A Combined Liquefied Natural Gas Routing and Deteriorating Inventory Management Problem

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Abstract. Liquefied Natural Gas (LNG) is becoming a more crucial source of energy due to its increased price competitiveness and environmental friendliness. We consider an inventory routing problem for inland distribution of LNG from storage facilities to filling stations. Here, an actor is responsible for the inventory management at the storage facilities and filling stations, as well as the routing and scheduling of a heterogeneous fleet of vehicles. A characteristic of the problem is that a constant rate of LNG evaporates each day at the storage facilities and filling stations. This is in contrast to maritime LNG inventory routing problems where the evaporation is considered at the ships only. The combined LNG routing and deteriorating inventory management problem is modelled with both an arc-flow and a path-flow formulation. Both models are tested and compared on instances motivated from a real-world problem.

Keywords: Inventory routing problem · Deteriorating inventory · Liquefied natural gas · Arc-flow model · Path-flow model

1 Introduction

Inventory-routing problems (IRPs) are receiving more and more attention from the research community due to the increasing demand for new applications. IRPs integrate routing and inventory decisions by taking into account the trade-off between holding costs and routing costs. For an overview of the literature of IRPs see [4].

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A subset of the IRP literature is dedicated to maritime transportation applications that address liquefied natural gas (LNG) distribution networks due to its fast growing market. In these networks, the natural gas is cooled down to a temperature of approximately (−162°C) where the gas reaches its liquid state and turns into LNG. This also reduces the volume of the gas by a factor of 610, which makes transportation and storage more efficient. The LNG is kept at a boiling state in the LNG distribution networks, so some of the LNG evaporates, and this gas, called boil-off, is used by some ships as fuel. A constant rate of the cargo capacity of the ship cargo tanks is boiling off each day during a voyage at sea, so some LNG is lost during transportation. As a general rule, about 0.15 % of storage content is lost every day. However, the boil-off rate depends on the vessel and the type of voyage [5]. This evaporation property is another main characteristic of LNG that makes it an interesting research topic. LNG is transported by ships from distant origins to ports close to a market from which inland distribution starts. Examples of maritime IRP applications for LNG can be found in [1], [8], [9], [10], [11], [13], [14], [15], and [16].

The existing literature of IRP for LNG distribution networks only addresses ship routing and scheduling and not inland distribution of LNG (see [3], and [4]). In addition, no evaporation of LNG in the storages at the ports are considered. In inland distribution, evaporation persists in storage facilities and filling stations. Current research on inland distribution operations shows that the evaporation of LNG in storage facilities and filling stations can be higher than experienced in LNG ships, and should be considered. However, the loss of LNG while it is transported between storage facilities and filling stations is almost zero due to short travelling time, and can therefore be disregarded.

In the literature, products like LNG are categorised as deteriorating inventory, an item of which a percentage of on-hand inventory is constantly lost due to, for instance, decay, evaporation, or spoilage. Reviews of deteriorating inventory are conducted by [2], [7], and [12]. According to the deteriorating inventory literature, deterioration rate varies and it can take relatively large values depending on the case (see [6]).

To our knowledge, no existing IRP model has considered deterioration in the storages. From a modelling point of view, incorporating deterioration at the storage facilities and filling stations into IRP does not add to the complexity of the problem. However, it introduces new trade-offs in the model that may have a large influence on the solution depending on the application.

The focus of this paper is the design of an LNG distribution network within the Netherlands. Such a system distributes LNG from storage facilities, usually located close to ports such as the Gate Terminal in Rotterdam, to a group of filling stations that are geographically scattered all over the country (or a larger area).

The remainder of this paper is organised as follows. Section 2 introduces the IRP application studied in this paper. The IRP is modelled with both arc-flow and path-flow formulations in Sections 3 and 4, respectively. In Section 5 some computational analyses are conducted to highlight the differences between the arc-flow and path-flow formulations for this specific application and also to show the effects of deterioration on the total cost function. This is followed by concluding remarks in Section 6.

