Doctoral theses at NTNU, 2024:2

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A case study of three lower secondary teachers', school management's, and three teacher educators' experiences of a practice-based development program in mathematics

Doctoral thesis

NTNU
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ISBN 978-82-326-7558-6 (printed ver.)
ISBN 978-82-326-7557-9 (electronic ver.)
ISSN 1503-8181 (printed ver.)
ISSN 2703-8084 (online ver.)

Doctoral theses at NTNU, 2024:2

Printed by NTNU Grafisk senter

Abstract

This thesis consists of three substudies, presented in articles 1-3, and a discussion based on the findings from these studies. The main research question that guided the investigation was formulated as follows: *How can a practice-based development program contribute to mathematics teachers' professional development?*

Where many studies on professional development of mathematics teachers emphasize the teachers and their development, this study attends to the roles the teachers, the school leader, and the teacher educators as the program leaders have as different actors in the teachers' professional development work. The aim is to contribute new knowledge to research on mathematics teachers' professional development, particularly related to how these actors act and interact to contribute to and support teachers' professional development.

The case examined in this study, a practice-based development program for mathematics teachers, called the Mastering Ambitious Mathematics teaching (MAM) program, aims to provide teachers with the opportunity to develop ambitious mathematics teaching. The MAM program was carried out in four lower secondary schools in Norway, where one of the schools constitutes the case for this study. The sources for the data material are interviews with the actors, both before and after the first year of the program period. Each substudy highlights the role of one of the three involved groups of participants. The constant comparative analysis method and Cultural Historical Activity Theory (CHAT), used as the theoretical framework for further analysis, identified tensions and contradictions that seemed to restrain the teachers' development work.

This study provides insight into the opportunities and challenges inherent in the interaction that unfolds in the encounter between a practice-based professional development program and the complex, dynamic setting of the school where the program has been carried out. The thesis discusses how these challenges in terms of tensions and contradictions can impede the implementation of the PD program, but also how tensions and contradictions can be the point of departure for change and development. It also discusses how the interaction provides opportunities for boundary crossing that enables the development of a partially shared object. The discussion also illustrates how a partially shared object can serve as a means of translation, helping teachers to incorporate ambitious mathematics teaching, as a new idea, into the context of their own practice.

Acknowledgment

I am deeply honored and very pleased to find myself in the position of writing the foreword for my doctoral thesis. Undertaking my Ph.D. work was a difficult and arduous journey that required immense effort and dedication, but the knowledge and personal growth that I gained from the process made the endeavor truly worthwhile. Although my name is written as the single author of this thesis, many have contributed to its birth and growth, and to whom I wish to express my gratitude. First of all, I would like to thank all the participating teachers, school management, and teacher educators. Without your generous sharing of experiences, this study would lack empirical evidence. Furthermore, I would like to extend my deepest appreciation to my two supervisors, Professor May Britt Postholm at the Norwegian University of Science and Technology (NTNU) in Trondheim and Professor Reidar Mosvold at the University of Stavanger. With your skillful blend of constructive criticism and realistic optimism you expertly guided me throughout the Ph.D. process, for which I am truly grateful.

I would like to extend a big thank you to the Norwegian Centre for Mathematics Education (NSMO) for providing me with the opportunity to carry out this doctoral project and for their financial support. To all my colleagues at NSMO, I am grateful for your support, everyday conversations, academic discussions, laughter, and friendly teasing throughout my daily work, especially during the darkest times.

I would also like to thank my family and friends for their invaluable support. Most importantly, I want to thank my partner, Ingrid E. Bruvoll, for coping with a man who has dedicated his time to this Ph.D. work and handball refereeing. I cannot thank you enough for carrying the heavy load and being a single parent during this period; it meant the world to me. Last, but not least, I want to thank my two-year-old son for all the love, laughter, and joy you have brought me. You reminded me that life is much more than a Ph.D. At the same time, with your smile and presence, you provided invaluable support when I needed it the most. I dedicate this thesis to you!

