NTNU – Trondheim
Norwegian University of
Science and Technology

Analysis of selected Multiphase metering concepts

Juliana Yotham Mwalyepelo

Petroleum Engineering
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Supervisor: Milan Stanko, IPT

Norwegian University of Science and Technology
Department of Petroleum Engineering and Applied Geophysics

Analysis of Selected Multiphase Metering Concepts

By

Juliana Yotham, Mwalyepelo

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Supervisor: Prof. Milan Stanko
Co - supervisors: Prof. A. Temu
Dr.E. Masanja
Miss G. Mwita

Norwegian University of Science and Technology
Faculty of Engineering Science and Technology
Department of Petroleum Engineering and Applied Geophysics

NTNU – Trondheim
Norwegian University of
Science and Technology

ABSTRACT

This thesis work aimed at evaluating existing choke models that are used in oil and gas industry and thereafter to develop a new choke model for predicting mass flow rate through restrictions. Evaluation of the existing model was done by using flow rate data provided by Schüller papers of 2003 and 2006. Individual data sets were simulated in HYSYS to get thermodynamics properties of the fluids to be used as inputs in the models. Only three existing choke models were selected for model evaluation i.e. Sachdeva, et al., 1986, Perkins, 1993 and Al-Safran, et al., 2007 models.

The three selected models were programmed in Excel using Visual Basic for Applications (VBA) and error analysis of each model was performed using three evaluation criteria i.e. average relative error, average absolute and standard deviation.

From the evaluation performed using Schüller data, Sachdeva, et al., 1986 model was found to have average relative error of 1.76%, average absolute error of 10.52% and standard deviation of 12.49%. Perkins, 1993 model gave an average relative error of -24.17%, average absolute error of 30.74% and standard deviation of 25.91%. Al-Safran, et al., 2007 model exihibited an average relative error of -10.10%, absolute error of 17.48% and standard deviation of 17.6%.

Based on the evaluation results Sachdeva model was found to be the best model for predicting mass flow rate through restriction. Sachdeva model was therefore modified and improved by introducing a slippgae factor. Error analysis of the modified Sachdeva model showed an average relative error of -0.4%, average absolute error of 6.12% and standard deviation of 7.66%. The modified Sachdeva model was calibrated using the concept of discharge coefficient (CD) and the best value of CD for model calibration was found to be 0.65.

The new choke model is proposed based on the model developed by Sachdeva, et al., 1986. The model improves the predictability considerably by reducing half the deviation from the original Sachdeva model, all the prediction seem to be within the range of 10% accurancy. However, it is recommended to evaluate the new model using more experimental data.

From the evaluation results of this thesis work it can be seen that different experimental data yields different evaluation results for the same model, Sachdeva, et al., 1986 evaluated his model using Pilehvari, 1980 and Ashford, 1974 data and obtained a standard deviation of 10.8% for Pilehvari critical data while Ashford data exihibited standard deviation of 13.8%. The same Sachdeva model showed a standard deviation of 12.49% when evaluated by using Schüller data.

Lastly the findings from this work recommend that more experimental data should be added for model evaluation in order to authenticate the accuracy of the model. It also recommends that data from other researchers who have done experiments on multiphase flow through chokes should be used to test the model.

ACKNOWLEDGMENTS

I would like to give much thanks to all whose suggestions and contribution helped me to carry out this thesis work. Indeed it is not possible to mention all but there are some who had significant contribution and a played greater role in the completion of this report and with respect to their contribution I must mention some of them.

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NOMECLATURE

Abbreviations

GVF Gas volume fraction
LHS Left hand side
LVF Liquid volume fraction
MFM Multiphase flow metering
MPFM Multiphase flow Meter
MPFMs Multiphase flow meters
PVT Pressure, volume and temperature
RHS Right hand side
VBA Visual Basic for Application
WC Water cut
WLR Water liquid ratio

Symbols

$\dot{m}_l$ Mass flow rate during two phase flow, (kg/sec)
$\dot{m}_{ls}$ Mass flow rate during a single phase flow with two phase pressure drop, (kg/sec)
ΔP_l Pressure drop when the liquid flows alone in Chisholm model ,(psi)
C_C Contraction coefficient
C_D Discharge coefficient
C_T Valve throating coefficient(A_T/A_1)
C_X (A_1/A_3), Inlet outlet ratio
C_p Heat capacity at constant pressure,(kJ/kgmol-c)
C_v Heat capacity at constant volume,(kJ/kgmol-c)
N_m(e) Measured count rate of the mixture at energy level (e)
N_o(e) Count rate of the empty pipe
R_i Logarithmic count rate of the component i
U_i Velocity of a phase component i, (m/sec)
d_i Pipe inner diameter, (m)
e_1 Energy level 1
e_2 Energy level 2
$\dot{m}$ Mass flow rate, (kg/sec)
p_r Pressure ratio (p_2/p_1) in Perkins model
x_i Phase fraction of component i
g_c Gravitational constant, 32.2 (lbmft/lbfsec^2)
α_l Constant defined in equation (21)
μ_i Linear attenuation coefficient of the component i
ρ_e Momentum Mixture density
ρ_i Density of a component i, (kg/sec)
ρ_m Density of the mixture, (kg/m^3)
Δp Pressure drop, (pa)
A Total Cross section area, (m^2)
h Elevation, m
B Geometrical and installation constant in Chilshom model
E Internal energy, (Joules, J)

F	Force resulting from fall of pressure, (newton,N)
G	Mass flux, (kg/m^2/sec)
M	Molecular weight,(kgmol)
N	Number of data points
R	Slip ratio
U	Velocity of the mixture, (m/sec)
W	Flow rate by weight, (newton/sec, N/sec)
Z	Gas compressibility factor
d	Diameter, (m)
f	Mass fraction in Perkins model
g	Acceleration due to gravity, (m/sec^2)
k	Ratio of specific heats (Cp/Cv)
n	Polytropic exponent of gas(-vdp/pdv)
p	Pressure, (pa)
r	Pressure ratio (p_2/p_1) in Al- Safran model
v	Specific volume,(m^3/kg)
y	Pressure ratio(p_2/p_1) in sachdeva model and improved sachdeva model
α	Constant defined in equation (28) and equation E - 20
β	Parameter in Selmer-Olsen slip model =5.0
ε	Error
λ	Constant defined in equation (19)
ξ	Parameter in Selmer-Olsen slip model =0.6
σ	Standard deviation

Subscripts

ε_1	Mean relative error
ε_2	Mean absolute error
1	Upstream condition
2	Choke (throat)position
$2'$	Position just downstream the choke
3	Choke's recovered condition
B	The back wall after sudden expansion in Selmer-Oslen model
$Pred$	Predicted value
c	Throat/choke/critical
g	Gas
gm	Gas at metering condition
l	Liquid
m	Mixture
$meas$	Measured value
o	Oil
p	Pipe
w	Water

CHAPTER 1

1 INTRODUCTION

Multiphase flow metering (MFM) is a measurement of flow rates of an individual phase in a multiphase flow. There are two main techniques used in measuring flow rates in multiphase flow, one is by using test separators and the second technique is by using multiphase flow meters (MPFMs).

Test separators happen to be the most common known method for metering multiphase flow in oil and gas industry. Metering principles applied by the test separators relies on the separation of the individual phases and then measuring the output of the separated fluids using conventional single phase techniques such as orifice plates for gas and turbine meters for oil. However, there are some disadvantages associated with test separators such as their bulkiness and therefore occupying a lot of space, the long time needed to stabilize the fluid so as to obtained reliable outputs and high installation and maintenance cost. In addition to that operation conditions sometimes prevent complete separation of the fluid phases, these conditions cause errors in separation instruments, which are designed to measure streams of single phase gas, oil or water.

MPFMs have also been used to measure multiphase flow in oil and gas industries for over 20 years now. The good thing about MPFMs is, they are capable of measuring the flow rates of each component directly without separation and they provide real time data. Principles applied by MPFMs differ from one manufacturer to another some MPFMs for example; Framo uses Venturi and dual gamma ray densitometers, and others for example Roxar and FlowSys use capacitance, inductance and venturi.

Dual gamma ray MPFMs give a lot of information, much more information than the information given by the test separators. All parameter measured are usually employed as input for numerical models based on the laws of conservation of energy, momentum and mass to predict flow rates.

The aim of this thesis is to develop a numerical model for predicating mass flow rate through restriction. Having the measured data from the MPFMs such as water cut (WC), Liquid volume fraction(LVF), Gas volume fraction(GVF) and pressure drop across the restriction a model for Multiphase flow through restriction can be employed to calculate either the total mass flow rate of the mixture or the common velocity of the mixture. The model can be used to validate the flow rate measurements given by the meter, it can also be used to calculate the mass flow rate when the MPFM is not available. The multiphase flow meter chosen to analyse Multiphase flow through restrictions is PhaseWatcherVx , the latest version of Framo MPFM.

Evaluations of different pressure drop models that are used in oil and gas industry to predict mass flow rate through restrictions have been done in this work. The results for model evaluation are as shown in CHAPTER 5. A new choke model was developed by modifying Sachdeva, et al., 1986 model following the results that Sachdeva model was found to be the best model for prerdicting mass flow rate through restriction in this study.

Section 2.2 describes the multiphase flow meter PhaseWatcherVx, its working principle, the variables measured and its attempt to clarify the computational steps that are taken in order to compute flow rates. The extra information that is needed for the model inputs like gas and liquid properties are found by simulating the flow rate data in HYSYS software. Explanation on how the flow can be simulated in HYSYS software and the procedures for HYSYS simulation are found in Appendix B.

CHAPTER 3 shows evaluation of different choke models that exits for predicting mass flow rates across restriction for multiphase flows. Only three models have been evaluated in this work, i.e. models of (Al-Safran & Kelkar, 2007), (Perkins, 1993) and that of (Sachdeva, et al., 1986). CHAPTER 4 shows the modified Sachdeva model that considers the slippage effect between gas phase and liquid phase for calculating mass flow rate through restriction.

For comparison reasons Perkins, 1993 model was also modified to see if it could perform better. mathematical derivation of the model and the results are found in Appendix E. This Appendix is included so as to justify that the improved Sachdeva model is the one which gives best prediction of mass flow rate through restrictions. All information concerning the modified Perkins model are found in Appendix E.

Explanation on how the flow can be simulated in HYSYS is found in section 3.3 and the simulation results are found in Appendix D, simulation procedures in HYSYS and a full HYSYS model are found in Appendix B, while the model derivations is shown in Appendix A.

1.1 Problem Description

Many of the existing flow measurements validation method during production test focus on specific applications, problem or field cases. Therefore it is necessary to develop a unique method of validating flow rate measurements in MPFMs. As pointed earlier, dual gamma ray MPFMs gives a lot of information, however data given by these meters are still approached and analyzed in the same way as data given by test separators, despites all the extra information. The aim of this work is to extract more information given by the PhaseWatcherVx meter and use these data to develop a choke model that can be used to validate flow rate measurements for PhaseWatcherVx by using the data provided by the meter itself.

1.2 Main Objectives

The main objective of this thesis work is to develop a numerical model for predicting mass flow rate through restrictions. This numerical model can be used to validate flow rate measurements given by the multiphase flow meter, it can also be used to calculate mass flow rate in the absence of multiphase flow meter.

1.3 Specific Objectives

i. To elucidate the general working mechanism of PhaseWatcher VX multiphase flow meter
ii. To evaluate the existing multiphase flow models through restrictions
iii. To modify (Sachdeva, et al., 1986) multiphase flow model through restrictions.

1.4 Methodology

This thesis work was carried out in three different phases. The first phase was to conduct literature review on the following three different areas of study.

i. Fundamental of Framo (PhaseWatcher VX) multiphase flow metering concept
ii. Mass, momentum and energy conservation equations applied in flow through restrictions
iii. Existing models for flow through restrictions

The second phase was to perform simulation in HYSYS so as to get thermodynamics properties of the gas and liquid phases using the data provided by Schüller , et al., 2003 and Schüller , et al., 2006.

The last phase was to evaluate the existing models for flow through restrictions so as to find which model gives the best predictions and thereafter modification of the best model was performed so as to have a new choke model for predicting mass flow rate through restriction. Model evaluation was done using Microsoft Excel with built in function of solver and VBA as presented in Appendix C.

1.5 Scope of the Project

There are many MPFM that are currently used in oil and gas industry employing different techniques in measuring the flow rate, but this thesis work focuses on PhaseWatcherVx multiphase flow meter only. PhaseWatcher Vx multiphase flow meter is selected because it is the most popular meter that is used in oil and gas industry and it is also the best MPFM in the market.

Choke models that can be used to validate flow rate measurements given by the multiphase flow meters can be developed empirically (obtained from curve fitting from experimental data) or theoretically (obtained by applying the law of conservation of mass, momentum and energy), this thesis work focuses on theoretical concepts only.

CHAPTER 2

2 LITERATURE REVIEW

The literature presented in this thesis work is divided into three sections

i. Fundamentals of Multiphase flow metering techniques
ii. PhaseWatcherVx MPFM as a meter which is selected for analysis of the metering concepts
iii. Different existing choke models that are currently used in oil and gas industry.

2.1 Fundamentals of Multiphase Flow Metering Techniques

The objective of multiphase flow metering is to determine the flow rates of the individual components, for example oil, water and gas. Unfortunately there is no single instrument, which will measure these parameters directly. Therefore, it is necessary to combine several devices in an instrument package to calculate the specific flow rates from the combined readings. As will be presented in this work there are many possible combinations, and the number of instruments required depends upon whether or not the three components can be mixed together upstream of the instrumentation (homogeneous flow) or not. If homogeneity of flow can be achieved, then only three measurements are required, one common velocity and the phase fractions of two components. If not then individual component velocities and phase fractions have to be determined.

There are different MPFM meter that exists in the market and each meter employs different techniques and technology in measuring the required parameters. Others use venturi and gamma ray attenuations while others use venturi and capacitance. Examples of these kinds of MPFM including their pros and cons are as presented in (Mwalyepelo, 2014)

Types of parameters monitored by multiphase metering system

i. Primary parameters

a) Phase fractions
b) Phase velocity
c) Phase density

ii. Secondary parameters

a) Flow regime
b) Phase viscosity
c) Phase salinity
d) Phase permittivity/conductivity

The main parameters needed by operators of MPFMs are the primary parameters. The density information can be obtained from other parts of production process such as estimations from PVT diagrams and densitometers readings. Therefore, the remaining unknown parameters will be the phase velocity and the phase fractions of the two components as the third fraction can be deduced from the fact that the sum of three phase fractions is equal to one.

Mathematically the mass flow rate of the individual phase (gas, oil and water) respectively can be expressed as

$$\dot{m}_g = \alpha_g \times U_g \times \rho_g \times A_{pipe}$$
$$\dot{m}_o = \alpha_o \times U_o \times \rho_o \times A_{pipe} \tag{1}$$
$$\dot{m}_w = \alpha_w \times U_w \times \rho_w \times A_{pipe}$$

And the mixture mass flow rate will be

$$\dot{m} = \alpha_g \times U_g \times \rho_g \times A_{pipe} + \alpha_o \times U_o \times \rho_o \times A_{pipe} + \alpha_w \times U_w \times \rho_w \times A_{pipe} \tag{2}$$

As presented in the equation (1) and (2) above the number of unknowns are six i.e. three unknown phase fractions and three unknown phase velocities. The number of required measurements can be reduced by separation or homogenisation. By separating the phases, the need for cross-sectional hold-up measurements disappears and the three volume flows can be established by conventional single-phase metering technology. However, it should be noted that a full separation of the three phases is difficult to achieve in many cases due to liquid carry over in the gas phase, or gas remaining trapped in the liquid phase, or formation of emulsions and foams. By homogenising the mixture, only one velocity needs to be measured and the total measurement requirement can be reduced to three. Homogenisation can be attained by inserting in-line mixing devices or flow conditioners, or by subjecting the stream to a sudden expansion and contraction. However, a full homogenisation of the mixture can also be very difficult to achieve in some cases, for example when there is substantial slippage between a heavy and a light fluid phase. (Falcone, et al., 2010)

2.1.1 Multiphase flow meter categories

Multiphase flow meters can be categorized in main two categories

i. In line meters
ii. Separation meters

In line meters

In this category of MPFMs all the measurements of the individual phase fractions and total or individual phase flow rates are performed directly in the multiphase flow line hence, no separation and/or sampling of the fluids is required (Sidel, et al., 2005).

The volume flow rate of each phase is represented by the area fraction multiplied by the velocity of each phase. This means that a minimum of six parameters has to be measured or estimated. Some MPFMs assume that either two or all three phases travel at the same velocity, thus reducing the required number of measurements. In this case either a mixer must be employed or a set of calibration factors established. (Falcone, et al., 2010)

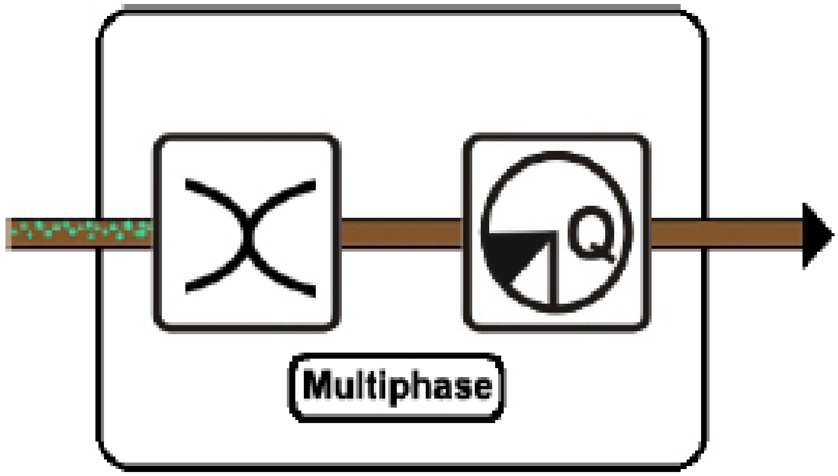

Figure 2- 1: Principle design of in line MPFM with a mixer **(Sidel, et al., 2005)**

Separation meters

This class of MPFMs is characterised by performing a complete or partial separation of the multiphase stream, followed by in-line measurement of each of the three phases. However, when a multiphase flow is split into two or more single-phase flows (assuming that the separation is 100% efficient), the need to refer to MPFMs ceases to exist (Falcone, et al., 2010). In case of partial separation of phases only part of the gas is separated into a secondary measurement loop around the main loop through MPFM. Since the separation is only partial, some liquid travel with the gas through the secondary measurement loop, which then calls for a "wet gas" measurement. The remaining multiphase stream will then have a reduced GVF and thereby operate within the designed envelope of the flow meter.

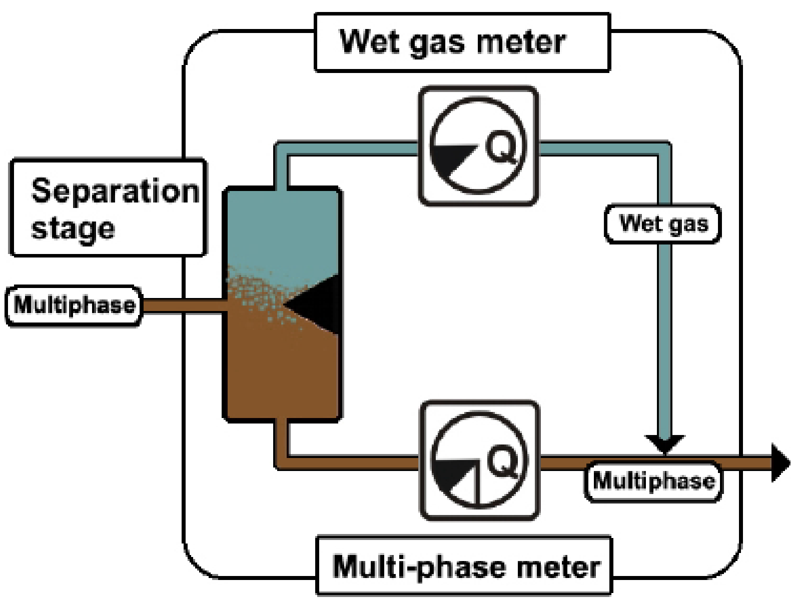

Figure 2- 2: Principle design of MPFM with partial separation **(Sidel, et al., 2005)**

However there is another type of separation MPFMs called Separation in sample line. In this kind of meter the separation is not performed in the total flow, but in the bypassed sample flow. The sample flow is separated into a gas and liquid flow, where the water-in-liquid ratio of the liquid sample stream can be determined using an on-line water fraction meter. Total gas/liquid flow rate and ratio must be measured in the main flow line, assuming the bypassed sample flow is representative of the main flow, the water in liquid ratio is based on the by-pass measurement of this parameter.

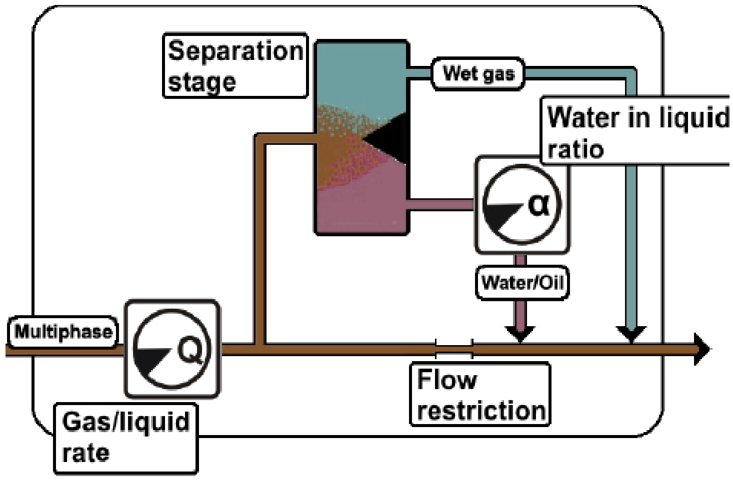

Figure 2- 3: Principle of a MPFM with separation in sample line **(Sidel, et al., 2005)**

2.1.2 Applications of MPFMs to oil and gas industry

Within Oil and gas industry MPFMs have greater benefits in terms of layout out of production facilities, well testing, reservoir management, production allocation, production monitoring, subsea / downhole metering and costs.

i. Layout of the production facilities

Removal of test separator for well testing applications using Multiphase flow meters minimizes space and load requirements for the well testing operations.

ii. Well testing

Traditionally the flow rates of well fluids have been measured by separating the phases by separators and measuring the output of the separated fluids by conventional single phase techniques. Conventional test separators are expensive, and take long time to monitor each well's performance because of the stabilized flow condition required. The use of MPFMs can eliminate the problem of stabilization and therefore performing measurements at the instatenous time.

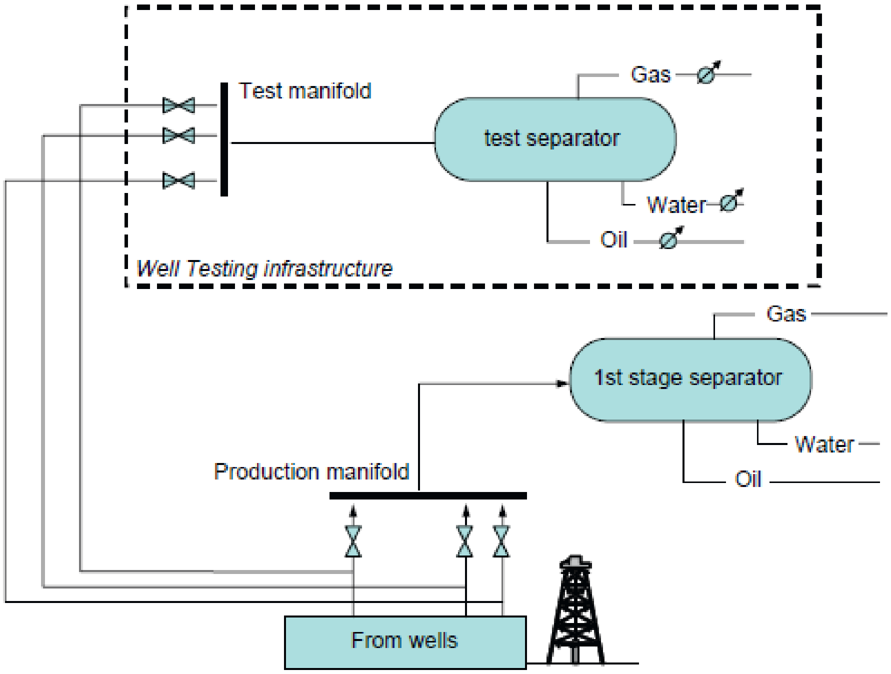

Figure 2- 4: Conventional well testing layout **(Falcone, et al., 2010)**

iii. Reservoir management

MPFMs provide real time, continuous production data which can help the operator to characterize the field and reservoir performance by monitoring pressure decline, water influx and increase in GOR, while traditional test separators only provide information on cumulative volumes at discrete points in time.

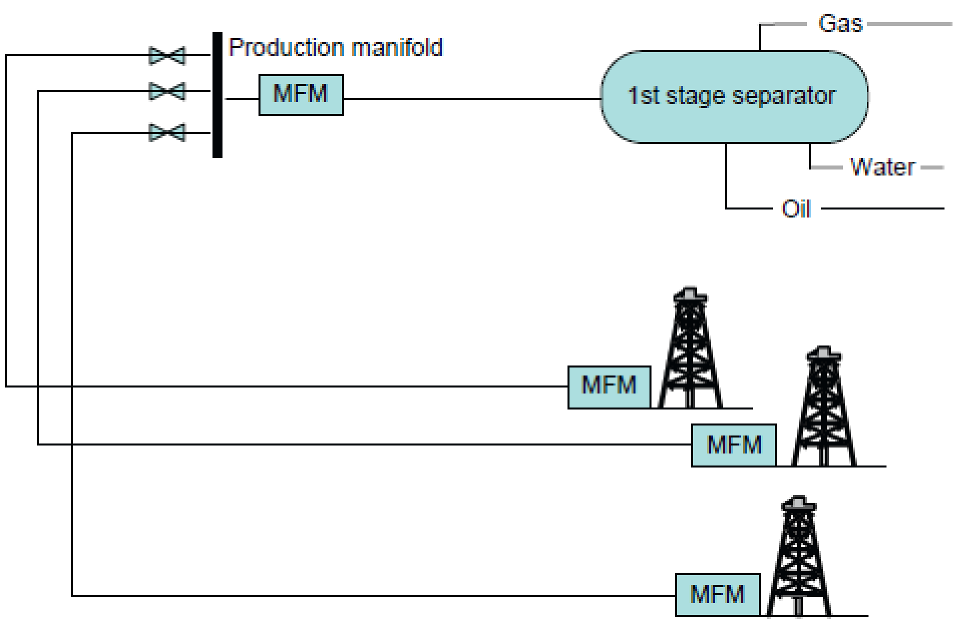

Figure 2- 5: MPFMs for reservoir management and allocation **(Falcone, et al., 2010)**

iv. Production Allocation

In any situation where production is from different wells/ fields owned by different operators is commingling in the same pipeline for export or to a common processing facility production allocation is required. Without MPFMs production of each well must flow through a test separator before commingling with the other produced streams. Accurate allocation of the fluids produced from different fields/Wells into a host facility is necessary to avoid litigations between the partners.

