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IFAC PapersOnLine 56-2 (2023) 2026–2031

Maximum Hands-Off Attitude Control of a Spacecraft Actuated by Thrusters ⋆

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Abstract: In this paper we investigate the use of maximum hands-off control for spacecraft attitude control of a spacecraft equipped with thrusters. Other than a more natural choice of actuators than in our previous paper, the definition of the L_0-norm for a vector of continuous-time signals most suited for this optimal control problem is refined. An extra term is introduced to the cost function to handle the actuator change. We introduce relative sparsity, a concept where sparsity is defined as a function of the control horizon. With this definition, comparing the sparsity of any given signal is easier. Finally, we show how the relative sparsity of an optimized state trajectory changes with the resolution and constraints of the optimal control problem.

Keywords: attitude control, optimal control, spacecraft attitude control, optimization

1. INTRODUCTION

The concept of maximum hands-off control, as shown in Nagahara et al. (2015), is an optimal control scheme where the objective is to minimize the time spent on actuation. The control scheme has a range of applications, including in the control of spacecraft, as was shown for a single-axis attitude control maneuver in Ikeda and Nagahara (2019). The concept was taken further by Schaanning et al. (2022) by demonstrating how maximum hands-off control would work for a fully nonlinear model for attitude control, using unit quaternions to represent the attitude of a CubeSat actuated by reaction wheels, a case inspired by the HYPSO-1 mission at NTNU, Grøtte et al. (2022).

Maximum hands-off control is a control scheme where the resulting torque is highly discontinuous. While Schaanning et al. (2022) showed that reaction wheels, a type of actuator that works well with continuous controllers, can be used with maximum hands-off control to control the attitude of a spacecraft, a more natural choice of actuators to couple with maximum hands-off control is reaction thrusters. Reaction thrusters, or simply thrusters, are actuators that are either completely on or completely off, giving a discontinuous input to the system. As opposed to reaction wheels, thrusters can be used for orbital maneuvers but also for attitude control. See for example Kristiansen et al. (2008); Jin et al. (2006).

There exists thrusters that are not off or on at their maximum magnitude, namely electric thrusters (see for example Rafalskyi et al. (2021)), which have been suggested for use for the attitude control of a spacecraft (Mier-Hicks and Lozano, 2017; King et al., 2021). The focus of this paper will be on/off thrusters. However, a brief discussion on generalizing the results in this paper for use with continuous thrust is included.

The contributions in this paper are the following: we show that the maximum hands-off control formulation demonstrated for a satellite with reaction wheels also works with attitude maneuvers for a satellite actuated with reaction thrusters. The formulation used in this paper differs from the one used previously, as the cost function is extended with an additional term. Furthermore, we justify using the union operator for optimizing based on the L_0-norm for a problem with multiple inputs. Finally, we defined the term relative sparsity as a way to compare the sparsity of different control strategies. The impact of the sampling rate and saturation limits on the inputs on the relative sparsity is discussed in some detail.

The remaining part of the article is structured as follows: the max hands-off control is defined in Section 2, both in general terms and with the relaxed formulation used for direct optimization in Schaanning et al. (2022). In the same section, we introduce the union operator and define relative sparsity. The optimal control problem, including the cost function based on the maximum hands-off formulation from Section 2, is presented in Section 3. The simulation setup is described in Section 4, which makes up three scenarios: a simple maneuver showing that the control strategy can accomplish the desired objectives and two different scenarios designed to show how the system parameters impact relative sparsity. The results are presented in Section 5 and discussed in Section 6, while the paper is concluded in Section 7.

⋆ The work is sponsored by the Research Council of Norway through the Centre of Excellence funding scheme, project number 223254, AMOS.

10.1016/j.ifacol.2023.10.1099

2. MAX HANDS-OFF CONTROL

In this section, we will introduce the theory behind the maximum hands-off formulation, as defined by Nagahara et al. (2015). The support of a function of time $u(t)$, a continuous-time signal, is defined by the closure of the set

$$\{t \in [0, T] : u(t) \neq 0\}, \tag{1}$$

which is the set of points where the function takes on non-zero values.

