New methodology of fatigue life evaluation for multiaxially loaded notched components based on two uniaxial strain-controlled tests

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Abstract

The present paper focuses on the application of the total strain energy density approach to assess fatigue life in notched samples subjected to multiaxial loading. This approach requires, as a starting point, a fatigue master curve defined in terms of total strain energy density versus number of cycles to failure, which is usually determined from a set of uniaxial fatigue tests using smooth standard specimens. In order to reduce the time and cost associated with the generation of the fatigue master curve, a straightforward methodology based on the outcomes of only two uniaxial strain-controlled tests is proposed. The methodology is applied to round bars with U-shaped notches subjected to proportional bending-torsion loading histories. A very good correlation between the experimental and the predicted life of the notched specimens is observed. In addition, the theoretical predictions are nearly independent of the pairs of uniaxial strain-controlled tests selected to obtain the fatigue master curve, which indicates a good robustness of the suggested methodology.

1. Introduction

A high competitiveness of the global market is one of the main challenges currently faced by many industries. It can be addressed through innovations and product diversity which require the development of effective engineering solutions. These solutions are expected to be delivered in a short timeframe with no additional cost to the customers. Another important aspect of the product competitiveness is its quality and reliability, which can be achieved by utilising advanced modelling and testing approaches.

With regard to engineering components subjected to multiaxial loading, the cost of the verification of the product specification, specifically, life expectancy could be quite large. This is because the complex loading conditions and, often, the presence of stress concentrators require the application of local fatigue life assessment approaches. The successful application of these approaches largely depends not only on an accurate knowledge of the stress-strain response at the geometric discontinuities, but also on an accurate determination of fatigue properties of the material.

In general, the determination of fatigue properties relies on time-consuming and costly testing procedures [1], [2], [3], [4]. These procedures need highly trained personnel, specialised testing equipment and sophisticated data acquisition systems, as well as the fabrication of large number of specimens of different geometries. The test outcomes, in turn, generate a significant amount of data, which requires a careful analysis. Without this analysis, the designers are likely to implement a greater safety factor, which can compromise the weight, cost and the efficiency of the final product. Therefore, effective solutions to reduce the overall costs of the product development are required.

In particular, innovative fatigue lifetime assessment procedures based on limited experimental testing programs have a potential to address the current challenges of the global market.

The combination of multiaxial loading and geometric discontinuities (i.e. stress concentrators), even in the case of small-scale plastic deformation, may result in complex and challenging problems for designers. One of the most well-known approaches to account for the notch and plasticity effects was proposed by Neuber [5]. Initially formulated for shear-strained prismatic bodies with hyperbolic-notches, the concept was later extended to other notch configurations, materials, and loading conditions, specifically to multiaxial fatigue loading [6], [7]. An alternative approach to deal with notches, suggested by Molski and Glinka [8], is the equivalent strain energy density (ESED) method. This method is based on the hypothesis that the elastic-plastic strain energy density of the material in the yielded zone is theoretically the same as the pseudo-strain energy assuming the material is in elastic state. The comparison of the two above-mentioned approaches indicates that the former generally overestimates the notch root strains, while the latter tends to underestimate the notch tip behaviour [9], [10]. Over the years, different energy-based formulations, which are able to estimate the local elastic-plastic stresses and strains, have been proposed. Among these, we can mention the total strain energy density approach suggested by Ellyin et al., which is based on the sum of the plastic and positive elastic strain energy density components [11], [12]. The material-related control volume technique introduced by Lazzarin et al. [13], initially developed for sharp notches and cracks under mode-I, and then extended to multiaxial fatigue loading conditions [14], [15], [16], [17], is another example of the strain-energy based approaches. Recent critical overviews of the latest developments in this area can be found in Refs. [18], [19].

In the context of fatigue, the so-called Theory of Critical Distances (TCD) is considered to be a very fruitful concept able to account for the weakening effect of notches [20]. It was first introduced by Neuber [21], in the middle of the past century. The key-idea behind the Line Method (LM) is the determination of an effective stress from a linear-elastic stress profile over a straight-line emanating from the notch root. A simplified approach, known as Point Method, was proposed by Peterson [22], who advocated that the averaged stress could be evaluated at a given distance from the notch tip. Due to the technological advance, and particularly the proliferation of the numerical methods, the TCD has gained an increased popularity, and its initial formulation has been considerably expanded [23]. In addition, this group of methods is able to incorporate different non-linear effects into the linear-elastic frameworks, which considerably reduces the time and costs associated with the analysis of the proposed engineering solutions [24].

