Evaluation and comparison of critical plane criteria for multiaxial fatigue analysis of ductile and brittle materials

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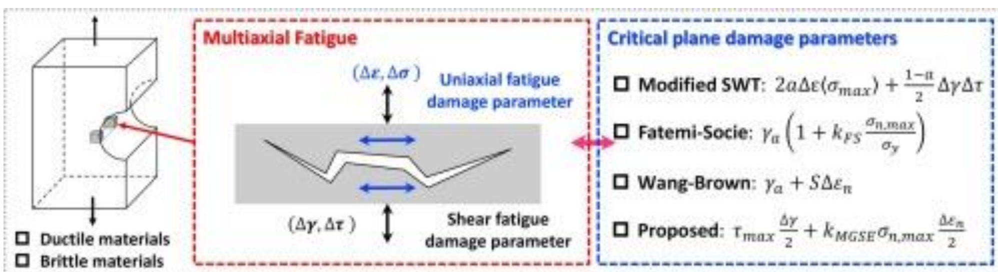
Abstract

This paper conducts a comparative evaluation on typical critical plane criteria, including Fatemi-Socie, Wang-Brown, modified Smith-Watson-Topper (MSWT) and proposed modified generalized strain energy (MGSE) criteria for multiaxial fatigue analysis of ductile/brittle materials. Experimental datasets of four materials under uniaxial tension, torsion and proportional/non-proportional multiaxial loadings are introduced for model comparison. This study results indicate that criteria with additional material constants yield robust life predictions for different materials. Moreover, the criteria with shear and uniaxial fatigue properties are respectively suitable for ductile and brittle materials, particularly the MGSE superior to others for ductile/brittle materials while MSWT only for brittle materials.

Graphical abstract



Keywords

Multiaxial fatigue
Critical plane
Strain energy
Life prediction
Damage parameter

1. Introduction

Engineering components like aero engine components and railway axles are often subjected to complex multiaxial stress and strain states during service loadings, leading to severe challenges for accurate fatigue life prediction [1], [2], [3], [4]. However, conventional uniaxial fatigue approaches usually overestimate fatigue life of these components, which might cause serious consequences. For the purpose of safe and reliable design, increasing researches on multiaxial fatigue analysis have been focused over the past few decades [1], [2], [3], [4], [5], [6]. Among them, the multiaxial fatigue criteria proposed by Gough [7], Sines [8] and Findley [9] laid the foundation for the development of multiaxial fatigue analysis [10]. Subsequently, various multiaxial fatigue criteria based on uniaxial/pure shear fatigue properties began to emerge and were carried out for different critical engineering applications [11]. The ideas of establishing these multiaxial fatigue criteria mainly include the equivalent thought, critical plane concept, strain energy-based thought and so on [12], [13], [14]. Based on these approaches, complex multiaxial loading can be degenerated into a general damage parameter by introducing a criterion to relate it with fatigue life. Among them, equivalent approaches have shown several limitations for multiaxial fatigue assessment since it cannot explain the observed cracking behavior of materials [15]. Both strain energy-based and critical plane approaches have shown corresponding physical explanation. In particular, the strain energy-based approach considers hysteresis loopscaused by cyclic deformation, while the critical plane approach explains processes of crack initiation and acceleration [16], [17]. Recently, Yu et al. [17] modified the generalized strain energy/amplitude (GSE/GSA) criterion of Ince-Glinka and presented a strain energy-critical plane fatigue criterion without any additional material constants, which provides comparable predictions comparing with the criterion of Fatemi and Socie. Until now, no particular approach has been considered to always give more accurate predictions than others, thus fatigue criterion applications need to be validated for different materials under different loading paths. From the viewpoint of ductility, materials can be described by ductility and brittleness. The early conventional understanding is that materials reflect ductile or brittle behavior depending on the ratio of theoretical shear strength to the theoretical tensile one [18]. However, the definition of distinction between ductility and brittleness of materials is not yet clarified except by experiments and/or experiences [19]. Materials with different properties exhibit different mechanical performances under the same uniaxial/multiaxial loadings resulting in the incomplete applicability of a criterion for various materials. The important material/mechanical parameters to be considered in multiaxial fatigue analysis of ductile materials include the shear fatigue properties and shear fatigue damage parameters, while for brittle materials, they are uniaxial fatigue properties and normal fatigue damage parameters [20]. Ellyin et al. [21] also indicate that there is currently no widely accepted criterion that predicts the multiaxial fatigue life of various types of materials due to the complexity of multiaxial fatigue and its dependence on the microstructure of different materials. Therefore, the necessity of evaluating and comparing multiaxial fatigue criteria for different types of materials is indubitable.

In this paper, the rest of this research is structured as follows: Section 2 presents three typical critical plane multiaxial fatigue criteria and the proposed criterion based on the strain energy concept; Section 3 introduces the materials and experimental data of various loading paths for multiaxial fatigue analysis; Section 4 performs model evaluation and comparison of the four critical plane criteria; Finally, Section 5 makes a summary of the current research.

2. Critical plane-based multiaxial fatigue criteria

The critical plane approach is developed from the experimental observations of nucleation and crack growth, which has been widely used in the prediction of multiaxial fatigue life and failure plane under various loadings [22]. Particularly, the critical plane is considered as the most likely failure plane of a material under fatigue loadings. The definition of the critical plane varies with different fatigue criteria, which can be summarized, but not limited as the shear strain plane, normal strain plane, and maximum damage parameter plane based on previous studies [12], [17], [22], [23], [24], [25], [26], [27], [28]. In this section, the proposed modified GSE criterion with an additional material constant and the commonly-used Fatemi-Socie (FS), Wang-Brown (WB) and modified Smith-Watson-Topper (MSWT) fatigue criteria are introduced in this analysis.

2.1. Wang-Brown criterion

Through considering the contribution of normal strain and shear strain for crack initiation and growth, Kandil, Brown and Miller (KBM) [29] presented a criterion under biaxial loadings. However, the early KBM criterion cannot characterize the effect of mean stress on fatigue life. In this regard, Wang and Brown [24], [25] performed a mean stress correction for the KBM criterion according to the mean stress approach of Morrow:

$$(1)\quad \gamma_a + S\Delta\varepsilon_n = A\frac{\sigma'_f - 2\sigma_{n,mean}}{E}(2N_f)^b + B\varepsilon'_f(2N_f)^c$$

and

$$(2) A=1+\nu_e+S(1-\nu_e)$$

$$(3) B=1+\nu_p+S(1-\nu_p)$$

where γ_a is the maximum shear strain amplitude; $\Delta\varepsilon_n$ is the normal strain range on the maximum shear strain plane; ν_e and ν_p are, respectively, the elastic and plastic Poisson's ratio; σ_f' and b are the fatigue strength coefficient and fatigue strength exponent, respectively; ε_f' and c are the fatigue ductility coefficient and fatigue ductility exponent, respectively; E is the Young modulus; $\sigma_{n,mean}$ is the normal mean stress on the critical plane; N_f is the number of cycles to failure. *S* is the model coefficient, which is normally derived by fitting fatigue data under uniaxial loadings or calculated by Eq. (4)[22]:

$$(4) S=\frac{\frac{\tau_f'}{G}(2N_f)^{b_0}+\gamma_f'(2N_f)^{c_0}-(1+\nu_e)\frac{\sigma_f'}{E}(2N_f)^{b}-(1+\nu_p)\varepsilon_f'(2N_f)^{c}}{(1-\nu_e)\frac{\sigma_f'}{E}(2N_f)^{b}+(1-\nu_p)\varepsilon_f'(2N_f)^{c}}$$

In reality, the material constant *S* is not an invariable value due to the scattered properties of material, like the TC4 alloy as shown in Fig. 1. Although the material constant *S* varies with fatigue life N_f, its slight change has shown little effect on fatigue life prediction accuracy based on the trial calculated results under different values of *S*, which is consistent with the results in [22]. In the current study, *S* is obtained by calculating the mean value within the fatigue life range of 5000–50,000 cycles referring to [26].

