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An overview on formulations and optimization methods for the unit-based short-term hydro scheduling problem

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ABSTRACT

The short-term hydro scheduling (STHS) problem aims at determining the optimal power generation schedules for either a single hydropower plant or an integrated system of cascaded watercourses during a time horizon from a single day to one week. Traditionally, an aggregated plant concept is usually adopted in the formulation of the STHS problem. The hydro-turbine generator units in a plant are aggregated as one equivalent unit. Nowadays, more and more hydro producers participate in both energy and capacity markets. It highlights the need for the precise calculation for energy conversion and available capacity of each unit. Formulating the STHS problem on individual units can accurately capture the physical and the operational characteristics of the unit. In this overview, a detailed classification of mathematical programming approaches to model and solve the unit-based STHS problem is presented. The various modeling techniques proposed in the publications since 2000 are categorized by their objectives and constraints. This provides a comprehensive comparison and discussion for each specific issue in the formulation of STHS. We anticipate this overview to be a starting point for finding more computationally solvable and effective methods to handle the challenges in the unit-based STHS problem.

1. Introduction

Traditionally, short-term hydro scheduling (STHS) aims at determining the optimal generation schedules for the available hydro resources for the coming hours and days by utilizing the water potential in the most economical way. STHS is usually employed to support spot bids in the day-ahead market and to provide a final dispatch plan after the market clearing process [1].

Nowadays, with the rapid development of wind and solar technology, non-dispatchable renewable energy (RE) sources play a notable part in the power production mix of many countries. Because of its storability, flexibility, and controllability, hydropower is of critical importance in ensuring system safety. A significant fraction of the capacity of the hydro units serves as an operating reserve to meet frequent fluctuations in the power system.

The basic hydro generation model is either plant-based or unit-based [2]. Most STHS problems in the literature [3–8], especially in the short-term scheduling of hydrothermal system [9] or hydro-thermal-RE interconnected power system [10–12], are plant-based. That is to say, an aggregated plant concept is adopted in the problem formulation, where the hydro-turbine generator units in a hydropower plant are aggregated as one equivalent unit. One advantage of using the aggregated plant concept is that it reduces the potential STHS problem size significantly.

However, participation in both energy and capacity markets highlights the need for the precise calculation for energy conversion and available capacity of each unit. It requires a more accurate and detailed representation of the hydropower generation, considering the impact of head variation, hydraulic losses, efficiency curves, and restricted operational zones on the power produced by each unit. The results from the model should not only indicate the dispatch of each unit but also the available capacity.

Abbreviations: AL, augmented lagrangian; B&B, branch and bound; DP, dynamic programming; FP, fixed speed pump; HPF, hydropower production function; HSC, hydraulic short-circuit; HUC, hydro unit commitment; IAL, inexact augmented lagrangian; I/O, input/output; LP, linear programming; LR, lagrangian relaxation; MCP, market clearing price; MILP, mixed integer linear programming; MINLP, mixed integer nonlinear programming; MIQP, mixed integer quadratic programming; N/A, not available; NLP, nonlinear programming; PSHP, pumped storage hydropower plant; PSO, particle swarm optimization; RE, renewable energy; RPG, Rosen's projected gradient; SQP, sequential quadratic programming; STHS, short-term hydro scheduling; UC, unit commitment; ULD, unit load dispatch; VP, variable speed pump

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Nomenclature

Sets and indexes[1]

T Set of time periods, index $t \in T$
K Set of reservoirs, index $k \in K$
U_k Set of all direct upstream hydraulic objects for reservoir k, index $u \in U_k$
S Set of hydropower plants, index $s \in S$
N_s Set of penstocks in plant s, index $n \in N_s$
I_s Set of hydro-turbine generator units in plant s, index $i \in I_s$
$I_{n,s}$ Set of units that connect to the same penstock n in plant s, index $i' \in I_{n,s}$

Parameters

$\bar{t}$ Number of the time periods of the scheduling problem
ΔT Length of each time period (hour, h)
$\tau_{u,k}$ Water delay time from upstream hydraulic object u to reservoir k (h)
$V_{k,0}^{INIT}$ Initial water storage of reservoir k (cubic meter, m^3)
$V_{k,\bar{t}}^{END}$ Target water storage of reservoir k at the end of scheduling horizon (m^3)
V_k^{MIN}, V_k^{MAX} Minimum and maximum water volume of reservoir k (m^3)
$Q_k^{BYPASS\_MAX}$ Maximum controllable spillage of reservoir k (m^3/s)
$Q_{k,t}^{NI}$ Forecasted natural inflow into reservoir k in period t (m^3/s)
E_s Energy conversion factor for plant s (megawatt-hour per cubic meter, MWh/m^3)
L_s Water level of outlet line of plant s (m)
G Conversion constant including the gravity acceleration and water density makes the appropriate unit conversions from (m) and (m^3/s) to (MW), the default setting is $9.81 \cdot 10^{-3}$ ($kg\ m^2/s^2$)
$\alpha_{n,s}$ Loss factor of penstock n in plant s, taking into account the length, diameter, curvature, and roughness of the penstock's inner walls (s^2/m^5)
$P_{i,s}^{MIN}$, $P_{i,s}^{MAX}$ Minimum and maximum production of unit i in plant s (MW)
P_s^{MIN}, P_s^{MAX} Minimum and maximum production of plant s (MW)
Q_s^{MIN}, Q_s^{MAX} Minimum and maximum discharge of plant s (MW)
$\Omega_{i,s,0}$ Initial status of unit i in plant s (1 on, 0 off)
$C_{i,s}$ Start-up cost of unit i in plant s (€)
D_t Load obligation in period t (MW)
M_t^{SELL} Forecasted market price of electricity in period t (€/MWh)
$W_{k,\bar{t}}^{END}$ Marginal water value of reservoir k at the end of the scheduling horizon $\bar{t}$ (€/MWh)

Variables

$\omega_{i,s,t} \in \{0,1\}$ Status of unit i in plant s in period t (1 on, 0 off)
$\mu_{i,s,t} \in \{0,1\}$ Start-up decision of unit i in plant s in period t (1 if it is started up in period t, 0 otherwise)
$v_{k,t}$ Water volume of reservoir k at the end of period t (m^3)
$q_{k,t}^{BYPASS}$ Water released via bypass gate of reservoir k in period t (m^3/s).
$q_{k,t}^{TOTAL}$ Total regulated water release of reservoir k in period t (m^3/s)
$h_{s,t}^{GROSS}$ Gross head of plant s in period t (m)
$h_{i,s,t}^{NET}$ Net head of unit i in plant s in period t (m)
$\Delta h_{i,s,t}^{PEN}$ Penstock head loss of unit i in plant s in period t (m)
$q_{i,s,t}$ Water discharge of unit i in plant s in period t (m^3/s)
$p_{i,s,t}$ Power output of unit i in plant s in period t (MW)
p_t^{SELL} Power sold to the market in period t (MW)

State-dependent functions

$l_{k,t-1}(v_{k,t-1})$ The water level of reservoir k as a function of the water storage of the reservoir (m)
$\Delta h_{s,t}^{INTAKE}(l_{k,t-1}, q_{k,t}^{TOTAL})$ Intake head loss of plant s as a function of the water level of upstream reservoir k and the total regulated water release of the upstream reservoir k (m)
$\Delta h_{s,t}^{TAIL}(l_{k+1,t-1}, q_{k,t}^{TOTAL})$ Tailrace head loss of plant s as a function of the water level of downstream reservoir $k+1$ and the total regulated water release of the upstream reservoir k (m)
$q_{k,t}^{OVER}(l_{k,t-1})$ Unregulated water release (overflow) of reservoir k in period t as a function of the water level of the reservoir (m^3/s)
$\eta_{i,s}^{GEN}(p_{i,s,t})$ Generator efficiency of unit i in plant s as a function of the production (%)
$\eta_{i,s}^{TURB}(h_{i,s,t}^{NET}, q_{i,s,t})$ Turbine efficiency of unit i in plant s as a function of the net head and water discharge of the unit (%)
$Q_{i,s,t}^{MIN}(h_{i,s,t}^{NET})$, $Q_{i,t}^{MAX}(h_{i,s,t}^{NET})$ Minimum and maximum water discharge of unit i in plant s in period t as a function of the net head (m^3/s)

Therefore, the traditional modeling method based on aggregated plant level must be supplemented by the unit-based modeling in the optimization ("unit-based" refers to "the individual hydro-turbine generator unit" in this paper). Unit-based STHS can accurately capture the physical as well as the operational characteristics of the unit. Furthermore, recent advances in hardware and software packages have significantly overcome earlier computational difficulties. In most situations, the STHS problems formulated by the sophisticated and detailed mathematical programming approaches can be solved within a reasonable time.

