ISBN 978-82-471-1616-6 (printed ver.)
ISBN 978-82-471-1617-3 (electronic ver.)
ISSN 1503-8181

NTNU
Norwegian University of
Science and Technology

Anne Christine Torp Handstanger

Doctoral Thesis

Doctoral theses at NTNU, 2009:116

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Thesis for the degree of
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Printed by Tapir Uttrykk

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Anne Christine Torp Handstanger

March 23, 2009

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Abstract

For the long term planning of railway infrastructure several analyses are necesarry. One of them is the prognosticated demand for future railway transportation. This information serves as input parameter for the capacity and benefit cost analysis. For the capacity analysis, timetable independent analytical methods that can estimate the scheduled waiting time in dependency of the number of trains running on the line is required. If the scheduled waiting time is too long, the operators risk losing customers and the infrastructure operators risk losing the operators as customers. Changes must be made in infrastructure or in train mixture to reduce the scheduled waiting time in an early planning stage.

There are only a few existing models concerning scheduled waiting time. The crossing situation has hardly been investigated in studies since the main focus has been on double track lines. The scheduled waiting time as a topic so far has not been paid enough attention to since unscheduled waiting time has been the main focus. Estimating the scheduled waiting time is even important than the estimation of the unscheduled waiting time. An overbooked railway line will seldom be as successful as a balanced railway line as a transportation offer or in operation. The planner's task is to estimate and design an infrastructure that will support the market on time in best way with regards to both scheduled and unscheduled waiting time.

There is a demand to develop and to improve models for the calculation of scheduled waiting time. This thesis is an attempt to meet a part of the demand of the research within this subject. In this thesis a deterministic analysis of the crossing situation of trains on single track lines has been performed. A new model for the calculation of the number of crossings, the expected waiting time for crossing and merging has been developed on the basis of the conditions formed for incuring a crossing. This analysis states that the conditions for a train to incur a crossing is similar to the conditions given in [Schw81] for an overtaking to take place.

The analytical model developed makes use of stochastics in order to estimate the expected waiting time from crossing for each train model. This ensures a timetable independent estimation. The model focuses on mixed train traffic in both directions with a strict hierarchical priority system. An exponential buffer time distribution between the requested train paths of higher priority is assumed to make it possible for trains of lower priority to merge in between the trains of higher priority. This phi-

losophy is based on a timetable construction process where different train models are given different priority due to their trackage rights achieved. The train model with highest priority will be included into the timetable first. The expected waiting time from crossing within a train model can then be estimated. In the model deduced, one direction is chosen to take priority over the opposite direction. The prior direction can, for example, be outward traffic. This means that the trains running in the direction of lower priority risks incuring waiting time from crossing.

A case study of the timetable characteristics on single track lines was carried out to investigate whether the timetables follow a cyclic or a stochatic pattern. The noncyclic timetables were further analysed by a χ^2-test of goodness of fit of the buffer time between the trains of higher priority if the buffer time distribution could be exponential, hyper-exponential or Erlang_2 distributed.

An asynchronous simulation tool has been used to control and evaluate the model. Random timetables were generated and served as input to the simulation tool. A dummy illustrating a railway single track line was constructed and served as the main study object. An existing train model was chosen to run the line in both directions. The number of crossings, the number of multiple crossings and the waiting times were recorded and compared with the results from the model established. The evaluation of the reliability of the model requires more research by simulation before a satisfactory statement can be fulfilled.

Two different priority strategies were simulated and compared. A priority strategy with equal priority between the trains of opposite directions tend to generate more crossings than a strategy with strict priority for one direction. On the other hand a strategy with one direction priori over the opposite direction tend to generated more multiple crossings relative to the number of crossings compared to the equal priority strategy.

An attempt has been made to use the same methodology for the derivation of a model for the estimation of the expected waiting time from crossing with constant buffer times between the requested train paths of higher priority. This model has some weaknesses and has therefore not been further analysed.

Finally, a sensitivity analysis of the model developed illustrates that the time gap necessary for a train to reach the next station before meeting an opposing train has the most influence on the estimated result. The model is therefore probably most suited for railway lines with less variation in the occupation time between the stations.

Acknowledgements

This work was carried out partly at the Norwegian University of Science and Technology (NTNU), Department of Civil and Transport Engineering, and the RWTH Aachen University, Department of Transport Science (VIA) in Germany.

My supervisors were Professor Asbjørn Hovd in Norway and Professor Ekkehard Wendler in Germany. Professor Hovd was my primary supervisor responsible for the administrative part of my study. Professor Wendler at the RWTH was my co-supervisor responsible for the theoretical part of my study. I am very thankful for the time I was allowed to spend at VIA and for the opportunity I had to learn the theories developed at their resepctive departments. I especially would like to thank Professor Wendler for being my doctoral dissertation supervisor, the time he spent and for the help he provided during my research.

I am very grateful to all of the staff at the VIA. I especially would like to thank Birgit Schwanenberg who taught me to operate the software tools developed at the institute. I would also like to thank Nils Niessen for the good advice he provided in the field of railway statistics.

This research was possible through a scholarship from the Norwegian railway infrastructure operator, Jernbaneverket, administrated by the NTNU. I also wish to thank Terje Eidsmoen at Jernbaneverket for his help in connecting me with the right persons for data collection at Jernbaneverket.

Contents

List of Symbols and Abbreviations

Symbols mentioned for the first time are listed in the order they appear in this thesis, sorted by chapter and section.

Chapter 2 Fundamentals of capacity assessment in railway operation research

2.1 Infrastructure based fundamentals

t_{form}	Time for route formation	$[min.]$
t_{view}	Time for visual distance	$[min.]$
t_{app}	Approaching time	$[min.]$
$t_{journey}$	Journey time	$[min.]$
t_{clear}	Clearing time	$[min.]$
$t_{release}$	Time for route release	$[min.]$
t_s	Minimum spacing time	$[min.]$
$min\ t_x$	Minimum time for a crossing	$[min.]$
t_r	Time supplement because of slowing down	$[min.]$
t_{ac}	Time supplement because of acceleration	$[min.]$
t_v	Running time with at a given speed	$[min.]$
$t_{v\rightarrow 0}$	Running time for a given speed down to halt	$[min.]$
$t_{0\rightarrow v}$	Running time from halt to a given speed	$[min.]$

2.3 Waiting time versus capacity

C_p	Practical capacity	$[trains/h]$
C_{theo}	Theoretical capacity	$[trains/h]$
t_w	Scheduled waiting time	$[min.]$
t_{halt}	Minimum time for halt	$[min.]$
t_{PD}	Primary delay	$[min.]$
t_{CD}	Source delay	$[min.]$
t_{SD}	Secondary delay	$[min.]$

t_{GD}	Gross delay	$[min.]$
t_{ND}	Net delay	$[min.]$
p	Probability	no units
λ	Rate of arrival	$[1/min.]$
A	Arrival process	no units
B	Service time process	no units
μ	Rate of service	$[1/min.]$
ρ	Traffic load	no units
$E[T_A]$	Expected interarrival time	$[min.]$
$E[T_S]$	Expected service time	$[min.]$
$E[L_{Nq}]$	Expected length of the queue	no units
$E[T_{Wq}]$	Expected waiting time in the queue	$[min.]$
$E[L_{Ns}]$	Expected length of the system	no units
$E[T_{Ws}]$	Expected waiting time in the system	$[min.]$
T	Time of survey	$[min.]$

Abbreviations in chapter 2

IRJ	Interlocking route junction
$CIRJ$	Complete interlocking route junction
M	Markovian process
E_k	Erlang process
D	Dirac process
GI	General-independent process
G	General process

Chapter 3 Past studies

3.1 Deterministic methods for describing capacity

C	Capacity	$[trains/h]$
t_b	Buffer time	$[min.]$
w	Difficulty	no units
$t_{b,\ sched}$	Scheduled buffer time	$[min.]$
S	Number of difficulty	no units
ρ_1	Traffic load in one direction	no units

3.2 Capacity described by fundamental diagrams

v_o	Optimal velocity	$[\frac{km}{h}]$
$E[Q]$	Expected traffic performance	$[\frac{trains \cdot km}{h^2}]$

$E[L_b]$	Expected train density	$[\frac{trains}{h}]$
$\overline{v}$	Mean velocity	$[\frac{km}{h}]$
$E[T_{journey}]$	Expected typical journey time	$[min.]$
$E[T_v]$	Expected time spent on the line	$[min.]$
N	Number of trains running during time of survey	$[\frac{trains}{h}]$
$E[T_{WF}]$	Expected scheduled waiting time	$[min.]$
$E[QN]$	Expected standard traffic performance	no units

3.3 Describing capacity by economical relations

N_p	Practical number of trains	no units
F_{theo}	Theoretical number of trains	no units
F_{Tr}	Transport force	$[N]$
m_{gt}	Gross train mass	$[t]$
v_{real}	Real journey velocity	$[\frac{km}{h}]$
$t_{journey}$	Typical journey time	$[min.]$
t_{WF}	Scheduled waiting time	$[min.]$
t_{WB}	Unscheduled waiting time	$[min.]$
l	Distance	$[m]$
C_{total}	Total Cost	$[euro]$
c_0, c_1, c_2	Constants	no units
m_Z	Mass of a train model	$[t]$
t_{real}	Real running time	$[min.]$
m_0/v_0	Equivalent for fixed costs	$[\frac{10^3 \cdot min.}{km}]$
C_{Fix}	Fixed costs	$[euro]$
C_{Var}	Variable costs	$[euro]$
P_Z	Profit of one train model	$[euro]$
R_Z	Revenue of one train model	$[euro]$
C_Z	Costs of one train model	$[euro]$
$c_{r,Z}$	Proportionality constant for revenue of one train model	$[\frac{euro}{t \cdot km}]$
$c_{r,PNV}$	Proportionality constant for revenue of passenger trains	$[\frac{euro}{t \cdot km}]$
$c_{k,Z}$	Proportionality constant for costs of one train model	$[\frac{euro}{t \cdot min.}]$
r_Z	Revenue equivalent of one train model	$[t \cdot km]$
c_Z	Cost equivalent of one train model	$[t \cdot min.]$
f_N	Function	no units
$ZB_{journey}$	Time evaluation factor for typical journey time	no units
ZB_{WB}	Time evaluation factor for unscheduled waiting time	no units
$c_{R,Z}$	Revenue ratio	no units
$c_{K,Z}$	Cost ratio	no units
$c_{r,PNV}$	Proportionality constant for passenger trains	$[t \cdot km]$
Δp	Transport momentum difference	$[euro \cdot t \cdot km - euro^2 \cdot t \cdot min.]$

3.4 Models for describing scheduled waiting time

d_x	Distance between two crossings	$[km]$
v	Velocity	$[\frac{km}{h}]$
n_x	Number of crossings	no units
n	Number of trains	no units
l_{line}	Length of entire line	$[km]$
$T_{journey,\ line}$	Typical journey time for entire line	$[min.]$
$\overline{\Delta t_{journey}}$	Mean typical journey time difference	$[min.]$
m	Numerator	no units
n_o	Number of overtakings	no units
n_S	Number of slow trains	no units
$max\ l$	Longest section	no units
n_F	Number of fast trains	no units
N	Total number of trains	no units
Δb	Time gap for reaching the next station	$[min.]$
$N_{train\ sequences}$	Total number of train sequences	no units
n_{ij}	The number of train sequence i followed by j	no units
ϕ	Chaining factor	no units
N_ϕ	Number of chained trips	no units
N_{all}	Complete number of trips	no units

3.5 Models for describing unscheduled waiting time

p_e	Probability of train sequence of equal rank	no units
m	Inverse of the mean primary delay	$[1/min.]$
t_l, t_i	Departure time train l and train i	$[min.]$

Abbreviations in chapter 3

$EMPF$	Relative timetable sensibility
LT	Light-Traffic-Model
HT	Heavy-Traffic-Model
LHT	Light-Heavy-Traffic-Model
$FIFO$	First-In-First-Out

Chapter 4 Scheduled waiting time from crossing on single track lines

4.2 Waiting time from crossing with exponential buffer time distribution

$\Delta t_x, \Delta t_o$	Time gap needed for crossing and overtaking	$[min.]$

$\Delta t_s, 1$	Addition of minimum spacing time for train of rank(1)	$[min.]$
twx	Waiting time for crossing	$[min.]$
twm	Waiting time for merging	$[min.]$
two	Waiting time for overtaking	$[min.]$
$min\ t_o$	Minimum time for an overtaking	$[min.]$
T_{WX}	Waiting time from crossing	$[min.]$
$N_{stations}$	Number of stations	$[min.]$

4.3 Special assumptions for constant buffer times

t_c	Cyclic time interval	$[min.]$
d_o	Distance between two overtakings	$[km]$

Chapter 5 Case study of single track lines

5.1 Timetable characteristics on single track lines

t_C	Cycle time	$[min.]$

5.2 Results of all three lines

α	Level og significance	no units
k	Parameter	no units
V_T	Coefficient of variation	no units

Abbreviations in chapter 5

Ft	Freight train
Nt	Night train
ICE	Inter-City-Express train
IC	Inter-City train
IR	Inter-Regional train
RB	Regional train

Chapter 1

Introduction and scope

1.1 Introduction

One of the major tasks of the infrastructure operator is dimensioning the infrastructure according to the rail transport demand. The rail transport demand is made up by two markets that need to be satisfied. The primary market is the one between passenger or loader and the railway operator. The secondary market is between the railway operator and the infrastructure manager. The infrastructure manager has to make a prognosis for both markets in order for them to be able to meet their requirements. Dimensioning future railway lines to meet the future traffic demand on time, is a long time process with large investments. Prognosis of the future traffic demand has to be made on the basis of expected development of society. For long term planning, the future railway timetable is not known, only an estimation on the number of trains and the type of trains according to the prognosticated traffic demand.

Once the traffic prognosis is established the dimensioning work can start. For the existing line investigated, the flow or capacity can be calculated. With the capacity number, the expected queue length and thus the waiting time can be estimated. The queue length or the waiting time is a measure of the timetable quality. An increased number of trains running on the line results in an increase in queue length and waiting time, which means less quality. Both scheduled and unscheduled waiting time can be estimated. The latter is also called delay during operation. The scheduled waiting time arises during timetable construction when two different trains prefer to occupy the same block or line segment at the same time. The train with lower priority has to make space for the train with higher priority and therefore incurs waiting time before entering the preferred line element. The scheduled waiting time is an additive to the typical journey time. The calculation of scheduled/unscheduled waiting time can be carried out in two ways:

- by simulation of timetables and operation

- by using mathematical analytical methods

The simulation method is very time consuming, which thus is limiting this method. Only a limited set of timetables is feasible with respect to time for the estimation of the scheduled waiting time. The same yields also for the calculation of the unscheduled waiting time. Only a selected set of timetables can be analysed. The analytical method saves time compared to simulation and saves disk space as well. The analytical method is able to estimate the expected waiting time without the need for a fixed timetable. Schwanhäußer [Schw74, Schw78, Schw81] was the first to develop mathematical analytical methods for the calculation of unscheduled and scheduled waiting time.

If the expected scheduled and unscheduled waiting time is too high, the operator will have to carry higher costs because of the loss of a customer. In a market with free competition an operator has to be financially sound to exist on the market. This gives the need to take economy into account when deciding the upper limit for scheduled and unscheduled waiting time and for the decision of the optimal capacity. For the infrastructure manager, the costs connected to the maintenance and investments of infrastructure must also be included. This can be carried out in a cost-benefit analysis. These results proide directions for the dimensioning of existing or new infrastructure.

The calculations of expected waiting time also has its use for short term planning. For changes in the timetable, for example due to temporary reparations, also leads to changes in the amount of waiting time. The same also applies for ad-hoc decisions that the train dispatcher meets when the traffic is disturbed. There are methods and tools developed to meet the challenge to find an optimum designed according to certain criteria. Still, there are several unfilled gaps within this area, and there is a big need for improvement of already existing models. Some of the main challenges for the improvement of the analytical models would be to increase the level of detail of input and output information. This especially yields models calculating on networks where graph theory is used. To make the model useable the graph might be too far reduced, so that important information is lost. On the other hand, models calculating on separate infrastructure elements, like for example a node or a line segment, might often have a higher degree of detail compared to network models. The drawback is that a high level of detail generates many constraints which makes these models less available for interconnection in order to make calculations as a network based model.

There are only a few models concerning the scheduled waiting time compared with the number of methods used for the investigation of the unscheduled waiting time. In an attempt to narrow the gap within this research area, this thesis develops a new model for scheduled waiting time from crossing on single track lines. In this thesis a new and detailed study of the crossing situation on single track lines where the dependencies between the trains of opposite direction are outlined. This work has been inspired by a model from Schwanhäußer [Schw81] concerning the estimation of the scheduled waiting time from overtaking on double track line.

The topic was chosen on behalf of Jernbaneverket (the Norwegian infrastructure operator). There is a need especially for Jernbaneverket to develop models for dimensioning single track lines since the Norwegian railway network consist mainly of single track line (about 95% of 4100 km track in total [Svin05]). This study is based on an asynchronous timetable construction methodology which is in accordance with the EU's demand for free trackage rights. Another important aspect is that single track lines are very sensitive for the generation of waiting time. So far there are very few studies concerning dimensioning and operating single track lines.

The mathematical analytical model in this study makes use of exponentially distributed buffer times between the trains of higher priority in the schedule. This means that there is stochastic demand for train paths. Then there will exist spacing times of different sizes. There also might be spaces long enough for irregular freight traffic to be included into the timetable. In less traffic dense periods of the day, this might be a possible solution to fill up the schedule with ad-hoc freight trains. The effort of this research, apart from dimensioning questions, will be to evaluate possibilities for transferring more freight transport from the road to the line by offering the freight operators a second attractive choice besides the road. The possibility of including freight trains into the schedule will not only depend on the number of trains in total and the infrastructure characteristics, but also how far the schedule chosen is from stochastic operation. The closer a cyclic timetable operation is, the more constraints have to be regarded. A cyclic timetable is therefore less flexible than timetables with stochastic operation. The spacing time between the trains in a cyclic timetable tends to be constant, and with high train density this will make it difficult to fit in slow and long freight trains.

1.2 Scope and outline of this study

This study focuses on the calculation of scheduled waiting time from crossing on single track line. Research has been done on relevant models for the calculation of scheduled waiting time. The theory arisen at the RWTH Aachen University makes a clear distinction between scheduled and unscheduled waiting time. The scheduled waiting time generated in an asynchronous timetable construction process is based on a train hierarchy with the use of minimum spacing time (visualized in time-way diagram as blocking time stairs). The asynchronous timetable construction process takes care of the free trackage rights as demanded by the EU. Therefore an asynchronous timetable construction methodology was chosen as the basis in this work. The model has been developed with the aim of extending the palette of scheduled waiting time models for dimensioning purposes. In addition to a dimensioning purpose, this study is basically working with stochastic running trains in the schedule as an attempt of introducing ad-hoc train traffic. A timetable study has been conducted for selected lines to investigate the timetable characteristics today. Finally, the model has been tested and evaluated in a discussion.

This study has been divided into these following main parts:

- In Chapter 2 some fundamentals necessary for the calculation of waiting time and capacity are briefly described.

- Chapter 3 presents and evaluates some of the past works that have importance to the waiting time calculation. The chapter starts with presenting some studies concerning the estimation of the optimal capacity since they provide an understanding of the importance of the waiting time calculation as a part of the capacity analysis. The models for capacity analysis are also commented.

- In Chapter 4 the model for the calculation of scheduled waiting time from crossing on single track line is developed. The model for scheduled waiting time from crossing uses exponential distributed buffer times between the trains of higher priority. According to the asynchronous timetable construction process, trains with highest priority are to be included first into the timetable. Second, the trains with the second highest priority are to be included into the timetable. This process is repeated until all trains are included into the timetable. In the end of the Chapter 4 an attempt has been made to adjust the model for constant buffer times between the trains of higher priority. This section 4.4 is kept short since the assumptions made in early stage of the model were not satisfying enough with respect to the level of detail.

- Chapter 5 provides a summary of the timetable analysis for three single track lines. This chapter starts with a description of timetable construction in a market with free competition of the trackage rights. The timetables analyzed are characterized whether the operation runs stochastically or follows a cyclic pattern. For the stochastic schedules a χ^2-test of goodness of fit of the buffer time between the trains of higher priority in the schedule were made to analyse if the buffer time distribution could be exponential, hyper-exponential or Erlang_2 distributed.

- In Chapter 6 the model for the calculation of scheduled waiting time from crossing with exponential buffer time distribution presented in Chapter 4 is analyzed and evaluated by using an asynchronous simulation tool.

- In Chapter 7 a summary of the most important results and factors influencing them are provided. Finally, a suggestion for further work is described.

Chapter 2

Fundamentals of capacity assessment in railway operations research

This chapter provides a brief introduction to some fundamentals necessary for the calculation of capacity and waiting time described in this thesis. The first section 2.1 concerns fundamentals assigned by the infrastructure characteristics. The second and the third section 2.2-2.3 describes capacity and waiting time and their relation.

2.1 Infrastructure based fundamentals

2.1.1 Decomposition of the railway line

There are several possibilities how to decompose the railway network into smaller fragments. In accordance with UIC codex 406 [UIC/406], a railway line is the distance between two nodes, figure 2.1. A node can be a station or a junction. The traffic mix and the train order should not change remarkably on the railway line, or else a further partition of the line is necessary. In a station overtaking, crossing and directions reversals are possible. In a junction at least two lines converge and neither overtaking, crossing nor direction reversals are possible.

A block section is the distance between two main signals. A section can consists of several blocks. In a section the traffic mix and the number of trains do not change fundamentally. In this study a section will be referred to as the distance between two neighboring stations, where crossing or overtaking can take place. The point at which the section is measured from can vary. The point must successively be chosen consistent for the following stations. For example a section can be measured from the outward signal in station (i) to the outward signal in station $(i{+}1)$, or from the clearance point station inwards in station (i) to the next clearance point inwards in station $(i{+}1)$.

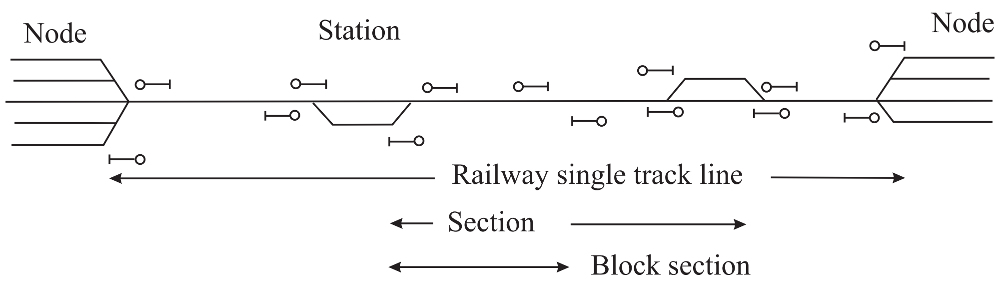

Figure 2.1: Decomposition of a single track railway line

Another possibility is to decompose a railway line or a railway node into single channels. This partition is useful in models where the waiting time is calculated with use of queueing theory. These results illustrates the bottleneck in a node or a railway line. Schwanhäußer proposes a decomposition of the railway line into single channels called *interlocking route junction* (IRJ) or "Teilfahrstrassenknoten" (TFK) in German, figure 2.2. An IRJ is a part of a *complete interlocking route junction* (CIRJ), or "Gesamtfahrstrassenknoten" (GFK) in German. An IRJ is defined as the longest carriage way where all carriage ways are mutually exclusive [Schw78].

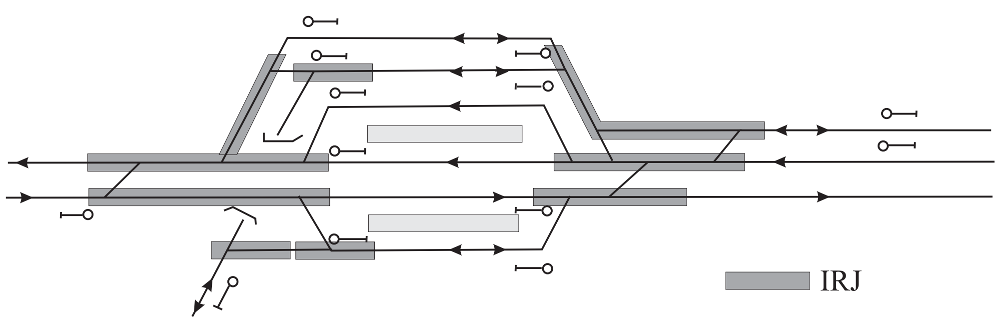

Figure 2.2: Decomposition of a station into interlocking route junctions

Another useful model is done by treating the railway network as a graph. The nodes are converted into vertices, and the distance between the vertices are edges. The graph theory is a useful tool for calculations on an entire network. Such calculations for example, can be for designing and optimizing cyclic timetables [Brak93, Eng02] and waiting time calculations [Huis02].

2.1.2 Occupation time in a block section

The smallest distance on a railway line is the block section. Happel [Happ59] was the first to define the occupation time in a block section. Only one train can occupy a block section at a time. The occupation time depends on block system, signaling system and safety technology [UIC/406]. The elementary occupation time consists of:

- Time for route formation: t_{form}
- Time the driver needs to discover the signal (visual distance): t_{view}
- Time for approach the main signal: t_{app}
- Time it takes to travel the entire block section (journey time): $t_{journey}$
- Time for clearing the block section left behind. The train has to pass a contact point or be at least one train length behind the signal: t_{clear}
- Time for route release: $t_{release}$

The occupation time is shown schematically in figure 2.3. The time-way-line is the front of the train. When all occupation times for a railway line are mapped in a time-way graph, they form blocking time stairs.

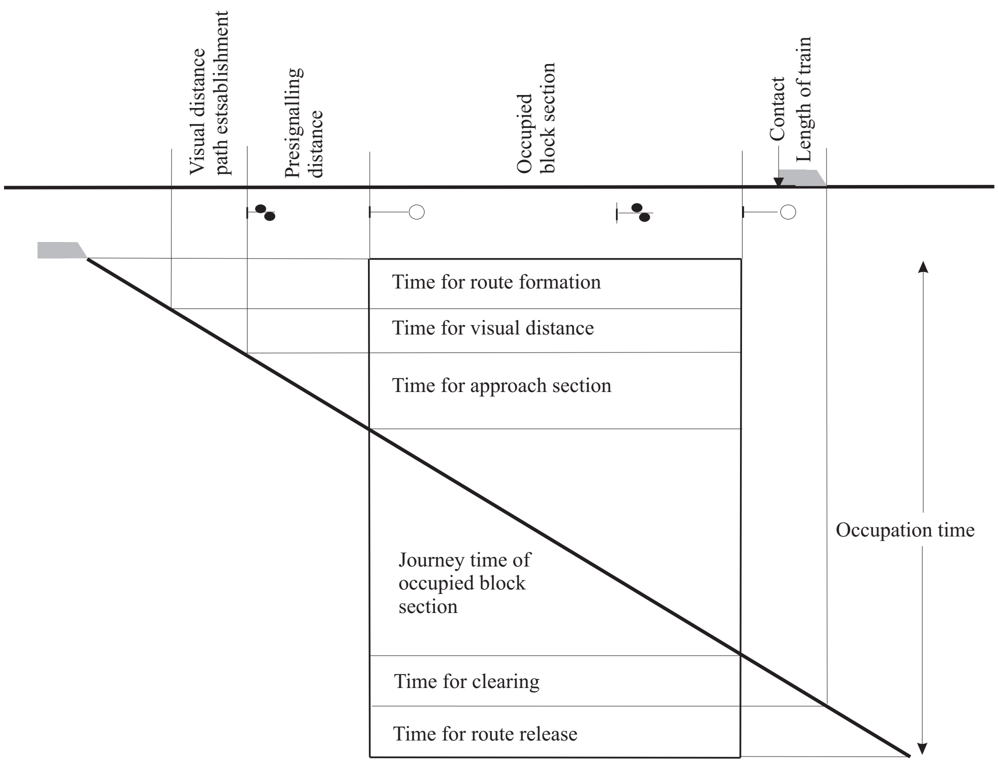

Figure 2.3: Elementary occupation time

2.1.3 Journey time

This thesis makes use of the *typical journey time* for the computation of the scheduled waiting time. The *typical journey time* is defined by UIC [UIC/406] to be the time it takes for a train to run a certain distance according to line and rolling stock characteristics plus a time supplement. This time supplement is usually about 3-5% depending on the train model.

UIC also defines the *timetable journey time* to be the time given in the final timetable. The *timetable journey time* includes the typical journey time, additional time which result from market requirements (for example synchronization time, prolonged travel time of night trains) and additional time which result from timetable construction constraints.

Some studies commented next in Chapter 3 use other definitions as mentioned by UIC.

Hertel [Hert92] makes use of the term *expected time spent on the line* which consists of the expected scheduled waiting time and the expected typical journey time.

Oetting [Oett05] uses the *Beförderungszeit* as the *real running time* which includes the typical journey time, the scheduled and unscheduled waiting time.

Schwanhäußer [Schw81] uses the difference in typical journey time for the calculation of scheduled waiting time from overtaking. Gast [Gast86] modifies this expression by including the signaling system and names this expression *Bereichszugfolgezeit*. This is the time gap necessary between two trains for a second train to merge in between them without generating any conflict for the following train. This is a better expression than the mean typical journey time difference since the signaling system is taken into account.

2.1.4 The minimum spacing time

The minimum spacing time, t_s, is the minimum time a train j can follow a train i without reaching into conflict on the first common entering line section. In a blocking time stair graph (time-way graph), the minimum spacing time can be found by moving the blocking time stair of the following train j until it touches, but not overlaps the blocking time stair of its descender train i, shown in figure 2.4.

On a single track line the minimum spacing time is measured at the point of the line where the crossing trains pass the main signal out of the station, as shown in figure 2.5 [Brün92]. Train j can first leave the station after train i has passed the clearance point and the green signal has been given. If train j is waiting next to the main signal time for approaching it will be skipped. Time for viewing the signal is mainly shorter than when running on the free line.

Figure 2.6 shows Happel's [Happ59] definition of minimum spacing time for crossing trains. This spacing time consists of following components:

- time for clearing after train i
- time for route release after train i
- time for route formation for train j

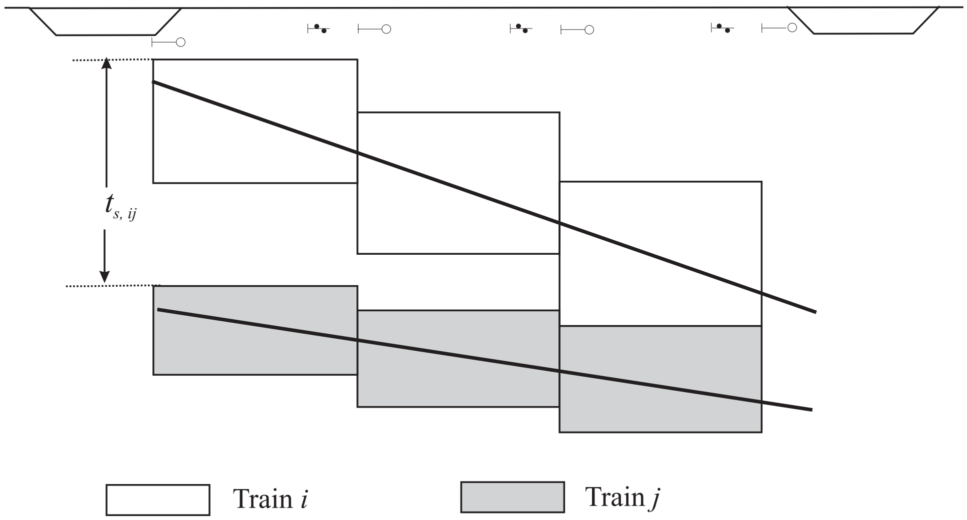

Figure 2.4: Minimum spacing time with unidirectional traffic

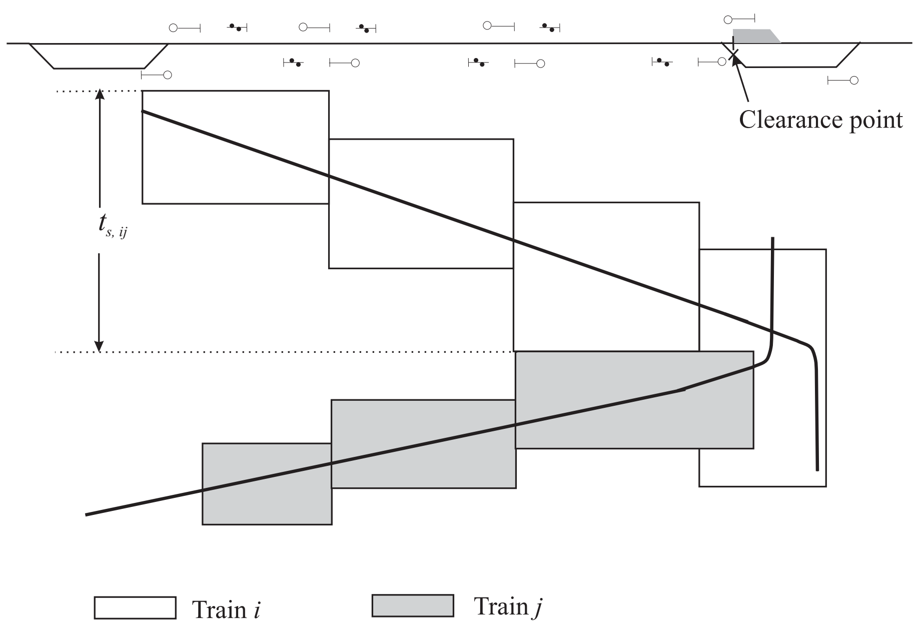

Figure 2.5: Minimum spacing time for single track line with bidirectional traffic

In this special case of a crossing Happel has skipped time for approaching the signal. This yields only for halting trains. In the time for route formation for train j a reaction time for the driver to register the signal is to be included. Happel's minimum spacing time explains the minimum time it takes before the halting train j can leave the station after train i has arrived.

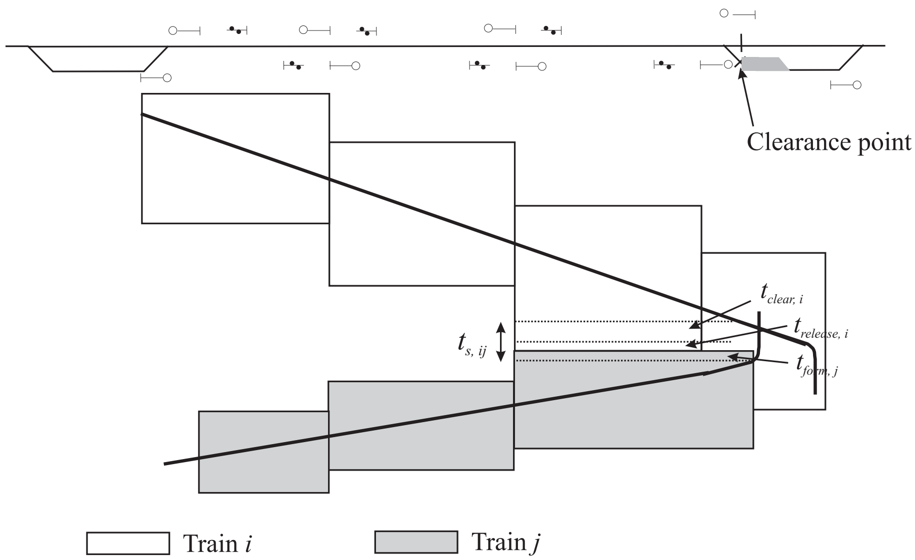

Figure 2.6: Minimum spacing time for crossing according to Happel

2.1.5 The minimum time necessary for a crossing

The minimum time necessary for a crossing ($min\ t_x$) is a technical quantity based on the infrastructure, signaling and safety system and rolling stock characteristics. This is the minimum time needed to carry out a crossing without incuring any waiting time from crossing. Figure 2.7 illustrates a conventionalized crossing where a train of rank(2) has lower priority than a train of rank(1). Train of rank(2) is running into the siding to meet the opposing train of rank(1).

The minimum time for a crossing consists of:
$t_{r,\ 2} = t_{v \to 0} - t_v$ is the addition in time for train of rank(2) because of slowing down to halt (which is made out of: $t_{v \to 0} - t_v$ the running time difference for a halting and not halting train)
$t_{release,\ 2}$ is the route release time for train of rank(2)
$t_{form+view,\ 1}$ is the time for route formation and time for visual distance for train of rank(1)
$t_{app,\ 1}$ is the time train of rank(1) needs for approaching the section
$t_{journey,\ 1}$ is the journey time for train of rank(1)
$t_{clear,\ 1}$ is the clearing time for train of rank(1)

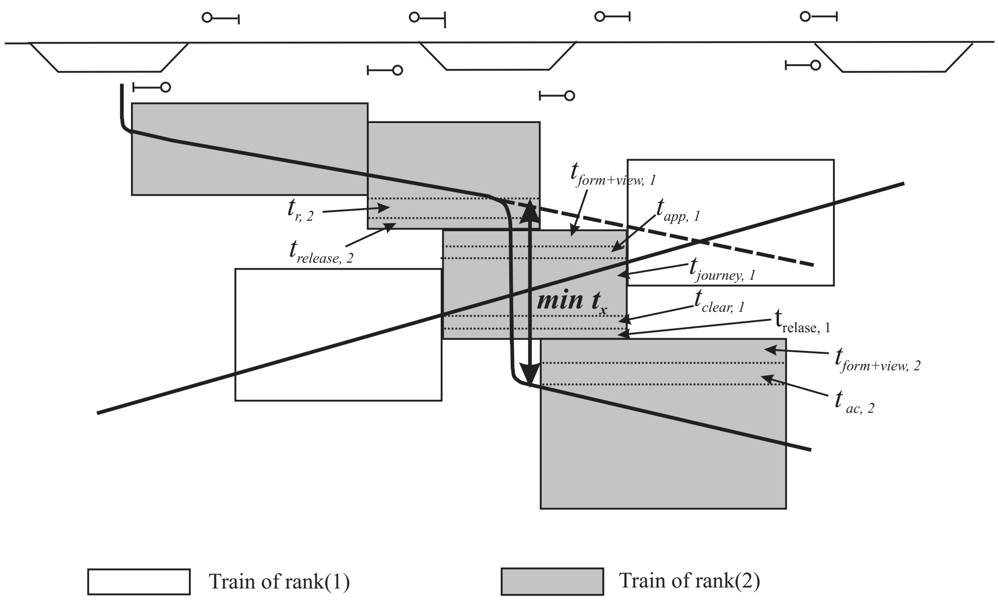

Figure 2.7: The minimum time necessary for a crossing

$t_{release,\ 1}$ is the route release time for train of rank(1)
$t_{form+view,\ 2}$ is the route formation time for train of rank(2)
$t_{ac,\ 2} = t_{0\to v} - t_v$ is the addition in time for train of rank(2) incurs because of the acceleration

Note that train of rank(2) has a reaction time for viewing the signal which is mainly shorter than the time for visual distance, since the train is halting in front of the signal. For manual operated signal cabins the operator could theoretically handle the switches to minimize the time each component needs. For an independent station a locking time for crossing replaces the time lost because of retardation and route release for train of rank(2). The locking time for crossing is the time the train of rank(2) needs for entering the siding and slowing down to halt. This quantity depends mainly on the length of the siding. According to Jernbaneverket this time is mostly 50-70 seconds [Skar98].

2.1.6 Train model

For the calculations of waiting time it is not only necessary to have information about the infrastructure, but also about the rolling stock that are supposed to run the line in the future. The model that will be presented in the next chapter makes use of blocking time stairs. The model input parameters from the different trains will be their minimum spacing time. To achieve the minimum spacing time, technical information about the train must be available such as haulage capacity, speed, mass, length and brake system (for the retardation). Together with the infrastructure characteristics and information about the halts of trains (station and minimum halting time), minimum spacing times can be generated. To make it more surveyable, trains with similar technical quantities builds the same train model.

2.2 Capacity definitions

There are several ways of defining capacity. UIC [UIC/406] refers to capacity as *the total number of paths in a defined window.* Capacity can be divided into theoretical and practical capacity of a line section. The theoretical capacity is a *theoretical maximum expressed in terms of the maximum number of trains that can be calculated by defining ideal circumstances* [UIC/406].

The practical capacity, according to Schwanhäußer, is *the most optimal for operation, where the profit is at its highest* [Schw94]. Hohnecker has described the practical capacity in this way: *The practical capacity expresses the number of trains that passes through a line section, according to the spacing time which depends on the train order and train mixture depending on density and direction, further their individual rank, the delays for the certain line and the quality level* [Hohn95]. Bär *et al.* [Bär88] describes railway line capacity to *the capacity of a system is the production or throughput of a special performance with respect to the available technology. The capacity is a stochastic quantity.* The maximum capacity is described as *the capacity of a system where all technical parameters are maximally exploitated. The theoretical capacity is a realization of the stochastic capacity quantity.*

Bär *et al.'s* definition fits Schwanhäußer's definition of theoretical capacity. This is the absolute maximum number of trains per time unit that is technically achievable regardless any operational qualities. Hohnecker's definition of the practical capacity covers a wide range of limiting factors. His definition is vague about the referred *quality level.* It could be the quality level with highest timetable stability or that quality level that gives the highest profit for the operator/manager. These two quality levels must not be the same. Schwanhäußer on the other hand defines operational quality to be monetary.

2.3 Waiting time versus capacity

As described in the previous section, capacity defined as trains per unit time does not express anything about the timetable or operational quality. Second, because of the difference in geometry, topology and rolling stock etc. of the lines it makes it difficult to compare the capacity number between the different railway lines. Capacity depends on several parameters/factors. The most important parameters/factors are the timetable quality and the operational quality. On one hand it is the scheduled waiting time which affects the customer when choosing a form of transportation. When the scheduled waiting time is increasing, the quality decreases. On the other hand it is the buffer time constructed in the timetable which should buffer the unscheduled waiting time. If the buffer time is not large enough, unscheduled waiting time will still be generated during operation. Both scheduled and unscheduled waiting times are parameters that can be estimated and calculated and serves as a measure for quality which makes it possible to make comparisons with other lines.

Scheduled waiting time arises during timetable construction whereas unscheduled waiting time is generated during operation. The unscheduled waiting time is all of the deviation compared to the timetable. Therefore unscheduled waiting time also contains arrivals that are too early.

There is a close connection between unscheduled waiting time and capacity (trains per time unit) on a railway line. Figure 2.8 illustrates the relation between the capacity and the unscheduled waiting time. As the traffic load increases (trains per time unit increases), the unscheduled waiting time increases rapidly. The closer the number of trains an hour it gets to the theoretical capacity, the more queues there will be until an infinite queue is reached.

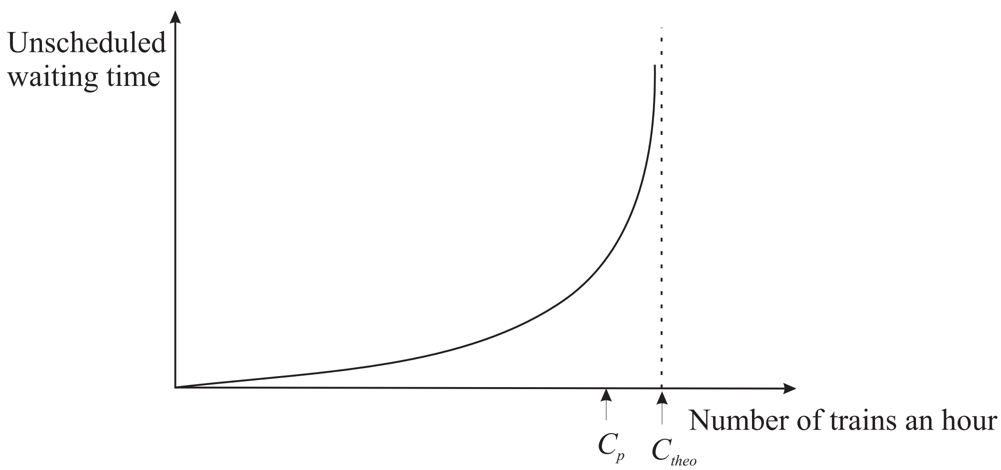

Figure 2.8: The dependency between the unscheduled waiting time and capacity

In the next two subsections the basics of scheduled and unscheduled waiting time will be explained.

2.3.1 Scheduled waiting time

Scheduled waiting time arises during time table constructions. In a market with several train operators, or in a market with several train services, there is an optimal time table for every train service/operator. This optimal timetable is the one that could give the operator highest profit. If all these optimal timetables are coordinated by the timetable constructor, several block occupational conflicts arise. After the decision has been made as to which operator/train service is given first priority, the conflict between the trains can be solved in different ways. The train not given first priority can be:

- placed in a siding if one is available.
- given longer journey time
- given earlier or later departure time than the train with first priority.
- dismissed if no other possibilities are available.

In some cases many operators do not know their optimal timetable. This might be because of lack of knowledge of their market. If there is a difference in the ordered timetable from the operator and their optimal timetable there will be loss of income for the operator. This has no affect on the timetable construction process. The method of timetable construction mentioned here follows a hierarchical procedure which characterizes the asynchronous timetable construction. It is also possible to use a partial hierarchical priority procedure by introducing certain criteria for when a priority order can be shifted during the timetable construction. The DB AG uses conversation as a method to solve conflicts between the different operators involved in addition to the strict hierarchical priority list. These conversations makes it possible to shift priority without following any certain criteria.

In many railway timetables international train routes and important connections and cyclic departures must be taken care of. The waiting time for designing a cyclic timetable or the waiting time for a connection between two train services is called *synchronization time* [DB405.0101]. As wll, environmental protection can cause waiting time. For example, a line can have restrictions for noise-intensive train movements. The timetable designer is restricted to an upper limit of the number of trains traveling during certain times of the day. The more constraints a time table has, the more waiting time there will be, and the lower the capacity becomes [UIC/406]. Wether the synchronization time and the waiting time are due to environmental or other constraints is not part of the general waiting time due to block occupational conflicts.

The following figures 2.9-2.13 illustrates common situations where scheduled waiting time occurs [DB405.0101].

In figure 2.9 train of rank(2) has a scheduled halt in the siding. An overtaking train of higher priority (rank(1)) has to pass before train of rank(2) can merge back into the line. Train of rank(2) is given a later departure time. The time difference from the requested departure time and the scheduled time is equal to the scheduled waiting time. The same condition yields if train of rank(2) has no halt in the siding. Then the scheduled waiting time increases compared to the halting situation.

Figure 2.10 illustrates a train of rank(2) that is about to traverse a junction. Train of rank(1) has a later departure time than train of rank(2). The departure time difference between the trains still leads to occupational conflicts of the first block ahead. Train of rank(2) is given a new and later departure time.

Another possible solution for the situation in figure 2.10 would be to let train of rank(1) incur a waiting time if the operator makes an agreement. In figure 2.11 train of rank(1) is given a later departure time. The scheduled waiting time in this case is smaller than is the first case.

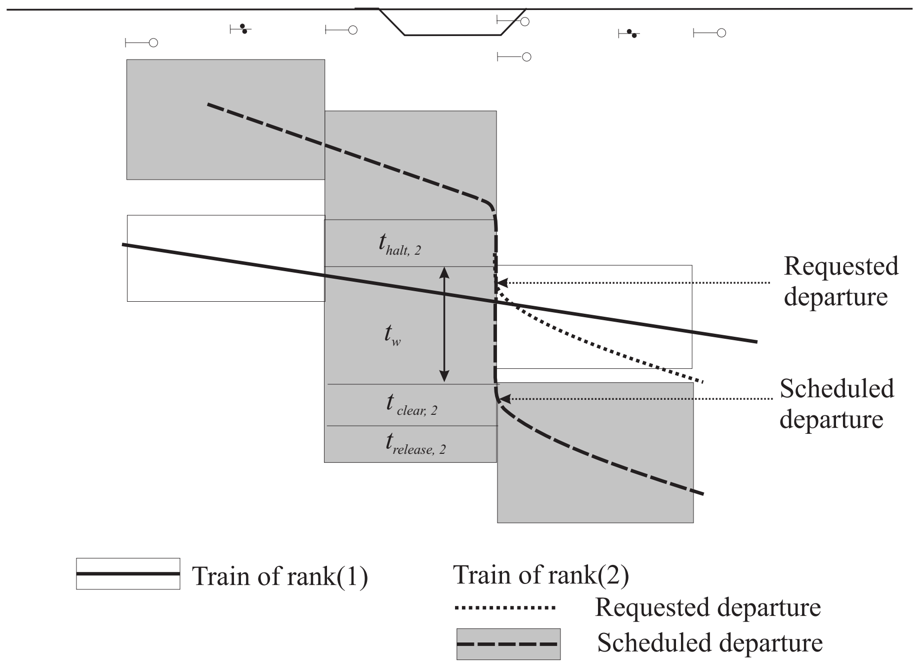

Figure 2.9: Scheduled waiting time from overtaking

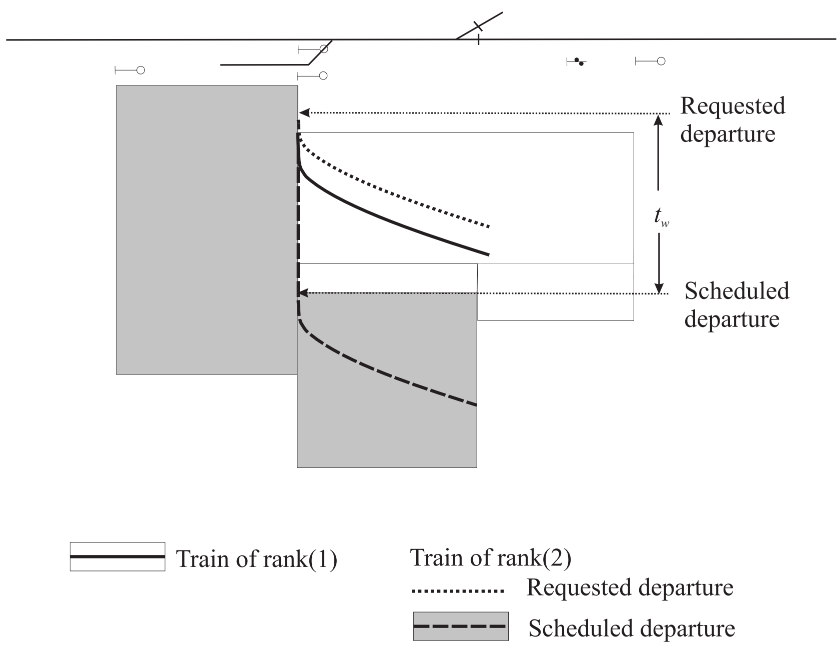

Figure 2.10: Train of rank(2) incurs later departure time

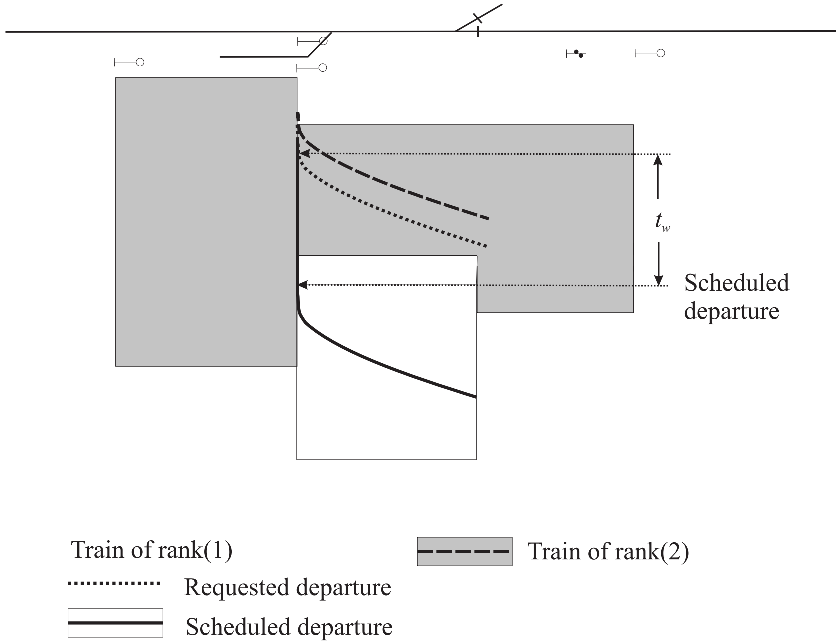

Figure 2.11: Train of rank(1) incurs later departure time

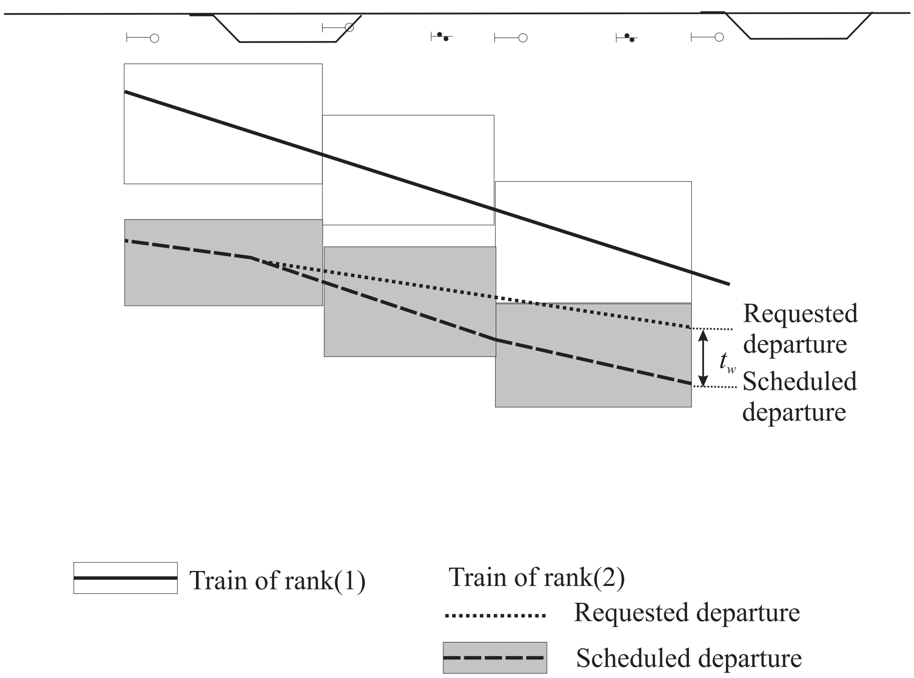

Figure 2.12: Train of rank(2) incurs longer journey time

In figure 2.12 a train of rank(2) catches up with train of rank(1) within a section. Train of rank(2) has a lower priority and will be slowed down. The time additive caused by the slowdown is in this case the scheduled waiting time.