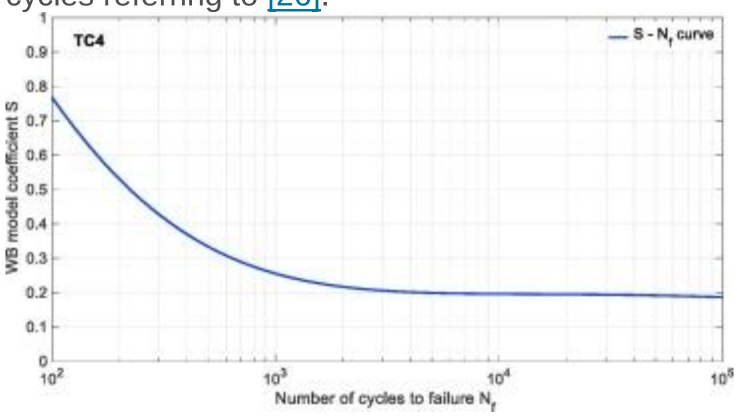


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Fig. 1. *S* vs. N_f curve of WB criterion for the TC4 alloy.

2.2. Fatemi-Socie criterion

Although the strain-based criterion of Wang and Brown offers satisfactory prediction of fatigue life, it lacks the ability to describe the effect of additional hardening caused by non-proportional loadings. Accordingly, Fatemi and Socie [27] developed a fatigue criterion by replacing the normal strain amplitude in the WB criterion with the maximum normal stress on the critical plane to reflect the effect of normal behavior on crack propagation:

$$\gamma_a\left(1+k_{FS}\frac{\sigma_{n,max}}{\sigma_y}\right)=\frac{\tau_f'}{G}(2N_f)^{b_0}+\gamma_f'(2N_f)^{c_0} \tag{5}$$

where $\sigma_{n,max}$ is the maximum normal stress on the critical plane, σ_y is the cyclic yield strength obtained from 0.05% offset rule [12]. τ_f' and b_0 are the shear fatigue strength coefficient and shear fatigue strength exponent, respectively; γ_f' and c_0 are the shear fatigue ductility coefficient and shear fatigue ductility exponent, respectively; G is the shear modulus; and k_{FS} is the normal stress sensitivity parameter, which presents the sensitive factor for normal stress on the critical plane, and can be obtained by referring the method of obtaining the material constant *S* of WB criterion. The relationship between k_{FS} and fatigue life can be expressed by [17], [22]:

$$k_{FS}=\left[\frac{\frac{\tau_f'}{G}(2N_f)^{b_0}+\gamma_f'(2N_f)^{c_0}}{(1+\nu_e)\frac{\sigma_f'}{E}(2N_f)^{b}+(1+\nu_p)\varepsilon_f'(2N_f)^{c}}-1\right]\frac{2\sigma_y}{\sigma_f'(2N_f)^{b}} \tag{6}$$

2.3. Proposed modified generalized strain energy (MGSE) criterion

Ince and Glinka [28] proposed generalized strain energy (GSE) and generalized strainamplitude (GSA) criteria based on the maximum damage plane. Recently, Yu et al. [17]pointed out that these two criteria have shown limited ability for pure shear fatigue and shear dominated multiaxial fatigue and then modified the two criteria based on the plane near the maximum shear strain plane, which shows good life prediction ability for ductile materials like GH4169 and TC4 alloys. However, the robustness of prediction accuracy is limited for various materials due to the lack of additional material constants, although the prediction results of MGSA and MGSE are acceptable. Therefore, additional material constants are introduced for robust model applications to different types of materials. Note from [17] that MGSE criterion apparently provides a better correlation of fatigue data especially for high cycle fatigue than the MGSA criterion. Similarly, the MGSE criterion modified by introducing an additional material constant is derived as follow:

$$\tau_{max}\frac{\Delta\gamma}{2}+k_{MGSE}\sigma_{n,max}\frac{\Delta\varepsilon_n}{2}=\frac{\tau_f'^2}{G}(2N_f)^{2b_0}+\tau_f'\gamma_f'(2N_f)^{b_0+c_0} \tag{7}$$

where τ_{max} is the maximum shear stress, $\Delta\gamma$ and $\Delta\varepsilon_n$ are the shear strain range and normal strain range on the critical plane, respectively; the added additional material constant, k_{MGSE} is the normal strain energy sensitivity parameter, which characterizes the contribution of normal strain energy on crack propagation, and has shown a detrimental effect for prediction accuracy of fatigue life.

For fully-reversed uniaxial fatigue, the normal strain amplitude on the maximum shear strain plane, $\frac{\Delta\varepsilon_n}{2}$ and the maximum shear strain amplitude, $\frac{\Delta\gamma}{2}$ can be estimated by uniaxial material properties [15], [30]:

$$\frac{\Delta\varepsilon_n}{2}=\frac{\sigma_f'}{E}(2N_f)^b+\varepsilon_f'(2N_f)^c \tag{8}$$

$$\frac{\Delta\gamma}{2}=(1+v)\varepsilon_a=(1+v_e)\frac{\sigma_f'}{E}(2N_f)^b+(1+v_p)\varepsilon_f'(2N_f)^c \tag{9}$$

Both the maximum shear stress, τ_{max} and the maximum normal stress on the maximum shear strain plane can be estimated by the axial stress amplitude, σ_a based on the Mohr's circle in the case of fully-reversed uniaxial fatigue as shown in Fig. 2. Moreover, they can be respectively expressed by a function of fatigue life [22], [30]:

$$\tau_{max}=\tau_a=\frac{\sigma_a}{2}=\frac{1}{2}\sigma_f'(2N_f)^b \tag{10}$$

$$\sigma_{n,max}=\sigma_{n,a}=\frac{\sigma_a}{2}=\frac{1}{2}\sigma_f'(2N_f)^b \tag{11}$$

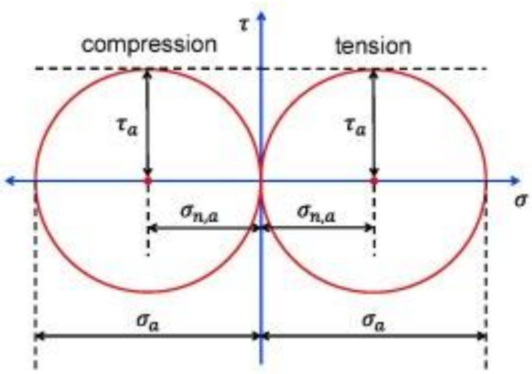

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Fig. 2. Mohr's circle presentation for fully-reserved uniaxial tension-compression fatigue.

