Masteroppgave

Mathematics didactics

Rose Mbewe

Understanding the meaning of the Equal Sign (=)

A qualitative case study of Grade 8 students' assigned meanings of the equal sign and how these assigned meanings affect their performance in solving equations

Trondheim, juli 2014

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A thesis submitted in partial fulfilment of the requirements for the degree of Masters of Science in mathematics didactics

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Dedication

This thesis is dedicated to God almighty for giving me strength and wisdom to forge ahead even when it looked impossible

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Abstract

Many research about students` understanding of the equal sign record that most students view the equal symbol as a signal to carry out a computation instead of a symbol expressing mathematical equivalence. The purpose of this study was to find out the meanings Zambian Grade 8 students assign to the equal sign and how their assigned meanings of the equal sign affect their performance in solving equations. A test and individual interviews were used to collect data and the assigned meanings were compared across students as they used the equal sign in solving equations with the aim of correlating between understanding the equal sign as a symbol representing equivalence and success in solving equations. It was found that students who demonstrated an ability to recognize a relational meaning for the equal sign scored higher in solving equations. However, it was also found that some students who defined or articulated a relational meaning of the equal sign were not able to demonstrate an understanding of an appropriate use of the symbol.

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1 Introduction

1.1 Introduction

Expertise in mathematics is seen as an essential tool to success in modern society as it is used in our daily life. In line with such thinking, mathematics has been considered as one of the most important subjects in the school curriculum. The students` under achievement in mathematics is a global concern (Pisa, 2003). This concern has led to research to finding out the reasons for students` low achievement in mathematics. Students` understanding of mathematical concepts in general have been problematic in most Zambian schools as observed in the mathematics pass percent at national junior secondary school leaving examination at Grade 9 level in most Basic schools in Zambia. In 2008, mathematics at middle basic level had a national mean performance of 39.8 % which is below pass mark of 40 % in the subject (Education, 2008).

Introduced by Recorde in the 16th century, the equal sign "=" symbol has become the universally recognized symbol to indicate mathematical equality (Cajori, 1928). Mathematical equality can be defined as the principle that two sides of an equation have the same value and are thus interchangeable (Kieran, 1981) It is among the symbols first met by students upon starting school and yet research show that many students have some misconceptions about the symbol even at secondary school level. A well-developed conception of the equal sign is characterized by relational understanding, realizing that the equal sign symbolizes the sameness of the expressions or quantities represented by each side of the equation (Baroody & Ginsburg, 1983). Students need to have this critical understanding of the equal sign in order to find logical conclusion when solving equations.

As a relational symbol, the equal sign gets different meanings dependent on the contexts in which it is used, For example, it can indicate an identity, define a function, at other times used with a placeholder and interpreted as a "do something" sign or as a sign that says "now follow the answer". While in algebraic equations, the symbol is an example of an equivalence relation and can be used to designate symmetric and transitive character between the left- hand and right-hand side of the equation, for instance in 6x + 2 = 8 + 4x (Kieran, 1992). Students` understanding of the equal sign has been extensively investigated before mostly in developed countries, where it was found that most middle school students (ages 11- 15), viewed the equal sign as an invitation to "do something" (Baroody & Ginsburg, 1983; Behr, Erlwanger, & Nichols, 1980; Kieran, 1981; Saenz-Ludlow & Walgamuth, 1998).

It has been noted however, that relatively little attention has been given to finding out students` conception about the equal sign in developing countries like Zambia. Do Zambian Grade 8 students (ages 11 – 15) also view the equal symbol as an invitation to "do something"? In my teaching career, I have noted that students at both junior and senior secondary school level have a problem with understanding and using the equal sign correctly. Due to their inability to understand the equal sign, students have had problems to interpret and use the symbol in context. I have noticed that they see it as "indicating the place where the answer should be" written when solving equations.

It was therefore, found prudent to investigate the meanings Grade 8 students assign to the equal sign and how the meanings they assign to the equal sign relate to their performance in solving equations. This knowledge may help in creating a general overview of how students understand the equal sign in Zambia specifically and add to existing knowledge worldwide in general.
I now present the background of the study from the Zambian perspective in section 1.1 below

1.2 From the Zambian Perspective

The Zambian education system in place today is not very different from the curricular structures that were put in place by the colonial masters. A major social characteristic of Zambia is that it is multi-ethnic and multicultural. There are seven major languages and seventy-three dialects. The diversity of ethnic groups entails existence of several traditions and cultural practices which have their implication on education. It also entails several indigenous education systems that these people practiced. Traces of indigenous education have remained in people`s way of life 50 years after independence, having an impact on the "western education" offered to Zambian students. The colonial concept of education taught that literacy and numeracy are about the ability to read the "word" and make simple additions. The introduction of English as a medium of instruction had its own challenges. It meant that a Zambian student had to learn English and use the English to understand the mathematics. Learning in an alien language means that for the majority of students, school is unrelated to real life. As observed by Kelly "*Rote learning was the only way to approach a situation where understanding was absent from school, with mindless repetition replacing problem solving and inventiveness*" (Kelly, 1995, p. 6). The above statement shows that most Zambian students learn mathematics basically by mastering concepts without the actual understanding. Contrary to formal learning of mathematics, children in Zambia demonstrate a relational understanding of the equal sign as early as 4 years of age. Children attach value to certain denominations of the Zambian currency (Kwacha), for example, a child of 4 years knows that a K1 coin buys 2 lollipop sweets and that it cannot buy a loaf of bread. Given countable items to share equally, children below school age are able to do so successfully. This type of understanding unfortunately is lost when they are introduced to formal education. When entering primary school, children bring with them their intuitions about basic arithmetic operations and their interpretations of symbols are based on these intuitions. Unfortunately, their intuitions get in conflict with formal symbols leading to failure to understand and use the equal sign correctly a fact which can be attributed to the medium of instruction, which in this case is English.

From my years of teaching mathematics at secondary school, there has been no time when the equal sign was considered to be among the concepts to be taught whether in Grade 8 or 9 curriculum. It was generally assumed that students had learned and understood its use when it was first introduced in early primary school level, hence no second thought was given to it in later years. The motivation to do this study stems from observations from my personal and professional point of view on students` increasing difficulties in solving equations as seen in various contexts in which the equal sign is used in equations. Students tend to hold misconceptions about the symbol. Kieran (1992) asserts that misconceptions relating to the equal sign make it difficult for students to transform and solve algebraic equations. The fact that algebra is an important gateway into higher education, the significance of building high quality relational understanding of the equal sign is of great importance.

In the next section I present a brief introduction to the theories that underpinned the study while an elaborate version will be given in chapter 2.

1.3 The theoretical framework that guided my study

I situated my study within the socio cultural theory of teaching and learning as it is rooted in the work of Lev Vygotsky (1978, 1981a, 1981b, 1934/1987) and his predecessors. There are three fundamental concepts in Vygotsky`s explanation of learning and development (Wertsch, 1991). Firstly is the genetic law of cultural development (Vygotsky, 1981b); followed by the zone of proximal development (Vigotsky, 1978) and lastly, concept formation as an outcome of the interplay between spontaneous and scientific concepts (Vygotsky, 1987). Vygotsky`s principle of scientific concepts as the content of school instruction is an elaboration of his general view of mediated learning as the major determinant of human development (Kozulin, 2003).

In the next section, I present a brief context of the study and the methods adopted.

1.4 Overview of the methods

This study addressed the research questions; what meanings do students assign to the equal sign and how do the meanings students assign to the equal sign affect their performance in solving equations, through a qualitative case study research design. 45 grade eight students were selected from 2 grade 8 classes using purposeful sampling (Creswell, 2012). High performing students in mathematics were selected with the help of the class teachers. Data sources include participants` answer scripts, individual participant`s interview transcripts and researcher`s reflective notes. Data were analyzed through the constant comparative method. Emergent themes for each participant were compared and synthesized across all participants. Credibility for the study was sought by using multiple data collection method thus, students` answer scripts, students` rich thick descriptions, and researcher`s reflectivity.

The purpose of the current study is outlined in the subsection below.

1.5 Purpose of the study

The equal sign concept is very important to understanding of algebra and without it no meaning would be made out of the symbols and letters that constitute algebra. The recognition of the importance of the equal sign concept is based on decades of research (e. g Ginsburg (1977) to

Falkner, Levi, & Carpenter (1999), and even more recently Sherman & Bisanz (2009)) in developed countries but not much work has been done in developing countries like Zambia. The purpose of this study therefore, was to investigate how Zambian Grade 8 students understood the mathematical concept of the equal sign and how their understanding impacts on their performance in solving equations.
To state the purpose of the study on its own would not suffice to warrant carrying out the study, I now therefore give details of how this study will be of benefit by looking at its significance in the next subsection.

1.6 Significance of Study

Through the study, I created opportunities to hear from students by listening to their thoughts and perceptions about the equal sign. This study has helped me, as a teacher trainer to better understand the difficulties students have in understanding the equal sign from their perspective. With this knowledge, I hope to improve my own practice and also by working with other teachers to improve their practice too. It is envisaged that the information that is generated by this study would provide awareness that in turn may be used to inform both students and teachers on the best approaches in using the equal sign in mathematics in a Zambian setting. Too often the blame for poor achievement in solving equations is placed upon students without seeking to understand the underlying issues. This information might be valuable to curriculum developers in Zambia so as to add the learning of fundamental mathematical concepts such as the equal sign at all levels of mathematics education in order to enhance the understanding of the concept among the students. While the rigor of the mathematics curriculum is not likely to change, developing an understanding of students` perception of the equal sign may shed some light on what needs to be done to support and make grade eight equation solving a more successful experience. It is also hoped that the findings would help develop a body of literature to which teachers and students of mathematics could refer in order to develop effective and efficient mathematical intervention programs that are sensitive to the unique needs of the subject consumers.

In the next subsection, I present my research questions.

1.7 Research Questions

The following are the research questions the study intends to address:

1. What meanings do Zambian Grade 8 students (aged 11 - 15) assign to the equal sign/equal symbol?
2. How do students` assigned meanings and use of the equal sign affect their performance in solving equations?

Next I define the key terms in my research question in section 1.7

1.8 Definition of Key Terms

1. **Equal sign/ Equal symbol** will be used interchangeably to mean "="
2. **Performance** will be used to mean ability or lack of it to solve equations correctly

I will now present the overview of the structure of this study in the following section.

1.9 The structure of the study

Following this introductory chapter, I present the theoretical framework that guided the study in chapter 2. This is followed by the research methodology in chapter 3. The data analysis and findings are presented in chapter 4. The thesis closes in chapter 5 with a synthesis of findings including discussion of strengths and reflections on potential pedagogical implications of the reported study.

In the next chapter, I present the theoretical framework that guided the research.

2 Theoretical framework and literature review

2.1 Introduction

In this chapter, I will explain the theoretical framework of my study and review related literature. I will start by presenting the sociocultural theory perspective on knowledge formation and learning. My study is situated within this theory because of the semiotic perspective. As I try to answer my research question; the meanings students assign to the equal sign and how these assigned meanings affect students` performance in solving equations. I will go on to explain the role of symbols in mathematics as the equal sign is an example of these symbols. The history of the equal sign and its significance will be discussed, and also the purpose of this study before I conclude.

2.2 Sociocultural perspective on learning

According to Wertsch (1991, p. 18), he states that "*a sociocultural approach to learning begins with the assumption that action is mediated and that it cannot be separated from the environment in which it is carried out*". The approach which Wertsch (1991) outlines, identifies three basic themes which are closely intertwined. First, he talks of reliance on genetic or developmental analysis. This means that the focus is on the processes through which the human consciousness is formed and not only the product of development. Second, Vigotsky (1978,1981a) claims that higher mental functioning in the individual derives from social interaction and third, that tools and signs mediate human action on both the social and individual planes. The power of these three intertwined themes comes from the way in which they presuppose each other for example, one must use the genetic analysis to understand how higher mental functions are the mediated, internalized results of social interaction (Wertsch, 1991).

The sociocultural theory emphasizes that knowledge is gained through interaction between people and the environment using tools or artefacts. Tools constitute what is called semiotics and they include "*language, various systems of counting, mnemonics techniques, algebraic symbols, works of art, writing, schemes, diagrams, maps and many more of conventional signs*" (Vygotsky, 1981, p. 137). These can be categorized into two groups thus; physical (such as axe, hoe, or knife) and psychological (such as language). By using various tools, we are able to master different social practices and solve problems of which without them, it would have been impossible. These tools are all products of human cultural historical activity, created and changed by societies over a period of time. Thus they are a product of socio-cultural evolution and individuals have access to these tools by being actively engaged in the practices of their communities (Daniels, 2001).

Of the tools used for mediation, language is deemed as an important element in socio-cultural theory. This tool is said to have many functions but it is its semiotic function that is of relevance to this study. Language is considered a resource to creating knowledge about the world (John-Steiner & Mahn, 1996). Vigotsky (1978)`s work postulates that a child uses the word for communication purposes before that child has a fully developed understanding of the word. As a result of this use of the word, the meaning of that word which is the concept evolves for the child (John-Steiner & Mahn, 1996). The student starts communicating with peers, teachers or the potential others when writing using the signs of the new mathematical object as in symbols and words before one fully comprehends the mathematical sign. Thus, the meaning of a concept as expressed by words or a mathematical symbol is "imposed" on the child and this meaning is not assimilated in a ready-made form, rather it undergoes substantial development for the child as she uses the language in her communication with more socialized others (Vygotsky, 1987). In mathematics, a mathematical language that is not natural to the students is used in the same way as a mother tongue. The mathematical written language stands out from other languages because of its symbolic notations and in that mathematics is mediated with written symbols (Radford, 2012). In addition to mathematical symbols, Radford (2002) postulates that other tools are also helping mediate students` activities such as the language of gestures, body language and physical tools such as computers and calculators. I will now go into details about the role of symbols in mathematics.

