GAS-FIRED POWER PLANTS: INVESTMENT TIMING, OPERATING FLEXIBILITY AND CO_2 CAPTURE

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We analyze investments in gas-fired power plants based on stochastic electricity and natural gas prices. A simple but realistic two-factor model is used for price processes, enabling analysis of the value of operating flexibility, the opportunity to abandon the capital equipment, as well as finding thresholds for energy prices for which it is optimal to enter into the investment. We develop a method to compute upper and lower bounds on plant values and investment threshold levels. Our case study uses representative power plant investment and operations data, and historical forward prices from well-functioning energy markets. We find that when the decision to build is considered, the abandonment option does not have significant value, whereas the operating flexibility and time-to-build option have significant effect on the building threshold. Furthermore, the joint value of the operating flexibility and the abandonment option is much smaller than the sum of their separate values, because both are options to shut down. The effects of emission costs on the value of installing CO_2 capture technology are also analyzed.

JEL Classifications: Q40, Q52, G13, G11

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1 Introduction

In the next 20 years, fossil fuels will account for 75% of all new electric power generating capacity, and 60% of this is assumed to come in the form of gas-fired power plants

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(see IEA, 2003). Thus many companies in the electricity and natural gas industries are considering investments in such plants. At the same time, the restructuring of electricity and gas markets has brought price transparency in the form of easily available spot- and forward prices. This article offers an approach to analyze gas-fired power plant investments, using the information available on electricity and natural gas futures and forward markets.

A gas-fired power plant may be interesting not only from the point of view of meeting increased power demand. Consider a company owning an undeveloped gas field at a distance to major gas demand hubs; most of the gas reserves in the world are in the category of stranded gas. Building natural gas pipelines is very costly, and the unit cost of gas transportation decreases rapidly with the capacity of the pipeline. This means that locating a gas-fired power plant at the end of a new pipeline improves the economy of scale in transmission of natural gas.

The research question addressed here is that of an energy company having an opportunity to build a gas-fired power plant.

- How high should electricity prices be compared to gas prices, before the company starts building the plant?
- Does it matter whether the plant is baseload, running whatever the level of electricity and gas prices, or cycling, running only when electricity price is above the fuel cost?
- How does the opportunity to abandon the plant influence the decision to invest?
- How do greenhouse gas emission costs affect profitability?

These questions differ from those in Näsakkäla and Fleten (2005), who use the same methodology and data, but a different model and analysis. Whereas that paper looks at investment and technology upgrade, the current one examines investment, operational flexibility, abandonment and CO2 capture technology installation.

Whether a new power plant will be run as a baseload plant, or ramped up and down according to current energy prices, depends more on the state of the local natural gas market than the technical design of the plant itself. New gas plants will often be the combined cycle gas turbine (CCGT) type, which can be operated both as baseload and cycling plants. The operating flexibility is often constrained by the flexibility of the gas inflow. If there is little local storage and/or alternative use of the natural gas, the plant operator will seldom find it profitable to ramp down the plant.

The operating cash flows in a gas-fired power plant depend on the spark spread, defined as the difference between the price of electricity and the cost of gas used for the generation of electricity. The long-maturity swaps on electricity and gas, e.g. three-year swaps, give the exact and certain market value of constant electricity and gas flow (disregarding the credit and liquidity risk). A baseload plant operates with constant electricity and gas flow, so a baseload plant can be valued with long-term swaps[2]. On the other hand, a cycle plant can react to short-term variations in the spark spread by ramping up and down, leading to non-constant electricity and gas flow. Therefore, the short-term dynamics of the spark spread are needed for the valuation of a cycling plant. The short-term dynamics can be estimated by using short-maturity swaps, for example.

Long-term investments, such as in power plants, are never undertaken due to non-persistent spikes in the spark spread. Rather, investment decisions are based on long-term price levels, called equilibrium prices here. Using a real options approach (or 'contingent claims approach' in the language of Dixit and Pindyck 1994), we compare the current equilibrium price estimate to a computed investment threshold, reflecting that at this threshold level of equilibrium price the value of waiting longer is equal to the net present value received if investment is commenced (McDonald and Siegel 1986). When the equilibrium price increases to the investment threshold, the implementation of the power plant project should be started. As it is difficult to characterize the ramping policy of a gas-fired power plant precisely, instead of giving an exact value of the plant, we give upper and lower bounds for the plant value. These bounds are used to calculate upper and lower bounds for the investment thresholds.

Brekke and Schieldrop (1999) and Abadie and Chamorro (2006) consider power plants which can burn two different types of fuel. Deng, Johnson and Sogomonian (2001) use electricity and fuel futures to value gas-fired peak load plants. Siddiqui and Maribu (2009) consider sequential vs direct investment in small gas-fired power and heating systems. Deng and Oren (2003) and Tseng and Lin (2007) take into account ramping, startup costs and non-constant operating efficiency, and the former show that the overvaluation made when ignoring these operational characteristics is small when operating efficiency is high. For this reason we abstract from the mentioned characteristics. This paper contributes, first, by presenting a case study of real option analysis that is hopefully interesting for many. Second, it provides upper and lower

[2] Swaps with maturities beyond a few years is typically not available. Long-term prices then have to be estimated or extrapolated. We use the approach suggested by Schwartz (1997) who consider i.a. long-maturity oil prices.

bounds for investment thresholds and plant values that depend on the degree of operating flexibility of the plant. Finally, our approach to modeling uncertainty, which is empirically realistic, reduces the dimension to just one. This greatly facilitates relatively simple real option analysis. Compared to Näsakkäla and Fleten (2005), who study technology choice among upgradable power plants, the second aspect is new.

We illustrate the use of our model by applying it to the energy markets in Scandinavia. The electricity markets there have been restructured since the late 1980s. Naturally, our model can be applied to other energy markets as well. Our case study indicates that the difference between cycling and baseload plant values is considerable, i.e. the value of being able to ramp up and down is significant. We also find that the addition of an abandonment option does not dramatically change the investment threshold. This means that when investments in gas-fired power plants are considered, a good overall view of the investment problem can be made by disregarding the abandonment option, whereas the operating flexibility and time-to-build options have significant effect on the investment threshold. In our case study, using investment cost data from 2000, we find that building a CO2 capture plant and piping CO2 off to permanent storage or in oil fields for increased recovery is not a cost-efficient way of reducing greenhouse gas emissions at carbon price levels of 25 $/tonne CO2.

The model generalizes beyond the case of gas-fired power plants. Any investment involving a relatively simple transformation of one commodity to another could be analyzed using this framework. The spread between output price and input costs is then an important source of uncertainty. Examples include the transformation of natural gas into liquefied natural gas, a methanol factory, and a biodiesel factory.

The paper is organized as follows. We present the model of price uncertainty in Section 2, where we also argue why it is important to incorporate information in swap prices into real options analyses. In Section 3 upper and lower bounds for the plant value are calculated, whereas in Section 4 the investment problem is studied. Section 5 illustrates the model using an example, and in Section 6 discusses the results of the example. Section 7 concludes the study.

2 The energy price process

As the indicator of the profitability of a gas-fired power plant and as the driver of uncertainty in our model, we use the spark spread. This is defined as the difference between the output price and the input cost

$$S = S_e - K_H S_g , \quad (1)$$

where S is the spark spread, S_e the electricity price per unit of energy (MWh), the heat rate K_H is the amount of gas required to generate one MWh of electricity, and S_g is the price of gas. The quantity of gas is measured in MWh gross caloric value. The heat rate, given in MWh_{gas}/MWh_{el}, measures the efficiency of the plant: the lower the heat rate, the more efficient the facility. A modern gas-fired power plant will typically be of the so-called combined cycle type (CCGT). The efficiency of such a plant wears down over time (but is restored and even improved with replacements and refurbishments), and is reduced when the plant is running on half capacity. Still, the use of a constant efficiency is considered plausible for long-term analyses (see Deng et al., 2001).

The spark spread is the contribution margin of a gas-fired power plant. It can be both positive and negative, and it may have a number of empirical properties including seasonality, mean reversion, jumps and/or spikes, and seasonality and/or stochasticity in the variance.

Seasonality is caused by the underlying seasonality in demand for electricity and gas, and in hydropower-rich systems also by seasonality in supply. Mean reversion is caused by time lags in the adjustments by energy producers to varying price levels: An increase in the spark spread attracts high cost producers to the market putting downward pressure on prices. Conversely, when prices decrease some high cost producers will withdraw capacity temporarily, putting upward pressure on prices. As these entries and exits are not instantaneous, prices may be temporarily high or low, but will revert toward a long-term spark spread level. Mean reversion can also be inherited from reversion in related energy commodities such as oil and coal. Possible jumps can occur in spark spread due to the sudden inflow of unexpected information regarding future supply or demand. Spikes, rapid large price movements followed quickly by large opposite movements, are due by the non-storable nature of electricity (and costly and capacitated storage of natural gas) causing tight market situations when demand is close to the system capacity.

