Short-term Production Optimization of Offshore Oil and Gas Production Using Nonlinear Model Predictive Control

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Abstract

The topic of this paper is the application of nonlinear model predictive control (NMPC) for optimizing control of an offshore oil and gas production facility. Of particular interest is the use of NMPC for direct short-term production optimization, where two methods for (one-layer) production optimization in NMPC are investigated. The first method is the *unreachable setpoints* method where an unreachable setpoint is used in order to maximize oil production. The ideas from this method are combined with the exact penalty function for soft constraints in a second method, named *infeasible soft-constraints*. Both methods can be implemented within standard NMPC software tools.

The case-study first looks into the use of NMPC for "conventional" pressure control, where disturbance rejection of time-varying disturbances (caused, e.g., by the 'slugging' phenomenon) is an issue. Then the above two methods for production optimization are employed, where both methods find the economically optimal operating point. Two different types of reservoir models are studied, using rate-independent and rate-dependent gas/oil ratios. These models lead to different types of optimums. The relative merits of the two methods for production optimization, and advantages of the two one-layer approaches compared to a two-layer structure, are discussed.

Keywords: Nonlinear model predictive control, production optimization, oil and gas production

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1. Introduction

In many industries there are strong incentives for dynamic process operation with improved profitability, enhanced flexibility, and reduced environmental footprints. As a response to this there has been a trend, at least in academic literature, towards closer integration of process control and economic process optimization [e.g., 1, 2, 3, 4] to address perceived shortcomings of traditional multi-layer control structures. Such integrated approaches are sometimes called *dynamic real-time optimization* (DRTO).

As oil and gas reserves become increasingly hard and expensive to explore and produce, there is a drive towards applying optimization strategies used in traditional chemical process industries to ensure profitable operation of oil and gas production [e.g., 5, 6].

The main objective in this paper is to study how dynamic real-time optimization in the form of nonlinear model predictive control (NMPC) can be used for short-term production optimization in an offshore oil and gas processing plant. While long-term production optimization strives to optimize net present value of the reservoir resources, short-term optimization is about optimizing daily production rate (throughput) given the injection and production strategies chosen by the long-term production optimization. Thus, short-term production optimization is similar to production optimization found in other process industries.

This study employs an industrial NMPC setup, and it is therefore focused on methods that can be implemented directly in such a package. It is assumed (with little loss of generality) that the production optimization objective can be cast as maximization of one (or more) of the controlled variables. Two methods are studied: the use of unreachable setpoints [7], and infeasible soft-constraints (see also [8]). To the authors' knowledge, the use of the latter approach in an economic optimization setting is new.

The NMPC formulation used is presented in Section 2, and approaches to direct production optimization are discussed, with emphasis on methods using unreachable setpoints and infeasible soft-constraints. In Section 3 a case study based on a fairly complex, realistic model of an offshore plant for oil and gas production is presented. We first study NMPC used for conventional pressure control in Section 4, where production optimization can be achieved by letting the NMPC setpoint be given from knowledge of active constraints of the economical optimum (similar to a two-layer approach). Disturbance rejection is also included in this case.

We then present the use of the one-layer approaches in two cases in Section 5: First using constant gas/oil ratios in the reservoir models (as in Section 4), where the optimum is characterized by active constraints, and then using variable gas/oil ratios, which results in an economic optimum that is not at a constraint. The latter case was also presented in [8]. We end the paper with some concluding remarks.

2. Nonlinear model predictive control and production optimization

In this section the NMPC formulation used in the paper is presented. Then production optimization is discussed, with emphasis on methods including economic objectives directly in the NMPC control objective function.

2.1. NMPC formulation

The nonlinear dynamic model representation used for NMPC prediction in this study is on discrete form

$$x_{k+1} = f(x_k, u_k, d_k), \tag{1a}$$
$$z_k = h(x_k, u_k), \tag{1b}$$

where $x_k \in \mathbb{R}^{n_x}$ are the states, $u_k \in \mathbb{R}^{n_u}$ are the inputs, $d_k \in \mathbb{R}^{n_d}$ are known (measured or estimated) disturbances, and $z_k \in \mathbb{R}^{n_z}$ are the controlled outputs.

We assume the following NMPC optimization formulation, where we include 'exact penalty'-type soft output constraints:

$$\min_{\{\Delta u_{k+i}\},\, \{\varepsilon_{k+i}\}} J(x_k, d_k, u_{k-1}) = \sum_{i=1}^{P} \|z_{k+i} - z_{\text{ref}}\|_Q + \sum_{i=0}^{N-1} \|\Delta u_{k+i}\|_S + \sum_{i=1}^{P} r^\top \varepsilon_{k+i} \tag{2a}$$

subject to

$$
\begin{aligned}
x_{k+i+1} &= f(x_{k+i}, u_{k+i}, d_k), && i = 0, \dots, P-1 && \text{(2b)}\\
z_{k+i} &= h(x_{k+i}, u_{k+i}), && i = 1, \dots, P && \text{(2c)}\\
u_{k+i} &= u_{k+i-1} + \Delta u_{k+i}, && i = 0, \dots, N-1 && \text{(2d)}\\
u_{k+i} &= u_{k+i-1}, && i = N, \dots, P-1 && \text{(2e)}\\
z_{\min} - \varepsilon_{k+i} \leq z_{k+i} &\leq z_{\max} + \varepsilon_{k+i}, && i = 1, \dots, P && \text{(2f)}\\
u_{\min} \leq u_{k+i} &\leq u_{\max}, && i = 0, \dots, N-1 && \text{(2g)}\\
\Delta u_{\min} \leq \Delta u_{k+i} &\leq \Delta u_{\max}, && i = 0, \dots, N-1 && \text{(2h)}\\
0 \leq \varepsilon_{k+i} &\leq \varepsilon_{\max}, && i = 1, \dots, P. && \text{(2i)}
\end{aligned}
$$

where $P > N$ are the prediction and control horizons, $Q \in \mathbb{R}^{n_z \times n_z}$ and $S \in \mathbb{R}^{n_u \times n_u}$ are the weighting matrices of output and input moves, respectively, and the vector $r \in \mathbb{R}^{n_z}$ is the penalty weight for constraint violations ε_{k+i}.

We believe the above formulation is fairly standard for industrial NMPC packages [9, 10]. Formulations employing soft output constraints via exact l_1 penalty functions are well-known from literature [11, 12] and are used in some industrial NMPC systems [10, 13]. The main rationale behind its use is to avoid feasibility problems related to hard output constraints. By allowing upper limits on the constraint violations ε_{k+i}, hard output constraints may also be included in this setup. Often, also quadratic (non-exact) penalties on the constraint violations are implemented, but for simplicity we do not include this here.

The above formulation is a state feedback policy, and must be coupled with a state- and parameter estimation scheme to obtain output feedback and integral control. This is not of direct relevance to the study herein, and therefore not discussed in more detail.

2.2. Production optimization

The control structure in a process plant is often divided into several layers separated by time scale, see, e.g., [10]. The MPC may be located in a control layer, responsible for multi-variable coordinating and constraint handling control, whereas a real-time optimization (RTO) system is a model-based system using steady-state models which can be located above the MPC in the control hierarchy, providing setpoints to the MPC.

