Methods of Reliability Analysis for Marine Structures

David Larsson davidla@stud.ntnu.no
Supervisor: Professor Bernt J. Leira

NTNU
Department of Marine Technology

Problem

The main task in a structural reliability analysis is to evaluate the probability of failure, which is defined by the fundamental integral:

$$p_f = \int \cdots \int_{g(\mathbf{x}) \leq 0} f_{\mathbf{X}}(\mathbf{x})\, d\mathbf{x}$$

where $f_{\mathbf{X}}(\mathbf{x})$ is the joint probability density function for load and resistance variables in the structural model. $g(\mathbf{x})$ is the safety margin expressing the relation between load,resistance and failure. By definition, the structure is in a safe state when $g(\mathbf{x}) > 0$ and in a failed state when $g(\mathbf{x}) < 0$. $g(\mathbf{x}) = 0$ is the limit state.

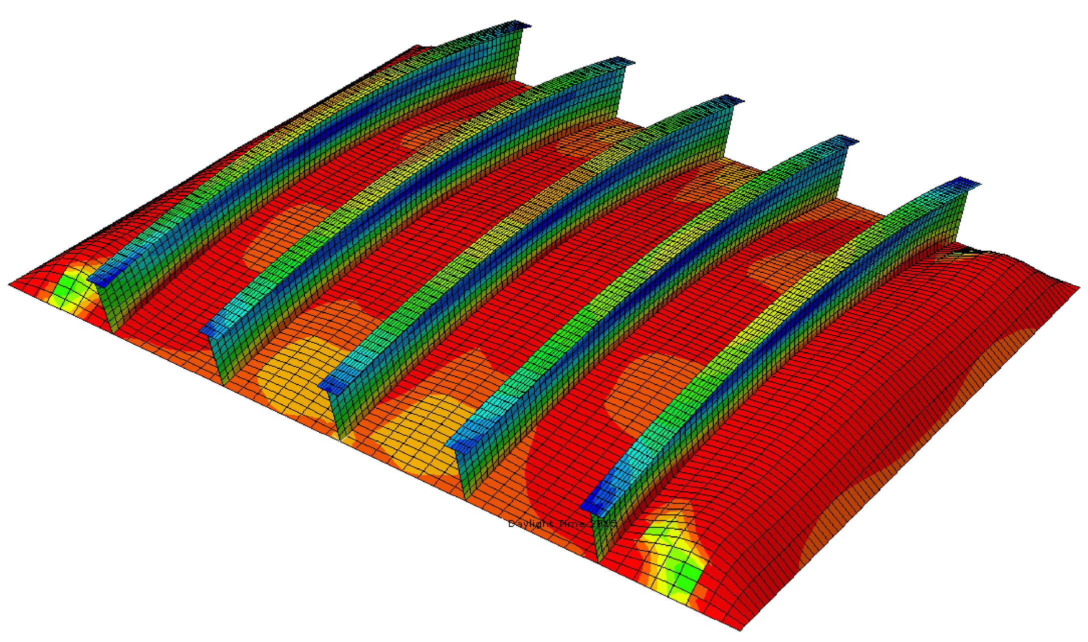
For most practical cases, obtaining the analytical solution of the above integral is impossible or very tedious for a number of reasons:

- $g(\mathbf{x}) = 0$ is unknown or implicit
- Joint density function is incomplete (only marginal distibutions and correlations are known)
- Computational difficulties due to mathematical complexity and high dimensionality.

Model and Methods

The example is a T-bar stiffened panel, a common structural component in ship hulls and offshore structures, subject to axial force and lateral pressure. 5 variables are used: Axial force (s_{ax}), lateral pressure (p_{lat}), plate thickness (t_p), shape error (w)and yield stress (s_f). Imperfection shape is taken as a sinusoidal half-wave, attempting to represent the global buckling mode for simply supported transverse edges. w represents the out-of-shape magnitude at midspan. Loads are Weibull-distributed, imperfection size and plate thickness are Gaussian and the yield stress is log-normal. The panel is modelled using scripting in FEM-software ABAQUS. The main programme consists of a MATLAB-code that manipulates the ABAQUS-script and performs the reliability analysis using the FEM-results. The element type is standard abaqus shell, S4R, and the material model is elastic-perfectly plastic. Mean and standard deviation of the variables are shown in the table below.

	μ	σ [MPa]
t_p	20 mm	0.4mm
s_{ax}	100 MPa	20 MPa
p_{lat}	0.1 MPa	0.01MPa
w	0 mm	0.4 mm
s_f	315 MPa	22.05



Basic Concepts

Structural reliability methods can be roughly divided into three different levels. With each level, the computational effort increases significantly.