Uncertainty in spark spread is caused by uncertainty in electricity and natural gas prices. There may be uncertainty not only in short-term spark spreads, but also in the average spark spread over a typical lifespan of a power plant. This long-term uncertainty is due to advances in gas exploration and production technology, changes in the discovery of natural gas, improved power plant technology, and political and regulatory effects. For example, unexpected development in the cost of alternative power generation technology, such as nuclear power, may lead to a persisting change in electricity prices.

We want to arrive at a model for spark spread that captures those of the above-mentioned properties that are important in investment evaluation and decision making. At the same time, the model must be parsimonious enough to facilitate actual investment and real option analysis. Since we do not aim to support hedging of risks in the cash flows of this project, the model does not have to map directly from prices on observable swap contracts as is done in forward curve models such as that of Heath, Jarrow and Morton (1992) (HJM). We finally arrive at the following model, which is based on Ross (1997), Pilipović (1998), and Schwartz and Smith (2000):

ASSUMPTION 1. *The spark spread is a sum of a short-term deviation and an equilibrium price*

$$S(t) = \chi(t) + \xi(t), \tag{2}$$

where the short-term deviation $\chi(t)$ *is assumed to revert toward zero, following an Ornstein-Uhlenbeck process*

$$d\chi(t) = -\kappa\chi(t)dt + \sigma_\chi dB_\chi(t). \tag{3}$$

The equilibrium price $\xi(t)$ *is assumed to follow an arithmetic Brownian motion process*

$$d\xi(t) = \mu_\xi dt + \sigma_\xi dB_\xi(t), \tag{4}$$

where κ, σ_χ, μ_ξ, *and* σ_ξ *are constants.* $B_\kappa(\cdot)$ *and* $B_\xi(\cdot)$ *are standard Brownian motions, with correlation* $\rho dt = dB_\chi dB_\xi$.

The modeled spark spread can be positive or negative, and it is mean reverting. The following corollary expresses the distribution of the future spark spread values.

COROLLARY 1. *When spark spread has the dynamics given in* (2)-(4), *prices are normally distributed, and the expected value and variance are given by*

$$E_t\left[S(T)\right] = e^{-\kappa(T-t)}\chi(t) + \xi(t) + \mu_\xi(T-t) \quad (5)$$

$$Var_t\left[S(T)\right] = \frac{\sigma_\chi^2}{2\kappa}\left(1 - e^{-2\kappa(T-t)}\right) + \sigma_\xi^2(T-t) + 2\left(1 - e^{-\kappa(T-t)}\right)\frac{\rho\sigma_\chi\sigma_\xi}{\kappa}. \quad (6)$$

PROOF: See Schwartz and Smith (2000).

Corollary 1 states that the spark spread is a sum of two normally distributed variables: short-term deviation and equilibrium price. The expected value of the short-term deviation converges to zero as the maturity $T-t$ increases and so the expected value of the spark spread converges to the expected value of the equilibrium price. The mean-reversion parameter κ describes the rate of this convergence. The maturity in which a short-term deviation is expected to halve is given by

$$T_{1/2} = -\frac{\ln(0.5)}{\kappa}. \quad (7)$$

The spark spread variance caused by the uncertainty in the equilibrium price increases linearly as a function of maturity, whereas the spark spread variance due to the short-term deviations converges toward $\sigma_\chi^2/2\kappa$. Note that the decreasing forward volatility structure, typical for commodities, is tied to the mean-reversion in the spot prices (see Schwartz, 1997).

This model has the advantage of avoiding the need for explicitly specifying the correlation between electricity and natural gas prices. On the other hand, neither the short-term deviation χ nor the equilibrium price ξ are directly observable, but must be estimated from electricity and gas swap prices. These swap prices provide the risk-adjusted expected future spark spread value, so swap prices can be used to infer the risk adjusted dynamics of short-term deviation and the equilibrium price. The expected short-term deviation decreases to zero when the maturity increases, so the long-maturity swaps give information about the equilibrium price. When the maturity is short, the short-term deviation has not yet converged to zero. Hence, the difference between long- and short-maturity swaps provides information about the short-term dynamics. Based on this simple idea Schwartz (1997) proposes a Kalman filter-based estimation for the parameters of multi-factor commodity price process. We use the procedure to estimate the spark spread process. The resulting model (2)-(4) becomes adjusted for risk, so that we can use risk-neutral pricing.

If there are no swap prices available, the short-term deviation and the equilibrium price dynamics must be estimated. One method is using a history of spot prices. However, when derivative prices are not available as spanning assets, finding the appropriate discount and growth rates for real option analysis is more challenging and tends to become more ad-hoc (see Section 4.3 of Dixit and Pindyck, 1994).

This model captures mean reversion, and short- and long-term uncertainty, but not seasonality, jumps/spikes and non-constant variance. We discuss each of the non-captured properties in turn.

Seasonality is present in both electricity and gas prices, and in some regions the peak prices of the two commodities may both be in the winter due to their use for heating. So for spark spread, the seasonality may to a degree be canceled out, since the spark spread is a difference and the seasonality of electricity and gas may follow similar patterns. This is found by Näsäkkälä and Fleten (2005). The estimated spark spread process, displayed in Figure 1 by a black line, supports the hypothesis of no seasonality in the data. We remark that introducing seasonality may help the decision maker to pinpoint the time of year the various investment and disinvestment decisions should be made. However, in practice there will be other concerns that determine the time of the year the construction and operational decisions will be made.

If jumps and/or spikes are introduced into the spark spread model, it would become more complex. Jumps and spikes is present in our data only to a small degree, and we have chosen to exclude it, instead we refer to Deng (2005), who performs model comparisons and finds that, although spikes are important for valuation in many cases, ignoring spikes leads to low valuation errors for efficient power plants and when the price processes exhibit mean reversion.

We have not included more sophisticated variance features as we opted for simplicity and also there was a lack of option price data to support such an approach. Thus, to the extent spark spread variance change when electricity or gas prices change, it cannot be captured by our model. This issue is discussed further in Näsäkkälä and Fleten (2005).

The traditional way of modeling spark spread is to use separate processes for electricity and natural gas prices, whereas this subsection has introduced a two-factor model for direct modeling of the spark spread. Direct modeling of the spark is also discussed by Eydeland and Wolyniec (2003). We use the spark spread model to provide value formulas for the power plant once it is installed.

3 Gas plant valuation

In this section we calculate upper and lower bounds for the value of the gas-fired power plant. The following assumption states the operational characteristics of such a plant.

ASSUMPTION 2. The degree to which the plant can or will be ramped up and down is not known. The costs associated with starting up and shutting down the plant can be amortized into fixed costs.

Although the operation and maintenance costs of a gas-fired power plant may vary from year to year, they do not vary much over longer time periods, so it is realistic to assume that the fixed costs are constant.

The ramping policy of a particular plant depends on local conditions associated with plant design and gas inflow arrangement. The degree to which the power plant can or will be ramped up is assumed unknown; there are unknown constraints on ramping. Instead of giving an exact specification of the ramping policy, we use upper and lower bounds for the gains associated with ramping. The lower bound V_L can be calculated by assuming that the plant cannot exploit unexpected changes in the spark spread, i.e. a baseload plant. The following lemma gives the value of that case.

LEMMA 1. At time t, the lower bound of the plant value $V_L(\chi,\xi) \le V(\chi,\xi)$ is given by the value of a baseload plant

$$\begin{aligned} V_L(\chi(t),\xi(t)) = \bar{C}\left(\frac{\chi(t)}{\kappa+r}+\frac{\xi(t)-E}{r}+\frac{\mu_\xi}{r^2}-\mathrm{e}^{-r(\bar{T}-t)}\left(\frac{\mathrm{e}^{-\kappa(\bar{T}-t)}\chi(t)}{\kappa+r}+\frac{\xi(t)-E}{r}+\frac{\mu_\xi\left(r(\bar{T}-t)+1\right)}{r^2}\right)\right) \\ -\frac{G}{r}\left(1-\mathrm{e}^{-r(\bar{T}-t)}\right) \end{aligned} \tag{8}$$

where $\bar{T}-t$ is the remaining lifetime of the plant, $\bar{C}$ is the capacity of the plant, E is the emission cost, and G are the fixed costs of running the plant.

PROOF: The value of a baseload plant is the present value of expected operating cash flows

$$\begin{aligned} V_L(\chi(t),\xi(t)) &= \int_t^{\bar{T}} e^{-r(s-t)}\left(\bar{C}\left(E_t\left[S(s)\right]-E\right)-G\right)ds = \\ &= \int_t^{\bar{T}} e^{-r(s-t)}\left(\bar{C}\left(e^{-\kappa(s-t)}\chi(t)+\xi(t)-E+\mu_\xi(s-t)\right)-G\right)ds \end{aligned} . \tag{9}$$

Integration gives (8). Q.E.D.