2 Problem Description

LNG is transported by tankers from its origins to storage facilities located close to its market. The scope of the inland distribution network studied in this paper includes these storage facilities and the downstream customers (filling stations).

Storage facilities receive LNG in large quantities from tankers and hold the commodity to meet the filling stations' demand. These replenishments induce large fixed costs to the distribution system due to the high fixed cost of operating tankers and also costly loading and unloading operations. In order to keep LNG in its liquid state, the temperature of storage facilities and filling stations should be held at a very low level which results in a variable holding cost. LNG is subject to constant evaporation while kept in tank (storage facility or filling station). Filling stations place orders to storage facilities for LNG quantities and incur a fixed ordering cost.

The infrastructure of LNG distribution network in the Netherlands is at its early stage and currently there are only a few operating filling stations. There are, however, industrial customers who use LNG as the main fuel for their business. From a modelling point of view these customers are the same as filling stations with a demand rate that should be met by the same distribution network.

In this distribution network two modes of transportation are used, namely road, and sea that can deliver LNG from storage facilities to filling stations. Accessibility and flexibility make road by far the dominating mode for transportation in this distribution network. Apart road, a large network of water canals all over the country makes sea (short-sea shipping) an efficient mode for transportation in this network. Due to limited accessibility, rail has not yet been part of this distribution system. In order to benefit from the low transportation cost that rail can offer, there is an ongoing research on *containerised* LNG. Only after the establishment of this concept, intermodal transportation will be an option for this distribution network where containers of LNG could be transferred from one mode to another.

We consider a group of vehicles that belong to road or sea. The distance between each pair of nodes in this network can be traversed with different costs since the two vertices may be connected by more than one vehicle. Due to accessibility limits, not all vehicles can operate between each pair of vertices in this network. In this case, the travel cost between the two nodes is considered to be suitably large. Each time that a filling station or storage facility is visited by a vehicle, a fixed cost is incurred due to loading and unloading operations. No boil-off gas is assumed when LNG is being transported by a vehicle between storage facilities and filling stations. Figure 1 depicts an example of an LNG inland distribution network in the Netherlands.

The goal of this LNG inland distribution problem is to maximise the total profit of the system by setting the inventory and routing policies for pick-up

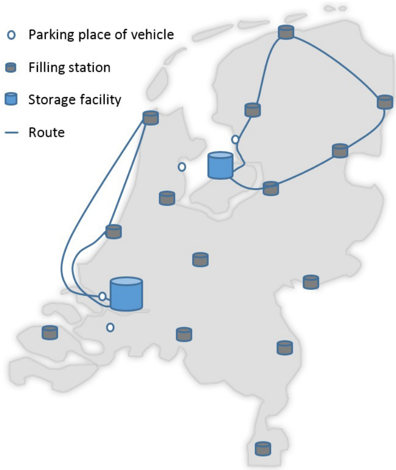

Fig. 1. LNG inland distribution network in the Netherlands

(storage facilities) and delivery (filling stations) points using the fleet of heterogeneous vehicles. The revenues of the network are earned by the filling stations that meet the demand of the final customers. The cost function of the system includes fixed purchasing cost at the filling stations and at the storage facilities, variable purchasing cost at the storage facilities, holding cost at all nodes, routing cost, and vehicle fixed visiting cost.

It should be noted that the variable purchasing cost paid by the filling stations is a revenue for the storage facilities, hence in the total profit function of the integrated system they cancel out. It is assumed that the demand at the filling stations is known and shortages are not allowed. This means that the revenue function of the system (earned at the filling stations) is independent of the decision variables. It then seems logical to discard the revenue functions and replace the profit maximisation objective with a cost minimisation objective.

