A simulated annealing algorithm for routing problems with fuzzy constrains

Mohammad Hossein Fazel Zarandi[a,*], Ahmad Hemmati[b], Soheil Davari[c] and I. Burhan Turksen[d,e]

[a]*Department of Industrial Engineering, Amirkabir University of Technology, Tehran, Iran*

[b]*Department of Industrial Economics and Technology Management, Norwegian University of Science and Technology, Norway*

[c]*Sabanci University, Manufacturing Systems and Industrial Engineering, Orhanli-Tuzla, Istanbul, Turkey*

[d]*TOBB Economics and Technology University, Ankara, Turkey*

[e]*Department of Mechanical and Industrial Engineering, University of Toronto, Toronto, ON, Canada*

Abstract. This paper puts forward a location-routing problem with fuzzy demands (LRPFD). A fuzzy chance constrained programming (CCP) model is presented and a simulation-embedded simulated annealing (SA) algorithm is proposed to solve it. Moreover, an initialization heuristic is presented which is based on the well-known fuzzy *c*-means clustering algorithm. Numerical examples clearly show the effectiveness of the proposed solution procedure. In addition, the sensitivity analysis of the objective function based on the dispatcher preference index is presented.

Keywords: Location-routing problem (LRP), uncertainty, fuzzy logic, simulation, simulated annealing, credibility theory

1. Introduction

Supply chain Management (SCM) is the process of planning, implementing and controlling the operations of the supply chain in an efficient way. It spans all movements and storage of raw materials, work-in-process inventory, and finished goods from the point-of-origin to the point-of-consumption [33]. There are many decisions to be made in a supply chain varying from locating facilities to determining the stock levels. These decisions are often categorized as strategic, tactical, and operational levels. Implementing a strategic or long-term decision does not take place on a regular basis and needs major capital investments. A tactical decision making is needed more often than a strategic decision. Finally, the operational decisions such as scheduling are those decisions that take place regularly. Facility location and vehicle routing problem (VRP) are examples of strategic and tactical decisions respectively. Location-routing problem (LRP) integrates the strategic (location) and tactical (routing) levels to form an integrated, extremely important decision to be made in many supply chains. LRP may be defined as a special case of VRP in which there is a need to determine the optimal number and location of depots simultaneously with finding distribution routes. There are numerous applications for LRP such as distribution of newspapers, medical evacuation, and waste collection.

The dynamic and complex nature of supply chain imposes a high degree of uncertainty in supply chain planning decisions and significantly influences the overall performance of the supply chain network [22]. Even though uncertainty is omnipresent in SCM problems, it receives relatively little attention from the operations research community. A problem under uncertainty may be modeled using various approaches such as random variables or fuzzy variables. Although many problems can be modeled using random variables, there are some instances in which it is almost impossible or irrational to use random variables, such as when:

- there are not enough data to model the problem
- the available data is not reliable

Besides, using scenario-based approaches which are employed in stochastic approaches, the large number of scenarios used in representing the uncertainty can lead to computationally challenging problems [38]. Hence, using fuzzy logic to model some real-world problems seems more reasonable. To put it in simpler terms, fuzzy variables can represent the uncertainty inherent in some problems in a better way and with less computational need. Fuzzy LRP (FLRP) arises whenever some elements of the problem are uncertain or ambiguous. For instance, the information about demand of a customer may be imprecise to some extent and may be estimated as "around 40 units" or "10 to 30 calls per day".

Considering the literature of LRP, this paper brings several new aspects to the literature. First, a chance-constrained programming (CCP) model of LRP with fuzzy demands is proposed. Moreover, a novel hybrid procedure is presented to initialize feasible solutions. In addition, a simulation-embedded simulated annealing (SESA) is proposed and its performance is compared with procedures in literature.

The rest of the paper is organized as follows. In the next section, the literature review of location-routing problems and credibility theory is presented. Fuzzy variables and basics of credibility theory are discussed in section 3. Section 4 is dedicated to the problem definition. The proposed methodology to solve the problem is discussed in section 5. Numerical experiments and analysis are given in section 6. Finally, section 7 presents several concluding remarks and future research directions.

2. Literature review

A survey of the LRP literature shows that the research on LRP has attracted relatively less attention compared to various VRP or location variants. There are several review papers dedicated to the study of LRP such as Nagy and Salhi [35]. Interested readers may refer to [4] and references therein in order to get a comprehensive review of LRP literature before 2007. In this paper, we touch on some pertinent literature of LRP with a focus on recent publications.