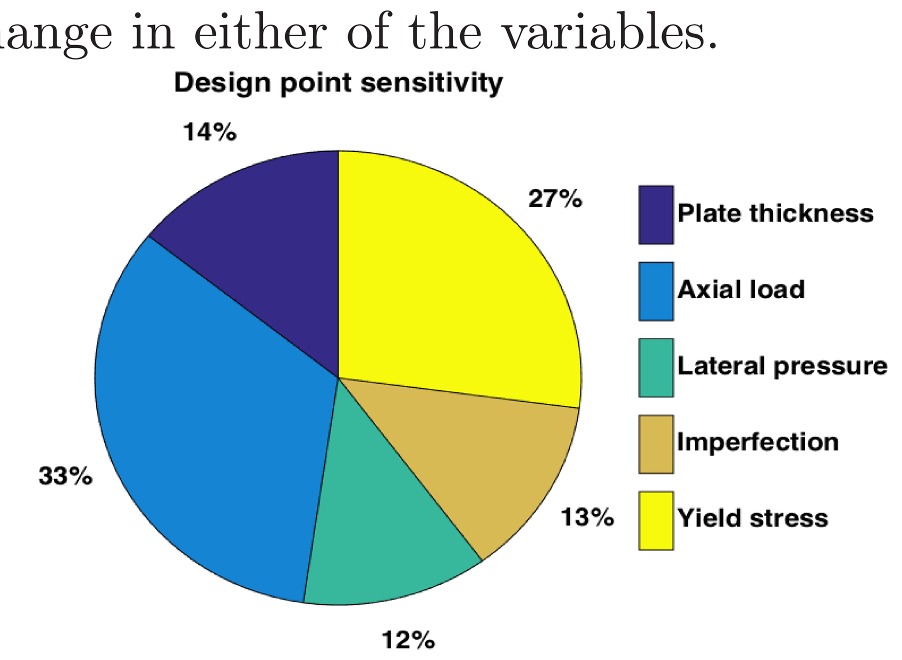
- **Level I**: Methods of partial coefficients
- **Level II**: Safety index methods
- **Level III**: Probability of failure methods

This thesis focuses on Level II and Level III. Safety index methods defines the probability of failure from the **design point**, which is the point on the limit state with largest probability of occurence. Hasofer and Lind defined the safety index, β, as the length of a vector from origo to the design point in standard gaussian space.
Level III - methods involve solving the fundamental integral directly. In practice, due to the reasons stated under "Problem", this boils down to simulation (Monte Carlo) techniques. The fundamental technique, **Crude Monte Carlo**, becomes computationally expensive or impossible for many practical problems. A number of techniques exist which utilizes a priori knowledge to rationalize the process, such as **Importance Sampling**. Here, sampling is focused on the most interesting region so that the ratio between failure and safe outcomes is nearly 1/1 (rather than $1/p_f^{-1}$), which makes convergence significantly faster when p_f is small.
When the safety margin is unknown, so-called **response surface methods** are employed. The response surface is an approximation of the safety margin, found from regression of discrete safety margin evaluations by e.g a FEM-model. Many methods exist in attempt to minimize the costly sampling (each regression point corresponds to 1 FE-run when a FEM-Model is used) required to reach a sufficiently accurate limit state description. In this thesis, two such methods are employed. The first method utilizes a 2^{nd} order polynomial without cross terms. A tentative design point is searched and used to locate new samples. From these, the final approximation is established and the probability of failure is evaluated by FORM and Monte Carlo.
The second procedure is called "response surface by vector projection", and uses a hyperplane as the approximation. By continously shifting sampling points based on vector projection the design point of the hyperplane converges towards the analytical design point. This response surface is thus only accurate in combination with FORM.

Reliability Analysis of Stiffened Panel

Presented below are the results from FORM when failure is defined as von-Mises stress exceeding the yield stress at the midspan. The table shows design point data and the pie chart illustrates the sensitivity of the safety index with respect to a change in either of the variables.

	Quadratic	Vector projection
t_p [mm]	19.667	19.671
s_{ax} [MPa]	135.21	134.91
p_{lat} [MPa]	0.1075	0.1075
w [mm]	3.0091	2.9530
s_f [MPa]	281.81	282.24
$\mathbf{p_f}$	**0.00217**	**0.00241**



From the quadratic limit state, simulations were performed by the Crude Monte Carlo and Importance Sampling method. The results were similar to the results using FORM.

Discussion

Both response surface methods yields similar results for all limit states that are evaluated. There are differences, but these are small and deemed insignificant from a decision-making perspective. The choice of response surface method should rather be made with respect to the effort of obtaining each safety margin sample. The vector projection method needs to be updated until convergence is reached, which might lead to an extensive number of FE-runs. The quadratic limit state is only sampled twice, so the number of samples required is known in advance. Another benefit of using a quadratic limit state is that the improved fit compared to a linear approximation enables the analyst to evaluate probability of failure with level III-methods, so that comparison can be made. Here, a purely quadratic polynomial is used, so that cross-terms are excluded. This limits the number of points required to uniquely determine the polynomial, but might give a significant lack-of-fit.
A weakness with the FORM-method, which is "inherited" by the response surface methods used here, is that the design point might not be the global one. The search is performed based on a first guess, and a local minimum in the vicinity of this point is located. If the analytical safety margin (which is unknown) is of higher order, there might be a global minimum located elsewhere which is overlooked. Since these response surfaces are established on the basis of such FORM-results, this error follows into e.g Monte Carlo simulations. Hence, a suitable start for further work would be to adress this problem by performing additional analysis with higher order polynomials and/or a search algorithm started at several locations to see if other design points can be located.