3 Arc-Flow Model

In this section we use an arc-flow formulation to model the problem described in Section 2. LNG is delivered from a set of storage facilities (pick-up points), $\mathcal{N}^P = \{1, ..., m\}$ to a set of filling stations (delivery points), $\mathcal{N}^D = \{m+1, ..., m+n\}$, using a set of vehicles, $\mathcal{V} = \{1, ..., k\}$. It is assumed that these vehicles belong to a set of vertices defined as $\mathcal{N}^V = \{m+n+1, ..., m+n+k\}$. Depending on the mode used for transporting LNG, the distance between two vertices may vary. The set of all storage facilities and filling stations in the network is given by $\mathcal{N}' = \mathcal{N}^P \cup \mathcal{N}^D$. A percentage ($\theta_i$) of LNG constantly evaporates while being kept in node i ($i \in \mathcal{N}'$). It is assumed that there is no evaporation while LNG is being transported by vehicles.

In order to model this IRP with an arc-flow formulation we consider a graph $\mathcal{G} = (\mathcal{N}, \mathcal{A}_v)$, where $\mathcal{N} = \mathcal{N}^P \cup \mathcal{N}^D \cup \mathcal{N}^V$ is the set of all vertices in the distribution network and $\mathcal{A}_v = \{(i,j) : i,j \in \mathcal{N}, i \neq j, v \in \mathcal{V}\}$ is the arc set using a specific vehicle. The set of periods in the planning horizon is given by $\mathcal{T} = \{1, ..., H\}$.

In each period we define a route as follows. A vehicle starts its trip from its parking place towards a storage facility. After loading LNG, it visits a series of filling stations on the route to unload some quantities of LNG and eventually returns to its parking place. Each vehicle can visit a maximum of one storage facility on a route, whereas one storage facility can be visited by multiple vehicles during one period. We define binary variables w_{ivt} equal to one if and only if vehicle v visits vertex i ($i \in \mathcal{N}'$) during period t to load (or unload) LNG. In its visit to vertex i during period t, vehicle v loads (unloads) a quantity of q_{ivt}. To construct a route we define binary variables x_{ijvt} ($(i,j) \in \mathcal{A}_v$) equal to one if and only if vehicle v visits vertex j immediately after vertex i in period t.

The storage facilities and filling stations have an inventory capacity of $\bar{S}_i$ and a minimum allowed inventory level $\underline{S}_i$ ($i \in \mathcal{N}'$). The inventory level at storage facility or filling station i at the end of period t is given by the variable s_{it}. The initial inventory level at the beginning of the planning horizon at storage facility or filling station i is given by S_{i0}. At the beginning of period t, storage facility $i \in \mathcal{N}^P$ places an order quantity of y_{it} to its upstream supplier and instantly receives the replenishment. We define binary variable z_{it} equal to one if and only if storage facility i replenishes its inventory at time period t. Storage facility i dispatches a total quantity of $\sum_{v \in \mathcal{V}} q_{ivt}$ to filling stations at the beginning of period t. Having dealt with upstream suppliers and filling stations at the start of period t, storage facility i is left with the remaining inventory that gets depleted with rate θ_i due to deterioration throughout period t before ending with an inventory level of s_{it}.

At the start of period t, filling station $i \in \mathcal{N}^D$ receives a total amount of $\sum_{v \in \mathcal{V}} q_{ivt}$ after which the inventory level decreases throughout the period due to the demand rate D_{it} and deterioration rate θ_i.

Vehicle $v \in \mathcal{V}$ has a capacity of $\bar{V}_v$ and costs C^T_{ijv} to operate between vertices i and j ($(i,j) \in \mathcal{A}_v$). A suitably large value is assigned to C^T_{ijv} whenever vehicle v cannot travel on arc (i,j). A fixed cost of C^{FV}_v is incurred when vehicle v visits a vertex. In each trip, due to practical limitations, vehicle v can visit a maximum number of $\bar{N}^D_v$ filling stations.