The authors of Nagahara et al. (2015) use (1) to define the L_0-norm of $u(t)$ as the length of the support using the Lebesgue measure on $\mathbb{R}$, This yields the definition of the L_0-norm

$$\|u\|_0 \triangleq \mu(\text{supp}(u(t))). \tag{2}$$

where $\mu(\cdot)$ is the mentioned Lebesque measure. In defining their optimal control problem, Nagahara et al. (2015) define a cost function based on the continuous time-signal $u(t)$ as

$$J_0(u(t)) \triangleq \sum_{i=1}^{m} \lambda_i \|u_i\|_0, \tag{3}$$

where m is the number of control inputs in the time domain, λ_i are positive weights for each control signal, and the L_0-norm is the one that was defined in (2).

As the formulation in (3) is only for a single continuous-time signal, the cost function should be changed to facilitate several continuous-time signals in a vector. With the maximum hands-off control scheme, we want to minimize the total amount of time any input is applied to the system. In other words: if one of the channels of the vector has support at a given time step, it should not cost anything for the other channels to have a non-zero signal at that time step. To achieve this, we reformulate the L_0 cost function in (3) as,

$$J_0(\mathbf{u}(t)) \triangleq \sum_{i=1}^{m} \bigcup_{j=1}^{n} \lambda_{j,i} \|\mathbf{u}_{j,i}\|_0, \tag{4}$$

where n is the number of channels in the input vectors, which in this paper will be the number of thrusters.

A longer discussion on different implementations of the maximum hands-off control and how the choices affect the amount of input signals the system receives can be found in Schaanning (2021).

The relative sparsity of a discrete signal can be defined as

$$\text{Relative sparsity} = \frac{\text{Time steps with actuation}}{\text{Total time steps}} \cdot 100\%. \tag{5}$$

The relative sparsity, defined this way, makes sense as a comparison measure between different control signals when it comes to how sparse the signal is as it measures how much of the available control horizon the control signal uses for actuation. This measure could give a better view of how sparse the actuation is compared to the view provided directly by the values from the L_0-norm, which will be a direct result of the point resolution of the simulation or the experiment.

The maximum hands-off control problem is discontinuous, so in order to use IPOPT we need to relax the problem. In this paper we use the same relaxation for the maximum hands-off control problem as in Schaanning et al. (2022). Formally, by defining the L_0-optimal control problem as

$$\begin{aligned} \min \quad & f(\mathbf{x}) + \gamma \|\mathbf{x}\|_0 & \text{(6a)} \\ \text{s.t.} \quad & c_i(\mathbf{x}) = 0,\ j \in \mathcal{E} & \text{(6b)} \\ & c_j(\mathbf{x}) \leq 0,\ j \in \mathcal{I}, & \text{(6c)} \end{aligned}$$

where $c_i(\mathbf{x})$ and $c_j(\mathbf{x})$ are constraints in the set of equality constraints, $\mathcal{E}$, and inequality constraints, $\mathcal{I}$, respectively. γ is an arbitrary, positive constant. the L_0-optimal control problem can be relaxed using a set of complementarity constraints (Feng et al., 2016)

$$\begin{aligned} \min \quad & f(\mathbf{x}) + \boldsymbol{\gamma}^\top (\mathbf{1}_N - \boldsymbol{\xi}) & \text{(7a)} \\ \text{s.t.} \quad & c_i(\mathbf{x}) = 0,\ i \in \mathcal{E} & \text{(7b)} \\ & c_i(\mathbf{x}) \leq 0,\ i \in \mathcal{I} & \text{(7c)} \\ & -\epsilon \mathbf{1} \leq \boldsymbol{\xi} \circ \mathbf{x} \leq \epsilon \mathbf{1} & \text{(7d)} \\ & \mathbf{0} \leq \boldsymbol{\xi} \leq \mathbf{1}, & \text{(7e)} \end{aligned}$$

where the vector $\boldsymbol{\xi}$ is a vector of the same size as the number of control intervals that indicates whether or not any control signal is spent at a given time step. The operator $\circ$ is the Hadamard product, which is the element-wise product of the two factors. ϵ is a small, positive number multiplied by a vector of ones, $\mathbf{1}$, of the same size as $\boldsymbol{\xi}$. The vector $\mathbf{0}$ is a vector of zeros of the same size as $\boldsymbol{\xi}$, similar to $\mathbf{1}$.