Such a two-layer approach has several advantages and is widely used. However, if a (nonlinear) process model with validity over the entire operational window is used, one might argue that the two-layer approach can be replaced by a one-layer approach by augmenting or replacing an MPC quadratic tracking cost function with an economic cost function. Advantages of such a scheme (see, e.g., [2]) include faster reaction to disturbances, exact constraints can be implemented for measured variables, inconsistencies between models are avoided, and all degrees of freedom can be used to optimize the process, also during transients. A disadvantage of using a one-layer approach is that the demands on the model used for dynamic optimization may be higher, typically implying increased computational demand.

An economic cost function in a one-layer approach may be expressed as the sum over the prediction horizon of an economic profit function $l(x, u)$,

$$J_{\text{eco}} = \sum_{i=0}^{P} l(x_{k+i}, u_{k+i}), \tag{3}$$

see, e.g., [2, 4, 14]. In many cases a linear profit function is used,

$$l(x, u) = a^\top x + b^\top u.$$

A practical objection to using purely economically motivated cost functions is that it may lead to overly aggressive control which might be unacceptable to operators, or (sometimes related) have robustness issues. Therefore, one is in practice often faced with a multi-objective optimization problem: that of optimizing economic performance together with (or subject to acceptable) control performance [15]. An example of this is given in [16], where a combined tracking and economic optimization criterion is used to optimize liquefied petroleum gas (LPG) production in an fluid catalytic cracking (FCC) process.

The two approaches to one-layer production optimization we study in this paper are two methods that are simple to implement in many "standard" MPC tools (cf. previous section). Both approaches allow a trade-off between economic and control performance (as measured by a NMPC tracking objective, cf. (2a)) based on tuning, although tuning this trade-off is not necessarily straightforward. In both cases we assume the controlled variables are either

- variables that should track a reference (setpoint) and/or stay within bounds, denoted z_{trac}, or
- variables to be optimized, denoted z_{opt},

such that we can partition $z = (z_{\text{trac}}, z_{\text{opt}})$. In the case of an economic profit function $l(x, u)$, one may use $z_{\text{opt}} = l(x, u)$.

Assumption 1. *The combination of model (1), initial states, input (2g), and hard output constraints (2i) are such that z_{opt} is upper bounded.*

2.3. Unreachable setpoints

The first method is the use of unreachable (infeasible) setpoints by selecting high/low unreachable setpoints for the variables that should be maximized/minimized (z_{opt}). This is a simple method that allows the inclusion of economic optimization in most standard MPC tools. Use of the method is related to the practice (in linear MPC, at least) of using target calculation and a priority hierarchy [17] for doing production optimization in a single MPC layer.

The use of unreachable setpoints is analyzed in [7] for linear MPC, and conditions for stability and convergence are established. Moreover, [7] compares MPC using unreachable setpoints to a standard two-layer structure with a target-tracking cost function in two examples, and considerable cost improvements are achieved under certain disturbance scenarios.

Let Q be partitioned conformly to z as

$$Q = \begin{bmatrix} Q_{\text{trac}} & 0 \\ 0 & Q_{\text{opt}} \end{bmatrix}.$$

The first part of (2a) can now be written as

$$\|z_{k+i} - z_{\text{ref}}\|_Q = \|z_{\text{trac},k+i} - z_{\text{trac,ref}}\|_{Q_{\text{trac}}} + \|z_{\text{opt},k+i} - z_{\text{opt,ref}}\|_{Q_{\text{opt}}},$$

where $z_{\text{opt,ref}}$ are the unreachable setpoints used to optimize the corresponding controlled variables.

A possible drawback of this method can be seen by examining the contribution of the unreachable setpoint to the sensitivity $\frac{\partial J}{\partial \Delta u_j}$ of (2):

$$\sum_{i=1}^{P} (z_{\text{opt},k+i} - z_{\text{opt,ref}})^\top Q_{\text{opt}} \frac{\partial z_{\text{opt},i}}{\partial \Delta u_j}. \tag{4}$$

The gradient (4) will be involved in the KKT conditions for optimality, and it is clear that the actual (likely somewhat arbitrary) value we choose for $z_{\text{opt,ref}}$ may affect the solution of the NMPC optimization. This might give difficulties in tuning, and can also cause unexpected behavior after retuning and/or reconfigurations and change of operating points.

Standard methods for ensuring stability of NMPC schemes like (2) consist of adding stability constraints and/or "quasi-expand" the horizon to infinity [18]. However, cost functions using unreachable setpoints are unbounded on infinite horizons, and thus standard analysis of stability and convergence employing cost functions as Lyapunov functions can no longer be used. This has been a subject of recent research, and is for linear MPC analysis provided in [7], while results towards nonlinear systems can be found in [14].

2.4. Infeasible soft-constraints

The exact penalty function and infeasible soft-constraints can be used as a method for production optimization in a similar manner as unreachable setpoints. The discussion here is based on the exact l_1 penalty function [11]. To keep the presentation simple we limit ourselves, without loss of generality, to maximization of z_{opt}.

The basic idea is to select the lower constraint $z_{\text{opt,min}}$ of z_{opt} larger than the maximum value that can occur. In other words, choosing $z_{\text{opt,min}}$ such that z_{opt} will always be infeasible with respect to this constraint. Note that this is always possible under Assumption 1.

Do a partition of r and ε_k conformal to z such that $r = (r_{\text{trac}}, r_{\text{opt}})$ and $\varepsilon_k = (\varepsilon_{\text{trac}}, \varepsilon_{\text{opt}})$. By choice of (infeasible) $z_{\text{opt,min}}$, we will always have

$$z_{\text{opt},k+i} < z_{\text{opt,min}} \leq z_{\text{opt,max}}.$$

This implies that the lower bound on the constraint (2f)

$$z_{\text{opt,min}} - \varepsilon_{\text{opt},k+i} \leq z_{\text{opt},k+i} \leq z_{\text{opt,max}} + \varepsilon_{\text{opt},k+i}$$

will always be active, that is, we will always have

$$\varepsilon_{\text{opt},k+i} = z_{\text{opt,min}} - z_{\text{opt},k+i} > 0.$$

The last term of the cost function J in (2) can now be written as

$$\sum_{i=1}^{P} r^\top \varepsilon_{k+i} = \sum_{i=1}^{P} r_{\text{opt}}^\top \left(z_{\text{opt,min}} - z_{\text{opt},k+i}\right) + \sum_{i=1}^{P} r_{\text{trac}}^\top \varepsilon_{\text{trac},k+i}.$$

Letting Q be partitioned conformly to z as

$$Q = \begin{bmatrix} Q_{\text{trac}} & 0 \\ 0 & 0 \end{bmatrix},$$

we have shown that in this case, an equivalent form of (2a) is

$$J(x_k, d_k, u_{k-1}) = \sum_{i=1}^{P} \|z_{\text{trac},k+i} - z_{\text{trac,ref}}\|_{Q_{\text{trac}}} - \sum_{i=1}^{P} r_{\text{opt}}^\top z_{\text{opt},k+i} + \sum_{i=0}^{N-1} \|\Delta u_{k+i}\|_S + \sum_{i=1}^{P} r_{\text{trac}}^\top \varepsilon_{\text{trac},k+i}, \quad (5)$$

that is, we have obtained a linear (economic) optimizing term in the objective.