The lower bound is just the discounted sum of expected spark spread values less emission and fixed costs. Thus, the lower bound is not affected by the short-term and equilibrium volatilities σ_χ and σ_ξ, and is hardly at all affected by the speed of mean reversion, κ.

An owner of a gas-fired power plant may be able react to adverse changes in the spark spread by temporarily shutting down the plant. The value of a cycling plant is the discounted sum of expected spark spread values less emission and fixed costs plus the option value of being able to ramp up and down. The value of the up and down ramping is dependent on the response times of the plant, and is maximized when ramping up and down can be done without delay. In other words, the upper bound V_U for the plant value can be calculated by assuming that the up and down ramping can be done without delay, i.e. by assuming that the plant produces electricity only when the spark spread exceeds emission costs.

LEMMA 2. At time t, the upper bound of the plant value $V_U(\chi,\xi) \geq V(\chi,\xi)$ *is given by the value of an ideal cycling plant*

$$V_U(\chi(t),\xi(t)) =$$
$$\bar{C}\int_t^{\bar{T}} e^{-r(s-t)}\left(\frac{\sqrt{Var_t\ S(s)}}{\sqrt{2\pi}}e^{\left(-\frac{(E[S(s)]-E)^2}{2Var\ S(s)}\right)} + \ E_t\left[S(s)\right]-E\ \ \Phi\left(\frac{E_t\left[S(s)\right]-E}{\sqrt{Var_t\ S(s)}}\right)\right)ds - \frac{G}{r}\ \ 1-\mathrm{e}^{-r(\bar{T}-t)} \tag{10}$$

where $\Phi\ \cdot$ *is the normal cumulative distribution function, and G are the fixed costs of running the plant. The expected value* $E_t\left[S(s)\right]$ *and variance* $Var_t\left(S(s)\right)$ *for the spark spread are given in Corollary 1.*

PROOF: See Appendix A.

The more the spark spread varies, the more valuable the option to ramp up and down is, and therefore the value of the cycling plant increases as a function of the variance of the spark spread. Increases in this variance can come about with increased short-term variance σ_χ^2, long-term variance σ_ξ^2 or correlation ρ, and decreased speed of mean reversion κ. The difference between the upper and lower bounds for the plant value is due to the option to temporarily shut down over the lifetime of the power plant. An increase in the starting level of the short-term deviation $\chi(0)$ will not affect plant values much, since its effect quickly fades off, but in principle the option to shut down temporarily becomes more out-of-the money, i.e. less valuable. For the same reason, the shut-down option also becomes less valuable if the start level of the equilibrium price $\xi(0)$ increases, or the emission costs E go down, or the growth rate of the equilibrium price

μ_ξ goes up. One can also see this by recognizing that such changes in the spark spread parameters increase the expected spark spread and will affect the value of a baseload plant more positively than a cycling plant, because the cycling plant is sometimes shut down.

It may be helpful to know the power plant value if the spark spread process is even simpler, e.g. a Brownian motion with drift.

ASSUMPTION 1'. *The spark spread process Z follows*

$$dZ = \alpha dt + \upsilon dW \qquad (3')$$

where dW is a Brownian motion.

Here α is the growth of the spark spread and υ is the standard deviation. By integration, the value of a baseload plant in this case is

$$V_L^B(Z(t)) = \frac{\bar{C}}{r}\left[\left(Z-E\right)\left(1-\mathrm{e}^{-r(\bar{T}-t)}\right)+\frac{\alpha}{r}\left(1-\mathrm{e}^{-r(\bar{T}-t)}\left(1+r(\bar{T}-t)\right)\right)\right]-\frac{G}{r}\left(1-\mathrm{e}^{-r(\bar{T}-t)}\right)$$

(8')

The value of a cycling plant consists of the baseload value plus the options to shut down and ramp up again. To conserve space, its formula will not be shown. However, its value is shown in Figure 1 below.

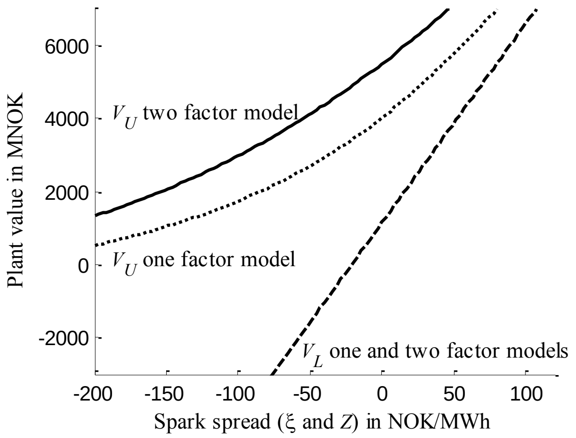

Figure 1 *Power plant value bounds, for the basic two-factor model and for a simplified model using Brownian motion with drift. The value of an idealized cycling plant is expressed as V_U, whereas a baseload plant has value V_L.*

We have used parameter estimates from Section 5. In Figure 1 the starting value of the spark spread $Z(0)$, the growth α and the standard deviation ν are chosen to match the long-term behavior of the data. The lower bound coincides with that of the two-factor model. As expected, the upper bound using a one-factor model is lower than the corresponding bound for the two-factor model, because the one-factor model is unable to capture the value of short-term variations.

To summarize: As we are not able to precisely characterize the shutdown/startup ability of the plant, we do not calculate the exact valuation formula for the gas-fired power plant, but provide bounds for the plant value. The lower bound is given by the baseload plant (Lemma 1) and the upper bound is given by the ideal cycling plant (Lemma 2). Decisions regarding the opportunity to invest and abandon the plant are analyzed in the next section.

4 Investment analysis

In this section we calculate bounds for the investment thresholds when the gas plant value has the bounds given by Lemma 1 and Lemma 2. The following assumption characterizes the state variables affecting the investment decisions.

ASSUMPTION 3. *The investment decisions are made as a function of equilibrium price. In the investment decisions the lifetime of the plant is assumed to be infinite, and construction occurs instantly.*

Assumption 3 states that when the gas plant investments, i.e. building and abandonment, are considered the decisions are made as a function of the equilibrium price ξ, i.e. the current short-term realization $\chi(0)$ is disregarded in investment decisions. In principle, investment decisions under two-factor dynamics depends on both factors, however, in practice short-term deviations fade away quickly, and have insignificant bearing on the decision to invest or to abandon. Assumption 3 makes this explicit.

The parameters governing the short-term dynamics, i.e. short-term volatility σ_χ and mean reversion κ, still affect the value of the plant, and thereby they also affect the investment decision. This means that the short-term parameters are important in the investment decision, even though the particular realization of the short-term deviation $\chi(0)$ does not matter when investment decisions are made. The omission of the short-term realization is motivated by the fact that gas-fired power plants are long-term investments, and a gas plant investment is never made due to a non-persistent spike in the price process. In valuing the plants for investment purposes we will therefore set the short-term deviation $\chi(0)$ to zero. The assumption that investment decisions are made as a function of equilibrium price is a realistic approximation of the investment decision process if the expected lifetime of the short-term deviation is considerably smaller than the expected lifetime of the plant. In Section 5 we use a speed of mean reversion of $\kappa = 2.6$, which means, with (7), that the short-term deviation is expected to halve in about three months. Since this is insignificant compared to the expected lifetime of the plant, the approximation obtained by omitting the short-term realization in the investment decision is realistic.

The infinite lifetime assumption is motivated by the fact that the lifetime of a plant is often increased by upgrading and reconstructions, and by downward shifts in the maintenance cost curve (see Ellerman, 1998). Once a power plant has been built, with very long-lived transmission lines and gas pipelines connecting to the rest of the system, it is often most economical to extend the lifetime of the power plant. This assumption also allows the analysis to be parsimonious. The upper and lower bounds for the plant value as a function of lifetime will be illustrated in Section 5.

Finally, in reality there is a time-lag between the investment decision and the time the plant can start to operate, around two years. The instantaneous construction assumption helps keep the exposition simple. The effects of time-lags is studied by e.g. Majd and Pindyck (1987).

Building the plant becomes optimal when the equilibrium price rises to a building threshold ξ_I. When waiting is optimal, i.e., when $\xi < \xi_I$, the investor has an option to postpone the building decision. The value of such a time-to-build option is given by the following lemma.

LEMMA 3. The value of an option to build a gas-fired power plant is

$$F_0(\xi) = A_1 e^{\beta_1 \xi} - \frac{W}{r}, \qquad \text{when} \quad \xi \leq \xi_I, \tag{11}$$

where A_1 is a positive parameter to be determined and W are constant payments that the firm faces to keep the build option alive. The parameter β_1 is given by

$$\beta_1 = \frac{-\mu_\xi + \sqrt{\mu_\xi^2 + 2\sigma_\xi^2 r}}{\sigma_\xi^2} > 0. \tag{12}$$

PROOF: See Appendix B.

Growth-related parameters of the option pricing formulas that come from the spark spread model, e.g. μ_ξ and κ, are adjusted for risk, since the spark spread dynamics is estimated using swap prices, which themselves reflect the price of risk.