Applications and numerous solution procedures varying from Lagrangean Relaxation (LR) to heuristic and metaheuristic approaches have been proposed in order to solve the LRP. Some location-routing models for realistic scenarios are reported in Ambrosino et al. [3], Alumur and Kara [2] and Caballero et al. [7]. Alumur and Kara [2] studied a multiobjective LRP for collection, transportation, treatment and disposal of hazardous materials. They presented a mixed integer programming model for such a problem and solved a real-world sample with 92 generation nodes. Cappanera et al. [8] presented an obnoxious facility location-routing (OFLR) problem in which LR was used to decompose the problem into two subproblems of location and routing and two Lagrangean heuristics were presented. Marinakis and Marinaki [32] combined Particle Swarm Optimization (PSO), Greedy Randomized Adaptive Search Procedure (GRASP), Expanding Neighborhood Search (ENS) and Path Relinking (PR) to solve LRP. Using a combination of GRASP and Evolutionary Local Search (ELS), Duhamel et al. [16] solved a capacitated LRP. Ambrosino et al. [3] considered a distribution network design problem and a two-phase heuristic is presented to solve it. Barreto et al. [5] considered integration of several hierarchical and non-hierarchical clustering procedures, in addition to several proximity measures to solve the LRP. Then, they compared the results of running their procedure on standard LRP datasets and results were analyzed. Stenger et al. [43] studied a real-world LRP of small package shippers and presented a hybrid of SA and variable neighborhood search to solve it. The planar single facility LRP was studied by Manzour-al-ajdad et al. [31] for which a hierarchical heuristic is proposed. Derbel et al. [14] combined genetic algorithm and iterated local search to solve a LRP with multiple capacitated depots and one uncapacitated vehicle per depot. The multi-period LRP with decoupled time series is another recent publication by Albareda-Sambola et al. [1] in which an approximation based on spanning trees is proposed in order to solve the problem. Another novel publication in the literature is Xu et al. [45] in which the objective is to minimize the maximum working time of the vehicles and to reach high levels of balancing in the network. The partitioning hub LRP was proposed by Catanzaro et al. [9] in which instances with up to 20 vertices were solved. Finally, Rath and Gutjahr [42] studied the warehouse LRP with three objectives and applied the Epsilon constraint method to find the Pareto frontier.

Table 1
Some problems solved in fuzzy environment using credibility theory

Author	Problem
Peng and Liu [37]	Parallel machine scheduling
Zhao and Liu [48]	Standby redundancy optimization
Zheng and Liu [49]	Vehicle routing problem
Liu and Li [27]	Quadratic assignment problem
Zhou and Liu [50]	Location-allocation problem
Erbao and Mingyong [17]	Vehicle routing problem
Lan et al. [23]	Multi-period production planning
Liu and Gao [28]	Multi-job assignment problem
Li et al. [25]	Portfolio selection
Ke and Liu [21]	Project scheduling
Lau et al. [24]	Distribution system design
Fazel Zarandi et al. [18]	Location-routing problem
Fazel Zarandi et al. [19]	Location-routing problem
Davari et al. [11]	Maximal covering location problem
Davari et al. [12]	Maximal covering location problem
Li et al. [26]	Trip distribution problem
Wang et al. [44]	Inventory control
Davari and Fazel Zarandi [13]	Hub location problem

Table 2
General information about the solution representation

	Range of values	Length
First section	$1, 2, \ldots, n$	n
Second section	Some values between 1 and n	m
Third section	Some values between 1 and d	m

In many real-world problems, it is almost impossible to describe the parameters as deterministic parameters. Some data such as customer demands, travel times, loading/unloading times, or return rates are unknown for decision makers. One may deal with such an uncertainty using stochastic or fuzzy variables based on the type of uncertainty in the problem. Fuzzy LRP (FLRP) arises whenever some elements of the problem are uncertain, ambiguous, or vague. Generally, fuzzy variables can be employed to deal with many uncertain parameters. Zheng and Liu [49], and Fazel Zarandi et al. [18, 19] surveyed routing variants with fuzzy travel times, and presented CCP models using credibility measure.

Credibility theory has been used in many problems with fuzzy parameters so far, in parallel with some metaheuristics. Table 1 gives a brief review of using credibility theory to solve various mathematical programming problems. In the following sections, the applicability of this theory to solve LRPFD will be presented.

From the above, it becomes clear that the LRP with fuzzy variables is still a relatively unexplored problem. Considering the different variants of LRP, it becomes clear that there is a large area for future research among which this paper addresses one. This paper contributes to the literature by proposal of an efficient simulation-embedded SA and a hybrid initialization algorithm for the proposed SA.

3. Credibility theory

The term "Fuzzy variable" was first used by Kaufmann [20] and then discussed in Zadeh [46] and Nahmias [36]. Later, possibility theory was proposed by Zadeh [47] and its extensions and developments were followed by Dubois and Prade (Interested readers may refer to [15]). A modification to possibility theory which is called credibility theory was founded by Liu [29] and recently have been studied by many scholars all over the world. Since a fuzzy version of LRP in credibility space will be considered in this paper, a brief introduction to basic concepts and definitions used in this paper are presented:

Definition 1. [29] Let Θ be a nonempty set, P the power set of Θ, and Cr a credibility measure. Then the triplet (Θ, P, Cr) is called a credibility space.

Definition 2. [29] A fuzzy variable is defined as a function from the credibility space (Θ, $P(\Theta)$, Cr) to the set of real numbers.

Definition 3. [29] Let ξ be a fuzzy variable on the credibility space (Θ, $P(\Theta)$, Cr). Then its membership function is derived from the credibility measure Cr:

$$\mu(x) = (2Cr\{\xi = x\}) \wedge 1, \quad x \in \Re \tag{1}$$

Definition 4. [29] Let ξ be a fuzzy variable on a possibility space (Θ,$P(\Theta)$,Pos). Then the set

$$\xi_\alpha = \{\xi(\theta)|\theta \in \Theta), \; Pos\{\theta\} \geq \alpha\} \tag{2}$$

is called the α-level set of ξ.

Definition 5. [29] Let (Θ, $P(\Theta)$, Pos) be a possibility space, and A be a set in $P(\Theta)$. Then the credibility measure of A is defined by $Cr\{A\}$ which is a self-dual measure (Possibility and necessity measures lack the self-duality property).

