# The peak stress method to calculate residual notch stress intensity factors in welded joints

M. Colussi  P. Ferro  F. Berto  G. Meneghetti

## Abstract

According to the recent literature, the intensity of linear elastic residual stress fields near the toe region of a welded joint can be quantified by the residual notch stress intensity factors (R-NSIFs). The computational effort required to compute the R-NSIFs implies strong limitations of applicability in practice, owing to the very refined meshes needed and to the non-linear transient nature of welding process simulations, especially in 3-dimensional numerical models of large structures. The peak stress method (PSM) is a design approach that takes care of the industrial needs of rapidity and ease of use. According to the PSM, it is possible to evaluate the R-NSIFs by using the peak stress calculated at the point of singularity with coarse finite element (FE) models. While the PSM was originally calibrated by using the Ansys FE code, in the present contribution, the PSM has been calibrated to rapidly estimate the R-NSIFs in the Sysweld FE environment.

## 1 INTRODUCTION

Residual stresses induced by welding processes could significantly affect high cycle fatigue life of welded joints1, 2: tensile residual stresses have unfavourable effects on high cycle fatigue, whereas in the presence of compressive residual stresses, the fatigue resistance is improved, which can be interpreted in terms of crack closure phenomenon.3-5 To assess the residual stress distribution, welding numerical simulation can be used.6-12 However, this approach is often limited by the required high computational time and cost that are related to very fine meshes near the weld bead, multipass operations, large 3-dimensional geometries, and non-linear and transient analyses.

In fatigue-susceptible welded structures, cracks usually initiate and propagate from the weld toe and the weld root regions, which are stress concentration points. According to the notch stress intensity approach, the weld toe is modelled as a sharp V notch (0 radius), as proposed in the literature in the last 2 decades.13-17 The mode I and mode II notch stress intensity factors (NSIFs) quantify the magnitude of the asymptotic stress distribution according to the Williams exact solution.18 In case of weld-like geometries, the NSIFs quantify the intensity of the elastic stress field near the weld toe and the weld root, which take into account the overall joint geometry (both local, ie, the weld shape effect, and global), the absolute dimensions, and the loading condition. A contribution to the NSIF analysis19 showed that near sharp V notches, thermal stresses induced by a steady thermal load have the same asymptotic nature of the stress fields induced by mechanical loads. As a consequence, new thermal load–induced NSIFs have been defined as the natural extension of those dependent on external mechanical loads; in case of transient thermal loads, the NSIFs related to residual stresses are identified as residual NSIFs (R-NSIFs).19, 20 In the literature,21 the R-NSIFs have been proposed to quantify the influence of residual stresses on the high cycle fatigue life of butt-welded joints, where a negligible stress redistribution due to local plasticity could be observed.

The main drawback in evaluating the R-NSIFs is that the asymptotic nature of the stress field close to the weld toe and root requires very refined finite element (FE) meshes, at least locally. The required degree of refinement is generally not easy to obtain in plane cases and very difficult in 3-dimensional cases.22 Additional computational effort is required by the transient and non-linear nature of the welding process simulations to compute R-NSIFs, compared with the linear elastic FE analyses to estimate the NSIFs13, 15, 23 induced by external applied loads. Therefore, strong limitations of applicability in practical design situations arise.

Nisitani and Teranishi24 showed that the linear elastic stress σI,peak, calculated at a crack tip through a linear elastic FE analysis characterized by a mesh pattern having a constant element size, can be used to estimate the mode I stress intensity factor KI for a crack initiating from an elliptical hole. In particular, they demonstrated that the KI-to-σI,peak ratio depends only on the FE size and type but does not depend on the crack size. As a consequence, the σI,peak value can be used to rapidly estimate the KI value, assuming that both the mesh pattern and the FE formulation have been previously calibrated on geometries for which the exact value of KI is known. Meneghetti and Lazzarin25 provided a theoretical justification to this approach, which was called peak stress method (PSM). Thereafter, aiming to rapidly assess the fatigue strength of fillet welded joints, the PSM has been extended to weld-like geometries and analytical expressions have been derived to estimate the NSIFs at the weld toe as well as the weld root.25-28

In a recent work, a preliminary investigation on the suitability of the PSM to rapidly estimate the R-NSIFs owing to the welding process has been performed and a good accuracy has been found.29 The present work is aimed to complete that investigation by first refining the PSM calibration in the Sysweld environment and then by deepening its applicability to R-NSIF calculation. Afterwards, a practical application of the PSM to evaluate the R-NSIFs is illustrated. The obtained results are in agreement with those reported previously and support the use of the PSM to evaluate the R-NSIFs.

