in the rebar

$$\sigma_{sd} := \frac{\Delta F_{td}}{A_s} = 126.247 \cdot \frac{N}{mm^2}$$

Necessary force insertion length

$$l_{b.rqd} := \frac{Ø_s}{4} \cdot \frac{\sigma_{sd}}{f_{bd}} = 281.298 \cdot mm$$

Coefficients,
EC2 -Tabel 8.2

$$\alpha_1 := 1 \qquad \alpha_2 := 1 - 0.15 \cdot \frac{(c_{nom} - Ø_s)}{Ø_s} = 0.94$$

$$\alpha_3 := 1 \quad \alpha_4 := 0.7 \qquad \alpha_5 := 1$$

Anchorig length

$$l_{bd} := \alpha_1 \cdot \alpha_2 \cdot \alpha_3 \cdot \alpha_4 \cdot \alpha_5 \cdot l_{b.rqd} = 185.094 \cdot mm$$

Min. anchoring length

$$l_{b.min} := \max(0.3 \cdot l_{b.rqd}, 10 \cdot Ø_s, 100mm) = 250 \cdot mm$$

Designed anchorage length at the support

$$l_{bd} := 250mm$$

The beam is illustrated in Figure

6.3.2 Beam A-2

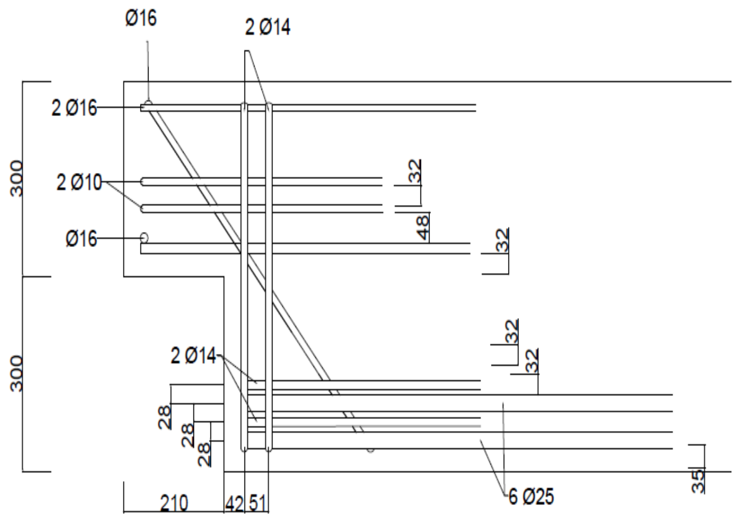

Figure 6.3: Reinforcement layout of beam A-2

Beam A-2 *-Slack-reinforced beam with dappend- ends*

Ultimate-and serviceability limit state calculation of the beam is similar as of the beam A-1. However, the dapped-end must be designed.

Support plate

Thickness $t_p := 10mm$

Width $w_p := 80mm$

Reinforcement layout in dapped end

Hanger stirrup $Ø_{sv} := 14mm$

Incline stirrup $Ø_{s\alpha} := 16mm$

Main rebar in nib $Ø_{sn} := 16mm$

Horizontal hoop $Ø_{sb} := 10mm$

Compression rebar $Ø'_{sn} := 10mm$

End anchoring hoop $Ø_{se} := 14mm$

Nib heigth $h_{nib} := \frac{h}{2} = 300 \cdot mm$

Nib length $l_{nib} := 0.7 \cdot h_{nib} = 210 \cdot mm$

Incline reinforcement

Largest aggregate size $d_g := 32mm$ $k_2 := 5mm$

Distance from center hanger stirrups to the nib edge $c_1 := c_{nom} + Ø_{sv} + \frac{d_g + k_2}{2} = 67.5 \cdot mm$

Angle of the inclined reinforcement $\alpha_{inc} := \text{atan}\left(\frac{h_{nib}}{l_{nib} + c_1}\right) = 47.231 \cdot °$

Incline reinforcement area $A_{s\alpha} := 2 \cdot Ø_{s\alpha}^2 \cdot \frac{\pi}{4} = 402.124 \cdot mm^2$

Design steel stress, Betongelementboka -Tabel C.6.5 $f_{yd'} := 380MPa$

Incline force $F_\alpha := f_{yd'} \cdot A_{s\alpha} = 152.807 \cdot kN$

Incline vertical load contribution $N_\alpha := F_\alpha \cdot \sin(\alpha_{inc}) = 112.176 \cdot kN$

Compression force $F_{c5} := \dfrac{N_\alpha}{\tan(\alpha_{inc})} = 103.762 \cdot kN$

Geometric eccentricity $e_u := 20mm$

Compression height to F_{c5} $x_{inc} := 2 \cdot \left(\dfrac{l_{nib}}{2} - e_u\right) = 170 \cdot mm$

Compressive stress in top of beam, due incline rein. $f_{c\alpha} := \dfrac{N_\alpha}{b \cdot \tan(\alpha) \cdot x_{inc} \cdot \tan(\alpha)} = 5.572 \cdot MPa$

Longitudinal main nib rebar

Load at support $N_{Ed} := V_{Ed} = 300.456 \cdot kN$

Hanger, contribution to the vertical $N_v := \max\left(N_{Ed} - N_\alpha, \dfrac{2}{3} \cdot N_{Ed}\right) = 200.304 \cdot kN$

$a_0 := \dfrac{l_{nib}}{2} - \dfrac{w_p}{2} - e_u = 45 \cdot mm$

Lever arm $a' := a_0 + c_1 = 112.5 \cdot mm$

Effective height, nib $d_{nib} := h_{nib} - c_{nom} - \dfrac{Ø_{sn}}{2} = 257 \cdot mm$

Internal lever arm in nib $z_n := d_{nib} - \dfrac{N_v \cdot a'}{\left[1.6 \cdot b \cdot d \cdot \left(0.8 \cdot f_{cd} - f_{c\alpha}\right)\right]} = 249.321 \cdot mm$

Upper limit for the internal lever arm: $z_{nib} := \min(2 \cdot a', z) = 225 \cdot mm$

Force of the tension nib rebar $F_s := \dfrac{N_v \cdot a'}{z_{nib}} = 100.152 \cdot kN$

Required main nib reinforcement $A_{sn} := \dfrac{F_s}{f_{yd'}} = 263.558 \cdot mm^2$

Choose two bars Ø16 as main nib reinforcement $A_{sn} := 2\left(\dfrac{Ø_{sn}{}^2 \cdot \pi}{4}\right) = 402.124 \cdot mm^2$

Min. reinforcement in main nib, EC2- 9.2.1.1(1) $A_{sn.min} := 0.26 \cdot \dfrac{f_{ctm} \cdot b \cdot d}{f_{yd}} = 340.908 \cdot mm^2$

Horizontal hoop reinforcement

No additional horizontal loads caused by neighbouring construction elements:
$F_n := F_s$

Required horizontal hoops in nib
$A_{sb} := 0.5 \cdot \frac{F_n}{f_{yd}} = 115.175 \cdot mm^2$

Choose two horizontal hoops Ø10
$A_{sb} := 2\left(2 \cdot \frac{Ø_{sb}^2 \cdot \pi}{4}\right) = 314.159 \cdot mm^2$

Hanger reinforcement

Force in the hanger
$F_v := N_v = 200.304 \cdot kN$

Required hanger reinforcement
$A_{sv} := \frac{N_v}{f_{yd'}} = 527.116 \cdot mm^2$

Choose two stirrups Ø16
$A_{sv} := 2\left(2 \cdot \frac{Ø_{sv}^2 \cdot \pi}{4}\right) = 615.752 \cdot mm^2$

End anchoring

Choose two stirrups Ø16 in end anchorage reinforcement
$A_{se} := A_{sv}$

Compression reinforcement

Required compression reinforcement
$A'_{sn} := 0.5 \cdot A_{sn.min} = 170.454 \cdot mm^2$

Choose two Ø10 reinforcements
$A'_{sn} := 2 \cdot \left(\frac{Ø'_{sn}{}^2 \cdot \pi}{4}\right) = 157.08 \cdot mm^2$

Compression fracture control

Compressive force capacity
$N_T := 0.25 \cdot b \cdot d_{nib} \cdot f_{cd} = 446.002 \cdot kN$

Requirement, compressive force capacity larger than hanger load
$N_v < N_T$

Anchoring

Incline reinforcement

Tension in the reinforcement
$\sigma_{sd.inc} := \frac{F_\alpha}{A_{s\alpha}} = 380 \cdot \frac{N}{mm^2}$

Required vigorously introduction length
$l_{b.rqd.A.s\alpha} := \frac{Ø_{s\alpha} \cdot \sigma_{sd.inc}}{4 \cdot f_{bd}} = 541.889 \cdot mm$

Design anchorage length $l_{bd.inc} := 0.7 \cdot l_{b.rqd.A.s\alpha} = 379.323 \cdot mm$

Min. anchorage length, EC2 - 8.4.4(1) $l_{bd.min} := \max\left(0.3 \cdot l_{b.rqd.A.s\alpha}, 10 \cdot Ø_{s\alpha}, 100mm\right) = 162.567 \cdot mm$

Longitudinal main nib rebar

Tension in the reinforcement $\sigma_{sd.sn} := \frac{F_s}{A_{sn}} = 249.058 \cdot \frac{N}{mm^2}$

Required vigorously introduction length $l_{b.rqd.A.sn} := \frac{Ø_{s\alpha} \cdot \sigma_{sd.sn}}{4 \cdot f_{bd}} = 355.162 \cdot mm$

Design anchorage length $l_{bd.sn} := 0.7 l_{b.rqd.A.sn} = 0.249 \, m$

Min. anchorage length, EC2 - 8.4.4(1) $l_{bd.min} := \max\left(0.3 \cdot l_{b.rqd.A.sn}, 10 \cdot Ø_{sn}, 100mm\right) = 160 \cdot mm$

Horizontal hoop

Tension in the reinforcement $\sigma_{sd.sb} := \frac{F_s \cdot 0.5}{A_{sb}} = 159.397 \cdot \frac{N}{mm^2}$

Required vigorously introduction length $l_{b.rqd.A.sb} := \frac{Ø_{s\alpha} \cdot \sigma_{sd.sb}}{4 \cdot f_{bd}} = 227.304 \cdot mm$

Design anchorage length $l_{bd.sb} := 0.7 l_{b.rqd.A.sb} = 159.113 \cdot mm$

Check of minimum requirement: EC2 - 8.4.4(1) $l_{bd.min} := \max\left(0.3 \cdot l_{b.rqd.A.sb}, 10 \cdot Ø_{sb}, 100mm\right) = 100 \cdot mm$

End anchoring

Tension in the reinforcement $\sigma_{sd.se} := \frac{N_v}{A_{se}} = 325.3 \cdot \frac{N}{mm^2}$

Required vigorously introduction length $l_{b.rqd.A.se} := \frac{Ø_{se} \cdot \sigma_{sd.se}}{4 \cdot f_{bd}} = 405.9 \cdot mm$

Design anchorage length $l_{bd.se} := 0.7 l_{b.rqd.A.se} = 0.284 \, m$

Check of minimum requirement: EC2 - 8.4.4(1) $l_{bd.min} := \max\left(0.3 \cdot l_{b.rqd.A.se}, 10 \cdot Ø_{se}, 100mm\right) = 140 \cdot mm$

6.3.3 Beam A-3

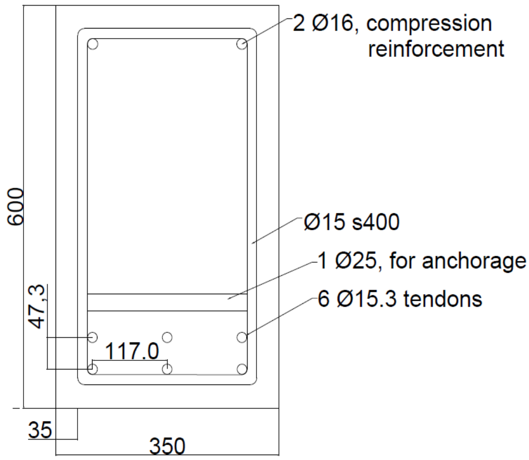

Figure 6.4: Reinforcement layout of beam A-3

Beam A-3 -*Prestressed -reinforced beam with straight -ends*

Data

15,3mm tension wire **-**Technical data, Spenncon

The tension in the tendons at 0,1 % inelastic strain	$f_{p0.1k} := 1636 \cdot MPa$
Strength of tendons	$f_{pk} := 1860MPa$
Material factor	$\gamma_s = 1.15$
Design yield strength	$f_{pd} := \frac{f_{p0.1k}}{\gamma_s} = 1.423 \times 10^3 \cdot MPa$
E-modulus of prestressing tendons	$E_p := 1.95 \cdot 10^5 \cdot MPa$
Loss of length ro tenstion strands due slippage at time of cutting	$\Delta l_{loss} := 4mm$
Maximum allowed stress allowed at time of prestressing, EC2 -5.10.2.1(1)	$\sigma_{p.0} := \min\left(0.8 \cdot f_{pk}, 0.9 \cdot f_{p0.1k}\right) = 1.472 \times 10^3 \cdot MPa$
Extended limit of allowed stress at time of prestressing, EC2 -5.10.2.1(2)	$\sigma_{p.max} := 0.95 \cdot f_{p0.1k} = 1.554 \times 10^3 \cdot MPa$
Initiel strain at time of prestressing:	$\varepsilon_{p0} := \frac{\sigma_{p.0}}{E_p} = 7.551 \times 10^{-3}$
Loss of strain due to loss of length at time of prestressing:	$\Delta\varepsilon_{loss} := \frac{\Delta l_{loss}}{l_b} = 6.667 \times 10^{-4}$
Increase in tension to substitute loss of stain:	$\Delta\sigma := \frac{\Delta\varepsilon_{loss}}{\varepsilon_{p0}} \cdot \sigma_{p.0} = 130 \cdot MPa$
Jack stress	$\sigma_{jekk} := \sigma_{p.0} + \Delta\sigma = 1.602 \times 10^3 \cdot MPa$
Not allowed	$\sigma_{jekk} > \sigma_{p.max}$
Allowed stress at time of prestressing. jack stress	$\sigma_{p.0} := \frac{0.95 \cdot \sigma_{p.0} \cdot f_{p0.1k}}{\sigma_{jekk}} = 1.428 \times 10^3 \cdot MPa$
Strain in tendons after cutting:	$\varepsilon_{p.0} := \frac{\sigma_{p.0}}{E_p} - \Delta\varepsilon_{loss} = 6.657 \times 10^{-3}$

Stress in stendons after cutting: $\sigma_{p.0} := \varepsilon_{p.0} \cdot E_p = 1.298 \times 10^3 \cdot MPa$

Assume 5% relaxtion $\sigma'_{p.0} := 0.95 \cdot \sigma_{p.0} = 1.233 \times 10^3 \cdot MPa$

Maximum allowed initiel prestressind force after cutting EC2 -5.10.3 $\sigma_{pm0} := \min(0.75 \cdot f_{pk}, 0.85 \cdot f_{p0.1k}) = 1.391 \times 10^3 \cdot MPa$

Diameter of the tendons $\text{Ø}_p := 15.3mm$

Ultimate limit state

Moment capacity

Effective height $d_{spenn} := h - c_{nom} - \text{Ø}_v - \text{Ø}_p - \frac{a_v}{2} = 518.7 \cdot mm$

Internal lever arm $z_{spenn} := 0.84 \cdot d_{spenn} = 435.708 \cdot mm$

Necessary tensile reinforcement: $A_p := \frac{M_{Ed}}{z_{spenn} \cdot f_{pd}} = 727.095 \cdot mm^2$

15,3mm tendon $A_{st} := 140mm^2$

Choose 6 tendons: $A_p := 6A_{st} = 840 \cdot mm^2$

Compression reinforcement: $\text{Ø}'_s := 16mm$

$d' := c_{nom} + \text{Ø}_v + 0.5 \cdot \text{Ø}'_s = 58 \cdot mm$

Choose 2ø16 compressice reinforcement $A'_s := 2 \cdot \frac{\text{Ø}'^2_s \cdot \pi}{4} = 402.124 \cdot mm^2$

Compressive height ratio $\alpha := \frac{A_p \cdot f_{pd} - A'_s \cdot f_{yd}}{\lambda \cdot f_{cd} \cdot b \cdot d_{spenn}} = 0.354$

Lever arm $z_{spenn} := d_{spenn} - 0.5 \cdot \lambda \cdot \alpha \cdot d_{spenn} = 445.219 \cdot mm$

Design moment capacity

$$M_{Rd} := \lambda \cdot \alpha \cdot d_{spenn} \cdot b \cdot f_{cd} \cdot z_{spenn} + f_{yd} \cdot A'_s \cdot (d_{spenn} - d') = 534.74 \cdot kN \cdot m$$

Min. reinforcement and ductility control

Balanced reinforcement cross section
$$\alpha_{pb} := \frac{\varepsilon_{cu}}{\frac{f_{pd}}{E_p} - \varepsilon_{p.0} + \varepsilon_{cu}} = 0.846$$

Critical reinforcement area in a balanced cross-section:
$$A_{pb} := 0.8 \cdot \frac{f_{cd}}{f_{pd}} \cdot \alpha_{pb} \cdot b \cdot d = 1.68 \times 10^3 \cdot mm^2$$

The beam in under-reinforced
$$A_p < A_{pb}$$

Minimum compression reinfrocement EC2 -9.2.1.1
$$A'_s := 0.26 \cdot \frac{f_{ctm}}{f_{yk}} \cdot b \cdot d_{spenn} = 302.091 \cdot mm^2$$

Compression reinforcement:
$$Ø'_s := 16mm$$

Choose 2 Ø16 rebars in the upper part of the beam:
$$A'_s := 2 \cdot \frac{Ø'_s{}^2 \cdot \pi}{4} = 402.124 \cdot mm^2$$

$$d' := c_{nom} + Ø_v + \frac{Ø'_s}{2} = 58 \cdot mm$$

Check if the compression reinforcement yield prior to concrete failure.
$$\varepsilon'_s := \frac{\alpha \cdot d_{spenn} - d'}{\alpha \cdot d_{spenn}} \cdot \varepsilon_{cu} = 2.395 \times 10^{-3}$$

Yielding in compression reinforcement
$$\varepsilon_{yd} < \varepsilon'_s$$

The designed locations of the tension tendons is in agreement with the EC2 -8.10.1.2

Controll at the moments of prestressing:
- 7 days after casting

Coefficient based on the age to the concrete
$$\beta_{cc} := e^{\left[0.25 \cdot \left[1 - \left(\frac{28}{7}\right)^{0.5}\right]\right]} = 0.779$$

Compressive strength to the concrete after seven days
$$f_{cm7} := \beta_{cc} \cdot f_{cm} = 33.488 \cdot MPa$$

$$f_{ck7} := f_{cm7} - 8MPa = 25.488 \cdot MPa$$

Dimensioning compressive strength
$$f_{cd7} := \alpha_{cc} \cdot \frac{f_{ck7}}{\gamma_c} = 14.443 \cdot MPa$$

Prestressing force: $P_0 := \sigma_{p.0} \cdot A_p = 1.09 \times 10^3 \cdot kN$

Designed prestressing force: $N_{Ed.s} := 1.1 \cdot P_0 = 1.199 \times 10^3 \cdot kN$

Lever arm, tendon $e_s := 218.7mm$

$d_s := 242mm$

Moment from the tension reinforcement: $M_{Ed.s} := N_{Ed.s} \cdot e_s = 262.321 \cdot kN \cdot m$

1) -Pure compression

Concrete compression force $T_{c1} := f_{cd7} \cdot b \cdot h = 3.033 \times 10^3 \cdot kN$

Strain in tendons is equal to bilinear strain in concrete $\varepsilon_p := \varepsilon_{c1} = 2.25 \times 10^{-3}$

Tendons, compression force $T_{p1} := \varepsilon_p \cdot E_p \cdot A_p = 368.55 \cdot kN$

Strain in comressive rebar is equal to bilinear strain in concrete $\varepsilon'_s := \varepsilon_{c1} = 2.25 \times 10^{-3}$

Force of the compressive rebar $T_{s1} := \varepsilon'_s \cdot E_s \cdot A'_s = 180.956 \cdot kN$

Total axial force $N_1 := T_{c1} + T_{p1} + T_{s1} = 3.583 \times 10^3 \cdot kN$

Bending moment $M_1 := T_{p1} \cdot e_s - T_{s1} \cdot d_s = 36.811 \cdot kN \cdot m$

2) -Balance point

Design concrete strain at failure $\varepsilon_{cu} = 3.5 \times 10^{-3}$

Design steel yielding strain $\varepsilon_{yd} = 2.174 \times 10^{-3}$

Compression height ratio $\alpha := \dfrac{\varepsilon_{cu}}{\varepsilon_{yd} + \varepsilon_{cu}} = 0.617$

Effective height, upper part in stress $d_1 := h - c_{nom} - Ø_v - \dfrac{Ø'_s}{2} = 542 \cdot mm$

Concrete compression force $T_{c2} := 0.8 \cdot \alpha \cdot d_1 \cdot b \cdot f_{cd} = 1.857 \times 10^3 \cdot kN$

Strain, tendons

$$\Delta\varepsilon_{p2} := \frac{\alpha \cdot d_1 - \left(c_{nom} + Ø_v + Ø_p + \dfrac{a_v}{2}\right)}{\alpha \cdot d_1} \cdot \varepsilon_{cu} = 2.649 \times 10^{-3}$$

Tendons compression force $T_{p2} := \Delta\varepsilon_{p2} \cdot E_p \cdot A_p = 433.892 \cdot kN$

Tensile force in the compressive rebars $S_2 := f_{yd} \cdot A'_s = 174.836 \cdot kN$

Total axial force $N_2 := T_{c2} + T_{p2} - S_2 = 2.116 \times 10^3 \cdot kN$

Bending moment

$$M_2 := T_{c2} \cdot (0.5 \cdot h - 0.4 \cdot \alpha \cdot d_1) + T_{p2} \cdot e_s + S_2 \cdot (0.5 \cdot h - d') = 445.905 \cdot kN \cdot m$$

3) Under reinforce

Strain in the compression rebar $\varepsilon_{sv} := 2 \cdot \varepsilon_{yk} = 5 \times 10^{-3}$

Compression height ratio $\alpha_3 := \dfrac{\varepsilon_{cu}}{2 \cdot \varepsilon_{yk} + \varepsilon_{cu}} = 0.412$

Concrete compression force $T_{c3} := 0.8 \cdot \alpha_3 \cdot d_1 \cdot b \cdot f_{cd} = 1.239 \times 10^3 \cdot kN$

Strain, tendon $\Delta\varepsilon_{p3} := \dfrac{\alpha_3 \cdot d_1 - 81.3mm}{\alpha_3 \cdot d_1} \varepsilon_{cu} = 2.225 \times 10^{-3}$

Tendon compression force $T_{p3} := \Delta\varepsilon_{p3} \cdot E_p \cdot A_p = 364.455 \cdot kN$

Compressive force of the rebar $S_3 := f_{yd} \cdot A'_s = 174.836 \cdot kN$

Total axial force $N_3 := T_{c3} + T_{p3} - S_3 = 1.429 \times 10^3 \cdot kN$

Bending moment

$$M_3 := T_{c3} \cdot (0.5 \cdot h - 0.4 \cdot \alpha \cdot d_1) + T_{p3} \cdot e_s + S_3 \cdot (0.5 \cdot h - d') = 328.081 \cdot kN \cdot m$$

Plot the three points in a moment-compression diagram and check it against the load situation.