The current paper is focused on the fatigue life assessment of notched round bars subjected to multiaxial loading histories using the total strain energy density approach. This approach relies on a fatigue master curve, usually determined from smooth specimens, which relates the total strain energy density quantified as the sum of both the plastic and the positive elastic strain components with the number of cycles to failure. Nevertheless, in order to reduce the time and costs associated with the determination of this master curve, a mixed numerical-experimental methodology based on only two uniaxial strain-controlled tests is proposed. This simplified methodology is applied to round bars with U-shaped notches subjected to pulsating bending-torsion loading histories. The selected specimens and loading allow to generate different orientations of the normal stress with respect to the notch root as well as a wide range of ratios of the normal stress to the shear stress.

The paper is organised as follows: Section 2 briefly describes the fatigue life prediction model. Sections 3 Experimental procedure, 4 Numerical procedure gather the main information on the experimental and numerical procedures. Section 5 tackles the determination of the fatigue master curve via the mixed experimental-numerical procedure proposed here; summarises the fatigue behaviour of the notched samples under multiaxial loading; and compares the experimental and predicted fatigue lives. The last section of this paper presents the summary and some concluding remarks.

2. Fatigue life prediction model

Fatigue life calculations are performed using the theoretical model proposed in Ref. [25]. This model is based on the premises that both the smooth and the notched samples accumulate the same damage and have the same lives if the stress-strain histories at the initiation sites are identical; and that the fatigue failure occurs when the total strain energy density at the initiation sites reaches a critical value. It is particularly suitable for applications undergoing proportional loading when there are no significant variations of stress ratios and stress intensity factor range thresholds.

The modus operandi consists of developing a fatigue master curve, relating the total strain energy density – defined as the sum of both the plastic and the positive elastic components – with the number of cycles to failure, from fully-reversed strain-controlled tests performed using smooth standard specimens, see Fig. 1(d), i.e.

(1)

where ΔWT is the total strain energy density, κt and αt are constants, Nf is the number of cycles to failure, and ΔW0t is the tensile elastic energy at the material fatigue limit. The use of the tensile elastic strain energy density makes this parameter sensitive to the mean stress effect [11], [12].

Fig. 1. Fatigue life prediction approach based on the total strain energy density evaluated at the initiation sites from hysteresis loops obtained through the ESED concept and an average stress given by the LM of the TCD [25]: (a) reduction of the multiaxial stress state to an equivalent uniaxial stress state; (b) computation of effective stress at the fatigue process zone; (c) calculation of the total strain energy density; (d) lifetime assessment using the fatigue master curve.

Then, the total strain energy density of the notched samples, under combined bending-torsion loading histories, is evaluated via a three-step procedure: (i) reduction of the multiaxial stress state to a uniaxial stress state by computing the equivalent von Mises stress range (Fig. 1(a)); (ii) calculation of an effective stress range (Fig. 1(b)) applying the LM of the TCD; and (iii) generation of a representative hysteresis loop using the ESED concept in conjunction with the effective stress range (Fig. 1(c)). Finally, the lifetime is estimated by inserting the total strain energy density of the notched sample into the fatigue master curve, as illustrated in Fig. 1(d).

In the present paper, as mentioned above, a simplified methodology to generate the fatigue master curve is proposed. This methodology, schematically shown in Fig. 2, only requires two uniaxial fully-reversed strain-controlled tests: one for a higher strain amplitude, termed here T1; and another for a lower strain amplitude, termed here T2 (Fig. 2(a)). These two tests also allow the definition of a strain-life relation given by the sum of both the Basquin and the Coffin-Manson equations, see Fig. 2(b). In parallel, the cyclic stress-strain responses obtained in the tests T1 and T2 are also utilised to develop two elastic-plastic numerical models, termed here N1 and N2, respectively (Fig. 2(c)). The total strain energy density for different strain amplitudes is computed through the numerical models (Fig. 2(d)). The former (N1) is applied in the simulations of higher strain amplitudes, and the latter (N2) is applied in the simulations of lower strain amplitudes. The number of cycles to failure, for each simulation, is estimated based on the strain-life relation, as illustrated in Fig. 2(b).

