Proceedings of 36th International Conference on Ocean, Offshore and Artic Engineering
OMAE2017
25-30 June, 2017, Trondheim, Norway

OMAE2017-62493

VALUE OF INFORMATION-BASED INSPECTION PLANNING FOR OFFSHORE STRUCTURES

Arifian Agusta
Department of Civil Engineering
Technical University of Denmark
Kongens Lyngby, Denmark 2800
Email:ariag@byg.dtu.dk

Sebastian Thöns
Department of Civil Engineering
Technical University of Denmark
Kongens Lyngby, 2800
Denmark

Bernt J. Leira
Department of Marine Technology
NTNU-Trondheim
Trondheim, NO-7491
Norway

ABSTRACT

Asset integrity and management is an important part of oil and gas industry especially for existing offshore structures. With declining oil price, the production rate is an important factor to be maintained that makes integrity of the structure one of the main concerns. Reliability based and risk- based inspection (RRBI) constitutes an efficient method to optimize inspection planning when it is based on the Bayesian decision analysis. A pre-posterior Bayesian decision analysis and especially a Value of Information analysis allows to explicitly quantify the expected benefits, costs and risks associated with each inspection strategy. A simplified and generic risk-based inspection planning utilizing pre-posterior Bayesian decision analysis had been proposed Faber et al. [1] and expanded within the field of offshore engineering by Straub [2]. This paper provides considerations on the theoretical background and a Value of Information analysis-based inspection planning. The paper will start out with a review of the state-of-art RBI planning procedure based on Bayesian decision theory and its application in offshore structure integrity management. With examples illustrating the full use of the Value of Information approach, it is pointed to further research challenges.

INTRODUCTION

Asset integrity management for offshore structure is important to ensure operational safety during hydrocarbon production. During its lifetime, an offshore structure is subjected to cyclic environmental loads i.e. wave, current, and wind which are the common causes of fatigue failure in the structures. There are several fatigue failures with catastrophic consequences in the 1980s with the most notable example being the Alexander L. Kielland accident [3].

Fatigue failure is essentially the consequence of deterioration of structural strength due to crack growth from initial defects in the material. The causes of the defects vary and involve complex interaction between micro-structures that caused by the welding process. The simplest way to describe the fatigue performance is to use S-N Curve and Miner's summation rule. However, an S-N curve approach with Miner's rule has limited comprehensiveness, which necessitates a fracture mechanics approach. The fracture mechanic approach then can be calibrated to the S-N curve approach based on equal fatigue reliability.

This paper first introduces the basic of pre-posterior decision analysis to plan the inspection and repair strategy with focus on the Value of Information (VoI) theory. Probability modelling of fatigue failure based on the fracture mechanics is presented. The uncertainties related to the fatigue crack growth and inspection outcomes are identified according to Folso et al [4] and Hong [5]. The updating methodology of the probability of fatigue failure is formulated according to the Bayes' Rule. Various inspection and repair strategies are described for the prior and the pre-posterior analysis in the form of decision tree. The extensive form analysis is used to find the optimum inspection and repair strategy based on VoI. The results from the extensive form analysis are then utilized to form the decision rules for normal form analysis.

INSPECTION PLANNING AS PRE-POSTERIOR DECISION ANALYSIS

Planning an inspection for offshore structures is an example of a decision problem. Solving the decision problem by means of Bayesian decision theory, including the pre-posterior decision analysis, had been introduced by Raiffa et al. [6] in the form of extensive and normal form analysis.

Inspection planning can be modeled using a decision tree as shown in Fig. 1. The inspection space, $\boldsymbol{e}$, consists of available inspection strategies that can be performed. The outcomes of the inspection are collectively stored in the inspection outcome space $\boldsymbol{Z}$. The repair action space, $\boldsymbol{a}$, consists of possible repair actions available to the decision maker. The repair action space and the inspection outcome space can be connected to form the decision rules $d(\boldsymbol{Z}) = \boldsymbol{a}$ that would be required in a normal form analysis. The state space, $\boldsymbol{\theta}$, describes the system states, i.e. failure or survival of the associated structural components, and may be affected by the outcomes of the decision rules $d(\boldsymbol{Z})$.

