# LEADER-FOLLOWER DYNAMIC SYNCHRONIZATION OF SURFACE VESSELS

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Abstract: This paper introduces a dynamic leader-follower synchronization scheme for surface vessels that specifies the behavior of the follower vessels as they approach their desired position in a formation, or in a docking operation. The synchronization reference dynamics are specified in terms of a reference filter, and the synchronization closed-loop errors are shown to be uniformly globally exponentially stable with respect to the dynamic synchronization reference.

Keywords: Synchronization, Nonlinear control, Ship control

## 1. INTRODUCTION

Leader-follower synchronization control of vessels in a formation, as for instance an underway replenishment operation, requires that the followers maintain a fixed static position relative to the leader vessel. However, in the phase when the follower vessels are approaching or changing their desired position in the formation, the behavior of the follower vessel is dynamic relative to the leader. This is also the case in docking operations when the target is moving, which can be considered a special case of leader-follower synchronization control. The behavior during this approach phase is usually specified through the tuning of static control gains, and in order to minimize overshoot or uncontrolled motions which could possibly lead to collisions, the control gains are often chosen conservatively low. This severely limits the performance of the control scheme at the same time as it does not guarantee a safe approaching behavior for different initial conditions in the nonlinear control schemes for marine formations.

Synchronization the theory of time conformity between systems, and is found as a natural phenomenon in nature as reviewed in Camazine *et al.* (2001) as well as the controlled synchronization of artificial systems as first reported by Huygens (1673) for a pair of pendulum clocks, and later revisited by Blekhman (1971) in works on vibromechanics. It can be seen as a type of state cooperation among two or more (sub)systems, and has received increasing attention in the control community (Fradkov *et al.*, 2000; Nijmeijer and Rodriguez-Angeles, 2003). Based on the results of Nijmeijer and Rodriguez-Angeles (2003) for synchronization of mechanical systems, Kyrkjebø and Pettersen (2003) proposed a leader-follower synchronization observer-controller scheme for formation control of ships in a underway replenishment operation. Related work on the coordination of marine vessels in formations can be found in Encarnacao and Pascoal (2001) and Skjetne *et al.* (2003).

Dynamical synchronization is defined in Efimov (2005) when the synchronization error obeys oscillatory differential equations, and is used in an adaptive scheme to synchronize two Lurie systems in oscillatory motion. In this paper, we utilize the idea of Efimov (2005) in the concept of *dynamic synchronization* where the

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synchronization error satisfies *some* differential equation, not necessarily oscillatory.

In Kyrkjebø and Pettersen (2005), the synchronization scheme of Kyrkjebø and Pettersen (2003) was extended using the sliding surface technique motivated by Slotine and Li (1987). The sliding surface technique is also utilized in the concept of dynamic surface control (DSC) in Swaroop *et al.* (2000) and Girard and Hedrick (2001) to avoid the explosion of terms associated with integrator backstepping techniques and the model differentiation required in the multiple sliding surface control approach. Both Kyrkjebø and Pettersen (2005) and Swaroop *et al.* (2000) address the regulation and tracking problem in a formation, but no special care is taken to specify the transient behavior of the vessels when they are approaching or changing their position in a formation, or when docking to a moving vessel. Girard and Hedrick (2001) address the transition between manoeuvres from a communication protocol view, but does not specify any dynamic behavior that guarantees the followers a stable approach to the leader.

Reference models have been used extensively throughout the literature to filter step inputs during course-changing manoeuvres in marine guidance systems, and its applications for marine crafts can be found in Fossen (2002). However, the reference model approach is not readily applicable to systems where the reference input is dynamic rather than a step input.

This paper introduces a dynamic synchronization scheme in order to specify the behavior of the follower vessels during the transient phase of approaching, or changing position, in a formation. The synchronization controller is based on the controller of Kyrkjebø and Pettersen (2005) utilizing a sliding surface to synchronize the followers to a leader vessel. Furthermore, to specify the dynamic synchronization behavior when approaching or changing position in a formation, a smooth reference model based on a first-order filter in cascade with a stable second-order mass-spring-damper system is used to filter the synchronization *error* of the closed-loop system. This imposes a controlled *dynamic synchronization* behavior of the follower relative to the leader in the approach phase. The dynamic synchronization control scheme is proven to be uniformly globally exponentially stabilized to the dynamic synchronization reference.