The sensitivity of the objective function with respect to the variables that we want to optimize is now independent of choice of $z_{\text{opt,min}}$ (as long as it is large enough) and the particular operating point, which can give easier tuning than the unreachable setpoint method. That is, the penalty weight r_{opt} is now a linear cost weight with a role corresponding to Q_{opt} for unreachable setpoints, but it can (in theory) be tuned independently of the value chosen for the infeasible constraint, and the effect is to some degree independent of operating point.

However, it should be noted that the choice of r_{opt} in relation to the choice of r_{trac} can be a delicate matter with significant impact on dynamics when some of the remaining output constraints are active. Also, strategies for complexity reduction might introduce 'blocking strategies' on ε_{k+i} which will have an impact on the solution. These tuning issues are subject to further investigation but will not be addressed in this paper.

Note that as in the previous section, the cost function will be unbounded on the infinite horizon, since by assumption $z_{\text{opt,min}} - z_{\text{opt},k+i} > 0$. In principle the methods of [14] can be used for stability and convergence analysis, if appropriate stability constraints are added.

3. Short-term production optimization of offshore oil and gas production

3.1. Offshore oil and gas production

The main objective of an offshore oil and gas processing plant is to transport and separate oil, gas, and water produced from a set of underground reservoirs. Oil is transported in pipes or stored in cargo tanks for export, and gas is compressed for re-injection, gas lift, and/or export. On the seabed there can be a large number of wells that have been drilled into the reservoirs, ordered in clusters. Pipes transport the streams from the different wells and clusters through a network on the seabed to a production manifold. The production manifold can route the stream (either to a test separator or) to the first stage of a production separator train, which separates oil, gas, and water. We assume in this paper that the main product in terms of revenue is oil, and that the gas has little direct value.

On a long term, the objective for process optimization is typically to maximize total recovery or net present value of the overall reservoir resources. This is achieved by deciding which wells to produce from (routing), and to what extent to use water flooding, gas injection, etc., to make optimal use of the reservoir resources. Herein, this problem is not considered and the routing and use of recovery methods are considered fixed.

The topic of this paper is thus optimization of production rate given a chosen long-term strategy. This is sometimes called a short-term optimization problem, where the problem can be considered time-independent [5, 19, 20]. A typical issue is how to maximize oil production from the producing wells, which have varying (production dependent) gas, oil, and water contents, under topside processing constraints (e.g., limited gas compression and water treatment capacity).

This study concentrates on optimization and control, while important issues like model updating (state- and parameter estimation) and robustness to modeling errors are considered outside the scope of this paper. Challenges with integrating large-scale models for online use are discussed in [21], where ways for state- and parameter estimation for a similar model as in this paper are presented.

3.2. Case description

The case is based on a model of a rather generic offshore oil production facility, including wells, pipelines and topside (platform). The model is developed using the Modelica-based advanced modeling software Dymola, using Cybernetica's in-house Modelica library for oil and gas production systems, named CyberneticaLib, along with components and interfaces from the Modelica Standard Library (in particular Modelica.Fluid). The scope of the model is integration in a real-time system. Thus, it is not as complex as some process simulators, but complex enough to capture key process dynamics in an offshore processing plant. The resulting model includes multi-phase flows through pipes and chokes, different gas/oil ratios (GOR) in near-well models of reservoirs, a multi-phase separator, and a polytropic head relationship in the compressors. The reservoir models are time-independent, and are thus limited to the short-term scope studied in this paper.

The model used in this case study has 249 dynamic states. Even though the model is fairly large, it is efficient enough to ensure that all NMPC optimization problems reported here are solved within real-time demands with a sample time of 1 minute. Since we do not consider model uncertainties we use the same model both in the NMPC and for simulation.

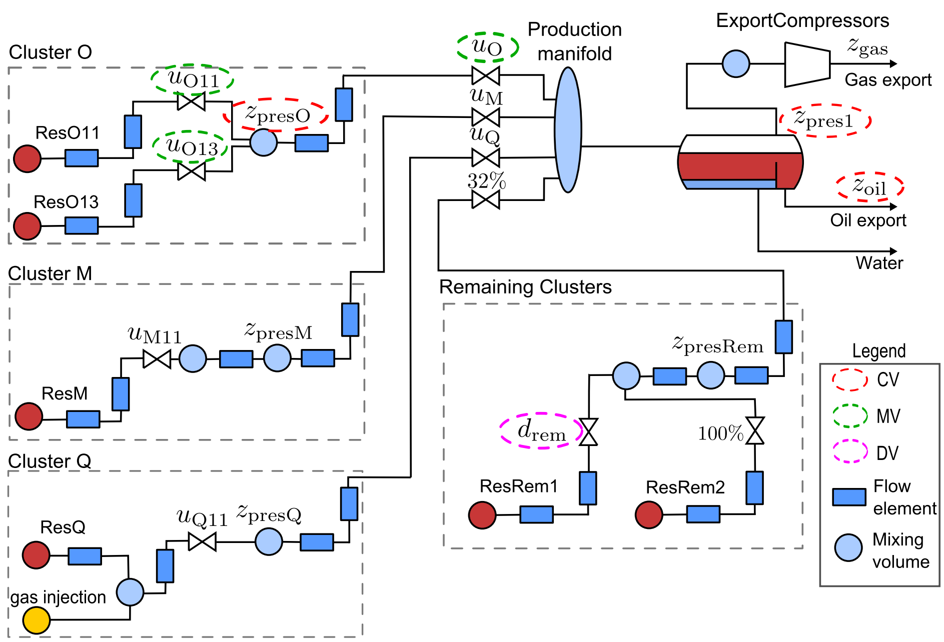

Figure 1: Process overview. CVs, MVs, and DVs are indicated.

A schematic overview of the model is shown in Figure 1. The overall structure is a set of well clusters which are mixed in a production manifold and fed to a separator train (topside process) that separates and processes gas, oil, and water. In each cluster there are one or two wells. Clusters O and M have similar types of wells, whereas gas injection is modeled in the well in Cluster Q. The 'Remaining Clusters'-part represents a lumped set of clusters each having several wells, but where we assume there is no control degrees of freedom. The conditions in this cluster are set such that the majority of the total production is produced here and that the typical total production is within typical values. The typical height from the clusters up to the manifold is 330 meters, and the depths from the well heads to the reservoir are set to be around 1200 meters.

The short-term economic objective is to maximize oil production within the plant's operating limits, by adjusting the production from the different active wells. That is, we define z_{oil} as the total production of oil, and choose $z_{\text{opt}} = z_{\text{oil}}$.

Depending on both the plant and the stream from the reservoirs, limited processing capacities for either oil, gas or water can limit the production, but in many cases, including this one, gas processing capacity is the main limiting factor. A consequence is often that the pressure in the inlet separator tank will be an active constraint under optimal production, since this typically maximizes gas processing capacity. A simple approach to optimizing control could thus be to use a 'squeeze and shift' [22] strategy using a single well as a 'swing producer' to keep the inlet separator pressure as close to the limit as safety allows. In our case the 'squeeze' is done by applying pressure control, reducing variations in the inlet separator pressure. It is then possible to 'shift' the pressure setpoint closer its upper limit. A dedicated well called a swing producer is used to control the pressure.