If the membership function of ξ is given as μ (u is an event), then the possibility, necessity, and credibility of the fuzzy event $\{\xi \geq r\}$ can be represented by:

$$\text{Pos}\{\xi \geq r\} = \sup_{u \geq r} \mu(u) \tag{3}$$

$$Nec\{\xi \geq r\} = 1 - \sup_{u<r} \mu(u) \tag{4}$$

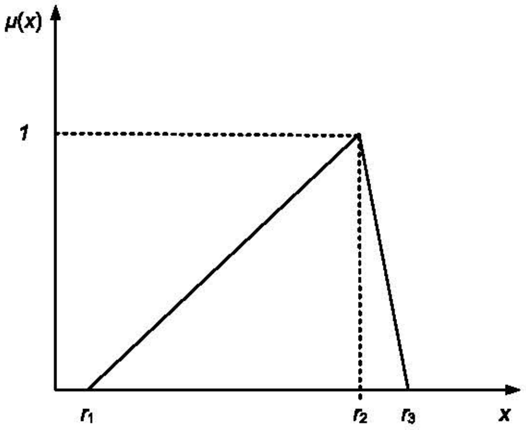

Fig. 1. A triangular fuzzy variable.

$$Cr\{\xi \geq r\} = \frac{1}{2}(Pos\{\xi \geq r\} + Nec\{\xi \geq r\}) \quad (5)$$

Considering Equation (5), the credibility of a fuzzy event is defined as the average of its possibility and necessity. The credibility measure is self-dual. A fuzzy event may fail even though its possibility achieves 1, and hold even though its necessity is 0. However, the fuzzy event must hold if its credibility is 1, and fail if its credibility is 0. Now, an example of a triangular fuzzy variable $\xi = (r_1, r_2, r_3)$ is considered as shown in Fig. 1 Then:

$$\mathrm{Pos}\{\xi \geq r\} = \begin{cases} 1 & \text{if } r \leq r_2 \\ \frac{r_3 - r}{r_3 - r_2} & \text{if } r_2 \leq r \leq r_3 \\ 0 & \text{if } r \geq r_3 \end{cases} \quad (6)$$

$$\mathrm{Nec}\{\xi \geq r\} = \begin{cases} 1 & \text{if } r \leq r_1 \\ \frac{r_2 - r}{r_2 - r_1} & \text{if } r_1 \leq r \leq r_2 \\ 0 & \text{if } r \geq r_2 \end{cases} \quad (7)$$

$$\mathrm{Cr}\{\xi \geq r\} = \begin{cases} 1 & \text{if } r \leq r_1 \\ \frac{2r_2 - r_1 - r}{2(r_2 - r_1)} & \text{if } r_1 \leq r \leq r_2 \\ \frac{r_3 - r}{2(r_3 - r_2)} & \text{if } r_2 \leq r \leq r_3 \\ 0 & \text{if } r \geq r_3 \end{cases} \quad (8)$$

4. Problem formulation

In this section, the fuzzy CCP model of LRPFD is presented with the following assumptions:

- The capacities of vehicles are identical and limited and denoted as C.
- Each vehicle is assigned to a single route.
- Each node must be served by only one vehicle.
- Each route must begin from and end at a single depot.
- Each potential depot has its distinct capacity QD_i.
- The number of depots to be located and vehicles to be used are variable. Each depot and vehicle has a fixed opening cost which are shown as ψ and η respectively.
- The unit cost of transportation is denoted as θ.
- The demands of customers are triangular fuzzy numbers which are shown as $d_i = (d_{1i}, d_{2i}, d_{3i})$.

For the last assumption, it is worth to be noted that after serving the first k customers, the available capacity of the vehicle will equal to $Q_k = C - \sum_{i=1}^{K} d_i$. Here, Q_k is also a triangular fuzzy numbers by using the rules of fuzzy arithmetic where $Q_k = (C - \sum_{i=1}^{k} d_{3i}, C - \sum_{i=1}^{k} d_{2i}, C - \sum_{i=1}^{k} d_{1i}) = (q_{1,k}, q_{2,k}, q_{3,k})$. The credibility that the next customer demand does not exceed the remaining capacity of the vehicle is obtained by:

$$\begin{aligned} Cr &= Cr\{d_{k+1} \leq Q_k\} = Cr\{(d_{1,k+1} - q_{3,k}, d_{2,k+1} - q_{2,k}, \\ &\quad d_{3,k+1} - q_{1,k}) \leq 0\} \\ &= \begin{cases} 0 & \text{if } d_{1,k+1} \geq q_{3,k} \\ \frac{q_{3,k} - d_{1,k+1}}{2\times(q_{3,k} - d_{1,k+1} + d_{2,k+1} - q_{2,k})} & \text{if } d_{1,k+1} \leq q_{3,k},\ d_{2,k+1} \geq q_{2,k} \\ \frac{d_{3,k+1} - q_{1,k} - 2\times(d_{2,k+1} - q_{2,k})}{2\times(q_{2,k} - d_{2,k+1} + d_{3,k+1} - q_{1,k})} & \text{if } d_{2,k+1} \leq q_{2,k},\ d_{3,k+1} \geq q_{1,k} \\ 1 & \text{if } d_{3,k+1} \leq q_{1,k} \end{cases} \end{aligned} \quad (9)$$

Obviously, the chance of being able to serve the next customer is higher, when the vehicle's remaining capacity is more and the demand of the next customer is less. We introduce $Cr \in [0, 1]$ as the preference index, which denotes the preference to send the vehicle to the next customer. When $Cr=0$, it is known that the vehicle is not able to serve the next customer and it must return to the depot. When $Cr=1$, the vehicle is definitely able to serve the next customer.