We will first present some preliminaries on models, coordinate systems and vessels in Sect. 2, and the dynamic synchronization approach in Sect. 3. Some simulations results are presented in Sec. 4, while conclusions and future work are commented in Sect. 5.

## 2. PRELIMINARIES

In this paper, we consider leader-follower synchronization control for fully actuated systems described

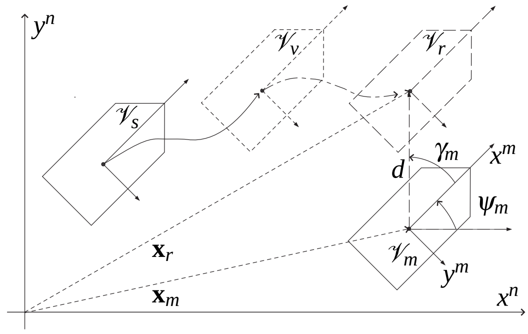


Fig. 1. Vehicles and coordinate frames

by a 3 degrees-of-freedom (3DOF) model. The systems considered are based upon the dynamic and kinematic model of a marine surface vessel, but the scheme can be applied to other *n*DOF mechanical systems (Kyrkjebø and Pettersen, 2005) with small modifications. First, we will introduce the reference frames and vehicles used in the text with their dynamic and kinematic model properties.

### 2.1 Vehicles definitions and reference frames

In the development of the output synchronization control scheme a number of reference frames, intermediate vehicles and dynamic and kinematic models will be used. A brief introduction to these concepts is given in this section, but for a more elaborate discussion see (Kyrkjebø *et al.*, 2006).

The synchronization control problem studied is as follows: Given the states of a leader vessel, we want the follower vessel to synchronize to the leader with its position shifted by a distance $d$ at an angle $\gamma_m$ relative to the leader. Furthermore, the transient behavior when approaching or changing position in the formation should be smooth and stable.

For this purpose we will utilize the concepts of a reference vehicle and a virtual vehicle, and we designate the following vehicles as illustrated in Fig. 1.

- $\mathcal{V}_m$ The leader vessel with position $\mathbf{x}_m$.
- $\mathcal{V}_r$ A reference vehicle shifted a distance $d$ in the direction given by $\gamma_m$ relative to the leader.
- $\mathcal{V}_v$ A virtual vehicle controlled to the reference vehicle $\mathcal{V}_r$ in a dynamic sychronization scheme.
- $\mathcal{V}_s$ The follower vessel synchronizing to the leader. The position $\mathbf{x}$ and velocity $\dot{\mathbf{x}}$ are considered measured, and the parameters of the dynamic model are known.

Note that the physical vehicles in the control scheme are the leader $\mathcal{V}_m$ and the follower $\mathcal{V}_s$ synchronizing to the leader. The reference vehicle is a mathematical reference constructed by shifting the position of the leader to the desired position of the follower in relation to the leader, and the virtual vehicle is a virtual reference vehicle that will be controlled to this shifted position. Note also that although we derive the control scheme for one follower, it can be easily extended to any number of followers providing the introduction of a collision avoidance scheme. A related issue on the extension of the scheme if defining followers to be leaders of others is to address the problem of string stability as in Swaroop and Hedrick (1996).

Kinematics and dynamics can be expressed in different reference frames, and the two essential reference frames used in this text are the earth-fixed North-East-Down (*NED*) reference frame defined relative to the Earth's reference ellipsoid, and the body-fixed *BODY* frame with origin in the centre of gravity of the vehicle and axes chosen as depicted in Fig. 1.

In the 3DOF case considered here, the vector of generalized coordinates $\mathbf{x}^n = [x, y, \psi]^T$ is defined in the *NED* frame, where $(x, y)$ is the position and $\psi$ is the heading angle of the vehicle. The velocities $\nu^b = [u, v, r]^T$ in the surge, sway and yaw directions are defined in the *BODY* frame of the vehicle. Superscripts $n$ and $b$ will be dropped from the notation when the reference frame is evident from the context. Subscripts $p \in \{m, r, v, s\}$ on these vectors will indicate their vehicle of origin.