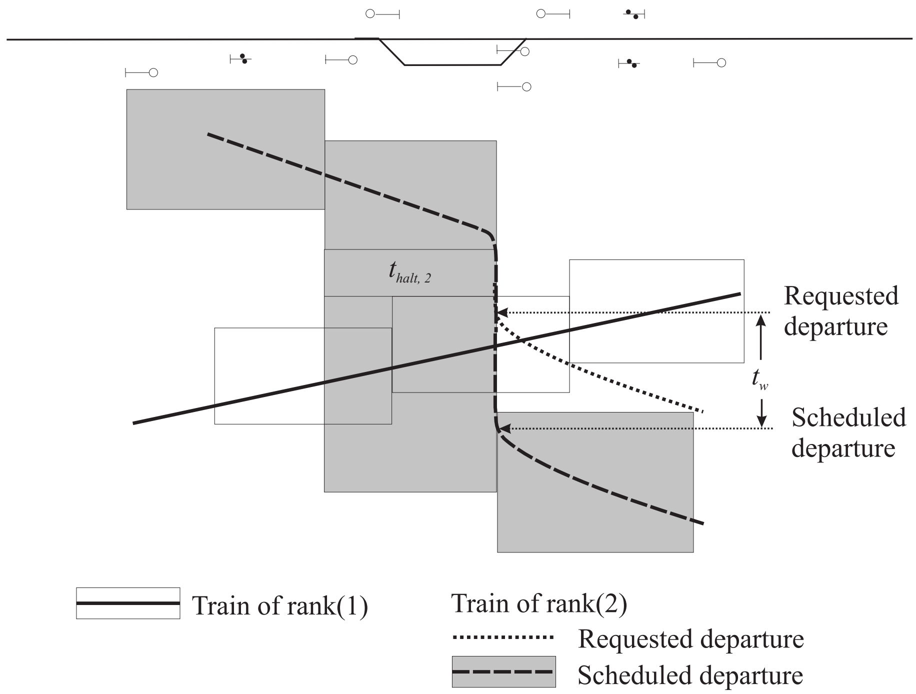

Figure 2.13: Scheduled waiting time from crossing

Figure 2.13 illustrates a train of rank(2) that incurs waiting time because of an opposing train of higher priority. Similarly to figure 2.9, the scheduled waiting time is the difference between the requested departure time and the scheduled departure time.

2.3.2 Unscheduled waiting time

Unscheduled waiting time is also referred to as a econdary delay which arises during operation. This waiting time is not incorporated in the timetable. Delay in general can be divided into different types as shown in figure 2.14:

- t_{PD}: Primary delay on arrival to the area of survey.
- t_{CD}: Source delay during the time spent in the area of survey. This is an external delay caused by for example trouble in signals, braks and other technical interruptions and commute hour traffic.
- t_{SD}: Secondary delay is the delay a train receives from a delayed train or from itself when it is delayed.

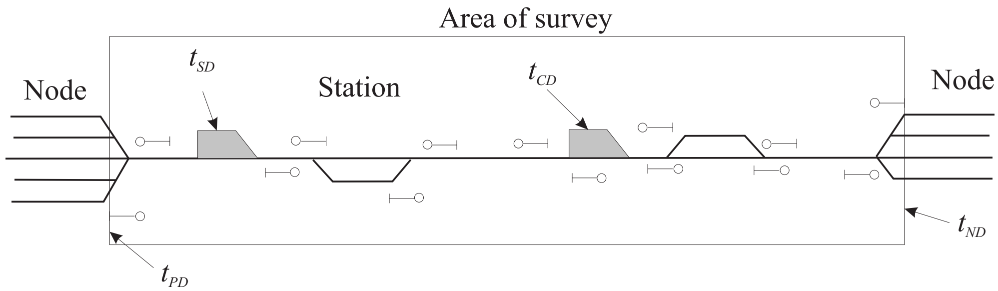

Figure 2.14: Delay in area of survey

- $t_{GD} = t_{PD} + t_{CD} + t_{SD}$: Gross delay is the sum of primary-, secondary- and source delay.
- t_{ND}: Net delay leaving the area of survey, is the gross delay reduced by the running and dwell time supplements during the time spent in the area of survey.

In other references [Higg98] secondary delay is referred to as *Knock-on-delay.* The term *Delay due to late connections* consists of synchronization time, delay caused by block occupational conflicts and time due to set allocation.

In 1974 Schwanhäußer investigated the distribution of primary delay. He assumed a combination of a one point distribution and an exponential distribution [Schw74]. His assumption is confirmed by [Eng02, Heis78].

The primary delay of a line is a random variable T_{PD} with a combined discrete and continuous distribution function. The amount of delayed trains is given by p_{PD} and the amount of non delayed trains by $1 - p_{PD}$. This forms a discrete distribution:

$$P(T_{PD} = 0) = 1 - p_{PD} \text{ and } P(T_{PD} > 0) = p_{PD}$$

The delay of the delayed trains is separately modeled by an exponential distribution with parameter λ. This is a continuous distribution on the form:

$$F_{T_{PD}}(t) = \begin{cases} 0 & ; t < 0, \\ 1 - p_{PD}\, e^{-\lambda \cdot t} & ; t \geq 0 \end{cases}$$

The expectation of the delay distribution function is:

$$E[T_{PD}] = \frac{p_{PD}}{\lambda} \tag{2.1}$$

This is the mean primary delay of all trains running the line. The mean primary delay of the delayed trains then becomes $1/\lambda$.

At $p_{PD} = 1$ all trains are primary delayed and the probability distribution function turns into a continuous exponential distribution function. In the opposite case, $p_{PD} = 0$ builds a discrete one point distribution since all trains are on time.

In figure 2.15 the probability distribution function for primary delay is illustrated.

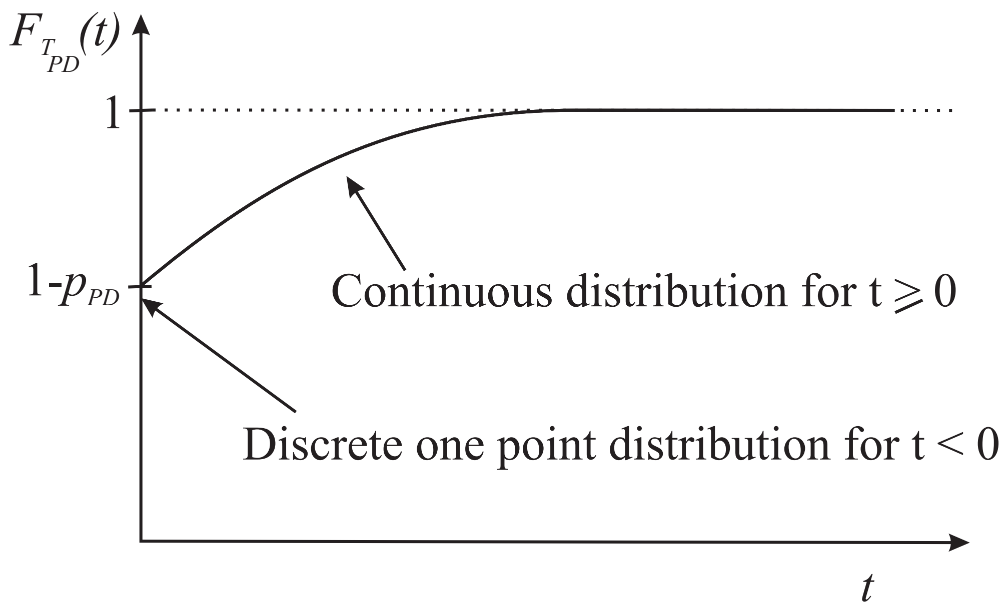

Figure 2.15: The probability distribution function for primary delay of delayed trains

The probability density function for primary delay of the dalyed trains is given by:

$$f(t) = \begin{cases} 0 & ; t < 0, \\ \lambda\, e^{-\lambda \cdot t} & ; t > 0 \end{cases}$$

The density function of the exponential distribution function is illustrated in figure 2.16.

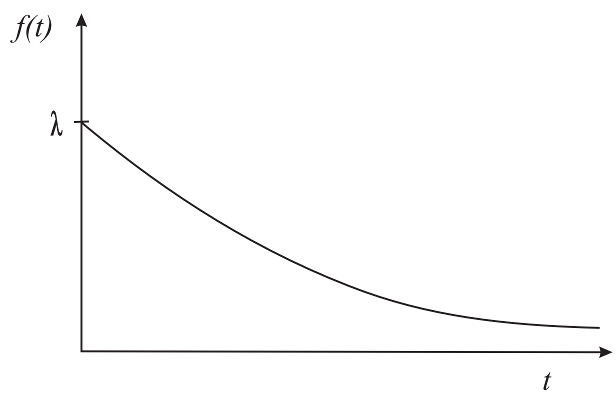

Figure 2.16: The probability density function for primary delay

The probability density function of the delayed trains illustrates that there are far fewer large primary delays than there are small primary delays, which makes sense for mixed railway traffic.

2.3.3 Queueing theory in the railway

Queueing theory is a method suitable for the calculation of both the scheduled and unscheduled waiting time. In general, a queueing system consists of an input and output stream. The input stream describes the interarrival time between the trains. Figure 2.17 illustrates an input stream of trains arriving in the waiting space, before getting served. The serving time is the minimum spacing time that makes the output stream.

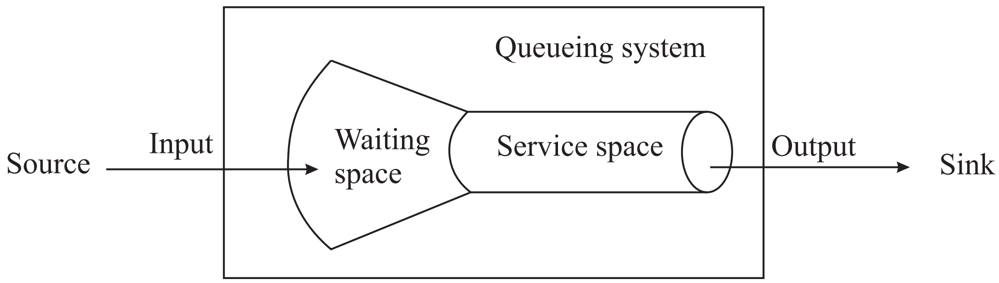

Figure 2.17: A queueing system

The most common way in describing a queueing system is done by use of the Kendall notation:

$A/S/s/m$

where

A : arrival process

S : service time process

s : the number of servants

m : size of the waiting space

Both the arrival process and the service time process can be treated as stochastic variables. The arrival and service process can be described differently:

M : Markowian process, the stochastic variables are independent exponential distributed with parameters λ (arrival rate) and μ (service rate). Often one uses the Poisson distribution to describe the interarrival time for mixed train traffic. Here the variance coefficients for arrival rate V_A and service rate V_S are equal to 1 [Schw74, FeHi96].

E_k : The Erlang process, is a combination of k independent exponential distributions, with parameters k and λ. The parameter k is a *natural* number and λ a *real* number. The Erlang distribution is also a stochastic process. If k takes a *real* number above zero, it becomes the general gamma $\Gamma(k)$ distribution.

D : In the Dirac process, the interarrival- and service time is constant point distribution. This means that the variances V_A or V_S are equal to 0. This process can be found in the arrival and departure of shuttle-trains, for example at airports (which has a homogeneous traffic flow).

GI : General-independent process, the independent stochastic variable has no dependencies to other processes [Wako85].

G : General process, the stochastic variable has no restriction at all. For the unscheduled waiting time calculation the arrival process is strongly dependent on the timetable relations.

The size of the waiting space decides how many trains are allowed to wait for service. In a railway queueing system the waiting space is set to be $m = \infty$. If the waiting space was a fixed number, (given number of available tracks or IRJ's) and the arrival stream was high, trains could be lost out of the system (system with loss, $m = k$) which is not possible. If the real waiting space at a node is not large enough, a train can wait at a previous node.

The rate of arrival λ, which is the number of trains by time or the inverse of the mean interarrival time $E[T_A]$ (same as *expected* inter arrival time):

$$\lambda = \frac{1}{E[T_A]}$$

The rate of service μ, which is the number of trains by time or the inverse of the mean service time $E[T_S]$:

$$\mu = \frac{1}{E[T_S]}$$

The traffic load ρ, which is the arrival rate divided by service rate, or the mean service time divided by the mean interarrival time:

$$\rho = \frac{\lambda}{\mu} = \frac{E[T_S]}{E[T_A]}$$

Queueing systems are only stable as long as $\frac{\lambda}{\mu} < 1$. If the load ρ reaches above 1, it will come to infinite queues. To solve traffic problems, one is most interested in calculating the queue. The queue is described by the mean waiting time for a customer in the queue $E[T_{Wq}]$, and the length of the queue $E[L_{Nq}]$ (average number of waiting customers in the queue). Both are treated as stochastic variables. Little's formula gives their relation:

$$E[T_{Wq}] = \frac{E[L_{Nq}]}{\lambda}$$

Little's formula can also be used for the entire queueing system:

$$E[T_{Ws}] = \frac{E[L_{Ns}]}{\lambda}$$

where $E[T_{Ws}]$ is the average amount of time a customer spends in the system, and $E[L_{Ns}]$ is the average number of customers in the system [Ross00, Fisc90]. The sum of waiting time spent by all customers during time of survey will be:

$$\sum T_{Wq} = E[L_{Nq}] \cdot T$$

Chapter 3

Past studies

This chapter is divided into six sections. In the first sections 3.1-3.3 some models describing capacity are introduced. These sections provide an insight that the capacity number alone is not sufficient enough for investigation of line capacity. Capacity is strongly related to scheduled and unscheduled waiting time, whose models are introduced in section 3.4 and 3.5 respectively. Finally a summary of the reference research is given in section 3.6.

3.1 Deterministic methods for describing capacity

How to achieve a fluid railway operation has always long been a current theme. In 1952 Dilli [Dill52] estimated the buffer time between steam locomotive trains. The actual capacity for a certain railway line can be found from the timetable. The capacity number is given by:

$$C = \frac{T}{\overline{t_b} + \overline{t_s}}$$

where
T = time of survey
$\overline{t_b}$ = the mean buffer time between the trains
$\overline{t_s}$ = the mean spacing time between the trains

The buffer time should be big enough to prevent delay. Dilli estimates the mean buffer time on different bottlenecks to minimize delay during operation. During operation the *difficulty*, w, for a train is recorded:

$$w = \frac{\overline{t_{PD}}}{t_{b,\ sched}}$$

where

$\overline{t_{PD}}$= mean primary delay
$t_{b,\ sched}$= scheduled buffer time

The number of difficulty S makes the sum of all train sequences through the bottleneck:

$$S = \sum_{train\ sequences} \frac{\overline{t_{PD}}}{t_{b,\ sched}}$$

If the number of difficulty:
$S = 1$; all delays are equal the scheduled buffer times
$S > 1$; delays are larger than the scheduled buffer times and queues arise
$S < 1$; delays are smaller than the scheduled buffer times

Dilli suggests that the number of difficulty must be somewhere between $0 - 1$ to satisfy operational quality. The closer to 0, the less delay there will be. He calls the buffer time the *tolerance* of the schedule. He explains the importance of calculating this number for the purpose of improving the existing timetable and for dimensioning of infrastructure. His investigation advises a buffer time of 5.5 minutes for a *tolerant* operation.

Dilli's method was a good and clear method for the registration of delays, yet it was also very time consuming. The day of survey should be representative for the whole year and therefore is chosen very carefully. This aspect makes Dilli's analysis risky.

Adler [Adle67] defines the chained exploitation rate in his dissertation. The chained exploitation rate is the amount of time that a railway line is occupied:

$$\text{chained exploitation rate} = \frac{T - \sum t_b}{T}$$

where
T = the time of survey
$\sum t_b$ = the sum of the buffer times within the time of survey

With means of time-way-graphs the exploitation rate can be increased by changing train orders. The buffer times can be monitored directly from the time-way graphs. The bottleneck of the line is identified for which section the sum of the buffer times are smallest and the exploitation rate highest.

Adler's method is also a manual time consuming empirical method. Which exploitation rate is acceptable for a given line and timetable is based on experience.

UIC [UIC/406] presents a method for the calculation of the capacity consumption. UIC's capacity consumption method is a simplification of the compression of railway paths which Adler used in his dissertation [Adle67]. This method takes into account that the level of capacity consumption is the only value that can be measured objectively. The method was developed with the aim of a common definition and methodology to express capacity on an international level between different countries.

Figure 3.1 illustrates the original timetable for a line.

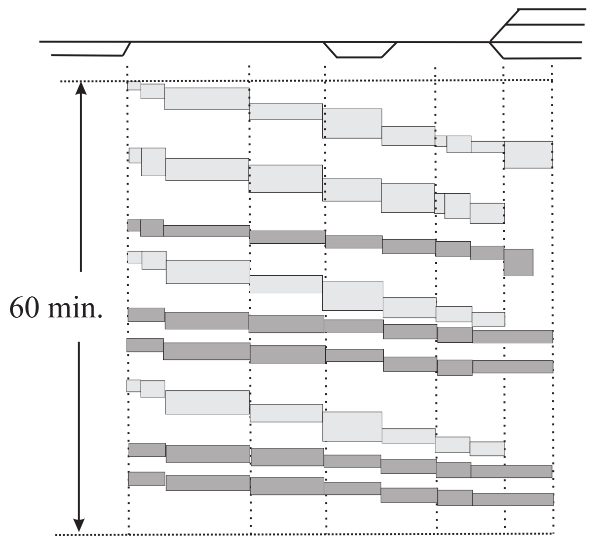

Figure 3.1: The original timetable for a line

The compressed form of this timetable is illustrated in figure 3.2. To compress a timetable means to *chain* the train paths in the schedule. Figure 3.2 illustrates the *capacity exploitation* for exactly this schedule and the *remaining time* which can be expressed in percent.

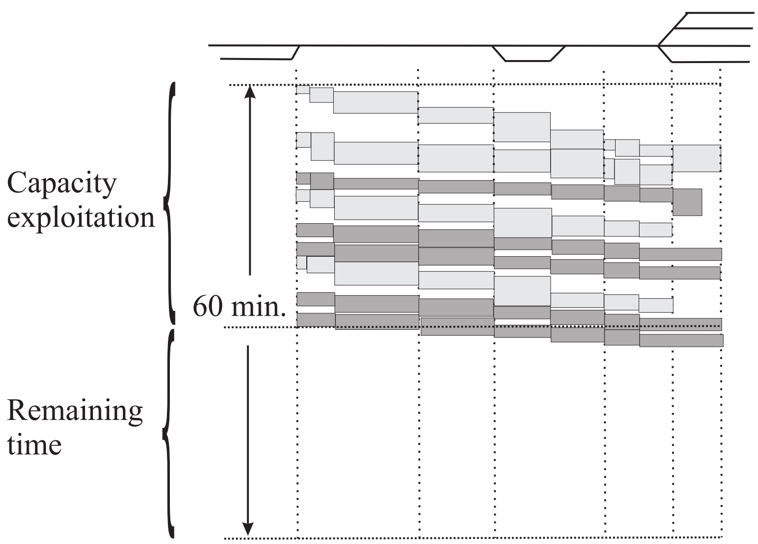

Figure 3.2: The original timetable has been compressed

The remaining time consists of buffer time, supplement for maintenance, available and lost capacity. It is possible to calculate the available capacity by adding trains into the original timetable. After compressing the enriched timetable, the difference between the new capacity exploitation and the original capacity exploitation makes the available capacity. The remaining time components cannot be separated into buffer time, supplement for maintenance and lost capacity with this method. Other methods must then be used, like for example the STRELE formula [Schw74] for the investigation of the required buffer time. The UIC capacity consumption method is easy to carry out, but it does not take any quality parameters into account.

Janić [Jani84] performed a capacity study on single track line. For a given train mix the rates of traffic in both directions are varied as complements. In figure 3.3 the traffic load ρ_1 of direction No. 1 is shown on the horizontal axis versus the capacity on the vertical axis. In this study the traffic load is interpreted as:

$\rho_1 = \frac{n_1}{N}$

where
n_1 =the number of trains scheduled in direction No. 1
N = the number of trains scheduled in both directions together

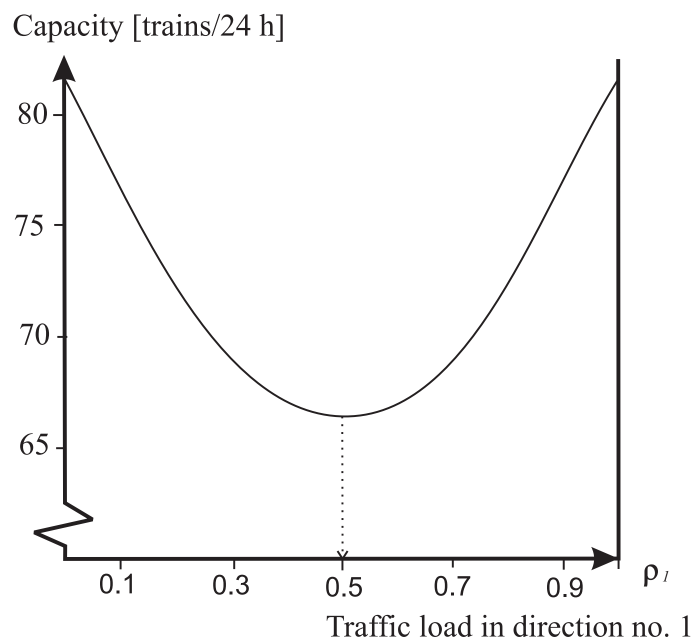

Figure 3.3: Capacity versus traffic load in one direction on a single track line

The capacity varies according to the traffic distribution of the particular directions. For example when $\rho_1 \to 1.0$ or $\rho_1 \to 0$ the trains run only in one direction, the capacity has the highest values for a given number of N. The lowest capacity is achieved with bidirectional traffic and equally distributed traffic flow in each direction at $\rho_1 = 0.5$.

Janić emphasizes that the capacity is very low on single track line compared to double track line with unidirectional traffic. The graph in figure 3.3 is based on an ideal operational situation with no delay. Lower numbers for capacity will probably be reached if delay will be taken into account.

3.2 Capacity described by fundamental diagrams

There are many ways how capacity can be measured and expressed. In road transportation, capacity can be described by use of fundamental diagrams.

In figure 3.4 *a* the flow[1] of vehicles an hour versus density is illustrated. By an increasing flow of vehicles, the density will also increase until a certain point. Here the capacity will drop as will the vehicle's speed. The speed will decrease until the vehicle density has reached its maximum and traffic will then stand still. In figure 3.4 *b* the vehicle velocity versus density is illustrated. By increasing vehicle density the slower the vehicles will have to drive. At a certain density queue formation starts, and velocity drops drastically [Scha06].

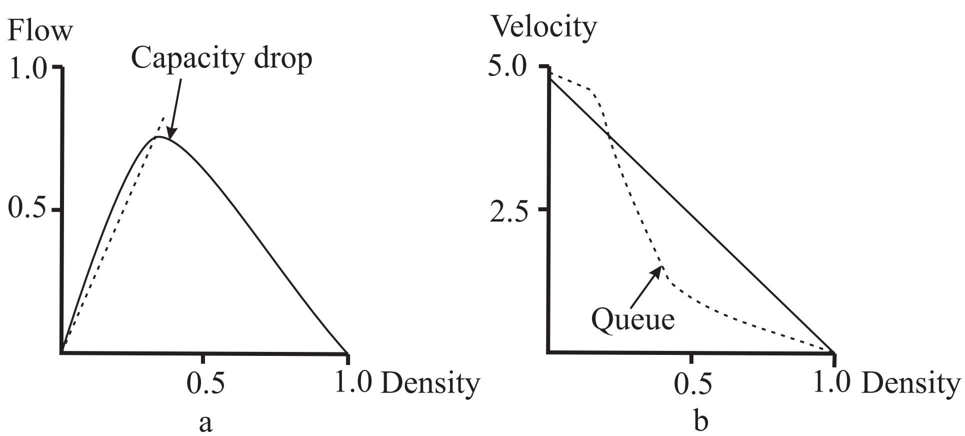

Figure 3.4: Fundamental diagram for road transportation. Figure a: Flow versus density. Figure b: Velocity versus density.

In figure *a* a linear dependency is made between the flow and density given by:

$$Flow = v_o \cdot Density \tag{3.1}$$

With this equation the optimal velocity for a maximum flow, V_o can be found.

[1] In fundamental diagrams the term flow is used. Both terms flow and capacity have the same units: vehicles/hour.

Potthoff [Pott80] makes use of the fundamental diagram from road transportation to estimate the maximum flow of trains on a certain railway line. Figure 3.5 illustrates a plot flow versus density on a railway line. The flow and density is measured every hour during a period of 24 hours. The spreading of the measuring is explained by the daily traffic profile that has two peak periods, one in the morning and one in the afternoon. Through linear regression a parable is placed through the mean flow and mean density, having a tangent, as illustrated in the graph. From this parable the maximum flow and density can be estimated. Potthoff points out that although this still is a rough estimation, it is useful for quick analysises. The main drawback with analysis of railway traffic flow expressed in fundamental diagrams is that the timetable quality in terms of waiting time is not explicitly expressed.

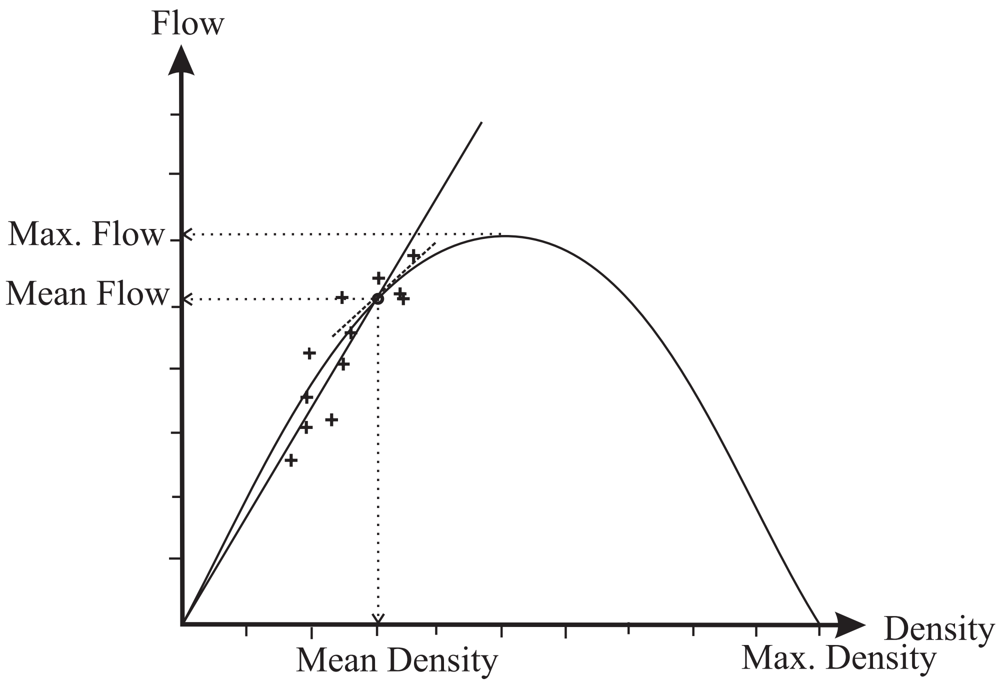

Figure 3.5: Fundamental diagram used by Potthoff

Hertel [Hert92] points out that railway traffic does not operate like road transport where vehicles run with distances equal to the relative braking distance. Instead, railway traffic operates with a separation distance to ensure safety and security. Therefore railway traffic does not obey the same relations as road traffic does. The relations valid for road traffic has to be modified to yield for railway operation. The fluidity of railway traffic in dependent on its density. With use of the approximation given in equation (3.1) Hertel describes this dependency with use of queueing theory. He describes the mean traffic performance with:

$$E[Q] = E[L_b] \cdot \overline{v} \tag{3.2}$$

where
$E[L_b]$ = mean train density
$\overline{v}$ = mean velocity

For the comparison of different lines, Hertel standardizes the equation (3.2) by using a unit length of 1 km. Equation (3.2) can then be expressed as:

$$E[Q] = E[T_{journey}] \cdot \frac{N}{T} \cdot \frac{unit\ length}{E[T_v]} \tag{3.3}$$

where
$E[T_{journey}]$ = the expected typical journey time
N = the number of trains running during the time of survey
T = the time of survey
$E[T_v]$ = the expected time spent on the line

The expected time spent on the line consists of the expected scheduled waiting time, $E[T_{WF}]$ and the expected typical journey time. Equation (3.3) can now be rearranged to:

$$E[Q] = \frac{E[T_{journey}] \cdot \frac{N}{T}}{E[T_{WF}] + E[T_{journey}]} \tag{3.4}$$

By the standardization to a unit length of 1 km in equation (3.4) is referred to as the mean standard traffic performance (*Mittlere normierte Verkehrsleistung*) in equation (3.5):

$$E[QN] = \frac{\rho}{1 + E[T_{WF}]/E[T_{journey}]} \tag{3.5}$$

Equation 3.5 gives:
$\rho = 0$ then $E[QN] = 0$
$\rho = 1$ then $E[QN] = 0$
$0 < \rho < 1$ then $0 < E[QN] < 1$

Figure 3.6 illustrates the mean standard traffic performance. The E[QN] takes values in the range $[0, 1]$. The shape of the graph for three different queueing systems are illustrated in figure a. Increased E[QN] is achieved for systems tending to cyclic behavior. When the traffic load increases above the maximum point of the graph, queues arise. By further increase (up to $\rho = 1$) of the traffic load, the traffic will run slower and finally stop. Then there is an infinite queue with infinite waiting time. For different lines investigated Hertel found an upper limit for the traffic load to be in the range of $0.5 < \rho < 0.7$.

The relative timetable sensibility (*Relative Fahrplanempfindlichkeit*) *EMPF* is also illustrated in figure 3.6. The EMPF is the changes to mean waiting time due to merging or overtaking for a given train model with dependency of the traffic load.

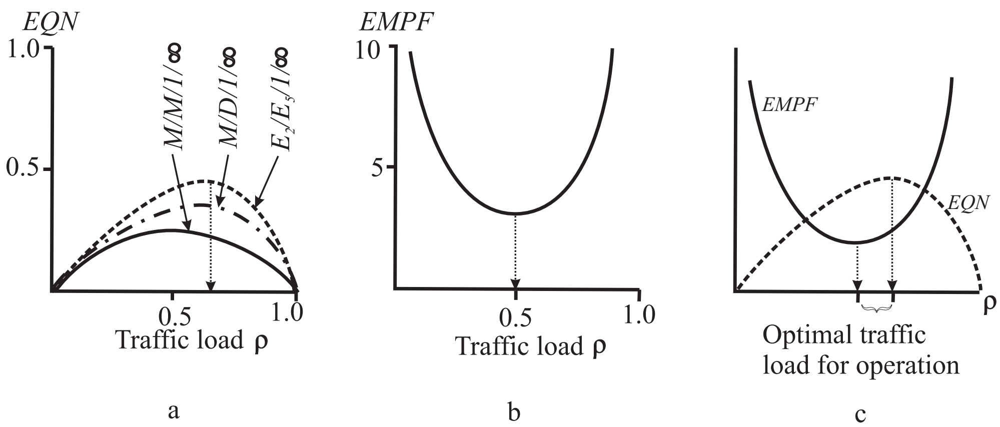

Figure 3.6: Graph a: mean traffic performance as a function of the traffic load. Graph b: the relative time table sensibility as a function of the traffic load. Graph c: The optimal traffic load for operation.

$$EMPF = \frac{\delta E[T_W]}{\delta\rho \cdot E[T_W]} \tag{3.6}$$

Figure 3.6b illustrates that there is an optimal traffic load that gives a stable timetable due to changes in waiting time. This is given as the minimum point of the graph. Hertel explains that beyond this traffic load very little timetable quality is gained. For an optimal train operation the traffic load is advised to be between the upper limit given by the $E[QN]$ and the lower limit given by the *EMPF*, as illustrated in figure 3.6c.

Hertel makes no clear distinction between scheduled and unscheduled waiting time in his model. In equation (3.5) and (3.6) only the scheduled waiting time is explicitly expressed as timetable quality. One possibility would be to include the unscheduled waiting time in the calculations. Which effect this change has on the model has not been investigated so far.

Schwanhäußer investigates the compatability of the fundamental diagram for road transportation with railway transportation [Schw90]. In figure 3.7 the most significant differences from road transportation are illustrated.

For railway traffic underlying a main- / distant signal system, a minimum distance must be held between the trains. By means of simulation the progress in flow versus density was investigated for a railway line. A maximum flow was not found, as illustrated in figure *a*. I figure *b* the linear dependency between the flow and velocity approximation from the road transportation given in equation (3.1) was investigated. Figure *b* illustrates that there is no linear dependency between the parameters. Herewith emphasizes Schwanhäußer that the railway traffic does not obey the same dependency between flow and density. Therefore it is impossible to establish the same sort of fundamental diagram for railway traffic underlying a main- / distant signal system. Schwanhäußer's statement is in accordance with Hertel's observation that

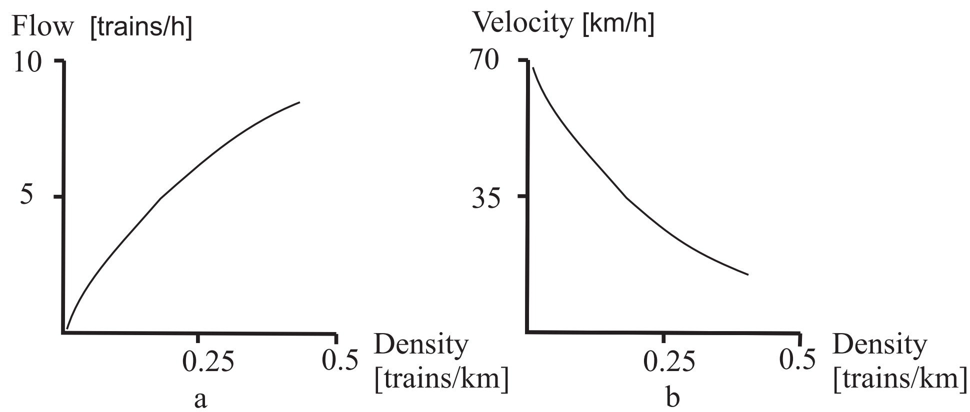

Figure 3.7: Fundamental diagram for railway transportation. Figure a: Flow versus density. Figure b: Velocity versus density.

railway traffic does not progress as road transportation. Schwanhäußer's statement also confirms that Hertel's approximation given in equation (3.2), where the linear dependency between flow and velocity is being modified, must be used with care.

3.3 Describing capacity by economical relations

The practical capacity, in the ideal case, might be set to the point where the profit is at its highest. Often compromises are made, and the traffic load is set higher than in an optimal situation. If the track is overloaded, the operator risks losing customers. In figure 3.8 [Schw99] costs and revenue is related to the number of trains an hour running on a line section. This relation was first time mentioned by Schwanhäußer in [Schw87]. The fixed costs are the investments in infrastructure. The variable costs are costs connected to the number of trains running on the line. These costs are maintainance of the infrastructure, trains and energy costs. In a market with high transport demand that is not saturated, the income increases linearly with the number of trains that are running on the line section. This under the assumption that all trains are filled equally with passengers. Is the number of trains low, there will probably be no queues. As the number of trains increases waiting time arises, and the variable costs increase progressively. The waiting time is a time dependent cost and consists of scheduled and unscheduled waiting time. The scheduled waiting time arises because the more train paths that are requested in a schedule, the more often a route has to be replaced. The unscheduled waiting time arises because the more trains running on the line, the higher the probability will be for delay. As the number of trains increases even more, the waiting time increases drastically until it reaches infinity. This is the theoretical maximum number of trains, which is not realistic. A certain amount of waiting time seems to be acceptable in the market, but if the waiting time becomes too high, the operator loses customers and income. There is an area of profit blended out in the figure. Where this area is at its widest, the profit is at its maximum. This point is referred to as the optimal number of trains. If the number of trains is lower, the railway line section is underloaded. If the number of trains is higher than optimal, the line section is overloaded [Schw99].

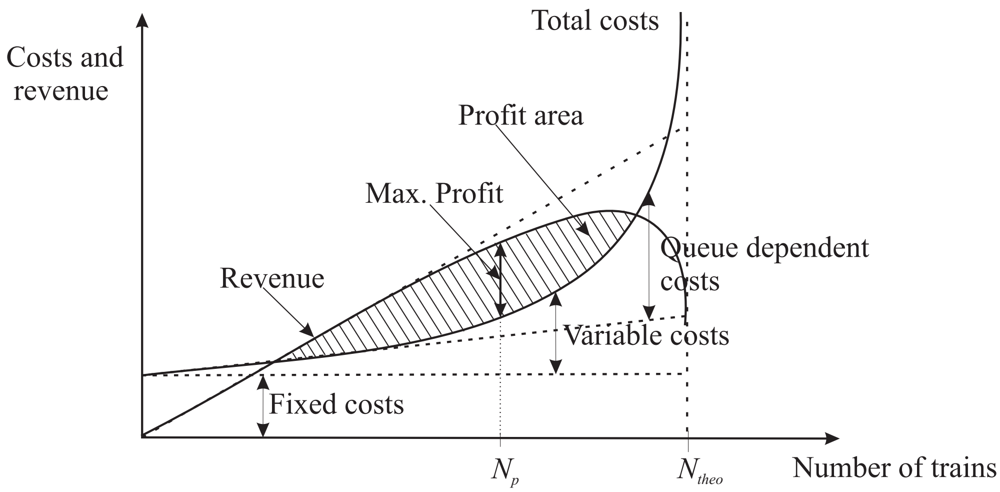

Figure 3.8: Costs and revenue related to the number of trains on a line section

Schwanhäußer defines the traffic flow in gross train mass an hour, m_{gt}, which is another expression of capacity, number of trains an hour. The product of the real journey velocity v_{real} and the traffic flow makes the *Transport force*:

$$F_{Tr} = \frac{m_{gt}}{T} \cdot v_{real} \tag{3.7}$$

The real journey velocity is based on the typical journey time, $t_{journey}$ and both scheduled t_{WF} and unscheduled t_{WB} waiting time and is given by:

$$v_{real} = \frac{1000}{60} \cdot \frac{l\ [m]}{(t_{journey} + t_{WF} + t_{WB})[s]} \tag{3.8}$$

Oetting [Oett05] goes further with Schwanhäußer's assessment and develops the model for the evaluation of railway line capacity. In her dissertation she relates physical quantities like length of the railway line, masses of the trains, typical journey time and waiting times to the profit. The profit is given as:

$$Profit = Revenue - Costs \tag{3.9}$$

The costs and revenue depending on the number of trains running on the line is illustrated in figure 3.8.

According to figure 3.8 the total costs for a line can be expressed as:

$$C_{total} = c_0 \cdot l_{line} + c_1 \cdot \sum_Z m_Z + c_2 \cdot \sum_Z (m_Z \cdot t_{real}) \tag{3.10}$$

where
m_Z = the mass of the train model
l_{line} = the length of the line
t_{real} = the real running time (equal the denominator in equation (3.8))

The first part of equation (3.10) illustrates the fixed costs depending of the length of the line. The second part is the sum of all trains running on the line which makes the linear costs. The third part is the variable costs which is time dependent.
In figure 3.9 Oetting illustrates the profit given in equation (3.9). The subtraction of the total costs from the revenue makes the profit, illustrated by the dotted function.

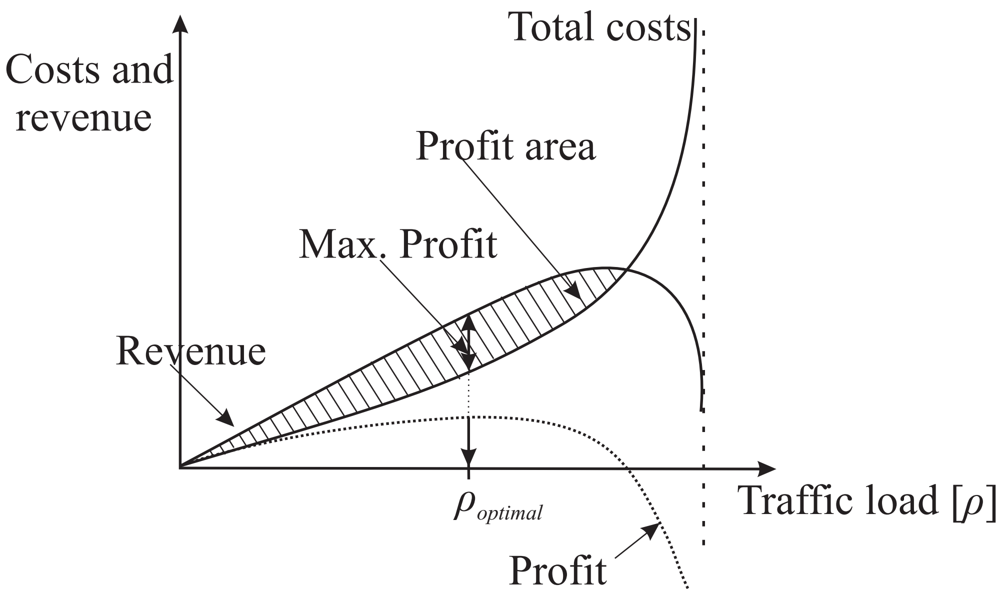

Figure 3.9: Profit of a railway line

The fixed costs (infrastructure) and the variable costs (number of trains and real running time) are modeled as a proportional ratio to the total costs given in equation (3.11):

$$C_{total} \sim \frac{m_0 \cdot l_{line}}{v_0} + \sum_{Z} (m_Z \cdot t_{real}) \tag{3.11}$$

Equation (3.11) does not directly express the linear costs (depending on the number of trains). Oetting concludes that the linear costs are small compared to the fixed and non linear costs and are therefore included into these two components. This assumption is a crucial point in her work.

The ratio $\frac{m_0}{v_0}$ in equation (3.11) is an equivalent for the fixed costs of the infrastructure given as:

$$\frac{m_0}{v_0} = \frac{C_{Fix}}{l_{line}} \cdot \frac{\sum_Z (m_Z \cdot t_{real})}{C_{Var}} \tag{3.12}$$

$$\frac{C_{Fix}}{C_{Var}} = \frac{\frac{m_0 \cdot l_{line}}{v_0}}{\sum_Z (m_Z \cdot t_{real})} \tag{3.13}$$

The real running time t_{real} of a train model on the a line is given in equation (3.14):

$$t_{real} = t_{journey} + t_{WF} + t_{WB} \tag{3.14}$$

The profit in equation (3.9) for one train model and for one specified traffic load can be described as:

$$P_Z = R_Z - C_Z \tag{3.15}$$

This can be split up into:

$$P_Z = c_{r,Z} \cdot r_Z - c_{k,Z} \cdot c_Z \tag{3.16}$$

where
$c_{r,Z}$ = proportionality constant for revenue of one train model given in $[\frac{euro}{t \cdot km}]$.
$c_{k,Z}$ = proportionality constant for costs of one train model given in $[\frac{euro}{t \cdot min.}]$.
r_Z = revenue equivalent of one train model given in $[t \cdot km]$.
c_Z = cost equivalent of one train model given in $[t \cdot min.]$.

The revenue R_Z of one train model is given by:

$$R_Z = c_{r,Z} \cdot m_Z \cdot l_{line} \cdot f_N \tag{3.17}$$

where f_N is a unit less function given in equation (3.18):

$$f_N = \frac{t_{journey} \cdot ZB_{journey}}{(t_{journey} + t_{WF}) \cdot ZB_{journey} + t_{WB} \cdot ZB_{WB}} \tag{3.18}$$

The time evaluation factor $ZB_{journey}$ for the typical journey time and the time evaluation factor ZB_{WB} for the delay during operation are subjective quantities.

Multiplying the first equation (3.16) with $\frac{c_{r,PNV}}{c_{r,PNV}}$ where $c_{r,PNV}$ is the proportionality constant for passenger trains and then multiplying only the second part with $\frac{c_{r,Z}}{c_{r,Z}}$ gives:

$$P_Z = c_{r,PNV} \cdot \left(\frac{c_{r,Z}}{c_{r,PNV}} \cdot r_Z - \frac{c_{k,Z}}{c_{r,PNV}} \cdot \frac{c_{r,Z}}{c_{r,Z}} \cdot c_Z \right) \tag{3.19}$$

The following ratio is introduced for the the revenue and costs for one train model:

$$c_{R,Z} = \frac{c_{r,Z}}{c_{r,PNV}} \text{ and } c_{K,Z} = \frac{c_{k,Z}}{c_{r,Z}}$$

Inserting $c_{R,Z}$ and $c_{K,Z}$ into equation (3.19) and rearranging it gives the profit for one train model:

$$P_Z = c_{r,PNV} [c_{R,Z} \cdot r_Z - c_{R,Z} \cdot c_{K,Z} \cdot c_Z] \tag{3.20}$$

The sum over all train models for the part of the product within the brackets in equation (3.20) is called the *transport momentum difference* Δp:

$$\Delta p = \sum_Z [c_{R,Z} \cdot r_Z - c_{R,Z} \cdot c_{K,Z} \cdot c_Z] \tag{3.21}$$

The transport momentum illustrates the maximum profit equivalent in figure 3.9. With this Oetting has shown how mass and real running time is related to the profit of a railway line without having exact information about the real costs and revenue of the operator. This relation is of interest when dimensioning infrastructure. The drawback of this model are the abstract parameters defined which makes the model less handable. The final question is whether the globalization of the parameters holds for any railway line to be investigated.

3.4 Models describing scheduled waiting time

All of the models mentioned in this section are suited for short- and long term planning of infrastructure. These models have the strength in being time table independent, compared to other timetable dependent models. This means that the future timetable is not known, which is an effort for long term planning. Models that need a known time table should be preferred in optimization of todays schedule and to solve smaller infrastructural problems. The time table independent models usually also work for known time tables. For future planning of infrastructure the quantity of waiting time generated is a useful measure for dimensioning infrastructure. There are not so many methods that clearly distinguish between the calculation of scheduled and unscheduled waiting time. The following two sections present some models that concentrate on scheduled waiting time.

3.4.1 Analytical models for scheduled waiting time calculation

Potthoff makes an estimation of the number of crossings for the entire line [Pott62]. The number of crossings builds the basis for the calculation of scheduled waiting time from crossing. At first Potthoff calculates the theoretical distance between two crossings.

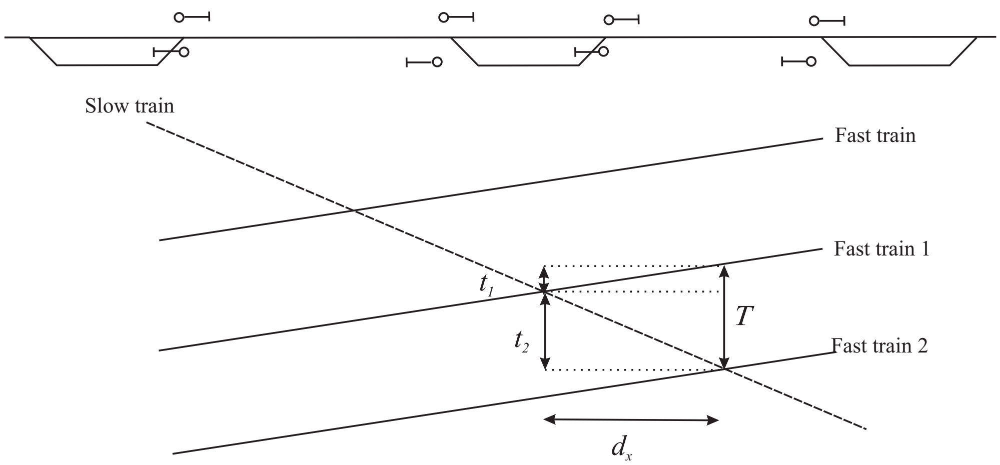

Figure 3.10: Theoretical distance between two crossings

Figure 3.10 illustrates the distance d_x with it's respective time T between two crossings. The time can be described by:

$$T = t_1 + t_2$$

Inserting the mean speed from train 1 (first crossing) and train 2 (second crossing) on the actual section with $t_1 = d_x/v_1$ and $t_1 = d_x/v_2$ gives:

$$T = \frac{d_x}{v_1} + \frac{d_x}{v_2} \tag{3.22}$$

Solving for d_x in equation (3.22) gives the theoretical distance between two crossings according to Potthoff:

$$d_x = T \cdot \left(\frac{v_1 \cdot v_2}{v_1 + v_2} \right) \tag{3.23}$$

The time T consists of the minimum spacing time and buffer time.

To obtain the mean distance between two crossings in equation (3.23) the mean values of the trains velocity and their buffer time for the entire line must be inserted. This is necessary for the calculation of the total number of crossings for the entire line. The mean distance between two crossings can be obtained by either taking the arithmetical mean for every section or by using global values for the entire line. According to Potthoff the number of crossings for the entire line is achieved by dividing the length of the entire line by the mean distance between two crossings and then multiplying it with the number of trains in one direction. Then the number of crossings can be expressed as:

$$n_x = n_2 \cdot \frac{l_{line}}{\overline{d_x}} \tag{3.24}$$

This function depends on the number of trains n_2 in one direction and the density of opposing trains from equation (3.23). The density is given by the sum of the minimum spacing time and buffer time.
The distance a train of rank(2) can run is expressed analogous to equation (3.23)

$$\overline{d_{x,2}} = \frac{\overline{t_{b,11}} + \overline{t_{s,11}}}{1/v_1 + 1/v_2} \tag{3.25}$$

Where v_1 and v_2 is the mean velocity for the entire line for train of rank(1) and rank(2) respectively. The mean minimum spacing time is applied when calculating the number of crossings on the entire line. Inserting $\overline{d_{x,2}}$ from equation (3.25) into equation (3.24) gives:

$$n_x = \frac{l_{line} \cdot n_2 \cdot (1/v_1 + 1/v_2)}{\overline{t_{b,11}} + \overline{t_{s,11}}} \tag{3.26}$$

Where the number of trains of rank(2) is given by $n_2 = T/(\overline{t_{b,22}} + \overline{t_{s,22}})$.