3. OPTIMAL CONTROL PROBLEM

The optimal control problem, using a relaxed formulation of the maximum hands-off control problem using the cost as it is defined in (4) for the attitude of a satellite actuated by thrusters is given by,

$$\begin{aligned} \min \quad & k_1 f(\boldsymbol{\omega}_{ib}^b) + k_2 g(\mathbf{q}_b^i) + k_3 (\mathbf{1} - \boldsymbol{\xi}) \mathbf{1}^\top & \text{(8a)} \\ \text{s.t.} \quad & & \\ & \dot{\mathbf{q}}_b^i = \frac{1}{2} \mathbf{T}(\mathbf{q}_b^i) \boldsymbol{\omega}_{ib}^b & \text{(8b)} \\ & \dot{\boldsymbol{\omega}}_{ib}^b = \mathbf{J}^{-1}(-\mathbf{S}(\boldsymbol{\omega}_{ib}^b)\mathbf{J}\boldsymbol{\omega}_{ib}^b + \mathbf{B}\boldsymbol{\tau}_u) & \text{(8c)} \\ & \mathbf{0} \leq \boldsymbol{\tau}_u \leq \boldsymbol{\tau}_{\text{limit}} & \text{(8d)} \\ & \mathbf{x}(0) = \mathbf{x}_0 & \text{(8e)} \\ & -\epsilon \mathbf{1} \leq \boldsymbol{\xi} \circ \boldsymbol{\tau}_{u,j}(t) \leq \epsilon \mathbf{1} & \text{(8f)} \\ & \mathbf{0} \leq \boldsymbol{\xi} \leq \mathbf{1}, & \text{(8g)} \end{aligned}$$

where $f(\cdot)$ and $g(\cdot)$ are costs imposed on the error in the angular velocity, $\boldsymbol{\omega}_{ib}^b$, and the quaternion, $\mathbf{q}_b^i$, with respect to the reference quaternion, respectively, given as

$$\begin{aligned} f(\boldsymbol{\omega}_{ib}^b) &= (\boldsymbol{\omega}_e^b)^\intercal \boldsymbol{\omega}_e^b, \quad \boldsymbol{\omega}_e^b = \boldsymbol{\omega}_{ib,\text{ref}}^b - \boldsymbol{\omega}_{ib}^b(T), \\ g(\mathbf{q}_b^i) &= 1 - \left| (\mathbf{q}_b^i(T))^\top \mathbf{q}_{b,\text{ref}}^i \right|, \end{aligned} \tag{9}$$

where T is the length of the control horizon, making an arbitrary vector $\mathbf{x}$ take the value $\mathbf{x}(T)$ at the final time. The function $g(\cdot)$ in (9) is chosen due to it being a metric on SO(3) (Huynh, 2009). The sub- and superscripts i and b denote the inertial and body frame, respectively. Note that the a rigid body model is assumed to be sufficient for the studied problem, i.e., the change in the inertia matrix is assumed to be either symmetric or small enough to be disregarded. $\mathbf{T}(\cdot)$ is given as (Egeland and Gravdahl, 2003)

$$\mathbf{T}(\mathbf{q}) = \begin{bmatrix} -\boldsymbol{\epsilon}^\intercal \\ \eta \mathbf{I}_{3\text{x}3} + \mathbf{S}(\boldsymbol{\epsilon}) \end{bmatrix}, \tag{10}$$

where η is the scalar part, and $\boldsymbol{\epsilon}$ is the vector part of the quaternion. The $\mathbf{S}(\cdot)$ matrix is a matrix that works like the three-dimensional cross product. k_1, k_2, and k_3 are positive constants that determine how much weight the optimization should put into getting closer to the reference in attitude or angular velocity or in order to keep the L_0-norm small, respectively. $\mathbf{J}$ is the satellite inertia matrix. $\mathbf{B}$ is the torque distribution matrix, mapping the torques from the thrusters, $\boldsymbol{\tau}_u$, into the body frame. $\boldsymbol{\tau}_u$ is a vector with one element for each thruster, showing only the sign and the magnitude of the torque the thruster provides. The torque from each separate thruster is accessed separately through $\boldsymbol{\tau}_{u,j}(t)$. The state, given by $\mathbf{x}$, is defined by $\mathbf{x} = [\mathbf{q}_b^i; \boldsymbol{\omega}_{ib}^b]$. The initial state variable, $\mathbf{x}(0)$, is given by $\mathbf{x}_0$. Note that the term $\boldsymbol{\tau}_{u,i}(t)$ in (8f) indicates that each element of the torque vector should be multiplied with the torque vector for a given time step, which satisfies the definition of the L_0-cost we want to use in this paper, as defined in (4).