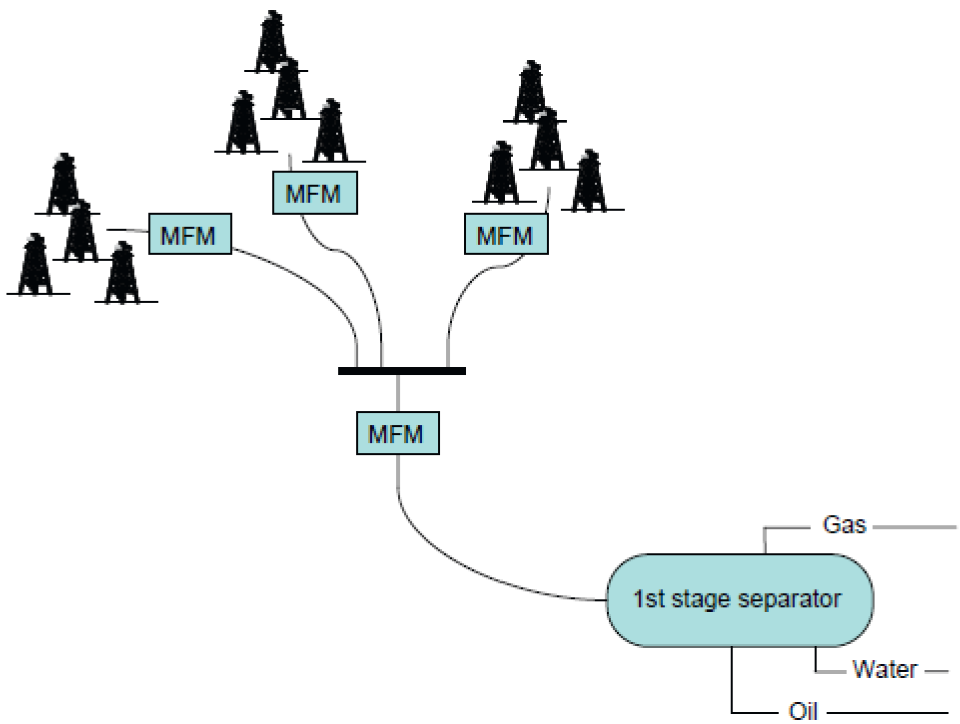

Figure 2- 6: MPFMs for production allocation **(Falcone, et al., 2010)**

v. Production Monitoring

Monitoring of the produced well refers to ability to tract in a real time any changes in fluid composition, flow rates, pressure and temperature profiles. Real time monitoring of the produced wells is recognized as the best way of optimizing field performance (Falcone, et al., 2010). Real-time production data from individual wells also allow to continuously update the drainage areas (and hence the reserves) associated to each well. This in turn helps the Operators plan work overs or infill drilling campaigns.

vi. Subsea / downhole Metering

Subsea/downhole MFM can be regarded as less challenging because of lower gas volume fraction (GVF), lower potential for hydrate, scale or asphaltene formation, and higher density contrast between oil and water. Downhole MFM is best suited for 'intelligent wells', where streams from different producing intervals need monitoring. This would otherwise require running wireline interventions. Downhole MFM also allows continuous optimisation of artificial lift systems (e.g. electrical submersible pumps and gas lift) by detecting any well performance change. (Falcone, et al., 2010)

2.2 PhaseWatcherVx Multiphase Flow Meter

Framo MPFM is one of the widely used MPFM in oil and gas industry. Recently Framo Engineering have joined with Schlumbeger and brought to market the latest version of MPFM called PhaseWatcherVx. Modifications which were done in this version of meter include change of a radioactive source in Framo MPFM which was replaced by Gd-153, which was kind of radioactive source which was used in Schlumbeger MPFM. Therefore the Version of Framo MPFM meter that is presented in this work is PhaseWatcherVx. The general overview of the meter and its component is as shown in Figure 2- **8**.

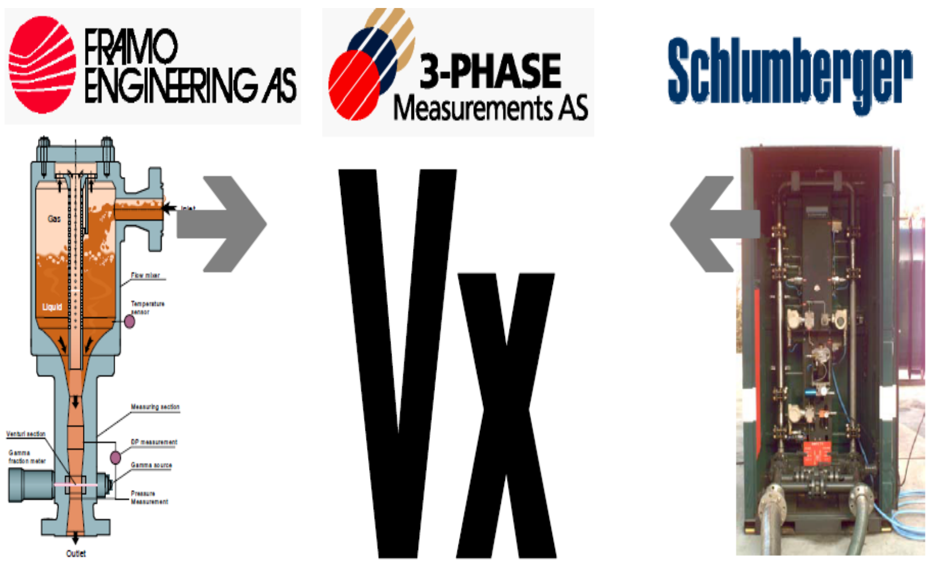

Figure 2- 7: Development of PhaseWatcherVx **(Framo Engineering As, 2009)**

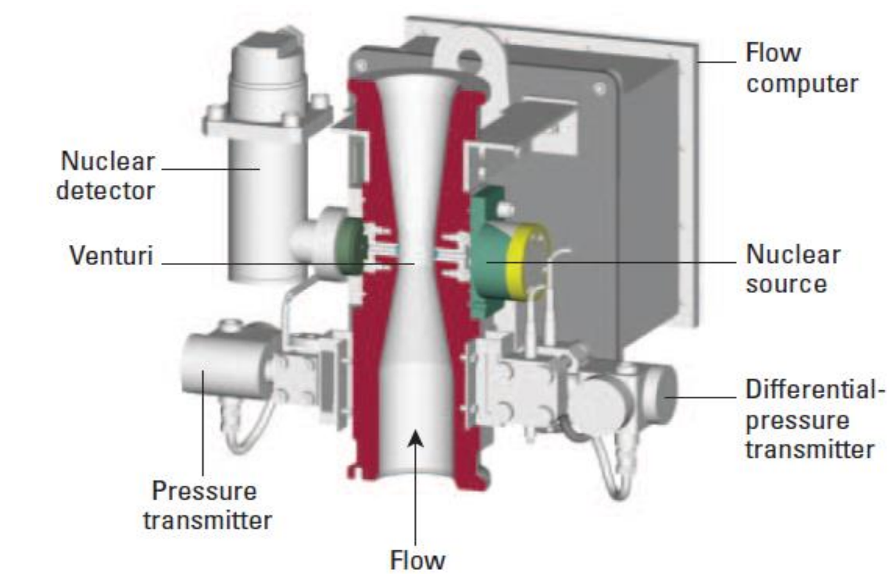

Figure 2- 8: General overview of PhaseWatcherVx **(Schlumberger, 2007)**

2.2.1 Working principles of PhaseWatcher VX

PhaseWatcherVx is an in-line MPFM that uses the measurement principle that combines Venturi and dual gamma ray measurements. The Venturi section is for measuring mass flow rate based on concept of common velocity for all the phases and the dual gamma ray densitometer for fully physical measurement of fractions. This meter is independent of the gas volume fractions, water cuts and emulsions the only inputs required for the meter is the fluid properties. Explanation on how the mass flow rate and phase fractions are measured is as presented in the section below.

i. Phase fraction measurements

Phase fraction measurement is done by the dual gamma ray attenuation technique which uses two energy levels ($e_1$and e_2). For a pipe with inner diameter d_i, containing water, oil and gas fractions (x_w, x_o and x_g) the measured count rate $N_m(e)$ is then given by the equation,

$$N_m(e) = N_o(e) \times \exp[-\sum_{i=1}^{3} x_i \times \mu_i(e) \times d_i] \tag{3}$$

Where by

$N_o(e)$is the count rate when the pipe is empty at the energy level (e)

μ_i is the linear attenuation coefficient for the three phases (μ_w, μ_o and μ_g)

For two energy levels $e_1$and e_2, two independent equations can be obtained, the third equation is simply from the fact that the sum of the three phase fractions equals to one. The three equations can be written in matrix form as:

$$\begin{bmatrix} R_w(e_1) & R_o(e_1) & R_g(e_1) \\ R_w(e_2) & R_o(e_2) & R_g(e_2) \\ 1 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} x_w \\ x_o \\ x_g \end{bmatrix} = \begin{bmatrix} R_m(e_1) \\ R_m(e_2) \\ 1 \end{bmatrix} \tag{4}$$

Where

$R_w,\ R_o, R_g$ and R_m represent the logarithmic counts of water, oil, gas and a mixture respectively

And the sum of three phase fractions is 1

$$x_w + x_o + x_g = 1 \tag{5}$$

The logarithmic counts of water, oil and gas are determined by filling the pipe with pure fluids i.e.; 100% water, 100% oil and 100% gas. Combining these count rates measured from the two energy levels then the phase fractions of the mixture can be calculated. Figure 2- 9 represent the graphical logarithmic count rates of the two energy levels, the corners of the triangle are the water, oil and gas calibrations, and any point inside this triangle represents a particular composition of water, oil and gas, for example a point half way on the water-gas line represents 50% water and 50% gas mixture.

The shape of the triangle depends mainly on the energy levels used (thus the specific radioactive source), pipe diameter and detector characteristics. However, fluid properties may also influence the triangular shape. If the energy levels are too close the triangle will transform into a line and obviously cannot be used for a three-phase composition measurement.

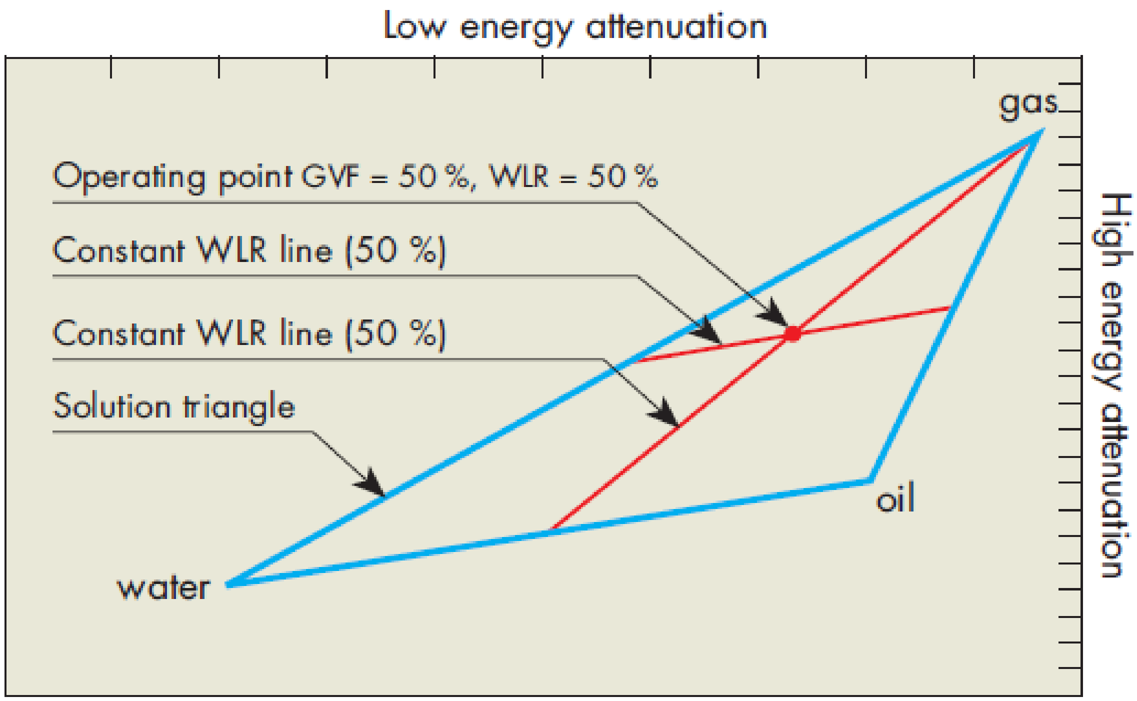

Figure 2- 9: Gamma ray attenuation for PhaseWatcherVx MPFM **(Framo Engineering As, 2003)**

ii. Mass flow rate measurements

Once the phase fractions are known the venturi meter is utilized to measure the total mass flow rate and the individual phase flow rates can then be calculated. Mass flow rate is derived from continuity equation and Bernoulli's principle, the venturi is built with differential pressure sensors and the modified equations are used to determine the total mass flow rate.

Volumetric flow rates can be calculated by using the following formulas.

$$q_m = \frac{\dot{m}_m}{\rho_g . GVF + \rho_l . (1 - GVF)}$$

$$q_g = GVF . q_m$$

$$q_l = \frac{\dot{m}_m - \dot{m}_g}{\rho_l} \tag{6}$$

$$q_o = (1 - WLR). q_l$$

$$q_w = WLR. q_l$$

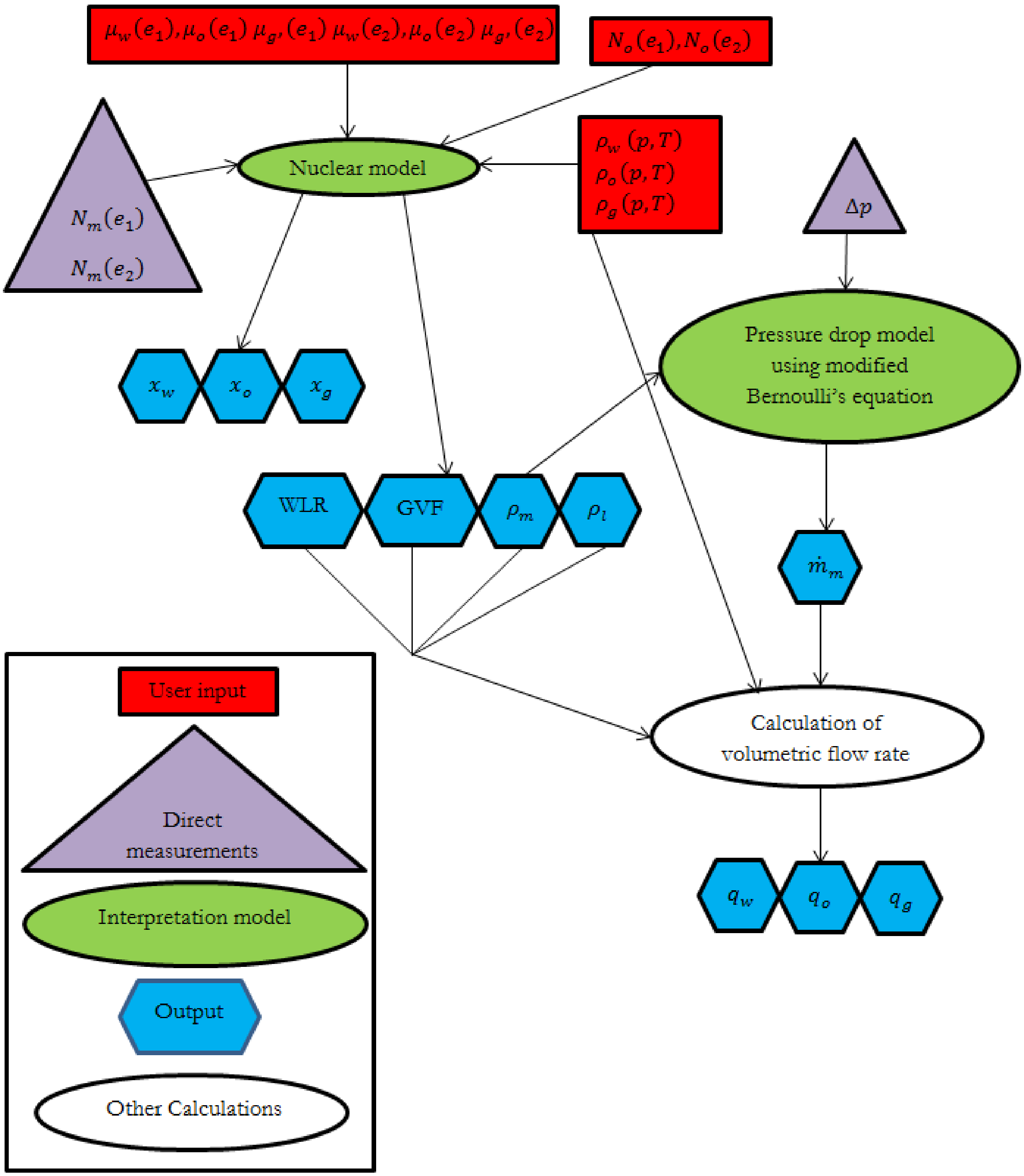

Figure 2- 10: Calculation model for PhaseWatcher VX

2.2.2 Meter's operating envelope

PhaseWatcherVx MPFM can operate in the following range

i. GVF 0-100%
ii. WLR 0-100%
iii. Temperature – 40˚C – 150 ˚C
iv. Pressure 0 - 5000 psi
v. Liquid viscosity 0 -2000 cp

Table 2 - 1: Typical uncertainty of Phase Watcher VX **(Schlumberger, 2012)**

Typical Uncertainty at line condition with average GVF<98%	
Total mass rate(relative error)	$\pm 2\%$
Liquid flow rate(relative error)	$\pm 2\%$
Gas flow rate(relative error)	$\pm 5 - 8\%$
Density liquid (relative error)	$\pm 1.5\%$
WLR (absolute error)	$\pm 2 - 3\%$

2.3 Choke Models for Predicting Mass Flow Rate through Restrictions

Having a correct value of flow rate is very important in relations to production control in oil and gas industry. Relating the change in pressure and temperature across the choke and correlating these with mass flow rate, the actuator position and properties of the well stream may lead to a mass flow rate meter that is simple and inexpensive compared to other designs. However to have this simple and inexpensive mass flow rate meter the knowledge of predictive ability and accuracy of available mass flow rate model is required to qualify such a solution. (Schüller, et al., 2003).

Types of flow through chokes

There are two types of flow that occurs when the fluid flows across the choke.

i. Critical flow
ii. Subcritical flow

Critical flow

When a flowing fluid at a given pressure and temperature passes through a restriction into a lower pressure environment the fluid velocity increases while the pressure downstream the restriction decreases which means the pressure upstream the restriction increases. Further increase in upstream pressure will cause the increase in the fluid velocity, the velocity will increase to a maximum point (sonic velocity) at this point any further increase in upstream pressure will not cause any increase in the velocity of the fluid and at this condition the flow is said to be at critical flow. Therefore in critical flow maximum flow rate of the fluid will be obtained with respect to the prevailing upstream condition.

Subcritical flow

When the actual velocity is below the critical one then it is said to be in the subcritical condition. In this case it means the flow will not reach maximum speed and therefore it will be changing as the upstream condition changes.

Critical/subcritical flow boundary

It is very important in production to operate with critical flow conditions so that the downstream effects cannot propagate backwards through the choke and damage the reservoir section. Therefore it is very important that the boundary between critical and subcritical flow is well defined in all cases where the flow passes through restrictions

2.3.1 Basic principles for modeling flow through restriction

Principles of modelling flow through restriction rely on;

i. Law of conservation of momentum i.e. considering momentum balance across the choke.

$$A_c dp_2 = d(U_2 \cdot \dot{m}_{L2} + U_2 \cdot \dot{m}_{g2}) \tag{7}$$

This concept was used by (Sachdeva, et al., 1986).

ii. Considering force balance across the choke like the model developed by (Chisholm, 1967)

$$(P_1 - P_2)A + F = \frac{W}{g}U \tag{8}$$

iii. Law of conservation of energy (Energy is neither created nor destroyed but can only be transformed from one form to another)

$$P_1 v_1 + \frac{1}{2}U_1^2 + gh_1 + E_1 + Q - WD = P_2 v_2 + \frac{1}{2}U_2^2 + gh_2 + E_2 \tag{9}$$

Calculation of mass flow rate from the equation (9) depends on the assumptions made by person developing a model through restriction.

2.3.2 Models for predicting mass flow rate through chokes

History of developing models for flow through restriction started back in 1940's and Tangren, et al., 1949 was the first person who did a significant study of two phase flow through restriction but his model was not successfully as he considered critical flow only and he assumed gas is uniformly dispersed in the mixture having liquid as a continuous phase. Ros, 1960 extended Tangren's work but he assumed liquid phase is homogenously dispersed as droplets in a continuous gas phase. Model of Forturnati, et al., 1972 defined the boundary between critical and subcritical flow graphically this model is only valid for a downstream pressure greater than 1.5 bars. Ashford, 1974 extended Ros, 1960 and Forturnati, et al., 1972 works and defined critical sub-critical boundary as a function of gas/ liquid ratio and fluid properties. Data generated by Ashford were also used by Sachdeva, et al., 1986 to evaluate his model. Among all model that have been developed the few successful models are now used in oil and gas industry as listed below.

2.4 Existing Choke Models Currently in Use in Oil and Gas Industry

i. Chisholm model

Chilsholm, 1983 developed a two phase multiplier model based on the following assumptions

a) Force balance across the orifice hole

$$(P_1 - P_2)A + F = \frac{W}{g}U \tag{10}$$

b) Mixture density remains constant

c) Density varies due to changes in velocity along the flow path

The proposed two phase multiplier (ϕ) for calculating pressure drop is based on pressure drop calculated for a single phase flow, and its mathematical representation is as follows.

$$\frac{\dot{m}_{ls}}{\dot{m}_l} = \left(\frac{\Delta P}{\Delta P_l}\right)^{0.5} = (\phi) = \left[1 + \left(\frac{v_g}{v_l} - 1\right)\{Bx(1-x) + x^2\}\right] \tag{11}$$

ii. Sachdeva model

Sachdeva, et al., 1986 used the concept of momentum and energy balances to define the boundary between critical and subcritical conditions. Sachdeva made the following assumptions in developing his model.

a) Flow is one dimension
b) Phase velocities are equal at the throat
c) Predominant pressure term is acceleration
d) The quality is constant for high speed process
e) The liquid phase is incompressible.

The boundary between critical and subcritical flow is calculated by using the following formula;

$$y = \left\{\frac{\frac{k}{k-1} + \frac{(1-x_1)v_l(1-y)}{x_1 v_{g1}}}{\frac{k}{k-1} + \frac{n}{2} + \frac{n(1-x_1)v_l}{x_1 v_{g2}} + \frac{n}{2}\left[\frac{(1-x_1)v_l}{x_1 v_{g2}}\right]^2}\right\}^{\frac{k}{k-1}} \tag{12}$$

For pure gas phase the value of y is termed as y_c and can be calculated by the formula;

$$y_c = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} \tag{13}$$

If the value of $y \leq y_c$ critical flow exists and if $y > y_c$ subcritical flow exists

Critical and subcritical flow rates

Once the boundary has been defined flow rate through the choke is calculated by using equation (14) below

$$G_2 = CD\left\{2\cdot p_1\rho_{m2}^2\left[\frac{(1-x_1)(1-y)}{\rho_l}\right] + \frac{x_1 k}{k-1}\left(v_{g1} - y v_{g2}\right)\right\}^{0.5} \tag{14}$$

Where,

$$G_2 = \frac{\dot{m}_{g2} + \dot{m}_{l2}}{A_c} \tag{15}$$

$$v_{g2} = v_{g1} y^{-1/k} \tag{16}$$

And,

$$\frac{1}{\rho_{m2}} = x_1 v_{g1} y^{-1/k} + (1 - x_1) v_l \tag{17}$$

Sachdeva used data from (Pilehvari, 1980) and (Ashford, 1974) to evaluate his model. He suggested a discharge coefficient of 0.75 for chokes with housing and 0.85 for chokes with no effect of housing.

iii. Perkins model

Perkins, 1993 derived a mathematical equation involving mass balance and energy balance for defining critical and subcritical boundary. He also used reduced compressibility of gas, oil and water properties correlations to predict critical/subcritical boundary. Perkins analyzed 1432 data points for obtaining the value of discharge coefficient. He found the best value of discharge coefficient to be 0.826 yielding a 15.41 % standard deviation

Assumptions made in developing his model are,

a) Isentropic flow(adiabatic with no friction)
b) The flow is one dimension
c) Temperature varies with position, but at any point, all phases are at the same temperature
d) Velocity varies with position, but at any point, all components are moving with the same velocity
e) The gas compressibility factor is constant
f) The liquids have a negligible compressibility compared to gas
g) Elevation changes are negligible

Boundary between critical and subcritical flow is calculated using equation (18).

$$\left[2\lambda\left(1-p_r^{\,n-1/n}\right)+2\alpha_l(1-p_r)\right]\cdot\left[\left\{1-\left(A_2/A_1\right)^2\left(\frac{f_g+\alpha_l}{f_g p_r^{-1/n}+\alpha_l}\right)^2\right\}\cdot\left\{\left(\frac{f_g}{n}p_r^{-(n+1)/n}\right)\right\}+\left(A_2/A_1\right)^2\cdot\frac{f_g}{n}\left(\frac{\left(p_r^{-(n+1)/n}\right).\{f_g+\alpha_l\}^2}{\left(f_g p_r^{-1/n}+\alpha_l\right)^2}\right)\right]=\left[\left\{1-\left(A_2/A_1\right)^2\left(\frac{f_g+\alpha_l}{f_g p_r^{-1/n}+\alpha_l}\right)^2\right\}\cdot\left\{f_g p_r^{-1/n}+\alpha_l\right\}\right]\cdot\left[\frac{(n-1)\lambda}{n}\left(p_r^{-1/n}\right)+\alpha_l\right] \tag{18}$$

Where,

$$\lambda=\left(f_g+\frac{\left(f_g C_{vg}+f_o C_{vo}+f_w C_{vw}\right)M}{ZR}\right) \tag{19}$$

$$n=\frac{K C_{vg} f_g\ +f_o C_{vo}+f_w C_{vw}}{f_g C_{vg}+f_o C_{vo}+f_w C_{vw}} \tag{20}$$

And,

$$\alpha_l=\frac{1}{v_1}\left(\frac{f_o}{\rho_o}+\frac{f_w}{\rho_w}\right) \tag{21}$$

The mass flow rate is calculated by equation (25) below after defining the boundary between critical and subcritical flow. The mass flow rate through restriction is found by using the following procedures

a) Calculating the pressure ratio p_r in equation (18) using solver in Excel.
b) Solving for P_2 using equation (22)

$$p_r={p_2}/{p_1} \tag{22}$$

c) Solve for $p_{2'}$ (pressure just downstream the choke) by using Perry relationship

$$p_{2'}=p_1-\frac{p_1-p_3}{\left[1-\left(d_c/d_p\right)^{1.85}\right]} \tag{23}$$

If $p_2 > p_{2'}$ the flow is critical and equation (22) is used to calculate the velocity and mass flow rate in equation (24) and (25) respectively, and if $p_2 < p_{2'}$ flow is subcritical and $p_r = p_{2'}/p_1$ is used to calculate the velocity and mass flow rate in equation (24) and (25) and if $p_2 = p_{2'}$ the flow is at the boundary condition and $p_r = p_{2'}/p_1 = p_2/p_1$ is used to calculate the velocity and mass flow rate.

The velocity of the flowing fluid is given by equation (24)

$$U_2 = \sqrt{\frac{2\left(\lambda p_1 v_1 \left(1 - p_r^{\,n-1/n}\right) + \left(\frac{f_o}{\rho_o} + \frac{f_w}{\rho_w}\right) p_1 (1 - p_r)\right)}{1 - \left({A_2}/{A_1}\right)^2 \left(\frac{f_g + \alpha_l}{f_g p_r^{-1/n} + \alpha_l}\right)^2}} \tag{24}$$

And the mass flow rate is calculated by using equation 24 below

$$\dot{m} = A_2 \rho_2 U_2 = \frac{(A_2/v_1) U_2}{f_g \left(p_r^{-1/n} + \frac{1}{v_1}\left(\frac{f_o}{\rho_o} + \frac{f_w}{\rho_w}\right)\right)} \tag{25}$$

iv. Al- Safran Model

Al-Safran, et al., 2007 developed a model based on one dimension balance equations of mass, momentum and energy. The model was developed from basis of Sachdeva, et al., 1986 and Perkins, 1993. This model considers the slippage effect between gas and liquid phase, slippage ratio used in development of this model comes from hydro models (Al-Safran & Kelkar, 2007). Al-Safran model showed a greater improvement in defining the boundary between critical and subcritical flow compared to models of Sachdeva and Perkins as shown in Figure 2-11.

Critical/subcritical flow boundary for Al-Safran model

The boundary between critical and subcritical flow is as presented in equation (26) below

$$(r_c)^{1-1/n} = \frac{\alpha(1 - r_c) + n/n - 1}{n/n - 1 + n/2\left(1 + \alpha r_c^{1/n}\right)^2} \tag{26}$$

Where,

$$n = \frac{K C_{vg} f_g + f_o C_{vo} + f_w C_{vw}}{f_g C_{vg} + f_o C_{vo} + f_w C_{vw}} \tag{27}$$

And

$$\alpha = \frac{R(1-x_g)v_l}{x_g v_{g1}} \tag{28}$$

The critical and subcritical mass flow rate is calculated by using equation (29) after calculating the pressure ratio in equation (26). If $r < r_c$, the flow is critical and r_c is used to calculate the mass flow rate and if $r > r_c$, the flow is subcritical and r is used to calculate the mass flow rate in equation (29).

$$\dot{m}^2 = \frac{CA_2^2 p_1 \left[\alpha(1-r) + {}^{n}/_{n-1}\left(1 - r^{n-1/n}\right)\right]}{x_g v_{g1}\left[\alpha + r^{-1/n}\right]^2 \left[x_g + \frac{1}{R}(1-x_g)\right]} \tag{29}$$

Where by, C is a constant that changes depending on the units used, ($C = 2 * C_D^2$) for SI units and ($C = 2 * C_D^2 * g_c * 144$) for field units, Al- Safran used a C_D value of 0.75 to calibrate his model for model imperfections and irreversible losses.