## 2 THE PSM ANALYTICAL BACKGROUND

In plane problems, the degree of singularity of linear elastic stress fields near sharp V notches (having 0 radius) has been given by Williams,18 for both mode I (opening) and mode II (sliding) loading, under the assumption of linear elastic conditions. Considering a polar coordinate system (r, θ) having its origin located at the sharp notch tip (Figure 1) and using only the first term of the Williams expansion series under mode I loading, the local stress field near the notch tip is described by where q is related to the opening angle 2α by the expression $2\alpha = \pi(2 - q)$.

The mode I NSIF quantifies the intensity of the local stress field, and it can be expressed, according to Gross and Mendelson, by means of

where the stress components $\sigma_{\theta\theta}$ and $\sigma_{r\theta}$ are evaluated along the notch bisector (angular coordinate θ = 0). The stress singularity exponent $\lambda_1$ refers to the notch opening angles 2α analysed in the present work, which are equal to 0°, 90°, and 135°, and their values are 0.500, 0.544, and 0.674, respectively; as a consequence, Equation 1 contains a singular term ( urn:x-wiley:8756758X:media:ffe12757:ffe12757-math-0005) when r → 0.

According to the PSM, there exists an analytical expression that permits us to estimate the mode I NSIF from the singular, linear elastic, mode I peak stress σI,peak computed at the point of singularity. In the case of welded joints, the points of singularity are both the weld toe and the weld root (see Figure 2). The PSM fundamental expression is25

Open in figure viewerPowerPoint

Schematization of weld toe and weld root as sharp V notches having opening angles of 2α = 135° and 2α = 0°, respectively. Definition of the mode I peak stresses σI,peak evaluated by means of linear elastic finite element analysis

where K1 is the exact mode I NSIF, σI,peak is the linear elastic peak stress, and d is the average edge length of the FE pattern generated by a free mesh generation algorithm, whereas the parameter K*FE is the non-dimensional mode I NSIF, which depends on (a) the adopted FE formulation; (b) the mesh pattern generated by the free meshing algorithm; and (c) the nodal stress extrapolation and averaging criteria. Equation 4 can be applied within the limitations of applicability listed in the following section. Considering mode I loading, at present, the PSM has been calibrated25, 31 in the Ansys FE environment for 4-node quadrilateral plane elements and 8-node brick elements and sharp V notches having opening angles 2α ranging from 0° to 135°. The mesh pattern generated automatically by the free mesh generation algorithm available in Ansys was adopted,25, 31 after setting the average element size d to be used. As a result, the typical mesh patterns at the notch tip are reported in Figure 3: 4 elements must share the node located at the notch tip if 0° ≤ 2α ≤ 90°; 2 elements must share the node at the notch tip if 90° < 2α ≤ 135°. If these conditions are fulfilled, the mode I PSM calibration constant is equal to K*FE = 1.38 and, once fixed an arbitrary average FE size d such that the mesh density ratio a/d is at least equal to 3, Equation 4 estimates K1 through the linear elastic peak stress σI,peak with a scatter band of ±3%. Recently, the PSM has been calibrated also for other FE software packages different from Ansys in case of plane FE models.32

In the following section, the PSM calibration in the Sysweld environment is presented, as well as the limitations of its applicability.

