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Andreas Kjelby og Øyvind Hansen

Modeling Volatility and Risk in the CO_2 Emissions Market

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HiST TRONDHEIM TØH

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Avdeling Trondheim økonomiske høgskole

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The master thesis has been challenging and demanding, but it has been interesting and instructive. It is a theoretical and empirical study where we analyze the volatility and risk in the carbon market.

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Author:
Andreas Kjelby

Author:
Øyvind Hansen

Abstract

In this master thesis various GARCH models for volatility and Value at Risk for ECX CO_2 futures contracts have been examined. This is of great importance for market participants such as CO_2 emitting companies, traders and risk managers. The first chapters of the thesis introduce the carbon market and the different futures contracts, which are later on tested. From the descriptive analysis, we found the futures contracts to be suitable for GARCH modeling, and we also found which specifications we had to include to model the returns and Value at Risk. The applied methods are different univariate and multivariate GARCH models, which have been tested based on evaluation criteria such as Akaike, Log likelihood and numbers of significant parameters.

Problem for discussion: *Analysis of time-varying volatility in the carbon market and time-varying binary correlation between the carbon market and other energy markets (electricity-, oil-, gas-, and coal market).*

There are several different CO_2 futures contacts being traded, but for this analysis we found the 2009 and 2010 EUA futures contracts to be most interesting and suitable. To find good models for volatility and Value at Risk, we compared the GARCH results with the results from the descriptive statistics of raw data. The estimation of Value at Risk is tested for each contract, with the Kupiec test. Monte Carlo simulations have also been implemented to support our findings. The datasets have been tested within the whole trading period, also after the structural break in 2006. Finally we have presented analysis of CO_2 futures contracts in bivariate portfolios with other different energy commodities.

The main focus of this thesis has been to find suitable GARCH models for volatility of returns and Value at Risk.

Sammendrag

I denne masteroppgaven har vi undersøkt ulike GARCH modeller for volatilitet og Value at Risk av ECX CO_2 futurekontrakter. Dette er av stor betydning for aktører som for eksempel: CO_2 utslippende selskaper, tradere og risikoanalytikere. De første kapitlene av oppgaven introduserer karbonmarkedet og de ulike futurekontraktene som er testet. Fra den beskrivende analysen, fant vi futurekontraktene til å være godt egnet for GARCH modellering, og hvilke spesifikasjoner vi måtte inkludere for å best modellere avkastning og Value at Risk. Vi har brukt ulike univariate og multivariate GARCH modeller, som har vært testet basert på evalueringskriterier som Akaike, Log likelihood og antall signifikante parametere.

Problemstilling: *Analyse av tidsvarierende volatilitet i karbonmarkedet og tidsvarierende bivariat korrelasjon mellom karbonmarkedet og andre energimarkeder (elektrisitet, olje, gass og kull).*

Det eksisterer en rekke forskjellige CO_2 futurekontakter som blir omsatt i markedet, men for denne analysen fant vi 2009 og 2010 EUA futurekontrakter til å være mest passende. For å finne gode modeller for volatilitet og value at risk, sammenlignet vi GARCH resultatene med resultatene fra den beskrivende analysen av rådata. Beregningene av Value at Risk er testet for hver kontrakt ved bruk av Kupiec testen. Monte Carlo simuleringer har også blitt brukt for å støtte opp om våre funn. Datasettene har blitt testet for hele handelsperioden, samt etter priskrasjet i 2006. Til slutt har vi analysert CO_2 futurekontrakter i bivariate porteføljer med andre ulike energikontrakter.

Hovedfokus i denne avhandlingen har vært å finne passende GARCH modeller for volatiliteten av avkastningen, samt Value at Risk.

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Chapter 1

Background

Liberalization of energy markets in Europe and the rest of the world have led to development of liquid future markets for oil, gas and coal. With the introduction of emission trading, completely new markets with highly specific characteristics have been created, such as CO_2 emission contracts. Oil, gas, and coal are inputs to electricity production and their prices are therefore related to electricity prices. In this project we are going to analyze the emergence of carbon emissions as a new commodity and asset class, and its interrelationships across the spectrum of energy commodities such as electricity, oil, gas and coal. Liquid markets for physical and financial products for electricity, fuels and CO_2 provide opportunities to control and manage risk. In energy markets, many risks are fundamentally related to each other. For example, electricity prices are not independent of fuel and CO_2 prices.[1]

This report uses daily data of CO_2 emissions allowances, valid for compliance under the EU Emissions Trading Scheme (EU ETS), exchanged on the European Climate Exchange (ECX) based in Amsterdam. We use futures contracts of maturity December 2009 and 2010 to examine the volatility and risk for these ECX CO_2 futures contracts. Some disadvantages with the datasets may be few observations and extreme price movements. Compared to other assets and commodities CO_2 futures have been traded for a short period of time, something that can effect the results of the modeling. To solve this problem Monte Carlo simulations will be applied. Extreme price movements can affect the descriptive statistics of the contracts' returns. Why is it important to model volatility and risk in this market? The understanding of the volatility properties of CO_2 returns can contribute to a better characterization of the relevant stochastic process to price derivatives, Chevallier and Sevi [2009]. It also appears of importance to hedge against different kinds of institutional, economic and financial risks.

The statistically properties of daily realized volatilities in futures markets have been investigated by, among others, Thomakos and Wang [2003]. They analyzed the D-Mark,

[1]Modelling and forecasting risk in electricity, carbon and related energy markets (Oil, Gas, Coal), Sjur Westgaard

E-Dollar, S&P500 and T-bonds and found that standard deviations exhibit long memory, while standardized returns are serially uncorrelated. Their analysis also showed that the unconditional distributions of daily returns' are leptokurtic and highly skewed to the right.

Chevallier and Sevi [2009] investigated realized volatility of the 2008 ECX CO_2 futures contract. They used one year tick-by-tick data from ECX CO_2 emissions futures. They found the unconditional distributions of daily returns to be near normal. Any attempt to standardize these returns using realized measures and to a lesser extent GARCH estimates did not lead the distribution to Gaussianity. Benz and Trueck [2009] examined the spot price dynamics of CO_2 emission allowances in the EU ETS. Their analysis showed that the log-returns exhibit skewness, excess kurtosis and different phases of volatility behavior coming from fluctuations in demand for CO_2 allowances. Their results strongly support the use of AR-GARCH or regime-switching models for modeling the returns of CO_2 emission allowances.

Our work differs from the analysis of the mentioned papers by also concentrating on multivariate interactions with other commodities. In particular, we evaluate price, returns and volatility for the different approaches that can be considered as a substantial issue in risk management. Risk managers and traders constantly hedge their positions against irregular and unexpected carbon price fluctuation. Hence, they are not only interested in the long-term perspective of emission allowance prices but also in short-term price dynamics of the assets. Having a wider understanding of pricing and volatility will allow companies, investors and traders to realize efficient trading strategies, risk management and investment decisions in the carbon market. The research methods we shall employ are fundamentally quantitative. Qualitative discussions are also necessary to complement and complete the analyses. We will use time series econometric approaches on daily data. Univariate and multivariate GARCH models will be used, as well as stochastic volatility models. The G@RCH module (see Laurent [2009]) in $OxMetrics_{TM}$, the leading software for GARCH models, will be applied.

There exists a great variety of GARCH models for volatility and risk estimation in many markets (stocks, bonds, FX, commodities). An excellent overview of modeling and forecasting GARCH models in these markets, is given in Poon and Granger [2003]. Although models of this type are now well established in financial markets, applications to electricity markets and particular carbon markets are still very sparse. Examples of some of the references on univariate GARCH models for various electricity, gas and oil markets are Aloui [2008], Giot and Laurent [2002] and Solibakke [2009]. Textbooks by Bunn [2004], Weron [2006], and Serletis [2007] list relevant work with respect to modeling the volatility of energy markets using GARCH models.

There are several reasons to model the volatility of CO_2 prices and returns, but for who is this of interest? The research questions developed in this report may be of precious use for risk management and market participants who have an interest in this field and require a careful understanding of the volatility of CO_2 returns and prices. We are now going to give a short list of reasons why these participants have an interest in this.

1.1 CO_2 emitters

Buyers and sellers of emission allowances are typically industrial companies that emit large amounts of CO_2. They will have to buy allowances if they emit more CO_2 than they have allowances for. On the other hand they may sell allowances if they have a surplus of allowances. Some of the reasons why they might be interested in this are, Løland and Aas [2009]:

1. Want an assumption about next years income and expenditures
2. Do not want to take greater risk than necessary (VaR)
3. Want to optimize trading
4. Have to enter in the accounts the market value of their contracts

By getting a better understanding of the volatility, returns and prices of the CO_2 market, industrial companies will be able to make a more correct assumption of the cost of emitting more or less than they have allowances for. With a better control of the emissions, companies get a clear overview over their income and expenditures regarding buying and selling new allowances. By entering into the futures markets, these companies will minimize price risk if they buy futures contracts for allowances.

1.2 Traders

Traders in the CO_2 market try to make profits by speculations in prices of allowances. Examples of traders in the market can be power companies, investment banks, hedge funds, pension funds, and private investors. Some of the reasons why they might be interested in this are:

1. Want an assumption about next years income and expenditures
2. Want to know the risk, but not necessarily minimize it (VaR)
3. Want to know if the market is pricing it correctly and take advantage of it if they are not
4. Want to optimize trading
5. Have to enter in the accounts the market value of their contracts

Power companies usually hold a portfolio of different commodities to hedge themselves against risk. CO_2 prices have an effect on the volatility of power prices. The reason for this is that both power and CO_2 prices react to some of the same fundamentals. For example, warm winters lead to lower power demand and hence fewer emissions, thus lower CO_2 prices. As a result, power prices will experience a double downturn effect directly and indirectly through the CO_2 pricing component. This means that an effective risk management strategy needs to consider the price risks in both CO_2 and power (and related fuels) markets. Many of the players in the carbon market have positions in power and fuels markets as well, and will therefore be able to use hedging tools in these markets to position themselves within a suitable risk perspective, PointCarbon [2007]. These groups are not only interested in the long-term perspective of the CO_2 emission allowance prices but also in the short-term price dynamics.

The reminder of this thesis is organized as follows: Chapter 2 and 3 provide an overview of the the CO_2 market and the futures contracts being traded. Chapter 4 studies the descriptive statistics, value at risk, and the monthly variations of the ECX futures contacts. In Chapter 5 we present the methods we will apply. Chapter 6 gives an overview of different factors that influence the volatility. Chapter 7, 8 and 9 investigate the results and Value at Risk form the GARCH modeling. Chapter 10 provides the multivariate modeling with portfolios of different energy commodities. Finally, Chapter 11 concludes our work.

Chapter 2

The Carbon Market

The rising concerns regarding climate change led to the establishment of the Kyoto Protocol. The Kyoto Protocol was negotiated in December 1997 and came into force the 16th of February 2005. The industrialized countries signed up to reduce their collective emissions of greenhouse gasses (GHGs) by 5.2% compared to the 1990 levels during the period 2008-2012, The Kyoto Protocol [2007]. Because of this, the European Union, whose target for reduction is 8%, has implemented the EU ETS. The scheme was officially opened on 1st of January 2005. The EU ETS gives the governments of the member countries permits to emit tons of CO_2-equivalent. These permits are then distributed to large CO_2 emitting installations in the respective countries. The permits can be traded in several spot, futures, and option markets, if they fulfill their targets at the scheduled time. Under the EU ETS, allowances can be traded internationally, and the purchase and holding of allowances is not restricted to companies signed up for the program. This means that market players, such as brokers and small investors, are free to trade allowances with any other party. One allowance traded on the EU ETS corresponds to one ton of CO_2 released into the atmosphere, and is called European Union Allowance (EUA), Paolella and Taschini [2006]. Each member country in the EU has to submit a National Allocation Plan (NAP). Here the member countries determine the quantity of CO_2 allowances granted each year to its companies for a specified period. According to the EU ETS, the first period lasted from 2005 to 2007. We are now in the second commitment period, which lasts from 2008 to 2012. The third commitment period will be 2013-2020, Bataller et al. [2006].

Under the first phase of the EU ETS only within-phase banking was allowed. This meant that allowances could be banked from one year to the next, but unused Phase I allowances were not valid during Phase II. The restriction of inter-phase banking became a subject of discussion because it could be a potential source of market distortion. When Phase II of the program started the inter-phase restriction was removed. It is now allowed to bank CO_2 emission allowances until the end of Phase III, Chevallier and Sevi [2009].
The EU ETS is today the largest emission-trading scheme in terms of both allowances distributed and in number of installations covered. Not all sectors producing CO_2 emissions are regulated by the 2003/87/EC Directive and thus do not participate in emission

trading. The directive applies for the following sectors: Combustion plants, oil refineries, coke ovens, iron and steel plants, and factories making cement, glass, lime, brick, ceramics, pulp and paper. These sectors are categorized as trading sectors, Bataller and Tornero [2008b].

The GHGs, listed in the Annex A of the Kyoto Protocol include, Carbon dioxide (CO_2), Methane (CH_4), Nitrous oxide (N_2O), Hydro fluorocarbons (HFCs), Per fluorocarbons (PFCs) and Sulphur hexafluoride (SF_6). The CO_2-equivalent tons are a measure unit constructed in order to indicate the global warming potential of the different GHGs, Bataller and Tornero [2008a]. An emission allowance is the right to emit one ton of the reference gas, CO_2, into the atmosphere. To legally emit CO_2 during a year, utilities must have enough allowances for the given year to cover all of its emissions. The affected utilities are legally obligated to have an emissions monitoring system that measures the true emissions. The emissions data have to be reported to the authorities. If a company does not have enough allowances to cover its emissions at the given time, a fine will be given. The level of the fines during Phase I of the EU ETS was €40 per ton CO_2. During Phase II the fines have increased to €100 per ton CO_2, Paolella and Taschini [2006].

The largest GHG emitter, the USA, did not sign the Kyoto Protocol, but in July 2004 the New York Attorney, General Eliot Spitzer, demanded the nations largest utilities to reduce their GHG emissions. Large states such as California and Massachusetts made plans to cut their emissions with 20% and 10% within 2020, Paolella and Taschini [2006]. This year will be decisive to the USA if they will get a new climate law. The three senators John Kerry, Lindsey Graham and Joseph Lieberman will soon present a draft for a new law. The target is to reduce the American GHG emissions by 17% from 2005 to 2020. The American president Barack Obama wanted to use emissions trading to reduce the GHG emissions before he was elected, but the emissions trading scheme was discredited the last year in the USA. The scheme has been labeled 'big government'. Most likely the new climate law will contain an emissions trading scheme. One of the reasons for this is the big success the USA has had with the emissions trading scheme to reduce SO_2 emissions which was introduced in 1990, Alstadheim [2010].

Recently, several large cases of fraud related to CO_2 emissions have been discovered. Trading fraud has been detected in several European countries, and is believed to be extensive. The characteristics of the fraud has been that some companies sell CO_2 allowances to legitimate businesses. The allowance seller collects sales tax, like supposed to, but the sales tax is not paid to the government. Europol estimated in a press release last winter that tax evasion relating to the EU CO_2 emissions system may have resulted in losses of €5 billion in domestic tax revenues. In some countries it is estimated that fraudulent motive is behind 90% of trade in emission allowances, Kristensen [2010].

2.1 Types of countries

As of 15th January 2008, 177 countries, plus the European Union (EU), have ratified the Kyoto Protocol.
The world has been divided into five groups, PointCarbon [2009]:

1. EU-15 countries Annex I (Industrialized countries)
2. European countries with economies in transition
3. Other countries with emission targets
4. Annex I countries that have not ratified the Kyoto Protocol
5. Non-Annex I countries (primarily developing countries)

2.1.1 The European Union

Members of the EU are Annex I countries, and there are 15 EU-members which have agreed upon a common commitment to reduce their average GHG emissions by 8% in the first period (2005-2008) compared to 1990 level. These 15 EU-countries emitted 23% of the global GHGs in 1990. Annex I countries are usually net buyers of emission permits. The EU ETS covers around 45% of the EU GHG emissions, PointCarbon [2009].

2.1.2 Countries in process of transition to market economy

There are several countries that are in a process of transition to a market economy. These countries are members of the EU, except Russia, Ukraine, and Croatia, and there are also a part of the EU ETS. In 1990 transition countries emitted 31% of the global GHGs, PointCarbon [2009].

2.1.3 Annex II non-EU countries

In 1990 these countries emitted 15% of the global GHGs. Countries in the Annex II group have compliance targets, but they are not members of the EU or not in a transition process. The last country to ratify the Kyoto Protocol was Australia in 2007, PointCarbon [2009].

2.1.4 Annex I countries not ratified

The USA was the only Annex I country that did not sign the Kyoto Protocol in 1997, and emitted 36,4% of the global GHGs in 1990, PointCarbon [2009].

2.1.5 Non-Annex I countries

These countries have ratified the Kyoto Protocol but they do not have emission caps (limitations). Such countries are potential host countries of Clean Development Mechanism (CDM) projects, PointCarbon [2009].

Chapter 3

Futures markets

Futures contracts represent a commitment to buy or sell an underlying asset at a given future date. Futures contracts are exchange-traded which means that they are standardized and have specified delivery dates, locations, and procedures. Each futures contract trade has an associated clearinghouse. The role of the clearinghouse is to match the buys and sells taking place during each day. It also keeps track of the obligations and payments required of the members, McDonald [2006].

Futures markets are derivative markets, they exist in relation to cash markets, and are the underlying markets in which actual physical commodities are bought and sold. Futures contracts do usually not lead to physical delivery, but are instead settled financially. This is in favor of speculators who neither have the capability nor the interest to handle a physical delivery of the commodity. Market participants using financially settled futures contracts to hedge a planned physical buy or sell, bear the risk that the actual price for the physical transaction differs from the final settlement price for the contract. Most futures contracts are not held until maturity but closed out in advance, Burger et al. [2007].

The reason for the success of energy futures and options contracts is that they present opportunities to reduce risk and increase profitability. An understanding of how energy futures and options markets work, and may be used in the energy business, will pay large dividends to those firms willing to invest time and money to master these techniques. Risk transfer and price discovery are two of the main roles in derivative markets. The purpose of risk transfer is to reduce the risk from risk adverse investors to those who are willing to accept the risks. In both futures and options markets there are large quantities of bid and asks that make the markets primary source of price discovery for the related commodities, EuropeanClimateExchange [a].

The Kyoto protocol created a new market for commodity trading in Europe, the market for EUAs. The EU ETS is the largest scheme worldwide, and the futures contracts and the underlying cash market are traded on different exchanges.

3.1 Types of futures contracts

The two main groups of carbon futures contracts are:

1. European Union Allowances
2. Certified Emissions Reductions

3.1.1 European Union Allowances

European Union Allowances are carbon credits issued under the EU ETS to CO_2-emitting installations. EUAs are held in electronic accounts and reached a total of 3.1 billion metric tons valued at €67bn.[1]

EUA futures contracts are based on underlying EU Allowances and provide the market with standardized contract terms and a benchmark for price discovery. One EUA futures contract represents 1,000 tons of CO_2 EU Allowances. This is standard for all the EUA futures contracts traded on the different platforms we will present. It is also standard for all of the futures contracts with different expiry dates. [2]

3.1.2 Certified Emissions Reductions

The Clean Development Mechanism projects generate Certified Emission Reduction (CER) credits to qualifying greenhouse gas reduction projects. The CERs are transferable to industrial countries, where they can be applied toward emissions reduction targets.

CER futures contracts are based on underlying CER units. One CER futures contract represents 1,000 CER units. The contracts ensure their liquidity by being highly standardized.

3.1.3 EUA & CER Futures

EUA and CER daily futures are exchange-traded cash contracts. Both EUA and CER daily futures contracts are physically delivered by transfer of either EU Allowances or Certified Emissions Reduction units from the seller to the buyer. The daily futures contracts provide flexibility for active companies in carbon markets to manage risks and monetise their allowances. EUA and CER Daily Futures offer next-day payment and delivery of the underlying units of trade[3]. In this thesis we are going to focus on the EUA futures contracts in our analysis, since these are the most liquid contracts.

[1] http://www.ecx.eu/EUA-Products
[2] http://www.ecx.eu/ECX-EUA-Futures-What-are-Futures
[3] http://www.ecx.eu/EUA-CER-Daily-Futures-Spot

We will now present some of the exchanges where EUA futures contracts are traded.

3.2 European Climate Exchange

The European Climate Exchange is located in Amsterdam, Netherlands, and began trading futures contracts in April 2005 with a starting daily volume of 300,000 CO_2 tons. The ECX CO_2 emission futures contract is a deliverable contract where each member with a position open at the date of contract expiration is obliged to take delivery of emission allowances from national registries. EUAs are the underlying commodities traded at ECX. The exchange added Certified Emission Reduction units in 2008 as another underlying commodity. In 2009, two spot-like contracts were added, the EUA and CER Daily Futures contracts, EuropeanClimateExchange [b]. ECX has developed a partnership with the Intercontinental Exchange (ICE) Futures Europe. ECX is responsible of managing the product development and marketing of its emissions contracts and the ICE is responsible of lists those contracts on its electronic trading platform, EuropeanClimateExchange [c].

3.3 Nord Pool

Nord Pool is located in Norway and started trading in CO_2 futures contracts in February 2005 with a daily volume of 150,000 CO_2 tons. Nord Pool was the first exchange in Europe to offer standardized contracts for emission allowances and carbon credits. The exchange is Europe‘s largest and most liquid marketplace for physical and financial power contracts. Nord Pool is also one of the largest exchanges in trading of European Union emission allowances and global certified emission reduction, NordPool.

3.4 European Energy Exchange (EEX)

The European Energy Exchange is located in Leipzig (Germany). They began trading in CO_2 futures contracts in April 2005 with a starting volume of 50,000 CO_2 tons. EEX trades emission rights derivatives. Regarding power trading EEX cooperates with the French Powernext, and EEX holds 50% of the shares in the joint venture EPEX Spot based in Paris. The power derivatives trading are concentrated within EEX Power Derivatives GmbH. That is an EEX subsidiary with headquarters located in Leipzig. The European Commodity Clearing (ECC) provides clearing and settlements of EEX's carbon futures contracts, EuropeanEnergyExchange.

3.5 Powernext

The French exchange started trading spot carbon contracts on 24th June 2005 with an initial 20,000 CO_2 tons per day. Today (2010) Powernext's carbon, spots and futures, contracts are published on EEC's website. Powernext hold stakes in EEX and the cooperation has several benefits such as price formation mechanism, centralized and increased liquidity, highly effective clearing and risk management services, and a more effective governance of market coupling. EEX Power Derivatives GmbH is responsible of the power trading, where Powernext holds a 20% stake.

3.6 SendeCO2

SendeCO2 in Spain started the carbon futures trading in 2005. This exchange has one main goal, to contribute significantly in the improvement of the environment through the reduction of the real CO_2 emissions. SendeCO2 has a web based electronic platform designed for small and medium companies that wish to access the market for EUA and CER trading, SendeCO2.

Since the underlying asset is equal on all exchanges, there are several similarities between the futures contracts that are traded on the different European exchanges. They are identical in terms of contract size, minimum tick (€0.01), and the trading days are from Monday to Friday. The ECX, however, offers a broader variety for expiry contracts dates. Nord Pool offers December and March contracts, while ECX offers contracts with monthly expiry dates, Bataller and Tornero [2008a]. EUA contracts differ from CER contracts in that the underlying commodity delivered is different.[4] For this reason we have chosen to continue our analysis by focusing on EUA futures traded on ECX.

[4] http://www.ecx.eu/ECX-EUA-Futures-What-are-Futures

Chapter 4

Descriptive Statistics

In this chapter we will focus on descriptive statistics of ECX futures contracts. We have chosen six of the most liquid futures contracts from the ECX. Throughout this analysis GARCH descriptive statistics will be applied as a tool to describe the contracts' differences. The descriptive statistics will consist of contracts prices and log-returns%. Chapter 7 presents a more thorough analysis of two main ECX futures contracts. In our dataset we have prices and returns from the contracts issued, to 31.12.09.

Dealing with financial data and time series we focus on log-returns in order to capture the feature of the data. The arithmetic rate of return is defined as the capital gain plus any interim payment such as dividend (D) divided by the initial price. Alternatively we can use the geometric rate of return, which is defined in terms of the logarithm of the price ratio. In our case the income payments D are zero, Jorion [2007].

$$r = ln\frac{S_t}{S_{t-1}}$$

There are several advantages with log-returns. Firstly, log-returns can be interpreted as continuously compounded returns, meaning that the frequency of compounding of the return does not matter. Secondly, continuously compounded returns are time-additive. Normally distributed geometric returns can never generate negative stock prices, in contrast to arithmetic returns, Brooks [2008].

There are also disadvantages with using continuously compounded returns. These returns are not additive across a portfolio, because the log sum is not the same as the sum of a log. A log operation constitutes of a non-linear transformation, Brooks [2008]. Since our models take care of the problem with non-linearity we will avoid problems using log-returns in our modeling.

Some financial time series exhibit certain cyclical behavior. Seasonal time series models are useful in pricing weather related derivatives and energy futures, because most environmental time series exhibit strong seasonal behavior.

4.1 Stationarity

Because econometric results may be unreliable if the dependent variable is non-stationary, we first need to test the stationarity for both returns series. Stationarity implies that the parameters are stable over time and can be inferred from historical data. For most series, the random variable is usually the rate of return on the financial instrument. In finance literature, it is common to assume that an asset return series is weakly stationary. In some studies series tend to be non-stationary, Brooks [2008].

To find out if the datasets have constant means, variances and covariances for each lag, we tested the stationarity. Table 4.1 presents a Augmented Dickey-Fuller test (ADF) with two lags.

	ADF
Logreturns09	-18.9016
Logreturns10	-18.9749

Table 4.1: ADF-test

The null hypothesis of a unit root is rejected in favor of the stationary alternative in each case if the test statistic is more negative than the critical value. With a 5% level of significance, the critical value is -3.406. According to the results in Table 4.1 the null hypothesis is rejected and the datasets are stationary.

4.2 Value at Risk

"Value at Risk summarizes the worst loss over a target horizon that will not be exceeded with a given level of confidence", Jorion [2007].

Value at Risk (VaR) is a well known method in risk management and it is getting more popular. There are four different types of financial market risks: Interest rate risk, exchange-rate risk, equity risk, and commodity risk. Risk can be measured by the standard deviation of unexpected outcomes, called volatility. Value at Risk captures the combined effect of underlying volatility and exposure to financial risks, Jorion [2007].

4.2.1 Conditional Value at Risk

Conditional Value at Risk (CVaR), also called Mean Excess Loss, Mean Shortfall, or Tail value at risk, tells us how much we could lose if we are "hit" beyond VaR, Jorion [2007]. To give an estimate on where the average returns will be when it hits the VaR level, we use the CVaR.

4.3 Overview of the main ECX's Futures Contracts

The European Climate Exchange has six main futures contracts with different expiration dates. Table 4.2 shows the descriptive statistics for the prices from the ECX futures contracts 2009-2014.

ECX Futures Contracts Prices					
Ex.Date	Min	Mean	Max	Std.dev	Obs
2009	8.20	19.728	32.90	4.6350	1212
2010	8.43	20.143	33.55	4.7218	1225
2011	8.90	20.695	34.20	4.7539	1225
2012	9.43	21.359	34.85	4.7824	1225
2013	11.30	21.318	36.43	6.3832	452
2014	12.30	22.433	37.78	6.3893	452

Table 4.2: ECX Futures Contracts Prices

There are some differences in the data observations. The 2009 contract has only 1212 observations because the contract expired 14.12.09. The 2013 and 2014 contracts were issued 09.04.08. What is worth noticing when it comes to the futures contracts in the table is the high standard deviation in the 2013 and 2014 contracts. This was an expected outcome of the use of a small set of data.

Figure 4.1: Price 2009

Figure 4.1 explains the price movements of the ECX futures contract 2009. The period covers both Phase I and Phase II. During this period of time the contract price has been very unpredictable with large variations. Celebi and Graves [2009] point out some factors that causes the unpredictability. First of all, it is the conditions of major drivers of CO_2 prices such as the price of natural gas or plant construction costs. Many of these drivers are themselves highly uncertain, something that will affect the CO_2 prices. Secondly, it is the uncertainty in construction costs for low CO_2 technologies.