The expected utility for a certain terminal action a, $E[C(a)]$, can be calculated as follows [7]:

$$E[C(a)] = \sum_{i=1}^{n_o} p(\theta_i|a)C(a,\theta_i) \quad (1)$$

where $i = 1,..,n_o$ is the number of possible state of natures, $\boldsymbol{\theta}$, $p(\theta_i|a)$ is the probability of a certain state of nature, θ_i given the terminal action a, and $C(a,\theta_i)$ is the utility of set (a,θ_i). As seen in Eqn. (1) the utility may be associated to monetary value i.e. to costs or benefits.

In structural reliability, the utility is often considered as the summation of cost of failure, repair, and inspection over the service life, T_{SL}, that may vary for different inspection and repair strategies as follows:

$$C_T(\boldsymbol{e},\boldsymbol{a}) = C_I(\boldsymbol{e}) + C_R(\boldsymbol{e},\boldsymbol{a}) + C_f(\boldsymbol{e},\boldsymbol{a}) \quad (2)$$

where $\boldsymbol{e} = [e_1, e_2, ..., e_{nI}]$ is the possible inspection strategies, $\boldsymbol{a} = [a_1, a_2, ..., a_{nR}]$ is the possible repair strategies that can be performed, $C_I(\boldsymbol{e})$ is the cost of inspections, $C_R(\boldsymbol{e},\boldsymbol{a})$ is the cost of repairs, and $C_f(\boldsymbol{e},\boldsymbol{a})$ is the cost of failures.

For inspection planning, the objective is to find the optimum repair and inspection strategy, which maximizes the utility calculated by Eqn. (1) and (2). The optimization problem can be formulated by utilizing extensive form decision analysis as follows [6]:

$$E[C_T(e^*,a^*,T_{SL})] = \arg\min_{e} E_Z[\arg\min_{a} E''_{\theta}[C_T(\boldsymbol{e},\boldsymbol{a},T_{SL})]] \quad (3)$$

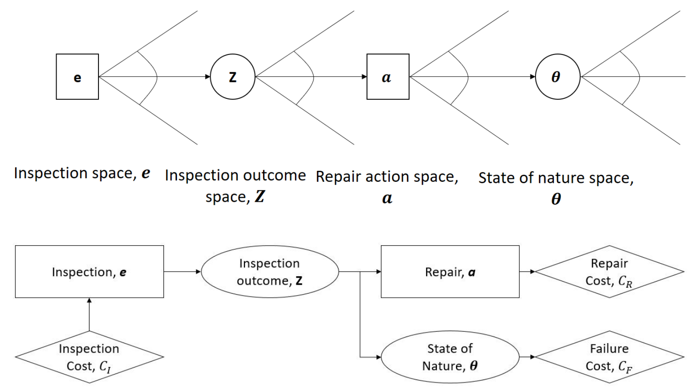

FIGURE 1: Decision tree (top) and influence diagram (bottom) to illustrate the pre-posterior decision analysis. The rectangular nodes are the decision nodes and the circular nodes are the chance nodes.

where e^* and a^* are the optimum inspection and the optimum repair plan, and $E''_{\theta}[C_T(\boldsymbol{e},\boldsymbol{a},T_{SL})]$ is the expected value over the posterior state of nature.

Value of Information (VoI)

VoI theory is one important concept building upon the pre-posterior decision analysis that describes the value of performing an inspection/experiment e.g. to reduce the expected costs and the risks. If we consider a^*_{prior} as the optimum repair action without any inspection performed (prior analysis), the Value of Information of performing an inspection can be calculated as follows:

$$VoI = E[C_T(e^*,a^*,T_{SL})] - E[[C_T(a^*_{prior},T_{SL})]] \quad (4)$$

In this paper, the VoI theory will be used as a basis for planning inspection and repair strategy.

PROBABILITY OF FATIGUE FAILURE

In inspection planning, the fatigue failure occurs if the crack depth, l, of the hotspot exceeds a certain critical crack depth, l_c. In mathematical terms, the fatigue failure can be modeled by a limit state function, g_f, as follows:

$$g_f(\boldsymbol{X},t) = l_c - l(\boldsymbol{X},t) \quad (5)$$

where $\boldsymbol{X}$ is a vector that contains random variables defining the crack size growth. Structural failure corresponds to the condition that $g_f(\boldsymbol{X},t) \leq 0$.