When $T_{journey1,line}$ and $T_{journey2,line}$ is the typical journey time for the entire line respectively for train of rank(1) and rank(2), then the number of crossings can be expressed as:

$$n_x = \frac{n_2 \cdot (T_{journey1,line} + T_{journey2,line})}{(\overline{t_{b,11}} + \overline{t_{s,11}})} \tag{3.27}$$

Inserting the number of trains of rank(1) $n_1 = T/(\overline{t_{b,11}} + \overline{t_{s,11}})$ equation (3.27) becomes:

$$n_x = \frac{n_1 \cdot n_2 \cdot (T_{journey1,line} + T_{journey2,line})}{T} \tag{3.28}$$

If the typical journey time for train of rank(1) and rank(2) is equal and the number of trains in each direction is equal, then equation (3.27) can be expressed as:

$$n_x = \frac{n^2 \cdot 2 \cdot T_{journey,\ line}}{T} \tag{3.29}$$

The number of crossings forms a function depending of n^2. The number of crossings can reach infinity according to Potthoff's estimation. In practice the number of crossings will never reach infinity since infinite queues will arise before this stage. Therefore Potthoff's estimation only yields for small numbers of trains in one direction.

Schwanhäußer presents in [Schw81] an analytical method for the calculation of scheduled waiting time from overtaking. On a double track line with mixed unidirectional traffic freight trains with lower speed and lower priority than passenger trains incur waiting time from overtaking. This method is based on an exponentially distributed buffer time between the faster passenger trains. Figure 3.11 illustrates the buffer time between two faster trains that is exactly necessary for a slow train to merge in between them. The buffer time in this case is equally the minimum spacing time for a fast train to follow a slow train ($t_{s,SF}$).

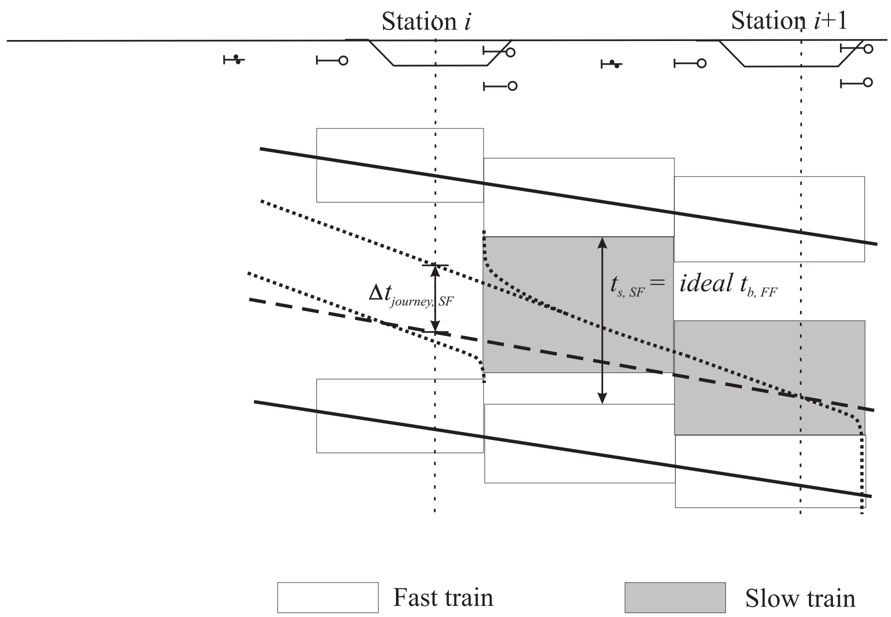

Figure 3.11: Ideal buffer time for overtaking

These sets of buffer times that makes a slow train reaching the next, second, third and so on station are called the ideal buffer times given by:

$$ideal\ t_{b,FF} = \overline{t_{s,SF}} + m \cdot \overline{\Delta t_{journey}}$$

where
$m = 0, 1, 2, 3, ...$
$\overline{\Delta t_{journey}}$ = the mean typical journey time difference between the slow and fast train ($t_{journey,SF}$) given by:

$$\overline{\Delta t_{journey}} = \frac{\bar{l}}{\overline{V_S}} - \frac{\bar{l}}{\overline{V_F}} \tag{3.30}$$

Schwanhäußer differs two cases from each other. First the case when the buffer time between the fast train is smaller than the minimum spacing time $t_{s,SF}$, is the condition for the calculation of the mean waiting time for merging. In the second case when the buffer time between the fast trains is larger than the minimum spacing time, this gives the condition for the calculation of the mean waiting time for when overtaking occurs. The entire waiting time for the entire line is then given by the product of the mean waiting time and the number of cases where an overtaking occurs. The number of overtakings on the entire line according to Schwanhäußer is:

$$n_o = n_S \cdot \left(\frac{l_{line-max\ l}}{\overline{l}} \right) \cdot \left(1 - e^{-\frac{\overline{\Delta t_{journey}}}{\overline{t_{b,FF}}}} \right) \tag{3.31}$$

where
n_S = the number of slow trains
l_{line} = the length of the entire line
$max\ l$ = the longest section of the line
$\overline{l}$ = the mean section length

The product in equation (3.31) consists of three factors:
The first factor is the number of slow trains to be included into the timetable.
The middle factor is the number of stations on the line.
The third factor is the probability for overtaking in one station.

Gast uses Schwanhäußer's assessment in his thesis [Gast86]. He substitutes the mean typical journey time difference by the *Bereichszugfolgezeit*, Δb, which is the time gap needed for a slower train to reach the next station. This is a better parameter than the mean typical journey time difference since the signaling system is taken into account. The number of overtakings according to Gast becomes:

$$n_o = \sum_{station} \frac{(n_S \cdot n_F)}{N} \cdot \left(1 - e^{-\frac{\overline{\Delta b}}{t_{b,FF}}} \right) \tag{3.32}$$

where
N = the total number of trains running on the line
n_F = the number of fast trains

The first part of the product in equation (3.32) is the probability for the train sequence slow train followed by a fast train.
The last part is the probability for overtaking in one station.
This is summated for every station on the line.

With this formula, Gast makes an attempt to determine the number of overtakings for each station, which indicates whether there are enough sidings available at a particular station on the line investigated.

The number of overtakings on a line according to Gast given in equation (3.32) is compared to the number of overtakings on a line according to Schwanhäußer given in equation (3.31). The factors building the number of stations on the line and the probability for overtaking in one station are approximately the same. This results in that:

$$n_S \neq \frac{(n_S \cdot n_F)}{N} \tag{3.33}$$

Gast's definition of the probability for the train sequence slow train followed by a fast train is not in accordance with the definition of the probability of train sequences for timetable independent calculations given in equation (3.34):

$$p_{ij} = p_i \cdot p_j = \frac{n_i}{N} \cdot \frac{n_j}{N} \tag{3.34}$$

where
p_{ij} = the probability for a train sequence train i followed by train j
n_i and n_j the number of train i and j respectively
N = the total number of trains

The expected number of train sequences can be described by:

$$n_{ij} = p_i \cdot p_j = \frac{n_i}{N} \cdot \frac{n_j}{N} \cdot N_{train\ sequences} \tag{3.35}$$

where
$N_{train\ sequences}$ = the number of all train sequences

Given that i is the slow train and j the fast train. If the number of train sequence is the same as the entire number of trains running the line, equation (3.35) becomes equal to the first part of the product in Gast's equation (3.32). The second part in the product is the probability for *a slow train* to incur an overtaking by a fast train. The probability for a slow train to be overtaken in one station is to be multiplied with *the number of slow trains* and not *the number of train sequences* to achieve the number of overtakings. This might be the point where Gast's assessment for the calculation of the number of overtakings on a line fails.

3.4.2 Queueing theory for scheduled waiting time calculation

In 1978 Schwanhäußer presented a method for dimensioning railway junctions [Schw78]. This method calculates the expected scheduled waiting time. He decomposes the junction into single channels, thus IRJ, as described in 2.1.1. He makes use of the Pollaczek/Chintschin's formula [Ross00] for waiting time for a $M/GI/1/$-queue:

$$E[T_W] = \frac{\lambda E[T_S^2]}{2(1 - \lambda E[T_S])} \tag{3.36}$$

where T_W is the waiting time in queue and T_S is the service time. Schwanhäußer modifies this formula to:

$$E[T_W] = \frac{\varphi \lambda E[T_S^2]}{2(1 - \varphi \lambda E[T_S])} \tag{3.37}$$

by introducing the chaining factor φ defined by Potthoff [Pott62].

$$\varphi = \frac{N_\varphi}{N_{all}} \tag{3.38}$$

where
φ = the probability of train sequences that rise conflicts
N_{φ} = number of chained trips
N_{all} = complete number of trips

The inverse of the chaining factor, $(1/\varphi)$, is the number of channels that give service at the same time in a junction.

Wakob goes further with Schwanhäußer's work [Schw78]. In his thesis, the scheduled waiting time has been calculated for an IRJ, using a $GI/GI/1/\infty$ -queueing model [Wako85]. This makes sense under the assumption that the different operators have ordered their routes independent of other operators. The arrival stream of trains and their service time then becomes stochastic and independent. This queueing model has no analytical exact solution. He uses four approximations to approach the solution for the $GI/GI/1/\infty$ system. He ends up with a *Erlang* distribution which is independent and gamma-distributed. He makes 5 approaches for estimating the parameters k and l in $E_k/E_l/1/\infty$-queue. Page has calculated the exact values for k and l, which Wakob uses [Page72]. Wakob also compares his result with the approximation formula of Gudehus for the same system [Gude76]. For a variation coefficient of the arrival stream V_A, equal 1, Wakob's estimation for the expected waiting time becomes equal to Gudehus approximation. Wakob's formula works best in the range where the traffic load ρ is between 0.3-0.7, where most train traffic usually is. Wakob's approximation is an improvement compared to Gudehus' approximation, which is only suitable when ρ is approaching 1, called a *Heavy-Traffic-Approximation.* Wakob's approximation makes it possible to identify the bottleneck in a rail junction or node.

Schwanhäußer's and Wakob's models calculate the waiting time that arises by solving the conflicts between two trains (train-couples) at a time on the basis of the minimum spacing time. Schwanhäußer's and Wakob's models hold for systems with low traffic load where $\rho << n$ and is referred to as *Light-Traffic-Model.* With high traffic load these models are no longer accurate. Wendler [Wend99] makes an attempt to improve these models by expanding to train-triples to solve some problems more exactly. For example, in branch-off points sometimes the inequality of the minimum spacing times of a train-triple is not fulfilled. In such a case the classic queuing models do not operate correctly. Wendler develops in his dissertation a *Light-Heavy-Traffic-Model* for the inclusion of heavy traffic load into the previous models. He develops solutions for the utilization of time gaps between high priority trains and for alternative routing of trains for the reduction of scheduled waiting time. Figure 3.12 illustrates a problem that can occur during a time table construction process. Here the classical queuing models using FIFO discipline do not hold[2].

In the branch-off point train k is supposed to merge in to the line. Considering the first block train i arrives before train j. In the second block train k lies after train j. This builds the train sequence $i - j - k$. In the second block train k is before train i,

[2]The blocking time stairs in figure 3.12 are moved a little bit to the right and left for better visualizing.

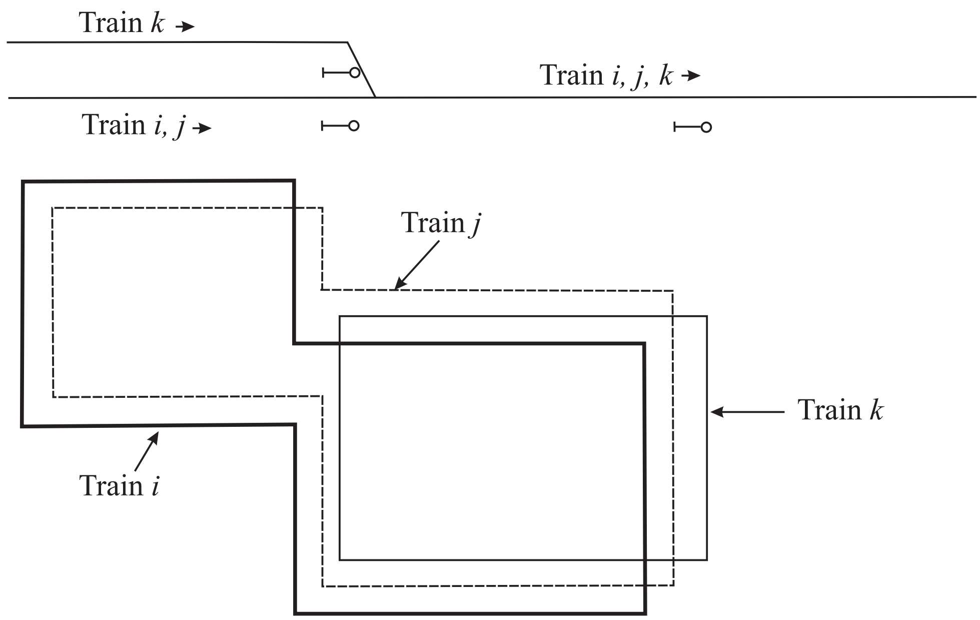

Figure 3.12: Occupational conflicts between train i and j and the merging train k

which would switch the train order to $j - k - i$.

Wendler has qualitatively improved the train-couple models by going into detail with a train-triple model. Especially important is that the potential reduction in waiting time by alternative routing has been solved. Wendler's train-triple model is very complex compared to the train-couple models where there are only three cases to be distinguished (equal rank, unequal rank with priority given to train 1 or train 2 respectively). In a train-triple model there are 35 cases to be handled. A model that would even go further into detail compared to the train-triple containing four trains would be too complex at the moment.

Potthoff developed a model for the calculation of the number of tracks within a node [Pott62]. He makes use of a $GI/D/n/0$ queue. The arrival process is Gamma distributed and the service time is constant. The queueing system estimates the number of tracks n necessary in the node. In this system there is no waiting space available, which is a system with loss. Since there are no waiting space available the probability for waiting is set equal to the probability for loss. This assumption holds for systems with low traffic load where $\rho << n$, a *Light-Traffic-Model*.

Hertel [Hert85] develops a model for the calculations of track groups which have fewer bounds compared to Potthoff's model. He uses a $GI/GI/n/\infty$ queue. Both arrival stream and service time are modeled as *general independent* processes. The waiting space is infinite. This means that if there are not enough tracks available a train will wait somewhere else in the network. This method is suited for high traffic load where $\rho \rightarrow n$ a *Heavy-Traffic-Model*.

Both models from Potthoff and Hertel are suitable for the dimensioning of track groups, since they also can be used for the calculation of unscheduled waiting time. The *general independent* arrival stream of trains is also valid in shunting yards where trains arrive independently of each other. For the arrival stream of passenger trains in a conflict free timetable, the arrival stream is no longer independent, and the *GI* is therefore not valid. The models should therefore not be used for regular interval timetables.

3.4.3 Simulation for scheduled waiting time calculation

The estimation of the scheduled waiting time by simulation follows an asynchronous time table construction process. Most known asynchronous simulation methods has been NSIM (Network simulation) and BABSI (Simulation of timetable construction and railway operation) developed at the RWTH Aachen. Asynchronous simulation is based on a hierarchical procedure visualized in a blocking time stair time-way graph. At first, a timetable has to be constructed. Trains are grouped into train models with a ranking or priority number. Trains of highest priority are included first into the time table. Trains of lower priority are included afterwards. For every train that is included, conflicts due to block occupation might arise. These conflicts are solved successively step by step within the line sections formed between the stations. An algorithm searches for the best solution by first investigating the possibility for using the siding, second the possibility for introducing a halt. If this does not solve the conflict, the third possibility is to move the entire train path to the next possible time gap between two trains. The conflict solution might give rise to a new conflict that then has to be solved the same way. This timetable construction process needs a predefined train hierarchy [Grög02, Grög04].

3.5 Models describing unscheduled waiting time

3.5.1 Analytical models for unscheduled waiting time calculation

In 1974 Schwanhäußer proposed an analytical model to calculate the secondary delay on a railway line [Schw74]. This model became a break-through since now it was possible to relate the line capacity to the quality parameter of secondary delay. His model is based on a schedule without conflicts. This schedule does not have to be known and can be considered as independent. He calculates the secondary delay between two trains of equal rank and of different rank. This model is meant as a short- and long term planning tool.

If the trains have different rank, thus train 1 has higher rank than train 2, train 2 will be secondary delayed if:

$$t_b + t_{PD,2} < t_{PD,1} < t_b + t_{PD,2} + t_{s,12} + t_{s,21} \tag{3.39}$$

where

$$\begin{aligned} t_b &= \text{the buffer time} \\ t_{s,12} &= \text{the minimum spacing time train 1 followed by train 2} \\ t_{s,21} &= \text{the minimum spacing time train 2 followed by train 1} \\ t_{PD,1} \text{ and } t_{PD,2} &= \text{primary delay of train 1 respectively train 2} \end{aligned}$$

The secondary delay illustrated in figure 3.13 becomes:

$$t_{SD,2} = t_{PD,1} - t_b - t_{PD,2} \tag{3.40}$$

In the opposite situation, train 1 has lower rank than train 2. Train 1 becomes secondary delayed if:

$$t_b + t_{PD,2} < t_{PD,1} < t_b + t_{PD,2} + t_{s,12} + t_{s,21} \tag{3.41}$$

The secondary delay of train 1 illustrated in figure 3.14 becomes:

$$t_{SD,1} = t_{s,12} + t_{s,21} + t_b + t_{PD,2} - t_{PD,1} \tag{3.42}$$

If the trains are equal in rank the successor, train 2, receives a secondary delay if the primary delay of train 1 lies within the following interval:

$$t_b + t_{PD,2} < t_{PD,1} < t_b + t_{PD,2} + t_{s,12} \tag{3.43}$$

The equation of secondary delay for train 2 becomes the same as in equation (3.40), where the rank is different.

If the primary delay of train 1 is bigger than:

$$t_{PD,1} > t_b + t_{PD,2} + t_{s,12} \tag{3.44}$$

then the trains will have to switch order. Train 1 receives a secondary delay if:

$$t_b + t_{s,12} + t_{PD,2} < t_{PD,1} < t_{s,12} + t_{s,21} + t_b + t_{PD,2} \tag{3.45}$$

In this situation the equation of secondary delay for train 1 becomes the same as in equation (3.42).

The overall secondary delay is calculated by the summation of the secondary delay of all four cases and multiplied with the frequency of primary delay. As described in Chapter 2.3.2, Schwanhäußer found the primary delay to be a combination of a one point Dirac distribution and a negative exponential distribution. This yields for traffic without long queues. If the primary delay is high, all successive trains will be secondary delayed. The operation is chaotic and trains arrive independently. To achieve the highest capacity in a queued situation, all trains should travel at the same speed. That means that the service time is constant. This description fits to a $M/D/1$ queue. The overall secondary delay with respect of both operating situations (little queue and long queue) becomes:

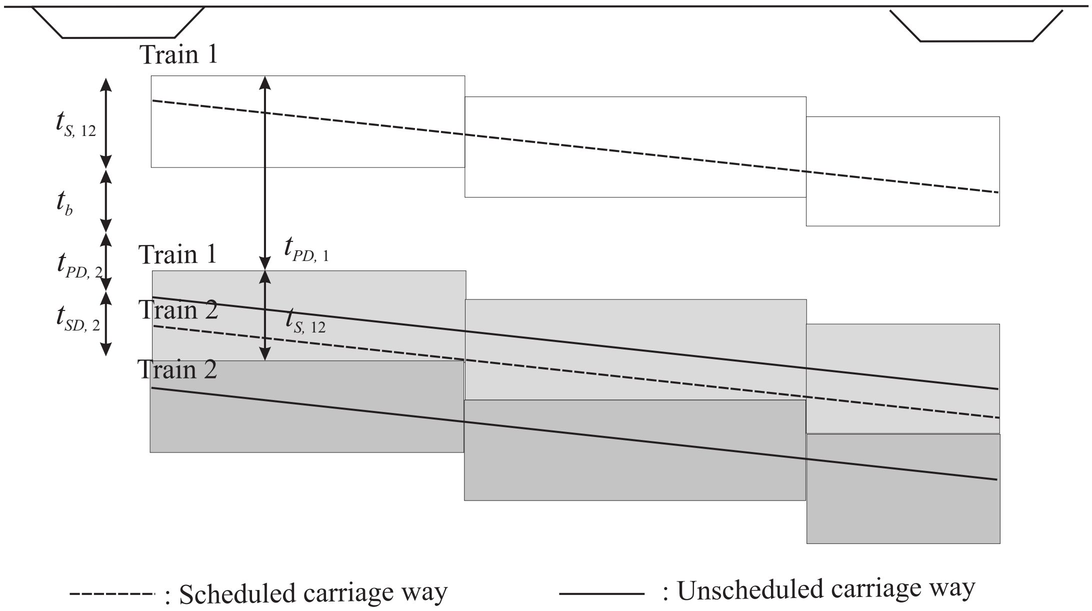

Figure 3.13: Secondary delay for train 2 if train 2 has equal or lower rank than train 1

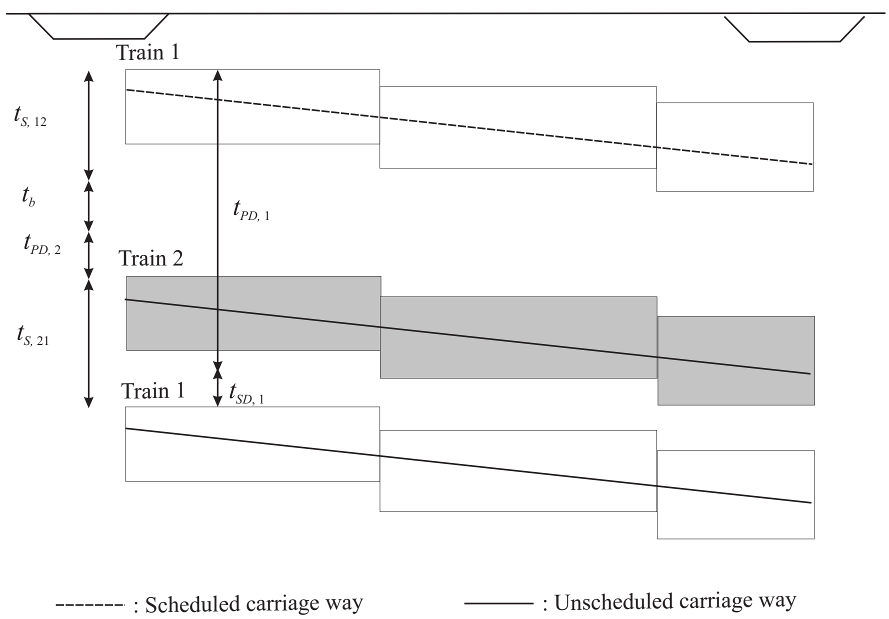

Figure 3.14: Secondary delay for train 1 if train 1 has equal or lower rank than train 2

$$E[T_{SD}] = \overline{t_{PD}}\,(1-\frac{p_{PD}}{2})\left[\frac{p_e(1-e^{-m\overline{t_{s,e}}})^2 + (1-p_e)m\overline{t_{s,p}}\,(1-e^{-2m\overline{t_{s,p}}}) + \frac{\overline{t_s}}{\overline{t_b}}(1-e^{-m\overline{t_s}})^2}{m\overline{t_b} + 1 - e^{m\overline{t_s}}}\right] \tag{3.46}$$

where
p_e = probability of a train sequence of equal rank
m = $\frac{1}{\overline{t_{PD}}}$; the inverse of the mean primary delay
$\overline{t_{s,e}}$ = the mean minimum spacing time between trains of equal rank
$\overline{t_{s,p}}$ = the mean minimum spacing time between trains of non equal rank
Schwanhäußer's formula is also referred to as the STRELE formula since it is implemented in the tool STRELE (*Strecken Leistungsfähigkeit*), which is in use of the DB AG [DS405/12].

The second part in Gast's thesis analyses the unscheduled waiting time from overtaking on a railway line. He distinguishes between three cases where waiting time arises due to occupation of the same block. The faster trains have priority over the slower trains in operation. The first case is illustrated in figure 3.15. The slower train has an obligation to halt in station (i) and the faster train was then scheduled to overtake the slow train. The faster train has a primary delay $t_{PD,F}$, large enough for the slower train to reach station (i+1). In the case that the slower train has no scheduled halt in station (i+1) it will incur waiting time from overtaking.

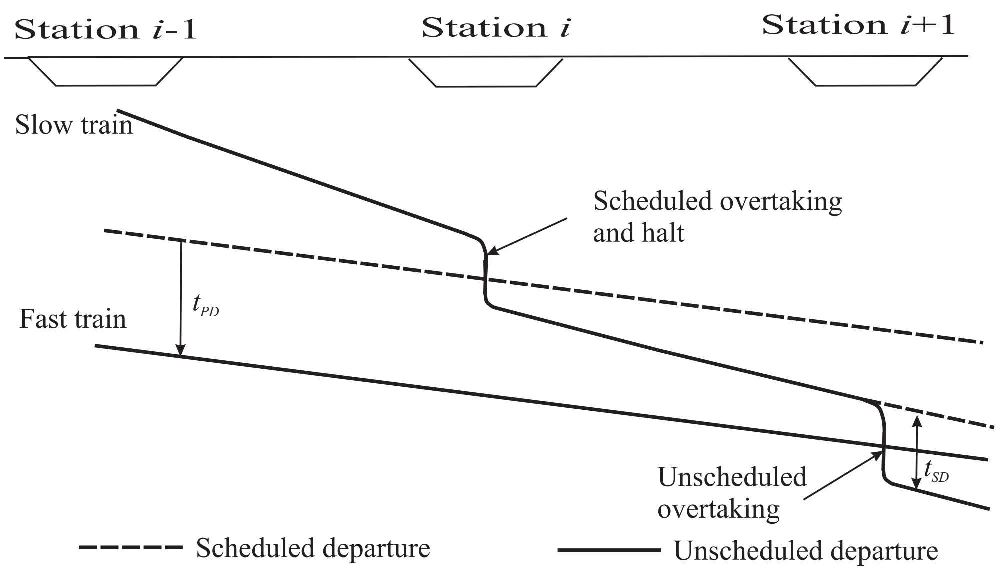

Figure 3.15: Case 1: The slower train incurs waiting time from overtaking in station (i+1)

The number of overtakings in station (i+1) becomes according to Gast in case 1:

$$n_o(i+1) = n_o(i) \cdot p_F(PD) \cdot (1 - p_S(PD)) \cdot \left(e^{-\frac{b(0,+)}{t_{PD,F}}} - e^{-\frac{b(0,+2)}{t_{PD,F}}} \right) \tag{3.47}$$

where
$n_o(i)$ = the number of of scheduled overtakings in station (i) given in equation (3.32)
$p_F(PD)$ = the probability for primary delay of the faster train
$p_S(PD)$ = the probability for primary delay of the slower train
$b(0,+)$ = the time gap from station (i) to station $(i{+}1)$
$b(0,+2)$ = the time gap from station (i) to station $(i{+}2)$

In case 2 both trains arrive the area of survey with a primary delay. Figure 3.16 illustrates that the slower train still has an obligatory halt in station (i). Since the primary delay of the slower train is less than the scheduled waiting time from overtaking, the slower train can leave on time station (i). If the slower train has no scheduled halt in station $(i{+}1)$, it will incur waiting time from overtaking in this station.

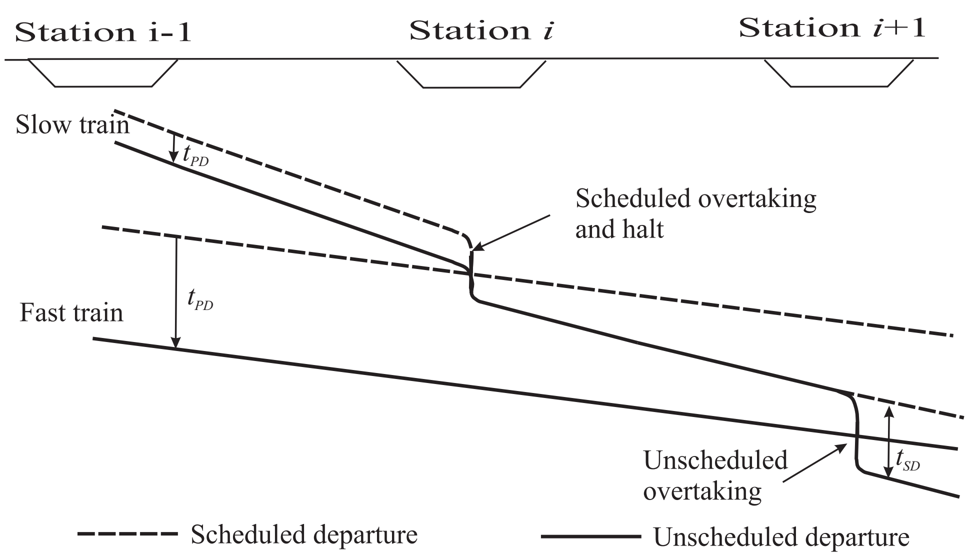

Figure 3.16: Case 2: Both trains are delayed. The slower train incurs waiting time from overtaking in station $(i{+}1)$.

The number of overtakings in station $(i{+}1)$ becomes according to Gast in case 2:

$$n_o(i+1) = n_o(i) \cdot p_F(PD) \cdot p_S(PD) \cdot (1 - e^{-\frac{t_{wo}(i)}{t_{PD,S}}}) \cdot \left(e^{-\frac{b(0,+)}{t_{PD,F}}} - e^{-\frac{b(0,+2)}{t_{PD,F}}} \right) \tag{3.48}$$

where
$t_{wo}(i)$ = the scheduled waiting time from one overtaking in station (i)

Equation (3.48) is different from equation (3.47) with respect to the probability of the primary delay of the slower train and the probability for this delay to be less than the scheduled waiting time from overtaking.

Figure 3.17 illustrates the 3rd case where both trains arrive the area of survey with a primary delay. This time the slow train has no halt in station (i). At this point the primary delay is reduced to $red\ t_{PD,S}$. The faster train is so much delayed that the scheduled overtaking is moved from station (i) to station $(i+1)$. In the case that the slower train exactly manages to reach station $(i+1)$, no extra waiting time (secondary delay) will be generated. The slower train will leave the area of survey with a net delay $t_{ND,S}$. If the slower train reaches station $(i+1)$ before the faster train, it will incur waiting time from overtaking.

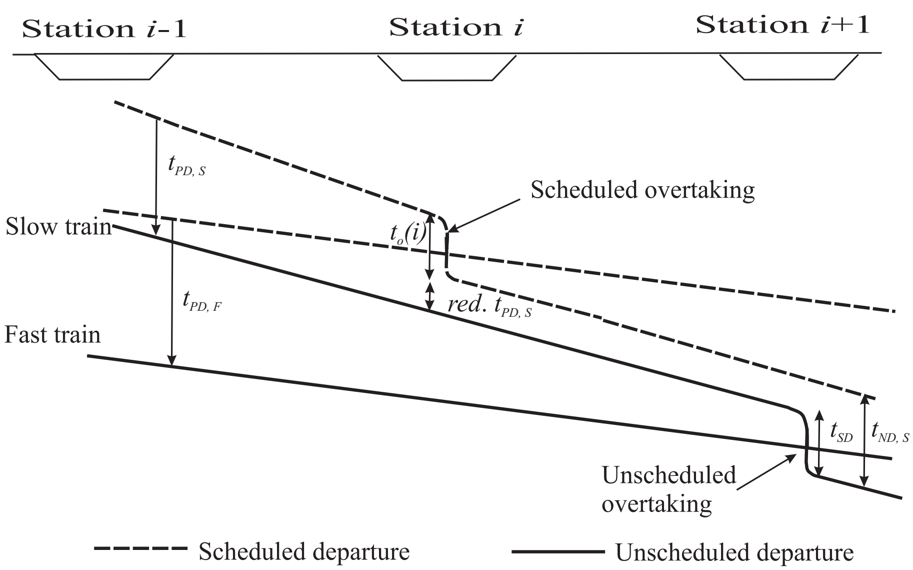

Figure 3.17: Case 3: Both trains are delayed. The scheduled overtaking in station (i) is moved to station $(i+1)$.

The number of overtakings in station $(i+1)$ becomes according to Gast in case 3:

$$
\begin{aligned}
n_o(i+1) &= n_o(i) \cdot p_F(PD) \cdot p_S(PD) \cdot e^{-\frac{t_{wo}(i)}{\overline{t_{PD,S}}}} \\
&\quad \cdot \frac{\overline{t_{PD,F}}}{\overline{t_{PD,F}} + red\ \overline{t_{PD,S}}} \cdot \left(e^{-\frac{b(0,+)}{\overline{t_{PD,F}}}} - e^{-\frac{b(0,+2)}{\overline{t_{PD,F}}}} \right)
\end{aligned} \tag{3.49}
$$

The last product in equation (3.49) is the probability for the difference in primary delay $t_{PD,F} - red\ t_{PD,S}$ to be within the interval $[\Delta b = b(0,+), b(0,+2)]$.

In equation (3.47)-(3.49) the scheduled number of overtakings from equation (3.32) is inserted. In the previous section it was pointed out that the probability of the train sequence in this equation was not in accordance with the theoretical definition. Gast's theory that unscheduled waiting time is generated from overtaking probably holds for passenger trains with scheduled halts. If the slow trains are freight trains without a schedule, then no waiting time from overtaking will be generated. A delaying passenger train will only move the overtaking from one station to the next station.

Higgins *et al.* [Higg98] estimate delay (unscheduled waiting time) analytically aimed for optimal timetable constructions. The authors separate source delay, knock-on delay (secondary delay) and delay due to late connections. In figure 3.18 an example is given for knock-on delay.

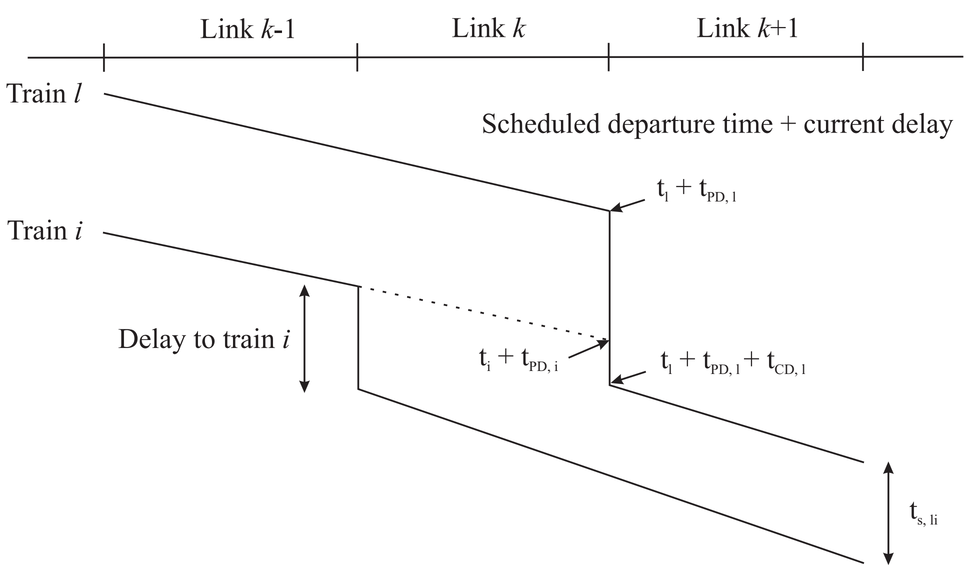

Figure 3.18: Knock-on delay to train *i*

The time distance graph in figure 3.18 illustrates the knock-on delay train i incurs from train l. The track is divided into links, for example block segments. Train l has scheduled departure time t_l with current primary delay $t_{PD,l}$ in link k. Train l incurs a source delay of duration $t_{CD,l}$. The successive train i also has a departure time t_i with current primary delay $t_{PD,i}$ in link k. The knock-on delay train i due to train l in link k becomes then:

$$t_l + t_{PD,l} + t_{CD,l} + t_{s,li} - t_i - t_{PD,i}$$

where $t_{s,li}$ is the minimum spacing time train l followed by train i in link k. The system forms a set of equations that is found using an iterative refinement algorithm.

The model was verified using stochastic simulation technique. Necessary input data was taken from earlier studies and historical data from Queensland Rail (Australia). The model slightly underestimated the simulated results. This model's inaccuracy may be due to the distribution of primary delay ($Erlang_3$), which is sensitive to long term knock-on delays such as those that occur in peak periods, being slightly different from the regular periods for some trains.

Higgins *et al.* [Higg97] developed a model to determine the number and positioning of sidings on single track lines for a given cyclic schedule. The sidings are positioned to minimize both the risk of delays and the delays caused by train conflicts. The risk of delay represents the likely amount of delay caused by three types of unexpected events, namely: those related to track problems; those caused by terminal/station problems; and those which are the result of rolling stock malfunctions. In order to estimate the likely risk of delay to each train caused by each delay type, it is necessary to have as an input the corresponding distributions of source delays. The model is subjected to various constraints to ensure safe operation, enforce speed restrictions and permit stops.

The siding location model is decomposed into two sub-models. One is solved for optimal track segment lengths and arrival and departure times given a fixed schedule. The other sub-model is solved for the optimal train schedule given the track segment lengths that are fixed. The process will iterate between the two sub-models until there is no more improvement.

The siding location model was tested on a real single track line corridor. The new computations showed a reduction in conflict and risk of delay compared to the original infrastructure and schedule. Next the model was tested out for designing the sidings on a new rail corridor. With a prefixed level of service reliability, the number of sidings and their position were defined for known departure times. Finally the results serves as an input to a full cost-benefit analysis of new track infrastructure.

The authors point out that there are many considerations to be made for a successful siding location that might be difficult to fit into the model. The model assumes that the costs of locating a siding is independent of its position on the corridor. In practice some parts of the track are less accessible than others, making the costs of building higher (for example boggy ground). Safety reasons also play an important role, l. e. it is not desirable to place a siding in a slope. Therefore the model should be used together with other considerations for the problem of siding location. An alternative is to use the model developed to estimate the minimum number of sidings needed for efficient and punctual train operation.

3.5.2 Queueing theory for unscheduled waiting time calculation

Huisman *et al.* [Huis02] use a $M/M/k/\infty$ - queueing model to calculate the unscheduled waiting time in an entire railway network of double tracks. The components of the network are modeled in a global way, where it is possible for more detailed analysis of components in isolation. The model is meant as a tool for first stages of design, to identify bottlenecks in the network, to compare alternative designs in a global way, or to analyze several traffic scenarios. The network is divided into stations, junctions and sections.

Stations: The stations are modeled as a first-come-first-served multiserever queueing system, in which each server represents a halting track and in which the server time corresponds to the occupation time of a platform track.

Juctions: The junction consists of two switches and a crossing. A train can enter the junction only if its entire route over the junction is free. If this is the case, it occupies its route during the time needed to cross the junction. If its route is occupied by another train, it has to wait on the section track leading to the junction until all elements of its route are released. Waiting trains are shifted from the section tracks to imaginary queues immediately before the junction. This is in confirmation with the theory in 2.3.3 that no trains can be lost in a network, and this applies for all components in the model.

Sections: A section consists of a number of parallel tracks that are not connected to each other. Section tracks can be used by multiple trains simultaneously if they consists of more than one block. Overtaking is physically impossible. The first and the last block on the section is modeled by two first-come-first-served single server queues, one at the beginning and one at the end of the track section. The remaining part of the track is modeled by a tandem of $M/M/1$-queues.

Network: All queueing systems in the model have exponential service time, and arrivals at the network occur according to a Poisson process. The network is modeled by a continuous-time Markov chain. The steady-state distribution of this Markov chain is found by observing that all the queues in the model are quasi-reversible. The total waiting time for a train is then the sum of the expected waiting time for each component that this train penetrates.

The model has been tested out on real data from the Netherlands which supports the practical value of the model. The most important problem with the model of Huisman is the assumption of exponential service times. Huisman points out that a general distribution for the service time might be better, but this gives no analytical solution for the stationary distribution of the network. Approximation methods must be used to solve this problem. On the other hand, this model shows very clearly one possibility how to decompose the network into smaller components for analysis.

One of the main efforts of the models made by Schwanhäußer, Gast and Huisman *et al.* is that they can calculate delay without knowing the future time table. This is ideal for dimensioning future infrastructure, where the time table is not known. These methods serve as short and long term infrastructure planning. There are methods that calculates delay from a known time table. These better serve as short term planning tool. If these models are used for long term planning, several time table scenarios must be evaluated. It requires an effort to compare these calculations together with a time table independent method before decisions are made.

3.5.3 Simulation for unscheduled waiting time calculation

Since computer aided tools havev become available, simulation of the existing schedules has become popular for the analysis of timetable stability. Timetable stability indicates the timetable's quality, how robust the timetable is against disturbance and how fast the disturbed traffic can be regenerated and become regular again. This is important knowledge for an optimal operation. The timetable stability does not tell us anything about the scheduled waiting time which is another important piece of information about the timetable.

The basics of this method consists of introducing primary delay to a given timetable. The primary delay generates secondary delay. By using algorithms, the secondary delay, timetable stability and recoverability of the timetable are some of the information that can be recorded. Timetable simulation can be used for long term planning for the evaluation of alternative infrastructure variants or to evaluate the effects on the waiting time by introduction of different signaling systems. The drawback of simulation for long term planning is that it can only be simulated on given timetables. It is time limited how many timetable alternatives that can be investigated. Therefore a timetable independent analytical method should also be preferred in the investigation. In the area of short term planning simulation is useful to optimize a preferred schedule or to adjust a schedule due to temporary maintenance work or small changes in the infrastructure. Simulation can also be used for ad-hoc decisions, for example as a support tool for the train dispatcher. The support tool is online for the current train graph. By use of given priority rules solutions for the further train dispatching can be suggested. Simulation is divided into two main groups:

A: Synchronous simulation

B: Asynchronous simulation

A general description of these two methods follows.

A: Synchronous simulation

The most famous synchronous simulation tool is RailSys developed at the University of Hannover. The area of survey (infrastructure of interest) is applicated as nodes and edges in a directed graph. The edges are in practice line sections with attributes like velocity, length, gradient etc. Only one train can occupy an edge at a time. Within the directed graph, trains can according to their schedule preblock their preferred route over several time steps. The arrival time is estimated for a certain point in the graph. This generates a time-way diagram. With use of the "First-in-first-out"' principle (FIFO) the trains are dispatched automatically by the algorithm that evaluates different routes and other options [Klah94]. With use of graph-theory in the synchronous simulation, it is possible to investigate large railway networks. The drawback is the reduction of the railway network infrastructure into a handling graph. In this step a lot of interesting data is lost. The results are of general character. A lot of work is still being invested to improve the details of the infrastructure reduction. For the investigation of smaller network parts, the synchron simulation has been succesful. Another problem to be mentioned is the preblocking over several time steps. Figure 3.19 provides an example.

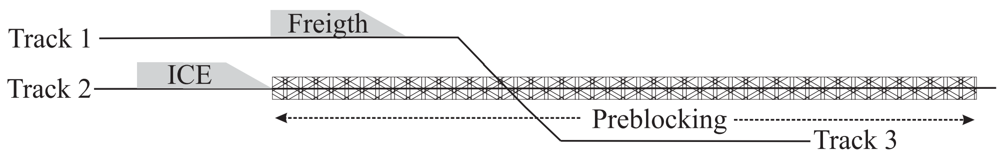

Figure 3.19: Preblocking of route leads to reduced line capacity

The ICE train on track 2 preblocks its route. The preblocking of the ICE train happens that early so that usually the freight train can be shunted over to track 3. Though the preblocking of the ICE train makes it impossible for the freight train to cross track 2. This results in a capacity loss where the real line capacity could actually be higher [Grög97]. An other problem with long distance preblocking is that large amounts of the simulations on single track line run into deadlocks.

Railway operation is based on the timetable. Theoretically one can assume that there will be no occupational conflicts during operation when the timetable is followed. In practice railway operation is influenced by many stochastic events. Some of these events can lead to a shift in the train order which can result in deadlocks. According to Pachl [Pach97] a deadlock is defined as: *A deadlock is a situation in a serving system where the proceding process runs in to a cyclic chain of prefered occupations.* Two examples of deadlocks on single track line are illustrated in figure 3.20 and 3.21.

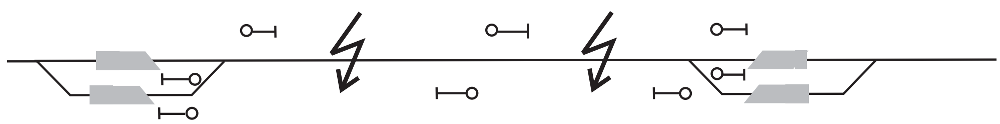

Figure 3.20: Deadlock on single track free line

In figure 3.20 all trains could theoretically run into the free block lying ahead of them. In every situation a deadlock will occur. In figure 3.21 train 1 and 2 have entered the line correctly, but run into a deadlock when entering the station in the middle of the line.

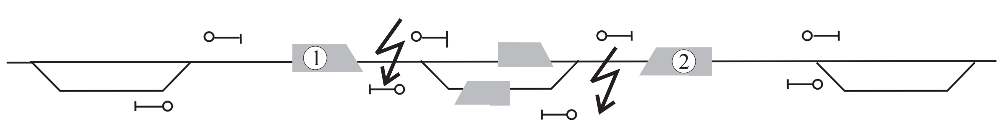

Figure 3.21: Deadlock in a station

B: Asynchronous simulation

Most known asynchron simulation methods has been NSIM (network simulation), STRESI (timetable construction and operation of double track lines) and BABSI developed at the RWTH Aachen. Asynchronous simulation is based on a hierarchical procedure visualized in a blocking time stair time-way graph. A primary delay is introduced when starting the simulation of the operation for the given and conflict free timetable. The delay generates conflicts that are solved with the same hierarchical principle as for the timetable construction process as described in 3.4.3. Different from the synchronous simulation is that the algorithm only looks at the first coming conflict and solves it (only one operation at a time within the section) [Grög02, Grög04].

The synchronous simulation with its preblocking decides on a longer distance which train will run into the FIFO queue first, and which trains will have to wait. The FIFO works hierarchically independent compared to the asynchronous method. The effort of step-by-step conflict solution is that situations as illustrated in figure 3.19 will not be registered as a conflict, and the freight train on track 1 can cross track 2 without incuring any waiting time. Another important notable effort is the simulation of timetable construction process, where the scheduled waiting time is recorded. A drawback is that so far it has been difficult to simulate large railway networks. The complexity rises within the nodes (which are the connections between the lines) within a network. Investigating each part separately has been successful so far.

3.6 Summary of the reference research

This reference research has shown that models describing railway capacity must be able to show the relation between the utilization (number of trains) and scheduled and unscheduled waiting time. Very few models are able to illustrate this relation at the same time. Dilli's [Dill52] and Potthoff's [Pott80] models are based on operation and are rather rough approximations. Hertel's model [Hert92] is based on scheduled waiting time and has no clear relation between capacity and waiting time. Schwanhäußer [Schw74] made for the first time in 1974 a direct relation between capacity and operational quality by calculating the unscheduled waiting time. His statement from 1994 [Schw94] confirms that the practical capacity is a monetary quantity. The capacity model presented in 1999 [Schw99] capacity is related to both scheduled and unscheduled waiting time. Oetting [Oett05] confirms this statement and develops Schwanhäußer's models further.

The research in railway capacity models has shown that there is a need to develop sub-models for scheduled and unscheduled waiting time calculation for the implementation into the global capacity models.

Already some models exist for the calculation of the scheduled waiting time on different track segments. The models mentioned are mainly designed for double track line. Schwanhäußer and Gast [Schw81, Gast86] calculate the scheduled waiting time from overtaking on double track line. After closer study, Gast's formula for the number of overtakings is not in accordance with the theoretical definition of probability of train sequences for independent schedules. Potthoff makes an estimation of the number of crossings on single track line. He does not explicitly make any model for the scheduled waiting time from crossing. The other models presented are mainly designed for nodes and junctions [Schw78, Wako85, Wend99]. Though they can also be used for line sections when treating the section as a single channel queuing system. Potthoff and Hertel [Pott62, Hert85] developed models for the dimensioning of track groups within nodes. The great effort of these models are that they are timetable independent which is useful for long term planning.

After Schwanhäußer's break through in 1974, other studies by Gast, Higgins *et al.* and Huisman *et al.* followed [Gast86, Higg97, Huis02]. Higgins developed a model for the optimization of schedules on single track lines [Higg98]. In another study Higgins *et al.* develops a model for the position and number of sidings on single track line. These models make use of a given timetable and are therefore suited for short-term planning. For the model of the siding positioning, a timetable independent model should be prefered. This is the effort of Schwanhäußer's, Gast's and Huisman's models. Huisman calculates the unscheduled waiting time in a network using a $M/M/k/\infty$-queue. The drawback in this model is the assumption of an exponentially distributed service time.

Another method for the calculation of waiting time is simulation. For the calculation of scheduled waiting time, a given timetable serves as input. At this point this method differs from the analytical methods for scheduled waiting time calculation, which focuses on timetable independency. This applies also for the simulation methods for the calculation of the unscheduled waiting time. They serve best for the investigation of given timetables for short-term planning.

The research has shown that there are many models for the calculation of waiting time. Rather few are directly designed for single track line since the focus has been on double track lines.

Chapter 4

Scheduled waiting time from crossing on single track lines

The buffer time distribution between the fast trains is the input parameter in the model for the calculations of scheduled waiting time. The buffer time distribution indicates what kind of timetable structure the fast trains follow, whether it is cyclic or stochastic. Different buffer time distribution will give different results in the amount of waiting time. The model introduced in this chapter calculates the scheduled waiting time from crossing following an asynchronous timetable construction process. With this timetable inclusion process the scheduled waiting time within the same train model is derivated.

First, in this chapter a brief introduction of the timetable construction methodology the model is based on is given in section 4.1. In section 4.2 the conditions for crossing are analysed. In section 4.3 the model for scheduled waiting time from crossing with exponential buffer time distribution is deduced. Special assumptions and derivation of the model with constant buffer time as given in section 4.4.

4.1 Asynchronous timetable construction process

The calculation of scheduled waiting time from overtaking by Schwanhäußer is based on a certain timetable construction methodology. The train operators can send in their traffic orders to a central timetable coordinator. Each train model is assigned a ranking number. If a train model has rank(1), it will be put first into the timetable. Which rank a train models receives depends on the ranking policy used. In Germany so far the train model with higher speed has priority over a slower train model. In Norway very often the commuter passenger train model has highest priority when creating time tables, since it occupies more time on the infrastructure than a train model with higher speed. If the train model with higher speed were included into the time table before the slow commuter train, it could be difficult to have enough buffer time between the fast trains to fit in the slow train between the fast trains. This is an important aspect for single track lines which dominate rail infrastructure in Norway.

This ranking policy can benefit the faster train if it is allowed to overtake the slower train in a siding. Then there is a partial priority between the fast and slow train.

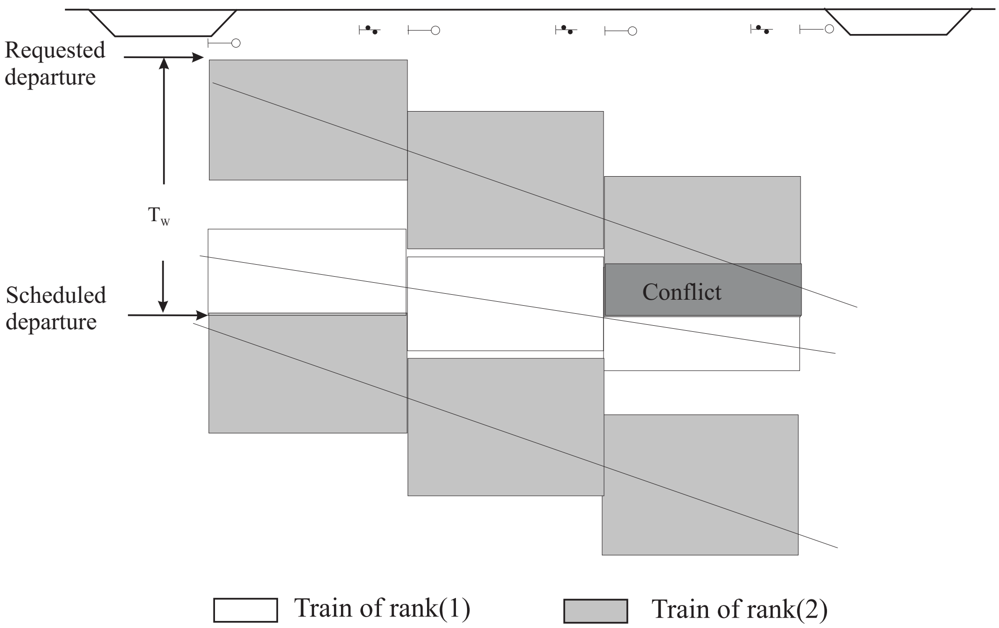

Figure 4.1: Overlapping occupation time. Solving the conflict generates waiting time.