STHS is related to two problems, which can be specified as (1) the unit commitment (UC) problem seeking to specify, for each period, the on/off status of the units; and (2) the unit load dispatch (ULD) problem trying to determine the respective dispatch of the committed units [13–16]. In the literature, these two problems can be combined and unified as a hydro unit commitment (HUC) problem [17–23].

From the perspective of a problem statement, STHS may be posed as a single problem, for example as for those generating companies with only hydropower plants, or it may be a subproblem integrated within a larger problem where thermal units and/or wind power generators are involved [24].

Mathematically, the STHS problem is formulated as a large-scale, discrete, nonlinear, and non-convex problem. A wide range of optimization techniques has been proposed for addressing this complex problem. These optimization methods can be generally divided into two main groups: exact methods and heuristic methods.

The authors of Ref. [25] presented a bibliographical survey and a methodology-based classification of the classical exact methods and modern heuristic algorithms applied to solve short-term thermal UC and economic power load dispatch problems together with hydro schedules. They also discussed the advantages and limitations of the methodologies. In Ref. [26], exact approaches for the HUC problem were summarized. Reference [9] offered a comprehensive review of the

[1] Note that the hydraulic objects in the cascaded watercourse are indicated in sequence. However, a reservoir can be associated with a plant or be interconnected by a junction/gate. Therefore, we separate the sets and indexes of reservoirs and plants. If not specifically mentioned, reservoir k always refers to the direct upstream reservoir of plant s and reservoir $k+1$ refers to the direct downstream reservoir of plant s.

application of heuristic methods to determine the optimal short-term scheduling of hydrothermal systems.

Moreover, the authors of Ref. [7] provided a chronological overview of STHS approaches proposed in the past 20 years, focusing on the main contributions of the methodology for each particular hydro system configuration. The review in Ref. [27] presented the operational aspects of reservoir-based as well as small run-of-river hydropower plants.

Though the STHS problem seeks to find the optimal dispatch plan within a relatively short period, it still faces uncertainties regarding the market price and natural inflow for the coming hours and days. Reference [28] outlined stochastic-programming-based formulations of the multi-market STHS and bidding problems. In addition, the penetration of the intermittent RE in the power system significantly increases the uncertainty in system operation. Authors in Ref. [12] summarized the latest short-term scheduling methods to model and evaluate the effect of RE for the safe and stable operation of the hydro-thermal-RE power system.

The overview in this paper principally addresses the modeling techniques for individual hydro units and the corresponding solution methodologies proposed in the papers since 2000. The focus of this paper is on the exact approaches, and heuristic methods are not discussed here. Uncertainties in the forecast of natural inflow and market price and the power generation of RE are not considered. The detailed explanation of the computationally efficient deterministic models will serve as a solid scientific reference for including the stochastic nature of the STHS problem as well as for the optimal scheduling in a hydro-thermal-RE interconnected power system.

Besides, the traditional layout of a literature review first presents a general set of objectives and constraints of the STHS problem and then enumerates the contributions of each paper; see for instance [7,9,25,26]. However, in this overview, the modeling techniques proposed in the literature are categorized by their objectives and constraints. This provides a comprehensive comparison and discussion for each specific issue in the formulation of STHS.

It is worth noting that, in contrast to the plant-based aggregation where water input and power output are formulated at plant level and each unit in the aggregated plant is assumed to have the same characteristics [3–6,8], another type of aggregation has recently been developed [21,23,29,30]. Instead of working directly with individual units in the model, combinations of units in operation are used. Since the formulations in these papers still keep the details of unit characteristics, they are taken as unit-based STHS problems. Furthermore, although papers [31–33] are modeled on the plant level, it is assumed that each plant operates with only one generating unit, or its equivalent, with a Hill chart given by turbine manufacturer. Therefore, they are included in this overview.

The remainder of the paper is organized as follows: Section 2 provides a comprehensive discussion of the various modeling techniques in the mathematical formulations of the STHS problem. Section 3 summaries the implementation of optimization methods and solvers for STHS. The conclusion is given in Section 4.

2. Mathematical formulations

This section contains a discussion of the various choices for modeling objectives and constraints in detail, with the definitions presented in Nomenclature.

2.1. Objectives

The objectives of the STHS problem are heavily dependent on the system characteristics and operational requirements. In a centralized system such as Brazil [34,35], the hourly generation target for each hydropower plant is given by the system operator, and there is no corresponding electricity price. The STHS problem focuses on searching for the most economical schedules among the generating units to meet the load demand while satisfying the physical and operational constraints of the system. In this context, the most economical schedules are presumed to be the efficient use of water and to minimize start-ups and shut-downs of units.

In deregulated power systems such as Scandinavia [1], Spain [33,36] and Canada [37,38], a market clearing process takes place to distribute electricity production among different producers by considering offers and demands from the participants. The market clearing price (MCP) is determined by the intersection of the supply and demand curves. All selling bids under the MCP and all purchase bids over the MCP are accepted. The hydro producers maximize their profits by trading electricity in a competitive electricity market under the assumption that they do not have the market power to influence the market price.

2.1.1. Objective 1: maximizing the total revenue

Revenue maximization is widely used in the competitive electricity market [6,18,24,31–33,36–40]. It is achieved by selling power in the market, as expressed in Eq. (1).

$$\mathrm{Max} f_1 = \sum_{t \in T} M_t^{SELL} \cdot \Delta T \cdot p_t^{SELL} \tag{1}$$

2.1.2. Objective 2: minimizing the total operational cost

When the value of stored water is ignored, the hydropower production costs are negligible [31]. The most significant costs having a real impact on STHS are the start-up and shut-down costs of the units [33]. Since both start-up and shut-down of units have a negative influence on the maintenance costs and service life of a machine, the economic STHS calls for a consideration of reducing the number of start-ups and shut-downs of the units [13,18,21,24,29,31,32,34,38–40]. Therefore, the start-up cost of each unit should be minimized (the shut-down cost can be added if needed), as presented in Eq. (2). The unit start-up cost can be calculated based on the history of expenses in maintenance and repairs concerning the number of start-ups [29], or be estimated as a function of the nominal output power of the unit [41]. In some cases, the start-up cost is not expressed in the form of monetary units (€) but in terms of water release (m^3) required to start up one unit [19].

$$\mathrm{Min} f_2 = \sum_{t \in T} \sum_{s \in S} \sum_{i \in I_s} C_{i,s} \cdot \mu_{i,s,t} \tag{2}$$

2.1.3. Objective 3: minimizing the value of energy used or spilled

If the electricity market is centrally controlled, the load obligation is predefined and no, or only a limited amount of, power can be sold to the spot market. In this case, the main objective is to minimize the value of water utilized by turbines or spilled [38], as expressed in Eq. (3a). Water value $W_{k,t}^{END}$ refers to the opportunity cost of storing water for later generation versus using it now (See Subsection 2.2.8 for details). In other words, the objective is to maximize the potential energy for future income, which leads to an alternative form as Eq. (3b). If water value is not considered in the STHS, then Eq. (3a) can be represented just by the volume of water released from storage [19] or Eq. (3b) can be expressed by water stored at the end of the planning horizon [23].

$$\mathrm{Min} f_{3a} = \sum_{k \in K} W_{k,\bar{t}}^{END} \cdot E_s \cdot (V_{k,0}^{INIT} - v_{k,\bar{t}}) \tag{3a}$$

$$\mathrm{Max} f_{3b} = \sum_{k \in K} W_{k,\bar{t}}^{END} \cdot E_s \cdot v_{k,\bar{t}} \tag{3b}$$

2.1.4. Other objectives

When determining optimal operation for a single hydropower plant with multiple available units, the objective can be to maximize the power generated for a given flow [14,17,29], to minimize the total discharge of the units for a given load [15,19,20,22,34,35], or to maximize the efficiency of the whole power plant energy conversion [16]. If the power loss is the focus, the objective becomes to minimize the losses in the hydro generation process due to tailrace elevation, penstock loss, and turbine-generator efficiency variations [13,21]. In Ref. [33], the authors presented an objective function to maximize the weighted technical efficiency where the power generated by each plant per period is taken as the weighting factor.