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List of abbreviations

CHAT	Cultural Historical Activity Theory
MAM	Mastering Ambitious Mathematics teaching
NSMO	Norwegian Center for Mathematics Education
NTNU	Norwegian University of Technology and Science
OECD	Organisation for Economic Co-operation and Development
PD	Professional Development
TE	Teacher Educator

List of articles

This thesis includes the following three articles:

Article 1

Braseth, E. A. (2022). Mathematics Teachers' Perceptions of Teaching Practices Alignment with Ambitious Teaching. *Mathematics Teacher Education and Development, 24*(1), 23–38.

Article 2

Braseth, E. A. (2021). Principals' leadership of mathematics teachers' professional development. *Frontiers in Education* (p. 361). Frontiers. https://doi.org/10.3389/feduc.2021.697231

Article 3

Braseth, E. A. (in review). Teacher educators' experiences of a failed practice-based development program in mathematics. Manuscript submitted for publication in NOMAD.

1.0 Introduction

This thesis examines mathematics teachers' professional development (PD). The research explores how practice-based development programs, here exemplified by the Mastering Ambitious Mathematics teaching (MAM) program, can contribute to the PD of mathematics teachers. The study focuses on the roles of teachers, principals, and teacher educators (TEs) as program leaders and facilitators, and how these roles act and interact to support teachers' PD in the first year of the program period.

Investment in teachers' PD is substantial and the requirement for continuous development of competence beyond the basic education degree has been widely accepted by most professional groups, including teachers. The PISA 2015 results (Volume II) show that most teachers in high-income countries participate in some sort of (PD) activities (OECD, 2016). Moreover, international surveys suggest that the average time teachers spend on PD activities is 10.5 days per year (Sellen, 2016). This substantial investment in teachers' PD also reflects my own experience of being a teacher and a TE. Since I started my work as a teacher in 2011 (primary school in 2011, and lower secondary school in 2012) and at the Norwegian Centre for Mathematics Education (NSMO) in 2013, I have experienced several PD efforts for mathematics teachers that have been initiated by schools, district education authorities, and the national government, some of which I have attended as a teacher or as a facilitator.

Despite the substantial investment in money, time, and effort to improve teachers' PD, I have come to question the effectiveness of such investments, particularly those initiated by education authorities, during my work as both a teacher and facilitator. While there are variations in who initiates PD efforts for mathematics teachers and for what reasons, research indicates that these efforts are primarily initiated by national governments and district authorities (e.g., Darling-Hammond et al., 2017; Kennedy, 2016). They are typically motivated by such reasons as poor student achievement results and changes in the curriculum. However, the same research shows that the effectiveness of these PD programs may be limited due to various factors, such as a lack of alignment between the PD content and teachers' needs, insufficient time, and a lack of sustained support and resources. In response to this, research on teacher PD across various fields has identified key characteristics that are more likely to improve PD outcomes (Cordingley et al., 2015; Desimone, 2009; King & Stevenson, 2017; Putman & Borko, 2000; Timperley et al., 2007). These characteristics include sustained duration over time,

collaboration, high teacher buy-in, subject-specific focus, coherence, involvement of outside expertise, and opportunities to apply what has been learned in practice. The consensus among researchers on these characteristics has led to their widespread adoption as guidelines for designing effective PD, and they have also had a significant impact on policymakers. Moreover, Cohen and Mehta (2017) argue that successful educational reforms require a deep understanding of the political, cultural, and social contexts in which they are carried out. They maintain that policymakers, administrators, and other stakeholders need to work together to create the conditions for successful reform and to develop strategies for sustaining reform over time.

Despite the extensive knowledge that research on teacher PD has provided when it comes to what works and what does not, many efforts to effect change still fail (e.g., Cohen & Mehta, 2017). There are still significant gaps in our understanding of how to effectively design and conduct PD programs that can lead to lasting improvements in teacher practice and student learning. These experiences, considerations, and what I learned from research literature aroused my interest in mathematics teachers' PD, and as such serve as the point of departure for my thesis.