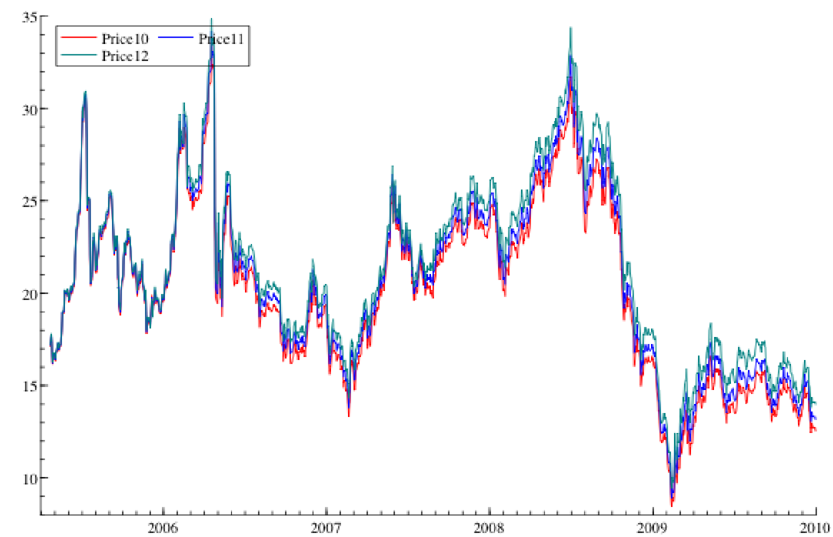


Figure 4.2: Price 2010-2012

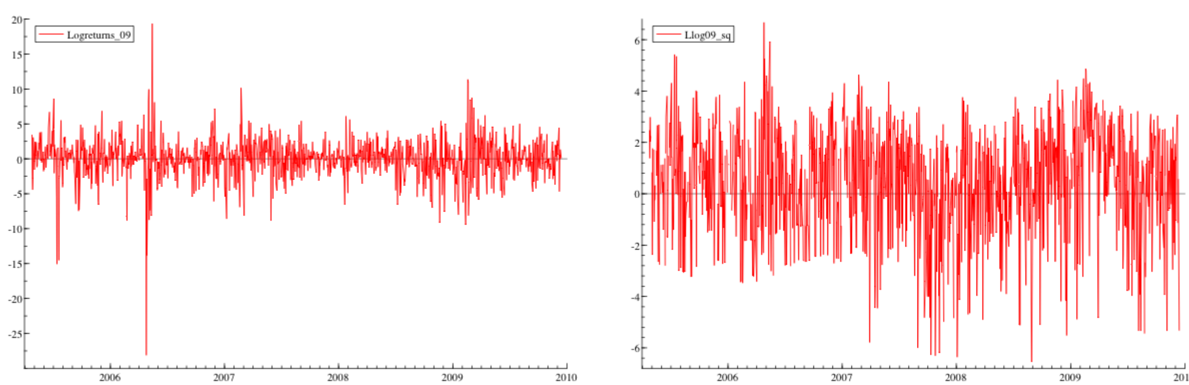
The price developments of the 2010-2012 contacts are described in Figure 4.2. In the beginning of the period prices and movements are similar. From year 2009, it is evident that the contracts' price differences are getting larger.

Referring to the introduction of this chapter, our main focus is on the contracts' returns. Since the 2010-2012 futures contracts in price and returns are very similar to each other (see Figure 4.2), we will focus on the 2009 and 2010 contracts. 2013 and 2014 have not been traded for a long time and the datasets have few observations, for that reason analysis with those contracts may contain errors.

ECX Futures Contracts Returns %					
Ex.Date	Min	Mean	Max	Std.dev	Obs
2009	-28.108	-0.014121	19.319	2.8392	1211
2010	-27.427	-0.025405	19.117	2.8175	1224
2011	-26.778	-0.021697	18.664	2.7867	1224
2012	-26.159	-0.017052	18.232	2.7723	1224
2013	-8.5632	-0.13854	9.8631	2.4234	451
2014	-8.1591	-0.13186	9.1076	2.2848	451

Table 4.3: ECX Futures Contracts Returns %

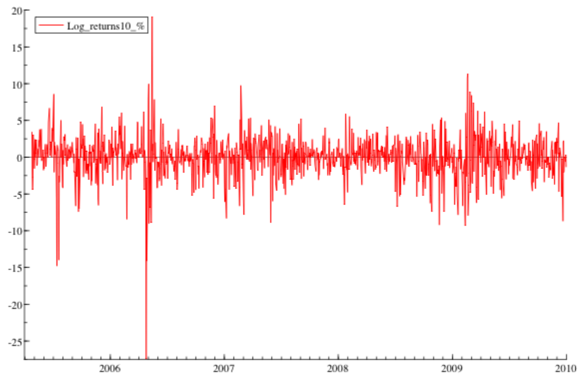
Descriptive statistics in returns are summarized in Table 4.3. The min-values are the highest negative price movement (%) during the period, and the max-values are the highest positive price movements. All contracts' means are negative or close to zero. Standard deviation for the 2009 contract is 2.8392 and 2.8175 for the 2010 contract.



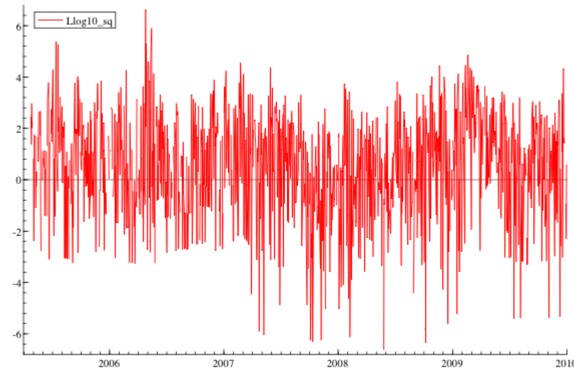
(a) Returns 2009

(b) Squared returns 2009

Figure 4.3: Returns and squared returns of the 2009 ECX EUA contract



(a) Returns 2010



(b) Squared returns 2010

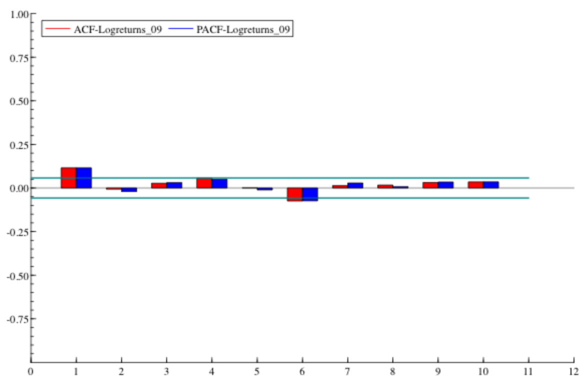
Figure 4.4: Returns and squared returns of the 2010 ECX EUA contract

There are obvious similarity between the contracts' volatility, showed in Figure 4.3 and 4.4, as expected from the price analysis above. Both Figure 4.3 and Figure 4.4 have volatility clusters. The results indicate higher volatility when the prices are falling. Because of the higher volatility in negative price movements, we can assume that the distribution is not normal distributed.

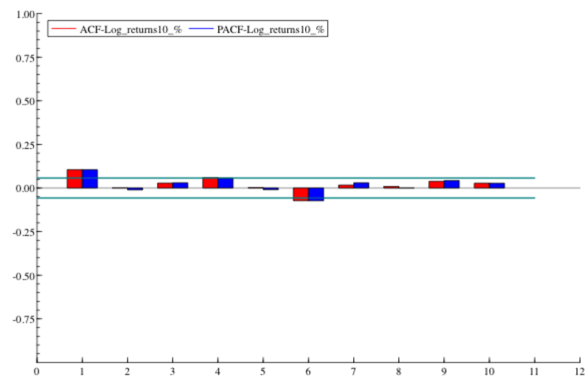
4.4 Autocorrelation

Before we start with the univariate GARCH modeling we have to check for autocorrelation in the raw series. Financial series typically exhibit little correlation, but the squared returns often indicate significant correlation. To check for autocorrelations of raw data we use autocorrelation function (ACF) and partial-autocorrelation (PACF) function in OxMetrics.

The autocorrelation function computes and displays the sample ACF of the returns, along with the upper and lower standard deviation confidence bounds, based on the assumption that all autocorrelations are zero beyond lag zero. The ACF can also be used to determine how many AR-lags we need to include in our model.[1]



(a) Autocorrelations of ECX Futures Contract 2009



(b) Autocorrelations of ECX Futures Contract 2010

Figure 4.5: ACF and PACF of returns

Figure 4.5 demonstrates the ACF and PACF of returns for the 2009 and 2010 contracts respectively. The returns are uncorrelated, except in lag 1 and 6. Since the autocorrelation function is significant in lag one, this might imply that one AR-lag should be included in the models. Following Benz and Trueck [2009], we specify the AR(1)-GARCH(1,1) model:

$$R_t = \beta_0 + \beta_0 R_{t-1} + \epsilon_t$$

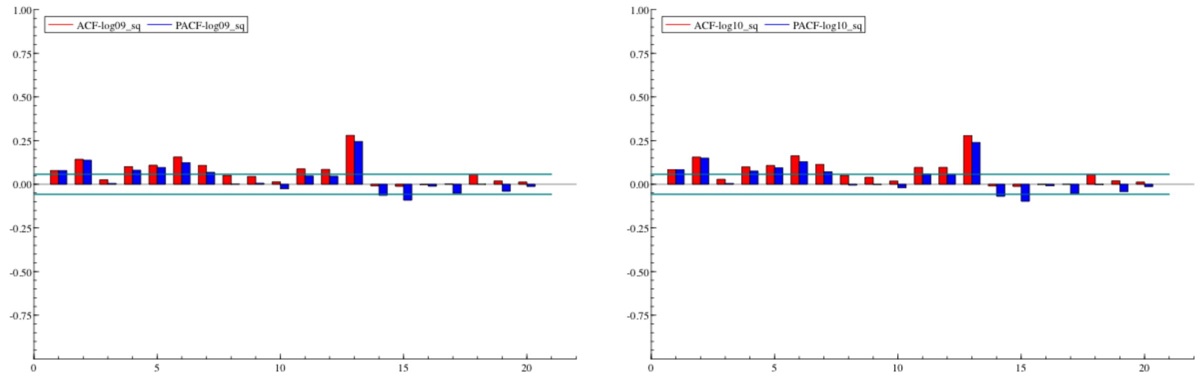
$$h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \alpha_2 h_{t-1}$$

With R_t the daily returns, and ϵ_t the error term.

While trying to develop the best model we will use models with both one and zero AR-lags. The benefit of few AR-lags is that it is easier to handle in a Monte Carlo simulation.

[1] www.mathworks.com/access/helpdesk_r13/help/toolbox/garch/overvi13.html

We know that there can be significant correlation and persistence in the ACF of the squared returns.



(a) ACF and PACF of the squared returns 2009 **(b)** ACF and PACF of the squared returns 2010

Figure 4.6: ACF and PACF of squared returns

From Figure 4.6 it can be seen that the squared returns are correlated. From the theory of financial time series, the result is not surprising. The ACF appears to die out in the end, something that indicates the possibility of a variance process close to being non-stationary.

4.5 Quantifying the correlation

There are several tests applicable to quantify the preceding qualitative checks for correlation. The most common tests are the Ljung-Box-Pierce Q-test and Engle's LM test for the presence of ARCH effects.

4.5.1 Q-test for the 2009 contract

Normally the Q-test is used as a post-estimation test applied to the fitted residuals. We use the test as a part of the pre-fit analysis. The reason for this is that the default model assumes that returns are a simple constant plus a pure innovations process. Under the null hypothesis of no autocorrelation, the Q-test statistic is asymptotically Chi-Square distributed, (see Box et al. [1994]).

Q-Statistics on Raw data 2009		
Lag	Statistic	P-value
Q(5)	18.8134	0.0020822
Q(10)	23.7332	0.0083410
Q(20)	50.6013	0.0001816

Table 4.4: Q-Statistics on Raw data 2009

Q-Statistics on Squared data 2009		
Lag	Statistic	P-value
Q(5)	143.898	0.0000000
Q(10)	263.401	0.0000000
Q(20)	369.076	0.0000000

Table 4.5: Q-Statistics on Squared data 2009

P-values in Table 4.4 and 4.5 confirm significant autocorrelation in our raw data for the 2009 futures contract. Later in the thesis, different GARCH-models will be used to try to reduce or eliminate the autocorrelation.

ARCH-test for the 2009 data		
Test	Statistic	P-value
ARCH 1-2 test:	F(2.1206)= 15.499	[0.0000]
ARCH 1-5 test:	F(5.1200)= 10.104	[0.0000]
ARCH 1-10 test:	F(10.1190)= 7.6451	[0.0000]

Table 4.6: ARCH-test for the 2009 data

Engle's LM ARCH test is conducted with ARCH 1-2, 1-5 and 1-10 lags. All lags are significant according to the LM ARCH test in Table 4.6. With significant ARCH effects in the dataset, it is reasonable to use ARCH and GARCH models.

4.5.2 Q-test for the 2010 contract

Significant autocorrelation, for the 2010 contract, is confirmed in Table 4.7 and 4.8. Later we will use different GARCH-models to try to reduce or eliminate the autocorrelation in the 2010 data as well.

Q-Statistics on Raw data 2010		
Lag	Statistic	P-value
Q(5)	19.0916	0.0018481
Q(10)	28.8874	0.0012994
Q(20)	52.3512	0.0001012

Table 4.7: Q-Statistics on Raw data 2010

Q-Statistics on Squared data 2010		
Lag	Statistic	P-value
Q(5)	66.0453	0.0000000
Q(10)	120.391	0.0000000
Q(20)	244.720	0.0000000

Table 4.8: Q-Statistics on Squared data 2010

ARCH-test for the 2010 data		
Test	Statistic	P-value
ARCH 1-2 test:	F(2.1219)= 18.457	[0.0000]
ARCH 1-5 test:	F(5.1213)= 11.046	[0.0000]
ARCH 1-10 test:	F(10.1203)= 8.3404	[0.0000]

Table 4.9: ARCH-test for the 2010 data

Engle's LM ARCH test is conducted for the 2010 data. P-values in Table 4.9 confirm significant ARCH effects. For this reason ARCH and GARCH models are reasonable to use.

4.6 Value at Risk

Value at risk ECX Futures Contracts		
	ECX Futures Contract 2009	ECX Futures Contract 2010
Value at risk 99%	6.80966	6.81072
Value at risk 1%	-8.01281	-8.39296
Value at risk 97.5%	5.15872	5.11976
Value at risk 2.5%	-6.01574	-6.22737
Value at risk 95%	3.94201	3.96521
Value at risk 5%	-4.54102	-4.45391
Conditional VaR 99%	9.43429	9.31940
Conditional VaR 1%	-12.10315	-12.07169
Conditional VaR 97.5%	7.31423	7.23684
Conditional VaR 2.5%	-8.92977	-8.89584
Conditional VaR 95%	5.92579	5.89814
Conditional VaR 5%	-7.01218	-7.03356

Table 4.10: Raw data empirical Value at Risk

The empirical Value at Risk and the Conditional Value at Risk are summarized in Table 4.10. Calculations can be found in the CD at the end of the thesis. The cutoff value in the 2009 contract is 5.15872% and 5.11976% in the 2010 contract with 95% confidence level. Additionally, the downside cutoff value is -6.01574% for the 2009 contract and -6.22737% for the 2010 contract. Since 95% confidence interval is the most common in risk management with financial data, it is employed as a standard in our further calculations. A two tailed test for VaR with a 95% confidence level is represented by 97.5% and 2.5% values. The conditional value at risk is 7.31423% (2009) and 7.23684% (2010) and the downside is -8.92977% (2009) and -8.89584% (2010).

In Chapter 7 we will look at univariate GARCH models and estimate VaR using Monte Carlo simulations (MCS).

In the next section a more detailed analysis of the 2009 and 2010 contracts will be conducted.

4.7 Analysis

There are several reasons for using returns in financial studies. The main reason is that return series are easier to handle than price series because the former have more attractive statistical properties. Many models that appear to be non-linear, can be made linear by making logarithmical transformations. However, many relationships in finance are intrinsically non-linear. As Campbell et al. [1997] state, the payoff to options are non-linear in some of the input variables, and investors' willingness to trade off returns and risks are also non-linear, Brooks [2008].

When looking at prices and returns in energy markets, it is important to provide a formal foundation for acceptance or rejection of distributions built into our pricing models. We will use the Jarque-Bera test of normality to answer the distribution question:

$$JB = T\left(\frac{\hat{\gamma}^2}{6} + \frac{(\hat{\delta}-3)^2}{24}\right)$$

Kurtosis describe the degree of flatness of a distribution. It is defined as:

$$\delta = \left\{\int_{-\infty}^{+\infty}[x - E(X)]^4 f(x)dx\right\}/\sigma^4$$

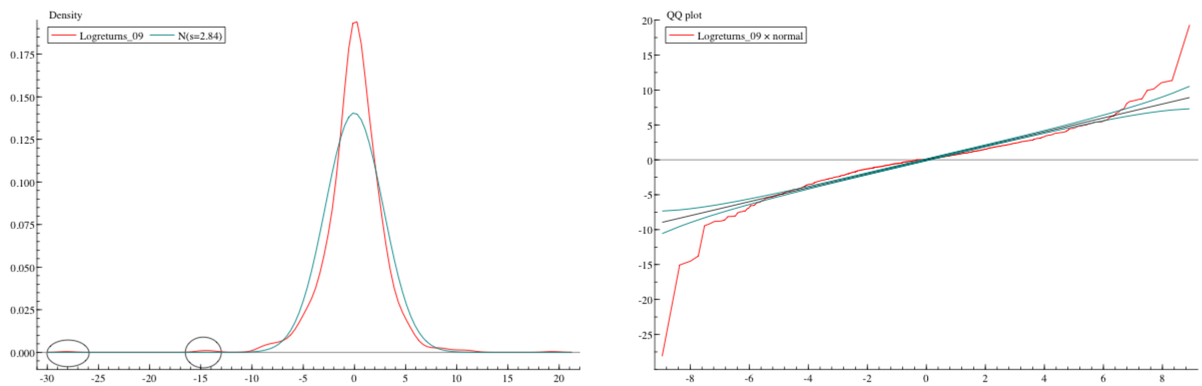
The kurtosis of a normal distribution is 3. A kurtosis coefficient greater than 3 indicates that the tails decay less quickly than for the normal distribution, implying a greater likelihood of large values, positive or negative. Such distribution is called leptokurtic, or fat-tailed, Brooks [2008].

Skewness describe departures from symmetry. It is defined as:

$$\gamma = \left\{\int_{-\infty}^{+\infty}[x - E(X)]^3 f(x)dx\right\}/\sigma^3$$

The skewness of a normal distribution is 0. Negative skewness indicates that the distribution has a long left tail and hence generates large negative values. The opposite yields for a positive skewness, Brooks [2008].

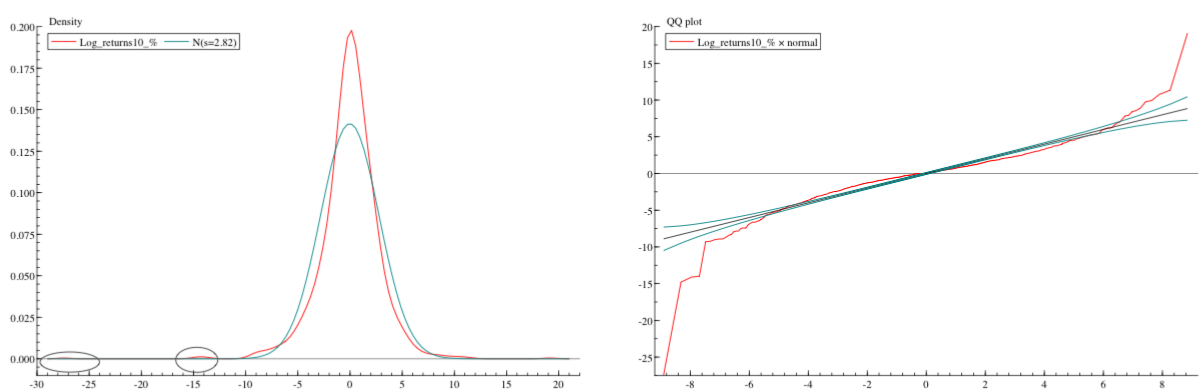
QQ plot is a plot of the percentiles of a standard normal distribution against the corresponding percentiles of the observed data. If the observations follow approximately a normal distribution, the resulting QQ plot should be roughly a straight line with a positive slope, Katenka [2008].



(a) Curve of normal distribution 2009 **(b)** QQ plot for the 2009 futures contract

Figure 4.7: Normal distribution and QQ-plot for the 2009 contract

Figure 4.7 (a) shows the 2009 contract with a normal distribution. Taking a closer look we can observe negative tails. The QQ plot in Figure 4.7 (b) shows indication of an underlying distribution that has heavier tails compared to those of a normal distribution.



(a) Curve of normal distribution 2010 **(b)** QQ plot for the 2010 futures contract

Figure 4.8: Normal distribution and QQ plot for the 2010 contract

According to Figure 4.8 (a) and (b) the 2010 contract prove the same pattern as the 2009 contract. Over the next pages we give a more thorough numeric analysis of these indications.

The summary of price statistics are presented in Table 4.11 and 4.12. Skewness exists, the kurtosis is larger than 3, and the results in the tables are clearly showing that the data have non-normality and asymmetry's features. The value of the Jarque-Bera test is 18.145 in the 2009 contract, and 19.803 in the 2010 contract. As expected from the figures above, the P-values are small and we reject the null hypothesis of normal distribution (the null hypothesis of JB test is normality), Zhang [2009].

ECX Futures Price 2009			
Factors	Statistic	t-test	P-value
Skewness	0.033733	0.47984	0.63134
Excess Kurtosis	-0.59586	4.2414	2.2217e-05
Jarque-Bera	18.145	.NaN	0.00011478

Table 4.11: ECX Futures Price 2009

ECX Futures Price 2010			
Factors	Statistic	t-test	P-value
Skewness	0.072968	1.0435	0.29673
Excess Kurtosis	-0.60580	4.3351	1.4569e-05
Jarque-Bera	19.803	.NaN	5.0102e-05

Table 4.12: ECX Futures Price 2010

The 2009 contract exhibits a negative skewness according to the Table 4.13. This tells us that the probability for extreme negative values are greater than for extreme positive values. The kurtosis is negative as well, implying that the probability is higher for large decreases in the price. With a high Jarque-Bera coefficient, the null hypothesis of normal distribution is rejected.

ECX Futures Returns % 2009			
Factors	Statistic	t-test	P-value
Skewness	-0.89729	12.763	2.6227e-37
Excess Kurtosis	11.655	82.959	0.0000
Jarque-Bera	7016.5	.NaN	0.0000

Table 4.13: ECX Futures Returns % 2009

ECX Futures Returns % 2010			
Factors	Statistic	t-test	P-value
Skewness	-0.87110	12.457	1.2810e-35
Excess Kurtosis	11.051	79.083	0.0000
Jarque-Bera	6383.5	.NaN	0.0000

Table 4.14: ECX Futures Returns % 2010

The returns from 2010 contract exhibit negative skewness according to Table 4.14. The kurtosis indicates that the probability for extreme increases in the returns, r_t, is higher compared to a normal distribution. From the Jarque-Bera there is no normal distribution.

It is widely known that financial asset returns do not follow a normal distribution, but they are almost always leptokurtic, or fat-tailed. This is also a fact for our data. This observation has implications for economic modeling. First, models are required to be robust to non-normal error distributions. Second, the risk of holding a particular security is probably no longer appropriately measured by its variance alone. In risk management, assuming normality when returns are leptokurtic will result in a systematic underestimation of the risk of the portfolio. Consequently, the use of a Student's t distribution has been employed to systematically allow for leptokurtosis in financial data, Brooks [2008]. Due to this, a Student's t distribution will be standard in our further modeling.

4.8 Monthly variations

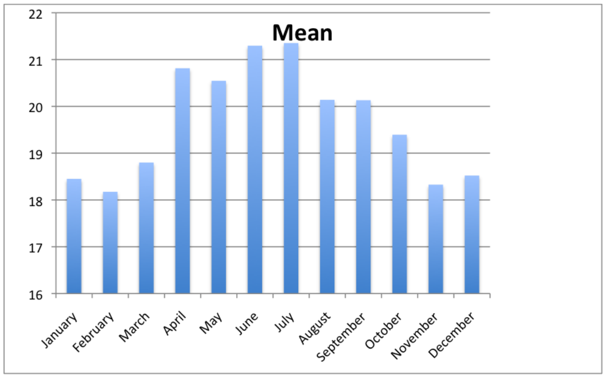
This section presents a closer look at the monthly variations for prices and returns. The data is daily prices and returns of the ECX futures contracts with expirations December 2009 and 2010.

4.8.1 Monthly variations for the ECX 2009 Futures Contract

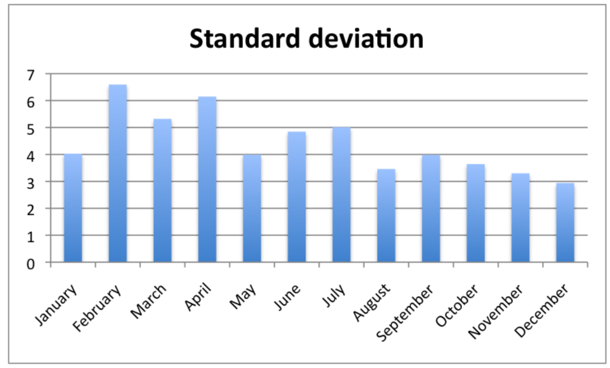
Monthly prices												
	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Mean	18.45	18.174	18.799	20.814	20.546	21.296	21.353	20.139	20.131	19.394	18.327	18.521
Std.dev	4.0295	6.5929	5.3194	6.147	3.9922	4.8446	5.0145	3.4577	3.9832	3.6424	3.2997	2.9408
Variance	16.2369	43.4663	28.2960	37.7856	15.9377	23.4701	25.1452	11.9557	15.8659	13.2671	10.8880	8.6483
Kurtosis	-0.616212	-0.79477	3.5446	-1.1765	-1.3691	-1.1735	-1.5932	-0.57248	-1.2567	-1.3460	-0.68887	-0.32766
Skewness	0.48822	-0.79484	2.1371	0.032871	0.47547	0.55790	0.035062	-0.65680	-0.093587	-0.11001	-0.64022	-0.48586
Min	11.5	8.2	10.06	12.14	14.08	12.39	13.03	14.18	12.79	13.05	12.63	13.49
Max	25	28.5	27.55	32.9	27.15	29.8	30.53	26.29	26.06	24.09	24.31	23.94
Jarque-Bera	1.2221	2.6323	29.548	0.34715	2.5472	2.4037	2.2252	1.9677	1.4798	1.6276	1.9379	0.96395
Count	90	81	88	90	111	108	110	111	107	111	107	110

Table 4.15: Descriptive statistics of prices

Table 4.15 gives an overview of the monthly variations for the ECX 2009 futures contract. Monthly differences are evident. March has a much higher positive kurtosis then any of the other months. For all the other months the kurtosis is slightly negative. The positive skewness in March indicates that there is a higher probability for extreme positive prices. As we can see form the Jarque-Bera, none of the months are totally normal distributed but all months except of March are close to a normal distribution. The highest variance is observed in the months February-April.



(a) Mean



(b) Standard deviation

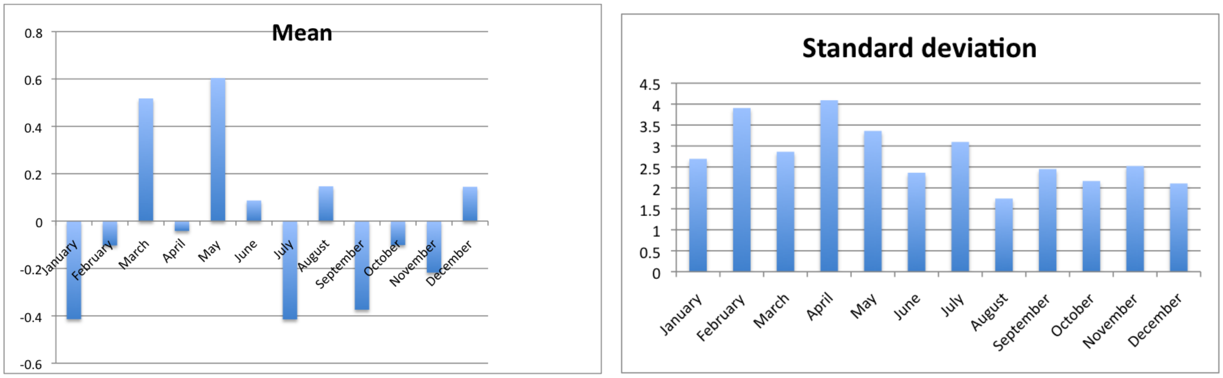
Figure 4.9: Mean and standard deviation for the price of the 2009 contract

Monthly means and standard deviations are drawn in Figure 4.9 (a) and (b). There is a tendency of higher prices in the spring and summer time of the period. The reason for this can be high energy prices. Another reason might be that the warm summers increases the use of electricity, which again increases the price for CO_2 allowances, (ref: Chapter 6, Price Dynamics). An example of this would be the hot and dry July of 2006 in Europe, which led to higher demand for electricity even as hydro resources were low and nuclear resources were off-line. This pushed the price of EUAs higher, Benz and Trueck [2009]. From Figure 4.9 (b) we can observe some variation in the standard deviation for each month. February-April have the highest standard deviation in prices.