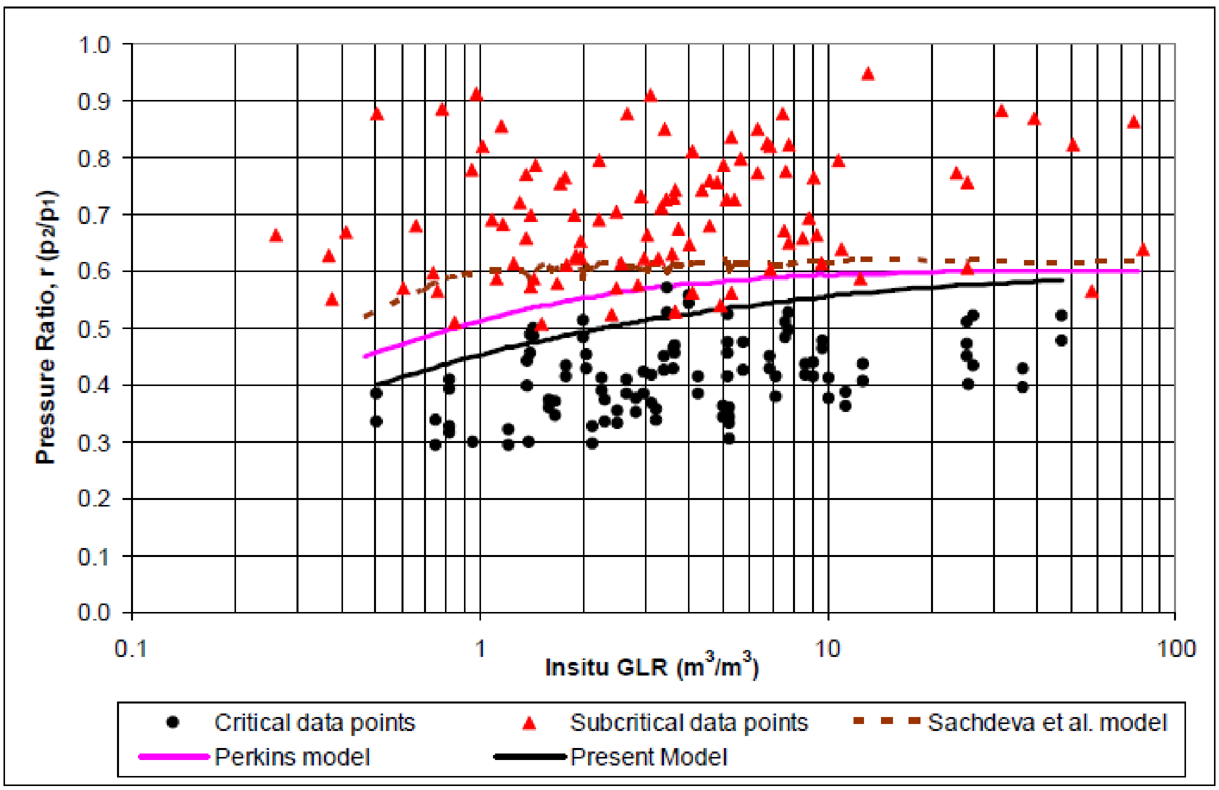

Figure 2-11: Comparison of predicted critical/subcritical flow boundary **(Al-Safran & Kelkar, 2007)**

iv. Selmer – Olsen (Hydro models)

Schüller, et al., 2003 used control volume approach, he used two volumes control which corresponds to two kinds of choke configurations (orifice type and cage type). The model is also termed as several models (Schüller, et al., 2003). Schüller described his model as hydro short and hydro long model.

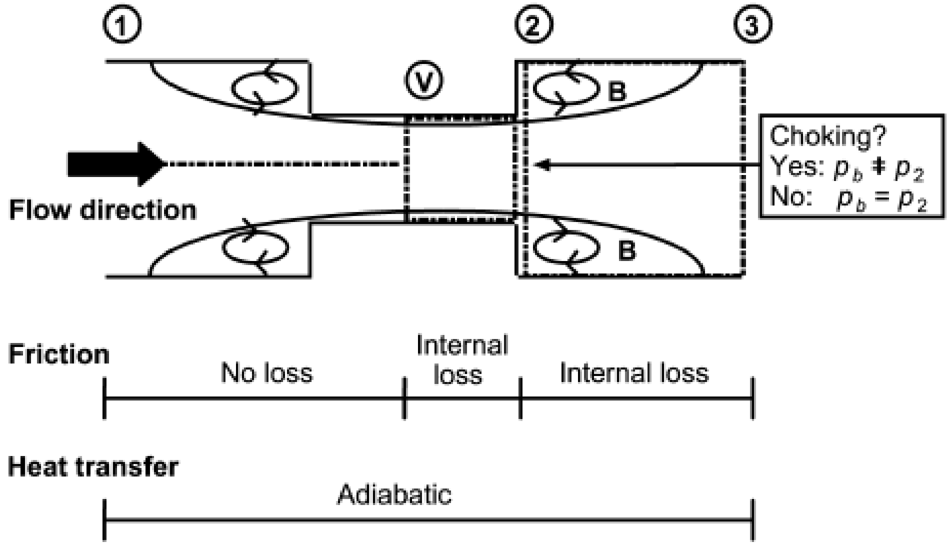

Figure 2-12: Basis for hydro models **(Schüller, et al., 2003)**

The two control volumes used by Schüller, et al., 2003 are two dotted boxes one box with dotted lines starting at V and the second is the dotted box starting at 2 as shown in Figure 2-12. His results suggested that the orifice type choke is best predicted by hydro short model with contraction coefficient (C_c) of 0.62 and the cage type choke is best predicted by hydro long model with contraction coefficient (C_c) of 0.42. The first hydro model didn't define clearly the boundary between critical and subcritical flow and therefore Selmer-Oslen collected more experimental data for subcritical flow and modified his model in 2005 (Schüller, et al., 2006).

For Hydro long model, from cross section 1 to 2 the mechanical energy balances is represented by equation (30) below

$$\int_1^2 \frac{\rho_{e1}}{\rho_{e2}} dp = \frac{\dot{m}^2}{2A_1^2\rho_{e1}}\left[1 - \left(\frac{\rho_{e1}}{\rho_{e2}}\right)^2 \frac{1}{C_T^2 C_C^2} + 2\left(\frac{\rho_{e1}}{\rho_{e2}}\right)^2 \frac{1}{C_T^2}\left(\frac{1}{C_C} - 1\right)\right] \tag{30}$$

Where,

$$\frac{1}{\rho_e} = \left[\frac{x_g}{\rho_g} + R\frac{1 - x_g}{\rho_l}\right]\left[x_g + \frac{1}{R}\left(1 - x_g\right)\right] \tag{31}$$

Similarly, from cross section 2 to 3

$$(p_3 - p_B) + C_T C_X (p_B - p_2) = \frac{\dot{m}^2}{2A_1^2\rho_{e1}}\left(\frac{C_X}{C_T}\frac{\rho_{e1}}{\rho_{e2}} - C_X^2\frac{\rho_{e1}}{\rho_{e3}}\right) \tag{32}$$

The critical mass flow rate of the mixture is given by equation (33)

$$\dot{m}_c^2 = -\left[\frac{A^2}{\frac{d}{dp}\left(\frac{1}{\rho_e}\right)}\right]_c = -\frac{A_1^2 A_T^2}{\frac{d}{dp}\left(\frac{1}{\rho_{e2}}\right)} \cdot \frac{1}{\left(\frac{1}{C_C} - 1\right)^2 + 1} \tag{33}$$

For Hydro short model, from cross section 1 to 2 the mechanical energy balances in equation (30) is replaced by equation (34) below

$$\int_1^2 \frac{\rho_{e1}}{\rho_{e2}} dp = \frac{\dot{m}^2}{2A_1^2 \rho_{e1}} \left[1 - \left(\frac{\rho_{e1}}{\rho_{e2}}\right)^2 \frac{1}{C_T^2 C_C^2}\right] \tag{34}$$

Similarly, from cross section 2 to 3 equation (32) is replaced by equation (35)

$$(p_3 - p_B) + C_T C_X C_C (p_B - p_2) = \frac{\dot{m}^2}{2A_1^2 \rho_{e1}} \left(\frac{C_X}{C_T C_C} \frac{\rho_{e1}}{\rho_{e2}} - C_X^2 \frac{\rho_{e1}}{\rho_{e3}}\right) \tag{35}$$

And the critical mass flow rate equation (33) is replaced by,

$$\dot{m}_c^2 = -\frac{A_1^2 C_T^2 C_C^2}{\frac{d}{dp}\left(\frac{1}{\rho_{e2}}\right)} \tag{36}$$

And the slip ratio (R) suggested by Selmer-Olsen is as given in the equation below

$$R = \sqrt{1 + x_g \left(\frac{\rho_l}{\rho_g} - 1\right)} \left[1 + \xi e^{-\beta x_g}\right] \tag{37}$$

CHAPTER 3

3 EVALUATION OF CHOKE MODELS USED IN OIL AND GAS INDUSTRY

Three different choke models i.e. Sachdeva, et al., 1986, Perkins, 1993 and Al-Safran, et al., 2007 have been evaluated in this work. These models are evaluated using 85 data points from Schüller papers, 60 from Schüller , et al., 2003 and 25 data point from Schüller , et al., 2006. During evaluation of Al-Safran, et al., 2007 some corrections were done on constant values suggested by Al-Safran, the value of a constant C was suggested to be ($C = 2000 * C_D$) for SI units but from the model derivation the value of C is supposed to be ($C = 2 * C_D^2$). Another correction was done in Perkins model were by the formula for calculating pressure ratio was missing a square on one part of the equation. The formula from Perkins, 1993 is as shown in equation (38) below and the correct formula is as shown in equation (18)

$$\left[2\lambda\left(1-p_r^{\,n-1/n}\right)+2\alpha_l(1-p_r)\right]\cdot\left[\left\{1-\left(A_2/A_1\right)^2\left(\frac{f_g+\alpha_l}{f_g p_r^{-1/n}+\alpha_l}\right)^2\right\}\cdot\left\{\left(\frac{f_g}{n}p_r^{\,-(n+1)/n}\right)\right\}+\left(A_2/A_1\right)\cdot\frac{f_g}{n}\left(\frac{\left(p_r^{\,-(n+1)/n}\right).\{f_g+\alpha_l\}^2}{\left(f_g p_r^{-1/n}+\alpha_l\right)^2}\right)\right]=\left[\left\{1-\left(A_2/A_1\right)^2\left(\frac{f_g+\alpha_l}{f_g p_r^{-1/n}+\alpha_l}\right)^2\right\}\cdot\left\{f_g p_r^{-1/n}+\alpha_l\right\}\right]\cdot\left[\frac{(n-1)\lambda}{n}\left(p_r^{-1/n}\right)+\alpha_l\right] \tag{38}$$

3.1 Evaluation Criteria

All the three existing choke models and the modified Sachdeva model has been analyzed and compared using average relative error, average absolute error and standard deviation. The formulas for error analysis are as represent below.

$$\varepsilon_1 = \left(\frac{1}{N}\sum\frac{\dot{m}_{pre}-\dot{m}_{meas}}{\dot{m}_{meas}}\right)\times 100 \tag{39}$$

$$\varepsilon_2 = \left(\frac{1}{N}\sum\left|\frac{\dot{m}_{pre}-\dot{m}_{meas}}{\dot{m}_{meas}}\right|\right)\times 100 \tag{40}$$

$$\sigma = \sqrt{\frac{1}{N}\sum_{N=1}^{N}\left(\frac{\dot{m}_{pre} - \dot{m}_{meas}}{\dot{m}_{meas}} - \frac{\varepsilon_1}{100}\right)} \times 100 \tag{41}$$

3.2 Evaluation Procedures

Evaluation of the three existing choke models and of the modified Sachdeva model was done by using multiphase flow data given in papers of Schüller , et al., 2003 and Schüller , et al., 2006 using the following procedure.

i. Calculating the mass flow rate of the individual phases using the given mass fractions of the individual phases and the total mass flow rate

$$\dot{m}_i = \dot{m} \times x_i \tag{42}$$

ii. Simulating the individual flow rate data point in HYSYS using the compositional data of each phase from Schüller papers to get thermodynamics properties of the fluids. Because HYSYS simulation involves a lot of procedures and some calculated inputs, section 3.3 explains on how the calculated values can be obtained and procedures in HYSYS simulation are shown in Appendix B. Simulation for each data point was performed from the data given in the molar compositions as shown in Table 3 - 1. The mass flow rate, pressure and temperature were adjusted to match with the given data in the flow rate data.

iii. Programming the selected choke models and the new modified Sachdeva model in VBA. Programmed VBA model are shown in Appendix C.

iv. Calculating the mass flow rates in each of the programmed model in VBA. Results for the mass flow rate calculation are shown in Appendix D.

v. Determination of error for each programmed model.

3.3 Inputs Data for HYSYS Simulation

Available data for HYSYS simulation are as listed in Table 3 - 1.

Table 3 - 1: Molar composition data for HYSYS simulation **(Schüller, et al., 2003)**

Molar Compositions, Viscosity and density 10 Bar and 50 ^{0}C

Component name	component symbol	Gas phase composition	Oil Phase Composition	Water phase composition
Nitrogen	N2	0.0156	0.0002	0.0000
Carbon dioxide	CO2	0.0079	0.0006	0.0000
Methane	C1	0.8304	0.0333	0.0001
Ethane	C2	0.0763	0.0147	0.0000
Propane	C3	0.0238	0.0140	0.0000
Iso- Butane	i-C4	0.0052	0.0066	0.0000
Butane	C4	0.0141	0.0254	0.0000
Iso- pentane	i-C5	0.0041	0.0163	0.0000
Pentane	C5	0.0048	0.0251	0.0000
C6+	C6+	0.0044	0.8624	0.0000
Water	H2O	0.0134	0.0014	0.9999
Total		**1.0000**	**1.0000**	**1.0000**
Visosity(mPas)	μ	0.0012	1.31	0.55
Density(Kg/m^3)	ρ	7.7	796	988

The C6+ in the molar composition was treated as a hypothetical component in HYSYS, for any Hypothetical component that is feed in HYSYS the molecular weight and density of the component have to been known, when the molecular weight and density of the component are feed to HYSYS then HYSYS can calculate other properties automatically.

Schüller, et al., 2006 provides the API of C6+ which can be used to calculate density of C6+ by using a formula below

$$\rho(kg/m^3) = \left(\frac{141.5}{^\circ API + 131.5}\right) \times \rho_w \tag{43}$$

The molecular weight is calculated by using Cragoe Correlation as suggested by (Whitson & Brule', 2000)

$$M_{\bar{o}} = \frac{6084}{\gamma_{API} - 5.9} \tag{44}$$

Using equation (43) and (44) density of C6+ was found to be 810.89 kg/m^3 and the molecular weight was 163.99 kgmol.

CHAPTER 4

4 MODIFIED SACHDEVA MODEL

This chapter explains about the new developed model which can be used to predict the mass flow rate of the mixture passing through the choke. Model derivations follow the same concept used by Sachdeva, et al., 1986, the only deference is the introduction of slippage factor which Sachdeva didn't consider when developing his model. The model derivation is explained in details in Appendix A.

4.1 Model Expressions

Critical/subcritical flow boundary

Before the calculations of predicting mass flow rate are performed it is very important to know the flow conditions. As mention in section 2.3, it is very important in production to operate with critical flow conditions so that the downstream effects cannot propagate backwards through the choke and damage the reservoir section. Therefore it is very important that the boundary between critical and subcritical flow is well defined in cases where the flow passes through restrictions. The expression for defining critical/subcritical boundary for this model is as shown in equation (45) below.

$$y = \left\{ \frac{\frac{k}{k-1} + \frac{R(1-x_g)v_l(1-y)}{x_g v_{g1}}}{\frac{k}{k-1} + \frac{n}{2} + \frac{nR(1-x_g)v_l}{x_g v_{g2}} + \frac{n}{2}\left(\frac{R(1-x_g)v_l}{x_g v_{g2}}\right)^2} \right\}^{0.5} \tag{45}$$

Polytropic Coefficient (n)

The polytropic coefficient of this model is the same as that used by Sachdeva, et al., 1986 as shown in equation (46)

$$n = 1 + \frac{x_g\left(C_{pg} - C_{vg}\right)}{x_g C_{vg} + \left(1 - x_g\right)C_l} \tag{46}$$

Slip ratio (R)

Slip model use in derivation of mass flow rate model is the general slip correlation suggested by Grolmes, et al., 1985 as shown in the equation (47)and the applicable values of the constants are as shown in Table 4 - 1.

$$R = a_0 \left(\frac{1 - x_g}{x_g}\right)^{(a_1 - 1)} \left(\frac{\rho_l}{\rho_g}\right)^{(a_2 + 1)} \left(\frac{\mu_l}{\mu_g}\right)^{a_3} \tag{47}$$

Table 4 - 1: Values for Use with Grolmes and Leung equation **(Schüller, et al., 2003)**

Model	a_o	a_1	a_2	a_3
Homogenous(no slip)	1	1	-1	0
Constant slip	k	1	-1	0
Fauske	1	1	-1/2	0
Moddy	1	1	-2/3	0
Simpson	1	1	-5/6	0
Thom	1	1	0.89	0.18
Baroczy	1	0.74	0.65	0.13
Lockhart- Martenelli	0.28	0.64	0.36	0.07

Relation between upstream pressure and recovered pressure

In data given by Schüller, et al., 2003 the experimentally measured p_3 is the recovered pressure further down the choke and is neither the pressure at the choke p_2 nor the pressure just downstream the choke $p_{2'}$. Because the critical pressure ratio is defined as the ratio between throat pressure (p_2) and upstream pressure p_1 only critical flow condition will be defined in this case. For subcritical flow condition (Perry, 1950) defined the expression which relates the upstream pressure and recovered pressure that can be used to calculate subcritical pressure ratio. Perry's expression is as shown in equation (48) below.

$$p_{2'} = p_1 - \left(\frac{p_1 - p_3}{1 - \left({d_c}/{d_p} \right)^{1.85}} \right) \tag{48}$$

Critical/subcritical mass flow rate

The final expression for predicting mass flow rate is as shown in equation (49).

$$\dot{m} = C_D A_2 \left\{ 2\rho_{m2}^2 P_1 \cdot \left(x_g + \left(\frac{1 - x_g}{R} \right) \right) \cdot \left[R\left(1 - x_g\right) v_l (1 - y) + \frac{k x_g}{k - 1} \left(v_{g1} - y v_{g2} \right) \right] \right\}^{0.5} \tag{49}$$

4.2 Procedure for Calculating Mass Flow Rate

When predicting mass flow rate using the improved Sachdeva the following procedures are to be followed.

i. Guess initial value of y.
ii. Use the initial guess to iterate for the critical pressure ratio y_c by employing equation (45) using solver in Excel.
iii. Calculate pressure downstream the choke $p_{2'}$ using Perry's relation in equation (48)
iv. Calculate the actual pressure ratio (y_{act}) which is $p_{2'}/p_1$
v. If $y_c > (y_{act})$, flow is critical then $y = y_c$ is used to calculate mass flow rate in equation (49) and if $y_c < (y_{act})$, flow is subcritical and $y = y_{act}$ is used to calculate the mass flow rate.
vi. Find the mass flow rate by multiplying with the discharge coefficient C_D

Calibration of the model

This model is calibrated using the discharge coefficient (C_D) concept. Three different values of C_D have been test to calibrate the model and the value of 0.65 was found appropriate to calibrate the model.

CHAPTER 5

5 RESULTS AND DISCUSSIONS

In this chapter evaluation results of the three existing choke models i.e. models Sachdeva, et al., 1986, Perkins, 1993 and Al-Safran, et al., 2007 and that of the new developed choke model are presented and discussed. Each model has been evaluated according to the evaluation criteria mention in section 3.1 of this report. In addition to that the graphical representation of every model is also presented in this chapter.

5.1 Results

i. Sachdeva, et al., 1986 model

Sachdeva model showed a relative error of 1.76%, absolute error of 10.52% and standard deviation of 12.49%

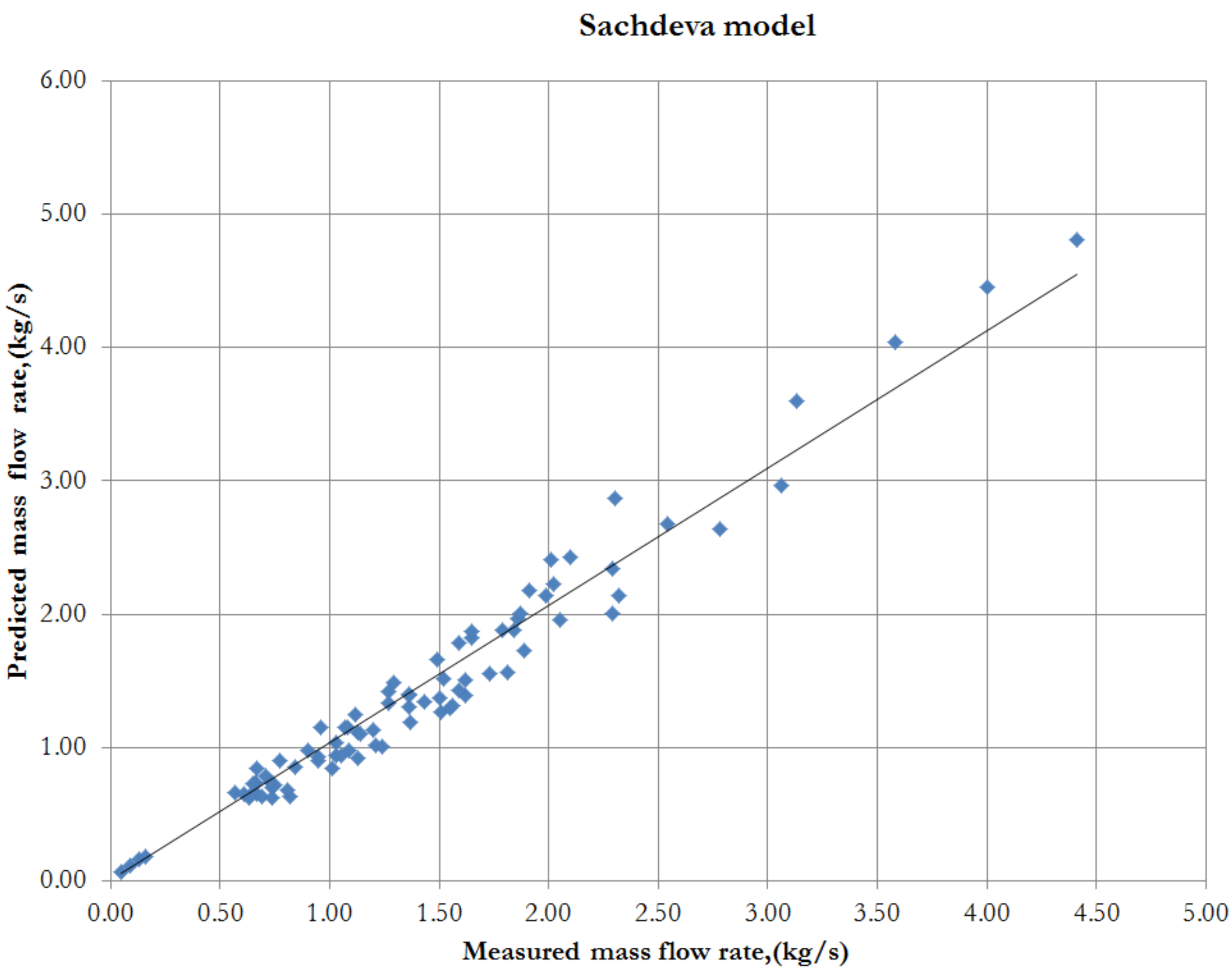

Figure 5 - 1: Results of Sachdeva's model

ii. (Perkins, 1993)Model

In this study Perkins model happened to be the model that exhibits largest errors, it showed a relative error of 24.17%, absolute error of 30.74% and standard deviation of 25.91%.

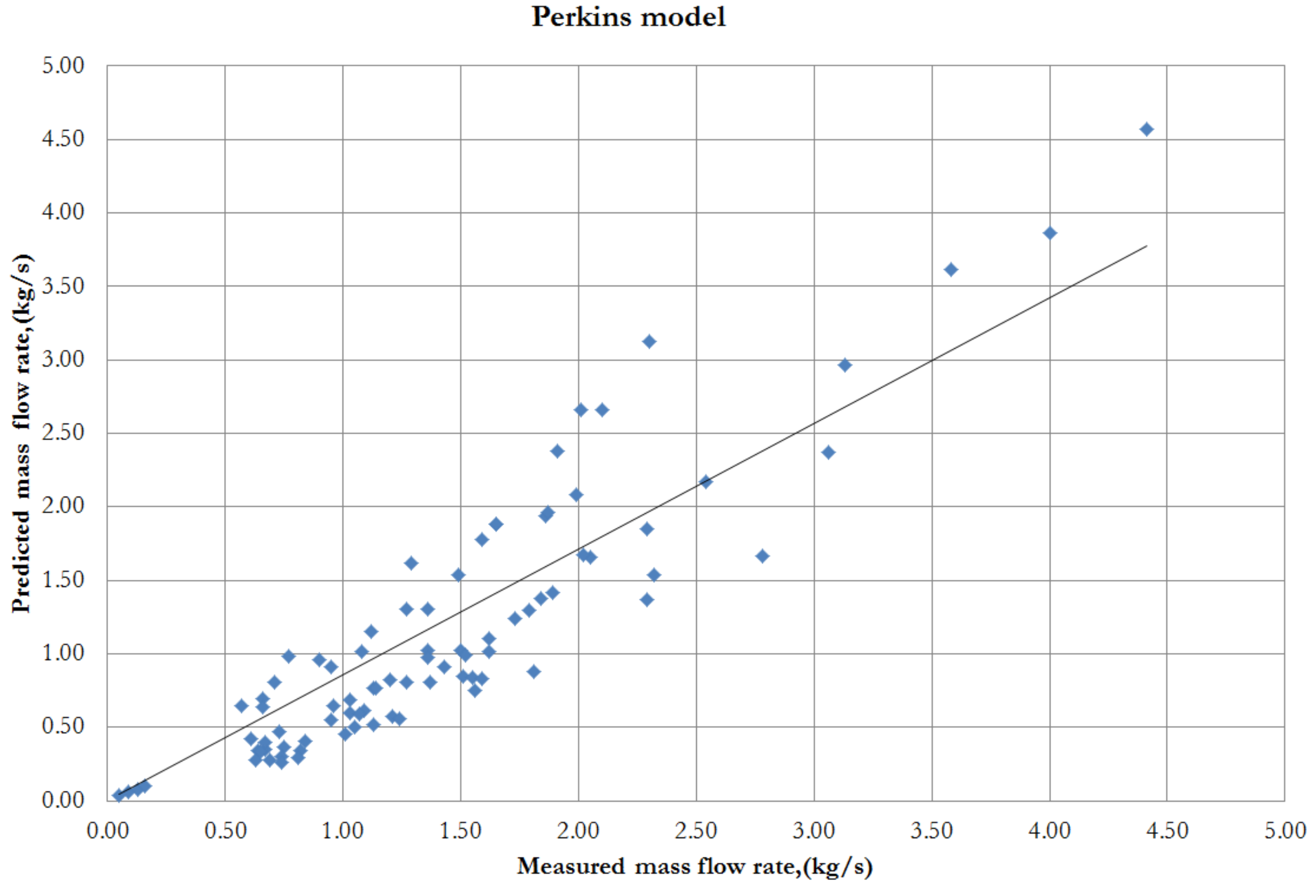

Figure 5 - 2: Results of Perkins' model

iii. (Al-Safran & Kelkar, 2007) Model

Al-Safran model showed a relative error of -10.10%, absolute error of 17.48% and standard deviation of 17.60%

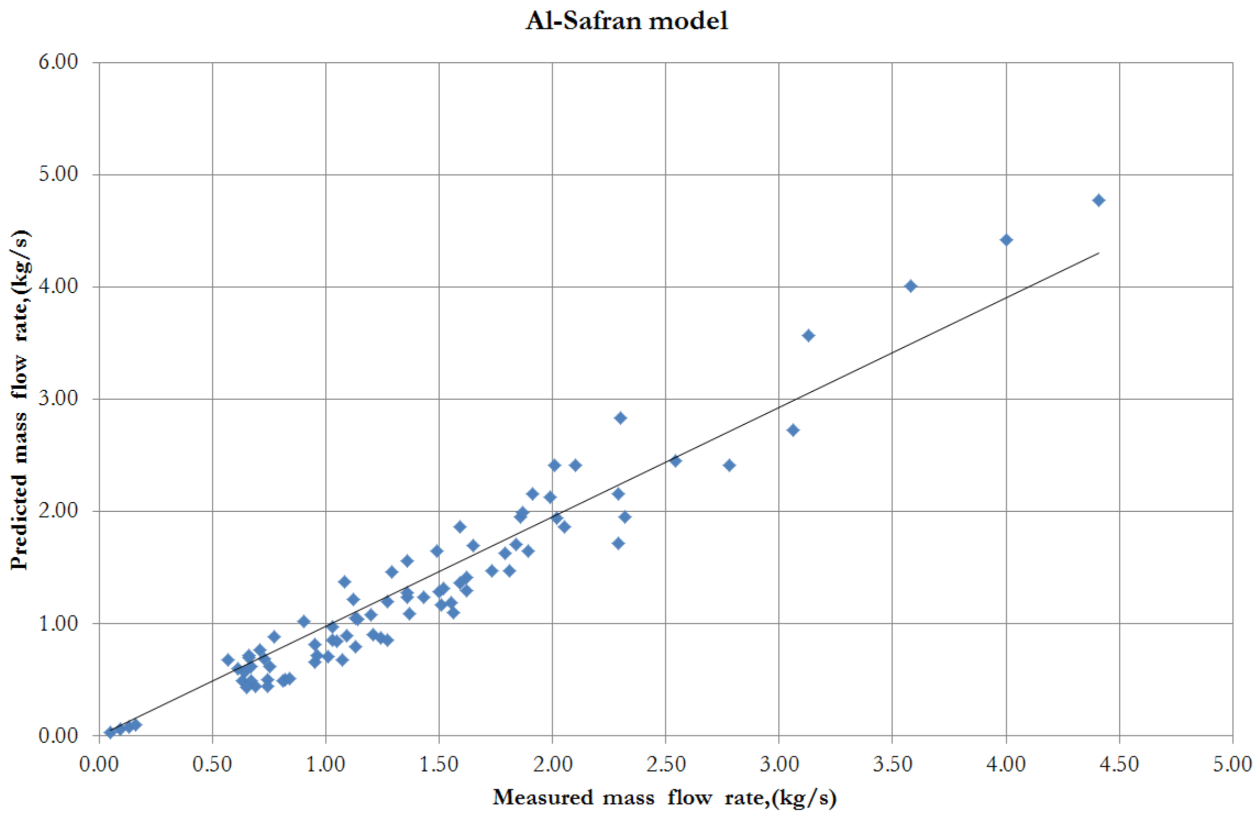

Figure 5 - 3: Results of Al-Safran model

iv. New Modified Sachdeva model

Results for this model include the results from model calibration which was done using three different discharge coefficients i.e. $C_D = 0.64$, 0.65 and 0.66. Discharge coefficient of 0.64 gave a relative error of -1.94% absolute error of 6.31% and standard deviation of 7.55% while a discharge coefficient of 0.65 gave a relative error of -0.4% absolute error of 6.12% and standard deviation of 7.66% and a discharge coefficient of 0.66 gave a relative error of 1.13% absolute error of 6.17% and standard deviation of 7.78. From these results a discharge coefficient of 0.65 was chosen to be a calibration factor for the new modified Sachdeva model as it gives the minimum error comparing to the other valve of discharge coefficient that there were tested to calibrate the model. Graphical representations of the model calibration are as presented below.