1.1 Background

Teachers' PD has been studied and presented in the literature in many ways and a consequence of this is the variety of definitions and descriptions that contribute to our understanding of what teachers' PD means and the work it involves. For example, teacher PD is defined by the Organisation for Economic Co-operation and Development (OECD) as "activities that develop an individual's skills, knowledge, expertise, and other characteristics as a teacher" (2009, p. 49). Elmore (2002) provides a similar definition, stating that PD comprises activities in which teachers and administrators engage to improve their practice. These activities contribute to their development of knowledge and skills which in turn will increase their ability to respond to external demands. Moreover, these activities take place in various settings, traditionally identified as official events, such as conferences, workshops, seminars, and degree programs (Burns & Darling-Hammond, 2014). Research has found that teachers' professional learning often takes place in formal settings, such as PD programs (Timperley, 2011). Avalos (2011) argues that the core of teacher PD can be defined as a threefold endeavor that involves teachers' learning, learning how to learn, and how their knowledge is used in efforts to support students' learning.

Research on teacher PD in mathematics teacher education has long had its focus on teacher properties, and much of this research argues that teacher's content knowledge is important (e.g., Hill et al., 2011). This discussion often stretches back to Shulman's (1986) construct of "pedagogical content knowledge" on which models of knowledge for teaching in mathematics have been built (e.g., Ball et al., 2008; Rowland et al., 2009). Scholars have also investigated what characterizes this body of knowledge, how it can be developed within teacher education, and the relationship between teacher knowledge, the quality of the teaching, and students' learning. (e.g., see Hoover et al., 2016 for a systematic review). In particular, the relationship between teaching quality and student learning, and how it is connected to teachers' practice, has received much attention recently. This attention to teachers' practice represents a shift in research focus from teacher knowledge to teaching practices (e.g., McDonald et al., 2013).

In relation to this shift in focus, researchers have attempted to identify key parts of the practice, so-called core practices or high-leverage practices (Forzani, 2014). To this end, an extensive body of research argues that teachers' PD should be connected to and contextualized within their practice and should enable them to develop their knowledge and ability to *use* new ideas (e.g., Ball & Bass, 2003; Ball & Even, 2009; Kennedy, 2016). Research on PD programs that reflect a practice-based pedagogy, such as the *Learning Teaching in, from, and for Practice* (LTP) project (e.g., Ghousseini, 2017; Lampert et al., 2013; Kazemi et al., 2016), Lesson Study (see Huang & Shimizu, 2016 for a systematic review), and the MAM program (e.g., Fauskanger & Bjuland, 2019; Wæge & Fauskanger, 2021, 2022) has become increasingly popular in studies on mathematics teacher education and teacher learning over the past two decades (e.g., Charalambous & Delaney, 2020). Such a focus on a practice-based pedagogy has been identified as an important characteristic of teacher PD. There is, however, less consensus among researchers as to what practice-based teacher education means, as the term has been used to underline how several programs differ from the traditional academic model of teacher education (Forzani, 2014). The research has, nevertheless, led to an understanding that teaching is a key part of the process of learning to teach (e.g., Ball & Forzani, 2009; Grossman et al., 2009; Lampert, 2010).

Even though research has provided knowledge about what is likely to make PD effective (e.g., Cordingley et al., 2015; Desimone, 2009; King & Stevenson, 2017), several evaluations of PD interventions have not found a positive impact, even when including all the key characteristics mentioned above. For example, Jacob et al. (2017) evaluated a PD program for in-service

mathematics teachers called the Maths Solution program. This evaluation found that the evidence of positive impact on teachers' mathematical knowledge for teaching was limited, and that the program had no effect on instructional practice or students' learning outcomes. Jacob et al. (2017) argued that a lack of leadership support, and that the PD itself was insufficient to change instructional practice as it failed to link knowledge to practice and fit the needs of the teachers, could be possible explanations for the findings. Furthermore, evaluations of mathematics PD programs designed to improve teachers' mathematical knowledge for teaching and students' mathematical thinking that did not make an impact suggest that more research is needed to improve what we know about which circumstances lead to PD yielding positive results (eg., Garet et al., 2016; Garet et al., 2011; Santagata et al., 2010). Kennedy (2016) finds the need to identify *how* to help teachers implement new ideas in their existing practice. She also claims that more attention needs to be placed on those people who lead PD in terms of what expertise they need as PD leaders and facilitators.