Certain combinations of the objectives are neither desirable nor feasible. The selection of objectives depends on the characteristics of the electricity market and the operating conditions. If there are two or more competing objectives in a model, it becomes a multi-objective optimization problem. In this case, no single solution exists that simultaneously optimize each objective. Instead, a set of Pareto optimal (also known as “non-dominated” or “Pareto efficient”) solutions exist. How to solve a multi-objective optimization problem mainly refers to two aspects: (1) how to find a representative set of Pareto optimal solutions that can approximate the entire Pareto front; and (2) how to select the most preferred solution among all Pareto optimal solutions [42].

The most popular method for the multi-objective optimization of the pure STHS problem is to convert the multi-objective problem into a single-objective optimization problem by using weighting coefficients [43,44] or by treating some of the objectives as constraints [45]. In Ref. [46], the authors developed an STHS model for peak shaving of multiple power grids. It is a typical multi-objective min-max problem. The objective function for one power grid has to compromise with the others, and a fuzzy optimization method is adopted to evaluate the Pareto optimal solutions. In Ref. [47], the authors included flood control in STHS and investigated a tri-objective optimization model. The multi-objective evolutionary algorithm based on the decision maker’s preference and the decomposition technique is developed.

In the short-term scheduling of hydrothermal systems [48] or hydro-thermal-RE hybrid systems [12], more advanced heuristic algorithms for solving multi-objective optimization are presented since more conflicting concerns such as environmental emissions are involved. However, the deeper discussion is out of the scope of this paper. The authors in Ref. [12] gave a thorough summary of the traditional and emerging heuristics for solving the multi-objective scheduling of the hybrid power systems.

Commonly seen objective function components for STHS in a competitive electric power system, as shown in Eq. (4), include the revenue from selling power, the start-up costs, and the value of energy stored in the reservoirs at the end of the study. The penalty costs associated with violation of various limits are optionally included in the objective function if necessary.

$$\mathrm{Max}\, F = f_1 - f_2 + f_{3b} \tag{4}$$

2.2. Constraints

The optimization problem is subjected to a variety of constraints, including

2.2.1. Constraint 1: water balance of the reservoirs

The hydrological balance of a cascaded reservoir k associated with plant s in each period t is formulated in Eqs. (5)–(7).

$$v_{k,0} = V_{k,0}^{INIT} \tag{5}$$

$$v_{k,t} = v_{k,t-1} + 3600 \cdot \Delta T \cdot \left(Q_{k,t}^{NI} + \sum_{u \in U_k} \left(q_{u,t-\tau_{u,k}}^{TOTAL} + q_{u,t-\tau_{u,k}}^{OVER}(l_{u,t-1}) \right) - q_{k,t}^{TOTAL} - q_{k,t}^{OVER}(l_{k,t-1}) \right) \tag{6}$$

$$q_{k,t}^{TOTAL} = \sum_{i \in I_s} q_{i,s,t} + q_{k,t}^{BYPASS} \tag{7}$$

These constraints are linearly coupled in time and space. The water storage of reservoir k at the end of period t is the storage at the beginning of the period plus the volume of inflow minus outflow in period t. The volume of flow is decided by the length of the time period ΔT (i.e. time resolution), and the constant “3600” represents 3600 s in one hour. In most recent works dealing with cascaded hydro systems, the water balance equality constraints are expressed in terms of volume (m^3). However, in the early publications such as [2,37,49], the constraints are represented based on flow (m^3/s).

The inflow includes the forecasted natural inflow and the water discharged from the upstream reservoirs or other hydraulic objects (e.g. gate, junction, and creek intake). Due to the cascaded hydraulic configuration, the fraction of water released upstream will contribute to the inflow of downstream reservoirs after a certain time delay. It is an important physical element of a cascaded watercourse, also known as "river routing effects" [38]. Water delay time can be a multiple of the time resolution [19], a real number constant [23,39], an integer variable [50], or a continuous variable [51].

The outflow consists of regulated and unregulated water release. The regulated water release refers to the total discharge of the units and the flow going through the bypass gates, as expressed in Eq. (7). In the literature, the flow that can be controlled precisely by adjusting gate openings is also called the spillage of the plant [22–24,52]. This flow can be regulated to balance the minimum outflow constraints and the transmission capacity limits [3]. The controlled spillage from a reservoir into a downstream area also occurs during the wet season [20]. However, spillage should be avoided as much as possible, since no electricity is produced in this case [38].

By contrast, the unregulated water release is associated with the uncontrollable flow, which occurs when a reservoir runs full, and the water spills over the top of the dam. The overflow description can be represented by a piecewise linear function [2,37,53], such that below the reservoir starting spill level, the spill flow is zero, while above the spill level the flow is proportional to the level over the full storage of the reservoir.

In some areas, the evaporation rate at reservoirs has to be accounted for. It depends on the surface area of the reservoir and the storage volume [3]. In addition, there is the use of reservoir water without the purpose of generating energy, such as urban water supply, irrigation, and navigation [54]. If necessary, these factors should be included in the reservoir balance constraint.

2.2.2. Constraint 2: storage limits of the reservoir and operational limits of controllable spillage

Eq. (8) restricts the allowable capacity of the reservoir [2,15,18–20,22,23,29,32–34,38–40]. The storage limits refer to the minimum operating level and the maximum flood level. Target operating limits can also be added for a designated period, typically the end of the study. In this context, it can be seen as a volume coupling to long/mid-term planning (See Subsection 2.2.8 for details). It is practical to add penalty variables and costs to avoid unregulated spillage or running out of water [2]. Eq. (9) represents the operational limits of the controllable spillage of the reservoir [18,20,38].

$$V_k^{\mathrm{MIN}} \le v_{k,t} \le V_k^{\mathrm{MAX}} \tag{8}$$

$$0 \le q_{k,t}^{BYPASS} \le Q_k^{BYPASS\_MAX} \tag{9}$$

2.2.3. Constraint 3: head variation and flow-related head losses

The net head of a turbine is primarily dependent on reservoir level variation (i.e. the gross head) and flow-related head losses (Fig. 1).

The gross head, as expressed in Eq. (10), is the difference between the water level of the upstream reservoir, i.e. the forebay level, and downstream reservoir (if the water level of downstream reservoir $k+1$ is higher than the outlet line of the plant s).

$$h_{s,t}^{GROSS} = l_{k,t-1}(v_{k,t-1}) - \mathrm{MAX}[l_{k+1,t-1}(v_{k+1,t-1}),\ L_s] \tag{10}$$

Whether the head variation caused by the change in water volume should be considered depends on the size of the reservoir. For large reservoirs, little change in the water level is observed during a short-term scheduling horizon (only a few days or even a single day) [13,21,52]. It is valid to assume that the variation on the water level is negligible. However, for the medium- or small-sized reservoirs with daily or hourly regulation capability, head variation should be considered [5,6,32]. The head variation (also known as "the head effect") has a direct impact on the unit's efficiency and operating limits, constituting one of the main difficulties in the modeling of the STHS problem [36]. In some large-scale optimization problems, for the sake of computational tractability, the head is assumed to be fixed irrespective of the characteristics of the reservoir [55].

The water level relies on the water stored in the reservoir and can be formulated as a (piecewise) linear function [19,33,37,40,53] or polynomial function [17,20,22–24,34,38,43] of the water storage. In addition, instead of utilizing the volume at the beginning of period t as shown in Eq. (10), it is common to use the average volume associated with the beginning and end of one period to compute the water level [19,22,36,39,56,57].