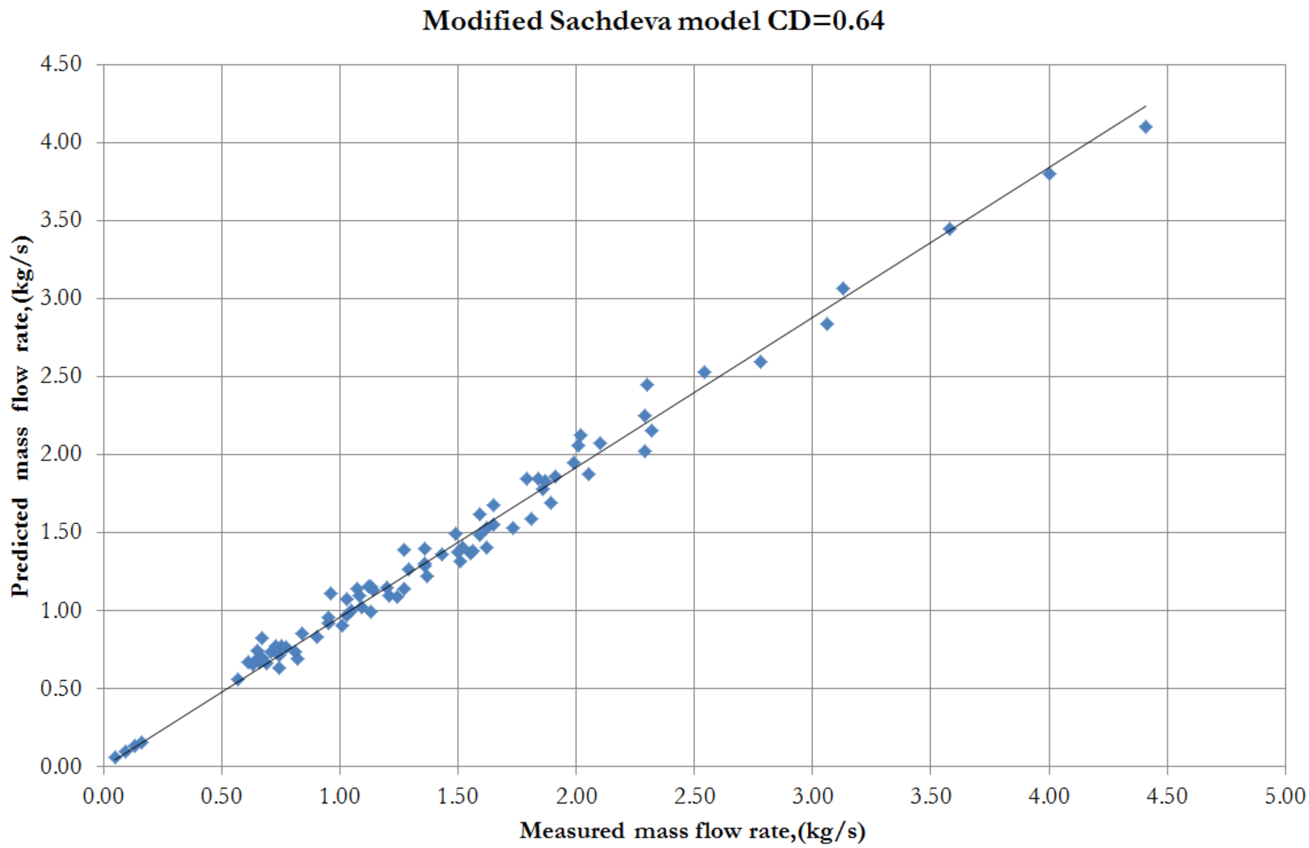

Figure 5 - 4: Results of modified Sachdeva model with CD=0.64

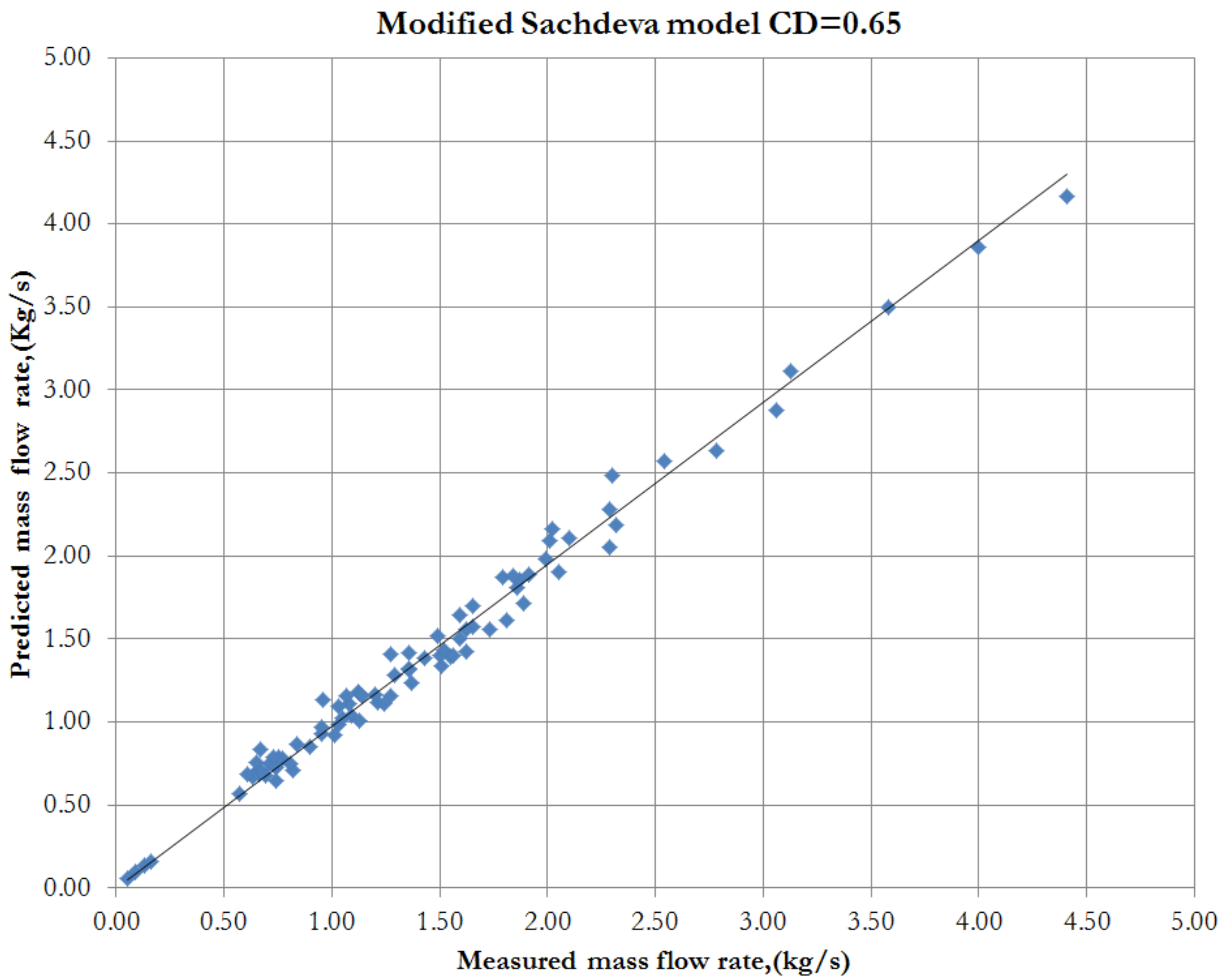

Figure 5 - 5: Results of modified Sachdeva model with CD=0.65

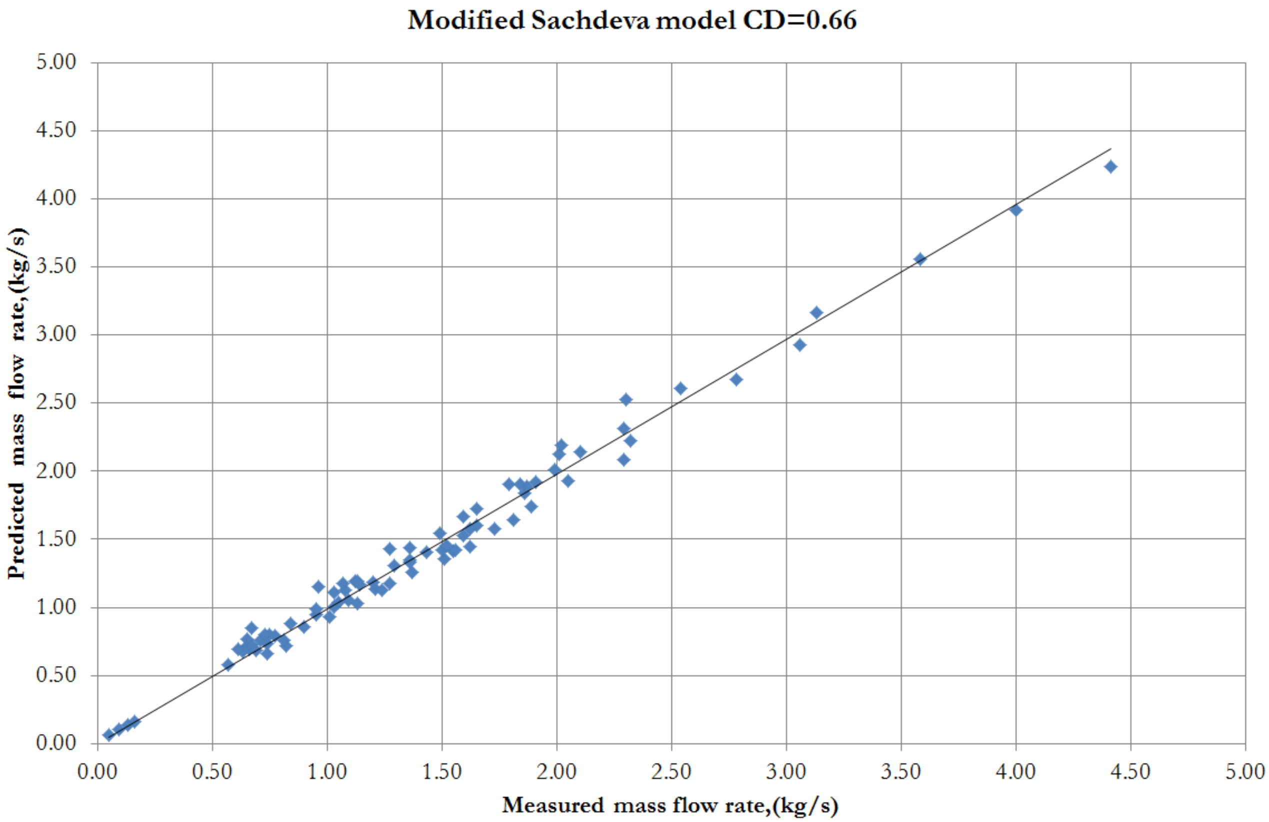

Figure 5 - 6: Results of modified Sachdeva model with CD=0.66

5.2 Discussions

Model evaluation gives different results depending on the data used to evaluate the model, note that model evaluation in this reports was done by using Schüller , et al., 2003 and Schüller , et al., 2006 data.

Sachdeva, et al., 1986 evaluated his model using a discharge coefficient of 0.75 by taking Pilehvari data and Ashford data, his error analysis showed an average relative error of 9.6%, absolute error of 11.5 % and standard deviation of 10.8% for Pilehvari critical data. The same model showed average relative error of 0.3%, absolute error of 8.0% and standard deviation of 12.8% for Pilehvari subctitical data. Ashford data showed evarage relative error of 12.5%, absolute error of 15.1% and standard deviation of 13.6%

On the other hand when Sachdeva, et al., 1986 model was evaluated using the same discharge coefficient of 0.75 by employing Schüller , et al., 2003 and Schüller , et al., 2006 data exhibited relative error of 1.76%, absolute error of 10.52% and standard deviation of 12.49%

Perkins, 1993 concluded that his model showed standard deviation of 15.41% using a discharge coefficient of 0.826 but error analysis using Schüller , et al., 2003 and Schüller , et al., 2006 data shows a standard deviation of 25.91%.

Al-Safran, et al., 2007 claimed that his model have a relative error of 5.2 % and standard deviation of 15.5% and that his model overperforms Sachdeva and Perknis model. Evaluation in this work shows that Al-Safran model exhibits an average relative error of -10.10% , absolute error of 17.48% and standard deviation of 17.60 %. In this case Al-Safran model just overperfoms perkins model and not Sachdeva model. However, during evaluation of Al-Safran model some of the calculation didn't converge when calculating for critical/ subcritical boundary. From the observed data Al-Safran, et al., 2007 seems not to converge in gas fractions >0.25.

The modified Sachdeva model which is the new model developed in this work overperfoms the three models that are evaluated in this report. The models uses the slip corelation suggested by (Grolmes & Leung, 1985). The model can predict the critical /subcritical mass flow rate based on the upstream conditions and measured pressure drop across the choke.

The new model shows average relative error of -0.4%, absolute error of 6.12% and standard deviation of 7.66%. These errors are found by using a dicharge coefficient of 0.65. Discharge coefficient of 0.64 gives a relative error of -1.94%, absolute error of 6.31% and standard deviation of 7.55%, while a discharge coefficient of 0.66 gives a relative error of 1.13%, absolute error of 6.17% and standard deviation of 7.78%. The best value of discharge coefficient in the modified Sachdeva model is choosen to be 0.65 due to the small error showed by this value of dischsrge coefficient

CHAPTER 6

6 CONCLUSIONS AND RECOMMENDATION

6.1 Conclusions

The main conclusions that can be pointed out from this thesis work are;

i. The choke models existing in the literature have different accuracy depending on the set of experimental data employed.
ii. Slippage phenomena between two phases at the choke entrance and at the throat is important parameter to improve predictability of the model.
iii. The new developed model over performs the models Sachdeva, Perkins and Al-Safran
iv. The new model is capable of predicting critical/subcritical mass flow rate with an average error of -0.4%, absolute error of 6.12% and standard deviation of 7.66%.
v. The best value of discharge coefficient for the new modified Sachdeva model is 0.65.

6.2 Recommendation

The work performed in this study used only 85 experimental data points from Schüller papers, it is therefore recommended that more data should be added for model evaluation. It is also recommended that other experimental data from other researchers who have done experiments on multiphase flow should be used to evaluate the model in order to authenticate the accuracy of the model.

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APPENDICES

Appendix A

Mathematical Derivation of the New Choke Model

In this appendix all the model expressions and assumptions mentioned in CHAPTER 4 are presented and explained step by step.

Assumptions

In deriving this model the following assumptions have been used

i. Flow is one dimension
ii. Predominant pressure term is acceleration
iii. The quality is constant for high speed process
iv. The liquid phase is incompressible.
v. The flow is adiabatic and frictionless
vi. Slippage effect exists between the gas phase and the liquid phase.

Concepts used in deriving this model are the same concepts used by Sachdeva, et al., 1986 where by the equations of conservation of mass, momentum and energy were used to determine the relationship between critical and subcritical flow. The difference arises from equation A - 16 when defining mixture density by introducing slippage factor.

Momentum equation at the throat

$$-A_2 dp_2 = d\left(U_2 \dot{m}_{l2} + U_2 \dot{m}_{g2}\right) \qquad \text{A - 1}$$

But we know,

$$\dot{m}_{l2} = \left(1 - x_{g2}\right)\dot{m} \qquad \text{A - 2}$$

And

$$\dot{m}_{g2} = x_{g2}\dot{m} \qquad \text{A - 3}$$

$$-A_2 dp_2 = d\left(U_2\left(1 - x_{g2}\right)\dot{m} + U_2 x_{g2}\dot{m}\right) \qquad \text{A - 4}$$

Dividing right hand side of equation A - 4 by G_2 and multiplying by G_2 where,

$$G_2 = {}^{\dot{m}}\!/_{A_2} \qquad \text{A - 5}$$

$$-A_2 dp_2 = d\left[G_2\left(\frac{U_2\left(1 - x_{g2}\right)\dot{m}}{G_2} + \frac{U_2 x_{g2}\dot{m}}{G_2}\right)\right] \qquad \text{A - 6}$$

$$-A_2 dp_2 = d\left[\, G_2 A_2 \left(U_2 \left(1 - x_{g2}\right) + U_2 x_{g2}\right)\right] \qquad \text{A - 7}$$

$$-A_2 dp_2 = d\left[\, G_2 A_2 U_2\right] \qquad \text{A - 8}$$

From equation A - 5 velocity of the moving fluid at the choke can be expressed as,

$$U_2 = {G_2}/{\rho_{m2}} = G_2 v_{m2} \qquad \text{A - 9}$$

Simplifying equation A - 8 we get,

$$-dp_2 = d\left[\, G_2 U_2\right] \qquad \text{A - 10}$$

Differentiating equation A - 10 with respect to p_2

$$-1 = G_2 \frac{dU_2}{dp_2} + U_2 \frac{dG_2}{dp_2} \qquad \text{A - 11}$$

And G_2 at the throat can be defined as

$$G_2 = \frac{\dot{m}_{g2} + \dot{m}_{l2}}{A_2} \qquad \text{A - 12}$$

For the fixed set of upstream conditions, during critical flow the mass flux reaches a maximum value with respect to downstream (throat) pressure. Therefore the change of mass flux with respect to throat pressure p_2 at the boundary can be defined as,

$$\frac{dG_2}{dp_2} = 0 \qquad \text{A - 13}$$

During critical flow condition equation A - 13 holds and equation A - 11 reduces to,

$$G_2 \frac{dU_2}{dp_2} = -1 \qquad \text{A - 14}$$

Substituting equation A - 9 into equation A - 14

$$G_2^2 \frac{dv_{m2}}{dp_2} = -1 \qquad \text{A - 15}$$

Defining momentum mixture density from Schüller, et al., 2003,

$$\frac{1}{\rho_m} = \left[\frac{x_g}{\rho_g} + R\frac{1 - x_g}{\rho_l}\right]\left[x_g + \frac{1}{R}\left(1 - x_g\right)\right] \qquad \text{A - 16}$$

$$v_{m2} = \left[x_{g2}v_{g2} + R(1 - x_{g2})v_l\right]\left[x_{g2} + \frac{1}{R}(1 - x_{g2})\right] \qquad \text{A - 17}$$

Differentiating equation A - 17 with respect to downstream pressure p_2

$$\frac{dv_{m2}}{dp_2} = \left[x_{g2}v_{g2} + R(1 - x_{g2})v_l\right].\left[\frac{dx_{g2}}{dp_2} - \frac{dx_{g2}}{Rdp_2}\right] + \left[x_{g2} + \frac{1}{R}(1 - x_{g2})\right] \\ \cdot \left[\frac{x_{g2}dv_{g2}}{dp_2} + \frac{v_{g2}dx_{g2}}{dp_2} + R(1 - x_{g2}).\frac{dv_l}{dp_2} - Rv_l.\frac{dx_{g2}}{dp_2}\right] \qquad \text{A - 18}$$

But with very high velocities at the throat, there is no time for mass transfer and therefore,

$$x_{g1} = x_{g2}, dx_g = 0 \qquad \text{A - 19}$$

Equation A - 18 is therefore reduced to,

$$\frac{dv_{m2}}{dp_2} = \left[x_g + \frac{1}{R}(1 - x_g)\right] \cdot \left[\frac{x_g dv_{g2}}{dp_2} + R(1 - x_g).\frac{dv_l}{dp_2}\right] \qquad \text{A - 20}$$

Considering the assumption of incompressible liquid phase $\frac{dv_l}{dp_2} = 0$, equation A - 20 is reduced to,

$$\frac{dv_{m2}}{dp_2} = \left[x_g + \frac{1}{R}(1 - x_g)\right] \cdot \left[\frac{x_g dv_{g2}}{dp_2}\right] \qquad \text{A - 21}$$

Substituting equation A - 21 into equation A - 15.

$$G_2^2.\left[x_g + \frac{1}{R}(1 - x_g)\right] \cdot \left[\frac{x_g dv_{g2}}{dp_2}\right] = -1 \qquad \text{A - 22}$$

$$G_2^2 = \frac{-dp_2}{\left[x_g + \frac{1}{R}(1 - x_g)\right].x_g dv_{g2}} \qquad \text{A - 23}$$

Considering polytropic expansion of gas,

$$p_2 v_{g2}^n = C \qquad \text{A - 24}$$

Differentiating equation A - 24 with respect to v_{g2}

$$np_2 v_{g2}^{n-1} + v_{g2}^n \frac{dp_2}{dv_{g2}} = 0 \qquad \text{A - 25}$$

$$\frac{dp_2}{dv_{g2}} = \frac{-np_2 v_{g2}^{n-1}}{v_{g2}^{n}} \qquad \text{A - 26}$$

$$\frac{dp_2}{dv_{g2}} = \frac{-np_2}{v_{g2}} \qquad \text{A - 27}$$

Substituting equation A - 27 into equation A - 23,

$$G_2^2 = \frac{np_2}{v_{g2} x_g \left[x_g + \frac{1}{R}\left(1 - x_g\right)\right]} \qquad \text{A - 28}$$

Now considering the general Bernoulli's equation,

$$\frac{p_1}{\rho_{m1}} + gz_1 + \frac{U_1^2}{2} = \frac{p_2}{\rho_{m2}} + gz_2 + \frac{U_2^2}{2} \qquad \text{A - 29}$$

With negligible elevation changes $gz_1 = gz_2$ equation A - 29 reduces to,

$$\frac{dp}{\rho_m} = d\left(\frac{U^2}{2}\right) \qquad \text{A - 30}$$

Assuming that the flow is adiabatic and $U_2^2 >>> U_1^2$, then integrating from p_1 to p_2 the term $\int \frac{dp}{\rho_m}$ can be deduced as,

$$p_2 v_{g1}^k = C, \ \ \rho = \left(\frac{p}{C}\right)^{1/k} \qquad \text{A - 31}$$

$$\int \frac{dp}{\rho_m} = \int \frac{dp}{\left(p/C\right)^{1/k}} = C^{1/k} \int p^{-1/k} dp \qquad \text{A - 32}$$

$$\int \frac{dp}{\rho_m} = C^{1/k} \left[\frac{p^{(k-1)/k}}{(k-1)/k}\right] = \frac{k}{k-1} C^{1/k} p^{(k-1)/k} = \frac{k}{k-1} \left(\frac{p}{\rho_m^k}\right)^{1/k} p^{(k-1)/k} \qquad \text{A - 33}$$

Then,

$$\int \frac{dp}{\rho_m} = \frac{k}{k-1} \frac{p}{\rho_m} \qquad \text{A - 34}$$

Therefore equation A - 30 can be written as

$$\frac{k}{k-1}\left(\frac{p_1}{\rho_{m1}} - \frac{p_2}{\rho_{m2}}\right) = \frac{U_2^2}{2} \qquad \text{A - 35}$$

Following the assumption of adiabatic flow, incompressible liquid phase and defining the mixture density equation A - 35 can be written as,

$$\left[x_g + \frac{(1-x_g)}{R}\right]\left[R(1-x_g)v_l(p_1 - p_2) + \frac{k}{k-1}\left(x_g v_{g1} p_1 - x_g v_{g2} p_2\right)\right] = \frac{U_2^2}{2} \qquad \text{A - 36}$$

Defining pressure ratio (y),

$$y = \frac{p_2}{p_1}, \qquad p_2 = y p_1 \qquad \text{A - 37}$$

$$\left[x_g + \frac{(1-x_g)}{R}\right]\left[R(1-x_g)v_l(p_1 - y p_1) + \frac{k}{k-1}\left(x_g v_{g1} p_1 - x_g v_{g2} y p_1\right)\right] = \frac{U_2^2}{2} \qquad \text{A - 38}$$

From equation A - 9 velocity (U_2) can be defined as,

$$U_2^2 = \frac{G_2^2}{\rho_{m2}^2} = G_2^2\left(\left[\frac{x_g}{\rho_g} + R\,\frac{1-x_g}{\rho_l}\right]\left[x_g + \frac{1}{R}(1-x_g)\right]\right)^2 \qquad \text{A - 39}$$

$$p_1\left[x_g + \frac{(1-x_g)}{R}\right]\left[R(1-x_g)v_l(1-y) + \frac{k}{k-1}\left(x_g v_{g1} - x_g y v_{g2}\right)\right] = \frac{G_2^2}{2\rho_{m2}^2} \qquad \text{A - 40}$$

Simplifying the equation above,

$$G_2^2 = 2\rho_{m2}^2 p_1\left[x_g + \frac{(1-x_g)}{R}\right] R(1-x_g)v_l(1-y) + \frac{x_g k}{k-1}\left(v_{g1} - y v_{g2}\right) \qquad \text{A - 41}$$

Therefore, the critical/subcritical mass flow rate formula can be written as,

$$G_2 = \left\{2\rho_{m2}^2 p_1\left[x_g + \frac{(1-x_g)}{R}\right] R(1-x_g)v_l(1-y) + \frac{x_g k}{k-1}\left(v_{g1} - y v_{g2}\right)\right\}^{0.5} \qquad \text{A - 42}$$

Calculating the critical/ subcritical boundary

Eliminating G_2 by substituting equation A - 28 into A - 40

$$p_1\left[x_g+\frac{(1-x_g)}{R}\right]\left[R(1-x_g)v_l(1-y)+\frac{k}{k-1}(x_g v_{g1}-x_g y v_{g2})\right] = \frac{np_2}{v_{g2}x_g\left[x_g+\frac{1}{R}(1-x_g)\right].2\rho_{m2}^2} \qquad \text{A - 43}$$