For discontinuous thrust, where the thrusters are either on at the maximum thrust or completely shut off, an extra term must be appended to the cost function (8a), yielding

$$\begin{aligned} \min \quad & k_1 f(\boldsymbol{\omega}_{ib}^b) + k_2 g(\mathbf{q}_b^i) + k_3(\mathbf{1} - \boldsymbol{\xi})\mathbf{1}^\intercal \\ & + k_4 \sum_{i=1}^{N} \boldsymbol{\xi}(i) \sum_{j=1}^{n_u} \tau_{u,(j,i)}(\tau_{\text{limit}} - \tau_{u,(j,i)}), \end{aligned} \tag{11}$$

where N is the number of control intervals, n_u is the number of thrusters, k_4 is a positive constant, and $\tau_{u,(j,i)}$ is the element of the torque vector $\boldsymbol{\tau}_u$ specified for thruster j at time step i. The added term provides the on/off behavior from the optimization: as $\boldsymbol{\xi}$ is a measurement for when a control signal is applied, this term should be close to zero when there is no control action at a given time step and one when control is applied. Thus, using $\boldsymbol{\xi}$ will only be an extra cost imposed based on control signals where at least one of the channels in the control vector, here signifying one of the thrusters, is active. The difference term $(\tau_{\text{limit}} - \tau_{u,(j,i)})$ is included to force the torque from each thruster to its maximum value, τ_{limit}. As we do not want to force all thrusters to fire when one of the thrusters is active, the maximizing terms need to be multiplied by the torque value from the given thruster, $\tau_{u,(j,i)}$. This term is necessary due to $\boldsymbol{\xi}$ having only one element shared between all torques, an artifact of the L_0-formulation as shown in (4).

For continuous thrust actuators, it seems reasonable to assume that the formulation in (8) is sufficient, making the extra term introduced in (11) redundant when the actuation can take a continuous range of values.

4. SIMULATION SETUP

The simulations in this paper are based on the ESEO satellite, Kristiansen et al. (2008). The inertia of the satellite is given by

$$\mathbf{J} = \begin{bmatrix} 4.350 & 0 & 0 \\ 0 & 4.3370 & 0 \\ 0 & 0 & 3.6640 \end{bmatrix} \text{kg} \cdot \text{m}^2. \tag{12}$$

The optimal control problem (8) is solved using IPOPT in CasADi. Runge-Kutta 4 is used as the numerical solver. Six thrusters actuate the satellite, each set to give torque along one body frame axis only. This gives the torque distribution matrix $\mathbf{B}$,

$$\mathbf{B} = \begin{bmatrix} 1 & 0 & 0 & \text{-}1 & 0 & 0 \\ 0 & 1 & 0 & 0 & \text{-}1 & 0 \\ 0 & 0 & 1 & 0 & 0 & \text{-}1 \end{bmatrix}, \tag{13}$$

with the negative signs signifying that all elements in $\boldsymbol{\tau}_u$ contain positive torque values.

Three simulation scenarios are run: one to show that the attitude control system can provide satisfactory results with maximum hands-off control with thrusters, and two scenarios where the relative sparsity metric's properties are explained. In the attitude control example, the satellite is given the initial conditions $\mathbf{x}(0) = [\mathbf{q}_b^i; \boldsymbol{\omega}_{ib}^b](0) = [0, 1, 0, 0, 0, 0, 0]^\intercal$, where the first four elements are the quaternion and the latter three are the angular velocity. The references, $\mathbf{q}_{b,\text{ref}}^i$ and $\boldsymbol{\omega}_{ib,\text{ref}}^b$, are given as $[0.5774, 0, -0.5774, -0.5774]^\intercal$ and $[0, 0, 0]^\intercal$. The control horizon T is set to be 30, and the number of steps is set as $N = 90$, yielding a time step of $h = \frac{T}{N} = \frac{30}{90}\text{s} = \frac{1}{3}$ s. The limit on the torques from the thrusters is set to be $\tau_{\text{limit}} = 0.195$ N · m, consistent with the thrusters in Kristiansen et al. (2008), where $\boldsymbol{\tau}_{\text{limit}} = \tau_{\text{limit}} \mathbf{1}_{n_u \text{x} 1}$. The torque limit, as well as the torque from the trusters themselves, take the distance between the place the force from the truster is applied, as well as the magnitude of the force from the trusters into account, and thus, the torque is expressed directly in this paper for simplicity. As discussed in Schaanning et al. (2022), the maximum hands-off attitude control problem is very sensitive to the choice of initial values. In this paper, the initial values are chosen to be the trajectory resulting from a PD controller modulated with a simple Schmitt trigger.