### 2.2 Ship model and properties

The equations of motions can be written in vectorial form in the *BODY* frame as in Fossen (2002)

$$\dot{\mathbf{x}} = \mathbf{J}(\mathbf{x})\,\nu \quad (1)$$

$$\mathbf{M}_\nu \dot{\nu} + \mathbf{C}_\nu(\nu)\,\nu + \mathbf{D}_\nu(\nu)\,\nu + \mathbf{g}_\nu(\mathbf{x}) = \tau_\nu \quad (2)$$

where $\mathbf{M}_\nu$ is a constant positive definite inertia matrix including added mass effects, $\mathbf{C}_\nu(\nu)$ is a skew-symmetric matrix of Coriolis and centripetal forces ($\mathbf{C}_\nu(\nu) + \mathbf{C}_\nu^T(\nu) = 0$), $\mathbf{D}_\nu(\nu)$ is a non-symmetric damping matrix, and gravitational/buoyancy forces are collected in $\mathbf{g}_\nu(\mathbf{x})$. The matrix $\mathbf{J}(\mathbf{x})$ is the transformation matrix from the *BODY* frame to the *NED* frame, and in a 3DOF application where pitch and roll motion are negligible, the matrix $\mathbf{J}(\mathbf{x})$ reduces to a simple rotation matrix around the $z^n$-axis as

$$\mathbf{J}(\mathbf{x}) = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3)$$

In this paper we will derive the control scheme using the model in the *NED* frame found by inserting (1) into (2)

$$\mathbf{M}(\mathbf{x})\ddot{\mathbf{x}} + \mathbf{C}(\mathbf{x}, \dot{\mathbf{x}})\dot{\mathbf{x}} + \mathbf{D}(\mathbf{x}, \dot{\mathbf{x}})\dot{\mathbf{x}} + \mathbf{g}(\mathbf{x}) = \tau \quad (4)$$

where the inertia matrix $\mathbf{M}(\mathbf{x})$ is positive definite and the Coriolis and centripetal matrix $\mathbf{C}(\mathbf{x}, \dot{\mathbf{x}})$ is defined in terms of Christoffel symbols. The dissipation vector $\mathbf{d}(\mathbf{x}, \dot{\mathbf{x}}) = \mathbf{D}(\mathbf{x}, \dot{\mathbf{x}})\dot{\mathbf{x}}$ collects friction and damping forces, while $\mathbf{g}(\mathbf{x})$ collects gravitational and buoyancy forces. The control input vector $\tau$ is generalized forces and moments acting on the system.

The dynamic model (4) in the NED frame has a number of properties similar to those of robotics systems (Ortega and Spong, 1989)

**P1** The positive definite inertia matrix satisfy $0 < \mathbf{M}_m \leq \mathbf{M}(\mathbf{x}) \leq \mathbf{M}_M < \infty$, where $\mathbf{M}_m$ and $\mathbf{M}_M$ are positive constants.

**P2** The inertia matrix $\mathbf{M}(\mathbf{x})$ is differentiable in $\mathbf{x}$ and $\mathbf{y}^T\left(\dot{\mathbf{M}}(\mathbf{x}) - 2\mathbf{C}(\mathbf{x}, \dot{\mathbf{x}})\right)\mathbf{y} = 0$, $\forall\, \mathbf{x}, \mathbf{y} \in \mathbb{R}^3$.

**P3** The Coriolis term in Christoffel symbols satisfies $\mathbf{C}(\mathbf{x}, \mathbf{y})\mathbf{z} = \mathbf{C}(\mathbf{x}, \mathbf{z})\mathbf{y}$, and also $\|\mathbf{C}(\mathbf{x}, \dot{\mathbf{x}})\| \leq \mathbf{C}_M \|\dot{\mathbf{x}}\|$.

We will also assume the following property of the dissipation vector $\mathbf{d}(\mathbf{x}, \dot{\mathbf{x}})$ for a marine vessel (Paulsen and Egeland, 1995)

**P4** The dissipation vector $\mathbf{d}(\mathbf{x}, \dot{\mathbf{x}}) = \mathbf{D}(\mathbf{x}, \dot{\mathbf{x}})\dot{\mathbf{x}}$ is continuously differentiable in $\mathbf{x}$ and $\dot{\mathbf{x}}$ and satisfies for some $k_d > 0$

$$\mathbf{y}^T \frac{\partial\, \mathbf{d}(\mathbf{x}, \dot{\mathbf{x}})}{\partial \dot{\mathbf{x}}} \mathbf{y} \geq k_d\, \mathbf{y}^T \mathbf{y}, \qquad \forall\, \mathbf{x},\ \dot{\mathbf{x}},\ \mathbf{y} \in \mathbb{R}^3 \quad (5)$$

and for a continuous function $\beta_d(\mathbf{s}) : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$

$$\left\| \frac{\partial \mathbf{d}(\mathbf{x}, \dot{\mathbf{x}})}{\partial \dot{\mathbf{x}}} \right\| \leq \beta_d(\|\dot{\mathbf{x}}\|). \quad (6)$$