In addition to identifying key characteristics of effective PD, research also indicates that good leadership is important in promoting teachers' learning through PD, especially in terms of meaningful support (e.g., Akiba et al., 2015; King & Stevenson, 2017). Desimone (2009) suggests that the school principal can support teacher learning by creating a learning culture, providing resources, time, and encouragement, and by creating learning opportunities. If they are to accomplish this, they need to acknowledge their role as facilitators of teachers' learning and ensure that proper learning conditions are established by developing professional learning environments and creating a culture for continuous improvement (e.g., Gibbons et al., 2021; Kazemi & Resnick, 2020). Furthermore, Matsumura et al. (2009) and Whitworth and Chiu (2017) found that the principal's support was a key factor in the implementation of PD programs. However, there is still much to learn about how to enact good leadership that supports teachers' learning and students' learning outcome. For example, Kazmi and Rensinck (2020) argue that we need to know more about how to organize schools and develop systems to support effective teacher and leader learning. Gibbons et al. (2019) suggest that further research needs to explore effective strategies for coordinating different types of leadership support in promoting instructional improvement, and determining how to tailor these strategies to the needs of the teachers and school. Kazemi et al. (2022) maintain that more knowledge is needed to better understand the role of the principal's leadership in promoting the school-wide transformation of mathematics teaching and how to effectively support principals in promoting teacher learning and PD.

This research aims to investigate how teachers, a principal, and TEs experience carrying out the Mastering Ambitious Mathematics teaching (MAM) program, a practice-based PD program for in-service mathematics teachers. The study aims to understand how different roles influence mathematics teachers' PD work. The findings will provide insights into how different actors influence and are influenced by each other's actions in a PD program. The study will shed new light on the circumstances and conditions that promote or impede teachers' PD and how they can integrate new ideas into their existing practice. Additionally, the research will contribute to the discussion on the knowledge and competencies TEs need as PD leaders and facilitators for mathematics teachers (Kennedy, 2016), thus adding to our knowledge in this field.

1.2 The Mastering Ambitious Mathematics teaching program

The Mastering Ambitious Mathematics teaching (MAM) program, a practice-based development program for in-service mathematics teachers developed by the Norwegian Centre for Mathematics Education (NSMO), is the background for this Ph.D. project. The MAM program has been developed and contextualized to the Norwegian context from the LTP project (e.g., Ghousseini, 2017; Kazemi, et al., 2016; Lampert et al., 2013), a course that aims to promote opportunities for novice teachers when learning to enact ambitious teaching in practice (e.g., Kazemi & Wæge, 2015; Lampert et al., 2013; Wæge & Fauskanger, 2021). The LTP project uses specific instructional activities to collectively engage novice teachers through these activities in cycles of enactment and investigation[1] (Lampert et al., 2010). The cycle has six steps: *observation*, *collective analysis*, *preparation*, *rehearsal*, *classroom enactment*, and *collective analysis* (e.g., Kazemi, 2017; Lampert et al., 2013). They are repeated routines that provide opportunities for novice teachers to both practice teaching and reflect on that practice. These cycles are then organized around a set of specific pedagogical teaching practices and principles[2] to support student learning and novice teachers when it comes to learning to teach. The practices are identified as key aspects for supporting students' learning (Grossman et al., 2009; McDonald et al., 2013). A core practice model for teacher education such as the cycle of enactment and investigation has the defining feature of focusing on specific pedagogical practices, which in turn are associated with ambitious mathematics teaching (Forzani, 2014; McDonald et al., 2013). The MAM program attempts to adapt this pedagogy of ambitious mathematics teaching to the Norwegian context. The work has led to the development of a

[1] A thorough description of the cycle of enactment and investigation is provided in section 4.1.
[2] A thorough description of both the practices and principles is provided in section 4.2.

model and related resources for practice-based PD for in-service teachers in Norway (e.g., Fauskanger & Bjuland, 2019; Wæge & Fauskanger, 2021).

The MAM program, briefly described, has been included in several PD projects since its inception and is currently included in several ongoing PD projects for mathematics teachers. Based on experience and research, the program has been refined and evolved throughout its existence. To avoid misconceptions, I will refer to the specific MAM program as "MAM 2019" in this study.

1.3 Objectives

Considering the background and the MAM program outlined above, this research project has been designed and developed according to the following two objectives that also highlight the relevance of the thesis.