Fig. 2. Mixed numerical-experimental determination of fatigue master curve: (a) experimental strain-controlled tests; (b) definition of a strain-life relation based on the two experimental tests; (c) development of two independent elastic-plastic models based on the two strain-controlled tests; (d) generation of the fatigue master curve.

3. Experimental procedure

3.1. Material

The test samples were fabricated from the DIN 34CrNiMo6 high strength steel, oil quenched and tempered, supplied in the form of 20-mm diameter round bars. Its main mechanical properties are listed in Table 1.

Table 1. Mechanical properties of the DIN 34CrNiMo6 high strength steel [2].

3.2. Strain-controlled fatigue tests of smooth specimens

The strain-controlled tests, first described in Refs. [26], [27], were performed using standard shape specimens, machined according to the specifications outlined in ASTM E606[1], with a gage-section measuring 8 mm in diameter and 15 mm in length. The tests were carried out under fully-reversed loading conditions ($R_\varepsilon = -1$), with a constant strain rate ($d\varepsilon/dt = 8 \times 10^{-3}\ s^{-1}$), and strain amplitudes between 0.4% and 2%. The stress-strain response was acquired through a digital data acquisition system connected to a 12.5 mm-long strain-gage extensometer mounted on the gage-section of the specimen. The tests were interrupted after complete failure of the specimen. In addition, fully-reversed stress-controlled tests ($R_\sigma = -1$) for stress amplitudes ranging from 540 to 635 MPa were also conducted. Table 2 summarises the main outcomes of the experimental tests, which will be used to validate the proposed methodology [26], [27].

Table 2. Summary of the strain-controlled fatigue tests (stresses and strains evaluated at the half-life cycle) [26].

Test reference	Total strain amplitude, $\Delta\varepsilon/2$ (%)	Elastic strain amplitude, $\Delta\varepsilon_e/2$ (%)	Plastic strain amplitude, $\Delta\varepsilon_p/2$ (%)	Stress amplitude, $\Delta\sigma/2$ (MPa)	Number of cycles to failure, N_f	Number of reversals to failure, $2N_f$
T1	2.00	0.425	1.578	891.8	131	262
T2	0.60	0.346	0.261	726.6	2523	5046
T3	0.50	0.332	0.180	697.5	5140	10,280
T4	0.40	0.322	0.091	675.3	13,378	26,756

3.3. Multiaxial fatigue tests of notched specimens

The multiaxial fatigue tests, first described in Ref. [28], were performed on a servo-hydraulic machine, connected to a custom made gripping system [28], [29], under constant-amplitude pulsating load ($R\sigma = 0$), with cyclic frequencies of 4 Hz, and sinusoidal waves. Two U-notched round bar specimen geometries (see Fig. 3) with two different stress concentration factors were subjected to six loading scenarios, more precisely: two bending moment (B) to torsion moment (T) ratios (B/T = 2, and B/T = 1), and three orientations of the bending moment (θ) with respect to the notch root (θ = 0°, θ = 45°, and θ = 90°). Three nominal stress amplitudes were applied for each loading case. The notch region was examined using a high-resolution digital camera connected to an optical device with a variable magnification. Images were acquired every 5000 cycles during the initial stage of the test. During the second stage, after crack initiation, images were captured every 2000 cycles. In the final part of the tests, the inspection intervals were reduced to 1000 cycles.

Fig. 3. Specimen geometries used in the multiaxial fatigue test program: (a) lateral U-notched round bar with 16-mm diameter cross-section; (b) lateral U-notched round bar with 14-mm diameter cross-section (dimensions in millimetres).

The number of cycles to crack initiation (N_i), first introduced in Ref. [28], were computed through the experimental a-N curves (i.e. crack length versus number of cycles) for a critical in-depth crack length (a_0) given by the El-Haddad parameter [30], i.e.

(2)

where ΔK_{th} is the range of the threshold value of the stress intensity factor, and $\Delta\sigma_0$ is the fatigue limit of the smooth specimen. The two above-mentioned constants should be calculated at the same stress ratio of the notched sample, i.e. $R\sigma = 0$ (please see Table 1). For pulsating load, based on the values listed in Table 1, a_0 = 123 µm. Table 3 presents a summary of the nominal normal stresses applied to the specimen as well as the fatigue life obtained from the tests.