Monthly returns												
	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Mean	-0.41393	-0.10256	0.51767	-0.0414	0.6039	0.087105	-0.4149	0.14687	-0.37398	-0.10126	-0.2174	0.1448
Std.dev	2.6928	3.906	2.8622	4.0914	3.3595	2.3615	3.0965	1.7452	2.4492	2.1641	2.5217	2.1057
Variance	7.2512	15.2568	8.1922	16.7396	11.2862	5.5767	9.5883	3.0457	5.9986	4.6833	6.3590	4.4340
Kurtosis	0.40250	2.5076	1.7696	-1.6985	-0.96655	0.95645	1.8525	1.1736	-0.25091	-0.82458	-0.45340	2.5426
Skewness	-0.38133	-0.96339	1.1959	0.22482	0.41599	0.94737	-1.3208	-0.84509	-59276	0.11831	-0.13990	1.1819
Min	-8.5903	-9.4346	-8.1126	-28.108	-8.8262	-5.6545	-15.062	-4.1847	-7.4627	-6.99	-9.1559	-7.5748
Max	6.0945	11.366	8.7508	6.2914	19.319	6.684	8.6094	4.9797	5.399	4.5445	5.4214	7.1826
Jarque-Bera	0.68169	8.3338	8.4830	0.64314	1.4909	4.1295	9.1087	4.0577	1.3461	0.64393	0.26020	11.048
Count	90	81	88	90	111	108	110	111	107	111	107	97

Table 4.16: Descriptive statistics for the months January-December 2009

Descriptive statistics for monthly returns are summarized in Table 4.16. The highest positive kurtosis is in February and December. There are no months of very positive or negative skewness. January, April, October and November are close to normally distributed, since the Jarque-Beras are close to zero. The highest variance is observed in February, April and May. The reason for this was large movements in the price during these months of 2006.



(a) Mean **(b)** Standard deviation

Figure 4.10: Mean and standard deviation for the returns of the 2009 contract

The highest average returns are observed in March and May, (see Figure 4.10 (a)). January, July and September have the greatest negative average returns. From Figure 4.10 (b), variation in the standard deviations for each month are evident. February, April and May possess the highest standard deviation in returns. One reason for the high standard deviation in both prices and returns in April is that in April 2006, it became clear that corporate participants had been granted around 10% more allowances than they actually needed, to cover their 2005 emissions. The consequent was that a surplus of EUAs flooded the market and prices crashed 60% within a week, from a high of around €30 per ton of CO_2 to €11, Benz and Trueck [2009].

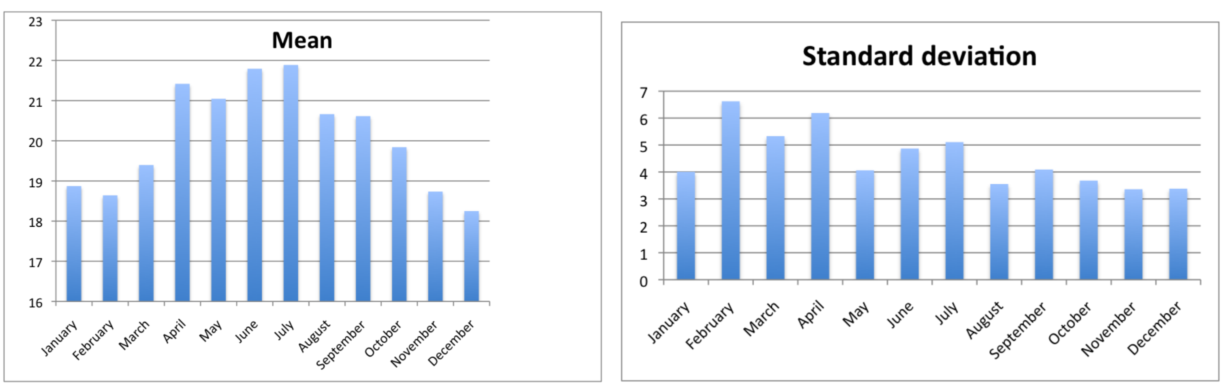
4.8.2 Monthly variations for the ECX 2010 Futures Contract

We will now take a look at the monthly variations for the 2010 futures contract, see Table 4.17. The results are expected to be similar to the ones of the 2009 contract, since both contracts cover the same period of time.

	Monthly prices											
	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Mean	18.872	18.642	19.397	21.418	21.045	21.793	21.888	20.664	20.614	19.84	18.735	18.249
Std.dev	4.0068	6.6223	5.3296	6.1864	4.0615	4.8697	5.1078	3.5522	4.0892	3.6796	3.3586	3.379
Variance	16.0544	43.8549	28.4046	38.2715	16.4958	23.7140	26.0896	12.6181	16.7216	13.5395	11.2802	11.4176
Kurtosis	-0.61258	-0.91145	3.1677	-1.1765	-1.3776	-1.1532	-1.5873	-0.53293	-1.2692	-1.3290	-0.70984	-0.045570
Skewness	0.48575	-0.60403	2.0383	0.032871	0.47096	0.56331	0.043464	-0.66409	-0.074228	-0.079420	-0.64232	-0.72374
Min	11.9	8.43	10.49	12.78	14.78	12.96	13.5	14.57	13.05	13.32	12.81	12.45
Max	25.15	29	28.1	33.55	27.79	30.79	31.71	27.27	27	24.85	24.91	24.55
Jarque-Bera	1.2091	1.9084	25.543	0.34715	2.5529	2.3827	2.2111	1.9627	1.4969	1.5675	1.9747	1.9225
Count	90	81	88	90	111	108	110	111	107	111	107	110

Table 4.17: Descriptive statistics of prices 2010

As expected, March differs from the other moths with a positive kurtosis (see Table 4.17), a high positive skewness and high Jarque-Bera. The other months have negative kurtosis, skewness close to zero, and a low Jarque-Bera. The highest variances are observed in February and April.



(a) Mean **(b)** Standard deviation

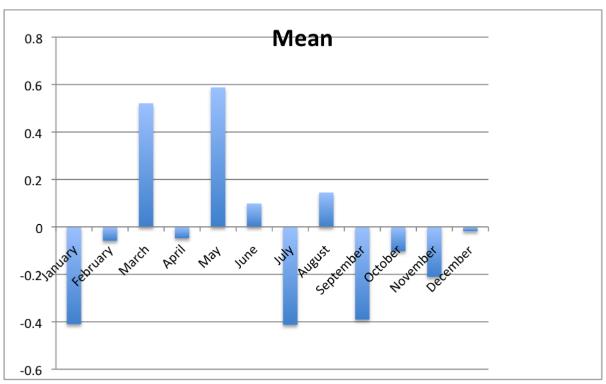
Figure 4.11: Mean and standard deviation for the prices of the 2010 contract

Monthly means and standard deviations for the 2010 contract are given in Figure 4.11 (a) and (b). The trend is the same as for the 2009 contract; higher prices in the spring and summer time of the period. The reason for this is discussed in Chapter 6, Price Dynamics. February-April have again the highest standard deviation in prices.

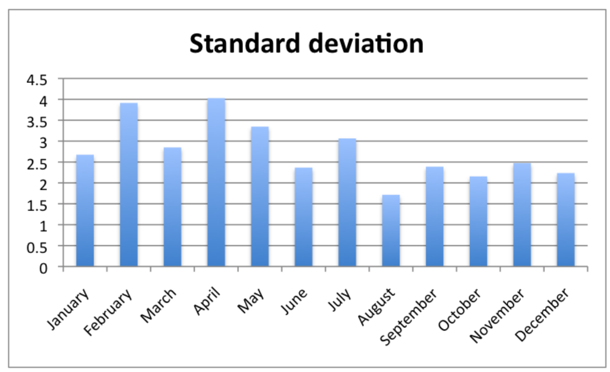
Monthly returns												
	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Mean	-0.41017	-0.0585	0.52131	-0.047514	0.58825	0.098825	-0.41288	0.14497	-0.39076	-0.10254	-0.21053	-0.017849
Std.dev	2.6742	3.9133	2.8478	4.028	3.3462	2.3659	3.0648	1.7159	2.3877	2.1551	2.4732	2.2353
Variance	7.1513	15.3139	8.1100	16.2248	11.1971	5.5975	9.3930	2.9443	5.7011	4.6445	6.1167	4.9966
Kurtosis	0.56402	1.5817	1.9132	-1.6985	-0.90100	0.91297	1.7167	1.3075	-0.26320	-0.63639	-0.42048	1.9669
Skewness	-0.35004	-0.51308	1.0819	0.22484	0.41675	0.93438	-1.2869	-1.0202	-0.58905	0.12034	-0.15005	1.0450
Min	-8.2997	-9.3014	-7.8216	-27.427	-8.9948	-5.9689	-14.8	-4.422	-7.428	-7.4184	-9.219	-8.6877
Max	5.8951	11.354	8.4068	6.2003	19.117	6.6543	8.5642	4.97	5.2032	4.5943	5.3879	6.9854
Jarque-Bera	0.74087	2.9625	7.9945	0.64312	1.3810	3.9653	8.3750	5.6276	1.3358	0.40505	0.24463	7.5505
Count	90	81	88	90	111	108	110	111	107	111	107	110

Table 4.18: Descriptive statistics for the months January-December 2010

Descriptive statistics for monthly returns in the 2010 contract, are presented in Table 4.18. The results are different from the 2009 contract in that the 2010 contract has the highest positive kurtosis in December. December and March have the highest positive skewness, and July has the highest negative skewness. The Jarque-Beras are highest in March, July and December. The largest variances are observed in February, April and May.



(a) Mean



(b) Standard deviation

Figure 4.12: Mean and standard deviation for the returns of the 2010 contract

According to Figure 4.12 (a) March and May obtain the highest average returns, while January, July and September achieve the greatest negative average returns, similar to the 2009 contract. When it comes to the standard deviations for the returns of the 2010 contract, they are the highest in February, April and May, corresponding confirmed by Figure 4.12 (b).

Chapter 5

Methods

5.1 Volatility

Risk can be measured by the standard deviation of unexpected outcomes, or sigma (σ), also called volatility. Volatility refers to fluctuations in some phenomenon over time. It can be seen as the uncertainty or risk about the size of changes in for example a security's value. High volatility indicates that the security's value can potentially be spread out over a larger range of values. High volatility means that the security's value can change dramatically in value either way, over a short period of time. If the volatility of the security is low, then it's value will not fluctuate as much. Volatility can be measured using either standard deviation or variance of the return from the security or market index.[1]

Modeling and forecasting market volatility have been subject of extensive empirical and theoretical research over the past decade. There are several reasons for this. Volatility is considered as one of the most important concepts in finance. Volatility, measured by the standard deviation or variance of returns, is often used as a measure of the total risk of financial assets, Brooks [2008]. Modern option pricing theory, beginning with Black and Scholes [1973], accords volatility a central role in determining the fair price of an option. While the returns volatility of the underlying asset is only one of five parameters in the basic Black and Scholes option pricing formula, its importance is magnified by the fact that it is the only one that is not directly observable. Both theorists and practitioners are concerned with the behavior of volatility and the construction of option pricing models in which volatility can change. To hedge volatility risk is also an important issue for market-makers, Jorion [2007].

Volatility is also important in portfolio management. Portfolio management involves deciding what assets to include in the portfolio, given the goals of the portfolio owner. The risk of a portfolio comprises systematic risk, also known as undiversifiable risk, and unsystematic risk which is known as diversifiable risk. Unsystematic risk can be diversified

[1] http://www.investopedia.com/terms/v/volatility.asp

away to smaller levels by including a greater number of assets in the portfolio. An optimal portfolio displays the lowest possible level of risk for it's level of return.

5.2 Historical volatility

The historical estimate is the simplest model for volatility. It involves calculating the variance or standard deviation of returns over some historical period. This becomes the volatility forecast for all future periods. The historical average variance or standard deviation was used as the volatility input to several options pricing models. There is growing evidence suggesting that the use of volatility predicted from more advanced time series models leads to more accurate option values. However, historical volatility is still used as a benchmark for comparing the forecasting ability of more advanced time models, Brooks [2008].

5.3 Correlation

Correlation between two variables measures the degree of linear association between the two of them. If two variables, x and y, are correlated, it means that the two variables are being treated completely symmetrical. Thus, it does not mean that changes in x cause the changes in y, or the other way around. It is only stated that there is evidence for a linear relationship between the two variables. The movements in the two are on average related to an extent given by the correlation coefficient, Brooks [2008]. The correlation coefficient, r, is a measure of the strength and direction of the linear relationship between the two variables. The range of the coefficient, r, is from -1 to +1, and the sign r indicates the direction of the correlation between the two variables. The two variables are perfectly positively correlated if r= +1, and perfectly negatively correlated if r= -1. The two variables are totally uncorrelated when r=0, Studenmund [2006].

5.4 EWMA

The exponentially weighted moving average (EWMA) is a simple extension of the historical volatility measure. The model allows more recent observations to have a stronger impact on the forecast of volatility than older observations. The latest observations carries the largest weight, and the weights of previous observations decline exponentially over time. Another advantage is that the effect on volatility of a single event declines at an exponential rate as weights attached to recent events fall. The EWMA model can be expressed like this:

$$\sigma_t^2 = (1-\lambda)\sum_{j=0}^{\infty} \lambda^j (r_{t-j}) - \bar{r})^2$$

Here σ_t^2 is the estimate of the conditional variance for the period t. $\bar{r}$ is the average return estimated over the observations and λ is the decay factor. The decay factor determines how much weight is given to recent versus older observations, Brooks [2008].

5.5 ARMA

An autoregressive model (AR) is one where the current value of a variable, y, depends only upon the values that the variable took in previous periods plus a added error term.

$$y_t = \mu + \sum_{i=1}^{t} \phi_i y_{t-i} + u_t$$

The simplest class of a time series model is the moving average (MA) process. MA(q) can be expressed as:

$$y_t = \mu + \sum_{i=1}^{q} \theta_i u_{t-i} + u_t$$

The moving average model is a linear combination of a white noise process, so that y_t depends on the current and previous values of a white noise disturbance term.

The autoregressive volatility model (ARMA) is a simple version of a stochastic volatility model. By combining the AR(p) and MA(q) models, the ARMA(p, q) process will be a combination of those two parts. The model states that the current value of a series y depends linearly on its own previous values. In addition a combination of current and previous values is added by a white noise error term, Brooks [2008].

5.6 Time-varying volatility

Earlier research on volatility shows that for many assets, there are periods of turbulence and periods of calm. If volatility is persistent over some time, a volatility measure should weight recent returns more heavily than older returns. ARCH and GARCH models, which we are going to use in our analysis, give more weight to recent returns, McDonald [2006].

5.7 Non-linear models

Many relationships in finance are non-linear. The payoffs to options are non-linear in some of the input variables, and investors' willingness to trade off returns and risks are non-linear. These relationships are clear motivations to consider non-linear models in order to capture better the relevant features of the data.

There are a number of different non-linear models, but only a small number of the models have been found useful for modeling financial data. The most popular and most used non-linear financial models are the ARCH and GARCH models. They are used for modeling and forecasting volatility, Brooks [2008].

5.8 ARCH

The Autoregressive Conditional Heteroscedasticity (ARCH) of Engle [1982] is an important and widely used volatility model. The model attempts to capture statistically the ebb and flow of volatility. The ARCH process allow the conditional variance to change over time as a function of past errors, leaving the unconditional variance constant. It was the first model that provided a systematic form for volatility modeling. The basic idea behind the ARCH model is that if volatility is high today it is more likely than average to be high tomorrow. In other words, the current level of volatility tend to be positively correlated to its levels during the preceding periods. This is called volatility clustering. A plausible explanation to the tendency for volatility in financial markets to appear in bunches regards information. The information arrivals that drive price changes occur in bunches, rather than being spread evenly. The assumption made under the classical linear regression model that the variance of the errors is constant is known as homoscedasticity. If the variance of the errors is not constant, than it is known as heteroscedasticity. In the context of financial time series, it is unlikely that the variance of the errors will be constant, Brooks [2008]. After Engle introduced the ARCH model, this type of models has been widely used in modeling economic phenomena and financial time series. However, after the ARCH process was appropriated, people found a lot of weaknesses in this new type of models, like the long lag length, a large number of parameters and it is not easy to control the existent of negative variance, in order to solve this, Bollerslev [1986] proposed the generalized ARCH, GARCH model.

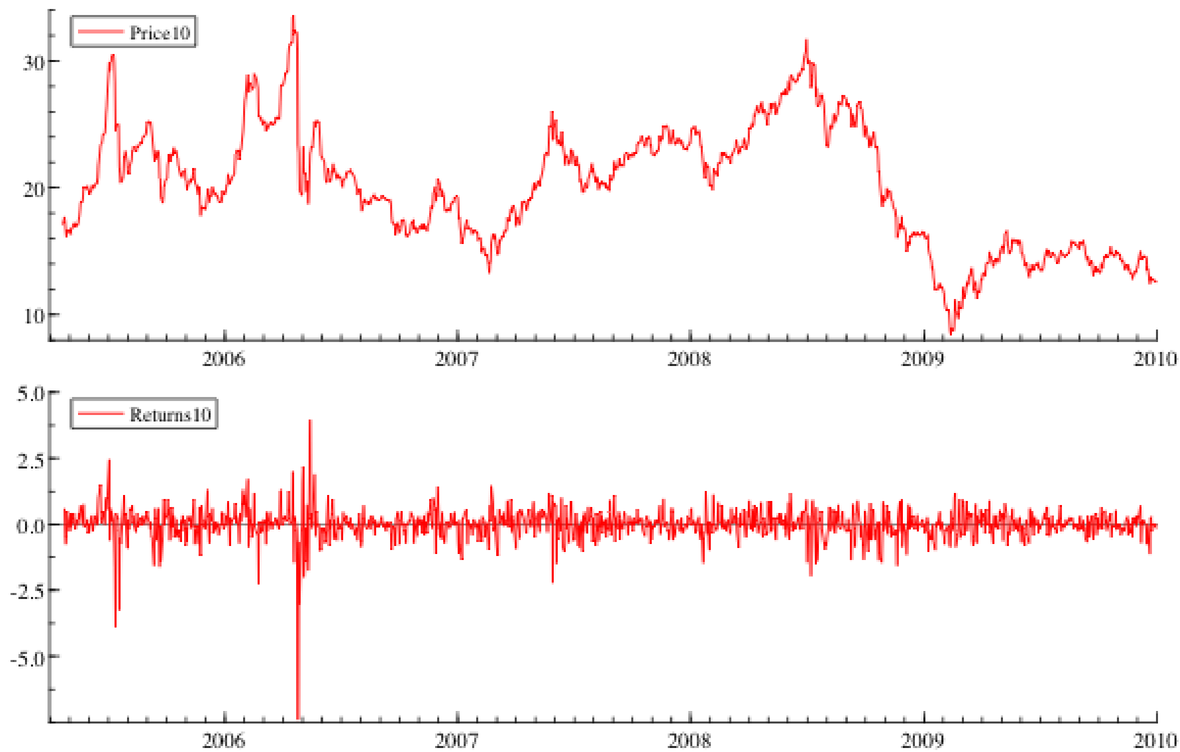


Figure 5.1: Futures prices and returns

Figure 5.1 shows the day to day futures prices of the ECX 2010 futures contract and its returns. We observe that the volatility is fluctuating and that volatility clustering is present.

5.8.1 ARCH effects

Before estimating a GARCH model, it is sensible to test for ARCH effects to make sure this type of models is appropriate. One can compute Engels LM ARCH test of Engle [1982] to test if there is ARCH effects in a series, as we did in Chapter 4, Descriptive Statistics. The test statistics is distributed $X^2(p)$ under the null hypothesis of no ARCH effect.

5.9 GARCH

The Generalized Autoregressive Conditional Heteroskedasticity of Bollerslev [1986] and Taylor [1986], has been very popular in the empirical literature as it usually provides a good fit with the empirical data. It allows the conditional variance to be dependent upon previous lags. The conditional variance equation also called GARCH (1,1) is:

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta\sigma_{t-1}^2$$

or the GARCH (p,q)

$$\sigma_t^2 = w + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$$

σ_t^2 is known as the conditional variance since it is a one-period ahead estimate for the variance calculated based on relevant past information. The GARCH model makes it possible to interpret the current fitted variance, as a weighted function of a long-term average value, which is dependent on α_0. $\alpha_1 u_{t-1}^2$ is information regarding volatility during the previous period, and $\beta\sigma_{t-1}^2$ is the fitted variance form the model during the previous period. Identification and estimation of GARCH models is performed by maximum likelihood. The method works by finding the most likely values for the parameters given the actual data. The maximum likelihood estimation is used to find parameter values for both linear and non-linear models, Brooks [2008].

5.9.1 The unconditional variance GARCH

The conditional variance is changing, but the unconditional variance of u_t is constant. It is given by:

$$var(u_t) = \frac{\alpha_0}{1 - \alpha_1 - \beta}$$

As long as $\alpha_1 + \beta < 1$. For $\alpha_1 + \beta \geq 1$, the unconditional variance of u_t is not defined, and will be termed "non-stationarity in variance". $\alpha_1 + \beta = 1$ will be known as a "unit root in variance", Brooks [2008].

It is known that volatility series are mean-reverting. This means that if the volatility currently are at a high level relatively to the historical average, it will fall back towards its average level. If the volatility is at a low level, it has a tendency to rise back towards the average. The time it takes for the volatility to move halfway back to its stationary level, given that it have moved away from it, is given by:

$$\lambda = \log(0.5)/\log(a + b)$$

5.10 New GARCH models

Since the GARCH model was introduced there have arrived extensions and new variants. Problems and restrictions are some of the reasons for this. One of the primary restrictions of the GARCH model is that it enforce a symmetric response of positive and negative shocks volatility shocks. It has been argued that in financial time series, negative shocks cause volatility to rise more than positive shocks of the same magnitude. Two models that take asymmetry into account are the GJR model and the exponential GARCH (EGARCH).

5.10.1 The GJR model

This model is a simple extension to the GARCH with an additional term added to account for asymmetries. The GJR model can be written as:

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta\sigma_{t-1}^2 + \gamma u_{t-1}^2 I_{t-1}$$

Where $I_{t-1} = 1$ if $u_{t-1} < 0$
=0 otherwise

For a leverage effect, we would see $\gamma > 0$. The condition for non-negativity will be: $\alpha_0 > 0$, $\alpha_1 > 1$, $\beta \geq 0$, and $\alpha_1 + \gamma \geq 0$, Brooks [2008].

5.10.2 EGARCH

There are several ways to express the exponential GARCH model, and one possible specification is this:

$$ln(\sigma_t^2) = w + \beta ln(\sigma_{t-1}^2) + \gamma\frac{u_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \alpha\left[\frac{|u_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}}\right]$$

One of the advantages this model has over the simple GARCH, is because of how the $ln(\sigma_t^2)$ is modeled. Because of this, then even if the parameters is negative, σ_t^2 will be positive. It also allows asymmetries, because if the relationship between volatility and returns is negative, γ , will also be negative, Brooks [2008].

5.10.3 ARFIMA-GARCH

Several studies have shown that the dependent variables (interest rate returns, exchange rate returns, etc.) may exhibit significant autocorrelation between observations widely separated in time. In such case, y_t is said to display long memory and is best modeled by a fractionally integrated ARMA process. This ARFIMA process was developed in Granger (1980) and Granger and Joyeux (1980). The ARFIMA(n,ζ, s) is given by:

$$\Psi(L)(1-L)^{\zeta}(y_t - \mu_t = \Theta(L)\epsilon_t$$

where the operator $(1-L)^{\zeta}$ accounts for the long memory of the process, Laurent [2009].

5.11 Multivariate GARCH models

Multivariate GARCH models are in general pretty similar to the univariate models, except that the multivariate models also specify equations for how the covariances move over time. There have been proposed several different multivariate GARCH formulations the literature, but we are going to concentrate on two of the most popular ones; the diagonal BEKK and RiskMetics.

5.11.1 RiskMetrics

J.P.Morgan (1996) uses the exponentially weighted moving average model (EWMA) to forecast variance and covariances. RiskMetrics define the variance and covariances as IGARCH type models. The RiskMetrics model is defined as:

$$H_t = (1-\lambda)\epsilon_{t-1}\epsilon'_{t-1} + \lambda H_{t-1}$$

or alternatively

$$H_t = \frac{(1-\lambda)}{(1-\lambda)^{t-1}} \sum_{i=1}^{t-1} \lambda^{t-1}\epsilon_{t-1}\epsilon'_{t-1}$$

The decay factor λ $(0 < \lambda < 1)$ proposed by RiskMetrics is equal to 0.94 for daily data which we will use. The decay factor is not estimated but suggested by RiskMetrics. In respect, this model is easy to work with in practice, Laurent [2009].

5.11.2 The diagonal BEKK

In the diagonal BEKK model both parameter matrices are diagonal. The number of parameters to be estimated is significantly lower while maintaining the main advantage of this specification, the positive definiteness of the conditional covariance matrix. The diagonal BEKK model is given by the following equations:

$$h_{11,t} = a_{11}^2 + b_{11}^2\epsilon_{1,t-1}^2 + c_{11}^2 h_{11,t-1}$$

$$h_{22,t} = a_{11}^2 + a_{22}^2 + b_{22}^2\epsilon_{2,t-1}^2 + c_{22}^2 h_{22,t-1}$$

$$h_{12,t} = h_{21,t} = a_{11}a_{22} + b_{11}b_{22}\epsilon_{1,t-1}\epsilon_{2,t-1} + c_{11}c_{22}h_{12,t-1}$$

$$h_{21,t} = h_{12,t}$$

The diagonal BEKK model exhibit essentially the same problem as the full BEKK model since there is no parameter in any equation that exclusively governs a particular covariance equation, Baur [2001].

5.12 Akaike's Information Criterion

Akaike's Information Criterion (AIC) is an index used to help us determine which of a number of alternative statistical models best fits the data by including or excluding competing factors. It is a measure of the information lost when a given model is used to describe reality. The AIC can be defined as:

$$AIC = ln(\hat{\sigma}^2) + \frac{2k}{T}$$

where $\hat{\sigma}^2$ is the residual variance, k=p+q+1 is the total number of parameters estimated and T is the sample size. The lowest value indicates the best model, while models with higher values are rejected, Everitt [1998].

The multivariate version of Akaike's information criterion can be defined as:

$$MAIC = ln|\hat{\sum}| + 2k'/T$$

where $\hat{\sum}$ is the variance-covariance matrix of residuals, T is the number of observations and k' is the total number of regressors in all equations, Brooks [2008].

Chapter 6

Price Dynamics

Brokers, traders and risk managers buy and sell emission allowances and derivatives. For these groups, the price behavior and dynamics of the CO_2 emission allowances is of great importance in order to realize trading strategies, risk strategies and investment decisions. They constantly have to hedge their positions against irregular and unexpected carbon price fluctuations. This section contains an overview of the main variables that influence the CO_2 emission allowance price.

Research is done on which variables influence the volatility of the CO_2 emission allowance prices. Research done by Springer [2003] and Christiansen et al. [2005] identified the carbon prices' main drivers to be economic growth, energy prices, weather conditions and policy issues. While allowance supply is fixed by each member state through the National Allocation Plans, allowance demand is a function of the level of CO_2 emissions, which again depends on the before mentioned factors.

Previous literature has pointed out energy prices to be the most important driver of the volatility of the carbon prices due to the ability of power generators to switch between fuel inputs (Kanen [2006], Christiansen et al. [2005], Bunn and Fezzi [2007], Convery and Redmond [2007]). The option to switch from natural gas to coal in their inputs is backed up by Bataller et al. [2006] who found this variable to be statistically significant. This idea is supported by, Lowrey [2006]:

"...if the price of gas increases relatively to the price of coal, then the cost of switching from gas to coal increases and - other things being equal - the demand for coal will increase. Therefore, the demand for carbon allowances to cover that generation will also rise, leading to a resultant increase in the emission allowance price."

High energy-prices contribute to an increase in carbon prices.

Weather conditions may influence the volatility of the EUA prices because it influences energy demand. Numerous studies have highlighted the effect of climate on energy prices.

These studies have indicated that the relationship between temperature and electricity is non-linear. Only increase and decrease, beyond a certain threshold, can lead to increased demand for power. Warm summers increase the demand for air conditioning, electricity, and derived coal. Cold winters increase the demand for natural gas and heating fuel. Because of the increase in input, power generators' CO_2 emissions will increase the demand for allowances, Alberola et al. [2007]. From their research Alberola et al. [2007] found that seasonal average matter more than temperatures themselves on CO_2 price changes during extreme weather events.

Political and institutional features impact the carbon price. The gap between initial allocation to industrials and their business-as-usual emission forecasts are problematic. During the last week of April 2006, prices collapsed dramatically when operators disclosed 2005 verified emissions data, which showed that the scheme was oversupplied. The EUA price fell 54% in four days after this happening, Alberola et al. [2007]. Changes in policy directives and regulations may have consequences for the demand and supply of emission allowances. The NAPs set the rules and reduction targets, Benz and Trueck [2009]. An example of this was the changes in rules from Phase I to Phase II when it was allowed to bank emission allowances.

Carbon prices may be affected by the economic activity of the various sectors covered by the EU ETS. Economic growth has a major impact on CO_2 emissions and therefore on allowances demand and supply from installations. Industrial sectors that experience higher production growth than their projections, are expected to be net buyers of allowances. Industrial sectors that experience lower production than their projections are expected to be net sellers of allowances, Alberola et al. [2007].