## 3 CALIBRATION OF THE PSM IN THE SYSWELD ENVIRONMENT

A number of 2-dimensional FE models subjected to mode I loading conditions have been considered to calibrate the PSM in the Sysweld environment. For the sake of generality, geometries involved in the calibration consist of cracks, in pointed V notches, and in welded joints. The investigated geometries have been taken from the original calibration of the PSM performed in the Ansys software25 under mode I loading and are summarized in Figure 4. They include cracks at the tip of a U notch (Figure 4A); edge cracks in a finite-width plate (Figure 4B); a plate with lateral open V notches (Figure 4C); and a full-penetration cruciform welded joint having a weld reinforcement angle equal to 135° (Figure 4D). The material is assumed to be a structural steel having a Young modulus of E = 206 000 MPa and a Poisson of ratio ν = 0.3. A nominal stress equal to 1 MPa has been applied to the gross section. Taking advantage of the double symmetry of the investigated geometries, only a quarter of each model has been analysed.

image

Figure 4

Open in figure viewerPowerPoint

Plane geometries investigated to calibrate the peak stress method in the Sysweld environment, according to the original calibration performed in Ansys.23 Dimensions are in millimetres

To calculate the peak stress values, linear elastic static analyses under generalized plane strain condition (the reason why the calibration has been performed under generalized plane strain is reported in Section 5) have been carried out in Sysweld (release 12.0), and free mesh patterns according to the standard ones of PSM shown in Figure 5 have been used. The free mesh generation algorithm provided by Visual-Mesh (release 9.0) has been adopted after setting the desired average FE size d. Four-node quadrilateral elements, ie, 2004 type elements in the Sysweld library, have been chosen in the FE analyses. The numerical integration scheme was set to 2 × 2 Gauss points (full-integration scheme), which is the default option in Sysweld. Such choice is widely used in the literature when simulating the welding process, because it has proven to deliver results in good agreement with experimental residual stresses measurements.33-35 By varying both the notch or crack size a and the FE size d, many different mesh density ratios a/d have been considered. If the mesh pattern generated by the free mesh algorithm was not the standard one (Figure 5), the mesh would be simply regenerated after setting a slightly different FE size d. Most of the times, the generated mesh fulfilled the standard requirements at the first attempt. In some cases, the mesh had to be regenerated once; in 10 out of the 61 analyses performed (about the 15%), the mesh had to be regenerated twice. The final d value has obviously been adopted in Equation 4 to calculate the nondimensional NSIF K*FE. Finally, the maximum principal stress σI,peak computed at the node located at the V-notch tip (Figure 2) has been considered. To apply the PSM correctly, nodal stress averaging must be activated, which is the default setting in Sysweld, and it consists in averaging the stress components extrapolated at FE nodes and afterwards in computing the nodal principal stresses from the nodal averaged components.

image

Figure 5

Open in figure viewerPowerPoint

Standard peak stress method mesh patterns generated by the Visual-Mesh free meshing algorithm

The exact values of mode I NSIFs K1, needed to compute K*FE according to Equation 4, were obtained from local stress fields computed through the Ansys software with very fine mesh patterns, where the size adopted in the FE mesh was on the order of 10−5 mm, according to Lazzarin and Tovo.13

The obtained results are shown in Figure 6 where the nondimensional ratio K*FE defined by Equation 4 is plotted as a function of the mesh density ratio a/d. Figure 6 shows that the computed values of K*FE are arranged in 2 separated scatter bands, depending on the notch opening angle. More precisely, the upper scatter band with K*FE = 1.90 ± 5% refers to the geometries represented in Figure 4A,B, having notch opening angle 2α = 0°, while the lower scatter band with K*FE = 1.64 ± 5% refers to the geometries showed in Figure 4C,D, having notch opening angles equal to 90° and 135°. In both cases, the convergence is guaranteed for a mesh density ratio a/d ≥ 4, which is slightly different from the original calibration,25 where convergence was achieved for a mesh density ratio a/d > 3.

image

Figure 6

Open in figure viewerPowerPoint

Nondimensional mode I notch stress intensity factor K*FE evaluated from the 61 calibration finite element analyses (see Figure 4) in the Sysweld environment. Full-integration scheme and stress averaging from nodal stress components are used. PSM, peak stress method

It is worth summarizing in the following the preprocessing and postprocessing options under which the results shown in Figure 6 are valid:

4-node quadrilateral element from the Sysweld library with a full-integration scheme;

generalized plane strain condition;

free meshing algorithm implemented in Visual-Mesh; and

nodal principal stresses calculated from averaged nodal components (default option in Sysweld).