Shear capacity:

Prestressing force: $F_{p0} := \sigma_{pm0} \cdot A_p = 1.168 \times 10^3 \cdot kN$

Partial safety factor $\gamma_p := 0.9$

Design compression force:
- Assume 15% loss of initial compression force

$N_{Ed} := \gamma_p \cdot 0.85 \cdot F_{p0} = 893.6 \cdot kN$

Bending moment at beam end $M_s := -N_{Ed} \cdot (0.5 \cdot h - 81.3mm) = -195.43 \cdot kN \cdot m$

Bending moment at midspan:

$M_{felt} := M_{Ed} + M_s = 255.254 \cdot kN \cdot m$

Compression, prestressing

$\sigma_{cp} := \min\left(\frac{N_{Ed}}{A_c}, 0.2 \cdot f_{cd}\right) = 3.967 \cdot \frac{N}{mm^2}$

Without reinforcement

Miscellaneous factors

$\rho_l := \frac{A_p}{b \cdot d} = 4.715 \times 10^{-3}$

$k := 1 + \sqrt{\frac{200}{\frac{d}{mm}}} = 1.627$

$k_1 := 0.15$

Shear capacity without reinforcement in cracked area
EC2 -6.2.2(1)

$$V_{Rd.c1} := \left[C_{Rd.c} \cdot k \cdot \left(100 \cdot \rho_l \cdot f_{ck} \cdot \frac{mm^2}{N}\right)^{\frac{1}{3}} \frac{N}{mm^2} + k_1 \cdot \sigma_{cp}\right] \cdot b \cdot d = 194.545 \cdot kN$$

With reinforcement

Need shear reinforcement. Same amount of reinforcement as in Beam A-1 is used, ø15s400

Shear capacity ,same as Beam A-1
EC2 -6.2.3

$V_{Rd} = 364.572 \cdot kN$

Transmission of prestressing force

Eurokode 2 [8.10.2.2]

Average value of tensile strength, after 7 days of curing EC2 [3.1.2] (9)

$f_{ctm7} := \beta_{cc} \cdot f_{ctm} = 2.492 \cdot \frac{N}{mm^2}$

$f_{ctd7} := \alpha_{ct} \cdot 0.7 \cdot \frac{f_{ctm7}}{\gamma_c} = 0.989 \cdot \frac{N}{mm^2}$

Adhesion tension at the interface between concrete and strands: EC2 [8.10.2.2] (1)

$\eta_{p1} := 3.2$ $\eta_1 := 1$

$f_{bpt} := \eta_{p1} \cdot \eta_1 \cdot f_{ctd7} = 3.163 \cdot \frac{N}{mm^2}$

Base value of the transmission length: EC2 [8.10.2.2] (2)

$$l_{pt} := 1.25 \cdot 0.19 \cdot Ø_p \cdot \frac{\sigma_{p.0}}{f_{bpt}} = 1.491 \cdot m$$

Design value of the transmission length: EC2 [8.10.2.2] (3)

$$l_{pt2} := 1.2 \cdot l_{pt} = 1.789\,m$$

Designed transmission length, EC2 [8.10.2.2] (4)

$$L_{disp} := \sqrt{l_{pt}^2 + d_{spenn}^2} = 1.579\,m$$

It is sufficient length according to requirements in ULS (EC2 [8.10.2.2](4) Note), Although, it does not satisfy the requirements related at the face when the tendons are cut. (EC2 [8.10.2.2](4).

Anchorage of strands in Ultimate Limit State

Anchorage length at support:

$$l_1 := 250mm + 0.5 \cdot 81.3mm = 290.65 \cdot mm$$

EC2 [8.10.2.3]

$$\eta_{p2} := 1.2$$

$$f_{bpd} := \eta_{p2} \cdot \eta_1 \cdot f_{ctd} = 1.496 \cdot \frac{N}{mm^2}$$

$$\sigma_{pd} := \frac{f_{p0.1k}}{\gamma_s} = 1.423 \times 10^3 \cdot \frac{N}{mm^2}$$

Neccessary anchorage length:

$$l_{bpd} := l_{pt2} + 0.19 \cdot 15.3mm \cdot \frac{\left(\sigma_{pd} - \sigma_{p.0}\right)}{f_{bpd}} = 2.031 \cdot m$$

Anchorage capacity of the 6 tendons, Betongelementboken volume C, Chapter C 9

$$F_{sp} := \frac{0.9 \cdot 6 \cdot P_0 \cdot l_1}{l_{bpd}} = 842.532 \cdot kN$$

Required area of reinforcement;

$$A_{se\_anchorage} := \frac{P_0 - F_{sp}}{f_{yd}} = 570.126 \cdot mm^2$$

Use 1 x Ø25 stirrups:

$$A_{se\_anchorage} := 2\,\frac{25^2 \cdot \pi}{4}\,mm^2 = 981.748 \cdot mm^2$$

The stirrups must have length of 1,4 m, ccording table C 8.8

Serviceability limit state

Uncracked cross- section (Stadium I)

Material stiffness ratio

$$\eta := \frac{E_p}{E_{cm}} = 5.735$$

$$\eta' := \frac{E_s}{E_{cm}} = 5.882$$

Transformed cross -section

$$A_t := A_c + (\eta - 1)\cdot A_p + (\eta' - 1)\cdot A'_s = 2.159 \times 10^5 \cdot mm^2$$

The distance for longitudinal reinforcement to concrete center of gravity

$$e_s := d_{spenn} - \frac{h}{2} = 218.7 \cdot mm$$

The distance for compressive rebar to concrete center of gravity

$$e'_s := 0.5 \cdot h - d' = 242 \cdot mm$$

Reduction distance to the center of gravity of reinforced cross -section.

$$y_t := \frac{(\eta - 1)\cdot A_p \cdot e_s - (\eta' - 1)\cdot A'_s \cdot e'_s}{A_t} = 1.828 \cdot mm$$

Second moment of area, contribution from concrete

$$I_{c1} := \frac{b \cdot h^3}{12} + h \cdot b \cdot y_t^2 = 6.301 \times 10^9 \cdot mm^4$$

Second moment of area, contribution from tendons

$$I_{p1} := A_p \cdot (e_s - y_t)^2 = 3.951 \times 10^7 \cdot mm^4$$

Second moment of area, contribution from steel bars

$$I_{s2} := A'_s \cdot (e'_s - y_t)^2 = 2.32 \times 10^7 \cdot mm^4$$

Cracking moment

$$M_{cr} := \frac{I_{c1} + \eta \cdot I_{s1} + \eta' \cdot I_{p1}}{0.5 \cdot h - y_t} \cdot f_{ctm} = 77.05 \cdot kN \cdot m$$

Cracking load

$$q_{cr} := \frac{M_{cr} \cdot 8}{l_b^2} = 17.122 \cdot \frac{kN}{m}$$

Beam stiffness, stadium I

$$EI_I := E_{cm} \cdot I_{c1} + E_p \cdot I_{p1} + E_s \cdot I_{s1} = 2.445 \times 10^5 \cdot kN \cdot m^2$$

$$M_{spenn} := -P_0 \cdot e_s = -238.473 \cdot kN \cdot m$$

Deflection, stadium I

$$\delta_I := \frac{5}{384} \cdot \frac{q_k \cdot l_b^4}{EI_I} + \frac{M_{spenn} \cdot l_b^2}{EI_I} \cdot \frac{1}{8} = 0.854 \cdot mm$$

Cracked cross- section, (Stadium II)

Dead load will be applied after 28 days, while live load will be applied after 50 days. 40% of the live load considered as permanent load. The beam is reside in indoor condition with relativ humidity = 50%

Calculation of the average E-modulus

Pre stress force is applied after 7 days. $t_{0.7} := 7$

Moment due load 1 $M_1 := \frac{g_k \cdot l_b^2}{8} = 206.82 \cdot kN \cdot m$

Moment due load 2 $M_2 := \frac{p_{lang} \cdot l_b^2}{8} = 54 \cdot kN \cdot m$

Moment due prestressing $M_{spenn} = -238.473 \cdot kN \cdot m$

$$\beta_{t.0.7} := \frac{1}{0.1 + t_{0.7}^{0.2}} = 0.635$$

$$\varphi_{0.7} := \varphi_{RH} \cdot \beta_{t.0.7} = 1.045$$

E -modulus caused by prestressing $E_{c.spenn} := \frac{E_{cm}}{1 + \varphi_{0.7}} = 1.663 \times 10^4 \cdot \frac{N}{mm^2}$

E -modulus caused by dead load and live load, respectively Similar to Bam A-1 and A-2

$$E_{c.1} = 1.111 \times 10^4 \cdot \frac{N}{mm^2}$$

$$E_{c.2} = 1.195 \times 10^4 \cdot \frac{N}{mm^2}$$

Mean E-modulus value

$$E_{c.mean} := \frac{M_1 + M_2 + |M_{spenn}|}{\frac{M_1}{E_{c.1}} + \frac{M_2}{E_{c.2}} + \frac{|M_{spenn}|}{E_{c.spenn}}} = 1.332 \times 10^4 \cdot MPa$$

Stiffness

Reinforcement ratio, strands $\rho := \frac{A_p}{b \cdot d_{spenn}} = 4.627 \times 10^{-3}$

Stiffness ratio, tendons compared to concrete

$$\eta_p := \frac{E_p}{E_{c.mean}} = 14.638$$

Reinforcement ratio, compressive reinforcement

$$\rho' := \frac{A'_s}{b \cdot d_{spenn}} = 2.215 \times 10^{-3}$$

Stiffness ratio, reinforcement compared to concrete

$$\eta_s := \frac{E_s}{E_{c.mean}} = 15.013$$

$$d' = 58 \cdot mm$$

$$e_s := d_{spenn} - \frac{h}{2} = 218.7 \cdot mm$$

Virtual lever arm

$$a_{virt} := \frac{M_k}{\sigma_{p.0} \cdot A_p} = 313.478 \cdot mm$$

Compression height ratio

$$\alpha_{II} := 0.532$$

$$\sigma_{ca} := \frac{\sigma_{p.0} \cdot A_p}{b \cdot d_{spenn}} \cdot \frac{1}{0.5 \cdot \alpha_{II} + \frac{\left(d_{spenn} - d'\right)}{\alpha_{II} \cdot d_{spenn}} \cdot \eta_s \cdot \rho' - \eta_p \cdot \rho \cdot \frac{1 - \alpha_{II}}{\alpha_{II}}} = 22.93 \cdot MPa$$

$$\sigma_{cM} := \frac{\sigma_{p.0} \cdot A_p}{b \cdot d_{spenn}} \cdot \frac{e_s + a_{virt}}{0.5 \cdot \left(1 - \frac{\alpha_{II}}{3}\right) \cdot \alpha_{II} \cdot d_{spenn} + \rho' \cdot \eta_s \cdot \frac{d_{spenn} - d'}{\alpha_{II} \cdot d_{spenn}} \cdot \left(d_{spenn} - d'\right)} = 22.982 \cdot MPa$$

Second moment of area, contribtution from concrete

$$I_{c2} := \frac{b \cdot \left(\alpha_{II} \cdot d_{spenn}\right)^3}{12} + b \cdot \alpha_{II} \cdot d_{spenn} \cdot \left(\frac{\alpha_{II} \cdot d_{spenn}}{2}\right)^2 = 2.451 \times 10^9 \cdot mm^4$$

Second moment of area, contribution from tendons

$$I_{p2} := A_p \cdot \left[\left(1 - \alpha_{II}\right) d_{spenn}\right]^2 = 4.95 \times 10^7 \cdot mm^4$$

Second moment of area, contribution from steel bars

$$I_{s2} := A'_s \cdot \left(\alpha_{II} \, d_{spenn} - d'\right)^2 = 1.91 \times 10^7 \cdot mm^4$$

Beam stiffness, stadium II

$$EI_{II} := E_{cm} \cdot I_{c2} + E_s \cdot I_{s2} + E_p \cdot I_{p2} = 9.682 \times 10^4 \cdot kN \cdot m^2$$

Deflection, stadium II

$$\delta_{II} := \frac{5.}{384} \cdot \frac{q_k \cdot l_b^4}{EI_{II}} + \frac{M_{spenn} \cdot l_b^2}{8 \cdot EI_{II}} = 2.155 \cdot mm$$

Moment at SLS

$$M_k := \frac{q_k \cdot l_b^2}{8} = 341.82 \cdot kN \cdot m$$

Long load duration

$$\beta := 0.5$$

Distrubution koeffisient

$$\zeta := 1 - \beta \cdot \left(\frac{M_{cr}}{M_k}\right)^2 = 0.975$$

Deflection, stadium I and II, EC2 - 7.4.3

$$\delta := \zeta \cdot \delta_{II} + (1 - \zeta) \cdot \delta_I = 2.122 \cdot mm$$

Crack Controll

Cover factor

$$k_c := 1.3$$

Max. accepted crack width Exposure class XC1,

$$w_{max} := 0.3 \cdot k_c\, mm = 0.39 \cdot mm$$

Tension in reinforcement, stadium II

$$\Delta\sigma_s := E_p \cdot \frac{M_k \cdot (1 - \alpha_{II}) \cdot d_{spenn}}{EI_{II}} = 167.114 \cdot MPa$$

Will satisfied requrements according to Tabel 7.2N and 7.3N, where greatest rebar dimension and rebar spacing is restrict to resulting rebar stress and limited crack width

Crack widt calculation

Factor based on duration to the load

$$k_t := 0.4$$

Concrete mean tensile strength value at time by cracking

$$f_{ct.eff} := f_{ctm}$$

Distance from neutral axis to pressure edge

$$x := \frac{A_c \cdot \frac{h}{2} + \eta \cdot A_p \cdot d_{spenn} + \eta' \cdot A'_s \cdot d'}{A_c + \eta \cdot A_s + \eta' \cdot A'_s} = 286.299 \cdot mm$$

Effective tensile height

$$h_{cef} := \min\left[2.5 \cdot (h - d_{spenn}) + 1.5 \cdot Ø_s, \frac{h}{2}, \frac{(h - x)}{3}\right] = 104.567 \cdot mm$$

Effective tensile area

$$A_{c.eff} := h_{cef} \cdot b = 3.66 \times 10^4 \cdot mm^2$$

$$\rho_{p.eff} := \frac{A_p}{A_{c.eff}} = 0.023$$

Modulus of elasticity ratio

$$\alpha_e := \frac{E_p}{E_{cm}} = 5.735$$

Mean strain in rebar drawn from mean strain in concrete between cracks

$$\Delta := \frac{\Delta\sigma_s - k_t \cdot \frac{f_{ct.eff}}{\rho_{p.eff}} \cdot \left(1 + \alpha_e \cdot \rho_{p.eff}\right)}{E_p} = 5.334 \times 10^{-4}$$

Miscellaneous factors -NA.7.3.4

$$k_3 := 3.4 \qquad k_4 := 0.425 \qquad k_1 := 0.8 \qquad k_2 := 0.5$$

Cover for longitudinal reabar

$$c_3 := c_{nom}$$

Upper value for crack width

$$S_{r.max} := k_3 \cdot c_3 + k_1 \cdot k_2 \cdot k_4 \cdot \frac{Ø_p}{\rho_{p.eff}} = 232.325 \cdot mm$$

Determined crack width

$$w_k := S_{r.max} \cdot \Delta = 0.124 \cdot mm$$

Shrinkages impact on beam deflection, EC2

With tendons in bottom and comressive rebars in top, the deflection caused by shrinkage can be neglected.