3.1 Inventory Level at Vertices

The inventory level at filling station $i \in \mathcal{N}^D$ at the beginning of period t is the sum of the inventory level at the end of the previous period and the quantities delivered by the vehicles at the start of the period:

$$s^D_{it}(t' = 0) = s_{i(t-1)} + \sum_{v \in \mathcal{V}} q_{ivt}. \tag{1}$$

This inventory level is depleted throughout the period due to demand and evaporation. The evaporation results in a continuous loss of θ_i percent of on-hand inventory. The following differential equation represents the changes in the inventory level during period t:

$$\frac{ds_{it}^D(t')}{dt'} = -\theta_i s_{it}^D(t') - D_{it}. \tag{2}$$

Solving this differential equation leads to the following inventory level at the filling station i in period t. Note that t is the unit of time of which we analyse the inventory level and t' is the time parameter of which the value changes from $t' = 0$ (the start of the period) to $t' = 1$ (the end of the period):

$$s_{it}^D(t') = -\frac{D_{it}}{\theta_i} + K\, e^{-\theta_i t'}, \tag{3}$$

where K is a positive constant. Considering the initial inventory of filling station i at period t presented in (1) as the boundary condition, the inventory level of this station throughout period t is obtained as

$$s_{it}^D(t') = -\frac{D_{it}}{\theta_i} + \left[s_{i(t-1)} + \sum_{v \in \mathcal{V}} q_{ivt} + \frac{D_{it}}{\theta_i} \right] e^{-\theta_i t'}. \tag{4}$$

Equation (4) gives the exact inventory level of the filling station at any moment during period t. The inventory level at the end of period t ($t' = 1$) is hence given by

$$s_{it} = \left[s_{i(t-1)} + \sum_{v \in \mathcal{V}} q_{ivt} \right] e^{-\theta_i} - \frac{D_{it}}{\theta_i} \left(1 - e^{-\theta_i}\right), \qquad \forall i \in \mathcal{N}^D,\ t \in \mathcal{T}. \tag{5}$$

In storage facility $i \in \mathcal{N}^P$ the inventory level changes according to a different pattern since the demand is realised in batches at the beginning of each period. Since all inventory transactions are performed at the start of each period, the inventory level decreases over the period due to evaporation only. The inventory level at storage facility i at the beginning of period t after the above-mentioned transactions is given by

$$s_{it}^P(t' = 0) = s_{i(t-1)} + y_{it} - \sum_{v \in \mathcal{V}} q_{ivt}. \tag{6}$$

The changes of inventory level at this storage facility over period t is shown by the following differential equation:

$$\frac{ds_{it}^P(t')}{dt'} = -\theta_i s_{it}^P(t'). \tag{7}$$

Considering the boundary condition shown in (6), solving differential equation (7) results in the following inventory level for storage facility i over time period t:

$$s_{it}^P(t') = \left[s_{i(t-1)} + y_{it} - \sum_{v \in \mathcal{V}} q_{ivt} \right] e^{-\theta_i t'}. \tag{8}$$

The inventory level at the end of this time period is hence given by

$$s_{it} = \left[s_{i(t-1)} + y_{it} - \sum_{v \in \mathcal{V}} q_{ivt} \right] e^{-\theta_i}, \qquad \forall i \in \mathcal{N}^P,\ t \in \mathcal{T}. \tag{9}$$

In the following section, we derive the objective function and the constrains using inventory levels of storage facilities and filling stations.

3.2 Objective Function and Constraints

Replenishment at filling station i in period t incurs a fixed purchasing cost of C_i^F. The sum of all these costs over the planning horizon is given by

$$FC^D = \sum_{i \in \mathcal{N}^D} \sum_{v \in \mathcal{V}} \sum_{t \in \mathcal{T}} C_i^F w_{ivt}. \tag{10}$$

At the start of period t, storage facility i receives a quantity y_{it} for which it pays a unit price of C_{it}^P. This replenishment also results in a fixed cost of C_i^F for the storage facility. The total (fixed and variable) purchasing cost incurred by all the storage facilities over the planning horizon is as follows:

$$FC^P = \sum_{i \in \mathcal{N}^P} \sum_{t \in \mathcal{T}} (C_i^F z_{it} + C_{it}^P y_{it}). \tag{11}$$