On the other hand, head losses lead to the reduction of the gross head, which in turn affects the power generated at the turbines. There are three main types of flow-related head losses, i.e. penstock/main tunnel head loss, canal intake head loss, and tailrace head loss. The first type of loss is caused by water friction and the latter two are due to the velocity of water flow [58]. Physically, these losses are not distances measured from water levels. However, they can be converted into adequate quantities expressed in meters (Eq. (11)).

$$\begin{aligned} h_{i,s,t}^{NET} = h_{s,t}^{GROSS} &- \sum_{n \in N_s | i \in I_{n,s}} \alpha_{n,s} \cdot \left(q_{i,s,t} + \sum_{i' \in I_{n,s} \setminus \{i\}} q_{i',s,t} \right)^2 \\ &- \Delta h_{s,t}^{INTAKE}(l_{k,t-1}(v_{k,t-1}),\ q_{k,t}^{TOTAL}) \\ &- \Delta h_{s,t}^{TAIL}(l_{k+1,t-1}(v_{k+1,t-1}),\ q_{k,t}^{TOTAL}) \end{aligned} \tag{11}$$

1) Penstock/main tunnel head loss is related to the friction of water on the penstock wall. It can be represented as a quadratic function of the flow going through the penstock [13]. Loss factor $\alpha_{n,s}$ depends on the length, diameter, curvature, and roughness of the penstock's inner walls [16,54]. In some simplified models, it is assumed that penstock head loss is either constant [19] or calculated as a percentage of the power output [3,24] or a percentage of the net head [59], regardless of the flow going through the tunnel. In most STHS problems where penstock head loss is considered as a quadratic function of the turbined flow [13,17,21–23,34,39,52,60], there is a common premise that the penstock loss for one specific unit only depends on the water flow processed by this unit. It is correct only if each unit is fed by an independent penstock from the reservoir.
However, in some areas, multi-level shared penstock (also known as "common penstock/tunnel/conduit") configuration often exists [1,61–63]. A hydropower plant consists of a main tunnel branching into several separate penstocks, through which the flowing water reaches multiple units. As illustrated in Fig. 1, the main tunnel and Penstock2 marked in red are shared penstocks, whereas Penstock1 is an independent penstock. Few approaches accounting for loss in shared penstocks in STHS have been presented in the literature. In Ref. [14], the authors considered the ULD problem that addresses the optimal distribution of the production among a set of online units for a given flow. Accounting for losses in the shared penstock introduces coupling net head variables. A decomposition algorithm is proposed and each subproblem is solved by dynamic programming (DP). In Ref. [16], the ULD problem is solved by maximizing the end pressure of the shared penstock. A gauge pressure sensor is used to measure the flow at the end of the penstock. In Ref. [64], a two-phase decomposition approach is presented. The UC subproblem is first solved by a hybrid algorithm that combines a heuristic searching method and a progressive optimal algorithm. The ULD subproblem is then solved by DP. In Ref. [40], three heuristics are proposed to incorporate the power loss in shared penstocks in the STHS problem. The nonlinear penstock loss can be effectively transformed into the formulation framework of mixed integer linear programming (MILP).
2) Canal intake head loss is associated with the water level of the upstream reservoir and the water flow passing through the plant. In contrast to the other two types of losses, it is seldom mentioned in the formulation of the STHS problem. In Ref. [35], the loss in canal intake is modeled as a quadratic function of the total flow to the plant. In Ref. [40,53], intake loss is expressed as a function of the water level of the upstream reservoir and the total regulated water release of the reservoir.
3) Tailrace elevation can vary considerably with an accumulation of the total water discharge of the plant. It leads to a decrease in the net head and has a negative effect on power generation. In the

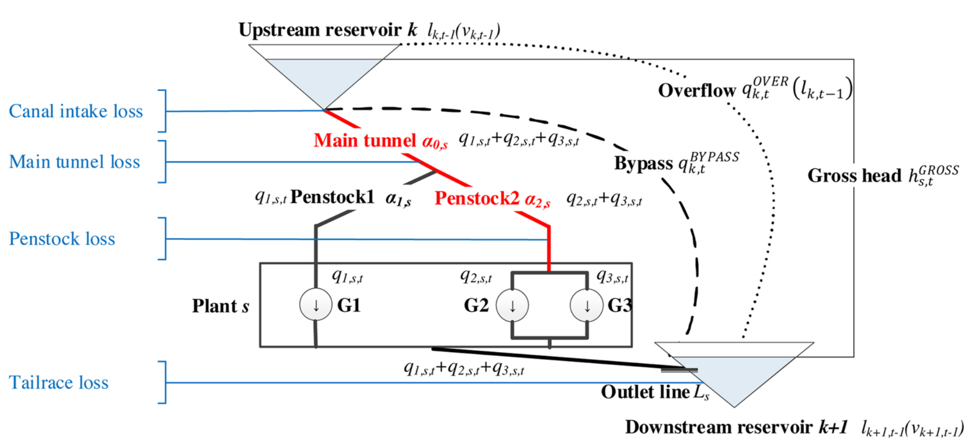

Fig. 1. Schematic illustration of head variation and flow-related head losses.

literature, the tailrace effect is usually included as the downstream head variation. No analytical relationship between tailrace level and total water release is available. It has to be determined from experimental or measurement data [21]. It can be described by a piecewise linear function [40,53,65] or polynomial function [13,14,17,19–24,35,36,43,52] of the total outflow from the plant. If the hydraulic cohesive relationship exists, the tailrace elevation is also influenced by the water level of the immediate downstream reservoir [13,19,40,53,60]. The authors of Ref. [8] presented a detailed model of tailrace elevation that has a nonlinear dependence on the downstream reservoir elevation and moves between two modes: encroached and not encroached.

Head losses are additive along a path in the hydraulic network, depending on the flow in the different sections (canal, main tunnel, penstock, outlet) of the network from the upstream to the downstream water surfaces. Therefore, the net head available at a turbine is a function of the flow of all the units. This multi-level configuration for the hydraulic network is represented as a tree structure presented in Ref. [14].

In addition, in some very detailed models, hydraulic efficiency and mechanical losses in the turbine, and mechanical and electrical losses in the generator are also taken into account [34,35]. These data can be obtained from meters and sensors. Reference [35] provides a thorough explanation of the measurement and calculation for these losses.

2.2.4. Constraint 4: hydropower production

The core of the STHS problem is how to model the relationship between water discharge (input) and electrical energy (output) [34]. This relationship is defined by the hydropower production function (HPF), as expressed in Eq. (12). It is a complex state-dependent, nonlinear, and non-convex function. This function has been referred to in the literature as hydro unit generating input/output (I/O) characteristic [2,14,36], I/O curve [32], and unit performance curves [19,22,31].

$$p_{i,s,t} = G \cdot \eta_{i,s}^{GEN}(p_{i,s,t}) \cdot \eta_{i,s}^{TURB}(h_{i,s,t}^{NET},\ q_{i,s,t}) \cdot h_{i,s,t}^{NET} \cdot q_{i,s,t} \tag{12}$$

The definition of the net head $h_{i,s,t}^{NET}$ has already been provided above. Its complexity is highlighted in the net head formulation in Eq. (11). The turbine efficiency is associated with converting the net head potential energy in the reservoir into mechanical energy in the turbine. Therefore, it depends on the net head and the turbined flow [66]. For a given net head, the turbine efficiency can be as low as 60% at minimum discharge and as high as 95% at the best efficiency point [57]. After reaching the best efficiency point, the turbine efficiency will decrease with the increase of discharge. In the same way, the generator efficiency is related to the conversion of mechanical energy into electrical energy in the generator. It is typically higher than 95% and monotonically increasing with the generator output [57]. Fig. 2 gives an illustration for the head-dependent turbine efficiency and generator efficiency. In some cases, the unit's global efficiency is introduced and defined by the product of the efficiencies of turbine and generator [21,29,33,52,67].

The level of detail included in the HPF depends on the time horizon, temporal resolution, system size, available data, and goals of the model [8]. It may have a significant impact on the economic performance of the system [24]. Considering the trade-off between the accurate representation of the HPF and the computational tractability of the problem, one approach is to simplify or even leave out the nonlinearity and state-dependency in the formal optimization methods. For example, the turbine efficiency is a fixed value [3,14,24,59] or only flow-dependent [29]; the generator efficiency is unchanging over a wide range of operation [17] or even not mentioned [23,24,39].

If the generator efficiency is included in the HPF, it is usually represented as a concave function of power output $\eta_{i,s}^{GEN}(p_{i,s,t})$ [13,40]. By contrast, the mathematical expression of the turbine efficiency is considerably more complicated. The turbine efficiency is typically described by Hill chart or Hill diagram, which is composed of a set of discrete triplets relating the turbine efficiency values, the net head, and the water discharge. These points are usually provided by the turbine manufacturer [17,34,40,52,68], obtained by in-site measurement [66,69], or taken from a real Hill chart presented in the literature [33,67].

However, the original number of points presented in a Hill chart is not sufficient to obtain precise results. The hydro producers need a continuous curve presenting turbine efficiency for the full working area of the unit. Curve fitting can be achieved by (1) applying a regression technique from the set of triplets to build a high order polynomial for the turbine efficiency (see next paragraph); (2) using an interpolation method to estimate the value from the set of triplets. Usually, linear interpolation is employed to calculate efficiency [23,57]. More precisely, in Ref. [35], spline interpolation is first performed to increase the number of points when composing the Hill chart. The interval of power output is reduced from 1 MW to 0.25 MW and the interval of the net head is reduced from 1 m to 0.25 m. Following this, in real-time operation, the actual value of efficiency is found through linear interpolation on the interpolated curve. This way, it is possible to obtain a precise value of the turbine efficiency with a low computational cost. The work in Ref. [68] evaluated the performance of linear interpolation and spline interpolation for turbine efficiency curves in terms of the dispatch schedules in the day-ahead market and the bidding strategy in the intraday market. The reported conclusion is that spline interpolation is a promising alternative to linear interpolation for obtaining actual turbine efficiency in STHS.