However, in addition to the dependence on the gas production capacity, the total oil production depends on the GOR and the marginal GOR (MGOR) in the different reservoirs. GOR is defined as

$$\text{GOR} = \frac{Q_G}{Q_O} \quad \left[\frac{\text{Sm}^3/\text{day}}{\text{Sm}^3/\text{day}}\right].$$

If the total field gas rate reaches the plant capacity in a gas-limited plant with gas coning wells and there are no other active constraints, maximal oil production is achieved when all the wells have the same MGOR [23]. That is,

$$\frac{\partial Q_{G,i}}{\partial Q_{O,i}} = \frac{\partial Q_{G,j}}{\partial Q_{O,j}} \quad i, j = 1, ..., n \tag{6}$$

where $Q_{G,i}$ [Sm^3/day] is the gas flow, $Q_{O,i}$ [Sm^3/day] is the oil flow of well i, and n is the number of wells. For wells without rate-dependent GOR, maximal oil production is achieved by producing as much as possible from the wells with the lowest GOR values, of course subject to process constraints.

In the following, a set of case studies are carried out with the goal of maximizing total oil production. In Section 4 we start by demonstrating how NMPC can be used to implement conventional pressure control, where increased oil production can be achieved through knowledge of the active constraints at optimum, but without any direct optimization. This approach could be seen as the MPC layer in a two-layer approach.

In Section 5.1 we look at how the unreachable setpoints method can be applied to optimize total oil production in a one-layer approach. For both these cases, there is a constant GOR model in the wells. In the case described in Section 5.2, the model is extended to have variable GOR in the wells in Cluster O, which (may, and does in this case) introduce optimums that are not characterized by active constraints. We then compare the unreachable setpoints method to the infeasible soft-constraints method as methods for production optimization.

4. Conventional pressure control using NMPC

An offshore oil and gas plant is conventionally controlled with PID controllers. A common problem is oscillations in the controlled variables (pressure, level, flow, etc.). Often these are caused by external disturbances like slugging, e.g., [24, 25], but also poorly tuned loops may be a cause. The oscillations will then typically propagate through the plant due to its multivariable nature. Good tuning of essential loops will reduce the problem, but due to the plant's size this job can be challenging. By using a multi-variable controller like (N)MPC, the problem is often reduced to selecting weighting values in a tracking cost, typically Q and S in (2a), but at the cost of developing and maintaining a process model. Both these control methods can be used in a two-layer RTO approach as described in Section 2.2, using optimal setpoints from a higher control level.

In this section a case with a periodic disturbance in the Remaining Clusters (see Figure 1) is studied. Such a disturbance can represent a multi-phase slug flow, which will affect the pressure z_{pres1} in the inlet separator, and thus the total oil production z_{oil}. For simplicity, a sine wave is used as disturbance d_{rem}. The principle of squeeze and shift by using a swing producer (mentioned in Section 3.2) is applied in order to operate closer to the pressure limit of the inlet separator. Such a pressure control scheme is conventionally implemented using PID control, but here it is demonstrated how this can be solved using NMPC. For this specific case in the same operating region it would be possible to find PID tunings which would perform similar to the NMPC, so to keep the discussion simple we omit any comparison study. An expected advantage with an NMPC solution is that retuning in case of changing operating conditions may be avoided, at the cost of keeping the model updated.

The well ResO13 is used as a swing producer to counteract the disturbance using choke u_{O13}. It is often desired that the process variables are kept at some value or within some range, often chosen by the operator and/or by some overlying optimization scheme as described in Section 2.2. With a well in Cluster O as a producer, the line pressure z_{presO} needs to be included as a controlled variable to not affect production in the other wells. There is a requirement of $n_z = n_u = 2$ degrees of freedom for control, and thus the line choke u_{O} is included as a manipulated variable. The controlled variables (CVs), manipulated variables (MVs) and disturbance variable (DV) of interest are then

$$z = \begin{bmatrix} z_{\mathrm{pres1}}, & z_{\mathrm{presO}} \end{bmatrix}^\top, \quad u = \begin{bmatrix} u_{\mathrm{O13}}, & u_{\mathrm{O}} \end{bmatrix}^\top, \quad d = d_{\mathrm{rem}}.$$

The NMPC setup is given by

$$\begin{bmatrix} 19\,\mathrm{bar} \\ 40\,\mathrm{bar} \end{bmatrix} \leq z \leq \begin{bmatrix} 21\,\mathrm{bar} \\ 80\,\mathrm{bar} \end{bmatrix},$$

$$Q = \mathrm{diag}\{10,\, 0.4\},\ S = \mathrm{diag}\{0.01, 0.01\},$$

$$r = \begin{bmatrix} 10^3 & 10^3 \end{bmatrix}^\top,$$

$$\begin{bmatrix} -20\,\% \\ -20\,\% \end{bmatrix} \leq \begin{bmatrix} \Delta u_{\mathrm{O13}} \\ \Delta u_{\mathrm{O}} \end{bmatrix} \leq \begin{bmatrix} 20\,\% \\ 20\,\% \end{bmatrix}, \ \begin{bmatrix} 10\,\% \\ 10\,\% \end{bmatrix} \leq \begin{bmatrix} u_{\mathrm{O13}} \\ u_{\mathrm{O}} \end{bmatrix} \leq \begin{bmatrix} 100\,\% \\ 100\,\% \end{bmatrix},$$

where the inlet separator pressure z_{pres1} is prioritized over the line pressure z_{presO} in Cluster O. A prediction horizon of $P = 30\,\mathrm{min}$ is used, which is longer than the dominant time constants in the system.

The model is first simulated with no control for 90 minutes with constant choke openings u_{O13} and u_{O} of 30 %. The disturbance d_{rem}, starting at $t = 8\,\mathrm{min}$, results in a sinusoidal response in z_{pres1} and z_{presO} as shown in Figure 2. Simulating again using NMPC with setpoints equal to open loop steady-state values $z_{\mathrm{pres1,ref}} = 20.00\,\mathrm{bar}$ and $z_{\mathrm{presO,ref}} = 57.07\,\mathrm{bar}$, it can be seen in the first 45 minutes of the simulation that the controller manages to reduce the effect of the disturbance ('squeeze') in the prioritized z_{pres1}. Utilizing this reduction, the setpoint $z_{\mathrm{pres1,ref}}$ is increased ('shifted') to 20.10 bar at $t = 45\,\mathrm{min}$, giving the same maximum value for z_{pres1} as without control. Pressure control of the inlet separator is achieved without any substantial deviation from setpoint in z_{presO}, and without having any excessive actuator use of u_{O13} and u_{O}. As described in Section 3.2, maximum oil production