Combining with Eqs. (7), (8), (9), (10), (11), the additional material constant of MGSE can be deduced as:

$$k_{MGSE}=\frac{\frac{\tau_f'^2}{G}(2N_f)^{2b_0}+\tau_f'\gamma_f'(2N_f)^{b_0+c_0}-\frac{1}{2}\left[(1+v_e)\frac{\sigma_f'^2}{E}(2N_f)^{2b}+(1+v_p)\sigma_f'\varepsilon_f'(2N_f)^{b+c}\right]}{\frac{1}{2}\frac{\sigma_f'^2}{E}(2N_f)^{2b}+\sigma_f'\varepsilon_f'(2N_f)^{b+c}} \tag{12}$$

Similarly, the criterion coefficient of MGSE k_{MGSE} can be obtained by fitting uniaxial fatigue test data or calculating the mean value of coefficient of a certain life interval according the relationship between criterion coefficient and fatigue life as shown in Eq. (12).

2.4. Modified SWT criterion

In order to overcome the inherent shortcomings of various criteria, Jiang [31], [32], [33]proposed a new damage parameter (DP) based on SWT criterion by using critical plane approach to consider the energy concept and material memory:

$$DP_{MSWT}=\sqrt{2a\Delta\varepsilon\langle\sigma_{max}\rangle+\frac{1-a}{2}\Delta\gamma\Delta\tau} \tag{13}$$

where DP_{MSWT} denotes the damage parameter of modified SWT (MSWT); $\Delta\varepsilon$ and $\sigma_{max}$are the normal strain range and the maximum normal stress on a material plane, respectively; $\Delta\gamma$ and $\Delta\tau$ are the shear strain range and shear stress range on the critical plane, respectively. The symbol $\langle\rangle$ is MacCauley bracket (i.e., $\langle x\rangle=0.5(x+|x|)$) which avoids negative damage. The criterion coefficient a presents the additional material constant whose range from 0 to 1.0 and its value varies with materials. Note from Eq. (13) that when $a=1$, Eq. (13) is reduced to the SWT criterion. When $0\leqslant a\leqslant 0.37$, the criterion applies shear crack behavior, when $a\geqslant 0.5$, the criterion predicts tensile crack behavior. Mixed crack behavior is predicted by choosing a between 0.37 and 0.5. The critical plane of MSWT is the plane with the maximum damage value.

Based on the above damage parameter of MSWT, Yu and Zhang et al. [34] indicate that the fatigue life can be calculated by:

$$(DP-DP_0)^{\xi}N_f=C \tag{14}$$

where DP_0, ξ and C are constants obtained by best fitting of experimental data. In order to obtain these constants, a large amount of additional experimental data are needed to ensure the accuracy of the fitting. Therefore, Ma and Markert [35] use the uniaxial cases of Coffin-Mason equation to express the relationship between fatigue life and damage parameter:

$$DP_{MSWT}=\sqrt{\frac{4a\sigma_f'^2}{E}(2N_f)^{2b}+4a\sigma_f'\varepsilon_f'(2N_f)^{b+c}} \tag{15}$$

Note that Eq. (15) makes fatigue life prediction more convenient and efficient due to all known fatigue parameters in this equation. All fatigue life for various materials predicted by MSWT criterion are calculated from Eq. (15) in this analysis.

3. Materials and experiments

In this section, experimental datasets of four materials under various loading paths [35], [36], [37], [38] are introduced for model validation and comparison. The static and fatigue properties of the four materials are shown in Table 1, in which the shear fatigue properties of sintered porous iron are estimated from uniaxial experimental data (see Section 4.1). The specimens and experimental conditions of these materials can be referred to [35], [36], [37], [38]. Particularly, the ductility/brittleness of these materials can be summarized as follows: TC4 and Wrought Ti-6Al-4V are ductile materials, GH4169 at room temperature is a semi-ductile material and sintered porous iron is brittle material according to [35], [36], [37], [38], [39], [40], [41]. Material-dependent failure criteria are needed to account for the differences in crack nucleation and early growth [23], [42], [43], [44], [45], [46], [47]. The failure modesare generally determined by materials, temperature and loadings, i.e. material properties and loading conditions [22], [48], [49]. In particular, the loading paths controlled by triangular and sine waves for these materials include uniaxial tension, torsion, multiaxial proportional and non-proportional loadings, which are shown in Fig. 3.

Table 1. Static and fatigue properties of the four materials.

Materials	E **(GPa)**	σ_f' **(MPa)**	b	ε_f'	c	K'	n'	ν_e	σ_y **(MPa)**
TC4	108.4	1116.9	−0.049	0.579	−0.679	1031	0.0478	0.25	716.9
GH4169	198.5	1815.5	−0.06	0.45	−0.63	1892.3	0.078	0.48	1083.1
Wrought Ti-6Al-4V	108.2	987	−0.034	0.569	−0.636	878	0.034	0.29	678
Sintered porous iron	162	289	−0.074	0.047	−0.406	466.5	0.172	0.3	126.6

Materials	*G* (GPa)	τ_f' (MPa)	b_0	γ_f'	c_0
TC4	43.2	716.9	−0.06	2.24	−0.8
GH4169	67	1091.6	−0.07	4.46	−0.77
Wrought Ti-6Al-4V	42	647	−0.044	0.352	−0.502
Sintered porous iron	\ \	\ \	\ \	\ \	\ \