Table 3. Summary of the combined bending-torsion fatigue test program [28].

4. Numerical procedure

4.1. Modelling of elastic-plastic behaviour

Low-cycle fatigue tests of smooth specimens under fully-reversed strain and stress-controlled loading conditions were modelled numerically with the DD3IMP implicit 3D FE code [31], [32] using a single trilinear 8-node hexahedral solid finite-element, as shown in Fig. 4. In this context, symmetry conditions were imposed in the planes x = 0, y = 0 and z = 0; cyclic solicitations were prescribed at the four nodes located in the plane x = 1 mm, along the direction parallel to the 0x axis, with 100 load sub-steps per half-cycle.

Fig. 4. Single element model of the cyclic uniaxial tension-compression test with initial dimensions 1 × 1 × 1 mm^3. The red arrows represent the displacements that are prescribed at the nodes (black dots) located in the plane x = 1 mm. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

An elastic-plastic model with purely kinematic hardening was used in all numerical simulations, assuming: (i) the isotropic elastic behaviour modelled by the generalized Hooke's law, with E = 209.8 GPa and ν = 0.296 (see Table 1); (ii) the plastic behaviour modelled by the von Mises yield criterion, coupled with Armstrong-Frederick non-linear kinematic hardening law under an associated flow rule.

The von Mises yield surface describes the onset of plastic yielding as follows:

(3)

where Σ_{11}, Σ_{22}, Σ_{33}, Σ_{12}, Σ_{13}, and Σ_{23} are the components of the effective Cauchy stress tensor, Σ ($\Sigma = \sigma' - X'$, where σ′ and X′ are the deviatoric component of the Cauchy stress tensor and the deviatoric part of the backstress tensor, respectively), and Y_0 is the initial yield stress. The Armstrong-Frederick law describes the nonlinear kinematic hardening as follows [33]:

(4)

where is the backstress rate, is the equivalent stress and C_X and X_{Sat} are material parameters, and is the equivalent plastic strain rate.

An optimization procedure was carried out to identify the set of material constants that best describe the cyclic plastic behaviour of DIN 34CrNiMo6. The identified set of the material parameters was obtained by minimisation of the least-squares objective function F(A):

(5)

where and are, respectively, the analytically fitted and experimentally measured values of true stress at data point i (which corresponds to a given equivalent plastic strain value); N is the total number of experimental data points and A is the set of material parameters Y0, CX and XSat to be identified. The fitting procedure was performed using a non-linear gradient-based optimisation algorithm implemented in Microsoft Excel SOLVER tool [34].

4.2. Modelling of elastic stress state at the notch

The typical finite-element mesh, created to evaluate the elastic stress state at the notch root, is presented in Fig. 5. It was developed in a parametric framework, with 8-node hexahedral isoparametric elements, and was the result of a trade-off between the accuracy and computational time. The notch region was meshed with ultra-refined finite elements, while the remaining volume was filled in with a coarse mesh. The bending moments were applied by a pair of forces (FB) parallel to the longitudinal axis of the specimen with identical magnitude but opposite directions. The points of application of those forces were selected in order to obtain θ angles equal to 0°, 45° and 90°. The torsion moments were applied by a couple of forces (FT) with the same magnitude and opposite directions on a plane normal to the longitudinal axis of the specimen. The B/T ratios were defined by changing the value of , i.e. the proportionality factor between FT and FB (see Fig. 5), which was equal to 1 and 1/2 for B = 2T and B = T, respectively. The loads were applied at one end, while the other end of the finite element model was fixed.

Fig. 5. Finite-element model developed in a parametric framework using 8-node hexahedral isoparametric elements.

5. Results and discussion

5.1. Fatigue master curve

The fatigue master curve, as explained in Section 2, was generated using a mixed numerical-experimental technique based on two uniaxial fully-reversed strain-controlled tests, and two elastic-plastic finite-element models. The loading range was $\Delta\varepsilon/2$ = 2% (T1), and $\Delta\varepsilon/2$ = 0.6% (T2), i.e. one for a higher strain amplitude, and another for a lower strain amplitude (see Table 2). As example, Fig. 6 shows the cyclic stress-strain response of the former case. For the sake of clarity, several cycles were intentionally omitted; and the colours of the first and half-life cycles were changed to grey and red, respectively. The results indicate a strain-softening behaviour throughout the entire life of the specimen, as the uncontrolled stress decreases progressively until the final failure. Nevertheless, after the first 40 cycles, the changes are tenuous and the shape of the hysteresis loops is maintained. Look, for example, the half-life hysteresis loop and the remaining circuits; no significant differences are distinguished.