The train model of rank(1) is put first into the timetable. Then the train model with rank(2) is included to the timetable. If there happens to be an overlapping occupation time between the train model of rank(1) and rank(2), the train model of rank(2) (which has a lower rank) is assigned to another departure time, slowed down or given another path until the conflict is solved. An example of an overlapping occupation time is illustrated in a time-way graph in figure 4.1. At this point scheduled waiting time is generated for train model of rank(2). This procedure is repeated until all conflicts are solved. In the end a new time table exists containing two train models. For train model of rank(3), the same procedure is applied, until all train models are fitted into the time table.

4.2 Condition analysis

In Schwanhäußer's model [Schw81] the possibility that a slow train can merge in between two fast trains with higher priority on a double track line with unidirectional traffic is calculated. His idea is based on following assumption illustrated in figure 4.2.

Schwanhäußer presents the relation between the minimum spacing time t_s between fast and slow trains with the buffer time $t_{b,FF}$ between the fast trains. His equality

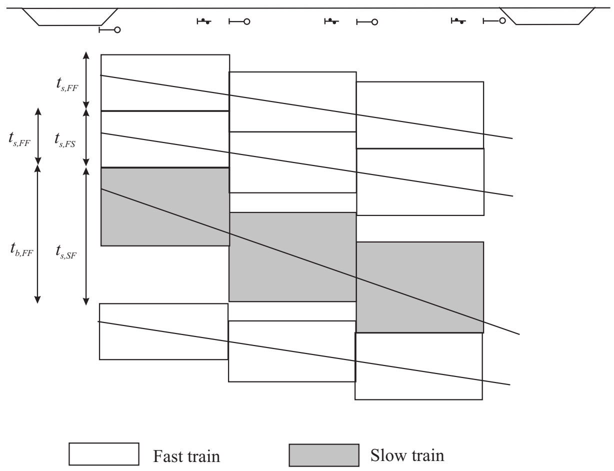

Figure 4.2: Condition for slow train to merge in between two fast trains for unidirectional traffic.

can be illustrated as:

$$t_{s,FS} + t_{s,SF} = t_{s,FF} + t_{b,FF}$$

The minimum spacing times are approximately constant quantities. This approximation holds when the scheduled waiting time calculations are based on precalculated occupation times. The occupation time is dependent on the accurancy of the journey time calculation and type of interlocking plant. The buffer time is a stochastic parameter, and can therefore become (in this case for merging) larger than illustrated in figure: 4.2:

$$\begin{aligned} t_{s,FS} + t_{s,SF} &\leq t_{s,FF} + t_{b,FF} \\ t_{b,FF} &\geq t_{s,FS} + t_{s,SF} - t_{s,FF} \end{aligned} \tag{4.1}$$

if

$$t_{s,FS} = t_{s,FF} \tag{4.2}$$

then

$$t_{b,FF} \geq t_{s,SF} \tag{4.3}$$

The last condition in (4.3) means that a slow train can merge in between two faster trains if the buffer time between the fast trains is at least as big as the minimum spacing time for the train sequence slow train followed by fast train.

The sub index $t_{s,first\ train,\ second\ train} =$ is meant as follows:

$first\ train$ means the first train arriving the block section of reference. The train can arrive from both directions.

second train means the second train arriving the same block section of reference. The train can arrive from both directions.

For example $t_{s,SF}$ in equation (4.3) means that the slower train S running in direction is arriving the block section first and the faster train F also running in direction arrives the same block section afterwards.

For bidirectional traffic on single track line, Schwanhäußer's condition for a slow train to merge in between two fast trains holds if there is only one block section between the station. This is illustrated in figure 4.3, where the slow train runs from the left to the right and the opposing faster trains run from the right to the left.

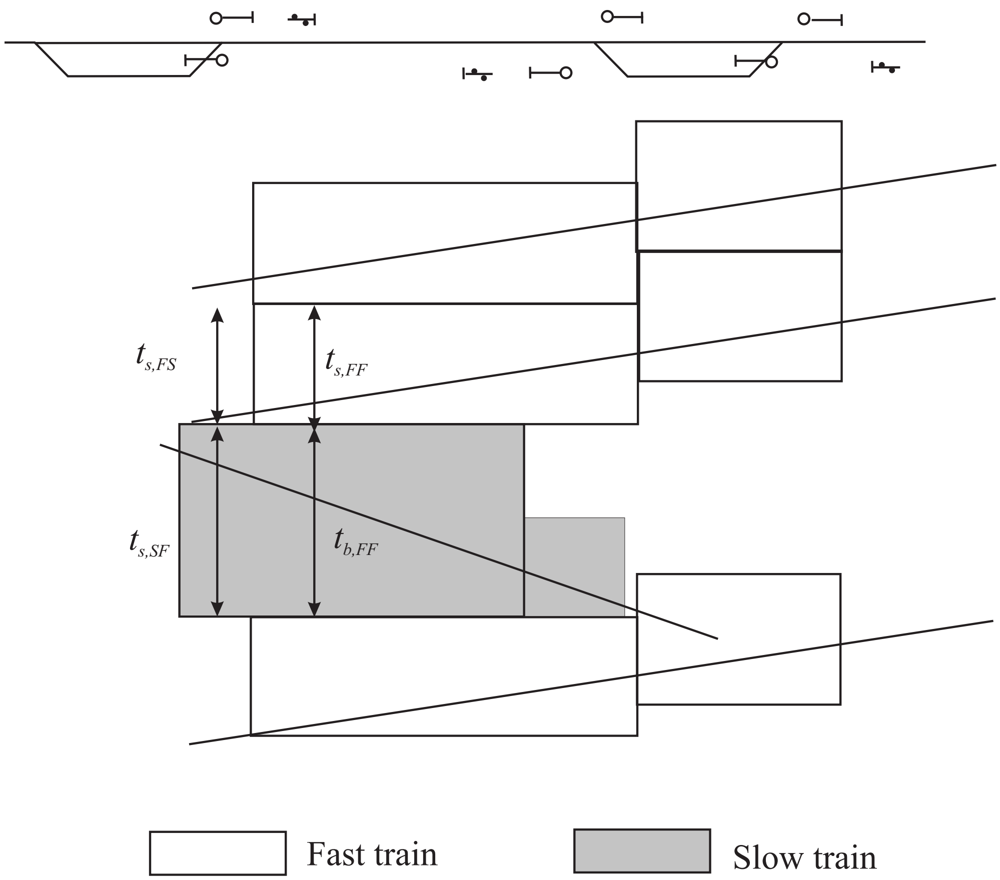

Figure 4.3: Condition for slow train to merge in between two fast trains for bidirectional traffic. Only one block section between the stations.

Figure 4.3 gives the merging condition:

$$t_{b,FF} \geq t_{s,SF} \tag{4.4}$$

The minimum spacing time and buffer time is measured in the first block section in the same section as defined in Chapter 2.1.4. Since there is only one block section between the stations, both the buffer time between the fast trains and the minimum spacing time between the trains can be measured in the same block section. If there are two or more block sections between the stations, the buffer time between the faster trains is not necessarily measured in the same block section as the minimum spacing time between a slow train followed by a fast train. Figure 4.4 illustrates a single track line with 3 block sections between the stations. The buffer time between the fast trains and the minimum spacing time between the fast trains is measured in block section c, which is the first block section when entering the section. The minimum spacing time for a fast train followed by a slow train and for a slow train followed by a fast train is measured by combining block section a and c.

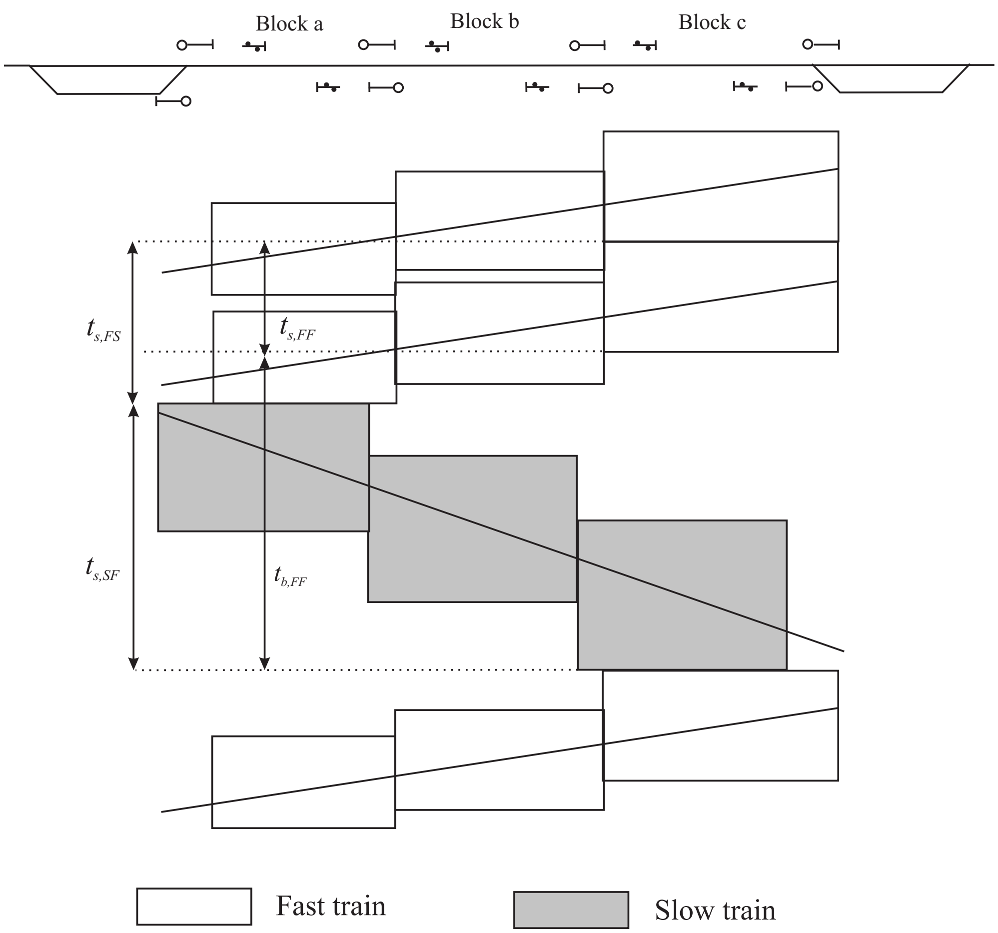

Figure 4.4: Condition for slow train to merge in between two fast trains for bidirectional traffic. Line with two or more block sections between the stations.

Schwanhäußer's assumption for unidirectional traffic is adjusted in equation (4.7) to bidirectional traffic

$$t_{s,FS} + t_{s,SF} = t_{s,FF} + t_{b,FF}$$

The main equation is analogous to equation (4.1), but the terms might have a different quantity.

$$\begin{aligned} t_{s,FS} + t_{s,SF} &\leq t_{s,FF} + t_{b,FF} \\ t_{b,FF} &\geq t_{s,FS} + t_{s,SF} - t_{s,FF} \end{aligned} \tag{4.5}$$

if

$$t_{s,FS} > t_{s,FF} \tag{4.6}$$

then

$$t_{b,FF} \geq t_{s,FS} + t_{s,SF} - t_{s,FF} \tag{4.7}$$

Equation (4.7) is the condition for a slow train to merge in between two fast trains. The buffer time between the fast trains must be the quantity $t_{s,FS} - t_{s,FF}$ bigger than in the case for unidirectional traffic.

4.3 Waiting time from crossing with exponential buffer time distribution

In this section an exponential buffer time distribution between the trains of higher priority will be assumed for the calculation of waiting time from crossing on single track lines. The asynchronous timetable construction process given in Chapter 4.1 is based on a ranking policy. Train model with the highest priority, referred to as rank(1) will be included first into the timetable and train model of lower priority, referred to as rank(2) will be included into the timetable afterwards. If a conflict between train model of rank(1) and rank(2) arises, the train model of rank(2) incurs a scheduled waiting time. Not only do conflicts between trains of different rank generate waiting time, but also do conflicts between trains of equal rank. For example on single track line, when trains of equal rank depart stochastically in each direction, scheduled crossings can arise on the free line. To solve these conflicts, the crossings have to be placed to one of its neighbouring stations. The train given lower priority (after certain criteria) in this case has to wait in the crossing station and incurs a waiting time from crossing.

Figure 4.5 illustrates a scheduled crossing on the free line between trains of rank(1). Within rank(1) a decision based on a chosen policy can be used for which of the trains that have to wait for crossing. In this case the train running from the left to the right has to wait in the siding for the opposing train. In the further calculations the waiting time from crossing for trains running in one direction will be calculated. These trains receive lower priority than the opposing trains. The opposing trains of superior direction will be treated as rank(1). Trains running in a non superior direction will be treated as rank(2). This suggestion to solve the problem of ranking within the same train model can be an effort for large nodes. The superior direction could for example be outbound of the node.

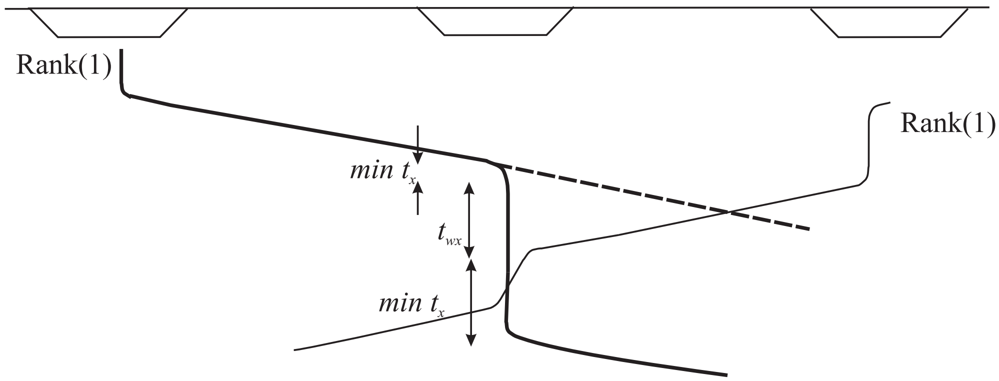

Figure 4.5: Waiting time from crossing for trains with the same ranking number

4.3.1 Expected waiting time for crossing

With an exponential buffer time distribution, there will exist small and large buffer times between the trains of rank(1). Figure 4.6 illustrates the ideal buffer time that is necessary between the trains of rank(1) $t_{b,11}$ for a train of rank(2) to merge in between the trains of opposing direction. With only one block section between the stations Schwanhäußer's assumption holds as described in Chapter 4.2. In this case 1 the ideal buffer time is equal the minimum spacing time for the train sequence train of rank(2) followed by train of rank(1). In case 2 the train of rank(2) needs an additional time gap $\Delta t(i, i+1)$ for exactly managing to run a further section before meeting the opposing train in station (i+1).

For running three sections, two time gaps are needed.

In case 3 a comparison is performed between the necessary time gap needed for respectively an overtaking and a crossing to take place in station (i+1). The blocking time stairs graph illustrates that the time gap needed in case of a crossing is larger than the time gap needed for an overtaking:

$$\Delta t_x > \Delta t_o$$

This means that the probability for a crossing is higher than the probability for an overtaking. The further derivation will concern only crossings, therefore the subindex x in the time gap will be skipped.

If there are two or more block sections between the stations on a single track line, Schwanhäußer's assumption given in equation (4.3) must be changed. The condition for a train of rank(2) to merge in between two trains of rank(1) becomes in analogy to equation (4.7):

$$t_{b,11} \geq t_{s,12} + t_{s,21} - t_{s,11} \tag{4.8}$$

To simplify the notation, let:

$$\Delta t_{s,1} = t_{s,12} - t_{s,11} \tag{4.9}$$

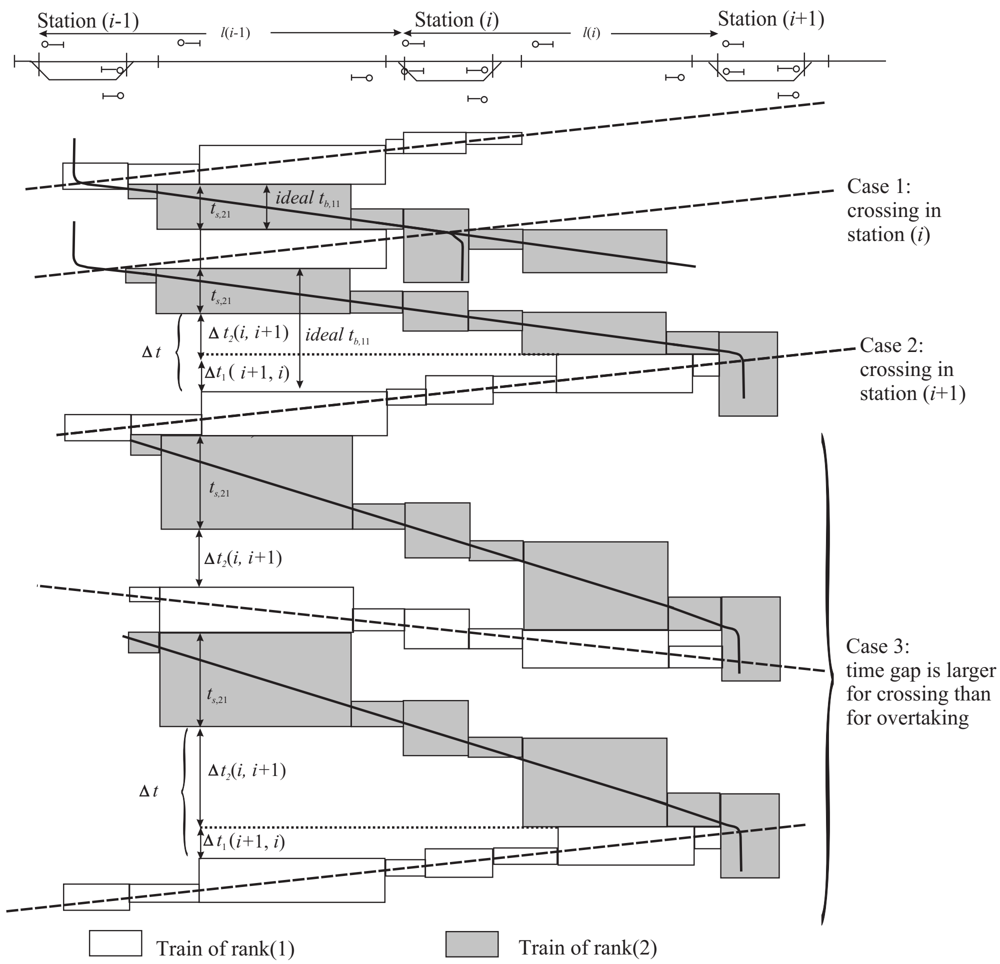

Figure 4.6: Condition for crossing in station (i) and station $(i{+}1)$. There is one block section between the stations.

Figure 4.7 illustrates a single track line with two block sections between the stations. The condition for merging changes from one block section to two or more block sections between the stations given in equation (4.8). This condition is viualized in figure 4.7. In the first case the train of rank(2) exactly manages to reach station (i). The ideal buffer time between the trains of rank(1) becomes for case 1:

$$ideal\ t_{b,11} = t_{s,21} + \Delta t_{s,1} \tag{4.10}$$

In the second case, an additional time gap $\Delta t_2(i,\ i+1) + \Delta t_1(i+1,\ i)$ is necessary for the train of rank(2) to reach station (i+1). The ideal buffer time is measured as the train of rank(2) enters the section between station (i-1) and station (i). The ideal buffer time between the trains of rank(1) becomes for case 2:

$$ideal\ t_{b,11} = t_{s,21} + \Delta t_{s,1} + \Delta t_2(i,\ i+1) + \Delta t_1(i+1,\ i) \tag{4.11}$$

In the third case, the train of rank(2) exactly manages to reach station (i+2). The time gap needed must be larger than the time gap needed in case 2. The ideal buffer time between the trains of rank(1) becomes for case 3:

$$ideal\ t_{b,11} = t_{s,21} + \Delta t_{s,1} + \Delta t_2(i,\ i+2) + \Delta t_1(i+2,\ i) \tag{4.12}$$

In general this becomes:

$$ideal\ t_{b,11} = \overline{t_{s,21}} + \overline{\Delta t_{s,1}} + m \cdot (\overline{\Delta t_2} + \overline{\Delta t_1}) \tag{4.13}$$

where
$\overline{\Delta t_2}$ = the mean of all time gaps necessary for train of rank(2) to reach the next station (i+1)
$\overline{\Delta t_1}$ = the mean of all time gaps necessary for train of rank(1) to reach the next station (i)
$m = 0, 1, 2, 3,$ ....number of sections

In equation (4.13) the mean of all time gaps like $(\Delta t_2(i,\ i{+}1){+}\Delta t_1(i{+}1,\ i))$, $(\Delta t_2(i+1,\ i+2)+\Delta t_1(i+2,\ i+1))$, $(\Delta t_2(i+2,\ i+3)+\Delta t_1(i+3,\ i+2))$ is used. The quantity of the time gap depends on the distance between the stations, the speed of the trains and the signaling system. The product of number of stations m and the mean time gap $\overline{\Delta t_2} + \overline{\Delta t_1}$ in equation (4.13) forms a generalization of the further development of case 1, 2, 3 illustrated in figure 4.7. This approximation holds for less variation in topology.

For great variations in the time gap, this assumption will be less true. With exponentially distributed buffer times and timetable independent calculations, one will not exactly know how large the biggest buffer time between the trains of rank(1) will be. Further more if the single track line is a part of a bigger railway network, the time gap might be large enough to run into the other parts of the network where other conditions and settings apply. Therefore the mean time gap for reaching one station further is used as an estimation for the calculations of the expected waiting time for

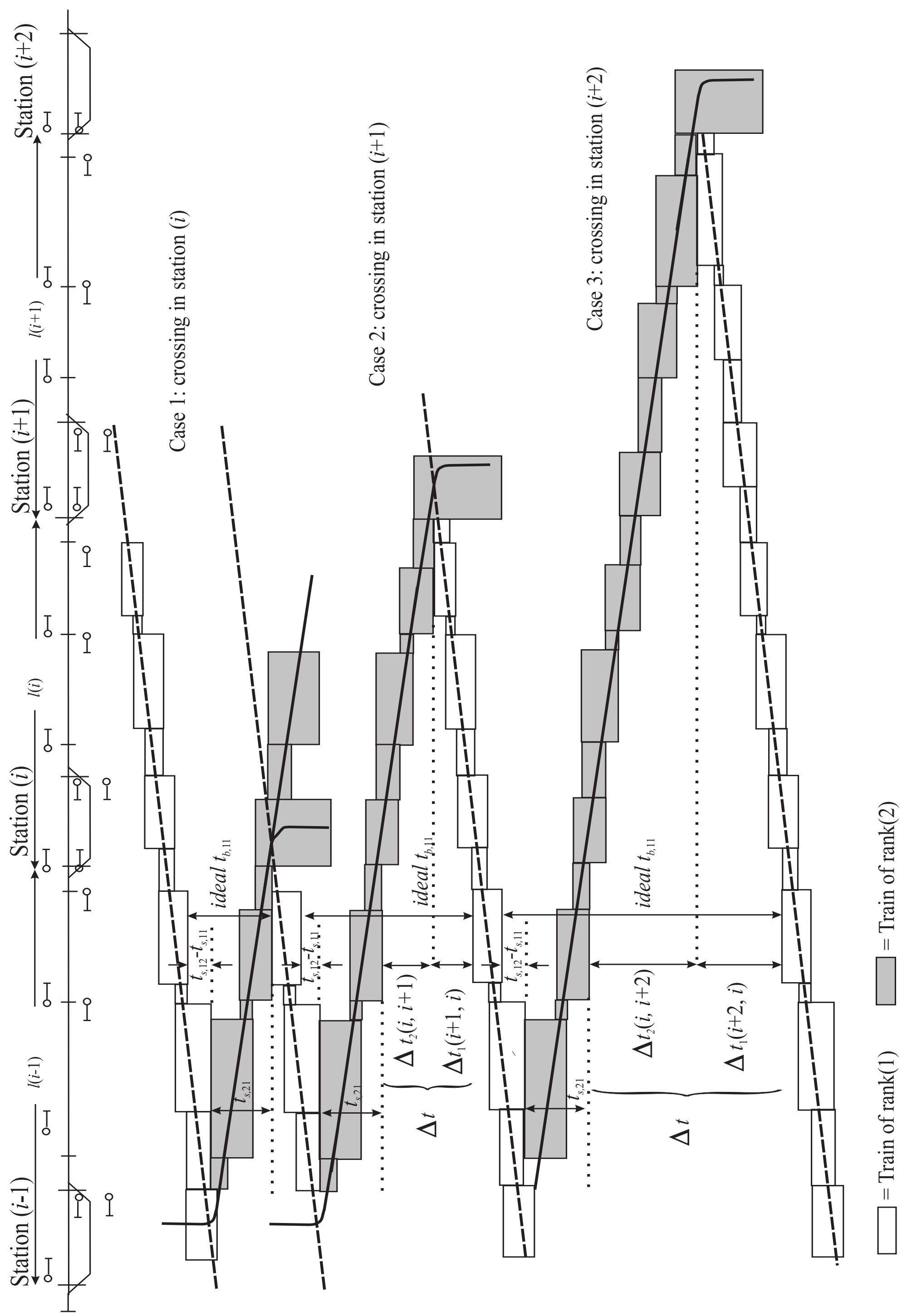

Figure 4.7: Condition for crossing in station (i), (i+1) and station (i+2). There are two or more block sections between the stations.

crossing.

To simplify equation (4.13) the mean time gap $\overline{\Delta t}$ will substitute $\overline{\Delta t_2} + \overline{\Delta t_1}$:

$$ideal\ t_{b,11} = \overline{t_{s,21}} + \Delta t_{s,1} + m \cdot (\overline{\Delta t}) \tag{4.14}$$

In this thesis the exponential distribution is assumed. The exponential distribution is a special case of the gamma disitribution. The probability density function of the gamma distribution has the form:

$$f(t_{b,11}) = \frac{\lambda^k}{\int_0^\infty t_{b,11}^{k-1} \cdot e^{-t_{b,11}} \cdot dt_{b,11}} \cdot t_{b,11}^{k-1} \cdot e^{-\lambda \cdot t_{b,11}} \quad ; t_{b,11} > 0$$

For $k = 1$ and $\lambda = 1/\overline{t_{b,11}}$ the probability gamma density function forms the exponential probability density function with parameter λ:

$$f(t_{b,11}) = \frac{1}{\overline{\overline{t_{b,11}}}} \cdot e^{-\frac{t_{b,11}}{\overline{\overline{t_{b,11}}}}} \qquad ; t_{b,11} > 0$$

where $\overline{t_{b,11}}$ is the mean buffer time between trains of rank(1) given by:

$$\overline{t_{b,11}} = \frac{T}{n_1} - \overline{t_{s,11}} \tag{4.15}$$

where
T = time of investigation, mostly 24 hours.
n_1 = the number of trains of rank(1) in one direction.
$\overline{t_{s,11}}$ = the mean minimum spacing time between the trains of rank(1) on the entire line investigated.

The exponential distribution is chosen because it is in accordance with stochastic departure times in both directions which is the focus in this study.

The probability density function of the exponential distribution is illustrated in figure 4.8. The graph shows that there is a high probability for a buffer time as large as the minimum spacing time $t_{s,21}$ which is needed for reaching the next station. As expected, the probability for buffer times big enough for reaching two or more stations decreases. There are also buffer times smaller or larger than the ideal buffer time given in equation (4.13). These buffer times generates waiting time for the train of rank(2). If the buffer time is smaller than $t_{s,21}$ waiting time for merging is generated. If the buffer time is larger than $t_{s,21}$ but not exactly equal to $t_{s,21} + m \cdot \overline{\Delta t}$, waiting time for crossing arises.

The further derivation will concern the situation with two or more block sections between the stations, since in this case the merging condition changes compared to those in Schwanhäußer's model. For the derivation of the situation with 1 block section between the stations, the reader is referred to [Schw81].

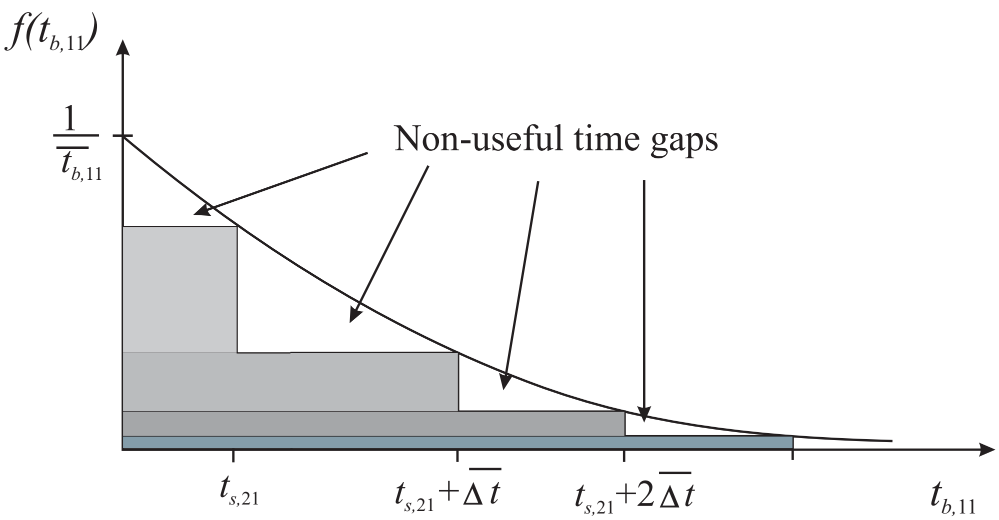

Figure 4.8: Probability density function of the buffer time between the trains of rank(1) with useful and non-useful time gaps

If the buffer time between the trains of rank(1) is bigger than the ideal buffer time for reaching the first station and smaller than the ideal buffer time for reaching the second station, then waiting time is generated:

$$t_{s,21} + \Delta t_{s,1} \leq t_{b,11} < t_{s,21} + \Delta t_{s,1} + \overline{\Delta t} \tag{4.16}$$

The expected waiting time for one crossing with buffer times given in equation (4.16) becomes, according to Schwanhäußer:

$$\begin{aligned}
E[twx1] &= \int_{t_{s,21}+\Delta t_{s,1}}^{t_{s,21}+\Delta t_{s,1}+\overline{\Delta t}} (t_{b,11} - t_{s,21} - \Delta t_{s,1}) \cdot f(t_{b,11}) \cdot dt_{b,11} \\
&= \int_{t_{s,21}+\Delta t_{s,1}}^{t_{s,21}+\Delta t_{s,1}+\overline{\Delta t}} \frac{1}{\overline{t_{b,11}}} \cdot (t_{b,11} - t_{s,21} - \Delta t_{s,1}) \cdot e^{-\frac{t_{b,11}}{\overline{t_{b,11}}}} \cdot dt_{b,11} \\
&= \left(-t_{b,11} \cdot e^{-\frac{t_{b,11}}{\overline{t_{b,11}}}} - \overline{t_{b,11}} \cdot e^{-\frac{t_{b,11}}{\overline{t_{b,11}}}} + t_{s,21} \cdot e^{-\frac{t_{b,11}}{\overline{t_{b,11}}}} + \Delta t_{s,1} \cdot e^{-\frac{t_{b,11}}{\overline{t_{b,11}}}} \right) \Bigg|_{t_{s,21}+\Delta t_{s,1}}^{t_{s,21}+\Delta t_{s,1}+\overline{\Delta t}} \\
&= e^{-\frac{(t_{s,21}+\Delta t_{s,1})}{\overline{t_{b,11}}}} \cdot \left(\overline{t_{b,11}} - (\overline{t_{b,11}} + \overline{\Delta t}) \cdot e^{-\frac{\overline{\Delta t}}{\overline{t_{b,11}}}} \right)
\end{aligned} \tag{4.17}$$

Buffer times large enough for the train of rank(2) to reach the second station and almost the third station is given in equation (4.18):

$$t_{s,21} + \Delta t_{s,1} \leq t_{b,11} + \overline{\Delta t} < t_{s,21} + \Delta t_{s,1} + 2\overline{\Delta t} \tag{4.18}$$

The integrating limits in equation (4.17) are enlarged by $\overline{\Delta t}$ given in equation (4.19):

$$
\begin{aligned}
E[twx2] &= \int_{t_{s,21}+\Delta t_{s,1}+\overline{\Delta t}}^{t_{s,21}+\Delta t_{s,1}+2\overline{\Delta t}} (t_{b,11} - t_{s,21} - \Delta t_{s,1} - \overline{\Delta t}) \cdot f(t_{b,11}) \cdot dt_{b,11} \\
&= \int_{t_{s,21}+\Delta t_{s,1}+\overline{\Delta t}}^{t_{s,21}+\Delta t_{s,1}+2\overline{\Delta t}} \frac{1}{\overline{t_{b,11}}} \cdot (t_{b,11} - t_{s,21} - \Delta t_{s,1} - \overline{\Delta t}) \cdot e^{-\frac{t_{b,11}}{\overline{t_{b,11}}}} \cdot dt_{b,11} \\
&= \Bigg(-t_{b,11} \cdot e^{-\frac{t_{b,11}}{\overline{t_{b,11}}}} - \overline{t_{b,11}} \cdot e^{-\frac{t_{b,11}}{\overline{t_{b,11}}}} + t_{s,21} \cdot e^{-\frac{t_{b,11}}{\overline{t_{b,11}}}} \\
&\quad + \Delta t_{s,1} \cdot e^{-\frac{t_{b,11}}{\overline{t_{b,11}}}} + \overline{\Delta t} \cdot e^{-\frac{t_{b,11}}{\overline{t_{b,11}}}} \Bigg) \Bigg|_{t_{s,21}+\Delta t_{s,1}+\overline{\Delta t}}^{t_{s,21}+\Delta t_{s,1}+2\overline{\Delta t}} \\
&= e^{-\frac{(t_{s,21}+\Delta t_{s,1})}{\overline{t_{b,11}}}} \cdot \left(\overline{t_{b,11}} - (\overline{t_{b,11}} + \overline{\Delta t}) \cdot e^{-\frac{\overline{\Delta t}}{\overline{t_{b,11}}}} \right) \cdot e^{-\frac{\overline{\Delta t}}{\overline{t_{b,11}}}}
\end{aligned}
\tag{4.19}
$$

Buffer times large enough for the train of rank(2) to reach the third station and almost the fourth station is given in equation (4.20):

$$
t_{s,21} + \Delta t_{s,1} \leq t_{b,11} + 2\overline{\Delta t} < t_{s,21} + \Delta t_{s,1} + 3\overline{\Delta t} \tag{4.20}
$$

The integrating limits in equation (4.19) are enlarged by $\overline{\Delta t}$ given in equation (4.21):

$$
\begin{aligned}
E[twx3] &= \int_{t_{s,21}+\Delta t_{s,1}+2\overline{\Delta t}}^{t_{s,21}+\Delta t_{s,1}+3\overline{\Delta t}} (t_{b,11} - t_{s,21} - \Delta t_{s,1} - 2\overline{\Delta t}) \cdot f(t_{b,11}) \cdot dt_{b,11} \\
&= \int_{t_{s,21}+\Delta t_{s,1}+2\overline{\Delta t}}^{t_{s,21}+\Delta t_{s,1}+3\overline{\Delta t}} \frac{1}{\overline{t_{b,11}}} \cdot (t_{b,11} - t_{s,21} - \Delta t_{s,1} - 2\overline{\Delta t}) \cdot e^{-\frac{t_{b,11}}{\overline{t_{b,11}}}} \cdot dt_{b,11} \\
&= \Bigg(-t_{b,11} \cdot e^{-\frac{t_{b,11}}{\overline{t_{b,11}}}} - \overline{t_{b,11}} \cdot e^{-\frac{t_{b,11}}{\overline{t_{b,11}}}} + t_{s,21} \cdot e^{-\frac{t_{b,11}}{\overline{t_{b,11}}}} \\
&\quad + \Delta t_{s,1} \cdot e^{-\frac{t_{b,11}}{\overline{t_{b,11}}}} + 2\overline{\Delta t} \cdot e^{-\frac{t_{b,11}}{\overline{t_{b,11}}}} \Bigg) \Bigg|_{t_{s,21}+\Delta t_{s,1}+2\overline{\Delta t}}^{t_{s,21}+\Delta t_{s,1}+3\overline{\Delta t}} \\
&= e^{-\frac{(t_{s,21}+\Delta t_{s,1})}{\overline{t_{b,11}}}} \cdot \left(\overline{t_{b,11}} - (\overline{t_{b,11}} + \overline{\Delta t}) \cdot e^{-\frac{\overline{\Delta t}}{\overline{t_{b,11}}}} \right) \cdot e^{-\frac{\overline{\Delta t}}{\overline{t_{b,11}}}} \cdot e^{-\frac{\overline{\Delta t}}{\overline{t_{b,11}}}}
\end{aligned}
\tag{4.21}
$$

For an independent timetable with exponentially distributed buffer times, the number of stations the train of rank(2) can run in the extreme case is assumed to be infinite. The further development of equation (4.17), (4.19) and (4.21) builds an infinite geometrical series:

$$
E[twx] = \sum_{i}^{\infty} E[twxi] \tag{4.22}
$$

$$\sum_{i=1}^{\infty} E[twxi] = e^{-\frac{t_{s,21}+\Delta t_{s,1}}{\overline{t_{b,11}}}} \cdot \left(\overline{t_{b,11}} - (\overline{t_{b,11}} + \overline{\Delta t}) \cdot e^{-\frac{\overline{\Delta t}}{\overline{t_{b,11}}}}\right) \cdot \left(1 + e^{-\frac{\overline{\Delta t}}{\overline{t_{b,11}}}} + (e^{-\frac{\overline{\Delta t}}{\overline{t_{b,11}}}})^2 + (e^{-\frac{\overline{\Delta t}}{\overline{t_{b,11}}}})^3 + \cdots + (e^{-\frac{\overline{\Delta t}}{\overline{t_{b,11}}}})^{i-1} + \cdots\right) \tag{4.23}$$

In general $i \to \infty$ an infinite geometrical serie on the general form has solution:

$$\sum_{i=1}^{\infty} aq^{i-1} = \frac{a}{1-q} \qquad ; |q| < 1 \tag{4.24}$$

In this case

$a = e^{-\frac{t_{s,21}+\Delta t_{s,1}}{\overline{t_{b,11}}}} \cdot \left(\overline{t_{b,11}} - (\overline{t_{b,11}} + \overline{\Delta t}) \cdot e^{-\frac{\overline{\Delta t}}{\overline{t_{b,11}}}}\right)$

and

$q = e^{-\frac{\overline{\Delta t}}{\overline{t_{b,11}}}}$

It is requested that $q < 1$. This holds if the following two conditions are satisfied:

Condition 1: There must be stations where crossings can take place. The time gap can then be measured.

Condition 2: There must be opposing trains of rank(1). Then the buffer time between the trains of rank(1) can be calculated.

The opposite situation when there are no crossings to estimate is given in the special case when:

$q = 1$

$e^{-\frac{\overline{\Delta t}}{\overline{t_{b,11}}}} = 1$

and the exponent:

$-\frac{\overline{\Delta t}}{\overline{t_{b,11}}} = 0$

The exponent can only become zero if either the nominator becomes zero or the denominator reaches infinity. This means that the time gap is only zero when the distance between the stations also is zero. On the other hand, if the buffer time between the trains of rank(1) goes to infinity it means that there are no opposing trains and no crossings will take place. Equation (4.24) is therefore valid for buffer times in the range: $(t_{s,21} + \Delta t_{s,1} \leq t_{b,11} < \infty)$.

Making use of equation (4.24) in equation (4.23) the expected waiting time for crossing becomes:

$$E[twx] = e^{-\frac{t_{s,21}+\Delta t_{s,1}}{\overline{\overline{t_{b,11}}}}} \cdot \left(\overline{t_{b,11}} - (\overline{t_{b,11}} + \overline{\Delta t}) \cdot e^{-\frac{\overline{\Delta t}}{\overline{\overline{t_{b,11}}}}}\right) \cdot \frac{1}{1 - e^{-\frac{\overline{\Delta t}}{\overline{\overline{t_{b,11}}}}}} \tag{4.25}$$

In the special case of equation (4.25) with one block between the station, $\Delta t_{s,1}$ becomes 0. The expected waiting time for crossing then becomes:

$$E[twx]_{\Delta t_{s,1}=0} = e^{-\frac{t_{s,21}}{\overline{\overline{t_{b,11}}}}} \cdot \left(\overline{t_{b,11}} - (\overline{t_{b,11}} + \overline{\Delta t}) \cdot e^{-\frac{\overline{\Delta t}}{\overline{\overline{t_{b,11}}}}}\right) \cdot \frac{1}{1 - e^{-\frac{\overline{\Delta t}}{\overline{\overline{t_{b,11}}}}}} \tag{4.26}$$

4.3.2 The number of crossings

The number of crossings one train receives for running the entire line can be described as:

$$n_x = \frac{l_{line} - \bar{l}}{\overline{d_x}} \tag{4.27}$$

where $\overline{d_x}$ is the conditional expected mean distance a train can run before meeting an opposing train. The entire length of the line is l_{line} and the average section[1] distance is $\bar{l}$.

The expected length between two crossings according to Schwanhäußer is:

$$\begin{aligned} E[\overline{d_x}] \quad = \quad & \bar{l} \int_{t_{s,21}+\Delta t_{s,1}}^{t_{s,21}+\Delta t_{s,1}+\overline{\Delta t}} \frac{1}{\overline{\overline{t_{b,11}}}}\, e^{-\frac{t_{b,11}}{\overline{\overline{t_{b,11}}}}}\, dt_{b,11} + \\ & 2 \cdot \bar{l} \int_{t_{s,21}+\Delta t_{s,1}+\overline{\Delta t}}^{t_{s,21}+\Delta t_{s,1}+2\overline{\Delta t}} \frac{1}{\overline{\overline{t_{b,11}}}}\, e^{-\frac{t_{b,11}}{\overline{\overline{t_{b,11}}}}}\, dt_{b,11} + \\ & 3 \cdot \bar{l} \int_{t_{s,21}+\Delta t_{s,1}+2\overline{\Delta t}}^{t_{s,21}+\Delta t_{s,1}+3\overline{\Delta t}} \frac{1}{\overline{\overline{t_{b,11}}}}\, e^{-\frac{t_{b,11}}{\overline{\overline{t_{b,11}}}}}\, dt_{b,11} + \cdots + \\ & m \cdot \bar{l} \int_{t_{s,21}+\Delta t_{s,1}+(m-1)\overline{\Delta t}}^{t_{s,21}+\Delta t_{s,1}+m\overline{\Delta t}} \frac{1}{\overline{\overline{t_{b,11}}}}\, e^{-\frac{t_{b,11}}{\overline{\overline{t_{b,11}}}}}\, dt_{b,11} \end{aligned} \tag{4.28}$$

After integration equation (4.28) becomes:

$$\begin{aligned} E[\overline{d_x}] \quad = \quad & \bar{l} \cdot e^{-\frac{t_{s,21}+\Delta t_{s,1}}{\overline{\overline{t_{b,11}}}}} \cdot \Bigg((1 - e^{-\frac{\overline{\Delta t}}{\overline{\overline{t_{b,11}}}}}) + \\ & 2 \cdot (e^{-\frac{\overline{\Delta t}}{\overline{\overline{t_{b,11}}}}} - e^{-\frac{2\overline{\Delta t}}{\overline{\overline{t_{b,11}}}}}) + \\ & 3 \cdot (e^{-\frac{2\overline{\Delta t}}{\overline{\overline{t_{b,11}}}}} - e^{-\frac{3\overline{\Delta t}}{\overline{\overline{t_{b,11}}}}}) + \cdots + \\ & m \cdot (e^{-\frac{(m-1)\overline{\Delta t}}{\overline{\overline{t_{b,11}}}}} - e^{-\frac{m\overline{\Delta t}}{\overline{\overline{t_{b,11}}}}}) \Bigg) \end{aligned} \tag{4.29}$$

[1]Recall that a section is the distance between two neighboring stations, Chapter 2.1.1.

Setting $q = e^{-\frac{\overline{\Delta t}}{\overline{t_{b,11}}}}$ equation (4.29) can be expressed as:

$$\begin{aligned} E[\overline{d_x}] &= \bar{l} \cdot e^{-\frac{t_{s,21}+\Delta t_{s,1}}{\overline{t_{b,11}}}} \cdot \Big((1-q) + \\ & \quad 2 \cdot (q - q^2) + \\ & \quad 3 \cdot (q^2 - q^3) + \cdots + \\ & \quad m \cdot (q^{(m-1)} - q^m) \Big) \end{aligned} \tag{4.30}$$

Equation (4.30) forms a geometrical serie. For $m \to \infty$ and $q < 1$ the sum of this series becomes:

$$\begin{aligned} E[\overline{d_x}] &= \bar{l} \cdot e^{-\frac{t_{s,21}+\Delta t_{s,1}}{\overline{t_{b,11}}}} \cdot \sum_{m=1}^{\infty} m \cdot (1-q) \cdot q^{m-1} \\ &= \bar{l} \cdot e^{-\frac{t_{s,21}+\Delta t_{s,1}}{\overline{t_{b,11}}}} \cdot \frac{(1-q)}{(q-1)^2} \\ &= \bar{l} \cdot e^{-\frac{t_{s,21}+\Delta t_{s,1}}{\overline{t_{b,11}}}} \cdot \frac{-1}{(q+1)} \\ &= \bar{l} \cdot e^{-\frac{t_{s,21}+\Delta t_{s,1}}{\overline{t_{b,11}}}} \cdot \frac{1}{(1-q)} \end{aligned} \tag{4.31}$$

Rearranging equation (4.31) gives:

$$E[\overline{d_x}] = \bar{l} \cdot e^{-\frac{t_{s,21}+\Delta t_{s,1}}{\overline{t_{b,11}}}} \cdot \frac{1}{1-q} = \bar{l} \cdot \frac{e^{-\frac{t_{s,21}+\Delta t_{s,1}}{\overline{t_{b,11}}}}}{1 - e^{-\frac{\overline{\Delta t}}{\overline{t_{b,11}}}}} \tag{4.32}$$

The probability for buffer times large enough for the train of rank(2) to merge back into the line is given by:

$$\begin{aligned} p(t_{s,21} + \Delta t_{s,1} \leq t_{b,11} < \infty) &= \int_{t_{s,21}+\Delta t_{s,1}}^{\infty} \frac{1}{\overline{t_{b,11}}} \, e^{-\frac{t_{b,11}}{\overline{t_{b,11}}}} \, dt_{b,11} \\ &= \left(e^{-\frac{t_{b,11}}{\overline{t_{b,11}}}}\right)\Big|_{t_{s,21}+\Delta t_{s,1}}^{\infty} \\ &= e^{-\frac{t_{s,21}+\Delta t_{s,1}}{\overline{t_{b,11}}}} \end{aligned} \tag{4.33}$$

Figure 4.9 illustrates the probability density function for buffer times between trains of rank(1) for buffer times large enough for merging. It makes sense that a train of rank(2) has to merge into the line before it can meet an opposing train. Therefore

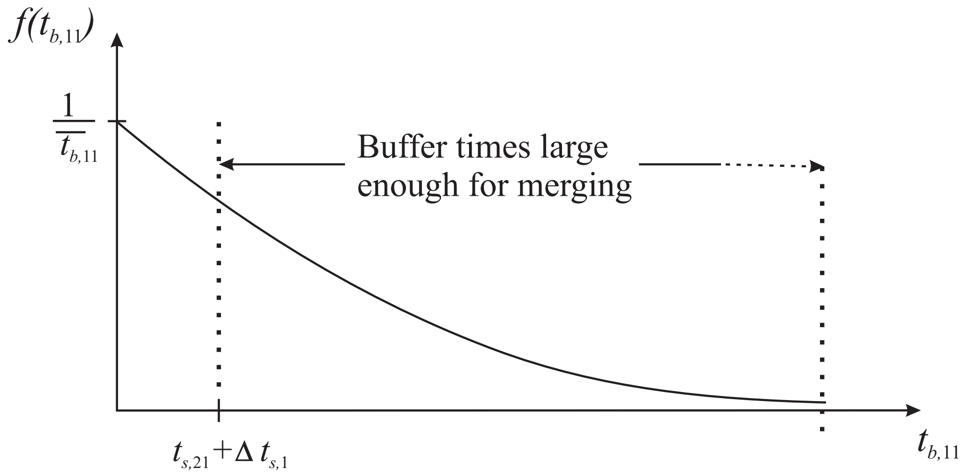

Figure 4.9: Probability density function of the buffer time between the trains of rank(1). Area for buffer times large enough for merging is indicated.

only the sum of all probabilites for buffer times larger than or equal $t_{s,21} + \Delta t_{s,1}$ can be taken into account.

The conditional expected mean distance between two crossings is given by:

$$\overline{d_x} = \frac{E[\overline{d_x}]}{\frac{1}{\overline{t_{b,11}}} \cdot \int_{t_{s,21}+\Delta t_{s,1}}^{\infty} e^{-\frac{t_{b,11}}{\overline{t_{b,11}}}} \, dt_{b,11}} \tag{4.34}$$

where $E[\overline{d_x}]$ is the expected length between two crossings. The denominator is the probability for buffer times large enough for merging. After inserting equation (4.32) and equation (4.33) into (4.34) the conditional expected mean distance between two crossings becomes:

$$\overline{d_x} = \frac{\overline{l}}{1 - e^{-\frac{\overline{\Delta t}}{\overline{t_{b,11}}}}} \tag{4.35}$$

Equation (4.35) is another manner to express equation (4.34). The average distance a train can run before it meets an opposing train is the mean section distance devided by the probability for buffer times large enough for reaching the next station(i), which is the probability for crossing in station(i). With decreasing probability for buffer times large enough to reach the next station(i), the probability increases for reaching station(i+1) and $\overline{d_x}$ increases.

Inserting equation (4.35) into equation (4.27) gives the number of crossings for one train on the entire line:

$$n_x = \frac{l_{line} - \overline{l}}{\overline{l}} \cdot (1 - e^{-\frac{\overline{\Delta t}}{\overline{t_{b,11}}}}) \tag{4.36}$$

The first term in equation (4.36) is equal the number of stations. The second term is equal the probability for buffer times large enough for merging to the next station. This is equal to the probability for crossing in one station.

4.3.3 Expected waiting time for merging

If the buffer time between the trains of rank(1) is smaller than the ideal buffer time necessary for train of rank(2) to reach the next station before meeting an opposing train, waiting time for merging arises. Figure 4.10 illustrates a train of rank(2) which cannot merge into the line because the buffer time is too small. The train has to wait $t_{s,12} + t_{b,11} - \Delta t_{s,1}$. If the second opportunity to merge does not succeed, the train has to wait an other $t_{s,12} + t_{b,11} - \Delta t_{s,1}$.