From the discussion above it is possible to specify a structural model of the factors that influence the volatility of the CO_2 prices:

$$\sigma_{co_2} = f(P_{coal}, P_{electricity}, P_{gas}, P_{oil}, T)$$

Where P denotes the price of the different commodities that influence the CO_2 price. T is the temperature variable which can affect the CO_2 price during extreme weather events.

Chapter 7

Results

In this chapter we will analyze our data with univariate GRACH models. We want to find the model that explains the volatility best without losing to much information.

7.1 GARCH-modeling of the 2009 Futures Contract

Several different GARCH models are used for modeling the 2009 futures contract. GARCH, GJR, EGARCH and ARFIMA models are conducted to find the best model for our data. The models were run with different p and q levels as well as different AR-lags, to find the best fit to our data.

Tables 7.1-7.4 display all parameters for some of the different GARCH models tested with t-values and p-values. We have chosen to only present the models that gave reasonable results. EGARCH and ARFIMA models did not give adequate results, and are therefore excluded from the further analysis.

	GARCH (1,1)			AR (1) GARCH		
	Coefficient	t-value	P-value	Coefficient	t-value	P-value
C (M)	0.123849	2.180	0.0295	0.123074	2.009	0.0448
C (V)	0.437152	3.950	0.0001	0.430182	3.962	0.0001
AR (1)				0.080942	2.717	0.0067
Alfa (1)	0.165850	4.867	0.0000	0.165660	4.982	0.0000
Beta (1)	0.791837	29.30	0.0000	0.791997	30.00	0.0000
Student (DF)	4.587160	7.070	0.0000	4.690047	6.785	0.0000

Table 7.1: GARCH (1,1) and AR(1) GARCH

	ARMA GARCH (1,1)			AR (1) GARCH (2,1)		
	Coefficient	t-value	P-value	Coefficient	t-value	P-value
C (M)	0.124654	2.070	0.0387	0.119518	1.935	0.0532
C (V)	0.430847	3.967	0.0001	0.204405	3.766	0.0002
AR (1)	-0.277746	-1.381	0.1675	0.082464	2.787	0.0054
MA (1)	0.362951	1.895	0.0584			
Alfa (1)	0.165149	4.974	0.0000	0.071572	3.666	0.0003
Beta (1)	0.792522	30.04	0.0000	1.515619	14.51	0.0000
Beta (2)				-0.607437	-6.970	0.0000
Student (DF)	4.673029	6.698	0.0000	4.704436	6.6680	0.0000

Table 7.2: ARMA GARCH (1,1) and AR (1) GARCH (2,1)

	GJR (1,1)			AR (1) GJR (1,1)		
	Coefficient	t-value	P-value	Coefficient	t-value	P-value
C (M)	0.106576	1.823	0.0686	0.100791	1.579	0.1146
C (V)	0.467901	3.960	0.0001	0.458773	4.005	0.0001
AR (1)				0.081504	2.718	0.0067
Alfa (1)	0.124214	3.467	0.0005	0.121427	3.393	0.0007
Beta (1)	0.785917	27.87	0.0000	0.787193	29.06	0.0000
Gamma (1)	0.078447	1.497	0.1346	0.082126	1.560	0.1191
Student (DF)	4.644064	7.015	0.0000	4.738763	6.744	0.0000

Table 7.3: GJR (1,1) and AR (1) GJR (1,1)

	GJR (2,1)			AR (1) GJR (2,1)		
	Coefficient	t-value	P-value	Coefficient	t-value	P-value
C (M)	0.099599	1.713	0.0870	0.096996	1.527	0.1271
C (V)	0.192636	3.472	0.0005	0.1934926	3.569	0.0004
AR (1)				0.081776	2.736	0.0063
Alfa (1)	0.043520	2.178	0.0296	0.044964	2.094	0.0365
Beta (1)	1.579890	12.31	0.0000	1.567651	12.42	0.0000
Beta (2)	-0.658874	-6.155	0.0000	-0.649353	-6.197	0.0000
Gamma (1)	0.029768	1.770	0.0769	0.031693	1.822	0.0687
Student (DF)	4.541308	6.950	0.0000	4.670708	6.633	0.0000

Table 7.4: GJR (2,1) and AR (1) GJR (2,1)

Test					
Model	Obs	Parameters	LL	AIC	JB
GARCH (1,1)	1211	5	-2785.134	4.607984	2091.6
AR (1) GARCH	1211	6	-2780.307	4.601663	1950.9
ARMA GARCH (1,1)	1211	7	-2779.324	4.601691	2116.9
AR (1) GARCH (2,1)	1211	7	-2778.203	4.599840	2042.3
GJR (1,1)	1211	6	-2783.734	4.607323	1779.1
AR (1) GJR (1,1)	1211	7	-2778.833	4.600880	1664.4
GJR (2,1)	1211	7	-2782.096	4.606270	2470.4
AR (1) GJR (2,1)	1211	8	-2776.607	4.598856	2209.8

Table 7.5: Test for GARCH models

Table 7.5 summarizes the different GARCH models. Our evaluation criteria to find the best model are the AIC value, log likelihood value and the number of significant parameters. The problem of autocorrelation is also taken into account. The best model based on our criteria is the AR (1) GARCH (1,1). All of the model's parameters are significant and the AIC is low. The model also reduced some autocorrelation in the data. Two other good models are the GARCH (1,1) and the AR (1) GARCH (2,1). These models have all their parameters significant, but the AIC level for the GARCH (1,1) is higher. AR (1) GARCH (2,1) is a much more complex model than the AR(1) GARCH (1,1). Because of this we have chosen the AR(1) GARCH (1,1) as the best model.

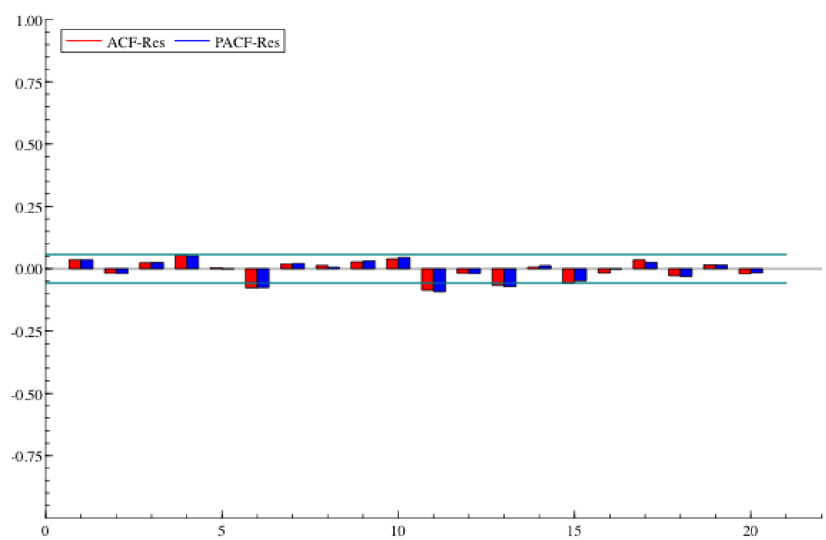


Figure 7.1: ACF and PACF of the residuals 2009

The GARCH model eliminates the autocorrelation in lag one (see Figure 7.1). The remaining autocorrelation in our data is not of any concern.

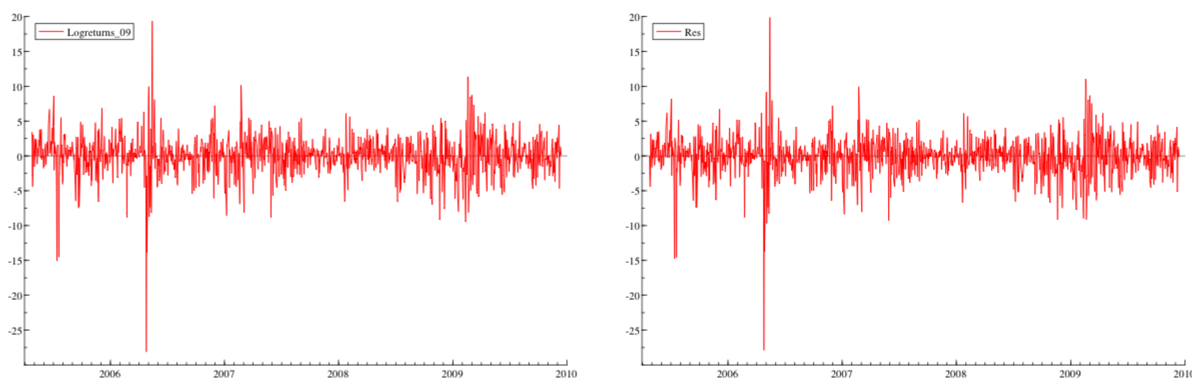
Q-Statistics on Standardized Residuals		
Lag	Statistic	P-value
Q(5)	17.6902	0.0014186
Q(10)	20.3770	0.0170735
Q(20)	35.2779	0.00993647

Table 7.6: Q-Statistics on AR (1) GARCH (1,1) 2009 futures

Q-Statistics on Squared Standardized Residuales		
Lag	Statistic	P-value
Q(5)	3.41200	0.3323556
Q(10)	10.1895	0.2519751
Q(20)	19.2400	0.3771874

Table 7.7: Q-Statistics on AR (1)GARCH (1,1) 2009

If we compare the results from the Q-statistics in Table 7.6 and 7.7 with the Q-statistics from the raw data, the autocorrelation in squared residuals is eliminated. Despite of that, there is still significant autocorrelation in the residuals.



(a) Log-returns **(b)** Residuals

Figure 7.2: Log-returns and residuals for the 2009 contract

From the Figure 7.2 it is evident that the volatility in the residuals corresponds to that of the raw data, and the model is suited for Monte Carlo simulation.

To eliminate the autocorrelation, dummy variables were tried out. A dummy variable is an artificial variable constructed such that it takes the value unity whenever the qualitative phenomenon it represents occurs, and zero otherwise. Dummy variables can also be used to capture changes in the intercept, changes in the slope, and changes in both the intercept and the slope. Most important for our study is the fact that dummy variables can minimize or eliminate the autocorrelation and capture the differences among more than two classifications, such as seasons and regions, Kennedy [2003].

We made two dummies, Monday and Friday, in OxMetrics. These dummies were included in several GARCH models to test the impact of dummies on our result. The results, which are presented in appendix, show that the dummies had little or no effect on our modeling. One of the reasons for this is because there is not a large autocorrelation problem in the AR (1) GARCH (1,1) model, which is the best model.

We will now take a closer look at the 2010 futures contract, and try to find the best model for this contract as well.

7.2 GARCH-modeling of the 2010 Futures Contract

The same types of GARCH models are used for the 2010 contract, as for the 2009 contract. The results are summarized in Table 7.8-7.11.

	GARCH (1,1)			AR (1) GARCH		
	Coefficient	t-value	P-value	Coefficient	t-value	P-value
C (M)	0.111368	2.002	0.0455	0.110830	1.864	0.0625
C (V)	0.440992	3.941	0.0001	0.435495	3.959	0.0001
AR (1)				0.071680	2.449	0.0145
Alfa (1)	0.163391	4.851	0.0000	0.163974	4.952	0.0000
Beta (1)	0.794672	29.34	0.0000	0.794257	29.87	0.0000
Student (DF)	4.355361	7.588	0.0000	4.428321	7.293	0.0000

Table 7.8: GARCH (1,1) and AR(1) GARCH

	ARMA GARCH (1,1)			AR (1) GARCH (2,1)		
	Coefficient	t-value	P-value	Coefficient	t-value	P-value
C (M)	0.112329	1.918	0.0554	0.108042	1.804	0.0715
C (V)	0.436563	3.962	0.0001	0.206456	3.900	0.0001
AR (1)	-0.282576	-1.403	0.1608	0.073403	2.533	0.0114
MA (1)	0.358028	1.866	0.0622			
Alfa (1)	0.163294	4.934	0.0000	0.071745	3.829	0.0001
Beta (1)	0.795073	29.92	0.0000	1.516873	14.97	0.0000
Beta (2)				-0.608623	-7.135	0.0000
Student (DF)	4.397059	7.220	0.0000	4.475985	7.161	0.0000

Table 7.9: ARMA GARCH (1,1) and AR (1) GARCH (2,1)

	GJR (1,1)			AR (1) GJR (1,1)		
	Coefficient	t-value	P-value	Coefficient	t-value	P-value
C (M)	0.097418	1.701	0.0892	0.092549	1.493	0.1356
C (V)	0.468969	3.896	0.0001	0.462853	3.949	0.0001
AR (1)				0.072914	2.471	0.0136
Alfa (1)	0.128446	3.580	0.0004	0.125639	3.492	0.0005
Beta (1)	0.788991	27.32	0.0000	0.789265	28.34	0.0000
Gamma (1)	0.066417	1.265	0.2060	0.071891	1.353	0.1763
Student (DF)	4.396382	7.544	0.0000	4.463447	7.254	0.0000

Table 7.10: GJR (1,1) and AR (1) GJR (1,1)

	GJR (2,1)			AR (1) GJR (2,1)		
	Coefficient	t-value	P-value	Coefficient	t-value	P-value
C (M)	0.092711	1.621	0.1054	0.090703	1.466	0.1428
C (V)	0.199378	3.409	0.0007	0.200607	3.660	0.0003
AR (1)				0.073822	2.529	0.0116
Alfa (1)	0.049704	2.193	0.0285	0.050965	2.282	0.0227
Beta (1)	1.559351	10.91	0.0000	1.550160	12.15	0.0000
Beta (2)	-0.642113	-5.343	0.0000	-0.635467	-5.948	0.0000
Gamma (1)	0.024476	1.377	0.1689	0.026698	1.442	0.1495
Student (DF)	4.364934	7.457	0.0000	4.456204	7.148	0.0000

Table 7.11: GJR (2,1) and AR (1) GJR (2,1)

Test					
Model	Obs	Parameters	LL	AIC	JB
GARCH (1,1)	1224	5	-2804.930	4.591388	2145.9
AR (1) GARCH	1224	6	-2800.784	4.586248	2025.6
ARMA GARCH (1,1)	1224	7	-2799.891	4.586423	2184.7
AR (1) GARCH (2,1)	1224	7	-2798.616	4.584340	1990.0
GJR (1,1)	1224	6	-2803.963	4.591398	1892.9
AR (1) GJR (1,1)	1224	7	-2799.667	4.586056	1783.0
GJR (2,1)	1224	7	-2802.411	4.590540	2319.0
AR (1) GJR (2,1)	1224	8	-2797.586	4.584291	2101.8

Table 7.12: Test for GARCH models 2010

Also the evaluation criteria for finding the best GARCH model are the same as for the 2009 contract. The best model is the AR (1) GARCH (1,1). For this model, all the parameters were significant and the AIC level is low. The model reduced some autocorrelation in our data. Two other good models were the GARCH (1,1) and the AR (1) GARCH (2,1). These models had all its parameters significant, but the AIC level was a bit higher for the GARCH (1,1). As pointed out earlier the AR(1) GARCH (2,1) is a very complex model, and this is the reason why we have chosen to appoint the AR(1) GARCH (1,1) as our best model.

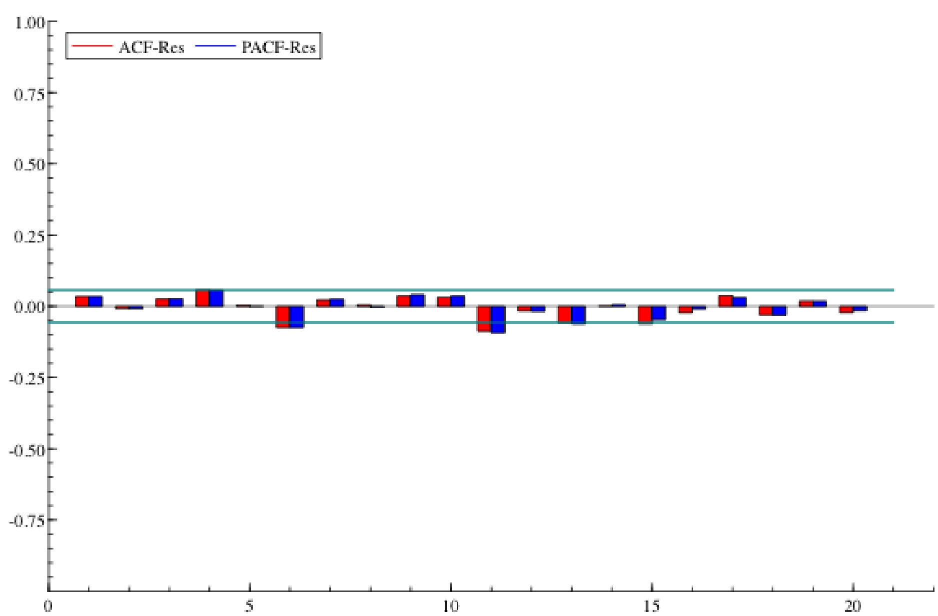


Figure 7.3: ACF and PACF of the residuals 2010

Q-Statistics on Standardized Residuals		
Lag	Statistic	P-value
Q(5)	18.2905	0.0010827
Q(10)	19.9433	0.0182659
Q(20)	36.1542	0.0101045

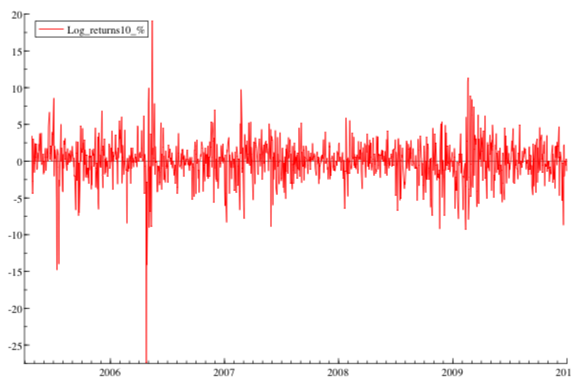
Table 7.13: Q-Statistics on AR (1) GARCH (1,1) 2010 futures

Q-Statistics on Squared Standardized Residuales		
Lag	Statistic	P-value
Q(5)	2.49631	0.4759583
Q(10)	8.58019	0.3789372
Q(20)	17.1013	0.5161497

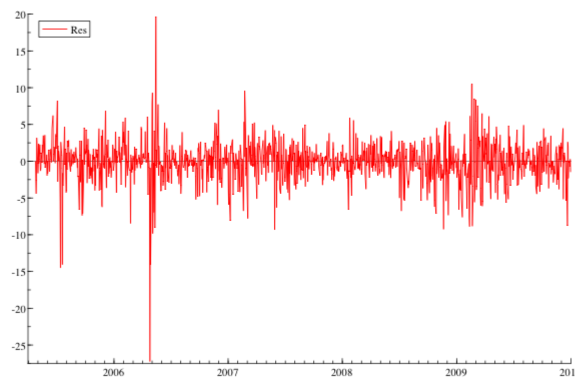
Table 7.14: Q-Statistics on AR (1)GARCH (1,1) 2010 futures

The Q-statistics in Table 7.13 and 7.14 show the same results as for the 2009 contract, which means that the autocorrelation in squared residuals are eliminated. There is still significant autocorrelation in the residuals.

Also for the 2010 contracts, we tried to reduce the level of autocorrelation by using dummy variables for Monday and Friday. This did not reduce the autocorrelation and the dummies did not affect the results.



(a) Log-returns



(b) Residuals

Figure 7.4: Log-returns and residuals for the 2010 contract

The figure above compares the volatility in the residuals to the raw data. The residuals follow the raw data good, indicating that Monte Carlo simulation can be applied to this model.

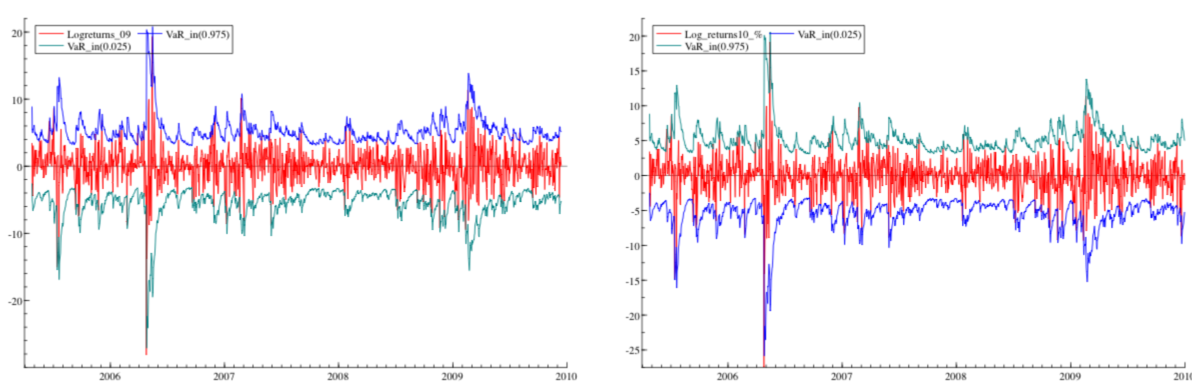
Chapter 8

Value at Risk

8.1 VaR of Univariate GARCH models

In this section we will analyze the Value at Risk in an AR (1) GARCH (1,1) model for ECX Future Contract 2009 and ECX futures contract 2010. From Chapter 10, we found that AR (1) GARCH (1,1) was the best model according to our evaluation criteria. The models' ability to calculate Value at Risk is tested using the Kupiec LR test. As a standard, the Student's t distribution is used, but according to Giot and Laurent [2001], student t distribution can cause some problems. This will be discussed later. Due to the problems, we will also test VaR with a skewed Student's t distribution.

A Monte Carlo simulation will be implemented after the models are tested, and the results from the simulation will be compared with the empirical Value at Risk (ref: Chapter 4).



(a) AR (1) GARCH (1,1) VaR 2009 **(b)** AR (1) GARCH (1,1) VaR 2010

Figure 8.1: VaR models

From Figure 8.1 we can observe the VaR in-sample with 95% confidence interval. If the graphs are correct, 2.5% of the observations are below the downside VaR and 2.5% above the upside VaR. The next step is to see if the stated probability is actually achieved.

8.2 Kupiec test

To test if the stated probability is actually achieved we use the Kupiec LR test in OxMetrics. Since the computed empirical failure rates define a sequence of yes/no observations, it is possible to test this hypothesis:

$$H_0 : f = \alpha \quad against \quad H_1 : f \neq \alpha$$

where f is the failure rate, Laurent [2009].

With use of the Kupiec LR test, we can test our hypothesis. The LR statistics for testing a hypothesis with the Kupiec test are:

$$LR = -2log\left(\frac{\alpha^N(1-\alpha)^{T-N}}{\hat{f^N}(1-\hat{f})^{T-N}}\right)$$

where N is the number of VaR violations, T is the total number of observations and α is the theoretical failure rate. Under the null hypothesis, that f is the true failure rate, the LR test statistics is asymptotically distributed as a $x^2(1)$, Laurent [2009].

The failure rate is widely applied in studying the effectiveness of VaR models. The definition of failure rate is the proportion of the number of times the observations exceed the forecasted VaR to the total number of observations. By assessing the differences between the pre-specified VaR level and the failure rate, it is possible to judge the performance of VaR models. If the failure rate is close to the pre-specified VaR level, we can conclude that the VaR model is well specified, Tang and Shieh [2006].

Kupiec LR test				
Quantile	Failure rate	Kupiec LRT	P-value	ESF1
0.95000	0.96449	5.9312	0.014875	5.8419
0.97500	0.97770	0.37686	0.53929	6.4032
0.99000	0.99587	5.4161	0.019951	9.8498

Table 8.1: Short positions 2009

Table 8.1 and 8.2 contain the results from the Kupiec LR test for 2009. Our main focus is on quantile 0.975 and 0.025 level since we want to test the 95% confidence interval. In short position we do not reject the null hypothesis because the P-value is not significant when the critical value is 5%. In the long position the null hypothesis is also retained.

Kupiec LR test				
Quantile	Failure rate	Kupiec LRT	P-value	ESF1
0.05000	0.063584	4.3481	0.037051	-5.8474
0.02500	0.031379	1.8727	0.17117	-7.2181
0.01000	0.013212	1.1465	0.28429	-9.3273

Table 8.2: Long positions 2009

Kupiec LR test				
Quantile	Failure rate	Kupiec LR	P-value	ESF1
0.95000	0.95670	1.2090	0.27152	5.6181
0.97500	0.97467	0.0053402	0.94175	6.0940
0.99000	0.99592	5.5705	0.018266	9.6757

Table 8.3: Short positions 2010

Kupiec LR test				
Quantile	Failure rate	Kupiec LR	P-value	ESF1
0.05000	0.064542	5.0114	0.025181	-5.8584
0.02500	0.031863	2.1791	0.13990	-7.2683
0.01000	0.012255	0.58652	0.44377	-9.4996

Table 8.4: Long positions 2010

Table 8.3 and 8.4 show the results for the 2010 futures contract. The results in quantile 0.975 (0.025) are satisfactory and the null hypothesis is retained in both short and long positions. High numbers are observed in quantile 0.99 (short position) and 0.05 (long position).

The Student's t distribution can cause difficulties with the Kupiec LR test, due to, in some cases, very large critical values of the Student's t distribution, Giot and Laurent [2001]. In our models we did not experience problems with the main quantiles (0.975,0.025), but some other quantiles showed high significant Kupiec observations. Since we are aware of the problem, we choose to run a model with skewed Student's t distribution to compare with the already used Student's t distribution.

8.2.1 Skewed Student's t distribution

The skewed Student's t distribution improves the results for both negative and positive returns. According to Giot and Laurent [2001], skewed Student's t distribution handles the problem with conservativeness of the symmetric Student t distribution, causing unusual high levels of long and short VaR.

Kupiec LR test				
Quantile	Failure rate	Kupiec LR	P-value	ESF1
0.95000	0.95706	1.3319	0.24847	5.6303
0.97500	0.97275	0.24452	0.62096	6.0560
0.99000	0.99174	0.39478	0.52980	8.3972

Table 8.5: Short positions 2009

Kupiec LR test				
Quantile	Failure rate	Kupiec LR	P-value	ESF1
0.05000	0.056152	0.92955	0.33498	-6.0659
0.02500	0.027250	0.24452	0.62096	-7.6585
0.01000	0.011561	0.28370	0.59428	-9.6654

Table 8.6: Long positions 2009

The results from Table 8.5 and 8.6 (the 2009 contract) indicate that the quantiles cannot be rejected. Therefore, the null hypothesis is kept in both long and short positions. The failure rates are low enough to not be significant.

Kupiec LR test				
Quantile	Failure rate	Kupiec LRT	P-value	ESF1
0.95000	0.95016	0.00068871	0.97906	5.4153
0.97500	0.97304	0.18833	0.66431	6.0230
0.99000	0.99183	0.44165	0.50632	7.7702

Table 8.7: Short positions 2010

Kupiec LR test				
Quantile	Failure rate	Kupiec LRT	P-value	ESF1
0.05000	0.058824	1.9033	0.16770	-6.0033
0.02500	0.026961	0.18833	0.66431	-7.6879
0.01000	0.012255	0.58652	0.44377	-9.4996

Table 8.8: Long positions 2010

From the 2010 results the quantiles are retained. The null hypothesis is kept in long and short positions with low failure rates and insignificant results, (see Table 8.7 and 8.8).

The Kupiec tests indicate that the AR (1) GARCH (1,1) model is well specified and the stated probability is achieved with a 95% confidence interval. Kupiec tests with skewed Student's t distribution give better results for all quantiles. Since our main focus is on the 0.975 and 0.025 quantiles, we choose to keep the Student's t distribution and run a Monte Carlo simulation for the AR (1) GARCH (1,1) model.

8.3 Monte Carlo simulation

Simulation experiments enable the determination of the effect of changing one factor of a problem, while leaving all other factors unchanged. In econometrics, simulation is very useful when models are complex or sample sizes are small, Brooks [2008].

The idea behind the Monte Carlo approach is to simulate repeatedly a random process for the financial variable of interest, covering a range of possible solutions. Variables are drawn from pre-specified probability distributions that are assumed to be known, Jorion [2007]. Monte Carlo simulation is used to investigate the properties and behavior of various statistics of interest. It is often applied when the properties of a particular estimation method are not known. Simulations are useful tools in finance, and Brooks [2008] points out situations such as:

- The pricing of exotic options, where an analytical pricing formula is unavailable

- Determining the effect on financial markets of substantial changes in the macroeconomic environment

- Stress testing risk management models to determine whether they generate capital requirements sufficient to cover losses in all situations

The simulation method is by far the most powerful approach for evaluating value at risk. It can potentially account for a wide range of risks, like price risk and volatility risk. Simulations can account for nonlinear exposures and complex pricing patterns. In addition, they can be extended to longer horizons, which is important for credit risk measurement and complex models of expected return, Brooks [2008].