Fig. 6. Stress-strain response of the DIN 34CrNiMo6 high-strength steel under fully-reversed strain-controlled conditions for $\Delta\varepsilon/2$ = 2% [27].

The strain-life relation required to correlate the number of fatigue cycles with the total strain energy density was established from the described above tests. This relation, which corresponds to the linear summation of the well-known Basquin and Coffin-Manson equations, can be written in the form [35], [36], [37]:

(6)

where σf′ is the fatigue strength coefficient, b is the fatigue strength exponent, εf′ is the fatigue ductility coefficient, c is the fatigue ductility exponent, and E is the Young's modulus. The constants were determined by the least square method and are reported in Table 4. Fig. 7(a) plots the fitted functions and those determined using the classical approach outlined in ASTM E739[4], i.e. a set of thirteen fully-reversed strain-controlled fatigue tests [26]. The experimental values of total, plastic, and elastic strain amplitudes measured in the tests are also shown. Overall, the fitted functions determined with the two tests are quite close to those obtained via the classical approach. The differences of these fitted functions relative to the experimental data (either the functions defined from the two tests, or the functions calculated through the classical approach) are presented in Fig. 7(b). As it can be seen, the errors have the same order of magnitude, and are essentially independent of the pair of tests considered in the fitting process. This observation is particularly interesting because it demonstrates the robustness of the proposed approach.

Table 4. Constants of Eq. (6) obtained from different combinations of uniaxial strain-controlled fatigue tests.

Fig. 7. (a) Strain-life relation, Basquin relation, and Coffin-Manson relation obtained from different combinations of strain-controlled fatigue tests; (b) differences between the strain-life relations and the experimental results.

In parallel to the computation of the strain-life relations, two independent elastic-plastic finite-element models are developed, also based on the two fully-reversed strain-controlled tests. For each test ($\Delta\varepsilon/2$ = 2%, $\Delta\varepsilon/2$ = 0.6%), the minimisation of the objective function, F(A), was performed for the values of true stress obtained from the cyclic stress-strain curve (with N = 138, see Eq. (5)), which represents about 80% of total life. As an example, Fig. 8 shows the stress versus plastic strain results of DIN 34CrNiMo6 and the fitted model for the test $\Delta\varepsilon/2$ = 2%. A good agreement was found between fitted and experimental data points of all the studied tests; in this context, the selected fully kinematic hardening model is considered adequate for describing the cyclic behaviour of the DIN 34CrNiMo6 high-strength steel. Table 5 lists the identified constants Y0, CX and XSat obtained for the selected strain-controlled tests.

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Fig. 8. Stress versus plastic strain results for the DIN 34CrNiMo6 high-strength steel ($\Delta\varepsilon/2$ = 2%) and the corresponding fitted model.

Fig. 9(a) and (c) display the comparison of the half-life experimental loops of the tests $\Delta\varepsilon/2$ = 2% (T1) and $\Delta\varepsilon/2$ = 0.6% (T2), respectively, with the half-life numerical hysteresis loops generated using the N1 and N2 models described previously. As can be observed in each figure, the numerical circuits simulated with the two models are different. Not surprisingly, the N1 model gives better results for $\Delta\varepsilon/2$ = 2%; while the N2 model simulates more accurately the loop shape for $\Delta\varepsilon/2$ = 0.6%. Such a fact suggests that the former model is more adequate for higher strain amplitudes, and the latter is more appropriate for lower strain amplitudes; and also shows the interest of developing two independent numerical models. Fig. 9(b) and (d) compare the half-life experimental and numerical loops for intermediate strain amplitudes, namely $\Delta\varepsilon/2$ = 1.5% and $\Delta\varepsilon/2$ = 0.4%. As expected, in the first case, the best performance is achieved by the N1 model; while, in the second case, the N2 model is more efficient. Therefore, in sum, the combination of the two numerical models allows a much more adequate simulation of the half-life hysteresis loops and, consequently, of the total strain energy density.