For the scenarios showcasing the properties of the relative sparsity metric, the initial conditions, and the references are kept identical as they are for the attitude demonstration example. In both of the scenarios, the thrusters are made less effective by reducing the torque limit by 50%, essentially weakening the actuators or moving them further from the center of rotation, while the number of control intervals have been decreased by 50% in the latter scenario.

The values for k_1, k_2, k_3, and k_4 for the different scenarios, numerated 1, 2, and 3, as introduced in this paper, can be found in Table 1.

Table 1. Cost function constants

k_i	Scenario 1	Scenario 2	Scenario 3
k_1	18.4	18.4	18.4
k_2	$3.92 \cdot 10^3$	$3.92 \cdot 10^3$	$3.92 \cdot 10^3$
k_3	$(N)^{-1} \cdot 4.5 \cdot 10^4$	$(N)^{-1} \cdot 4.5 \cdot 10^4$	$(N)^{-1} \cdot 1.35 \cdot 10^5$
k_4	$\frac{2.295 \cdot 10^5}{N \cdot n_u}$	$\frac{2.295 \cdot 10^5}{N \cdot n_u}$	$\frac{6.885 \cdot 10^5}{N \cdot n_u}$

5. RESULTS

5.1 *Attitude control maneuver*

Figure 1 shows the attitude response of the system with the trajectory based on the maximum hands-off control formulation solved using IPOPT. As can be seen from the figure, at the end of the simulation, the optimal trajectory reaches the reference attitude, as the quaternion covers SO(3) twice, making $\mathbf{q}$ and $-\mathbf{q}$ represent the same attitude. The corresponding angular velocities are shown in

Figure 3. The torques from the thrusters are plotted in Figure 2. Note that thruster one is fixed in the opposite direction of thruster four, thruster two in the opposite direction of thruster five, and thruster three in the opposite direction of thruster six, as indicated by the distribution matrix in (13). The output from the optimal solver IPOPT, showing the time it takes to solve the problem and the number of iterations, is shown in Table 2. The relative sparsity for this scenario, calculated using the definition in (5), is 11.11% for this scenario.

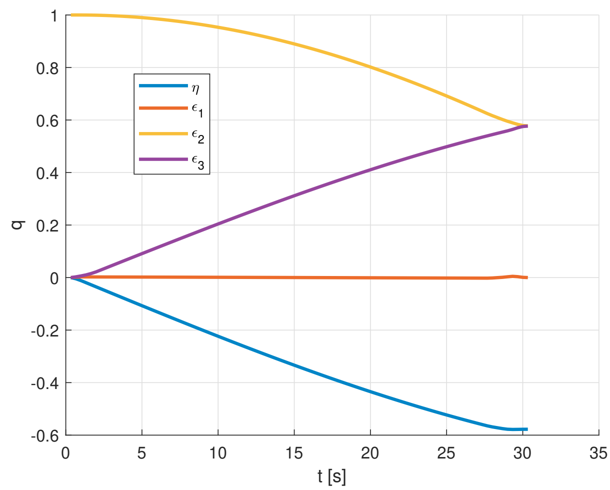

Fig. 1. Quaternion response with maximum hands-off control.

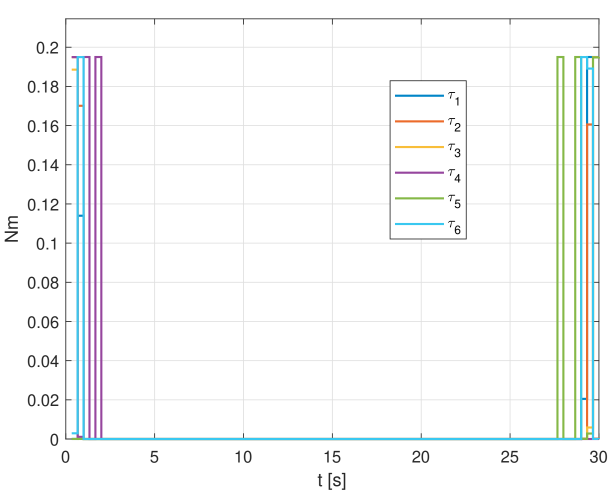

Fig. 2. Torque from the trusters with maximum hands-off control.