In Refs. [17,20,21,33,34,52], the turbine efficiency $\eta_{i,s}^{TURB}(h_{i,s,t}^{NET},\ q_{i,s,t})$ is described by a second order polynomial of the net head and turbined flow (or, equivalently, power output [21]), whereas in Ref. [67] the terms are added up to the fourth order. However, due to

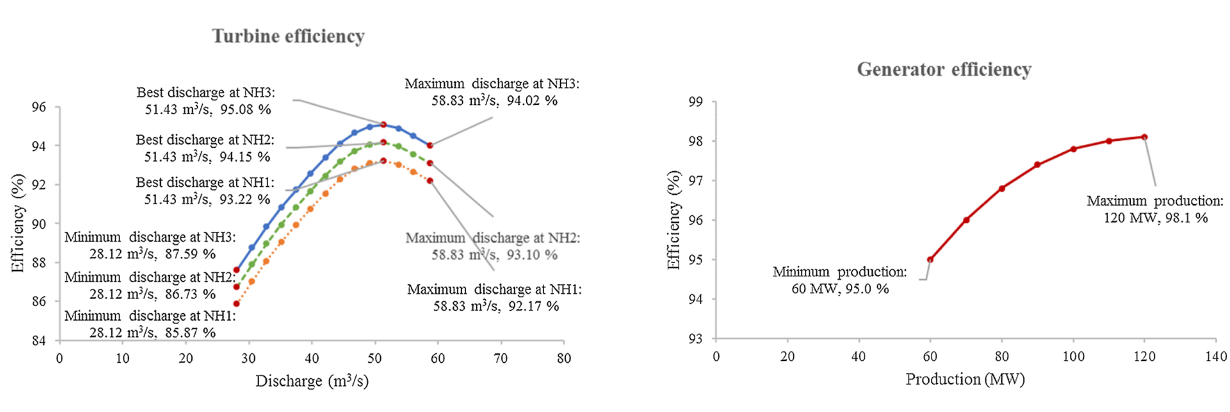

Fig. 2. Illustration for the head-dependent turbine efficiency and generator efficiency.

the nonlinearities of the original surface of the Hill chart, the nonlinear regression for the coefficients can give high relative errors, which are not acceptable in the optimization. Therefore, the authors of Ref. [67] suggested assigning the points different weights to balance the distribution of the available data, especially near the higher efficiency region. Furthermore, in Ref. [35], the authors divided the Hill chart into many segments and the regressions are performed individually in each segment. In this context, the HPF is represented as a high order nonconcave-nonconvex polynomial.

In contrast to explicitly considering the turbine efficiency and the net head as in Eq. (12), the HPF can also be formulated as a quadratic function of water release and reservoir volume. This formulation is common when the STHS problem is modeled on an aggregated plant level [4–6,9]. However, it may lead to significant inaccuracies in the cases where there are multiple units in the plant [3], and, therefore, is not recommended when the unit-based STHS problem is considered.

Another conventional technique to incorporate the nonlinearity of the HPF is to use a piecewise linear approximation [2,18,19,22,31,32,38,39]. The breakpoints (i.e. the given pairs of water discharge and power output) of the function are usually specified in advance and represented by a two-dimensional table for fixed-head (i.e. a single I/O curve [2,38]) or a three-dimensional matrix for head-sensitive (i.e. a family of I/O curves [18,19,22,31,32,39]). The determination of the breakpoints in the three-dimensional convex hull before optimization was presented in Refs. [3,23,29]. DP in Ref. [29] and UC heuristic algorithms in Ref. [23] are used to compute the maximum power output for a discretization grid of water discharge and reservoir volume, respectively. In Ref. [40], the authors proposed a new method to dynamically update the breakpoints considering unit efficiency, head variation, and hydraulic losses, but requiring only one binary variable per unit and period.

2.2.5. Constraint 5: limits of power production and water discharge of the unit

Eq. (13) determines the lower and upper output power limits of the generator, whereas Eq. (14) corresponds to the permissible discharge range of the turbine (minimal and maximal flows). The physical limits of unit output power are usually constant [14–16,18–22,24,35,40] since dissipated power does not depend on the net head, and the rotational speed of turbines is constant [14]. Nevertheless, the discharge range can be fixed [15,18,19,22,24,33,39,51] or head-dependent [13,19,20,23,32,34,35,40].

$$P_{i,s}^{\mathrm{MIN}} \bullet \omega_{i,s,t} \le p_{i,s,t} \le P_{i,s}^{\mathrm{MAX}} \bullet \omega_{i,s,t} \tag{13}$$

$$Q_{i,s,t}^{\mathrm{MIN}}\,(h_{i,s,t}^{NET}) \bullet \omega_{i,s,t} \le q_{i,s,t} \le Q_{i,s,t}^{\mathrm{MAX}}\,(h_{i,s,t}^{NET}) \bullet \omega_{i,s,t} \tag{14}$$

In real life, the operating limits for the hydro-turbine generator unit are complex, associated with the net head, turbine discharge, and generator output. If the net head is lower than certain value (e.g. the nominal value), the turbine is unable to make the generator achieve its maximum power output. If this is the case, the operating limit is decided by the variable maximum discharged outflow as a function of the net head $Q_{i,s,t}^{\mathrm{MAX}}(h_{i,s,t}^{NET})$. On the other hand, if the net head is higher than the nominal value, the turbine could easily reach the power level beyond the maximum output. If so, the operating limit should be imposed by the power limit on the generator capabilities $P_{i,s}^{\mathrm{MAX}}$ [34]. The unit minimum operating limit has the reverse behavior.

It is possible to set bounds on operational limits and the total water flow released in the plant [18,19,23,29,32,37–39], as expressed in Eqs. (15) and (16), respectively. Although the limits are static as presented, it is possible to modify the constraints to vary with time.

$$P_s^{\mathrm{MIN}} \le \sum_{i \in I_s} p_{i,s,t} \le P_s^{\mathrm{MAX}} \tag{15}$$

$$Q_s^{\mathrm{MIN}} \le \sum_{i \in I_s} q_{i,s,t} + q_{k,t}^{BYPASS} \le Q_s^{\mathrm{MAX}} \tag{16}$$

For some hydro units, mechanical vibration, cavitation phenomena, and efficiency loss will result in certain operational forbidden zones. Units will not be allowed to operate in these specific ranges. When modeling the regularly shaped forbidden zone that is independent of the net head, one more index indicating the operating zone should be added to the status binary variable and lower and upper bounds on the turbined water or generated power. A constraint establishing the coherent relationship between operating zone and the on/off status of the unit should also be added, implying that only a unique operating zone for each unit can be selected [17,23,34,39,52]. In contrast to the aforementioned medium- and small-size units with a single regularly shaped forbidden zone, large-size units have multiple and irregularly shaped forbidden zones varying with the net head, which can be approximated by several simple polygons [22].

2.2.6. Constraint 6: operation status of the unit

Eqs. (17) and (18) reflect the start-up decision of the unit, based on the commitment status of the units during two consecutive periods [2,18,19,31,32,40]. Shut-down decisions can be straightforwardly included [39]. If necessary, the constraint associated with the minimum and maximum numbers of online units for one plant at one period can be added [2].

$$\omega_{i,s,0} = \Omega_{i,s,0} \tag{17}$$

$$\mu_{i,s,t} \ge \omega_{i,s,t} - \omega_{i,s,t-1} \tag{18}$$

Since frequent start-up shortens the lifetime of the unit as a result of mechanical stress, corresponding start-up costs are introduced to discourage frequent on/off operation of the unit, as expressed in Eq. (2). In addition, logic constraints can be added to force each unit to remain online for at least a certain time after it is switched on [19,20,22,39]. Alternatively, the authors of Ref. [30] chose to penalize the variations of turbined flow in each plant. The penalization choice accelerates the MILP solving time by limiting the impact of the binary variables on the objective.

2.2.7. Constraint 7: power balance

In a competitive electricity market, the power generated can be sold to the market (and the power consumed can be bought from the market if there are pumped storage hydropower plants (PSHPs) in the system considered). This relationship is represented by the energy balance constraint for the plant in Eq. (19).

$$\sum_{s \in S} \sum_{i \in I_s} p_{i,s,t} = p_t^{SELL} \tag{19}$$

2.2.8. Constraint 8: coupling to long-term/mid-term strategy

STHS should implement the long-term/mid-term strategy in the best possible way during operation of the watercourse. Adequate signals must be transferred from the strategic level to the operational level. There are three main coupling signals. Note that it is not necessary to give all the coupling signals at the same time. If the final reservoir contents are defined by a mid-term planning procedure, the future value of stored water can be set to 0 [31].