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Fig. 9. Comparison between the experimental and numerical mid-life hysteresis loops obtained using the elastic-plastic finite-element models for strain amplitudes: (a) $\Delta\varepsilon/2$ = 2%; (b) $\Delta\varepsilon/2$ = 1.5%; (c) $\Delta\varepsilon/2$ = 0.6%; (d) $\Delta\varepsilon/2$ = 0.4%.

The fatigue master curve obtained using the two numerical models and the strain-life relation defined from the strain-controlled tests of $\Delta\varepsilon/2$ = 2% (T1) and $\Delta\varepsilon/2$ = 0.6% (T2) is presented in Fig. 10. The N2 model was used in the simulations of strain amplitudes lower than 1.2% (i.e. lower than two times the strain amplitude of the experimental test considered in the development of the model); and the N1 model was used for higher strain amplitudes. Overall, the numerical calculations of the total strain energy density, defined as the sum of both the plastic and positive elastic components, agree very well with the experimental observations, for both high and low strain amplitudes. With respect to the fitted functions, the same conclusions can be drawn; clearly, the differences are not significant, which shows the effectiveness of the proposed approach. The constants of Eq. (1) for this combination, termed here C1 (experimental tests T1 and T2, and numerical models N1 and N2), were determined using the least square method, and are compiled in Table 6. The experimental value of ΔW0t was determined for a fatigue life of 1x106 cycles and corresponds to a fatigue limit stress range of 537 MPa, which is satisfactorily close to the values reported by Rabb [38] for smooth specimens subjected to fully-reversed tension-compression loading (526 ± 15 MPa). Numerical values were defined for the same fatigue life using the model of lower strain amplitude (N2) of each combination.

Fig. 10. Fatigue master curves obtained from the experimental tests and from the mixed numerical-experimental approach proposed in the paper for the DIN 34CrNiMo6 high-strength steel.

5.2. Experimental fatigue behaviour of the notched specimens

Fig. 11 shows typical surface crack trajectories observed in the experiments for the different loading scenarios. In this notch geometry, as can be inferred from the figure, crack trajectories are the reflex of the normal stress to shear stress (σ/τ) ratio, and the bending moment angle (θ). With regard to the former variable, the analysis shows that either the surface crack curvature, or the degree of inflection at the centre of the notch are smaller for higher σ/τ ratios (see, for instance, Fig. 11(a) and (d)). In relation to the latter variable, although the effect is more tenuous than the normal stress to shear stress ratio, it can be clearly concluded that the lower the θ angle, the lower the crack curvature and the degree of inflection (see, for example, Fig. 11(a) and (b)).

Fig. 11. Examples of crack paths and initiation sites detected in the experiments for different loading paths: (a) B = 2T (0°); (b) B = 2T (45°); (c) B = 2T (90°); (d) B = T (0°); (e) B = T (45°); (f) B = T (90°).

Moreover, as expected, the experimental crack initiation sites, represented by the circles, are also affected by both the normal stress to shear stress (σ/τ) ratio, and the bending moment angle (θ). The effect of the σ/τ ratio is clearly perceptible by comparing Fig. 11(a) and (d). In the former case, the crack initiated close to the notch centre; but, in the latter case, the crack initiation occurred close to the curved edge of the notch. This suggests that the reduction of the σ/τ ratio (i.e. increase of the shear stress level) moves the crack initiation sites to the position closer to the curved edge of the notch. Regarding the θ angle, lower values of this variable promotes the crack nucleation in sites closer to the centre of the notch (see Fig. 11(a)–(c) or Fig. 11(d)–(f)). As demonstrated in previous studies, these locations can be predicted numerically, in a very accurate way, from the nodes of maximum first principal stress. Predictions, represented by the squares, are relatively close to the experimentally detected sites for the different loading paths. A more systematic investigation on the effect of both the B/T ratio and the θ angle on crack paths and crack initiation locations can be found in Refs. [28], [39].