The expected waiting time if the first possibility to merge fails becomes:

$$
\begin{aligned}
E[twm1] &= \int_0^{t_{s,21}+\Delta t_{s,1}} (t_{b,11} + t_{s,12} - \Delta t_{s,1}) \cdot f(t_{b,11}) \cdot dt_{b,11} \\
&= \int_0^{t_{s,21}+\Delta t_{s,1}} \cdot \frac{1}{\overline{t_{b,11}}} (t_{b,11} + t_{s,12} - \Delta t_{s,1}) \cdot e^{-\frac{t_{b,11}}{\overline{t_{b,11}}}} \cdot dt_{b,11} \\
&= \left(-t_{b,11} \cdot e^{-\frac{t_{b,11}}{\overline{t_{b,11}}}} - \overline{t_{b,11}} \cdot e^{-\frac{t_{b,11}}{\overline{t_{b,11}}}} - t_{s,12} \cdot e^{-\frac{t_{b,11}}{\overline{t_{b,11}}}} + \Delta t_{s,1} \cdot e^{-\frac{t_{b,11}}{\overline{t_{b,11}}}} \right) \Bigg|_0^{t_{s,21}+\Delta t_{s,1}} \\
&= -(t_{s,21} + \Delta t_{s,1}) \cdot e^{-\frac{(t_{s,21}+\Delta t_{s,1})}{\overline{t_{b,11}}}} - \overline{t_{b,11}} \cdot e^{-\frac{(t_{s,21}+\Delta t_{s,1})}{\overline{t_{b,11}}}} - t_{s,12} \cdot e^{-\frac{(t_{s,21}+\Delta t_{s,1})}{\overline{t_{b,11}}}} \\
&\quad + \Delta t_{s,1} \cdot e^{-\frac{(t_{s,21}+\Delta t_{s,1})}{\overline{t_{b,11}}}} + \overline{t_{b,11}} + t_{s,12} - \Delta t_{s,1} \\
&= \overline{t_{b,11}} - \overline{t_{b,11}} \cdot e^{-\frac{t_{s,21}+\Delta t_{s,1}}{\overline{t_{b,11}}}} - (t_{s,21} + \Delta t_{s,1}) \cdot e^{-\frac{t_{s,21}+\Delta t_{s,1}}{\overline{t_{b,11}}}} \\
&\quad + (t_{s,12} - \Delta t_{s,1}) - (t_{s,12} - \Delta t_{s,1}) \cdot e^{-\frac{t_{s,21}+\Delta t_{s,1}}{\overline{t_{b,11}}}} \\
&= (1 - e^{-\frac{t_{s,21}+\Delta t_{s,1}}{\overline{t_{b,11}}}}) \cdot (\overline{t_{b,11}} + t_{s,12}) - t_{s,21} \cdot e^{-\frac{t_{s,21}+\Delta t_{s,1}}{\overline{t_{b,11}}}} - \Delta t_{s,1}
\end{aligned}
\tag{4.37}
$$

The probability for buffer times between the trains of rank(1) smaller than $t_{s,21} + \Delta t_{s,1}$ is:

$$
p(t_{b,11} < t_{s,21} + \Delta t_{s,1}) = 1 - e^{-\frac{t_{s,21}+\Delta t_{s,1}}{\overline{t_{b,11}}}} \tag{4.38}
$$

If the second possibility to merge back into the line fails, the waiting time for merging becomes:

$$
\begin{aligned}
E[twm2] &= E[twm1] \cdot p(t_{b,11} < t_{s,21} + \Delta t_{s,1}) \\
&= \left((1 - e^{-\frac{t_{s,21}+\Delta t_{s,1}}{\overline{t_{b,11}}}}) \cdot (\overline{t_{b,11}} + t_{s,12}) - t_{s,21} \cdot e^{-\frac{t_{s,21}+\Delta t_{s,1}}{\overline{t_{b,11}}}} - \Delta t_{s,1} \right) \cdot (1 - e^{-\frac{t_{s,21}+\Delta t_{s,1}}{\overline{t_{b,11}}}})
\end{aligned}
\tag{4.39}
$$

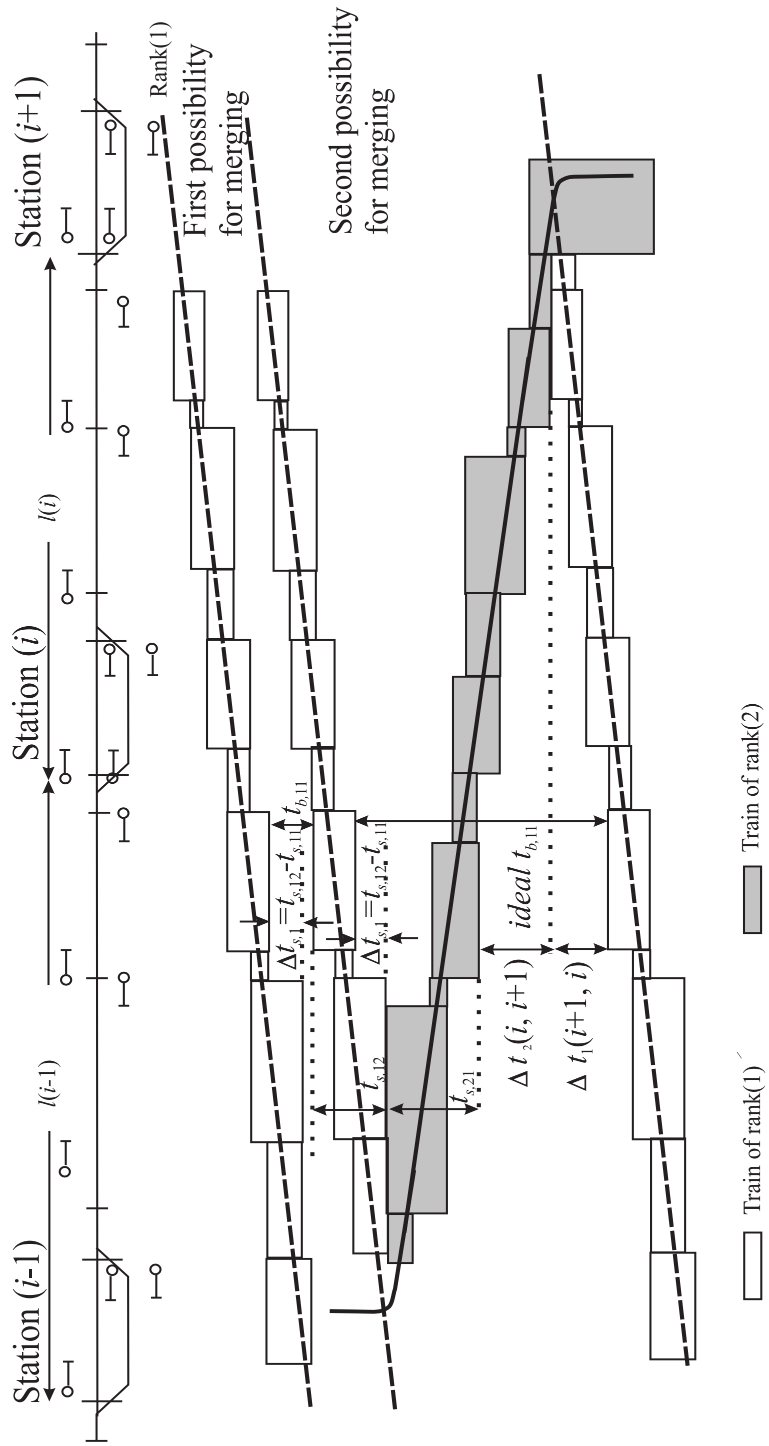

Figure 4.10: Train of rank(2) incurs waiting time for merging. There are two or more block sections between the stations.

The buffer time between the trains of rank(1) and the waiting time for merging are stochastic variables. In equation (4.39) the expected waiting time for merging into the first possibility and the probability for the next buffer time to be too small for merging back into the line forms a product. This product can be formed since the waiting time generated for not being able to merge back into the line is independent of the buffer time between the trains of rank(1) forming the next possibility for merging. Whether the buffer time between the trains of rank(1) is large enough for merging or not is a stochastic event. The buffer time is independent of the earlier or later buffer times in the same schedule.

If also the third possibility to merge back into the line fails, the expected waiting time becomes:

$$\begin{aligned} E[twm3] &= E[twm1] \cdot p(t_{b,11} < t_{s,21} + \Delta t_{s,1}) \cdot p(t_{b,11} < t_{s,21} + \Delta t_{s,1}) \\ &= \left((1 - e^{-\frac{t_{s,21}+\Delta t_{s,1}}{\overline{t_{b,11}}}}) \cdot (\overline{t_{b,11}} + t_{s,12}) - t_{s,21} \cdot e^{-\frac{t_{s,21}+\Delta t_{s,1}}{\overline{t_{b,11}}}} - \Delta t_{s,1} \right) \cdot (1 - e^{-\frac{t_{s,21}+\Delta t_{s,1}}{\overline{t_{b,11}}}})^2 \end{aligned} \tag{4.40}$$

The expected waiting time for merging for i possibilities then becomes:

$$\begin{aligned} E[twm] &= \sum_{i}^{\infty} E[twmi] \\ \sum_{i=1}^{\infty} E[twmi] &= E[twm1] \cdot \sum_{i=1}^{\infty} \left(p(t_{b,11} < t_{s,21} + \Delta t_{s,1} \right)^{(i-1)} \\ &= \left((1 - e^{-\frac{t_{s,21}+\Delta t_{s,1}}{\overline{t_{b,11}}}}) \cdot (\overline{t_{b,11}} + t_{s,12}) - t_{s,21} \cdot e^{-\frac{t_{s,21}+\Delta t_{s,1}}{\overline{t_{b,11}}}} - \Delta t_{s,1} \right) \\ &\quad \cdot \sum_{i=1}^{\infty} (1 - e^{-\frac{t_{s,21}+\Delta t_{s,1}}{\overline{t_{b,11}}}})^{(i-1)} \end{aligned} \tag{4.41}$$

The second term in the product in equation (4.41) builds a geometrical serie. Letting the quotient be:

$$q = 1 - e^{-\frac{t_{s,21}+\Delta t_{s,1}}{\overline{t_{b,11}}}} \tag{4.42}$$

Then equation (4.41) can be expressed as:

$$\begin{aligned} \sum_{i=1}^{\infty} E[twmi] &= \left((1 - e^{-\frac{t_{s,21}+\Delta t_{s,1}}{\overline{t_{b,11}}}}) \cdot (\overline{t_{b,11}} + t_{s,12}) - t_{s,21} \cdot e^{-\frac{t_{s,21}+\Delta t_{s,1}}{\overline{t_{b,11}}}} - \Delta t_{s,1} \right) \\ &\quad \cdot \sum_{i=1}^{\infty} (1 + q + q^2 + q^3 \cdots q^{i-1}) \end{aligned} \tag{4.43}$$

The geometrical serie in (4.43) converges when $i \to \infty$:

$$\sum_{i=1}^{\infty} q^{i-1} = \frac{1}{1-q} \quad ; |q| < 1 \tag{4.44}$$

Note that equation (4.44) is not the sum of all probabilities for buffer times $t_{b,11} < t_{s,21} + \Delta t_{s,1}$. The sum of equation (4.44) is larger than 1. The probability for not merging back into the line in the first possibility is included into equation (4.37), respectively in the first part of the product in equation (4.43). The sum of all probabilities for not merging back into the line is thus given in equation (4.43). This partition of equation (4.41) into a product is necessary for the formation of a converging series of the second part of the product in equation (4.43). The sum of all probabilities for not merging back into the line would include the probability for not merging into the first possibility into equation (4.44): $\sum p(t_{b,11} < t_{s,21} + \Delta t_{s,1}) = \frac{q}{(1-q)}$ which satisfies the condition $\frac{q}{(1-q)} \leq 1$.

Inserting for q from equation (4.42) into equation (4.44) makes :

$$\begin{aligned} \frac{1}{1-q} &= \frac{1}{1-(1-e^{-\frac{t_{s,21}+\Delta t_{s,1}}{\overline{t_{b,11}}}})} \\ &= e^{\frac{t_{s,21}+\Delta t_{s,1}}{\overline{t_{b,11}}}} \end{aligned} \tag{4.45}$$

Inserting the sum of equation (4.44) given in (4.45) into equation (4.43) gives the overall expected waiting time for merging for one train:

$$\begin{aligned} E[twm] &= \left((1-e^{-\frac{t_{s,21}+\Delta t_{s,1}}{\overline{t_{b,11}}}}) \cdot (\overline{t_{b,11}} + t_{s,12}) - t_{s,21} \cdot e^{-\frac{t_{s,21}+\Delta t_{s,1}}{\overline{t_{b,11}}}} - \Delta t_{s,1} \right) \cdot e^{\frac{t_{s,21}+\Delta t_{s,1}}{\overline{t_{b,11}}}} \\ &= \overline{t_{b,11}} \cdot e^{\frac{t_{s,21}+\Delta t_{s,1}}{\overline{t_{b,11}}}} + t_{s,12} \cdot e^{\frac{t_{s,21}+\Delta t_{s,1}}{\overline{t_{b,11}}}} - \overline{t_{b,11}} - t_{s,12} - t_{s,21} - \Delta t_{s,1} \cdot e^{\frac{t_{s,21}+\Delta t_{s,1}}{\overline{t_{b,11}}}} \\ &= (\overline{t_{b,11}} + t_{s,12})(e^{\frac{t_{s,21}+\Delta t_{s,1}}{\overline{t_{b,11}}}} - 1) - t_{s,21} - \Delta t_{s,1} \cdot e^{\frac{t_{s,21}+\Delta t_{s,1}}{\overline{t_{b,11}}}} \end{aligned} \tag{4.46}$$

In analogy to equation (4.46) the expected waiting time for merging in the special case of one block section between the stations becomes:

$$E[twm]_{\Delta t_{s,1}=0} = (\overline{t_{b,11}} + t_{s,12})(e^{\frac{t_{s,21}}{\overline{t_{b,11}}}} - 1) - t_{s,21} \tag{4.47}$$

4.3.4 Expected waiting time from crossing

To obtain the entire scheduled waiting time from crossing for the entire line, the expected waiting time for crossing and merging respectively, must be multiplied by the number of trains n_2 that are to be included into the schedule and the number of crossings, as described in formula (4.48):

$$T_{WX} = n_2 \cdot n_x \cdot (min\ t_x + E[twx] + E[twm]) \tag{4.48}$$

The term $min\ t_x$, which is a constant, is the minimum time necessary for one crossing decribed in Chapter 2.1.5. Inserting for the number of crossings from equation (4.36) into equation (4.48):

$$T_{WX} = n_2 \cdot \frac{l_{line} - \bar{l}}{\bar{l}} \cdot (1 - e^{-\frac{\overline{\Delta t}}{\overline{t_{b,11}}}}) \cdot (min\ t_x + E[twx] + E[twm]) \tag{4.49}$$

Inserting the expected waiting time for crossing and merging from equation (4.25) and (4.46) gives the waiting time from crossing for the entire line for all the trains of rank(2) that are to be included.

$$\begin{aligned} T_{WX} \quad = \quad & n_2 \cdot \frac{l_{line} - \bar{l}}{\bar{l}} \cdot (1 - e^{-\frac{\overline{\Delta t}}{\overline{t_{b,11}}}}) \\ & \cdot \Bigg(min\ t_x + \\ & e^{-\frac{t_{s,21}+\Delta t_{s,1}}{\overline{t_{b,11}}}} \cdot \left(\overline{t_{b,11}} - (\overline{t_{b,11}} + \overline{\Delta t}) \cdot e^{-\frac{\overline{\Delta t}}{\overline{t_{b,11}}}} \right) \cdot \frac{1}{1 - e^{-\frac{\overline{\Delta t}}{\overline{t_{b,11}}}}} \\ & + (\overline{t_{b,11}} + t_{s,12})(e^{\frac{t_{s,21}+\Delta t_{s,1}}{\overline{t_{b,11}}}} - 1) - t_{s,21} - \Delta t_{s,1} \cdot e^{\frac{t_{s,21}+\Delta t_{s,1}}{\overline{t_{b,11}}}} \Bigg) \end{aligned} \tag{4.50}$$

For the special case where there are only one block section between the stations, $\Delta t_{s,1}$ is set equal zero in equation (4.50). For the special case of a single track line with mixed numbers of block sections between the stations, equation (4.50) can be fractionated:

$$\begin{aligned} T_{WX} \quad = \quad & n_2 \cdot \frac{n_{\geq 1\ block}}{N_{stations}} \cdot (1 - e^{-\frac{\overline{\Delta t}}{\overline{t_{b,11}}}}) \cdot (min\ t_x + E[twx] + E[twm]) + \\ & n_2 \cdot \frac{n_{\Delta t_{s,1}=0}}{N_{stations}} \cdot (1 - e^{-\frac{\overline{\Delta t}}{\overline{t_{b,11}}}}) \cdot (min\ t_x + E[twx]_{\Delta t_{s,1}=0} + E[twm]_{\Delta t_{s,1}=0}) \end{aligned} \tag{4.51}$$

where

$\frac{n_{\geq 1\ block}}{N_{stations}}$ = the proportion of sections with two or more block sections between the stations.
$\frac{n_{1\ block}}{N_{stations}}$ = the proportion of sections with one block section between the stations.

In the next step of the timetable construction, another train model of lower priority with ranking number 2 is to be included into the existing timetable of train model of rank(1). In this case equation (4.51) must be calculated for the number of trains that are to be included for both directions seperately.

4.4 Special assumption for constant buffer times

Figure 4.11 illustrates three different cyclic timetables on a fictitous single track railway line. They are cyclic since the cycle time, t_c, between the departures for one direction is constant. Timetable A has traffic only in one direction. On single track lines, there is mostly traffic in both directions. The cyclic timetable in A is not a common timetable for single track. Timetable A only serves as an example.

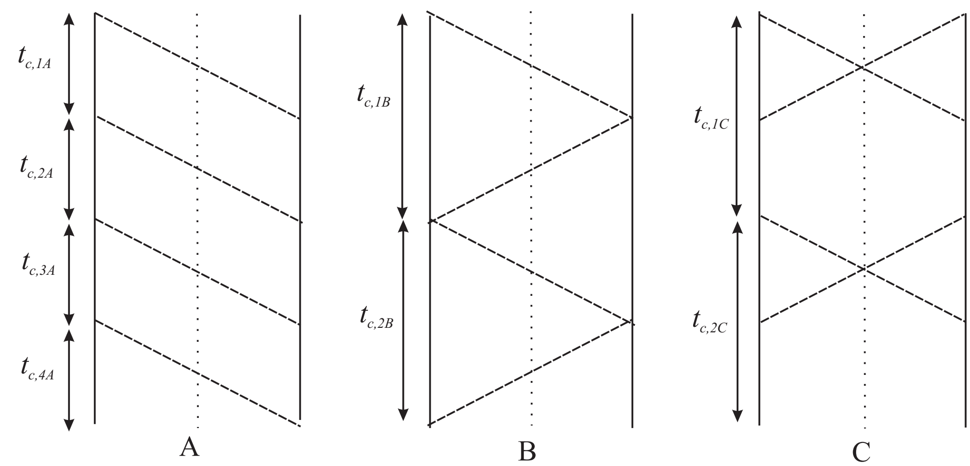

Figure 4.11: Three examples of cyclic timetables

Timetable B and C have traffic in both directions. In timetable C the trains are crossing.

When the buffer time between the trains of rank(1) is treated as a constant, only the number of trains in superior direction needs to be known for the calculation of the mean buffer time given in equation (4.15). A constant buffer time between the trains of rank(1) is only an assumption. With constant train velocity but variation in the distance between the stations, it gives different minimum spacing time which influences the buffer time in equation (4.15). With additional variation in the train velocity, it gives even larger variation in the minimum spacing time between the trains of rank(1) and the buffer time between the trains of rank(1). The larger the deviation in buffer time, the less the assumption of constant buffer time holds.

In the following derivation there are few variations in topology and distance between the stations assumed. The train velocity is kept constant over the entire line to ensure an approximately constant buffer times between the trains of rank(1) on the entire line. In practise there will always be some variation in the buffer time between the trains of rank(1), therefore the mean quantity of the buffer times between the trains of rank(1) wil be used as an approximation for a constant buffer time denoted with $\overline{t_{b,11}}$. The constant buffer time is given in equation (4.15). For the estimation of the waiting time from crossing and overtaking, it is assumed that one direction is prior over the opposite direction, as in section 4.3.1.

4.4.1 Waiting time from crossing and overtaking with constant buffer time

Figure 4.12 illustrates the buffer time needed for a crossing to take place in station (i), case 1, and station (i+1), case 2. In the first case, the train of rank(2), running in one direction, manages to reach station (i). The ideal buffer time between the trains of rank(1) is equal to the minimum spacing time $t_{s,21}$. In case 2 the train of rank(2) manages exactly to reach the station (i+1). In this case the ideal buffer time needed between the trains of rank(1) is analogous to equation (4.11):

$$ideal\ t_{b,11} = t_{s,21} + \Delta t_{s,1} + \Delta t_2(i,\ i+1) + \Delta t_1(i+1,\ i)$$

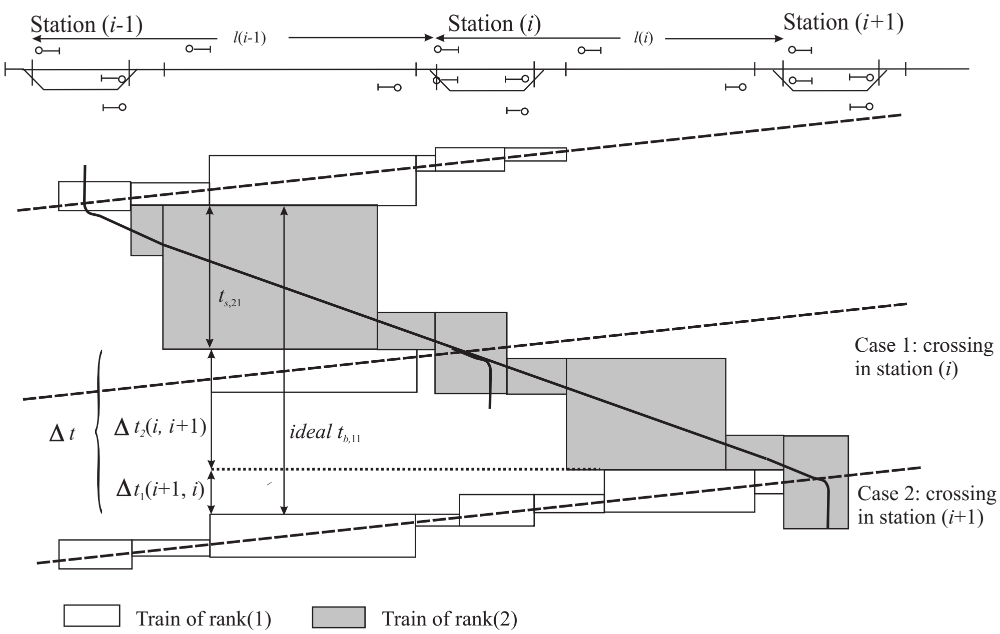

Figure 4.12: Ideal buffer time between trains of rank(1) for crossing with train of rank(2) in station (i) and (i+1)

Figure 4.13 shows a situation for an overtaking. A train of rank(2) starting from station (i-1), running in one direction can exactly reach the next station (i), or even one or more stations further, if the constant buffer time between the trains of rank(1) is big enough. In the ideal case, the train of rank(2) manages to reach the next station without generating any waiting time in addition to the minimum time needed for overtaking.

In case 1 a train of rank(2) is being overtaken in station (i) or station (i+1) which is case 2, depending on how big the constant buffer time is between the trains of rank(1). In the first case, the buffer time between the trains of rank(1) must at least equal the minimum spacing time, $t_{s,21}$, on the section $l(i-1)$. In the second case, if the train

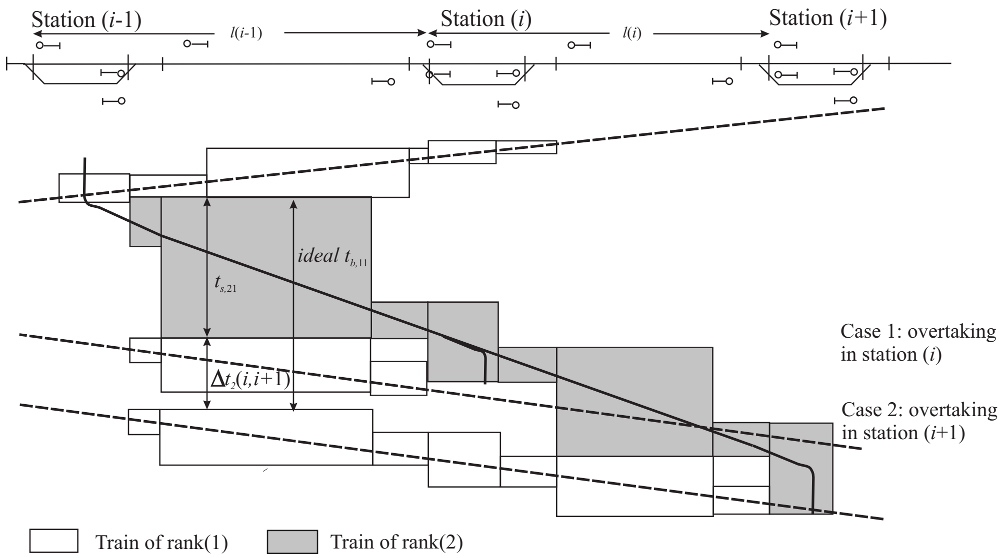

Figure 4.13: Ideal buffer time between trains of rank(1) for overtaking the train of rank(2) in station (i) and (i+1)

of rank(2) were exactly to reach one more station (i+1), the ideal buffer time between the trains of rank(1) becomes:

$$ideal\, t_{b,11} = t_{s,21} + \Delta t_2(i, i+1)_o \tag{4.52}$$

where
$\Delta t_2(i, i+1)_o =$ is the time gap needed in addition to the minimum spacing time for reaching station (i).

The same derivation for the ideal buffer time for crossing and overtaking follows the same principles as given in equation (4.11)-(4.13).

When using the same train model as in the example for overtaking, given in figure 4.13, the time gap needed for the train of rank(2) to reach the next station for a crossing is bigger than the time gap needed in the case of an overtaking on the same line sections:

$$\begin{aligned} \Delta t(i, i+1)_o &< \Delta t_2(i, i+1)_x + \Delta t_1(i+1, i)_x \\ \Delta t_o &< \Delta t_x \end{aligned} \tag{4.53}$$

This is the same observation as in section 4.3.1.

If the trains of rank(1) have a constant buffer time bigger than the mean minimum spacing time, the train of rank(2) will manage to run at least one station further:

$$\overline{t_{b,11}} > \overline{t_{s,21}}$$

This is the buffer time referred to in equation (4.15).

If the buffer time between the trains of rank(1) is constant and the distance between the stations is equal, then the waiting time for every crossing becomes:

$$t_{wx} = \overline{t_{b,11}} - \overline{t_{s,21}} - m \cdot \overline{\Delta t_x} \tag{4.54}$$

and for every overtaking:

$$t_{wo} = \overline{t_{b,11}} - \overline{t_{s,21}} - m \cdot \overline{\Delta t_o} \tag{4.55}$$

where
$m = 0, 1, 2, 3...$

For small variations in the buffer time between the trains of rank(1) and the distance between the stations is spreading, following approximation for the waiting time can be used for every crossing:

$$t_{wx} \approx \frac{\overline{\Delta t_x}}{2} \tag{4.56}$$

and for every overtaking:

$$t_{wo} \approx \frac{\overline{\Delta t_o}}{2} \tag{4.57}$$

When the buffer time between the trains of rank(1) is spreading the quantity of time gap needed for the train of rank(2) to reach the next station will normally also spread. In this case the waiting time will be $\frac{\overline{\Delta t}}{2}$.

Figure 4.14 serves as an example for the explanation of equation (4.56). Four variants of one schedule with constant buffer time is illustrated. In this example the opposing trains of rank(1) have priority over trains running from the left to the right. The requested crossing is marked by a circle. In schedule 1 the train of rank(2) and lower priority almost manages to reach the next station (i+1). This train incurs waiting time from crossing in station(i). In the second schedule the requested crossing is situated in the middle of the section. In this schedule the train of rank(2) also incurs waiting time from crossing which is a little larger as in schedule 1. In the third schedule the train of rank(2) is not running far before meeting the opposing train. It incurs a small amount of waiting time for crossing in station(i). In the last example, schedule 4, the train of rank(2) exactly manges to reach the next station (i+1) before meeting the opposing train. This train incurs no waiting time for crossing. These 4 example schedules illustrate that the requested departure for the trains of lower priority can run into a crossing with the opposing train at any point on the section when timetable independent estimation is used. The same explanantion yields for the occurence of waiting time for overtaking.

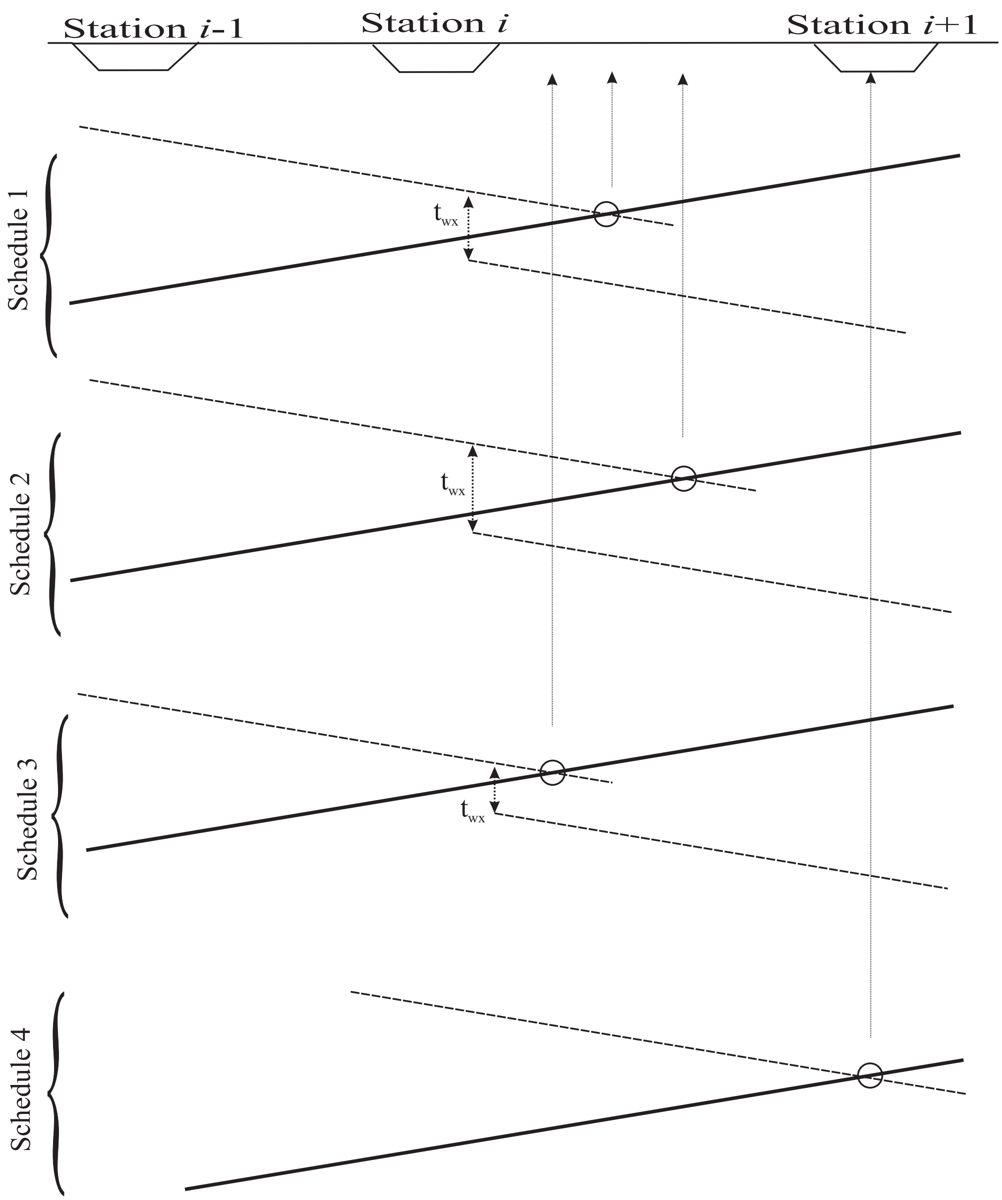

Figure 4.14: Four examples of equal probability of requested timetables

If $\overline{t_{b,11}} \geq \overline{t_{s,21}}$, then the mean distance $\overline{d_o}$ a train of rank(2) can run on average before an overtaking takes place becomes:

$$\overline{d_o} = \overline{l} \cdot (\frac{\overline{t_{b,11}} - \overline{t_{s,21}}}{\overline{\Delta t_o}} + 1) \tag{4.58}$$

The mean distance $\overline{d_x}$ a train of rank(2) can run on average before a crossing takes place becomes:

$$\overline{d_x} = \overline{l} \cdot (\frac{\overline{t_{b,11}} - \overline{t_{s,21}}}{\overline{\Delta t_x}} + 1) \tag{4.59}$$

The number of overtakings for one train on the entire line will then be:

$$n_o = \frac{(l_{line} - \overline{l})}{\overline{d_o}} \tag{4.60}$$

Analogous with the number of overtakings, the number of crossings for one train on the entire line becomes:

$$n_x = \frac{(l_{line} - \overline{l})}{\overline{d_x}} \tag{4.61}$$

where
$\overline{l}$ = the mean section distance between the stations.

Inserting for $\overline{d}$ from equation (4.59), the number of overtakings and crossings for one train on the entire line becomes, respectively:

$$n_o = \frac{(l_{line} - \overline{l})}{\overline{l}} \cdot \frac{\overline{\Delta t_o}}{(\overline{t_{b,11}} - \overline{t_{s,21}} + \overline{\Delta t_o})} \tag{4.62}$$

and

$$n_x = \frac{(l_{line} - \overline{l})}{\overline{l}} \cdot \frac{\overline{\Delta t_x}}{(\overline{t_{b,11}} - \overline{t_{s,21}} + \overline{\Delta t_x})} \tag{4.63}$$

where
l_{line} = the entire length of the line investigated.
The first term in equation (4.62) and (4.63) represents the number of stations on the line.

In the first round of inclusion of the timetable the waiting time between trains of equal rank with trains in the opposite direction being superior becomes:

$$T_{W,d,\ 1-incl.} = n_{2,d} \cdot \left(n_x \cdot (t_{wx} + min\ t_x) \right) \tag{4.64}$$

where $n_{2,d}$ = the number of trains of rank(2) to be included in one direction
$min\ t_x$ = the minimum time theoretical needed for a crossing described in Chapter 2.1.5
$min\ t_o$ = the minimum time theoretical needed for an overtaking

In the first round of inclusion there is no waiting time from overtaking, since all trains within the same train model are assumed to run with the same speed. In the second round of inclusion of trains into the timetable waiting time from overtaking might also occur. If the velocity of the including trains is lower than the velocity of the already included trains of rank(1), then the waiting time from crossing and overtaking becomes in the second inclusion:

$$T_{W,d,\ 2-incl.} = n_{2,d} \cdot \Big(n_o \cdot (t_{wo} + min\ t_o) + n_x \cdot (t_{wx} + min\ t_x) \Big) \tag{4.65}$$

Inserting equation (4.62) and (4.63) for the number of overtakings and crossings, and the equations (4.57) and (4.56) for the waiting time for one overtaking and crossing into equation (4.65) gives:

$$\begin{aligned} T_{W,d,\ 2-incl.} \quad &= \quad n_{2,d} \cdot \frac{(l_{line} - \bar{l})}{\bar{l}} \cdot \Bigg(\frac{\overline{\Delta t_o}}{(\overline{t_{b,11}} - \overline{t_{s,21}} + \overline{\Delta t_o})} \cdot \Big(\frac{\overline{\Delta t_o}}{2} + min\ t_o \Big) \\ &\quad + \frac{\overline{\Delta t_x}}{(\overline{t_{b,11}} - \overline{t_{s,21}} + \overline{\Delta t_x})} \cdot \Big(\frac{\overline{\Delta t_x}}{2} min + t_x \Big) \Bigg) \end{aligned} \tag{4.66}$$

Note that from the second round of inclusion new quantities must be recorded for $t_{b,11}$, $t_{s,21}$ and $\overline{\Delta t_x}$. The number of trains running in the opposite direction treated as trains of rank(1) is the sum of trains running in the opposite direction from the first and second round of inclusion. If the sum of the number of trains running in the opposite direction in the second round of inclusion is larger than the number of trains running in the opposite direction in the first round of inclusion, the buffer time between the trains of rank(1) in the second round of inclusion is smaller than in the first round of inclusion. This means that the trains running in direction might incur waiting time from crossing with trains from the first and second round of inclusion.

For the third inclusion into the timetable equation (4.65) is repeated with new quantites for $t_{b,11}$, $t_{s,21}$ and $\overline{\Delta t_x}$. For the overall waiting time from crossing and overtaking for all rounds of m inclusions in one direction becomes:

$$T_{W,d} = n_{2,d} \cdot \Big(n_x \cdot (t_{wx} + min\ t_x) \Big) + \sum_{i=2-icl.}^{m} T_{W,d,\ i-incl.} \tag{4.67}$$

Trains of the opposite direction might incur waiting time from overtaking and crossing. This can occur from the second round of inclusion into the timetable. The trains running in the opposite direction can only incur waiting time from crossing when crossing with a train from the first round of inclusion with higher priority, running in direction. Trains running in the opposite direction will not incur waiting time from crossing when crossing with a train running in direction from the second round of inclusion. The train running in direction from the second round of inclusion already incured this waiting time given in equation (4.65).

The trains running in the opposite direction cannot incur waiting time from crossing because of their priority over trains running in one direction. The waiting time from crossing and overtaking for trains running in the opposite direction for the second round of inclusion becomes:

$$T_{W,od,\ 2-incl.} = n_{2,od} \cdot \Bigg(n_o \cdot (t_{wo} + min\ t_o) + n_x \cdot (t_{wx} + min\ t_x) \Bigg) \tag{4.68}$$

where
$n_{2,od}$ = the number of trains of rank(2) to be included in opposite direction

For the third inclusion into the timetable equation (4.68) is repeated.

The overall waiting time from crossing for all rounds of m inclusions in the opposite direction becomes:

$$T_{W,od} = \sum_{i=2-incl.}^{m} T_{W,od,\ i-incl.} \tag{4.69}$$

The overall waiting time for the entire line for both direction is the sum of equation (4.67) and (4.69):

$$\begin{aligned} T_W &= n_{2,d} \cdot \Bigg(n_x \cdot (t_{wx} + min\ t_x) \Bigg) + \sum_{i=2-incl.}^{m} T_{W,d,\ i-incl.} \\ &\quad + \sum_{i=2-incl.}^{m} T_{W,od,\ i-incl.} \end{aligned} \tag{4.70}$$

Chapter 5

Case study of single track lines

In this chapter a case study of the timetable characteristics of three single track lines is performed. The study is built up in three parts I-III as described in section 5.1. Section 5.2 lists the results of each line. Section 5.3 summarizes the results of case study I, where section 5.4 gives the result of case II and section 5.5 gives the results of case III.

5.1 Timetable characteristics on single track lines

In this section the timetable characteristics for three single track lines were investigated. This case study serves the purpose of providing a rough picture to what extent the schedules in this study are stochastic or cyclic. At first the main timetable characteristics for each train model were investigated as to whether they were cyclic or stochastic. Second, for the stochastic schedules a buffer time analysis was conducted to determine whether an exponential, hyper-exponential or an Erlang distribution can be accepted as distribution. This analysis was carried out with a χ^2-test of goodness of fit. This will be a rough approximation compared with the study made by Kaas [Kaas98], where the distribution of the arrival rate at Glostrup station were analysed in detail. Glostrup station is situated between Copenhagen and the national airport on a double track line. Since Schwanhäußer showed that the buffer times between fast trains on double track lines with mixed traffic in most cases is approximately negative exponentially distributed [Schw74], it is obvious to check whether this also applies to single track lines.

A definition of a cyclic timetable is made by Dirmeier [Dirm77]:

A cyclic schedule has a repeating cycle time t_C between two trains travelling in the same direction on the same line.

For example a cycle time of 60 minutes means that the same train model departs every 60 minutes from the same point. For single track line, this definition also holds. In most cases the cycle time is the same for both directions within the same schedule.

The following three single track lines were investigated:

- Line A: Eidsvoll-Hamar (Norway). Day of survey was Thursday, April 18^{th}, 2002.
- Line B: Day of survey was Thursday, April 18^{th}, 2002
- Line C: Day of survey was Thursday, April 17^{th}, 2003

Upon request from the infrastructure operator, no details regarding the track line B and line C will be disclosed in this paper. Common for all three single track lines is that four to five different train models operate, where one of the train models is a freight train model. On the mentioned week days, 51-75 passengers and freight trains frequented each line over 24 hours. For these lines this is a high traffic load.

The following data were collected for different line sections on the three lines:

I: The timetable characteristics for each train model individually were extracted from the time tables

II: The timetable characteristics for a selection of pairs of two train models together were extracted from the time tables. The buffer time between the trains was analyzed.

III: The buffer time was extracted from the time tables for all train models together

All three timetables were or had to be built in a time-way graph that illustrated the occupation time of each train individually. In this time-way graph it is possible to extract the buffer time between the trains. Details are given in Appendix B.

5.2 Results of all three lines

5.2.1 Line A: Eidsvoll-Hamar

One of the lines investigated was Eidsvoll-Hamar, which is a single track line north of the national Norwegian Gardermoen Airport. This line is about 55 km long, and has 9 stations for overtaking and crossing between the nodes Eidsvoll and Hamar. The single track line is a part of an entire line from Oslo to Trondheim in the north, as illustrated in figure 5.1. A conventionalized track diagram of line A from Eidsvoll to Hamar is illustrated in figure 5.2.

On April 18^{th}, 2002, 62 passengers and freight trains (Ft) were scheduled on this line over 24 hours. Figure 5.3 illustrates the timetable structure of two of three passengers train models running on the single track line Eidsvoll-Hamar. The third train model is a passenger train running only at night (Nt) with one departure in each direction. The Inter-City-Express train model (ICE, called *Signatur* in Norway) has

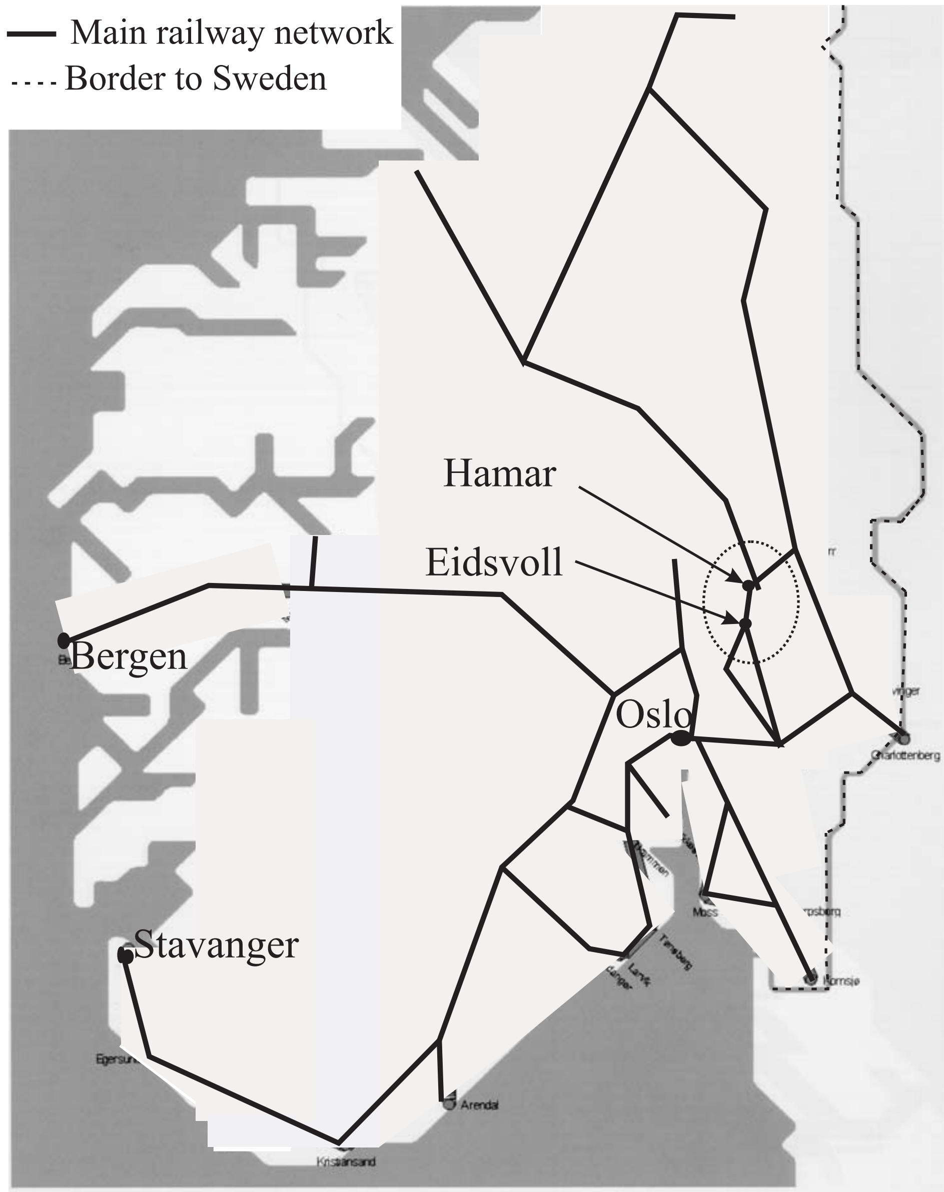

Figure 5.1: Localization of line A: Eidsvoll-Hamar in Norway

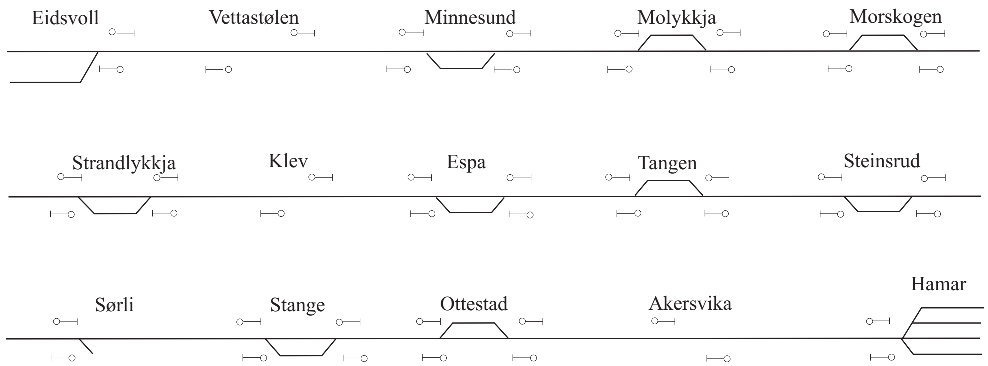

Figure 5.2: Conventionalized track diagram of line A: Eidsvoll-Hamar

four departures during the day in each direction. The departure times of the ICE trains are not regular. The trains travel two by two with a time distance of 2 hours in between. Their origin and destination is Oslo and Trondheim respectively. The other train model is an Inter-City train (IC) with regular departures of 1 hour from Eidsvoll and Hamar. This timetable is cyclic. The ICE and IC train do not operate during the night from 12am to 5am.

The 17 scheduled freight trains (Ft) on the line Eidsvoll-Hamar run irregularly. There were no freight trains scheduled between 12pm and 3am. Their timetable is stochastic. A χ^2-test of goodness of fit of the buffer time between the freight trains on the section between station Tangen and Steinsrud gave conformity with an exponential distribution and a hyper-exponential distribution with a significance level of $\alpha = 0.05$.

A buffer time analysis of the IC trains and freight trains together in the same schedule on the same section provided no conformity with the wanted distributions. There are 35 IC trains that operate cyclically, and 17 freight trains that operate stochastically. The cyclic schedule has a dominating influence on the buffer time distribution. This can be seen by the high number of observations in the same time interval.

The analysis of the buffer time between all trains in the timetable on the section between station Tangen and Steinsrud did not conform to any of the investigated distributions. For the section Molykkja-Morskogen the buffer time distribution conforms with an exponential and hyper exponential distribution with a significance level of $\alpha = 0.05$. Since the coefficient of variation $V_T = 1.35 > 1.2$, the investigated distribution is closer to a hyper-exponential distribution than to an exponential distribution. The buffer times from section Espa-Tangen did not conform to any of the investigated distributions.

There is no clear priority detected in the schedules between the passenger train models. For some crossings, the ICE train model uses the siding in a station, and other times does not. The same holds for the IC train model. Comparing the freight

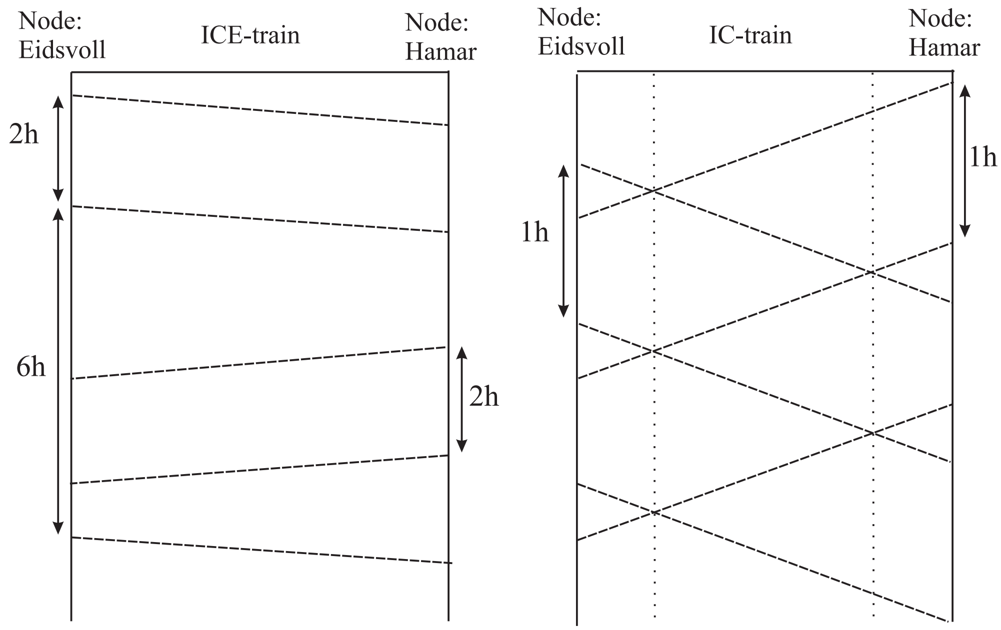

Figure 5.3: Timetable characteristics of the ICE train and the IC train on the single track line Eidsvoll-Hamar

trains with the passenger trains, a priority is clearly detected. All passenger trains have priority over the freight trains.

A summary of line A is given in table 5.1

Train model	Section	Number of trains	Buffer time distribution	α	V_T
Nt	Tangen-Steinsrud	2	not defined		
ICE	Tangen-Steinsrud	8	not defined		
IC	Tangen-Steinsrud	35	cyclic		
Ft	Tangen-Steinsrud	17	$Expo., H.-expo.$	0.05	1.35
ICE+IC	Tangen-Steinsrud	43	cyclic domination		
IC+Ft	Tangen-Steinsrud	52	not defined		1.64
All	Tangen-Steinsrud	62	not defined	0.05	1.41
All	Molykkja-Morskogen	62	$Expo., H.-expo.$	0.05	1.66
All	Espa-Tangen	62	not defined	0.05	1.47

Table 5.1: Summary of line A: Eidsvoll-Hamar

5.2.2 Line B

On line B there are four different train models operating. The three passenger trains models operate between 4am and 11pm, and the freight train between 3am and 11pm.

One of the passenger train models contains only one IC train. For this train model it cannot be decided whether this train model follows a cyclic or a stochastic schedule. A model using only one train is not sufficient for such a description. The other two train models are a Inter-Regional train model (IR) and a Regional train model (RB). The IR train model has in total 15 trains in both directions, and the Regional train model has in total 25 trains in both directions on section 2, 3 and 4. On section 1 41 Regional trains operate. Sixteen of them terminate at the first station. Figure 5.4 illustrates the timetable characteristics of the IR train model and the RB train model. They have a cyclic schedule with departures every 2 hours in each direction. In the timetable, the cyclic behavior of both schedules is recognized as a systematic pattern in the time-way-graph with crossing on the same station. The time-way graph illustrates that the IR train model has higher priority than the RB train model, since the latter uses the siding during the crossing.

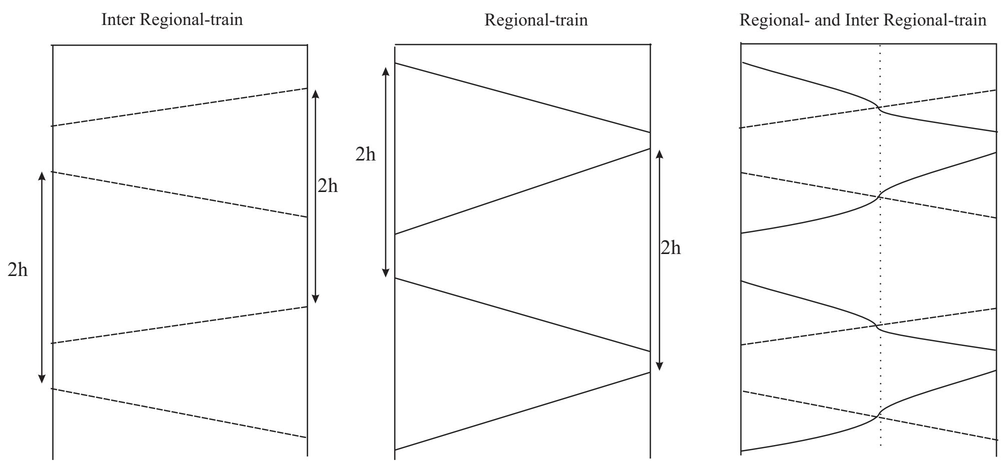

Figure 5.4: Timetable characteristics of the IR train and the RB train on the single track line B

There were in total 11 freight trains operating in both directions together. Their schedule is stochastic. A χ^2-test of goodness of fit of the buffer time between the freight trains conforms with an exponential and $Erlang_2$ distribution with a significance level of 0.05. Since the coefficient of variation is $V_T = 0.76$, the investigated distribution is closer to an Erlang distribution than an exponential distribution.