Table 2. IPOPT output

Output	Value
Number of iterations	5635
Total CPU secs in IPOPT (w/o function evaluations)	58.893
Total CPU secs in NLP function evaluations	929.226

5.2 Relative sparsity simulations

Figure 6 shows the torque with stricter torque limits than in Section 5.1. As seen from the quaternion trajectory in Figure 4 and the angular velocity trajectories in Figure 5, the satellite still manages to reach the reference values.

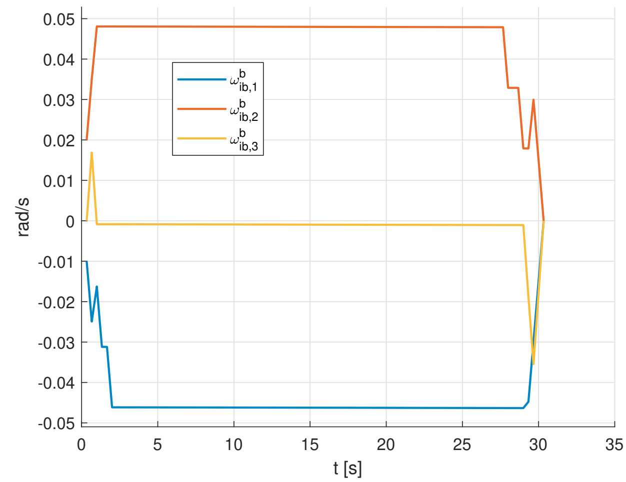

Fig. 3. Angular velocity response with maximum hands-off control.

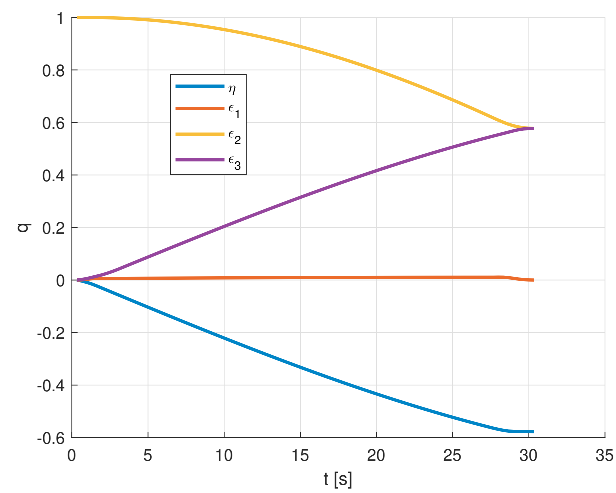

Fig. 4. Quaternion response with maximum hands-off control with lower torque saturation limits.

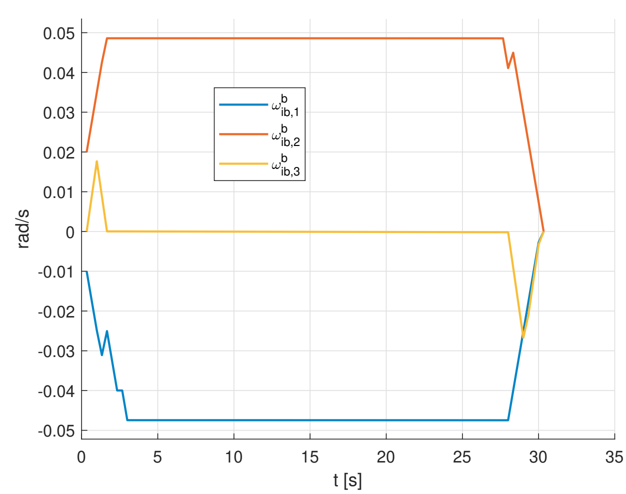

Fig. 5. Angular velocity response with maximum hands-off control with lower torque saturation limits.