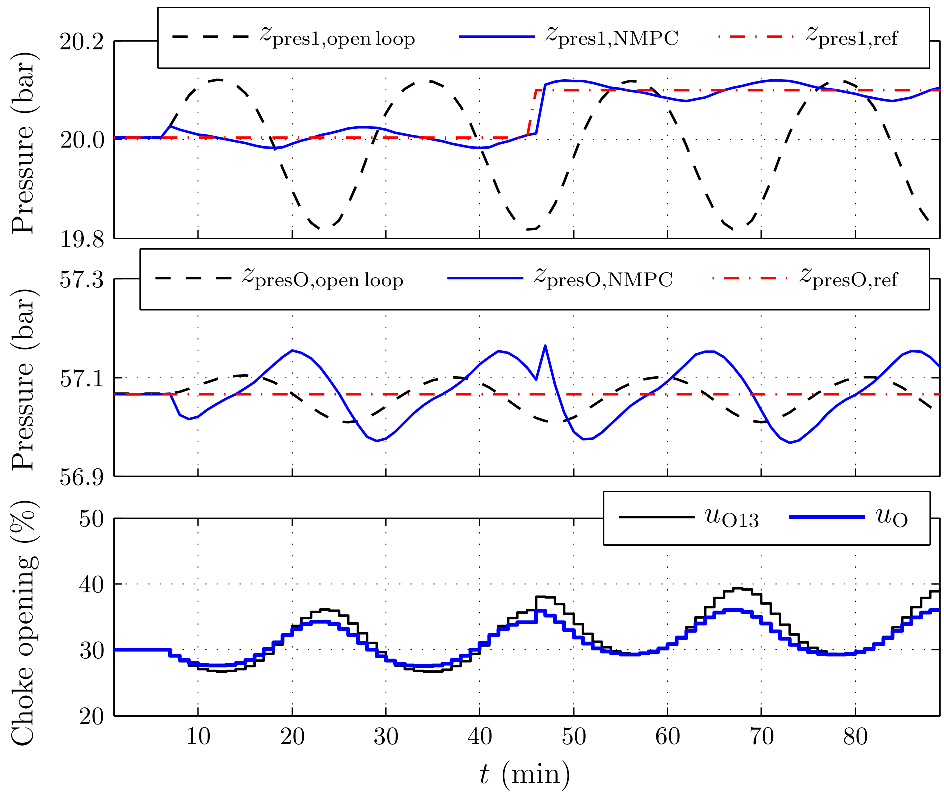

Figure 2: Pressure control with sine disturbance d_{rem}.

is achieved with the inlet pressure being as high as possible. Increasing this pressure, the total gas production is increased due to a decreased compressor head. With a limitation in gas processing capabilities in the plant, the total oil production is then increased as shown in Figure 3. Calculating the mean value of the total oil production, there is a mean production increase of 57 Sm^3/day using NMPC. Then by increasing the setpoint, the production increase compared to open loop is 143 Sm^3/day.

5. Production optimization using NMPC

In the rest of the cases in this paper, the unreachable setpoints method and the infeasible soft-constraints methods presented in Section 2 are used to manipulate chokes in Cluster O to optimize total oil production. Indirectly, this implies finding points where the GOR in the different wells are optimal,

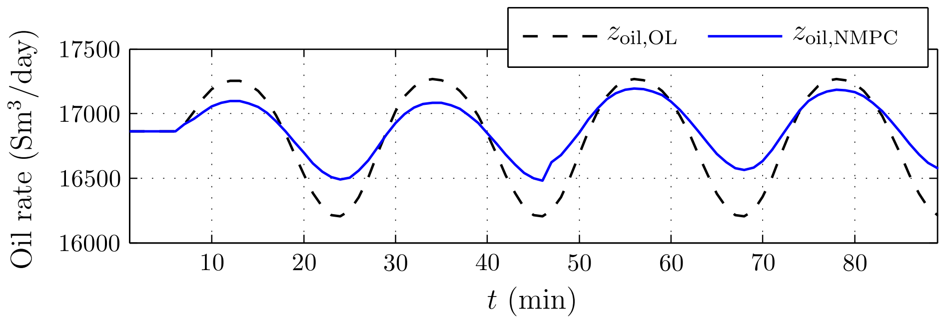

Figure 3: Total oil production with sine disturbance d_{rem}.

as explained in Section 3.2. To best illustrate the effect of the optimization methods, no disturbances are present. This is still a realistic scenario, since within the time frame of minutes or hours a real plant can often have more or less constant process variables.

5.1. Constant GOR

We begin using the unreachable setpoints method on the same model as in the previous case, with constant GOR in all the wells. The CVs included in the MPC objective function are the line pressure z_{presO} in Cluster O, the inlet separator pressure z_{pres1} and the total oil production z_{oil}, which represents the economic variable which should be maximized. In the given initial production setup z_{pres1} is considered to be lying on the maximum safe operating point, which cannot be further increased. The objective is thus to alter the choke configuration

$$u = \begin{bmatrix} u_{\text{O13}} & u_{\text{O}} \end{bmatrix}^\top \tag{7}$$

in order to further increase production, while not violating this constraint.

With u from (7) there are two degrees of freedom for optimizing oil production and for setpoint tracking. There are no specifications on setpoints for z_{pres1} and z_{presO} other than that they must lie within some limits, giving two degrees of freedom for maximizing oil production. Note that these setpoints in Section 4 were set to be at the initial process values, without any consideration of their optimality.

The setup used is given by

$$z = \begin{bmatrix} z_{\text{trac}} \\ \hdashline z_{\text{opt}} \end{bmatrix} = \begin{bmatrix} z_{\text{pres1}} \\ z_{\text{presO}} \\ \hdashline z_{\text{oil}} \end{bmatrix}, \quad \begin{bmatrix} 19\,\text{bar} \\ 40\,\text{bar} \\ 100\,\text{kg/s} \end{bmatrix} \leq z \leq \begin{bmatrix} 20.01\,\text{bar} \\ 80\,\text{bar} \\ 300\,\text{kg/s} \end{bmatrix},$$

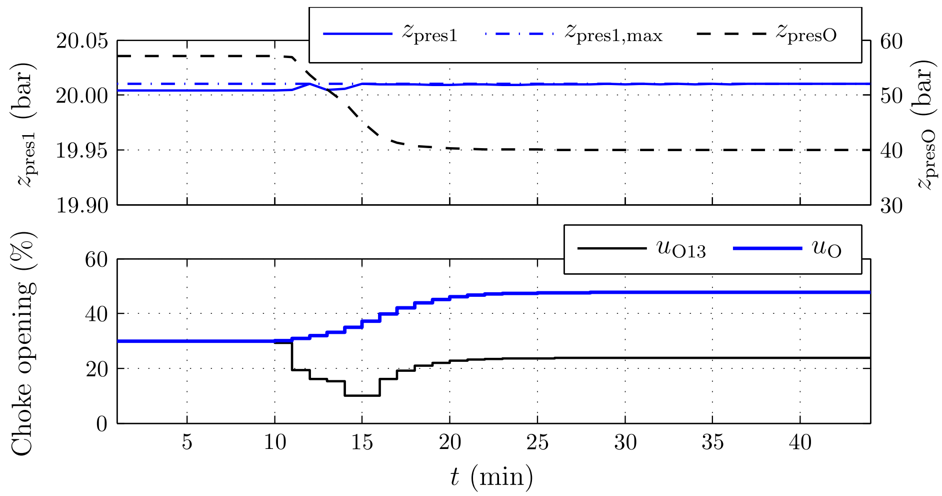

Figure 4: Pressures and choke openings with constant GOR, using unreachable setpoints.

$$Q_{\text{trac}} = \text{diag}\{0,\ 0\}, \quad Q_{\text{opt}} = 0.01, \quad S = \text{diag}\{0.01, 0.01\},$$

$$r = \begin{bmatrix} r_{\text{pres1}} & r_{\text{presO}} & r_{\text{oil}} \end{bmatrix}^\top = \begin{bmatrix} 10^3 & 10^3 & 10^3 \end{bmatrix}^\top,$$

$$\begin{bmatrix} -20\,\% \\ -20\,\% \end{bmatrix} \leq \begin{bmatrix} \Delta u_{\text{O13}} \\ \Delta u_{\text{O}} \end{bmatrix} \leq \begin{bmatrix} 20\,\% \\ 20\,\% \end{bmatrix}, \quad \begin{bmatrix} 10\% \\ 10\% \end{bmatrix} \leq \begin{bmatrix} u_{\text{O13}} \\ u_{\text{O}} \end{bmatrix} \leq \begin{bmatrix} 100\% \\ 100\% \end{bmatrix}.$$