2.3 The role symbols play in mathematics

Symbols are very significant in mathematics because without them it is not possible to think or perform mental generalizations. Thinking is not just something that takes place mentally, but which is realized through semiotic activity (Otte, 2005). Symbols are useful in understanding structures and also help us to carry out some operations (Pimm, 1991; Tall, 1995). Saenz-Ludlow & Walgamuth (1998) states that children are "born" into a world with already well defined mathematical symbol systems which they adopt without understanding. They argue that symbols do not express all their meaning without the interpreting activity of individuals. For mathematicians, the making of a symbol for a mathematical concept and the interpreting of the mathematical symbol become conventional and students are acculturated into the school mathematics. Meanwhile, research by Branford, Brown, & Cocking (2000) show that if knowledge is to be versatile and adaptable, it should be "organized" around a key concept and this key concept should be developed from students` informal understanding. This argument entails that if students are to learn the meaning of the mathematical symbols correctly, it is important to build this knowledge around their existing intuitions which they come with as they come to formal instruction. The interpretation of mathematical symbols and the interplay between natural language and symbolic marks in order to express mathematical ideas become a complex intellectual activity for students. Part of this complexity is the lack of one – to – one relationship between a particular mathematical symbol and the verbal expressions describing the meanings showed in a particular mark on paper. For instance, `6 – 3` can describe more than one process for example, 6 minus 3; 6 take away 3, the difference between 6 and 3, 3 less than 6, 3 more than 3 and so on. Building up mathematical meanings from symbols is a complex cognitive process. This process is what is called mathematizing. Freudenthal (1991) suggests that "*learners should reinvent mathematizing instead of mathematics; abstracting rather than abstractions; schematizing in place of schemes; formalizing instead of formulas; algorithmising instead of algorithms and verbalizing rather than language*" (Freudenthal, 1991, p. 49).

In most instances, symbols are perceived as being abstract as students fail to associate them with what they already know in their day to day life. Because of this lack of connection most students believe that symbols are only mathematical objects used to learn mathematics. Unlike other subjects, in mathematics one has no direct access to the objects for example the number 3 is abstract and cannot be understood without using the word and symbol 3. A student is often expected to construct the properties of the object from the definition where neither diagrams nor examples of the mathematical object are presented together with the definition, making access to the object only through the various signs such as words and symbols (Tall, 1995). Steinbring (2006) argues that mathematical symbols have no meaning in themselves; they are seen mainly as instruments to interpret, describe and communicate mathematical knowledge. He goes on to say that they can simultaneously help us to operate this mathematical knowledge and generalize it.

The written language of mathematics is often seen as the core for mathematical activities. (Pimm, 1991) The shift from verbal mathematical language to written language presents many challenges to most students. They tend to adopt and use symbols without understanding what they really are (Skemp, 2002). Experience has shown that most students think that the symbols are the objects themselves for example the concept of 3 and the symbol 3, hence using the symbols without a correct understanding. For example, students are asked to find the area of a garden measuring 30cm by 20m they come up with the answer 600 cm^2. Asked why it is cm^2 and not m^2, their answer is that area should be in cm^2. These students have mastered the rule for finding area but have no understanding that the rule can only work for same units. Skemp (1989) says that to create an understanding of symbols in students is a huge and significant task on the part of the teacher. He urges teachers not to overlook the problems students have with symbols as they have to understand these symbols in order to excel in mathematics. He describes two types of structures; deep structures and surface structures. He states that the problems students have with symbols is due to a conflicting relationship between surface structures and deep structures. He describes surface structures as what we use to communicate with symbols (symbol structures) that is the algorithms such as the symbol 3 and deep structures as containing what we are trying to communicate (conceptual structures) such as the value that the symbol 3 represent, which is the "why" part of the algorithms (Skemp, 1989). In mathematics, it calls for an insight into both levels of structures through extensive use of symbols in the manipulation of mathematical concepts. If these structures are not in place it can cause problems in students` learning of mathematics concepts.

Skemp (1989) argues that most students` structures are not organized and because of this they learn to manipulate empty symbols without content reducing mathematics to only act on symbol manipulation. For example, when students are introduced to subtraction of numbers like 200 – 91 the concept of "borrowing" comes in which means that they are working with surface structures and if they don't understand the principle behind the "borrowing" then solving arithmetic problems of this nature becomes a big problem. Given an equation say for example, 5x + 7 = x -

5, students with deep structures will add and subtract the same amounts on both sides of the equation to maintain the balance. Skemp (1989) argues that many students only work on surface level. He postulates that teachers must help students to build deeper structures and a variety of meanings of the symbols.

Much of the power of mathematics lies in understanding its structures; while access to them is through symbols (Skemp, 1989). He further argues that it is important to not only use surface structures but also to understand deep structures of the mathematical symbol. Steinbring (2006) describes two approaches to a characterization of the role of mathematical symbols; Semiotic function and Epistemological function. The semiotic function is that symbol that stands for something else and is called the object of reference; it expresses a referential relationship in which emphasis is on the representational character of the sign. For example;

The reference object for these stars is 3. The epistemological function on the other hand is the role of a mathematical symbol in the frame of the epistemological constitution of mathematical knowledge. It indicates the possibility to function as an aid in knowing or understanding the concepts such as the role of the mathematical symbol in the development of mathematical knowledge (Steinbring, 2006), to understand the relationship between the object of reference and the symbol for instance the stars and the symbol 3 in the previous example will include the concept of equivalence.

Duval (2006), states that mathematics differs from other sciences in that a mathematician does not have access to their mathematical objects in the same way as for example biologists do. He says that the only access one has of the mathematical objects is through symbols, signs, words or drawings. He further argues that mathematical objects should not be confused with the semiotic representations, but that one must understand the mathematical concepts in the process. Wagner (1981) says that symbols, symbolic relationships and the introduction into the use and the reading of symbols are essential aspect for the formation of every culture and the mathematics culture give social and communicative meaning to letters, signs and diagrams during the learning process. The equal sign is an example of a mathematical symbol.

2.4 The equal sign

Efforts to measure students` understanding of the equal sign date as far back as the early 80`s. Even though there is more than 30 years of history in the studying of students` understanding and use of the equal sign there is very little difference in the way students understand and use the symbol over the years. Baroody & Ginsburg (1983) and Behr et al. (1980) carried seminal work on the students` understanding of the equal sign and found that students viewed the equal symbol as a "do something" signal. Drawing on these two works, many researchers have further refined and developed models to better describe and measure students` understanding of the equal sign (e.g. Molina & Ambrose (2008); Knuth, Stephens, McNeil, & Alibali (2006); Essien & Setati (2006); Alibali, Knuth, Hattikudur, McNeil, & Stephens (2007)).

The following section provides a rationale for analyzing students` meanings of the equal sign. This includes an analysis of the equal symbol and a brief history of the equal sign, thus how the symbol evolved and the confusion attached to it, a socio-cultural perspective on learning as a theory on which the study is anchored, students` understanding of the equal sign, their misunderstandings and the effect of symbols in learning mathematics in general and the equal sign in particular.

2.4.1 A short history of the equal sign

The equal symbol (=) is used in different mathematical contexts for example in arithmetic, algebra, trigonometry, set theory and so on. Freudenthal (1986) pointed at several of these uses such as, a computational result, as in $2 + 2 = 4$, an identity, as in $(x + y)(x – y) = x^2 – y^2$, a definition as in $i = \sqrt{-1}$, a function, as in $f(x) = \text{Cos } x$, a substitution as in $x = ½$. He asserts that generally in mathematics, the use of the equal symbol is often a matter of how it is defined. History shows that the equality symbol evolved together with the symbols for arithmetic operations thus, +, - , x, ÷. The two parallel horizontal lines that we use today to symbolize "quantitative sameness" for example in equations or "existence" as in definitions are taken for granted because they form an already well-established world of mathematical symbols which children have to learn in school. The socio-cultural struggle to standardize this symbol as it stands today lasted for several centuries. This struggle is well documented by Cajori (1928) who walks us through the times and presents us with a historical analysis and evidence of how mathematicians as individuals and also as cultures, as holistic and evolving social entities, came together to capture abstract concepts and meanings in written marks.

The equal symbol (=) as we know it today is accredited to Robert Recorde, a Welsh mathematician who came up with the symbol in 1557. It traces its first use in his work *The Whetstone of Witte*, to indicate equality. Recorde argues that "*no two things can be more equal than 2 parallel straight lines (═)*" (Groza, 1968, p. 214). History shows how several attempts by mathematicians to represent the equality relationship were made. Early writings on the Papyrus reveal that equality was represented by the symbol ╤, Diophantus a Greek mathematician represented the equal sign with the symbol i and after him, Pacioli and others indicated equality by æ, from the Latin word aequales for `equal`. In 1540, Recorde in his book Grounde of Artes had used the symbol Z to represent equality before adopting the current symbol a few years later (Ball, 1908).

Recorde`s symbol was not immediately adopted for use in mathematics. Groza (1968), says that in 1637, Descartes used the symbol α (or ∞) to denote equality which is well after Recorde had invented the equal sign. Even after its adoption the equal sign symbol (=) was used with different meanings, for example it was used to designate arithmetical difference by Francis Vieta or to designate plus or minus (±). According to Saenz-Ludlow & Walgamuth (1998), Caramuel and Euclides used it to designate parallel lines. In general, up until the time of Newton, the equal sign was more often represented by ∞ than any other symbol (Ball, 1908). Newton and Liebniz`s use of Recorde`s symbol is believed to be what led to its general adoption (Cajori, 1928). The equality sign ranks among the few mathematical symbols that met universal adoption. This brief summary of the evolution of the equal symbol gives us an insight of the nature of the mathematicians` symbolic activity through the course of time and across civilizations in which ingenuity, creativity, negotiation, modification and agreement were necessary for them to reach a consensus on the equal symbol (=).

From the above historical account of how the equal sign evolved, it clearly shows the symbol meaning confusion that exists in mathematics. Saenz-Ludlow & Walgamuth (1998, p. 156) argue that the historical evolution of the equal sign, like other mathematical symbols brings out not only;

> *"…… the complexity involved in marking a mathematical concept with a particular symbol and the adoption of symbolic conventions, but also exemplifies for us that mathematical meanings are not directly conveyed by symbols without the interpreting activity of the individual*".

This confusion has continued reproducing itself in the classroom with limited freedom for students as they construct mathematical meanings for this conventional symbol.

Many researchers have endeavored to understand the meanings students assign to the equal sign as evidenced from literature. The next section looks at students` understanding of the equal sign

2.4.2 Students` understanding of the equal sign

Students` understanding of the equal sign in the early stage of their education has been well documented and dates as far back as the 70s. Research shows that many students have a shallow understanding of equivalence. While equivalence is a very general relation in mathematics, the focus in this study is narrowed specifically to the equal sign in the context of equations. Studies have shown that primary school students view the equal sign as an automatic invitation to write the answer (thus, they hold an operational meaning) rather than as a symbol of mathematical equivalence (for example, Behr et al. (1980); Carpenter, Franke, & Levi (2003); Demonty & Vlassis (1999); Falkner et al. (1999); Herscovics & Kieran (1980); Kieran (1981); Lubinksi & Otto (2002); Saenz-Ludlow & Walgamuth (1998); Stacey & MacGregor (1997)). This interpretation of the equal sign is considered as a "misinterpretation" as it does not indicate the general understanding that the symbol expresses an equivalence relation between two quantities.

The equal sign should be understood as a relational symbol, indicating that a balanced relationship exists between numbers on the two sides of the equal symbol (Jacobs, Franke, Carpenter, Levi, & Battey, 2007). Young children show an understanding of equivalence by counting two sets and stating whether the sets are the same (Kieran, 1981). Unfortunately, students come to misunderstand the equal sign as they implicitly develop ideas about addition and subtraction after entering school (Seo & Ginsburg, 2003) and as they experience early primary school teaching that present equations in a typical form (e.g. 1 + 2 =---, (Capraro, Ding, Matteson, Capraro, & Li, 2007)). Such a focus on typical equations in textbooks and teacher instruction leads students to an operational understanding of the equal sign, which signals them to "do something" (Saenz-Ludlow & Walgamuth, 1998) or "find the answer" (McNeil & Alibali, 2005).

A correct understanding of the equal sign is cardinal due to its important role in higher level mathematics, including algebra, trigonometry and calculus. Skemp (1976), describes two types understanding, thus instrumental and relational understanding. To exemplify Skemp`s description of the two understandings I will use a visitor to a new city. Imagine you are in a new city and you ask for directions from someone else on how to get to point B from Point A. You move back and forth and learn how to get from point A to point B and even be able to move faster from one point to another but once you step off the known roads, you are completely lost and you might even develop a fear of losing your way. You never really develop an overall understanding of the city. This is an example of instrumental understanding. On the other hand, instead of asking for specific roads, you get to wander all over the city, though at certain points you may be guided, but through most parts you just wander aimlessly. In time you develop an overall picture of the city and might even discover the shortest routes from point A to point B and might understand how each point is related to each other. If someone showed you a shortcut you would probably understand why it worked and why it was faster but you wouldn't worry of getting lost since even if you did, you would be able to use your overall understanding of the city to come to a place you know. This is an example of relational understanding. Skemp (1976) describes relational understanding as "*knowing both what to do and why*" (Skemp, 1976, p. 2). In case of algebra, a relational understanding of the equal sign helps students solve equations correctly as they consider the cardinal issue of balancing the left hand side and the left hand side of the equation. When students believe that the equal sign means performing an operation, then equations in the form 5 = 5 are seen as being incorrect as there is no plus sign (for example (Carpenter & Levi, 2000)) and tend to solve atypical equations incorrectly (e.g. Lindvall & Ibarra (1980) and Weaver (1973)). Students who interpret the equal sign in a relational manner are more likely to solve algebraic equations successfully (Knuth et al., 2006). If an operational understanding of the equal sign compromises understanding of equations as well as solution accuracy, then word problem performance will also be affected (Carpenter et al., 2003). Understanding the equal sign in a relational manner is very important if students are going to solve equations correctly more especially when they begin to solve equations with missing addends, minuends, subtrahends, factors, dividends, or divisors when the equal sign is not in a standard position (e.g. 3 = 7 - _; 3 x _ = 9). If students believe the equal sign to mean perform an operation, equations where the missing part is not the answer will most likely be answered incorrectly.

From primary through to secondary school, students often have misconceptions about the equal sign as found from previous studies for example Kieran (1981), Behr et al. (1980) and Knuth et al. (2006). Students` difficulties in understanding the equal sign has been noted as early as kindergarten (Falkner et al., 1999). Researchers hypothesize that as students see and work with typical teacher, or textbook presented equations, where the answer always requires to be computed after the equal sign, students come to understand the equal sign as an operational indicator directing them to perform a calculation (McNeil et al., 2006). Most time students are introduced to solving equations where only a sum or difference is computed (Behr et al., 1980). These equations are simple to solve and despite students perceiving the equal sign as operational, they still get correct answers. Misconceptions are most likely to be seen in equations that are not in the standard form.