Fig. 12 displays some examples of the relations between the surface crack length (2b) and the numbers of loading cycles (N) registered in the experiments which can be fitted by power laws (see full lines). The criterion defined to stop the analysis was the instant at which one end of the crack reached the curved edge of the notch. In general, the data collected agree with the expectations, i.e. for the same loading conditions, higher stress levels lead to lower lives; and for the same crack length, higher stresses lead to higher slopes of the curves (see, for the sake of clarity, the slopes plotted in the figure for 2b = 1.5 mm). Nevertheless, some exceptions were detected; one example relative to the discrepancies in terms of fatigue life, for the same loading conditions, is identified in Fig. 12(b) by the labels 1 and 2. This can be explained by the occurrence of multi-crack initiation in the former test. The occurrence of multi-crack initiation, in this steel, is recurrent in the literature [39]; an example of incongruences concerning the slopes is identified, in Fig. 12(c), by the label 3. A plausible explanation of this phenomenon is that the crack nucleation in a position slightly distant from the theoretical site due to the presence of inclusions which act as local stress raisers, changing the stress fields and, predictably, the crack growth rates. The susceptibility of this high-strength steel to non-metallic inclusions is well-known [39], [40], [41].

Fig. 12. Surface crack length (2b) versus number of cycles (N) for [28]: (a) B = 2T (0°); (b) B = T (0°); (c) B = T (45°).

A comparative analysis of Fig. 12(a) and (b) shows a clear effect of the σ/τ ratio on fatigue life. The cases for higher nominal stress amplitudes (σa = 298.4 MPa) lead to 2b-N curves substantially different. It can be distinguished a clear increase in fatigue life for higher B/T ratios in virtue of the lower levels of shear stresses. The effect of the θ angle can be inferred from the 2b-N curves of similar stress amplitude (σa = 223.8 MPa) plotted in Fig. 12(b) and (c). As can be seen, for the same crack length, the higher the angle, the higher the fatigue life. Another interesting difference observed in the 2b-N curves caused by the bending moment angle is the maximum surface crack length measured in the experiments. The maximum value of 2b, for the same B/T ratio, reduces gradually with the increase of θ (see, for example, the cases displayed in Fig. 12(b) and (c)). This is coherent with the conclusions drawn on the initiation sites; cracks nucleate closer to the curved edge of the notch for higher angles, irrespective of the B/T ratio, which results in lower propagation distances up to the border of the notch.

5.3. Fatigue life prediction

The assessment of fatigue life, as referred to in Section 2, was performed using the methodology proposed in Ref. [25]. The first step (see Fig. 1(a)) encompasses the reduction of the multiaxial bending-torsion state acting at the notch root to an equivalent uniaxial stress state which is done here by computing the von Mises stress range at the theoretical initiation site using the linear-elastic finite-element models developed for the various loading scenarios. Fig. 13 plots the variation of the von Mises stress range ($\Delta\sigma_{vM}$) over a straight-line (d) emanating from the notch root for the case B2T45-1. As expected, the stress range is maximum at the notch surface, and then gradually diminishes over the distance to an asymptotical value. The second step (see Fig. 1(b)) is devoted to the calculation of an effective stress over a critical distance (DLM) ahead of the notch, which is carried out via the LM (i.e. DLM = 2a0) of the TCD. The effective stress of the example represented in Fig. 13 is equal to 941 MPa. Next, the procedure consists of generating a representative hysteresis loop (see Fig. 1(c)) using the effective stress range in conjunction with the Equivalent Strain Energy Density concept [42]. Fig. 14 shows representative stress-strain circuits obtained for B = 2T (45°). For each hysteresis loop, firstly it is calculated the coordinates of Point A (i.e. maximum stress and maximum strain) and then the coordinates of Point B (by computing the stress range and the strain range with respect to an auxiliary coordinate system with origin at Point A). Finally, the total strain energy density of the notched sample is calculated and inserted into the fatigue master curve to estimate the fatigue life.

Fig. 13. Plot of local von Mises equivalent stress range over a straight-line emanating from the notch root at the theoretical initiation site for B = 2T (45°).

Fig. 14. Hysteresis loops generated from the average stress using the ESED concept for B = 2T (45°) under three different nominal stress amplitudes.

The fatigue life predictions (NP1) based on the fatigue master curve defined from the experimental tests T1 and T2, and numerical models N1 and N2 (i.e. combination C1) are summarised in Table 7. The comparison between the experimental (Ni) and the predicted (Np1) fatigue lives is presented in Fig. 15(a). As can be seen in the figure, predictions are very well correlated with the experiments, with all points within a factor of 2. For B = 2T, results are, in general, conservative (only a single case is on the unsafe side). For B = T, there is a mixed trend, i.e. conservative predictions for θ = 0°, and non-conservative results in the remaining cases. Overall, the level of accuracy is similar to that obtained via the experimental-based fatigue master curve, as can be inferred from Fig. 15(b). This fact strongly supports the conclusion that the proposed methodology is sufficiently accurate to predict the fatigue life of notched components subjected to bending-torsion loading scenarios. This in turn allows a substantial reduction of costs and labour associated with the determination of material properties.