1) Load coupling D_t by meeting load obligation, either for the whole system (especially in the ULD problem as the total load) [2,15,16,19,35,37], for different regions [23] or for each plant [20–22,34]. In a centralized system, the purpose of the objective function is to maximize the efficient use of water resources. The target power defined by mid-term planning must be satisfied. Usually, Eqs. (19) and (20) can be combined as one constraint.

$$\sum_{s \in S} \sum_{i \in I_s} p_{i,s,t} = D_t \tag{20}$$

2) Volume coupling $V_{k,t}^{END}$ by keeping the storage volume of each reservoir at the end of scheduling horizon as close as possible to the target volume [2,32,36,37,51,52]. The final target storage can also be set within a certain range [39,59]. In some cases, it requires the end water storage to be the same as the initial storage [6,24,31].

$$v_{k,\bar{t}} = V_{k,t}^{END} \tag{21}$$

3) Price coupling $W_{k,t}^{END}$ by adding the reservoir value at the end of scheduling horizon into the objective function [7,30,33,37,38,40], as expressed in Eq. (3b). It is worth mentioning that if the unit of $W_{k,t}^{END}$ is given as €/MWh rather than €/m^3, an energy conversion factor E_s is needed to build the ending condition for each reservoir, indicating the electricity that can be produced with each m^3 of water. How to define the energy conversion factor is crucial. The higher up the reservoir is located, the larger this value should be because more electricity can be produced with the same amount of water due to the cascade effect [38]. Nevertheless, the assumption of the constant energy conversion factor is acceptable as shown in

Table 1
Summary of model formulation and solution method for unit-based STHS problems

Author	STHS problem	Model formulation	Solution method	Hydro system	Country of case study
Shawwash et al. [37]	ULD	LP	CPLEX[a] solver, an iterative procedure	Cascade	Canada
Arce et al. [13]	ULD	NLP	DP	One plant	Brazil
Breton et al. [14]	ULD	NLP	Decomposition algorithm, DP	One plant	Canada
Cheng et al. [15]	ULD	NLP	1) DP; 2) PSO[b]	One plant	China
Perez-Diaz et al.[36]	UC + ULD	NLP	Preprocessing: DP; DP, an iterative procedure	One plant	Spain
Bortoni et al. [16]	ULD	NLP	Steepest Ascent Hill Climbing heuristic	One plant	Brazil
Chang et al. [2]	UC + ULD	MILP	CPLEX solver	Cascade with pumped storage	New Zealand, Switzerland
Conejo et al. [31]	UC + ULD	MILP	CPLEX solver	Cascade	Spain
Garcia-Gonzalez et al. [32]	UC + ULD	MILP	CPLEX solver, an iterative procedure	Cascade	Spain
Borghetti et al. [18]	UC + ULD	MILP	CPLEX solver	One plant with pumped storage	N/A
De Ladurantaye et al. [38]	UC + ULD	MILP	CPLEX solver, an iterative procedure	Cascade	Canada
Tong et al. [39]	UC + ULD	MILP	CPLEX solver	Cascade	China
Li et al. [19]	UC + ULD	MILP	LINGO[c] (B&B[d]), an iterative procedure	One plant	China
Seguin et al. [29]	UC + ULD	1) NLP & MILP 2) MINLP	Preprocessing: DP; 1) IPOPT[e], Xpress[f] solvers 2) BONMIN[g] solver	Cascade	Canada
Cheng et al. [22]	UC + ULD	MILP	LINGO (B&B)	One plant	China
Guedes et al. [23]	UC + ULD	MILP	Preprocessing: UC heuristic algorithm; GUROBI[h] solver	Cascade	Brazil
Marchand et al. [30]	UC + ULD	MILP decomposed to LP & MILP	LR (CPLEX solver)	Cascade	Canada
Skjelbred et al. [40]	UC + ULD	MILP	CPLEX solver, an iterative procedure	Cascade	Norway
Finardi and Silva [17]	UC + ULD	MINLP	B&B, RPG[i] method	One plant	Brazil
Finardi and Silva [52]	UC + ULD	MINLP decomposed to LP, MILP, & NLP	LR (CPLEX solver, SQP[j] algorithm)	Cascade	Brazil
Diaz et al. [33]	UC + ULD	MINLP	CPLEX solver	Cascade	Spain
Finardi and Scuzziato [34]	UC + ULD	MINLP decomposed to LP & MINLP	LR, IAL[k]	Cascade	Brazil
Lima et al. [24]	UC + ULD	MINLP	1) Spatial B&B; 2) CPLEX, DICOPT[l], BARON[m] solvers	Cascade	Brazil
Cordova et al. [35]	UC + ULD	MINLP	N/A[n]	One plant	Brazil
Finardi et al. [20]	UC + ULD	1) MINLP decomposed to LP & MINLP; 2) MINLP; 3) MILP	1) LR (CPLEX solver, AOA[o] solver, IAL); 2) AOA solver; 3) CPLEX solver	Cascade	Brazil
Santo and Costa [21]	UC + ULD	MINLP	DICOPT solver	Cascade	Brazil

[a] Solver for linear programming/mixed integer programming/quadratic programming from IBM ILOG [94].
[b] Particle swarm optimization.
[c] Optimization modeling software with built-in solvers for linear programming/nonlinear programming/quadratic programming/integer programming from LINDO Systems Inc. [95].
[d] Branch and Bound.
[e] Open source "Interior Point OPTimizer" solver for large-scale nonlinear optimization from COIN-OR [96].
[f] Linear programming/mixed integer programming/nonlinear solver from FICO optimization [97].
[g] Open source "Basic Open-source Nonlinear Mixed INteger programming" solver for mixed integer nonlinear programming from COIN-OR [98].
[h] Solver for linear programming/mixed integer programming/quadratic programming from Gurobi optimization [99].
[i] Rosen's Projected Gradient.
[j] Sequential Quadratic Programming.
[k] Inexact Augmented Lagrangian.
[l] "DIscrete and Continuous OPTimizer" solver for mixed integer nonlinear programming from the Engineering Design Research Center (EDRC) at Carnegie Mellon University [100].
[m] "Branch-And-Reduce Optimization Navigator" solver for nonlinear programming/mixed integer nonlinear programming from University of Illinois at Urbana-Champaign [101].
[n] Not Available.
[o] White box "AIMMS Outer Approximation" module for mixed integer nonlinear programming from AIMMS [102].

Table 2
Summary of the characteristics of optimization model for unit-based STHS problems.

Author	Objective	Penstock structure	Head variation	Head loss	Coupling to long/mid-term strategy
Shawwash et al. [37]	Maximize the total profit	N/A	No	Tailrace	Target volume, water value
Arce et al. [13]	Minimize the total power losses	Independent	No	Penstock, Tailrace	N/A
Breton et al. [14]	Maximize the power generated for a given flow	Independent & Shared	No	Penstock, Tailrace	Target volume
Cheng et al. [15]	Minimize the total discharge of units for a given load	N/A	No	No	Target load
Perez-Diaz et al. [36]	Maximize the revenue	N/A	Yes	Penstock, Tailrace	Target volume
Bortoni et al. [16]	Maximize the penstock end pressure (net head)	Shared	No	Penstock	Target load
Chang et al. [2]	Maximize the final water storage in the reservoirs	N/A	No	No	Target load, Target volume, water value
Conejo et al. [31]	Maximize the total profit	N/A	Yes	No	Target volume, water value
Garcia-Gonzalez et al. [32]	Maximize the total profit	N/A	Yes	No	Target volume
Borghetti et al. [18]	Maximize the total profit	N/A	Yes	No	Target volume
De Ladurantaye et al. [38]	Maximize the total profit	N/A	Yes	No	Water value
Tong et al. [39]	Maximize the total profit	Independent	Yes	Penstock, Tailrace	Target volume
Li et al. [19]	Minimize the total discharge of units for a given load	N/A	Yes	Constant penstock, Tailrace	Target load
Seguin et al. [29]	Maximize the power generated for a given flow	Independent	Yes	Penstock, Tailrace	Target volume
Cheng et al. [22]	Minimize the total discharge of units for a given load	Independent	Yes	Penstock, Tailrace	Target load
Guedes et al. [23]	Maximize the final water storage in the reservoirs	N/A	Yes	Penstock, Tailrace	Target load
Marchand et al. [30]	Maximize the final value of water stored in the reservoirs	N/A	Yes	Tailrace	Water value
Skjelbred et al. [40]	Maximize the total profit	Independent & Shared	Yes	Intake, Penstock, Tailrace	Water value
Finardi and Silva [17]	Maximize the power generated for a given flow	Independent	Yes	Penstock, Tailrace	Target load
Finardi and Silva [52]	Maximize the total profit	Independent	No	Penstock, Tailrace	Target volume
Diaz et al. [33]	1) Maximize the total profit; 2) Maximize the efficiency	Independent	Yes	No	Water value
Finardi and Scuzziato [34]	Minimize the total discharge of units for a given load	Independent	Yes	Penstock, Tailrace, Mechanical, Generator loss	Target load
Lima et al. [24]	Maximize the total profit	N/A	Yes	Constant penstock, Tailrace	Target volume
Cordova et al. [35]	Minimize the total discharge of units for a given load	Independent	No	Intake, Penstock, Tailrace, Mechanical, Generator loss	Target load
Finardi et al. [20]	Minimize the total discharge of units for a given load	Independent	Yes	Penstock, Tailrace, Mechanical, Generator loss	Target load
Santo and Costa [21]	Minimize the total power losses	Independent	No	Penstock, Tailrace	Target load