The unreachable setpoint for z_{oil} is selected to be $z_{\text{opt,ref}} = 208.3\,\text{kg/s} = 20\ 000\,\text{Sm}^3/\text{day}$ which is large enough to always be infeasible, but not too large since together with the weight Q_{opt}, the difference $|z_{\text{oil}} - z_{\text{opt,ref}}|$ will affect the gain, cf. (4). The MPC is activated at $t = 10\,\text{min}$ and the responses are shown in Figures 4 and 5. By the choice of penalty weight r, the pressure z_{pres1} never violates the upper constraint, meaning that the priority of optimizing oil production never exceeds the objective of keeping the process within its limits. In Cluster O the GOR value for well ResO13 is higher than for ResO11, hence will it always be more beneficial to produce from ResO11. This can be seen in the decrease of choke opening u_{O13}, while the line choke u_{O} is opened until the constraint in z_{presO} is met. In other words, the solution is found where both of the constraints for z_{pres1} and z_{presO} are active. The result can be seen in Figure 5 where there is a larger increase in z_{oil11} than the decrease in z_{oil13}, giving a net increased oil production z_{oil}.

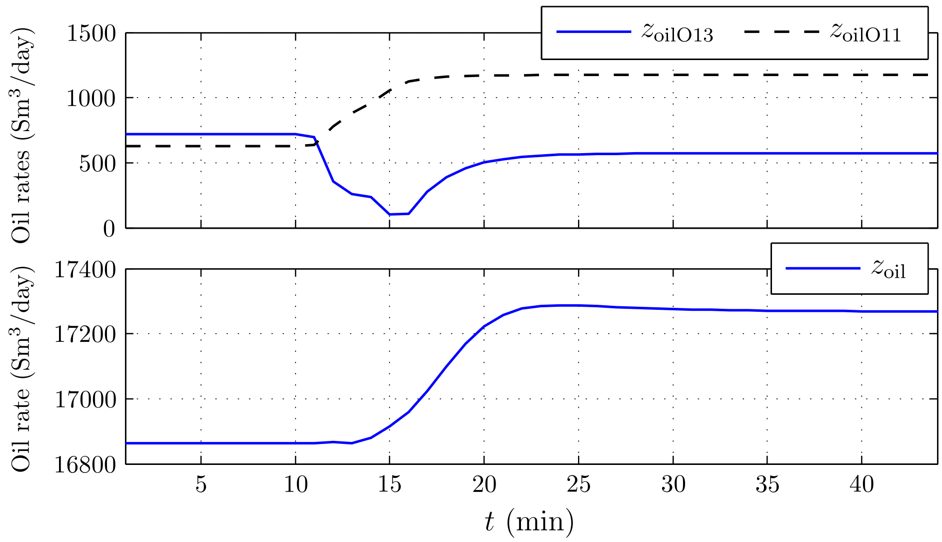

Figure 5: Oil rates with constant GOR, using unreachable setpoints.

5.2. Variable GOR

The model is now extended to have rate-dependent GOR in the wells in Cluster O, and the unreachable setpoints method and the infeasible soft-constraints method are used to maximize oil production. The GOR curves used in the model is shown in Figure 6, where linear GOR curves have been used to represent gas coning, and a constant GOR value of 58 (Sm^3/day)/(Sm^3/day) is used to represent non-coning conditions, see, e.g., [26]. With variable GOR, an optimal choke configuration can not be intuitively chosen, where in general only the maximum constraint for the inlet separator pressure will be active.

5.2.1. Using the unreachable setpoints method

The setup is the same as in Section 5.1, except that both the choke openings in Cluster O are used as MVs,

$$u = \begin{bmatrix} u_{\mathrm{O13}} & u_{\mathrm{O11}} \end{bmatrix}^{\top}. \tag{8}$$

The maximum value for z_{pres1} is set to be 20.34 bar, just above the initial value. The MPC is activated at $t = 10$ min and the responses are shown in Figures 7 and 8. With the smooth changes in choke openings u_{O13} and u_{O11} shown in Figure 7, the corresponding changes in oil flow rates from the

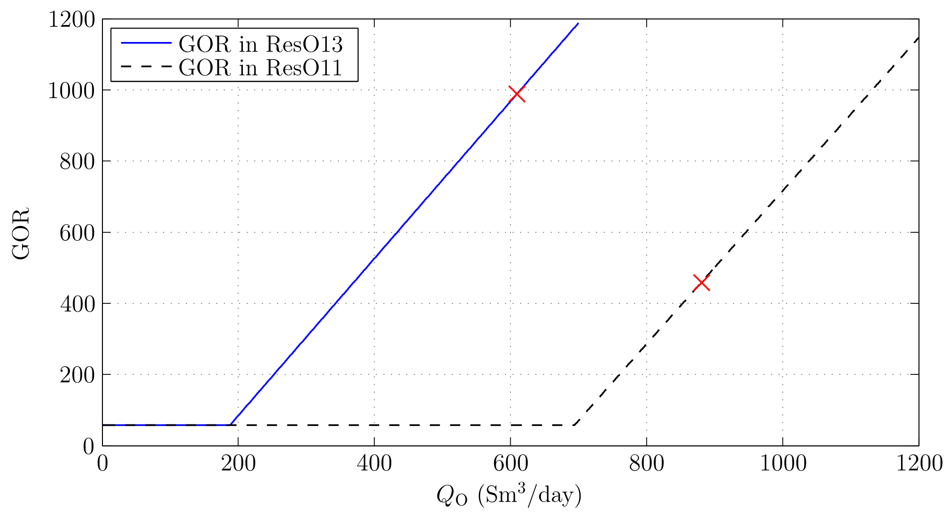

Figure 6: GOR curves for the wells in Cluster O. The red crosses denote the GOR values at the unknown optimal operating point.

wells in Cluster O are shown in Figure 8. Balancing total gas production, the decrease in z_{oilO13} is lower than the increase in z_{oilO11}, giving a net increase in total oil production z_{oil}. Except from the limit on the inlet separator pressure, no other constraints are active. In contrast to the constant GOR case in Section 5.1, the line pressure z_{presO} now settles at a new intermediate value. At the same time there is no intuitive way for adjusting choke openings u to increase oil production. This solution would therefore be difficult to find without any form of optimization.