The cost of routing includes fixed and variable costs. The total value of all fixed costs of vehicle v in period t is $\sum_{i \in \mathcal{N}'} C_v^{FV} w_{ivt}$. The variable cost of a route is obtained by summing up the transportation cost between each two nodes on the route: $\sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}, (i,j) \in \mathcal{A}_v} C_{ijv}^T x_{ijvt}$. The total routing cost is hence given by

$$\begin{aligned} RC^A = & \sum_{v \in \mathcal{V}} \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}, (i,j) \in \mathcal{A}_v} C_{ijv}^T x_{ijvt} \\ & + \sum_{v \in \mathcal{V}} \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{N}'} C_v^{FV} w_{ivt}. \end{aligned} \tag{12}$$

In order to obtain the total holding cost at filling station $i \in \mathcal{N}^D$ over the planning horizon, the holding cost of each period is first calculated using the inventory level presented in (4). It is assumed that a unit holding cost of C_i^H is incurred per unit of time when keeping LNG at vertex $i \in \mathcal{N}^P$:

$$\begin{aligned} HC_{it}^D &= \int_0^1 C_i^H s_{it}^D(t')dt' \\ &= \frac{C_i^H \left[1 - e^{-\theta_i}\right]}{\theta_i} \left[s_{i(t-1)} + \sum_{v \in \mathcal{V}} q_{ivt} + \frac{D_{it}}{\theta_i} \right] - \frac{C_i^H D_{it}}{\theta_i}. \end{aligned} \tag{13}$$

The total holding cost of all filling stations is hence given by

$$HC^D = \sum_{i \in \mathcal{N}^D} \sum_{t \in \mathcal{T}} HC_{it}^D. \tag{14}$$

The inventory holding cost at the storage facilities is obtained in a similar way to filling stations. Using the inventory level at storage facility $i \in \mathcal{N}^P$ presented in (8), the inventory holding cost over period t at the storage facility is

$$\begin{aligned} HC_{it}^P &= \int_0^1 C_i^H s_{it}^P(t')dt' \\ &= \frac{C_i^H \left[1 - e^{-\theta_i}\right]}{\theta_i} \left[s_{i(t-1)} + y_{it} - \sum_{v \in \mathcal{V}} q_{ivt} \right]. \end{aligned} \tag{15}$$

The total holding cost incurred at all the storage facilities is then

$$HC^P = \sum_{i \in \mathcal{N}^P} \sum_{t \in \mathcal{T}} HC_{it}^P. \tag{16}$$

Considering all the costs obtained in this section, the objective function of the model is

$$\text{Minimise } TC^{ARC} = FC^D + FC^P + RC^A + HC^D + HC^P. \tag{17}$$

The constraints on the inventory levels at the filling stations and the storage facilities are presented in (5) and (9), respectively. The remaining constraints are as follows:

$$s_{i(t-1)} + \sum_{v \in \mathcal{V}} q_{ivt} \leq \bar{S}_i, \qquad \forall i \in \mathcal{N}^D,\ t \in \mathcal{T}, \tag{18}$$

$$s_{i(t-1)} + y_{it} - \sum_{v \in \mathcal{V}} q_{ivt} \leq \bar{S}_i, \qquad \forall i \in \mathcal{N}^P,\ t \in \mathcal{T}, \tag{19}$$

$$s_{it} \geq \underline{S}_i, \qquad \forall i \in \mathcal{N}',\ t \in \mathcal{T}, \tag{20}$$

$$\sum_{i \in \mathcal{N}^P} w_{ivt} \leq 1, \qquad \forall v \in \mathcal{V},\ t \in \mathcal{T}, \tag{21}$$

$$\sum_{i \in \mathcal{N}^D} w_{ivt} \leq \bar{N}_v^D, \qquad \forall v \in \mathcal{V},\ t \in \mathcal{T}, \tag{22}$$

$$y_{it} \leq (\bar{S}_i - \underline{S}_i) z_{it}, \qquad \forall i \in \mathcal{N}^P,\ t \in \mathcal{T}, \tag{23}$$