Comparing descriptive statistics				
	Contract 09	Contract 10	MCS Contract 09	MCS Contract 10
Min	-28.108	-27.427	-20.4808	-21.5408
Mean	-0.014121	-0.025405	0.1362	0.1216
Max	19.319	19.117	20.7808	21.7915
Std.dev	2.8392	2.8175	3.0718	3.1100
Kurtosis	11.655	11.051	10.705	12.0825
Skewness	-0.89729	-0.87110	0.00274	0.00941

Table 8.9: MCS ECX Futures Contracts

The Monte Carlo simulation is run in OxMetrics and then sorted in Excel. Our data consists of 1000 observations and 5000 simulations. If we compare the minimum, maximum, mean and standard deviation with the descriptive statistics there are relations between the raw data and the simulated data. Our goal using Monte Carlo simulation is to test if the AR (1) GARCH (1,1) is a good model for volatility. The model is well specified if the results of Monte Carlo simulations are equal, or almost equal, to the raw data (see Table 8.9).

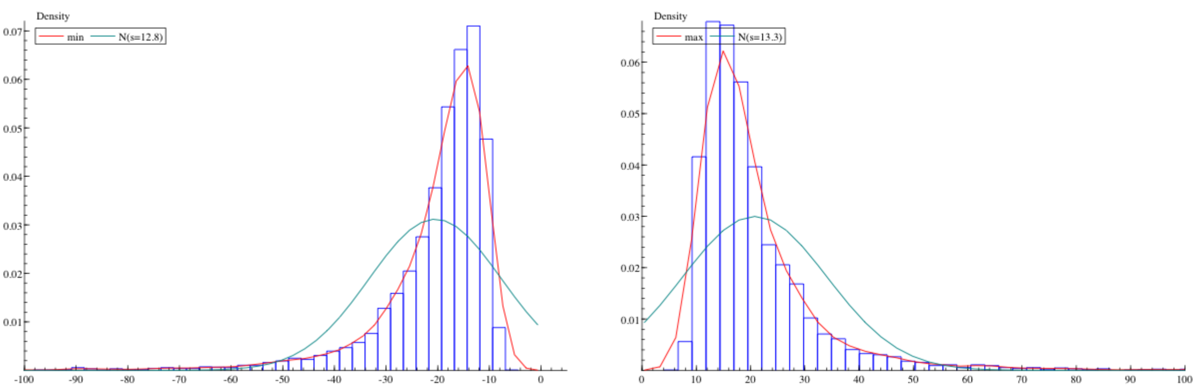
Comparing Value at Risk				
	VaR 97,5%	VaR 2,5%	CVaR 97,5%	CVaR 2,5%
Contract 09	5.15872	-6.01574	7.31423	-8.92977
Contract 10	5.11976	-6.22737	7.23684	-8.89584
MCS Contract 09	6.3173	-5.7299	9.4831	-8.8108
MCS Contract 10	6.3329	-5.7663	9.6382	-8.9827

Table 8.10: Comparing Value at Risk

We have compared the levels of Value at Risk in Table 8.10. The two first rows are raw data from both contracts with full dataset. The third and fourth rows represent the VaR with Monte Carlo simulation. We observe that the Monte Carlo simulation gives us higher VaR in short and long position for both contracts. Since the difference in the results are not very different we can conclude that the AR (1) GARCH (1,1) is a good model for volatility for the chosen CO_2 futures contracts.

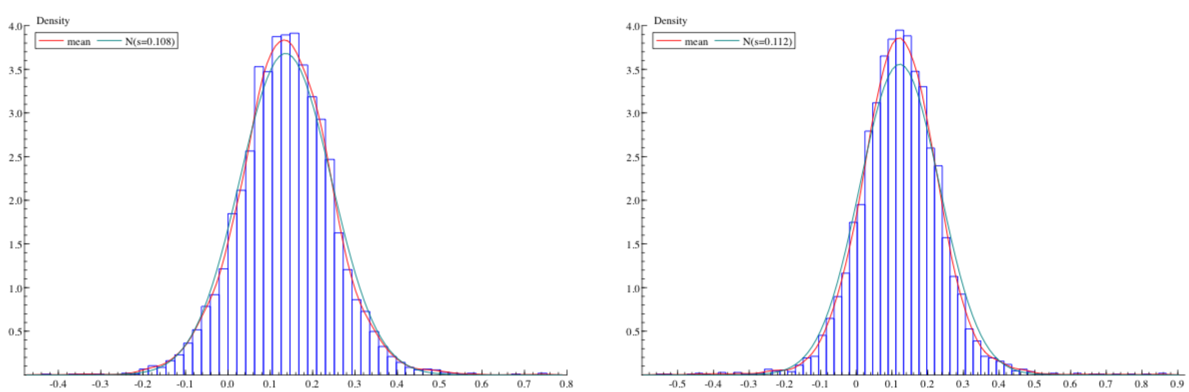
8.3.1 Distributions

The following distributions are drawn from the Monte Carlo simulations we have computed in Excel.



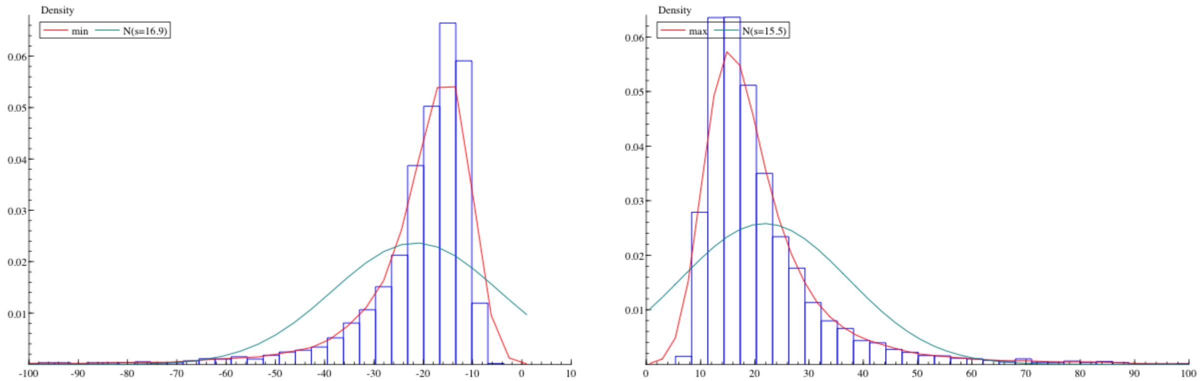
(a) Minimum returns 2009 **(b)** Maximum returns 2009

Figure 8.2: Distributions for minimum and maximum returns 2009



(a) Mean returns 2009 **(b)** Mean returns 2010

Figure 8.3: Distributions for average returns 2009 and 2010



(a) Minimum returns 2010 **(b)** Maximum returns 2010

Figure 8.4: Distributions for minimum and maximum returns 2010

Above we have presented the distributions for minimum, maximum and average returns for the 2009 and 2010 contracts. More distributions of other parameters are found in the appendix at the end of the thesis.

Chapter 9

After the structural break

At the start of the new CO_2 emissions market the price of one EUA was traded at €8 on January 1 2005. The EUA price rose to €25-30 until the release of 2005 verified emissions on April 24, 2006. This had a depressive effect on EUA prices as shown by the sharp break in the EUA futures December 2009 price. Verified emissions were about 80 million tons of 4% lower than the amount of allowances distributed to installations for 2005 emissions, Chevallier and Sevi [2009].

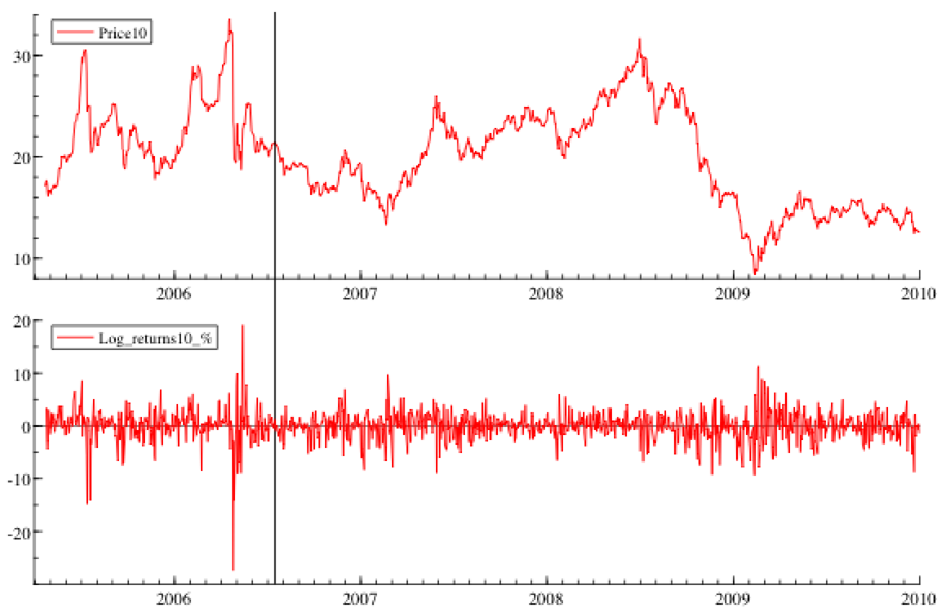


Figure 9.1: The cutoff line for price and returns 2010

As displayed in the figure, we have divided the dataset into two periods due to the presence of one structural break following the simultaneous releases of 2005 verified emissions. We will now take a closer look at the volatility form the period after this structural break.

The reason for this is that before the price adjustment on April, 2006 allowance trading may be characterized as uncertain with heterogeneous expectations related to the EUA price pattern. The release of the 2005 verified emissions gave industrials a hint about their net long/shot positions. Since the end of 2007, the liquidity of the EU ETS has been increasing, Chevallier and Sevi [2009].

Descriptive statistics for both contracts with the new datasets are found in appendix. The 2009 contract has a minimum value of -9.4346%, the average return is -0.039931%, and a maximum of 11.366%. The 2010 contract contains a minimum value of -9.3014%, the average return is -0.057864%, and a maximum of 11.354%. If we compare these results we find them to be, as expected, almost equal. The standard deviation for the 2009 contract is 2.5222 and 2.5092 for the 2010 contract. The futures contracts have significant autocorrelations and ARCH effects in the raw data. As earlier we will try to eliminate the autocorrelation and find the best GARCH-models for both contracts. Since the Jarque-Beras are high 145.13 (2009) and 163.78 (2010) we will use a Student's t distribution.

9.1 Univariate GARCH-modeling

We will now run some univariate GARCH-models with the new dataset. In the light of previous modeling (ref: Chapter 7) we choose to present the four best GARCH-models.

9.1.1 GARCH-modeling of ECX Futures Contract 2009

	GARCH (1,1)			AR (1) GARCH (1,1)		
	Coefficient	t-value	P-value	Coefficient	t-value	P-value
C (M)	0.056450	0.8755	0.3815	0.054079	0.7864	0.4318
C (V)	0.266940	2.300	0.0217	0.267762	2.321	0.0205
AR (1)				0.066983	1.935	0.0533
Alfa (1)	0.140572	3.872	0.0001	0.141551	3.955	0.0001
Beta (1)	0.827442	19.58	0.0000	0.825878	19.71	0.0000
Student (DF)	6.388634	5.056	0.0000	6.584866	4.862	0.0000

Table 9.1: GARCH (1,1) and AR(1) GARCH (1,1)

	AR (1) GARCH (2,1)			GJR (1,1)		
	Coefficient	t-value	P-value	Coefficient	t-value	P-value
C (M)	0.054863	0.7927	0.4281	0.024764	0.3695	0.7118
C (V)	0.207062	3.047	0.0024	0.255000	2.198	0.0282
AR (1)	0.071736	2.063	0.0394			
Alfa (1)	0.089754	4.200	0.0000	0.083888	2.310	0.0211
Beta (1)	1.404807	8.994	0.0000	0.836604	18.03	0.0000
Beta (2)	-0.520732	-3.551	0.0004			
Gamma (1)				0.088476	2.073	0.0384
Student (DF)	6.875672	4.585	0.0000	6.700141	4.838	0.0000

Table 9.2: AR (1) GARCH (2,1) and GJR (1,1)

Test					
Model	Obs	Parameters	LL	AIC	JB
GARCH (1,1)	908	5	-2043.776	4.512723	58.865
AR (1) GARCH (1,1)	908	6	-2041.810	4.510594	59.068
AR (1) GARCH (2,1)	908	7	-2040.500	4.509911	58.430
GJR (1,1)	908	6	-2041.314	4.509502	49.524

Table 9.3: Test for GARCH models

To find the best model we use the same evaluation criteria as in Chapter 7. AR(1) GARCH (1,1) was found to be the best model. The Jarque-Bera indicates that the residuals follows a normal distribution better, compared to the earlier results. From the QQ plot in Figure 9.1 we can observe that the residuals are not totally normal distributed. The significant P-values in Table 9.4 indicates some autocorrelation.

Q-Statistics of the 2009 contract		
Lag	Statistic	P-value
Q(5)	11.9681	0.0175903
Q(10)	14.0239	0.1214782
Q(20)	30.4034	0.0468835

Table 9.4: Q-Statistics of the 2009 contract

9.1.2 GARCH-modeling of ECX Futures Contract 2010

	GARCH (1,1)			AR (1) GARCH (1,1)		
	Coefficient	t-value	P-value	Coefficient	t-value	P-value
C (M)	0.043502	0.6956	0.4869	0.041936	0.6344	0.5260
C (V)	0.272010	2.213	0.0272	0.273139	2.248	0.0248
AR (1)				0.057406	1.677	0.0940
Alfa (1)	0.138253	3.657	0.0003	0.139909	3.748	0.0002
Beta (1)	0.830011	18.46	0.0000	0.827767	18.63	0.0000
Student (DF)	5.641427	5.677	0.0000	5.796820	5.453	0.0000

Table 9.5: GARCH (1,1) and AR(1) GARCH (1,1)

	AR (1) GARCH (2,1)			GJR (1,1)		
	Coefficient	t-value	P-value	Coefficient	t-value	P-value
C (M)	0.043295	0.6523	0.5144	0.018162	0.2795	0.7799
C (V)	0.210878	2.946	0.0033	0.265342	2.125	0.0338
AR (1)	0.060885	1.776	0.0760			
Alfa (1)	0.091738	3.776	0.0002	0.089465	2.380	0.0175
Beta (1)	1.372364	7.027	0.0000	0.835909	17.02	0.0000
Beta (2)	-0.490206	-2.704	0.0070			
Gamma (1)				0.078227	1.765	0.0778
Student (DF)	6.010098	5.153	0.0000	5.848700	5.509	0.0000

Table 9.6: AR (1) GARCH (2,1) and GJR (1,1)

Test					
Model	Obs	Parameters	LL	AIC	JB
GARCH (1,1)	921	5	-2065.353	4.495881	77.873
AR (1) GARCH (1,1)	921	6	-2063.852	4.494794	77.906
AR (1) GARCH (2,1)	921	7	-2062.806	4.494692	83.018
GJR (1,1)	921	6	-2063.551	4.494140	65.179

Table 9.7: Test for GARCH models

GARCH (1,1) is the best model for the 2010 contract. The other models have a slightly lower AIC level. The AR-lag for the AR(1) GARCH (1,1) is not significant, and the gamma parameter for the GJR is not significant either. Compared to the results in Chapter 7 we observe lower Jarque-Bera. Q-statistics in Table 9.8 show significant autocorrelation.

Q-Statistics of 2010 contract		
Lag	Statistic	P-value
Q(5)	17.9396	0.0030231
Q(10)	19.3637	0.0358793
Q(20)	36.0353	0.0152347

Table 9.8: Q-Statistics of 2010 contract

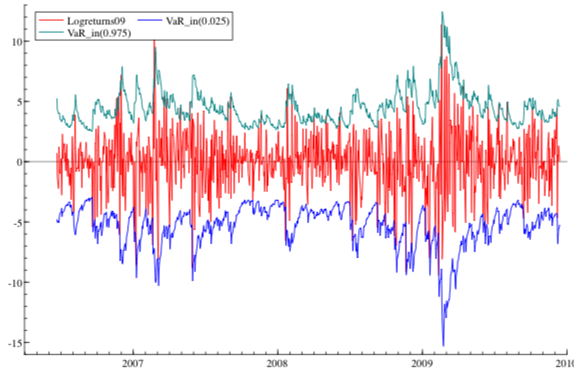
9.2 Value at Risk

In this section Value at Risk is calculated and compared to the results with the results in Chapter 8. A Monte Carlo simulation is also conducted and compared with the results form the previous simulations.

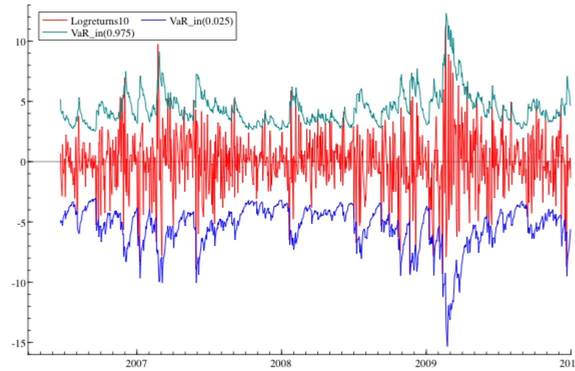
Value at risk ECX Futures Contracts		
	ECX Futures Contract 2009	ECX Futures Contract 2010
Value at Risk 97.5%	4.89258	4.90619
Value at Risk 2.5%	-5.47686	-5.64067
Conditional VaR 97.5%	6.43376	6.46766
Conditional VaR 2.5%	-7.04483	-7.23793

Table 9.9: Value at Risk

From the figures on the next page the Value at Risk limits are calculated with a AR(1) GARCH (1,1) model for the 2009 contract, and a GARCH (1,1) for the 2010 contract.



(a) AR (1) GARCH (1,1) VaR 2009



(b) GARCH (1,1) VaR 2010

Figure 9.2: VaR models

9.2.1 Kupiec test

The Kupiec LR tests are presented in Table 9.10 and 9.11.

Kupiec LR test				
Quantile	Failure rate	Kupiec LR	P-value	ESF1
0.97500	0.97797	0.34292	0.55815	5.6705
0.02500	0.035242	3.4738	0.062349	-5.9519

Table 9.10: Short and long positions 2009

Kupiec LR test				
Quantile	Failure rate	Kupiec LR	P-value	ESF1
0.97500	0.97937	0.76663	0.38126	5.6702
0.02500	0.039088	6.4179	0.011297	-5.9852

Table 9.11: Short and long positions 2010

The results from the Kupiec tests states that the null hypothesis are retained for both short and long positions for the 2009 contract. For the 2010 contract the long position is significant with a P-value of 0.011297, and the null hypothesis is rejected. Which means that 3.9% is the empirical limit for the long position.

9.3 Monte Carlo simulation

Monte Carlo Simulations				
	Contract 09	Contract 10	MCS Contract 09	MCS Contract 10
Min	-9.4346	- 9.3014	-21.5408	-16.9826
Mean	-0.039931	-0.057864	0.0575	0.0456
Max	11.366	11.354	21.7915	17.0083
Std.dev	2.5222	2.5092	2.8108	2.8228
Kurtosis	1.9497	2.0493	5.9070	6.81807
Skewness	-0.092939	-0.13082	0.00401	-0.00734

Table 9.12: MCS ECX Futures Contracts

All our simulations have 1000 observations and 5000 simulations. If we compare the minimum, maximum, mean and standard deviation with the descriptive statistics, we observe the relations between the raw data and the simulated data.

Comparing Value at Risk				
	VaR 97,5%	VaR 2,5%	CVaR 97,5%	CVaR 2,5%
Contract 09	4.89258	-5.47686	6.43376	-7.04483
Contract 10	4.90619	-5.64067	6.46766	-7.23793
MCS Contract 09	5.8518	-5.4694	8.2840	-7.8666
MCS Contract 10	5.8295	-5.4676	8.3834	-8.0018

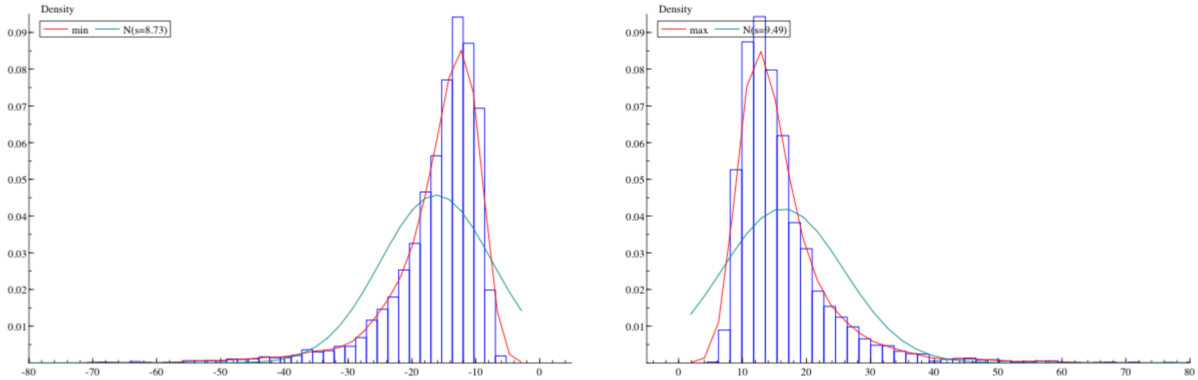
Table 9.13: Comparing Value at Risk

In Table 9.13 we have compared the levels of Value at Risk. The new data shows a higher VaR in short positions and about the same VaR in long positions. Since the deviations between the results are not very different we conclude that AR(1) GARCH (1,1), and GARCH (1,1) are good models for volatility.

Even though we did not manage to eliminate all autocorrelations, with the new datasets, the Monte Carlo simulations indicates that AR(1) GARCH (1,1) is a good model for VaR and volatility.

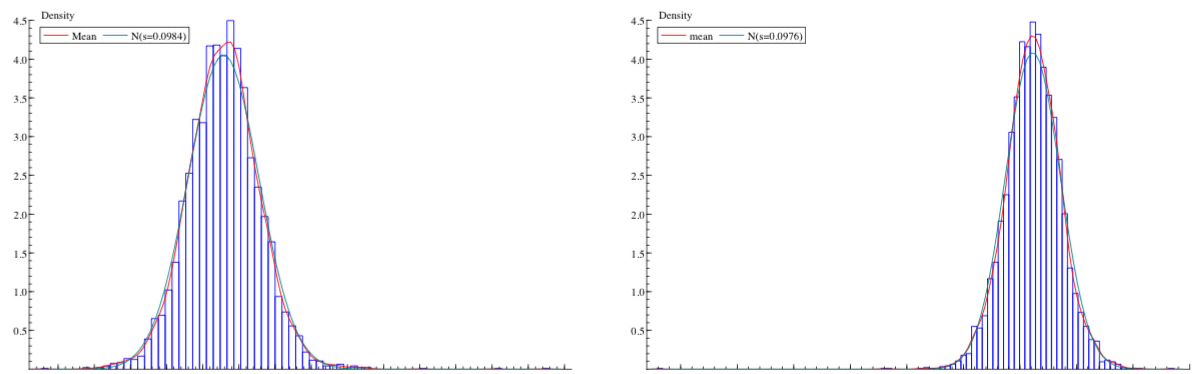
9.3.1 Distributions

The following distributions are drawn form the Monte Carlo simulations we have computed in Excel.



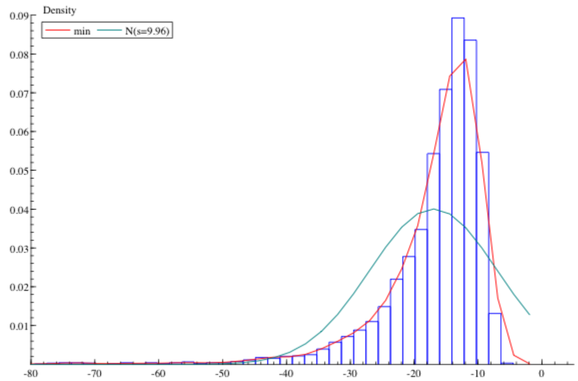
(a) Minimum returns 2009 **(b)** Maximim returns 2009

Figure 9.3: Distributions for minimum and maximum returns 2009

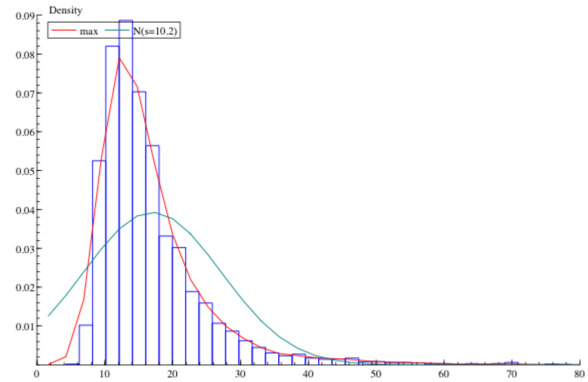


(a) Mean returns 2009 **(b)** Mean returns 2010

Figure 9.4: Distributions for average returns 2009 and 2010



(a) Minimum returns 2010



(b) Maximim returns 2010

Figure 9.5: Distributions for minimum and maximum returns 2010

Above we have presented the distributions of minimum, maximum and average returns for the new dataset. More distributions of other parameters are possible to find in the appendix at the end of the thesis.

Chapter 10

Multivariate modeling

Multivariate modeling framework leads to more relevant empirical models than working with separate univariate models. From a financial point of view, it opens the door to better decision tools in various areas, such as asset pricing, portfolio selection, option pricing, hedging, and risk management, Laurent [2009].

We are going to model the dependence between the prices of carbon and oil, carbon and coal, carbon and electricity and carbon and gas. With several time-series it is important to model the dependence between these in a good way. The dependence can take different shapes. Changes in the time-series can fluctuate symmetrically or correlate. The trends in the time-series can move in the same way, also called co integration. Earlier values of one time-series can explain the future value of another, also called causality. The most obvious application of multivariate GARCH models is the study of the relations between the volatilities and co-volatilities of several markets.

10.1 Data

The data under consideration are daily log-returns series of futures on electricity, coal, oil, natural gas and carbon emission allowances. Since we will establish the joint behavior of commodities , the analysis consists of monthly and yearly futures. In this section we briefly discuss the specifics of each time series. The data analysis starts at the beginning of 2006 and all the different returns are converted into Euro to easier compare it to the CO_2 returns.

10.1.1 Electricity

Electricity has the unique feature that it is not storable. A second main feature is the necessity for a transmission network, which prevents a global market. These characteristics of electricity have strong implications on the trading products and their prices. The non-storability of electricity causes high price movements on the spot market. In the forward and futures market with delivery dates in the far future the price movements are much smaller, because the availability of power and weather dependent demand are still unknown. The time series of electricity prices are taken from the ICE-traded futures. The futures prices are quoted in EUR/MWh. For the analysis of volatility and correlation, we will use price series of monthly futures, Borger et al. [2007].

10.1.2 Coal

For some time now coal futures have been traded at several exchanges, among them the ICE in London and the EEX. They offer trades in coal with different points of delivery. In the following we have picked Rotterdam as an example. The futures prices are quoted in US$/t, but we have converted it to EUR/t, Borger et al. [2007].

The coal contracts were introduced at the ICE in March 2006. This is the shortest series of our data. We have chosen to start all of the series in 2006 to compare them to the same period of time.

10.1.3 Natural Gas

Natural gas is one of the most important primary energy sources covering about 25% of worldwide consumption. It is primarily used as a fuel for electricity generation, transportation and heating. The worldwide natural gas consumption has a higher growth rate then oil consumption. With growing demand for primary energy sources, gas prices have risen and large investments have been made, Borger et al. [2007].

The only exchange in Europe offering sufficient historical price data for futures on natural gas is the ICE. Available products are monthly, quarterly and seasonal gas futures. Seasonal contracts start delivery in October and April. Futures prices are quoted in GBP, we convert all prices to EUR, Borger et al. [2007].

10.1.4 Oil

The crude oil market is the largest commodity market in the world. The most important trading exchanges are New York, London and Singapore. Available products are monthly futures contracts for at least the next 12 months quoted in US$/barrel, we have converted it to EUR/barrel. Crude oil comes in a variety of grades, determined by it's gravity and sulfur content. The world benchmark crude oil is the Brent crude form the North Sea. We will use the Brent price in our analysis, Eydeland and Wolyniec [2003].

Crude oil needs to be refined in order to yield products that can be directly consumed. The most popular refined product is gasoline, as well as heating oil and fuel oil used by utilities.

10.1.5 CO_2 emissions

Earlier in the thesis we have described the CO_2 market. In this multivariate analysis we will use the 2010 futures contract and compare it to the other commodities.