5.2.2. Using infeasible soft-constraints

The infeasible soft-constraints method is used to get a linear part in the cost function, representing the objective of maximizing total oil production, by choosing $z_{\text{oil,min}}$ larger than the maximum oil production. Using $z_{\text{oil,min}} = 208.3\,\text{kg/s} = 20\,000\,\text{Sm}^3/\text{day}$ (the same value as the unreachable setpoint in Sections 5.1 and 5.2.1) the linear (economic) part of cost in (5) is now

$$-\sum_{i=1}^{P} r_{\text{oil}} z_{\text{oil},k+i}, \tag{9}$$

representing the objective of maximizing total oil production. As discussed in Section 2.4, the value of $z_{\text{oil,min}}$ is not important as long it is large enough to

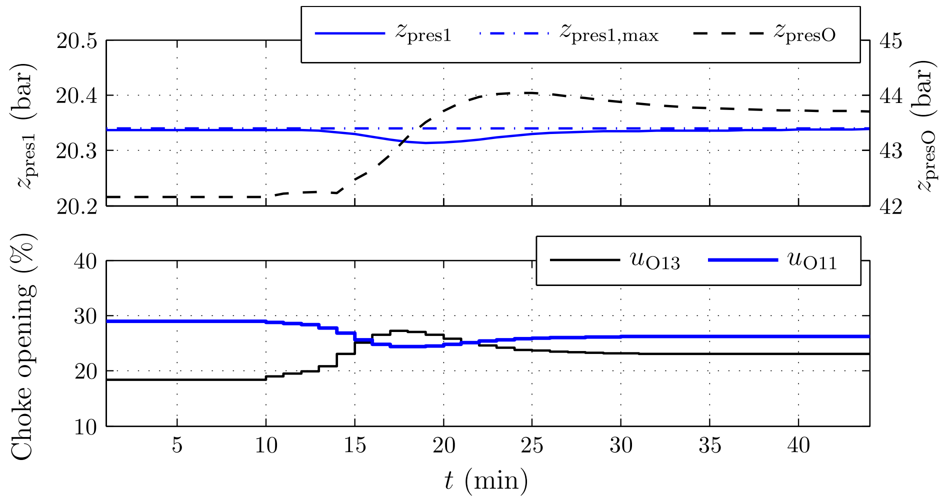

Figure 7: Pressures and choke openings with variable GOR, using unreachable setpoints.

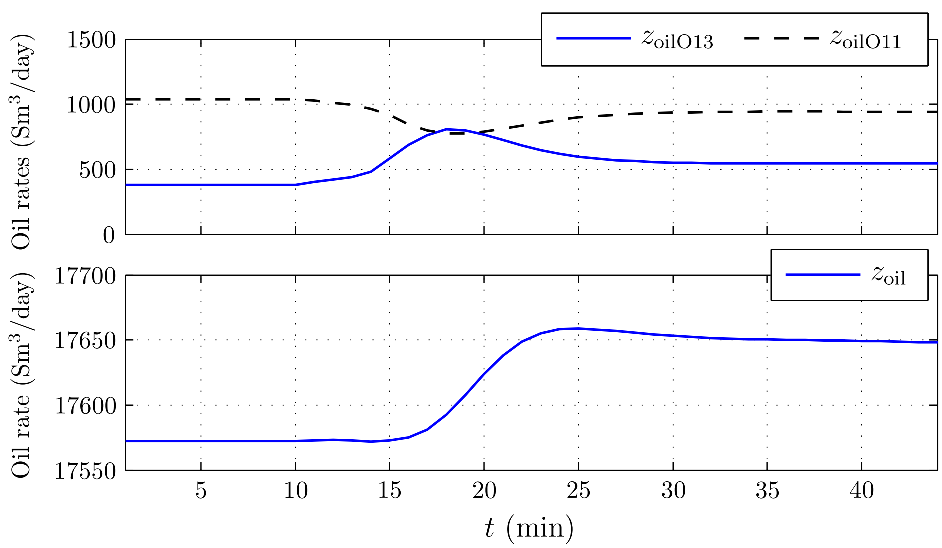

Figure 8: Oil rates with variable GOR, using unreachable setpoints.

always be infeasible and not too large, possibly creating numerical problems. The penalty weights are chosen to be

$$r = \begin{bmatrix} r_{\text{trac}} \\ \hdashline r_{\text{opt}} \end{bmatrix} = \begin{bmatrix} r_{\text{pres1}} \\ r_{\text{presO}} \\ \hdashline r_{\text{oil}} \end{bmatrix} = \begin{bmatrix} 10^3 \\ 10^3 \\ \hdashline 1 \end{bmatrix},$$

where optimizing total oil production is given a lower priority than operating within the constraints for the remaining CVs. Apart from choosing $z_{\text{oil,min}}$, r, and using $Q = \text{diag}\{0, 0, 0\}$, the MPC setup is equal to the one in Section 5.2.1.

The simulated responses shown in Figures 9 and 10 are similar to the ones obtained with the unreachable setpoints method in Section 5.2.1, giving approximately the same steady-state result. The difference is that the controller seems to be somewhat more aggressive, shown in the plots of u_{O13} and u_{O11} in Figure 9. This gives a less smooth response in z_{presO}, whereas the inlet separator pressure z_{pres1} is closer to it's constraint without violating it. The result is that the oil rates from the wells, and thus the total oil rate, settle faster at a new steady state, shown in Figure 10. The aggressiveness may be attributed to the use of a linear cost compared to the quadratic cost in Section 5.2.1. However, aspects related to the numerical implementation of the soft constraints may also come into play here (cf. discussion at the end of Section 2.4).

6. Concluding remarks

In this paper we have studied methods for maximizing total oil production on a short time scale, in a synthetic but realistic case study. We began with showing how NMPC can be used for conventional pressure control, e.g., during slugging, possibly in combination with a higher level of optimization in a two-layer control structure. Next it was demonstrated how a one-layer approach can be used for production optimization with constant GOR in the wells, using the unreachable setpoints method. We ended the study by comparing this method with the infeasible soft-constraints method on a model with rate-dependent GOR, maximizing total oil production. Both of these one-layer methods can be implemented within standard NMPC software tools. A one-layer approach has several potential advantages compared to two-layer approaches, including that only a single model requires updating, disturbances can be counteracted as they appear, and economics can

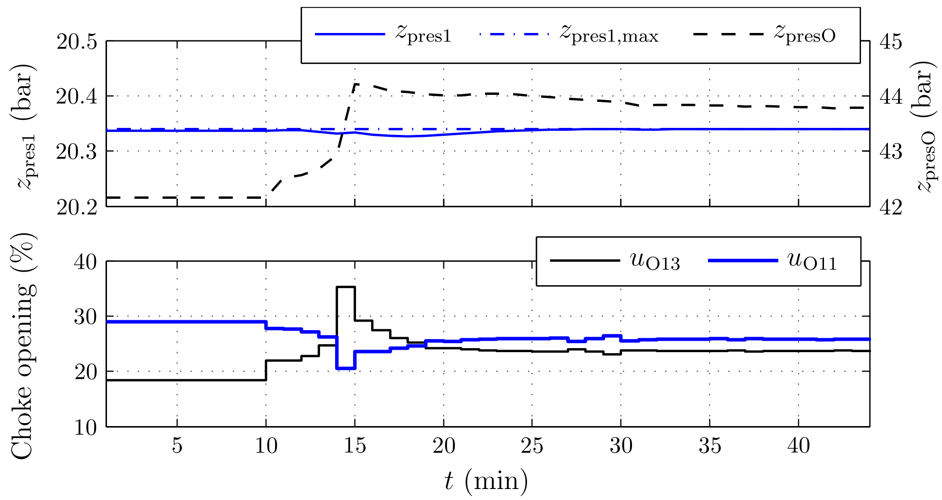

Figure 9: Pressures and choke openings with variable GOR, using infeasible soft-constraints.