The decision to be made in each step is whether to send a vehicle to serve the next customer or return to the depot and dispatch another. The decision is made using the dispatcher preference index which is denoted as $Cr^* \in [0, 1]$. If $Cr \geq Cr^*$, the vehicle should be sent to the next customer. Otherwise, the vehicle returns to its depot. Clearly, if the decision maker opts to take a risk, they prefer a lower value for Cr^* and the capacity of the vehicle is used more efficiently. This policy may lead to an increase in the number of cases when a vehicle arrives at a customer without enough capacity to serve it. On the contrary, while a higher value for Cr^* leads to a less efficient use of vehicle capacity, it mitigates the number of cases without enough capacity to serve customers. Hence, the selection of Cr^* is of utmost importance in reaching a solution.

In this paper, we consider an additional cost to calculate the "failure" of the planned route. This idea derives from Erbao and Mingyong [17] which considers the additional distance that the vehicle makes due to "failure" arising in some customers along the route when evaluating the planned route. The value of Cr^* which is subjectively determined has a great impact on both the total length of the planned routes and on the additional distance covered by vehicles due to "failures" at some customers. The optimal value for Cr^* which is denoted as CrV^* leads to the least total routing costs. Moreover, the parameter CrD^* determines the confidence level of visiting all the customers within the depot capacities. In order to determine the additional distances due to route failures, a simulation approach is employed. Now, the notations, parameters, and variables of the proposed model are presented.

Indices

$i=1, 2, \ldots, d$: Depots

$i=d+1, d+2, \ldots, d+n$: Customers

$k=1, 2, \ldots, m$: Vehicles

Parameters

d_i: The fuzzy demand of customer i

C: The physical capacity of vehicles

QD_i: The physical capacity of depot i; $1 \leq i \leq d$

D_{ij}: The travel distance from i to j; $i, j=1, 2, \ldots, d+n$

Variables

Each operational plan can be encoded using three decision vectors x, y and z, where

$X=(x_1, x_2, \ldots, x_n)$ is an integer vector of decision variables representing n customers as a rearrangement of $\{1, \ldots, n\}$ $1 \leq x_i \leq n$; $x_i \neq x_j (i \neq j)$; $i, j=1, 2, \ldots, n$

$Y=(y_1, y_2, \ldots, y_m)$ is an integer vector of decision variables where $y_0 \equiv 0 \leq y_1 \leq y_2 \leq \ldots \leq y_{m-1} \leq n \equiv y_m$

$Z=(z_1, z_2, \ldots, z_m)$ is a vector of integer decision variables concerning depots $1 \leq z_k \leq d$; $k=1, 2, \ldots, m$

$$t_{ki} = \begin{cases} 1, & \text{if vehicle } k \text{ is assigned to depot } i \\ 0, & \text{otherwise} \end{cases}$$

$$U_i = \begin{cases} 1, & \text{If depot } i \text{ is used} \\ 0, & \text{otherwise} \end{cases}$$

Objective function

Let $g(x, y, z)$ be the total travel cost of vehicles. Then, we have:

$$g(x, y, z) = (m \times \eta) + \left(\theta \times \sum_{k=1}^{m} g_k(x, y, z)\right) + \left(\sum_{i=1}^{d} (U_i \times \psi_i)\right) \quad (10)$$

$$g_k(x, y, z) = D_{z_k, d+x_{(y_{k-1}+1)}} + \sum_{j=y_{k-1}+1}^{y_k-1} D_{d+x_j, d+x_{j+1}} + D_{d+x_{(y_k)}, z_k} \quad (11)$$

Constraints

$$Cr\left(\sum_{j=y_{k-1}+1}^{y_k} d_{x_j} \leq C\right) \geq CrV^*, k = 1, 2, \ldots, m \quad (12)$$

$$Cr\left(\sum_{k=1}^{m} \sum_{j=y_{k-1}+1}^{y_k} d_{x_j}.t_{ki} \leq QD_i\right) \geq CrD^*, i = 1, 2, \ldots, d \quad (13)$$

$$\sum_{i=1}^{d} t_{ki} = 1 \text{ k} = 1, \ldots, \text{m} \quad (14)$$

$$t_{ki} \leq U_i \; i = 1, 2, \ldots, d; k = 1, 2, \ldots, m \quad (15)$$

The chance constraint (12) assures that all customers are visited within the vehicle capacity with a pre-determined confidence level. The chance constraint (13) assures that all routes are visited within their depot capacity with another pre-determined confidence level. Equation (14) holds that each vehicle is assigned to one and only one depot. Constraint (15) states that a vehicle must be assigned to a depot, if and only if the depot is opened. To minimize the total travel cost of vehicles, one should solve the following model:

$$\begin{cases} \min g(x,\ y,\ z) \\ \min c' \\ \text{subject to} \\ Cr\left(\sum_{j=y_{k-1}+1}^{y_k} d_{x_j} \le C\right) \ge CrV^*, k = 1, 2, \ldots, m, \\ Cr\left(\sum_{k=1}^{m}\sum_{j=y_{k-1}+1}^{y_k} d_{x_j}.t_{ki} \le QD_i\right) \\ \quad \ge CrD^*, i = 1, 2, \ldots, d, \\ 1 \le x_i \le n, \mathrm{i} = 1, 2, \ldots, n, \\ x_i \ne x_j, i \ne j, i,\ j = 1, 2, \ldots, n, \\ 0 = y_0 < y_1 < y_2 < \ldots < y_m = n, \\ 1 \le z_k \le d, \mathrm{k} = 1, 2, \ldots, m, \\ \sum_{i=1}^{d} t_{ki} = 1, k = 1, 2, \ldots, m, \\ t_{ki} \le U_i, \mathrm{i} = 1, 2, \ldots, \mathrm{d}, k = 1, 2, \ldots, m, \\ x_i,\ y_j,\ z_k, \mathrm{i} = 1, 2, \ldots, n, j, \\ \quad k = 1, 2, \ldots, m, \text{integer}, \\ t_{ki},\ U_i = \{0,\ 1\}. \end{cases}$$

While the objective function $g(x,y,z)$ seeks to minimize the total planned travel distance, the objective function c' minimizes the total additional travel distance due to route failures. The value of c' is determined using a simulation algorithm which will be elaborated in the next section.

LRP is easily reducible to VRP considering one located depot. Therefore, knowing that VRP is a NP-Hard problem, LRP is proven to be more combinatorial and is NP-Hard. This means that for larger instances of LRP, exact solution procedures are handicapped to solve the problem efficiently. Hence, one should resort to heuristics and metaheuristics to solve the problem. In this paper, an SA is proposed to solve the crisp version of the problem. Then, a fuzzy simulation procedure is embedded within the proposed SA in order to form a SESA. This combined procedure is employed to solve the fuzzy version of the problem.

5. Solution algorithm

The problem in this paper is not of deterministic nature, due to the fuzziness of demands. Therefore, a fuzzy simulation algorithm is embedded within the proposed SA to form a SESA.

SA is a local search procedure which is capable of exploring the solution space stochastically and effectively trying to escape from being trapped into local minima. To escape local minima, SA accepts worse solutions during its search with a probability which is monotonically decreasing by temperature. SA was first introduced by Metropolis et al. [34] in 1953 and has been applied to various combinatorial optimization problems as well as real world problems, such as vehicle routing, scheduling, and facility location variants. In the following sections, the proposed SA of our paper will be discussed in detail, including solution representation, neighborhood generation, tuning SA parameters, and simulation algorithm.

5.1. *Fuzzy simulation*

In the real world, the actual demand of a customer is known when the vehicle reaches the customer. Hence, there is a need to use some simulation algorithms for determination of additional distances (c') which are caused by route failures. The proposed simulation algorithm is as follows:

Repeat steps 1 and 2 for M times.

Step 1. For each customer, estimate the additional distances by simulating actual demands. To generate the actual demands, first a real number x is generated randomly in the interval between the left and right boundaries of the triangular fuzzy number representing demand of the customer and its membership u is computed. Then, a random number $\omega \in [0, 1]$ is generated and its value is compared with u. If ω is less than u, then the actual demand of the customer is adopted as being equal to x. On the contrary, if the value of ω is greater than or equal to u, x and ω are generated again and again until the relation $\omega \le u$ is satisfied. These steps are followed to find the values of demands for each customer.

Step 2. Find the additional costs of each customer by moving along the designed route.

Step 3. Compute the average value of additional distances to find the additional distances.

5.2. Solution representation and initial solution generation

In this paper, a solution is represented using a string of numbers. A solution representation must determine the assigned customers to each vehicle, the depots to be established and the sequence of customers to be served by a specific vehicle starting and ending at a depot. Considering n customers, m vehicles and d candidate locations for depots, our proposed solution representation is comprised of $n + 2\,m$ elements which incorporates three sections. While the first two sections of the solution representation (first $n + m$ elements) should be used together to decode the solution, the final m elements representing the depots to be established should be decoded separately. The first n elements show the sequence of customers to be served by vehicles. The second section (m elements) determines the customer indices to be served by a vehicle and the third section (m elements) shows the vehicles to start from each of the established depots.

To clarify the encoding, a simple example of eight customers, four vehicles and three candidate locations is presented. The representation and its counterpart are shown in Fig. 2. The first section of the string shows that customers should be served according to the order [1–8]. The second section determines which customers are served by each vehicle. Since there are four vehicles in this example, the second section is comprised of four elements. In the proposed representation, the customers with indices lying between the values of ith and $(i + 1)$th elements in the second section are served by a single vehicle. Clearly, the last element of the second section must be equal to n, considering n customers. In addition, the values in the second section must be ordered from smallest to the largest. In our sample solution representation, the customers with indices 1 and 2 (first and seventh customers) are served by a single facility, the customer with index 3 is served by a different vehicle, demand nodes with indices 4, 5, and 6 (second, third, and fourth customers) in the first section are served separately, and finally demands of customers

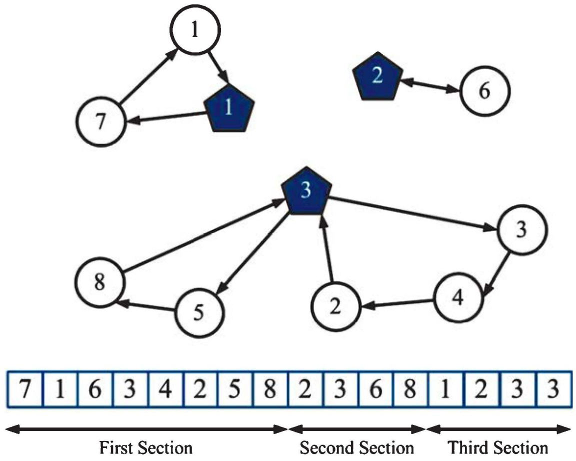

Fig. 2. A sample solution representation of eight customers and three available depots.

with indices 7 and 8 in the first section (fifth and eighth customers) are served by the fourth vehicle. Moreover, the third section shows that vehicle 1 starts from the first depot, the second vehicle starts from the second depot, and the last two vehicles start serving customers from the third depot. It is easy to validate that the proposed representation is effective, short and easily decodable.