Fig. 15. Experimental versus estimated fatigue lifetime via: (a) numerical-based fatigue master curve; (b) experimental-based master curve.

One important aspect that has not been yet discussed is the sensitivity of the proposed methodology to the experimental strain-controlled fatigue tests used in the calculations. In order to clarify this subject, other combinations of tests were considered, namely $\Delta\varepsilon/2$ = 2% (T1) and $\Delta\varepsilon/2$ = 0.5% (T3), termed here combination C2; and $\Delta\varepsilon/2$ = 2% (T1) and $\Delta\varepsilon/2$ = 0.4% (T4), termed here combination C3. Therefore, in sum, two additional strain-life relations were established from the pairs of tests T1-T3 and T1-T4 (the fatigue strength and fatigue ductility constants are summarised in Table 4). In parallel, two additional elastic-plastic finite-element models (N3 and N4), whose constants are listed in Table 5, were developed using the information collected, respectively, in the tests of $\Delta\varepsilon/2$ = 0.5% (T3) and $\Delta\varepsilon/2$ = 0.4% (T4). Then, two new fatigue master curves, termed here C2 and C3, whose constants are reported in Table 6, were generated. Finally, new fatigue life predictions, based on the above-mentioned fatigue master curves, were performed. The results of such calculations are presented in Table 7. Predictions for combination C2 are termed Np2 and for combination C3 are termed Np3.

Table 8 compares the ratios of the experimental to predicted fatigue lives for the three combinations studied here, i.e. C1, C2, and C3, which are identified as Ni/Np1, Ni/Np2 and Ni/Np3, respectively. As can be seen, the statistical variables presented in the table are quite close. In all circumstances, the predicted fatigue lives are within the scatter bands, since the minimum Ni/Npi values are greater than 0.5 and the maximum Ni/Npi values are lower than 2. A more detailed analysis shows that the predictions of combination C1 are slightly better than those of combinations C2 and C3 (which is in line with the conclusions drawn with respect to the data of Fig. 7). Therefore, in conclusion, the predicted fatigue lives are nearly independent of the pairs of uniaxial strain-controlled fatigue tests selected in the process. This denotes high soundness and high robustness of the proposed approach, which is a key aspect.

Table 8. Statistical data for the Ni/Npi ratios obtained from different combinations of elastic-plastic numerical models.

6. Conclusions

The main objective of the paper was the development of a simplified methodology based on total strain energy density curves able to assess the fatigue life in notched samples undergoing bending-torsion cyclic loading. It was demonstrated that the developed methodology can substantially reduce the cost and other necessary investments associated with the determination of the material properties. Briefly, the methodology consists of determining a fatigue master curve defined in terms of total strain energy density versus number of cycles to failure from only two uniaxial fully-reversed strain-controlled fatigue tests. These tests are then used to establish a strain-life relation, as well as to develop two independent elastic-plastic finite-element models. The strain-life relation in conjunction with the two numerical models enable the generation of a fatigue master curve that relates the total strain energy density defined as the sum of the elastic and positive elastic components with the number of cycles to failure. Finally, the strain energy density is determined at the notch root for the different loading scenarios and the fatigue life is estimated on the basis of the fatigue master curve generated for smooth specimens. The following concluding remarks can be drawn based on the results presented in this paper:

- The fatigue master curve can be efficiently generated from only two uniaxial strain-controlled tests, and a set of numerical simulations performed via single-element elastic-plastic models;

- Although the hysteresis loops computed with the numerical models generates small differences in comparison with the experimental data, the differences in terms of the total strain energy density are quite small;

- The fatigue life predictions of the three sets of experimental tests selected to validate the proposed methodology were similar, which denotes that the methodology is nearly independent of the pairs of tests selected;

- The use of two independent numerical models, one for higher strain amplitudes, and other for lower strain amplitudes, attenuates the differences between the experimental and numerical hysteresis loops making the methodology more robust;

- The significant reduction of the overall cost and effort associated with the determination of fatigue properties needed for the evaluation of fatigue life shows a great potential for adapting this methodology by the industry.