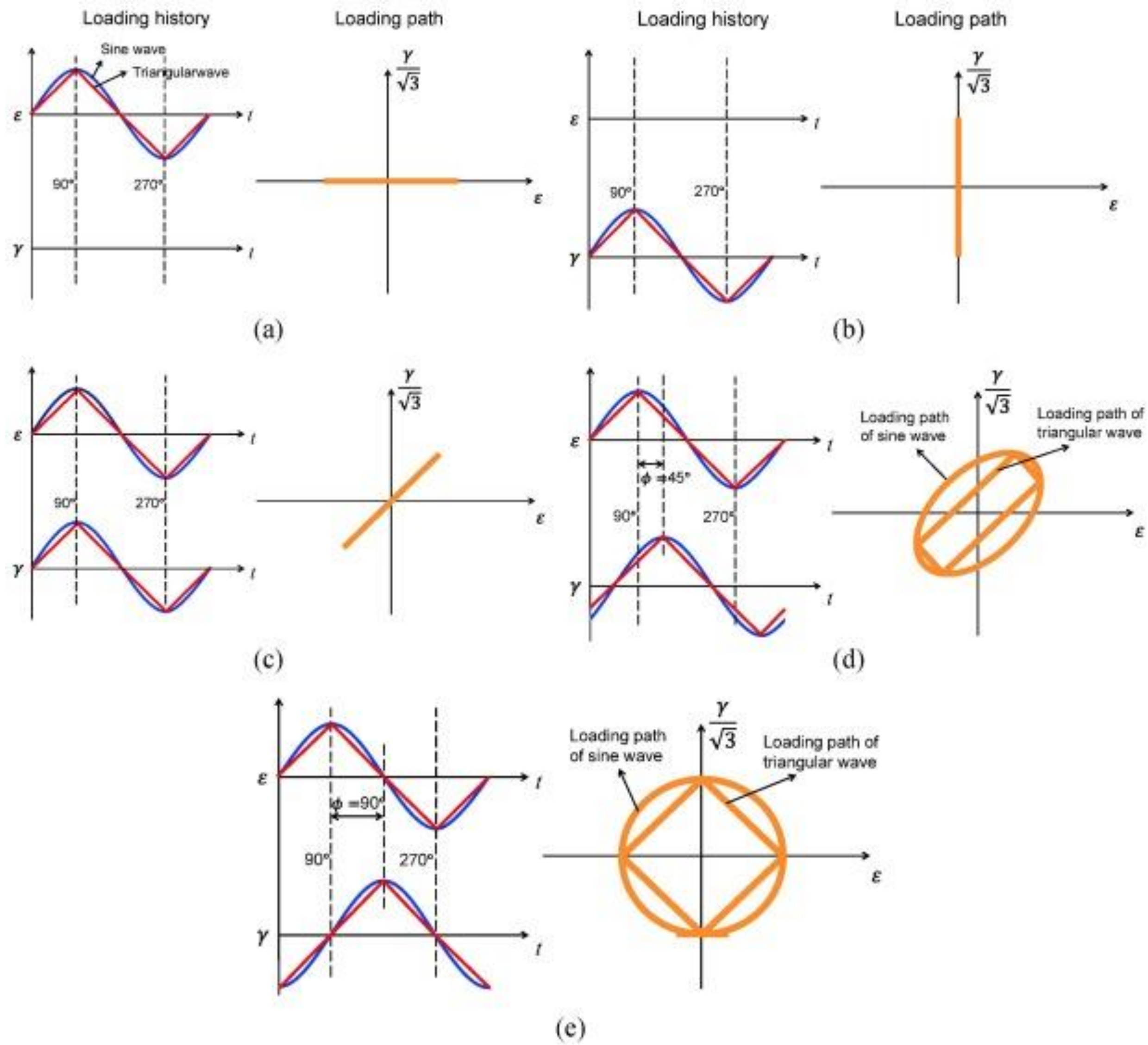

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Fig. 3. Schemes for strain history and loading paths: (a) Pure axial; (b) pure torsional; (c) axial-torsional proportional; (d) axial-torsional non-proportional loading with path sine and triangular waves for $\phi=45°$; (e) axial-torsional non-proportional loading with path sine and triangular waves for $\phi=90°$.

4. Criteria evaluation and comparison

4.1. Material constants analysis

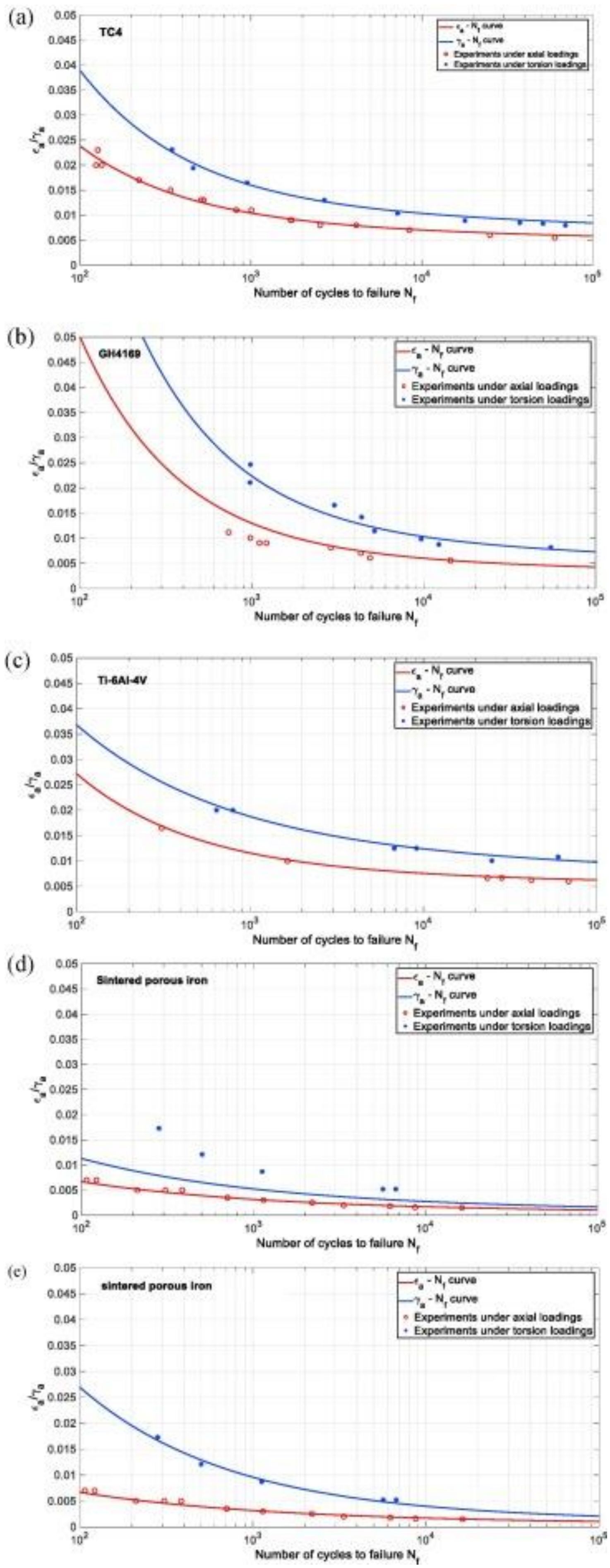
Only uniaxial fatigue properties were obtained for sintered porous iron as shown in Table 1. For the case of lacking of shear fatigue properties, they can be generally obtained by fitting pure torsional fatigue experimental data based on the Coffin-Manson equation in shear form, namely experimental fitting method, or roughly estimating from corresponding uniaxial fatigue properties based on the von Mises' criterion, namely empirical estimation method [12], [22], [30]:

$$(16)\quad \tau_f' = \frac{\sigma_f'}{\sqrt{3}};\ \gamma_f' = \sqrt{3}\varepsilon_f';\ b_0 = b;\ c_0 = c;$$

The shear fatigue properties calculated from the two abovementioned methods for the sintered porous iron are shown in Table 2, in which the shear fatigue properties in the first row were empirically estimated by using Eq. (16) under limited data conditions, while that in the second row were fitted from the pure torsional fatigue data according to the shear form of Coffin-Manson equation. In order to evaluate shear fatigue properties derived by Eq. (16)and fit test data of sintered porous iron, the Coffin-Manson curves and test data in the case of uniaxial and pure shear loadings are compared as shown in Fig. 4. As it can be seen from Fig. 4 that the Coffin-Manson curve based on shear fatigue properties estimated from Eq. (16) deviates further from the test data, while the fitted shear fatigue parameters performed better as expected.