Students should have a correct understanding of the equal sign in order to work on higher level mathematical problems such as non-standard equations and algebraic equations (Herscovics & Kieran (1980); Powell & Fuchs (2010)). Links have been proposed between students` interpretation of the equal sign and their success in working with equations. Most time students are taught to read word problems and then to develop a mathematical equation to assist in solving the word problem (Lindvall & Ibarra (1980); Nathan & Koedinger (2000)). Lack of a correct understanding of the equal sign will lead to students` wrong interpretation of the word problem. For example given the word problem; I have 5 apples and I add one more. Then I add again 2 more apples. How many apples do I have now? Students with a wrong interpretation of the equal sign will transform this sentence as 5 + 1 = 6 + 2 = 8. These students do not take into consideration the relationship between the two sides of the equal sign but will use the equal sign as if they are punching numbers into a calculator. If students have difficulties solving simple equations in primary school and do not learn the correct interpretation of the equal sign, solving equations like $x + 20 = 25$ or algebraic equations like $2y = x – 5$ and performing other higher level mathematics calculations will become more difficult as they advance in mathematics education (Carpenter et al., 2003).

Knuth, Alibali, McNeil, Weinberg, & Stephens (2005) provide evidence of a correlation between middle school students` understandings of the equal sign and their abilities to recognize equivalent equations. Students who defined the equal sign as a symbol of mathematical equivalence were more likely to recognize not only that the equation 2 x □ + 15 = 31 and 2 x □ + 15 – 9 = 31 – 9, had the same solutions, but also that this could be determined without actually solving the two equation (Knuth et al., 2005). These studies suggest not only that students` correct interpretation of the equal sign is cardinal but also that a relational understanding of the equal sign will enhance students` proficiency in solving equations.

Given that passing mathematics is compulsory requirement at junior secondary school leaving examination in the Zambian curriculum, and algebra being an integral part of the mathematics offered at this level, a proper foundation for understanding the equal sign is necessary.

2.4.3 Misinterpretation of the equal sign

The operational and relational understanding of the equal sign has been rather contentious among scholars, who advance that there are variables at play which may hamper or harness the understanding of the operations. One school of thought contends that students` early exposure to arithmetic operations related to addition, subtraction, multiplication and division has a bearing on how they interpret, use and understand the equal sign (Kieran (1981), 1981; Saenz-Ludlow & Walgamuth (1998); Baroody & Ginsburg (1983)). This school of thought gives rise to a relational view that suggests equivalence (Wheeler, 2010). Another school of thought is of the view that the way individual students learn may have an underlying obstruction of having a relational interpretation of the equal sign (Molina & Ambrose (2008); Warren (2003)). In essence, it would not be wrong to assume that both cognition and instructional factors do contribute to students` perception of the equal sign as a symbol operator (Baroody & Ginsburg, 1983).

Due to their perception of the equal sign as an operational symbol, most primary students assume that the equal sign signals them to “do something” or to “find the total” or that “the answer comes next". Students viewing the equal sign as a signal to "do something", look to the right side of the equal sign (Cobb, 1987), the same as those who view the equal sign as a prompt to find the total even when finding the total is not appropriate (McNeil & Alibali, 2005a), for instance, students may solve this equation $5 + 4 = \_ + 3$ by adding all the three numbers and getting 12 as their answer. Meanwhile for students who interpret the equal sign as meaning "the answer comes next" will put 9 as the answer in the blank space as found by Sherman & Bisanz (2009). Falkner et al. (1999) study of students` attempt to solve the equation $8 + 4 = \square + 5$ got answers of 12, 17 or both 12 and 17 as the missing addend. Evidence of operational understanding has been recorded even among secondary school students. Kieran (1981, p. 324), for example, found 12 and 13 year-olds had difficulty assigning meaning to expressions like 3a or a + 3 because, as one student stated, "there is no equal sign with a number after it" . McNeil & Alibali (2005a) found this poor understanding of the equal sign even among college students. These studies show that misinterpretation of the equal sign does not get better with students` progress through grades and also that the understanding of mathematical equivalence is worthy of direct instructional attention.

Jones (2009) affirms that the do something interpretation of the equal sign seems to hold as he agrees with Kieran (1981) and most scholars that have studied students` understanding of the equal sign. However, his argument is whether this is a true claiming for all students, hence my interest to investigate on Zambian grade 8 students of mathematics. He further observed that Renwick (1932) reported that 12 year olds used the equal sign (=) to simply separate an expression from its answer. He also cited Knuth et al. (2006)`s definition of the equal sign as:

a) What the sum of the two numbers is.

b) A sign connecting the answer to the problem

c) How much the numbers added together equal. (Jones, 2009, p. 6)

These are what have been formed as operational conceptions because of their nature to give a result of an arithmetic operation. He adds that Renwick (1932) also envisioned that in a child`s mind the function of the equal sign appeared to separate rather than to bridge mathematical sentences

Misinterpretation of the equal sign may also stem from a misunderstanding of symbols. Without formal instruction, children perform relatively well with story problems as found by Seo & Ginsburg (2003). For example, a kindergarten 4 year old student was able to share 12 sweets equally between the two of them. Seo & Ginsburg (2003) argue that children have a relational understanding of the equal sign but tend to lose it when they receive formal instruction as they start formal school. This indicates that symbolic representations and problem structure may hinder students` ability to solve problems correctly (Carpenter, Hiebert, & Moser, 1981).

In section 2.6 below, I present the purpose of the current study.

2.5 Purpose of the present study

Based on this body of research, it appears that without formal relational instruction of the equal sign, students view the equal sign as an operator symbol. Thus, the equal sign invites students to "do something" or to "find the total" or that "the answer comes next" (Cobb, 1987; McNeil & Alibali, 2005; Sherman & Bisanz, 2009). Viewing the equal sign as an operational symbol may be a result of early exposure to standard equations (e.g. $1 + 1 = 2$) where the equal sign always mean to add or write an answer (Baroody & Ginsburg, 1983; Herscovics & Kieran, 1980) or from misunderstanding the symbol that represent equivalence (Moss & Beatty (2010), ;Sherman & Bisanz (2009)). The purpose of this study was to extend the work done by (Baroody & Ginsburg, 1983; Behr et al., 1980; Essien & Setati, 2006; McNeil et al., 2006), to determine whether Zambian grade 8 students demonstrate similar trends in terms of how they understand the equal sign and their success solving equations.

In Chapter three I present the methodology used in this study. I looked at the Research design, methods of collecting data and methods of how the data was analyzed.

3 Methodology

3.1 Introduction

This chapter presents the research methodology that was used to collect data, and the means of analyzing it to arrive at the study's conclusions. Among other important components in this chapter are research design, methods of data collection and methods of data analysis.

3.2 Research Design

3.2.1 Qualitative Research design

The purpose of my study was to identify students` assigned meanings of the equal sign and how these meanings impacted on their performance in solving equations. I had to make a choice on how I was going to collect my data beforehand. As the purpose of my study is to provide a holistic, in-depth account of the case under study, extensive, multiple sources of data are needed (Erickson, 1986). Triangulation is the term used to indicate the use of multiple pieces of evidence to claim a result with confidence. This increases the credibility or trustworthiness of the findings (Johnson & Christensen, 2008). For example in my study, I used students` written work, interview transcripts, and researcher`s notes to triangulate the data and arrive at valid conclusions about students` understanding of the equal sign.

A qualitative research design was adopted for this study for exploring students` understanding of the equal sign. Maxwell (2012) identifies five particular research purposes for which qualitative studies are especially suited. These are; to understand the meaning of the events, situations and actions involved to understand the particular context within which the participants act, to identify unanticipated phenomenon and to generate new grounded theories, to understand the process by which events and actions take place and to develop casual explanations. In my study, the qualitative data are used for explanatory as well as for exploratory purposes. This type of design is suitable when you seek to understand the phenomenon and meanings students have constructed. The interest of qualitative researchers is to understand how their research subjects make sense of their world and their experiences. Here the interest is to understand the phenomenon of interest from the participants` perspective by highlighting their voice. My voice is also revealed as I present the story about the research process as one of the features of qualitative research is the intertwining of data collection and data analysis throughout the whole process.

In qualitative research it is imperative to be aware of the subjectivity of the process. Experiences as well as choice of theoretical framework guide the process in trying to understand and create meaning out of the collected data. According to Creswell (2012), a qualitative researcher sees that her research can never be objective or devoid of prejudice. As I have already presented my theoretical framework for this study in chapter 2, this could compromise the way I view the data and according to Merriam (1998), every study has one or all aspects of the research affected by it this bias. She states that the framework can be identified through our vocabulary, concepts and theories. "*The disciplinary orientation is the lens through which you view the world and that determines what you are curious about, what puzzles you and hence, what questions you ask that in turn begin to form your investigation*" (Merriam, 1998, p. 45). Students were tested and interviewed in their natural setting without controlling any aspect of the research situation. As this study explored students` assigned meanings of the equal sign and how the assigned meanings relate to performance in solving equations, these are questions concerned with the process of phenomenon which are best answered through qualitative paradigm(Creswell, 2012).

3.2.2 Case study

Creswell (1998) defines case study research as an exploration of a system bounded in time and place. Yin (2014) suggests that case studies are preferred strategy when asking "how" and "why" questions where the investigator has little or no control over events and when the focus is on the phenomenon within some real life context. Yin (2014) suggestion is backed by Merriam (1998) who says case studies are particularly suitable designs if you are interested in the process. This method was preferred because it allowed me to get in-depth understanding of the students` conception of the equal sign and exploring how their understanding of the equal sign affected their performance in solving equations which in itself is a process. Miles & Huberman (1994, p. 25) define a "case" as "*a phenomenon of some sort occurring in a bounded context*". They describe a phenomenon (or case) to be a program, group, institution, process or person. I used the case study method as I considered individual students as cases.

I present the targeted population in section 3.3

3.2.3 Target population

The target population was 50 grade 8 students (ages 11 - 15) at a basic School in Livingstone in the Southern Province of Zambia.

The sample size of the participants and the sampling procedure for this study are described in the section that follows.

3.3 Sample size and Sampling procedure

The study enlisted a total number of 45 students from two grade 8 classes with age range from 11 to 15 years old regardless of gender representation. The school has two grade 8 classes with an average of 50 students in each class. This sample was fairly representative of the larger population of grade 8 students in this school because almost half of the total number of the grade 8 students participated in the study. The 50 best performing students in mathematics (25 from each class) according to the class teachers` recommendation were targeted but 5 could not obtain written consent from their parents/guardians and hence could not participate in the study exercise. Participants were drawn from this school because it is a demonstration school for the Teachers` College where the researcher works and also because the Head teacher is a personal friend and this gave me easier access to the school. The purposive sampling technique was used for this study in choosing the school and the participants for the written task. As the school`s large number of grade 8 students could not be all recruited for the study. Maxwell (2012) suggests using a purposive sampling when persons are "*selected deliberately in order to provide important information that cannot be obtained as well from other choices*" (p.70). This sample of students was hoped to provide a better picture of students` understanding of the equal sign considering their being ranked as high achievers in mathematics and I hypothesized that if these had a problem in understanding the equal sign then it was worse with the low achievers. To get an adequate and a manageable number of students for interviewing, I used the theoretical sampling strategy while analyzing students` answers to the test. Bui (2013) describes theoretical sampling as starting with the data, constructing tentative ideas about the data and then examining these ideas through further empirical inquiry. After the field work, I made a brief overview of possible solutions to the task on solving equations. In order to engage with the data, I felt a need to do more detailed analyses of the mathematics content in the task given to students. I tried to get a sense of what was going on and how with respect to students` engagement with the written task. I examined how students interpreted and solved the task and what was challenging for them in terms of solving the test items. After I had read the answer scripts, I made a brief over view of each solution to the test item. I came up with three distinct groups for the students` ability in solving equations thus the able, the unstable (Not very able but trying) and the not-able. Therefore, I selected my interviewees by thoroughly examining their answers to the test.

I present the instruments I used to collect data for this study in the section that follows.

3.4 Data collection instruments

As a researcher, I acted as an instrument for collecting data by making decisions, choosing from among many alternatives and exercising judgments throughout the process (Merriam, 1998). This will be backed by other voices from literature, experience and from the environment. A written test and individual interviews were the other voices during data collection. Essien and Setati (2006) say that while errors in the use of the equal sign may not be visible in verbal mathematics; they become prominent in written form. A written record of how students understand and use the equal sign was therefore important to have at hand to answer the research question. In addition to a written test, semi structured interviews organized around the six test items (adapted from existing research studies, on students` understanding of the equal sign for instance (Baroody & Ginsburg, 1983; Behr et al., 1980; Essien & Setati, 2006; Knuth et al., 2006) were conducted with 3 students.

Below, I present the test items with a rationale to why each item was used.

3.4.1 The test

Item 1

1. 3 + 4 = 7
   ↑

(a) The arrow above points to a symbol, what is the name of the symbol?

(b) What does the symbol mean?
(c) Can the symbol mean anything else? If yes, explain.

Item 1 was designed to assess students` assigned meanings of the equal symbol. (a) name of the symbol (b) to provide a statement regarding the symbol`s meaning and (c) if possible, provide a statement regarding an alternative meaning.

The rationale for (a), above is to restrict students from using the symbol in their answer to (b) for example "the symbol means equals". Then (c) was based on research by Rittle-Johnson & Alibali (1999), where they found that students give more than one interpretation of the equal sign if given the opportunity. This measure of the equal sign understanding is based on the seminal works of Behr et al. (1980) and Baroody & Ginsburg (1983) and it has been used widely in previous studies since.

Item 2 was posted to examine students` attention to underlying equation structure. Write the missing number in each square box below.

a) 14 x 3 = □ – 3
b) 9 – 5 = □ – 9
c) 24 + □ = 27 + 31
d) 100 ÷ 5 = □ + 5
e) $\frac{169}{13}$ = 13 - □

Item 2 constituted of arithmetic equations and was preferred because of its previous use on primary school students` understanding of the equal sign.(e.g. Baroody & Ginsburg (1983); Kieran (1981); Saenz-Ludlow & Walgamuth (1998); Essien & Setati (2006)). While some students might recognize the preservation of a relationship from one equation to another, others might yet identify the tasks as ones concerned with arithmetic computations. The equal sign was presented in different contexts and this lead to finding out whether grade 8 students would interpret the symbol correctly in these contexts. And by *correct interpretation* here it is assumed that students would recognize an equivalence relationship between the left hand side and the right hand side of the equal sign.