The time-to-build option value increases exponentially as a function of the equilibrium price. The parameter A_1 depends on the value of the plant and on the investment cost *I*. As we are not able to state the gas plant value exactly, we cannot state the exact building threshold, but the following proposition provides a method to calculate upper and lower bounds $\xi_{IL} \le \xi_I \le \xi_{IU}$ for the building threshold.

PROPOSITION 1. The lower bound of the building threshold $\xi_{IL} \le \xi_I$ is given by

$$F_0(\xi_{IL}) = V_U(0, \xi_{IL}) - I \tag{13}$$

$$\frac{dF_0(\xi_{IL})}{d\xi} = \frac{\partial V_U(0, \xi_{IL})}{\partial \xi}, \tag{14}$$

whereas the upper bound $\xi_I \le \xi_{IU}$ is given by

$$F_0(\xi_{IU}) = V_L(0, \xi_{IU}) - I \tag{15}$$

$$\frac{dF_0(\xi_{IU})}{d\xi} = \frac{\partial V_L(0, \xi_{IU})}{\partial \xi}. \tag{16}$$

PROOF: This is a special case of Proposition 2 and the proof will be omitted.

The equations in Proposition 1 cannot be solved analytically but a numerical solution can be attained. For example, to find the lower bound one substitutes (10) and (11) into (13) and (14) and solve the latter two nonlinear equations for A_1 and ξ_{IL}.

Note that the short-term deviation is set to zero in Proposition 1. The reason is, as we have argued, that its starting value is unimportant since its effect is quickly faded away due to mean reversion. One cannot know its value when the equilibrium price reaches the building threshold, and its value has arbitrarily been set to zero.

The more valuable the plant becomes, the more eager the firms are to invest, thus the lower bound for the building threshold is given by the upper bound of the value of the plant and vice versa. In particular, the upper and lower bounds are calculated for the building threshold by finding the prices that satisfy the value-matching and smooth-pasting conditions under the most pessimistic and optimistic scenarios, respectively. The upper threshold uses V_L because it assumes that the plant is completely inflexible, and therefore requires the highest possible price to entice investment. By contrast, the lower threshold uses V_U because it assumes an ideal cycling plant; hence it requires a lower price to entice investment.

Next we will consider how the investment decision changes if there is an opportunity to abandon the gas plant and realize the salvage value of the plant *J*. In this case, when a decision to build is made the investor receives both the gas plant and an option to abandon the plant. As the lifetime of the plant is assumed to be infinite, there is a constant threshold value ξ_A for the abandonment, i.e. abandoning is not optimal when $\xi_A < \xi$. The following Lemma states the value of such an abandonment option.

LEMMA 4. The value of an abandonment option is

$$F_1(\xi) = D_2 e^{\beta_2 \xi} \qquad \text{when} \quad \xi_A \le \xi \tag{17}$$

where D_2 *is a positive parameter to be determined. The parameter* β_2 *is given by*

$$\beta_2 = \frac{-\mu_\xi - \sqrt{\mu_\xi^{\,2} + 2\sigma_\xi^{\,2} r}}{\sigma_\xi^{\,2}} < 0 . \tag{18}$$

PROOF: The proof is similar to that of the build option (Appendix B), but now the option becomes less valuable as the spark spread increases. Q.E.D.

The abandonment option value decreases exponentially as a function of the equilibrium price. The parameter D_2 depends on the salvage value *J*. Again we are not able to state the exact building and abandonment thresholds, but the following Proposition gives upper and lower bounds for the thresholds, i.e. $\xi_{IL} \le \xi_I \le \xi_{IU}$ and $\xi_{AL} \le \xi_A \le \xi_{AU}$.

PROPOSITION 2. The lower bounds for the building and abandonment thresholds $\xi_{IL} \le \xi$ *and* $\xi_{AL} \le \xi$ *are given by*

$$F_0(\xi_{IL}) = V_U(0, \xi_{IL}) + F_1(\xi_{IL}) - I \tag{19}$$

$$F_1(\xi_{AL}) + V_U(0, \xi_{AL}) = J \tag{20}$$

$$\frac{dF_0(\xi_{IL})}{d\xi} = \frac{\partial V_U(0, \xi_{IL})}{\partial \xi} + \frac{dF_1(\xi_{IL})}{d\xi} \tag{21}$$

$$\frac{dF_1(\xi_{AL})}{d\xi} + \frac{\partial V_U(0, \xi_{AL})}{\partial \xi} = 0 , \tag{22}$$

whereas the upper bounds $\xi \le \xi_{IU}$ *and* $\xi \le \xi_{AU}$ *are given by*

$$F_0(\xi_{IU}) = V_L(0, \xi_{IU}) + F_1(\xi_{IU}) - I , \tag{23}$$

$$F_1(\xi_{IU}) + V_L(0, \xi_{IU}) = J , \tag{24}$$

$$\frac{dF_0(\xi_{IU})}{d\xi} = \frac{\partial V_L(0, \xi_{IU})}{\partial \xi} + \frac{dF_1(\xi_{IU})}{d\xi} \tag{25}$$

$$\frac{dF_1(\xi_{AU})}{d\xi} + \frac{\partial V_L(0,\xi_{AU})}{\partial \xi} = 0. \tag{26}$$

PROOF: See Appendix C.

The equations in Proposition 2 cannot be solved analytically either, but a numerical solution can be attained. The less valuable the plant is, the more eager the firms are to abandon the plant. Thus the upper bound of the abandonment threshold is given by the lower bound of the plant value, and vice versa.

To summarize: in this section we have derived a method to calculate the lower and upper bounds for the building and abandonment thresholds. If the abandonment option is ignored the building threshold is given by Proposition 1. When both building and abandonment are studied the thresholds are given by Proposition 2. Next we present the case study.

5 Application

It is estimated that over the period 2001-2030 about 2000 *GW* of new natural gas-fired power plant capacity will be built (see IEA, 2003). Our method can be used to estimate benchmark values of such investments. In this example we concentrate on the possibility to build a natural gas-fired power plant in Norway. The main reason to concentrate on this particular case is the availability of good spark spread and investment cost data. Norwegian energy and environmental authorities have given a number of licenses to build gas-fired power plants and we take the view of an investor having one of these licenses.

5.1 Data and estimation

The costs of building and running a natural gas-fired power plant in Norway are estimated by Undrum et al. (2000). With an exchange rate of 7 *NOK/USD,* a combined cycle gas turbine (CCGT) plant costs approximately 1620 *MNOK*, and the maintenance costs G are approximately 50 *MNOK/year*. We estimate that the costs of holding the license W are 5% of the fixed costs of a running a plant. In Undrum et al. (2000) approximately 35% of the investment costs are used for capital equipment. We assume that if the plant is abandoned all the capital equipment can be realized on the second-hand market, i.e. the salvage value of the plant J is 567 *MNOK*. The estimated parameters are for a gas plant whose maximum capacity is 415 *MW*. We assume that the capacity factor of the plant is 90%, thus we use a production capacity of 3.27 *TWh/year*. Table 1 contains a summary of the gas plant parameters.

Table 1: Characteristics of the gas-fired power plant.

Parameter	W	$\bar{C}$	G	I	J
Unit	*MNOK/year*	*TWh/year*	*MNOK/year*	*MNOK*	*MNOK*
Value	2.5	3.27	50	1620	567

Näsäkkälä and Fleten (2005) use electricity data from Nord Pool (The Nordic Power Exchange) and gas data from International Petroleum Exchange (IPE) to estimate spark spread dynamics for a combined cycle gas turbine plant whose efficiency is 58.1%, i.e. the heat rate is $K_H = 1.72$ $\mathrm{MWh_{gas}/MWh_{el}}$, which corresponds to 5.9 Btu/kWh. The spark spread parameters are summarized in Table 2.

Table 2: Spark spread parameter estimates.

Parameter	r	κ	μ_ξ	ρ	σ_χ	σ_ξ	χ_0	ξ_0
Unit			*NOK/MWh*		*NOK/MWh*	*NOK/MWh*	*NOK/MWh*	*NOK/MWh*
Value	0.06	2.6	2.18	-0.21	382.2	47.8	52.9	62.3

For short-maturity swaps, giving information about the short-term dynamics, they use monthly swap contracts with 1-month swap term/tenor. For long-maturity contracts, giving information about the equilibrium price dynamics, they use contracts with 1-year swap term and 1 to 3 years to maturity. One could consider using finer granularity to capture short-term variations. If one could obtain daily or even hourly spot price data, the large variations in the very short run would mean that the estimate of the upper bound would increase significantly, because these large variations would be absorbed by increased estimates on short-term variance. However, we do not have spot price data for natural gas, and the shortest-maturity product is the nearest month. Furthermore, using the shortest maturity futures/forward as a proxy for the spot price has been common practice in empirical investigations on commodity prices, see Schwartz (1997). Last but not least, we find it unrealistic to go too far toward the idealized peaking plant given our choice of a CCGT – a single cycle gas turbine is the preferred technology for an idealized peaking plant. A cycling CCGT plant will probably be held on or off for several days (or weeks) in a row.