Table 2. Shear fatigue properties obtained by two methods for the sintered porous iron.

	τ_f' **(MPa)**	b_0	γ_f'	c_0
Empirical estimation method	166.85	−0.074	0.0814	−0.406
Experimental fitting method	206.134	−0.0754	0.3588	−0.5049



(a)
TC4
ε_a - N_f curve
γ_a - N_f curve
Experiments under axial loadings
Experiments under torsion loadings
ε_a/γ_a
Number of cycles to failure N_f
(b)
GH4169
ε_a - N_f curve
γ_a - N_f curve
Experiments under axial loadings
Experiments under torsion loadings
ε_a/γ_a
Number of cycles to failure N_f
(c)
Ti-6Al-4V
ε_a - N_f curve
γ_a - N_f curve
Experiments under axial loadings
Experiments under torsion loadings
ε_a/γ_a
Number of cycles to failure N_f
(d)
Sintered porous iron
ε_a - N_f curve
γ_a - N_f curve
Experiments under axial loadings
Experiments under torsion loadings
ε_a/γ_a
Number of cycles to failure N_f
(e)
sintered porous iron
ε_a - N_f curve
γ_a - N_f curve
Experiments under axial loadings
Experiments under torsion loadings
ε_a/γ_a
Number of cycles to failure N_f

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Fig. 4. Axial/pure-shear strain amplitude vs. fatigue life curves for (a) TC4, (b) GH4169, (c) wrought Ti-6Al-4V, (d) sintered porous iron based on estimated shear fatigue properties and (e) sintered porous iron based on fitted shear fatigue properties.

During fatigue analysis, material constants of a criterion often play a key role for the fatigue life prediction accuracy. Methods for determining material constants of different criteria are also different. For the FS and WB criteria, the criterion coefficient can be obtained from the mean value of k_{FS} or S in the fatigue life range $5\times10^3 \leqslant N_f \leqslant 5\times10^4$ cycle based on Eqs. (4), (6). However, for TC4, GH4169 and wrought Ti-6Al-4V alloys, it was found that the criterion coefficient of MGSE criterion calculated from the mean value of k_{MGSE} in the same fatigue life range based on Eq. (12) is more appropriate for correlation of fatigue data through some trials and errors. For brittle material sintered porous iron, k_{MGSE} is determined by the lower fatigue life ranges based on Eq. (12), which implies hysteresis loop under low cycle fatigue can better describe the material deformation and energy dissipation processes. Therefore, the effect of normal strain energy on the fatigue crack growth should be described by the k_{MGSE} calculated from lower fatigue life ranges based on Eq. (12). The criterion coefficients of the four critical plane criteria for different materials are given in Table 3, from which note that the value of k_{MGSE} deviates further from 1 for brittle materials sintered porous iron with fitted shear fatigue properties, indicating that additional material constants provide a guarantee for ensuring fatigue life prediction accuracy of different materials. For ductile materials, k_{MGSE} is close to 1. In the absence of test data or materialproperties, k_{MGSE} is suggested to be 1 during fatigue life prediction of ductile materials [17].

Table 3. Criterion coefficients of the four critical plane criteria for different materials.

Materials	S_{WB}	k_{FS}	k_{MGSE}	a_{MSWT}
TC4	0.194	0.245	0.84	0.35
GH4169	0.355	0.28	0.583	0.38
Wrought Ti-6Al-4V	0.439	0.459	0.64	0.32
Sintered porous iron (estimated)	0.349	0.3	0.496	0.5
Sintered porous iron (fitted)	1.29	1.1	8.33	0.5

For the MSWT criterion, its criterion coefficient cannot be obtained by a specific formula. Three types of cracking behavior guide the range of criterion coefficient *a*. Zhao and Jiang [33] summarized the method of judging the crack behavior from experiments. In fact, the range of *a* can be easily determined by the properties of a material. However, the cracking behavior is load-dependent for mixed crack failure mode, in which makes *a* difficult to be determined [34]. Although criterion coefficient a is not constant under different loadings, it can be determined by making the two curves of tension-compression and torsion in the form of FP-Nf as close together as possible [50]. The determined criterion coefficient a of MSWT for TC4, GH4169, Wrought Ti-6Al-4V and sintered porous iron are 0.35, 0.38, 0.32 and 0.5, respectively as shown in Table 3.

4.2. Results comparison and analysis

The comparisons of fatigue lives predicted by using the four criteria mentioned above and tested lives for the evaluated materials are shown in Fig. 5, Fig. 6, Fig. 7, Fig. 8, Fig. 9, respectively, in which the abscissas are for the tested fatigue lives of specimens and the ordinates are for the model predicted fatigue lives by using the abovementioned criteria; the black solid diagonal represents the best prediction; the two red dotted lines represent the life factor-of-two boundaries; the two blue dashed lines represent the life factor-of-three boundaries. For the TC4, wrought Ti-6Al-4V and GH4169 alloys, most of predictions by the four criteria are within the life factor-of-three bands. However, for sintered porous iron with shear fatigue properties estimated from Eq. (16), only the MSWT criterion provides a satisfactory correlation with tested fatigue life due to no-dependence of MSWT criterion for shear fatigue properties. The WB, FS and MGSE criteria yield good predictions for uniaxial fatigue and perform poorly for fatigue life prediction of multiaxial and pure torsional fatigue, which indicate that shear fatigue properties have a significant impact on fatigue life prediction for WB, FS and MGSE criteria. The abovementioned four criteria all generally perform well for multiaxial fatigue life predictions under proportional loadings, however, yield several poor life predictions for multiaxial fatigue under 90° non-proportional loadings as shown in Fig. 5, Fig. 7. For the TC4 alloy, although both the MGSE and WB criteria as well as the FS criterion provide several poor life predictions for multiaxial fatigue under 90° non-proportional loadings, the MGSE criterion is not inferior to the performance of the FS and WB criteria, and even better as shown in Fig. 5. For the wrought Ti-6Al-4V alloy, both the FS and MGSE criteria give more accurate life predictions for multiaxial fatigue under 90° non-proportional loadings than the WB and MSWT criteria as shown in Fig. 7.