The crack size is a function of time as can be seen in Eqn. (5). The crack growth is calculated using the widely applied Paris Law for a crack in an infinite size plate as follows:

$$\frac{dl(n)}{dn} = C\left(\Delta S\sqrt{\pi l(n)}\right)^m \tag{6}$$

where l is the crack depth, n is the number of cycles, ΔS is the stress range, and C and m are the empirical parameters. The differential equation in Eq. (6) can be solved to find the crack depth as a function of time, $l(\boldsymbol{X},t)$ [4,8]:

$$l(\boldsymbol{X},t) = \left(\left(1-\frac{m}{2}\right)CB_{SIF}^m B_{\Delta S}^m \Delta S^m \pi^{\left(\frac{m}{2}\right)} vt - l_o^{\left(1-\frac{m}{2}\right)}\right)^{\left(\frac{1}{1-m/2}\right)} \tag{7}$$

where l_0 is the initial crack size, v is the annual cycle rate, B_{SIF} and $B_{\Delta S}$ are the model uncertainties in the stress intensity factor and the stress range calculation as shown by Folso et al. [4]. The random variable space is defined as $\boldsymbol{X} = [\Delta S, v, l_o, C, m, B_{SIF}, B_{\Delta S}]$.

The probability of fatigue failure therefore can be calculated by using the following equation:

$$P_f(t) = \int_{\Omega_{(t)}} f(\boldsymbol{X})dx \tag{8}$$

where $f(\boldsymbol{X})$ is the joint distribution of all random variables contained in vector $\boldsymbol{X}$ and $\Omega_f(t) = g_f(\boldsymbol{X},t) \leq 0$.

Eqn. (8) can be solved using Monte Carlo Simulation (MCS), i.e. by creating an indicator function, $I_f(\boldsymbol{X})$, where:

$$\begin{aligned} I_f(\boldsymbol{X},t) &= 1, g_f(\boldsymbol{X},t) \leq 0 \\ I_f(\boldsymbol{X},t) &= 0, g_f(\boldsymbol{X},t) > 0 \end{aligned} \tag{9}$$

Eqn. (8) can therefore be written as follows based on application of the MCS method:

$$P_f(t) \approx \frac{1}{n_{MCS}} \sum_{n=1}^{n_{MCS}} I_f(\boldsymbol{X},t) \tag{10}$$

where n_{MCS} is the number of simulation performed by using the Monte Carlo method.

PROBABILITY OF INSPECTION OUTCOMES

The inspection outcomes, $\boldsymbol{Z}$, are affected by many factors such as the inspection technique, the inspector performance, the environmental condition, and the inspection equipment. The probability of detection (PoD) is the measure of detection success taking the uncertain factors into account and may be monotonically increasing from 0 to 1 [2]. The one dimensional log-logistic PoD model accounting for the mean rate of success for given crack depth, l, is defined as follows [2]:

$$PoD(l) = \frac{exp(\alpha + \beta_D ln(l))}{1 + exp(\alpha_D + \beta_D ln(l))} \tag{11}$$

where α_D,β_D, are the model parameters that depend on the applied inspection techniques.

The limit state function for an inspection resulting in undetection, Z_1, is defined as follows [5]:

$$g_{Z_1}(\boldsymbol{X},t_I) = \Phi^{-1}(PoD(l(\boldsymbol{X},t_I))) - \tilde{Z} \tag{12}$$

where $\tilde{Z}$ is a random variable with Standard Normal distribution and t_I is the time when the inspection is performed.

Therefore, the probability of undetection, $P_{Z_1}(t_I)$ can be calculated as follows:

$$P_{Z_1}(t_I) = \int_{\Omega_{Z_1(t_I)}} f(\boldsymbol{X})dx \tag{13}$$

where $\boldsymbol{X}$ is a vector containing the random variables in Eqn. (7) and $\Omega_{Z_1(t_I)}$ is the domain where $g_{Z_1}(\boldsymbol{X},t_I) \leq 0$. The probability of detection can easily be found with $P_{Z_2} = 1 - P_{Z_1}$.