6.3.4 Beam B-1

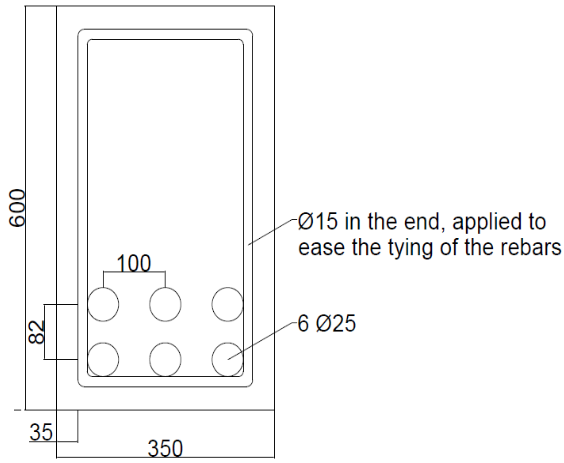

Figure 6.5: Reinforcement layout of beam B-1

Beam B-1 - *1 vol-% fibre, slack reinforced straight-ended beam.*

Ultimate limit state

Moment capacity:

Design

For compressive strengths larger than 50 MPa:	$\lambda_1 := 0.8 = 0.8$
	$\eta_1 := 1.0 = 1$
Normal reinforced Table 4.3, Sørensen	$\alpha := 0.4$
	$K := \lambda_1 \cdot \eta_1 \cdot (1 - 0.5 \cdot \lambda_1 \alpha)\alpha = 0.269$
Effective height	$d_f := \sqrt{\frac{M_k}{K \cdot f_{cd} \cdot b}} = 428.008 \cdot mm$
Internal lever arm	$z_f := (1 - 0.5\lambda_1 \cdot \alpha) \cdot d_f = 359.527 \cdot mm$
Required tensile reinforcement	$A_s := \frac{M_k}{f_{yd} \cdot z_f} = 2.187 \times 10^3 \cdot mm^2$
Use 6ø25	$A_s := 6 \cdot \frac{Ø_s^2 \cdot \pi}{4} = 2.945 \times 10^3 \cdot mm^2$
Effective height	$d := h - c_{nom} - 15mm - Ø_s - \frac{a_v}{2} = 509 \cdot mm$

Min. reinforcement and ductility control:

Check of minimum reinforcement [EC2 NA.9.2.1.1.(1)]	$A_{s.min} := 0.26 \cdot \frac{f_{ctm}}{f_{yk}} \cdot b \cdot d = 296.442 \cdot mm^2$
Balanced reinforcement cross section	$\alpha_b := \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{yd}} = 0.617$
Amount of reinforcement at ballanced cross-section	$A_{s.b} := \lambda_1 \cdot \frac{f_{cd}}{f_{yd}} \cdot b \cdot d \cdot \alpha_b = 4.01 \times 10^3 \cdot mm^2$
Check reinforcement strain	$\varepsilon_s := \frac{1 - \alpha}{\alpha} \cdot \varepsilon_{cu} = 5.25 \times 10^{-3}$
	$\varepsilon_s < \varepsilon_{ud}$

Moment capacity- *according to Norwegian Concrete Association (2015)*

Characteristic compressive strength $f_{ck.fibre} := \alpha_{cc} \cdot f_{ck} = 29.75 \cdot MPa$

Compressive height ratio, based on characteristic value.

$$\alpha_k := \frac{f_{yk} \cdot A_s}{\lambda_1 \cdot f_{ck} \cdot b \cdot d} = 0.295$$

$$z := \left(1 - 0.5\lambda_1 \cdot \alpha_k\right) \cdot d = 448.893 \cdot mm$$

Characteristic moment capacity requirement [4.2.2]

$$M_{Rd.k} := \lambda_1 \cdot \alpha_k \cdot z \cdot f_{ck.fibre} \cdot b \cdot d = 561.892 \cdot kN \cdot m$$

Compression height

$$x := \frac{A_s \cdot f_{yd} + h \cdot b \cdot f_{ftd.res2.5}}{\lambda_1 \cdot \eta_1 \cdot f_{cd} \cdot b + f_{ftd.res2.5} \cdot b} = 269.302 \cdot mm$$

$$\alpha := \frac{A_s \cdot f_{yd} + h \cdot b \cdot f_{ftd.res2.5}}{\left(\lambda_1 \cdot \eta_1 \cdot f_{cd} \cdot b + f_{ftd.res2.5} \cdot b\right) \cdot d} = 0.529$$

Momement capacity, included fibre contribution

$$M_{Rd} := (h - x) \cdot b \cdot f_{ftd.res2.5} \cdot (0.5h + 0.1x) + A_s \cdot f_{yd}(h - 0.4x) = 700.668 \cdot kN \cdot m$$

Shear Strength

Without shear reinforcement -*according to Norwegian Concrete Association (2015)*

Data
EC2 -6.2

$$\rho_l := \frac{A_s}{b \cdot d} = 0.017$$

$$k := 1 + \sqrt{\frac{200mm}{d}} = 1.627$$

$$C_{Rd.c} := \frac{0.18}{1.5} = 0.12$$

$$v_{min} := 0.035 \cdot k^{\frac{2}{3}} \cdot \left(f_{ck} \cdot \frac{mm^2}{N}\right)^{0.5} = 0.286$$

Shear capacity of the concrete

$$V_{Rd.ct} := \left[C_{Rd.c} \cdot k \cdot \left(100\rho_l \cdot f_{ck} \cdot \frac{mm^2}{N}\right)^{\frac{1}{3}}\right] \cdot b \cdot d \cdot \frac{N}{mm^2} = 134.518 \cdot kN$$

Shear capacity, contribution from the fibre: $V_{Rd.cf} := 0.6 \cdot f_{ftd.res2.5} \cdot b \cdot h = 234.032 \cdot kN$

Shear capacity of the beam: $V_{Rd.c} := V_{Rd.ct} + V_{Rd.cf} = 368.55 \cdot kN$

Anchoring

Internal lever arm $z := (1 - 0.5\lambda_1 \cdot \alpha) \cdot d = 401.279 \cdot mm$

$$\Delta F_{td} := \frac{V_{Ed}}{\tan(22grad)} - 0.5 \cdot \frac{f_{ftd.res2.5} \cdot b \cdot z}{\sin(45 \cdot grad)} \cdot \left(\frac{\cos(45 \cdot grad)}{\tan(45 \cdot grad)} + \sin(45 \cdot grad) \right) = 482.787 \cdot kN$$

Factors which depend on the adhesion condition and size of the rebars. $\eta_1 := 1$

$\eta_2 := 1$

Adhesion strength, EC2 -8.4.2(2) $f_{bd} := 2.25 \cdot \eta_1 \cdot \eta_2 \cdot f_{ctd} = 2.805 \cdot \frac{N}{mm^2}$

Tension in the rebar $\sigma_{sd} := \frac{\Delta F_{td}}{A_s} = 163.921 \cdot \frac{N}{mm^2}$

Base value of the transmission length $l_{b.rqd} := \frac{\text{Ø}_s}{4} \cdot \frac{\sigma_{sd}}{f_{bd}} = 365.243 \cdot mm$

Coefficients, EC2 -Tabel 8.2 $\alpha_1 := 1$ $\alpha_2 := 1 - 0.15 \cdot \frac{(c_{nom} - \text{Ø}_s)}{\text{Ø}_s} = 0.94$

$\alpha_3 := 1$ $\alpha_4 := 0.7$ $\alpha_5 := 1$

Anchorig length $l_{bd} := \alpha_1 \cdot \alpha_2 \cdot \alpha_3 \cdot \alpha_4 \cdot \alpha_5 \cdot l_{b.rqd} = 240.33 \cdot mm$

EC2 [8.4.4](8.5) $l_{b.min} := \max(0.3 \cdot l_{b.rqd}, 10 \cdot \text{Ø}_s, 100mm) = 250 \cdot mm$

Designed anchorage length: $l_{bd} := \max(l_{b.min}, l_{bd}) = 250 \cdot mm$

Service Limit State

Minimum reinforcement - *according COIN (2011)*

Minimum flexural reinforcement

$$A_{smin} := \max\left[0.26 \cdot \frac{f_{ctm} - 2.1 f_{ftk.res2.5}}{f_{yk}} \cdot b \cdot d, 0.0013 \cdot \left(1 - \frac{f_{ftk.res2.5}}{f_{ctm}}\right) \cdot b \cdot d\right] = 29.955 \cdot mm^2$$

Minimum shear reinforcement

$$\rho_{w.min} := \left(\frac{0.1 \cdot \sqrt{f_{ck} \cdot \frac{mm^2}{N}} \cdot \frac{N}{mm^2} - 0.3 \cdot f_{ftk.res2.5}}{f_{yk}}\right) = -4.884 \times 10^{-4}$$

No minimum requirement

Minimum reinforcement - *according to Norwegian Concrete Association (2015)*

Minimum flexural reinforcement

$$f_{Ftsm} := \frac{f_{Ftsk}}{0.7} = 3.928 \cdot \frac{N}{mm^2}$$

$$A_{smin} := \max\left[0.26 \cdot \frac{f_{ctm} - f_{Ftsm}}{f_{yk}} \cdot b \cdot d, 0.0013 \cdot \left(1 - \frac{f_{Ftsm}}{f_{ctm}}\right) \cdot b \cdot d\right] = -5.268 \times 10^{-5} m^2$$

No minimum requirement

Minimum shear reinforcement

$$\rho_{w.min} := \left(\frac{0.1 \cdot \sqrt{f_{ck} \cdot \frac{mm^2}{N}} \cdot \frac{N}{mm^2} - 0.2 \cdot f_{Ftsm}}{f_{yk}}\right) = -3.879 \times 10^{-4}$$

No minimum requirement

Crack widt calculation *-according to COIN (2011)*

Factor based on the duration of the load $k_t := 0.4$

Concrete mean tensile strength value at time of cracking $f_{ct.eff} := f_{ctm}$

The multi-layer procedure was performed in aim to find the tensile stress in the reinforcement, when the beam was subjected to service load. The model had to be run two times, to get the curvature and strains of the examined fibre reinforced beam. The strain value was found by interpolation between the two iteration steps.

Bending moment at SLS $M_k = 341.82 \cdot kN \cdot m$

Calculated bending moment and strain value at iteration step 1 and 2.

$M_{SLS1} := 237.59kN \cdot m$

$M_{SLS2} := 369.93kN \cdot m$

$\varepsilon_{cc1} := -0.0003476$

$\varepsilon_{cc2} := -0.0005931$

$\varepsilon_{cs1} := 0.0005001$

$\varepsilon_{cs2} := 0.0010001$

Meassured strain values, of interest in tension and compression

$$\varepsilon_{cc} := \varepsilon_{cc1} + \frac{\varepsilon_{cc2} - \varepsilon_{cc1}}{M_{SLS2} - M_{SLS1}} \cdot \left(M_k - M_{SLS1}\right) = -5.41 \times 10^{-4}$$

$$\varepsilon_{cs} := \varepsilon_{cs1} + \frac{\varepsilon_{cs2} - \varepsilon_{cs1}}{M_{SLS2} - M_{SLS1}} \cdot \left(M_k - M_{SLS1}\right) = 8.939 \times 10^{-4}$$

Compression height ratio $\alpha_{SLS} := \frac{h}{\left(\left|\varepsilon_{cc}\right| + \varepsilon_{cs}\right) \cdot d} \cdot \left|\varepsilon_{cc}\right| = 0.444$

Strain in rebars $\varepsilon_s := \frac{\left|\varepsilon_{cc}\right|}{\alpha_{SLS} \cdot d} \cdot \left(1 - \alpha_{SLS}\right) \cdot d = 6.763 \times 10^{-4}$

Stress in rebars $\sigma_s := E_s \cdot \varepsilon_s = 135.255 \cdot MPa$

Calculated stress in the reinforcement, based on the meassured strains from the multi-layer procedure

Compression height $x := \alpha_{SLS} \cdot d = 226.206 \cdot mm$

Effective tensile height $h_{cef} := \min\left[2.5 \cdot (h - d) + 1.5 \cdot Ø_s, \frac{h}{2}, \frac{(h - x)}{3}\right] = 124.598 \cdot mm$

Effective tensile area $A_{c.eff} := h_{cef} \cdot b = 4.361 \times 10^4 \cdot mm^2$

$$\rho_{p.eff} := \frac{A_s}{A_{c.eff}} = 0.068$$

Modulus of elasticity ratio $\alpha_e := \frac{E_s}{E_{cm}} = 5.882$

The strain in the rebsrs minus the strain in the concrete

$$\Delta := \frac{\sigma_s - k_t \cdot \frac{f_{ct.eff}}{\rho_{p.eff}} \cdot \left(1 + \alpha_e \cdot \rho_{p.eff}\right)}{E_s} = 5.439 \times 10^{-4}$$

Miscellaneous factors -NA.7.3.4 $k_3 := 3.4$ $k_4 := 0.425$ $k_1 := 0.8$ $k_2 := 0.5$

Cover of the longitudinal reabar $c_3 := c_{nom}$

$$k_5 := \left(1 - \frac{f_{ftk.res2.5}}{f_{ctm}}\right) = 0.129$$

Upper value of the crack spacing $S_{r.max} := k_3 \cdot c_3 + k_1 \cdot k_2 \cdot k_4 \cdot k_5 \cdot \frac{Ø_s}{\rho_{p.eff}} = 127.139 \cdot mm$

Calculated crack width $w_k := S_{r.max} \cdot \Delta = 0.069 \cdot mm$

Crack widths *-according to Norwegian Concrete Association (2015)*

Four different stages shall be considered. Crack formation stage at short and long term, and stabilized cracking stage at both short and long term

Constants:

Height of the concrete cover	$c_{nom} = 35 \cdot mm$
Diameter of the rebar	$Ø_s = 25 \cdot mm$
Cross -sectional area of the concrete at tension, EC2- figure 7.1.	$\rho_{s.ef} := \dfrac{A_s}{\min\left[2.5(h-d) + 1.5 \cdot Ø_s, \dfrac{(h-x)}{3}, \dfrac{h}{2}\right] \cdot b} = 0.068$
Averrage tensilte strength, of the cocnrete	$f_{ctm} = 3.2 \cdot MPa$
Contribution from the fibers	$f_{Ftsm} = 3.928 \cdot MPa$
Modulus of elasticity of the reinforcement	$E_s = 2 \times 10^5 \cdot MPa$
Strain, caused by shrinkage	$\varepsilon_{sh} := \varepsilon_{cs}$
Tension in the rebars	$\sigma_s = 135.255 \cdot MPa$

$$\sigma_{sr} := \frac{(f_{ctm} - f_{Ftsm})}{\rho_{s.ef}} \cdot \left(1 + \frac{E_s}{E_{cm}} \cdot \rho_{s.ef}\right) = -15.059 \cdot MPa$$

Crack fomation stage

Short term, instantaneous loading

$\tau_{bm} := 1.8 \cdot f_{ctm} = 5.76 \cdot MPa$ Table 7.6-2

$\beta := 0.6$

$\eta_r := 0$

$$w_d := 2 \cdot \left[k \cdot c_{nom} + \frac{1}{4} \cdot \frac{Ø_s}{\rho_{s.ef}} \cdot \frac{(f_{ctm} - f_{Ftsm})}{\tau_{bm}}\right] \cdot \frac{1}{E_s} \cdot (\sigma_s - \beta \cdot \sigma_{sr} + \eta_r \cdot \varepsilon_{sh} \cdot E_s) = 0.065 \cdot mm$$

Long term, repeated loading

$\tau_{bm} := 1.35 \cdot f_{ctm} = 4.32 \cdot MPa$ Table 7.6-2

$\beta := 0.6$

$\eta_r := 0$

$$w_d := 2 \cdot \left[k \cdot c_{nom} + \frac{1}{4} \cdot \frac{Ø_s}{\rho_{s.ef}} \cdot \frac{(f_{ctm} - f_{Ftsm})}{\tau_{bm}}\right] \cdot \frac{1}{E_s} \cdot (\sigma_s - \beta \cdot \sigma_{sr} + \eta_r \cdot \varepsilon_{sh} \cdot E_s) = 0.06 \cdot mm$$

Stabilized cracking stage

Short term, instantaneous loading — Same crack width as in the Crack formation stage

Long term, repeated loading

$\tau_{bm} := 1.8 \cdot f_{ctm} = 5.76 \cdot MPa$ — Table 7.6-2

$\beta := 0.4$

$\eta_r := 1$

$$w_d := 2 \cdot \left[k \cdot c_{nom} + \frac{1}{4} \cdot \frac{\varnothing_s}{\rho_{s.ef}} \cdot \frac{\left(f_{ctm} - f_{Ftsm}\right)}{\tau_{bm}} \right] \cdot \frac{1}{E_s} \cdot \left(\sigma_s - \beta \cdot \sigma_{sr} + \eta_r \cdot \varepsilon_{sh} \cdot E_s\right) = 0.145 \cdot mm$$

Uncracked cross- section (Stadium I)

Material stiffness ratio

$$\eta := \frac{E_s}{E_{cm}} = 5.882$$

Transformed cross -section

$$A_t := A_c + (\eta - 1) \cdot A_s = 2.244 \times 10^5 \cdot mm^2$$

The distance from the longitudinal reinforcement to concrete center of gravity

$$e_s := d - \frac{h}{2} = 209 \cdot mm$$

Reduction distance to the center of gravity of reinforced cross -section.

$$y_t := \frac{(\eta - 1) \cdot A_s \cdot e_s}{A_t} = 13.394 \cdot mm$$

Second moment of area, contribution from concrete

$$I_{c1} := \frac{b \cdot h^3}{12} + h \cdot b \cdot y_t^2 = 6.338 \times 10^9 \cdot mm^4$$

Second moment of area, contribution from steel bars

$$I_{s1} := A_s \cdot \left(e_s - y_t\right)^2 = 1.127 \times 10^8 \cdot mm^4$$

Cracking moment

$$M_{cr} := \frac{I_{c1} + \eta \cdot I_{s1}}{0.5 \cdot h - y_t} \cdot f_{ctm} = 78.162 \cdot kN \cdot m$$

Cracking load

$$q_{cr} := \frac{M_{cr} \cdot 8}{l_b^2} = 17.369 \cdot \frac{kN}{m}$$

Beam stiffness, stadium I

$$EI_I := E_{cm} \cdot I_{c1} + E_s \cdot I_{s1} = 2.38 \times 10^5 \cdot kN \cdot m^2$$

Deflection, stadium I

$$\delta_I := \frac{5}{384} \cdot \frac{q_k \cdot l_b^4}{EI_I} = 5.385 \cdot mm$$

Cracked corss- section, (Stadium II)

Reinforcement ratio $\rho := \frac{A_s}{b \cdot d} = 0.017$

Compression height ratio $\alpha_{SLS} = 0.444$

Computed bending moment with the given curvature in itersion 1 and 2, respectively.

$M_{SLS1} = 237.59 \cdot kN \cdot m$

$M_{SLS2} = 369.93 \cdot kN \cdot m$

$\kappa_1 := 1.41 \cdot 10^{-6} \cdot \frac{1}{mm}$

$\kappa_2 := 2.66 \cdot 10^{-6} \cdot \frac{1}{mm}$

Resultinfg bending moment in SLS $M_k = 3.418 \times 10^5 \, J$

$$\kappa_{SLS} := \kappa_1 + \frac{\kappa_2 - \kappa_1}{(M_{SLS2} - M_{SLS1})} \cdot (M_k - M_{SLS1}) = 2.394 \times 10^{-3} \frac{1}{m}$$

$$EI_{II} := \frac{M_k}{\kappa_{SLS}} = 1.428 \times 10^{14} \cdot N \cdot mm^2$$

Deflection, stadium II $\delta_{II} := \frac{5.}{384} \cdot \frac{q_k \cdot l_b^4}{EI_{II}} = 8.979 \cdot mm$

Moment at SLS $M_k := \frac{q_k \cdot l_b^2}{8} = 341.82 \cdot kN \cdot m$

Load, long duration $\beta := 0.5$

Distribution koeffisient $\zeta := 1 - \beta \cdot \left(\frac{M_{cr}}{M_k}\right)^2 = 0.974$

Deflection, stadium I and II, EC2 - 7.4.3 $\delta := \zeta \cdot \delta_{II} + (1 - \zeta) \cdot \delta_I = 8.885 \cdot mm$

Stress limitation

Maximum compressive stress in concrete, based on stadium II behaviour

$$\sigma_c := \frac{2\cdot\left[M_k + f_{ftk.res2.5}\cdot b\cdot\left(h - \alpha_{SLS}\cdot d\right)\cdot\left[0.5\left(h - \alpha_{SLS}\cdot d\right) - (h - d)\right]\right]}{\alpha_{SLS}\cdot b\cdot d^2\cdot\left(1 - \frac{\alpha_{SLS}}{3}\right)} = 21.951\cdot MPa$$

Max. compressive stress, EC2 -7.2.(2) $0.6\cdot f_{ck} = 21\cdot\frac{N}{mm^2}$

Tensile stress in the rebar $\sigma_s = 135.255\cdot MPa$

Max. tensile stress in rebar, EC2- 7.2.(5): $0.8\cdot f_{yk} = 400\cdot\frac{N}{mm^2}$

6.3.5 Beam B-2

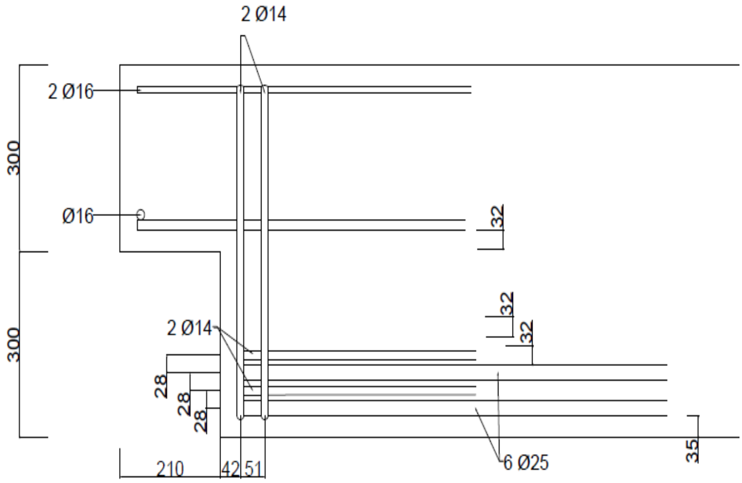

Figure 6.6: Reinforcement layout of beam B-2

Beam B-2 *- 1 vol-% fibre, slack-reinforced beam with dapped -ends*

Capacity calculation in dapped-end Model -1

Inclined contribution

Largest aggregate size $d_g := 32mm \qquad k_2 := 5mm$

Distance from center hanger stirrups to the nib edge $c_1 := c_{nom} + Ø_{sv} + \frac{d_g + k_2}{2} = 67.5 \cdot mm$

Virtual inclination angle, used to calculate the strength contribution offered by the fibres $\alpha := 45°$

Effektiv fibre length $l_{eff.fibre} := \frac{h_{nib}}{2} = 150 \cdot mm$

Contribution offered by the fibres $F_{f\alpha} := f_{ftd.res2.5} \cdot b \cdot l_{eff.fibre} = 97.514 \cdot kN$

Support force $N_{f\alpha} := F_{f\alpha} \cdot \sin(\alpha) = 68.952 \cdot kN$

It is possible to exlude the Ø16 incline reinforcement if the contribution from fibres is accounted for . Although, 2 vertical Ø14 hanger reinforcment, as used in example for B35 without fibre concrete, will then be fully utilized.