The first objective is to contribute to what we know about how principals and program leaders can support lower secondary school teachers' learning of ambitious mathematics teaching through a practice-based PD program, here exemplified by the MAM program. Even though the original design of the LTP project was to support teacher students in learning to enact practices of ambitious mathematics teaching (e.g., Kazemi & Wæge, 2015; Lampert et al., 2013), efforts have been made to adapt this pedagogy to in-service teachers' PD (Gibbons et al., 2017). However, evaluation of these efforts suggests that further research is needed to explore how to support in-service teachers in learning to enact ambitious mathematics teaching, and what in-service teachers can learn from participating in such PD programs. The current research on the MAM program and the LTP project has mainly focused on primary school teachers and their development, leaving a gap in knowledge about how other actors involved can support and contribute to the teachers' learning. This thesis aims to close this gap by exploring how the MAM program can contribute to mathematics in-service teachers at the lower secondary school level, and also by providing new knowledge on the support in-service teachers need to learn ambitious mathematics teaching through the program, how this support can be facilitated, and the factors that promote this learning. This can then contribute new insights into the ongoing discussion on which specific competencies PD leaders and facilitators need, and how to help teachers to implement new ideas in their existing practice (Kennedy, 2016).

The second objective of this study is to examine how the MAM program can contribute to the PD of lower secondary mathematics teachers in the context of the Norwegian government's new model for competence development. The new model, introduced in 2017, emphasizes decentralized competence development, where the aim is to strengthen collective competence in schools based on local needs. This new model represents a departure from previous approaches where the national authorities were responsible for initiating and governing development measures. Instead, the school owner (local authority) is now responsible for identifying local development needs in collaboration with universities and implementing development measures based on those needs. These measures often include teacher PD programs, such as the MAM program, which is currently used by schools as a mathematics teacher PD activity and will likely continue to be used in the future. This thesis focuses on the various actors involved in practice-based PD programs for mathematics teachers, such as the MAM program. It investigates how these actors, both individually and in interaction, support and facilitate teachers' PD within their contexts. By improving our knowledge on this issue, this thesis can contribute to understanding the conditions required for the interaction between practitioners and PD leaders to blossom. This will also provide information about how this interaction can promote teachers' PD in relation to the new model for competence development.

1.4 Aim, focus, and the design of the study

In my Ph.D. project I explore how the MAM program can contribute to mathematics teachers' PD. More specifically, the study focuses on the roles the teachers, the principal, and the TEs as the program leaders occupy as different actors in the teachers' PD work, and how these roles act and interact to contribute to the teachers' PD during the first year of the program. My main problem statement and the subsidiary research questions addressed in this study provide the framework for the study:

How can a practice-based development program contribute to mathematics teachers' professional development?

Research questions:

1. What perceptions do three Norwegian lower secondary mathematics teachers have about classroom practice and students' learning?
2. How does school management support and facilitate mathematics teachers' professional development as they participate in a practice-based development program?
3. How do teacher educators experience a practice-based development program for mathematics teachers?

The study is part of a mathematics teacher development project in Norway consisting of ten primary and four lower secondary schools located in the same district. This mathematics teacher development project has its origins in an enquiry from the school owner, on the behalf of the schools, who requested the NSMO to contribute to their development work. NSMO offered them the MAM program, which was found interesting. The decision was made to start the MAM 2019 development project in the fall of 2019. A qualitative case study (Creswell, 2013) was designed to study the first year of MAM 2019, in which one of the participating lower secondary schools was chosen as the context of the study. Two data-collection periods were planned, each lasting three weeks. The first was planned for the start of the program in the fall of 2019, and the second after the last plenary session in the first year of the program, in the spring of 2020. The plan was for me as a researcher to be present during the three weeks to become acquainted with the school and get a grasp of its daily life. It is important to have this acquaintance to be able to give a thick description of the context where the research is conducted (Stake, 1995), and it can help to better understand what is really going on when analyzing the data material (Corbin & Strauss, 2008; Strauss & Corbin, 1990, 1998). The data collection was

designed to include multiple methods: observations of a teacher's classroom practice, individual semi-structured interviews (Brinkmann & Kvale, 2015) with three mathematics teachers, and focus-group interviews (Kamberelis & Diamitiadis, 2011) with mathematics teachers, the school management, and the TEs, separate for each group.