RELIABILITY UPDATING OF PROBABILITY OF FATIGUE FAILURE

By performing the inspection, the probability of fatigue failure can be updated by following Bayes' Rule as follows:

$$P(F(t)|Z_1(t_I)) = \frac{P_f(t) \bigcap P_{Z_1}(t_I)}{P_{Z_1(t_I)}} \tag{14}$$

where t_I is the time when the inspection is performed, and $P_{Z_1(t_I)}$ is the probability of undetection at time t_I. By inserting Eqn. (8) and (13), Eqn. (14) can be written as:

$$P(F(t)|Z_1(t_I)) = \frac{\int_{\Omega_{f(t)} \cap \Omega_{Z_1(t_I)}} f(\boldsymbol{X})dx}{\int_{\Omega_{Z_1(t_I)}} f(\boldsymbol{X})dx} \tag{15}$$

ANNUAL PROBABILITY OF FATIGUE FAILURE

In structural engineering context, the probability of failure of interest most often is the annual probability of fatigue failure. The annual probability of fatigue failure given survival before time t, $\Delta P_f(t)$, is calculated as follows:

$$\Delta P_f(t) = \frac{P_f(t) - P_f(t - \Delta t)}{\Delta t(1 - P_f(t - \Delta t))} \tag{16}$$

where Δt is taken as 1. The probability of fatigue failure, $P_f(t)$, in Eqn. (16) may refer to the updated probability of failure in Eqn. (15) if $t \geq t_I$ or Eqn. (8) if otherwise.

REPAIR STRATEGY

The main objective of the inspection planning is to find the optimum inspection and repair strategy. After an inspection, the decision maker could decide to perform the repair immediately or in the future. A repair strategy can be modeled with the time of repair and/or the repair location. The repair performance depends on many factors such as repair techniques, difficulties of repair location, and costs of performing the repair. An example of repair strategies decision tree based on the time of repair can be seen in top figure of Fig. 2.

There are two assumptions that widely utilized for the behavior of the repaired element [2]:

1. The repaired element behaves like new element,
2. The repaired element behaves like an element that has no indication during inspection.

In this paper, the first assumption is used for the repaired element. A new initial crack size is generated after the repair is performed. It is assumed that the repair shall be performed by welding, and thus the crack size may be calculated as follows:

$$l_r = B_Q l_o \tag{17}$$

where B_Q is uncertainty of weld quality [4].