Compressive force in top of the beam, caused by the fibre contribution

Compression force in the top of the beam $F_{c5} := \frac{N_{f\alpha}}{\tan(\alpha)} = 68.952 \cdot kN$

Geometric eccentricity of the load, EC2 -6,1(4) $e_u := 20mm$

The height of the compression zone, "betongelementboka, bind C" $x := 2 \cdot \left(\frac{l_{nib}}{2} - e_u \right) = 170 \cdot mm$

Stress in the concrete . $f_{c\alpha} := \frac{N_{f\alpha}}{b \cdot x \cdot \tan(\alpha)^2} = 1.159 \cdot \frac{N}{mm^2}$

Hanger stirrups contribution

Effective height in the nib $d_{nib} := 257mm$

$$a_0 := \frac{l_{nib}}{2} - \frac{w_p}{2} - e_u = 45 \cdot mm$$

Lever arm $a' := a_0 + c_1 = 112.5 \cdot mm$

Force carreid by the hanger stirrups $N_v := V_{Ed} - N_{f\alpha} = 231.504 \cdot kN$

Internail lever arm in the nib $$z_{nib} := d_{nib} - \frac{N_v \cdot a'}{1.6 \cdot b \cdot d_{nib} \cdot \left(0.8 \cdot f_{cd} - f_{c\alpha}\right)} = 244.696 \cdot mm$$

Full utilization of the compression zone $z_{nib} := 2 \cdot a' = 225 \cdot mm$

$$F_s := \frac{N_v \cdot a'}{z_{nib}} = 115.752 \cdot kN$$

Compressive force in top of the beam, to regard for the hanger stirrups

Compression zone height $x_h := 2\left(d_{nib} - z_{nib}\right) = 64 \cdot mm$

Stress in compression zone due hanger reinforcement $F_{c2} := F_s = 115.752 \cdot kN$

Stress in the compression zone due hanger force $$f_{cv} := \frac{F_{c2}}{x_h \cdot b} = 5.167 \cdot MPa$$

Total stress in the compressive zone $f_c := f_{cv} + f_{c\alpha} = 6.326 \cdot MPa$

Design compressive strength of the concrete $f_{cd} = 19.833 \cdot MPa$

Capacity calculation in dapped-end Model-2

Fibre contribution $F_{fv} := l_{eff.fibre} \cdot b \cdot f_{ftd.res2.5} = 97.514 \cdot kN$

Force into the support $N_{fv} := F_{fv} = 97.514 \cdot kN$

The fibres offer a significant increase in the load bearing capacity. It is possible to exclude inclined reinforcemnet. Allthough, two Ø14 is still necessary.

Compression force in top of the beam

Internal lever arm in the nib $z_{nib} := d_{nib} - \dfrac{V_{Ed} \cdot a'}{1.6 \cdot b \cdot d_{nib} \cdot \left(0.8 \cdot f_{cd} - f_{c\alpha}\right)} = 241.031 \cdot mm$

Full utilization of compression zone $z_{nib} := 2 \cdot a' = 225 \cdot mm$

Force in the longitudinal nib rebars $F_s := \dfrac{V_{Ed} \cdot a'}{z_{nib}} = 150.228 \cdot kN$

Compressive zone height $x_h := 2 \cdot \left(d_{nib} - z_{nib}\right) = 64 \cdot mm$

Compressice force, top of the beam $F_{c2} := F_s$

$$f_{cw} := \frac{F_{c2}}{x_h \cdot b} = 6.707 \cdot MPa$$

Splitting tensile forces caused by consentrated loads

Incoming compressive zone, Figure 5.9 $a_1 := w_p = 80 \cdot mm$

Outcoming compressive zone $a_2 := l_{nib} = 210 \cdot mm$

Resulting splitting tensile forces, equation 5.28 $Z_{s1} := 0.25 \cdot V_{Ed} \cdot \left(1 - \dfrac{a_1}{a_2}\right) = 46.499 \cdot kN$

$$d' := c_{nom} + Ø_{sv} + 0.5 \cdot Ø'_{sn} = 54 \cdot mm$$

Effective height $d_t := d_{nib} - d' = 203 \cdot mm$

Effective width $b_t := b - 2 \cdot c_{nom} = 280 \cdot mm$

Splitting tensile force capacity, equation 5.30 $SP_{std} := f_{ftd.res2.5} \cdot b_t \cdot d_t = 105.575 \cdot kN$

Requirement according to equation 4.19 $SP_{std} > 0.5 \cdot F_s$

6.3.6 Beam B-3

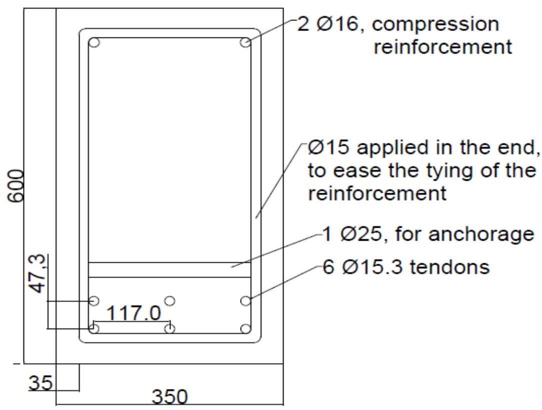

Figure 6.7: Reinforcement layout of beam B-3

Beam B-3 - *1 vol-% fibre, prestressed -reinforced beam with straight -ends*

Data

15,3mm tension wire -Technical data, Spenncon

The tension in the tendons at 0,1 % inelastic strain	$f_{p0.1k} := 1636 \cdot MPa$
Strength of tendons	$f_{pk} := 1860MPa$
Material factor	$\gamma_s = 1.15$
Design yield strength	$f_{pd} := \frac{f_{p0.1k}}{\gamma_s} = 1.423 \times 10^3 \cdot MPa$
E-modulus of the prestressing tendons	$E_p := 1.95 \cdot 10^5 \cdot MPa$
Loss of length ro tenstion tendons due slippage at time of cutting	$\Delta l_{loss} := 4mm$
Maximum allowed stress allowed at time of prestressing, EC2 -5.10.2.1(1)	$\sigma_{p.0} := \min(0.8 \cdot f_{pk}, 0.9 \cdot f_{p0.1k}) = 1.472 \times 10^3 \cdot MPa$
Extended limit of allowed stress at time of prestressing, EC2 -5.10.2.1(2)	$\sigma_{p.max} := 0.95 \cdot f_{p0.1k} = 1.554 \times 10^3 \cdot MPa$
Initiel strain at time of prestressing:	$\varepsilon_{p0} := \frac{\sigma_{p.0}}{E_p} = 7.551 \times 10^{-3}$
Loss of strain due to loss of length at time of prestressing:	$\Delta\varepsilon_{loss} := \frac{\Delta l_{loss}}{l_b} = 6.667 \times 10^{-4}$
Increase in tension to substitute loss of stain:	$\Delta\sigma := \frac{\Delta\varepsilon_{loss}}{\varepsilon_{p0}} \cdot \sigma_{p.0} = 130 \cdot MPa$
Jack stress	$\sigma_{jekk} := \sigma_{p.0} + \Delta\sigma = 1.602 \times 10^3 \cdot MPa$
Not allowed	$\sigma_{jekk} > \sigma_{p.max}$
Allowed stress at time of prestressing. jack stress	$\sigma_{p.0} := \frac{0.95 \cdot \sigma_{p.0} \cdot f_{p0.1k}}{\sigma_{jekk}} = 1.428 \times 10^3 \cdot MPa$
Strain in tendons after cutting:	$\varepsilon_{p.0} := \frac{\sigma_{p.0}}{E_p} - \Delta\varepsilon_{loss} = 6.657 \times 10^{-3}$

Stress in tendons after cutting: $\sigma_{p.0} := \varepsilon_{p.0} \cdot E_p = 1.298 \times 10^3 \cdot MPa$

Assume 5% relaxtion $\sigma'_{p.0} := 0.95 \cdot \sigma_{p.0} = 1.233 \times 10^3 \cdot MPa$

Maximum allowed initiel prestressind force after cutting EC2 -5.10.3 $\sigma_{pm0} := \min(0.75 \cdot f_{pk}, 0.85 \cdot f_{p0.1k}) = 1.391 \times 10^3 \cdot MPa$

Diameter of the tendons $\text{Ø}_p := 15.3mm$

Ultimate Limit State

Miscellaneous data

Effective height $d_{spenn} := h - c_{nom} - \text{Ø}_v - \text{Ø}_p - \frac{a_v}{2} = 518.7 \cdot mm$

Internal lever arm $z_{spenn} := 0.84 \cdot d_{spenn} = 435.708 \cdot mm$

Necessary tensile reinforcement: $A_p := \frac{M_{Ed}}{z_{spenn} \cdot f_{pd}} = 727.095 \cdot mm^2$

15,3mm tendon $A_{st} := 140mm^2$

Choose 6 tendons: $A_{p} := 6 \cdot A_{st} = 840 \cdot mm^2$

Compression reinforcement: $\text{Ø}'_s := 18mm$

Choose 2 Ø16 rebars in the upper part of the beam: $A'_s := 2 \cdot \frac{\text{Ø}'^2_s \cdot \pi}{4} = 508.938 \cdot mm^2$

Moment capacity- *according to Norwegian Concrete Association (2015)*

Characteristic compressive strength $f_{ck.fibre} := \alpha_{cc} \cdot f_{ck} = 29.75 \cdot MPa$

$$\alpha_k := \frac{A_p \cdot f_{pk} - A'_s \cdot f_{yk}}{\lambda_1 \cdot \eta_1 \cdot f_{ck.fibre} \cdot b \cdot d_{spenn}} = 0.303$$

$$z := (1 - 0.5\lambda_1 \cdot \alpha_k) \cdot d_{spenn} = 455.894 \cdot mm$$

Characteristic moment capacity requirement [4.2.2]

$$M_{Rd.k} := \lambda_1 \cdot \alpha_k \cdot z \cdot f_{ck.fibre} \cdot b \cdot d_{spenn} + f_{yk} \cdot (d_{spenn} - d') \cdot A'_s = 714.53 \cdot kN \cdot m$$

Compressive height ratio

$$\alpha := \frac{A_p \cdot f_{pd} - A'_s \cdot f_{yd} + h \cdot b \cdot f_{ftd.res2.5}}{\left(\lambda_1 \cdot \eta_1 \cdot f_{cd} \cdot b + f_{ftd.res2.5} \cdot b\right) \cdot d_{spenn}} = 0.424$$

$$x := \frac{A_p \cdot f_{pd} - A'_s \cdot f_{yd} + h \cdot b \cdot f_{ftd.res2.5}}{\lambda_1 \cdot \eta_1 \cdot f_{cd} \cdot b + f_{ftd.res2.5} \cdot b} = 219.841 \cdot mm$$

$$z_{spenn} := d_{spenn} - 0.5 \cdot \lambda_1 \cdot \alpha \cdot d_{spenn} = 430.764 \cdot mm$$

Momement capacity, included fibre contribution. The calculation is split in two parts, because it was not enough pace to make it fit at one line

$$M_{Rd1} := \lambda_1 \cdot x \cdot b \cdot \eta_1 \cdot f_{cd} \cdot z_{spenn} + f_{yd} \cdot A'_s \cdot \left(d_{spenn} - d'\right)$$

$$M_{Rd2} := -\left[(h - x) \cdot b \cdot f_{ftd.res2.5} \cdot \left[0.5 \cdot (h - x) - \left(h - d_{spenn}\right)\right]\right]$$

$$M_{Rd\_total} := M_{Rd1} + M_{Rd2} = 601.842 \cdot kN \cdot m$$

Min reinforcement and ductility control

Balanced reinforcement cross section

$$\alpha_{pb} := \frac{\varepsilon_{cu}}{\frac{f_{pd}}{E_p} - \varepsilon_{p.0} + \varepsilon_{cu}} = 0.846$$

Critical reinforcement area in a balanced cross-section:

$$A_{pb} := 0.8 \cdot \frac{f_{cd}}{f_{pd}} \cdot \alpha_{pb} \cdot b \cdot d_{spenn} = 1.712 \times 10^3 \cdot mm^2$$

Minimum compression reinfrocement EC2 -9.2.1.1

$$A'_s := 0.26 \cdot \frac{f_{ctm}}{f_{yk}} \cdot b \cdot (h - d') = 317.99 \cdot mm^2$$

Compression reinforcement:

$$Ø'_s := 18mm$$

Choose 2 Ø16 rebars in the upper part of the beam:

$$A'_s := 2 \cdot \frac{Ø'^2_s \cdot \pi}{4} = 508.938 \cdot mm^2$$

$$d' := c_{nom} + Ø_v + \frac{Ø'_s}{2} = 59 \cdot mm$$

Check if the compression reinforcement yield prior to concrete failure

$$\varepsilon'_s := \frac{\alpha_b \cdot d_{spenn} - d'}{\alpha_b \cdot d_{spenn}} \cdot \varepsilon_{cu} = 2.855 \times 10^{-3}$$

Yielding in compression reinforcement

$$\varepsilon_{yd} < \varepsilon'_s$$

The designed locations of the tension tendons is in agreement with the EC2 - [8.10.1.2]

Controll at the moments of prestressing:
- 7 days after casting

$$\beta_{cc} := e^{\left[0.25\cdot\left[1-\left(\frac{28}{7}\right)^{0.5}\right]\right]} = 0.779$$

Average value of compressive strength of the cocnrete, after 7 days of curing:

$$f_{cm7} := \beta_{cc}\cdot f_{cm} = 33.488\cdot MPa$$ EC2 [3.1.2](6)

$$f_{ck7} := f_{cm7} - 8MPa = 25.488\cdot MPa$$

$$f_{cd7} := \alpha_{cc}\cdot\frac{f_{ck7}}{\gamma_c} = 14.443\cdot MPa$$

Prestressing force:

$$P_0 := \sigma_{p.0}\cdot A_p = 1.09\times 10^3\cdot kN$$

$$e_s := 218.7mm$$

$$d_s := 242mm$$

Designed prestressing force:

$$N_{Ed.s} := 1.1\cdot P_0 = 1.199\times 10^3\cdot kN$$

Moment from the tension reinforcement:

$$M_{Ed.s} := N_{Ed.s}\cdot e_s = 262.321\cdot kN\cdot m$$

1) -Pure compression

Concrete compression force

$$T_{c1} := f_{cd7}\cdot b\cdot h = 3.033\times 10^3\cdot kN$$

Strain in tendon is equal to bilinear strain in concrete

$$\varepsilon_p := \varepsilon_{c1} = 2.25\times 10^{-3}$$

Tendon copression force

$$T_{p1} := \varepsilon_p\cdot E_p\cdot A_p = 368.55\cdot kN$$

Strain in comressive rebar is equal to bilinear strain in concrete

$$\varepsilon'_s := \varepsilon_{c1} = 2.25\times 10^{-3}$$

Compressive rebar compressive force

$$T_{s1} := \varepsilon'_s\cdot E_s\cdot A'_s = 229.022\cdot kN$$

Total axial force

$$N_1 := T_{c1} + T_{p1} + T_{s1} = 3.631\times 10^3\cdot kN$$

Bending moment

$$M_1 := T_{p1}\cdot e_s - T_{s1}\cdot d_s = 25.179\cdot kN\cdot m$$

2) -Balance point

Design concrete strain at failire $\varepsilon_{cu} = 3.5 \times 10^{-3}$

Design steel yielding strain $\varepsilon_{yd} = 2.174 \times 10^{-3}$

Compression height ratio $\alpha := \frac{\varepsilon_{cu}}{\varepsilon_{yd} + \varepsilon_{cu}} = 0.617$

Effective height,
upper part in stress $d_1 := h - c_{nom} - Ø_v - \frac{Ø'_s}{2} = 541 \cdot mm$

Concrete compression force $T_{c2} := \lambda_1 \cdot \alpha \cdot d_1 \cdot b \cdot \eta_1 \cdot f_{cd} = 1.853 \times 10^3 \cdot kN$

Strain the tendon

$$\Delta\varepsilon_{p2} := \frac{\alpha \cdot d_1 - \left(c_{nom} + Ø_v + Ø_p + \frac{a_v}{2}\right)}{\alpha \cdot d_1} \cdot \varepsilon_{cu} = 2.647 \times 10^{-3}$$

Tendon compression force $T_{p2} := \Delta\varepsilon_{p2} \cdot E_p \cdot A_p = 433.634 \cdot kN$

Tensile force in comressive rebars $S_2 := f_{yd} \cdot A'_s = 221.277 \cdot kN$

Tensile force contributed from fibre $S_{2.Fibre} := \left(h - \alpha \cdot d_1\right) \cdot b \cdot f_{ftd.res2.5} = 173.106 \cdot kN$

Total axial force $N_2 := T_{c2} + T_{p2} - S_2 - S_{2.Fibre} = 1.893 \times 10^3 \cdot kN$

Bending moment

$$M_2 := T_{c2} \cdot \left(0.5 \cdot h - 0.4 \cdot \alpha \cdot d_1\right) + T_{p2} \cdot e_s + S_2 \cdot (0.5 \cdot h - d') + S_{2.Fibre} \cdot 0.5 \cdot \left(h - \alpha \cdot d_1\right) = 479.801 \cdot kN \cdot m$$

3) Under reinforce

Strain in the comression rebar $\varepsilon_s := 2 \cdot \varepsilon_{yk} = 5 \times 10^{-3}$

Compression height ratio $\alpha_3 := \frac{\varepsilon_{cu}}{2 \cdot \varepsilon_{yk} + \varepsilon_{cu}} = 0.412$

Concrete compression force $T_{c3} := \lambda_1 \cdot \alpha_3 \cdot d_1 \cdot b \cdot \eta_1 \cdot f_{cd} = 1.237 \times 10^3 \cdot kN$

Strain in the tendon $\Delta\varepsilon_{p3} := \frac{\alpha_3 \cdot d_1 - 81.3mm}{\alpha_3 \cdot d_1} \varepsilon_{cu} = 2.223 \times 10^{-3}$

Tendon compression force $T_{p3} := \Delta\varepsilon_{p3} \cdot E_p \cdot A_p = 364.069 \cdot kN$

Compressive rebar tensile force $S_3 := f_{yd} \cdot A'_s = 221.277 \cdot kN$

Tensile force contributed from fibre $S_{3.Fibre} := (h - \alpha_3 \cdot d_1) \cdot b \cdot f_{ftd.res2.5} = 245.237 \cdot kN$

Total axial force $N_3 := T_{c3} + T_{p3} - S_3 - S_{3.Fibre} = 1.135 \times 10^3 \cdot kN$

Bending moment

$$M_3 := T_{c3} \cdot (0.5 \cdot h - 0.4 \cdot \alpha \cdot d_1) + T_{p3} \cdot e_s + S_3 \cdot (0.5 \cdot h - d') + S_{3.Fibre} \cdot 0.5 \cdot (h - \alpha_3 \cdot d_1) = 385.195 \cdot kN \cdot m$$

Plot the three points in a moment-compression diagram and check it against the load situation.

Shear Strength

Without shear reinforcement -*according to Norwegian Concrete Association (2015)*

Prestressing force: $P'_0 := \sigma'_{p.0} \cdot A_p = 1.036 \times 10^3 \cdot kN$

$\gamma_p := 0.9$

Design compression force:
- Assume 15% loss of initial compression force.