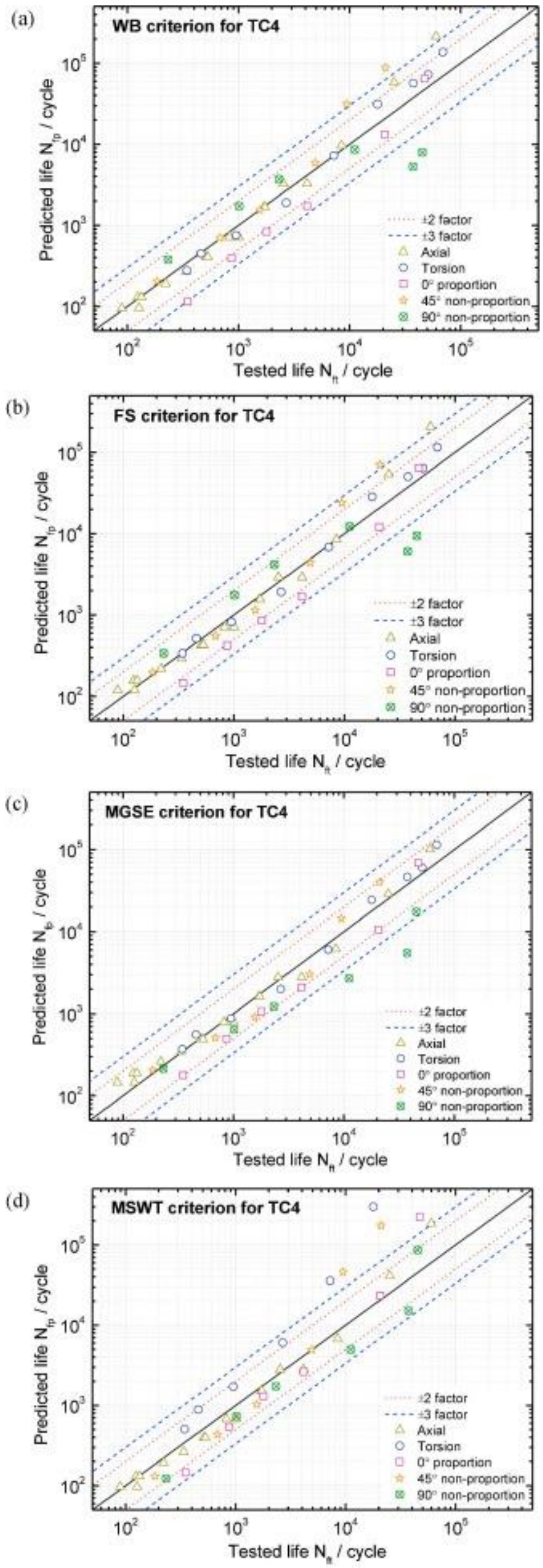

Fig. 5. Fatigue life prediction for TC4 by using (a) WB, (b) FS, (c) MGSE and (d) MSWT criteria.

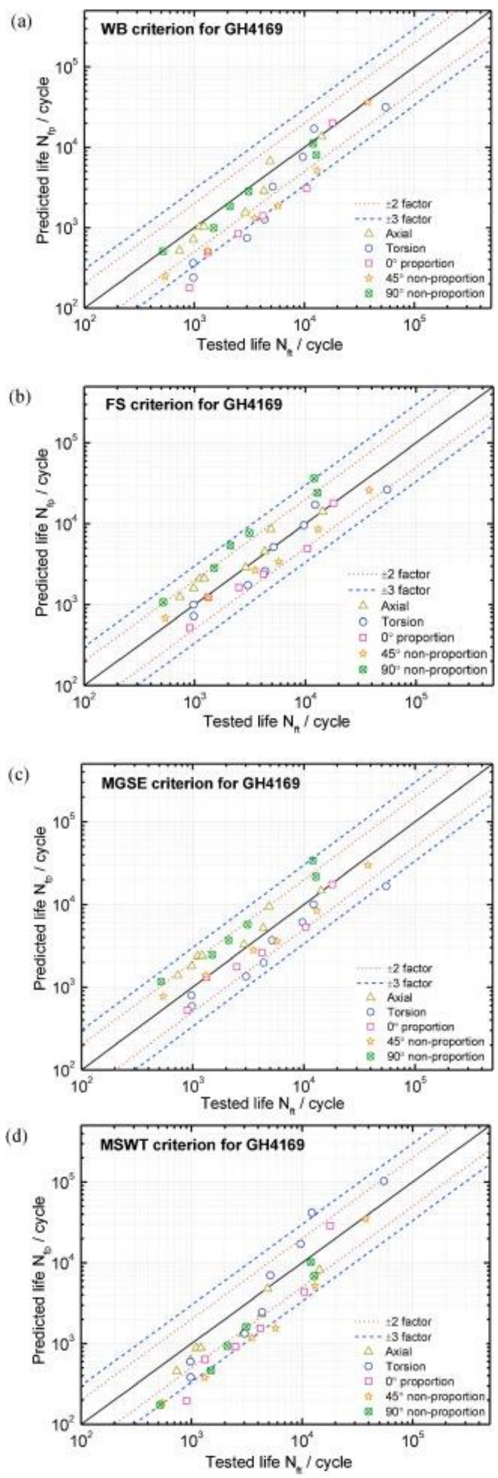

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Fig. 6. Fatigue life prediction for GH4169 by using (a) WB, (b) FS, (c) MGSE and (d) MSWT criteria.

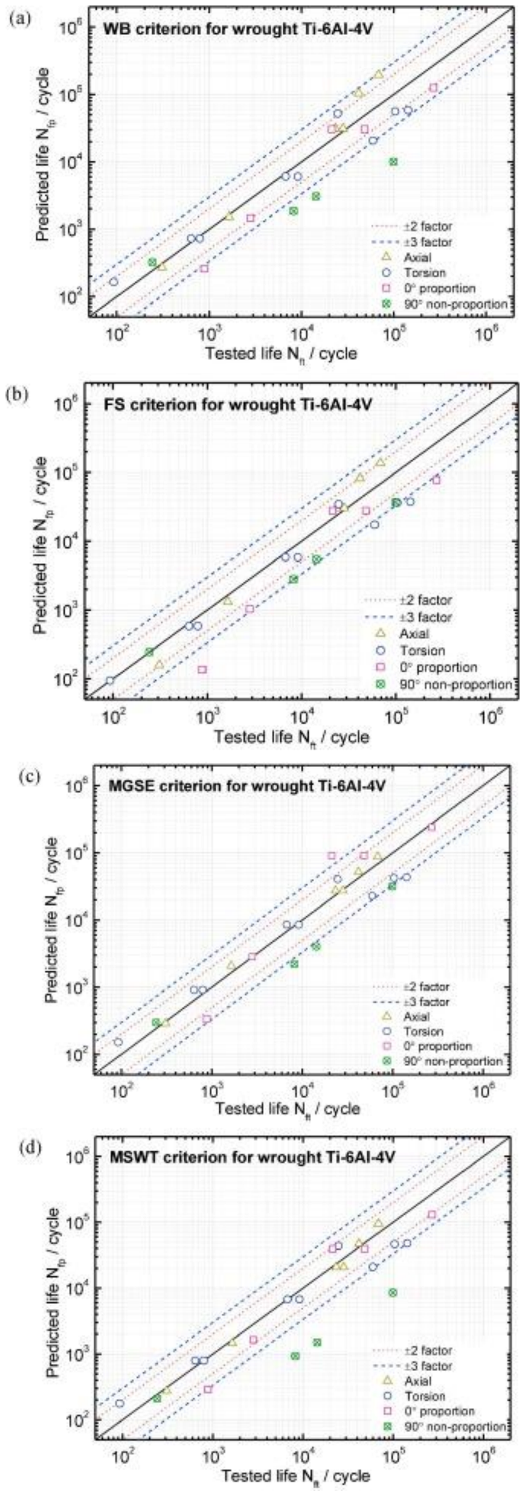

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Fig. 7. Fatigue life prediction for wrought Ti-6Al-4V by using (a) WB, (b) FS, (c) MGSE and (d) MSWT criteria.