3. Is the mathematical statement below correct? If yes, say why, if no say why.

   5 + 9 = 14 ÷ 2 = 7 x 3 = 21

4. I have 5 apples and I add 1 more. Then I add again 2 more apples. How many apples do I have now? Show all your working (show all the given information in your solution)

Item 3 and 4 were nearly identical in structure; these tasks were posed to assess students` awareness of correct mathematical syntax. These items tested the verbal – written dichotomy which is attributed to most misconceptions students have of the equal sign.

The last part constituted of question 5 and 6 below

5. What value of x will make the following number sentence true?, show each step of your solution clearly

   $4x + 7 = 15$

6. Solve for x,

   $12 - 2x = 2 + 3x$

Question 5 and 6 were algebraic equations. The first equation was in one variable while the second equation had variables on both side. Does students` assigned meaning of the equal sign have an effect in the way they solve these equations?

The validity of a study is very important and if a study`s research questions are not answered then it is not valid. I present how I took care of the validity concern of this study in subsection 3.4.2.

3.4.2 Validity

Qualitative research does not seek to capture an objective truth but all the same, it is important to ensure the credibility of the findings. One of the best known methods for increasing internal validity of a qualitative study is triangulation. Various methods for ensuring the quality and rigor in qualitative research are described in the research literature (Gall, Gall, & Borg, 2007). Validity for this study was ensured by using multiple data collection methods as in task examination, students` rich descriptions and researcher`s reflexivity. Data were triangulated among the various forms of data that was collected among them students` answer transcripts, individual interviews and researcher`s personal notes. Rich thick description was achieved by presenting the participants` voices as direct quotations and by providing a detailed description of each participant. I also kept a record of my thoughts and reactions in a reflective journal during the course of the study in order to be aware of my personal biases related to the study.

I now present the procedure that was taken in order to collect the data in section 3.6

3.5 Data collection procedure

I got access into the targeted school with the help of introductory letters from the District Education Board Secretary's Office (DEBS) and the Principal for David Livingstone College of Education. With approval to carry out the research, in the school, from the DEBS office, I sought consent from the school head and gave out written consent forms to the students for their parents/guardian in the presence of their mathematics teachers.

The written test and interviews were only administered to students whose parents/guardian signed the consent forms (see appendix C). The study was conducted in regular school hours. 45 students were asked to write the test during one of their preparation periods in the afternoon. This was done in order not to disturb the normal learning hours. The test was taken within 50 minutes and the researcher conducted the test in person while class teachers took observer roles and did not participate in the study in any way. Students` questions concerning how to answer the test items were answered promptly by the researcher. Confidentiality of the research participants was maintained in each case by assigning each subject a random number in the data collection. Students wrote these numbers on their answer scripts instead of their names.

Individual interview sessions were also conducted during the same preparation (prep) periods in the afternoon. Since interviews were not conducted on the same day as the written test, it was possible that some students would not remember what they did during the test. To avoid this scenario, students` answer scripts were photo copied. During the interview process, participants were given back their answer scripts while I had remained with the photocopies and were encouraged to explain their thinking during the time of solving the task in order to give them an opportunity to explain their reasoning during the test. No corrective feedback was given at the time. At the start of each interview session, I explained to the participants the objective of the study and what was expected from them. I made it clear that I was not interested in their answers per se but the thinking that underlies them. From a constructivist point of view, reflective ability is a major source of knowledge on all levels of mathematics. Students should be allowed to articulate their thoughts and to verbalize their actions which will ensure insights into their thinking processes. During such mental operations, insufficiencies, contradictions or irrelevancies are likely to be spotted (Essien & Setati, 2006). Students` thoughts enabled me to have a clue on how they understood the equal symbol. Communicating one`s rationale and reasoning processes to another person simultaneously shapes and transforms one`s reflexive thinking and schemes of internalized actions (Confrey, 1998). I prompted students with "explain more", `go ahead` and "why" or "how" questions whenever necessary. These Interviews lasted between 45 and 60 minutes each session. According to Silverman (2000) interviews provide a tool for accurate interpretations as they produce situated understanding grounded in specific interactional episodes. The interviews were audio recorded and later transcribed.

The methods of how data was analyzed are discussed in the next section.

3.6 Methods of data analysis

Cohen, Manion, & Morrison (2007) describe qualitative data analysis as involving organizing, accounting and explanation of the data or in short to understand the data in terms of participants` definitions of the situation. He says that one needs to look for patterns, themes and categories and regularities in the data. He further argues that the researcher`s approach to data analysis must be adapted to the study. In my study, I will move forward and backward in my quest to create a link between the observed trends and the data. In addition to the socio-cultural approach within which my study is anchored, other theories on students` understanding of the equal sign will influence my analysis and interpretation.

Cohen et al. (2007) discusses a comparative method that scientists use to compare the newly acquired data with the existing data categories and theories that have been constructed with the purpose of obtaining insight between these two. They describe studies that challenge the existing categories or theories causing them to modify their work until they satisfy all the data. The process here is not chronological but comparing constantly the new discoveries in the study of the data, theory and findings that are already there in order to have codes that satisfy all the data. This analysis will take this format.

I will present how I transcribed and categorized the data in the next subsection.

3.7 Transcribing and categorizing

Raw data according to Cohen et al. (2007) is the type of data one has after conducting interviews or an observation. This data must be transcribed if it is to be useful. As I was interviewing I also did a rough transcription by noting down interesting points that were said within the interview. I chose not to transcribe the recorded interviews immediately in the school due to a lot of noise which would distract my attention. This task was done at home where I had all the time dedicated to the task at hand. I wrote detailed notes of each interview from my audio recordings and I also went through students` answer scripts and noted for example, how they solved the equations, how their meaning of the equal symbol related to the way they solved equations. I also wrote the emerging themes from the tasks. I went through my data over and over again, always writing any new theme that I discovered. Eventually, I had a general overview of my data. I then began to transcribe excerpts from audio recordings that I thought would be useful for my data analysis. I listened to the recorded interviews again and again while comparing them to the answer scripts. Transcriptions subsequently were assigned with numbers and pseudonyms were adopted for the 3 students.

After my transcription, work began of looking for categories for my data. The data was divided into segments based on the research questions to be explored. The segments therefore, were named as follows; students` interpretation of the equal sign; and students` understanding and use of the equal sign in relation to solving equations. I first started working with just the transcripts, as I went through all the transcripts; I selected all the words and statements expressing the concept of equality. These statements and word were critically examined for commonality and written down. I then called these statements and words "codes". I went ahead and grouped them according to their common themes and came up with two groups which I called categories. Codes therefore were contained in categories. One of the categories I created contained among other codes words like "gives", "answer", or statements like "Put the answer", "It means we have found the answer" in the students` definition of the equal sign. This category I called "Operational understanding" as the words and statements imply that the equal sign signifies that one should carry out a computation "do something". The second category I came up with was the category which contained for example words like "same as", "balancing", "equal on both sides", "two things are equal". In some cases students used the word equivalent as they had just been looking at sets in mathematics. This category I called “relational understanding”, as these terms imply that the equal sign signals equal-ness on both sides hence reflecting an understanding of a relationship between two quantities.

As I went through the transcripts I came across statements which were not in any of the two categories. In this study, I had to make some choices which could affect the results in that some codes had a double meaning that could create confusion in the analysis, for example, “equals” was used both as a reference to the symbol and also as the meaning of the symbol. In answer to question 1(a) for the name of the symbol, a student wrote “equals” and in giving the symbol meaning in question 1 (b) this student still wrote equals

As I started to transcribe the data, I thought of various categories which kept on being transformed as I transcribed until finally I ended up with two. I went ahead and looked at students` methods of solving equations. These I categorized in the same two categories thus operational understanding and relational understanding. For instance, a student who solved the equation $14 \times 3 = \square - 3$ as finding 42 as the missing number in the box was assumed to view the equal sign as a signal to put the answer, compute the left hand side and put the answer on the right immediately after the equal sign. This student was put in the “*operational understanding category*”. A student who solves the same equation and gets 39 for the missing number was also categorized as having an operational understanding. This student went on to compute all the numbers in the equation viewing the symbol as “do something”. While a student who solved the same equation and put 45 in the box was put in the *relational understanding* category. It was assumed that this student was able to recognize the relationship between the left hand of the equal sign in the equation and the right hand that it signifies balancing, sameness or equivalence.

Since I also wanted to find out whether there was any connection between these categories and how students solved the equations, I went deeper by selecting 6 test items and analyzed them in view of the categories and relevant theory. I used these 6 test items to conduct the interviews with the 3 students. Task selection for the individual interviews was based on concepts identified in the literature as being problematic for many students when solving equations; thus, interpretation and use of the equal sign in different contexts. The test items were purposefully chosen to facilitate the generation and testing of hypotheses about students` unanticipated ways and means of understanding and using the equal sign in different contexts. This enabled me to create models of students` thinking that could be examined.

The data collected were transcribed and coded to enable qualitative analysis. The qualitative data was analyzed thematically based on common themes.

In the next section I present how I addressed the ethical issues

3.8 Ethical Considerations

Approval to conduct this research was obtained from the District Education Board Secretary`s office (DEBS) as it is a requirement demanded when doing studies of this kind in Livingstone district schools. Three basic principles are important in research with "human subjects"; informed consent in recruitment; avoidance of harm in fieldwork; and confidentiality in reporting (Flinders, 1992). Informed consent is referred to as "*the procedures in which individuals choose whether to participate in an investigation after being informed of facts that would be likely to influence their decisions*" (Cohen et al., 2007, p. 52). I informed administration of the school from which participants were to be recruited about the project. Information about the research project style, data collection methods, time needed, which people would have access to the data and facts about confidentiality was given. After this information, participants were recruited for the study with the help of the class teachers. The aim of seeking approval according to Cohen et al. (2007) is to protect participants to a study against physical and psychological harm. Hence, the researcher was aware and respectful of students' limitations, level of cognitive development, social and emotional needs. Informed consent was sought from the students` parents/guardians through a written consent form and those who did not return the consent form did not participate in the study. The purpose of the study was explained to the participants before and debrief was done.

The researcher made sure that every respondent wrote the test and those selected for interviews answered the interview questions with their dignity unthreatened. The participants were made aware that they were free to discontinue participating at any time should they feel uncomfortable.

In the next chapter I present my data analysis.

4 Data analysis

4.1 Introduction

In this chapter, I will present the findings from the data and analyze them. The purpose of this study was to explore students` assigned meanings of the equal sign and to find out how these assigned meanings affect their performance when solving equations. The primary source of data for this study was 45 students answer scripts and interview transcripts for three students. The test items for interview questions were organized into two segments to address each of the research questions. The first segment consists of question 1 a, b and c meant to capture students` assigned meanings of the equal sign and the second segment consists of equations with the equal sign appearing in varied positions. During the individual student interview, each participant was encouraged to think aloud as she/he explained how they solved the mathematical problems in the test.

4.2 Equal sign meaning

For the first segment of data related to students` meanings of the equal sign, the following codes emerged;

- It means that to guide us to the answer
- Put the answer after adding
- It represents the answer or show that you have found the answer
- It is for example equal treatment at home
- Equal to
- Equal treatment of boys and girls at home
- Put the equal sign to show the answer
- Same as
- Equal rights to education
- It shows balancing of equations.

After going through individual student`s answer scripts and think-aloud interview transcripts and coding the data as described above, I compared the codes across students to look for patterns and codes with similar patterns were put together into categories. As I critically examined the data from the first segment which looked at students` meanings of the equal sign in question 1 (b) and (c), I came up with 4 distinct categories where these meanings fell and these were; ***Relational***, ***Operational***, **Not defined equal** and **Other .** A response was coded as relational if a student expressed the general idea that the sign means, “same as”, “same value”, or “balancing”. A response was coded as Operational if a student expressed the general idea that the equal sign means, “the answer”, or “where to put the answer”. A response was coded as Not defined equal if a student gave a response using the word equal but not elaborating any further to suggest a more specific understanding. The Other category included definitions that had no mathematical meaning of the symbol “equal treatment for boys and girls at home” or “boys and girls are equal”.

Table 4.1 Students’ response to question 1b

Category	**Number of students` responses**	**Percentage**
Relational	2	4.4
Operational	42	93.3
Not defined equal	1	2.2
Other	0	0
Total	45	100

Source: Fieldwork data, 2013.

The table above shows that 93.3% of the participants gave an operational meaning of the equal sign compared with only 4.4 % who gave a relational meaning of the equal sign. Given a chance for a second opinion on the meaning of the equal sign, some students changed their earlier meaning as evidenced by table 4.2 below.

Table 4.2 Students` response to question 1c

Category	Number of students` responses	Percentage
Relational	3	7
Operational	40	89
Not defined equal	0	0
Other	2	4
Total	45	100

Source: Fieldwork data, 2013.

Students who gave an operational meaning of the equal sign declined to 89% in table 2 compared to 93.3% in table 4.1 above. From the two tables, it can be seen that an average of 90% of the students gave an operational meaning of the equal sign. There was evidence that some students have a dual meaning for the equal sign as seen from table 4.2 where the number of responses recorded as relational increased by 1.

A response was put under the *relational* category if a student expressed an idea that the equal sign means for example, same as, same value, balancing. For example, Mary gave her meaning of the equal sign as “same” for the answer to question 1b and “the same value” for the answer to question 1 c as seen in figure 1 below and her meanings were coded as relational.