$$\sum_{i \in \mathcal{N}^P} q_{ivt} = \sum_{j \in \mathcal{N}^D} q_{jvt}, \qquad \forall v \in \mathcal{V},\ t \in \mathcal{T}, \tag{24}$$

$$q_{ivt} \leq \min\{\bar{S}_i - \underline{S}_i, \bar{V}_v\} w_{ivt}, \qquad \forall i \in \mathcal{N}',\ v \in \mathcal{V},\ t \in \mathcal{T}, \tag{25}$$

$$\sum_{j \in \mathcal{V}, j \neq v} w_{(m+n+j)vt} = 0, \qquad \forall v \in \mathcal{V},\ t \in \mathcal{T}, \tag{26}$$

$$x_{(m+n+v)ivt} = w_{ivt}, \qquad \forall i \in \mathcal{N}^P, v \in \mathcal{V},\ t \in \mathcal{T}, \tag{27}$$

$$\sum_{i \in \mathcal{N}^P} w_{ivt} = \sum_{j \in \mathcal{N}^D} x_{j(m+n+v)vt}, \qquad \forall v \in \mathcal{V},\ t \in \mathcal{T}, \tag{28}$$

$$\sum_{j\in\mathcal{N},(j,i)\in\mathcal{A}_v} x_{jivt} + \sum_{j\in\mathcal{N},(i,j)\in\mathcal{A}_v} x_{ijvt} = 2w_{ivt}, \qquad \forall i\in\mathcal{N},\ v\in\mathcal{V},\ t\in\mathcal{T}, \quad (29)$$

$$\sum_{i\in\mathcal{M}}\sum_{j\in\mathcal{M},i\neq j} x_{ijvt} \leq \sum_{i\in\mathcal{M}} w_{ivt} - w_{kvt}, \qquad \forall\mathcal{M}\subseteq\mathcal{N}^D,\ k\in\mathcal{M},\ v\in\mathcal{V},\ t\in\mathcal{T}, \quad (30)$$

$$q_{ivt} \geq 0, \qquad \forall i\in\mathcal{N}^{'},\ v\in\mathcal{V},\ t\in\mathcal{T}, \quad (31)$$

$$y_{it} \geq 0, \qquad \forall i\in\mathcal{N}^P,\ t\in\mathcal{T}, \quad (32)$$

$$z_{it} \in \{0,1\}, \qquad \forall i\in\mathcal{N}^P,\ t\in\mathcal{T}, \quad (33)$$

$$w_{ivt} \in \{0,1\}, \qquad \forall i\in\mathcal{N}^{'},\ v\in\mathcal{V},\ t\in\mathcal{T}, \quad (34)$$

$$x_{ijvt} \in \{0,1\}, \qquad \forall (i,j)\in\mathcal{A}_v,\ v\in\mathcal{V},\ t\in\mathcal{T}. \quad (35)$$

Constraints (18) and (19) keep the inventory level at vertex i at the start of period t less than or equal to the available capacity, while constraints (20) impose the minimum acceptable inventory level at vertex i during period t. Constraints (21) and (22) limit the number of storage facilities and filling stations visited by vehicle v in period t to one and $\bar{N}_v^D$, respectively. Constraints (23) guarantee that order quantities received by storage facilities stay within the allowed limits. Constraints (24) control that the sum of the amounts delivered to filling stations by a vehicle equals the amount picked up by the vehicle from assigned storage facility. Constraints (25) link the routing variables to the quantities delivered to the filling stations. Each vehicle has a designated parking place, which is imposed by constraints (26). Constraints (27) and (28) ensure that a vehicle (if assigned) starts its route from its parking place and at the end of the route traverses an arc from one of the filling stations to it parking place. Constraints (29) and (30) represent degree constraints and subtour eliminations constraints, respectively. Constraints (31)–(35) impose non-negativity and integrality conditions to the relevant decision variables.

4 Path-Flow Model

In order to model this distribution system with a path-flow formulation, we consider all sets defined in Section 3 except for arc set $\mathcal{A}_v$ which is replaced with path set $\mathcal{R}_v$. Here a path is defined as the shortest route that consecutively connects a parking place of a particular vehicle, a storage facility, a group of filling stations and finally the same parking place.