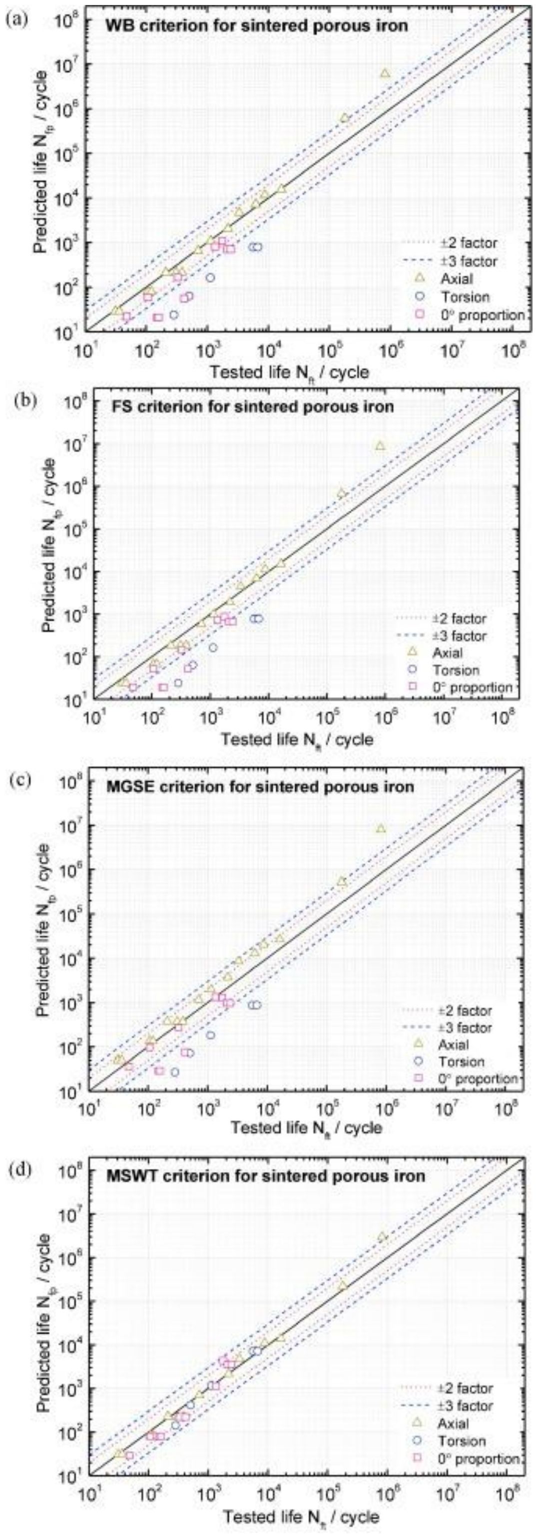

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Fig. 8. Fatigue life prediction for sintered porous iron based on estimated shear fatigue properties by using (a) WB, (b) FS, (c) MGSE and (d) MSWT criteria.

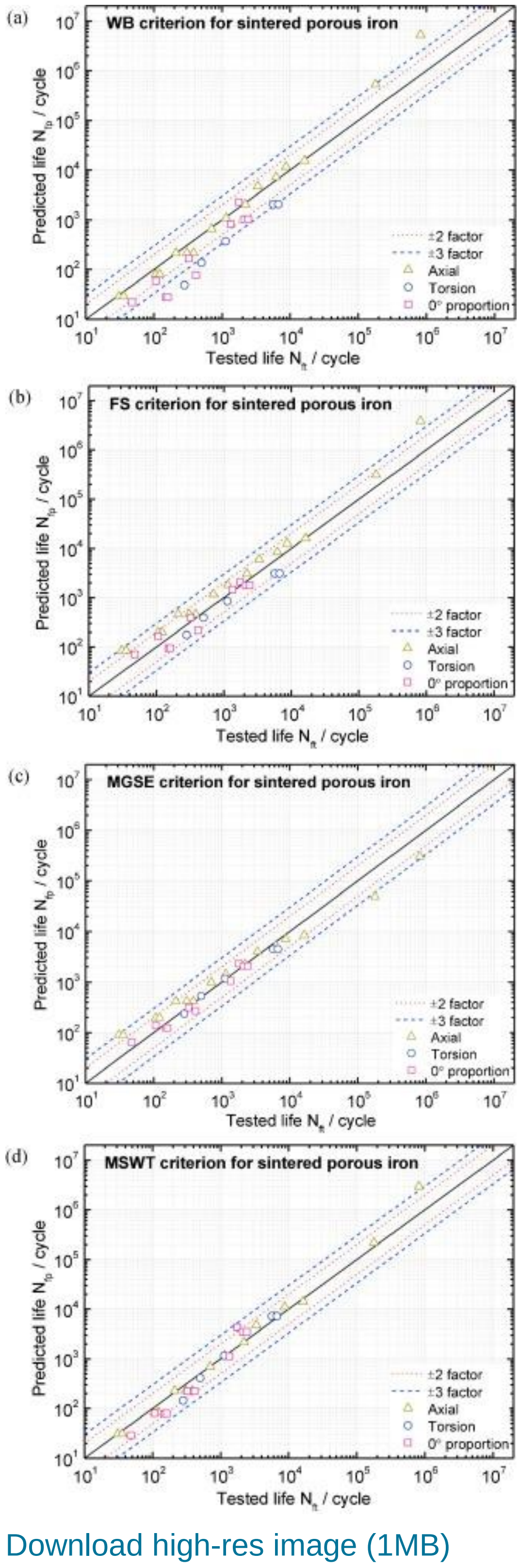

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Fig. 9. Fatigue life prediction for sintered porous iron based on fitted shear fatigue properties by using (a) WB, (b) FS, (c) MGSE and (d) MSWT criteria.