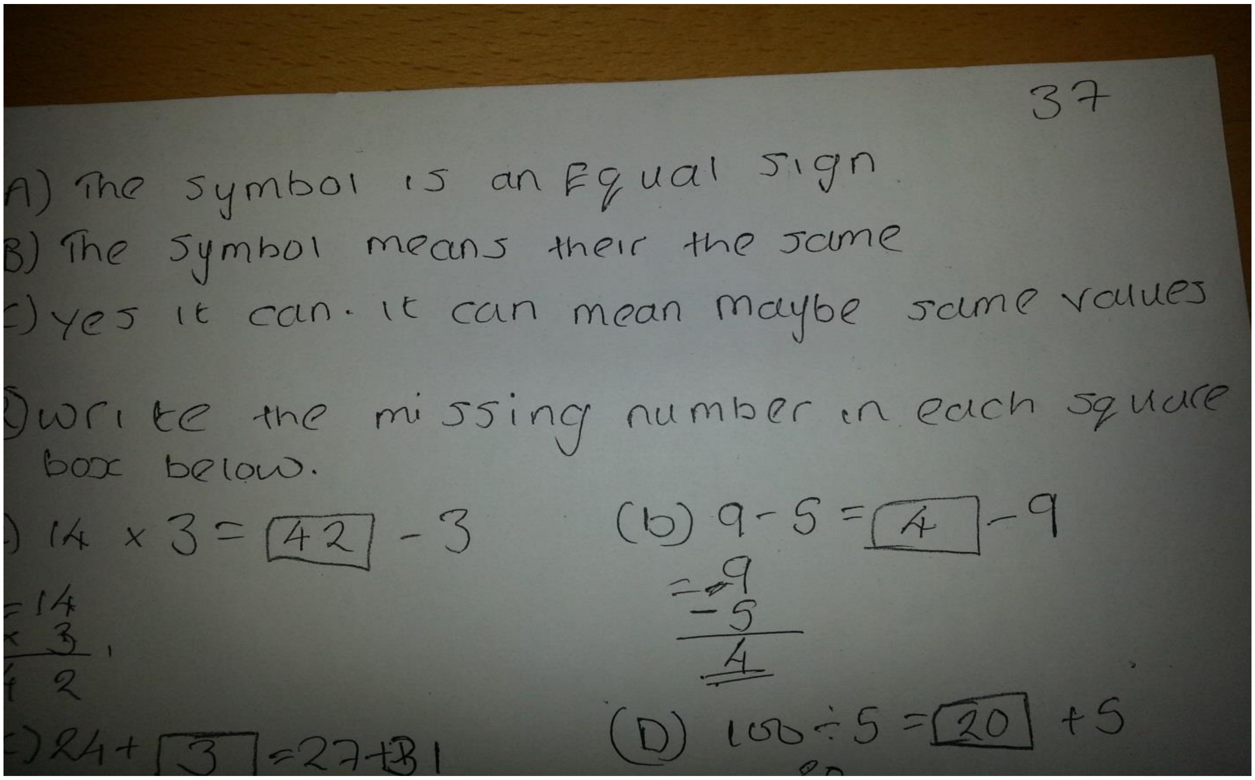

Figure 4.1 Mary´s answer script

Source: Fieldwork data, 2013.

A response was put in the *Operational* category if a student expressed an idea that the equal sign means for example “the answer”, “where to put the answer”. For example, James` answer where he writes “equals” to answer question 1 b and “it means to guide us to the answer…..” to answer question 1 c as seen in figure 2 below. James` answers were put under the operational category.

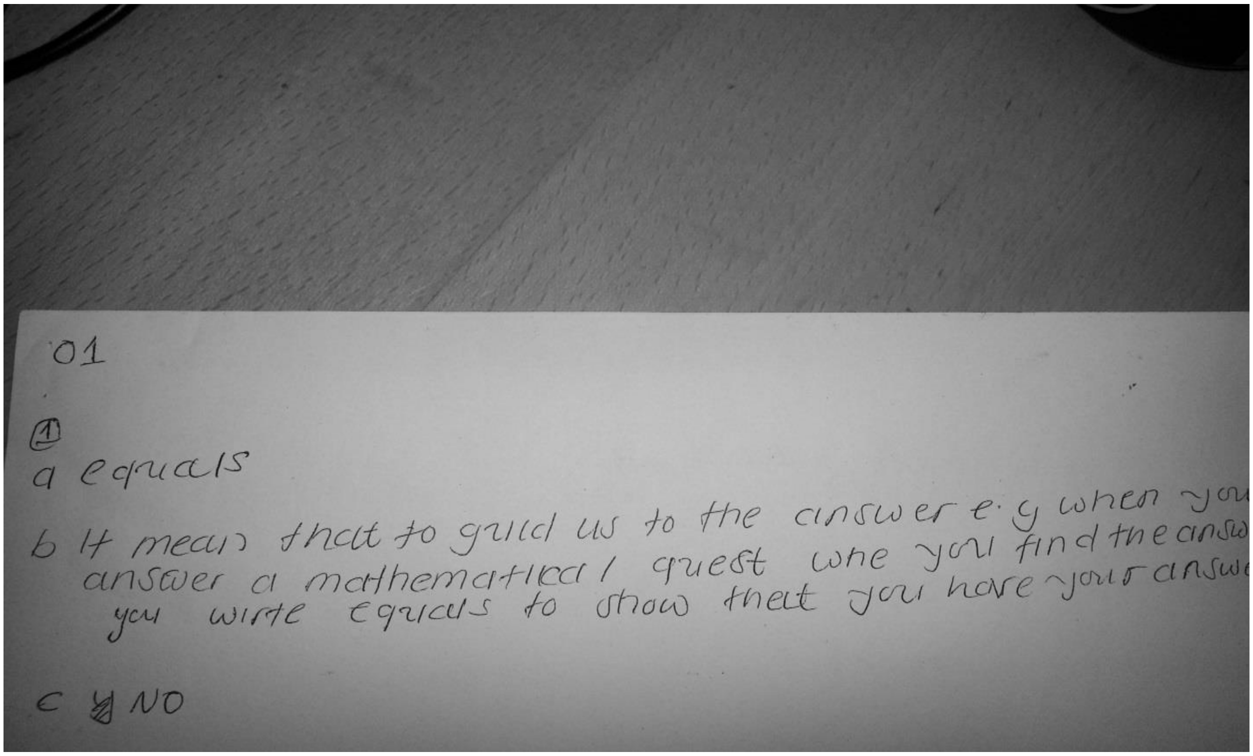

Figure 4.2 James` answer script.

Source: Fieldwork data, 2013.

A response was put into the *Not defined equal* if a student gave a response using the word equal but does not elaborate any further to suggest a more specific understanding. An example of a meaning that was put in this category was John`s answer to question 1 (b) where he writes that the symbol`s meaning is "equal".

The *Other* category included definitions that had no mathematical meaning of the symbol for instance John gave the meaning of the equal sign in question 1 (c) as "equal treatment for boys and girls at home".

Students` responses to the meaning of the equal symbol were put in categories as exemplified in Table 3 below.

Table 4.3 Examples for the equal sign meanings categories

Definition	Category
"It means that both sides of the equation are equal"	Relational
"It shows the answer"	Operational
"Equal treatment at home by parents"	Other
"The values on both sides are the same"	Relational
"What you get after adding"	Operational
"Equal"	Not defined equal
"Equal rights to education"	Other
"Equals"	Not defined equal

Source: Fieldwork data, 2013.

4.2.1 Students` assigned meanings of the equal sign in equations

The second segment of data consisted of students` responses from their written test items 2 through to 6, which were there to help answer the second research question which was; how do students` assigned meanings of the equal sign affect their performance in solving equations

Students` responses to questions 2 through to 6 were scored as right if a student solved the test item correctly, wrong if the test item was not solved correctly and no response if the test item was not attempted. The way students performed in solving the equations is seen in table 4 below.

Table 4.4 Students` performance on individual test items 2, 3, 4, 5 and 6

Question	Right	Wrong	No response
2(a) 14 x 3 = □ - 3	2	43	0
(b) 9 – 5 = □ - 9	0	45	0
(c) 24 + □ = 27 + 31	1	44	0
(d)100 ÷ 5 = □ + 5	0	45	0
(e) $\frac{169}{13}$ = 13 - □	2	42	1
3. 5 + 9 = 14 ÷ 2 = 7 x 3 = 21	0	45	0
4. I have 5 apples and I add 1 more. Then I add again 2 more apples. How many apples do I have now?	6	39	0
5. 4x + 7 = 15	10	23	12
6. 12 – 2x = 2 + 3x	1	13	31

Source: Fieldwork data, 2013.

From the analysis of the written test, only 4% of the students got question 2(a) correct. 96% of the students put 42 in the box as the missing number in the mathematical sentence 14 x 3 = □ – 3. 100% of the students got question 2(c) wrong. The prominent answers were 4 and 9 for the mathematics sentence, 9 – 5 = □ – 9. 2% of the students got question 2(c) correct. 98% of the students settled for 3 as the answer.

For question 2(d), no student got it right. The mathematical sentence 100 ÷ 5 = □ was treated the same as these other question, a, and b where the left hand side had the operation and the answer comes just after the equal sign. All the students put 20 in the box.

In question 2(e), 2% of the students did not attempt it, 4% got it correct while the rest got it wrong. This question was most interesting because of the wrong answers, 2 answers were most prominent and these were 13 and 169. The mathematical sentence was $\frac{169}{13}$ = 13 - □. Student who wrote 13 in the box divided 13 into 169 and got 13 which were put into the box. Meanwhile those who wrote 169 used the cancellation law in equations by cancelling the 13`s both sides leaving them with 169 as the answer.

All students failed to answer question 3 correctly as they saw the mathematical statement 5 + 9 = 14 ÷ 2 = 7 x 3 = 21 to be true. Students interpreted the equal sign as to mean 5 + 9 →14, ÷ 2→7, x 3 = 21.

For question 4, only 13% were able to translate the word problem into a mathematical sentence correctly. 22% solved the equation in question 5 correct with all of them using the same method of subtracting 7 from both sides and later dividing both sides by 4 in the equation 4x + 7 = 15. The 51% who got it wrong showed lack of knowledge of solving equations, some seriously so. 27% did not attempt the question

Question 6 being an equation with unknowns on both sides proved a challenge to most students seeing the number of correct answers dropping drastically to 1 with a higher percentage of those who didn't attempt the question.

I then critically analyzed students` solutions to establish their understanding and use of the equal sign as they solved equations and I identified the following codes;

- Operator symbol
- Call for action "do something"
- Equivalence
- Balancing

Additional codes that emerged from think-aloud interviews included;

- Difficulty solving equations
- Misconceptions
- Overgeneralisation of some rules

As I analyzed this data, I recognized some similarities in their responses and decided to put together those that fell in the same group. I came up with three distinct groups of their meanings of the equal sign which I assigned codes. The first code was where the equal sign was viewed as an **operator**. Under this code, were students` responses which suggested that the number on the right hand side immediately after the equal sign was the answer to a computation of an expression which is written on its left and only readable from left to right? Students put their answers to the computation of what is on the left side in the box immediately after the equal sign and ignored the other numbers on the right side or considered the number after the equal sign as the answer and used it to find the missing addend. For example in question 2(a). 14 x 3 = □ – 3, James put 42 as the answer in the box. The other scenario where James still considered the number after the equal sign as an answer was when answering question 2(c) 24 + □ = 27 + 31 . Despite the missing addend being before the equal sign, James used 27 to determine what goes in the box and hence he wrote 3 as evidenced from his answer script in figure 3 below. From his work with the different orientations of the equal sign, James considered the answer to be placed on the right side of the equal symbol and not on the left (See James` answer script in figure 3). I put all the related meanings that referred to the use of the equal sign to indicate the answer to a computation readable only from left to right under the same code. These students were viewed as giving an operational understanding of the equal sign.

The second code contained meanings which suggested that an action needs to be carried out or a command to do something, in this group of meanings of the equal sign students calculated answers from either side of the equal sign. This code was made up of students` meanings that were not consistent with one side only for example the answer on the right. These students considered both sides of the equal sign to be potential places for putting the answer. For instance, in sentences like 14 x 3 = □ – 3 (left) and 24 + □ = 27 + 31 (right). Students regarded the equal sign to mean that the answer must come next regardless of the direction they looked at it from. These students wrote 42 as an answer for the first equation read from left to right while they had 58 as an answer for the second equation read from right to left. Here the answer was the answer to the computation that came right before the equal sign regardless of the direction (either left or right). Under this code, it can be seen that some meanings which qualify to be put under the operator code were put here because of the instability of the operator use. A student who was not consistent with viewing the equal sign as an operator was regarded as having a do something understanding of the equal sign. With this scenario the equal sign in some equations was viewed both as operator and do something symbol. The equal sign was viewed as a command to carry out the calculation, and did not represent a relation between the numbers represented by the equations 14 x 3 = □ – 3 and 24 + □ = 27 + 31. The other group of answers that fell under this code was where students computed all the numbers in the equation, for example the two equations above had answers 39 and 82 with a second equal sign introduced at the end of the equation before they wrote their answers.

Using the two codes of Operator and Do something as a lens, I analyzed students` answers to questions 2 through to 4 and Table 5 shows the students` responses as they used the two operational meanings.

Table 4.5 Students` use of the two operator meanings

Types of responses	Operator	Do something
Ignore one of the terms and consider only part of the sentence e.g. 14 x 3 = □ – 3, 42 as the answer. Computed from left to right	X	X
Ignore one of the terms and consider only part of the sentence e.g. 24 + □ = 27 + 31, 58 as the answer. Computed from right to left		X
Combine two of the terms in the sentences by interpreting the sentence from left to right with no regard for the structure of the sentence e.g. 24 + □ = 27 + 31, answer; 55 from adding 24 and 31.	X	X
Combine two of the terms in the sentences by interpreting the sentence from right to left with no regard for the structure of the sentence e.g. 9 – 5 = □ – 5, answer; 0 as 5 – 5 = 0.		X
Compute all the numbers in the sentence together, e.g. 14 x 3 = □ – 3, answer; 39, as in 14 x 3 = 42 – 3 = 39	X	X
Building a chain of operations like say; the mathematical statement 5 + 9 = 14 ÷ 7 = 7 x 3 = 21, considered to be true or transforming the word problem; I have 5 apples and I add one more. Then I add again 2 more apples. How many apples do I have now? Transformed as 5 + 1 = 6 + 2 = 8	X	X

Source: Fieldwork data, 2013.

The third code had meanings that suggested an equivalence relationship between the numbers on both sides of the equal sign**,** under this code were meanings that indicated a representation of a relation between the same mathematical object. These students were viewed as exhibiting relational thinking where relational thinking refers to students` recognition and use of the relationship between elements in number sentences and expressions to construct their solving strategies (Carpenter et al, 2003). As they used relational thinking, students were able to consider sentences and expressions as "wholes" instead of as processes to carry out step by step, analyze them, discern some details and recognize some relations and finally exploit these relations to construct a solution strategy as evidenced from the conversation with Mary in the excerpt below line 2.1 from Mary`s answer.

These three codes were later put into two categories, thus, *Operational understanding* of the equal sign and *Relational understanding* of the equal sign. The first and second codes above were put in the Operational Understanding category, while the third code was put under the Relational understanding category.