In the original PSM calibration performed in the Ansys environment,25 the nondimensional NSIF K*FE was found to be equal to 1.38 under mode I loading condition, with a scatter of ±3% for the same geometries investigated here. Conversely, different mean values of K*FE for different notch opening angles and slightly wider scatter bands have been found in the present work, as reported in Figure 6. This outcome is justified by the dependence of the PSM on the FE formulation of a given software package and FE analysis settings, which have previously mentioned: (1) the adopted FE formulation; (2) the mesh pattern generated by the free meshing algorithm; and (3) nodal stress extrapolation and averaging criteria.

Let us first mention that the nodal stress extrapolation and averaging criteria are the same in Sysweld and in Ansys. In summary, nodal stresses in the element are extrapolated from the integration points of FEs, and then nodal stress components are calculated by averaging the nodal stresses of the elements sharing that node. Principal stresses are eventually calculated from averaged nodal stress components. In Ansys, this method is called average from components (AVPRIN,0 setting).

The reason for the differences of K*FE values between Sysweld and Ansys is primarily the FE formulation (ie, the integration scheme) of the adopted 4-node quadrilateral elements and, less importantly, the mesh pattern. The original calibration of the PSM was performed by setting the so-called simple enhanced strain element formulation, which is a particular 2 × 2 full-integration scheme provided by Ansys to avoid shear locking phenomena; however, with the aforementioned integration scheme being not available in Sysweld, the 2 × 2 standard full-integration scheme has been adopted. It is the default setting in Sysweld.

It could be verified that, if a different nodal principal stress evaluation method had been available in Sysweld, then a unique scatter band could have been obtained, independently of the notch opening angle. Such different evaluation method consists in calculating first the principal stresses at the nodes of each FE and afterwards in averaging the principal stresses at the node shared by the FEs. In Ansys, such a procedure is called average from principals (AVPRIN,1 setting). Several software packages make both the average from components technique and the average from principals technique available, one of them being the default setting (in Ansys, it is the average from components technique) and the other one being able to be activated by the FE analyst.32

Therefore, the 61 calibration analyses have been repeated by importing in the Ansys environment the same FE meshes generated through Visual-Mesh and previously used to calibrate Sysweld. Obviously, material elastic constants and boundary conditions have been set the same. Linear, 4-node FEs (named Plane182 in the Ansys library) with a standard 2 × 2 full-integration scheme have been used, and generalized plane strain condition has been set to exactly match Sysweld in the preparation of the numerical analysis. After the FE models in Ansys are solved, the average from components technique (AVPRIN,0 setting) has been activated in the postprocessor environment with the purpose of fully matching the analysis procedure performed in Sysweld, including the nodal stress averaging option. Figure 7 summarizes the results and shows that the nondimensional ratios K*FE are arranged in 2 separated bands, which have the same scatter and mean value as those computed with Sysweld in Figure 6. Then, the average from principals technique (AVPRIN,1 setting) has been activated in the Ansys postprocessor environment, and all results have been re-evaluated. By so doing, the analysis procedure performed in Ansys matched again the one conducted in Sysweld, the sole exception being the nodal stress averaging option. Figure 8 reports the results and shows that the nondimensional ratio K*FE becomes equal to 1.55 with a scatter of ±5%. The improvement achieved in the summary of all results can be appreciated by comparison with Figure 6. That said, calibrations shown in Figures 7 and 8 have not been used further, Sysweld being the FE software that the present paper is focused on.

image

Figure 7

Open in figure viewerPowerPoint

Nondimensional mode I notch stress intensity factor K*FE evaluated by using Ansys and the same free meshes adopted in Figure 6. Full-integration scheme and stress averaging from nodal stress components are used. PSM, peak stress method

image

Figure 8

Open in figure viewerPowerPoint

Nondimensional mode I notch stress intensity factor K*FE evaluated by using Ansys from results reported in Figure 7 by activating stress averaging from nodal principal stresses. PSM, peak stress method

To conclude this section, it should be mentioned that, according to the preliminary calibration performed recently,29 a constant ratio K*FE = 1.71 was proposed. This value falls inside the scatter band K*FE = 1.64 ± 5% suggested in this work by performing more extensive numerical investigations.