## 3. DYNAMIC SYNCHRONIZATION

We consider the dynamic synchronization problem of synchronizing a follower to a leader, and imposing a dynamic relation between the follower and the leader in the form of a differential equation

$$\dot{\varepsilon} = \mathbf{f}_\varepsilon(\varepsilon) \quad (7)$$

to control the behavior of the follower while approaching the leader in a replenishment or formation operation. Note that dynamic synchronization is particularly suited for docking operations to a moving leader, where the control scheme can be developed by observing that the reference vehicle is simply the leader vessel.

In this paper we restrict the reference vehicle to a motion parallel to the motion of the leader, and thus we can choose the heading angle of the reference vehicle equal to the heading angle of the leader; $\psi_r = \psi_m$, and $\gamma_m = \pm\frac{\pi}{2}$. See Kyrkjebø *et al.* (2006) for a model of the reference vehicle kinematics when $\psi_r \neq \psi_m$, and $\gamma_m \neq \pm\frac{\pi}{2}$.

The kinematic model (1) of the leader vehicle $\mathscr{V}_m$ with the position/heading vector $\mathbf{x}_m$ can be written as

$$\dot{\mathbf{x}}_m = \mathbf{J}(\mathbf{x}_m)\,\nu_m \quad (8)$$

Taking $\gamma_m = \frac{\pi}{2}$, we can find the position of the reference vehicle in the *NED* frame through (8)

$$\mathbf{x}_r = \mathbf{x}_m + \mathbf{J}(\mathbf{x}_m)\,\mathbf{d}_r^m \quad (9)$$

where

$$\mathbf{d}_r^m = \begin{bmatrix} d\cos\gamma_m \\ d\sin\gamma_m \\ 0 \end{bmatrix} \tag{10}$$

is a vector of the distance $d$ in the *BODY*-frame of the leader. Note that this reduces to

$$\mathbf{d}_r^m = \begin{bmatrix} 0 \\ d \\ 0 \end{bmatrix} \tag{11}$$

for the choice of $\gamma_m = \frac{\pi}{2}$. Through the time derivative we obtain the velocity of the reference vehicle in the *NED*-frame

$$\dot{\mathbf{x}}_r = \dot{\mathbf{x}}_m + \mathbf{J}(\mathbf{x}_m)\,\mathbf{S}(r_m)\,\mathbf{d}_r^m \tag{12}$$

where

$$\mathbf{S}(r_m) = \begin{bmatrix} 0 & -r_m & 0 \\ r_m & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{13}$$

Note that in terms of *BODY* fixed velocities, we can write (12) in component form as

$$\begin{aligned} \dot{x}_r &= u_r\cos\psi_m - v_r\sin\psi_m \\ \dot{y}_r &= u_r\sin\psi_m + v_r\cos\psi_m \\ \dot{\psi}_r &= r_r \end{aligned} \tag{14}$$

where we have defined $u_r = u_m + dr_m$, $v_r = v_m$ and $r_r = r_m$. It is easy to see from (14) that in this particular case only the reference forward velocity is changed ($u_r = u_m + dr_m$) with respect to that of the leader. Note that this is necessary for the follower to maintain its position parallel to the leader during turns, due to the difference in turn radius. The kinematic model (14) of the reference vehicle can now be written as

$$\dot{\mathbf{x}}_r = \mathbf{J}(\mathbf{x}_m)\,\nu_r \tag{15}$$

where $\nu_r = [u_m + dr_m,\ v_m,\ r_m]^T$.

We can define the dynamic behavior of the synchronization error by defining

$$\varepsilon = \mathbf{x}_v - \mathbf{x}_r, \qquad \dot{\varepsilon} = \dot{\mathbf{x}}_v - \dot{\mathbf{x}}_r, \qquad \ddot{\varepsilon} = \ddot{\mathbf{x}}_v - \ddot{\mathbf{x}}_r \tag{16}$$

and introduce a 1st order low-pass filter cascaded with a stable mass-damper-spring system (Fossen, 2002) as the dynamic synchronization reference system

$$\varepsilon^{(3)} + (2\mathbf{\Delta}+\mathbf{I})\,\mathbf{\Omega}\ddot{\varepsilon} + (2\mathbf{\Delta}+\mathbf{I})\,\mathbf{\Omega}^2\dot{\varepsilon} + \mathbf{\Omega}^3\varepsilon = \mathbf{\Omega}^3\varepsilon_r \tag{17}$$

for designed filter constants $\mathbf{\Delta} > 0$ and $\mathbf{\Omega} > 0$, and where $\varepsilon_r$ is the desired value for $\varepsilon$ since

$$\lim_{t\to\infty}\varepsilon(t) = \varepsilon_r \tag{18}$$

Note that (17) guarantees that (16) are smooth signals.