Substituting the mixture density into RHS equation A - 43

$$=\frac{np_2\left(\left[x_{g2}v_{g2}+R(1-x_{g2})v_l\right]\left[x_{g2}+\frac{1}{R}(1-x_{g2})\right]\right)^2}{2.v_{g2}x_g\left[x_g+\frac{1}{R}(1-x_g)\right]} \qquad \text{A - 44}$$

$$=\frac{np_2\left[x_{g2}v_{g2}+R(1-x_{g2})v_l\right]^2.\left[x_{g2}+\frac{1}{R}(1-x_{g2})\right]}{2.x_g v_{g2}} \qquad \text{A - 45}$$

Combining equation A - 45 as RHS of the equation and LHS of equation A - 43 ,

$$p_1\left[x_g+\frac{(1-x_g)}{R}\right]\left[R(1-x_g)v_l(1-y)+\frac{kx_g}{k-1}(v_{g1}-yv_{g2})\right] = \frac{nyp_1\left[x_{g2}v_{g2}+R(1-x_{g2})v_l\right]^2.\left[x_{g2}+\frac{1}{R}(1-x_{g2})\right]}{2.x_g v_{g2}} \qquad \text{A - 46}$$

Simplifying equation A - 46

$$R(1-x_g)v_l(1-y)+\frac{kx_g v_{g1}}{k-1}=\frac{kx_g y.y^{-1/k}v_{g1}}{k-1}+\frac{ny\left[x_{g2}v_{g2}+R(1-x_{g2})v_l\right]^2}{2.x_g v_{g2}} \qquad \text{A - 47}$$

Considering LHS of equation A - 47,

$$x_g v_{g1}\left[\frac{k}{k-1}+\frac{R(1-x_g)v_l(1-y)}{x_g v_{g1}}\right] \qquad \text{A - 48}$$

Now considering RHS of equation A - 47

$$\frac{kx_g y^{(k-1)/k}.v_{g1}}{k-1}+\frac{ny\left[x_{g2}v_{g2}+R(1-x_{g2})v_l\right]^2}{2.x_g v_{g2}} \qquad \text{A - 49}$$

$$\frac{kx_g v_{g1}y^{(k-1)/k}}{k-1}+\frac{n}{2}\frac{(x_g v_{g1})^2 y^{-2/k}}{x_g v_{g1}y^{-1/k}}+\frac{nR(1-x_g)v_l x_g v_{g1}.y^{(k-1)/k}}{x_g v_{g2}} + \frac{ny}{2x_g v_{g2}}\left(R(1-x_g)v_l\right)^2 \qquad \text{A - 50}$$

$$x_g v_{g1}\left[\frac{ky^{(k-1)/k}}{k-1}+\frac{ny^{(k-1)/k}}{2}+\frac{nR(1-x_g)v_l y^{(k-1)/k}}{x_g v_{g2}}+\frac{n}{2}\left(\frac{R(1-x_g)v_l}{x_g v_{g2}}\right)^2 y^{(k-1)/k}\right] \quad \text{A - 51}$$

Combining equation A - 48 as LHS of the equation and equation A - 51 as the RHS of the equation,

$$x_g v_{g1}\left[\frac{k}{k-1}+\frac{R(1-x_g)v_l(1-y)}{x_g v_{g1}}\right] = x_g v_{g1}\left[\frac{ky^{(k-1)/k}}{k-1}+\frac{ny^{(k-1)/k}}{2}+\frac{nR(1-x_g)v_l y^{(k-1)/k}}{x_g v_{g2}}+\frac{n}{2}\left(\frac{R(1-x_g)v_l}{x_g v_{g2}}\right)y^{(k-1)/k^{2}}\right] \quad \text{A - 52}$$

Simplify equation A - 52 then the critical /subcritical boundary formula can be written as

$$y=\left\{\frac{\frac{k}{k-1}+\frac{R(1-x_g)v_l(1-y)}{x_g v_{g1}}}{\frac{k}{k-1}+\frac{n}{2}+\frac{nR(1-x_g)v_l}{x_g v_{g2}}+\frac{n}{2}\left(\frac{R(1-x_g)v_l}{x_g v_{g2}}\right)^2}\right\}^{\frac{k}{k-1}} \quad \text{A - 53}$$

Appendix B

HYSYS Simulation Procedures

From the data available, HSYSY simulation procedures depend on the pressure and temperature conditions, if the pressure is below the pressure given at the molar composition of the phases, then only valve is to be used to reduce the pressure to the required value. On the other hand if the pressure is above the pressure given at the given molar compositions of the phases then compressors and pumps are used to obtain the required pressure. To obtain the temperature required a cooler or a heater is used. The mass flow rate of the fluid is to be changed until the same mass flow rate that was obtained experimentally is reached. The pictorial presentation of the procedures is as shown in the figures below.

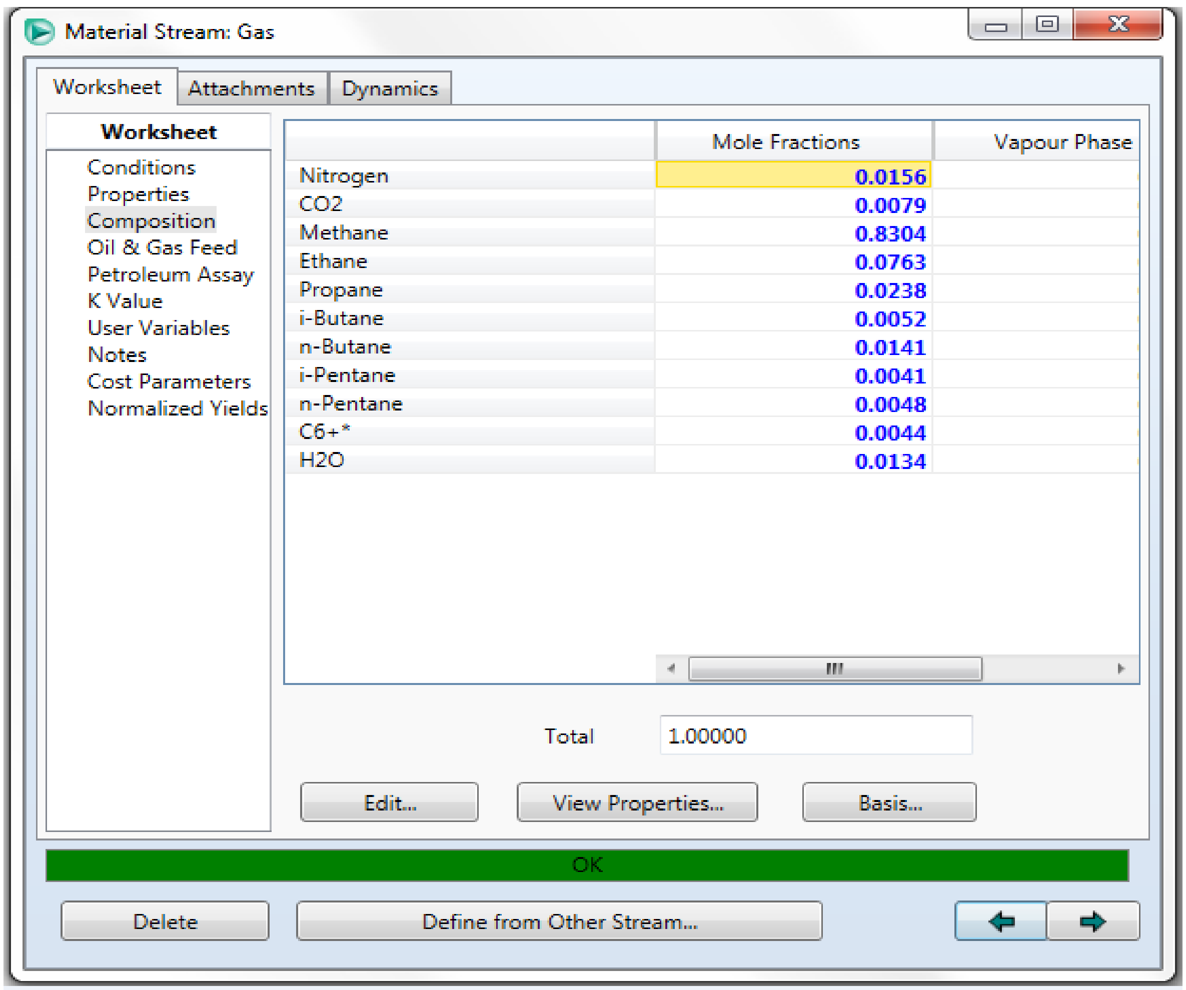


Figure B - 1: Defining molar compositions of the gas stream

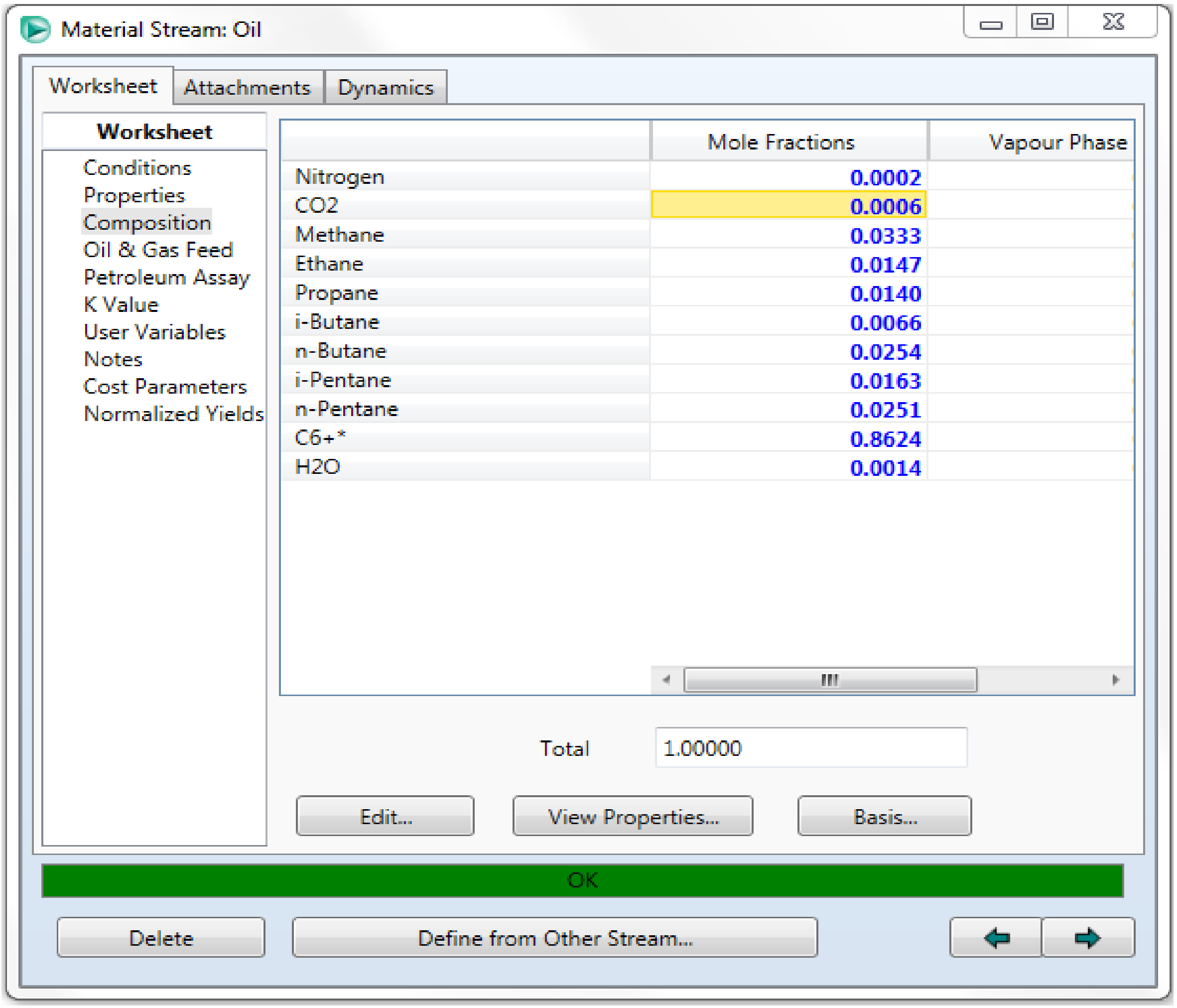


Figure B - 2: Defining molar compositions of the oil stream

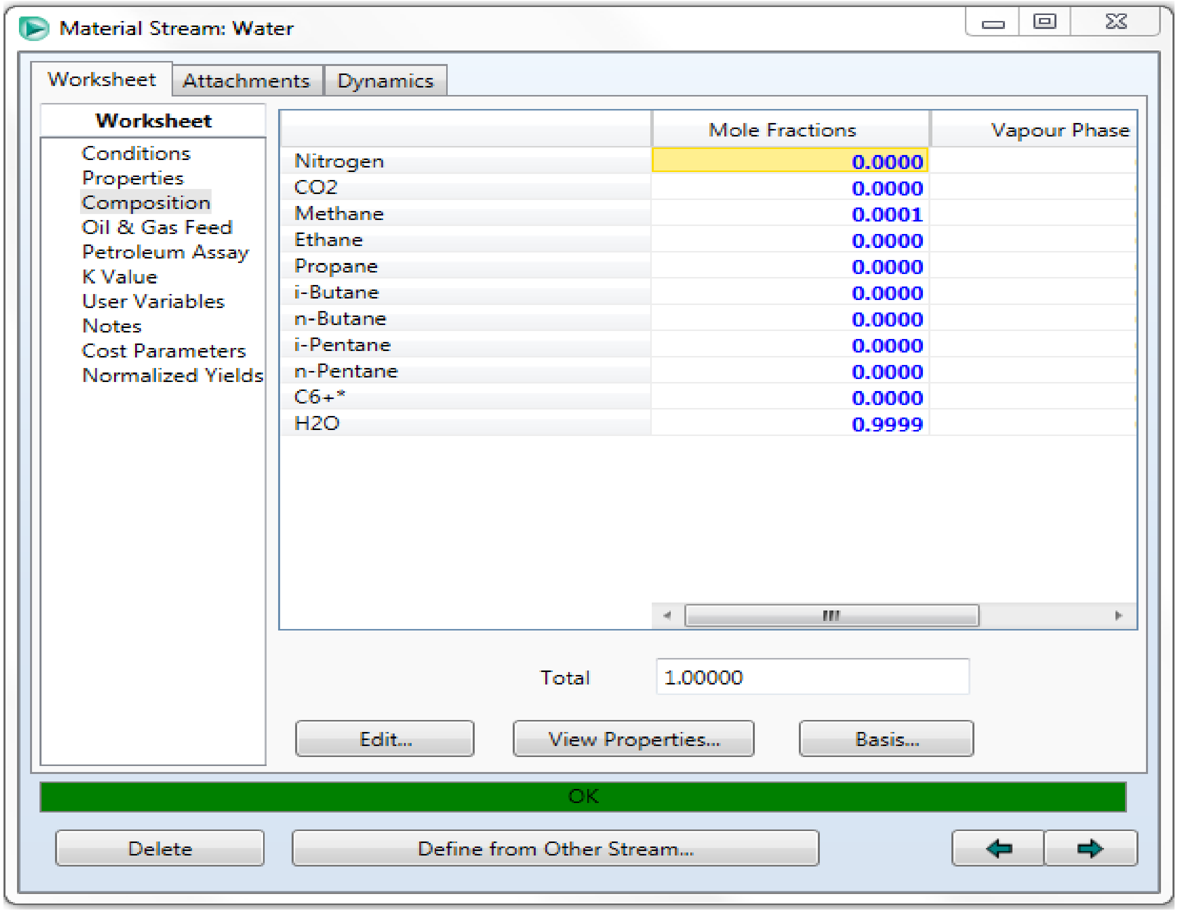
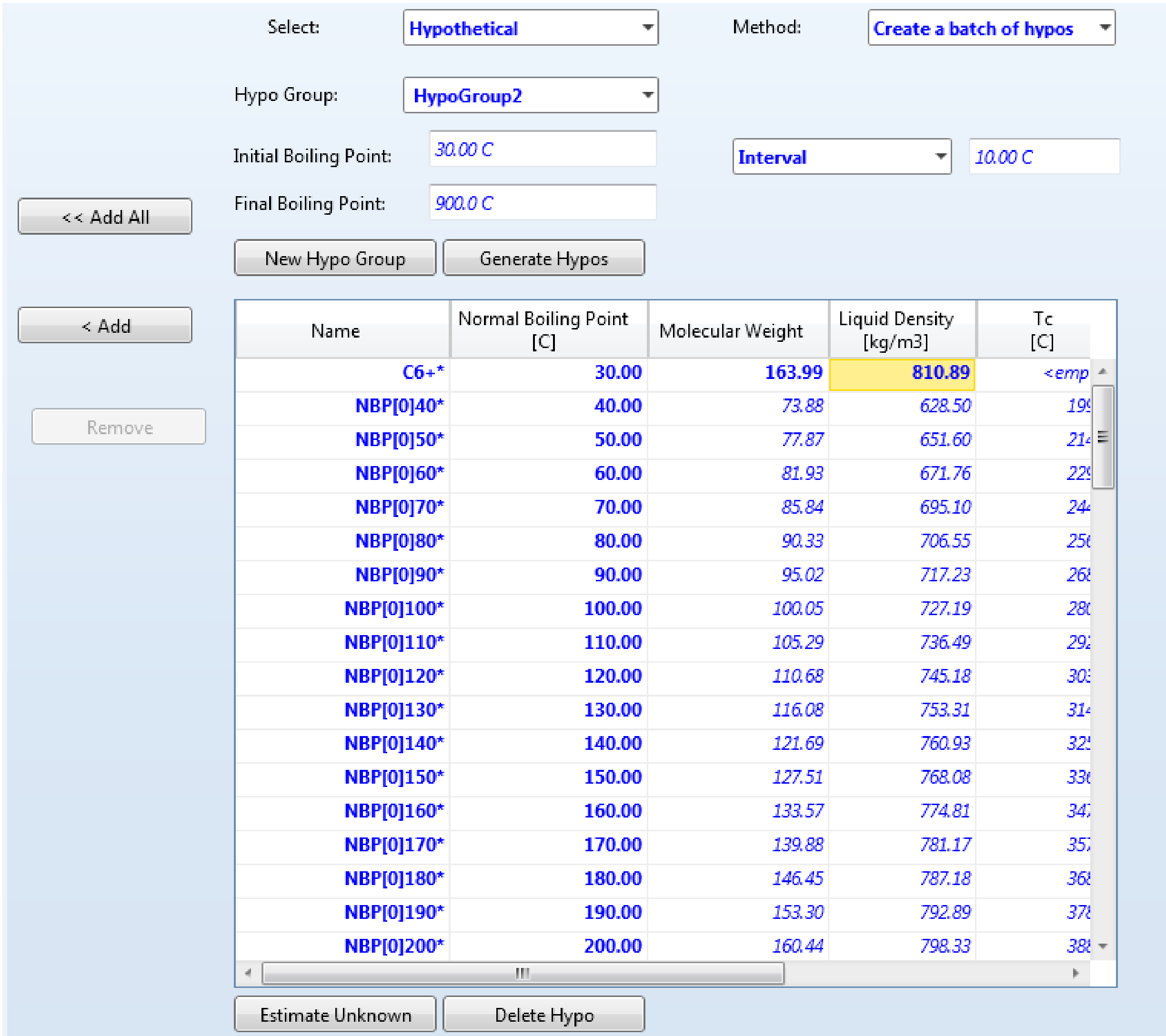


Figure B - 3: Defining molar compositions of the water stream



Name	Normal Boiling Point [C]	Molecular Weight	Liquid Density [kg/m3]	Tc [C]
C6+*	30.00	163.99	810.89	<emp
NBP[0]40*	40.00	73.88	628.50	19
NBP[0]50*	50.00	77.87	651.60	21
NBP[0]60*	60.00	81.93	671.76	22
NBP[0]70*	70.00	85.84	695.10	24
NBP[0]80*	80.00	90.33	706.55	25
NBP[0]90*	90.00	95.02	717.23	26
NBP[0]100*	100.00	100.05	727.19	28
NBP[0]110*	110.00	105.29	736.49	29
NBP[0]120*	120.00	110.68	745.18	30
NBP[0]130*	130.00	116.08	753.31	31
NBP[0]140*	140.00	121.69	760.93	32
NBP[0]150*	150.00	127.51	768.08	33
NBP[0]160*	160.00	133.57	774.81	34
NBP[0]170*	170.00	139.88	781.17	35
NBP[0]180*	180.00	146.45	787.18	36
NBP[0]190*	190.00	153.30	792.89	37
NBP[0]200*	200.00	160.44	798.33	38

Figure B - 4: Adding C6+ as hypothetical components

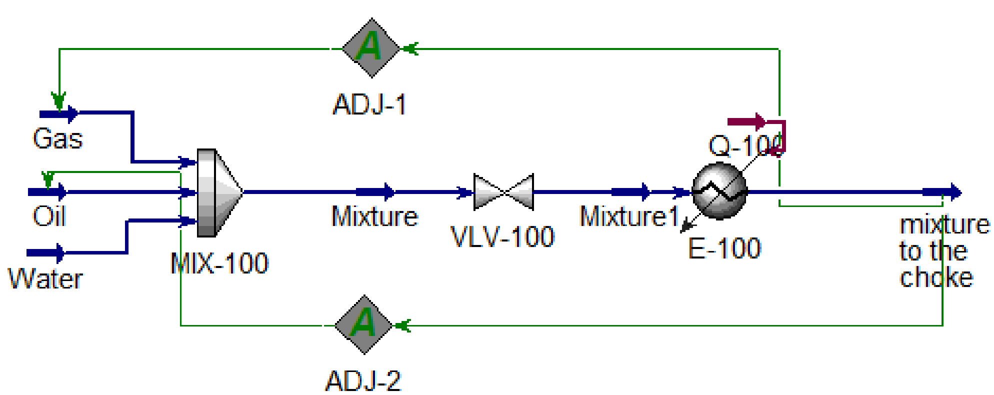


Figure B - 5: Adjusting mass flow rate for pressures lower than the pressure at given molar composition

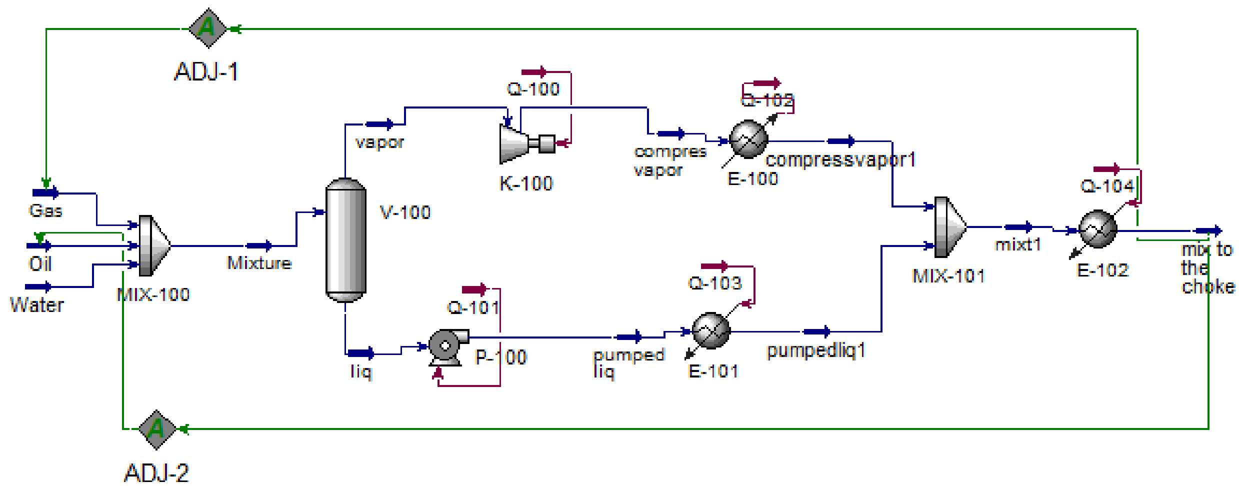


Figure B - 6: Adjusting mass flow rate for pressures higher than the pressure at given molar composition

Appendix C

Programmed Choke Models

Table C- 1: Programmed VBA code for Sachdeva model

```
Function Sachdeva_n(x_1, Cvg, fo, Cvo, fw, cvw, k)
cpg = Cvg * k
firstpart = x_1 * (cpg - Cvg)
Cvl = (fo * Cvo) + (fw * cvw)
secondpart = (x_1 * Cvg) + ((1 - x_1) * Cvl)
Sachdeva_n = 1 + ((firstpart / secondpart))
End Function

Function VG_2(vg1, y, k)
VG_2 = vg1 * y ^ -(1 / k)
End Function

Function SachdevaLHS(k, x1, vL, y, vg1)
part_1 = k / (k - 1)
part_2 = (1 - x1) * vL * (1 - y)
part_3 = x1 * vg1
SachdevaLHS = (part_1 + (part_2 / part_3)) ^ part_1
End Function

Function sachdevaRHS(y, k, n, x1, vL, VG2)
part_1 = k / (k - 1)
part_2 = n / 2
part_3 = (n * (1 - x1) * vL) / (x1 * VG2)
part_4 = (((1 - x1) * vL) / (x1 * VG2)) ^ 2
sachdevaRHS = y * (part_1 + part_2 + part_3 + (part_2 * part_4)) ^ part_1
End Function

Function Sachdeva_massflowrate(x1, vg1, VG2, y, k, vL, density_liquid, CD, p1)
densitymixture = 1 / (x1 * vg1 * y ^ (-1 / k) + (1 - x1) * vL)
part_1 = ((1 - x1) * (1 - y) / density_liquid) + (x1 * k * (vg1 - y * VG2) / (k - 1))
Sachdeva_massflowrate = CD * (2 * p1 * (densitymixture ^ 2) * part_1) ^ 0.5
End Function
```

Table C- 2: Programmed VBA code for Perkins' model

```
Function polytropic_nPerkins(k, Cvg, fg, Cvo, fw, cvw, fo)
polytropic_nPerkins = (k * Cvg * fg + fo * Cvo + fw * cvw) / (fg * Cvg + fo * Cvo + fw * cvw)
End Function

Function Perkins_RHS(v1, M, Z, R, k, fg, fo, fw, Cvg, Cvo, cvw, density_oil, density_water, area2,
area1, pr)
alpha = 1 / v1 * ((fo / density_oil) + (fw / density_water))
lambda = fg + (((fg * Cvg) + (fo * Cvo) + (fw * cvw)) * M / (Z * R))
coef = ((k * Cvg * fg) + (fo * Cvo) + (fw * cvw)) / ((fg * Cvg) + (fo * Cvo) + (fw * cvw))
AreaT = (area2 / area1) ^ 2
Part1 = fg + alpha
Part2 = fg * (pr ^ (-1 / coef)) + alpha
part3 = (Part1 / Part2) ^ 2
Part4 = ((((coef - 1) / coef)) * lambda) * (pr ^ (-1 / coef)) + alpha
Perkins_RHS = (1 - (AreaT * part3)) * Part2 * Part4
 End Function

Function perkins_LHS(v1, M, Z, R, k, fg, fo, fw, Cvg, Cvo, cvw, density_oil, density_water, area2,
area1, pr)
alpha = 1 / v1 * ((fo / density_oil) + (fw / density_water))
lambda = fg + (((fg * Cvg + fo * Cvo + fw * cvw) * M) / (Z * R))
coef = ((k * Cvg * fg) + (fo * Cvo) + (fw * cvw)) / ((fg * Cvg) + (fo * Cvo) + (fw * cvw))
AreaT = (area2 / area1) ^ 2
Part1 = fg + alpha
Part2 = fg * (pr ^ (-1 / coef)) + alpha
part3 = (Part1 / Part2) ^ 2
Part4 = (fg / coef) * (pr ^ (-(coef + 1) / coef))
part5 = 2 * lambda * (1 - pr ^ ((coef - 1) / coef)) + (2 * alpha * (1 - pr))
part6 = ((1 - (AreaT * part3)) * Part4) + (AreaT * Part4 * part3)
perkins_LHS = part5 * part6
End Function

Function p_2(pr, P_1)
p_2 = pr * P_1
End Function

Function P_3(P_1, pressure_drop, choke_diameter, pipe_diameter)
```

```
P_3 = P_1 - (pressure_drop / 1 - (choke_diameter / pipe_diameter) ^ 1.85)
End Function

Function specificv2(k, fg, fo, fw, Cvg, Cvo, cvw, v1, pr)
coef = ((k * Cvg * fg) + (fo * Cvo) + (fw * cvw)) / ((fg * Cvg) + (fo * Cvo) + (fw * cvw))
specificv2 = v1 * (pr ^ (-1 / coef))
End Function

Function velocity_2(v1, M, Z, R, k, fg, fo, fw, Cvg, Cvo, cvw, density_oil, density_water, area2, area1,
p1, pr)
lambda = fg + (((fg * Cvg) + (fo * Cvo) + (fw * cvw)) * M / (Z * R))
coef = ((k * Cvg * fg) + (fo * Cvo) + (fw * cvw)) / ((fg * Cvg) + (fo * Cvo) + (fw * cvw))
part_A = (lambda * p1 * v1) * (1 - pr ^ ((coef - 1) / coef))
Part_B = ((fo / density_oil) + (fw / density_water)) * p1 * (1 - pr)
alpha = 1 / v1 * ((fo / density_oil) + (fw / density_water))
part_C = fg + alpha
part_D = fg * (pr ^ (-1 / coef)) + alpha
part_E = (part_C / part_D) ^ 2
AreaT = (area2 / area1) ^ 2
part_F = 1 - (AreaT * part_E)
velocity_2 = ((2 * (part_A + Part_B)) / part_F) ^ 0.5
End Function

Function perkins_mass_flow_rate(area2, VelocityV2, specificvol2, fg, fo, fw, density_oil,
density_water)
Part1 = area2 * VelocityV2
Part2 = (fg * specificvol2) + ((fo / density_oil) + (fw / density_water))
perkins_mass_flow_rate = Part1 / Part2
End Function
```