## 4 RAPID R-NSIF ESTIMATION BY USING THE PSM

To illustrate the use of the PSM to evaluate the R-NSIFs, a 6-mm-thick butt-welded joint has been analysed using the FE code Sysweld, under generalized plane strain hypothesis. It was shown that this assumption better describes the out-of-plane stress values in the 2-dimensional cross-sectional model of the welding process.36

The welded joint geometry and the assumed dimensions are shown in Figure 9. According to the NSIF-based approaches, the weld toe has been modelled as a sharp V-shaped notch. The notch opening angle 2α has been chosen equal to 135°.

image

Figure 9

Open in figure viewerPowerPoint

Schematic representation of the butt-welded joint considered in the present work and the polar coordinate system centred at the V-notch tip. Dimensions are in millimetres

The adopted carbon steel has chemical composition according to the ASTM SA 516 Standard (Grades 65 and 70), and the corresponding thermomechanical and thermometallurgical properties have been taken from the Sysweld database.

In the metallurgical analysis, the following phases have been included: martensite, bainite, and ferrite-pearlite. The metallurgical transformations mainly depend on thermal history, with this dependence described by continuous cooling transformation diagrams, which plot the start and the end transformation temperatures as a function of cooling rate or cooling time. In the present work, the diffusion-controlled phase transformations and the displacive martensitic transformation have been modelled according to the Leblond and Devaux37 and Koistinen and Marburger38 kinetic laws, respectively.

Radiative and convective heat losses have been applied at the boundary (external surfaces) of the plates to be joined. The former by using the Stefan-Boltzmann law, the latter by using a convective heat transfer coefficient equal to 25 W/m2·K. The thermal gradient in the out-of-plane direction cannot be taken into account in a 2-dimensional cross-sectional model because of its intrinsic formulation. However, it is supposed that the higher the welding speed, the lower the out-of-plane thermal gradient.

The thermal energy flow into the material during the welding process represents the sole computational load modelled in the welding simulation. The amount of thermal energy flow into the material is determined by the welding parameters (including welding speed) and by the welding technology used. In this work, the heat source has been modelled using a double-ellipsoid power density distribution function39 described by Equation 5, which has been widely used in literature for arc welding simulations.40

urn:x-wiley:8756758X:media:ffe12757:ffe12757-math-0007(5)

The double-ellipsoid heat source and the meaning of the symbols used in Equation 5 are shown in Figure 10, whereas the adopted numerical values are summarized in Table 1.

image

Figure 10

Open in figure viewerPowerPoint

Double-ellipsoid heat source configuration together with the power distribution function along the $\xi$ axis of the moving coordinate system $(x, y, \xi)$. The transformation relating the fixed and moving coordinate system is $\xi = z + v(\tau - t)$

By taking advantage of the symmetry, only one-half of the joint has been modelled. In Figure 11, the very refined mesh pattern used in the calculation of R-NSIF from the local stress field (Figure 11A) and 2 different meshes used for the PSM evaluation (Figure 11B,C) are compared. The FE model used to compute R-NSIFs from local stress fields had the minimum size of the elements at the notch tip equal to about 5 · 10−5 mm, according to Lazzarin and Tovo.13 The FE models used to estimate R-NSIFs by means of the PSM were meshed by means of the free meshing algorithm provided by Visual-Mesh according to the PSM calibration rules. Four-node 2004 quadrilateral elements from the Sysweld library have been used, and the numerical integration scheme was set to 2 × 2 Gauss points. Even though an average FE size d = 6/4 → 1.5 mm could have been adopted according to Figure 5; the average FE size d imposed to the free mesh generation algorithm was indeed much lower and equal to 0.2 mm, which translates into a mesh density ratio a/d equal to 30 (see Table 2). Such FE size was necessary to obtain a temperature field in agreement with that obtained with the very refined mesh pattern. More precisely, to establish the appropriate d value, a difference in nodal temperatures of few percentage points was allowed between the very refined and the PSM coarse meshes. Two different preprocessing approaches have been considered to apply the PSM. In the first case (Figure 11B), the free mesh pattern has been generated without involving any guiding line or surface division in the weld toe region, according to the PSM calibration reported in the previous section. In the second case (Figure 11C), a mesh guiding line was introduced, which divides the surface to be meshed into 2 parts along the V-notch bisector, starting from the weld toe. As a consequence, a row of FE nodes is forced to be located along the V-notch bisector line. This case may be of interest and has been investigated here because it is a typical preprocessing technique in welding simulations to consider 2 different material models (parent material and filler material) or to simulate multipass welding processes.