The relational understanding of the equal sign were evidenced by students who were able to correctly solve the equations an example of Mary and those who could not correctly solve the equations due to computational errors but showed an understanding of a relationship between the left hand side and the right hand side of the equations like John. Both Mary and John were comfortable with expressions involving addition or other operations on either side of the equal sign. While research by Kieran (1981) attest that most students believe that number sentences should follow a certain pattern with an operation on the right, both John and Mary had no problem with the missing addend being on the left side of the equal sign. For example question 2(c), 24 + □ = 27 + 31 received the answer 34 from both Mary and John. When asked to explain her answer, Mary`s answer is in the excerpt below:

1.1 Rose: Mary, what was your answer?

1.1 Mary: I got 34

1.2 Rose: how did you do that?

1.2 Mary: This one was simple, since the equation has to balance; I first looked at the right side where all the numbers are given. I looked for something common on both sides then I realized that 27 was 3 more than 24 then what goes in the box should be 3 more than 31 to make the equation balance.

1.3 Rose: What did you do to achieve that?

1.3. Mary: I then subtracted 24 from 27 and got 3 which I then added to 31 and got 34 and that is the number which should be in the box for the equation to balance.

From the excerpt above, it can be seen that Mary viewed the equal sign as expressing a relation between numbers. She considered the expressions on both sides of the equation to represent the same number as seen in line 2.1 above where she says "*This one was simple, since the equation has to balance; I first looked at the right side where all the numbers are given. I looked for something common on both sides then I realized that 27 was 3 more than 24 then what goes in the box should be 3 more than 31 to make the equation balance*". She realized that she was looking for a relation between the two sides of the equation and that a relation among the numbers in the two expressions made it unnecessary for her to actually carry out the calculations. Mary showed greater understanding of the equal sign by considering the two addition expressions in the equation, not just the relation between the answers to the two calculations.

John also put 34 in the box and when asked to defend his answer this is what he said:

1.4. Rose: How did you get 34?

1.4. John: I looked at both sides of the equation and saw that on the right hand we had all the numbers, so I went ahead and added them to get 58. Knowing that both sides should have the same value, I then subtracted 24 from 58 and got 34, which is what I put in the box

John looked for the sameness of value on both sides for the equation to balance.

The other instance where a relational meaning was reflected was in question 2b, 9 – 5 = □ – 9 where John put 5 in the box. When asked to explain his answer, he said:

1.5. Rose: How did you get the 5 John?

1.5 John: I looked at the question and knew that both sides should be equal therefore, since on the left hand side there was 9 and 5 and the operation is minus, there should also be a 9 and a 5 on the right hand side since the operation is also minus and because 9 is already there, then what is missing is 5.

John understood that the equation should be equal on both sides by considering numerical sameness. He looked for some resemblance or sameness on both sides of the equation but did not consider being important the relative position of the terms, the operational sign that relate them or their position in relation to the equal sign. From the discussion, John thought that for the equation to be in balance, it had to be exactly "the same" on both sides of the equation with identical numbers or symbols. John showed an over- generalized meaning of the commutative law.

All the three students utilized the three meanings of the equal sign mentioned above to make sense of the test items in the task. In many instances, students had different meanings depending on the context the equal sign was used. For example a "do something" meaning of the equal sign was seen in two different sentences, 14 x 3 = □ – 3, and 24 + □ = 27 + 31 with answers 42 and 58 respectively given by John, while Mary gave 45 for the first equation and 34 for the second equation. However, James was consistent with his belief of operation on the left and the answer on the right as a result when encountered with different forms of number sentences such as 24 + □ = 27 + 31, he gave the answer as 3 as adapted his rules to respond to an unfamiliar context. His adaptations often involved calculating answers instead of looking for relations. James read both sentences from left to right giving answers 42 for the first sentence and 3 for the second sentence. The action was to put the answer after the equal sign.

James also tended to use an operator meaning of the equal sign whenever he encountered sentences he considered difficult as evidenced from this dialogue for solving the equation

4 x + 7 = 15:

1.6 Rose: What value of X makes the sentence true in the equation above, what did you do?

1.6 James: I got 4

1.7 Rose: How did you get the 4?

1.7 James: I did 4 + 4 = 8, plus 7 gives 15

1.8 Rose: Where is this 4 coming from? Are you adding 4 to itself?

1.8 James: Since I have 4 (pointing at the 4 with an x) I had to find a number which when I add to it I will get 8 since 15 – 7 = 8, that is how I got 4

1.9 Rose: From where did you get this 4?

1.9 James: Oh! I subtracted 4 from 8.

1.10 Rose: But why did you have to subtract from 8 and where did You get the 8 from?

1.10 James: So that I can get 15 when I add to 7. 8 + 7 is 15

1.11 Rose: What does 4 x mean? Does it mean 4 plus x?

1.11 James: I don't know, but I think so

From the excerpt above of the dialogue with James, what comes out clearly is that James had a limited understanding concerning manipulating equations. He could not explain his strategy in a way that highlights the critical role of the equal sign in the equation which is the equivalence relation as evidenced by his answer by his answer in line 1.8 where he is answering as to what he did to get 4 and he says "Since I have 4 (pointing at the 4 with an x) I had to find a number which when I add to it I will get 8 since 15 – 7 = 8, that is how I got 4". James had a "do something" understanding of the equal sign and hence he had to do something regardless of whether it made sense or not. James` understanding of the equal sign was limited to arithmetic making it difficult for him to transfer it to algebra as can be seen in line 1.7 where he says "I did 4 + 4 = 8, plus 7 gives 15". He could not explain correctly the relation between 4 and the symbol x. James` mind was fixed on getting the answer 15 hence pasting 4 in place of x and introduced an operator symbol plus to get his 8 which when added to 7 gives him the 15.

The results from this study suggests that when these Grade 8 students were confronted with an equation they were not familiar with, they proceeded to solve it according to what they thought they were "supposed to do" and the equal sign did little to deter them from their pattern of thought. Rather than misunderstanding the meaning of an equivalence designation in an equation, students simply ignored it in order to proceed as they felt they were supposed to.

All the three students accepted that the number sentence 5 + 9 = 14 ÷ 2 = 7 x 3 = 21 was true. Here it is assumed that students appeared to have only paid attention to the movement from left to right and not vice versa. Students interpreted the equal sign as to mean 5 + 9 →14, ÷ 2→7, x 3 = 21. In this case, the sentence was not regarded as an expression of relations but as a list of numbers and operational symbols and hence they applied these operations to the numbers as they considered it possible or most convenient. The equal sign was considered as unidirectional (one way). Students exhibited lack of mathematical syntax as they could not see the error of symbolization in the mathematical sentence. They solved the problem as if they were punching numbers into a calculator. Students did not think of both sides of the equality symbol as a relationship, for instance $5 + 9 \neq 14 \div 2$. To them what mattered was the answer to the problem.

4.2.2 Understanding of the equal sign reflected by students

The three students` conception of the equal sign based on the meanings above were put in three categories of Operational understanding; Unstable understanding and Relational understanding. The operational understanding category was characterized by the consistent reflection of the two codes of the equal sign meanings thus operator and do something. The unstable understanding was characterized by fluctuating understanding of the equal sign thus sometimes operational and other times relational. Lastly, the relational understanding category was characterized by meanings that took into consideration the structure of the equations by considering the relationships between the two sides of the equal sign.

Figure 3 below gives an example of evidence of an operational understanding of the equal sign in solving equations. James` answers to questions 2(a) through to 2(e) considered the number immediately after the equal sign to be the answer and read from left or right.

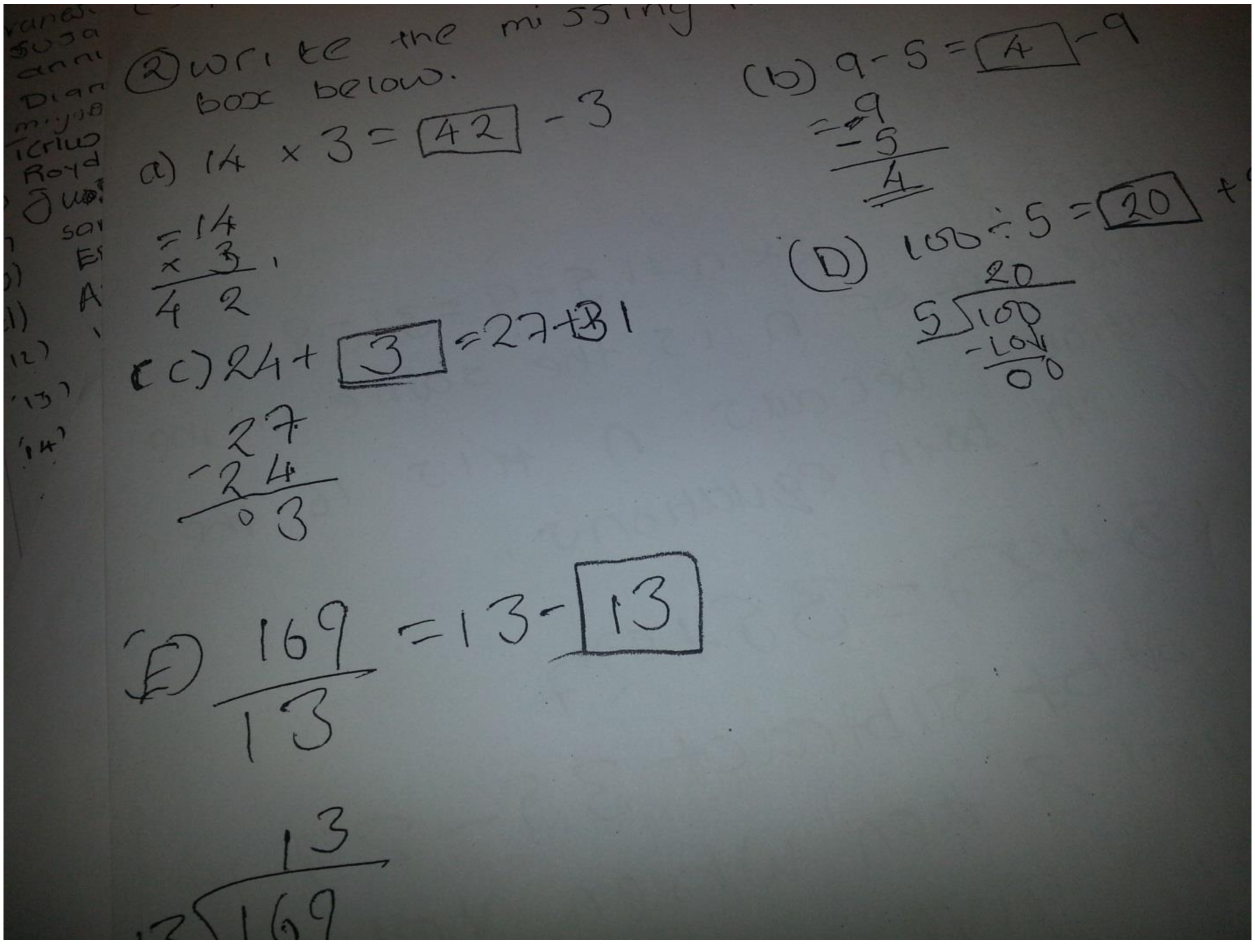

Figure 4.3 James` answer script

Source: Fieldwork data, 2013.

The figure above shows how James solved problems on question 2 in the test. James reflected an operational understanding of the equal sign for he was consistent in computing what was on the left and putting the answer on the right. The equal sign here was viewed as a "do something" symbol. Here is an excerpt from interview with James on his answer to question 2e which had the box on the right but not right after the equal sign. $\frac{169}{13}$ = 13 - □.

Here is James` argument.

2.1 Rose: What was your answer for this question?

2.1 James: 13

2.2 Rose: Explain

2.2 James: I did 13 into 169 and I got 13

2.3 Rose: Which 13 is this, is it the 13 just after the equal sign or you put 13 into the box?

2.3 James: Mmmmm! [long pause], actually I didn't know what to do here but I just divided and got 13

2.4 Rose: Where did you take the 13 that you got?

2.5 James: I put it in the box

2.6 Rose: How about the 13 that is already there?

2.7 James: It is also the answer

James found himself in a dilemma because the unknown was not just after the equal sign but on the right of the equal sign anyway. His admission that 13 was also an answer showed his serious limited understanding of the equal sign. Most students were found in this category of equal sign understanding where James was a typical example. James had to "do something" as he admitted in line 2.3 above and he did without regarding the structure of the equation. It is interesting to note that James was willing to confess that he didn't know what to do but instead of leaving the question unsolved, he had to do something, whether it made sense or not was not his problem.

The second category, I called unstable. Students in this category exhibited both operational and relational understanding when solving the test items and John was in this category. John gave me a problem to figure out which category to place him because during the interviews, he displayed a relational understanding of the equal sign meaning at first but shortly afterwards, he was not able to defend his earlier meaning and chose to switch to an operational understanding. John was able to solve some atypical equations correctly but not consistently leading me to hypothesize that he did it by trial and error. He occasionally showed some difficulties in recognizing the sentences as expressions of relations; an example of John`s interview excerpt for

2 b. 9 – 5 = □ – 9

3.4 Rose: What did you put in the box for question 2 b

3.4 John: I put 5

3.5 Rose: Can you explain why 5?

3.5 John: yes, I looked at the equation and saw that the two sides must be the same and this side (pointing to the left side) has 9 – 5 and this side (pointing to the left side) has a box and – 9, then in the box I put 5 since 9 is already there.

3.6 Rose: why did you have to put 5?

3.6 John: To make the equation to balance

John exhibited some recognition of the relationship which the equal sign represents for he was consistent in using the word "balance" in his explanation for his answer to the equation above even though his thinking was not correct. John looked for numerical sameness in the equation as seen by his answer in line 3.5 but his problem was that of over generalizing the commutative law. On the contrary to his relational understanding, John`s answer to question 2c, which had the box just before the equal sign as in 24 + □ = 27 + 31 evidenced an operational understanding. As seen in line 3.8 below, John used 27 as an answer to the computation on the left to determine what goes in the box. Here is his justification of his answer to question 2c in this interview excerpt.

3.7 Rose: What did you write in the box?

3.7 John: I wrote 3

3.8 Rose: Why?

3.8 John: I looked for a number which will give me 27 when I add it to 24. I found 3 and then added 27 + 31 and got 58

3.9 Rose: So what is your answer then?

3.9 John: 58

3.10 Rose: You mean you added everything starting on your left, jumped the equal sign and went on to add what is on the right to get 58?