Table C- 3: Programmed VBA code for Al-Safranmodel

```
Function polytropic_nAlsafran(k, Cvg, xg, Cvl)
polytropic_nAlsafran = (k * Cvg * xg + (1 - xg) * Cvl) / (Cvg * xg + (1 - xg) * Cvl)
End Function

Function slip_ratiosimpson(density_liquid, density_gas)
slip_ratiosimpson = (density_liquid / density_gas) ^ 0.17
End Function
```

```
Function Alpha_coefficient(slip_ratio, xg, liquid_specificV, vg1)
Alpha_coefficient = (slip_ratio * (1 - xg) * liquid_specificV) / (xg * vg1)
End Function

Function Al_SafranLHS(Alpha_coefficient, rc, Alsafran_n)
Al_SafranLHS = (Alpha_coefficient * (1 - rc)) + (Alsafran_n / Alsafran_n - 1)
End Function

Function Al_SafranRHS(Alpha_coefficient, rc, Alsafran_n)
part_1 = rc ^ (1 - (1 / Alsafran_n))
part_2 = Alsafran_n / Alsafran_n - 1
part_3 = Alsafran_n / 2
part_4 = (1 + Alpha_coefficient * rc ^ (1 / Alsafran_n)) ^ 2
Al_SafranRHS = part_1 * (part_2 + (part_3 * part_4))
End Function

Function Alsafran_massflowrate(area2, p1, Alpha_coefficient, R, Alsafran_n, xg, vg1, slip_ratio, CD)
part_1 = Alpha_coefficient * (1 - R)
part_2 = Alsafran_n / Alsafran_n - 1
part_3 = 1 - R ^ ((Alsafran_n - 1) / Alsafran_n)
part_4 = part_1 + (part_2 * part_3)
part_5 = 2 * CD ^ 2 * area2 ^ 2 * p1 * part_4
part_6 = (R ^ (-1 / Alsafran_n) + Alpha_coefficient) ^ 2
part_7 = xg + ((1 / slip_ratio) * (1 - xg))
part_8 = xg * vg1 * part_6 * part_7
Alsafran_massflowrate = (part_5 / part_8) ^ 0.5
End Function
```

Table C- 4:Programmed VBA code for Modified Sachdeva model

```
Function julianym_n(x_1, Cvg, fo, Cvo, fw, cvw, k)
cpg = Cvg * k
firstpart = x_1 * (cpg - Cvg)
Cvl = (fo * Cvo) + (fw * cvw)
secondpart = (x_1 * Cvg) + ((1 - x_1) * Cvl)
julianym_n = 1 + ((firstpart / secondpart))
End Function
```

```
Function julianymLHS(k, x1, vL, y, vg1, slip_R)
part_1 = k / (k - 1)
part_2 = ((1 - x1) * vL * (1 - y)) * slip_R
part_3 = x1 * vg1
julianymLHS = (part_1 + (part_2 / part_3)) ^ part_1
End Function

Function julianymRHS(y, k, n, x1, vL, VG2, slip_R)
part_1 = k / (k - 1)
part_2 = n / 2
part_3 = (n * slip_R * (1 - x1) * vL) / (x1 * VG2)
part_4 = (((1 - x1) * vL * slip_R) / (x1 * VG2)) ^ 2
julianymRHS = y * (part_1 + part_2 + part_3 + (part_2 * part_4)) ^ part_1
End Function

Function julianym_massflowrate(x1, vg1, VG2, y, k, vL, density_liquid, p1, slip_R)
densitymixture = 1 / ((x1 * vg1 * y ^ (-1 / k) + (1 - x1) * vL * slip_R) * (x1 + (1 - x1) / slip_R))
part_1 = ((1 - x1) * slip_R * (1 - y) / density_liquid) + (x1 * k * (vg1 - y * VG2) / (k - 1))
julianym_massflowrate = (2 * p1 * (x1 + ((1 - x1) / slip_R)) * (densitymixture ^ 2) * part_1) ^ 0.5
End Function
```

Appendix D

Results of Mass Flow Rates Calculations

Table D - 1: Results of Sachdeva model

Test point	Flow type	Measured mass flow rate	Predicted mass flow rate	Error analysis		
		kg/s		E1	E2	E3
G-OR-11-01	subcritical	0.05	0.08	0.56	0.56	1.98E-03
G-OR-11-02	subcritical	0.09	0.13	0.46	0.46	1.11E-03
G-OR-11-03	subcritical	0.13	0.18	0.38	0.38	5.84E-04
G-OR-11-04	critical	0.16	0.21	0.30	0.30	2.37E-04
O-OR-11-01	subcritical	0.57	0.75	0.31	0.31	2.80E-04
O-OR-11-02	subcritical	0.90	1.11	0.23	0.23	7.27E-05
O-OR-11-03	subcritical	1.27	1.51	0.19	0.19	1.82E-05
O-OR-11-04	subcritical	1.65	2.06	0.25	0.25	1.08E-04
O-OR-11-05	subcritical	2.01	2.73	0.36	0.36	5.05E-04
W-OR-11-01	subcritical	0.77	1.02	0.32	0.32	3.40E-04
W-OR-11-02	subcritical	1.29	1.68	0.30	0.30	2.64E-04
W-OR-11-03	subcritical	1.91	2.47	0.29	0.29	2.35E-04
W-OR-11-04	subcritical	2.30	3.25	0.41	0.41	7.97E-04
GOW-OR-11-01	subcritical	0.66	0.83	0.26	0.26	1.36E-04
GOW-OR-11-02	subcritical	0.95	1.02	0.07	0.07	8.14E-05
GOW-OR-11-03	subcritical	1.36	1.59	0.17	0.17	3.18E-06
GOW-OR-11-04	subcritical	1.65	2.12	0.28	0.28	2.00E-04
GOW-OR-11-05	subcritical	1.86	2.23	0.20	0.20	2.48E-05
GOW-OR-11-06	subcritical	0.66	0.83	0.25	0.25	1.19E-04
GOW-OR-11-07	subcritical	1.08	1.31	0.21	0.21	3.67E-05
GOW-OR-11-08	subcritical	1.49	1.88	0.26	0.26	1.43E-04
GOW-OR-11-09	subcritical	1.87	2.28	0.22	0.22	4.77E-05
GOW-OR-11-10	subcritical	0.71	0.89	0.26	0.26	1.28E-04
GOW-OR-11-11	subcritical	1.12	1.41	0.26	0.26	1.29E-04
GOW-OR-11-12	subcritical	1.59	2.02	0.27	0.27	1.70E-04
GOW-OR-11-13	subcritical	1.99	2.42	0.22	0.22	4.67E-05

GOW-OR-11-14	subcritical	0.64	0.73	0.13	0.13	4.39E-06
GOW-OR-11-15	subcritical	1.03	1.06	0.03	0.03	1.77E-04
GOW-OR-11-16	subcritical	1.37	1.35	-0.02	0.02	3.45E-04
GOW-OR-11-17	subcritical	1.62	1.58	-0.03	0.03	3.85E-04
GOW-OR-11-18	subcritical	0.67	0.74	0.11	0.11	2.19E-05
GOW-OR-11-19	subcritical	1.09	1.11	0.02	0.02	2.23E-04
GOW-OR-11-20	subcritical	1.51	1.43	-0.05	0.05	4.88E-04
GOW-OR-11-21	critical	1.81	1.78	-0.02	0.02	3.47E-04
GOW-OR-11-22	subcritical	0.75	0.81	0.08	0.08	5.50E-05
GOW-OR-11-23	critical	1.21	1.16	-0.04	0.04	4.61E-04
GOW-OR-11-24	critical	1.55	1.46	-0.05	0.05	5.10E-04
GOW-OR-11-25	critical	0.63	0.70	0.11	0.11	1.83E-05
GOW-OR-11-26	critical	1.01	0.96	-0.05	0.05	4.85E-04
GOW-OR-11-27	critical	0.74	0.80	0.08	0.08	7.00E-05
GOW-OR-11-28	critical	1.13	1.05	-0.07	0.07	6.01E-04
GOW-OR-11-29	critical	0.82	0.71	-0.13	0.13	9.56E-04
GOW-OR-11-30	critical	1.24	1.15	-0.08	0.08	6.20E-04
GOW-OR-11-31	critical	0.69	0.71	0.04	0.04	1.62E-04
GOW-OR-11-32	critical	0.74	0.71	-0.04	0.04	4.55E-04
GOW-OR-11-33	critical	0.81	0.77	-0.05	0.05	4.79E-04
GOW-OR-11-34	subcritical	0.61	0.74	0.21	0.21	3.88E-05
GOW-OR-11-35	subcritical	1.13	1.26	0.12	0.12	1.54E-05
GOW-OR-11-36	subcritical	1.36	1.48	0.09	0.09	4.81E-05
GOW-OR-11-37	subcritical	1.73	1.76	0.02	0.02	2.09E-04
GOW-OR-11-38	subcritical	1.03	1.17	0.14	0.14	2.61E-06
GOW-OR-11-39	subcritical	1.20	1.28	0.07	0.07	8.83E-05
GOW-OR-11-40	subcritical	1.50	1.55	0.03	0.03	1.66E-04
GOW-OR-11-41	subcritical	1.89	1.96	0.03	0.03	1.66E-04
GOW-OR-11-42	subcritical	0.73	0.84	0.15	0.15	7.36E-07
GOW-OR-11-43	subcritical	1.14	1.25	0.09	0.09	4.23E-05
GOW-OR-11-44	subcritical	1.62	1.70	0.05	0.05	1.21E-04
GOW-OR-11-45	subcritical	2.05	2.22	0.08	0.08	6.05E-05
C2-GOW-OR-11-162	critical	0.67	0.95	0.42	0.42	8.43E-04
C2-GOW-OR-11-163	critical	0.96	1.30	0.36	0.36	4.93E-04

C2-GOW-OR-11-165	critical	0.65	0.83	0.27	0.27	1.67E-04
C2-GOW-OR-11-166	subcritical	0.84	0.96	0.15	0.15	5.69E-07
C2-GOW-OR-11-167	critical	1.07	1.30	0.22	0.22	4.67E-05
C2-GOW-OR-11-168	critical	1.27	1.61	0.27	0.27	1.63E-04
C2-GOW-OR-11-169	subcritical	0.95	1.05	0.11	0.11	2.42E-05
C2-GOW-OR-11-170	critical	1.52	1.71	0.13	0.13	7.96E-06
C2-GOW-OR-11-171	critical	1.79	2.13	0.19	0.19	1.48E-05
C2-GOW-OR-11-172	critical	2.02	2.52	0.25	0.25	1.09E-04
C2-GOW-OR-11-173	subcritical	1.05	1.07	0.02	0.02	2.15E-04
C2-GOW-OR-11-174	subcritical	1.59	1.62	0.02	0.02	2.21E-04
C2-GOW-OR-11-175	critical	2.29	2.27	-0.01	0.01	3.04E-04
C2-GOW-OR-11-177	subcritical	1.36	1.58	0.16	0.16	4.75E-07
C2-GOW-OR-11-178	subcritical	1.84	2.13	0.16	0.16	2.16E-07
C2-GOW-OR-11-179	subcritical	2.29	2.65	0.16	0.16	4.13E-07
C2-GOW-OR-11-180	critical	2.54	3.03	0.19	0.19	1.92E-05
C2-GOW-OR-11-181	subcritical	1.43	1.52	0.06	0.06	1.00E-04
C2-GOW-OR-11-182	critical	2.32	2.42	0.04	0.04	1.39E-04
C2-GOW-OR-11-183	critical	2.78	2.99	0.08	0.08	7.00E-05
C2-GOW-OR-11-184	critical	3.06	3.36	0.10	0.10	3.69E-05
C2-GOW-OR-11-190	critical	1.56	1.48	-0.05	0.05	4.77E-04
C2-W-OR-11-251	subcritical	2.10	2.75	0.31	0.31	2.94E-04
C2-W-OR-11-252	subcritical	3.13	4.07	0.30	0.30	2.59E-04
C2-W-OR-11-253	subcritical	3.58	4.58	0.28	0.28	1.85E-04
C2-W-OR-11-254	subcritical	4.00	5.05	0.26	0.26	1.40E-04
C2-W-OR-11-255	subcritical	4.41	5.45	0.24	0.24	8.09E-05
				0.15	0.17	0.0200385
				15.33	**16.96**	**14.16**

Table D - 2: Results of Perkins model

Test point	Flow type	Measured mass flow rate	Predicted mass flow rate	Error analysis		
		(kg/s)		E1	E2	E3
G-OR-11-01	subcritical	0.05	0.0368	-0.2648	0.2648	6.28E-06
G-OR-11-02	subcritical	0.09	0.0623	-0.3082	0.3082	5.20E-05
G-OR-11-03	subcritical	0.13	0.0774	-0.4045	0.4045	3.12E-04
G-OR-11-04	critical	0.16	0.0999	-0.3756	0.3756	2.11E-04
O-OR-11-01	subcritical	0.57	0.6502	0.1408	0.1408	1.72E-03
O-OR-11-02	subcritical	0.90	0.9631	0.0702	0.0702	1.14E-03
O-OR-11-03	subcritical	1.27	1.3027	0.0258	0.0258	8.41E-04
O-OR-11-04	subcritical	1.65	1.8825	0.1409	0.1409	1.72E-03
O-OR-11-05	subcritical	2.01	2.6576	0.3222	0.3222	3.74E-03
W-OR-11-01	subcritical	0.77	0.9803	0.2731	0.2731	3.12E-03
W-OR-11-02	subcritical	1.29	1.6173	0.2537	0.2537	2.89E-03
W-OR-11-03	subcritical	1.91	2.3795	0.2458	0.2458	2.80E-03
W-OR-11-04	subcritical	2.30	3.1284	0.3602	0.3602	4.26E-03
GOW-OR-11-01	subcritical	0.66	0.6968	0.0558	0.0558	1.04E-03
GOW-OR-11-02	subcritical	0.95	0.9104	-0.0417	0.0417	4.71E-04
GOW-OR-11-03	subcritical	1.36	1.3038	0.1475	0.0413	4.72E-04
GOW-OR-11-04	subcritical	1.65	1.8837	0.1417	0.1417	1.73E-03
GOW-OR-11-05	subcritical	1.86	1.9408	0.0434	0.0434	9.56E-04
GOW-OR-11-06	subcritical	0.66	0.6380	-0.0333	0.0333	5.11E-04
GOW-OR-11-07	subcritical	1.08	1.0181	-0.0573	0.0573	4.00E-04
GOW-OR-11-08	subcritical	1.49	1.5355	0.0306	0.0306	8.72E-04
GOW-OR-11-09	subcritical	1.87	1.9619	0.0492	0.0492	9.95E-04
GOW-OR-11-10	subcritical	0.71	0.8072	0.1369	0.1369	1.69E-03
GOW-OR-11-11	subcritical	1.12	1.1518	0.0284	0.0284	8.58E-04
GOW-OR-11-12	subcritical	1.59	1.7754	0.1166	0.1166	1.51E-03
GOW-OR-11-13	subcritical	1.99	2.0855	0.0480	0.0480	9.87E-04
GOW-OR-11-14	subcritical	0.64	0.3404	-0.4681	0.4681	6.03E-04
GOW-OR-11-15	subcritical	1.03	0.5976	-0.4198	0.4198	3.73E-04
GOW-OR-11-16	critical	1.37	0.8040	-0.4132	0.4132	3.46E-04
GOW-OR-11-17	critical	1.62	1.0180	-0.3716	0.3716	1.99E-04

GOW-OR-11-18	subcritical	0.67	0.3490	-0.4790	0.4790	6.63E-04
GOW-OR-11-19	subcritical	1.09	0.6123	-0.4383	0.4383	4.55E-04
GOW-OR-11-20	critical	1.51	0.8448	-0.4405	0.4405	4.65E-04
GOW-OR-11-21	critical	1.81	0.8825	-0.5124	0.5124	8.62E-04
GOW-OR-11-22	subcritical	0.75	0.3686	-0.5086	0.5086	8.38E-04
GOW-OR-11-23	critical	1.21	0.5737	-0.5259	0.5259	9.50E-04
GOW-OR-11-24	critical	1.55	0.8384	-0.4591	0.4591	5.56E-04
GOW-OR-11-25	critical	0.63	0.2814	-0.5533	0.5533	1.14E-03
GOW-OR-11-26	critical	1.01	0.4512	-0.5532	0.5532	1.14E-03
GOW-OR-11-27	critical	0.74	0.3004	-0.5940	0.5940	1.46E-03
GOW-OR-11-28	critical	1.13	0.5201	-0.5398	0.5398	1.05E-03
GOW-OR-11-29	critical	0.82	0.3415	-0.5836	0.5836	1.38E-03
GOW-OR-11-30	critical	1.24	0.5610	-0.5476	0.5476	1.10E-03
GOW-OR-11-31	critical	0.69	0.2769	-0.5988	0.5988	1.50E-03
GOW-OR-11-32	critical	0.74	0.2647	-0.6423	0.6423	1.89E-03
GOW-OR-11-33	critical	0.81	0.2946	-0.6363	0.6363	1.83E-03
GOW-OR-11-34	subcritical	0.61	0.4206	-0.3105	0.3105	5.58E-05
GOW-OR-11-35	subcritical	1.13	0.7669	-0.3214	0.3214	7.47E-05
GOW-OR-11-36	subcritical	1.36	1.0237	-0.2473	0.2473	3.71E-07
GOW-OR-11-37	critical	1.73	1.2411	-0.2826	0.2826	1.97E-05
GOW-OR-11-38	subcritical	1.03	0.6905	-0.3296	0.3296	9.09E-05
GOW-OR-11-39	subcritical	1.20	0.8248	-0.3127	0.3127	5.93E-05
GOW-OR-11-40	subcritical	1.50	1.0234	-0.3177	0.3177	6.81E-05
GOW-OR-11-41	subcritical	1.89	1.4194	-0.2490	0.2490	6.27E-07
GOW-OR-11-42	subcritical	0.73	0.4700	-0.3562	0.3562	1.54E-04
GOW-OR-11-43	subcritical	1.14	0.7689	-0.3256	0.3256	8.28E-05
GOW-OR-11-44	critical	1.62	1.1032	-0.3190	0.3190	7.04E-05
GOW-OR-11-45	subcritical	2.05	1.6554	-0.1925	0.1925	2.85E-05
C2-GOW-OR-11-162	critical	0.67	0.4014	-0.4009	0.4009	2.98E-04
C2-GOW-OR-11-163	critical	0.96	0.6476	-0.3254	0.3254	8.26E-05
C2-GOW-OR-11-165	critical	0.65	0.3315	-0.4900	0.4900	7.26E-04
C2-GOW-OR-11-166	critical	0.84	0.4067	-0.5158	0.5158	8.84E-04
C2-GOW-OR-11-167	critical	1.07	0.5919	-0.4469	0.4469	4.95E-04
C2-GOW-OR-11-168	critical	1.27	0.8062	-0.3652	0.3652	1.80E-04

C2-GOW-OR-11-169	subcritical	0.95	0.5530	-0.4178	0.4178	3.65E-04
C2-GOW-OR-11-170	critical	1.52	0.9923	-0.3472	0.3472	1.31E-04
C2-GOW-OR-11-171	critical	1.79	1.2966	-0.2756	0.2756	1.36E-05
C2-GOW-OR-11-172	critical	2.02	1.6704	-0.1731	0.1731	5.54E-05
C2-GOW-OR-11-173	subcritical	1.05	0.5019	-0.5220	0.5220	9.24E-04
C2-GOW-OR-11-174	critical	1.59	0.8346	-0.4751	0.4751	6.41E-04
C2-GOW-OR-11-175	critical	2.29	1.3693	-0.4021	0.4021	3.03E-04
C2-GOW-OR-11-177	subcritical	1.36	0.9753	-0.2829	0.2829	2.00E-05
C2-GOW-OR-11-178	critical	1.84	1.3727	-0.2540	0.2540	1.78E-06
C2-GOW-OR-11-179	critical	2.29	1.8513	-0.1916	0.1916	2.95E-05
C2-GOW-OR-11-180	critical	2.54	2.1713	-0.1452	0.1452	1.10E-04
C2-GOW-OR-11-181	subcritical	1.43	0.9141	-0.3608	0.3608	1.67E-04
C2-GOW-OR-11-182	critical	2.32	1.5394	-0.3365	0.3365	1.06E-04
C2-GOW-OR-11-183	critical	2.78	1.6615	-0.4023	0.4023	3.04E-04
C2-GOW-OR-11-184	critical	3.06	2.3751	-0.2238	0.2238	3.75E-06
C2-GOW-OR-11-190	critical	1.56	0.7498	-0.5193	0.5193	9.07E-04
C2-W-OR-11-251	subcritical	2.10	2.6570	0.2653	0.2653	3.02E-03
C2-W-OR-11-252	subcritical	3.13	2.9659	-0.0524	0.0524	4.21E-04
C2-W-OR-11-253	critical	3.58	3.6131	0.0092	0.0092	7.41E-04
C2-W-OR-11-254	critical	4.00	3.8649	-0.0338	0.0338	5.09E-04
C2-W-OR-11-255	critical	4.41	4.5665	0.0355	0.0355	9.04E-04
				-0.2417	0.3074	0.07
				-24.17	**30.74**	**25.91**

Table D - 3: Results of Al-Safran model

Test point	Flow type	Measured mass flow rate	Predicted mass flow rate	Error analysis		
		kg/s		E1	E2	E3
G-OR-11-01	subcritical	0.05	0.03518	-0.296	0.2964	4.49E-04
G-OR-11-02	subcritical	0.09	0.06095	-0.323	0.3228	5.78E-04
G-OR-11-03	subcritical	0.13	0.08557	-0.342	0.3418	6.82E-04
G-OR-11-04	subcritical	0.16	0.10117	-0.368	0.3677	8.36E-04
O-OR-11-01	subcritical	0.57	0.67564	0.1853	0.1853	9.65E-04
O-OR-11-02	subcritical	0.90	1.01888	0.1321	0.1321	6.39E-04
O-OR-11-03	subcritical	1.27	1.20317	-0.053	0.0526	2.76E-05
O-OR-11-04	subcritical	1.65	1.70187	0.0314	0.0314	2.06E-04
O-OR-11-05	subcritical	2.01	2.41255	0.2003	0.2003	1.07E-03
W-OR-11-01	subcritical	0.77	0.88970	0.1555	0.1555	7.74E-04
W-OR-11-02	subcritical	1.29	1.46787	0.1379	0.1379	6.72E-04
W-OR-11-03	subcritical	1.91	2.15973	0.1308	0.1308	6.32E-04
W-OR-11-04	subcritical	2.30	2.83947	0.2346	0.2346	1.33E-03
GOW-OR-11-01	subcritical	0.66	0.71815	0.0881	0.0881	4.21E-04
GOW-OR-11-02	subcritical	0.95	0.66051	-0.305	0.3047	4.88E-04
GOW-OR-11-03	subcritical	1.36	1.56058	0.1475	0.1475	7.27E-04
GOW-OR-11-04	subcritical	1.65	1.85925	0.1268	0.1268	6.11E-04
GOW-OR-11-05	subcritical	1.86	1.95236	0.0497	0.0497	2.67E-04
GOW-OR-11-06	subcritical	0.66	0.70017	0.0609	0.0609	3.08E-04
GOW-OR-11-07	subcritical	1.08	1.37274	0.2711	0.2711	1.63E-03
GOW-OR-11-08	subcritical	1.49	1.65325	0.1096	0.1096	5.22E-04
GOW-OR-11-09	subcritical	1.87	1.99426	0.0664	0.0664	3.30E-04
GOW-OR-11-10	subcritical	0.71	0.76584	0.0787	0.0787	3.80E-04
GOW-OR-11-11	subcritical	1.12	1.22124	0.0904	0.0904	4.31E-04
GOW-OR-11-12	subcritical	1.59	1.86103	0.1705	0.1705	8.67E-04
GOW-OR-11-13	subcritical	1.99	2.12843	0.0696	0.0696	3.42E-04
GOW-OR-11-14	subcritical	0.64	0.57185	-0.106	0.1065	3.47E-07
GOW-OR-11-15	subcritical	1.03	0.85699	-0.168	0.168	5.27E-05
GOW-OR-11-16	subcritical	1.37	1.09381	-0.202	0.2016	1.19E-04

GOW-OR-11-17	subcritical	1.62	1.29784	-0.199	0.1989	1.13E-04
GOW-OR-11-18	subcritical	0.67	0.61899	-0.076	0.0761	7.30E-06
GOW-OR-11-19	subcritical	1.09	0.89623	-0.178	0.1778	6.93E-05
GOW-OR-11-20	subcritical	1.51	1.16691	-0.227	0.2272	1.87E-04
GOW-OR-11-21	subcritical	1.81	1.47515	-0.185	0.185	8.29E-05
GOW-OR-11-22	subcritical	0.75	0.61770	-0.176	0.1764	6.68E-05
GOW-OR-11-23	subcritical	1.21	0.90129	-0.255	0.2551	2.79E-04
GOW-OR-11-24	subcritical	1.55	1.19099	-0.232	0.2316	2.01E-04
GOW-OR-11-25	subcritical	0.63	0.49487	-0.214	0.2145	1.51E-04
GOW-OR-11-26	subcritical	1.01	0.71100	-0.296	0.296	4.47E-04
GOW-OR-11-27	subcritical	0.74	0.50137	-0.322	0.3225	5.77E-04
GOW-OR-11-28	subcritical	1.13	0.79428	-0.297	0.2971	4.52E-04
GOW-OR-11-29	subcritical	0.82	0.50617	-0.383	0.3827	9.33E-04
GOW-OR-11-30	subcritical	1.24	0.88013	-0.29	0.2902	4.21E-04
GOW-OR-11-31	subcritical	0.69	0.44267	-0.358	0.3584	7.79E-04
GOW-OR-11-32	subcritical	0.74	0.44039	-0.405	0.4049	1.09E-03
GOW-OR-11-33	subcritical	0.81	0.49665	-0.387	0.3868	9.61E-04
GOW-OR-11-34	subcritical	0.61	0.60226	-0.013	0.0127	9.19E-05
GOW-OR-11-35	subcritical	1.13	1.04838	-0.072	0.0722	9.77E-06
GOW-OR-11-36	subcritical	1.36	1.24020	-0.088	0.0881	1.98E-06
GOW-OR-11-37	subcritical	1.73	1.47790	-0.146	0.1457	2.35E-05
GOW-OR-11-38	subcritical	1.03	0.96948	-0.059	0.0588	2.10E-05
GOW-OR-11-39	subcritical	1.20	1.07791	-0.102	0.1017	5.62E-09
GOW-OR-11-40	subcritical	1.50	1.29193	-0.139	0.1387	1.67E-05
GOW-OR-11-41	subcritical	1.89	1.65029	-0.127	0.1268	7.82E-06
GOW-OR-11-42	subcritical	0.73	0.68976	-0.055	0.0551	2.48E-05
GOW-OR-11-43	subcritical	1.14	1.03841	-0.089	0.0891	1.68E-06
GOW-OR-11-44	subcritical	1.62	1.41693	-0.125	0.1254	6.95E-06
GOW-OR-11-45	subcritical	2.05	1.86322	-0.091	0.0911	1.16E-06
C2-GOW-OR-11-162	subcritical	0.67	0.49186	-0.266	0.2659	3.20E-04
C2-GOW-OR-11-163	subcritical	0.96	0.71464	-0.256	0.2556	2.81E-04
C2-GOW-OR-11-165	subcritical	0.65	0.43743	-0.327	0.327	6.01E-04
C2-GOW-OR-11-166	subcritical	0.84	0.50912	-0.394	0.3939	1.01E-03
C2-GOW-OR-11-167	subcritical	1.07	0.68175	-0.363	0.3629	8.06E-04