$N_{Ed} := \gamma_p \cdot 0.85 \cdot P_0 = 834.166 \cdot kN$

Bending moment at beam end: $M_s := -N_{Ed} \cdot (0.5 \cdot h - 81.3mm) = -182.432 \cdot kN \cdot m$

Bending moment at midspan: $M_{felt} := M_{Ed} + M_s = 268.252 \cdot kN \cdot m$

Compression casued due to the prestressing force $\sigma_{cp} := \min\left(\frac{N_{Ed}}{A_c}, 0.2 \cdot f_{cd}\right) = 3.967 \cdot \frac{N}{mm^2}$

Data
EC2 -6.2

$$\rho_1 := \frac{A_p}{b \cdot d} = 4.715 \times 10^{-3}$$

$$k := 1 + \sqrt{\frac{200mm}{d}} = 1.627$$

$$C_{Rd.c} := \frac{0.18}{1.5} = 0.12$$

$$k_1 := 0.15$$

$$v_{min} := 0.035 \cdot k^{\frac{2}{3}} \cdot \left(f_{ck} \cdot \frac{mm^2}{N}\right)^{0.5} = 0.286$$

$$V_{Rd.ct} := \left[C_{Rd.c} \cdot k \cdot \left(100 \cdot \rho_l \cdot f_{ck} \cdot \frac{mm^2}{N} \right)^{\frac{1}{3}} \frac{N}{mm^2} + k_1 \cdot \sigma_{cp} \right] \cdot b \cdot d = 194.545 \cdot kN$$ EC2 [6.2.2]

$$V_{Rd.cf} := 0.6 \cdot f_{ftd.res2.5} \cdot b \cdot h = 234.032 \cdot kN$$ COIN (2015) [4.2.5]

$$V_{Rd.c} := V_{Rd.ct} + V_{Rd.cf} = 428.577 \cdot kN \quad > \quad V_{Ed} = 300.456 \cdot kN$$ Approved, no need of shear stirrups

The designed locations of the tension tendons is in agreement with the EC2 [8.10.1.2]

Transmission of prestressing force

Eurokode 2 [8.10.2.2]

Average value of tensile strength, after 7 days of curing EC2 [3.1.2] (9)

$$f_{ctm7} := \beta_{cc} \cdot f_{ctm} = 2.492 \cdot \frac{N}{mm^2}$$

$$f_{ctd7} := \alpha_{ct} \cdot 0.7 \cdot \frac{f_{ctm7}}{\gamma_c} = 0.989 \cdot \frac{N}{mm^2}$$

Adhesion tension at the interface between concrete and tendons: EC2 [8.10.2.2] (1)

$$\eta_{p1} := 3.2 \qquad \eta_1 := 1$$

$$f_{bpt} := \eta_{p1} \cdot \eta_1 \cdot f_{ctd7} = 3.163 \cdot \frac{N}{mm^2}$$

Base value of the transmission length: EC2 [8.10.2.2] (2)

$$l_{pt} := 1.25 \cdot 0.19 \cdot \text{Ø}_p \cdot \frac{\sigma_{p.0}}{f_{bpt}} = 1.491 \cdot m$$

Design value of the transmission length: EC2 [8.10.2.2] (3)

$$l_{pt2} := 1.2 \cdot l_{pt} = 1.789\, m$$

Designed transmission length, EC2 [8.10.2.2] (4)

$$L_{disp} := \sqrt{l_{pt}^2 + d_{spenn}^2} = 1.579\, m$$

It is sufficient length according to requirements in ULS (EC2 [8.10.2.2](4) Note), Although, it does not satisfy the requirements related at the face when the tendons are cut. (EC2 [8.10.2.2](4).

Anchorage of strands in Ultimate Limit State

Anchorage length at support: $l_1 := 250mm + 0.5 \cdot 81.3mm = 290.65 \cdot mm$

EC2 [8.10.2.3] $\eta_{p2} := 1.2$

$$f_{bpd} := \eta_{p2} \cdot \eta_1 \cdot f_{ctd} = 1.496 \cdot \frac{N}{mm^2}$$

$$\sigma_{pd} := \frac{f_{p0.1k}}{\gamma_s} = 1.423 \times 10^3 \cdot \frac{N}{mm^2}$$

Neccessary anchorage length: $l_{bpd} := l_{pt2} + 0.19 \cdot 15.3mm \cdot \frac{(\sigma_{pd} - \sigma_{p.0})}{f_{bpd}} = 2.031 \cdot m$

Anchorage capacity of the 6 tendons, Betongelementboken volume C, Chapter C 9

$$F_{sp} := \frac{0.9 \cdot 6 \cdot P_0 \cdot l_1}{l_{bpd}} = 842.532 \cdot kN \qquad < P_0 = 1.09 \times 10^3 \cdot kN$$

Required area of reinforcement; $A_{se\_anchorage} := \frac{P_0 - F_{sp}}{f_{yd}} = 570.126 \cdot mm^2$

Use 1 x Ø25 stirrups: $A_{se\_anchorage} := 2 \frac{25^2 \cdot \pi}{4} mm^2 = 981.748 \cdot mm^2$

The stirrups must have length of 1,4 m, ccording table C 8.8

Ultimate Limit State

Unfortunately, there has not been executed multi-layer analysis for determining the compressive zone height in SLS.

6.3.7 Beam C-1

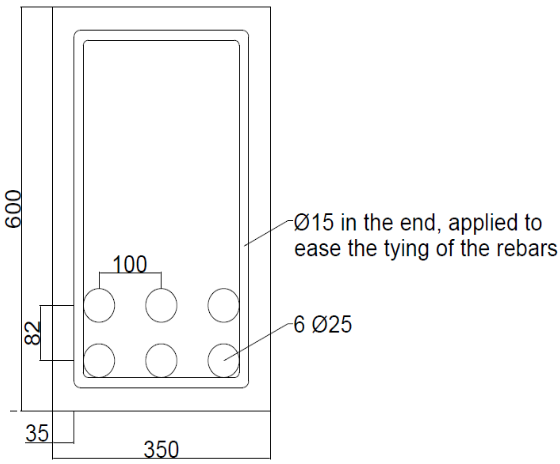

Figure 6.8: Reinforcement layout of beam B-3

Beam C-1 *- 2 vol-% fibre, slack reinforced straight-ended beam.*

Ultimate limit state

Moment capacity:

Design

For compressive strengths larger than 50 MPa:

$$\lambda_1 := 0.8 - \frac{\left(f_{ck} \cdot \frac{mm^2}{N} - 50\right)}{400} = 0.763$$

$$\eta_1 := 1.0 - \frac{\left(f_{ck} \cdot \frac{mm^2}{N} - 50\right)}{200} = 0.925$$

Normal reinforced
Table 4.3, Sørensen

$$\alpha := 0.342$$

$$K := \lambda_1 \cdot \eta_1 \cdot (1 - 0.5 \cdot \lambda_1 \alpha)\alpha = 0.21$$

Effective height

$$d_f := \sqrt{\frac{M_k}{K \cdot f_{cd} \cdot b}} = 0.356\,m$$

Internal lever arm

$$z_f := (1 - 0.5\lambda_1 \cdot \alpha) \cdot d_f = 0.309\,m$$

Required tensile reinforcement

$$A_s := \frac{M_k}{f_{yd} \cdot z_f} = 2.543 \times 10^3 \cdot mm^2$$

Use 6ø25

$$A_s := 6 \cdot \frac{\text{Ø}_s^{\,2} \cdot \pi}{4} = 2.945 \times 10^3 \cdot mm^2$$

Effective height

$$d := h - c_{nom} - 15mm - \text{Ø}_s - \frac{a_v}{2} = 509 \cdot mm$$

Min. reinforcement and ductility control:

Check of minimum reinforcement
[EC2 NA.9.2.1.1.(1)]

$$A_{s.min} := 0.26 \cdot \frac{f_{ctm}}{f_{yk}} \cdot b \cdot d = 416.871 \cdot mm^2$$

Balanced reinforcement cross section

$$\alpha_b := \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{yd}} = 0.563$$

Amount of reinforcement at ballanced cross-section

$$A_{s.b} := \lambda_1 \cdot \frac{f_{cd}}{f_{yd}} \cdot b \cdot d \cdot \alpha_b = 6.478 \times 10^3 \cdot mm^2$$

Moment capacity- *according to Norwegian Concrete Association (2015)*

Characteristic compressive strength $f_{ck.fibre} := \alpha_{cc} \cdot f_{ck} = 55.25 \cdot MPa$

Compressive height ratio, based on characteristic $\alpha_k := \frac{f_{yk} \cdot A_s}{\lambda_1 \cdot f_{ck} \cdot b \cdot d} = 0.167$

$z := (1 - 0.5\lambda_1 \cdot \alpha_k) \cdot d = 476.635 \cdot mm$

Characteristic moment capacity requirement [4.2.2] $M_{Rd.k} := \lambda_1 \cdot \alpha_k \cdot z \cdot f_{ck.fibre} \cdot b \cdot d = 596.617 \cdot kN \cdot m$

Compression height $x := \frac{A_s \cdot f_{yd} + h \cdot b \cdot f_{ftd.res2.5}}{\lambda_1 \cdot \eta_1 \cdot f_{cd} \cdot b + f_{ftd.res2.5} \cdot b} = 191.971 \cdot mm$

$$\alpha := \frac{A_s \cdot f_{yd} + h \cdot b \cdot f_{ftd.res2.5}}{(\lambda_1 \cdot \eta_1 \cdot f_{cd} \cdot b + f_{ftd.res2.5} \cdot b) \cdot d} = 0.377$$

Momement capacity, included fibre contribution

$$M_{Rd} := (h - x) \cdot b \cdot f_{ftd.res2.5} \cdot (0.5h + 0.1x) + A_s \cdot f_{yd}(h - 0.4x) = 818.417 \cdot kN \cdot m$$

Check reinforcement strain $\varepsilon_s := \frac{1 - \alpha}{\alpha} \cdot \varepsilon_{cu} = 4.624 \times 10^{-3}$

$\varepsilon_s < \varepsilon_{ud}$

Shear Strength

Without shear reinforcement -*according to Norwegian Concrete Association (2015)*

Data
EC2 -6.2

$\rho_l := \frac{A_s}{b \cdot d} = 0.017$

$k := 1 + \sqrt{\frac{200mm}{d}} = 1.627$

$C_{Rd.c} := \frac{0.18}{1.5} = 0.12$

$v_{min} := 0.035 \cdot k^{\frac{2}{3}} \cdot \left(f_{ck} \cdot \frac{mm^2}{N}\right)^{0.5} = 0.39$

Shear capacity of the concrete

$$V_{Rd.ct} := \left[C_{Rd.c}\cdot k\cdot\left(100\rho_l\cdot f_{ck}\cdot\frac{mm^2}{N}\right)^{\frac{1}{3}}\right]\cdot b\cdot d\cdot\frac{N}{mm^2} = 165.346\cdot kN$$

Shear capacity, contribution from the fibre:

$$V_{Rd.cf} := 0.6\cdot f_{ftd.res2.5}\cdot b\cdot h = 410.256\cdot kN$$

Shear capacity of the beam:

$$V_{Rd.c} := V_{Rd.ct} + V_{Rd.cf} = 575.602\cdot kN$$

Anchoring

Internal lever arm

$$z := \left(1 - 0.5\lambda_1\cdot\alpha\right)\cdot d = 0.436\,m$$

$$\Delta F_{td} := \frac{V_{Ed}}{\tan(22grad)} - 0.5\cdot\frac{f_{ftd.res2.5}\cdot b\cdot z}{\sin(45\cdot grad)}\cdot\left(\frac{\cos(45\cdot grad)}{\tan(45\cdot grad)} + \sin(45\cdot grad)\right) = 247.005\cdot kN$$

Factors which depend on the adhesion condition and size of the rebars.

$$\eta_1 := 1$$

$$\eta_2 := 1$$

Adhesion strength, EC2 -8.4.2(2)

$$f_{bd} := 2.25\cdot\eta_1\cdot\eta_2\cdot f_{ctd} = 3.953\cdot\frac{N}{mm^2}$$

Tension in the rebar

$$\sigma_{sd} := \frac{\Delta F_{td}}{A_s} = 83.866\cdot\frac{N}{mm^2}$$

Base value of the transmission length

$$l_{b.rqd} := \frac{Ø_s}{4}\cdot\frac{\sigma_{sd}}{f_{bd}} = 132.615\cdot mm$$

Coefficients, EC2 -Tabel 8.2

$$\alpha_1 := 1 \qquad \alpha_2 := 1 - 0.15\cdot\frac{\left(c_{nom} - Ø_s\right)}{Ø_s} = 0.94$$

$$\alpha_3 := 1 \qquad \alpha_4 := 0.7 \qquad \alpha_5 := 1$$

Anchorig length

$$l_{bd} := \alpha_1\cdot\alpha_2\cdot\alpha_3\cdot\alpha_4\cdot\alpha_5\cdot l_{b.rqd} = 87.261\cdot mm$$

EC2 [8.4.4](8.5)

$$l_{b.min} := \max\left(0.3\cdot l_{b.rqd}, 10\cdot Ø_s, 100mm\right) = 250\cdot mm$$

Designed anchorage length:

$$l_{bd} := \max\left(l_{b.min}, l_{bd}\right) = 250\cdot mm$$

Service Limit State

Minimum reinforcement *- according COIN (2011)*

Minimum flexural reinforcement

$$A_{smin} := \max\left[0.26\cdot\frac{f_{ctm} - 2.1f_{ftk.res2.5}}{f_{yk}}\cdot b\cdot d, 0.0013\cdot\left(1 - \frac{f_{ftk.res2.5}}{f_{ctm}}\right)\cdot b\cdot d\right] = -19.763\cdot mm^2$$

No minimum requirement

Minimum shear reinforcement

$$\rho_{w.min} := \left(\frac{0.1\cdot\sqrt{f_{ck}\cdot\frac{mm^2}{N}}\cdot\frac{N}{mm^2} - 0.3\cdot f_{ftk.res2.5}}{f_{yk}}\right) = -1.318\times 10^{-3}$$

No minimum requirement

Minimum reinforcement *- according to Norwegian Concrete Association (2015)*

Minimum flexural reinforcement

$$f_{Ftsm} := \frac{f_{Ftsk}}{0.7} = 7.071\cdot\frac{N}{mm^2}$$

$$A_{smin} := \max\left[0.26\cdot\frac{f_{ctm} - f_{Ftsm}}{f_{yk}}\cdot b\cdot d, 0.0013\cdot\left(1 - \frac{f_{Ftsm}}{f_{ctm}}\right)\cdot b\cdot d\right] = -1.323\times 10^{-4}\, m^2$$

No minimum requirement

Minimum shear reinforcement

$$\rho_{w.min} := \left(\frac{0.1\cdot\sqrt{f_{ck}\cdot\frac{mm^2}{N}}\cdot\frac{N}{mm^2} - 0.2\cdot f_{Ftsm}}{f_{yk}}\right) = -1.216\times 10^{-3}$$

No minimum requirement

Crack widt calculation *-according to COIN (2011)*

Factor based on the duration of the load $k_t := 0.4$

Concrete mean tensile strength value at time of cracking $f_{ct.eff} := f_{ctm}$

The multi-layer procedure was performed in aim to find the tensile stress in the reinforcement, when the beam was subjected to service load. The model had to be run two times, to get the curvature and strains of the examined fibre reinforced beam. The strain value was found by interpolation between the two iteration steps.

Bending moment at SLS $M_k = 341.82 \cdot kN \cdot m$

Calculated bending moment and strain value at iteration step 1 and 2.

$M_{SLS1} := 331.93 kN \cdot m$

$M_{SLS2} := 474.82 kN \cdot m$

$\varepsilon_{cc1} := -0.0003125$

$\varepsilon_{cc2} := -0.0005045$

$\varepsilon_{cs1} := 0.0005001$

$\varepsilon_{cs2} := 0.0010001$

Meassured strain values, of interest in tension and compression

$$\varepsilon_{cc} := \varepsilon_{cc1} + \frac{\varepsilon_{cc2} - \varepsilon_{cc1}}{M_{SLS2} - M_{SLS1}} \cdot \left(M_k - M_{SLS1}\right) = -3.258 \times 10^{-4}$$

$$\varepsilon_{cs} := \varepsilon_{cs1} + \frac{\varepsilon_{cs2} - \varepsilon_{cs1}}{M_{SLS2} - M_{SLS1}} \cdot \left(M_k - M_{SLS1}\right) = 5.347 \times 10^{-4}$$

Compression height ratio

$$\alpha_{SLS} := \frac{h}{\left(\left|\varepsilon_{cc}\right| + \varepsilon_{cs}\right) \cdot d} \cdot \left|\varepsilon_{cc}\right| = 0.446$$

Strain in rebars

$$\varepsilon_s := \frac{\left|\varepsilon_{cc}\right|}{\alpha_{SLS} \cdot d} \cdot \left(1 - \alpha_{SLS}\right) \cdot d = 4.042 \times 10^{-4}$$

Stress in rebars

$$\sigma_s := E_s \cdot \varepsilon_s = 80.84 \cdot MPa$$

Calculated stress in the reinforcement, based on the meassured strains from the multi-layer procedure

Compression height $x := \alpha_{SLS} \cdot d = 227.164 \cdot mm$

Effective tensile height $h_{cef} := \min\left[2.5 \cdot (h - d) + 1.5 \cdot Ø_s, \frac{h}{2}, \frac{(h - x)}{3}\right] = 124.279 \cdot mm$

Effective tensile area $A_{c.eff} := h_{cef} \cdot b = 4.35 \times 10^4 \cdot mm^2$

$$\rho_{p.eff} := \frac{A_s}{A_{c.eff}} = 0.068$$

Modulus of elasticity ratio $\alpha_e := \frac{E_s}{E_{cm}} = 5$

Strain in the rebars minus strain in the comcrete

$$\Delta := \frac{\sigma_s - k_t \cdot \frac{f_{ct.eff}}{\rho_{p.eff}} \cdot (1 + \alpha_e \cdot \rho_{p.eff})}{E_s} = 2.263 \times 10^{-4}$$

Miscellaneous factors -NA.7.3.4 $k_3 := 3.4$ $k_4 := 0.425$ $k_1 := 0.8$ $k_2 := 0.5$

Cover of the longitudinal reabar $c_3 := c_{nom}$

$$k_5 := \left(1 - \frac{f_{ftk.res2.5}}{f_{ctm}}\right) = -0.085$$

Upper value of the crack spacing $S_{r.max} := k_3 \cdot c_3 + k_1 \cdot k_2 \cdot k_4 \cdot k_5 \cdot \frac{Ø_s}{\rho_{p.eff}} = 113.644 \cdot mm$

Calculated crack width $w_k := S_{r.max} \cdot \Delta = 0.026 \cdot mm$

Crack widths *-according to Norwegian Concrete Association (2015)*

Four different stages shall be considered. Crack formation stage at short and long term, and stabilized cracking stage at both short and long term

Constants:

Height of the concrete cover	$c_{nom} = 35 \cdot mm$
Diameter of the rebar	$Ø_s = 25 \cdot mm$
Cross -sectional area of the concrete at tension, EC2- figure 7.1.	$\rho_{s.ef} := \dfrac{A_s}{\min\left[2.5(h-d) + 1.5 \cdot Ø_s, \dfrac{(h-x)}{3}, \dfrac{h}{2}\right] \cdot b} = 0.068$
Averrage tensilte strength, of the cocnrete	$f_{ctm} = 4.5 \cdot MPa$
Contribution from the fibers	$f_{Ftsm} = 7.071 \cdot MPa$
Modulus of elasticity of the reinforcement	$E_s = 2 \times 10^5 \cdot MPa$
Strain, caused by shrinkage	$\varepsilon_{sh} := \varepsilon_{cs}$

Tension in the rebars $\sigma_s = 80.84 \cdot MPa$

$$\sigma_{sr} := \frac{(f_{ctm} - f_{Ftsm})}{\rho_{s.ef}} \cdot \left(1 + \frac{E_s}{E_{cm}} \cdot \rho_{s.ef}\right) = -50.834 \cdot MPa$$

Crack fomation stage:

Short term, instantaneous loading

$\tau_{bm} := 1.8 \cdot f_{ctm} = 8.1 \cdot MPa$ Table 7.6-2

$\beta := 0.6$

$\eta_r := 0$

$$w_d := 2 \cdot \left[k \cdot c_{nom} + \frac{1}{4} \cdot \frac{Ø_s}{\rho_{s.ef}} \cdot \frac{(f_{ctm} - f_{Ftsm})}{\tau_{bm}}\right] \cdot \frac{1}{E_s} \cdot (\sigma_s - \beta \cdot \sigma_{sr} + \eta_r \cdot \varepsilon_{sh} \cdot E_s) = 0.031 \cdot mm$$

Long term, repeated loading

$\tau_{bm} := 1.35 \cdot f_{ctm} = 6.075 \cdot MPa$ Table 7.6-2

$\beta := 0.6$

$\eta_r := 0$

$$w_d := 2 \cdot \left[k \cdot c_{nom} + \frac{1}{4} \cdot \frac{Ø_s}{\rho_{s.ef}} \cdot \frac{(f_{ctm} - f_{Ftsm})}{\tau_{bm}}\right] \cdot \frac{1}{E_s} \cdot (\sigma_s - \beta \cdot \sigma_{sr} + \eta_r \cdot \varepsilon_{sh} \cdot E_s) = 0.02 \cdot mm$$

Stabilized cracking stage:

Short term, instantaneous loading — Same crack width as in the Crack formation stage

Long term, repeated loading

$\tau_{bm} := 1.8 \cdot f_{ctm} = 8.1 \cdot MPa$ — Table 7.6-2

$\beta := 0.4$

$\eta_r := 1$

$$w_d := 2 \cdot \left[k \cdot c_{nom} + \frac{1}{4} \cdot \frac{\varnothing_s}{\rho_{s.ef}} \cdot \frac{\left(f_{ctm} - f_{Ftsm}\right)}{\tau_{bm}} \right] \cdot \frac{1}{E_s} \cdot \left(\sigma_s - \beta \cdot \sigma_{sr} + \eta_r \cdot \varepsilon_{sh} \cdot E_s\right) = 0.058 \cdot mm$$

Uncracked cross- section (Stadium I)

Material stiffness ratio $\eta := \frac{E_s}{E_{cm}} = 5$

Transformed cross -section $A_t := A_c + (\eta - 1) \cdot A_s = 2.218 \times 10^5 \cdot mm^2$

The distance from the longitudinal reinforcement to concrete center of gravity $e_s := d - \frac{h}{2} = 209 \cdot mm$

Reduction distance to the center of gravity of reinforced cross -section. $y_t := \frac{(\eta - 1) \cdot A_s \cdot e_s}{A_t} = 11.102 \cdot mm$

Second moment of area, contribution from concrete $I_{c1} := \frac{b \cdot h^3}{12} + h \cdot b \cdot {y_t}^2 = 6.326 \times 10^9 \cdot mm^4$

Second moment of area, contribution from steel bars $I_{s1} := A_s \cdot \left(e_s - y_t\right)^2 = 1.153 \times 10^8 \cdot mm^4$

Cracking moment $M_{cr} := \frac{I_{c1} + \eta \cdot I_{s1}}{0.5 \cdot h - y_t} \cdot f_{ctm} = 107.518 \cdot kN \cdot m$

Cracking load $q_{cr} := \frac{M_{cr} \cdot 8}{{l_b}^2} = 23.893 \cdot \frac{kN}{m}$

Beam stiffness, stadium I $EI_I := E_{cm} \cdot I_{c1} + E_s \cdot I_{s1} = 2.761 \times 10^5 \cdot kN \cdot m^2$

Deflection, stadium I $\delta_I := \frac{5}{384} \cdot \frac{q_k \cdot {l_b}^4}{EI_I} = 4.643 \cdot mm$

Cracked corss- section, (Stadium II)

Reinforcement ratio $\rho := \frac{A_s}{b \cdot d} = 0.017$

Compression height ratio $\alpha_{SLS} = 0.446$

Computed bending moment with the given curvature in itersion 1 and 2, respectively.