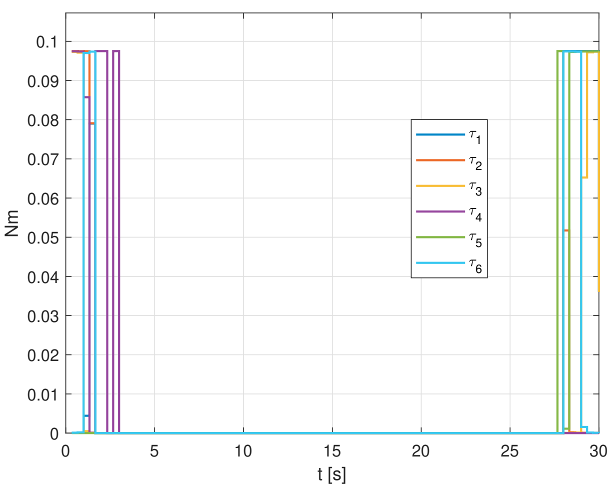

Fig. 6. Torque from the trusters with maximum hands-off control with lower torque saturation limits.

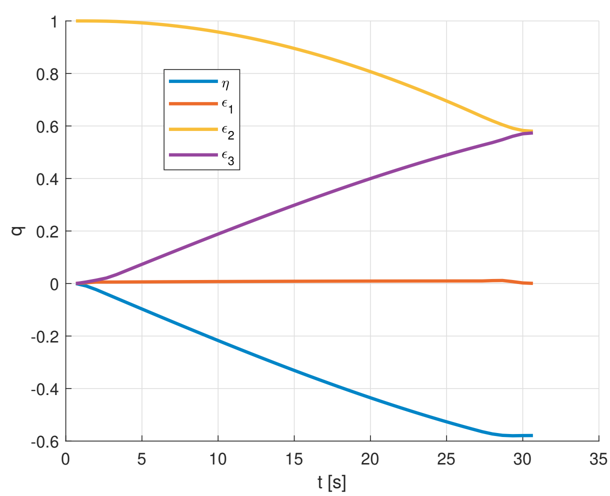

Fig. 7. Quaternion response with maximum hands-off control with fewer control intervals.

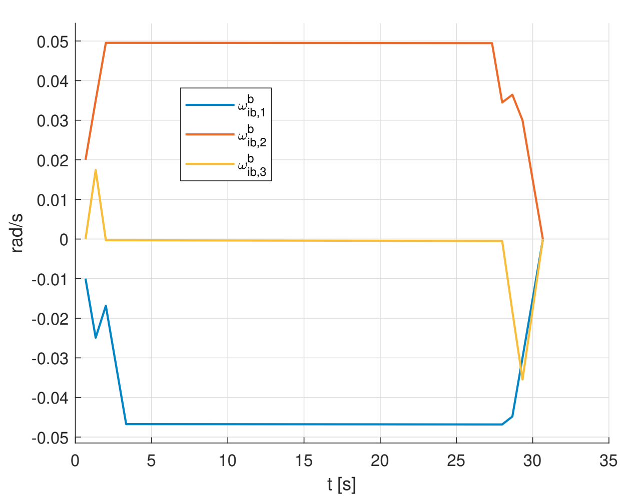

Fig. 8. Angular velocity response with maximum hands-off control with fewer control intervals.

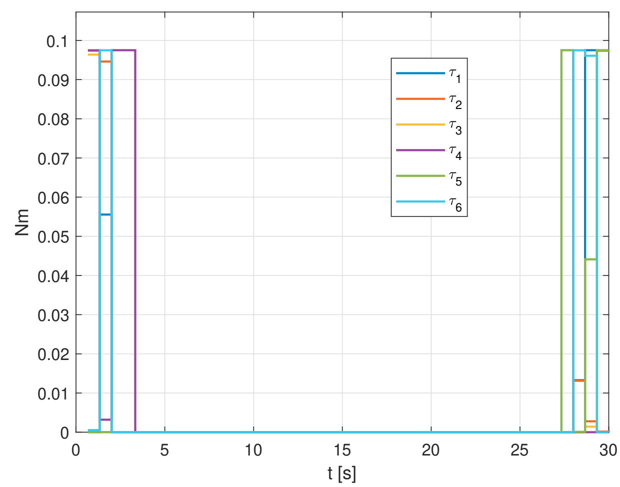

Fig. 9. Torque from the thrusters with maximum hands-off control with fewer control intervals.

Keeping the torque limit low and reducing the time step gives the quaternion and angular velocity trajectories, as shown in Figure 7 and Figure 8, respectively. The torques this scenario requires are shown in Figure 9. The relative sparsity of the two simulations are shown in Table 3.