Eq. (3b) for large reservoirs while it incurs errors for small ones [49].
In addition, water value in Eq. (3b) is assumed to be an independent fixed value. In practice, this value can be expressed as a concave piecewise linear function of the water volume in the cascaded reservoirs (known as “cuts”) [38]. This is usually provided by a long-/mid-term hydro scheduling model that would integrate the stochastic nature of inflows, electricity prices, load, and non-dispatchable RE in the power system [70,71].

In Ref. [33], the authors showed in the case study that, in a free ending volume model with water values, higher total profit can be achieved. The water can be discharged freely in an optimal way and be stored in reservoirs with better water values. Therefore, a price coupling signal is expected to be more accurate and comprehensive than a volume coupling signal.

2.2.9. Other constraints

Environmental constraints imposed on reservoir management and hydropower plant operation are usually given in the form of minimum environmental flows and, in some cases, in the form of maximum and minimum rates of change of flows, or ramping rates [72]. Ramping constraints restrict the change of reservoir volume and plant discharge between two successive time intervals [2,18]. They can be imposed to reduce the rapid change in water level of reservoirs to avoid excessive wear of the shores of the reservoirs [73].

If one wants to keep the reservoirs/gates releasing the water or plants/units running at a certain level, schedules or committed-to-run constraints can be activated. However, the use of this type of constraints requires a high level of knowledge of the entire hydro system. Too many schedules could lead to a problem without feasible solutions.

The STHS problem for a PSHP is analogous to that of a pure power generating plant [74]. For a reversible pump-turbine unit, one more binary variable should be introduced to indicate the operation status of the pump. The exclusive constraints should be added to forbid simultaneously pumping and generating [2,18]. For a given head, the rate of pumped water flow for a fixed speed pump (FP) is fixed, hence the consumed power is fixed. It is somehow easier to model the FP in STHS. Head variation is usually neglected in pumping mode [18,75]. By contrast, a variable speed pump (VP) can be operated within a certain range in both generating and pumping modes. Therefore, a plant with VPs faces the same challenges in STHS as a pure power generating plant confronts [76]. Authors in Ref. [77] reviewed the trends and challenges in the scheduling models for PSHPs.

When a hydro producer participates in both energy and capacity markets, an optimal decision involves not only the optimal generation scheduling of available units but also the reserved capacity for various types of ancillary services. Adding operational reserve requirements into the STHS problem is prevalent. Initially, the reserve is simply expressed as constraints such that the sum of power produced and reserved should be no more than the maximum production, either on the plant level [17,19,52] or the entire system [2,30]. Nowadays, more detailed constraints should be added to the optimization problem for distributing reserve obligations considering response time, reserve types, and market regulations [38,78–82]. In the context of reserve-driven operation strategies, one of the trends in operation of the PSHPs is to employ hydraulic short-circuit (HSC) scheme as a means to provide spinning reserves [77]. HSC occurs in a plant when at least one unit operates in pumping mode and the others in generating mode, which increases the operational flexibility of the plant [83–85].

Aside from the above-mentioned reservoirs, plants, and units, other hydraulic objects such as gates, junctions [86], and pressure links [87] are also important components in the hydro system. Gates are used to connect reservoirs (in this case they are also referred to as tunnels [49,88]), create a bypass of plants, and add routes for the spill. Junctions and pressure links are used to model the topology where a plant is draining from two reservoirs at the same time. Necessary constraints should be introduced to guarantee the flexibility in the network as well as to obey the law of physics.

3. Optimization methods

Exact methods based on mathematical programming take advantage of the analytical properties of the optimization problem to generate a sequence of points that converge to a globally optimal solution [89]. The efficiency in solving optimization problems, the solid mathematical foundation, and the availability of commercial solvers make exact methods used by the majority of reported implementations [25].

Exact methods primarily include linear programming (LP), nonlinear programming (NLP), MILP, mixed integer quadratic programming (MIQP), mixed integer nonlinear programming (MINLP), DP, and Lagrangian relaxation (LR). The first five methods focus on the mathematical formulation of the problem, whereas the last two relate to solution strategies to solve the problem.

The unit-based STHS problems solved by the exact methods and published in the scientific journals since 2000 are summarized below. Due to the page layout, the summary is split into two tables. Table 1 lists the type of STHS problem, model formulation, solution methods, and hydro system the papers focus on. Table 2 outlines the characteristics of the optimization model. For the authors who published many papers regarding the same mathematical formulation or solution methodology, the paper which includes the most features of the STHS problem is chosen. For example, Finardi and co-authors have brought many contributions in modeling the STHS problem as well as the unit-based scheduling problem of hydrothermal systems [90–92] or hydro-thermal-RE hybrid systems [93] with a highly detailed level in the framework of MINLP. Five of their papers focusing on pure hydro scheduling and representing the typical evolution of the mathematical formulation and solution strategy are presented in Tables 1 and 2.

3.1. Linear, nonlinear and dynamic programming

Limited by the computing environment, the earlier mathematical formulation for the STHS problem is mostly LP [37,49]. Commercial solvers such as MINOS [103] and OSL [104] based on the classical simplex method and the interior point method are widely used to solve the LP model. However, LP formulation does not fully represent the physical characteristics of the hydro generation [105].

The nonlinearity and non-convexity of the hydro generation, referring to the HPF in Eq. (12), can be handled well in the framework of NLP [13–16,36], especially for solving the ULD problem that addresses the optimal distribution of the production among a set of online units within a plant. In that context, a DP algorithm is adopted to solve the problem [13–15,36]. Actually, DP has been one of the most popular optimization techniques used to solve the STHS problem for a single hydropower plant with a small number of units and low installed capacity [15]. In Ref. [29], DP is used in the preprocessing of optimization to maximize total power output for given water discharge, a reservoir volume, and a given number of units. There is no approximation of the data.

However, DP has limitations when the number of discrete states becomes very large (e.g. the unit start-up and shut-down status). This difficulty can be avoided if the number of units in operation is taken as a state variable [13]. In Ref. [36], the authors used a state diagram defined by carefully discretizing the feasible region to effectively include units' start-up and shut-down in the STHS problem.

Another disadvantage of DP is the so-called "curse of dimensionality". If there is a large number of units with huge installed generating capacity in a hydropower station, it is hard to apply DP to find the solution [15]. Moreover, if there are two or more cascaded reservoirs in the system, using DP-based methods to solve the STHS problem becomes enormously challenging [36,39].

3.2. Mixed integer linear programming

With the development of the mathematical and computational techniques, additional details of the problem have been addressed. The discrete nature of the STHS problem such as the on/off status of the units or operational forbidden zones cannot be ignored. Binary integer variables, also called 0/1 variables, are introduced to the mathematical programming method. On the other hand, if the nonlinearity of the HPF is approximated by piecewise linear functions, together with the discrete nature, the STHS problem is formulated as an MILP model [2,18,19,22,23,30–32,38–40].