$M_{SLS1} = 331.93 \cdot kN \cdot m$

$M_{SLS2} = 474.82 \cdot kN \cdot m$

$\kappa_1 := 1.35 \cdot 10^{-6} \cdot \frac{1}{mm}$

$\kappa_2 := 2.51 \cdot 10^{-6} \cdot \frac{1}{mm}$

Resultinfg bending moment in SLS $M_k = 3.418 \times 10^5 J$

$$\kappa_{SLS} := \kappa_1 + \frac{\kappa_2 - \kappa_1}{(M_{SLS2} - M_{SLS1})} \cdot (M_k - M_{SLS1}) = 1.43 \times 10^{-3} \frac{1}{m}$$

$$EI_{II} := \frac{M_k}{\kappa_{SLS}} = 2.39 \times 10^{14} \cdot N \cdot mm^2$$

Deflection, stadium II $\delta_{II} := \frac{5.}{384} \cdot \frac{q_k \cdot l_b^4}{EI_{II}} = 5.364 \cdot mm$

Moment at SLS $M_k := \frac{q_k \cdot l_b^2}{8} = 341.82 \cdot kN \cdot m$

Load, long duration $\beta := 0.5$

Distribution koeffisient $\zeta := 1 - \beta \cdot \left(\frac{M_{cr}}{M_k}\right)^2 = 0.951$

Deflection, stadium I and II, EC2 - 7.4.3 $\delta := \zeta \cdot \delta_{II} + (1 - \zeta) \cdot \delta_I = 5.328 \cdot mm$

Stress limitation

Maximum compressive stress in concrete, based on stadium II behaviour

$$\sigma_c := \frac{2 \cdot \left[M_k + f_{ftk.res2.5} \cdot b \cdot (h - \alpha_{SLS} \cdot d) \cdot \left[0.5(h - \alpha_{SLS} \cdot d) - (h - d)\right]\right]}{\alpha_{SLS} \cdot b \cdot d^2 \cdot \left(1 - \frac{\alpha_{SLS}}{3}\right)} = 23.376 \cdot MPa$$

Max. compressive stress, EC2 -7.2.(2)	$0.6 \cdot f_{ck} = 39 \cdot \frac{N}{mm^2}$
Tensile stress in the rebar	$\sigma_s = 80.84 \cdot MPa$
Max. tensile stress in rebar, EC2- 7.2.(5):	$0.8 \cdot f_{yk} = 400 \cdot \frac{N}{mm^2}$

6.3.8 Beam C-2

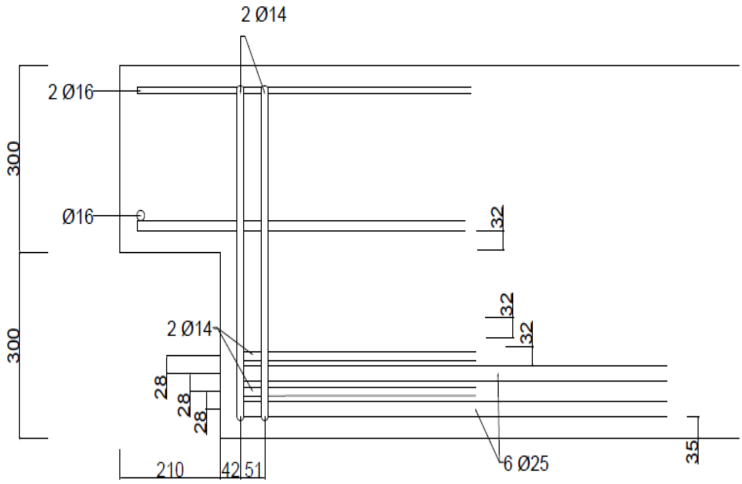

Figure 6.9: Reinforcement layout of beam C-2

Beam C-2 - *2 vol-% fibre, slack-reinforced beam with dapped -ends*

Capacity calculation in dapped-end Model -1

Inclined contribution

Largest aggregate size $d_g := 32mm \qquad k_2 := 5mm$

Distance from center hanger stirrups to the nib edge $c_1 := c_{nom} + Ø_{sv} + \frac{d_g + k_2}{2} = 67.5 \cdot mm$

Virtual inclination angle, used to calculate the strength contribution offered by the fibres $\alpha := 45°$

Effektiv fibre length $l_{eff.fibre} := \frac{h_{nib}}{2} = 150 \cdot mm$

Contribution offered by the fibres $F_{f\alpha} := f_{ftd.res2.5} \cdot b \cdot l_{eff.fibre} = 170.94 \cdot kN$

Support force $N_{f\alpha} := F_{f\alpha} \cdot \sin(\alpha) = 120.873 \cdot kN$

It is possible to exlude the Ø16 incline reinforcement, if the contribution from fibres is accounted for . Although, it would be hard to reduce the amount of hanger reinforcement. Approximately 180 kN has to be carried by the hanger rein. 2Ø14 were used in the design of beam A-2, recommend 2Ø14 in this design as well.

Compressive force in top of the beam, caused by the fibre contribution

Compression force in the top of the beam $F_{c5} := \frac{N_{f\alpha}}{\tan(\alpha)} = 120.873 \cdot kN$

Geometric eccentricity of the load, EC2 -6,1(4) $e_u := 20mm$

The height of the compression zone, "betongelementboka, bind C" $x := 2 \cdot \left(\frac{l_{nib}}{2} - e_u\right) = 170 \cdot mm$

Stress in the concrete . $f_{c\alpha} := \frac{N_{f\alpha}}{b \cdot x \cdot \tan(\alpha)^2} = 2.031 \cdot \frac{N}{mm^2}$

Hanger stirrups contribution

Effective height in the nib $d_{nib} := 257mm$

$a_0 := \frac{l_{nib}}{2} - \frac{w_p}{2} - e_u = 45 \cdot mm$

Lever arm $a' := a_0 + c_1 = 112.5 \cdot mm$

Force carreid by the hanger stirrups $N_v := V_{Ed} - N_{f\alpha} = 179.583 \cdot kN$

Internail lever arm in the nib $z_{nib} := d_{nib} - \frac{N_v \cdot a'}{1.6 \cdot b \cdot d_{nib} \cdot (0.8 \cdot f_{cd} - f_{c\alpha})} = 251.883 \cdot mm$

Full utilization of the compression zone $z_{nib} := 2 \cdot a' = 225 \cdot mm$

$F_s := \frac{N_v \cdot a'}{z_{nib}} = 89.792 \cdot kN$

Compressive force in top of the beam, to regard for the hanger stirrups

Compression zone height $x_h := 2(d_{nib} - z_{nib}) = 64 \cdot mm$

Stress in compression zone due hanger reinforcement $F_{c2} := F_s = 89.792 \cdot kN$

Stress in the compression zone due hanger force $f_{cv} := \frac{F_{c2}}{x_h \cdot b} = 4.009 \cdot MPa$

Total stress in the compressive zone $f_c := f_{cv} + f_{c\alpha} = 6.04 \cdot MPa$

Design compressive strength of the concrete $f_{cd} = 36.833 \cdot MPa$

Capacity calculation in dapped-end Model-2

Fibre contribution $F_{fv} := l_{eff.fibre} \cdot b \cdot f_{ftd.res2.5} = 170.94 \cdot kN$

Force into the support $N_{fv} := F_{fv} = 170.94 \cdot kN$

The fibres offer a significant increase in the load bearing capacity. It is possible to decrease the dimmension of the hanger reinforcement.

Necessary hanger reinforcement $A_{sv} := \dfrac{V_{Ed} - N_{fv}}{f_{yd'}} = 340.832 \cdot mm^2$

Hanger stirrups $Ø_{sv} := 12mm$

2Ø12 gives $A_{sv} := 2 \cdot \left(2 \dfrac{Ø_{sv}^2 \cdot \pi}{4}\right) = 452.389 \cdot mm^2$

Compression force in top of the beam

Internal lever arm in the nib $z_{nib} := d_{nib} - \dfrac{V_{Ed} \cdot a'}{1.6 \cdot b \cdot d_{nib} \cdot (0.8 \cdot f_{cd} - f_{c\alpha})} = 248.439 \cdot mm$

Full utilization of compression zone $z_{nib} := 2 \cdot a' = 225 \cdot mm$

Force in the longitudinal nib rebars $F_s := \dfrac{V_{Ed} \cdot a'}{z_{nib}} = 150.228 \cdot kN$

Compressive zone height $x_h := 2 \cdot (d_{nib} - z_{nib}) = 64 \cdot mm$

Compressice force, top of the beam $F_{c2} := F_s$

$f_{cv} := \dfrac{F_{c2}}{x_h \cdot b} = 6.707 \cdot MPa$

Splitting tensile forces caused by consentrated loads

Incoming compressive zone, Figure 5.9 $a_1 := w_p = 80 \cdot mm$

Outcoming compressive zone $a_2 := l_{nib} = 210 \cdot mm$

Resulting splitting tensile forces, equation 5.28 $Z_{s1} := 0.25 \cdot V_{Ed} \cdot \left(1 - \dfrac{a_1}{a_2}\right) = 46.499 \cdot kN$

$d' := c_{nom} + Ø_{sv} + 0.5 \cdot Ø'_{sn} = 52 \cdot mm$

Effective height $d_t := d_{nib} - d' = 205 \cdot mm$

Effective width $b_t := b - 2 \cdot c_{nom} = 280 \cdot mm$

Splitting tensile force capacity, equation 5.30 $SP_{std} := f_{ftd.res2.5} \cdot b_t \cdot d_t = 186.894 \cdot kN$

Requirement according to equation 4.19 $SP_{std} > 0.5 \cdot F_s$

6.3.9 Beam C-3

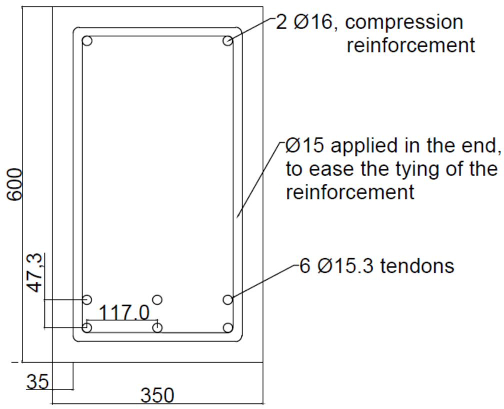

Figure 6.10: Reinforcement layout of beam B-1

Beam C-3 - *2 vol-% fibre, prestressed -reinforced beam with straight -ends*

Data

15,3mm tension wire -Technical data, Spenncon

The tension in the tendons at 0,1 % inelastic strain	$f_{p0.1k} := 1636 \cdot MPa$
Strength of tendons	$f_{pk} := 1860MPa$
Material factor	$\gamma_s = 1.15$
Design yield strength	$f_{pd} := \dfrac{f_{p0.1k}}{\gamma_s} = 1.423 \times 10^3 \cdot MPa$
E-modulus of the prestressing tendons	$E_p := 1.95 \cdot 10^5 \cdot MPa$
Loss of length ro tenstion tendons due slippage at time of cutting	$\Delta l_{loss} := 4mm$
Maximum allowed stress allowed at time of prestressing, EC2 -5.10.2.1(1)	$\sigma_{p.0} := \min(0.8 \cdot f_{pk}, 0.9 \cdot f_{p0.1k}) = 1.472 \times 10^3 \cdot MPa$
Extended limit of allowed stress at time of prestressing, EC2 -5.10.2.1(2)	$\sigma_{p.max} := 0.95 \cdot f_{p0.1k} = 1.554 \times 10^3 \cdot MPa$
Initiel strain at time of prestressing:	$\varepsilon_{p0} := \dfrac{\sigma_{p.0}}{E_p} = 7.551 \times 10^{-3}$
Loss of strain due to loss of length at time of prestressing:	$\Delta\varepsilon_{loss} := \dfrac{\Delta l_{loss}}{l_b} = 6.667 \times 10^{-4}$
Increase in tension to substitute loss of stain:	$\Delta\sigma := \dfrac{\Delta\varepsilon_{loss}}{\varepsilon_{p0}} \cdot \sigma_{p.0} = 130 \cdot MPa$
Jack stress	$\sigma_{jekk} := \sigma_{p.0} + \Delta\sigma = 1.602 \times 10^3 \cdot MPa$
Not allowed	$\sigma_{jekk} > \sigma_{p.max}$
Allowed stress at time of prestressing. jack stress	$\sigma_{p.0} := \dfrac{0.95 \cdot \sigma_{p.0} \cdot f_{p0.1k}}{\sigma_{jekk}} = 1.428 \times 10^3 \cdot MPa$
Strain in tendons after cutting:	$\varepsilon_{p.0} := \dfrac{\sigma_{p.0}}{E_p} - \Delta\varepsilon_{loss} = 6.657 \times 10^{-3}$

Stress in tendons after cutting: $\sigma_{p.0} := \varepsilon_{p.0} \cdot E_p = 1.298 \times 10^3 \cdot MPa$

Assume 5% relaxtion $\sigma'_{p.0} := 0.95 \cdot \sigma_{p.0} = 1.233 \times 10^3 \cdot MPa$

Maximum allowed initiel prestressind force after cutting EC2 -5.10.3 $\sigma_{pm0} := \min\left(0.75 \cdot f_{pk}, 0.85 \cdot f_{p0.1k}\right) = 1.391 \times 10^3 \cdot MPa$

Diameter of the tendons $\varnothing_p := 15.3mm$

Ultimate Limit State

Miscellaneous data

Effective height $d_{spenn} := h - c_{nom} - \varnothing_v - \varnothing_p - \frac{a_v}{2} = 518.7 \cdot mm$

Internal lever arm $z_{spenn} := 0.84 \cdot d_{spenn} = 435.708 \cdot mm$

Necessary tensile reinforcement: $A_p := \frac{M_{Ed}}{z_{spenn} \cdot f_{pd}} = 727.095 \cdot mm^2$

15,3mm tendon $A_{st} := 140mm^2$

Choose 6 tendons: $A_p := 6 \cdot A_{st} = 840 \cdot mm^2$

Compression reinforcement: $\varnothing'_s := 18mm$

Choose 2 Ø16 rebars in the upper part of the beam: $A'_s := 2 \cdot \frac{\varnothing'^2_s \cdot \pi}{4} = 508.938 \cdot mm^2$

Moment capacity- *according to Norwegian Concrete Association (2015)*

Characteristic compressive strength $f_{ck.fibre} := \alpha_{cc} \cdot f_{ck} = 55.25 \cdot MPa$

$$\alpha_k := \frac{A_p \cdot f_{pk} - A'_s \cdot f_{yk}}{\lambda_1 \cdot \eta_1 \cdot f_{ck.fibre} \cdot b \cdot d_{spenn}} = 0.171$$

Characteristic moment capacity requirement [4.2.2] $z := \left(1 - 0.5\lambda_1 \cdot \alpha_k\right) \cdot d_{spenn} = 484.881 \cdot mm$

$$M_{Rd.k} := \lambda_1 \cdot \alpha_k \cdot z \cdot f_{ck.fibre} \cdot b \cdot d_{spenn} + A'_s \cdot f_{yk} \cdot \left(d_{spenn} - d'\right) = 752.952 \cdot kN \cdot m$$

Compressive height ratio

$$\alpha := \frac{A_p \cdot f_{pd} - A'_s \cdot f_{yd} + h \cdot b \cdot f_{ftd.res2.5}}{\left(\lambda_1 \cdot \eta_1 \cdot f_{cd} \cdot b + f_{ftd.res2.5} \cdot b\right) \cdot d_{spenn}} = 0.291$$

$$x := \frac{A_p \cdot f_{pd} - A'_s \cdot f_{yd} + h \cdot b \cdot f_{ftd.res2.5}}{\lambda_1 \cdot \eta_1 \cdot f_{cd} \cdot b + f_{ftd.res2.5} \cdot b} = 151.098 \cdot mm$$

$$z_{spenn} := d_{spenn} - 0.5 \cdot \lambda_1 \cdot \alpha \cdot d_{spenn} = 0.461\,m$$

Momement capacity, included fibre contribution. The calculation is split in two parts, because it was not enough pace to make it fit at one line

$$M_{Rd1} := \lambda_1 \cdot x \cdot b \cdot \eta_1 \cdot f_{cd} \cdot z_{spenn} + f_{yd} \cdot A'_s \cdot \left(d_{spenn} - d'\right)$$

$$M_{Rd2} := -\left[(h - x) \cdot b \cdot f_{ftd.res2.5} \cdot \left[0.5 \cdot (h - x) - \left(h - d_{spenn}\right)\right]\right]$$

$$M_{Rd\_total} := M_{Rd1} + M_{Rd2} = 714.893 \cdot kN \cdot m$$

Min reinforcement and ductility control

Balanced reinforcement cross section

$$\alpha_{pb} := \frac{\varepsilon_{cu}}{\frac{f_{pd}}{E_p} - \varepsilon_{p.0} + \varepsilon_{cu}} = 0.814$$

Critical reinforcement area in a balanced cross-section:

$$A_{pb} := 0.8 \cdot \frac{f_{cd}}{f_{pd}} \cdot \alpha_{pb} \cdot b \cdot d_{spenn} = 3.062 \times 10^3 \cdot mm^2$$

Minimum compression reinfrocement EC2 -9.2.1.1

$$A'_s := 0.26 \cdot \frac{f_{ctm}}{f_{yk}} \cdot b \cdot (h - d') = 448.812 \cdot mm^2$$

Compression reinforcement:

$$\text{Ø}'_s := 18mm$$

Choose 2 Ø16 rebars in the upper part of the beam:

$$A'_s := 2 \cdot \frac{\text{Ø}'^2_s \cdot \pi}{4} = 508.938 \cdot mm^2$$

$$d' := c_{nom} + \text{Ø}_v + \frac{\text{Ø}'_s}{2} = 59 \cdot mm$$

Check if the compression reinforcement yield prior to concrete failure

$$\varepsilon'_s := \frac{\alpha_b \cdot d_{spenn} - d'}{\alpha_b \cdot d_{spenn}} \cdot \varepsilon_{cu} = 2.234 \times 10^{-3}$$

Yielding in compression reinforcement

$$\varepsilon_{yd} < \varepsilon'_s$$

The designed locations of the tension tendons is in agreement with the EC2 - [8.10.1.2]

Controll at the moments of prestressing:
- 7 days after casting

$$\beta_{cc} := e^{\left[0.25\cdot\left[1-\left(\frac{28}{7}\right)^{0.5}\right]\right]} = 0.779$$