A cycling plant is valued as a sum of operating options. These options are typically not traded, so an exact replicating portfolio cannot be set up. However, the traded swaps do provide information about risk adjusted values and serve as "spanning assets" in the Dixit and Pindyck (1994) terminology. This issue is discussed by Deng, Johnson and Sogomonian (2001). The valuation estimates from our model are as consistent as possible with the observed swap prices.

5.2 Plant and option values, and decision thresholds

When emission costs E are assumed to be zero, and the lifetime of the plant $\bar{T}$ is assumed infinite, the lower bound for the plant value V_L, given by Lemma 1, is 4542 *MNOK*. Correspondingly, the upper bound for the plant value V_U, given by Lemma 2, is 7539 *MNOK*. The plant value as a function of the lifetime $\bar{T}$ is illustrated in Figure 2. Figure 2 indicates how the plant value gradually stabilizes to a given level as the lifetime increases. In traditional engineering economic analyses, the lifetime of such a plant is often around 25 years, however as argued earlier, in practice power plans tend to be upgraded and refurbished, greatly extending the effective project life.

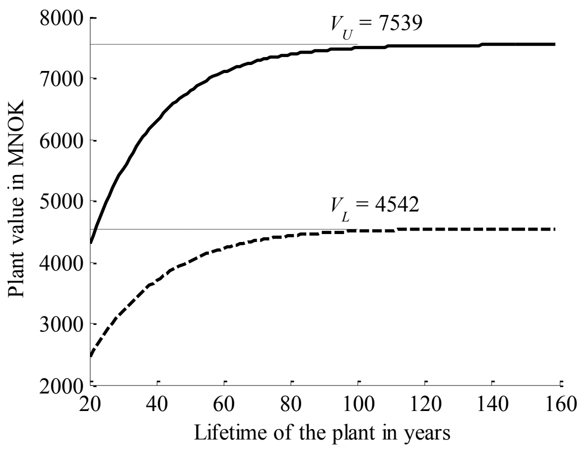

Figure 2 *Plant value as a function of the lifetime of the plant.*

We consider the investment decision next. Solving the equations in Proposition 1 gives that the building threshold ξ_I, when abandonment is not considered, is somewhere between [46.3; 165.3] *NOK/MWh*. When also the abandonment option is taken into account the building threshold ξ_I^A is in the interval [43.8; 134.3] *NOK/MWh*, and the abandonment threshold ξ_A^A is between [-362.8; -131.6] *NOK/MWh*. In the latter case the thresholds are calculated by solving the equations in Proposition 2. Note that when solving these nonlinear equations numerically, we simultaneously determine the constants in the option value functions, e.g. A_1 in (11) and D_2 in (17). If there is an option to abandon, some of the investment costs can be re-couped if the investment turns unprofitable, so the addition of an abandonment option makes earlier investment more favorable. The abandonment option also narrows the gap between the upper and lower bounds of the building threshold. The abandonment makes the flexibility in the plant less valuable because the possibility to abandon partly provides the same kind of hedge against low spark spreads as the option to shut down temporarily. This is an option interaction effect and the result is found to be robust against a change in the spark spread model toward Assumption 1' (Brownian motion).

The bounds of the plant value and investment thresholds are summarized in Table 3. In both cases the current equilibrium price ξ_0, given in Table 2, is within the building interval, so the building decision depends on the ramping policy.

Table 3: *Plant value and investment thresholds.*

Variable	$V(0,\xi_0)$	ξ_I	ξ_I^A	ξ_A^A
Unit	*MNOK*	*NOK/MWh*	*NOK/MWh*	*NOK/MWh*
Value	[4540; 7537]	[46.3; 165.3]	[43.8; 134.3]	[-362.8; -131.6]

For comparison we calculate the thresholds with a net present value method, i.e. we assume that the plant is built when the expected value of the plant is equal to investment costs and the abandonment is done when the plant value is equal to the salvage value. In this case only the options to postpone the investment decisions are ignored, and thus the uncertainty in the spark spread process still affects the investment decisions by changing the value of operating flexibility. This method gives that the investment threshold ξ_I^{NPV} is in the interval [-178.2; 8.7] *NOK/MWh* and the abandonment threshold ξ_A^{NPV} is in the interval [-271.8; -10.6] *NOK/MWh.* The options to postpone have positive value, so the building threshold increases and the abandonment threshold decreases when the options to postpone are included. The net present value calculations indicate that it is optimal to invest with the current equilibrium price, whatever the ramping policy is.

Figure 3 illustrates the option values F_0 and F_1 and the plant value V as a function of equilibrium price ξ. The black lines are the bounds of the plant value, and the gray lines are the option values. Abandonment option values are indicated by dashed lines, whereas the bounds of the building option are gray solid lines. Bounds for the investment thresholds are indicated by vertical lines; the solid vertical lines are the bounds of the building threshold, and the dashed vertical lines are bounds of the abandonment threshold. The value of the build option increases exponentially as a function of the equilibrium price until it is optimal to build the plant. The abandonment option value decreases exponentially as a function of equilibrium price. There are four pairs of value-matching and smooth-pasting conditions, and the easiest to spot is perhaps for the investment trigger for the cycling plant (upper bound), where the investment option slope and value equals the plant slope and value less investment cost. For the cycling plant the abandonment option value and slope is close to zero, but for the base load plant the abandonment option has a bearing in the contact conditions. Note also that the abandonment option value slope at the trigger point equals the negative of the slope of the plant value. The gap between the bounds of the build option is small compared to the gap between bounds of the abandonment option. This is explained as follows: The cycling plant can react to decreasing prices by ramping down the plant. Therefore, the wedge between the bounds of the plant value increases as the equilibrium price decreases. As the bounds for the option values are determined by the bounds of the plant value, the upper and lower bounds of the abandonment option diverge when equilibrium price decreases.

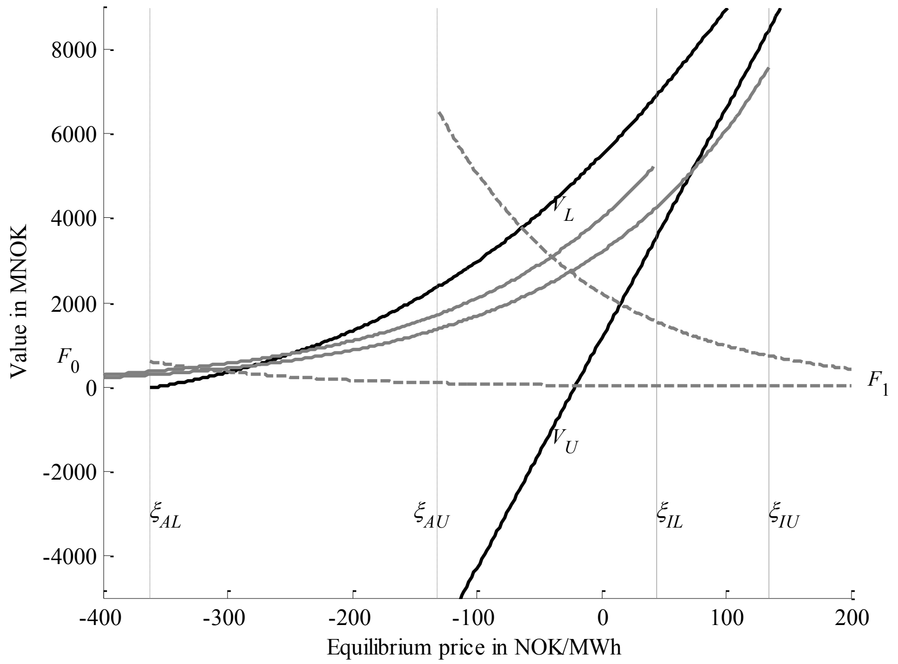

Figure 3 *Plant and option values. The solid black lines display the upper and lower bounds for a power plant that has been put online. Before the investment has taken place, one holds the option to invest, whose value is F_0 and is indicated with solid gray lines. These option values are valid for equilibrium prices below the investment threshold which is in the interval [ξ_{IL}; ξ_{IU}]. Note how e.g. the upper option bound curve becomes parallel to the V_U line at $\xi = \xi_{IL}$ (smooth pasting). Dashed gray lines indicate the value of the abandonment option, F_1, valid for equilibrium prices above the abandonment threshold [ξ_{AL}; ξ_{AU}].*

5.3 Sensitivity analyses

Next we study how the thresholds change as a function of some key parameters. In Figure 4 the thresholds are illustrated as a function of equilibrium volatility σ_ξ. The gray lines are the bounds of the building threshold and the black lines are the bounds of the abandonment threshold. An increase in the equilibrium volatility increases the building threshold, but at the same time the abandonment threshold decreases, i.e. uncertainty makes waiting more favorable. In Figure 4 the gap between the bounds of the abandonment threshold increases as function of uncertainty. An increase in the equilibrium volatility does not change the value of a baseload plant, but it increases the value of a cycling plant. When the equilibrium price is small and the market becomes more volatile, the more valuable the cycling plant is compared to the baseload plant, and the broader the gap between the bounds of the abandonment thresholds becomes. On the other hand, when the equilibrium price is high, the difference between peak and baseload plant values is not sensitive to changes in equilibrium volatility, so the gap between upper and lower bounds of the building threshold does not increase much as a function of equilibrium volatility.