C2-GOW-OR-11-168	subcritical	1.27	0.85198	-0.329	0.3292	6.12E-04
C2-GOW-OR-11-169	subcritical	0.95	0.81378	-0.143	0.1434	2.11E-05
C2-GOW-OR-11-170	subcritical	1.52	1.31287	-0.136	0.1363	1.46E-05
C2-GOW-OR-11-171	subcritical	1.79	1.62679	-0.091	0.0912	1.15E-06
C2-GOW-OR-11-172	subcritical	2.02	1.94657	-0.036	0.0364	4.92E-05
C2-GOW-OR-11-173	subcritical	1.05	0.84912	-0.191	0.1913	9.59E-05
C2-GOW-OR-11-174	subcritical	1.59	1.36693	-0.14	0.1403	1.81E-05
C2-GOW-OR-11-175	subcritical	2.29	1.72016	-0.249	0.2488	2.57E-04
C2-GOW-OR-11-177	subcritical	1.36	1.27804	-0.06	0.0603	1.96E-05
C2-GOW-OR-11-178	subcritical	1.84	1.70493	-0.073	0.0734	8.99E-06
C2-GOW-OR-11-179	subcritical	2.29	2.15464	-0.059	0.0591	2.07E-05
C2-GOW-OR-11-180	subcritical	2.54	2.45461	-0.034	0.0336	5.35E-05
C2-GOW-OR-11-181	subcritical	1.43	1.23667	-0.135	0.1352	1.37E-05
C2-GOW-OR-11-182	subcritical	2.32	1.95394	-0.158	0.1578	3.79E-05
C2-GOW-OR-11-183	subcritical	2.78	2.41804	-0.13	0.1302	1.00E-05
C2-GOW-OR-11-184	subcritical	3.06	2.72600	-0.109	0.1091	7.72E-07
C2-GOW-OR-11-190	subcritical	1.56	1.09956	-0.295	0.2952	4.43E-04
C2-W-OR-11-251	subcritical	2.10	2.41170	0.1484	0.1484	7.32E-04
C2-W-OR-11-252	subcritical	3.13	3.56910	0.1403	0.1403	6.85E-04
C2-W-OR-11-253	subcritical	3.58	4.01134	0.1205	0.1205	5.77E-04
C2-W-OR-11-254	subcritical	4.00	4.42455	0.1061	0.1061	5.05E-04
C2-W-OR-11-255	subcritical	4.41	4.77766	0.0834	0.0834	4.00E-04
				-0.101	0.1748	0.03097
				-10.10	**17.48**	**17.60**

Table D - 4: Results of new modified Sachdeva model

		Mass flow rates (kg/s)					Error analysis								
		Measured	Predicted	Calibrated mass flow rate											
				CD=0.64	**CD=0.65**	**CD=0.66**	**CD=0.64**			**CD=0.65**			**CD=0.66**		
Test point	Flow type			**0.64**	**0.65**	**0.66**	E1	E2	E3	E1	E2	E3	E1	E2	E3
G-OR-11-01	subcritical	0.05	0.09	0.06	0.06	0.06	0.18	0.18	0.00	0.20	0.20	0.00	0.21	0.21	0.00
G-OR-11-02	subcritical	0.09	0.15	0.10	0.10	0.10	0.10	0.10	0.00	0.12	0.12	0.00	0.13	0.13	0.00
G-OR-11-03	subcritical	0.13	0.21	0.13	0.14	0.14	0.04	0.04	0.00	0.05	0.05	0.00	0.07	0.07	0.00
G-OR-11-04	critical	0.16	0.24	0.16	0.16	0.16	-0.02	0.02	0.00	-0.01	0.01	0.00	0.01	0.01	0.00
O-OR-11-01	subcritical	0.57	0.88	0.56	0.57	0.58	-0.02	0.02	0.00	0.00	0.00	0.00	0.02	0.02	0.00
O-OR-11-02	subcritical	0.90	1.30	0.83	0.85	0.86	-0.07	0.07	0.00	-0.06	0.06	0.00	-0.04	0.04	0.00
O-OR-11-03	subcritical	1.27	1.78	1.14	1.16	1.18	-0.10	0.10	0.00	-0.09	0.09	0.00	-0.07	0.07	0.00
O-OR-11-04	subcritical	1.65	2.43	1.55	1.58	1.60	-0.06	0.06	0.00	-0.04	0.04	0.00	-0.03	0.03	0.00
O-OR-11-05	subcritical	2.01	3.22	2.06	2.09	2.12	0.02	0.02	0.00	0.04	0.04	0.00	0.06	0.06	0.00
W-OR-11-01	subcritical	0.77	1.20	0.77	0.78	0.79	0.00	0.00	0.00	0.01	0.01	0.00	0.03	0.03	0.00
W-OR-11-02	subcritical	1.29	1.98	1.27	1.29	1.31	-0.02	0.02	0.00	0.00	0.00	0.00	0.01	0.01	0.00
W-OR-11-03	subcritical	1.91	2.91	1.86	1.89	1.92	-0.03	0.03	0.00	-0.01	0.01	0.00	0.01	0.01	0.00
W-OR-11-04	subcritical	2.30	3.82	2.45	2.49	2.52	0.06	0.06	0.00	0.08	0.08	0.00	0.10	0.10	0.00
GOW-OR-11-01	subcritical	0.66	1.05	0.67	0.68	0.69	0.02	0.02	0.00	0.04	0.04	0.00	0.05	0.05	0.00
GOW-OR-11-02	subcritical	0.95	1.44	0.92	0.93	0.95	-0.03	0.03	0.00	-0.02	0.02	0.00	0.00	0.00	0.00
GOW-OR-11-03	subcritical	1.36	2.04	1.31	1.33	1.35	-0.04	0.04	0.00	-0.03	0.03	0.00	-0.01	0.01	0.00
GOW-OR-11-04	subcritical	1.65	2.62	1.67	1.70	1.73	0.02	0.02	0.00	0.03	0.03	0.00	0.05	0.05	0.00
GOW-OR-11-05	subcritical	1.86	2.79	1.78	1.81	1.84	-0.04	0.04	0.00	-0.03	0.03	0.00	-0.01	0.01	0.00

GOW-OR-11-06	subcritical	0.66	1.08	0.69	0.70	0.72	0.05	0.05	0.00	0.07	0.07	0.00	0.08	0.08	0.00
GOW-OR-11-07	subcritical	1.08	1.71	1.09	1.11	1.13	0.01	0.01	0.00	0.03	0.03	0.00	0.05	0.05	0.00
GOW-OR-11-08	subcritical	1.49	2.34	1.50	1.52	1.54	0.00	0.00	0.00	0.02	0.02	0.00	0.04	0.04	0.00
GOW-OR-11-09	subcritical	1.87	2.86	1.83	1.86	1.89	-0.02	0.02	0.00	-0.01	0.01	0.00	0.01	0.01	0.00
GOW-OR-11-10	subcritical	0.71	1.16	0.74	0.75	0.76	0.04	0.04	0.00	0.06	0.06	0.00	0.07	0.07	0.00
GOW-OR-11-11	subcritical	1.12	1.81	1.16	1.18	1.20	0.04	0.04	0.00	0.05	0.05	0.00	0.07	0.07	0.00
GOW-OR-11-12	subcritical	1.59	2.53	1.62	1.64	1.67	0.02	0.02	0.00	0.03	0.03	0.00	0.05	0.05	0.00
GOW-OR-11-13	subcritical	1.99	3.05	1.95	1.98	2.01	-0.02	0.02	0.00	0.00	0.00	0.00	0.01	0.01	0.00
GOW-OR-11-14	subcritical	0.64	1.06	0.68	0.69	0.70	0.06	0.06	0.00	0.08	0.08	0.00	0.09	0.09	0.00
GOW-OR-11-15	subcritical	1.03	1.52	0.97	0.99	1.00	-0.06	0.06	0.00	-0.04	0.04	0.00	-0.03	0.03	0.00
GOW-OR-11-16	subcritical	1.37	1.91	1.22	1.24	1.26	-0.11	0.11	0.00	-0.10	0.10	0.00	-0.08	0.08	0.00
GOW-OR-11-17	subcritical	1.62	2.20	1.41	1.43	1.45	-0.13	0.13	0.00	-0.12	0.12	0.00	-0.11	0.11	0.00
GOW-OR-11-18	subcritical	0.67	1.10	0.70	0.71	0.72	0.05	0.05	0.00	0.06	0.06	0.00	0.08	0.08	0.00
GOW-OR-11-19	subcritical	1.09	1.60	1.03	1.04	1.06	-0.06	0.06	0.00	-0.04	0.04	0.00	-0.03	0.03	0.00
GOW-OR-11-20	subcritical	1.51	2.06	1.32	1.34	1.36	-0.13	0.13	0.00	-0.12	0.12	0.00	-0.10	0.10	0.00
GOW-OR-11-21	subcritical	1.81	2.48	1.59	1.61	1.64	-0.12	0.12	0.00	-0.11	0.11	0.00	-0.09	0.09	0.00
GOW-OR-11-22	subcritical	0.75	1.21	0.78	0.79	0.80	0.03	0.03	0.00	0.05	0.05	0.00	0.07	0.07	0.00
GOW-OR-11-23	subcritical	1.21	1.72	1.10	1.12	1.14	-0.09	0.09	0.00	-0.08	0.08	0.00	-0.06	0.06	0.00
GOW-OR-11-24	subcritical	1.55	2.14	1.37	1.39	1.41	-0.12	0.12	0.00	-0.10	0.10	0.00	-0.09	0.09	0.00
GOW-OR-11-25	subcritical	0.63	1.03	0.66	0.67	0.68	0.04	0.04	0.00	0.06	0.06	0.00	0.08	0.08	0.00
GOW-OR-11-26	critical	1.01	1.41	0.91	0.92	0.93	-0.10	0.10	0.00	-0.09	0.09	0.00	-0.08	0.08	0.00
GOW-OR-11-27	critical	0.74	1.11	0.71	0.72	0.73	-0.04	0.04	0.00	-0.02	0.02	0.00	-0.01	0.01	0.00
GOW-OR-11-28	subcritical	1.13	1.56	1.00	1.01	1.03	-0.12	0.12	0.00	-0.11	0.11	0.00	-0.09	0.09	0.00
GOW-OR-11-29	critical	0.82	1.09	0.70	0.71	0.72	-0.15	0.15	0.00	-0.14	0.14	0.00	-0.12	0.12	0.00

GOW-OR-11-30	critical	1.24	1.70	1.09	1.11	1.12	-0.12	0.12	0.00	-0.11	0.11	0.00	-0.09	0.09	0.00
GOW-OR-11-31	critical	0.69	1.04	0.67	0.68	0.69	-0.04	0.04	0.00	-0.02	0.02	0.00	-0.01	0.01	0.00
GOW-OR-11-32	critical	0.74	1.00	0.64	0.65	0.66	-0.14	0.14	0.00	-0.13	0.13	0.00	-0.11	0.11	0.00
GOW-OR-11-33	critical	0.81	1.15	0.74	0.75	0.76	-0.09	0.09	0.00	-0.08	0.08	0.00	-0.06	0.06	0.00
GOW-OR-11-34	subcritical	0.61	1.05	0.67	0.68	0.69	0.10	0.10	0.00	0.12	0.12	0.00	0.14	0.14	0.00
GOW-OR-11-35	subcritical	1.13	1.80	1.15	1.17	1.19	0.02	0.02	0.00	0.04	0.04	0.00	0.05	0.05	0.00
GOW-OR-11-36	subcritical	1.36	2.02	1.29	1.31	1.33	-0.05	0.05	0.00	-0.03	0.03	0.00	-0.02	0.02	0.00
GOW-OR-11-37	subcritical	1.73	2.39	1.53	1.56	1.58	-0.11	0.11	0.00	-0.10	0.10	0.00	-0.09	0.09	0.00
GOW-OR-11-38	subcritical	1.03	1.69	1.08	1.10	1.11	0.05	0.05	0.00	0.06	0.06	0.00	0.08	0.08	0.00
GOW-OR-11-39	subcritical	1.20	1.79	1.15	1.16	1.18	-0.04	0.04	0.00	-0.03	0.03	0.00	-0.01	0.01	0.00
GOW-OR-11-40	subcritical	1.50	2.15	1.38	1.40	1.42	-0.08	0.08	0.00	-0.07	0.07	0.00	-0.05	0.05	0.00
GOW-OR-11-41	subcritical	1.89	2.64	1.69	1.72	1.74	-0.11	0.11	0.00	-0.09	0.09	0.00	-0.08	0.08	0.00
GOW-OR-11-42	subcritical	0.73	1.21	0.78	0.79	0.80	0.06	0.06	0.00	0.08	0.08	0.00	0.10	0.10	0.00
GOW-OR-11-43	subcritical	1.14	1.78	1.14	1.15	1.17	0.00	0.00	0.00	0.01	0.01	0.00	0.03	0.03	0.00
GOW-OR-11-44	subcritical	1.62	2.39	1.53	1.56	1.58	-0.05	0.05	0.00	-0.04	0.04	0.00	-0.03	0.03	0.00
GOW-OR-11-45	subcritical	2.05	2.93	1.87	1.90	1.93	-0.09	0.09	0.00	-0.07	0.07	0.00	-0.06	0.06	0.00
C2-GOW-OR-11-162	critical	0.67	1.29	0.82	0.84	0.85	0.23	0.23	0.00	0.25	0.25	0.00	0.27	0.27	0.00
C2-GOW-OR-11-163	critical	0.96	1.74	1.11	1.13	1.15	0.16	0.16	0.00	0.18	0.18	0.00	0.20	0.20	0.00
C2-GOW-OR-11-165	critical	0.65	1.16	0.74	0.75	0.77	0.14	0.14	0.00	0.16	0.16	0.00	0.18	0.18	0.00
C2-GOW-OR-11-166	subcritical	0.84	1.34	0.86	0.87	0.88	0.02	0.02	0.00	0.03	0.03	0.00	0.05	0.05	0.00
C2-GOW-OR-11-167	critical	1.07	1.78	1.14	1.16	1.18	0.07	0.07	0.00	0.08	0.08	0.00	0.10	0.10	0.00
C2-GOW-OR-11-168	critical	1.27	2.17	1.39	1.41	1.43	0.09	0.09	0.00	0.11	0.11	0.00	0.13	0.13	0.00
C2-GOW-OR-11-169	subcritical	0.95	1.50	0.96	0.97	0.99	0.01	0.01	0.00	0.02	0.02	0.00	0.04	0.04	0.00

C2-GOW-OR-11-170	subcritical	1.52	2.20	1.41	1.43	1.45	-0.07	0.07	0.00	-0.06	0.06	0.00	-0.05	0.05	0.00
C2-GOW-OR-11-171	subcritical	1.79	2.88	1.85	1.87	1.90	0.03	0.03	0.00	0.05	0.05	0.00	0.06	0.06	0.00
C2-GOW-OR-11-172	critical	2.02	3.32	2.13	2.16	2.19	0.05	0.05	0.00	0.07	0.07	0.00	0.09	0.09	0.00
C2-GOW-OR-11-173	subcritical	1.05	1.57	1.01	1.02	1.04	-0.04	0.04	0.00	-0.03	0.03	0.00	-0.01	0.01	0.00
C2-GOW-OR-11-174	subcritical	1.59	2.32	1.48	1.51	1.53	-0.07	0.07	0.00	-0.05	0.05	0.00	-0.04	0.04	0.00
C2-GOW-OR-11-175	critical	2.29	3.16	2.02	2.05	2.08	-0.12	0.12	0.00	-0.10	0.10	0.00	-0.09	0.09	0.00
C2-GOW-OR-11-177	subcritical	1.36	2.19	1.40	1.42	1.44	0.03	0.03	0.00	0.04	0.04	0.00	0.06	0.06	0.00
C2-GOW-OR-11-178	subcritical	1.84	2.89	1.85	1.88	1.91	0.01	0.01	0.00	0.02	0.02	0.00	0.04	0.04	0.00
C2-GOW-OR-11-179	subcritical	2.29	3.51	2.25	2.28	2.32	-0.02	0.02	0.00	0.00	0.00	0.00	0.01	0.01	0.00
C2-GOW-OR-11-180	critical	2.54	3.96	2.53	2.57	2.61	0.00	0.00	0.00	0.01	0.01	0.00	0.03	0.03	0.00
C2-GOW-OR-11-181	subcritical	1.43	2.13	1.36	1.38	1.40	-0.05	0.05	0.00	-0.03	0.03	0.00	-0.02	0.02	0.00
C2-GOW-OR-11-182	subcritical	2.32	3.36	2.15	2.19	2.22	-0.07	0.07	0.00	-0.06	0.06	0.00	-0.04	0.04	0.00
C2-GOW-OR-11-183	subcritical	2.78	4.05	2.59	2.63	2.67	-0.07	0.07	0.00	-0.05	0.05	0.00	-0.04	0.04	0.00
C2-GOW-OR-11-184	subcritical	3.06	4.43	2.84	2.88	2.93	-0.07	0.07	0.00	-0.06	0.06	0.00	-0.04	0.04	0.00
C2-GOW-OR-11-190	critical	1.56	2.16	1.38	1.40	1.43	-0.11	0.11	0.00	-0.10	0.10	0.00	-0.09	0.09	0.00
C2-W-OR-11-251	subcritical	2.10	3.24	2.07	2.11	2.14	-0.01	0.01	0.00	0.00	0.00	0.00	0.02	0.02	0.00
C2-W-OR-11-252	subcritical	3.13	4.79	3.07	3.12	3.16	-0.02	0.02	0.00	0.00	0.00	0.00	0.01	0.01	0.00
C2-W-OR-11-253	subcritical	3.58	5.39	3.45	3.50	3.55	-0.04	0.04	0.00	-0.02	0.02	0.00	-0.01	0.01	0.00
C2-W-OR-11-254	subcritical	4.00	5.94	3.80	3.86	3.92	-0.05	0.05	0.00	-0.03	0.03	0.00	-0.02	0.02	0.00
C2-W-OR-11-255	subcritical	4.41	6.41	4.10	4.17	4.23	-0.07	0.07	0.00	-0.05	0.05	0.00	-0.04	0.04	0.00
							-0.02	0.06	0.01	0.00	0.06	0.01	0.01	0.06	0.01
							-1.94	**6.31**	**7.55**	**-0.40**	**6.12**	**7.66**	**1.13**	**6.17**	**7.78**

Appendix E

Modified Perkins model

Mathematical derivation
Considering the general energy equation for compressible flow,

$$\frac{dp}{\rho} + UdU + gdz = 0 \qquad \text{E - 1}$$

Integrating both sides of the equation,

$$\int \frac{dp}{\rho} + \frac{U^2}{2} + gz = C(constant) \qquad \text{E - 2}$$

Assuming adiabatic flow, and dealing with the term $\int \frac{dp}{\rho}$, only

$$p_2 v_{g1}^k = C, \;\; \rho = \left(\frac{p}{C}\right)^{1/k} \qquad \text{E - 3}$$

$$\int \frac{dp}{\rho_m} = \int \frac{dp}{\left(p/C\right)^{1/k}} = C^{1/k} \int p^{-1/k} dp \qquad \text{E - 4}$$

$$\int \frac{dp}{\rho_m} = C^{1/k}\left[\frac{p^{(k-1)/k}}{(k-1)/k}\right] = \frac{k}{k-1} C^{1/k} p^{(k-1)/k} = \frac{k}{k-1}\left(\frac{p}{\rho_m^k}\right)^{1/k} p^{(k-1)/k} \qquad \text{E - 5}$$

$$\int \frac{dp}{\rho_m} = \frac{k}{k-1}\frac{p}{\rho_m} \qquad \text{E - 6}$$

Substituting equation E - 6 into E - 2 and assuming negligible elevation changes,

$$\frac{k}{k-1}\frac{p_1}{\rho_{m1}} + \frac{U_1^2}{2} = \frac{k}{k-1}\frac{p_2}{\rho_{m2}} + \frac{U_2^2}{2} \qquad \text{E - 7}$$

Defining Mixture density,

$$\frac{1}{\rho_{m1}} = \frac{\frac{x_g}{\rho_g} + R\frac{1-x_g}{\rho_l}}{x_g + (1-x_g)R} \qquad \text{E - 8}$$

$$\frac{1}{\rho_{m1}} = \frac{x_g v_{g1} + R(1 - x_g) v_{l1}}{x_g + (1 - x_g) R} \qquad \text{E - 9}$$

Taking equation E - 7

$$\frac{k}{k-1}\left[\frac{p_1}{\rho_{m1}} - \frac{p_2}{\rho_{m2}}\right] = \frac{U_2^2}{2} - \frac{U_1^2}{2} \qquad \text{E - 10}$$

Consider the LHS of equation E - 10,

$$\frac{k}{k-1}\left[\frac{p_1 x_{g1} v_{g1} + R(1 - x_g) v_{l1}}{x_{g1} + (1 - x_{g1}) R} - \frac{p_2 x_{g2} v_{g2} + R(1 - x_g) v_{l2}}{x_{g2} + (1 - x_{g2}) R}\right] \qquad \text{E - 11}$$

But with very high velocities at the throat, there is no time for mass transfer and therefore

$$x_{g1} = x_{g2}, dx_g = 0 \qquad \text{E - 12}$$

$$\frac{k x_g}{k-1}\left[\frac{p_1 v_{g1}}{x_g + (1 - x_g) R} - \frac{p_2 v_{g2}}{x_g + (1 - x_g) R}\right] + \frac{R(1 - x_g) v_l (p_1 - p_2)}{x_g + (1 - x_g) R} \qquad \text{E - 13}$$

Assuming polytropic expansion of gas,

$$p_1 v_{g1}^n = p_2 v_{g2}^n \qquad \text{E - 14}$$

$$v_{g2} = v_{g1}\left(\frac{p_2}{p_1}\right)^{-1/n} \qquad \text{E - 15}$$

Let$\frac{p_2}{p_1} = p_r$,

$$p_2 = p_1 p_r \qquad \text{E - 16}$$

Substituting equation E - 15 and E - 16 into E - 13,

$$\frac{k x_g}{k-1}\left[\frac{p_1 v_{g1}}{x_g + (1 - x_g) R} - \frac{p_1 p_r . v_{g1} (p_r)^{-1/n}}{x_g + (1 - x_g) R}\right] + \frac{R(1 - x_g) v_l (p_1 - p_1 p_r)}{x_g + (1 - x_g) R} \qquad \text{E - 17}$$

$$\frac{k x_g p_1 v_{g1}}{k - 1. (x_g + (1 - x_g) R)}\left[1 - {p_r}^{(n-1)/n}\right] + \frac{p_1 . R(1 - x_g) v_l (1 - p_r)}{x_g + (1 - x_g) R} \qquad \text{E - 18}$$

Multiplying each term of equation E - 18 by $\left(x_g + (1 - x_g) R\right)$ and dividing by $x_{g1} v_{g1}$

$$\frac{kp_1}{k-1}\left[1-p_r{}^{(n-1)/n}\right]+\frac{p_1.R\left(1-x_g\right)v_l(1-p_r)}{x_{g1}v_{g1}} \qquad \text{E - 19}$$

Let,

$$\frac{R\left(1-x_g\right)v_l}{x_{g1}v_{g1}}=\alpha \qquad \text{E - 20}$$

Finally the LHS of the equation will be,

$$p_1\left[\frac{k}{k-1}\left(1-p_r{}^{(n-1)/n}\right)\right]+\alpha(1-p_r) \qquad \text{E - 21}$$

Now, considering the RHS of equation E - 11,

From the continuity equation,

$$U^2=\frac{\dot{m}^2}{\rho^2A^2} \qquad \text{E - 22}$$

$$RHS=\frac{\dot{m}^2}{2\rho_{m2}^2A_2^2}-\frac{\dot{m}^2}{2\rho_{m1}^2A_1^2} \qquad \text{E - 23}$$

$$\frac{\dot{m}^2}{2\rho_{m2}^2A_2^2}\left(1-\frac{\rho_{m2}^2A_2^2}{\rho_{m1}^2A_1^2}\right) \qquad \text{E - 24}$$

Now, substituting the mixture density into equation E - 24,

$$\frac{\dot{m}^2}{2A_2^2}\left[\frac{x_gv_{g2}+R\left(1-x_g\right)v_l}{x_g+\left(1-x_g\right)R}\right]^2\left[1-\left(\frac{x_gv_{g1}+R\left(1-x_g\right)v_l}{x_gv_{g2}+R\left(1-x_g\right)v_l}\right)^2\left(\frac{A_2}{A_1}\right)^2\right] \qquad \text{E - 25}$$

Substituting equation E - 15 into E - 25

$$\frac{\dot{m}^2}{2A_2^2}\left[\frac{x_gv_{g1}p_r{}^{-1/n}+R\left(1-x_g\right)v_l}{x_g+\left(1-x_g\right)R}\right]^2\left[1-\left(\frac{x_gv_{g1}+R\left(1-x_g\right)v_l}{x_gv_{g1}p_r{}^{-1/n}+R\left(1-x_g\right)v_l}\right)^2\left(\frac{A_2}{A_1}\right)^2\right] \qquad \text{E - 26}$$

Just as LHS, multiply each term of equation E - 26 by $\left(x_g+\left(1-x_g\right)R\right)$ and divide by x_gv_{g1}

$$\frac{\dot{m}^2}{2A_2^2}\left[\alpha+p_r{}^{-1/n}\right]^2\left[\frac{x_g+\left(1-x_g\right)R}{x_gv_{g1}}-\left(\frac{1+\alpha}{\alpha+p_r{}^{-1/n}}\right)^2\left(\frac{A_2}{A_1}\right)^2.\frac{\left(x_g+\left(1-x_g\right)R\right)}{x_gv_{g1}}\right] \qquad \text{E - 27}$$

$$\frac{x_g v_{g1}.\dot{m}^2}{2A_2^2\left(x_g+\left(1-x_g\right)R\right)}\left[\alpha+p_r^{-1/n}\right]^2\left[1-\left(\frac{1+\alpha}{\alpha+p_r^{-1/n}}\right)^2\left(\frac{A_2}{A_1}\right)^2\right] \qquad \text{E - 28}$$

Now, combining equation E - 21 and E - 28 as LHS and RHS of the equation E - 10

$$p_1\left[\frac{k}{k-1}\left(1-p_r^{(n-1)/n}\right)\right]+\alpha(1-p_r) = \frac{x_g v_{g1}.\dot{m}^2}{2A_2^2\left(x_g+\left(1-x_g\right)R\right)}\left[\alpha+p_r^{-1/n}\right]^2\left[1-\left(\frac{1+\alpha}{\alpha+p_r^{-1/n}}\right)^2\left(\frac{A_2}{A_1}\right)^2\right] \qquad \text{E - 29}$$

And the formula for mass flow rate can be written as,

$$\dot{m}^2=\frac{2A_2^2 p_1\left(x_g+\left(1-x_g\right)R\right).\left[\left(\frac{k}{k-1}\left(1-p_r^{(n-1)/n}\right)\right)+\alpha(1-p_r)\right]}{x_g v_{g1}\left[\alpha+p_r^{-1/n}\right]^2\left[1-\left(\frac{1+\alpha}{\alpha+p_r^{-1/n}}\right)^2\left(\frac{A_2}{A_1}\right)^2\right]} \qquad \text{E - 30}$$

At optimal flow rate $\frac{d\dot{m}}{dp_r}=0$

$$\frac{d}{dp_r}\left(\frac{\dot{m}x_g v_{g1}}{2A_2^2.p_1\left(x_g+\left(1-x_g\right)R\right)}\right)=0 \qquad \text{E - 31}$$

Then,

$$\frac{\left[\left(\left(1-p_r^{(n-1)/n}\right)\right)+\alpha(1-p_r)\right]}{\left[\alpha+p_r^{-1/n}\right]^2\left[1-\left(\frac{1+\alpha}{\alpha+p_r^{-1/n}}\right)^2\left(\frac{A_2}{A_1}\right)^2\right]}=0 \qquad \text{E - 32}$$

Assuming that $\left(\frac{A_2}{A_1}\right)^2\approx 0$ then Differentiating equation E - 32 and rearranging to get the formula for critical/subcritical flow boundary,

$$\left[\alpha+p_r^{-1/n}\right]^2\left[\frac{k}{k-1}\frac{n-1}{n}\left(p_r^{(-1/n)}-\alpha\right)\right] = \left[\frac{k}{k-1}\left(1-p_r^{(n-1)/n}\right)+\alpha(1-p_r)\right]\left[\frac{2}{n}p_r^{-(n+1)/n}.\left(\alpha+p_r^{-1/n}\right)\right] \qquad \text{E - 33}$$

Procedures for Calculating Mass Flow Rate

i. Guess initial value of p_r

ii. Use the initial guess to iterate for the critical pressure ratio p_{rc} by employing equation E - 33(45) using solver in Excel.

iii. Calculate pressure downstream the choke $p_{2'}$ using Perry's relation in equation (48)

iv. Calculate the actual pressure ratio $(p_{r(act)})$ which is $p_{2'}/p_1$

v. If $p_{rc} > p_{r(act)}$, flow is critical then $p_r = p_{rc}$ is used to calculate mass flow rate in equation E - 30(49) and if $p_{rc} < p_{r(act)}$ flow is subcritical and $p_r = p_{r(act)}$ is used to calculate the mass flow rate.

vi. Find the mass flow rate by multiply with the discharge coefficient C_D.