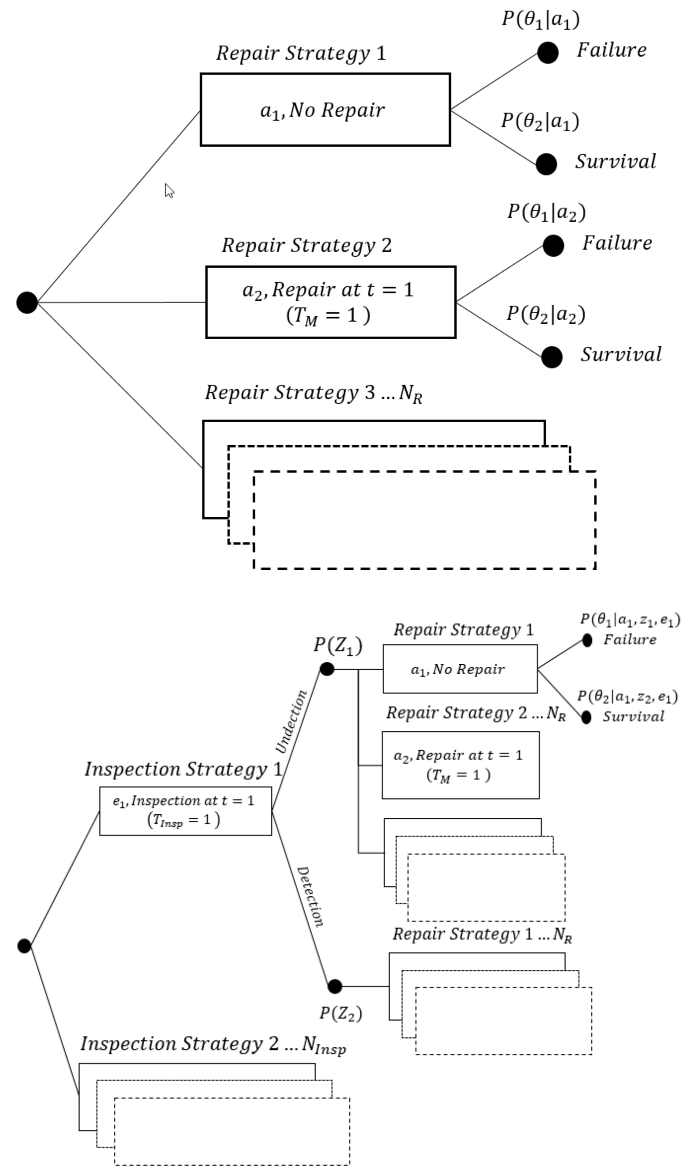

FIGURE 2: Decision tree of inspection and repair strategy for prior (top) and pre-posterior analysis (bottom).

APPLICATION

In this section, an example of inspection planning for a typical fixed offshore platfrom is presented. The objective is to optimize the inspection strategies, $\boldsymbol{e}$, and the repair strategies, $\boldsymbol{a}$, by the application of the VoI theory as shown in Eqn. (4). The extensive form decision analysis is used to find the optimum strategies.

Problem Description

A structural joint in an offshore structure with the lifetime, T_{SL}, of 30 years will be investigated. The joint is subjected to cyclic loading due to waves and current, and deteriorates over time due to fatigue damage. A repair and inspection strategy is to be determined through out structure's lifetime. The decision tree for the repair and inspection strategy is shown in Fig.2. The failure limit state function, g_f, is given by Eqn. (5) where the critical crack depth, l_c, is equal to 40 mm. The considered random variables are the stress range, ΔS, the initial crack depth, l_o, the material parameter, C, the stress intensity uncertainty, B_{SIF}, and the stress range uncertainty, $B_{\Delta S}$. The inspection is assumed performed by MPI (Magnetic Particle Inspection) with the associated probability of detection is given by Eqn. (11). The stress range is assumed to be Weibull distributed. A summary of the parameters used in this example is presented in Tab. 1 according to Straub [2] and Thöns et al. [9].

Each inspection strategy is characterized by the inspection time over the service life, $t_I = [1, 2, ..., 29]$, while the repair can be performed at the same time or after the inspection, $t_M = [t_I, ..., T_{SL}]$, depending on the inspection strategy. The decision tree for the inspection and repair strategies can be seen in Fig. 2. In total, 930 combinations of inspection and repair strategies are investigated.

The cost model considered used in this example contitutes a typical fixed offshore structure [10]. The unit cost of an inspection, C_I, is equal to \$2000 and is subjected to the discount rate, r, which depends on the financial strategy of the decision maker. In this example, the discount rate is assumed equal to 0.03. The expected cost of inspection over the service life is as follows:

$$E[C_I(\boldsymbol{e}, T_{SL})] = \sum_{t=t_I}^{t_{nI}} C_I(\boldsymbol{e}) \frac{1}{(1+r)^t} \tag{18}$$

where t_I is the inspection time.

The unit repair cost, C_R, is equal to \$20'000 and also subjected to the discount rate, r. The expected repair cost over the service life is as follows:

$$E[C_R(\boldsymbol{e}, \boldsymbol{a}, T_{SL})] = \sum_{t=t_I}^{t_{nR}} C_R(\boldsymbol{e}, \boldsymbol{a}) \frac{1}{(1+r)^t} \tag{19}$$

where t_M is the repair time. It is assumed that the repaired element behaves as new element according to Eqn. (17).

The cost of structural failure, C_F, is assumed equal to \$30'000'000 and the expected cost of failure over the lifetime is as follows [10]:

TABLE 1: Summary of random variables.

Par.	Description	Distribution	μ	σ
lnC	Material parameters	Normal	-29.9	0.5
B_{SIF}	Uncertainty of stress intensity factor	Log-Normal	1.0	0.3
$B_{\Delta S}$	Uncertainty in stress range calculation	Log-Normal	0.95	0.285
B_Q	Uncertainty in weld quality	Log-Normal	1.10	0.22
h	Shape parameter	Normal	1.2	0.18
λ	Scale parameter	Deterministic	2.3	-
l_o	Initial crack depth	Exponential	0.11	-
m	Material parameter	Deterministic	3.0	-
v	Annual stress cycle	Deterministic	$5 \cdot 10^6$	-