Average value of compressive strength of the cocnrete, after 7 days of curing:

$$f_{cm7} := \beta_{cc}\cdot f_{cm} = 56.852\cdot MPa$$ EC2 [3.1.2](6)

$$f_{ck7} := f_{cm7} - 8MPa = 48.852\cdot MPa$$

$$f_{cd7} := \alpha_{cc}\cdot\frac{f_{ck7}}{\gamma_c} = 27.683\cdot MPa$$

Prestressing force:

$$P_0 := \sigma_{p.0}\cdot A_p = 1.09\times 10^3\cdot kN$$

$$e_s := 218.7mm$$

$$d_s := 242mm$$

Designed prestressing force:

$$N_{Ed.s} := 1.1\cdot P_0 = 1.199\times 10^3\cdot kN$$

Moment from the tension reinforcement:

$$M_{Ed.s} := N_{Ed.s}\cdot e_s = 262.321\cdot kN\cdot m$$

1) -Pure compression

Concrete compression force

$$T_{c1} := f_{cd7}\cdot b\cdot h = 5.813\times 10^3\cdot kN$$

Strain in tendon is equal to bilinear strain in concrete

$$\varepsilon_p := \varepsilon_{c1} = 2.65\times 10^{-3}$$

Tendon copression force

$$T_{p1} := \varepsilon_p\cdot E_p\cdot A_p = 434.07\cdot kN$$

Strain in comressive rebar is equal to bilinear strain in concrete

$$\varepsilon'_s := \varepsilon_{c1} = 2.65\times 10^{-3}$$

Compressive rebar compressive force

$$T_{s1} := \varepsilon'_s\cdot E_s\cdot A'_s = 269.737\cdot kN$$

Total axial force

$$N_1 := T_{c1} + T_{p1} + T_{s1} = 6.517\times 10^3\cdot kN$$

Bending moment

$$M_1 := T_{p1}\cdot e_s - T_{s1}\cdot d_s = 29.655\cdot kN\cdot m$$

2) -Balance point

Design concrete strain at failire $\varepsilon_{cu} = 2.8 \times 10^{-3}$

Design steel yielding strain $\varepsilon_{yd} = 2.174 \times 10^{-3}$

Compression height ratio $\alpha := \dfrac{\varepsilon_{cu}}{\varepsilon_{yd} + \varepsilon_{cu}} = 0.563$

Effective height, upper part in stress $d_1 := h - c_{nom} - Ø_v - \dfrac{Ø'_s}{2} = 541 \cdot mm$

Concrete compression force $T_{c2} := \lambda_1 \cdot \alpha \cdot d_1 \cdot b \cdot \eta_1 \cdot f_{cd} = 2.994 \times 10^3 \cdot kN$

Strain the tendon

$$\Delta\varepsilon_{p2} := \frac{\alpha \cdot d_1 - \left(c_{nom} + Ø_v + Ø_p + \dfrac{a_v}{2}\right)}{\alpha \cdot d_1} \cdot \varepsilon_{cu} = 2.053 \times 10^{-3}$$

Tendon compression force $T_{p2} := \Delta\varepsilon_{p2} \cdot E_p \cdot A_p = 336.205 \cdot kN$

Tensile force in comressive rebars $S_2 := f_{yd} \cdot A'_s = 221.277 \cdot kN$

Tensile force contributed from fibre $S_{2.Fibre} := \left(h - \alpha \cdot d_1\right) \cdot b \cdot f_{ftd.res2.5} = 336.696 \cdot kN$

Total axial force $N_2 := T_{c2} + T_{p2} - S_2 - S_{2.Fibre} = 2.772 \times 10^3 \cdot kN$

Bending moment

$$M_2 := T_{c2} \cdot \left(0.5 \cdot h - 0.4 \cdot \alpha \cdot d_1\right) + T_{p2} \cdot e_s + S_2 \cdot (0.5 \cdot h - d') + S_{2.Fibre} \cdot 0.5 \cdot \left(h - \alpha \cdot d_1\right) = 710.01 \cdot kN \cdot m$$

3) Under reinforce

Strain in the comression rebar $\varepsilon_s := 2 \cdot \varepsilon_{yk} = 5 \times 10^{-3}$

Compression height ratio $\alpha_3 := \dfrac{\varepsilon_{cu}}{2 \cdot \varepsilon_{yk} + \varepsilon_{cu}} = 0.359$

Concrete compression force $T_{c3} := \lambda_1 \cdot \alpha_3 \cdot d_1 \cdot b \cdot \eta_1 \cdot f_{cd} = 1.909 \times 10^3 \cdot kN$

Strain in the tendon $\Delta\varepsilon_{p3} := \dfrac{\alpha_3 \cdot d_1 - 81.3mm}{\alpha_3 \cdot d_1} \varepsilon_{cu} = 1.628 \times 10^{-3}$

Tendon compression force $T_{p3} := \Delta\varepsilon_{p3} \cdot E_p \cdot A_p = 266.64 \cdot kN$

Compressive rebar tensile force $S_3 := f_{yd} \cdot A'_s = 221.277 \cdot kN$

Tensile force contributed from fibre $S_{3.Fibre} := (h - \alpha_3 \cdot d_1) \cdot b \cdot f_{ftd.res2.5} = 462.444 \cdot kN$

Total axial force $N_3 := T_{c3} + T_{p3} - S_3 - S_{3.Fibre} = 1.492 \times 10^3 \cdot kN$

Bending moment

$$M_3 := T_{c3} \cdot (0.5 \cdot h - 0.4 \cdot \alpha \cdot d_1) + T_{p3} \cdot e_s + S_3 \cdot (0.5 \cdot h - d') + S_{3.Fibre} \cdot 0.5 \cdot (h - \alpha_3 \cdot d_1) = 545.62 \cdot kN \cdot m$$

Plot the three points in a moment-compression diagram and check it against the load situation.

Shear Strength

Without shear reinforcement -*according to Norwegian Concrete Association (2015)*

Prestressing force: $P'_0 := \sigma'_{p.0} \cdot A_p = 1.036 \times 10^3 \cdot kN$

$\gamma_p := 0.9$

Design compression force:
- Assume 15% loss of initial compression force.

$N_{Ed} := \gamma_p \cdot 0.85 \cdot P_0 = 834.166 \cdot kN$

Bending moment at beam end: $M_s := -N_{Ed} \cdot (0.5 \cdot h - 81.3mm) = -182.432 \cdot kN \cdot m$

Bending moment at midspan: $M_{felt} := M_{Ed} + M_s = 268.252 \cdot kN \cdot m$

Compression casued due to the prestressing force

$$\sigma_{cp} := \min\left(\frac{N_{Ed}}{A_c}, 0.2 \cdot f_{cd}\right) = 3.972 \cdot \frac{N}{mm^2}$$

Data
EC2 -6.2

$$\rho_l := \frac{A_p}{b \cdot d} = 4.715 \times 10^{-3}$$

$$k := 1 + \sqrt{\frac{200mm}{d}} = 1.627$$

$$C_{Rd.c} := \frac{0.18}{1.5} = 0.12$$

$$k_1 := 0.15$$

$$v_{min} := 0.035 \cdot k^{\frac{2}{3}} \cdot \left(f_{ck} \cdot \frac{mm^2}{N}\right)^{0.5} = 0.39$$

$$V_{Rd.ct} := \left[C_{Rd.c} \cdot k \cdot \left(100 \cdot \rho_l \cdot f_{ck} \cdot \frac{mm^2}{N} \right)^{\frac{1}{3}} \frac{N}{mm^2} + k_1 \cdot \sigma_{cp} \right] \cdot b \cdot d = 214.986 \cdot kN$$ EC2 [6.2.2]

COIN (2015) [4.2.5]

$$V_{Rd.cf} := 0.6 \cdot f_{ftd.res2.5} \cdot b \cdot h = 410.256 \cdot kN$$

$$V_{Rd.c} := V_{Rd.ct} + V_{Rd.cf} = 625.242 \cdot kN \quad > V_{Ed} = 300.456 \cdot kN$$ Approved, no need of shear stirrups

The designed locations of the tension tendons is in agreement with the EC2 [8.10.1.2]

Transmission of prestressing force

Eurokode 2 [8.10.2.2]

Average value of tensile strength, after 7 days of curing EC2 [3.1.2] (9)

$$f_{ctm7} := \beta_{cc} \cdot f_{ctm} = 3.505 \cdot \frac{N}{mm^2}$$

$$f_{ctd7} := \alpha_{ct} \cdot 0.7 \cdot \frac{f_{ctm7}}{\gamma_c} = 1.39 \cdot \frac{N}{mm^2}$$

Adhesion tension at the interface between concrete and tendons: EC2 [8.10.2.2] (1)

$$\eta_{p1} := 3.2 \qquad \eta_1 := 1$$

$$f_{bpt} := \eta_{p1} \cdot \eta_1 \cdot f_{ctd7} = 4.449 \cdot \frac{N}{mm^2}$$

Base value of the transmission length: EC2 [8.10.2.2] (2)

$$l_{pt} := 1.25 \cdot 0.19 \cdot Ø_p \cdot \frac{\sigma_{p.0}}{f_{bpt}} = 1.06 \cdot m$$

Design value of the transmission length: EC2 [8.10.2.2] (3)

$$l_{pt2} := 1.2 \cdot l_{pt} = 1.272\,m$$

Designed transmission length, EC2 [8.10.2.2] (4)

$$L_{disp} := \sqrt{l_{pt}^2 + d_{spenn}^2} = 1.18\,m$$

It is sufficient length according to requirements in ULS (EC2 [8.10.2.2](4) Note), Although, it does not satisfy the requirements related at the face when the tendons are cut. (EC2 [8.10.2.2](4).

Anchorage of strands in Ultimate Limit State

Anchorage length at support:

$$l_1 := 250\text{mm} + 0.5\cdot 81.3\text{mm} = 290.65\cdot\text{mm}$$

EC2 [8.10.2.3]

$$\eta_{p2} := 1.2$$

$$f_{bpd} := \eta_{p2}\cdot\eta_1\cdot f_{ctd} = 2.108\cdot\frac{N}{mm^2}$$

$$\sigma_{pd} := \frac{f_{p0.1k}}{\gamma_s} = 1.423\times 10^3\cdot\frac{N}{mm^2}$$

Neccessary anchorage length:

$$l_{bpd} := l_{pt2} + 0.19\cdot 15.3\text{mm}\cdot\frac{\left(\sigma_{pd} - \sigma_{p.0}\right)}{f_{bpd}} = 1.444\cdot\text{m}$$

Anchorage capacity of the 6 tendons, Betongelementboken volume C, Chapter C 9

$$F_{sp} := \frac{0.9\cdot 6\cdot P_0\cdot l_1}{l_{bpd}} = 1.185\times 10^3\cdot\text{kN} \quad > P_0 = 1.09\times 10^3\cdot\text{kN}$$

Approved

Ultimate Limit State

Unfortunately, there has not been executed multi-layer analysis for determining the compressive zone height in SLS.

Chapter 7

Discussion

The calculations with conventional concrete is a well known and covered concept. Available design methods of fiber reinforcement are to the authors' knowledge more limited, which has provided challenges in the work. Hence, it is in some cases necessary in the design process to make some new design approaches to calculate required reinforcement. A discussion and evaluation of design of beams are presented in this chapter.

7.1 Moment Capacity

The design method is well explained in the Norwegian Concrete Association's proposed guideline. The guideline [9, 4.2.2] state that elements where failure can lead to loss of life, large social issues or economic loss, should be made sure to have enough flexural strength by just regarding the slack-or prestressed reinforcement. In other words, the additional strength offered by the fibres should not be accounted. Nevertheless, there is an advantage because material factors and load factors for material safety and the loads can be set equal 1.0.

This means that it must be ensured that the beam have enough moment capacity in both cases. Firstly, (design check 1) were the fibres strength are utilized and the safety concept developed in Eurocode 2 [1] is applied. Secondly, (design check 2) the case where no addition from the fibres is taken into account and the safety concept from Eurocode 2 is excluded. This approach seems to be valid, but it sets a limit to how much effect the fibres will have on the flexural strength as a consequence to the fact that the fibre volume is not accounted for in design check 1.

Beam	M_k [kNm]	M_{Rd} [kNm]
Beam A-1	-	533.7
Beam A-3	-	534.7
Beam B-1	561.9	700.7
Beam B-3	714.5	601.8
Beam C-1	596.6	818.4
Beam C-3	752.9	714.9

Table 7.1: Characteristics and designed moment capacity

Table 7.1 displays different values of moment capacity with respect to concrete quality, fibre contents and slack- or prestressed reinforcement. The first column illustrates the maximum characteristics capacity without fibre contribution. The second column shows the designed moment capacity value, (M_{Rd}), with supplement from fibre where the safety factors is taken into consideration. Beam B-1 and B-3 (B35 1 vol-% fibre, conventional and prestressed reinforced, respectively) shows an increment in the capacity compared to the plain concrete designed beams, A-1 and A-3 (B35). Further will the beams C-1 and C-3 (B65 2 vol-% fibre, conventional and prestressed reinforced, respectively) provide an even greater capacity, based on the calculation method from Norwegian Concrete Association. Although, it appears that the requirement to self-bearing capacity without fibre contribution will be determinative for the conventional reinforced beams, B-1 and C-1.

7.2 Shear Design

Concrete elements do often have to resist heavy loads, which require a great amount of shear reinforcement at the supporting ends. Available space for reinforcement are often limited and make the placement complicated.

Hence, the shear strength of the designed elements was of great interest of this thesis. The design proposal of the Norwegian Concrete Association was easy to use, and gave a substantial benefit from the fibres. However, the draft have no approach to combine the contribution offered from fibres and shear stirrups. Designing a structural element without stirrups can potentially be dangerous for the structural safety, since the stirrups make the structures more ductile.

The capacity of the nine designed beams is presented in Figure 7.1. The blue bars represent the contribution offered by the conventional reinforcement, while the red bars represent the contribution offered by the fibres.

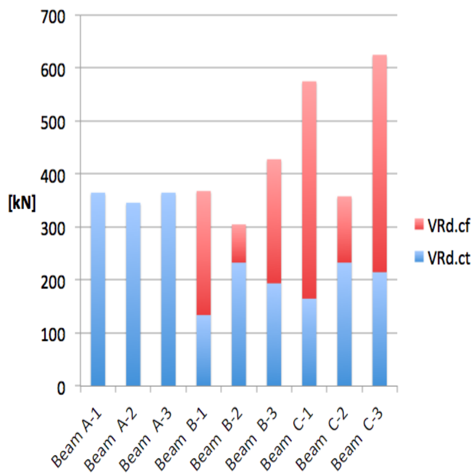

Figure 7.1: Shear capacity of the designed beams

The fibres provided a significant increase in the shear capacity. In all design cases offered the fibres the opportunity to drop some of the conventional shear reinforcement. There were no need of shear stirrups in beam B-1, B-3, C-1 and C-3. The fibres also allowed some of the conventional reinforcement in the dapped-end to be skipped, which is discussed more detailed in Chapter 7.4.

Beam A-3 had the same shear capacity as beam A-1, this is because they were designed with the same amount of shear reinforcement. The positive effect offered by the prestressed reinforcement disappears in cracked condition. This is because in order to activate the shear stirrups, the beam has to crack. In other words, the benefits from the compressive force disappears and the shear reinforcement has to carry the entire load.

7.3 Anchorage of Tensile Reinforcement

The anchorage of the slack tensile reinforcement has been done according to chapter [8.4] Eurocode 2, while it has been done according to to chapter [8.10.2.3] for the prestressed reinforcement. The guideline draft for fibre reinforced concrete, given by the Norwegian Concrete Association [9] does not say anything about the anchoring capacity of FRC. However, the proposed guideline from COIN [19, 6.4.6] state: -" When fibre- and conventional reinforcement are used together, shall still the design rules given in Eurocode 2 for anchorage of reinforcement be applied."

Thereby, have the design rules given in Eurocode 2 been applied in all the beams in this thesis. Some people would say this is conservative, since the fibres will increase the tensile strength of concrete, contributing to larger forces in order to pull out the tensile reinforcement. However, conducted cast tests with FRC have showed that the concrete do not always properly float out and fill the formwork. This would hurt the anchorage capacity.

7.4 Dapped-End Design

7.4.1 Comparison of Calculation Method 1 and 2

The principles behind Model 1 are based on the calculation method for dapped-end beam with inclined reinforcement [30]. By replacing the inclined reinforcement with a tensile fibre stress path is the contribution from fibre taken into account. With a fibre stress path orientation of 45 degrees with respect to the horizontal axis, the corresponding crack propagation is expected to occur in the beam nose with an angel of 45 degrees, orthogonally to the fibre stress path orientation.

Backe-Hansen and Hamstad [7] proposed a second calculation method in their Master‘s Thesis. In similarity to Model 1, was the inclined reinforcement also for this proposed method skipped. In contrast were the fibre stress path considered to attack parallel with the hanger reinforcement.

There was conducted capacity calculations for both of the methods. However, Model 1 is considered more relabel since the intended crack path propagated with an angel of 45 degrees. Although, Method 2 was assessed as an upper limit for fibre contribution.

7.4.2 Effective Fibre Length and its Orientation

The effective fibre area is quite essential in the determination of the contribution from fibre into dapped-end capacity. The effective fibre width is normally set equal the beam‘s width. The fibre length is on the other hand more comprehensive to decide. Since there has not been conducted any experiment researches in this thesis, the evaluation of the fibre length is based on literary research.

Backe-Hansen and Hamstad [7] conducted capacity tests of FRC dapped-ended beams. They also made some numerical analysis where the stress development within the beam were examined. The length and orientation of the stress path where optimized to fit the results. The outcome, was a fibre length of 170 mm and an angle of 56 degrees with respect to the horizontal axis. Hence, the crack will propagate with an angle of 25 degrees, orthogonally to the stress path. In the Master‘s Thesis of Kittelsen et al. [20] was a slightly different approach used. The value to the fibre length was given as the half of the beam height with a path orientation of 45 degrees. A path orientation angle of 45 degrees is more conservative than 65 degrees, which was used in Backe-Hansen and Hamstad Master‘s Thesis. Kittelsen et al. [20] documented the crack development in their beams with pictures, during failure propagation. After examining the pictures, should a crack angle smaller than 45 degrees be considered in dapped-ended beams with hanger and main nib reinforcement. It seemed like the crack angle increased in line with larger amounts of hanger reinforcement. It was assumed by the authors that the fibres would counteract the hangers effect. A path orientation of 45 degrees was therefore chosen in this thesis.

An effective fibre length of half the nib height was chosen in Kittelsen et al. [20] Master‘s thesis. This assumption is uncertain, but was applied in this thesis due to limited material found on the subject.

7.4.3 Discussion of Capacity

The intention of this was to utilize the fibres in order to reduce the amount of conventional reinforcement. This was successfully accomplished.

Beam / Vertical force into the support	*Incline* (stirrup or fibre)	*Hanger reinforcement*
Beam A-2, B35	112.18kN (1$\phi_{s\alpha}$16 stirrup)	233.4kN (2ϕ_{sv}14 stirrup)
Beam B-2, B35 1 vol-% fibre	71.58kN (fibre)	233.4kN (2ϕ_{sv}14 stirrup)
Beam C-2, B65 2 vol-% fibre	125.49kN (fibre)	233.4kN (2ϕ_{sv}14 stirrup)

Table 7.2: Model-1, Support-load capacity

Calculation based on Model 1 gave good results in the light of high fibre contribution. As it is depicted in Table 7.3, the fibre contribution will in both B35 1 vol-% and B65 2 vol-% FRC manage to replace the inclined reinforcement. Nevertheless, it should be mention that "Betongelementboken" [30] recommends that the hanger reinforcement should manage to carry two-thirds of the reaction force. This requirement state a minimum limit of required hanger reinforcement. However, this requirement was disregarded to allow a higher contribution from the fibres.

Beam / Vertical force into the support	*Fibre* (parallel to the hanger)	*Hanger reinforcement*
Beam B-2, B35 1 vol-% fibre	170,94kN	233.4kN (2ϕ_{sv}14 stirrup)
Beam C-2, B65 2 vol-% fibre	125.49kN	233.4kN (2ϕ_{sv}14 stirrup)

Table 7.3: Model-2, Support-load capacity

Table 7.3 illustrates the capacity in beam- B-2 and C-2 based on Model 2 calculation. It is obvious that this gives greater results, since it use the same fibre length as in model 1 and the fibre stress path is orientated vertical to the horizontal axes.

In accordance with the principles of Mohr‘s circle, would a 90 degree orientated main crack with respect to the horizontal axis be unlikely for FRC. The fibres increase the tensile capacity of the concrete, which provide tensile resistance in the lower cross-sectional area of the beam. This resistance will decrease the crack angle propagation.