Fatigue life prediction results and distribution of life on scatter band for different materials are shown in Fig. 5, Fig. 6, Fig. 7, Fig. 8, Fig. 9, respectively. In order to further quantitatively compare and analyze the prediction ability of the four criteria for different materials, a probability analysis is carried out according to [12], [51], [52], [53], [54]:

$$P_{error} = \log_{10}(N_{fp}) - \log_{10}(N_{ft}) \quad (17)$$

where P_{error} presents the criterion prediction error, N_{fp} and N_{ft} are the predicted lives and tested lives, respectively. The negative value and positive value of P_{error} indicate underestimation and overestimation of fatigue life. The distribution of P_{error} values can quantitatively reflect the prediction errors of different criteria. However, not all data sets satisfy the normal distribution based on the K-S test due to the limited number of tested samples, thus box plots are created for criteria comparison of different materials as shown in Fig. 10. Particularly, the point and horizontal lines in the box represent the mean and median respectively, the size of the box reflects the size of the standard deviation and the data points marked by the 'star' symbols represent the min/max outliers of prediction error sample data. It’s worth mentioning that the mean values of prediction errors closer to 0 and the lower standard deviation indicate the better prediction accuracy of a criterion.

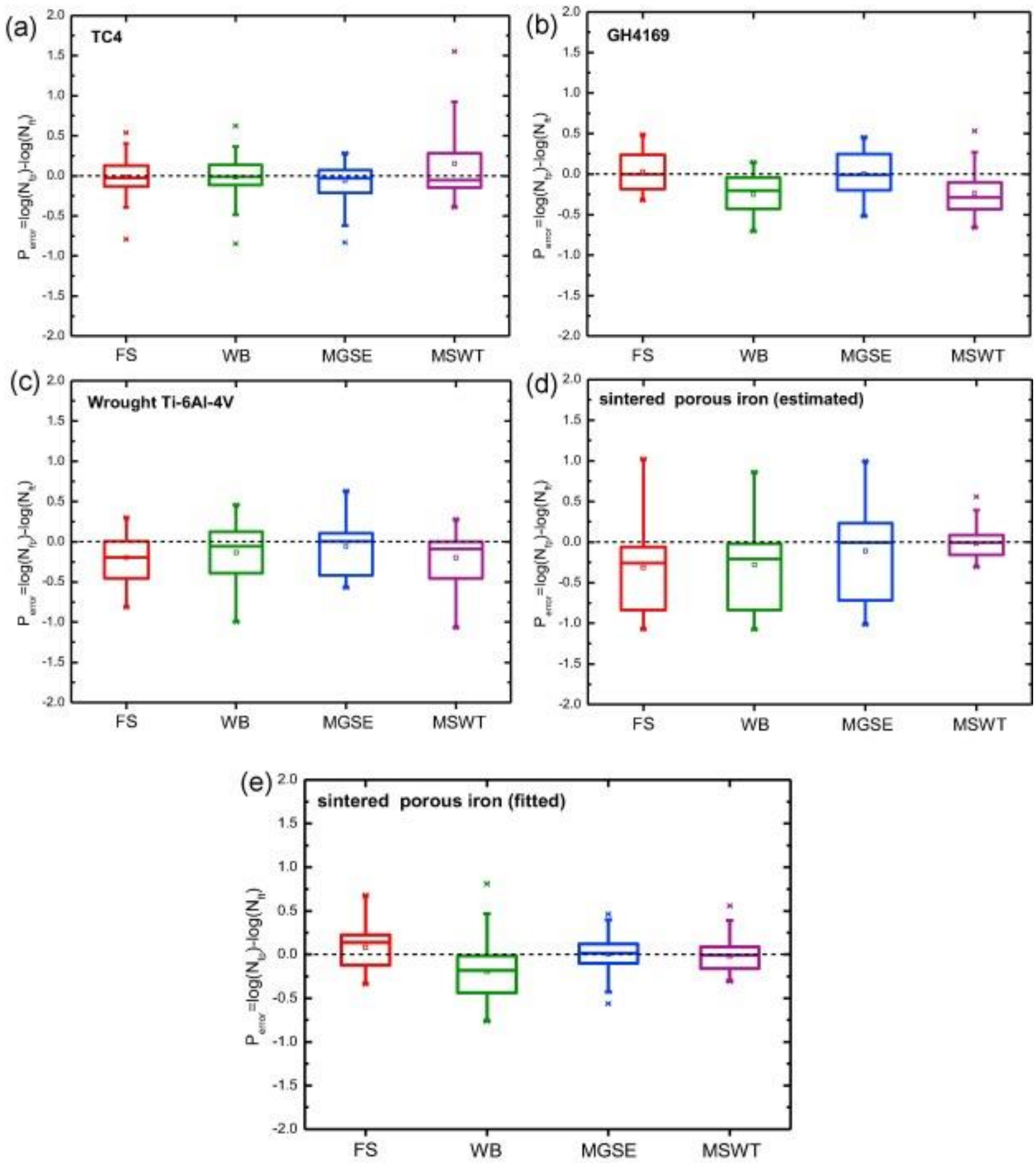

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Fig. 10. Box plot of criteria prediction errors for (a) TC4, (b) GH4169, (c) wrought Ti-6Al-4V, (d) sintered porous iron with estimated shear properties and (e) sintered porous iron with fitted shear properties.

Note from Fig. 10 that FS, WB and MGSE criteria give the most accurate predictions for TC4 and the worst predictions for sintered porous iron based on the estimated shear fatigue properties. However, the MSWT criterion offers a better correlation between predicted lives and tested lives for brittle material sintered porous iron than other ductile and semi-ductile materials due to no-dependence of shear fatigue properties as shown in Fig. 10(d), which is consistent with the conclusion of Li et al. [20]. It also reflects that the shear fatigue properties estimated by Eq. (16) are inappropriate for fatigue life prediction of the sintered porous iron. The prediction results obtained by the abovementioned four criteria for brittle and ductile materials are all acceptable, which have shown certain robustness in fatigue life prediction ability for different types of materials. Moreover, the proposed MGSE criterion with an additional material constant provide better correlation with tested lives by a lower mean prediction error and standard deviation for the four materials.

5. Conclusions

In the current study, four critical plane criteria with additional material constants were evaluated for ductile and brittle materials under uniaxial/multiaxial fatigue loadings. The following conclusions can be summarized from the current research:

(1)

A modified generalized strain energy (MGSE) criterion is proposed for multiaxial fatigue analysis, in which an additional material constant is introduced to describe the effect of normal stress associated with the critical plane on crack propagation for different materials.

(2)

Through a comparative study on critical plane criteria, the shear fatigue propertiesestimated by using the empirical equation cannot correlate well the fatigue data and its usage in fatigue life prediction, which might cause significant errors for criteria depending on shear fatigue properties such as WB, FS and MGSE criteria. Experimental fitted fatigue properties are more accurate than the estimated fatigue properties for brittle materials.

(3)

For the four types of materials, WB, FS, MGSE and MSWT criteria all provide satisfactory fatigue life predictions, especially the MGSE criterion superior to others with lower model prediction errors. While MSWT as a criterion based on uniaxial fatigue properties provide more accurate life predictions for brittle materials rather than ductile materials, which implies that the criterion based on uniaxial conditions is more suitable for brittle materials and the criterion based on shear conditions for ductile materials.