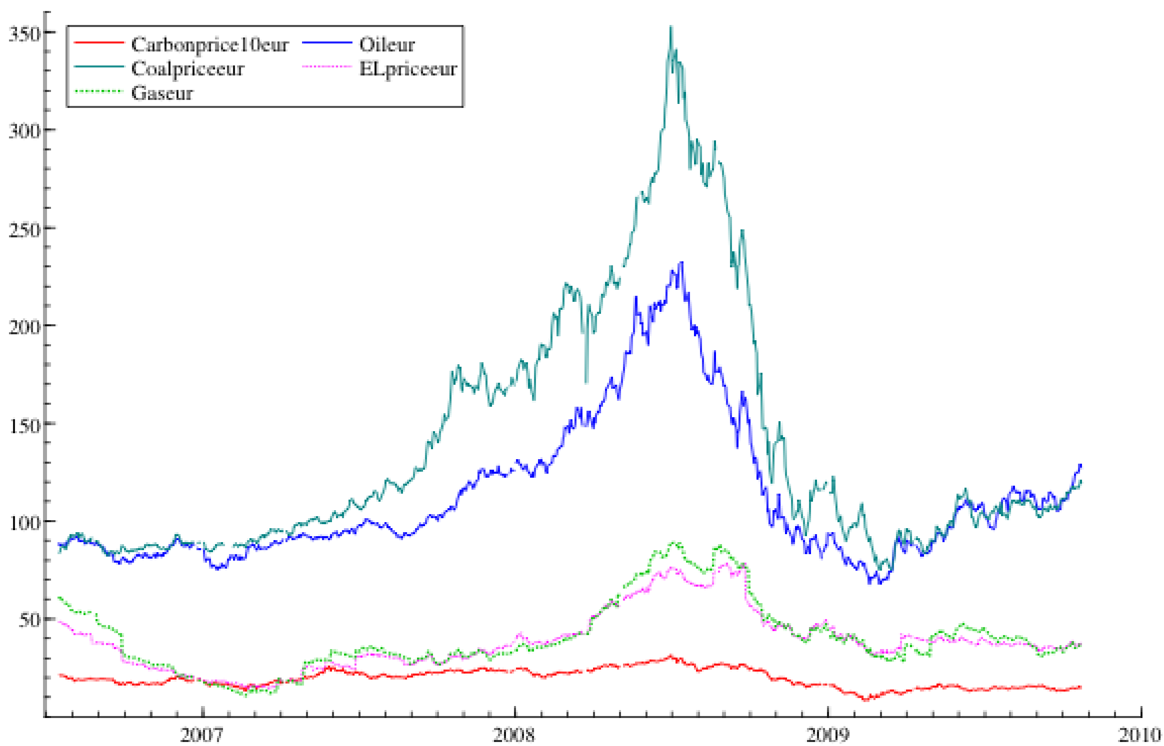


Figure 10.1: Prices for the different commodities

The figure shows the price development for the different commodities from 2006 until the end of 2009.

10.2 Results

10.2.1 QQ plots

There is a variety of procedures for testing for normality in our data. The most prominent visual test is a QQ plot comparing quantiles of the empirical distribution with quantiles from a normal distribution. In case of normality this plot should give a straight line. The method can be applied to multivariate time series as well, since each linear combination of the time series should be normal, in particular each time series itself, Borger et al. [2007].

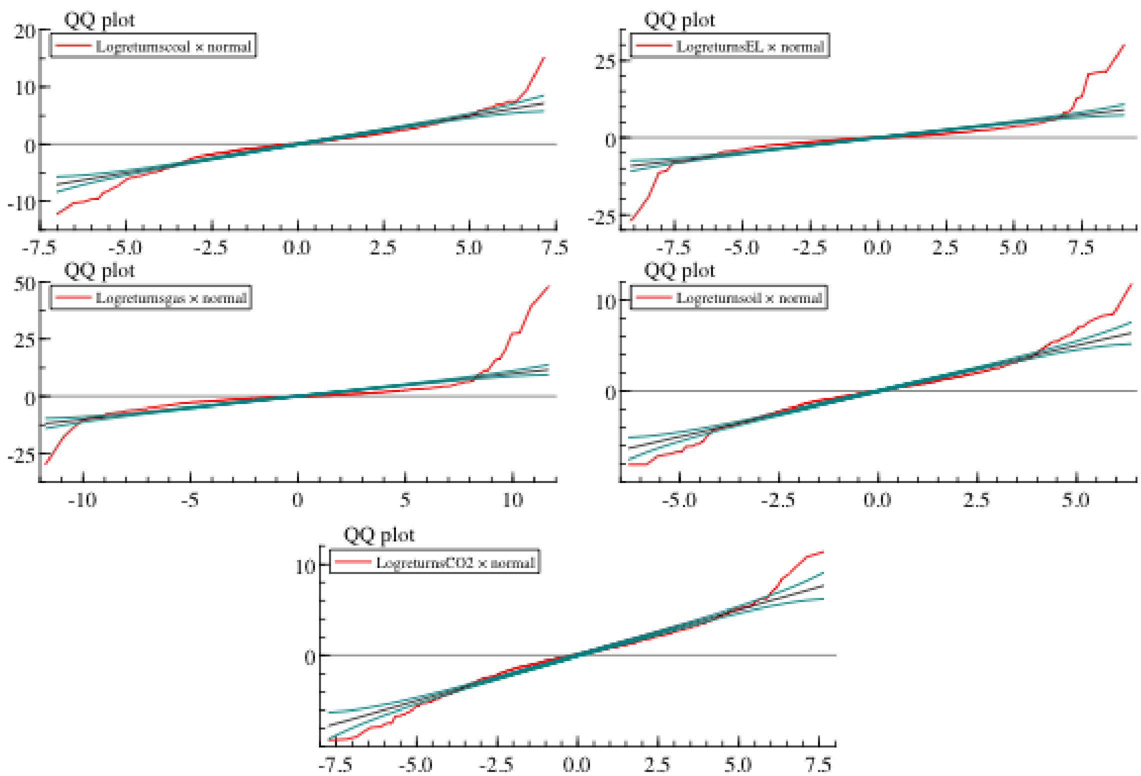


Figure 10.2: QQ plot for the different commodities

Figure 10.2 shows the QQ plot of log-returns of gas, oil, coal, CO_2 and electricity futures contracts. The oil and CO_2 contracts are the contracts with distribution closest to normal distribution. This is also confirmed by the Jarque-Beras in the descriptive statistics at the next page.

10.2.2 Descriptive Statistics

Descriptive Statistics					
	Carbon	Oil	Gas	EL	Coal
Min	-9.3014	-8.0471	-29.803	-26.961	-12.183
Mean	-0.04241	0.040972	-0.043918	-0.030446	0.063194
Max	11.354	11.79	48.183	30.29	15.295
Std.dev	2.5278	2.0829	3.8469	2.9875	2.3306
Skewness	-0.096949	0.19418	4.0541	1.2945	-0.35092
Excess kurtosis	2.0396	3.2892	52.276	30.575	5.2435
Jarque-Bera	149.37	390.34	99582	33503	995.85
Q (5)	24.9407	14.2878	1.81083	0.900264	28.7970
P-value	0.0001431	0.0138812	0.8746521	0.9702025	0.0000254
$Q^2(5)$	185.883	155.994	0.120807	0.559426	140.939
P-value	0.0000000	0.0000000	0.9997415	0.9897855	0.0000000
Q (10)	33.5666	18.1235	4.40344	6.74460	33.2247
P-value	0.0002187	0.0529163	0.9273176	0.7493163	0.0002497
$Q^2(10)$	312.717	328.876	0.322822	1.00401	226.158
P-value	0.0000000	0.0000000	0.9999992	0.9998247	0.0000000
ADF	-12.5872	-12.5006	-12.5708	-13.2576	-12.2884

Table 10.1: Descriptive Statistics

Carbon and coal have negative skewness, which means higher possibility for large negative values. Gas and Electricity have much higher Jarque-Beras then the rest, and does not follow a normal distribution. We can observe significant autocorrelations in carbon, oil and coal, which have the highest Q-value in both 5 and 10 lags. All of the five future contracts are stationary according to the ADF-test.

10.2.3 Volatility plots

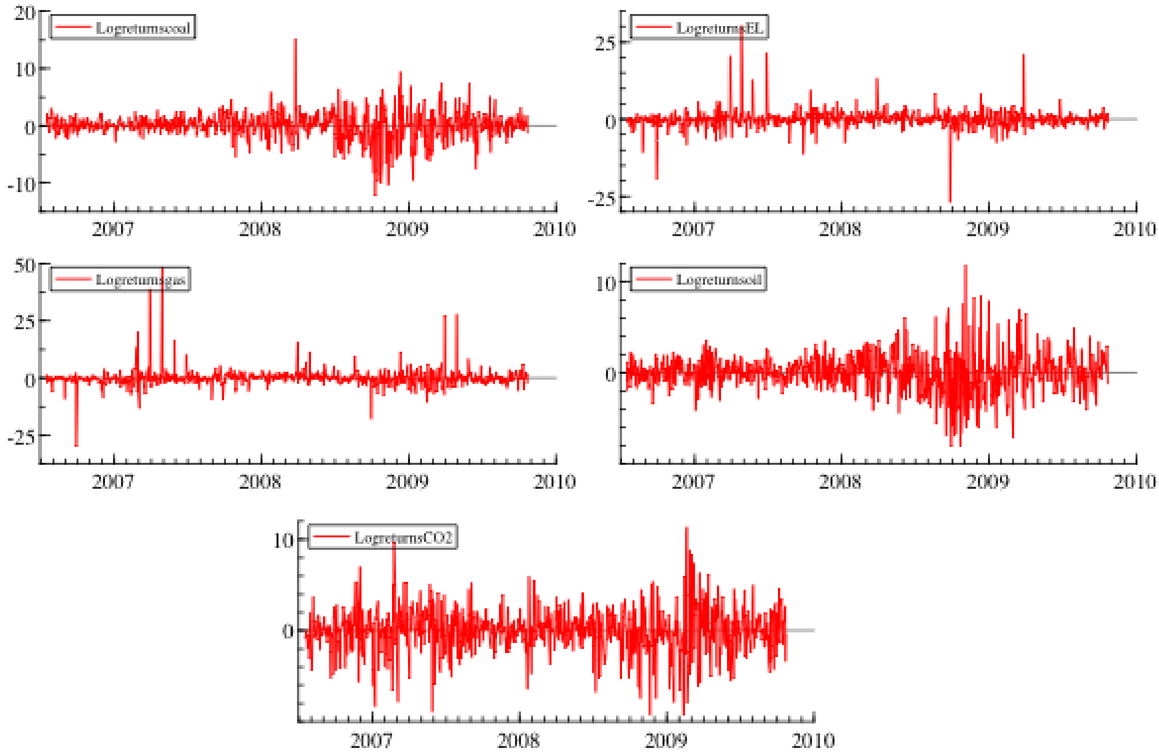


Figure 10.3: Volatility plots for the different commodities

The volatility plots indicate large volatility clusters in coal, oil and carbon. Volatility clusters occurs often when the dataset contains autocorrelation.

10.2.4 ACF and PACF plots

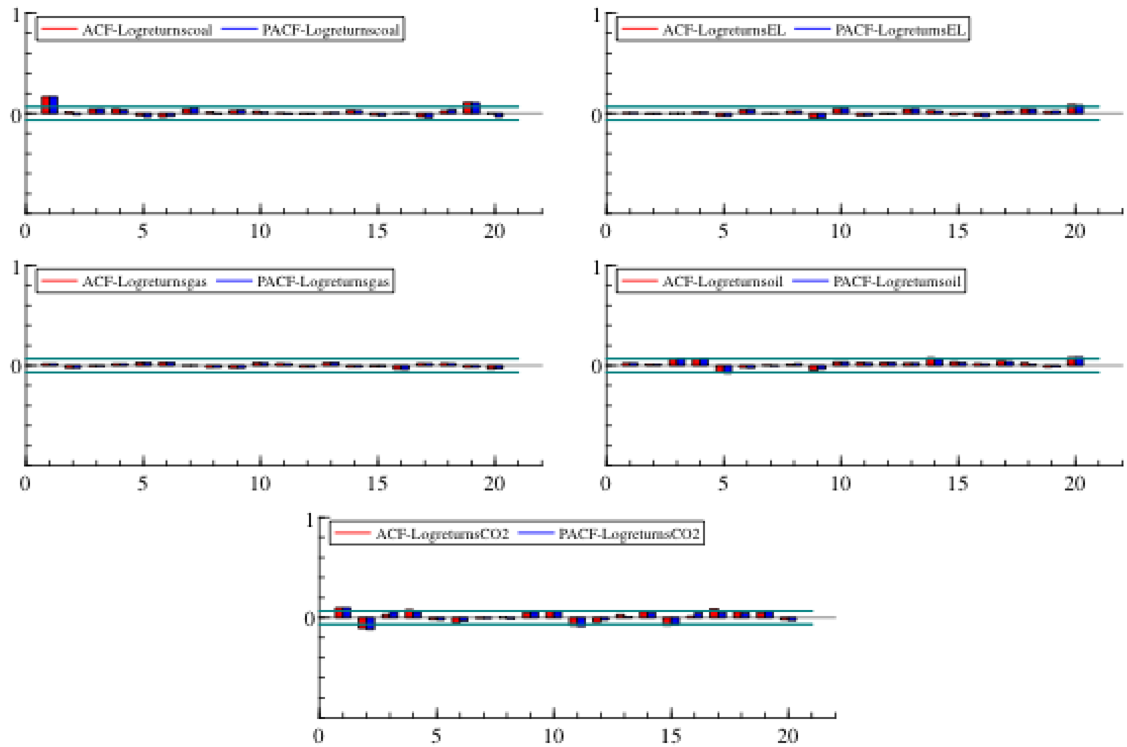


Figure 10.4: ACF and PACFplots for the different commodities

The ACF and PACF plots of log-returns indicate autocorrelation in oil, carbon and coal. For coal and carbon an AR(1) lag may be reasonable to use in the GARCH model. From the univariate analysis we used one AR-lag for CO_2 and it will be reasonable to do this in the multivariate analysis as well.

The next sections consist of two multivariate GARCH models, RiskMetrics and diagonal BEKK. After a discussion with our tutor we found those models most relevant for this thesis.

10.3 Test of models

Test for MGARCH models							
Model	Derivatives	Obs	Parameters	LL	AIC	JB(CO_2)	JB(Derivatives)
RiskMetrics (1,1)	CO_2/Oil	854	3	-3596.468	8.429668	134.62	8.9287
AR (1) RiskMetrics (1,1)	CO_2/Oil	854	5	-3593.087	8.426434	138.64	9.6810
Diagonal BEKK (1,1)	CO_2/Oil	854	10	-3623.014	8.508230	93.292	7.6879
AR (1) Diagonal BEKK (1,1)	CO_2/Oil	854	12	-3620.168	8.506248	91.328	8.1458
RiskMetrics (1,1)	CO_2/Gas	854	3	-3942.433	9.239890	169.93	82068
AR (1) RiskMetrics (1,1)	CO_2/Gas	854	5	-3937.835	9.233806	165.46	85455
Diagonal BEKK (1,1)	CO_2/Gas	854	10	-3842.544	9.022350	65.558	1.2322e+05
AR (1) Diagonal BEKK (1,1)	CO_2/Gas	854	12	-3836.637	9.013201	64.491	1.5113e+05
RiskMetrics (1,1)	CO_2/El	854	3	-3822.525	8.959075	189.02	33879
AR (1) RiskMetrics (1,1)	CO_2/El	854	5	-3821.814	8.962095	191.13	33510
Diagonal BEKK (1,1)	CO_2/El	854	10	-3745.148	8.794259	60.881	40263
AR (1) Diagonal BEKK (1,1)	CO_2/El	854	12	-3744.687	8.797861	60.441	40317
RiskMetrics (1,1)	CO_2/Coal	854	3	-3614.155	8.471089	123.93	303.27
AR (1) RiskMetrics (1,1)	CO_2/Coal	854	5	-3597.741	8.437332	118.71	422.20
Diagonal BEKK (1,1)	CO_2/Coal	854	10	-3597.814	8.449214	79.607	372.23
AR (1) Diagonal BEKK (1,1)	CO_2/Coal	854	12	-3637.292	8.546351	96.139	739.71

Table 10.2: Test for MGARCH models

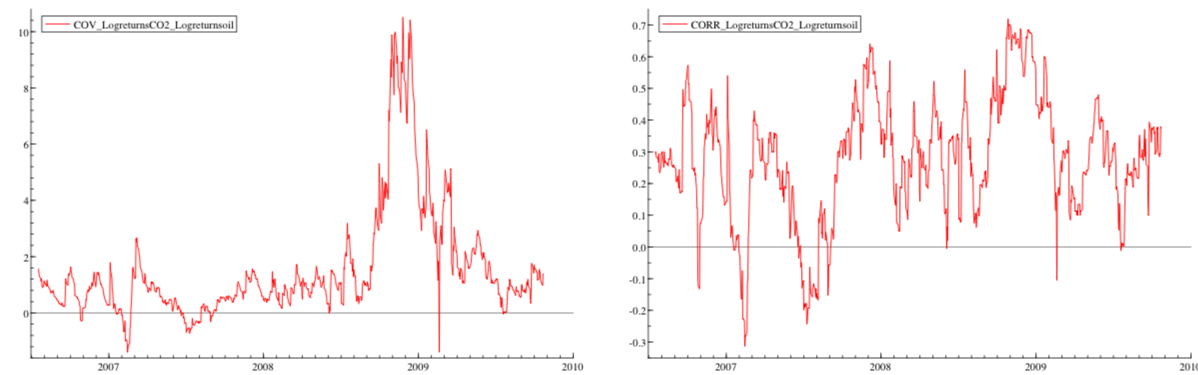
Four different multivariate GARCH models have been tested to find the best bivariate model. RiskMetrics (1,1) and diagonal BEKK (1,1) came out as the best models. For the two AR (1) models, the AR lags were not significant (see appendix). Now a closer analysis of RiskMetrics (1,1) and diagonal BEKK (1,1) will be presented.

10.3.1 RiskMetrics

RiskMetrics								
	Carbon	Oil	Carbon	Gas	Carbon	EL	Carbon	Coal
Cst1	0.012623		0.011773		0.001283		0.023138	
P-value	0.8681		0.8825		0.9868		0.7473	
Cst2		0.109935		-0.069903		-0.000693		0.136427
P-value		0.0656		0.3203		0.9915		0.0159
Skewness	-0.47845	-0.038072	-0.41849	-1.0197	-0.32715	-0.55119	-0.45978	0.29241
Excess kurtosis	1.6934	0.49510	2.0187	47.981	2.2100	30.837	1.6240	2.8602
Jarque-Bera	134.62	8.9287	169.93	82068	189.02	33879	123.93	303.27
Q (5)	14.1073	2.77992	10.5016	5.95495	10.2203	0.658196	17.7988	31.7833
P-value	0.0149418	0.7338711	0.0622082	0.3106268	0.0692281	0.9851839	0.0032095	0.0000066
Q^2 (5)	26.6495	6.23237	30.3996	0.521803	26.6889	0.936347	27.2766	6.97354
P-value	0.0000667	0.2842603	0.0000123	0.9913032	0.0000656	0.9675347	0.0000504	0.2226157
Q (10)	16.3999	3.43471	15.3694	7.24575	14.2174	2.09075	21.2621	34.7445
P-value	0.0887431	0.9692690	0.1191628	0.7020591	0.1633058	0.9955961	0.0193384	0.0001380
Q^2 (10)	31.8064	7.73086	39.4264	0.960862	31.1673	1.90645	31.9481	8.31438
P-value	0.0004313	0.6551077	0.0000214	0.9998567	0.0005505	0.9970081	0.0004085	0.5981585

Table 10.3: RiskMetrics

Table 10.3 contains the results form RiskMetrics. We have done a bivariate analysis with carbon as the main y-variable in every estimation. First is carbon versus oil. The results shows negative skewness in both contracts. Kurtosis is lower than 3, which indicates a more normal distribution than in the descriptive statistics. The Jarque-Bera is still high for carbon, but oil has a lower JB. This indicates a more normal distribution for the oil contract. From the Q-statistics carbon has significant autocorrelations in lag 5 and lag 20. There are no problems with autocorrelations for the oil contract.

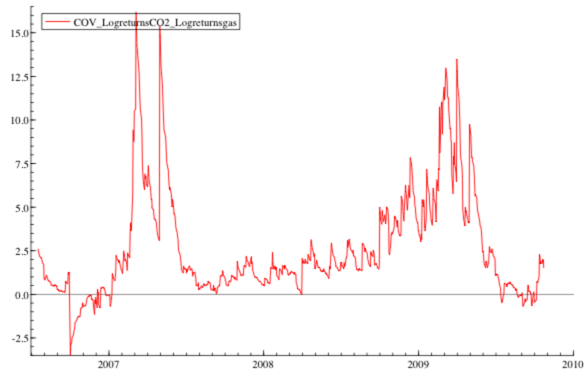


(a) Covariance between carbon and oil **(b)** Correlation between carbon and oil

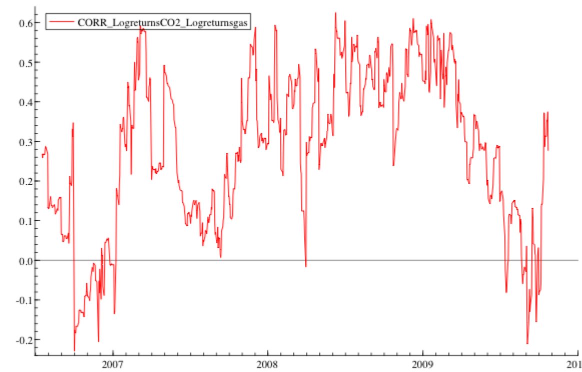
Figure 10.5: Covariance and correlation between carbon and oil

Covariance measures if two variables move linearly together. If the two variables are independent form each other, their covariance is equal to zero. Positive covariance means that the two variables tend to move in the same direction, while a negative covariance indicates that they move in opposite directions, Jorion [2007].

Carbon and gas are also negatively skewed. The kurtosis is lower for the carbon contract compared to descriptive statistics (Table 10.1), while the gas contract still contains high kurtosis and Jarque-Bera.



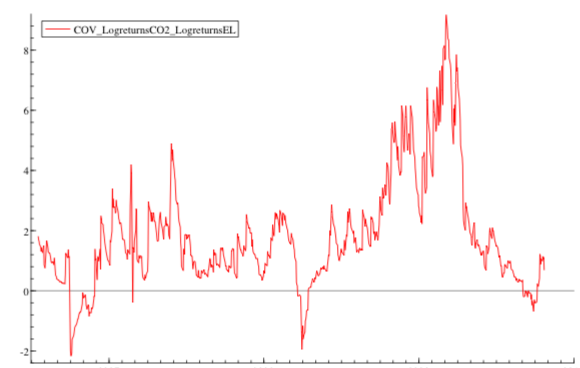
(a) Covariance between carbon and gas



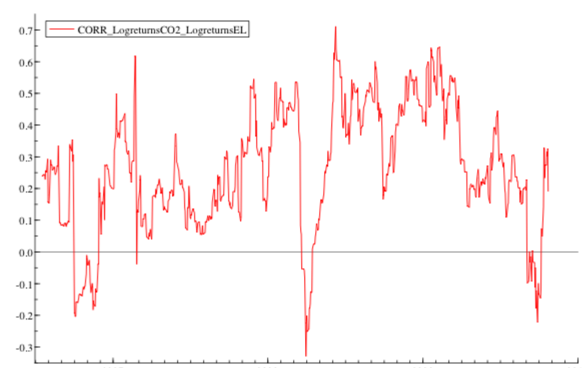
(b) Correlation between carbon and gas

Figure 10.6: Covariance and correlation between carbon and gas

Carbon and electricity are negative skewed and again carbon has a kurtosis lower then 3. Electricity contains high kurtosis and Jarque-Bera after the modeling. Lag 5 and lag 20 shows significant autocorrelation in the carbon data. There is significant autocorrelation in electricity.



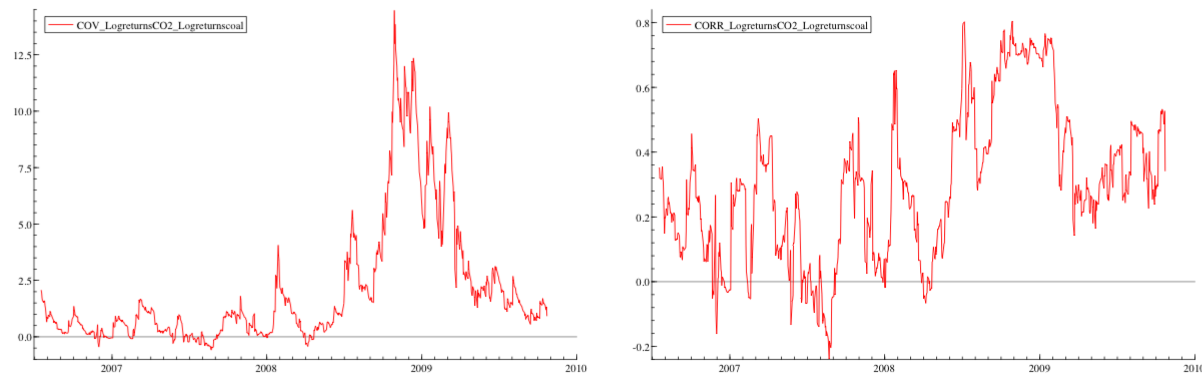
(a) Covariance between carbon and El



(b) Correlation between carbon and El

Figure 10.7: Covariance and correlation between carbon and el

Finally we take a look at carbon and coal. Carbon is negatively skewed while the coal contract is positively skewed. Both contracts have low kurtosis, but the Jarque-Bera is too high to conclude normality in the data. There are big differences in the q-statistics. While the other contracts had small significant autocorrelations, the carbon and coal contracts have large significant autocorrelations all lags. This will obvious affect the result in further analysis.



(a) Covariance between carbon and coal

(b) Correlation between carbon and coal

Figure 10.8: Covariance and correlation between carbon and coal

The Figures 10.5-10.8 indicates that the conditional covariance between carbon and the other commodities tends to bee positive, which indicates that they are not independent form each other. The correlation between them is changing rapidly over time, but tends to be positive.

10.3.2 Diagonal BEKK

$$h_{11,t} = a_{11}^2 + b_{11}^2 \epsilon_{1,t-1}^2 + c_{11}^2 h_{11,t-1}$$

$$h_{22,t} = a_{11}^2 + a_{22}^2 + b_{22}^2 \epsilon_{2,t-1}^2 + c_{22}^2 h_{22,t-1}$$

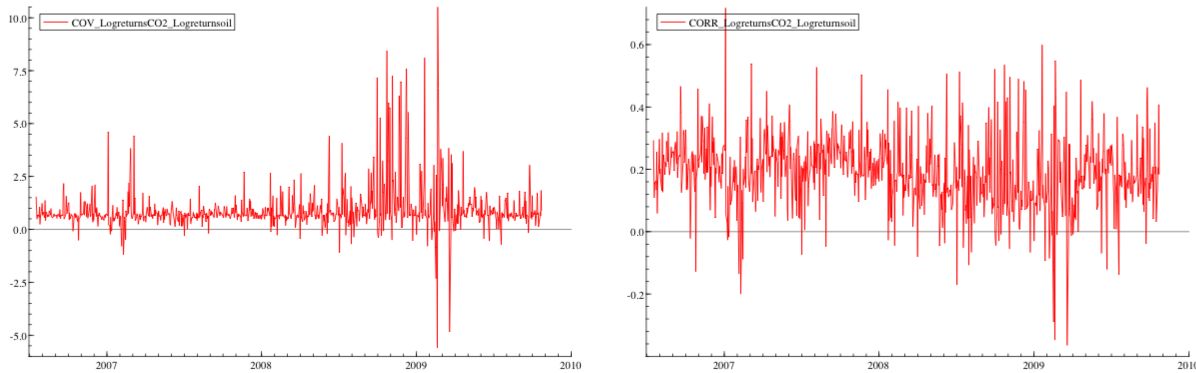
$$h_{12,t} = h_{21,t} = a_{11} a_{22} + b_{11} b_{22} \epsilon_{1,t-1} \epsilon_{2,t-1} + c_{11} c_{22} h_{12,t-1}$$

$$h_{21,t} = h_{12,t}$$

Diagonal BEKK								
	Carbon	Oil	Carbon	Gas	Carbon	EL	Carbon	Coal
Cst1	0.022914		0.016364		0.021946		0.038133	
P-value	0.7492		0.8000		0.7351		0.5404	
Cst2		0.100484		-0.142014		-0.021502		0.152759
P-value		0.0552		0.0197		0.7088		0.0029
$C_1 1$	2.158080		0.905149		0.610230		0.444454	
P-value	0.0000		0.0000		0.0213		0.0000	
$C_1 2$	0.304054	0.304054	2.812927	2.812927	2.056167	2.056167	0.049809	0.049809
P-value	0.0000	0.0000	0.0000	0.0000	0.0470	0.0470	0.1157	0.1157
$C_2 2$		0.000000		0.000008		0.818416		0.171286
P-value		0.0000		0.9977		0.7111		0.0011
$b_1.11$	0.000000		0.896506		0.929020		0.925605	
P-value	1.0000		0.0000		0.0000		0.0000	
$b_1.22$		-0.943721		-0.078487		0.397326		0.961134
P-value		0.0000		0.6354		0.1867		0.0000
$a_1.11$	0.509712		0.443019		0.370016		0.350194	
P-value	0.0000		0.0000		0.0000		0.0000	
$a_1.22$		0.298979		0.257495		0.124496		0.269391
P-value		0.0000		0.6793		0.5022		0.0000
Skewness	-0.033343	0.078302	-0.34250	4.2363	-0.34890	1.3711	-0.42314	0.30189
Excess kurtosis	1.6178	0.43764	1.1718	58.232	1.1063	33.526	1.2333	3.1775
Jarque-Bera	93.292	7.6879	65.558	1.2322e+05	60.881	40263	79.607	372.23
Q (5)	18.4786	4.48175	12.3298	1.22349	10.2145	1.04833	16.5404	30.1559
P-value	0.0024027	0.4823283	0.0305375	0.9426036	0.0693808	0.9585741	0.0054591	0.0000137
Q^2 (5)	43.4172	3.63747	10.6375	0.212003	12.1008	0.572925	10.2796	7.78250
P-value	0.0000000	0.6026966	0.0590600	0.9989792	0.0334325	0.9892093	0.0676889	0.1686369
Q (10)	25.6268	6.34892	15.7154	4.03107	14.2962	5.74311	20.4564	32.7677
P-value	0.0042759	0.7851467	0.1080753	0.9459343	0.1599047	0.8363648	0.0252199	0.0002980
Q^2 (10)	87.0259	11.1564	14.4478	0.415939	16.1720	1.03630	14.4947	9.31240
P-value	0.0000000	0.3454639	0.1535301	0.9999973	0.0948126	0.9997973	0.1515982	0.5027349

Table 10.4: Diagonal BEKK

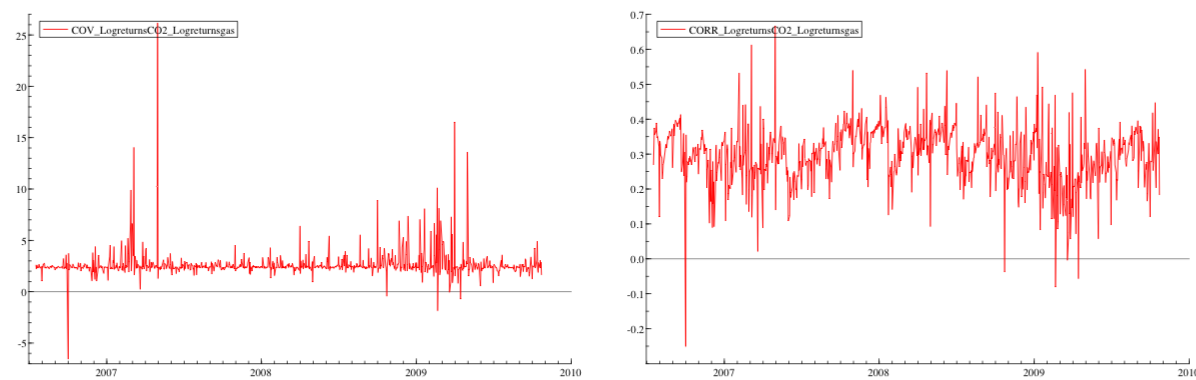
From Table 10.4 we can observe negative $b_1.22$ values for oil and gas. This may cause more unreliable results in the following analysis.