A buffer time analysis between the RB trains and freight trains together in the same schedule on the same section conforms with an exponential and hyper-exponential distribution with a significance level of 0.05. The coefficient of variation is $V_T = 1.24$ indicates that the investigated distribution is closer to a hyper-exponential distribution than to an exponential distribution.

The buffer time analysis between all trains in the time table on section 1 did not fit into any of the investigated distributions. On section 2, the buffer time distribution between all trains gave conforms with an Erlang$_2$ distribution with a significance level $\alpha = 0.05$. Both section 3 and 4 conforms with an exponential and hyper-exponential distribution with a significance level of $\alpha = 0.05$. Since the coefficient of variation in both cases was above 1.20, it is assumed that the buffer time distributions investigated are closer to a hyper-exponential distribution than to an exponential distribution. The time-way-graph of the entire timetable also illustrated that the passenger trains all have priority over the freight trains.

A summary for line B is given in table 5.2.

Train model	Section	Number of trains	Buffer time distribution	α	V_T
IC	all	1	not defined		
IR	all	15	cyclic		
RB	1, (2, 3), 4	41, (25), 23	cyclic		
Ft	3	11	$Expo., Erlang_2$	0.05	0.76
IR+RB	all	40	cyclic domination		
RB+Ft	3	36	$Expo., H.-expo.$	0.05	1.24
All	1	71	not defined		1.48
All	2	55	$Erlang_2$	0.05	1.07
All	3	55	$Expo., H.-expo.$	0.05	1.36
All	4	53	$Expo., H.-expo.$	0.05	1.21

Table 5.2: Summary of line B

5.2.3 Line C

On line C 5 different train models operate. These operate between 4am and 11pm. Two of the four passenger train models count only one or two trains within the model. As in the case of line B, this is too little information to make any conclusion about the type of schedule. The other two passenger train models are, like line B, an IR train model and a RB train model. Their timetable characteristics are illustrated in figure 5.5. The IR train model has in total 14 trains, thus 7 trains in each direction depart every second hour. This is a cyclic schedule. The RB train model has in total 37 trains in both directions together. This schedule consists of two integrated cyclic schedules with departure every second hour respectively. One of these schedules has trains that run the entire line with a crossing in the same station. The other schedule alternates with the first. The trains here travel only to the crossing station before returning back in the other direction. The last illustration in figure 5.5 shows both train models together in the same schedule. A repeating pattern is still to be observed when adding these two cyclic schedules together. It is not clear if the IR train model has priority over the RB train model, since there are no overtakings and crossings scheduled in the time table.

The last train model in this timetable is the schedule of the freight trains. There were 21 freight trains operating between 2am and 11am. This schedule is stochastic like the other freight train schedules investigated so far. A buffer time analysis with the χ^2-test of goodness of fit conforms with an exponential distribution and a hyper-exponential distribution with a significance level of 0.05. Since the coefficient of

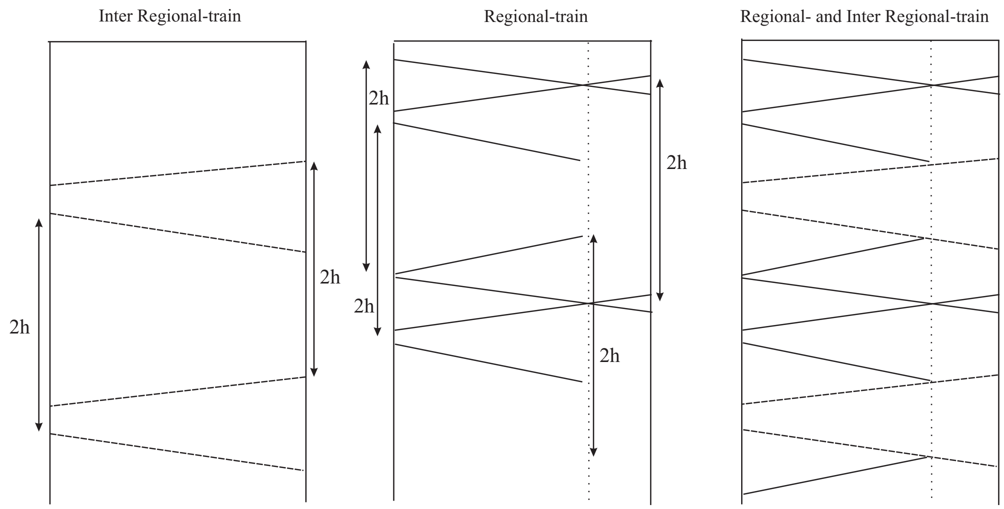

Figure 5.5: Timetable characteristics of the IR train and the RB train on the single track line C

variation , $V_T = 1.07$, is close to 1.00, the exponential distribution will be the better estimate than the hyper-exponential distribution.

As for line B, a buffer time analysis between the RB trains and freight trains together in the same schedule was investigated. On section 1 the χ^2-test of goodness of fit did not conform with any of the compared distributions.

The χ^2-test of goodness of fit of the buffer time between all trains in the timetable on section 1 did not conform with any of the investigated distributions. On section 2, a conformity with the exponential distribution was observed. In this case a too high coefficient of variation was observed, $V_T = 1.53$. This distribution might fit to a hyper-exponential distribution, but the analysis gave no conformity with the hyper-exponential distribution. On section 3 both conformity with an exponential, a hyper-exponential and an Erlang$_2$ distribution was observed. The coefficient of variation was close to 1.00, therefore it is assumed that the investigated distribution better suits an exponential distribution. All tests were made with a significance level of $\alpha = 0.05$. As for line A and B, the time-way-graph of the entire timetable also illustrated that the passenger trains all have priority over the freight trains.

A summary for line C is given in table 5.3.

Train model	Section	Number of trains	Buffer time distribution	α	V_T
Nt	all	1	not defined		
IC	all	2	not defined		
IR	all	14	cyclic		
RB	1,2, (3)	37, (21)	cyclic		
Ft	1	21	$Expo., H.-expo.$	0.05	1.07
IR+RB	1	51	cyclic domination		
RB+Ft	1	58	not defined		1.50
All	1	77	$Erlang_2$	0.05	1.48
All	2	77	$Expo.$	0.05	1.53
All	3	61	$Expo., H.-expo., Erlang_2$	0.05	1.03

Table 5.3: Summary of line C

More details are given in Appendix B.

5.2.4 Summary of all lines

5.3 Case study I: Time table characteristics for each train model individually extracted from the time-tables

For the three single track lines investigated in general, a cyclical schedule was observed among the passenger trains. For the freight trains a stochastic schedule was observed. This is in accordance to the principle of the freight train operation. Operating freight is mostly a short term decision compared to operating passenger trains, which does not change too much over the longer term. It is not clear which stochastic distribution should be preferred. The previous analysis of the buffer time distribution between the three freight trains schedule confirmed an exponential, a hyper-exponential and an Erlang_2 distribution separately.

5.4 Case study II: Time table characteristics for a selection of pairs of two train models together extracted from the timetables

The analysis of pairs of cyclic passenger train schedules still showed a cyclical domination. If several cyclic schedules are added on the same line, the schedule will be less cyclical in total. This is illustrated in figure 5.6 for a double track line. The cycle time is denoted with t_C. Dirmeier [Dirm77] showed that the inter-arrival time between the cyclic trains in a node is stochastic. This is possible if two or more cyclic timetables (with several cyclic schedules) arrive in the node. This example illustrates that several cyclic schedules in the timetable makes the cyclic pattern in the timetable less dominant. However, there must be many overlapping cyclic schedules to make a stochastic timetable, as is possible in a node. For a single track line the number of cyclic schedules are limited.

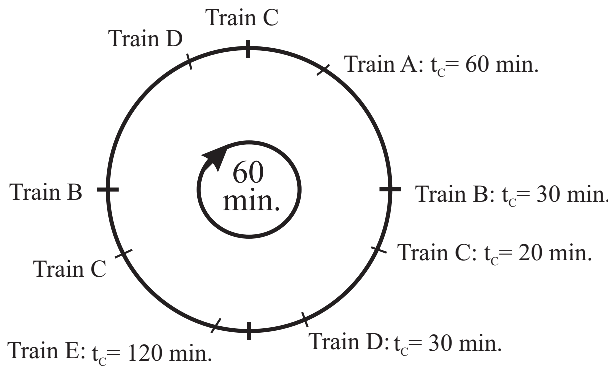

Figure 5.6: Several different cyclical schedules in the same timetable

The buffer time distribution of a cyclic passenger train schedule combined with one stochastic freight train schedule provides no confirmity with the examined distributions for line A and line C. For line B, a hyper-exponential distribution was accepted.

5.5 Case study III: The buffer time extracted from the timetables for all train models together

The χ^2-test of goodness of fit confirms that the three complete timetables have stochastic buffer time distributions. Table 5.4 summarizes the χ^2-test of goodness of fit for all three lines investigated.

Line	*Expo.*	*Erlang*$_2$	*Hyper − expo.*
A	$V_T = 1.66 > 1.2, !$		$V_T = 1.66$
B		$V_T = 1.07 > 0.8, !$	
B	$V_T = 1.36 > 1.2, !$		$V_T = 1.36$
B	$V_T = 1.21$		$V_T = 1.21$
C	$V_T = 1.53 > 1.2, !$		
C	$V_T = 1.03$	$V_T = 1.03 > 0.8, !$	$V_T = 1.03 < 1.2, !$
Sum	2	0	3

Table 5.4: Summary of line A, B and C. The coefficient of variation is given for the corresponding analysis.

For line A the test confirmed an exponential distribution, but the coefficient of variation is too high. For an exponential distribution the variation coefficient should be between $0.8 < V_T < 1.2$. For an Erlang$_2$ distribution the coefficient of variation should be between $0.3 < V_T < 0.8$, and for a hyper-exponential distribution the coefficient of variation should be $V_T > 1.2$. If these criteria are determined, then there are two analysis that correspond to an exponential distribution and three analysis that correspond to a hyper-exponential distribution.

Chapter 6

Results and discussion

In this chapter the model will be compared with the results from the simulation. In the first section 6.1 line D (dummy of Eidsvoll-Hamar) and line A (Eidsvoll-Hamar) used in the simulation are presented. In section 6.2 a failure in the simulation algorithm is documented. In the following sections 6.3 and 6.4 the number of crossings and multiple crossings are analyzed and compared. In section 6.5 the expected waiting time for crossing, merging and the overall waiting time from crossings is analyzed. Finally a discussion followed by a sensitivity analysis is carried out in section 6.6 and section 6.7 respectively .

6.1 Lines investigated by simulation

For the investigation of the number of crossings between trains of equal rank on the entire line, 100 stochastic timetables were simulated and the number of crossings registered for two different single track lines. Line A is the single track line running from Eidsvoll to Hamar described in Chapter 5.2.1. Line A has 9 stations with one or two block sections between the stations. Line D is a dummy version of line A where only one block section is situated between the stations. The 13 block sections in the dummy between Eidsvoll and Hamar are equally spaced. The distance between Eidsvoll and Hamar is not changed from the original Line A. Second, the analysis of line D also serves to demonstrate another type of line compared to line A. A schematic illustration of line D is given in figure 6.1.

For line D and A two simulations were carried out with different priority between the trains of different direction:

- Simulation alternative 1: trains running from Eidsvoll to Hamar have lower priority than trains running from Hamar to Eidsvoll (also denoted with "alternative 1")
- Simulation alternative 2: trains running from both directions have equal priority (also denoted with "alternative 2")

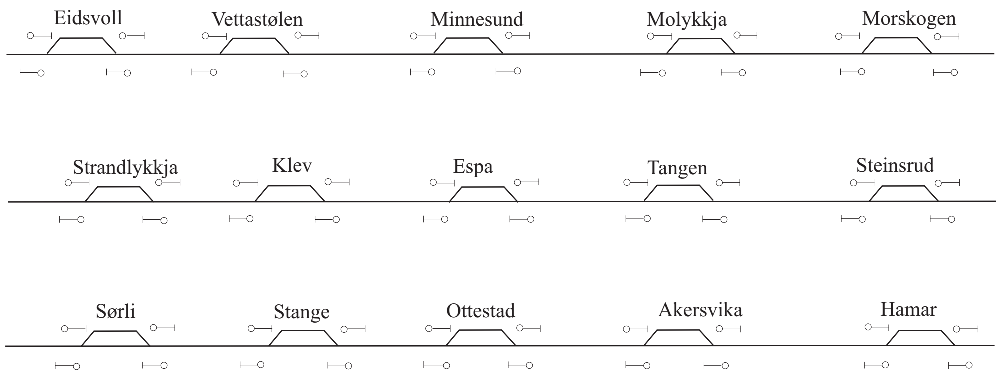

Figure 6.1: Schematic illustration of the dummy of Eidsvoll-Hamar

The main comparing study is between the results from the model established and simulation alternative 1, where the priority strategy is the same. Simulation alternative 2 is introduced to illustrate the effect of a strategy with equal priority between the trains of different direction. Also here a comparing analysis between the results from the model and the simulation is performed. In this case the time of survey and the buffer time between the fast trains are those from simulation alternative 2.

The 100 stochastic timetables were generated in Excel with use of a random function. Both simulation alternatives were carried out with the same timetables. Only trains of rank(1) were included in the timetables. These trains arrived with exponential distributed buffer times. The simulations were carried out with a locomotive of type EL16 with no wagons attached to it[1]. The train runs at its permitted speed at the original line A. In line D the train runs at a speed 70 km/h on the entire line. Six freight trains were scheduled in each direction separately during a survey time of 3 hours. The freight trains run non-stop on the entire line. A stop is only required to solve a conflict. It is assumed that it is possible to stop on every station and siding if necessary. The departure times generated in Excel were constructed in BABSI (Version 6.0, from now on BABSI). These original timetables contained conflicts. The simulation program solved the conflicts by using asynchronous algorithms.

For every simulated schedule the number of crossings between the nodes were recorded. The time of survey became in most cases larger than 3 hours because of the conflict resolution. In the simulation conflicts are solved and the train incurs waiting time which is a positive addition on the time. Therefore a timetable of 3 hours duration with conflicts becomes a timetable with solved conflicts with longer duration. The new survey time after simulation was recorded for every timetable. This will be the time of survey for both the model established and for the simulation. Making use of the same input parameters in the model as in the simulation is necessary for a comparing analysis. The necessary input parameters for the model were recorded in BABSI for all both lines. Details are given in Appendix A.

[1]Note that the original timetable presented in Chapter 5.2.1 is not used. This analysis concerns only the number of crossings between trains of equal rank. Therefore only one train model was chosen.

6.2 Comments to simulation alternative 1

A 100 stochastic timetables were simulated for line D and A respectively. Simulation alternative 1 has the same priority strategy as the model deduced in this thesis. The results from these simulations should be compared with the results from the model. The simulation of these timetables for alternative 1 did not succeed in BABSI. Figure 6.2 is a screen shot from BABSI from a simulated file of line D which illustrates a typical failure BABSI makes during the simulation.

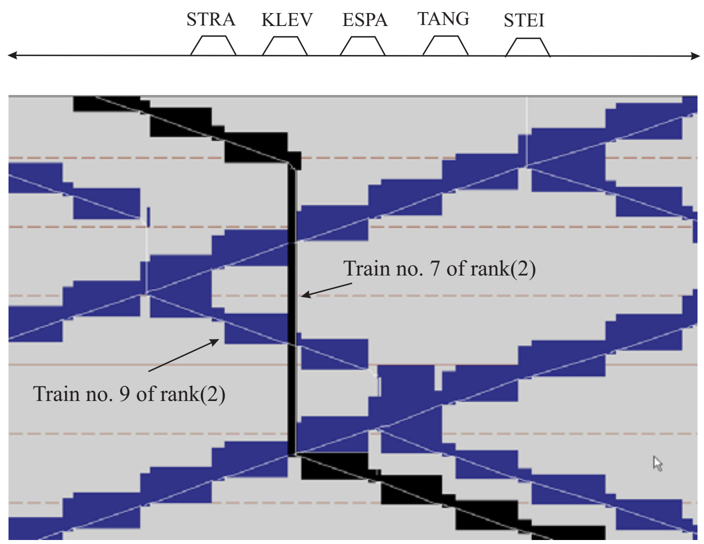

Figure 6.2: Screen shot in BABSI of line D alternative 1

The crossing in station KLEV between train no. 7 of rank(2) and an opposing train of rank(1) illustrates several aspects where BABSI operates differently from the properties in the model. In alternative 1, the trains of rank(2) have lower priority than the opposing trains of rank(1). At this point there is no distinction between the simulation and the model. In figure 6.3 the same file has been adjusted to the condition of the model. The model calculates the probability the distance a train of rank(2) can run before meeting an opposing train. In this case the train no. 7 of rank(2) actually could run to station STEI (Steinsrud) before meeting an opposing train. This action increases the number of crossings calculated compared to the number recorded from the simulation.

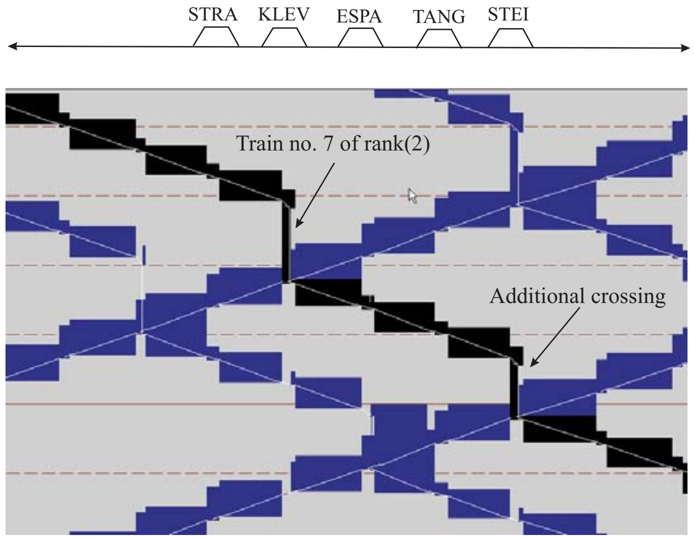

Figure 6.3: Screen shot in BABSI of line D alternative 1 adjusted to model

Figure 6.2 also illustrates that train no. 7 is overtaken by a train of equal rank running in the same direction. According to the model an overtaking cannot take place in the first round of inclusion into the timetable. An overtaking is only possible within the model from the second round of inclusion into the timetable when there is different speed and priority between trains running in the same direction. BABSI treats train no. 7 with lower priority than train no. 9.

The algorithm in BABSI is supposed to solve the conflicts which occurs successively as they appear. If the algorithm in BABSI had operated properly, no overtaking of train no. 7 by train no. 9 in figure 6.2 would occur. Train no. 7 would continue running after the crossing with the opposing train in station KLEV as illustrated in figure 6.3. These failures were found in almost every simulated file. Fifty of the simulated files of line D were adjusted manually to operate as the algorithm was supposed to work. These 50 files will serve as basis for the comparing analysis between simulation alternative 1 and the model deduced. This problem was not detected in simulation alternative 2.

6.3 Comparing analysis of the number of crossings

6.3.1 Analysis of the number of crossings on line D: dummy of Eidsvoll-Hamar

The number of crossings on the entire line with the corresponding time of survey was recorded for each timetable established in BABSI. The number of crossings for the entire line versus frequency for the 50 respectively 100 timetables for simulation alternative 1 and 2 are illustrated in figure 6.4. The peak for simulation alternative

1 is 12 crossings per timetable, whereas the peak for alternative 2 is in the range of 14-17 crossings per timetable.

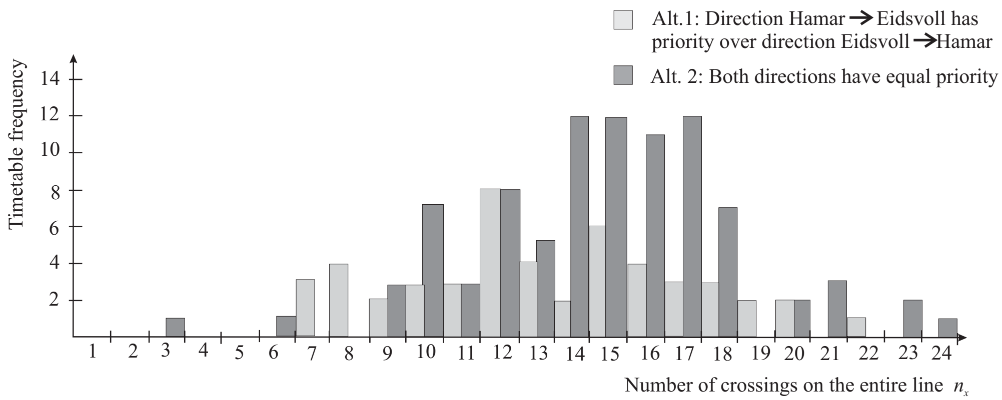

Figure 6.4: Frequency versus the number of crossings for alternative 1 and 2 on line D: dummy of Eidsvoll-Hamar

The number of crossings simulated will be compared to the number of crossings calculated in the next two subsections.

Alternative 1: Direction Hamar to Eidsvoll has priority over direction Eidsvoll to Hamar

The arithmetical mean becomes 13.4 crossings per timetable with variance 14.6 and standard deviation 3.8. The arithmetical mean time of survey was 230.8 min.

The arithmetical mean of 13.4 crossings for the entire line gives:

$$\frac{13.4 \text{ crossings per timetable}}{6 \text{ trains}} = 2.2 \text{ crossings per train}$$

This result is compared with the result from equation (4.36):

$$n_x = \frac{l_{line} - \bar{l}}{\bar{l}} \cdot (1 - e^{-\frac{\overline{\Delta t}}{t_{b,11}}}) \tag{6.1}$$

With $\overline{t_{s,11}} = 4.3$ min., the mean buffer time between the trains of rank(1) becomes:

$$\overline{t_{b,11}} = \frac{\overline{T}}{n_1} - \overline{t_{s,11}} = \frac{230.8 \text{ min.}}{6 \text{ trains}} - 4.3 \text{ min.} = 34.2 \text{ min.}$$

where
n_1 = the number of trains of rank(1) in one direction

The mean time gap necessary for reaching the next station was recorded:
$\overline{\Delta t} = 11.8$ min.

After inserting into equation (6.1) the number of crossings for one train on the entire line gives:

$$n_x = 13 \text{ stations} \cdot (1 - e^{-\frac{11.8 \text{ min.}}{34.2 \text{ min.}}}) = 3.8 \text{ crossings per train}$$

Comparing the estimated result with the simulated result makes a deviation of 1.6 crossings per train. This is 73% larger than the simulated result.

Alternative 2: Both directions have equal priority

The arithmetical mean becomes 15.2 crossings per timetable with variance 13.2 and standard deviation 3.6. The arithmetical mean time of survey was 227.7 min.

The arithmetical mean of 15.2 crossings for the entire line gives:

$$\frac{15.2 \text{ crossings per timetable}}{6 \text{ trains}} = 2.5 \text{ crossings per train}$$

This result is compared with the calculated result. The same mean spacing time is used as in alternative 1. The mean buffer time between the trains of rank(1) becomes:

$$\overline{t_{b,11}} = \frac{\overline{T}}{N_{11}} - \overline{t_{s,11}} = \frac{227.7 \text{ min.}}{6 \text{ trains}} - 4.3 \text{ min.} = 33.7 \text{ min.}$$

After inserting into equation (6.1) the number of crossings for one train on the entire line gives:

$$n_x = 13 \text{ stations} \cdot (1 - e^{-\frac{11.8 \text{ min.}}{33.7 \text{ min.}}}) = 3.8 \text{ crossings per train}$$

where the mean time gap is the same as used in alternative 1.

Comparing the estimated result with the simulated result makes a deviation of 1.3 crossings per train. This is 52% larger than the simulated result.

Comparing the simulation results from alternative 1 with alternative 2 illustrates the effect of difference in priority strategy. With equal priority between the trains from both directions the number of crossings increases compared to the strategy with one superior direction.

6.3.2 Analysis of the number of crossings on line A: Eidsvoll-Hamar

The number of crossings on the entire line with the corresponding time of survey was recorded for each timetable established in BABSI. The number of crossings for the entire line versus frequency is illustrated in figure 6.5. The peak is on 11 crossings recorded 17 times for alternative 2.

The number of crossings from the simulation will be compared to the calculated result in the next subsection.

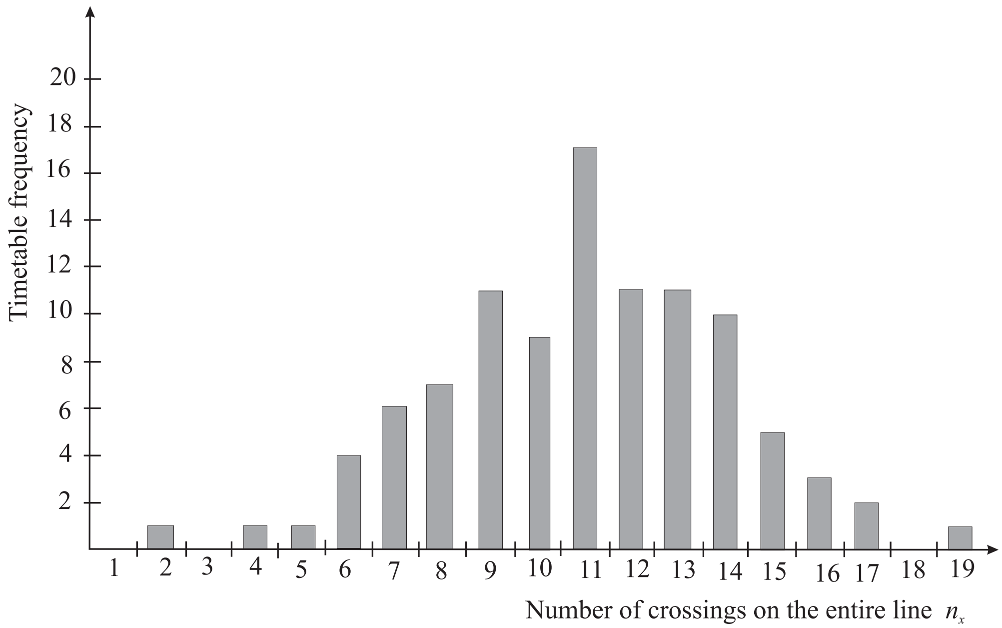

Figure 6.5: Frequency versus the number of crossings for alternative 2 on line A: Eidsvoll-Hamar

Alternative 2: Both directions have equal priority

The 100 registrations give a mean of 11 crossings per timetable with variance 9.2 and standard deviation 3.0. The arithmetical mean time of survey was 217.7 min. The arithmetical mean of 11 crossings for the entire line gives:

$$\frac{\text{11 crossings per timetable}}{\text{6 trains}} = 1.8 \text{ crossings per train}$$

This result is compared with the number of crossings estimated given in equation (6.1).

With $\overline{t_{s,11}} = 3.7$ min. The mean buffer time between the trains of rank(1) becomes:

$$\overline{t_{b,11}} = \frac{\overline{T}}{N_{11}} - \overline{t_{s,11}} = \frac{217.7 \text{ min.}}{6 \text{ trains}} - 3.7 \text{ min.} = 32.6 \text{ min.}$$

The mean time gap necessary for reaching the next station was recorded:
$\overline{\Delta t} = 11.7$ min.

After inserting into equation (4.36) the number of crossings for one train on the entire line gives:

$$n_x = 9 \text{ stations} \cdot (1 - e^{-\frac{11.7 \text{ min.}}{32.6 \text{ min.}}}) = 2.7 \text{ crossings per train}$$

Comparing the estimated result with the simulated result makes a deviation of 0.9 crossing per train. This is 50% larger than the simulated result.

6.4 Comparing analysis of the number of multiple crossings

When a train incurs waiting time because of a crossing it might incur waiting time for merging, too. The number of crossings (multiple crossings) where waiting time for merging occurred, (n_m) were recorded from timetables simulated for line D and A. For every multiple crossing the number of additional passing trains ($n_{1(passing)}$) was registered. In figure 6.6 the terms n_m and $n_{1(passing)}$ are explained.

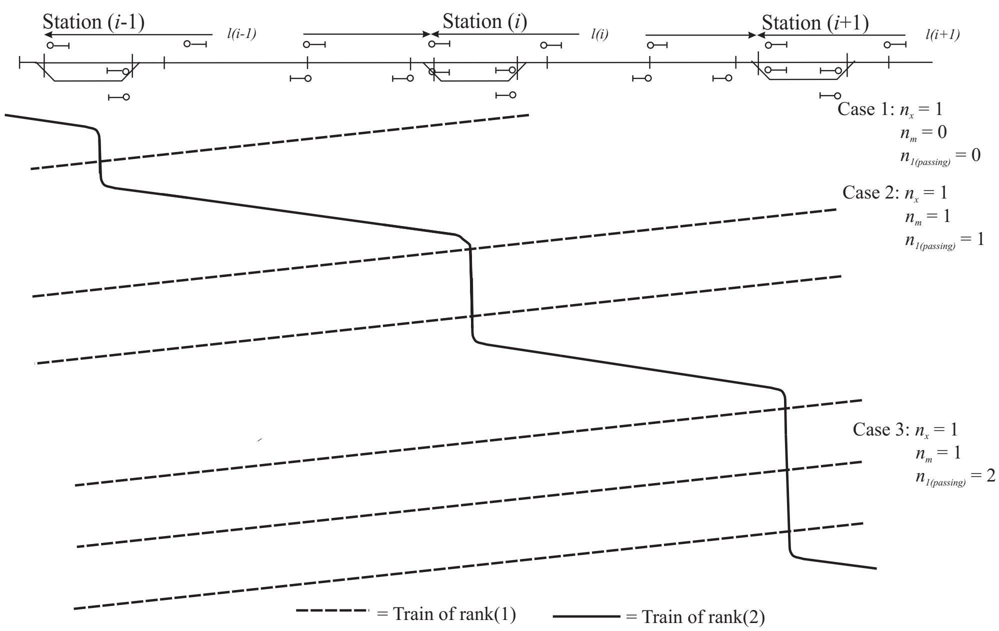

Figure 6.6: Explaination of the terms n_m and $n_{1(passing)}$

In case 1 train of rank(2) only incurs waiting time for crossing. In case 2 train of rank(2) incurs both waiting time for crossing and merging. In this case the train has to wait for only one additional train to pass in station (i). In the 3rd case the train has to wait for two additional passing trains, thus $n_{1(passing)}$ is equal 2 but n_m still remains equal to 1.

6.4.1 Analysis of the number of multiple crossings on line D: dummy of Eidsvoll-Hamar

There were in total 669 crossings recorded among the 50 timetables simulated in alternative 1. Of these 192 were multiple crossings. In alternative 2 there were in total 1520 crossings recorded among the 100 timetables simulated. Of these 402 were multiple crossings. Table 6.1 summarizes the number of multiple crossings with the correspondent number of additional passing trains from the simulated timetables in both alternatives.

	Alt.1 : Non equal priority		Alt. 2: Equal priority	
$n_{1(passing)}$	n_m	$\sum n_{1(passing)}$	n_m	$\sum n_{1(passing)}$
1	160	160	280	280
2	32	64	87	174
3	0	0	28	84
4	0	0	7	28
Sum all timetables	192	224	402	565
Sum/timetable	3.84	4.48	4.02	5.65
Sum/train	0.64	0.75	0.67	0.94

Table 6.1: The distribution of passing trains for every multiple crossing for both alternatives

In alternative 1 there were 160 multiple crossings with only one additional train passing. Only 32 multiple crossings occured with two additional passing trains.

For alternative 2 with equal priority between the directions there were 280 multiple crossings with one additional train passing. In 87 of the multiple crossings there were two additional passing trains. In 28 of the multiple crossings there were three additional passing trains. There were only 7 multiple crossings with four additional passing trains.

For both alternatives the number of additional passing trains drop exponentially in accordance with the exponentially distributed buffer time. Figure 6.7 illustrates the exponential drop in number of passing trains in dependence to the number of multiple crossings.

Figure 6.8 illustrates the frequency of the number of multiple crossings for every timetable recorded for both alternatives. Alternative 1 has its peak at four multiple crossings recorded in 12 timetables. In alternative 2 there were two peaks recorded in 19 timetables for two and five multiple crossings each, respectively.

In figure 6.9 the frequency of additional passing trains for all timetables simulated is illustrated. Alternative 1 has the largest peak at four additional passings trains recorded in 13 timetables. The second largest peak is at two additional passings trains recorded 8 times. In alternative 2 there are several peaks in the interval of 1-6 additional passings trains recorded up to 14 times. This is in accordance with figure 6.8 where the main interval for the number of multiple crossings is 2-5 for alternative 2. The peak at one additional passing train recorded once in alternative 1 can be explained because there are many small time gaps where a train cannot merge back into the line, but still large enough for merging back into the line after one additional

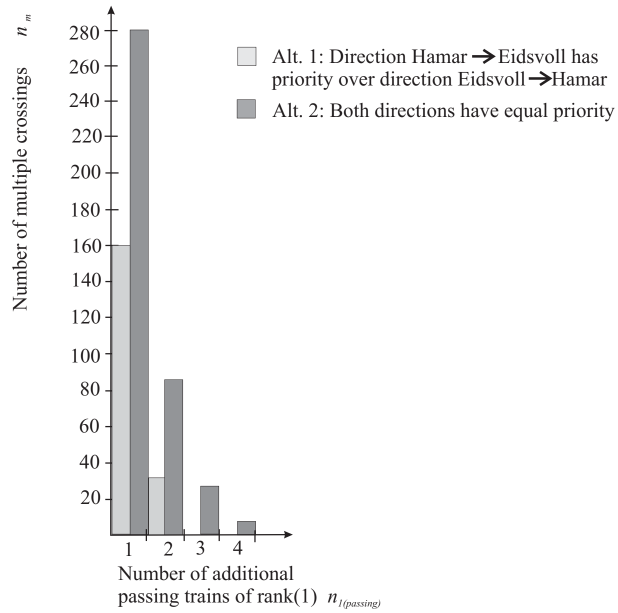

Figure 6.7: Frequency of the number of additional passing trains for every crossing including waiting time for merging for 50 and 100 timetables simulated for alternative 1 and alternative 2

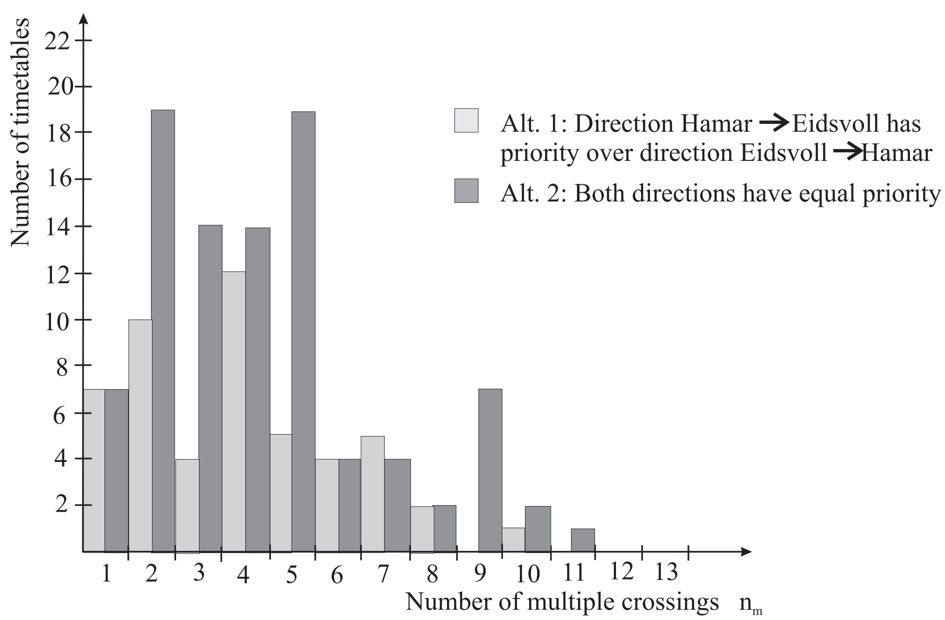

Figure 6.8: Frequency of the number of multiple crossings for every timetable recorded for both alternatives

train has passed. There are fewer large time gaps. There were rather few registrations with the number of additional passing trains larger than 10. This means that after several small time gaps a larger time gap appears which makes it possible for the waiting train to merge back into the line. These occurances are due to the exponential distributed buffer time.

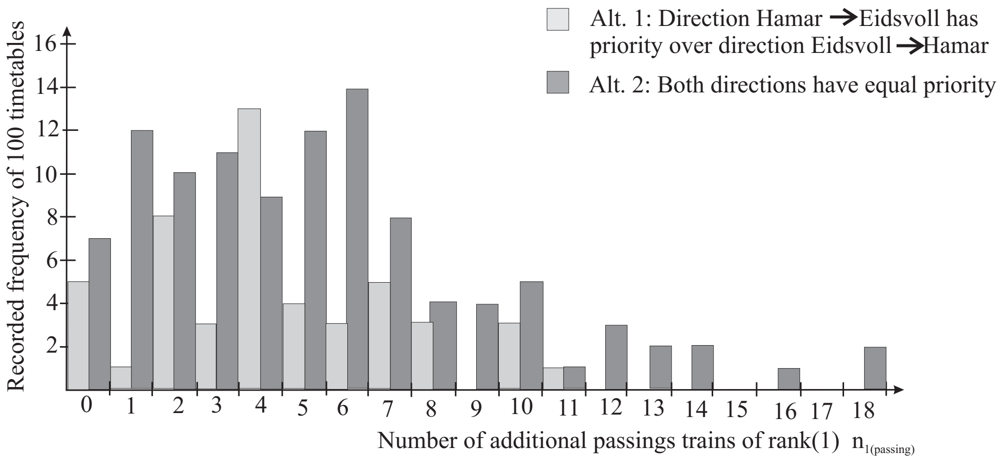

Figure 6.9: Frequency of the number of additional passing trains for every timetable recorded for both alternatives

Comparing the number of multiple crossings on line D

The number of multiple crossings from the simulation is compared with the estimated number of multiple crossings for alternative 1 and 2 on line D.

The sum of incuring waiting time for merging is given by:

$$\sum p(t_{b,11} < t_{s,21} + \Delta t_{s,1}) = (1 - e^{-\frac{t_{s,21}+\Delta t_{s,1}}{t_{b,11}}}) \cdot e^{\frac{t_{s,21}+\Delta t_{s,1}}{t_{b,11}}} \tag{6.2}$$

Inserting data into equation (6.2) with 5.7 min. for $t_{s,21}$ gives for alternative 1:

$$\sum p(t_{b,11} < t_{s,21} + \Delta t_{s,1}) = (1 - e^{-\frac{5.7 \text{ min.}}{34.2 \text{ min.}}}) \cdot e^{\frac{5.7 \text{ min.}}{34.2 \text{ min.}}} = 0.18$$

For every crossing there is a probability of 0.18 for incuring waiting time for merging. For alternative 2 there is also a probability of 0.18 for incuring waiting time for merging. In section 6.4.1 the number of crossings per timetable was estimated to be 22.8 for both alternatives. The estimated number of multiple crossings per timetable then becomes for alternative 1:

$$n_m = 22.8 \text{ crossings/timetable} \cdot 0.18 = 4.1 \text{ multiple crossings per timetable}$$

There were 3.8 multiple crossings per timetable simulated. This gives a deviation of 0.3 which is 8% larger than the simulated result. For alternative 2 there were 4.0

multiple crossings per timetable simulated. The estimated result gives a deviation of 0.1 which is only 3% larger than the simulated result.

6.4.2 Analysis of the number of multiple crossings on line A: Eidsvoll-Hamar

In alternative 2 there 1100 crossings registered of the 100 timetables generated. Of them 330 were multiple crossings. Table 6.2 summarizes the number of multiple crossings with the corresponding number of passing trains from the 100 timetables simulated.

	Alt. 2: Equal priority	
$n_{1(passing)}$	n_m	$\sum n_{1(passing)}$
1	228	228
2	81	162
3	16	48
4	5	20
Sum all timetables	330	458
Sum/timetable	3.3	4.58
Sum/train	0.55	0.76

Table 6.2: The distribution of passing trains for every multiple crossing for alternative 2

In alternative 2 there were 228 multiple crossings with only one additional train passing. There were 81 multiple crossings with two additional passing trains. Alternative 2 further has only 21 multiple crossings with three to four additional passing trains. As for line D the number of additional passing trains drops exponentially in accordance with the exponentially distributed buffer time. Figure 6.10a illustrates the drop in number of passing trains in dependence to the number of multiple crossings.

Figure 6.10b illustrates the frequency of the number of multiple crossings for every timetable simulated. The peak is at three multiple crossings recorded in 22 timetables. There were only 7 timetables without any multiple crossing. There were few timetables with more than four multiple crossings.

Figure 6.11 illustrates the frequency of the number of additional passing trains. Alternative 2 has three peaks in the interval with one to three additional passing trains recorded up to 19 times each. The histogram of line A shows the same effect of the exponential distributed buffer time as for line D.

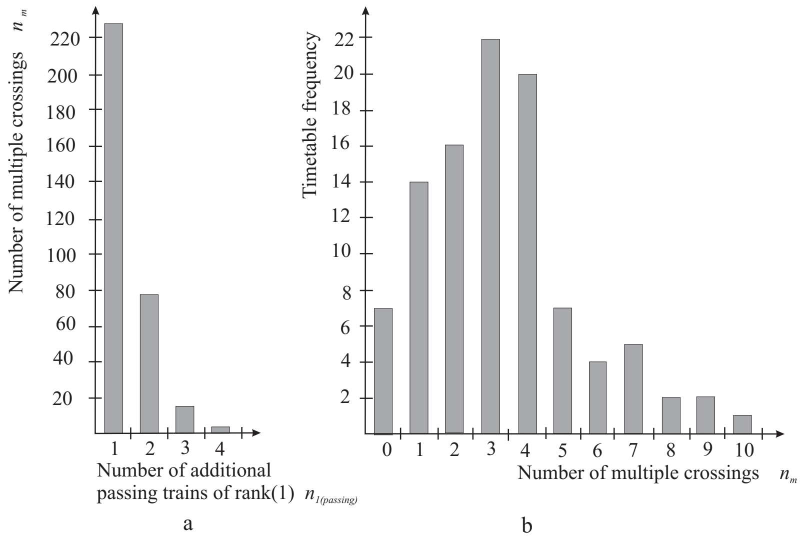

Figure 6.10: A: Frequency of the number of additional passing trains for every multiple crossing for 100 simulated timetables. B: Frequency of the number of multiple crossings for 100 simulated timetables

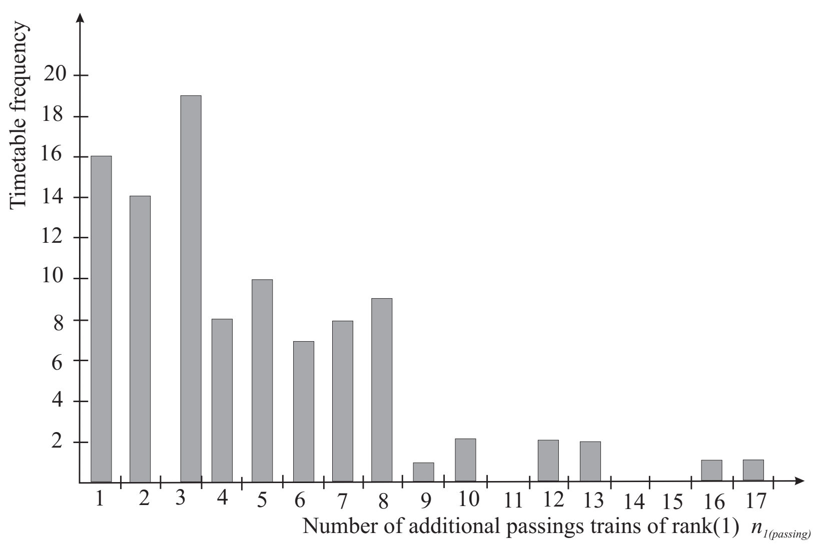

Figure 6.11: Frequency of the number of additional passing trains for 100 simulated timetables

Comparing the number of multiple crossings on line A

Only the number of multiple crossings simulated in alternative 2 could be compared with the estimated result. Inserting 6.5 min. for the mean of $t_{s,21}$ and 2.0 min. for the mean of $\Delta t_{s,1}$ into equation (6.2) gives the sum of incuring waiting time for merging:[2]

$$\sum p(t_{b,11} < t_{s,21} + \Delta t_{s,1}) = (1 - e^{-\frac{6.5 \text{ min.}+2.0 \text{ min.}}{32.6 \text{ min.}}}) \cdot e^{\frac{6.5 \text{ min.}+2.0 \text{ min.}}{32.6 \text{ min.}}} = 0.30$$

For every crossing there is a probability of 0.30 for incuring waiting time for merging. In section 6.3.2 the number of crossings per timetable were estimated to be 16.2 in alternative 2. The estimated number of multiple crossings per timetable becomes:

$$n_m = 16.2 \text{ crossings/timetable} \cdot 0.30 = 4.9 \text{ multiple crossings per timetable}$$

There were 3.3 multiple crossings per timetable simulated. This gives a deviation of 1.6 which is 48% larger than the simulated result.

6.5 Comparing analysis of the overall waiting time

To illustrate the difference in waiting time between the model and BABSI, 50 of the simulated timetables of line D for both alternatives were recorded separately with respect to the generated waiting time. Due to the rather low number of timetables analyzed, the results given are a rough estimation which serves for the discussion of the waiting time calculated. Details are given in Appendix A.2.1. The waiting time of the 50 timetables was registered in three groups:

- Waiting time for crossing including the minimum waiting time necessary for crossing.

- Waiting time for merging when a crossing had occurred.

- Waiting time from overtaking.

The recorded results are summarized in table 6.3.

[2]Since the occupation time between the stations is spreading mean quantities are inserted for $t_{s,21}$ and $\Delta t_{s,1}$.

	Alt. 1	Alt. 2
Waiting time	[min./timetable]	[min./timetable]
$T_{wx} + min\ t_x$	125	143
T_{wm}	36	38
$T_{wx} + min\ t_x + T_{wm}$	161	181
$\sum T_{ox} + T_{OMom} + min\ t_o$	0	25
$\sum T_{wx} + min\ t_x + T_{wm} + T_{ox} + T_{om} + min\ t_o$	161	206

Table 6.3: Mean waiting time for 50 simulated timetables for both alternatives

In both alternatives the waiting time for crossing is much larger than the waiting time for merging. The waiting time for crossing in alternative 2 is larger than in alternative 1. This is due to the difference in priority strategy. An equal priority strategy generates more crossings than with a non-equal priority strategy. The waiting time for merging is almost equal in both alternatives. In a non-equal priority strategy a train of lower rank cannot continue to the next station if an opposing train is entering the common section. A train in simulation alternative 1 therefore risks longer waiting time for merging compared to a flexible priority strategy as in alternative 2. On the other hand simulation alternative 2 has more crossings than simulation alternative 1. For every crossing there is a risk of incuring waiting time for merging. In simulation alternative 2 on average every 3.8 crossing is a multiple crossing, whereas for simulation alternative 1 on average every 3.5 crossing a multiple crossings takes place. This confirms that alternative 1 generates more waiting time for merging relative to the number of crossings than alternative 2.

Simulation alternative 2 also generates waiting time for overtaking, which is not possible in alternative 1. The overall waiting time from crossing is larger in alternative 2 than in alternative 1. This is due to the larger waiting time for crossing and the waiting time for overtaking in alternative 2. This relation is illustrated in figure 6.12.

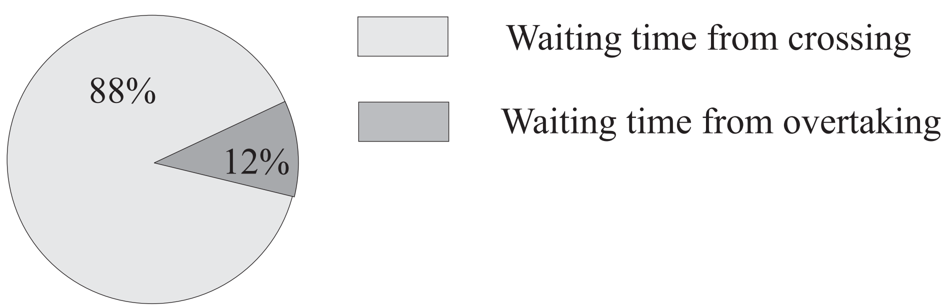

Figure 6.12: Waiting time from crossing is dominating waiting time from overtaking for alternative 2 on line D

From table 6.3 the waiting time for one crossing can be calculated:

$$125 \text{ min.}/13.4 \text{ crossings/timetable} - 5.80 \text{ min.} = 9.3 \text{ min.} - 5.80 \text{ min.} = 3.5 \text{ min.}$$

The quantity 5.80 min. is the minimum time necessary for one crossing. The waiting time for merging becomes:

$$36 \text{ min.}/13.4 \text{ crossings/timetable} = 2.7 \text{ min.}$$

These numbers for both alternatives are listed in table 6.4-6.5 and compared with the simulated results.

Waiting time	Simulation	Calculation	Deviation	Deviation [%]
$E[t_{wx}]$	3.5	4.7	1.2	34
$E[t_{wm}]$	2.7	1.5	-1.2	-44
$\sum min\ t_x + E[t_{wx}] + E[t_{wm}]$	12.0	12.0	0	0

Table 6.4: Comparing the simulated result for one crossing with the calculated result for alternative 1 on line D. The units are given in [min./crossing].

Waiting time	Simulation	Calculation	Deviation	Deviation [%]
$E[t_{wx}]$	3.6	4.7	1.1	31
$E[t_{wm}]$	2.5	1.5	-1.0	-40
$\sum min\ t_x + E[t_{wx}] + E[t_{wm}]$	11.9	12.0	0.1	1

Table 6.5: Comparing the simulated result for one crossing with the calculated result for alternative 2 on line D. The units are given in [min./crossing].

Table 6.4 illustrates the deviations between the estimated and the simulated quantities. The model estimates an expected waiting time for crossing which is 34% larger than the simulated quantity. The expected waiting time for merging is estimated as 44% lower than the simulated result. These deviations are large. The overall waiting time from crossing does not deviate from the simulated result. The same trend is observed when comparing simulation alternative 2 with the model.

6.6 Discussion

6.6.1 Discussion of the number of crossings and multiple crossings

Table 6.6 compares the results for the number of crossings and multiple crossings respectively from simulation alternative 1 with the estimations from the model developed.

	Simulation	Calculation	Deviation [%]
n_x[per train]	2.2	3.8	73
n_m[per train]	0.64	0.68	6

Table 6.6: Summary of results from simulation of line D, alternative 1, and calculation for the number of crossings and multiple crossings

The estimation of the number of crossings from the model is 73% larger than the simulated quantity. This deviation is very large. The number of crossings forms the product with the expected waiting time from one crossing. Therefore the number of crossings is an important factor in the equation for the calculation of the entire waiting time from crossing.
The estimated number of multiple crossings deviates by 6% from the simulated result. A deviation of 6% is acceptible. Almost the same deviations between the model and the simulation in alternative 2 is registered and given in table 6.7. The deviations between the results of the model and simulation alternative 2 is a little less than for simulation alternative 1.

Table 6.7 compares the results from simulation alternative 2 with the results from the model.

	Simulation	Calculation	Deviation [%]
n_x[per train]	2.5	3.8	52
n_m[per train]	0.67	0.68	1

Table 6.7: Summary of results from simulation of line D, alternative 2, and calculation for the number of crossings and multiple crossings

The small deviation between the estimated number of multiple crossings and the simulated result can have its origin in the stochastic generation of buffer times when creating the requested timetables for simulation. This explaination does not hold for the deviation between the estimated number of crossing and the simulated result. These deviations are far too large. The most reasonable explaination for the large deviations of the number of crossings between the model and the simulated result is that only 50 timetables investigated is far too small to serve as a basis for a comparative study. Calculating the chained exploitation rate [UIC/406] for some of the simulations in alternative 1 gives a chained exploitation rate between 0.8-1.0. Opposing traffic on single track lines generally generates larger chained exploitation rates compared to lines with unidirectional traffic. In general large chained exploitation rates require many simulations to achieve a stable result.

In simulation alternative 1 the mean number of crossings were calculated to be 13.4 crossings per timetable with a variance of 14.6. The large variance illustrates that that there is a large spreading of the data registered. This indicates that the number of crossings calculated from the registrations is not a final number.

Comparing simulation alternative 1 with simulation alternative 2 illustrates that an equal priority strategy between the trains of opposing directions generates more crossings and multiple than a strategy with strict priority between the trains. This observation seems logical, and would probably not be affected by an increased number of simulations.