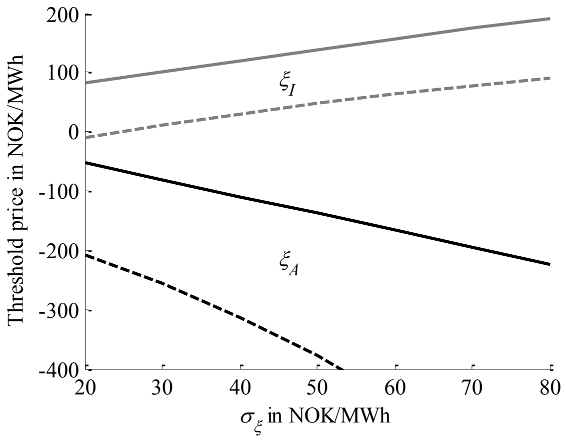

Figure 4 *Investment thresholds as a function of equilibrium volatility. The solid lines correspond to build (gray) and abandon (black) thresholds for a baseload plant, and the dashed lines correspond to build (gray) and abandon (black) thresholds for a cycling plant.*

Increasing the equilibrium volatility increases option value, as can be seen in Figure 5. One might think that the investment option value of a cycling plant increases more than the option value of a base load plant since a cycling plant value increases while a base load plant value is unchanged. However, there is an effect balancing this, namely the change in the abandonment option value. For a cycling plant the abandonment value does not change much (due to the interaction with the operating flexibility), but for a base load the abandonment option is sensitive to increases in equilibrium volatility.

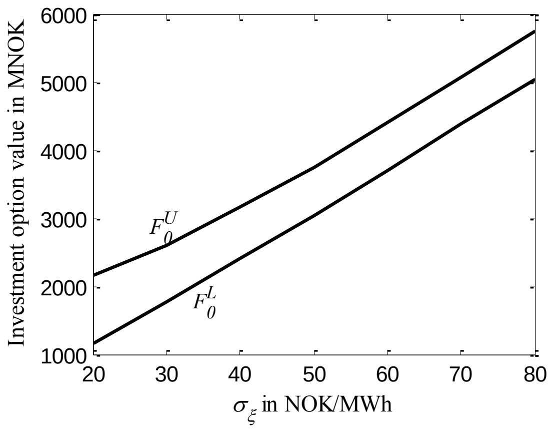

Figure 5 *Investment option value as a function of equilibrium volatility. The option value bounds are evaluated at ξ_0 = -15 NOK/MWh, for which the investment option is kept open even for the lowest investment threshold.*

Figure 6 illustrates the thresholds as a function of emission costs *E*. In Figure 6 the unit of emission cost is *NOK/MWh*, whereas it usually is quoted in *USD/tonne*. The CO_2 production of the gas-fired power plant is 363 kg/MWh_{el}. With an exchange rate of 7 *NOK/USD*, an emission cost of 10 *NOK/MWh* corresponds to 3.94 *USD/tonne*. In Figure 6 the thresholds increase linearly, with slope one, as a function of emission costs. So, if the emission costs are increased by one *NOK/MWh,* both thresholds are also increased by one *NOK/MWh*. This is a consequence of a normally distributed equilibrium price. Change in emission costs can be seen as a change in initial value of the equilibrium price. Even though we have used constant emission costs, there is uncertainty in future levels of emission costs. An easy way to model the uncertainty in the emission costs is to increase the equilibrium uncertainty. This means that not just an increase in the expected value of emission costs, but also uncertainty in emission costs postpones investment decisions, i.e. it increases the building threshold and decreases the abandonment threshold.

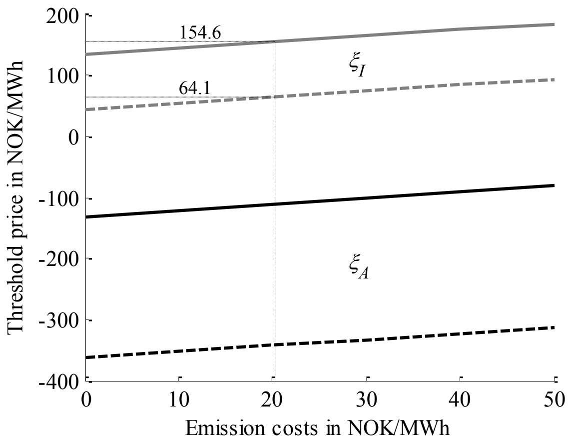

Figure 6 *Investment thresholds as a function of emission costs. The solid lines correspond to build and abandon thresholds for a baseload plant, and the dashed lines correspond to build and abandon thresholds for a cycling plant.*

5.4 The implied value of CO_2 capture technology

Undrum et al. (2000) evaluate different alternatives to capture CO_2 from gas turbine power cycles. They estimate that the costs of installing equipment to capture CO_2 from flue gas using absorption by amine solutions are 2140 *MNOK*. Given the investment costs in Table 1, the cost of a low-carbon-emitting gas power plant is 3760 *MNOK*. Figure 7 illustrates the thresholds as a function of investment costs when the salvage value is 35% of the investment costs, i.e. $J = 0.35I$. The resale value of a plant with CO_2 capture technology is 1316 *MNOK*.

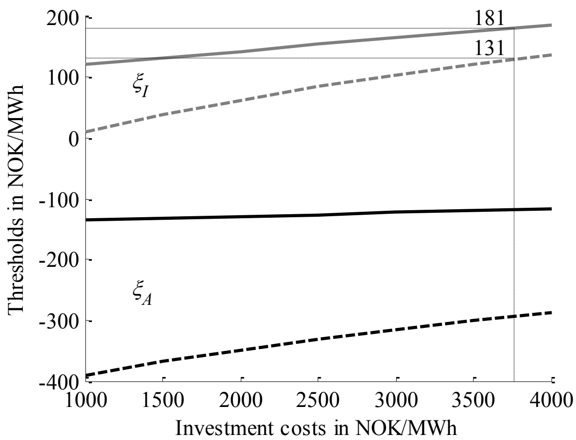

Figure 7 *Equilibrium price thresholds as a function of investment costs. The investment threshold is within the gray lines, and the abandonment threshold is within the black lines. The solid lines correspond to build and abandon thresholds for a baseload plant, and the dashed lines correspond to build and abandon thresholds for a cycling plant. When the gas fired power plant is to be built with CO2 capture technology in place, the total investment is 3760 MNOK, and the investment threshold are as high as [131;181] NOK/MWh.*

In Figure 7 it is indicated that the threshold to build a gas turbine with CO_2 capture equipment is in the interval [131.0; 181.0] *NOK/MWh*. In Table 2 the current equilibrium price is estimated at 62.3 *NOK/MWh*. Therefore, with the current costs of CO_2 capture equipment it is not optimal to invest in such equipment. To simplify the following analyses, let us assume that the building threshold is in the middle of its upper and lower bounds, i.e. at 156 *NOK/MWh*. An ordinary gas-fired power plant needs to pay emission costs, whereas a low-carbon plant does not. To find the level of emission cost that makes the energy manager indifferent between the two alternatives, we find the emission cost that is so high as to make the building threshold equal in both cases. Once the emission costs are around 65 *NOK/MWh,* the average of the upper and lower bounds of the building threshold, for a plant *without* CO_2 capture equipment, is 156 *NOK/MWh*. By assuming that all emission costs are caused by CO_2, and by ignoring the reduced efficiency of the plant when the greenhouse gas capture equipment is in place and uncertainty in CO_2 emission costs, we find that it is optimal to install the CO_2 capture equipment when emission costs are greater than 65 *NOK/MWh*, i.e. 25.6 *USD/tonne*.

Next we consider how much the investment costs need to be lowered in order to make the energy manager want to choose to install carbon capture equipment, using the current emission cost level. At the time of analysis the carbon emission market has not begun its activity, however, the estimate is that emission costs will be somewhere between 5 *USD/tonne* and 20 *USD/tonne*, where the lower range is most likely. Figure 5 indicates that when emission costs are 8 *USD/tonne,* i.e. 20.3 *NOK/MWh,* the threshold to build a plant without CO_2 capture equipment is in the interval [64.1; 154.6] *NOK/MWh*. By assuming again that the building threshold is the average of the upper and lower bounds, we see that building threshold for a gas plant without CO_2 capture equipment is 109.4 *NOK/MWh*. Considering Figure 7, the (average) building threshold for the plant with CO_2 capture equipment is lowered from 156 *NOK/MWh* to 109.4 *NOK/MWh* if the investment costs are lowered to 2215 *MNOK*. Therefore, if the costs of building a gas plant with CO_2 capture equipment are lowered by 1540 *MNOK,* it is optimal to build a gas plant with such equipment.