Initialization of solutions plays a pivotal role in reaching good solutions using local search procedures. Here, a procedure is introduced to generate feasible initial solutions (fulfilling the relation $Cr \geq Cr^*$) before allocating next customer to the current route. To do so, a fuzzy c-means (FCM) algorithm is employed to cluster the customers. In addition, the sweep procedure is used to generate the customer arrangements.

5.3. Fuzzy c-means

Fuzzy clustering has been a valuable tool in various fields such as data mining, medicine, etc. Contrary to hard clustering algorithms, in a soft clustering algorithm such as FCM, gradual membership functions of data points to clusters are possible. FCM was first proposed by Bezdek [6] and has been used in many applications around the world. In FCM, there are n data to be allocated to c clusters; m is a number greater than 1 (often equals 2), x_i is the ith data, c_j is the center of the jth cluster, and $\|*\|$ is a norm representing the similarity of two vectors. The main virtue of FCM variants is the softness incorporated in assigning degrees of memberships.

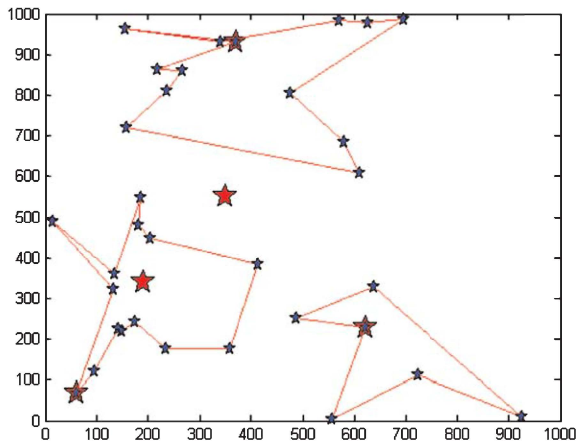

Fig. 3. A sample of using the sweep procedure for clustered data.

5.4. *The proposed sweep algorithm*

In this procedure, the result of previous clustering of the customers is used. For this purpose, in each cluster we follow these steps:

- Set the cluster center as the core of sweeping.
- Set the sweep line in zero degree.
- For each customer in the current cluster, calculate the angle between the zero line (the line from cluster center through the zero degree) and a line from customer to cluster center.
- Sort the customers by the angles in an ascending order.
- Sweep the customer with the sweep line from the low angles to high.

A sample output of the sweep algorithm is shown in Fig. 3.

Then, all routes are checked to find out whether the chance constraints are satisfied. To do so, the first customer of sequence is selected according to the customer demand and the remaining capacity of the vehicle. Then, the estimated credibility of vehicle capacity Cr_1 and the estimated depot capacity Cr_2 are found. For dispatcher preference indices CrV^* and CrD^*, if $Cr_1 \geq CrV^*$ and $Cr_2 \geq CrV^*$, the customer is assigned to the current vehicle and depot. Otherwise, another vehicle (but the same depot) is used to serve this customer. Finally, if $Cr_2 < CrD^*$, another vehicle and depot are used to serve the customer. Then, this customer is removed from the sequence and the process is followed to obtain a feasible solution.

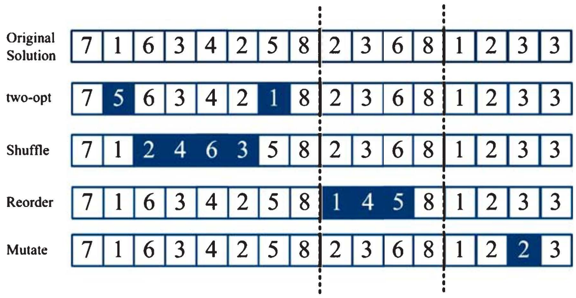

Fig. 4. The four moves used in this paper.

5.5. *Neighborhood search structure*

A neighborhood search structure (NSS) is a mechanism to obtain new solutions by slightly changing the current solution. An efficient set of NSS types should guarantee the accessibility of all the solutions and lead to a balance in diversification and intensification. In this paper, four NSS types are used which are called as two-opt, shuffle (used for first section), reorder (used for the second section), and mutate (used for the third section). A sample for each of these four moves is shown in Fig. 4. Generally, in an r-opt move, the values of r randomly selected elements are substituted. A solution is r-optimal when it cannot be improved by any r-opt move and is shown as r-opt*. In this paper, a two-opt move follows the same rule. In a shuffle move, two random indices are selected and the values between these two are shuffled randomly to get a new solution. Such a move has a stochastic character and is used in order to diversify the solutions. Moreover, whenever a solution is reordered, the second part is modified considering the rule that the last element of this section must be equal to the number of customers. In other words, while the last element does not change at all, the other sections are modified. Finally, to change the allocation of vehicles to depots, a mutation is used. To mutate, the value of one element is changed to get a new allocation plan of vehicles to depots. While mutation does not change the number of vehicles in the solution, the solution space is searched for a better utilization of vehicles. It is worth to note that in each iteration of the algorithm, one of these moves is employed based on a Monte-Carlo approach. Figure 4 shows the four moves which are mentioned above.