In this formulation all the feasible paths are generated a priori. In order to generate a feasible path for a specific vehicle in the path-flow model, it should visit one storage facility, and a maximum number of filling stations, and eventually its parking place from which it starts the trip. To do so we generate all the subsets of $\mathcal{N}^D$ that include a maximum of $\bar{N}_v^D$ filling stations. We then complete each generated path by adding different combinations of "vehicle-storage facility". In order to guarantee the shortest path, we optimise the order of the filling stations on the path.

Binary parameter A_{ivr} equals to one if and only if vehicle $v \in \mathcal{V}$ visits vertex $i \in \mathcal{N}^{'}$ on path $r \in \mathcal{R}_v$. We define binary variable λ_{vtr} equal to one if and only if vehicle v traverses path r in period t. Parameter C^T_{vr} includes the transportation cost (C^T_{ijv}) and a fixed visiting cost (C^{FV}_v) incurred when vehicle v follows path r. This parameter also includes the set-up cost that each filling station should pay when visited by a vehicle (C^F_i, $i \in \mathcal{N}^D$).

In the path-flow model the inventory level calculations of storage facilities and filling stations stay the same as presented in Section 3.1.

4.1 Objective Function and Constraints

In this section, we include the same costs in the objective function as in Section 3.2, however due to changes in the decision variables the cost formulations are modified accordingly.

The sum of all fixed and variable transportation costs and set-up costs at the filling stations is obtained as follows:

$$RC^P = \sum_{v \in \mathcal{V}} \sum_{t \in \mathcal{T}} \sum_{r \in \mathcal{R}_v} C^T_{vr} \lambda_{vtr}. \tag{36}$$

This cost function is equivalent to the sum of costs presented in (10) and (12). The cost functions (11), (14), and (16) remain unchanged. The objective function of the path-flow formulation is hence given by

$$\text{Minimise } \; TC^{PATH} = FC^P + RC^P + HC^D + HC^P. \tag{37}$$

The constraints of the path-flow formulation are as follows. The inventory levels at the filling stations and storage facilities are as presented in (5) and (9), respectively. Constraints (18) and (19) define upper bounds on the inventory levels at vertices while constraints (20) set lower bounds. Constraints (23) are to limit the batch sizes that are received by storage facilities while constraints (24) guarantee the sum of delivered quantities in a trip is equal to the amount picked up from the storage facility. The following constraints link the routing and quantity variables:

$$q_{ivt} \leq \sum_{r \in \mathcal{R}_v} \min\{\bar{S}_i - \underline{S}_i, \bar{V}_v\} A_{ivr} \lambda_{vtr}, \qquad \forall i \in \mathcal{N}^{'},\ v \in \mathcal{V},\ t \in \mathcal{T}. \tag{38}$$

We define the following constraints to ensure that in time period t, vehicle v can travel on at most one path:

$$\sum_{r \in \mathcal{R}_v} \lambda_{vtr} \leq 1, \qquad v \in \mathcal{V},\ t \in \mathcal{T}. \tag{39}$$

Finally, non-negativity and integrality conditions are imposed by constraints (31)–(33) together with the following:

$$\lambda_{vtr} \in \{0, 1\}, \qquad \forall r \in \mathcal{R}_v,\ v \in \mathcal{V},\ t \in \mathcal{T}. \tag{40}$$

Having all the paths determined a priori, there is no need for the routing constraints (21)–(22) and (26)–(30).

In the next section we analyse two numerical examples to compare the two formulations when the size of an instance changes and also to study the effect of deterioration on the objective function.

5 Computational Results

In this section we present two numerical examples. In the first one we conduct a comparison between the two formulations introduced in this paper to see how they perform when the size of the network increases. In the second example we show how deterioration rate can influence the optimal solution.