Since we have ignored the reduced efficiency of the power plant when CO_2 capture equipment is installed, this latter figure (1540 *MNOK*) can be seen as a lower bound for the amount of subsidies needed to entice investment in this greenhouse gas technology. Another reason for 1540 *MNOK* to be an underestimation of the subsidies needed is the fact that emission costs are uncertain, and the attachment of a CO_2 capture plant can be postponed indefinitely beyond the investment in the power plant itself. Attaching green technology is hence a real option that will not be triggered before the net present value is well above the investment cost.

6 Discussion

To make this model work as decision support, one must run the estimation process regularly to update the parameters to the current market prices and recent dynamics. This includes getting information on what is the current long-term equilibrium price to monitor whether a (dis)investment is to be triggered. Of course, the equilibrium price is almost observable via the prices of long-term swaps.

We find that the gap between upper and lower bounds of the investment thresholds is rather large. This indicates that the cycling plant value differs considerably from the baseload plant value. Our case study also indicates that the addition of an abandonment option does not dramatically change the building threshold. Therefore, as a first approximation for the investment decision it is plausible to ignore the abandonment option, but the operating flexibility should not be disregarded.

In our case study, even with zero emission costs, it is not optimal to build a baseload plant. However, it is optimal to build a rather efficient cycling plant. If the postponement option is also omitted, i.e. building is commenced when the expected value of the plant is equal to investment costs, the situation changes. In this case it is also optimal to build a baseload plant. Thus, the option to postpone has a significant effect on the building decision.

There are some issues that have been disregarded in the modeling, but should be considered when the Norwegian case is analyzed more thoroughly. First, we have used the UK market as a reference for gas. There is lot of natural gas available in the Norwegian continental shelf. Due to the physical distance from the Norwegian coastline to the UK, the gas price at a Norwegian terminal will be equal to the UK price less some transportation costs. It is estimated that this adjustment is around $0.10 NOK/Sm^3$, where one Sm^3 is equal to 9.87 *kWh*, this means that by using price quotes from IPE, we underestimate the spark spread by around 17 *NOK/MWh.* This issue is clogged by the fact that pipeline capacity is fully utilized in the winter season. Second, there is also a possible tax effect that has not been considered. Oil and gas companies operating on the Norwegian shelf have a 78% tax rate, while onshore activities are taxed at 28%. If a gas producer invests in a gas power plant, it can sell the gas at a loss with offshore taxation, and buy the same gas, now in the form of electricity, as a power plant owner with onshore taxation. Finally, we have assumed that building a power plant occurs instantly. Analyzing these issues is left for future work.

The theory developed rests on an assumption that the energy company has an exclusive license, i.e. a monopoly right to invest. One may be concerned with how competition or other forms of market failure in the electricity or gas markets affect the results. However, as long as the information in efficient market prices of derivatives contracts is incorporated in the analysis, these concerns are unfounded. Efficient swap prices will reflect any market failure. Of course, in practical cases there will be basis risk, for example due to electricity or gas being delivered or purchased at a different location or due to the quality of the gas that is underlying the forward contracts. Another problem is that long-term contracts may not be available. For a discussion of these issues, see e.g. Fama and French (1987).

We have modeled the CO2 emission cost as a constant. In Europe, where the emission trading scheme is in effect, it would have been pertinent to model this as a stochastic process. However, adding more factors to the model would erode the simplicity of the valuation and decision rule calculations. This argument is also valid for more complicated commodity price models involving e.g. oil and coal prices for explaining electricity price dynamics.

7 Conclusions

We use real options theory to analyze gas-fired power plant investments. Our valuation is based on electricity and gas forward prices. We have derived a method to compute upper and lower bounds for the plant value and investment thresholds when the spark spread follows a two-factor model, capturing both the short-term mean-reversion and long-term uncertainty.

In our case study we take the view of an investor having a license to build a gas-fired power plant. Our results indicate that the abandonment option and the operating flexibility interact so that their joint value is less than their separate values, because an option to permanently shut down overlaps with the option to temporarily shut down and vice versa. However, the case study indicates that the addition of the abandonment option does not dramatically change the bounds of the building threshold. On the other hand, the difference between the upper and lower bounds of the investment thresholds is considerable, so the operating flexibility has a significant effect on the building decision. When investments in gas-fired power plants are considered, a good overall view of the investment problem can be made by ignoring the abandonment option, whereas the operating flexibility and time-to-build option should not be disregarded.

Appendix A

A cycling plant operates only when the spark spread exceeds emission costs. The value of the plant, at time *t,* is the expected cash flows less operational costs G

$$V_U\left(\chi(t),\xi(t)\right)=\int_t^{\bar{T}} e^{-r(s-t)}\left(\bar{C}c\left(\chi(s),\xi(s)\right)-G\right)ds\,, \tag{A1}$$

where $\bar{T}$ is the lifetime of the plant, $\bar{C}$ is the capacity of the plant, and $c\left(\chi(s),\xi(s)\right)$ is the expected value of spark spread exceeding emission costs at time *s*, i.e.

$$c\left(\chi(s),\xi(s)\right)=E\left[\max\left(S(s)-E,0\right)\right]=\int_E^{\infty}\left(y-E\right)h(y)dy\,. \tag{A2}$$

In (A2) $h(y)$ is the density function of a normally distributed variable *y*, whose mean and variance are the mean and variance of the spark spread at time *s*, given in Corollary 1. A spark spread process that is different from that of Assumption 1 will lead to different statistical moments or a different distribution. For clarity we rewrite the mean and variance here

$$E_s\left[S(T)\right]=\mathrm{e}^{-\kappa(T-s)}\chi(s)+\xi(s)+\mu_\xi(T-s) \tag{A3}$$

$$Var_t\left(S(T)\right)=\frac{\sigma_\chi^2}{2\kappa}\left(1-\mathrm{e}^{-2\kappa(T-s)}\right)+\sigma_\xi^2(T-s)+2\left(1-\mathrm{e}^{-\kappa(T-s)}\right)\frac{\rho\sigma_\chi\sigma_\xi}{\kappa}\,. \tag{A4}$$

Integration gives

$$c\left(\chi(s),\xi(s)\right)=\frac{\sqrt{Var_s\left(S(s)\right)}}{\sqrt{2\pi}}e^{\left(-\frac{(E[S(s)]-E)^2}{2Var\left(S(s)\right)}\right)}+\left(E_s\left[S(s)\right]-E\right)\Phi\left(\frac{E_s\left[S(s)\right]-E}{\sqrt{Var_s\left(S(s)\right)}}\right), \tag{A5}$$

where $\Phi(\cdot)$ is the normal cumulative distribution function. Equations (A1) and (A5) give the value of the cycling plant

$$\begin{aligned}&V_U\left(\chi(t),\xi(t)\right)\\&=\bar{C}\int_t^{\bar{T}} e^{-r(s-t)}\left(\frac{\sqrt{Var_t\left(S(s)\right)}}{\sqrt{2\pi}}e^{\left(-\frac{(E[S(s)]-E)^2}{2Var\left(S(s)\right)}\right)}+\left(E_t\left[S(s)\right]-E\right)\Phi\left(\frac{E_t\left[S(s)\right]-E}{\sqrt{Var_t\left(S(s)\right)}}\right)\right)ds\\&-\frac{G}{r}\left(1-\mathrm{e}^{-r(\bar{T}-t)}\right)\end{aligned} \tag{A6}$$

Appendix B

When it is not optimal to exercise the build option, i.e. when $\xi < \xi_I$, the option to build F_0 must satisfy following Bellman equation[3]

$$rF_0(\xi)dt = E\left[dF_0(\xi)\right] - Wdt, \qquad when\, \xi < \xi_I. \tag{B1}$$

Using Itô's lemma and taking the expectation we get following differential equation for the option value

$$\frac{1}{2}\sigma^2 \frac{d^2F_0(\xi)}{d\xi^2} + \mu_\varepsilon \frac{dF_0(\xi)}{d\xi} - rF_0(\xi) - W = 0, \qquad when\, \xi < \xi_I. \tag{B2}$$