3.10 John: Yes

3.11 Rose. Great thinking, so what goes in the box?

3.11 John: 3 eeeh! No, 58…… . . . . . I`m confused

John struggled to appreciate that the equal sign designated a relation for the two expressions; as evidenced in line 3.11 where he did not know whether 3 or 58 was the answer hence his getting confused as to what the actual answer should be for the question above. John used the equal sign as a signal for writing an answer after the equal sign as seen from his answer script where he introduced a second equal sign in the equation and wrote 58 at the end of it so that the equation read 24 + 3 = 27 + 31 = 58.

In the next excerpt however, John reflected a relational meaning again when solving the equation in question 5. 4 x + 7 = 15 as evidenced in this excerpt:

3.14 Rose: How did you solve the equation in question 5?

3.14 John: I first moved 7 to the other side.

3.15 Rose: How did you do that?

3.15 John: I had to bring the like terms to the same side

3.16 Rose: And which are the like terms in this case?

3.16 John: 7 and 15, I moved 7 and since it has to cross the equal sign then the sign changed to – 7 so that on my left I had 4x and on my right I had 15 – 7. Subtracting 7 from 15 gave me 8. Then now I had 4 x = 8, I then divided both sides by 4 to get the value of x and I got 2.

3.17 Rose: Why does the sign have to change when 7 crosses the equal sign?

3.17 John: That is what we were taught

3.18 Rose: Why did you have to divide both sides by 4

3.18 John: That is what you do to get x

John was able to employ the algebraic strategy to solve the equation by first moving 7 to the other side and later divided 4 on both sides but could not explain why he had to do so. His response in line 3.17 and 3.18 does not show that John knows why he is doing what he is doing. John`s admission to doing what he did because of how he was taught was an interesting aspect of the interview. John could not explain how changing the sign when a quantity is taken the other side occurs or why he was dividing the same quantities both sides. There was evidence that John had mastered the “rules” of manipulating equations without the actual understanding. He attributes his knowledge to how he has been taught which indicates that he was just mimicking his teacher. While teachers may use examples to illustrate definitions and exemplify the use of a particular rule, students may focus on the specific details of examples which may restrict their thinking as evidenced by John in this case.

The third category was that of a relational understanding of the equal symbol. Under this category, students who reflected this kind of understanding of the equal sign solved correctly all types of sentences except for computational errors. Mary was classified as an example of a student having a relational understanding of the equal sign as evidenced from the following excerpts of interviews with her:

Here is an excerpt of Mary`s response to the questions on 2b, 9 – 5 = □ – 9

4.1 Rose: What goes in the box?

4.1 Mary: I found 13

4.2 Rose: Explain why you put 13 in the box

4.2 Mary: Since this side (pointing to the left side of the equation) must be equal to this side (pointing to the right), then the number I put in the box should make the equation to balance. If 9 – 5 = 4, then something – 9 should also equal to 4.
I added 4 to 9 to get 13.

For question 2e, $\frac{169}{13}$ = 13 - □. Mary continued to show a relational understanding as she explains her answer in the following excerpt;

4.3 Rose: what did you put in the box?

4.3 Mary: I wrote 13 but then I know that it is wrong

4.4 Rose: How do you know that it is wrong?

4.4 Mary: It just doesn't seem right to put 13 in the box

4.5 Rose: What happens when you put 13 in the box?

4.5 Mary: Then the answer is 0

4.6 Rose: Does that give a correct answer?

4.6 Mary: No that is why I got confused; the two sides are not balancing

4.7 Rose: So what was your answer?

4.7 Mary: 13

Mary again exhibited a relational understanding of the equal sign though her answer was not correct. From her response in line 4.4, it can be seen that she was not comfortable with her answer but she had to write it anyway. She was convinced early in her working that she was not doing the right thing as she examined the condition of the equation on both sides. She admits that putting 0 in the box does not seem right because the equation does not balance which is a very important factor to consider when solving equations. This ability to reflect on relations among mathematical expressions such as 9 – 5 and □ – 5 is critical for students to think more generally about arithmetic and to extend their knowledge of arithmetic to algebra.

4.3 Relationship between the understanding of the equal symbol and solving equations

From the above analysis, I looked at the relation between students` definition of the equal sign and their performance in solving equations. Did students solve the equations correctly? Based on previous work by Kieran (1981) where it was found that students who lack a relational understanding of the equal sign mighty have difficulties understanding the steps involved in solving equations. Mary, because of her reflecting understanding of the equal sign in a relational manner, she solved problems by recognizing equivalence and was able to solve the equations correctly showing and explaining all steps. On the other hand, James had difficulty solving the equations correctly. Reflecting an operational understanding of the equal sign, James got all the sub questions in question 2 wrong due to regarding the equal sign as an invitation to "do something".

The last question appeared to be a big challenge for all the three students. Question 6 of the test was 12 – 2 x = 2 + 3 x. James did not attempt this question and when he was asked for an explanation he said he didn't know what to do and therefore decided to just leave the question un-attempted. John on the other had struggled to solve this equation as can be seen from this excerpt of this conversation;

5.7 Rose: John, how did you solve this equation?

5.7 John; Aaah! This confuses me with this equal sign here and there`s two xs in this problem that is more confusing.

John made a number of errors with evidence of a lot of cancellations on his answer script

5.8 Rose: In your confusion what did you do then?

5.8 John: I subtracted 2 x on both sides of the equation and subtracted 2 x from 2 to get 1 x.

When he wrote - 2 x on the right side of the equation, he wrote it underneath the 2 and subtracted 2 x from 2 to get x.

5.9 John: (…thinking loudly), maybe I should do the 12 and the 2 because they have no x on them, because those two are like, eeh! What is that word again? Well the x`s go together and the numbers go together

5.10 Rose: what did you do?

5.10 John: I subtracted 12 on both sides and I got 2 x = 10 - 3 x and then I thought I should do 2 x and the 10, I don't know

5.11 Rose: Why 2x and 10 and not 2x and 3x?

5.11 John: That will mean having two x`s together but what would I do with them, I know that I would add 2 plus 3 to get 5 but what is x plus x?

5.12 Rose: How would you solve this equation, x – 2 = 4?

5.12 John: I would take 2 to the other side and then I will have x = 4 – 2, x = 2

5.13 Rose Having found the value of x here could you do the same for the other equation?

5.13 John: I can`t, which x do I find since there are two?

5.14 Rose: What is the purpose of solving an equation John?

5.14 John: That equal sign is getting me mixed up, it makes me very confused

5.15 Rose: According to you, what is the meaning of the equal sign then?

5.15 John: I don't know in this equation how all those (Pointing to the terms on both sides of the equal sign in 12 – 2 x = 2 + 3 x are supposed to equal each other but the one I know means that it equals something like 2 + 2 equals 4.

John struggled to do something with this equation as evidenced by his instability on what to do with the terms in the equation. He kept changing his mind of which terms should be operated on as seen in line 5.8 and 5.9. He seems to have the algorithms which he was not able to use at the moment. He correctly say that like terms should be collected together and correctly identifies which terms in the equation are like but went ahead and subtracted 2 x from 2 which are not like. John`s "like terms" in this matter was the numbers 2 on the right hand side and the 2 on the left hand side despite being a coefficient of x. John realized that what he did was not correct in line 5.9 he says "maybe I should do 12 and 2 because they have no x". By this statement, I hypothesize that John was not sure of what he was doing but he had to do something to get the answer. The presence of x on both sides of the equal sign unsettled him. He correctly subtracted

12 from both sides but neglected to take into consideration the symbol that went with 2x. I introduced the equation x – 2 = 4, to try and see if he could connect it to the original equation. Despite solving this equation successfully, he couldn't see how he could end up with a single x. When I asked John on what the purpose of solving an equation was, I expected to hear something like "in order to separate x" but instead he says the equal sign is messing him up. When he was asked about the meaning of the equal sign, the example he gave was that of operation on the left and answer on the right. I realized that his confusion could have stemmed from thinking of the equal sign as signaling the answer on the right side and the operation should be on the left. Having variables on both sides truly messed him up.

Throughout development of mathematics, symbols have crept their way into the manner in which we write and communicate conceptually and procedurally. For some a symbol is a mathematical object, a thing that can be manipulated in the mind, for others, it signifies a procedure to be carried out. Students who concentrate on procedure may be able to develop a set of skills which allow them to do computational arithmetic and succeed in the short term but in long term; they may lack the flexibility that will give them ultimate success in learning mathematical concepts (Carpenter et al., 2003). John in the conversation above used the equal symbol as to signify an action to be carried out. His skills were not able to get him through the process of solving the equation correctly as his knowledge was limited to solving typical equations as evidenced from the excerpt above but could not apply it on atypical equation. He admits that the equal symbol made him confused, more so because of the presence of the variables on both sides of the equation. His confusion is articulated by Steinbring (2006) who says that the school most times focus on teaching the algorithms as expressed in formulas as a procedure of calculating without care as to whether students have an understanding or not. John knew that he had to move like terms to the same side of the equation showing that he had learned the algorithms but didn't know what to do with them when he has moved them hence he ended up subtracting 2 x from 2 and got x.

Ernest (1997) suggests that mathematical knowledge cannot be revealed by a mere reading of the mathematical signs, symbols and principles, but that signs have to be interpreted and that this interpretation requires experience and understanding of the symbols. The equal sign is an example of a mathematical symbol often seen in relation with other operation symbols for example, +, -, x, ÷ and often interpreted as meaning to write an answer (Kieran, 1981). John failed to interpret the relational structures that the equal sign contained as he tried to solve the equation, he treated the negative (-) symbol attached to 2x as if it didn't exist as his focus was on getting the answer regardless of whether it was reasonable to subtract 2x from 2. John can be perceived as having mastered the algorithms but not understood them. When a student encounters a symbol for the first time and is instructed as to its meaning, the hope is that he connects that symbol with some prior conceptual understanding of what it represents, as postulated by Saenz-Ludlow & Walgamuth (1998) "*A mathematical symbol should evoke a particular thought that will help to unveil a conceptual object and point to specific features that belong to the significant field of meaning of such a conceptual object*" (Saenz-Ludlow & Walgamuth, 1998, p. 154). Students who see these symbols as computational directives with no connection as to how to use them functionally will read the symbol but not connect it to anything practical or relational as evidenced by John.

4.4 Summary

Analysis of the data from this study revealed that most of the 45 students hold an operational understanding of the equal sign. From their assigned meanings of the equal sign, students see the equal sign as an invitation for action, thus, to do something or to compute the answer. Individual interviews of the three students also revealed that some students had a dual understanding of the equal sign for example both operational and relational. It was not always that once a student display an understanding of either operational or relational continued to do so. It seems that students were relying on a set of memorized rules or steps to solve equations rather than developing an understanding of concepts as a result when a student forgets the rules or steps, they were unable to proceed to find a correct answer.

Data also indicated that students who displayed a relational understanding of the equal sign had greater competence in solving equations. But it was also found that on the contrary, some students who appeared to have an understanding of the steps involved in finding a solution had little or no understanding of what that solution represented.

5 Discussion and conclusion

5.1 Introduction

In this chapter, I will look at my research questions; what meanings do grade eight (aged 11 - 15) students assign to the equal sign and how do these assigned meanings affect students` performance in solving equations? These questions will now be answered with the help of my analysis in Chapter 4. I will further consider any weaknesses in my analysis method that could have affected the results of this study. I will also reflect on the implication of these results on the teacher before I conclude.

5.2 Understanding the meaning of the equal sign

Asking Zambian grade 8 students for a formal definition of a concept poses some challenge due to language constraints, but asking them for the meaning of a concept becomes relatively easy to handle because then, their insights are being considered instead of formal sentences. Through asking a meaning question, one can easily have an insight into students` misunderstandings. In this study, I found that students understand and use the equal sign in various ways. Their meaning of the equal sign revealed students` limited understanding of the equal sign concept as evidenced by very poor results from the test with less that 10% of the participants getting half or more of test items correct. This poor performance is not surprising given the lack of explicit focus on the equal sign in grade 8 mathematics curricula in Zambia. From my experience as a student and also as a teacher of mathematics at secondary school level, at no point in time was the equal sign given attention at any level in secondary school curriculum as a concept that needed to be revisited or learned. Generally it is assumed that once students have been introduced to the concept during early primary school mathematics, no revisiting was required. These findings are a cause for concern because I also found a strong relation between the equal sign understanding and success in solving equations. Students who showed a relational understanding of the equal sign were more able to solve equations correctly.

Although all students were able to use their equal sign meanings in solving equations, not all of them used them with understanding. By looking at the way students solved their tasks, I got an insight into how they understood the equal symbol. What emerged from the discussion with my interviewees is that they do not necessarily understand the contexts of the various meanings. From the test answer scripts, I could see that most of the participants in this research understood the equal sign operationally, suggesting that the symbol signals a command for an arithmetic computation to be carried out. Viewed through this lens, the equal sign is taken to be an instruction for carrying out an operation “do something” and consequently, writing the answer. For example in the mathematical statements 14 x 3 = □ – 3, James wrote 42 as the answer in the box which was read from left to write, while the mathematical statement 24 + □ = 27 + 31, received 58 from John as the answer, as he added all the numbers in the equation and introduced another equal sign at the end of it and wrote 58. Most students in this study failed to recognize the extent of sameness given by an equation because their focus was on solving, evaluating or “finding the answer”. They failed to recognize the contribution of the equal sign as indicating a relation in a given context.

This study`s findings are consistent with previous results by Alibali et al. (2007) that suggest that students` misconceptions may be due to instruction and textbooks. When students are exposed to same procedure of solving equation, this maybe ingrained in them to think that it is the only way to do it. If instruction or textbook examples are not varied, students may believe that unless they follow the laid down procedure, they may not be doing the right thing. In a context where a procedural interpretation of an equation is consistent with a student`s perception of how they should interact with the equation, they will most likely fail to interpret the equal relation. An example is that of James` answers to question 2 (a) through to (e) in figure 3 in the previous chapter where he treated all the equations the same and ended up with wrong answers. It has been noted from previous studies that students are said to have an operator meaning for the equal sign when they focus on processing what is on the left and putting the answer on the right (e.g. (Behr et al., 1980; Knuth et al., 2006; Saenz-Ludlow & Walgamuth, 1998; Stacey & MacGregor, 1997). This study concurs with the previous studies as students` strategies for solving equations revealed that they applied their knowledge of standard addition problems when trying to solve new problems. Most students used incorrect strategies in solving atypical equations such as operating on all the terms in the sentence, for instance in 14 x 3 = □ – 3, some students multiplied 14 by 3 and got 42 and went on to subtract 3 and introduced another equal sign at the end of the statement to equal 39, these students wanted to take into account of the – 3 after the box. While yet others just multiplied 14 and 3 to get 42 and ignored – 3 on the right hand side of the equal sign. Being able to apply existing knowledge in new situations is a sign of developing conceptual understanding but procedural knowledge however, is limited to executing solution steps in situations resembling those in which the procedure was learned as evidenced by these findings. It can be hypothesized that the students` methods were not unreasonable attempts to deal with unfamiliar problem because maybe their previous number sentences had answers right after the equal sign. They generalized from their experience that one of the “rules” for writing number sentences was that the answer comes right after the equal sign.