(a) Covariance between carbon and oil

(b) Correlation between carbon and oil

Figure 10.9: Covariance and correlation between carbon and oil

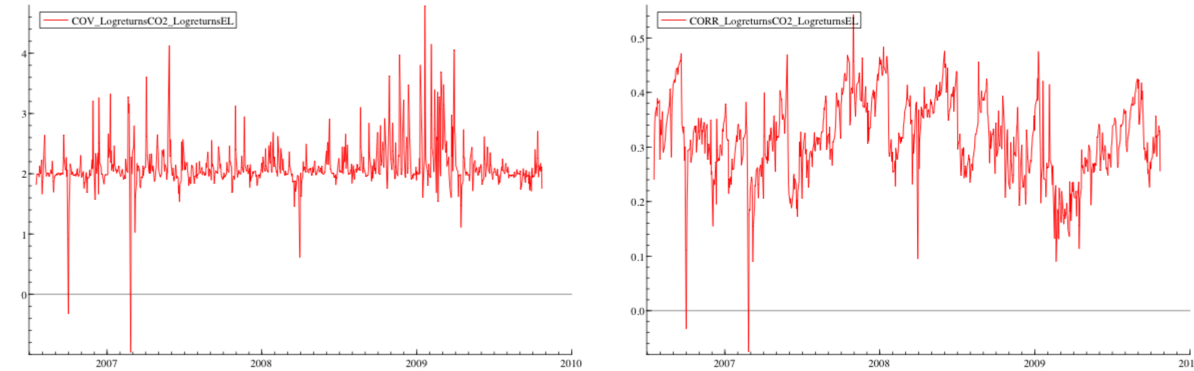


(a) Covariance between carbon and gas

(b) Correlation between carbon and gas

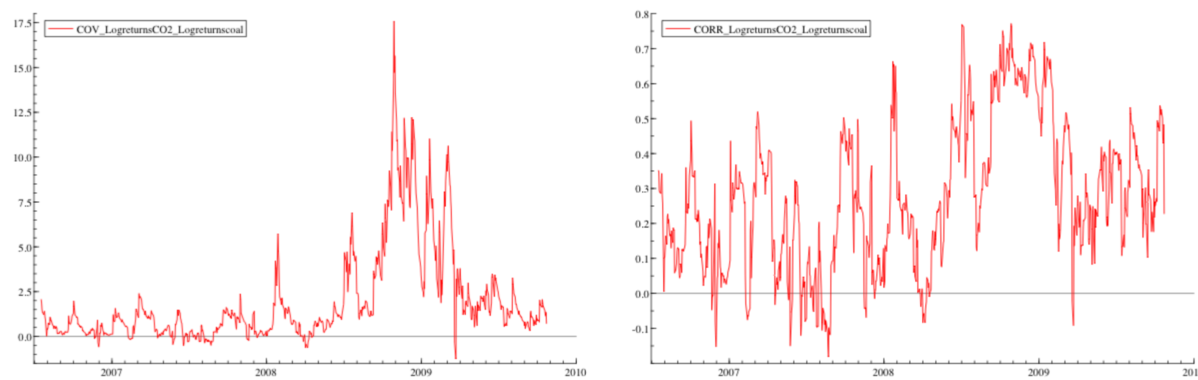
Figure 10.10: Covariance and correlation between carbon and gas

We can observe high fluctuations in both covariance and correlation for carbon/oil and carbon/gas. The reason for this is the negative $b_1.22$ values in the diagonal BEKK model.



(a) Covariance between carbon and El **(b)** Correlation between carbon and El

Figure 10.11: Covariance and correlation between carbon and el



(a) Covariance between carbon and coal **(b)** Correlation between carbon and coal

Figure 10.12: Covariance and correlation between carbon and coal

The Figures 10.9-10.12 from the diagonal BEKK indicates that the conditional covariance between carbon and the other commodities tends to be positive. Covariance form the diagonal BEKK model looks more unstable compared to the results form the RiskMetrics model. The correlation between the them is changing rapidly over time, but tends to be positive.

We compare the modeling for RiskMetrics and diagonal BEKK to find the best estimation of volatility. The RiskMetrics model gave the best results for CO_2/oil, and CO_2/electricity, while the diagonal BEKK best modeled the CO_2/gas, and CO_2/coal contracts. Based on these results we want to test the Value at Risk for CO_2/oil, and CO_2/gas with a RiskMetrics, and CO_2/electricity, CO_2/coal with diagonal BEKK.

10.4 Portfolio Value at Risk

"A portfolio can be characterized by positions on a certain number of constituent assets. If the positions are fixed over the selected horizon, the portfolio rate of return is a linear combination of the returns on underlying assets, where the weights are given by the relative amounts invested at the beginning of the period," Jorion [2007].

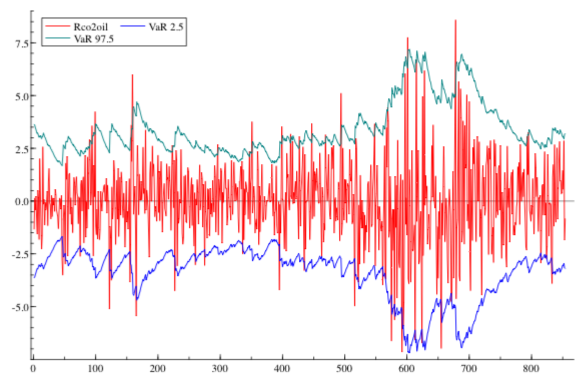
Previously we have looked at univariate Value at Risk for CO_2 contracts. In this section we will focus on CO_2 contracts in a portfolio with other energy commodities. The reason for this is that many market participants use CO_2 contracts as a hedge in portfolios. Therefore, it will be interesting to look at Value at Risk for CO_2 contracts compared to other energy contracts.

The portfolio's calculations are presented on the CD. To limit the thesis, portfolios with two commodities are made, with 50% allocation between the commodities. Log-returns, squared log-returns, conditional variances, and conditional correlation are all included to execute the variance-covariance matrix multiplications.

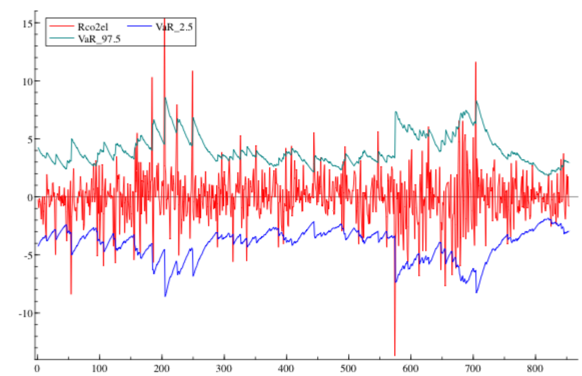
Predicted variance and realized variance are calculated, and multiplied with 0,5 to get 50% in CO_2 and commodity (x). This section contains of four different portfolios; CO_2 and oil, CO_2 and gas, CO_2 and electricity, and CO_2 and coal.

For many portfolios, the assumption of normally distributed returns does not apply. Fat tailed distributions are rule rather than exception for financial market factors and the inclusion of non-linear derivative instruments in the portfolio gives rise to distributional asymmetries. Whenever these deviations from normality are expected to cause serious biases in VaR calculations, one has to resort either to alternative distribution specifications (like the t-distribution) or to historical and Monte Carlo simulation methods, Hallerbach [1999]. A value at risk measure with student's t distribution is complex because of the uncertainty of calculation with the degrees of freedom. We have tried to translate the variances to a VaR measure with normal distribution. We are aware of the problem with these transformations.

The results are presented, as earlier, with figures consisting of portfolio's returns and VaR limits (97.5%, 2.5%). Kupiec tests are used to test the VaR limits.

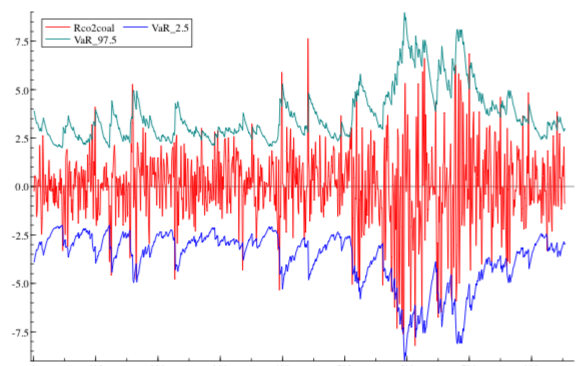


(a) VaR carbon and oil

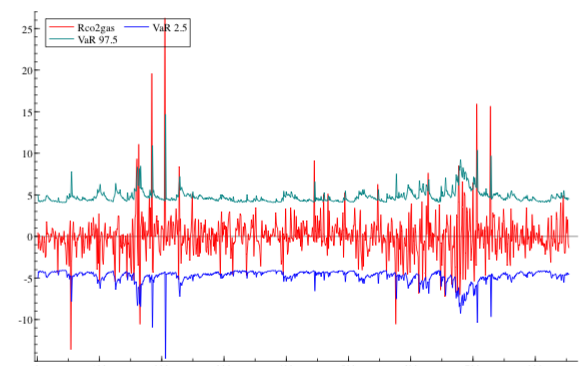


(b) Var carbon and el

Figure 10.13: VaR RiskMetrics



(a) VaR carbon and coal



(b) Var carbon and gas

Figure 10.14: VaR Diagonal BEKK

Kupiec LR test				
Contracts	Quantile	Failure rate	Kupiec LRT	P-value
CO_2/Oil	0.975	0.9742	0.0200	0.8872
CO_2/Oil	0.025	0.0351	3.1989	0.0736
CO_2/Gas	0.975	0.9777	0.2753	0.5997
CO_2/Gas	0.025	0.0234	0.0894	0.7649
CO_2/El	0.975	0.9718	0.3245	0.5688
CO_2/El	0.025	0.0328	1.9378	0.1639
CO_2/Coal	0.975	0.9765	0.0894	0.1115
CO_2/Coal	0.025	0.0340	2.5326	0.7649

Table 10.5: Kupiec test

The Kupiec test has significant P-values for all positions. The null hypothesis is retained, which means that the log-returns lies within a two tailed 95% confidence interval. This indicates that RiskMetrics and diagonal BEKK are good models for modeling Value at Risk for their respective portfolios.

Chapter 11

Conclusion

This thesis presented an understanding of ECX CO_2 futures contracts returns' volatility, and value at risk for these derivatives. The thesis is relevant for the future development of the carbon markets. This is a new market, so predictable models for the volatility in returns and value at risk are essential to effectively be able to get a better overview.

Through various empirical statistical analyses we have been able to make conclusions for modeling univariate GARCH, multivariate GARCH, Monte Carlo simulations, and value at risk for the 2009 CO_2 futures contract and the 2010 CO_2 futures contract.

Prices and returns fluctuations for the CO_2 futures derivatives show very similar characteristics for the period they have been traded. The datasets are stationary and significant autocorrelations are found, as well as volatility clusters and non normal distributions. Monthly variations show that March is a special month compared with the others.

From the univariate modeling, we found the AR (1) GARCH (1,1) to be the best model for both contracts according to our evaluation criteria. The model contains most information, has most significant parameters, and reduce autocorrelations in the dataset. We conclude that value at risk, based on the Kupiec test, gives us a good indication of the cutoff line and the conditional value at risk. Results from Monte Carlo simulations confirm our conclusions to be trustworthy, since those results do not contain major differences from the GARCH results. We tried to eliminate all autocorrelations with dummy variables, but the new results were not significant.

The price adjustments that took place in April 2006, due to the release of the verified emissions, led to extreme volatility movements in returns. We did a new research with data after this structural break, to check if the results were better after the changes. The new results found the GARCH (1,1) to be the best model for the 2010 contract, and, still the AR (1) GARCH (1,1) as the best model for the 2009 contract. The results reduced, but did not eliminate, the autocorrelations as we hoped. Value at risk is well predicted in short positions, but it contains more uncertainty in long positions. Monte Carlo simulations give grounds for concluding that AR (1) GARCH (1,1) and GARCH (1,1) are good models for the returns' volatility and value at risk.

We have tested CO_2 in bivariate portfolios with oil, gas, electricity, and coal. The datasets from all derivatives are stationary and contain significant autocorrelations. RiskMetrics and diagonal BEKK are used with different specifications, and we found the RiskMetrics (1,1) to be the best model for CO_2/oil and CO_2/electricity. The diagonal BEKK model was the best model for CO_2/gas and CO_2/coal. The value at risk has been studied in portfolios with allocation of 50% in CO_2 futures and 50% in the other commodities. Results from covariances and correlations between the different contracts have been used to calculate the value at risk. Our findings show that the null hypotheses are retained and that both models gives us a good measure of value at risk for the futures contracts.

Chapter 12

Critique and further research

The results form our modeling have proven it difficult to find a perfect GARCH model with all parameters significant and no autocorrelation.
In this thesis we looked exclusively at European Climate Exchange futures. It may be of interest to compare the same contracts from various exchanges, to see if arbitrage opportunities can arise. Out of sample forecasting and tick-by-tick data are suggestions for further work in the univariate analysis.

It also may be of interest to test all the different energy derivatives in the same multivariate GARCH model. It will be natural to test various multivariate GARCH models, such as the CCC (Constant Conditional Correlation) model and the DCC (Dynamic Conditional Correlation) model, to better describe the correlations between the derivatives.

Our portfolios are only bivariate portfolios with CO_2 futures as the common feature. For further work it will be interesting to look at portfolios existing of (1) several energy futures contracts, for example 20% in each of our contracts (CO_2, oil, gas, electricity, and coal), and (2) CO_2 contracts versus other assets such as stocks, bonds and currency.

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Chapter 13

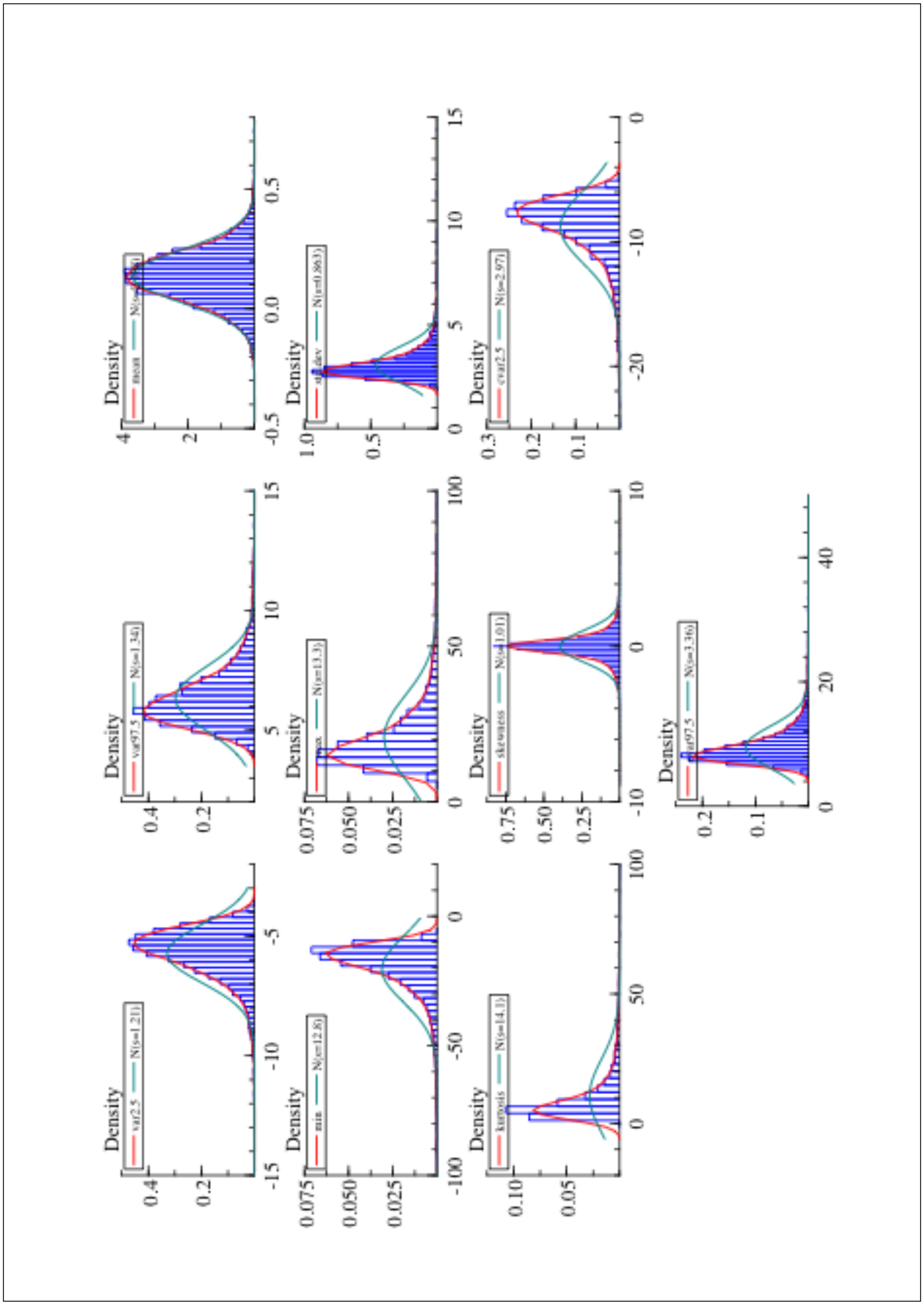
Appendix

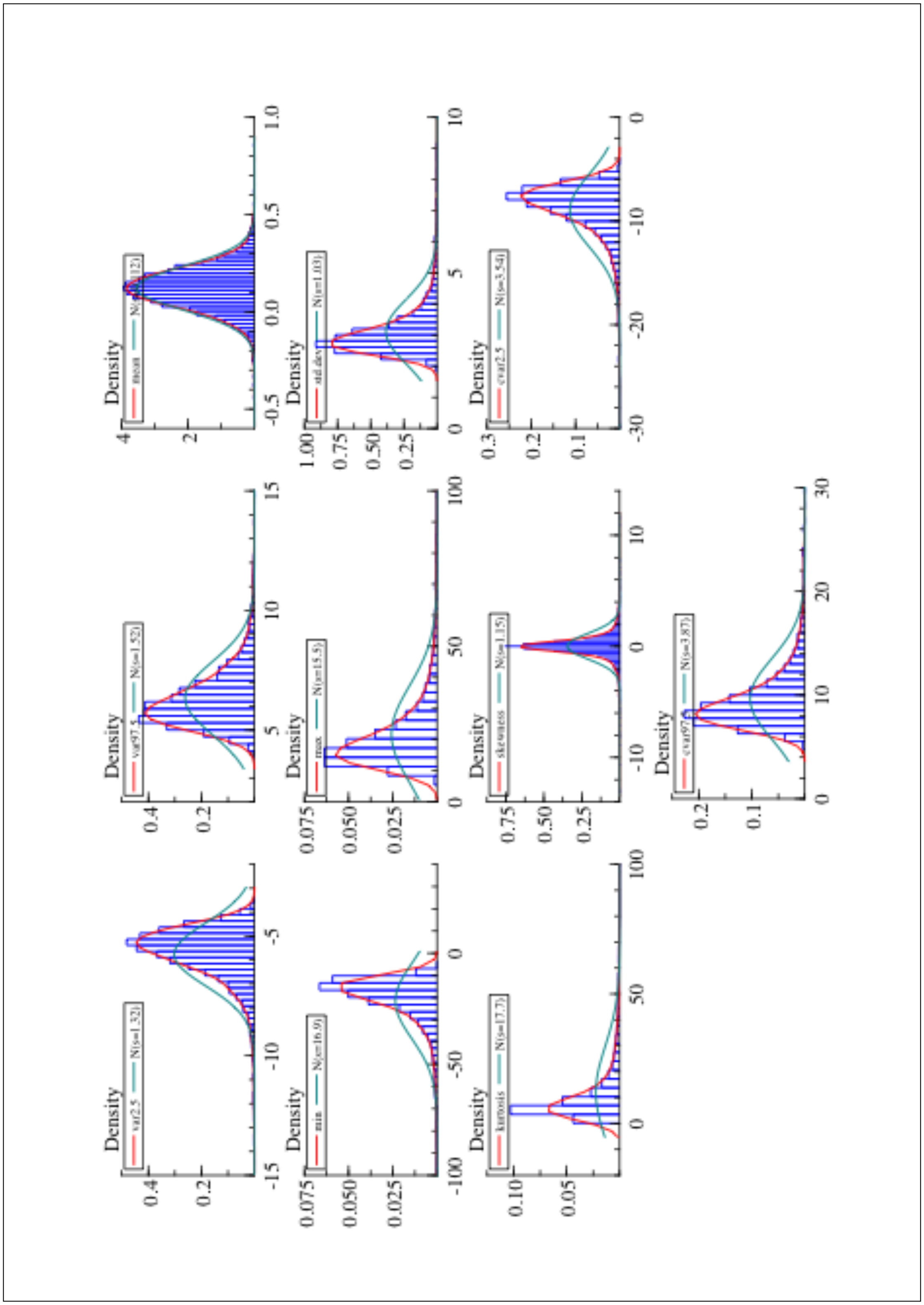
```
*****************************
 ** G@RCH( 3) SPECIFICATIONS **
 *****************************
Dependent variable : Logreturns_09
Mean Equation : ARMA (1, 0) model.
1 regressor(s) in the conditional mean.
Variance Equation : GARCH (1, 1) model.
No regressor in the conditional variance
Student distribution, with 4.62265 degrees of freedom.

Strong convergence using numerical derivatives
Log-likelihood = -2778.51
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
               Coefficient  Std.Error  t-value  t-prob
Cst(M)              0.068359   0.067367    1.015  0.3104
Monday (M)          0.272080    0.14612    1.862  0.0628
AR(1)               0.081051   0.029752    2.724  0.0065
Cst(V)              0.426469    0.10945    3.896  0.0001
ARCH(Alpha1)        0.166036   0.033372    4.975  0.0000
GARCH(Beta1)        0.792834   0.026797    29.59  0.0000
Student(DF)         4.622645    0.67219    6.877  0.0000

No. Observations :      1211  No. Parameters  :         7
Mean (Y)         :  -0.01412  Variance (Y)    :   8.06079
Skewness (Y)     :  -0.89729  Kurtosis (Y)    :  14.65479
Log Likelihood   : -2778.512  Alpha[1]+Beta[1]:   0.95887

TESTS :
---------
Information Criteria (to be minimized)
Akaike          4.600349  Shibata         4.600283
Schwarz         4.629824  Hannan-Quinn    4.611447
---------------

Normality Test

                Statistic       t-Test      P-Value
Skewness          -0.88193       12.545   4.2426e-36
Excess Kurtosis     5.9972       42.688       0.0000
Jarque-Bera         1971.8         .NaN       0.0000
---------------

Q-Statistics on Standardized Residuals
  --> P-values adjusted by 1 degree(s) of freedom
  Q(  5) =  17.9425   [0.0012665]**
  Q( 10) =  20.2112   [0.0166528]*
  Q( 20) =  36.7810   [0.0084504]**
  Q( 50) =  61.6668   [0.1057453]
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------
```

```
Q-Statistics on Squared Standardized Residuals
  --> P-values adjusted by 2 degree(s) of freedom
  Q(  5) =  3.23815   [0.3563436]
  Q( 10) =  10.0134   [0.2640862]
  Q( 20) =  18.9430   [0.3953422]
  Q( 50) =  51.4743   [0.3394023]
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------
ARCH 1-2 test:    F(2,1204) =  0.25619 [0.7740]
ARCH 1-5 test:    F(5,1198) =  0.64241 [0.6674]
ARCH 1-10 test:   F(10,1188)=   1.0227 [0.4216]
```

```
 ******************************
 ** G@RCH(11) SPECIFICATIONS **
 ******************************
Dependent variable : Log_returns10_%
Mean Equation : ARMA (1, 0) model.
1 regressor(s) in the conditional mean.
Variance Equation : GARCH (1, 1) model.
No regressor in the conditional variance
Student distribution, with 4.36899 degrees of freedom.

Strong convergence using numerical derivatives
Log-likelihood = -2799.38
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                Coefficient  Std.Error  t-value  t-prob
Cst(M)              0.063205   0.065618   0.9632  0.3356
Monday (M)          0.237265    0.14325    1.656  0.0979
AR(1)               0.071416   0.029265    2.440  0.0148
Cst(V)              0.435745    0.11148    3.909  0.0001
ARCH(Alpha1)        0.164655   0.033437    4.924  0.0000
GARCH(Beta1)        0.794363   0.027080    29.33  0.0000
Student(DF)         4.368985    0.59276    7.371  0.0000

No. Observations :      1224  No. Parameters  :         7
Mean (Y)         :  -0.02540  Variance (Y)    :   7.93840
Skewness (Y)     :  -0.87110  Kurtosis (Y)    :  14.05134
Log Likelihood   : -2799.377  Alpha[1]+Beta[1]:   0.95902


TESTS :
---------
Information Criteria (to be minimized)
Akaike          4.585584  Shibata         4.585519
Schwarz         4.614807  Hannan-Quinn    4.596581
---------------

Normality Test

                 Statistic       t-Test      P-Value
Skewness           -0.89324       12.774   2.2994e-37
Excess Kurtosis      6.0531       43.316       0.0000
Jarque-Bera          2031.4         .NaN       0.0000
---------------

Q-Statistics on Standardized Residuals
  --> P-values adjusted by 1 degree(s) of freedom
  Q(  5) =  18.4852   [0.0009917]**
  Q( 10) =  20.0161   [0.0178130]*
  Q( 20) =  36.3624   [0.0095243]**
  Q( 50) =  58.2016   [0.1727426]
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
```

```
---------------

Q-Statistics on Squared Standardized Residuals
  --> P-values adjusted by 2 degree(s) of freedom
  Q(  5) =  2.34338   [0.5042620]
  Q( 10) =  8.42544   [0.3930519]
  Q( 20) =  16.7674   [0.5391469]
  Q( 50) =  41.3559   [0.7399749]
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------
ARCH 1-2 test:    F(2,1217) = 0.074033 [0.9286]
ARCH 1-5 test:    F(5,1211) =  0.46459 [0.8028]
ARCH 1-10 test:   F(10,1201)=  0.84199 [0.5880]
```