7.5 Serviceability Limit State

Tensile reinforcement

The minimum reinforcement rules are given to prevent brittle failures. In other words, ensuring that the beam maintains capacity after cracking (the moment resistance is larger than the cracking moment). Two design methods were presented in Chapter 5.2. The value of $\left(1 - 2.1 \times \frac{f_{tk.res2.5}}{f_{ctm}}\right)$ is an important factor in Equation 5.7. If the value of this factor is negative, is no minimum conventional reinforcement required in the ultimate limit state according COIN's [19] design approach. The same applies for the factor $\left(1 - \frac{f_{Ftsm}}{f_{ctm}}\right)$ given by the Norwegian Concrete Association [9] (the factor is found in Equation 5.10).

Table 7.4 shows the values of the two factors discussed above, for the fibre reinforced straight-ended beams.

				COIN	Norwegian Concrete Association
Concrete Mix	f_{ctm} [MPa]	f_{Ftsm} [MPa]	$f_{ftk.res2.5}$ [MPa]	$\left(1 - 2.1 \times \frac{f_{tk.res2.5}}{f_{ctm}}\right)$	$\left(1 - \frac{f_{Ftsm}}{f_{ctm}}\right)$
B35, 1 vol-%	3.2	3.9	2.8	-1.2	-0.2
B65, 2 vol-%	4.5	7.1	4.9	-1.7	-0.6

Table 7.4: Factors in the Equations for the minimum tensile reinforcement

As seen in Table 7.4, are no minimum tensile reinforcement required in the designed fibre reinforced beams. The fibres provide enough capacity to ensure ductility of the beam.

Shear Reinforcement

Table 7.5 present the shear reinforcement ratio of the two FRC mixtures, which is used in Equation 5.8 to calculate the maximum center distance between the stirrups. The negative values expresses that no stirrups are required.

				COIN	Norwegian Concrete Association
Concrete Mix	f_{ctm} [MPa]	f_{Ftsm} [MPa]	$f_{ftk.res2.5}$ [MPa]	$\rho_{w.min} = \frac{0.1\sqrt{f_{ck}}-0.3f_{ftk.res2.5}}{f_{yk}}$	$\rho_{w.min} = \frac{0.1\sqrt{f_{ck}}-0.2f_{Fstm}}{f_{yk}}$
B35, 1 vol-%	3.2	3.9	2.8	-5.0×10^{-4}	-3.8×10^{-4}
B65, 2 vol-%	4.5	7.1	4.9	-1.3×10^{-3}	-1.2×10^{-3}

Table 7.5: Factors in the Equations for the minimum shear reinforcement

Multi-Layer Procedure

It was a bit tricky to program the multi-layer procedure in Excel 2013. After some trial-and-error, it become clear that it would require too much time to compute the program. We were very fortunate to hear that Elena Vidal Sarmiento, a Ph.d. student here at NTNU already had developed a well working excel sheet of the procedure. Many methods have been developed based on different material properties. Sarmiento‘s excel sheet was based on the following material stress-strain relationships:

- Parabola-rectangle stress-strain relationship of the FRC in compression.
- Elastic-plastic stress-strain relationship of the steel in tension.
- Rigid-plastic stress-strain relationship of the FRC in tension. Figure 7.2 illustrate the stress-strain deformation of the 2 vol-% fibre reinforced concrete.

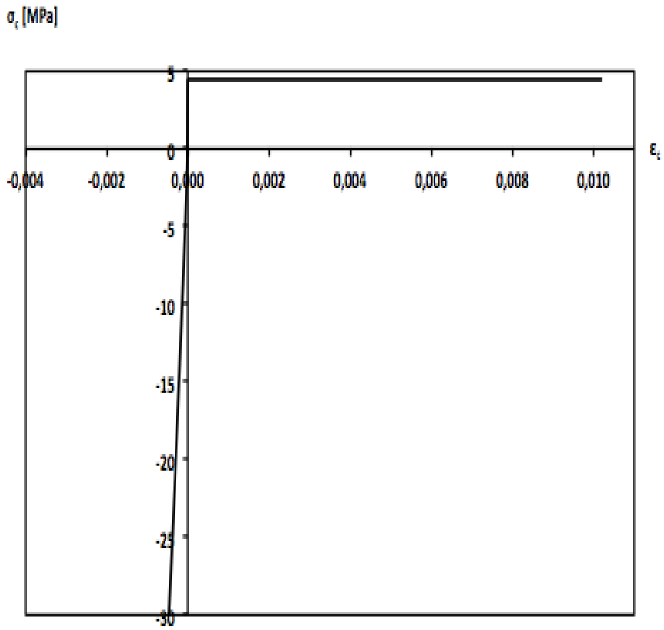

Figure 7.2: Illustration of the B65, 2 vol-% fibre reinforced concrete in tension

The multi-layer procedure was performed to calculate the tensile stress in the reinforcement, when the beam was subjected to service load. The model ran two times, to get the curvature and strains of the two slack reinforced FRC beams. The strain at the bottom of the beam increased by 0.0005 for each iterative step. The service bending moment of the FRC designed beams were passed in iterative step two. The strain and curvature for both examined beams had to be interpolated between iterative step one and two. The curvature of the FRC beams showed very low values. This indicate that the FRC beams would have minimal deflection in cracked condition ($Stadium$ 2).

7.6 Crack Width Calculation

Implementation of fibres into the concrete is well suited to limit the crack widths. The maximum calculated cracks widths are presented in Table 7.6. The results from both COIN and Norwegian Concrete Association's guideline were compared to check their accordance. The crack width calculation for the plain concrete designed beams, A-1 and A-3, were based on design formulas from Eurocode2 [1].

Beam / crack width	w_d COIN (2011) mm	w_d NCA (2015) mm
Beam A-1, B35	0.197 (EC2)	-
Beam A-3, B35 prestressed	0.124 (EC2)	-
Beam B-1, B35 1 vol-%	0.069	0.145
Beam C-1, B65 2 vol-% fibre	0.026	0.058

Table 7.6: Calculated crack widths

Comparison of the results of beam A-1 and B-2, revealed the fibres significant effectiveness to limit crack widths. COIN‘s design approach is based on the Eurocode 2 [1], while the Norwegian Concrete Association‘s approach is based on the next generation of Eurocode.

Crack width calculation based on Norwegian Concrete Association‘s method showed a slightly larger value. This might be due to conservative values for the factors that regulate the influence to the load situation. The calculated crack width for the 2 vol-% fibre reinforced was less than a half of calculated crack with for the 1 vol-% FRC beam. Although, this was casted width a B65 concrete. This made it hard to tell the fibres effect to limit crack width

7.7 Stiffness

The beams stiffness were examined in the research. The multi-layer procedure were run for both beam B-1 and B3 to find the bending moment to curvature ratio. This stiffness were then used to calculate the deflection of the two beams. The results is presented in Table 7.7.

Beam	δ mm
Beam A-1, B35	14.507
Beam A-3, B35 prestressed	2.155
Beam B-1, B35 1 vol-%	8,885
Beam C-1, B65 2 vol-% fibre	5.328

Table 7.7: Deflection in cracked cross-section (stadium II)

Table 7.7 show a significant reduction in beam deflection for FRC beams. Furthermore, it show that the fibres increase the beams stiffness.

7.8 Splitting Tensile Forces

The splitting tensile forces were examined in the FRC dapped-ended beams. A effective area in the plane of the cross section, perpendicular to the tensile force (Z_{s1} in Equation 5.28) was assumed to carry the tensile stress. This assumption gave a substantial increase in the capacity of the beam. Allowing us to skip the horizontal hoops in the nib of the beam. The results of the design is summed up in Table 7.8:

Beam	B-2	C-2
Splitting tensile force Z_{s1} [kN]	47	47
Capacity SP_{std} [kN]	106	186

Table 7.8: Splitting tensile force and capacity

For Beam C-3, the fibre reinforcement provided the same splitting tensile strength as almost 2 Ø12 hoops.

Conclusion

This thesis reveals several properties that can be improved by use of fibre reinforced concrete in design of structural elements. Among them was a significant increase of beam stiffness and shear- and moment capacity. Although, perhaps the most promising of they all was the crack width limitation, which confirm FRC's huge potential in acidic environments.

The fibres allowed some of the traditional reinforcement in the dapped-ended beams to be omitted. However, the fibres efficiency varied at extent. The hanger reinforcement was hard to replace in the designed beams. It had to take the majority of the force down to the support. The fibres allowed the frequently used inclined reinforcement to be skipped. The splitting tensile forces were also sufficiently taken care of by the fibres, it allowed the horizontal hoops to be skipped.

All the fibre reinforced beams demonstrated a significant increase in shear strength capacity. In fact, all the shear reinforcement was skipped in the straight-ended beams.

FRC is expansive to manufacture compared to conventional concrete. In addition is fibre reinforced concrete very sensitive in its fresh state, requiring highly skilled workers.

Therefore, FRC may have the greatest potential in structural elements which are hard to design with conventional reinforcement. For instance, heavy reinforced structural elements where there are limited space to put the reinforcement. In these cases the fibres can contribute with additional required strength.

Further Work

Due to limited time when writing this thesis, it was not possible to conduct all the research that initially were intended. Furthermore, many interesting aspects were discovered during the work of the thesis. The studies listed below could be subjects for further work:

- Casting and testing of the author‘s designed beams. How does the failure correspond with the design load?
- The Concrete Association‘s guideline draft provide only design rules for calculating the shear strength of FRC beams without shear stirrups. It would be interesting to research the shear capacity of FRC beams with shear stirrups. A combination of those would provide great shear capacity.
- More knowledge about the designed method based on effective fibre length ($L_{eff.fiber}$) as a design parameter. A more precise and applicable definition of the $L_{eff.fiber}$ would be necessary in order to standardize the method.
- The benefits offered by the fibre reinforcement is revealed in this thesis. However, it come at a price. It would be interesting to research the economic expenses related to casting of the author‘s designed beams.

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Load scenario

Load values for the load scenario is fetched from EC1-load on structure, where value of the live load is based on an office situation. Values for hollowcore is fetched from Spenncon Consolis.

Beam design

Width of beam: $b := 350mm$

Height of beam: $h := 600mm$

Corss- section area: $A_c := h \cdot b = 0.21 \cdot m^2$

Length of the beam: $l_b := 6m$

HD 265 hollow core:

Span of hollow core $l_h := 10m$

Load values:

HD 265 hollow core: $g_{hk} := 3.95 \frac{kN}{m^2}$

5cm floor screed: $g_{sk} := 0.1 \frac{kN}{m^2}$

Density of concrete: $\rho_c := 26 \frac{kN}{m^3}$

Self-weight load, beam: $g_{bk} := \rho_c \cdot A_c = 5.46 \cdot \frac{kN}{m}$

Total of dead loads: $g_k := l_h \cdot \left(g_{sk} + g_{hk}\right) + g_{bk} = 45.96 \cdot \frac{kN}{m}$

Office load (live load): $p_k := l_h \cdot 3 \frac{kN}{m^2} = 30 \cdot \frac{kN}{m}$

Serviceability limit state

Characteristic load: $q_k := g_k + p_k = 75.96 \cdot \frac{kN}{m}$

Characteristic moment: $M_k := \frac{q_k \cdot l_b^2}{8} = 341.82 \cdot kN \cdot m$

Characteristic shear force: $V_k := q_k \cdot \frac{l_b}{2} = 227.88 \cdot kN$

Ultimate limit state

Design load $q_{Ed} := 1.2 \cdot g_k + 1.5 \cdot p_k = 100.152 \cdot \frac{kN}{m}$

Design moment $M_{Ed} := \frac{q_{Ed} \cdot l_b^2}{8} = 450.684 \cdot kN \cdot m$

Design shear force $V_{Ed} := q_{Ed} \cdot \frac{l_b}{2} = 300.456 \cdot kN$

B35

Data

Concrete B35

Compressive strength: $f_{ck} := 35MPa$

Safety factor: $\gamma_c := 1.5$

Reduction factor: $\alpha_{cc} := 0.85$ $\alpha_{ct} := 0.85$

Design compressive strength: $f_{cd} := \alpha_{cc} \cdot \frac{f_{ck}}{\gamma_c} = 19.833 \cdot MPa$

5 % fractile of tensile strength: $f_{ctk.0.05} := 2.2MPa$

Design tensile strength: $f_{ctd} := \alpha_{ct} \cdot \frac{f_{ctk.0.05}}{\gamma_c} = 1.247 \cdot MPa$

Average compressive strength: $f_{cm} := 43MPa$

Average tensile strength: $f_{ctm} := 3.2MPa$

Modulus of elasticity: $E_{cm} := 34000 \cdot MPa$

Design strain at failure: $\varepsilon_{cu} := 0.0035$

Bilinear strain $\varepsilon_{c1} := 0.00225$

Steel B500C

Yield strength	$f_{yk} := 500 \cdot MPa$
Safety factor	$\gamma_s := 1.15$
Design strength	$f_{yd} := \frac{f_{yk}}{\gamma_s} = 434.783 \cdot MPa$
Modulus of elasticity	$E_s := 2 \cdot 10^5 \cdot MPa$
Characterisic yield strain	$\varepsilon_{yk} := 2.5 \cdot 10^{-3}$
Design yield strain:	$\varepsilon_{yd} := \frac{\varepsilon_{yk}}{\gamma_s} = 2.174 \times 10^{-3}$
Maximume value of allowed strain:	$\varepsilon_{ud} := 0.03$

Concrete B35 1% fibre reinforced concrete

Data

Compressive strength: $f_{ck} := 35 \cdot MPa$

Safety factor: $\gamma_c := 1.5$

Reduction factor: $\alpha_{cc} := 0.85$

Design compressive strength: $f_{cd} := \alpha_{cc} \cdot \frac{f_{ck}}{\gamma_c} = 19.833 \cdot MPa$

Average compressive strength: $f_{cm} := 43 \cdot MPa$

Residual flexural tensile strength(2,5 mm crack width): $f_{R.3} := 7.53MPa$

Residual flexural tensile strength(2,5 mm crack width) $f_{R1k} := 6.11 \cdot MPa$

Residual tensile strength: $f_{ftk.res2.5} := 0.37 \cdot f_{R.3} = 2.786 \cdot MPa$

Safetcy factor of FRC: $\gamma_{cf} := 1.5$

Design tensile strength: $f_{ftd.res2.5} := \frac{f_{ftk.res2.5}}{\gamma_{cf}} = 1.857 \cdot MPa$

5 % fractile of tensile strength, without fibers: $f_{ctk.0.05} := 2.2MPa$

$\alpha_{ct} := 0.85$

Design tensile strength, without fibers: $f_{ctd} := \alpha_{ct} \cdot \frac{f_{ctk.0.05}}{\gamma_c} = 1.247 \cdot \frac{N}{mm^2}$

Average tensile strength: $f_{ctm} := 3.2 \cdot MPa$

Modulus of Elasticity: $E_{cm} := 34000MPa$

$\varepsilon_{su} := 0.025$

$\varepsilon_{c1} := 0.00225$

Design strain at failure: $\varepsilon_{cu} := 0.0035$

Characteristic compressive stregth at CMOD=0.5 $f_{Ftsk} := 0.45 \cdot f_{R1k} = 2.749 \cdot MPa$

Concrete cover $c_{nom} := 35mm$

Steel B500C

Yield strength $f_{yk} := 500 \cdot MPa$

Safety factor $\gamma_s := 1.15$

Design strength $f_{yd} := \frac{f_{yk}}{\gamma_s} = 434.783 \cdot MPa$

Modulus of elasticity $E_s := 2 \cdot 10^5 \cdot MPa$

Characterisic yield strain $\varepsilon_{yk} := 2.5 \cdot 10^{-3}$

Design yield strain: $\varepsilon_{yd} := \frac{\varepsilon_{yk}}{\gamma_s} = 2.174 \times 10^{-3}$

Maximume value of allowed strain: $\varepsilon_{ud} := 0.03$

$f_{yd'} := 380MPa$

Beam

Width $b := 350mm$

Height $h := 600mm$

Nib length $l_{nib} := 210mm$

Nib height $h_{nib} := 300mm$

Support plate

Thickness $t_p := 10mm$

Width $w_p := 80mm$

Reinforcement layout in dapped end

Hanger stirrup $\text{Ø}_{sv} := 14mm$

Incline stirrup $\text{Ø}_{s\alpha} := 16mm$

Main rebar in nib $\text{Ø}_{sn} := 16mm$

Horizontal hoop $\text{Ø}_{sb} := 10mm$

Compression rebar $\text{Ø}'_{sn} := 10mm$

End anchoring hoop $\text{Ø}_{se} := 14mm$

Nib heigth $h_{nib} := \frac{h}{2} = 300 \cdot mm$

Nib length $l_{nib} := 0.7 \cdot h_{nib} = 210 \cdot mm$

Miscellaneous data

Covering, [EC2 4.4.1] $c_{nom} := 35mm$

Vertical spacing $a_v := 32mm$

Horizontal spazing $a_h := 70mm$

Horizontal rebar $\text{Ø}_s := 25mm$

Shear reinforcement $\text{Ø}_v := 15mm$

Effective height $d := h - c_{nom} - \text{Ø}_v - \text{Ø}_s - \frac{a_v}{2} = 509 \cdot mm$

Concrete B65 2% fibre reinforced concrete

Data

Compressive strength: $f_{ck} := 65\frac{N}{mm^2}$

Safety factor: $\gamma_c := 1.5$

Reduction factor: $\alpha_{cc} := 0.85$

Design compressive strength: $f_{cd} := \alpha_{cc}\cdot\frac{f_{ck}}{\gamma_c} = 36.833\cdot\frac{N}{mm^2}$

Average compressive strength: $f_{cm} := 73\frac{N}{mm^2}$

Residual flexural tensile strength(2,5 mm crack width): $f_{R.3} := 13.2\frac{N}{mm^2}$

Residual flexural tensile strength(2,5 mm crack width) $f_{R1k} := 11.0\cdot\frac{N}{mm^2}$

Residual tensile strength: $f_{ftk.res2.5} := 0.37\cdot f_{R.3} = 4.884\cdot\frac{N}{mm^2}$

Safetcy factor of FRC: $\gamma_{cf} := 1.5$

Design tensile strength: $f_{ftd.res2.5} := \frac{f_{ftk.res2.5}}{\gamma_{cf}} = 3.256\cdot\frac{N}{mm^2}$

5 % fractile of tensile strength, without fibers: $f_{ctk.0.05} := 3.1\frac{N}{mm^2}$

$\alpha_{ct} := 0.85$

Design tensile strength, without fibers: $f_{ctd} := \alpha_{ct}\cdot\frac{f_{ctk.0.05}}{\gamma_c} = 1.757\cdot\frac{N}{mm^2}$

Average tensile strength: $f_{ctm} := 4.5\frac{N}{mm^2}$

Modulus of Elasticity: $E_{cm} := 40\frac{kN}{mm^2}$

$\varepsilon_{su} := 0.025$

$\varepsilon_{c1} := 0.00265$

Design strain at failure: $\varepsilon_{cu} := 0.0028$

Characteristic compressive stregth at CMOD=0.5 $f_{Ftsk} := 0.45 \cdot f_{R1k}$

Concrete cover $c_{nom} := 35mm$

Steel B500C

Yield strength $f_{yk} := 500 \cdot MPa$

Safety factor $\gamma_s := 1.15$

Design strength $f_{yd} := \frac{f_{yk}}{\gamma_s} = 434.783 \cdot MPa$

Modulus of elasticity $E_s := 2 \cdot 10^5 \cdot MPa$

Characterisic yield strain $\varepsilon_{yk} := 2.5 \cdot 10^{-3}$

Design yield strain: $\varepsilon_{yd} := \frac{\varepsilon_{yk}}{\gamma_s} = 2.174 \times 10^{-3}$

Maximume value of allowed strain: $\varepsilon_{ud} := 0.03$

$f_{yd'} := 380MPa$

Beam

Width $b := 350mm$

Height $h := 600mm$

Nib length $l_{nib} := 210mm$

Nib height $h_{nib} := 300mm$

Support plate

Thickness $t_p := 10mm$

Width $w_p := 80mm$

Reinforcement layout in dapped end

Hanger stirrup	$Ø_{sv} := 14mm$
Incline stirrup	$Ø_{s\alpha} := 16mm$
Main rebar in nib	$Ø_{sn} := 16mm$
Horizontal hoop	$Ø_{sb} := 10mm$
Compression rebar	$Ø'_{sn} := 10mm$
End anchoring hoop	$Ø_{se} := 14mm$
Nib heigth	$h_{nib} := \frac{h}{2} = 300 \cdot mm$
Nib length	$l_{nib} := 0.7 \cdot h_{nib} = 210 \cdot mm$

Miscellaneous data

Covering, [EC2 4.4.1]	$c_{nom} := 35mm$
Vertical spacing	$a_v := 32mm$
Horizontal spazing	$a_h := 70mm$
Horizontal rebar	$Ø_s := 25mm$
Shear reinforcement	$Ø_v := 15mm$
Effective height	$d := h - c_{nom} - Ø_v - Ø_s - \frac{a_v}{2} = 509 \cdot mm$