From this study, it is interesting to note that Zambian grade 8 students hold this operator meaning as a necessity in order to be seen to be doing something even when they know that they are not doing the right thing. From my analysis of their understanding of the equal sign, I came to a conclusion that actually for these students, the operator notion was not a misconception but rather a favored notion over relational meaning when not sure of what to do. Students had to do something in order to move on regardless of whether it made sense or not. For example in question 5 with the equation $4x + 7 = 15$, James found the value of x as 4. Asked as to how he found 4, he said he added $4 + 4$ to get 8 so that when he adds the 8 to 7 it will give him 15. Asked as to why he had to add $4 + 4$, he said he was looking for a number which when subtracted from 15 will give him 7 and he found 8. Since there was already a 4, then the value of x should also be 4. Asked as to what relationship was there between 4 and x, James said he didn’t know. It is clear that for this student, his focus was on getting what was after the equal sign, in this case 15 by all means.

There was evidence that some students` understanding of the symbol fluctuated between operational and relational. In this study, some students that offered a relational understanding for the equal sign in question 1 (b), later offered an Operational understanding in question 1 (c), and vice versa. Some students who recognized equivalence in the first two equations in question 2 failed to do so in other contexts in the equations that followed. During the interviews, I had problems determining which category to put their understanding of the equal sign because at some point they indicated a good understanding of the equal sign with a relational understanding but shortly afterwards, they gave me an operational understanding of the equal sign. This maybe a result of a weak relational understanding and hence they continue to hold both the relational and operational understandings. Molina & Ambrose, (2008) argue that students on the road from an operational and towards a relational understanding of the equal sign will not have a clear understanding of the equal sign. In this study for instance John`s answer to the equation 9 – 5 = □ – 9 was 5. He thought it had to be exactly the same on both sides of the equation with identical numbers or symbols. John looked at the "sameness" of the numbers instead of sameness in value in the equation thereby without any calculation; he thought both sides should have the same numbers to maintain the equation balance, hence introduced a 5 in the box. For this student, it can be assumed that for all practical purposes, the equal sign was irrelevant. As teachers, we must realize that when we tell or show students how to do any mathematical process, that process is interpreted in as many different ways as there are students. When students tell us the procedure, we must not be fooled into thinking that they know the exact thing. Only by giving each student a voice in mathematics instruction and listening to it can we truly see the depth of their understanding.

5.3 Links between equal sign understanding and success in solving equations

Knuth et al. (2006) argue that students` understanding of the equal sign is cardinal to being successful with algebra. They state that students who have a relational understanding of the equal sign succeed in solving algebraic equations better than students who do not have a relational understanding because they are able to correctly judge that two expressions were equivalent. Kieran (1992) says that "*one of the requirements for generating and adequately interpreting structural representations such as equations is a conception of the symmetric and transitive character of equality – sometimes referred to as the "left – right equivalence" of the equal sign"* (p.398). A relational view of the equal sign sanctions students to interpret equations appropriately and appropriate interpretations can guide judgments about the equivalence of equations. By varying where the unknown was in the equations used in my study, I was able to determine the nature of students` conception, the consistency of their application of them and their behavior when their conception did not apply. Students with a relational understanding of the equal sign like Mary adopted algebraic strategies of solving equations by either adding or subtracting the same amounts from both sides of the equations in order to constantly maintain the balance between the left hand side and right hand side of the equal sign as suggested by Knuth et al. (2006). This can be seen from her answer in line 1. 2 where she explains that she had to do something both sides of the equal sign in order to keep the equation balanced.

It was observed that most of the students` errors were due to ignoring the structure of the equations. Data obtained in this study suggests that when some grade 8 students are confronted with an equation, they proceed according to what they are "supposed to do" with the equal sign doing little to sway them from their answer pattern. This type of thinking indicates that students` misinterpretation is entrenched in their conditioning that the purpose of mathematics is to evaluate expressions and has less to do with misunderstanding the equal relation itself. Instead of misunderstanding the meaning of an equivalence relation in an equation, students simply ignored the equal sign in order to proceed as they are supposed to. When solving equations, most of the students went straight to do some computation without looking at the whole sentence. Their focus was on finding the answer, giving no attention to the structure of the whole sentence. I attributed this to their orientation to computation in arithmetic especially in earlier grades. This computational mind-set, also interfere with students work in solving algebraic equations where analysis of the whole equation is cardinal for one to manipulate it. Some answers revealed that students were unsure of what they were to do for example, when a student ignored one item or added all of them an example of John when solving the equation $14 \times 3 = \square - 3$. He multiplied 14 and 3 to get 42 but went on to subtract 3 and introduced another equal sign at the end of the sentence so he could "write the answer" 39. This finding is in agreement with Lindvall & Ibarra (1980), who observed that for many students the sentences were not expressions of relations but a list of numbers and operational symbols and students applied these operations on numbers as they considered it possible or most convenient.

Students` conception of concepts in mathematics can be attributed to the way they interact in form of socio-mathematical norms in the classroom where they actively interact with each in the social processes. For example in this study, more than 80% of the participants answered question 2 with the same wrong answers indicating that they had a collective understanding of the equal sign stemming from a knowledgeable other in this instance either from the teacher or the textbooks they used. Teachers` role in this situation is to look for opportunities to engage students in conversations about the equal sign during classroom interactions as well as to create opportunities intentionally by providing students with equations to solve in which the equal sign is presented in different contexts which may help to promote more appropriate interpretations and uses of the equal sign (Carpenter et al., 2003). Unfortunately, during my data collection, I did not observe any classroom interaction to be able to tell for sure that students` understanding of the equal sign is as a result of the socio-mathematical norms. However, I can hypothesize that the classroom interaction had an impact on the way the students understood the equal sign concept going by the way they answered their test because more than 80% exhibited an operational understanding of the symbol. Powell & Fuchs (2010) believe that exercises that students are given to work on will help define what mathematics is and what it is all about. Students working with typical equations only will not be able to transfer their knowledge to atypical equations successfully as was the case with James and John in some instances.

5.4 Limitation

A primary limitation to this study is that data was collected from a small group of students from one basic school. Generalizability of the results is limited but it is important for the students` voices to be heard, Patton (2002) notes that "while one cannot generalize from a single case or a small sample, one can learn from them and learn a great deal, often opening up new territories for further research" (p.46). It is worth to note that assessment tools used in this study may affect students` propensity to activate relational or operational meanings of the equal sign. I believe that if other test items were used rather than these to assess students` understanding of the equal sign could have elicited different types of responses from the same students at the same time. Since this study was not concerned with the instruction in the classroom nor the textbooks used in the classroom, its findings were limited to students only though it would have been a richer study if teachers` instruction and textbook uses of the equal sign were explored to find out how they impact on the students` understanding of the equal sign.

5.5 Implications for teaching

In order to get a student to a point of really understanding what a symbol like the equal sign is communicating, teachers need to provide meaningful contextual and coherent experiences to nurture and develop this progressive understanding. The concept of equivalence with respect to the equal sign is a concept that students must have a chance to work with in variety of contexts. If students are not exposed to appropriate multiple forms and representations of the equal sign, they will never be able to interpret and use it correctly. Several researchers have written about how students can easily develop a procedural do something notion of the equal symbol if the teachers do not provide varied contexts of the equal sign (Behr et al., 1980; Carpenter et al., 2003; Falkner et al., 1999).

Cobb (1987) reports that the teachers he surveyed believed that their curriculum was a collection of isolated facts and skills that had to be mastered separately. The teachers` role as they saw it was to train students to use particular skills, in turn students learned to just mimic their teacher` s talk and behavior and never learned to understand and make connections within the mathematics, more especially with respect to the equal sign. The language a teacher uses to instruct can reinforce the notion of procedure versus understanding in mathematics for example using phrases like “do the same thing on both sides of the equation” without elaborating the reason why can be confusing. John in chapter 4 and last dialogue was able to articulate that he needed to take the like terms to one side but didn’t demonstrate an understanding of which terms were like as he subtracted 2x from 2. A teacher who believes this way and teaches this way will produce students who are not mathematical thinkers because of lack of classroom experience. The concept of the equal sign and understanding of the equal symbol is not as intuitive as most teachers think. It is a concept that should be learned and understood before any formal algebraic concepts (Falkner et al., 1999; Knuth et al., 2006).

If mathematics is taught as a collection of isolated facts and skills that have to be mastered separately it may lead students to decide at some point that understanding was not necessary but that learning the procedure was more appropriate and hence start looking for answers instead of understanding. A very disturbing outcome that should be noted in students who take this stance is that they rarely look for reasonableness of results (Lindvall & Ibarra, 1980). This is one of the most frustrating behaviors for teachers to overcome and for me; it has always been a difficult endeavor to focus my students on considering the importance of “reasonableness of results”. As Mathematics teachers, the overall goal should be to cultivate students` abilities to reason. Carpenter et al. (2003) postulates that putting students in a position to challenge their existing conceptions was productive. They advocate for engaging them in discussions in which different conceptions about equality emerge and must be resolved to broaden their views.

5.6 Conclusion

Although students` difficulties in learning algebra have been attributed to several factors, My study agrees with Carpenter et al. (2003)`s contention that “a limited conception of the meaning of the equal sign is a major stumbling block in learning algebra, virtually all manipulations on equations require understanding that the equal sign represents a relation” (p.22). The present study`s findings highlight the need for continued attention to mathematical equivalence in general and more specifically, the equal sign in Junior secondary school mathematics curriculum and teaching in the Zambian curriculum. Previous research has attributed much of the difficulties students have in their early algebra learning to the urge to “*do something”* (Stacey & MacGregor, 1997, p. 15) developed during students` primary school mathematics. Therefore, it is prudent to inculcate a correct understanding of the equal sign in the students at primary level before they adopt the symbol as a command to do something. Students` compulsion to calculate, often foster an operational view of the equal sign (Baroody & Ginsburg, 1983). Thus, students` success in early algebra learning may require providing them with opportunities to move beyond an operational view of the equal sign and towards a relational view which will in turn facilitate the developing of an understanding of the symmetric nature of an equation thus, the equivalence relationship between expressions on the left – hand and right – hand side of an equation.

Much has been said about students` operational perception of the equal sign both at primary and secondary levels. It will be useful to develop and implement an intervention measure which will focus on developing and mastering the important mathematical concept of equivalence in general and with an example of the equal sign in particular. If algebraic thinking is to be fostered in junior secondary school curriculum, it is necessary to emphasize the structural characteristics of an equation. When students understand and recognize these characteristics, they will be able to generalize their thinking to new situations. Students with a limited understanding of the equal sign may fail to apply their knowledge in all contexts in which it would be required therefore, continued attention to the equal sign in junior secondary school curriculum and teaching should include a variety of opportunities for students to develop a relational understanding. For example, teachers in junior secondary school can build on students` arithmetic experience and help them to move beyond computation by giving them non – standard equations and emphasizing on attending to symmetry of the equation.

In this study, I examined grade 8 students` understanding of the equal sign and how their understanding relates to success in solving equations. The findings show that very few Grade 8 students reflect a relational view of the equal sign. The generally poor performance on solving equations does not come as a surprise to me given the lack of explicit focus on the equal sign in Grade 8 curricula in Zambian education system. The findings are of great importance because they show that students have a better understanding of how to solve equations if they have a relational understanding of the equal sign which is a pivotal aspect to success in solving equations.

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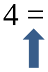
7 Appendices

A Test

2. 3 + 4 = 7



a) The arrow above points to a symbol, what is the name of the symbol?
b) What does the symbol mean?
c) Can the symbol mean anything else? If yes, explain.

3. Write the missing number in each square box below.

f) 14 x 3 = ☐ – 3

g) 9 – 5 = ☐ – 9

h) 24 + ☐ = 27 + 31

i) 100 ÷ 5 = ☐ + 5

j) $\frac{169}{13}$ = 13 - ☐

4. Is the mathematical statement below correct? If yes, say why, if no say why.
   5 + 9 = 14 ÷ 2 = 7 x 3 = 21
5. I have 5 apples and I add one more. Then I add again 2 more apples. How many apples do I have now? Show all your working (show all the given information in your solution)
6. What value of x will make the following number sentence true?, show each step of your solution clearly
   4 x + 7 = 15
7. Solve for x, 12 – 2 x = 2 + 3 x.

B CONSENT FORM

Dear Parent or Guardian,

<u>Parent/Guardian consent letter</u>

I am a third year student at HiST My thesis supervisor is Professor Ole Enge. For my research project, I am hoping to conduct a research study which examines grade 8 students' understanding of the equal sign and how the meanings they ascribe to the equal sign relates to performance in solving equations. I have selected your child's school as the school I will use to collect data for this study.

The purpose of this study is to identify student difficulties in interpreting the equal sign in problems. In order to examine student errors and misconceptions, I wish to administer a test instruments to 3 students from grade 8 mathematics classrooms. Your child will be asked to participate in this written test during the data collection. The test will take approximately 40 minutes. Based on the results, your child will be asked to participate in an interview to identify his or her understanding of the equal sign in the test items. This interview will take not more than 30 minutes.

I would like to request the participation of your child in this study. Participation in this study is voluntary and will not affect your child's attendance in class or his/her evaluation by the school. All information collected will be anonymous. In a way, the results of this study may help the school as well to identify students' ascribed meanings of the equal sign and how these meanings relate to performance in solving equations and hence pay more attention to the teaching of the concept.

Please indicate on the attached form whether you permit your child to take part in this study. Your cooperation will be very much appreciated. If you have any questions or would like more information, please contact me by phone at +26096731053 or by e-mail at mbewerose@yahoo.com.

Thank you for your consideration.
Yours sincerely,

Rose Mbewe

Parent/Guardian Consent Form

I agree/disagree to allow my child________________________________to participate
(son/daughter's name)

In the test ☐

In the interview ☐

Parent's/Guardian's signature:________Date: __