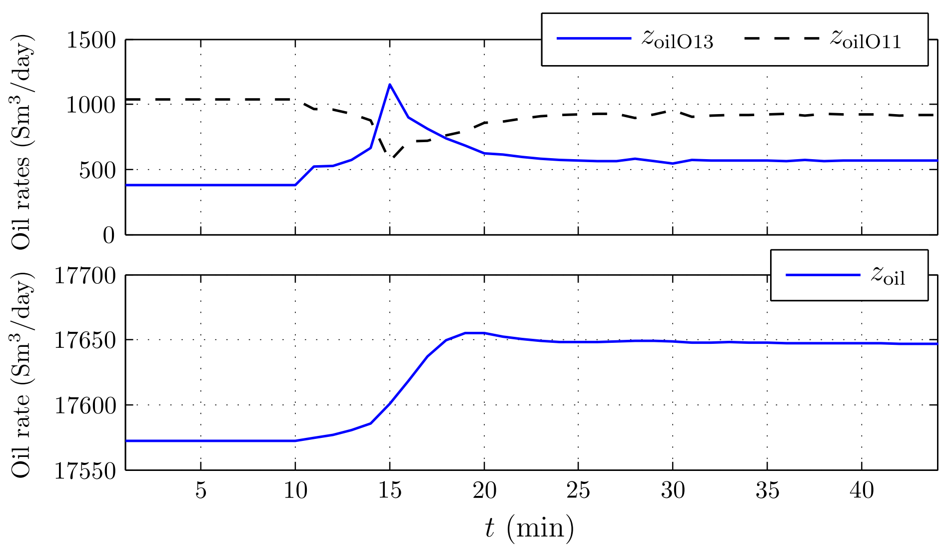

Figure 10: Oil rates with variable GOR, using infeasible soft-constraints.

be optimized also during transients. The infeasible soft-constraints method gives fewer tuning parameters and is in principle easier to set up compared to the unreachable setpoints method, since the only tuning parameter for optimization is the penalty weight r. Not unexpectedly, our study showed that by using infeasible soft-constraints the controller becomes more aggressive due to the resulting linear cost, which may make it harder to tune if smooth responses are a requirement. For increasing system size, choosing penalty weights that properly trade off the "soft" process constraints deviations against the economic objective, may become a challenge. Nevertheless, we believe that the use of infeasible constraints can be a valuable addition to the toolbox for the NMPC control engineer that uses an industrial type NMPC package.

6.1. Conventional pressure control

The case in Section 4 showed how pressure control could be used to increase oil production without any optimization, other than knowledge of the active constraint of the inlet separator pressure. The effect of a disturbance on the inlet separator pressure z_{pres1} was reduced by using NMPC. It was then possible to increase the setpoint closer to the active constraint, resulting in an increased oil production of $143\,\text{Sm}^3/\text{day}$ compared to no control. This case could be solved using PID controllers, but by using NMPC we have inherent multi-variable control and constraint handling, making it possible to operate even closer to process limits.

6.2. Constant GOR

In the case described in Section 5.1, the unreachable setpoints method was used to maximize total oil production. To avoid repetition, we limited the study to only the unreachable setpoints method in the constant GOR case. Comparison between this method and the infeasible soft-constraints method was done in Section 5.2, using variable GOR models. With constant GOR, the optimal solution will lie on some active constraint combination, as it will always pay to increase production from the wells with lowest GOR. (This is in addition to the active constraint for the inlet separator pressure, as was considered in Section 4.) Here the new active constraint was that z_{presO} reached its minimum value. If this minimum value would have been decreased, the active constraint would at some point change to u_{O} at fully open. The initial conditions were quite far from the optimal operating point, resulting in an increased oil production of $406\,\text{Sm}^3/\text{day}$.

Of course, the conventional pressure control structure in Section 4 would have reached the same steady-state optimum within a two-layer structure using a RTO system to calculate the corresponding optimal setpoints. This would require model updating and disturbance estimation at the RTO level, as well as on the NMPC level.

6.3. Variable GOR

In Sections 5.2.1 and 5.2.2 variable GOR models are used, and the optimums are characterized by (6) rather than active constraints (except the inlet separator pressure constraint). The total oil production was increased with $71\,\mathrm{Sm}^3/\mathrm{day}$ using unreachable setpoints and $73\,\mathrm{Sm}^3/\mathrm{day}$ using infeasible soft-constraints (the difference being due to numerical inaccuracies), compared to the initial situation where the separator inlet pressure was on its maximum constraint. The increase in production is dependent on the choice of initial producing state that in this case was somewhat arbitrary (but not unrealistic). However, in oil and gas production in general, even small increases in production rates can imply significantly increased income; with an oil price of USD $100 per barrel, the increases reported represent a yearly increased revenue of over USD $16 million.

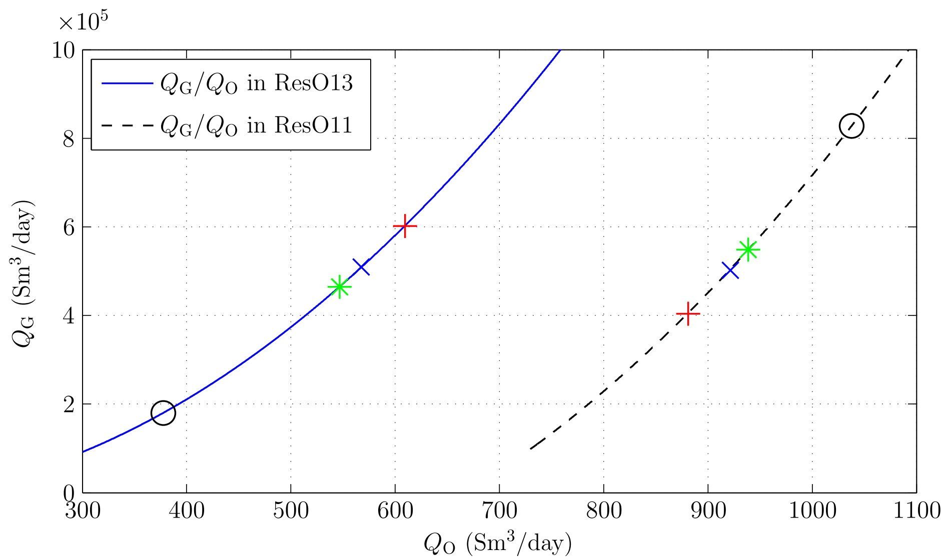

Figure 11: $Q_G - Q_O$ relationships at the start ($\circ$) and end of the simulation, using unreachable setpoint ($*$) and infeasible soft-constraint ($\times$). The optimum is marked with '+'.

Since the separator inlet pressure already was at its constraint, the increase came from exploiting the different gas/oil relationships in the wells (see Figure 11) where the MGOR is the tangent line of the curves. Using (6), the maximum oil production from the two wells is achieved when the MGOR values are equal, giving parallel tangent lines in the points marked '+', where the total increase of oil production is $75\,\mathrm{Sm}^3/\mathrm{day}$. With the two methods used almost equal MGOR values were found. The gap was due to numerical inaccuracies caused by the tangent lines which become almost parallel when approaching the optimum. Note that we have used the MGOR relationships to verify the solutions found, but these relationships are not included in the optimization problem in any form other than in the objective of maximizing total oil production.

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