Table 3. Relative sparsity comparison

Controller	Relative sparsity (%)
With stricter torque limit	16.6667
With lower N	20.000

6. DISCUSSION

6.1 *Attitude control maneuver*

The results in Section 5.1 shows that the maximum hands-off control algorithm, as described in this paper, works for controlling the attitude of a satellite towards a desired attitude and angular velocity.

The main issue with the direct optimization approach to the maximum hands-off control problem using a solver based on derivatives like IPOPT is that the problem is discontinuous. This would be an issue for the maximum hands-off control problem even if the reaction thrusters with their on/off actuation were not in use, as the L_0-norm is not differentiable and motivates the approximation given in (7). With the relaxation parameter ϵ set above zero, some of the values in the optimized trajectory will be slightly above zero, which is where they would be if the solver managed to find the "true" optimal solution without the relaxed formulation. Furthermore, there are four different objectives the optimization should accomplish: the attitude quaternion and the angular velocity should reach their references, the torque should be applied for as little time as possible, and the torque should always be on/off. The first three are not opposing criteria for optimization: by defining a region around the references, there will be a minimum number of (discrete) torque signals required to reach this region. Most control signals will be as high as possible, probably reaching the saturation limits for all or all but one control signal to minimize the control signals in the optimal scenario. The problem with the fourth criterion arises if the control horizon is too coarse relative to the desired region around the references, i.e., N is large relative to T, so it might be hard for the solver to find a good trajectory without careful tuning. This explains why even the optimal solution here has some non-maximized values, as the solver prioritizes reaching the end attitude and angular velocities. Saturating the values from this optimization based on a threshold would give the true on/off behavior the actuators require, but the values for the other objectives, particularly the quaternion and the angular velocity, will naturally end up further from the references than they are in Figure 1 and Figure 3.

A point was made in Section 4 about the importance of initial guesses for the maximum hands-off control problem. The last term in the cost function, which was introduced in (11), $k_4 \sum_{i=1}^{N} \boldsymbol{\xi}(i) \sum_{j=1}^{n_u} \tau_{u,(j,i)}(\tau_{\text{limit}} - \tau_{u,(j,i)})$, is introduced to penalize the solver if the control signal is not on or off, as required by the actuators. Optimizing without this term, using a trajectory generated by a regular PD controller, does not yield satisfactory results as the resulting control signal is significantly less sparse than what is shown in Section 5.1. This suggests that the term would help create the maximum hands-off control in situations where the torque does not have to reach its maximum magnitude each time the actuators are used, as it seems to improve the solver's resilience against poor initial guesses. The term could, however, if the maximum control torque is not called for at any point, lead to a less sparse solution than the maximum hands-off control scheme calls for, as it will force the control signal at a given point toward the maximum value, requiring more torque to counteract the motion when moving towards the reference value. Additionally, the added term and the maximum hands-off term work in the opposite direction: the maximum hands-off term drags the values toward zero, while the added term drags the terms toward the maximum value. This could introduce an extra error into the system since the vector $\boldsymbol{\xi}$ cannot be exactly zero, meaning that the added term will often be in effect when it should not be.

6.2 Relative sparsity simulations

The relative sparsity metric for the two scenarios with stricter torque limits and longer time steps, given in Table 3, shows a significant increase compared to the 11.11% relative sparsity in the nominal case in Section 5.1. For the stricter torque limit case, this consequence is always to be expected when lowering the torque limit as long as the maximum hands-off torque naturally reaches the saturation limit before it is lowered, which is what is shown in the figures here. For the decrease in control intervals, i.e., the decrease in N, the case is the opposite: if the control signal is naturally saturated before N is decreased the relative sparsity will not necessarily increase. An example that illustrates this is a control signal with four saturated control signal points before the decrease in N. If N is then divided by four, there will be one saturated control signal point, thus preserving the relative sparsity. If N, for some reason, is divided by three, two new signal points will be required to represent the same control signal, which in turn will span a more extended area on $\mathbb{R}$, thus increasing the relative sparsity. Due to these concerns, it is important only to compare two different control schemes with respect to relative sparsity if the underlying conditions, such as the control limits, the control horizon, and the number of control signals, differ between the schemes.

7. CONCLUSION

Maximum hands-off control, as formulated by Schaanning et al. (2022), also works with thrusters, although the on/off nature of the actuators makes the problem harder to solve. Using the union operator is reasonable for the L_0 norm and relative sparsity can be used to distinguish between sparse control signals, provided the sampling rate, saturation limits, and control horizon stay identical.

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