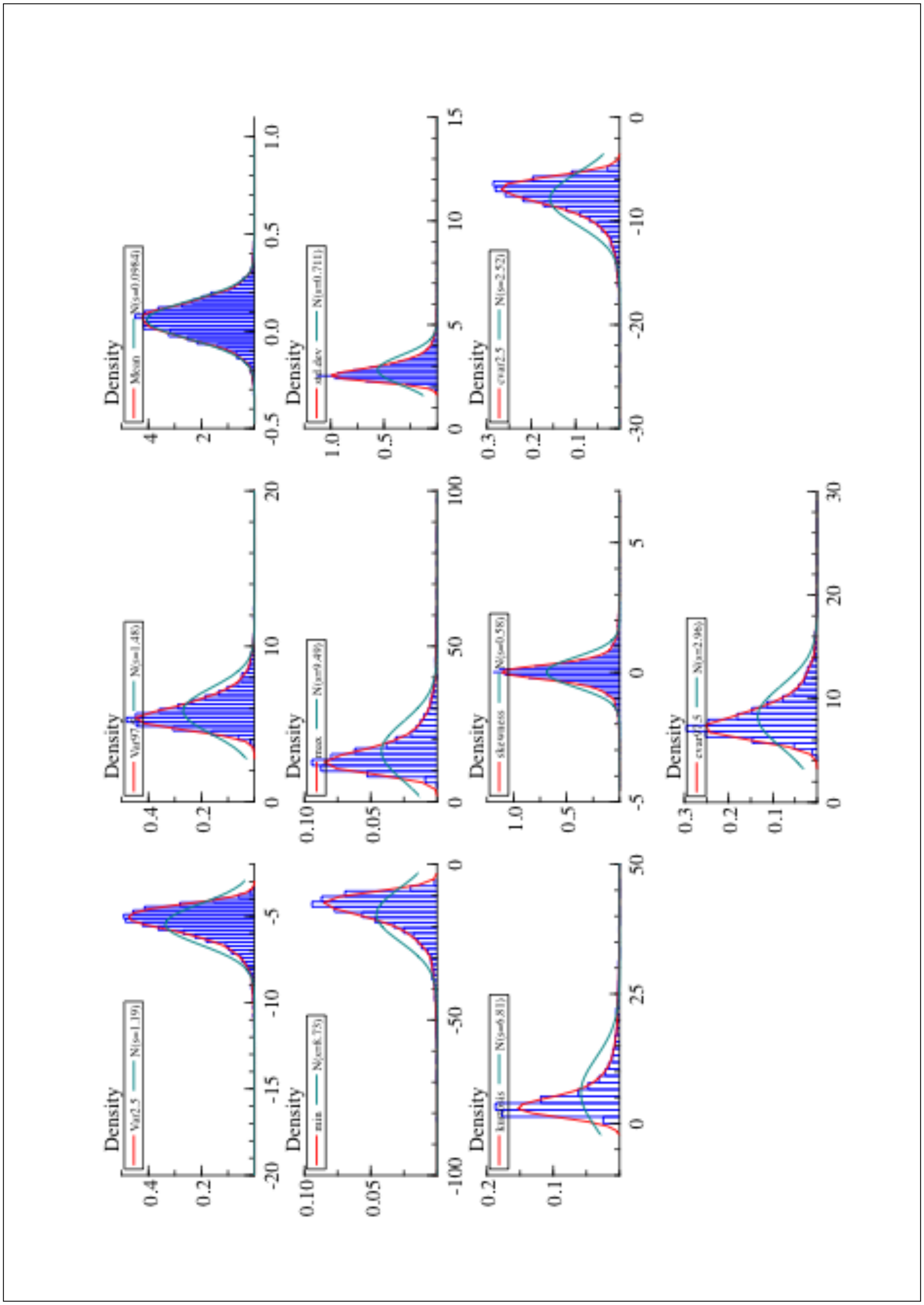

Density
var2.5
var97.5
mean
min
max
std.dev
kurtosis
skewness
cvar2.5
cvar97.5

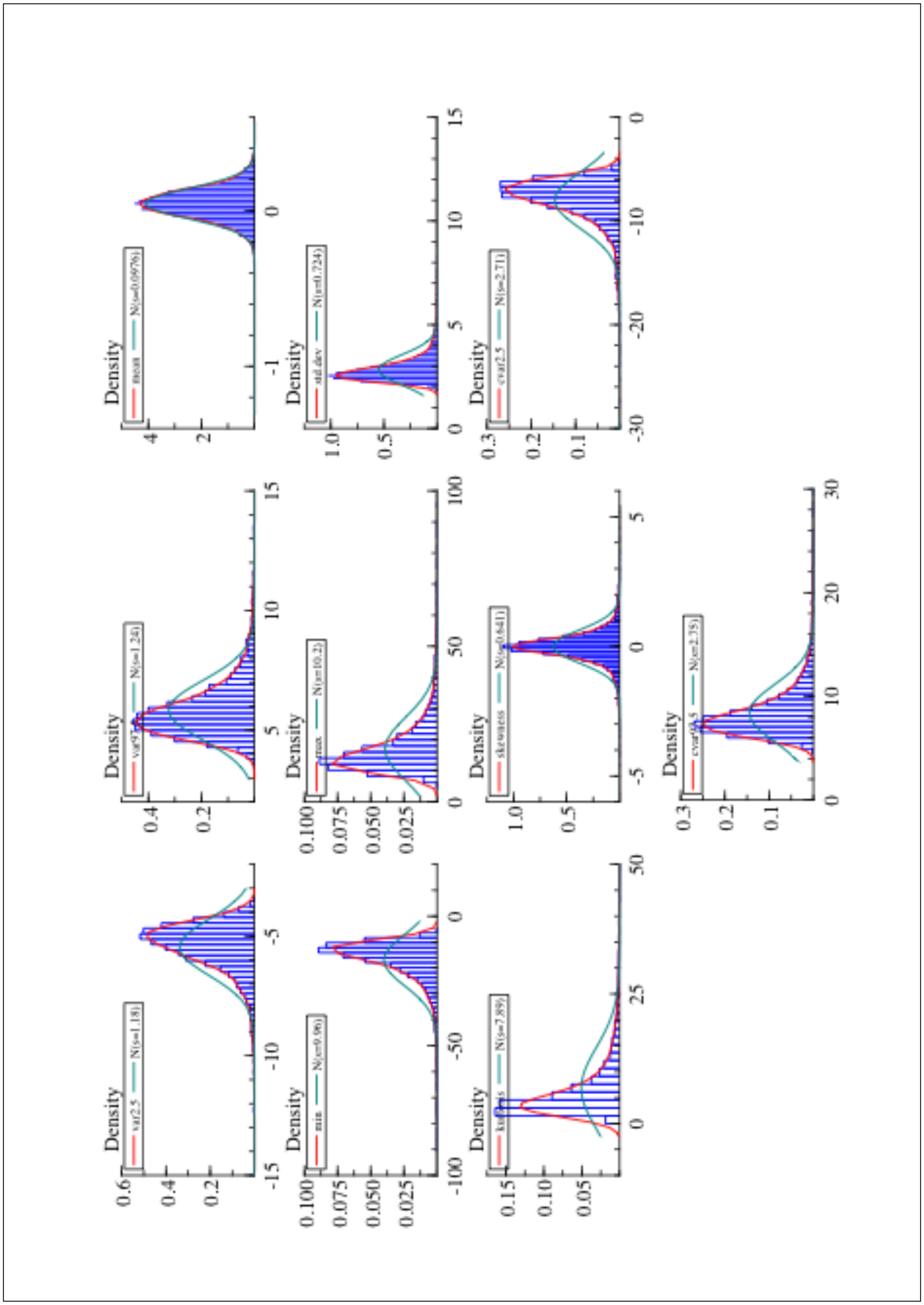
```
TESTS :
=======


---- Database information ----
Sample:    2006-06-22 - 2009-12-14 (908 observations)
Frequency: 1
Variables: 4

Variable        #obs  #miss    type          min         mean          max      std.dev
Date             908      0    date       2006-06-22          2009-12-14
Logreturns09     908      0  double      -9.4346    -0.039931       11.366       2.5222
Constant         908      0  double            1            1            1            0
Trend            908      0  double            1        454.5          908       262.12

Series #1/1: Logreturns09
---------
Normality Test

                 Statistic       t-Test      P-Value
Skewness          -0.092939       1.1452      0.25212
Excess Kurtosis      1.9497       12.026   2.6081e-33
Jarque-Bera          145.13         .NaN   3.0563e-32
---------------
ARCH 1-2 test:    F(2,903)  =   37.767 [0.0000]**
ARCH 1-5 test:    F(5,897)  =   25.650 [0.0000]**
ARCH 1-10 test:   F(10,887) =   17.796 [0.0000]**
---------------
Q-Statistics on Raw data
  Q(  5) =  24.9281   [0.0001439]**
  Q( 10) =  32.2308   [0.0003665]**
  Q( 20) =  57.5264   [0.0000171]**
  Q( 50) =  82.8729   [0.0023859]**
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------
Q-Statistics on Squared data
  Q(  5) =  187.157   [0.0000000]**
  Q( 10) =  321.848   [0.0000000]**
  Q( 20) =  418.963   [0.0000000]**
  Q( 50) =  531.179   [0.0000000]**
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
--------------------------------------------

ADF Test with 2 lags
No intercept and no time trend
H0: Logreturns09 is I(1)

ADF Statistics: -17.0852

Asymptotic critical values, Davidson, R. and MacKinnon, J. (1993)
```

```
       1%        5%       10%
  -2.56572  -1.94093  -1.61663

OLS Results
                 Coefficient    t-value
y_1                -0.945788    -17.085
dy_1                0.063136     1.4251
dy_2               -0.053055    -1.5963
RSS              5627.080871
OBS               905.000000

Information Criteria (to be minimized)
Akaike          4.671924  Shibata         4.671902
Schwarz         4.687861  Hannan-Quinn    4.678010
```

```
TESTS :
=======

---- Database information ----
Sample:    2006-06-22 - 2009-12-31 (921 observations)
Frequency: 1
Variables: 4

Variable        #obs  #miss    type         min         mean         max      std.dev
Date             921      0    date       2006-06-22          2009-12-31
Logreturns10     921      0  double      -9.3014    -0.057864      11.354      2.5092
Constant         921      0  double           1            1           1           0
Trend            921      0  double           1          461         921      265.87

Series #1/1: Logreturns10
---------
Normality Test

               Statistic       t-Test      P-Value
Skewness          -0.13082       1.6235      0.10449
Excess Kurtosis     2.0493       12.729   4.0813e-37
Jarque-Bera         163.78        .NaN   2.7252e-36
---------------
ARCH 1-2 test:    F(2,916)  =   36.003 [0.0000]**
ARCH 1-5 test:    F(5,910)  =   24.437 [0.0000]**
ARCH 1-10 test:   F(10,900) =   16.273 [0.0000]**
---------------
Q-Statistics on Raw data
  Q(  5) =  23.6848   [0.0002496]**
  Q( 10) =  30.0221   [0.0008495]**
  Q( 20) =  57.3067   [0.0000184]**
  Q( 50) =  80.3205   [0.0041835]**
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
---------------
Q-Statistics on Squared data
  Q(  5) =  183.718   [0.0000000]**
  Q( 10) =  308.078   [0.0000000]**
  Q( 20) =  407.607   [0.0000000]**
  Q( 50) =  517.542   [0.0000000]**
H0 : No serial correlation ==> Accept H0 when prob. is High [Q < Chisq(lag)]
--------------------------------------------

ADF Test with 2 lags
No intercept and no time trend
H0: Logreturns10 is I(1)

ADF Statistics: -17.1599

Asymptotic critical values, Davidson, R. and MacKinnon, J. (1993)

       1%        5%       10%
```

```
  -2.56572  -1.94093  -1.61663

OLS Results
              Coefficient    t-value
y_1              -0.947972    -17.160
dy_1              0.054424     1.2306
dy_2             -0.055283    -1.6757
RSS            5664.531488
OBS             918.000000

Information Criteria (to be minimized)
Akaike         4.664200  Shibata          4.664179
Schwarz        4.679959  Hannan-Quinn     4.670215
```


Density


Density
mean
min
max
std.dev
skewness

```
 *************
 ** SERIES  **
*************
#1: LogreturnsCO2
#2: Logreturnsoil

 ******************************
 ** MG@RCH(13) SPECIFICATIONS **
 ******************************
Conditional Mean : ARMA (1, 0) model.
No regressor in the conditional mean.
Conditional Variance : Diagonal BEKK (1, 1).
No regressor in the conditional variance
Multivariate Student distribution, with 6.45124 degrees of freedom.

Strong convergence using numerical derivatives
Log-likelihood = -3620.17
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                Coefficient  Std.Error  t-value  t-prob
Cst1               0.019664   0.075524   0.2604  0.7946
Cst2               0.101887   0.053521    1.904  0.0573
AR_1-1             0.062695   0.038040    1.648  0.0997
AR_1-2             0.030385   0.035380   0.8588  0.3907
C_11               2.162439   0.094391    22.91  0.0000
C_12               0.302529   0.058970    5.130  0.0000
C_22               0.000002 1.2977e-06    1.481  0.1389
b_1.11             0.000000    0.17284     0.00  1.0000
b_1.22            -0.944114   0.011290   -83.63  0.0000
a_1.11             0.505281   0.067899    7.442  0.0000
a_1.22             0.298676   0.029335    10.18  0.0000
df                 6.451235    0.97303    6.630  0.0000
No. Observations :       854  No. Parameters  :        12
No. Series       :         2  Log Likelihood  : -3620.168
Elapsed Time : 4.82 seconds (or 0.0803333 minutes).

TESTS:
------------


 ***********
 ** TESTS **
***********
Information Criteria (to be minimized)
Akaike          8.506248  Shibata         8.505860
Schwarz         8.572992  Hannan-Quinn    8.531809
---------------

Individual Normality Tests
-------------------------
```

```
 Series: LogreturnsCO2

                 Statistic      t-Test     P-Value
Skewness          -0.025837     0.30878     0.75749
Excess Kurtosis     1.6012      9.5795   9.7538e-22
Jarque-Bera         91.328        .NaN   1.4732e-20

 Series: Logreturnsoil

                 Statistic      t-Test     P-Value
Skewness           0.079877     0.95463     0.33976
Excess Kurtosis    0.45100      2.6981    0.0069727
Jarque-Bera         8.1458        .NaN     0.017028

Vector Normality test:   Chi^2(4)  =   71.562 [0.0000]**

Starting estimation process...


  *************
 ** SERIES  **
*************
#1: LogreturnsCO2
#2: Logreturnsgas

 ********************************
 ** MG@RCH(14) SPECIFICATIONS **
 ********************************
Conditional Mean : ARMA (1, 0) model.
No regressor in the conditional mean.
Conditional Variance : Diagonal BEKK (1, 1).
No regressor in the conditional variance
Multivariate Student distribution, with 2.87796 degrees of freedom.

Strong convergence using numerical derivatives
Log-likelihood = -3836.64
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                Coefficient  Std.Error  t-value  t-prob
Cst1              0.016386   0.067175   0.2439  0.8073
Cst2             -0.137474   0.064325   -2.137  0.0329
AR_1-1            0.045203   0.035718    1.266  0.2060
AR_1-2            0.091877   0.037683    2.438  0.0150
C_11              0.870017    0.14105    6.168  0.0000
C_12              2.687970    0.25221    10.66  0.0000
C_22              0.004092   0.032837   0.1246  0.9009
b_1.11            0.904168   0.020284    44.58  0.0000
b_1.22           -0.027097   0.048802  -0.5552  0.5789
a_1.11            0.427166   0.057948    7.372  0.0000
a_1.22            0.443043    0.15724    2.818  0.0050
df                2.877960    0.23517    12.24  0.0000
No. Observations :      854  No. Parameters  :       12
```

```
No. Series       :         2  Log Likelihood  : -3836.637
Elapsed Time : 7.63 seconds (or 0.127167 minutes).

TESTS:
------------


  ***********
 ** TESTS **
***********
Information Criteria (to be minimized)
Akaike          9.013201  Shibata         9.012814
Schwarz         9.079945  Hannan-Quinn    9.038762
---------------

Individual Normality Tests
-------------------------

 Series: LogreturnsCO2

                 Statistic       t-Test      P-Value
Skewness           -0.33098       3.9557   7.6314e-05
Excess Kurtosis      1.1723       7.0131   2.3304e-12
Jarque-Bera          64.491         .NaN   9.9066e-15

 Series: Logreturnsgas

                 Statistic       t-Test      P-Value
Skewness             4.5220       54.044       0.0000
Excess Kurtosis      64.540       386.11       0.0000
Jarque-Bera       1.5113e+05        .NaN       0.0000

Vector Normality test:   Chi^2(4)  =   640.37 [0.0000]**

Starting estimation process...


  *************
 ** SERIES  **
*************
#1: LogreturnsCO2
#2: LogreturnsEL

 *******************************
 ** MG@RCH(15) SPECIFICATIONS **
 *******************************
Conditional Mean : ARMA (1, 0) model.
No regressor in the conditional mean.
Conditional Variance : Diagonal BEKK (1, 1).
No regressor in the conditional variance
Multivariate Student distribution, with 3.22886 degrees of freedom.

Strong convergence using numerical derivatives
```

```
Log-likelihood = -3744.69
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                Coefficient  Std.Error  t-value  t-prob
Cst1               0.023893   0.066316   0.3603  0.7187
Cst2              -0.021254   0.058338  -0.3643  0.7157
AR_1-1             0.021718   0.034677   0.6263  0.5313
AR_1-2             0.006012   0.024388   0.2465  0.8053
C_11               0.614247    0.26877    2.285  0.0225
C_12               2.010525     1.0275    1.957  0.0507
C_22               0.907042     1.9221   0.4719  0.6371
b_1.11             0.928446   0.040555    22.89  0.0000
b_1.22             0.403203    0.30289    1.331  0.1835
a_1.11             0.371455   0.077327    4.804  0.0000
a_1.22             0.129663    0.18768   0.6909  0.4898
df                 3.228860    0.28893    11.18  0.0000
No. Observations :       854  No. Parameters  :        12
No. Series       :         2  Log Likelihood  : -3744.687
Elapsed Time : 6.17 seconds (or 0.102833 minutes).

TESTS:
------------


  ***********
 ** TESTS **
***********
Information Criteria (to be minimized)
Akaike          8.797861  Shibata         8.797474
Schwarz         8.864605  Hannan-Quinn    8.823422
---------------

Individual Normality Tests
-------------------------

 Series: LogreturnsCO2

                 Statistic       t-Test      P-Value
Skewness           -0.34772       4.1557   3.2425e-05
Excess Kurtosis      1.1022       6.5942   4.2748e-11
Jarque-Bera          60.441        .NaN   7.5059e-14

 Series: LogreturnsEL

                 Statistic       t-Test      P-Value
Skewness             1.3718       16.394   2.0938e-60
Excess Kurtosis      33.548       200.71       0.0000
Jarque-Bera          40317.        .NaN       0.0000

Vector Normality test:   Chi^2(4)  =   2606.6 [0.0000]**

Starting estimation process...
```

```
 *************
 ** SERIES  **
*************
#1: LogreturnsCO2
#2: Logreturnscoal

 ******************************
 ** MG@RCH(16) SPECIFICATIONS **
 ******************************
Conditional Mean : ARMA (1, 0) model.
No regressor in the conditional mean.
Conditional Variance : Diagonal BEKK (1, 1).
No regressor in the conditional variance
Multivariate Student distribution, with 5.11378 degrees of freedom.

Strong convergence using numerical derivatives
Log-likelihood = -3637.29
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                Coefficient  Std.Error  t-value  t-prob
Cst1              0.045087   0.073306   0.6151  0.5387
Cst2              0.155094   0.060010    2.584  0.0099
AR_1-1            0.053829   0.036855    1.461  0.1445
AR_1-2            0.196039   0.035785    5.478  0.0000
C_11              2.250507    0.10302    21.85  0.0000
C_12              0.314017   0.084878    3.700  0.0002
C_22              0.000000     0.0000    +.Inf  0.0000
b_1.11            0.000000    0.32245     0.00  1.0000
b_1.22           -0.934682   0.021887   -42.70  0.0000
a_1.11            0.470922   0.070684    6.662  0.0000
a_1.22            0.330513   0.055566    5.948  0.0000
df                5.113782    0.66290    7.714  0.0000
No. Observations :       854  No. Parameters  :        12
No. Series       :         2  Log Likelihood  : -3637.292
Elapsed Time : 4.27 seconds (or 0.0711667 minutes).

TESTS:
------------


 ***********
 ** TESTS **
***********
Information Criteria (to be minimized)
Akaike         8.546351  Shibata        8.545963
Schwarz        8.613095  Hannan-Quinn    8.571912
---------------

Individual Normality Tests
-------------------------
```

Series: LogreturnsCO2

	Statistic	t-Test	P-Value
Skewness	-0.019929	0.23818	0.81174
Excess Kurtosis	1.6432	9.8307	8.3011e-23
Jarque-Bera	96.139	.NaN	1.3297e-21

Series: Logreturnscoal

	Statistic	t-Test	P-Value
Skewness	0.43979	5.2560	1.4718e-07
Excess Kurtosis	4.4738	26.765	8.3488e-158
Jarque-Bera	739.71	.NaN	2.3616e-161

Vector Normality test: Chi^2(4) = 341.45 [0.0000]**

```
 *************
 ** SERIES  **
*************
#1: LogreturnsCO2
#2: Logreturnsoil

 ******************************
 ** MG@RCH( 5) SPECIFICATIONS **
 ******************************
Conditional Mean : ARMA (1, 0) model.
No regressor in the conditional mean.
Conditional Variance : RiskMetrics with lambda = 0.94.
No regressor in the conditional variance
Multivariate Student distribution, with 9.14087 degrees of freedom.

Strong convergence using numerical derivatives
Log-likelihood = -3593.09
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
              Coefficient  Std.Error  t-value  t-prob
Cst1             0.013857   0.081662   0.1697  0.8653
Cst2             0.111978   0.060453    1.852  0.0643
AR_1-1             0.070711   0.039130    1.807  0.0711
AR_1-2             0.017578   0.039553   0.4444  0.6569
df               9.140871     1.4627    6.249  0.0000
No. Observations :       854  No. Parameters  :         5
No. Series       :         2  Log Likelihood  : -3593.087
Elapsed Time : 0.63 seconds (or 0.0105 minutes).

TESTS:
------------


  ***********
 ** TESTS **
***********
Information Criteria (to be minimized)
Akaike          8.426434  Shibata         8.426366
Schwarz         8.454244  Hannan-Quinn    8.437084
---------------

Individual Normality Tests
-------------------------

 Series: LogreturnsCO2

                Statistic       t-Test      P-Value
Skewness           -0.47829       5.7161   1.0899e-08
Excess Kurtosis       1.7266       10.330   5.1733e-25
Jarque-Bera          138.64        .NaN   7.8377e-31

 Series: Logreturnsoil
```

```
                 Statistic       t-Test      P-Value
Skewness          -0.041719      0.49860      0.61806
Excess Kurtosis      0.51488       3.0803    0.0020678
Jarque-Bera           9.6810         .NaN    0.0079030

Vector Normality test:   Chi^2(4)  =   68.017 [0.0000]**

Starting estimation process...


  *************
 ** SERIES  **
*************
#1: LogreturnsCO2
#2: Logreturnsgas

 ******************************
 ** MG@RCH( 6) SPECIFICATIONS **
 ******************************
Conditional Mean : ARMA (1, 0) model.
No regressor in the conditional mean.
Conditional Variance : RiskMetrics with lambda = 0.94.
No regressor in the conditional variance
Multivariate Student distribution, with 3.57785 degrees of freedom.

Strong convergence using numerical derivatives
Log-likelihood = -3937.84
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                Coefficient  Std.Error  t-value  t-prob
Cst1                0.018519   0.082109   0.2255  0.8216
Cst2               -0.064131   0.075431  -0.8502  0.3955
AR_1-1              0.027674   0.042642   0.6490  0.5165
AR_1-2              0.075520   0.027969    2.700  0.0071
df                 3.577856    0.17838    20.06  0.0000
No. Observations :       854  No. Parameters  :         5
No. Series       :         2  Log Likelihood  : -3937.835
Elapsed Time : 0.58 seconds (or 0.00966667 minutes).

TESTS:
------------


  ***********
 ** TESTS **
***********
Information Criteria (to be minimized)
Akaike          9.233806  Shibata         9.233738
Schwarz         9.261616  Hannan-Quinn    9.244456
---------------
```

```
Individual Normality Tests
--------------------------

 Series: LogreturnsCO2

                  Statistic       t-Test      P-Value
Skewness           -0.41752       4.9899   6.0406e-07
Excess Kurtosis      1.9881       11.894   1.2734e-32
Jarque-Bera          165.46         .NaN   1.1799e-36

 Series: Logreturnsgas

                  Statistic       t-Test      P-Value
Skewness            -1.0894       13.020   9.4548e-39
Excess Kurtosis      48.957       292.89       0.0000
Jarque-Bera          85455.         .NaN       0.0000

Vector Normality test:   Chi^2(4)  =   4734.9 [0.0000]**

Starting estimation process...


  *************
 ** SERIES  **
*************
#1: LogreturnsCO2
#2: LogreturnsEL

 *******************************
 ** MG@RCH( 7) SPECIFICATIONS **
 *******************************
Conditional Mean : ARMA (1, 0) model.
No regressor in the conditional mean.
Conditional Variance : RiskMetrics with lambda = 0.94.
No regressor in the conditional variance
Multivariate Student distribution, with 3.70946 degrees of freedom.

Strong convergence using numerical derivatives
Log-likelihood = -3821.81
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                 Coefficient  Std.Error  t-value  t-prob
Cst1                0.002742   0.080207  0.03419  0.9727
Cst2               -0.001440   0.065660 -0.02193  0.9825
AR_1-1              0.032139   0.040708   0.7895  0.4300
AR_1-2              0.000571   0.026037  0.02193  0.9825
df                  3.709469    0.18889    19.64  0.0000
No. Observations :       854  No. Parameters  :         5
No. Series       :         2  Log Likelihood  : -3821.814
Elapsed Time : 0.53 seconds (or 0.00883333 minutes).

TESTS:
```

```
------------


  ***********
 ** TESTS **
***********
Information Criteria (to be minimized)
Akaike          8.962095  Shibata         8.962027
Schwarz         8.989905  Hannan-Quinn    8.972745
---------------

Individual Normality Tests
--------------------------

 Series: LogreturnsCO2

                 Statistic       t-Test      P-Value
Skewness           -0.33132       3.9597   7.5042e-05
Excess Kurtosis      2.2209       13.286   2.7757e-40
Jarque-Bera          191.13         .NaN   3.1410e-42

 Series: LogreturnsEL

                 Statistic       t-Test      P-Value
Skewness           -0.53054       6.3406   2.2887e-10
Excess Kurtosis      30.669       183.48       0.0000
Jarque-Bera          33510.         .NaN       0.0000

Vector Normality test:   Chi^2(4)  =   3096.4 [0.0000]**

Starting estimation process...


  *************
 ** SERIES  **
*************
#1: LogreturnsCO2
#2: Logreturnscoal

 *******************************
 ** MG@RCH( 8) SPECIFICATIONS **
 *******************************
Conditional Mean : ARMA (1, 0) model.
No regressor in the conditional mean.
Conditional Variance : RiskMetrics with lambda = 0.94.
No regressor in the conditional variance
Multivariate Student distribution, with 6.75445 degrees of freedom.

Strong convergence using numerical derivatives
Log-likelihood = -3597.74
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
```

```
               Coefficient  Std.Error  t-value  t-prob
Cst1              0.018470   0.075121   0.2459  0.8058
Cst2              0.137113   0.067653    2.027  0.0430
AR_1-1             0.050040   0.039620    1.263  0.2069
AR_1-2             0.182507   0.038539    4.736  0.0000
df               6.754456    0.78371    8.619  0.0000
No. Observations :       854  No. Parameters  :         5
No. Series       :         2  Log Likelihood  : -3597.741
Elapsed Time : 0.58 seconds (or 0.00966667 minutes).

TESTS:
------------


 ***********
 ** TESTS **
***********
Information Criteria (to be minimized)
Akaike          8.437332  Shibata         8.437264
Schwarz         8.465142  Hannan-Quinn    8.447982
---------------

Individual Normality Tests
--------------------------

 Series: LogreturnsCO2

               Statistic       t-Test      P-Value
Skewness           -0.44661       5.3376   9.4203e-08
Excess Kurtosis       1.5932       9.5315   1.5497e-21
Jarque-Bera          118.71        .NaN   1.6668e-26

 Series: Logreturnscoal

               Statistic       t-Test      P-Value
Skewness            0.36478       4.3595   1.3033e-05
Excess Kurtosis       3.3664      20.140   3.2919e-90
Jarque-Bera          422.20        .NaN   2.0870e-92

Vector Normality test:   Chi^2(4)  =   233.41 [0.0000]**
```