$$E[C_F(\boldsymbol{e}, \boldsymbol{a}, T_{SL})] = \sum_{t=1}^{T_{SL}} C_F \Delta P_f(\boldsymbol{e}, \boldsymbol{a}, t)(1 - P_f(\boldsymbol{e}, \boldsymbol{a}, t - \Delta t)\frac{1}{(1+r)^t} \tag{20}$$

where $\Delta P_f(\boldsymbol{e}, \boldsymbol{a}, t)$ is the annual probability of failure in time t for every combination of the inspection and repair strategies. The expected total cost over the service life can then be calculated as follows:

$$E[C_T(\boldsymbol{e}, \boldsymbol{a}, T_S L)] = E[C_I(\boldsymbol{e}, T_S L)] + E[C_R(\boldsymbol{e}, \boldsymbol{a}, T_{SL}) + E[C_F(\boldsymbol{e}, \boldsymbol{a}, T_S L)] \tag{21}$$

Prior and Pre-Posterior Analysis

The results of the prior analysis may be seen in Fig. 3. The top left diagram shows the total expected costs for different repair times, t_M, and no repair. The optimum repair time was found at

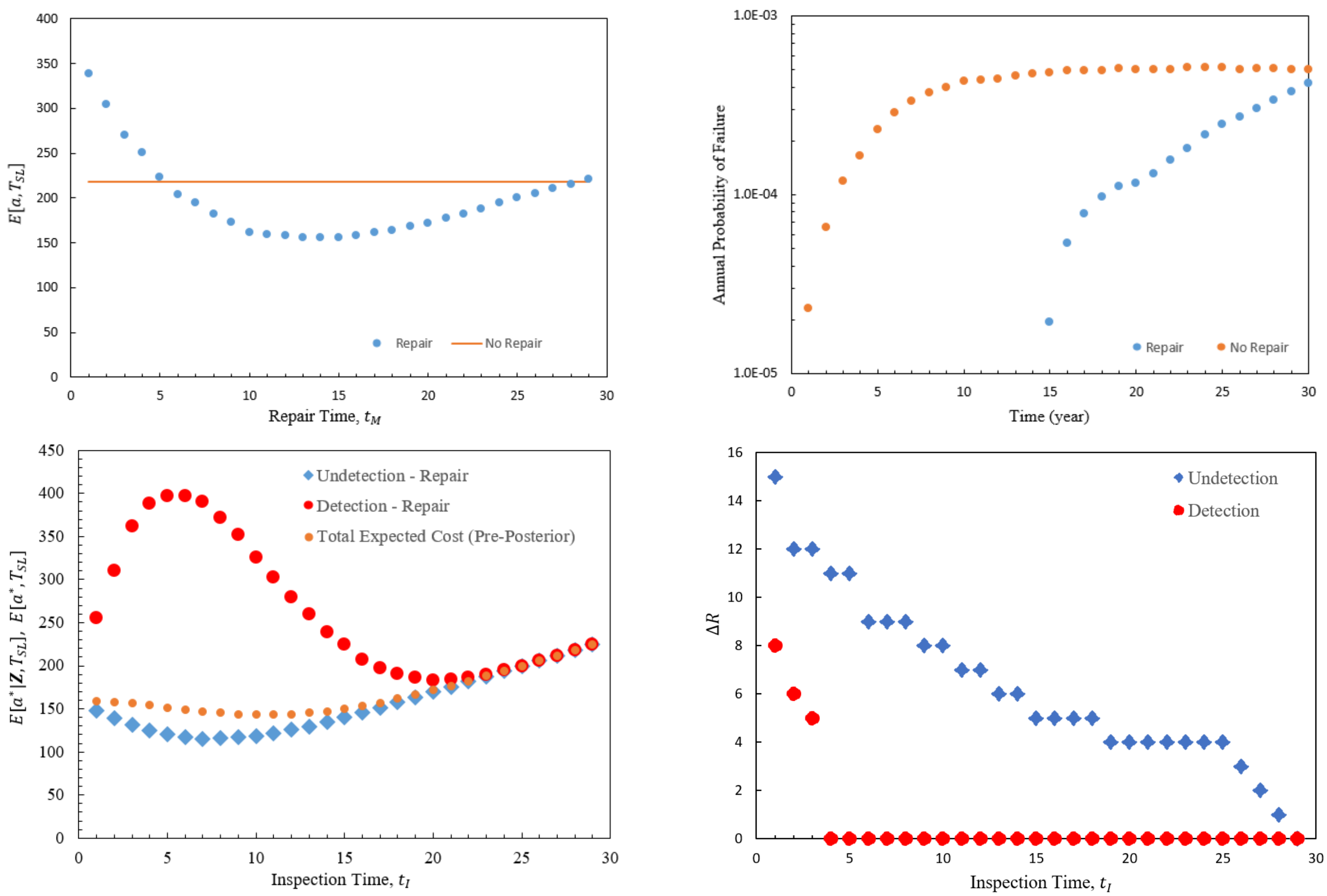

FIGURE 3: Total expected costs for prior analysis in thousand USD (top left), annual probability of fatigue failure for $t_M = 14$ in prior analysis (top right), total expected costs for the pre-posterior analysis in thousand USD (bottom left), and the optimum repair and inspection time difference, ΔR (bottom right).

$t = 14$ years. The top right diagram shows the annual probability of fatigue failure for the optimum repair strategy.

The pre-posterior analysis was carried out for every branch of decision tree in Fig. 2 by utilizing the extensive form analysis. The results of the analysis may be seen in bottom left and right diagrams in Fig. 3. The bottom left diagram shows the expected total costs over the service life for the optimum repair strategy given the inspection outcomes, while the bottom right diagram shows the difference between the optimum repair and the inspection time, ΔR, for each inspection strategy. From both diagrams, it can be seen that the optimum inspection strategy is to perform inspection at $t = 10$ years while the optimum repair time is conditional on the inspection outcomes. For the undetection branch, the repair should be performed at $t = 18$ years while for detection branch the repair should be performed in the same year as the inspection.

Value of Information

Figure 4 shows the Value of Information for each inspection strategy. It can be seen that the maximum VoI is $12'500 and occurs if the inspection is performed at $t = 10$ years. The VoI is zero if the inspection is performed below t=4 years and after t=16 years.

DISCUSSION