We can write (17) as a linear time invariant system

$$\dot{\bar{\varepsilon}} = \mathbf{A}\bar{\varepsilon} + \mathbf{B}\varepsilon_r, \qquad \bar{\varepsilon} = \begin{bmatrix} \varepsilon & \dot{\varepsilon} & \ddot{\varepsilon} \end{bmatrix}^T \tag{19}$$

where

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \\ -\mathbf{\Omega}^3 & (2\mathbf{\Delta}+\mathbf{I})\,\mathbf{\Omega}^2 & -(2\mathbf{\Delta}+\mathbf{I})\,\mathbf{\Omega} \end{bmatrix} \tag{20}$$

and

$$\mathbf{B} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{\Omega}^3 \end{bmatrix}^T \tag{21}$$

We now have a virtual vehicle $\mathscr{V}_v$ defined by

$$\mathbf{x}_v = \mathbf{x}_r + \varepsilon, \qquad \dot{\mathbf{x}}_v = \dot{\mathbf{x}}_r + \dot{\varepsilon}, \qquad \ddot{\mathbf{x}}_v = \ddot{\mathbf{x}}_r + \ddot{\varepsilon} \tag{22}$$

where the dynamic behavior of the synchronization errors is in terms of $\varepsilon$. Defining the synchronization control errors as

$$\mathbf{e} = \mathbf{x} - \mathbf{x}_v, \qquad \dot{\mathbf{e}} = \dot{\mathbf{x}} - \dot{\mathbf{x}}_v, \qquad \ddot{\mathbf{e}} = \ddot{\mathbf{x}} - \ddot{\mathbf{x}}_v \tag{23}$$

and using a sliding surface motivated by (Slotine and Li, 1987) as a passive filtering of the virtual vehicle states, we can design a virtual reference trajectory as

$$\dot{\mathbf{y}}_v = \dot{\mathbf{x}}_v - \mathbf{\Lambda}\mathbf{e}, \qquad \ddot{\mathbf{y}}_v = \ddot{\mathbf{x}}_v - \mathbf{\Lambda}\dot{\mathbf{e}} \tag{24}$$

where $\mathbf{\Lambda} > 0$ is a design parameter. Defining a measure of tracking as

$$\mathbf{s} = \dot{\mathbf{x}} - \dot{\mathbf{y}}_v = \dot{\mathbf{e}} + \mathbf{\Lambda}\mathbf{e} \tag{25}$$

and using the relationship $\dot{\mathbf{x}} = \mathbf{s} - \dot{\mathbf{y}}_v$ we can rewrite (4) as

$$\begin{aligned} &\mathbf{M}(\mathbf{x})\,\ddot{\mathbf{e}} + \mathbf{C}(\mathbf{x},\dot{\mathbf{x}})\,\dot{\mathbf{e}} + \mathbf{D}(\mathbf{x},\dot{\mathbf{x}})\,\dot{\mathbf{e}} = \\ &\quad \tau - \mathbf{M}(\mathbf{x})\,\ddot{\mathbf{y}}_v - \mathbf{C}(\mathbf{x},\dot{\mathbf{x}})\,\dot{\mathbf{y}}_v - \mathbf{D}(\mathbf{x},\dot{\mathbf{x}})\,\dot{\mathbf{y}}_v - \mathbf{g}(\mathbf{x}) \end{aligned}$$

and propose the synchronization control law

$$\begin{aligned} &\tau = \\ &\mathbf{M}(\mathbf{x})\,\ddot{\mathbf{y}}_v + \mathbf{C}(\mathbf{x},\dot{\mathbf{x}})\,\dot{\mathbf{y}}_v + \mathbf{D}(\mathbf{x},\dot{\mathbf{x}})\,\dot{\mathbf{y}}_v + \mathbf{g}(\mathbf{x}) - \mathbf{K}_d\mathbf{s} - \mathbf{K}_p\mathbf{e} \end{aligned} \tag{26}$$

where $\mathbf{K}_p$ and $\mathbf{K}_d$ are symmetric positive gain matrices.