6.6.2 Discussion of the expected waiting time for crossing and merging

The deviation in the expected waiting time for crossing and merging in table 6.4 is very large. The expected waiting time for crossing estimated is larger than the simulated quantity. Contradictionally the expected waiting time for merging estimated is far to low. The estimated overall waiting time from crossing is equal to the simulated result. This means that the low quantity of the expected waiting time for merging compensates for the too large estimated quantity for the expected waiting time for crossing. For a fixed number of trains within a specified time and area of survey, a dependency seems to exist between the waiting time for crossing and merging. The more crossings a schedule has, the less number of multiple crossings are expected and vice versa. The large deviations for both expected waiting time for crossing and merging can only be explained by the low number of simulations carried out. For this reason it might be a coincidence that the overal waiting time from crossing estimated is equal to the simulated result.

In the first round of inclusion into timetable according to the model established, waiting time from overtaking is not possible. In the second round of inclusion into the timetable overtakings are possible. The model deduced makes a clear distinction between waiting time for merging after a crossing, i. e. illustration A in figure 6.13 or after an overtaking, i. e. illustration B. Figure 6.13 also illustrates two other variants for the number of multiple crossings and overtakings, variant C and D, which occured in simulation alternative 2. In variant C a train has to wait because of a crossing followed by an overtaking train. Variant D illustrates the opposite situation, an overtaking followed by a crossing. In the model developed, only the variants A and B are possible. In fact, variant C and D are realistic cases in the second round of inclusion.

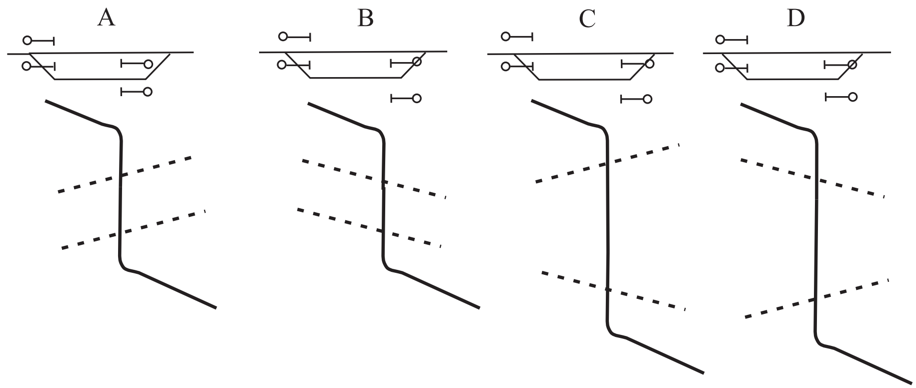

Figure 6.13: Variants of multiple crossings and overtakings

The model deduced is a two-train model where one model of lower rank is included into the timetable with a train model of higher rank. For example, in the inclusion of the third round train model with rank(1) and rank(2) are grouped together as rank(1) when train model of rank(3) is included into the timetable as rank(2). When grouping two train models into one train model, new mean quantites for each parameter used in the model must be calculated. The new time gap for train of rank(1) in the third round of inclusion is based on the previously used time gaps in inclusion round 2. This leads to larger variances of the parameters, and the accurancy of the estimations decreases. One opportunity to reduce the variance when calculating a new mean quantity for the second round of inclusion would be to weight the time gaps with respect to the number of trains of the respective train model.

6.6.3 Discussion of the model with constant buffer time

Variation in infrastructure, velocity and rolling stock characteristics influences the minimum spacing time $t_{s,21}$ and the time gap between the stations. Together these form the buffer time between the trains of rank(1). A variation of these parameters might result in a variation of the buffer time. The larger the variation of the minimum spacing time and the time gap between the trains of rank(1) is, the more a variation of the buffer time between the trains of rank(1) is expected. The approximation of constant buffer time will not hold for large variations of these parameters given in equation (4.54)- (4.55). In the model there is assumed that only few variations of the parameter quantity might be acceptible. Under this assumption the model could be derived. In reality variations of the parameters can be expected. Due of this weakness this model has not been further analyzed by simulation. With small deviations in the buffer time between the trains of rank(1), the model might serve as an estimation for the prediction of the number of crossings and the waiting time from crossing. Therefore the approximated waiting time from crossing and overtaking given in equation (4.56) and (4.57) where the distance between the station is spreading is a better approximation in most cases. There already exist several models for the purpose of optimization of cyclic timetables, for example by using graph theory [Brak93].

6.6.4 Discussion of the timetable characteristics on single track lines

The investigation of the buffer time distribution of the three different timetables of the three different lines illustrates that cyclic and stochastic timetable patterns are combined in the same schedule. For the part of the schedule following a stochastic pattern, two buffer time distributions conformed with an exponential distribution and three buffer time distributions conformed with a hyper-exponential distribution. The analysis of only three single track lines is too small to make a general statement about which distribution the buffer time between the trains conforms with. This analysis illustrates that stochastic buffer time distributions were found in parts of these timetables. This supports the assumption of exponential buffer time distribution betwen the trains of rank(1) made in the model.

6.7 Sensitivity analysis

A sensitivity analysis was carried out on the model developed with the parameters from simulation alternative 1 on line D. The effect of increasing each parameter quantity separately with 10% on the number of crossings and the waiting time was studied. Table 6.8 lists the results.

Parameter	Alt. 1 [min.]	+10% [min.]	n_x[%]	$E[t_{wx}]$[%]	$E[t_{wm}]$[%]	T_{WX} [%]
$\overline{t_{b,11}}$	34.2	37.6	-7.7	2.1	-9.7	-8.1
$\overline{\Delta t}$	11.8	13.0	8.2	9.3		12.2
$\overline{t_{s,21}}$	5.7	6.3		-1.7	14.6	1.1
$\overline{t_{s,12}}$	5.3	5.8			6.6	1.0

Table 6.8: The effect on the waiting time with an increase of 10% to the parameter quantity

The mean time gap and the mean buffer time between the trains of rank(1) have almost both equal influence on the number of crossings. The time gap has remarkably more influence on the expected waiting time for crossing than the mean buffer time and the mean minimum spacing time $\overline{t_{s,21}}$ have. In the analysis of the expected waiting time for merging the mean minimum spacing time $\overline{t_{s,21}}$ has the greatest effect. On the overall waiting time from crossing both the mean time gap and the mean buffer time between the trains of rank(1) have a large effect. The mean minimum spacing time $\overline{t_{s,12}}$ hardly generates any change to the overall waiting time.

A change in the mean time gap probably has the greatest influence apart from the mean buffer time between the trains of rank(1) on the waiting time from crossing. A change in an exponent generates large effects on the output. A change in the buffer time is due to a change in the number of trains. A change in the time gap is controlled by the infrastructure (especially the signaling system), the rolling stock characteristics and the train velocities. On this information the blocking time stairs are generated and the time gap is calculated.

On line D the stations are equally spaced and the maximum permitted train velocity on the line stays constant. The rolling stock characteristics is the same within the same train model. In this special case a variation in the time gap is not reasonable. In most real cases the velocity on a line shifts, and the stations are generally not exactly equally spaced. Line A is a real case. The time gap represents the spacing of the stations. In table 6.9 a summary of the variance analysis of the mean time gap on line A is given.

Parameter	Direction	Variance	Smallest reg. [min.]	Largest reg. [min.]
$\overline{\Delta t}$	Eidsvoll→ Hamar	2.6	8.9	14.2
$\overline{\Delta t}$	Hamar→ Eidsvoll	4.4	8.5	14.5
$\overline{\Delta t}$	Both directions	3.5	8.5	14.5

Table 6.9: Summary of the variance and spreading of the time gap on line A

The variation of the time gap on line A is smaller in direction Eidsvoll→Hamar, which is simulation alternative 1, than for the opposite direction. Simulation alternative 2 with equal priority uses the mean time gap of both directions together. The smallest registration of Δt in direction Eidsvoll→Hamar is 15% lower than the mean time gap calculated ($\overline{\Delta t} = 7.5$ min.). The largest registration is 25% larger than the mean quantity. The results in table 6.8 illustrate that even a deviation of 10% in Δt gives about 12% change in the overall waiting time from crossing. This is probably the most sensitive point in the model. When calculating the mean time gap for both directions together the spreading in Δt increases.

Single track railway lines with variations in the distance between the stations generally results in variations in the time gap. Long distances between the stations can give longer occupation times than shorter distances between the stations. Long distances can be divided into several block sections. Line A is an example where some sections between the stations contain one block section whereas other setions contain two block sections. Figure 6.14 illustrates that the time gap for line A becomes larger when there are two block sections (between station Strandlykkja and Espa) than when there are one block section (between station Tangen and Steinsrud). The variance of 4.4 for direction Hamar to Eidsvoll confirms that there are large variations in the time gap for this direction.

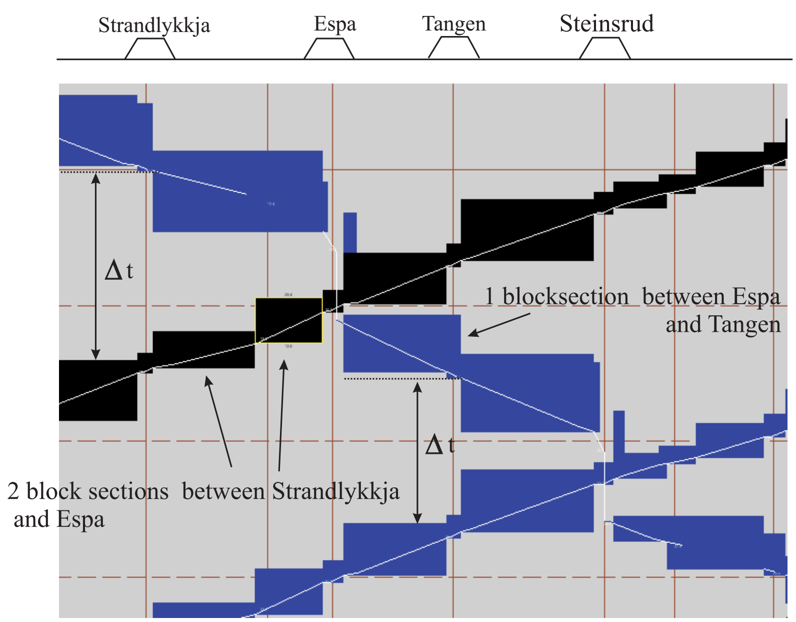

Figure 6.14: Screen shot in BABSI of line A alternative 1

Chapter 7

Conclusion

7.1 Conclusions

During this work a new model for the calculation of the expected scheduled waiting time from crossing with exponential buffer time distribution on single track lines has been developed.

At first the assumption made by Schwanhäußer [Schw81] for the overtaking case was analyzed in detail and compared with the crossing situation on single track line. Similarities and dissimiliarities have been detected between the overtaking and crossing situation. New assumptions had to be developed for the crossing situation. On this basis it has been possible to deduce the expected waiting time for one crossing and the expected waiting time for merging. The mean number of crossings one train incurs has also been derived. Finally, the model has was compared with the results from an asynchronous simulation tool (BABSI).

A model for the estimation for the scheduled waiting time from crossing with constant buffer time between the trains was also derived.

7.1.1 Conditions

It has been shown that the condition for merging back into the line after a crossing given in (4.7) follows the similar principle as for merging back into the line after an overtaking given in equation (4.1). In the crossing case the buffer time between the fast trains must be the quantity $t_{s,FS} - t_{s,FF}$ larger than in the overtaking case.

In the model an exponential buffer time distribution between the trains of rank(1) is assumed. An independent timetable request from operators with different train models generates a stochastic arrival process of trains to the timetable coordinator. The exponential distribution is one suitable stochastic distribution. In reality it is so far not known which stochastic distribution the buffer time follows. The main focus in this study was to show that is was possible to deduce a model for the estimation of the scheduled waiting time from crossing with a stochastic arrival process. The exponential distribution is therefore assumed to be satisfactory for this study.

7.1.2 Factors influencing the expected waiting time from crossing with exponential buffer time distribution

During the derivation of the expected waiting time for crossing it has been shown that the time gap necessary for a train to reach the next station (i) before being overtaken or meeting an opposing train must be larger in a crossing situation than in an overtaking situation.

The calculation of the expected waiting time for crossing and the number of crossings the time gap a train needs for reaching the next station has the largest influence, and the buffer time between the trains of rank(1) has the second largest influence on the result. The model uses the mean time gap as input parameter. Large variations in the time gap on the line will reduce the reliability of the calculations. An analysis of the single track line from Eidsvoll to Hamar has shown that the distance between the stations is spreading. For a model train of type EL16 a mean time gap of 11.7 min. with variance 3.5 was calculated for both directions together. The line from Eidsvoll to Hamar serves as an example that the time gap in reality can spread. If the variations in the time gap is too large, then the model is not recommended.

The same statement applies for the calculation of the expected waiting time for merging. Large variations of the minimum spacing time train of rank(2) followed by a train of rank(1) has the largest influence on the expected waiting time for merging. For the calculation of the overall waiting time from crossing, the waiting time for merging is small compared to the overall waiting time. The time gap and the buffer time between the trains of rank(1) has the largest influence on the overall waiting time from crossing since they are included in both number of crossing and the expected waiting time for crossing.

7.1.3 Factors influencing the expected waiting time from crossing with constant buffer time

A model for the estimation of the scheduled waiting time from crossing with constant buffer time between the trains of rank(1) has been deduced. The mean time gap has also in this case the most influence on the waiting time from crossing. Variation in the time gap generates variations in the buffer time which is a contradiction to the assumed constant buffer time between the trains of rank(1). With small deviations in the buffer time the model might serve as an estimation for the prediction of the number of crossings and the waiting time from crossing. Due to this weakness other existing models for the optimization of cyclic timetables should be preferred.

7.1.4 Priority strategy has influence on the scheduled waiting time

The results from the model were compared with the results from randomly generated timetables simulated in BABSI. There were two different simulation alternatives investigated. Simulation alternative 1 had the same priority strategy as the model deduced where trains of one direction took priority over trains running in the opposite direction. Simulation alternative 2 had equal priority between the trains of opposite directions. This simulation alternative served to illustrate the effect of an equal priority strategy compared to a superior strategy.

Comparing the simulation results from alternative 1 with alternative 2 in this study illustrates that an equal priority strategy generates more crossings than a strategy with superior priority for one direction. In this case alternative 2 generated 12% more crossings compared to alternative 1. On the other hand a strategy with superior priority generates more multiple crossings per crossing compared to an equal priority strategy. In this case there were about 12% more multiple crossings per crossing compared to alternative 2. These observations are probably independent of the number of timetables investigated.

7.2 Reliability of the model

The accuracy of the model decreases with increasing number of inclusions in the timetable. In the model deduced trains of rank(2) running in one direction have lower priority than trains of rank(2) running in the opposite direction. In the first round of inclusion into the timetable, these trains are of the same train model. In the second round of inclusion a second train model is included into the conflict free timetable of the first train model. New mean quantities of the input parameters of the model must be calculated. The variance of the parameters will probably increase.

The simulations results from simulation alternative 1 illustrates that there is large spreading of the results. The mean number of crossings simulated became 13.4 crossings/timetable with variance 14.6. Comparing the simulated result with the calculated number of crossings showed a deviation of 73% which is very large. The number of crossings is a very important part of the formula for the calculation of the overall waiting time since it forms a product with the expected waiting time for one crossing including merging.

The main reasons for the large deviation is probably that the investigation of 50 random timetables is not enough to make an accurate statement. Analysis of the chained exploitation rate expresses a system which is highly exploitated, between 0.8-1.0. Exploitations rates which are that high indicates that there might be many simulations necessary to generate a stable result. The simulations in this analysis were performed manually since there were no other suitable tool available during the study period. Comprehensive simulation work needs to be carried out to achieve a more precise eval-

uation statement.
The other reason is an asynchronous simulation algorithm that solves the existing conflicts successively by moving the non-priority train to a later departure time than requested. This results in that trains are moved out of the time of survey window to later departures than 3 hours from start. After these 3 hours there are no requested train departures, therefore these trains do not run into any further conflicts or crossings. This contributes to underestimating the number of crossings and number of multiple crossings. A lower number of crossing incurs lower waiting time from crossing.

7.3 Suggestions for further research

The study of crossing situations on single track lines is very complex with many parameters involved. Within the scope of this thesis the most important aspects concerning the generation and computation of the scheduled waiting time from crossing has been prioritized.

For a better evaluation of the model there must be investments made in improving and developing the simulation tools for the generation of stochastic timetables followed by a simulation algorithm for the generation of conflict free timetables with the necessary output information. With this strategy, a very large number of timetables could be investigated which would improve the basis for the evaluation of the model. If the simulation results should confirm the model as reliable, further work can be invested into expanding the model to other conditions.

In future work the most interesting option would be to expand the model to apply for equal priority between trains of different directions. This could be done by introducing a point between the sections where the train paths would cross if the trains started from the station at the same time. The train first passing this point can continue to the next station whereas the other opposing train has to wait in the previous station. The probability for passing the point for a train of one direction can be calculated using the same principle as in the model for strict priority for one direction. Another option could be to separate the estimation of the number of multiple crossings into different situations where a crossing is followed by an overtaking. Finally, the buffer time distribution between the trains of rank(1) can be analyzed and substituted by an other distribution.

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Appendix A

Simulation results and quantities recorded

In section A.1 the simulation preparations and conditions are listed. In section A.2 the simulation results recorded are listed in tables. In section A.3 - A.5 the quantities recorded from BABSI serving as input data for the computations are presented.

A.1 Simulation preparations

A.1.1 Train characteristics

The train model 100 SBB Re 4/4 III in the train model database of Faktus.451 was manipulated to an EL 16 (Norwegian freight train locomotive). This locomotive has maximum velocity of 140 km/h and a mass of 80 metric tons. The haulage capacity graph had to be found for every 2.5 km/h, given in table A.1.

Velocity [km/h]	Estimated force [kN]	Given force [kN]
0-45	320 (const.)	320 (const.)
47.5	309.3	
50	298.8	298
52.2	288.7	
55	278.8	277
57.5	269.3	
60	260.0	260
62.5	251.0	
65	242.4	243
67.5	234.0	
70	225.9	227
72.5	218.1	
75	210.7	212
77.5	203.5	
80	196.6	198
82.5	190.0	
85	183.7	184
87.5	177.7	
90	172.0	172
92.2	166.6	
95	161.3	162
97.5	156.3	
100	151.5	152
102.5	146.9	
105	142.5	143
107.5	138.3	
110	134.3	134
112.5	130.6	
115	127.0	127
117.5	123.7	
120	120.5	120
122.5	117.6	
125	114.9	113
127.5	112.4	
130	110.1	109
132.5	108.0	
135	106.1	105
137.5	104.5	
140	103.0	103

Table A.1: Haulage capacity for EL16 estimated every 2.5 km/h

A.1.2 Creation of stochastic timetables

The timetables were generated in Excel. With the random generator, buffer times were generated following an exponential distribution. To make the simulation feasible a survey time of 3 hours was chosen. In this analysis six trains run in a time window of 3 hours in each direction. The six buffer times for each direction was generated in Excel. In some cases the trains in a direction and opposite direction did not match within the same time window of 3 hours. This can happen when generating large buffer times for one directions and small buffer times for the opposite direction. For making an analysis on the number of crossings it made sense to adjust the buffer time (still following an exponential distribution) to fit into the same time window. This adjustment holds if the operator making the order of train paths plans to order 6 trains in each direction within the time window, which would be natural in reality. The 3 hours represent the 24 hours of a day. Both software tools Faktus and BABSI are created for a 24 hour analysis. In the simulation, BABSI does not realize the time window of 3 hours. BABSI treats the schedule as it was to fit into a 24 hour schedule. BABSI solves the conflicts between the trains and moves the trains "'downwards"' and exceeds 3 hours. Therefore a new time of survey had to be recorded after the simulation to establish the same basis for the further computations.

A.1.3 Simulation parameters

Figure A.1 and A.2 illustrates the parameters chosen in BABSI. Figure A.1 illustrates that the simulation is to be carried out without any buffer time between the trains when solving a conflict. This is chosen to make the analysis more easy to handle and has no influence on the analysis result.

Figure A.2 illustrates the standard options BABSI suggests. The only change is done in the last window. Option 3 is chosen instead of option 4. This makes it possible for freight trains of equal rank to stop if necessarry.

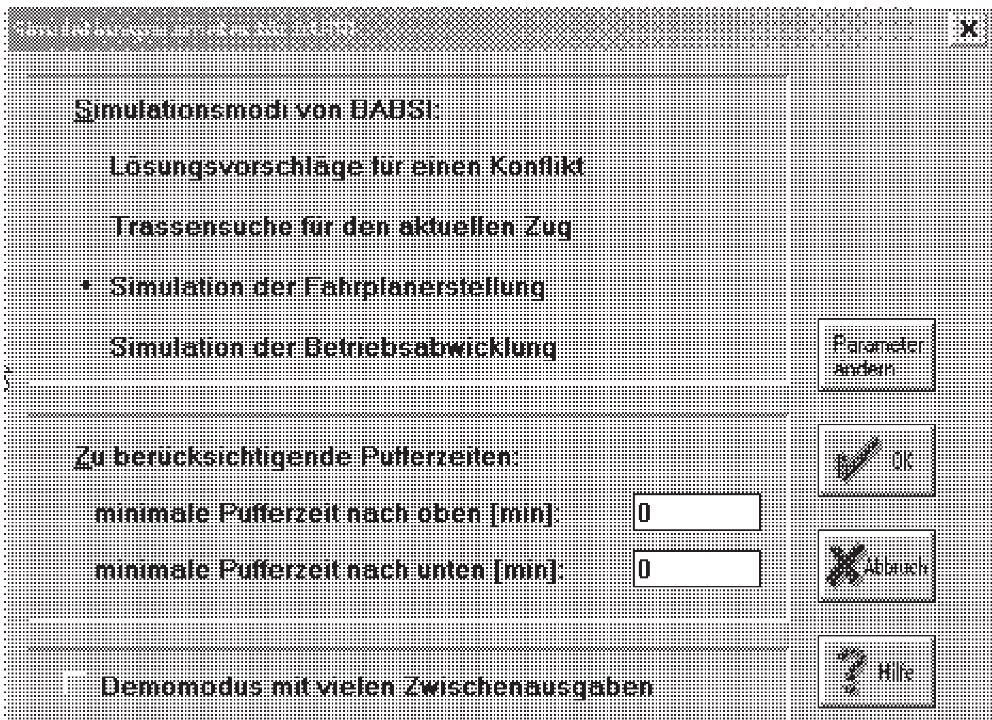

Figure A.1: Simulation modus in BABSI with chosen options

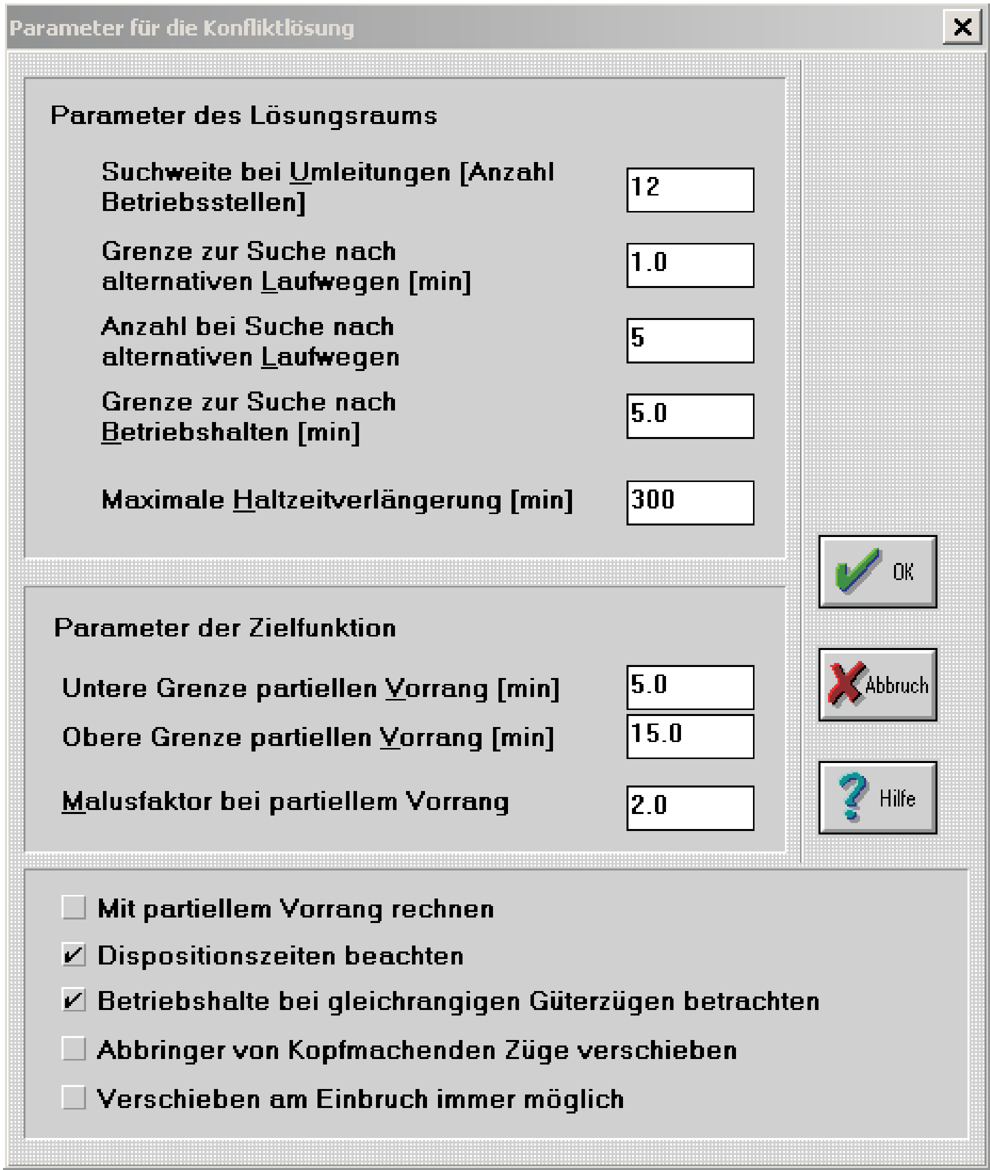

Figure A.2: Parameter options chosen for the simulation of timetables in BABSI

A.2 Simulation results

In this section the simulation results are listed. First follows the result from line D: dummy of Eidsvoll-Hamar. Afterwards follows the results from line A: Eidsvoll-Hamar.

A.2.1 Results recorded for line D: dummy of Eidsvoll-Hamar

Alternative 1, modified files: Direction Hamar to Eidsvoll has priority over direction Eidsvoll to Hamar

Table A.2 lists the number of crossings, multiple crossings with the respective number of additional passing trains.

File No.	n_x	n_m	$n_{1(p.)} = 1$	$n_{1(p.)} = 2$	$\sum n_{1(p.)}$
8601/02	15	4	4	0	4
8603/04	12	4	0	4	8
8605/06	15	6	6	0	6
8607/08	13	7	7	0	7
8609/10	9	2	0	2	4
8613/14	8	3	3	0	3
8615/16	18	0	0	0	0
8617/18	19	2	2	0	2
8619/20	11	3	0	3	6
8623/24	7	4	4	0	4
8625/26	14	4	4	0	4
8627/28	14	7	7	0	7
8633/34	15	3	3	0	3
8635/36	12	2	2	0	2
8637/38	12	2	2	0	2
8641/42	9	6	2	4	10
8643/44	17	2	2	0	2
8649/50	17	4	4	0	4
8651/52	13	7	7	0	7
8655/56	20	5	5	0	5
8657/58	16	4	4	0	4
8659/60	10	1	0	2	2
8661/62	10	5	5	0	5
8665/66	17	4	4	0	4
8667/68	16	2	2	0	2
8669/70	15	1	1	0	1
8671/72	7	5	5	0	5
8673/74	11	4	4	0	4
8675/76	12	7	7	0	7
8677/78	12	7	7	0	7
8685/86	13	8	6	2	10
8687/88	8	2	0	2	4
8689/90	7	6	4	2	8
8691/92	15	4	0	4	8
8695/96	15	4	4	0	4
8697/98	12	2	2	0	2

File No.	n_x	n_m	$n_{1(p.)} = 1$	$n_{1(p.)} = 2$	$\sum n_{1(p.)}$
8699/00	22	0	0	0	0
8701/02	20	0	0	0	0
8705/06	12	5	5	0	5
8707/08	12	2	0	2	4
8709/10	10	10	10	0	10
8711/12	13	8	5	3	11
8713/14	18	4	4	0	4
8715/16	19	3	3	0	3
8717/18	6	4	4	0	4
8719/20	11	0	0	0	0
8721/22	8	2	2	0	2
8723/24	16	0	0	0	0
8725/26	16	6	6	0	6
8727/28	8	5	2	3	8
Mean	13.38	3.84	3.20	1.28	4.48

Table A.2: Number of crossings, multiple crossings and additional passing trains simulated in BABSI for modified alternative 1

Statistical analysis of the number of crossings:
Sample: 50 timetables
Mean time of survey: $\overline{T} = 230.8$ min.
Largest registration: 22 crossings/timetable
Smallest registration: 7 crossings/timetable
Sample mean: $\bar{t} = \frac{1}{n} \cdot \sum_{i=1}^{n} t_i = 13.4$ crossings/timetable
Sample variance: $\sigma^2 = \frac{1}{n-1} \cdot \sum_{i=1}^{n} (t_i - \bar{t})^2 = 14.60$
Sample standard deviation: $\sigma = \sqrt{\sigma^2} = 3.82$

Table A.3 illustrates the recorded scheduled waiting time for crossing, merging, overtaking and minimum time necessary for crossing.

File No.	$T_{wx} + min\ t_x$[min.]	T_{wm}[min.]	$\sum T_w$ [min.]
8601/02	119.7	39.6	159.3
8603/04	110.3	91.3	210.6
8605/06	147.8	54.6	202.4
8607/08	129.5	38.3	167.8
8609/10	102.3	20.4	122.7
8613/14	82.0	18.4	100.4
8615/16	160.1	0	160.1
8617/18	184.1	20.4	204.5
8619/20	109.2	51.9	161.1
8623/24	76.5	38.3	114.8
8625/26	127.5	24.6	152.1
8627/28	149.7	61.8	211.5

File No.	$T_{wx} + min\ t_x$[min.]	T_{wm}[min.]	$\sum T_w$ [min.]
8633/34	156.9	22.0	178.9
8635/36	107.1	10.3	117.4
8637/38	124.5	10.2	134.7
8641/42	90.1	67.8	157.9
8643/44	169.4	21.1	190.6
8649/50	149.7	24.6	174.3
8651/52	135.2	48.2	183.4
8655/56	200.1	55.4	255.5
8657/58	130.7	22.3	153.0
8659/60	104.0	12.4	116.4
8661/62	89.4	48.4	137.7
8665/66	164.3	44.8	209.1
8667/68	161.4	17.8	179.2
8669/70	156.1	8.5	164.6
8671/72	64.9	33.7	98.6
8673/74	117.6	43.0	160.6
8675/76	108.3	50.1	158.4
8677/78	113.0	59.2	172.2
8685/86	82.9	110.2	193.1
8687/88	74.7	27.6	102.3
8689/90	70.0	56.4	126.4
8691/92	130.5	49.4	179.9
8695/96	133.5	36.1	169.6
8697/98	107.0	18.6	125.6
8699/00	211.0	0	211.0
8701/02	175.2	0	175.2
8705/06	112.5	34.1	146.6
8707/08	115.0	24.8	139.8
8709/10	104.8	71.7	176.5
8711/12	106.7	98.1	204.8
8713/14	178.1	19.6	197.7
8715/16	158.6	41.9	200.5
8717/18	59.0	41.9	100.9
8719/20	126.5	0	126.5
8721/22	79.2	17.3	96.5
8723/24	134.2	0	134.2
8725/26	144.1	27.9	172.0
8727/28	80.5	62.7	143.2
Mean	124.5	36.0	160.5

Table A.3: Waiting time for crossing, merging and overtaking simulated in BABSI for modified alternative 1

In table A.4 a summary of a statistical evaluation of the data given in table A.3 is presented.

T_w	Max [min]	Min [min]	Mean [min]	Variance	Standard deviation
$T_{wx} + mint_x$	211.0	59.0	124.5	1288.4	35.9
T_{wm}	110.2	0	36.0	662.5	25.7
$\sum T_w$	255.5	96.5	160.5	1334.8	36.5

Table A.4: Statistical evaluation of the data given in table A.3

Alternative 2: Both directions have equal priority

Table A.5 lists the number of crossings and survey time recorded for each of the 100 files of alternative 2. Of the 100 simulations 14 files marked with a * in table A.5 and A.6 are simulations where the algorithm ran in an infinite loop. This loop was manipulated manually in BABSI, and the simulation could continue.

File No.	n_x	T[min.]	File No.	n_x	T[min.]
9007/08	14	357.6	9107/08	15	239.3
9009/10	10	236,3	9109/10	17	197.0
9011/12*	12	247.9	9111/12	16	235.1
9013/14	16	233.4	9113/14	16	247.1
9015/16	19	224.7	9115/16	14	195.3
9017/18	16	201.4	9117/18	17	213.7
9019/20	21	242.8	9119/20	13	234.0
9021/22*	19	233.5	9121/22	12	227.2
9023/24	10	227.2	9123/24	17	224.9
9025/26	19	232,8	9125/26	17	227.2
9027/28	15	238.6	9127/28	15	234.6
9029/30*	14	227.2	9129/30	15	243.7
9031/32*	13	224.1	9131/32	19	233.8
9033/34	15	238.5	9133/34	13	254.6
9035/36	18	238.8	9135/36	16	233.1
9037/38	17	233.3	9137/38	12	248.8
9039/40*	3	184.7	9139/40	14	239.9
9041/42	19	227.2	9141/42*	14	210.6
9043/44	14	234.0	9143/44	15	227.2
9045/46*	16	230.8	9145/46	16	187.7
9047/48	9	264.3	9147/48	12	227.2
9049/50	11	227.2	9149/50	23	227.2
9051/52	14	227.2	9151/52	20	218.0
9053/54*	18	227.2	9153/54	10	227.2
9055/56	23	232.0	9155/56	15	227.2
9057/58	15	227.2	9157/58	21	227.2
9059/60	18	227.2	9159/60	18	227.2
9061/62	16	227.2	9161/62*	17	209.0

File No.	n_x	T[min.]	File No.	n_x	T[min.]
9063/64*	12	227.7	9163/64	19	193.7
9065/66	16	227.2	9165/66	13	253.0
9067/68	10	241.8	9167/68	19	211.5
9069/70	18	235.1	9169/70	16	201.0
9071/72	9	227.2	9171/72	14	237.7
9073/74	18	244.2	9173/74	21	216.0
9075/76	17	249.7	9175/76	15	227.2
9077/78	17	243.0	9177/78	13	227.3
9079/80*	11	227.2	9179/80	16	194.5
9081/82*	12	230.9	9181/82	14	227.2
9083/84	17	183.7	9183/84	6	251.1
9085/86	15	193.1	9185/86	17	235.1
9087/88	15	233.4	9187/88	10	227.2
9089/90	12	227.2	9189/90	14	213.0
9091/92	17	218.7	9191/92	9	227.2
9093/94*	15	181.4	9193/94	10	227.2
9095/96	18	271.7	9195/96	24	215.1
9097/98	19	239.0	9197/98	20	234.8
9099/00	19	221.8	9199/00	12	227.2
9101/02	10	186.2	9001/02	14	234.9
9103/04*	17	254.7	9003/04	14	249.5
9105/06	11	240.4	9005/06	19	237.8

Table A.5: Number of crossings simulated in BABSI with the corresponding time of survey for alternative 2

Statistical analysis of the number of crossings:
Sample: 100 timetables
Mean time of survey: $\overline{T} = 227.7$ min.
Largest registration: 24 crossings/timetable
Smallest registration: 3 crossings/timetable
Sample mean: $\overline{t} = \frac{1}{n} \cdot \sum_{i=1}^{n} t_i = 15.2$ crossings/timetable
Sample variance: $\sigma^2 = \frac{1}{n-1} \cdot \sum_{i=1}^{n} (t_i - \overline{t})^2 = 13.19$
Sample standard deviation: $\sigma = \sqrt{\sigma^2} = 3.63$

Table A.6 lists the number of multiple crossings with the corresponding additional passing trains recorded for each of the 100 files.

File No.	n_m	$n_{1(p.)} = 1$	$n_{1(p.)} = 2$	$n_{1(p.)} = 3$	$n_{1(p.)} = 4$	$\sum n_{1(p.)}$
9001/02	3	3	0	0	0	3
9003/04	3	3	0	0	0	6
9005/06	2	2	0	0	0	2
9007/08	6	5	1	0	0	7
9009/10	2	1	0	1	0	4

File No.	n_m	$n_{1(p.)} = 1$	$n_{1(p.)} = 2$	$n_{1(p.)} = 3$	$n_{1(p.)} = 4$	$\sum n_{1(p.)}$
9011/12*	4	3	1	0	0	5
9013/14	2	2	0	0	0	2
9015/16	2	0	2	0	0	4
9017/18	5	4	1	0	0	6
9019/20	2	2	0	0	0	2
9021/22*	5	4	1	0	0	6
9023/24	4	0	2	2	0	8
9025/26	3	3	0	0	0	3
9027/28	4	1	3	0	0	7
9029/30*	9	9	0	0	0	9
9031/32*	5	2	2	1	0	9
9033/34	5	5	0	0	0	5
9035/36	2	2	0	0	0	2
9037/38	1	1	0	0	0	1
9039/40*	3	1	0	2	0	7
9041/42	10	10	0	0	0	10
9043/44	4	4	0	0	0	4
9045/46*	4	1	3	0	0	7
9047/48	5	4	0	1	0	7
9049/50	4	1	0	3	0	10
9051/52	7	7	0	0	0	7
9053/54*	8	5	3	0	0	11
9055/56	3	3	0	0	0	3
9057/58	2	1	1	0	0	3
9059/60	5	5	0	0	0	5
9061/62	7	7	0	0	0	7
9063/64*	4	2	0	1	1	9
9065/66	2	2	0	0	0	2
9067/68	2	0	2	0	0	4
9069/70	2	2	0	0	0	2
9071/72	5	5	0	0	0	5
9073/74	5	5	0	0	0	5
9075/76	2	2	0	0	0	2
9077/78	5	4	1	0	0	6
9079/80*	4	2	2	0	0	6
9081/82*	2	0	0	2	0	6
9083/84	11	8	3	0	0	14
9085/86	8	8	0	0	0	8
9087/88	0	0	0	0	0	0
9089/90	5	5	0	0	0	5
9091/92	5	4	1	0	0	6
9093/94*	9	6	3	0	0	12
9095/96	1	1	0	0	0	1

File No.	n_m	$n_{1(p.)} = 1$	$n_{1(p.)} = 2$	$n_{1(p.)} = 3$	$n_{1(p.)} = 4$	$\sum n_{1(p.)}$
9097/98	0	0	0	0	0	0
9099/00	4	4	0	0	0	4
9101/02	5	0	0	2	3	18
9103/04*	2	1	1	0	0	3
9105/06	3	3	0	0	0	3
9107/08	1	1	0	0	0	1
9109/10	5	3	1	1	0	8
9111/12	9	6	3	0	0	12
9113/14	7	6	1	0	0	8
9115/16	2	0	0	2	0	6
9117/18	3	3	0	0	0	3
9119/20	2	1	1	0	0	3
9121/22	0	0	0	0	0	0
9123/24	0	0	0	0	0	0
9125/26	5	4	1	0	0	6
9127/28	5	4	1	0	0	6
9129/30	9	4	5	0	0	14
9131/32	1	0	1	0	0	2
9133/34	4	0	4	0	0	8
9135/36	2	1	1	0	0	3
9137/38	3	0	3	0	0	6
9139/40	5	5	0	0	0	5
9141/42*	6	4	2	0	0	8
9143/44	0	0	0	0	0	0
9145/46	6	0	2	4	0	16
9147/48	4	4	0	0	0	4
9149/50	0	0	0	0	0	0
9151/52	0	0	0	0	0	0
9153/54	5	5	0	0	0	5
9155/56	2	1	1	0	0	3
9157/58	3	0	3	0	0	6
9159/60	6	6	0	0	0	6
9161/62*	10	7	3	0	0	13
9163/64	9	8	1	0	0	10
9165/66	4	2	2	0	0	6
9167/68	7	4	3	0	0	10
9169/70	9	5	4	0	0	13
9171/72	1	0	1	0	0	2
9173/74	2	2	0	0	0	2
9175/76	1	1	0	0	0	1
9177/78	3	3	0	0	0	3
9179/80	3	1	2	0	0	5
9181/82	9	9	0	0	0	9

File No.	n_m	$n_{1(p.)} = 1$	$n_{1(p.)} = 2$	$n_{1(p.)} = 3$	$n_{1(p.)} = 4$	$\sum n_{1(p.)}$
9183/84	3	0	0	0	3	12
9185/86	4	4	0	0	0	4
9187/88	6	0	0	6	0	18
9189/90	3	1	2	0	0	5
9191/92	4	4	0	0	0	4
9193/94	5	3	2	0	0	7
9195/96	1	1	0	0	0	1
9197/98	5	5	0	0	0	5
9199/00	2	0	2	0	0	4
Mean	4.03	2.83	0.85	0.28	0.07	5.65

Table A.6: Number of multiple crossings and additional passing trains simulated in BABSI for alternative 2

Table A.7 illustrates the recorded scheduled waiting time for crossing, merging, overtaking and minimum time necessary for crossing. Of the 100 timetables simulated 50 were selected for this partition. Only the timetables where the simulation did not stop were analyzed starting from the beginning of the list.

File No.	$T_{wx} + mint_x$[min.]	T_{wm}[min.]	$T_{wo} + mint_o$ [min.]	$\sum T_w$ [min.]
9001/02	122.57	9.60	42.43	174.60
9003/04	120.76	41.40	0	162.16
9005/06	168.54	4.40	43.43	216.37
9007/08	169.15	39.10	0	208.25
9009/10	79.30	22.91	76.07	178.28
9013/14	149.90	11.90	0	161.80
9015/16	172.28	28.80	15.30	216.38
9017/18	171.06	54.80	38.93	264.79
9019/20	208.70	15.60	0	224.30
9023/24	92.23	63.20	31.23	186.66
9025/26	169.46	11.80	57.80	239.06
9027/28	136.21	67.60	24.46	228.27
9033/34	145.76	25.20	23.22	194.18
9035/36	165.08	21.40	14.31	200.79
9037/38	148.42	5.90	0	154.32
9041/42	190.16	87.30	0	277.46
9043/44	178.30	34.20	0	212.50
9049/50	100.23	74.00	21.81	196.04
9051/52	153.39	57.80	0	211.19
9055/56	222.45	24.40	58.34	305.19
9057/58	126.24	23.80	55.95	205.99
9059/60	149.35	36.40	0	185.75
9061/62	151.96	58.40	0	210.36
9065/66	159.65	5.60	21.64	186.89

File No.	$T_{wx} + mint_x$[min.]	T_{wm}[min.]	$T_{wo} + mint_o$ [min.]	$\sum T_w$ [min.]
9067/68	93.50	38.10	14.50	146.10
9069/70	118.99	17.60	34.10	170.69
9071/72	84.81	28.40	0	113.21
9073/74	132.23	52.00	34.02	218.25
9075/76	155.80	22.40	15.85	194.05
9077/78	176.63	47.10	12.30	236.03
9085/86	134.00	51.00	24.30	209.30
9087/88	110.28	0	16.93	127.21
9089/90	114.86	46.40	17.21	178.47
9091/92	145.01	37.80	34.33	217.14
9095/96	178.30	5.10	0	183.40
9097/98	148.21	0	0	148.21
9099/00	198.20	33.80	35.21	267.21
9101/02	88.37	104.30	92.23	284.90
9105/06	100.96	23.60	0	124.56
9107/08	133.33	6.20	0	139.53
9109/10	129.99	65.90	102.94	298.83
9111/12	145.80	142.50	55.52	343.82
9113/14	208.70	129.70	0	338.40
9115/16	104.80	31.50	61.20	197.50
9117/18	135.38	21.90	46.83	204.11
9119/20	123.50	34.90	15.60	174.00
9121/22	111.32	23.00	0	134.32
9123/24	160.29	0	0	160.29
9125/26	117.86	42.50	66.66	227.02
9127/28	136.05	44.00	23.30	203.35
9109/10	129.99	65.90	102.94	298.83
9111/12	145.80	142.50	55.52	343.82
Mean	142.77	37.50	24.56	204.83

Table A.7: Waiting time for crossing, merging and overtaking simulated in BABSI

In table A.8 a summary of a statistical evaluation of the data given in table A.7 is presented.

T_w	Max [min]	Min [min]	Mean [min]	Variance	Standard deviation
$T_{wx} + mint_x$	222.45	79.30	142.77	1140.2	33.8
T_{wm}	142.50	0	37.50	946.6	30.8
$T_{wo} + mint_o$	102.94	0	24.56	695.5	26.4
$\sum T_w$	343.82	113.21	204.8	2669.6	51.7

Table A.8: Statistical evaluation of the data given in table A.7

A.2.2 Results recorded for line A: Eidsvoll-Hamar

Alternative 2: Both directions have equal priority

Table A.9 lists the number of crossings and survey time recorded for each of the 100 files. Of the 100 simulations 19 files marked with a * in table A.9 and A.10 are simulations where the algorithm ran in an infinite loop. This loop was manipulated manually in BABSI, and the simulation could continue.

File No.	n_x	T[min.]	File No.	n_x	T[min.]
4007/08	7	235.8	4107/08*	11	213.0
4009/10	15	186.4	4109/10	16	229.4
4011/12*	15	180.0	4111/12	12	192.2
4013/14	10	205.5	4113/14	11	214.0
4015/16	9	180.0	4115/16	7	251.8
4017/18	14	214.0	4117/18	8	241.5
4019/20	7	235.8	4119/20	12	214.0
4021/22*	13	180.0	4121/22	8	218.7
4023/24*	11	246.1	4123/24	17	213.0
4025/26	14	213.0	4125/26	13	220.9
4027/28	11	187.2	4127/28*	11	201.0
4029/30	11	237.9	4129/30*	9	213.0
4031/32	16	185.3	4131/32	15	213.2
4033/34	12	214.0	4133/34	14	231.0
4035/36	12	237.1	4135/36	11	224.1
4037/38	12	222.1	4137/38	10	209.3
4039/40	8	240.1	4139/40	10	222.0
4041/42	9	220.3	4141/42	9	222.1
4043/44	13	198.5	4143/44	11	220.6
4045/46	15	232.5	4145/46	9	214.0
4047/48	14	213.0	4147/48	11	214.0
4049/50	10	230.3	4149/50	12	206.6
4051/52	10	214.0	4151/52*	7	214.0
4053/54	9	229.0	4153/54	14	223.5
4055/56	12	228.6	4155/56*	6	222.1
4057/58*	10	222.1	4157/58	8	244.7
4059/60*	16	229.8	4159/60*	13	201.6
4061/62	14	223.2	4161/62	13	223.3
4063/64	10	253.3	4163/64	14	242.1
4065/66	11	230.3	4165/66	12	214.0
4067/68	8	214.0	4167/68	5	213.0
4069/70	13	200.6	4169/70	13	219.7
4071/72	7	233.7	4171/72*	15	202.6
4073/74	11	247.7	4173/74	13	191.3
4075/76	11	231.2	4175/76	13	206.8

File No.	n_x	T[min.]	File No.	n_x	T[min.]
4077/78*	10	217.8	4177/78	11	214.0
4079/80*	19	180.0	4179/80*	6	209.1
4081/82	13	229.8	4181/82	13	229.4
4083/84	6	237.5	4183/84	17	225.7
4085/86	6	206.9	4185/86	14	214.0
4087/88	12	196.3	4187/88*	11	213.0
4089/90	11	214.0	4189/90*	9	214.0
4091/92	14	246.5	4191/92	14	220.9
4093/94	11	213.0	4193/94	9	222.1
4095/96	9	222.8	4195/96	10	214.0
4097/98	4	218.1	4197/98	2	213.0
4099/00	9	214.0	4199/00*	8	213.0
4101/02	12	237.8	4201/02	9	213.0
4103/04	11	214.0	4203/04	7	226.8
4105/06	8	230.3	4205/06*	12	180.0

Table A.9: Number of crossings simulated in BABSI with the corresponding time of survey for alternative 2

Statistical analysis of the number of crossings:
Sample: 100 timetables
Mean time of survey: $\overline{T} = 217.7$ min.
Largest registration: 19 crossings/timetable
Smallest registration: 2 crossings/timetable
Sample mean: $\overline{t} = \frac{1}{n} \cdot \sum_{i=1}^{n} t_i = 11$ crossings/timetable
Sample variance: $\sigma^2 = \frac{1}{n-1} \cdot \sum_{i=1}^{n} (t_i - \overline{t})^2$=9.23
Sample standard deviation: $\sigma = \sqrt{\sigma^2}$=3.04

Table A.10 lists the number of multiple crossings with the correspondent passing trains recorded for each of the 100 files.

File No.	n_m	$n_{1(p.)} = 1$	$n_{1(p.)} = 2$	$n_{1(p.)} = 3$	$n_{1(p.)} = 4$	$\sum n_{1(p.)}$
4007/08	4	2	2	0	0	6
4009/10	8	6	2	0	0	10
4011/12*	6	5	1	0	0	7
4013/14	1	0	1	0	0	2
4015/16	2	2	0	0	0	2
4017/18	1	0	1	0	0	2
4019/20	3	3	0	0	0	3
4021/22*	10	5	3	2	0	17
4023/24	7	4	3	0	0	10
4025/26	3	3	0	0	0	3
4027/28	4	3	0	1	0	6
4029/30	1	1	0	0	0	1

File No.	n_m	$n_{1(p.)} = 1$	$n_{1(p.)} = 2$	$n_{1(p.)} = 3$	$n_{1(p.)} = 4$	$\sum n_{1(p.)}$
4031/32	8	8	0	0	0	8
4033/34	2	2	0	0	0	2
4035/36	1	1	0	0	0	1
4037/38	1	1	0	0	0	1
4039/40	3	2	1	0	0	4
4041/42	2	2	0	0	0	2
4043/44	3	3	0	0	0	3
4045/46	3	3	0	0	0	3
4047/48	3	3	0	0	0	3
4049/50	2	2	0	0	0	2
4051/52	1	1	0	0	0	1
4053/54	2	2	0	0	0	2
4055/56	3	3	0	0	0	3
4057/58*	3	2	1	0	0	4
4059/60*	2	2	0	0	0	2
4061/62	2	2	0	0	0	2
4063/64	3	3	0	0	0	3
4065/66	1	1	0	0	0	1
4067/68	3	3	0	0	0	3
4069/70	4	2	1	1	0	7
4071/72	4	1	3	0	0	7
4073/74	3	3	0	0	0	3
4075/76	2	2	0	0	0	2
4077/78*	6	5	1	0	0	7
4079/80*	9	6	2	1	0	13
4081/82	0	0	0	0	0	0
4083/84	3	3	0	0	0	3
4085/86	3	0	3	0	0	6
4087/88	4	4	0	0	0	4
4089/90	4	4	0	0	0	4
4091/92	1	1	0	0	0	1
4093/94	6	6	0	0	0	6
4095/96	0	0	0	0	0	0
4097/98	1	0	0	1	0	3
4099/00	2	0	2	0	0	4
4101/02	3	3	0	0	0	3
4103/04	3	3	0	0	0	3
4105/06	5	5	0	0	0	5
4107/08*	7	7	0	0	0	7
4109/10	0	0	0	0	0	0
4111/12	7	6	1	0	0	8
4113/14	4	3	1	0	0	5
4115/16	4	1	2	1	0	8

File No.	n_m	$n_{1(p.)} = 1$	$n_{1(p.)} = 2$	$n_{1(p.)} = 3$	$n_{1(p.)} = 4$	$\sum n_{1(p.)}$
4117/18	2	2	0	0	0	2
4119/20	3	2	1	0	0	4
4121/22	4	3	1	0	0	5
4123/24	1	1	0	0	0	1
4125/26	4	0	4	0	0	8
4127/28*	7	7	0	0	0	7
4129/30*	3	1	2	0	0	5
4131/32	2	1	1	0	0	3
4133/34	0	0	0	0	0	0
4135/36	1	1	0	0	0	1
4137/38	4	3	1	0	0	5
4139/40	4	1	3	0	0	7
4141/42	5	5	0	0	0	5
4143/44	3	0	3	0	0	6
4145/46	2	0	0	2	0	6
4147/48	5	5	0	0	0	5
4149/50	4	4	0	0	0	4
4151/52*	4	1	3	0	0	7
4153/54	1	1	0	0	0	1
4155/56*	2	0	0	0	2	8
4157/58	4	0	4	0	0	8
4159/60*	2	1	1	0	0	3
4161/62	3	3	0	0	0	3
4163/64	2	2	0	0	0	2
4165/66	5	1	4	0	0	9
4167/68	1	0	0	1	0	3
4169/70	0	0	0	0	0	0
4171/72*	5	3	1	1	0	8
4173/74	6	6	0	0	0	6
4175/76	4	3	1	0	0	5
4177/78	2	2	0	0	0	2
4179/80*	3	1	2	0	0	5
4181/82	3	3	0	0	0	3
4183/84	4	3	1	0	0	5
4185/86	4	4	0	0	0	4
4187/88*	5	1	2	1	1	12
4189/90*	4	0	0	4	0	12
4191/92	0	0	0	0	0	0
4193/94	5	2	3	0	0	8
4195/96	7	3	3	0	1	13
4197/98	0	0	0	0	0	0
4199/00*	4	2	1	0	1	8
4201/02	3	3	0	0	0	3

File No.	n_m	$n_{1(p.)} = 1$	$n_{1(p.)} = 2$	$n_{1(p.)} = 3$	$n_{1(p.)} = 4$	$\sum n_{1(p.)}$
4203/04	1	0	1	0	0	2
4205/06*	9	2	7	0	0	16
Mean	3.3	2.28	0.85	0.12	0.05	4.58

Table A.10: Number of mergings and passing trains simulated in BABSI for alternative 2

A.3 Quantities for the calculation of the number of crossings

First in this section the conditions of recording the minimum spacing time and the time gap used in the calculations in this thesis are presented. The records of these two parameters follows afterwards for both lines D and A. All quantities were recorded in BABSI (VO.60).