From Fig. 3, it can be seen that it is not beneficial to perform the repair at the early stage of the structure's lifetime. It is because the repair will not sufficiently reduce the probability of failure and is more expensive to perform. This behavior can be seen both in the prior and the pre-posterior analysis (given the detection outcome). In the pre-posterior analysis, the optimum repair time is conditional on the inspection outcome. If a crack is detected during the inspection, generally the repair should be performed immediately after the inspection unless the inspection is performed very early. If no crack is detected during the inspection, the optimum repair strategy is to perform the repair several years after the inspection. From Fig. 3, it can be seen that the distance between the optimum repair and the inspection time, (ΔR), for the undetection branch had inverse relationship with the time of the inspection. This is because the remaining structure's lifetime after the inspection is lower with increasing inspection time; therefore the available repair time after the inspection decreases. The ΔR is constant for several inspection times in the undetection branch before the optimum repair time changes due to the lower remaining lifetime after the inspection.

It should be noted that the VoI becomes zero after $t = 16$. Therefore, it can be said that little to zero value is gained if the inspection is performed very late in the structure's lifetime. It also can be seen that there is no value in performing the inspection very early due to the low probability of detection.

In the extensive form analysis, the entire decision tree shown in Fig. 2 is evaluated. In practice, the computational cost of the analysis can be very high if more optimization parameters are included in the analysis, i.e. multiple inspection times and different repair methods.

The normal form analysis requires the decision rules which connect the inspection outcome, $\boldsymbol{Z}$, to the repair strategies, $\boldsymbol{a}$, in the form $d(\boldsymbol{Z}) = \boldsymbol{a}$. Depending on the formulation of the decision rules, significantly fewer branches of the decision tree may then be assessed if the decision rules cover the branches containing the optimal inspection strategy and action. From the previous discussion, it can be noticed that if a crack is detected, immediate repair is required. If a crack is undetected, the optimum repair strategy is to perform repair after the inspection. Therefore, the decision rule for the normal form analysis can be formulated as shown in Tab. 2. By utilizing the decision rules, the decision tree branches can be reduced significantly but still retain the important branches that govern the optimum inspection and repair strategy. Further elimination of the branches may be done by discarding branches that have inspection time higher than half of the the structure's lifetime due to the low VoI.