Considering the Lyapunov function candidate

$$V(t\mathbf{s},\mathbf{e}) = \frac{1}{2}\mathbf{s}^T\mathbf{M}(\mathbf{x})\,\mathbf{s} + \frac{1}{2}\mathbf{e}^T\mathbf{K}_p\mathbf{e} \tag{27}$$

and differentiating along the closed-loop trajectories we get

$$\dot{V}(t) = -\mathbf{s}^T\left(\mathbf{D}(\mathbf{x},\dot{\mathbf{x}}) + \mathbf{K}_d\right)\mathbf{s} - \mathbf{e}^T\mathbf{K}_p\mathbf{\Lambda}\mathbf{e} \tag{28}$$

Since $V(t)$ is positive definite, and $\dot{V}(t)$ is negative definite through P4 (see Kyrkjebø and Pettersen (2005)), it follows that the equilibrium $(\mathbf{e},\mathbf{s}) = (\mathbf{0},\mathbf{0})$ is uniformly globally exponentially stable (UGES), and from convergence of $\mathbf{s} \to \mathbf{0}$ and $\mathbf{e} \to \mathbf{0}$ that $\dot{\mathbf{e}} \to \mathbf{0}$.

## 4. SIMULATIONS

Simulations were performed in Matlab with the model ship Cybership II as the follower vessel. For simplicity, and to illustrate that the follower $\mathscr{V}_s$ converge to the reference vehicle $\mathscr{V}_r$, simulations were performed for a docking situation where the follower is docking to a moving leader vessel. In this situation, the reference position $\mathbf{x}_r$ coincides with the leader position $\mathbf{x}_m$, and the convergence of the follower $\mathbf{x}$ to the leader vessel is illustrated when the synchronization errors $(\mathbf{e}, \dot{\mathbf{e}})$ goes to zero. Note that for a formation change or an approach operation, the position of the reference vehicle would simply be a shifted replica of the leader position/heading through the definition of $d$ and $\gamma_m$ in (9), and thus $\mathbf{x} \to \mathbf{x}_r \neq \mathbf{x}_m$.

The surface ship model of Cybership II from Skjetne *et al.* (2004) is given in the body frame ($\mathbf{g}(\mathbf{x}) = \mathbf{0}$ for surface vessels)

$$\mathbf{M}\dot{\nu} + \mathbf{C}(\nu)\,\nu + \mathbf{D}(\nu)\,\dot{\nu} = \tau_\nu \tag{29}$$

and is a function of the body fixed velocities $\nu = [u, v, r]^T$ in surge, sway and yaw, respectively. The inertia matrix $\mathbf{M}$, Coriolis and centrifugal matrix $\mathbf{C}(\nu)$, and the nonlinear damping matrix $\mathbf{D}(\nu) = \mathbf{D} + \mathbf{D}_n(\nu)$ are defined as

$$\mathbf{M} = \begin{bmatrix} 25.8 & 0 & 0 \\ 0 & 33.8 & 1.0115 \\ 0 & 1.0115 & 2.76 \end{bmatrix}$$

$$\mathbf{C}(\nu) = \begin{bmatrix} 0 & 0 & -33.8v - 1.0115r \\ 0 & 0 & 25.8u \\ 33.8v + 1.0115r & -25.8u & 0 \end{bmatrix}$$

$$\mathbf{D} = \begin{bmatrix} 0.72 & 0 & 0 \\ 0 & 0.8896 & 7.25 \\ 0 & 0.0313 & 1.90 \end{bmatrix}$$

$$\mathbf{D}_n(\nu) = \begin{bmatrix} 1.33|u| + 5.87u^2 & 0 & 0 \\ 0 & 36.5|v| + 0.805|r| & 0.845|v| + 3.45|r| \\ 0 & 3.96|v| - 0.130|r| & -0.080|v| + 0.75|r| \end{bmatrix}$$

Initial conditions in the simulations were chosen as $\mathbf{x}_m(0) = \left[4,\ 8,\ -\frac{\pi}{2}\right]^T$ for the leader ship position vector, and the leader ship tracks a sine wave reference trajectory $\sin(\omega\, t)$ with frequency $\omega = 1/45$ with heading angle $\psi_m$ along the tangent line.