A.3.1 Recording of the minimum spacing time between trains of rank(1)

The minimum spacing time was recorded on the first block when entering the section according to description in Chapter 2.1.4. The train does not halt and runs through the main track with the speed permitted for the actual space.

A.3.2 Recording of the time gap

The time gap necessary for a train to reach the next station was recorded. The recording is based on a halt in the siding in station (i-1) and station (i+1) for the train of rank(2) with lower priority. An example from BABSI is illustrated in figure A.3.

A.3.3 Line D: dummy of Eidsvoll-Hamar

In line D every block and station is equal which means that the following quantities are the same for every section in both directions.
The minimum spacing time between the trains of rank(1) in one direction:
$\overline{t_{s,11}} = 4.3$ min.
The time gap necesarry for a train to reach the next station:
$\overline{\Delta t} = 11.8$ min.

A.3.4 Line A: Eidsvoll-Hamar

The minimum spacing time $t_{s,11}$ for each section in both directions between Eidsvoll and Hamar was recorded and illustrated in table A.11.

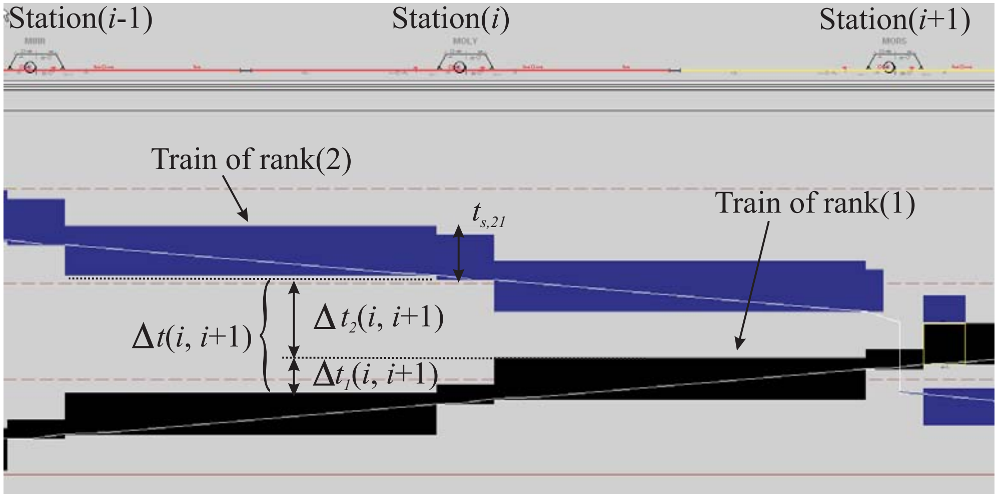

Figure A.3: Example of a time gap recorded in BABSI with locomotive EL16

Section in Eidsvoll → Hamar	$t_{s,11}$ [min]	Section in Hamar → Eidsvoll	$t_{s,11}$ [min]
Eidsvoll-Minnesund	3.4	Hamar-Ottestad	3.1
Minnesund-Molykkja	4.0	Ottestad-Stange	3.1
Molykkja-Morskogen	3.7	Stange-Steinsrud	4.3
Morskogen-Strandlykkja	4.6	Steinsrud-Tangen	4.6
Strandlykkja-Espa	3.3	Tangen-Espa	3.8
Espa-Tangen	3.9	Espa-Strandlykkja	3.4
Tangen-Steinsrud	4.5	Strandlykkja-Morskogen	4.4
Steinsrud-Stange	2.6	Morskogen-Molykkja	3.9
Stange-Ottestad	3.1	Molykkja-Minnesund	4.0
Ottestad-Hamar	3.2	Minnesund-Eidsvoll	3.5
Mean	3.63	Mean	3.81

Table A.11: The minimum spacing time between train of rank(1) for each section for both directions

The mean minimum spacing time for both directions together becomes:

$$\overline{t_{s,11}} = \frac{3.63 \text{ min.} + 3.81 \text{ min.}}{2} = 3.72 \text{ min.} \tag{A.1}$$

The time gap Δt necessary for reaching the next station $(i+1)$ was recorded for each station (i) and illustrated in table A.12.

Section in Eidsvoll → Hamar	Δt [min]	Section in Hamar → Eidsvoll	Δt [min]
Eidsvoll-Minnesund-Molykkja	10.2	Hamar-Ottestad-Stange	8.5
Minnesund-Molykkja-Morskogen	11.5	Ottestad-Stange-Steinsrud	13.0
Molykkja-Morskogen-Strandlykkja	11.3	Stange-Steinsrud-Tangen	13.7
Morskogen-Strandlykkja-Espa	13.8	Steinsrud-Tangen-Espa	10.5
Strandlykkja-Espa-Tangen	14.2	Tangen-Espa-Strandlykkja	14.5
Espa-Tangen-Steinsrud	10.8	Espa-Strandlykkja-Morskogen	14.3
Tangen-Steinsrud-Stange	10.9	Strandlykkja-Morskogen-Molykkja	10.6
Steinsrud-Stange-Ottestad	8.9	Morskogen-Molykkja-Minnesund	10.7
Stange-Ottestad-Hamar	12.9	Molykkja-Minnesund-Edisvoll	9.4
Mean	11.7	Mean	11.7

Table A.12: Time gap necessary for reaching the next station $(i+1)$ for both directions

The mean time gap for both directions together becomes:

$$\overline{\Delta t} = \frac{11.7 \text{ min.} + 11.7 \text{ min.}}{2} = 11.7 \text{ min.} \tag{A.2}$$

Variance of Δt for the entire registrations for both directions together: 3.5

A.4 Quantities for the calculation of the number of multiple crossings

First in this section the conditions of recording the minimum spacing time between train of rank(2) and rank(1) and vice versa used in the calculations in this thesis is presented. The records follows afterwards for both lines D and A. All quantities were recorded in BABSI.

A.4.1 Recording of the minimum spacing time between train of rank(2) and rank(1)

The mininum spacing time for a train of rank(2) running in one direction and train of rank(1) running in opposite direction is illustrated in figure A.4.

A.4.2 Recording of the minimum spacing time between train of rank(1) and rank(2)

The mininum spacing time for a train of rank(1) running in one direction and train of rank(2) running in opposite direction is illustrated in figure A.5.

A.4.3 Line D: dummy of Eidsvoll-Hamar

The minimum spacing time between train of rank(2) and rank(1) and vice versa for line D was only recorded once since every section was equally constructed.

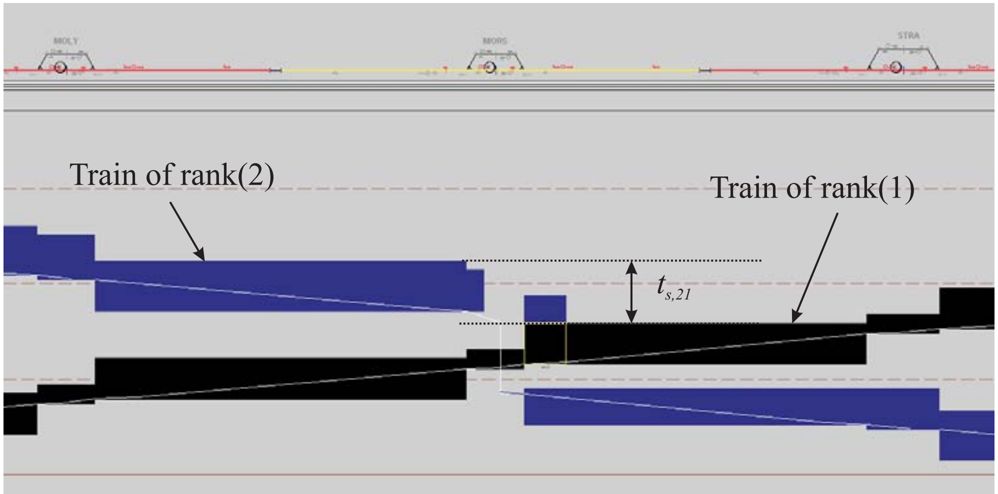

Figure A.4: Example of minimum spacing time between train of rank(2) and rank(1) in BABSI with locomotive EL16

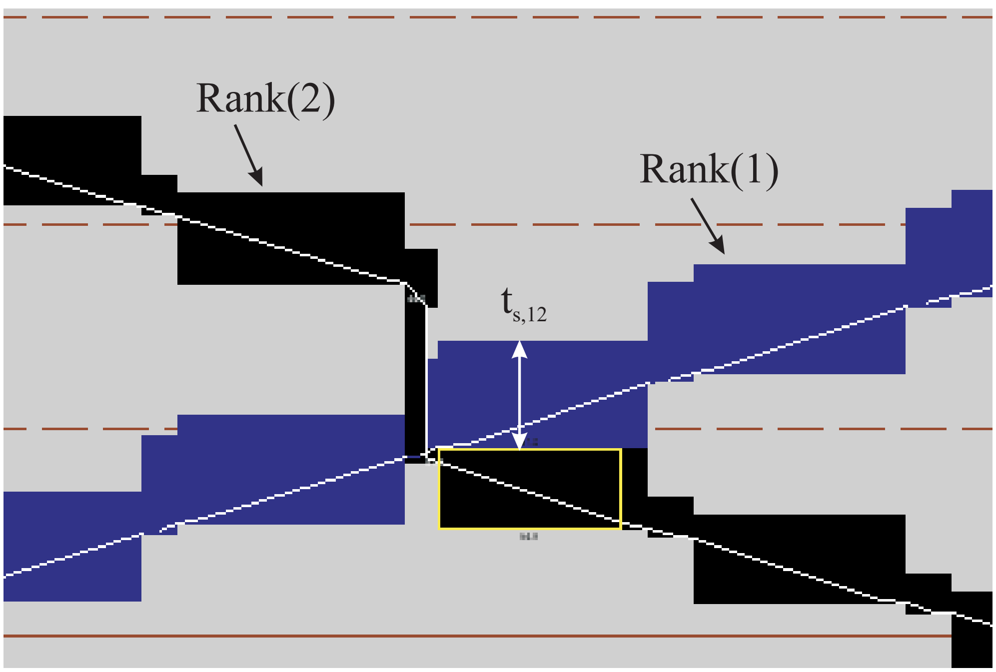

Figure A.5: Example of minimum spacing time between train of rank(1) and rank(2) in BABSI with locomotive EL16

$\overline{t_{s,21}} = 5.7$ min.
$\overline{t_{s,12}} = 5.3$ min.

A.4.4 Line A: Eidsvoll-Hamar

The minimum spacing time $t_{s,21}$ for each section in both directions between Eidsvoll and Hamar was recorded and illustrated in table A.13.

Section in Eidsvoll → Hamar	$t_{s,21}$ [min]	Section in Hamar → Eidsvoll	$t_{s,21}$ [min]
Eidsvoll-Minnesund	7.5	Hamar-Ottestad	7.3
Minnesund-Molykkja	6.3	Ottestad-Stange	5.5
Molykkja-Morskogen	6.3	Stange-Steinsrud	5.0
Morskogen-Strandlykkja	6.9	Steinsrud-Tangen	6.8
Strandlykkja-Espa	7.5	Tangen-Espa	6.3
Espa-Tangen	6.1	Espa-Strandlykkja	7.7
Tangen-Steinsrud	7.4	Strandlykkja-Morskogen	6.9
Steinsrud-Stange	6.1	Morskogen-Molykkja	5.8
Stange-Ottestad	5.4	Molykkja-Minnesund	6.8
Ottestad-Hamar	6.2	Minnesund-Eidsvoll	6.5
Mean	6.57	Mean	6.46

Table A.13: The minimum spacing time between train of rank(1) for each section for both directions

The mean minimum spacing time for both directions together becomes:

$$\overline{t_{s,21}} = \frac{6.57 \text{ min.} + 6.46 \text{ min.}}{2} = 6.5 \text{ min.} \tag{A.3}$$

The minimum spacing time addition $\Delta t_{s,1}$ for each section in both directions between Eidsvoll and Hamar was recorded and illustrated in table A.14. For values of $\Delta t_{s,1}$ equal zero means that there is only one block section between the stations.

Section in Eidsvoll → Hamar	$\Delta t_{s,1}$ [min]	Section in Hamar → Eidsvoll	$\Delta t_{s,1}$ [min]
Eidsvoll-Minnesund	1.5	Hamar-Ottestad	2.3
Minnesund-Molykkja	0	Ottestad-Stange	0
Molykkja-Morskogen	0	Stange-Steinsrud	2.0
Morskogen-Strandlykkja	0	Steinsrud-Tangen	0
Strandlykkja-Espa	1.9	Tangen-Espa	0
Espa-Tangen	0	Espa-Strandlykkja	2.3
Tangen-Steinsrud	0	Strandlykkja-Morskogen	0
Steinsrud-Stange	1.6	Morskogen-Molykkja	0
Stange-Ottestad	0	Molykkja-Minnesund	0
Ottestad-Hamar	2.1	Minnesund-Eidsvoll	2.5
Mean	1.78	Mean	2.28

Table A.14: The minimum spacing time between train of rank(1) for each section for both directions

The mean minimum spacing time addition $\Delta t_{s,1}$ for both directions together becomes:

$$\overline{\Delta t_{s,1}} = \frac{1.78 \text{ min.} + 2.28 \text{ min.}}{2} = 2.03 \text{ min.} \tag{A.4}$$

The minimum spacing time $t_{s,12}$ for each section in direction Eidsvoll → Hamar was recorded and illustrated in table A.15. It is assumed that the qunatity for $t_{s,12}$ in direction Hamar → Eidsvoll is equal the quantity of $t_{s,12}$ in direction Eidsvoll → Hamar.

Section in Eidsvoll → Hamar	$t_{s,12}$ [min]
Eidsvoll-Minnesund	4.7
Minnesund-Molykkja	4.4
Molykkja-Morskogen	5.0
Morskogen-Strandlykkja	5.8
Strandlykkja-Espa	4.5
Espa-Tangen	5.1
Tangen-Steinsrud	4.8
Steinsrud-Stange	3.6
Stange-Ottestad	5.2
Mean	4.78

Table A.15: The minimum spacing time between train of rank(1) followed by train of rank(2) for each section in direction Eidsvoll → Hamar

A.5 Quantities for the calculation of the scheduled waiting time

Figure A.6 is a screenshot from BABSI illustrating the recording of the minimum time necessary for one crossing.

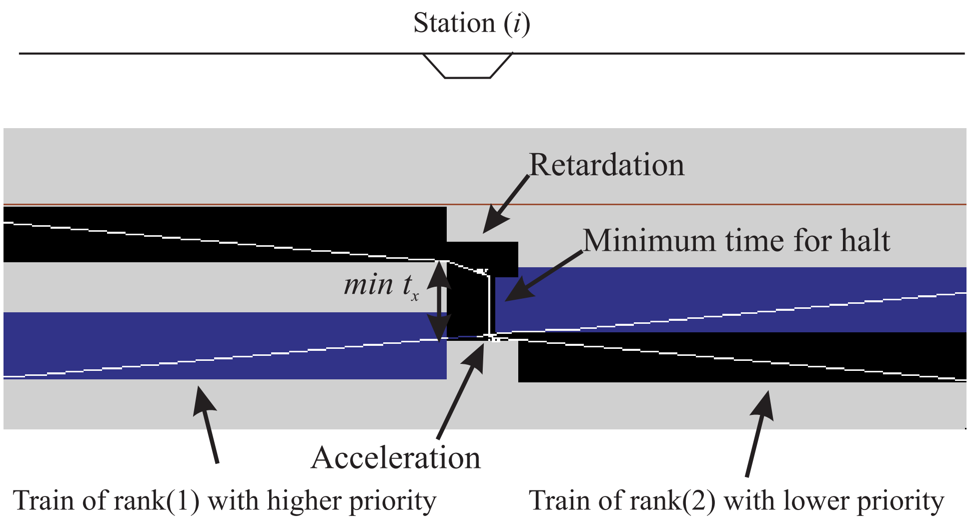

Figure A.6: Minimum time for a crossing between train of rank(2) and rank(1) in BABSI with locomotive EL16

The minimum time necessary for one crossing $min\ t_x$ was determined in BABSI:

Time lost because of slowing down to halt:	0.84 min.
Minimum time for a halt when crossing:	4.8 min.
Time lost because of acceleration:	0.16 min.
$min\ t_x$:	5.80 min.

Appendix B

Buffer time analysis on single track lines

In section B.1 data from the χ^2-test of goodness of fit of the buffer times in schedule from Line A is listed. In section B.2 and B.3 follow the data recorded from the χ^2-test of goodness of fit for schedules from line B respectively line C .

Parameters:

Sample mean: $\bar{t} = \frac{1}{n} \cdot \sum_{i=1}^{n} t_i$
Sample variance: $\sigma^2 = \frac{1}{n-1} \cdot \sum_{i=1}^{n} (t_i - \bar{t})^2$
Sample standard deviation: $\sigma = \sqrt{\sigma^2}$
Coefficient of variation: $V_T = \frac{\sigma}{\bar{t}}$

Estimated parameters:

Exponential distribution: $\lambda = \frac{1}{\bar{t}}$
Hyper-exponential distribution: $\lambda = \frac{1}{\bar{t}}$ and $\xi = \frac{1}{2} \cdot (1 - \sqrt{\frac{V^2-1}{V^2+1}})$
Erlang$_2$ distribution: $\lambda = \frac{1}{\bar{t}}$ and $k = 2$

B.1 Line A: Eidsvoll-Hamar

B.1.1 χ^2-test of the buffer times between the freight trains on section Tangen-Steinsrud.

Day of investigation: Thursday 18.04.2002
Sample: 17 freight trains
Largest registration: 395 minutes
Smallest registration: 4 minutes
Sample mean $\bar{t} = 78.41$

Sample variance $\sigma^2 = 11198.00$
Sample standard deviation $\sigma = 105.82$
Coefficient of variation $V_T = 1.35$
Estimated parameter: $\lambda = \frac{1}{\bar{t}} = 0.01$
Estimated parameter $\xi = 0.23$

Class no.	Time interval [min]	Obs. frequency h_i	Theo. frequency y_i	$\frac{(h_i-y_i)^2}{y_i}$
1	25	6	4.64	0.398
2	50	4	3.37	0.116
3	75	3	2.45	0.122
4	100	0	1.78	1.783
5	400	4	4.64	0.090
Sum		17	16.897	2.509

Table B.1: χ^2-test of an exponential buffer time distribution between the freight trains on section Tangen-Steinsrud

Table B.1 lists the values for a χ^2-test for an exponential distribution.
Number of classes $z = 5$
Number of estimated parameters $r = 1$
Degree of freedom $z - r - 1 = 3$
Level of significance $\alpha = 0.05$
Since $2.51 < 7.81$ the exponential distribution can be accepted.

Class no.	Time interval [min]	Obs. frequency h_i	Theo. frequency y_i	$\frac{(h_i-y_i)^2}{y_i}$
1	25	6	5.61	0.027
2	50	4	3.57	0.052
3	75	3	2.30	0.213
4	100	0	1.51	1.508
5	400	4	3.63	0.037
Sum		17	16.621	1.837

Table B.2: χ^2-test of a hyper-exponential buffer time distribution between freight trains on the Tangen-Steinsrud section

Table B.2 lists the values for a χ^2-test for a hyper-exponential distribution.
Number of classes $z = 5$
Number of estimated parameters $r = 2$
Degree of freedom $z - r - 1 = 2$
Level of significance $\alpha = 0.05$
Since $1.84 < 5.99$ the hyper-exponential distribution can be accepted.

B.1.2 χ^2-test of the buffer times between the freight trains and the IC trains on section Tangen-Steinsrud

Day of investigation: Thursday 18.04.2002
Sample: 17 freight trains and 35 IC trains
Largest registration: 264 minutes
Smallest registration: 0 minutes
Sample mean $\bar{t} = 22.35$
Sample variance $\sigma^2 = 1339.90$
Sample standard deviation $\sigma = 36.60$
Coefficient of variation $V_T = 1.64$
Estimated parameter: $\lambda = \frac{1}{\bar{t}} = 0.04$
Estimated parameter $\xi = 0.16$

Class no.	Time interval [min]	Obs. frequency h_i	Theo. frequency y_i	$\frac{(h_i-y_i)^2}{y_i}$
1	6	6	16.49	6.674
2	12	9	10.71	0.273
3	18	14	7.01	6.971
4	24	11	4.63	8.744
5	30	1	3.11	1.428
6	36	6	2.12	7.109
7	270	5	7.76	1.150
Sum		52	52	32.349

Table B.3: χ^2-test of a hyper-exponential buffer time distribution between the freight and IC trains on section Tangen-Steinsrud

Table B.3 lists the values for a χ^2-test for a hyper-exponential distribution.
Number of classes $z = 7$
Number of estimated parameters $r = 2$
Degree of freedom $z - r - 1 = 4$
Level of significance $\alpha = 0.05$
Since $32.35 > 9.49$ the hyper-exponential distribution cannot be accepted as buffer time distribution beteween the freight and IC trains.

B.1.3 χ^2-test of the buffer times between all of the trains on section Tangen-Steinsrud

Day of investigation: Thursday 18.04.2002
Sample: 17 freight trains, 35 IC trains, 8 ICE trains and 2 Night trains (Nt)
Largest registration: 196 minutes
Smallest registration: 0 minutes
Sample mean $\bar{t} = 18.26$
Sample variance $\sigma^2 = 660.71$
Sample standard deviation $\sigma = 25.70$

Coefficient of variation $V_T = 1.41$
Estimated parameter: $\lambda = \frac{1}{\bar{t}} = 0.05$
Estimated parameter $\xi = 0.21$

Class no.	Time interval [min]	Obs. frequency h_i	Theo. frequency y_i	$\frac{(h_i-y_i)^2}{y_i}$
1	4	7	15.40	4.582
2	8	6	11.15	2.378
3	12	7	8.12	0.153
4	16	17	5.95	20.538
5	20	7	4.39	1.546
6	24	8	3.28	6.807
7	28	1	2.47	0.876
8	32	0	1.89	1.887
9	36	6	1.46	14.105
10	200	3	7.77	3.055
Sum		62	62	55.927

Table B.4: χ^2-test of a hyper-exponential buffer time distribution between all of the trains on section Tangen-Steinsrud. Time interval 4 minutes.

Table B.4 lists the values for a χ^2-test for a hyper-exponential distribution.
Number of classes $z = 10$
Number of estimated parameters $r = 2$
Degree of freedom $z - r - 1 = 7$
Level of significance $\alpha = 0.05$
Since $14.07 > 55.93$ the hyper-exponential distribution can not be accepted.

B.1.4 χ^2-test of the buffer times between all of the trains on section Molykkja-Morskogen

Day of investigation: Thursday 18.04.2002
Sample: 17 freight trains, 35 IC trains, 8 ICE trains and 2 night trains
Largest registration: 231 minutes
Smallest registration: 0 minutes
Sample mean $\bar{t} = 19.15$
Sample variance $\sigma^2 = 1013.92$
Sample standard deviation $\sigma = 31.84$
Coefficient of variation $V_T = 1.66$
Estimated parameter: $\lambda = \frac{1}{\bar{t}} = 0.05$
Estimated parameter $\xi = 0.16$

Class no.	Time interval [min]	Obs. frequency h_i	Theo. frequency y_i	$\frac{(h_i-y_i)^2}{y_i}$
1	5	20	14.25	2.320
2	10	9	10.97	0.355
3	15	4	8.45	2.345
4	20	11	6.51	3.097
5	25	3	5.01	0.809
6	30	3	3.86	0.192
7	35	2	2.97	0.319
8	235	10	9.96	0
Sum		62	62	9.438

Table B.5: χ^2-test of an exponential buffer time distribution between all of the trains on section Molykkja-Morskogen

Table B.5 lists the values for a χ^2-test for an exponential distribution.
Number of classes $z = 8$
Number of estimated parameters $r = 1$
Degree of freedom $z - r - 1 = 6$
Level of significance $\alpha = 0.05$
Since $9.44 < 12.59$ the exponential distribution can be accepted.

Class no.	Time interval [min]	Obs. frequency h_i	Theo. frequency y_i	$\frac{(h_i-y_i)^2}{y_i}$
1	5	20	19.36	0.021
2	10	9	12.68	1.070
3	15	4	8.37	2.279
4	20	11	5.57	5.295
5	25	3	3.75	0.151
6	30	3	2.57	0.071
7	35	2	1.80	0.023
8	235	10	7.69	0.695
Sum		62	61.796	9.606

Table B.6: χ^2-test of a hyper-exponential buffer time distribution between all of the trains on section Molykkja-Morskogen

Table B.6 lists the values for a χ^2-test for a hyper-exponential distribution.
Number of classes $z = 8$
Number of estimated parameters $r = 2$
Degree of freedom $z - r - 1 = 5$
Level of significance $\alpha = 0.05$
Since $9.61 < 11.07$ the hyper-exponential distribution can be accepted.

B.1.5 χ^2-test of the buffer times between all of the trains on section Espa-Tangen

Day of investigation: Thursday 18.04.2002
Sample: 17 freight trains, 35 IC trains, 8 ICE trains and 2 night trains
Largest registration: 204 minutes
Smallest registration: 0 minutes
Sample mean $\bar{t} = 18.20$
Sample variance $\sigma^2 = 712.88$
Sample standard deviation $\sigma = 26.70$
Coefficient of variation $V_T = 1.47$
Estimated parameter: $\lambda = \frac{1}{\bar{t}} = 0.05$
Estimated parameter $\xi = 0.20$

Class no.	Time interval [min]	Obs. frequency h_i	Theo. frequency y_i	$\frac{(h_i-y_i)^2}{y_i}$
1	5	11	14.90	1.019
2	10	9	11.32	0.475
3	15	12	8.60	1.346
4	20	5	6.53	0.359
5	25	15	4.96	20.300
6	30	6	3.77	1.319
7	35	1	2.86	1.214
8	205	3	9.06	4.050
Sum		62	61.9992	30.082

Table B.7: χ^2-test of an exponential buffer time distribution between all trains on section Espa-Tangen

Table B.8 lists the values for a χ^2-test for an exponential distribution.
Number of classes $z = 8$
Number of estimated parameters $r = 1$
Degree of freedom $z - r - 1 = 6$
Level of significance $\alpha = 0.05$
Since $30.08 > 12.59$ the exponential distribution cannot be accepted.

Class no	Time interval [min]	Obs. frequency h_i	Theo. frequency y_i	$\frac{(h_i-y_i)^2}{y_i}$
1	4	9	12.24	0.856
2	8	10	9.82	0.003
3	12	5	7.88	1.054
4	16	10	6.33	2.132
5	20	3	5.08	0.851
6	24	12	4.08	15.403
7	28	4	3.27	0.162
8	32	6	2.63	4.335
9	270	3	10.68	5.523
Sum		62	62	30.319

Table B.8: χ^2-test of a hyper-exponential buffer time distribution between all trains on section Espa-Tangen

Table B.8 lists the values for a χ^2-test for a hyper-exponential distribution.
Number of classes $z = 9$
Number of estimated parameters $r = 2$
Degree of freedom $z - r - 1 = 6$
Level of significance $\alpha = 0.05$
Since $30.32 > 14.07$ the hyper-exponential distribution cannot be accepted.

B.2 Line B

B.2.1 χ^2-test of the buffer times between the freight trains on section 3

Day of investigation: Thursday 18.04.2002
Sample: 11 freight trains
Largest registration: 266 minutes
Smallest registration: 1 minutes
Sample mean $\bar{t} = 122.25$
Sample variance $\sigma^2 = 8707.26$
Sample standard deviation $\sigma = 93.31$
Coefficient of variation $V_T = 0.76$
Estimated parameter: $\lambda = \frac{1}{\bar{t}} = 0.01$
Estimated parameter $k = 2$

Class no.	Time interval [min]	Obs. frequency h_i	Theo. frequency y_i	$\frac{(h_i-y_i)^2}{y_i}$
1	60	3	4.27	0.376
2	120	4	2.61	0.738
3	180	1	1.60	0.224
4	270	3	1.31	2.160
Sum		11	9.791	3.498

Table B.9: χ^2-test of an exponential buffer time distribution between the freight trains on section 3

Table B.9 lists the values for a χ^2-test for an exponential distribution.
Number of classes $z = 4$
Number of estimated parameters $r = 1$
Degree of freedom $z - r - 1 = 2$
Level of significance $\alpha = 0.05$
Since $3.498 < 5.99$ the exponential distribution can be accepted.

Class no.	Time interval [min]	Obs. frequency h_i	Theo. frequency y_i	$\frac{(h_i-y_i)^2}{y_i}$
1	60	3	2.83	0.376
2	120	4	3.59	0.738
3	180	1	2.29	0.224
4	270	3	1.56	2.160
Sum		11	10.281	2.105

Table B.10: χ^2-test of an Erlang$_2$ buffer time distribution between the freight trains on section 3

Table B.10 lists the values for a χ^2-test for an Erlang$_2$ distribution.
Number of classes $z = 4$
Number of estimated parameters $r = 2$
Degree of freedom $z - r - 1 = 1$
Level of significance $\alpha = 0.05$
Since $2.11 < 3.84$ the Erlang$_2$ distribution can be accepted.

B.2.2 χ^2-test of the buffer times between the freight trains and the RB trains on section 3

Day of investigation: Thursday 18.04.2002
Sample: 11 freight trains and 25 RB trains
Largest registration: 237 minutes
Smallest registration: 1 minutes
Sample mean $\bar{t} = 34.57$
Sample variance $\sigma^2 = 1828.78$
Sample standard deviation $\sigma = 42.76$
Variation coefficient $V_T = 1.24$

Estimated parameter: $\lambda = \frac{1}{t} = 0.03$
Estimated parameter $\xi = 0.27$

Class no.	Time interval [min]	Obs. frequency h_i	Theo. frequency y_i	$\frac{(h_i-y_i)^2}{y_i}$
1	5	5	4.71	0.017
2	10	5	4.08	0.208
3	15	3	3.53	0.079
4	20	4	3.05	0.293
5	25	2	2.64	0.156
6	30	1	2.29	0.724
7	240	15	14.66	0.008
Sum		35	34.966	1.486

Table B.11: χ^2-test of an exponential buffer time distribution between the freight and RB trains on section 3

Table B.11 lists the values for a χ^2-test for an exponential distribution.
Number of classes $z = 7$
Number of estimated parameters $r = 1$
Degree of freedom $z - r - 1 = 5$
Level of significance $\alpha = 0.05$
Since $1.49 < 11.07$ the exponential distribution can be accepted as buffer time distribution between the freight and RB trains.

Class no.	Time interval [min]	Obs. frequency h_i	Theo. frequency y_i	$\frac{(h_i-y_i)^2}{y_i}$
1	5	5	5.57	0.057
2	10	5	4.59	0.037
3	15	3	3.79	0.166
4	20	4	3.14	0.234
5	25	2	2.61	0.142
6	30	1	2.17	0.634
7	240	15	12.91	0.339
Sum		35	34.779	1.610

Table B.12: χ^2-test of a hyper-exponential buffer time distribution between the freight and RB trains on section 3

Table B.12 lists the values for a χ^2-test for a hyper-exponential distribution.
Number of classes $z = 7$
Number of estimated parameters $r = 2$
Degree of freedom $z - r - 1 = 4$
Level of significance $\alpha = 0.05$
Since $1.61 < 9.49$ the hyper-exponential distribution can be accepted as buffer time distribution between the freight and RB trains.

B.2.3 χ^2-test of the buffer times between all of the trains on section 1

Day of investigation: Thursday 18.04.2002
Sample: 1 IC train, 15 IR trains, 41 RB trains, 11 freight trains and 3 non defined locomotives = 71 trains
Largest registration: 109 minutes
Smallest registration: 1 minutes
Sample mean $\overline{t} = 13.08$
Sample variance $\sigma^2 = 376.90$
Sample standard deviation $\sigma = 19.41$
Coefficient of variation $V_T = 1.48$
Estimated parameter: $\lambda = \frac{1}{\overline{t}} = 0.08$
Estimated parameter $\xi = 0.19$

Class no.	Time interval [min]	Obs. frequency h_i	Theo. frequency y_i	$\frac{(h_i-y_i)^2}{y_i}$
1	3	6	18.86	8.772
2	6	20	13.29	3.382
3	9	12	9.43	0.703
4	12	7	6.73	0.011
5	15	4	4.85	0.149
6	18	10	3.54	11.820
7	21	0	2.61	2.611
8	24	8	1.96	18.651
9	180	4	9.66	3.386
Sum		71	71	49.485

Table B.13: χ^2-test of a hyper-exponential buffer time distribution between all of the trains on section 1

Table B.13 lists the values for a χ^2-test for a hyper-exponential distribution.
Number of classes $z = 9$
Number of estimated parameters $r = 2$
Degree of freedom $z - r - 1 = 6$
Level of significance $\alpha = 0.05$
Since $49.49 > 12.59$ the hyper-exponential distribution cannot be accepted as buffer time distribution between all of the trains.

B.2.4 χ^2-test of the buffer times between all of the trains on section 2

Day of investigation: Thursday 18.04.2002
Sample: 1 IC train, 15 IR trains, 25 RB trains, 11 freight trains and 3 not defined locomotives = 55 trains
Largest registration: 111 minutes

Smallest registration: 1 minutes
Sample mean $\overline{t} = 21.21$
Sample variance $\sigma^2 = 519.34$
Sample standard deviation $\sigma = 22.79$
Coefficient of variation $V_T = 1.07$
Estimated parameter: $\lambda = \frac{1}{\overline{t}} = 0.05$
Estimated parameter $k = 2$

Class no.	Time interval [min]	Obs. frequency h_i	Theo. frequency y_i	$\frac{(h_i-y_i)^2}{y_i}$
1	5	5	11.55	3.715
2	10	10	9.13	0.084
3	15	12	7.21	3.185
4	20	11	5.69	4.942
5	25	2	4.50	1.388
6	30	2	3.55	0.679
7	35	5	2.81	1.712
8	270	8	10.56	0.621
Sum		55	55	16.327

Table B.14: χ^2-test of an exponential buffer time distribution between all of the trains on section 2

Table B.15 lists the values for a χ^2-test for an exponential distribution.
Number of classes $z = 7$
Number of estimated parameters $r = 1$
Degree of freedom $z - r - 1 = 5$
Level of significance $\alpha = 0.05$
Since $16.33 > 12.59$ the exponential distribution cannot be accepted as buffer time distribution between all of the trains.

Class no.	Time interval [min]	Obs. frequency h_i	Theo. frequency y_i	$\frac{(h_i-y_i)^2}{y_i}$
1	4	3	3.05	0.001
2	8	6	6.57	0.049
3	12	8	7.57	0.025
4	16	10	7.29	1.005
5	20	11	6.44	3.223
6	24	2	5.41	2.147
7	180	15	18.67	0.721
Sum		55	55	7.171

Table B.15: χ^2-test of an Erlang_2 buffer time distribution between all of the trains on section 2

Table B.15 lists the values for a χ^2-test for an Erlang_2 distribution.

Number of classes $z = 7$
Number of estimated parameters $r = 2$
Degree of freedom $z - r - 1 = 4$
Level of significance $\alpha = 0.05$
Since $7.17 < 9.49$ the $Erlang_2$ distribution can be accepted as buffer time distribution between all of the trains.

B.2.5 χ^2-test of the buffer times between all of the trains on section 3

Day of investigation: Thursday 18.04.2002
Sample: 1 IC train, 15 IR trains, 25 RB trains, 11 freight trains and 3 non defined locomotives = 55 trains
Largest registration: 107 minutes
Smallest registration: 0 minutes
Sample mean $\bar{t} = 18.07$
Sample variance $\sigma^2 = 603.84$
Sample standard deviation $\sigma = 24.57$
Coefficient of variation $V_T = 1.36$
Estimated parameter: $\lambda = \frac{1}{\bar{t}} = 0.05$
Estimated parameter $\xi = 0.23$

Class no.	Time interval [min]	Obs. frequency h_i	Theo. frequency y_i	$\frac{(h_i-y_i)^2}{y_i}$
1	6	18	15.54	0.390
2	12	15	11.15	1.330
3	18	3	8.00	3.124
4	24	6	5.74	0.012
5	30	1	4.12	2.360
6	36	1	2.95	1.293
7	42	4	2.12	1.668
8	180	7	5.38	0.488
Sum		55	55	10.665

Table B.16: χ^2-test of an exponential buffer time distribution between all of the trains on section 3

Table B.16 lists the values for a χ^2-test for an exponential distribution.
Number of classes $z = 8$
Number of estimated parameters $r = 1$
Degree of freedom $z - r - 1 = 6$
Level of significance $\alpha = 0.05$
Since $10.67 < 12.59$ the exponential distribution can be accepted as buffer time distribution between all of the trains.

Class no.	Time interval [min]	Obs. frequency h_i	Theo. frequency y_i	$\frac{(h_i-y_i)^2}{y_i}$
1	6	18	18.82	0.035
2	12	15	11.72	0.919
3	18	3	7.41	2.622
4	24	6	4.77	0.316
5	30	1	3.15	1.464
6	36	1	2.13	0.602
7	42	4	1.49	4.216
8	180	7	5.38	0.487
Sum		55	54.864	10.663

Table B.17: χ^2-test of a hyper-exponential buffer time distribution between all of the trains on section 3

Table B.17 lists the values for a χ^2-test for a hyper-exponential distribution.
Number of classes $z = 8$
Number of estimated parameters $r = 2$
Degree of freedom $z - r - 1 = 5$
Level of significance $\alpha = 0.05$
Since $10.66 < 12.59$ the hyper-exponential distribution can be accepted as buffer time distribution between all of the trains.

B.2.6 χ^2-test of the buffer times between all of the trains on section 4

Day of investigation: Thursday 18.04.2002
Sample: 1 IC train, 15 IR trains, 25 RB trains and 11 freight trains = 52 trains
Largest registration: 133 minutes
Smallest registration: 1 minutes
Sample mean $\bar{t} = 23.06$
Sample variance $\sigma^2 = 773.02$
Sample standard deviation $\sigma = 27.80$
Coefficient of variation $V_T = 1.21$
Estimated parameter: $\lambda = \frac{1}{\bar{t}} = 0.04$
Estimated parameter $\xi = 0.29$

Class no.	Time interval [min]	Obs. frequency h_i	Theo. frequency y_i	$\frac{(h_i-y_i)^2}{y_i}$
1	8	15	15.53	0.018
2	16	17	10.98	3.299
3	24	4	7.76	1.824
4	32	7	5.49	0.417
5	40	1	3.88	2.137
6	48	1	2.74	1.107
7	180	8	6.59	0.301
Sum		53	52.9784	9.103

Table B.18: χ^2-test of an exponential buffer time distribution between all of the trains on section 4

Table B.18 lists the values for a χ^2-test for an exponential distribution.
Number of classes $z = 7$
Number of estimated parameters $r = 1$
Degree of freedom $z - r - 1 = 5$
Level of significance $\alpha = 0.05$
Since $9.10 < 11.07$ the exponential distribution can be accepted as buffer time distribution between all of the trains.

Class no.	Time interval [min]	Obs. frequency h_i	Theo. frequency y_i	$\frac{(h_i-y_i)^2}{y_i}$
1	8	15	17.53	0.364
2	16	17	11.25	2.942
3	24	4	7.32	1.506
4	32	7	4.84	0.959
5	40	1	3.27	1.573
6	48	1	2.25	0.694
7	180	8	6.37	0.418
Sum		53	52.823	8.457

Table B.19: χ^2-test of a hyper-exponential buffer time distribution between all of the trains on section 4

Table B.19 lists the values for a χ^2-test for a hyper exponential distribution.
Number of classes $z = 7$
Number of estimated parameters $r = 2$
Degree of freedom $z - r - 1 = 4$
Level of significance $\alpha = 0.05$
Since $8.46 < 9.49$ the hyper-exponential distribution can be accepted as buffer time distribution between all of the trains.

B.3 Line C

B.3.1 $\chi^2 - test$ of the buffer times between all of the freight trains on section 1

Day of investigation: Thursday 17.04.2003
Sample: 21 freight trains
Largest registration: 232 minutes
Smallest registration: 1 minutes
Sample mean $\bar{t} = 61.96$
Sample variance $\sigma^2 = 4406.82$
Sample standard deviation $\sigma = 66.38$
Coefficient of variation $V_T = 1.07$
Estimated parameter: $\lambda = \frac{1}{\bar{t}} = 0.02$
Estimated parameter $\xi = 0.37$

Class no.	Time interval [min]	Obs. frequency h_i	Theo. frequency y_i	$\frac{(h_i - y_i)^2}{y_i}$
1	30	8	8.06	0.000
2	60	7	4.97	0.833
3	90	1	3.06	1.387
4	120	1	1.89	0.416
5	350	4	2.95	0.370
Sum		21	20.926	3.007

Table B.20: χ^2-test of an exponential buffer time distribution between all of the freight trains on section 1

Table B.20 lists the values for a $\chi^2 - test$ for an exponential distribution.
Number of classes $z = 5$
Number of estimated parameters $r = 1$
Degree of freedom $z - r - 1 = 3$
Level of significance $\alpha = 0.05$
Since $3.01 < 7.82$ the exponential distribution can be accepted as buffer time distribution between all of the freight trains.

Class no.	Time interval [min]	Obs. frequency h_i	Theo. frequency y_i	$\frac{(h_i-y_i)^2}{y_i}$
1	30	8	8.39	0.018
2	60	7	4.92	0.882
3	90	1	2.92	1.266
4	120	1	1.77	0.332
5	350	4	2.88	0.440
Sum		21	20.869	2.938

Table B.21: χ^2-test of a hyper-exponential buffer time distribution between all of the freight trains on section 1

Table B.21 lists the values for a χ^2-test for a hyper-exponential distribution.
Number of classes $z = 5$
Number of estimated parameters $r = 2$
Degree of freedom $z - r - 1 = 2$
Level of significance $\alpha = 0.05$
Since $2.94 < 5.99$ the hyper-exponential distribution can be accepted as buffer time distribution between all of the freight trains.

B.3.2 χ^2-test of the buffer times between the freight trains and the RB trains on section 1

Day of investigation: Thursday 17.04.2003
Sample: 37 RB trains and 21 freight trains =58 trains
Largest registration: 232 minutes
Smallest registration: 1 minutes
Sample mean $\bar{t} = 20.65$
Sample variance $\sigma^2 = 963.99$
Sample standard deviation $\sigma = 31.05$
Coefficient of variation $V_T = 1.50$
Estimated parameter: $\lambda = \frac{1}{\bar{t}} = 0.05$
Estimated parameter $\xi = 0.19$

Class no.	Time interval [min]	Obs. frequency h_i	Theo. frequency y_i	$\frac{(h_i-y_i)^2}{y_i}$
1	6	7	18.82	7.422
2	12	7	12.06	2.120
3	18	27	7.80	47.232
4	24	4	5.12	0.245
5	30	5	3.42	0.732
6	36	3	2.33	0.190
7	42	0	1.64	1.635
8	235	5	6.67	0.566
Sum		58	58	60.143

Table B.22: χ^2-test of a hyper-exponential buffer time distribution between the freight trains and the RB trains on section 1

Table B.22 lists the values for a χ^2-test for a hyper-exponential distribution.
Number of classes $z = 8$
Number of estimated parameters $r = 2$
Degree of freedom $z - r - 1 = 5$
Level of significance $\alpha = 0.05$
Since $60.14 > 11.07$ the hyper-exponential distribution cannot be accepted as buffer time distribution between freight trains and the RB trains on section 1.

B.3.3 χ^2-test of the buffer times between all of the trains on section 1

Day of investigation: Thursday 17.04.2003
Sample: 1 night train (Nt), 2 IC trains, 14 IR trains, 37 RB trains, 21 freight trains (Ft) and 2 single locomotives = 77 trains
Largest registration: 172 minutes
Smallest registration: 1 minutes
Sample mean $\bar{t} = 13.45$
Sample variance $\sigma^2 = 398.56$
Sample standard deviation $\sigma = 19.96$
Coefficient of variation $V_T = 1.48$
Estimated parameter: $\lambda = \frac{1}{\bar{t}} = 0.07$
Estimated parameter $\xi = 0.19$

Class no.	Time interval [min]	Obs. frequency h_i	Theo. frequency y_i	$\frac{(h_i-y_i)^2}{y_i}$
1	6	16	34.21	9.690
2	12	22	17.50	1.156
3	18	27	9.23	34.201
4	24	8	5.09	1.663
5	30	1	2.98	1.315
6	36	1	1.87	0.406
7	42	0	1.26	1.265
8	48	0	0.91	0.913
9	54	0	0.70	0.695
10	60	1	0.55	0.370
11	300	1	2.69	1.069
Sum		77	77	52.742

Table B.23: χ^2-test of a hyper-exponential buffer time distribution between all of the trains on section 1

Table B.23 lists the values for a χ^2-test for a hyper-exponential distribution.
Number of classes $z = 11$
Number of estimated parameters $r = 2$
Degree of freedom $z - r - 1 = 8$
Level of significance $\alpha = 0.05$
Since $52.74 > 15.51$ the hyper-exponential distribution cannot be accepted as buffer time distribution between all of the trains on section 1.

B.3.4 χ^2-test of the buffer times between all of the trains on section 2

Day of investigation: Thursday 17.04.2003
Sample: 1 night train (Nt), 2 IC trains, 14 IR trains, 37 RB trains, 21 freight trains (Ft) and 2 single locomotives = 77 trains
Largest registration: 131 minutes
Smallest registration: 1 minutes
Sample mean $\bar{t} = 11.01$
Sample variance $\sigma^2 = 281.74.56$
Sample standard deviation $\sigma = 16.79$
Coefficient of variation $V_T = 1.53$
Estimated parameter: $\lambda = \frac{1}{\bar{t}} = 0.09$
Estimated parameter $\xi = 0.18$

Class no.	Time interval [min]	Obs. frequency h_i	Theo. frequency y_i	$\frac{(h_i-y_i)^2}{y_i}$
1	5	36	35.07	0.025
2	10	10	17.52	3.227
3	15	8	9.03	0.118
4	20	9	4.88	3.474
5	25	7	2.82	6.221
6	30	5	1.76	6.000
7	35	0	1.19	1.186
8	40	0	0.86	0.861
9	180	2	3.85	0.923
Sum		77	77	22.034

Table B.24: χ^2-test of a hyper-exponential buffer time distribution between all of the trains on section 2

Table B.25 lists the values for a χ^2-test for a hyper-exponential distribution.
Number of classes $z = 9$
Number of estimated parameters $r = 2$
Degree of freedom $z - r - 1 = 6$
Level of significance $\alpha = 0.05$
Since $22.03 > 12.59$ the hyper-exponential distribution cannot be accepted as buffer time distribution between all of the trains on section 2.

Class no.	Time interval [min]	Obs. frequency h_i	Theo. frequency y_i	$\frac{(h_i-y_i)^2}{y_i}$
1	5	36	28.11	2.212
2	10	10	17.85	3.452
3	15	8	11.33	0.980
4	20	9	7.19	0.453
5	25	7	4.57	1.295
6	30	5	2.90	1.521
7	35	0	1.84	1.841
8	40	0	1.17	1.169
9	180	2	2.03	0.001
Sum		77	77	12.923

Table B.25: χ^2-test of an exponential buffer time distribution between all of the trains on section 2

Table B.25 lists the values for a χ^2-test for an exponential distribution.
Number of classes $z = 9$
Number of estimated parameters $r = 1$
Degree of freedom $z - r - 1 = 7$
Level of significance $\alpha = 0.05$
Since $12.92 < 14.07$ the exponential distribution can be accepted as buffer time distribution between all of the trains on section 2.

B.3.5 χ^2-test of the buffer times between all of the trains on section 3

Day of investigation: Thursday 17.04.2003
Sample: 1 Night tra (Nt), 2 IC trains, 14 IR trains, 21 RB trains, 21 freight trains (Ft) and 2 single locomotives = 61 trains
Largest registration: 123 minutes
Smallest registration: 1 minutes
Sample mean $\bar{t} = 18.24$
Sample variance $\sigma^2 = 350.58$
Sample standard deviation $\sigma = 18.72$
Coefficient of variation $V_T = 1.03$
Estimated parameter: $\lambda = \frac{1}{\bar{t}} = 0.05$
Estimated parameter $\xi = 0.42$
Estimated parameter $k = 2$

Class no.	Time interval [min]	Obs. frequency h_i	Theo. frequency y_i	$\frac{(h_i - y_i)^2}{y_i}$
1	5	3	9.25	4.222
2	10	10	7.85	0.591
3	15	5	6.66	0.412
4	20	6	5.65	0.022
5	25	7	4.79	1.018
6	30	6	4.06	0.921
7	35	3	3.45	0.058
8	180	21	19.29	0.151
Sum		61	61	7.397

Table B.26: χ^2-test of an exponential buffer time distribution between all of the trains on section 3

Table B.26 lists the values for a χ^2-test for an exponential distribution.
Number of classes $z = 8$
Number of estimated parameters $r = 1$
Degree of freedom $z - r - 1 = 6$
Level of significance $\alpha = 0.05$
Since $7.40 < 12.59$ the exponential distribution can be accepted as buffer time distribution between all of the trains on section 3.

Class no.	Time interval [min)]	Obs. frequency h_i	Theo. frequency y_i	$\frac{(h_i-y_i)^2}{y_i}$
1	5	3	9.45	4.222
2	10	10	7.96	0.591
3	15	5	6.70	0.412
4	20	6	5.65	0.022
5	25	7	4.77	1.018
6	30	6	4.02	0.921
7	35	3	3.40	0.058
8	180	21	19.04	0.202
Sum		61	60.993	7.648

Table B.27: χ^2-test of a hyper-exponential buffer time distribution between all of the trains on section 3

Table B.27 lists the values for a χ^2-test for a hyper-exponential distribution.
Number of classes $z = 8$
Number of estimated parameters $r = 2$
Degree of freedom $z - r - 1 = 5$
Level of significance $\alpha = 0.05$
Since $7.65 < 11.07$ the hyper-exponential distribution can be accepted as buffer time distribution between all of the trains on section 3.

Class no.	Time interval [min]	Obs. frequency h_i	Theo. frequency y_i	$\frac{(h_i-y_i)^2}{y_i}$
1	5	3	2.66	0.044
2	10	10	5.96	2.740
3	15	5	7.20	0.673
4	20	6	7.28	0.225
5	25	7	6.75	0.009
6	30	6	5.94	0.001
7	35	3	5.06	0.838
8	180	21	20.15	0.036
Sum		61	61	4.565

Table B.28: χ^2-test of an Erlang_2 buffer time distribution between all of the trains on section 3

Table B.28 lists the values for a χ^2-test for an Erlang_2 distribution.
Number of classes $z = 8$
Number of estimated parameters $r = 2$
Degree of freedom $z - r - 1 = 5$
Level of significance $\alpha = 0.05$
Since $4.57 < 11.07$ the Erlang_2 distribution can be accepted as buffer time distribution between all of the trains on section 3.

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