5.1 Example 1

The two formulations described in Sections 3 and 4 were implemented in Java using CPLEX. The code was run on a personal computer with Intel V 2.00GHz processor and 8.00 GB RAM. In the computational analysis, one storage facility is assumed to serve a group of filling stations using a fleet of four vehicles over a two-day period.

In order to construct all feasible paths in the path-flow model, we first generate all the subsets of $\mathcal{N}^D$. In the next step we assign one vehicle and one storage facility to the subsets. Since all the nodes of the path are determined, we have a travelling salesman problem. We solve this problem using the tabu search algorithm, coding in Java, to minimise the travelling cost of the path.

Having all the paths generated, we run the two models for different instances. The result of this analysis is reported in Table 1. It should be noted that for the path-flow formulation, the runtime does not include the time used for path generation.

The initial results show that the path-flow formulation solves the same problem much faster than the arc-flow formulation. Enumerating paths can take a relatively long time, however, it is a one-off task to perform. This means for

Table 1. Computational results when the number of filling stations varies

n	m	k	H	Arc-flow				Path-flow			
				Runtime	Number of			Runtime	Number of		
					Variables	Constraints	Nodes		Variables	Constraints	Nodes
5	1	4	2	0.4	865	822	176	0.3	313	78	487
7	1	4	2	2.0	1237	3802	414	1.0	1101	98	1263
9	1	4	2	109.2	1673	18686	2719	5.5	4193	118	2705
10	1	4	2	1025.1	1915	41232	8546	94.0	8299	128	37739
11	1	4	2	18111.2	2173	90402	21958	716.9	16501	138	215330
12	1	4	2	100418.3	2447	196916	23269	9751.0	32895	148	1384076

a given number of filling stations we need to generate the paths once and the path-flow model could be solved very frequently (i.e. on a daily basis) using these generated paths. This can continue until a new filling station is added to the network which necessitates generating the new set of all paths. This however is not applicable to the arc-flow model as we should include all the subtour elimination constraints each time that we run the model.

5.2 Example 2

The deterioration rate for LNG in deep-sea shipping is around 0.15%. LNG inland distribution networks are evolving. Therefore, it is hard to give a specific rate for the boil-off. However, based on the current practices, we can say that the rate is higher than what is the case in sea shipping operations. Here we run the IRP developed in this paper for deterioration rate changing from 0 to 2%.

In order to illustrate the effect of the evaporation we analyse two different networks and obtain the change in the objective function when the deterioration is taken into account. The results of this analysis are illustrated in Table 2.

Table 2. The effects of the deterioration on the total cost function

	Deterioration rate (%)				
	0.0	0.5	1.0	1.5	2.0
n m k H	Total cost	Increase in total cost* (%)			
4 1 3 6	9239	1.03	1.67	2.27	2.91
6 1 3 6	14063	2.99	3.88	4.54	5.23

*Compared with the case with no deterioration

Table 2 shows that a network of four filling stations, one storage facility, and three vehicles over a six-day planning horizon incurs a total cost of 9239 when no deterioration is taken into account. This experiment shows that if for instance the real deterioration rate is as high as 2%, the accurate total cost is 2.91% more than the case when the deterioration is not modelled.

We examine the same network with an increase in the number of the filling stations while the number of vehicles remains unchanged. The initial results show that in cases where the transportation resources are tight the model tends to keep more inventory in filling stations which results in more deterioration. The analysis of this network shows that for example the total cost of the system increases by 5.23% when there is a deterioration rate of 2%.

6 Conclusion

We have analysed an IRP for LNG inland distribution network taking into account the evaporation property of the item. We have modelled the distribution network with both arc-flow and path-flow formulations. The basic variant of each formulation has been derived and solved by CPLEX.

The results of the computational analysis conducted in this paper show that the path-flow formulation can solve the problem faster compared with the arc-flow model. Moreover, the analysis indicates that disregarding the deterioration rate even in a small instance of this model could result in a relatively large underestimation in the total cost. These initial results suggest that the deterioration property should be incorporated into the model as the underestimation may be significant depending on the instance. The computational analysis also shows that including deterioration in the model does not add to the complexity of the problem.

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