Gains were chosen as $\mathbf{K}_p = \mathrm{diag}[50,\ 150,\ 50]$, and $\mathbf{K}_d = \mathrm{diag}[14,\ 14,\ 14]$ for the synchronization control law of the follower vessel, and the filter constants of (17) were chosen as $\mathbf{\Delta} = \mathrm{diag}[0.77,\ 0.77,\ 0.77]$ and $\mathbf{\Omega} = \mathrm{diag}[0.2,\ 0.2,\ 0.2]$. The sliding surface parameter of (24) were chosen as $\mathbf{\Lambda} = \mathrm{diag}[0.8,\ 0.8,\ 0.8]$.

The position errors $\mathbf{e}$ of the simulations in Fig. 2 and the velocity errors $\dot{\mathbf{e}}$ in Fig. 3 show that $(\mathbf{e}, \dot{\mathbf{e}}) \to (\mathbf{0}, \mathbf{0})$.

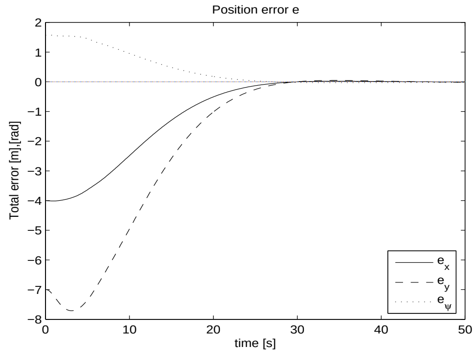


Fig. 2. Position errors $\mathbf{e} = \mathbf{x} - \mathbf{x}_r\,(\mathbf{x}_m)$

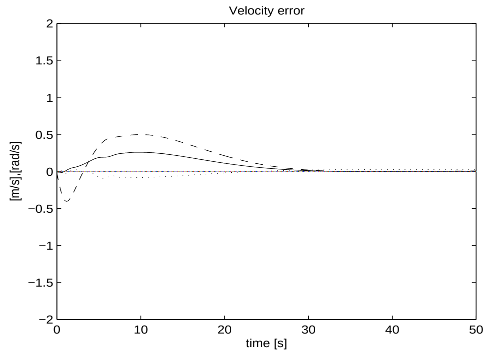


Fig. 3. Velocity errors $\dot{\mathbf{e}} = \dot{\mathbf{x}} - \dot{\mathbf{x}}_r\,(\dot{\mathbf{x}}_m)$

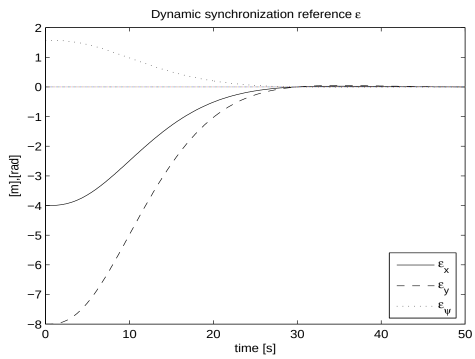


Fig. 4. Dynamic synchronization reference $\varepsilon(t)$

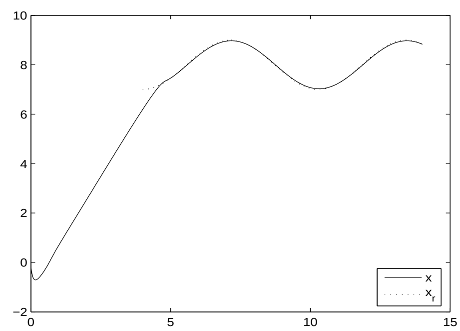


Fig. 5. XY-plot of the follower $\mathbf{x}$ and leader $\mathbf{x}_r\,(\mathbf{x}_m)$

The dynamic synchronization reference is shown in Fig. 4 and an *xy*-plot of the leader and follower vessel is shown in Fig. 5.

## 5. CONCLUSIONS AND FUTURE WORK

We have presented a leader-follower dynamic synchronization control scheme to specify and guarantee the behavior of marine vessels when approaching or changing positions in a formation, including the special case of docking operations where the target is moving. We specified the dynamic behavior of the synchronization errors through a third-order reference filter to ensure smooth position, velocity and acceleration reference signals for the synchronization error, and showed that the closed-loop errors from the follower to the dynamic reference states are uniformly globally exponentially stable.

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