Results of the modified Perkins Model

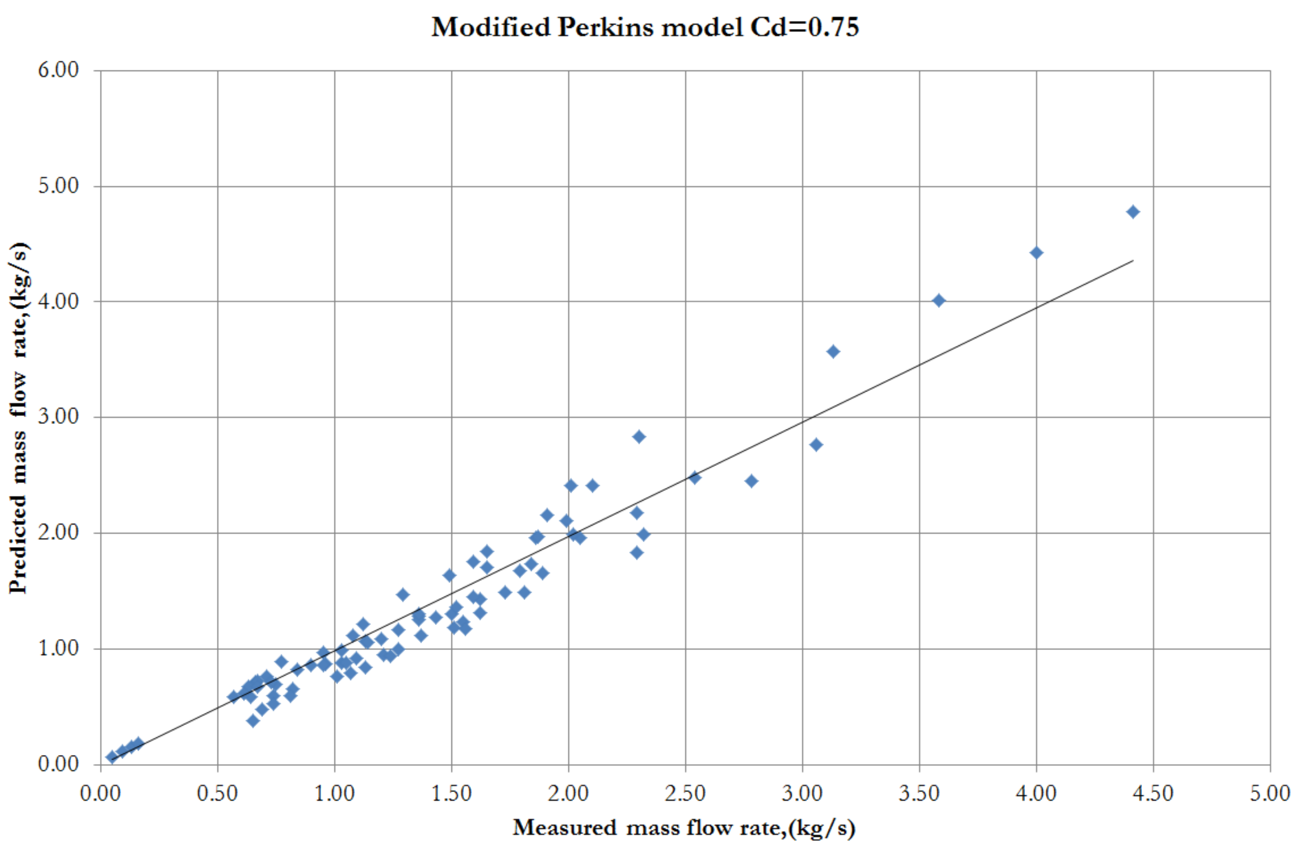


Figure E - 1: Results of modified Perkins model with CD=0.75

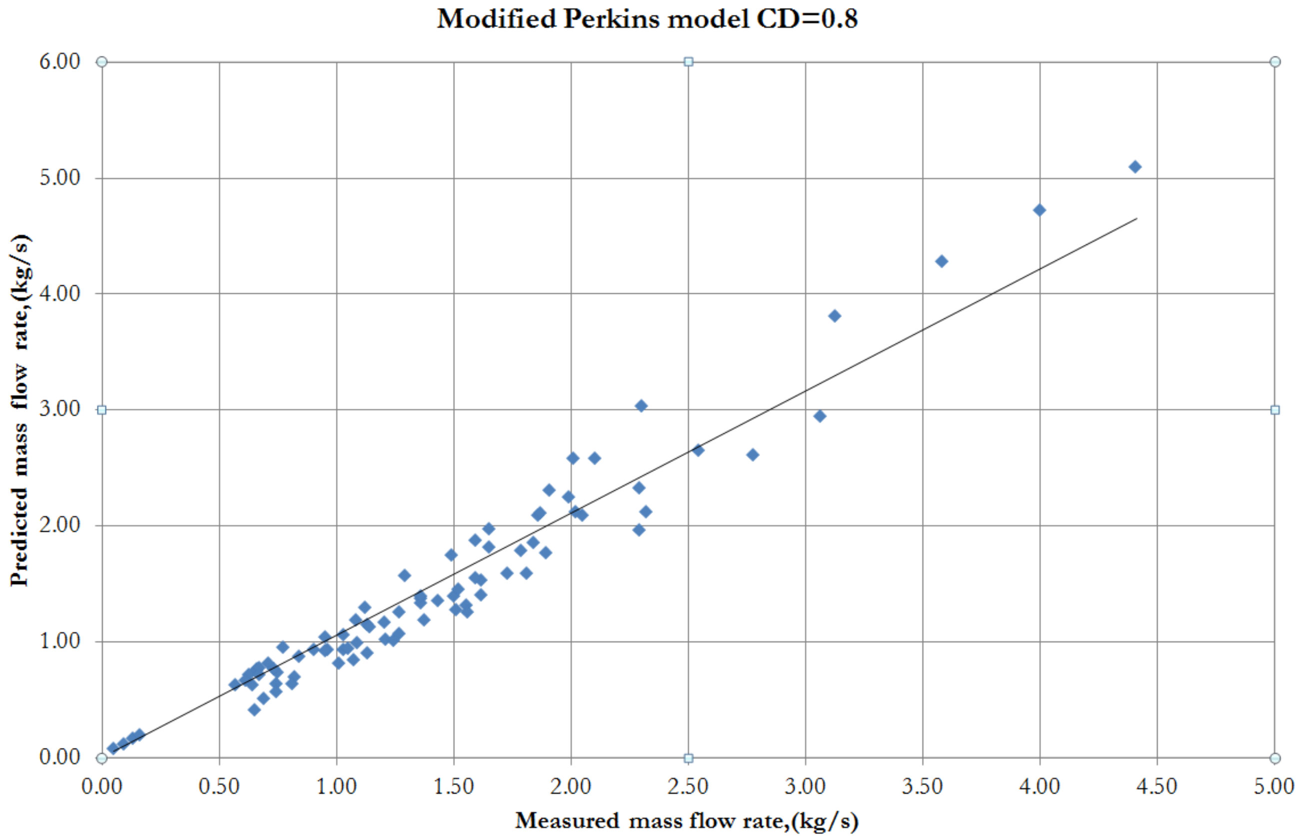


Figure E - 2: Results of modified Perkins model with CD=0.8

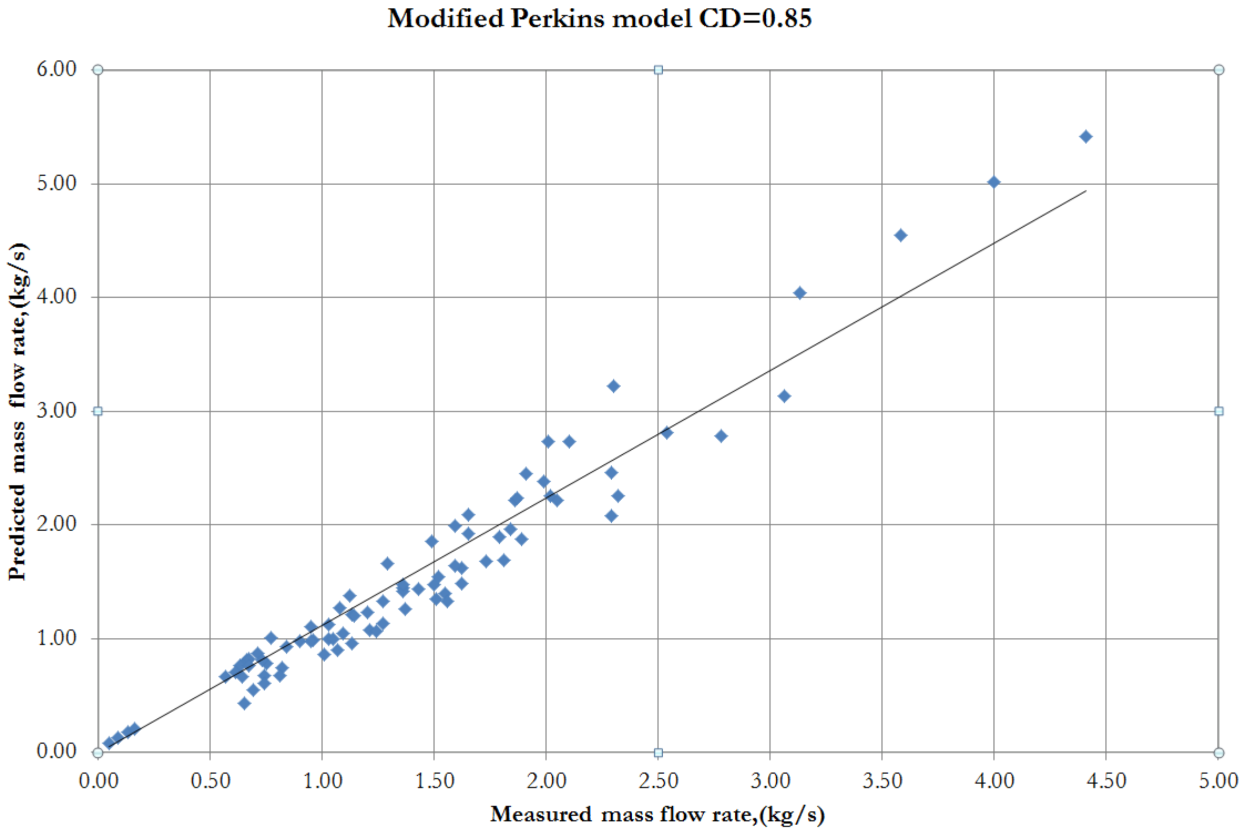


Figure E - 3: Results of modified Perkins model with CD=0.85

Table E - 1: Results of new modified Perkins model

		Mass flow rates(kg/s)													
		Measured	Predicted	Calibrated mass flow rate			Error analysis								
				CD=0.75	**CD =0.8**	**CD=0.85**	**CD=0.75**			**CD=0.8**			**CD=0.8**		
Test point	Flow type			0.75	0.8	0.85	E1	E2	E3	E1	E2	E3	E1	E2	E3
G-OR-11-01	subcritical	0.05	0.09	0.07	0.07	0.08	0.38	0.38	0.00	0.47	0.47	0.00	0.56	0.56	0.00
G-OR-11-02	subcritical	0.09	0.15	0.12	0.12	0.13	0.29	0.29	0.00	0.37	0.37	0.00	0.46	0.46	0.00
G-OR-11-03	subcritical	0.13	0.21	0.16	0.17	0.18	0.21	0.21	0.00	0.30	0.30	0.00	0.38	0.38	0.00
G-OR-11-04	subcritical	0.16	0.24	0.18	0.19	0.21	0.14	0.14	0.00	0.22	0.22	0.00	0.29	0.29	0.00
O-OR-11-01	subcritical	0.57	0.78	0.58	0.62	0.66	0.03	0.03	0.00	0.09	0.09	0.00	0.16	0.16	0.00
O-OR-11-02	subcritical	0.90	1.15	0.87	0.92	0.98	-0.04	0.04	0.00	0.03	0.03	0.00	0.09	0.09	0.00
O-OR-11-03	subcritical	1.27	1.56	1.17	1.25	1.33	-0.08	0.08	0.00	-0.02	0.02	0.00	0.05	0.05	0.00
O-OR-11-04	subcritical	1.65	2.27	1.70	1.82	1.93	0.03	0.03	0.00	0.10	0.10	0.00	0.17	0.17	0.00
O-OR-11-05	subcritical	2.01	3.22	2.41	2.57	2.73	0.20	0.20	0.00	0.28	0.28	0.00	0.36	0.36	0.00
W-OR-11-01	subcritical	0.77	1.19	0.89	0.95	1.01	0.16	0.16	0.00	0.23	0.23	0.00	0.31	0.31	0.00
W-OR-11-02	subcritical	1.29	1.96	1.47	1.57	1.66	0.14	0.14	0.00	0.21	0.21	0.00	0.29	0.29	0.00
W-OR-11-03	subcritical	1.91	2.88	2.16	2.30	2.45	0.13	0.13	0.00	0.21	0.21	0.00	0.28	0.28	0.00
W-OR-11-04	subcritical	2.30	3.79	2.84	3.03	3.22	0.23	0.23	0.00	0.32	0.32	0.00	0.40	0.40	0.00
GOW-OR-11-01	subcritical	0.66	0.95	0.72	0.76	0.81	0.08	0.08	0.00	0.16	0.16	0.00	0.23	0.23	0.00
GOW-OR-11-02	subcritical	0.95	1.30	0.97	1.04	1.10	0.02	0.02	0.00	0.09	0.09	0.00	0.16	0.16	0.00
GOW-OR-11-03	subcritical	1.36	1.71	1.28	1.37	1.45	-0.06	0.06	0.00	0.00	0.00	0.00	0.07	0.07	0.00
GOW-OR-11-04	subcritical	1.65	2.46	1.84	1.97	2.09	0.12	0.12	0.00	0.19	0.19	0.00	0.27	0.27	0.00
GOW-OR-11-05	subcritical	1.86	2.61	1.96	2.09	2.22	0.05	0.05	0.00	0.12	0.12	0.00	0.19	0.19	0.00

GOW-OR-11-06	subcritical	0.66	0.94	0.71	0.75	0.80	0.07	0.07	0.00	0.14	0.14	0.00	0.21	0.21	0.00
GOW-OR-11-07	subcritical	1.08	1.49	1.12	1.19	1.27	0.04	0.04	0.00	0.10	0.10	0.00	0.17	0.17	0.00
GOW-OR-11-08	subcritical	1.49	2.18	1.64	1.74	1.85	0.10	0.10	0.00	0.17	0.17	0.00	0.24	0.24	0.00
GOW-OR-11-09	subcritical	1.87	2.63	1.97	2.10	2.24	0.06	0.06	0.00	0.13	0.13	0.00	0.20	0.20	0.00
GOW-OR-11-10	subcritical	0.71	1.02	0.77	0.82	0.87	0.08	0.08	0.00	0.15	0.15	0.00	0.22	0.22	0.00
GOW-OR-11-11	subcritical	1.12	1.62	1.21	1.29	1.37	0.08	0.08	0.00	0.15	0.15	0.00	0.23	0.23	0.00
GOW-OR-11-12	subcritical	1.59	2.35	1.76	1.88	1.99	0.11	0.11	0.00	0.18	0.18	0.00	0.25	0.25	0.00
GOW-OR-11-13	subcritical	1.99	2.81	2.11	2.25	2.39	0.06	0.06	0.00	0.13	0.13	0.00	0.20	0.20	0.00
GOW-OR-11-14	subcritical	0.64	0.79	0.59	0.63	0.67	-0.08	0.08	0.00	-0.01	0.01	0.00	0.05	0.05	0.00
GOW-OR-11-15	subcritical	1.03	1.17	0.88	0.94	1.00	-0.15	0.15	0.00	-0.09	0.09	0.00	-0.03	0.03	0.00
GOW-OR-11-16	subcritical	1.37	1.48	1.11	1.19	1.26	-0.19	0.19	0.00	-0.13	0.13	0.00	-0.08	0.08	0.00
GOW-OR-11-17	subcritical	1.62	1.75	1.31	1.40	1.49	-0.19	0.19	0.00	-0.14	0.14	0.00	-0.08	0.08	0.00
GOW-OR-11-18	subcritical	0.67	0.97	0.73	0.77	0.82	0.08	0.08	0.00	0.16	0.16	0.00	0.23	0.23	0.00
GOW-OR-11-19	subcritical	1.09	1.23	0.92	0.99	1.05	-0.15	0.15	0.00	-0.10	0.10	0.00	-0.04	0.04	0.00
GOW-OR-11-20	subcritical	1.51	1.58	1.19	1.27	1.35	-0.21	0.21	0.00	-0.16	0.16	0.00	-0.11	0.11	0.00
GOW-OR-11-21	subcritical	1.81	1.98	1.49	1.59	1.69	-0.18	0.18	0.00	-0.12	0.12	0.00	-0.07	0.07	0.00
GOW-OR-11-22	subcritical	0.75	0.92	0.69	0.74	0.78	-0.08	0.08	0.00	-0.02	0.02	0.00	0.04	0.04	0.00
GOW-OR-11-23	critical	1.21	1.27	0.95	1.02	1.08	-0.21	0.21	0.00	-0.16	0.16	0.00	-0.11	0.11	0.00
GOW-OR-11-24	critical	1.55	1.64	1.23	1.31	1.39	-0.21	0.21	0.00	-0.15	0.15	0.00	-0.10	0.10	0.00
GOW-OR-11-25	critical	0.63	0.90	0.67	0.72	0.76	0.07	0.07	0.00	0.14	0.14	0.00	0.21	0.21	0.00
GOW-OR-11-26	critical	1.01	1.02	0.76	0.81	0.86	-0.25	0.25	0.00	-0.19	0.19	0.00	-0.14	0.14	0.00
GOW-OR-11-27	critical	0.74	0.79	0.59	0.63	0.67	-0.20	0.20	0.00	-0.14	0.14	0.00	-0.09	0.09	0.00
GOW-OR-11-28	critical	1.13	1.13	0.84	0.90	0.96	-0.25	0.25	0.00	-0.20	0.20	0.00	-0.15	0.15	0.00
GOW-OR-11-29	critical	0.82	0.88	0.66	0.70	0.75	-0.20	0.20	0.00	-0.14	0.14	0.00	-0.09	0.09	0.00

GOW-OR-11-30	critical	1.24	1.26	0.94	1.00	1.07	-0.24	0.24	0.00	-0.19	0.19	0.00	-0.14	0.14	0.00
GOW-OR-11-31	critical	0.69	0.64	0.48	0.51	0.55	-0.30	0.30	0.00	-0.26	0.26	0.00	-0.21	0.21	0.00
GOW-OR-11-32	critical	0.74	0.71	0.53	0.57	0.60	-0.28	0.28	0.00	-0.23	0.23	0.00	-0.18	0.18	0.00
GOW-OR-11-33	critical	0.81	0.80	0.60	0.64	0.68	-0.26	0.26	0.00	-0.21	0.21	0.00	-0.16	0.16	0.00
GOW-OR-11-34	subcritical	0.61	0.83	0.62	0.66	0.70	0.02	0.02	0.00	0.09	0.09	0.00	0.15	0.15	0.00
GOW-OR-11-35	subcritical	1.13	1.43	1.07	1.14	1.21	-0.05	0.05	0.00	0.01	0.01	0.00	0.07	0.07	0.00
GOW-OR-11-36	subcritical	1.36	1.67	1.25	1.33	1.42	-0.08	0.08	0.00	-0.02	0.02	0.00	0.04	0.04	0.00
GOW-OR-11-37	subcritical	1.73	1.98	1.49	1.59	1.68	-0.14	0.14	0.00	-0.08	0.08	0.00	-0.03	0.03	0.00
GOW-OR-11-38	subcritical	1.03	1.33	0.99	1.06	1.13	-0.03	0.03	0.00	0.03	0.03	0.00	0.09	0.09	0.00
GOW-OR-11-39	subcritical	1.20	1.45	1.09	1.16	1.24	-0.09	0.09	0.00	-0.03	0.03	0.00	0.03	0.03	0.00
GOW-OR-11-40	subcritical	1.50	1.74	1.31	1.39	1.48	-0.13	0.13	0.00	-0.07	0.07	0.00	-0.01	0.01	0.00
GOW-OR-11-41	subcritical	1.89	2.21	1.66	1.77	1.88	-0.12	0.12	0.00	-0.06	0.06	0.00	-0.01	0.01	0.00
GOW-OR-11-42	subcritical	0.73	0.95	0.72	0.76	0.81	-0.02	0.02	0.00	0.05	0.05	0.00	0.11	0.11	0.00
GOW-OR-11-43	subcritical	1.14	1.41	1.06	1.13	1.20	-0.07	0.07	0.00	-0.01	0.01	0.00	0.05	0.05	0.00
GOW-OR-11-44	subcritical	1.62	1.91	1.43	1.53	1.62	-0.11	0.11	0.00	-0.06	0.06	0.00	0.00	0.00	0.00
GOW-OR-11-45	subcritical	2.05	2.61	1.96	2.09	2.22	-0.04	0.04	0.00	0.02	0.02	0.00	0.08	0.08	0.00
C2-GOW-OR-11-162	critical	0.67	0.90	0.67	0.72	0.76	0.00	0.00	0.00	0.07	0.07	0.00	0.14	0.14	0.00
C2-GOW-OR-11-163	Critical	0.96	1.16	0.87	0.93	0.98	-0.09	0.09	0.00	-0.03	0.03	0.00	0.03	0.03	0.00
C2-GOW-OR-11-165	critical	0.65	0.51	0.38	0.41	0.43	-0.41	0.41	0.00	-0.37	0.37	0.00	-0.33	0.33	0.00
C2-GOW-OR-11-166	critical	0.84	1.09	0.82	0.87	0.93	-0.03	0.03	0.00	0.04	0.04	0.00	0.10	0.10	0.00
C2-GOW-OR-11-167	critical	1.07	1.06	0.79	0.84	0.90	-0.26	0.26	0.00	-0.21	0.21	0.00	-0.16	0.16	0.00
C2-GOW-OR-11-168	critical	1.27	1.33	1.00	1.07	1.13	-0.21	0.21	0.00	-0.16	0.16	0.00	-0.11	0.11	0.00
C2-GOW-OR-11-169	subcritical	0.95	1.15	0.86	0.92	0.98	-0.09	0.09	0.00	-0.03	0.03	0.00	0.03	0.03	0.00
C2-GOW-OR-11-170	subcritical	1.52	1.81	1.36	1.45	1.54	-0.11	0.11	0.00	-0.05	0.05	0.00	0.01	0.01	0.00

C2-GOW-OR-11-171	critical	1.79	2.23	1.67	1.78	1.90	-0.07	0.07	0.00	0.00	0.00	0.00	0.06	0.06	0.00
C2-GOW-OR-11-172	critical	2.02	2.65	1.99	2.12	2.25	-0.02	0.02	0.00	0.05	0.05	0.00	0.12	0.12	0.00
C2-GOW-OR-11-173	subcritical	1.05	1.18	0.88	0.94	1.00	-0.16	0.16	0.00	-0.10	0.10	0.00	-0.05	0.05	0.00
C2-GOW-OR-11-174	subcritical	1.59	1.94	1.45	1.55	1.65	-0.09	0.09	0.00	-0.03	0.03	0.00	0.04	0.04	0.00
C2-GOW-OR-11-175	Critical	2.29	2.45	1.84	1.96	2.08	-0.20	0.20	0.00	-0.14	0.14	0.00	-0.09	0.09	0.00
C2-GOW-OR-11-177	subcritical	1.36	1.73	1.30	1.39	1.47	-0.04	0.04	0.00	0.02	0.02	0.00	0.08	0.08	0.00
C2-GOW-OR-11-178	subcritical	1.84	2.31	1.73	1.85	1.96	-0.06	0.06	0.00	0.00	0.00	0.00	0.07	0.07	0.00
C2-GOW-OR-11-179	subcritical	2.29	2.90	2.18	2.32	2.47	-0.05	0.05	0.00	0.01	0.01	0.00	0.08	0.08	0.00
C2-GOW-OR-11-180	subcritical	2.54	3.31	2.48	2.64	2.81	-0.02	0.02	0.00	0.04	0.04	0.00	0.11	0.11	0.00
C2-GOW-OR-11-181	subcritical	1.43	1.69	1.27	1.35	1.44	-0.11	0.11	0.00	-0.05	0.05	0.00	0.01	0.01	0.00
C2-GOW-OR-11-182	Critical	2.32	2.65	1.99	2.12	2.25	-0.14	0.14	0.00	-0.09	0.09	0.00	-0.03	0.03	0.00
C2-GOW-OR-11-183	Critical	2.78	3.27	2.45	2.61	2.78	-0.12	0.12	0.00	-0.06	0.06	0.00	0.00	0.00	0.00
C2-GOW-OR-11-184	subcritical	3.06	3.68	2.76	2.95	3.13	-0.10	0.10	0.00	-0.04	0.04	0.00	0.02	0.02	0.00
C2-GOW-OR-11-190	critical	1.56	1.57	1.18	1.26	1.33	-0.25	0.25	0.00	-0.20	0.20	0.00	-0.14	0.14	0.00
C2-W-OR-11-251	subcritical	2.10	3.22	2.41	2.57	2.73	0.15	0.15	0.00	0.22	0.22	0.00	0.30	0.30	0.00
C2-W-OR-11-252	subcritical	3.13	4.76	3.57	3.81	4.04	0.14	0.14	0.00	0.22	0.22	0.00	0.29	0.29	0.00
C2-W-OR-11-253	subcritical	3.58	5.35	4.01	4.28	4.55	0.12	0.12	0.00	0.20	0.20	0.00	0.27	0.27	0.00
C2-W-OR-11-254	subcritical	4.00	5.90	4.42	4.72	5.01	0.11	0.11	0.00	0.18	0.18	0.00	0.25	0.25	0.00
C2-W-OR-11-255	subcritical	4.41	6.37	4.78	5.10	5.41	0.08	0.08	0.00	0.16	0.16	0.00	0.23	0.23	0.00
							-0.04	0.13	0.02	0.02	0.13	0.02	0.08	0.15	0.03
							-4.41	**12.81**	**14.75**	**1.97**	**12.73**	**15.73**	**8.34**	**14.81**	**16.71**

Table E - 2: Programmed VBA code for modified Perkins model

```
Function julie_polytropic_n(k, Cvg, fg, Cvo, fw, cvw, fo)
julie_polytropic_n = (k * Cvg * fg + fo * Cvo + fw * cvw) / (fg * Cvg + fo * Cvo + fw * cvw)
End Function

Function julie_alpha_coeff(xg, liquid_specificV, vg1, slip_R)
julie_alpha_coeff = ((1 - xg) * slip_R * liquid_specificV) / (xg * vg1)
End Function

Function julieLHS(pr, n, alpha_coeff, julie_k)
part_1 = (pr ^ (-1 / n) + alpha_coeff) ^ 2
part_5 = ((julie_k / (julie_k - 1)) * (((n - 1) / n) * pr ^ (-1 / n))) - alpha_coeff
julieLHS = part_1 * part_5
End Function

Function julieRHS(pr, n, alpha_coeff, julie_k)
part_1 = (-2 / n) * (pr ^ (-((n + 1) / n)))
part_4 = pr ^ (-1 / n) + alpha_coeff
part_3 = (julie_k / (julie_k - 1)) * (1 - (pr ^ ((n - 1) / n)))
part_5 = alpha_coeff * (1 - pr)
julieRHS = (part_3 + part_5) * part_1 * part_4
End Function

Function julie_massflowrate(area2, p1, Alpha_coefficient, pr, julie_n, xg, vg1, julie_k, slip_R)
part_1 = Alpha_coefficient * (1 - pr)
part_2 = julie_k / (julie_k - 1)
part_3 = 1 - pr ^ ((julie_n - 1) / julie_n)
part_4 = part_1 + (part_2 * part_3)
part_5 = 2 * area2 ^ 2 * p1 * part_4 * (xg + (1 - xg) * slip_R)
part_6 = (pr ^ (-1 / julie_n) + Alpha_coefficient) ^ 2
part_8 = xg * vg1 * part_6
julie_massflowrate = (part_5 / part_8) ^ 0.5
End Function
```