5.6. *Initial temperature and cooling schedule*

Determination of initial temperature and type of cooling schedule are of utmost importance in design of a SA algorithm. There exist several types of cooling schedule such as linear or nonlinear procedures. Three of these procedures are linear, exponential, and hyperbolic cooling schedules which are stated below. Further details can be obtained in Lundy & Mees [30]:

- Linear cooling rate:

$$T_l = T_0 - l\frac{T_0 - T_f}{N};\ l = 1, 2, \ldots, N$$

- Exponential cooling rate

$$T_l = \frac{A}{l+1} + B;\ A = \frac{(T_0 - T_f)(N+1)}{N};$$
$$B = T_0 - A;\ l = 1, 2, \ldots, N$$

- Hyperbolic

$$T_l = \frac{1}{2}(T_0 - T_f)\left(1 - \text{tgh}\left(\frac{101}{N} - 5\right)\right)$$
$$+T_f;\ l = 1, 2, \ldots, N$$

where T_0, T_f, and T_l represent initial temperature, stopping temperature, and temperature of iteration l, respectively. Moreover, N is the number of temperatures between T_0 and T_f, and tgh is the tangent hyperbolic function

To set the initial temperature, we have used the procedure of Crama and Schyns [10]. The aim of this procedures is to get roughly equal probabilities of acceptance ($\chi_0 = 0.8$ in this paper) during the first L steps of SA. Therefore, in a preliminary phase, SA is run for L steps without rejecting any move at all. Then, the average deterioration of the objective function over this period is calculated and noted as Δ. So, the initial temperature is set equal to:

$$T_0 = \frac{\Delta}{\ln\ \chi_0} \quad (16)$$

6. Numerical experiment and discussions

This section is devoted to the computational experiments. Here, we present an example to show models that we have discussed before and how SESA works. It should be stated that all the experiments were coded and run on a 2.53 GHz laptop with 4 GB of RAM. To validate our solution approach, we first show its performance on five crisp problems in [51]. To do some tuning of the proposed SA, we first used various combinations of move ratios. To this end, the chance of shuffle was set to 5% and then excluding the infeasible cases, the

Table 3
The ratio of moves used in the proposed model

Two-opt	Shuffle	Reorder	Mutate
0.15	0.05	0.3	0.50

Table 4
Comparing some solution procedures using the 20-5-1 dataset [51]

GRASP*	MAPM**						
Cost	*Cd****	*Cr*****	Gap	Cost	*Cd*	*Cr*	Gap
55021	25549	29472	0.42	54793	25549	29244	0
LRGTS*****	Our algorithm						
Cost	*Cd*	*Cr*	Gap	Cost	*Cd*	*Cr*	Gap
55131	25549	29582	0.62	54793	25549	29244	0

*"Greedy adaptive search procedure" [40]; **"Memetic algorithm with population management" [40]; ***The cost corresponding to the setup of the depots; ****The cost corresponding to the routing; *****"Lagrangean Relaxation-granular Tabu Search" [41].

value of two-opt and reorder moves were changed from zero to one with steps of 0.05. Among all the 210 possible combinations, the one with the best fitness was selected to solve test problems. Table 3 represents the results of this step and the ratio of moves to be used. Results of this step clearly show that changing the location of depots should be regarded as the most significant factor to reach better solutions. Furthermore, the next effective change of a solution is changing the allocation of customers to vehicles.

Table 4 shows the performance of the proposed SA for a test instance of [51] compared with three other procedures. Results show that the proposed procedure is able to reach optimal solutions.

Afterwards, instances from Prins and Prodhon [39] and Barreto [4] were solved as reported in Table 5. These instances were solved using a random initialization approach and also the proposed FCM-based initialization. Each sample problem was solved for ten times and the best, average, and worst solutions are reported. Table 5 shows that the proposed solution approach is able to solve problems with negligible errors. Moreover, results verify that the FCM-based initialization approach contributes to getting better solutions in all the cases solved.

From above, it becomes clear that the proposed SA is able to solve instances to optimality or with negligible errors. Hence, we embed the already presented fuzzy simulation in the proposed SA to solve fuzzy instances. In order to solve the fuzzy version, we generated some test problems as follows. We assumed that $N = 30$ and $D = 5$. In other words, there are thirty customers and five depots. Besides, the coordinates of all the customers and

Table 5
Comparing results of the proposed approach and the exact solutions of two well-known datasets

Problem name		Initialization				Gap
		Random		Heuristic		
		Min	Max	Min	Max	
Prins et al. [39]	20-5-1a	55835	60159	5493	57396	0.0
	20-5-1b	41478	48858	39253	43439	0.38%
	20-5-2a	49199	53272	48908	50362	0.00%
	20-5-2b	37936	43095	37542	41096	0.00%
	50-5-1	99524	115317	94084	100908	4.40%
Barreto [4]	21 × 5	443.7	586.9	430.5	500.4	1.33%
	22 × 5	599.6	801.2	586.7	636.7	0.27%
	32 × 5	623.8	758.3	589.4	684.4	3.10%
	32 × 5b	576.9	674.6	510.6	629.1	1.26%