MILP has good performance with respect to adding constraints and solution efficiency. It has been widely applied to solve large-scale STHS problems with continuous and discrete variables [2]. In the earlier work [49], the non-convex part in the low power operating region is omitted, and the HPF is approximated by a two-segment piecewise linear I/O curve, with breakpoints at the best efficiency point and the maximum discharge limit. In Ref. [2], binary variables are introduced to indicate the unit on/off status, and hence the first point of the two-segment linear curve starts from the minimum discharge limit. Head variation effect is first successfully modeled in Ref. [106] by developing a piecewise linear approximation of the nonlinear and non-convex I/O surface of water discharge and reservoir volume. Almost at the same time, the authors in Ref. [31] used a pre-defined number of piecewise linear non-concave I/O curves to represent the head effect. In both papers, the whole non-concave curve and head effect are considered through the use of binary variables. In Ref. [18], the authors proposed a tight representation of the head effect in which the linearization is enhanced through two-dimensional considerations of both water storages and flow. Instead of including all the candidate I/O curves, an iterative procedure is introduced in Ref. [32] to update the head until convergence is achieved. In Ref. [38], the nonlinear head effect is evaluated using a successive linear programming method. The authors of [19] used a case study of the Three Gorges Project, the world's largest and most complex hydropower system with 32 heterogeneous generating units in operation, to demonstrate an accurate representation of the unit HPF by a three-dimensional piecewise linear approximation. In Ref. [39], the authors discussed the effects of piecewise linearization of the nonlinear functions and presented a method to ensure the solution feasibility for the original nonlinear formulation of the HPF. In Ref. [22], based on the discretized net head intervals, the HPF is approximated as a set of piecewise curves and the head-sensitive characteristic of multiple irregularly shaped forbidden zones is effectively handled by several simple polygons.

B&B and cutting plane are among the most common methods used to solve the MILP problem. In the most real-world application, commercial solvers such as CPLEX, GUROBI, and Xpress are employed to directly solve the problem. The authors of Ref. [107] discussed the convexity issues of some standard commercial mixed integer programming solvers.

The main drawback of the mixed integer programming approach is the high computational burden, especially when it is applied to the large-size STHS problem. Although the commercial solvers can obtain solutions at any desired precision level, the results are achieved at the cost of higher calculational times. In some cases, it may even be very hard with direct approaches and standard commercial solvers [30]. Providing an initial feasible integer solution to the problem can improve the B&B search methods and reduce the solution time [108].

Furthermore, the effect of linearization of nonlinear functions will result in deviations between calculated values from MILP model and the real values [24] or infeasibility of the original problem [39]. An increase in the number of breakpoints can overcome the deviation between the piecewise linear model and the true nonlinear function, but also leads to increasing computational time. A dynamic approach in which breakpoints are included dynamically as the solving procedure evolves was presented in Ref. [109]. This approach drastically decreases the solution time. However, this method is based on the existence of a "complete" piecewise linear model with a very dense discretization grid (1000 breakpoints in the discharge axis) [3].

3.3. Mixed integer nonlinear programming

If the STHS problem is modeled with more details, e.g. to find schedules for cascaded plants with multiple units considering head variation in both forebay and tailrace, varying penstock, and mechanical and electrical losses in turbine and generator, it becomes a MINLP model [17,20,21,24,33–35,52].

If the number of units in a hydropower plant is not too large, the MINLP problem can be solved in an enumerative way [35]. That is to say, for each feasible combination of units that meets the power target requirement, the system solves the associated continuous nonlinear optimization problem by means of an optimization subroutine and determines the combination of units with the lowest water consumption. The authors of [29] developed a two-phase optimization method. First, it solves the relaxation of a MINLP program to find the volume, water discharge, and a number of active units at each period. Then it solves a MILP model to find the exact combination of units that maximizes the total production but also penalizes the start-up of units. The comparison between the outputs of the two-phase method and the MINLP model shows that the proposed method solves all the test cases in a computational time that is significantly lower than in the one-phase optimization process. The authors of [24] gave a clear and concise introduction of available MINLP solvers that can address the deterministic global optimization of MINLP problems and presented a spatial B&B algorithm to solve a detailed MINLP model.

3.4. Lagrangian relaxation

Large-scale optimization problems are usually decomposed into a set of smaller subproblems that are mathematically solvable and computationally efficient. Decomposition methods are used to solve this type of problem using an iterative-based methodology. Among the various decomposition techniques found in the literature, LR is one of the most successful methods to solve the large-scale STHS problems, especially within the MINLP formulation framework [26]. Since the units are usually coupled through power target requirements [30] and/or water balance equations [20,34], these coupling constraints can be relaxed by including them into the objective function and weighted by Lagrangian multipliers. Following this, the corresponding dual problem is decomposed into independent subproblems.

In Ref. [20], the authors compared the solution quality and computational performance when the problem is solved by LR and a MINLP solver using the AIMMS outer approximation (AOA) algorithm. In Ref. [30], the authors presented an effective three-phase solution method that takes advantage of both MILP and LR. This method obtains a good solution much more quickly than commercial solvers and gives a measure of the solution quality.

LR decomposition technology comprises three major steps: (1) dualization, (2) finding a solution to the dual subproblems, and (3) finding a feasible optimal or near-optimal solution to the primal problem. In the first step, two dualization methods are extensively used [110]. One is to dualize complicating constraints that tie together the problem. For the STHS problem, it is to dualize the spatial and temporal coupling constraints, i.e. water balance expressed in Eq. (6) [52]. The other is to duplicate the common variables involved in two (or more) subproblems and then to relax the equality constraints. In the second step, the dual subproblems are resolved and the Lagrangian multipliers are updated in each iteration by using a subgradient approach such as Bundle algorithm [52,111]. If the subproblem is still a MINLP problem, new decomposition can be introduced. In the last step, augmented Lagrangian (AL) or inexact augmented Lagrangian (IAL) [34] is generally used to find a primal feasible solution.

The problem for the LR approach is that although some complex linking constraints can be relaxed by LR, the nonlinear and non-convex HPF makes it very difficult to obtain the true dual function. If this is the case, the algorithm convergence cannot be guaranteed [39]. In addition, the processes to find a primal feasible solution are often based on heuristics, depending on the particular problem structure [52].

Given the problem complexity, the specific purpose, and computational capacities, each method has its own advantages and disadvantages and is generally efficient in a particular operational context. However, there is no guarantee that any solution will be globally optimal [20]. Comparing results from different methods is further complicated when the models have different constraints and levels of accuracy. For example, in [20], the authors tried to convert the MINLP formulation for the STHS problem into an MILP formulation by using a triangulation technique of piecewise linear approximation. Then the quadratic penstock head loss in the original formulation was assumed to be a constant value in the framework of MILP. It is thus neither fair nor adequate to ascertain that one method outperforms another.

4. Conclusions

Compared to the widespread publications of the STHS problem formulated on an aggregated plant level, significantly fewer works in the literature address optimal scheduling for individual hydro-turbine generator units. However, it is important to consider the representation of each unit separately in order to model its efficiency accurately and obtain practical solution. Moreover, the participation of hydro producers in both energy and capacity markets and the quick adaptation to the changeable markets emphasize the necessity for and importance of unit-based modeling.

In this overview, a detailed classification of different approaches to model and solve the unit-based STHS problem is presented. Given the possible objectives and constraints, various modeling techniques proposed in the publications since 2000 are discussed. The most common mathematical programming methods are summarized.

Although it is widely admitted that the realistic and detailed representation of the HPF is critical for obtaining reliable results, simplification cannot be avoided in most of the works reported in the literature, even in those already highlighting the importance of developing accurate models to describe the HPF. For example, the water level is kept static within the planning horizon irrespective of the characteristics of the reservoir [55]; the turbine efficiency is a fixed value [3,14,24,59] or only flow-dependent [29]; the generator efficiency is unchanging over a wide range of operation [17] or even not mentioned [23,24,39]; the penstock head loss is either constant [19] or calculated as a percentage of the power output [3,24] or a percentage of the net head [59], regardless of the flow going through the tunnel; and the units are always fed by independent penstocks [13,17,21–23,34,39,52,60]. These types of simplification neither reflect enough of the complexity of real-world operations nor match the requirements from the hydro producers. Therefore, finding a computationally solvable and effective way to handle these issues will be crucial in the future work for the unit-based STHS problem. It will also be valuable to conduct a quantitative comparison to indicate the effect of usual simplifications on the optimal results through a real-world case study.

Declaration of interests

The authors declare that they have no competing interests.

CRediT authorship contribution statement

Jiehong Kong: Conceptualization, Writing - review & editing, Supervision. **Hans Ivar Skjelbred:** Conceptualization, Writing - original draft. **Olav Bjarte Fosso:** Supervision, Writing - review & editing, Supervision.

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