TABLE 2: Decision rule for normal form analysis.

Decision Rule	Description
$d(Z_1) = a_1^{'}$	Repair after the inspection is performed/Do not repair during the inspection,
$d(Z_2) = a_2$	Repair at the same time as the inspection.

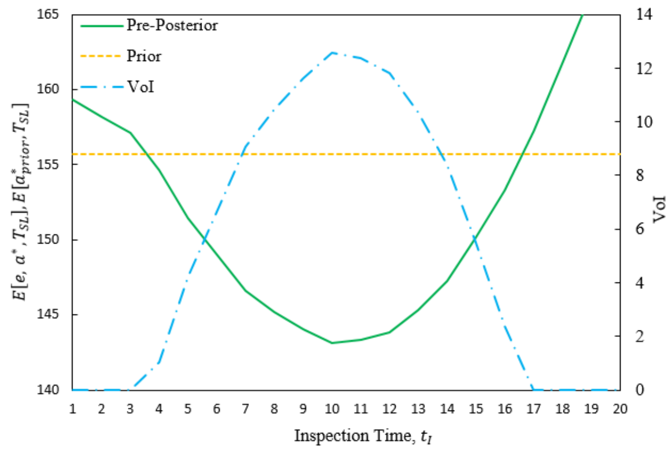

FIGURE 4: VoI for each inspection strategy in thousand USD

CONCLUSION

This paper contains a formulation of the decision theoretical basis for inspection planning applying the Bayesian decision theory. The inspection planning is formulated as a Value of Information problem where additional information about the fatigue performance is provided by inspections. The fatigue performance is modeled with a fracture mechanics approach and the limited precision of inspections is accounted for. The inspections are performed to optimize the maintenance and repair planning.

The introduced formulation facilitates that an optimal inspection strategy can be identified with an analysis in extensive form based on the Value of Information it provides. Consecutively, it becomes possible to derive decision rules which lead to the optimal inspection strategy and optimal maintenance and repair action. These decision rules can then be applied in future analyses leading to a significant reduction of the computational efforts by utilizing a normal form analysis.

The approach is applied to the inspection and repair planning of one hot spot. By considering all branches of the decision tree, the optimal inspection and repair times are determined. It has been found VoI optimal to repair immediately when a crack is detected.

REFERENCES

[1] Bloch, A., J.D.Sorensen, and Madsen, H., 2000. "Simplified approach to inspection planning". In PMC2000, 8th ASCE Specialty Conference on Probabilistic Mechanics and Structural Reliability.
[2] Straub, D., 2004. "Generic Approaches to Risk Based Inspection Planning for Steel Structures". PhD Thesis, ETH Zurich, Zurich, June.
[3] OGP, 2010. Risk Assessment Data Directory. Report No. 434-17, OGP, London, March 2010.

[4] Folso, R., Otto, S., and Permentier, G., 2002. "Reliability-based calibration of fatigue design guidelines for ship structures". *Marine Structures,* ***15***(6), November-December, pp. 627–651.
[5] Hong, H., 1997. "Reliability analysis with nondestructive inspection". *Structural Safety,* ***19***(4), January, pp. 383–395.
[6] Raiffa, H., and Schlaifer, R., 1991. *Applied Statistical Decision Theory*. Clinton Press, Inc, Boston.
[7] von Neumann, J., and Morgenstern, O., 1953. *Theory of Games and Economic Behavior*. Princeton University Press, Princeton.
[8] Madsen, H., Krenk, S., and Lind, N., 1986. *Methods of Structural Safety*. Prentice-Hall, Inc, New Jersey.
[9] Thöns, S., Schneider, R., and Straub, D., 2012. "Reliability-based inspection planning with application to orthotropic bridge deck structures subjected to fatigue". Life-Cycle and Sustainability of Civil Infrastructure Systems.
[10] Straub, D., Goyet, J., Dalsgaard, J., and Faber, M., 2006. "Benefits of risk based inspection planning for offshore structures". In 25th International Conference on Offshore Mechanics and Arctic Engineering, Vol. 3 of *Reliability and Ultimate Strength of Marine Structures*, ASME, pp. 59–68.