image

Figure 11

Open in figure viewerPowerPoint

Finite element models: A, very fine pattern used to compute residual notch stress intensity factors from local stress fields and detail of the notch tip refinement (order of magnitude of the smallest element equal to 10−4 mm); B, completely free generated mesh pattern with average element size d = 0.2 mm; C, guided free mesh pattern with average element size d = 0.2 mm (a row of finite element nodes is forced to be along the notch bisector line) [Colour figure can be viewed at wileyonlinelibrary.com]

Table 2. Comparison between the values of the residual notch stress intensity factors evaluated with very fine meshes and coarse meshes, both using linear isoparametric elements, taking advantage of Equation 4 linking the peak stress and the mode I residual notch stress intensity factor

Finally, uncoupled thermomechanical analyses have been conducted. The molten effect has been simulated by using a function that clears the history of an element whose temperature exceeds the melting temperature. During welding, the plates were supposed to be free of constraints: therefore, only the symmetry boundary condition shown in Figure 9 was considered.

## 5 RESULTS AND DISCUSSION

Figure 12A shows the temperature distribution when the melted zone has reached its maximum extension. In the same Figure, the residual stress field distributions near the toe region are reported according to a cylindrical coordinate system centred at the notch tip. The stress distribution near the weld toe is linear in a log-log plot (Figure 13), and its slope matches the analytical solution given by Equation 1. The intensity of such residual stress field has therefore been given in terms of R-NSIFs.

Figure 12

Open in figure viewerPowerPoint

A, Temperature distribution at the instant of maximum width of the fusion zone (in red). B, σθθ component of residual stress distribution near the notch tip. C, σrr component of residual stress distribution near the notch tip. Temperature is in degrees Celsius and stress is in megapascals [Colour figure can be viewed at wileyonlinelibrary.com]

image

Figure 13

Open in figure viewerPowerPoint

Asymptotic σθθ component of the residual stress field near the notch tip along the notch bisector, ie, θ = 0°. FEM, finite element method

The results are summarized in Table 2. It is possible to notice a good agreement between the K1 values obtained from the local stress field computed with very fine meshes and the ones estimated by means of the coarse meshes, by using the PSM with K*FE = 1.64. The investigation performed here demonstrated that the PSM can be used for a rapid R-NSIF evaluation, saving meshing and solving time. In particular, the solution time associated with the very refined meshes was about 1 minute for thermal analysis and 4 minutes for mechanical analysis, whereas the one associated with the PSM mesh pattern was about few seconds for thermal analysis and 1 minute for mechanical analysis.

In summary, the following main advantages can be outlined if the R-NSIFs are estimated by means of the PSM rather than computed directly from local stress fields: (a) only 1 nodal stress value calculated at the point of singularity is sufficient to compute the R-NSIF, the whole stress distribution along the notch bisector being no longer required; and (b) 3 orders of magnitude more coarse meshes could be used by using the PSM, compared with the very refined meshes required to evaluate the local stress field directly. In the authors' opinion, both reasons make the PSM easy and fast to apply in industrial and research applications. Future developments are envisaged to compute R-NSIFs by using 3-dimensional FE models of the welding process coupled with the PSM.

## 6 CONCLUSIONS

The PSM has been calibrated in the Sysweld FE code, with the aim to rapidly evaluate the mode I R-NSIFs by welding process numerical simulation. The obtained results are summarized as follows:

the calibration constant K*FE has been found equal to 1.64 in case of V notches with opening angles ranging from 90° to 135° and equal to 1.90 in case of cracks (0° notch opening angle);

the R-NSIF has been estimated in good agreement with the results of a very refined FE mesh;

by using the PSM, a 3 orders of magnitude more coarse mesh could be adopted as compared with that required to evaluate directly the asymptotic stress distribution.

The advantages observed in the simple example analysed in the present paper seem to encourage future developments of the PSM to analyse the R-NSIFs in 3-dimensional cases.