A solution to the differential equation is a linear combination of two independent solutions plus any particular solution (see Dixit and Pindyck, 1994). Thus, the value of the build option is

$$F_0(\xi) = A_1\, \mathrm{e}^{\beta_1 \xi} + A_2\, \mathrm{e}^{\beta_2 \xi} - \frac{W}{r}, \qquad when\, \xi < \xi_I, \tag{B3}$$

where A_1, A_2 are unknown non-negative parameters to be determined, and β_1 and β_2 are the roots of the fundamental quadratic equation. This fundamental quadratic is found by substituting the general solution $F(\xi) = A\mathrm{e}^{\beta\xi} - W/r$ into (B2), and is given by

$$\frac{1}{2}\sigma^2\beta^2 + \mu_\varepsilon\beta - r = 0 \tag{B4}$$

This gives

$$\beta_1 = \frac{-\mu_\xi + \sqrt{\mu_\xi^{\ 2} + 2\sigma_\xi^{\ 2} r}}{\sigma_\xi^{\ 2}} > 0 \tag{B5}$$

$$\beta_2 = \frac{-\mu_\xi - \sqrt{\mu_\xi^{\ 2} + 2\sigma_\xi^{\ 2} r}}{\sigma_\xi^{\ 2}} < 0. \tag{B6}$$

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[3] Risk neutral pricing is employed, called contingent claims analysis by Dixit and Pindyck (1994). The right hand side is the fair return from holding the option over dt, whereas the left hand side is the sum of the capital gain and (negative) dividends from holding the same option. This approach assumes that the underlying market is arbitrage-free and complete. In this context, it means assuming that there are no arbitrage opportunities among the traded energy contracts, and that these contracts span all relevant spark spread risks. Although the former is realistic, the latter clearly does not hold in an absolute sense, e.g. one cannot perfectly hedge a monthly (much less hourly) spark spread operational option that expires three years from now. However, traded swap prices hint at where the unique pricing measure (following from completeness) may lie. It becomes the role of the pricing model (2)-(4) to fill in the missing information. In essence we assume that the pricing model describes the spark spread dynamics adjusted for risk.

The build option value approaches zero as the spark spread decreases, i.e. A_2 must be equal to zero, so

$$F_0(\xi) = A_1 e^{\beta_1 \xi} - \frac{W}{r}, \qquad when\, \xi < \xi_I . \tag{B7}$$

Appendix C

It is optimal to exercise the build option when the option value becomes equal to the values gained by exercising the option (*I* is investment cost)

$$F_0(\xi_I) = V(0,\xi_I) - I + F_1(\xi_I) . \tag{C1}$$

Correspondingly, it is optimal to abandon when values gained by abandoning (the salvage value *J*) are equal to values lost

$$F_1(\xi_A) + V(0,\xi_A) = J . \tag{C2}$$

The smooth-pasting conditions must also hold when the options are exercised (for an intuitive proof see Dixit and Pindyck, 1994 and for a rigorous derivation see Samuelson, 1965)

$$\frac{dF_0(\xi_I)}{d\xi} = \frac{\partial V(0,\xi_I)}{\partial \xi} + \frac{dF_1(\xi_I)}{d\xi} \tag{C3}$$

$$\frac{dF_1(\xi_A)}{d\xi} + \frac{\partial V(0,\xi_A)}{\partial \xi} = 0 . \tag{C4}$$

The building and abandonment thresholds ξ_I and ξ_A as well as the option parameters A_1 and D_2 for all plant values V must satisfy (C1)- (C4). It remains to show that an increase in the plant value decreases the investment and abandonment thresholds. Let us denote

$$G^U\left(\xi_I, A_1, D_2\right) = F_0(\xi_I) - V(0,\xi_I) - F_1(\xi_I) + I \tag{C5}$$

$$G^L\left(\xi_A, D_2\right) = F_1(\xi_A) + V(0,\xi_A) - J , \tag{C6}$$

where A_1 and D_2 are the parameters of investment and abandonment options and ξ_I and ξ_A are the investment thresholds when the plant value is V. By denoting the partial derivatives with subscripts, the value-matching and smooth-pasting conditions for plant value V are

$$G^U\left(\xi_I, A_1, D_2\right) = 0 \tag{C7}$$

$$G^L\left(\xi_A, D_2\right) = 0 \tag{C8}$$

$$G^U_{\xi_I}\left(\xi_I, A_1, D_2\right) = 0 \tag{C9}$$

$$G^L_{\xi_A}\left(\xi_A, D_2\right) = 0 . \tag{C10}$$

When the plant value V is changed by df, differentiation gives

$$G^U_{A_1}\left(\xi_I, A_1, D_2\right) dA_1 + G^U_{D_2}\left(\xi_I, A_1, D_2\right) dD_2 + G^U_{\xi_I}\left(\xi_I, A_1, D_2\right) d\xi_I = df \tag{C11}$$

$$G^L_{D_2}\left(\xi_A, D_2\right) dD_2 + G^L_{\xi_A}\left(\xi_A, D_2\right) d\xi_A = -df\ . \tag{C12}$$

Differentiation of the smooth-pasting condition gives

$$G^U_{\xi_I\xi_I}\left(\xi_I, A_1, D_2\right) d\xi_I + G^U_{\xi_I A_1}\left(\xi_I, A_1, D_2\right) dA_1 + G^U_{\xi_I D_2}\left(\xi_I, A_1, D_2\right) dD_2 = 0 \tag{C13}$$

$$G^L_{\xi_A\xi_A}\left(\xi_A, D_2\right) d\xi_A + G^L_{\xi_A D_2}\left(\xi_A, D_2\right) dD_2 = 0\,. \tag{C14}$$

Equations (C10), (C12), and (C14) give, for the change of the abandonment threshold

$$d\xi_A = \frac{G^L_{\xi_A D_2}\left(\xi_A, D_2\right) df}{G^L_{\xi_A\xi_A}\left(\xi_A, D_2\right) G^L_{D_2}\left(\xi_A, D_2\right)} = \frac{\beta_2 df}{G^L_{\xi_A\xi_A}\left(\xi_A, D_2\right)}\,. \tag{C15}$$

The second equality is obtained by calculating the derivatives of the abandonment option given in (17). Before abandonment, in the value-matching condition, $G^L\left(\xi_A, D_2\right)$ approaches zero from above (starting e.g. from $\xi = \xi_I, G^L\left(\xi_I, D_2\right) = F_0(\xi_I) + I - J > 0$), thus $G^L\left(\xi, D_2\right)$ must be convex in ξ. When the plant value is increased by a positive amount, i.e. $df > 0$, we get

$$d\xi_A < 0\,. \tag{C16}$$

Hence when the plant value increases the abandonment threshold decreases.

Equations (C9), (C11), (C13) and (C15) give the change in the building threshold

$$d\xi_H = \frac{G^U_{\xi_I A_1}\left(\xi_I, A_1, D_2\right)\left(\dfrac{df + G^U_{D_2}\left(\xi_I, A_1, D_2\right)\dfrac{df}{G^L_{D_2}\left(\xi_A, D_2\right)}}{G^U_{A_1}\left(\xi_I, A_1, D_2\right)}\right) + df\,\dfrac{G^U_{\xi_I D_2}\left(\xi_I, A_1, D_2\right)}{G^L_{D_2}\left(\xi_A, D_2\right)}}{G^U_{\xi_I\xi_I}\left(\xi_I, D_2, A_1\right)}, \tag{C17}$$

$$= \frac{-\beta_1\left(1 + e^{\beta_2(\xi_I - \xi_A)}\right) + \beta_2 e^{\beta_2(\xi_I - \xi_A)}}{G^U_{\xi_I\xi_I}\left(\xi_I, D_2, A_1\right)} df$$

where the second equality is obtained by calculating the derivatives of the build and abandonment options given in (11) and (17). Before building, in the value-matching condition, $G^U\left(\xi_I, A_1, D_2\right)$ approaches zero from above (e.g. $G^U\left(\xi_A, A_1, D_2\right) = F_0(\xi_A) + I - J > 0$), thus $G^U\left(\xi, A_1, D_2\right)$ must be convex in ξ near the threshold. When the plant value is increased with a positive amount, i.e. $df > 0$, we get

$$d\xi_I < 0\,. \tag{C18}$$

Q.E.D.

Figure 8 illustrates, for the case reported in Section 5, the bounds for the value functions G in (C5) and (C6).

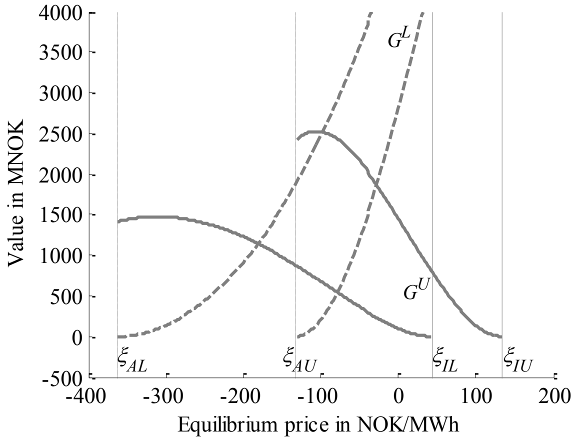

Figure 8 *Bounds for "excess" option value.* G^U *is investment option value less its underlying, i.e.* $F_0 - V - F_1 + I$, *and is convex near the investment threshold.* G^L *is abandonment option value less its underlying, i.e.* $F_1 + V - J$, *and is convex. Note all smooth pastes at an excess value of 0.*

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