Table 6
The parameters of model moves used in this paper

N	*D*	*C*	*M*	*CrD*	Vehicle cost
30	5	10	100	1	1000

Table 7
The average results with different CrV*

*CrV**	Planned distance	Additional distance	Total
0	23200.1	26996.7	50196.8
0.1	36177.1	13807	49984.1
0.2	37157.3	13301.9	50459.2
0.3	40708.2	8662.6	49370.8
0.4	**45738.3**	**3413.5**	**49151.8**
0.5	46852.2	3670.7	50522.9
0.6	50441	608.1	51049.1
0.7	56244.4	0	56244.4
0.8	59693.4	0	59693.4
0.9	59693.4	0	59693.4
1	59693.4	0	59693.4

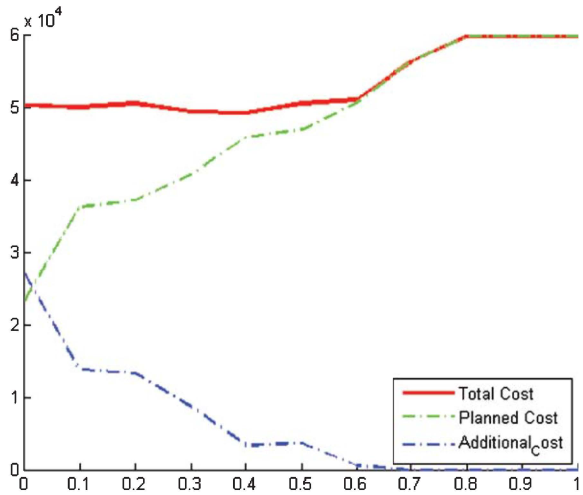

Fig. 5. The costs of the problem with different *CrV**.

depots are generated randomly in [1000×1000]. Owing to the strategic nature of depots, we assume that *CrD** is equal to 1. In other words, the depot has sufficient capacity to serve a route. The fuzzy demands of customers that are triangular fuzzy numbers which are generated randomly within vehicle capacity *C*. We obtain the additional distances due to routes failure by the simulation algorithm which has been explained before to obtain the planned distances and total distances by the SA algorithm. The remaining parameters of the problem are listed in Table 6.

The value of dispatcher preference index for vehicles *CrV** varied with the interval of 0 to 1 with a step of 0.1. The average computational results of ten runs are given in Table 7. Figure 5 shows the tendencies about the planned distances, additional distances due to failures, and the total distances of the problem with different dispatcher preference indices for vehicles.

Figure 5 shows that when dispatcher preference index for vehicles *CrV** is increased, a strictly upward trend in the planned routes and a strictly downward trend are observed in the additional distances. Moreover, when the dispatcher preference index *CrV** is equal to 0.4, the value of total distance is the lowest. Besides, lower values of *CrV** correspond to routes with shorter planned distances. On the other hand, lower values of *CrV** increase the number of cases in which vehicles arrive at a customer and are unable to serve that customer, thereby increasing the total additional distance they cover due to the “failure”. In addition, higher values of *CrV** are characterized by less utilization of vehicle capacity along the planned routes and less additional distance to cover due to failures. Figure 6 shows the output of the proposed SESA for a sample problem and its output.

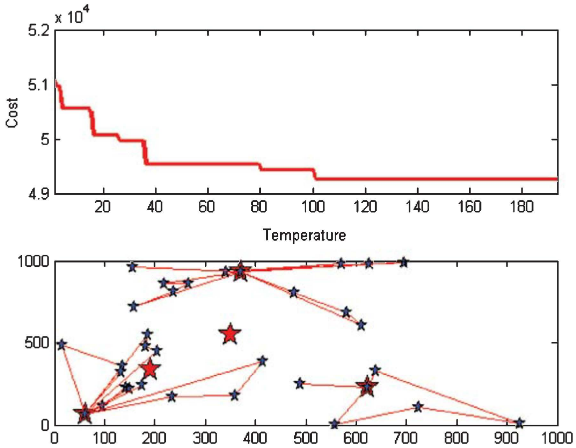

Fig. 6. The performance of the proposed SESA and final routes for a LRPFD.

7. Conclusions and future research

In this paper, an LRP is presented assuming fuzzy demands and a fuzzy chance-constrained programming formulation is given for it. To solve the problem, a SESA algorithm is proposed and its effectiveness is shown by a numerical example. The paper contributes to the knowledge pool of LRP in the following aspects:

- Proposing a CCP model of LRP
- Using FCM and the sweep procedure to initialize feasible solution for proposed algorithm which satisfies the chance constraints.
- Determining the best value of dispatcher preference index for vehicles considering the preference index for depots.

It should be noted that the fine tuning of both SA and the fuzzy simulation are critical in success of the algorithms proposed. Our experiments clearly showed that the results are highly sensitive to the tuning of algorithms, especially the cooling schedule in the SA phase.

The paper has several potential future works. It is possible to solve some other variants of the problem considering some assumptions such as LRPs with backhauls or even LRP with pickup/delivery. Also some other parameters may be considered as fuzzy variables such as travel times. Moreover, replacing SA with some other solution algorithms such as tabu search seems to be a good research area. Moreover, the model may be enriched assuming heterogeneous vehicles. Finally, there is the possibility of using some other hybrid systems to solve the problem or finding alternative procedures to simulate results.

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