The thesis comprises three articles that aim to answer the three research questions. The first article focuses on mathematics teachers' perceptions on teaching and students' learning before they enter a practice-based development program. In this article I report from individual semi-structured interviews (Brinkmann & Kvale, 2015) of three mathematics teachers conducted in the fall of 2019 before the program period started. Based on the findings, the article discusses how the three teachers' perceptions of teaching and student learning align with the principles of ambitious mathematics teaching, and how these perceptions might influence their participation in MAM 2019.

The second article focuses on the role of the principal in supporting and facilitating teachers' PD during their participation in MAM 2019, and how teachers experience this support. The data material in this article has been taken from four focus-group interviews (Kamberelis & Diamitiadis, 2011), two with five mathematics teachers, and the other two with the school management. The constant comparative analysis method was used to structure and conduct an initial analysis of the data material (Charmaz, 2014). With a focus on the principals' role, Cultural Historical Activity Theory (CHAT) and the activity system (Engeström, 1987, 1999, 2001) were used as the theoretical framework to further analyze and discuss the findings.

The focus of the third article is on the TEs' role and their experiences in leading the first year of a practice-based development program, here represented by MAM 2019. This article reports from a focus-group interview (Kamberelis & Diamitiadis, 2011) and a follow-up interview of three TEs after the first year of the program. The activity system and a network of systems (Engeström, 1987, 1999, 2001) were used as the theoretical framework for analyzing and discussing the findings.

While focusing on the main research question, I have used CHAT[3] (Engeström, 1987, 1999, 2001) as the theoretical framework for analyzing and discussing the findings reported in the three articles. CHAT, and particularly its third generation, which involves interacting networks of activity systems, provides opportunities to analyze various activity systems where the various subjects act on the object that is partially shared between the systems (Engeström, 1987, 2015). As this study focuses on the various actors' roles in the MAM program and their contributions to the teachers' PD work through their actions, these actors can be seen as subjects acting in different systems. As the program leaders, the TEs contribute knowledge on ambitious mathematics teaching and activities for developing core practices, the principal supports and facilitates the teachers' development work as they participate in the program, and the teachers themselves contribute to their own and their colleagues' PD by actively taking part in the program. In other words, all three actors contribute to the teachers' PD through their goal-directed actions.

To investigate how the MAM program can contribute to teachers' PD, it is important to identify and study aspects that promote or restrain the development process. One of the key features of CHAT as a tool for analyzing human activity is its capacity to identify potential starting points for development and change. Engeström and Miettinen (1999) argue that tensions and contradictions can arise and constitute the foundation for development and change. By using the activity system and a network of systems as the theoretical framework for analyzing the findings from the three substudies, tensions and contradictions both within and between the activity systems can be discovered (Engeström, 2001). Thus, CHAT provided a powerful framework for analyzing how the three TEs contributed to the teachers' PD when participating in MAM 2019.

[3] CHAT and its concepts are elaborated on in section 3.2.

2.0 Literature review

This chapter presents a review of the existing literature on the particular topic related to this thesis, which is situated within the research on PD for teachers. In addition to providing an overview of research on teachers' PD, the literature review will synthesize research on practice-based PD, specifically related to the MAM program. Moreover, the review will address the leadership of teachers' learning to examine what is known about supporting teachers' PD. Along with providing a synthesis of related research on these topics, the review will also include a historical overview of key terms and concepts that have shaped the current state of the available research. This overview will show how my study fits into a broader context within the field.

2.1 Teachers' professional development

In general, researchers have long recognized that teachers' PD is vital to successful school reform and student learning (e.g., Desimone, 2009; Darling-Hammond, 2017), and is essential for successful change in classroom practice (Borko, 2004; Sztajn et al., 2017). To this end, scholars have found several factors important to take into consideration to ensure that these efforts directly or indirectly lead to teacher learning and change. For example, in a review of teachers' PD in school, Postholm (2018) points to several studies that underscore the importance culture, structure, and practice have for teacher learning and the development of collaborative teacher learning processes, both isolated and in interaction, which in turn can ensure school improvement. Moreover, research has shown that developing teachers' PD involves determining both what to develop and how to develop it. According to Postholm et al. (2013), the development of content and process should go hand in hand and be integrated into the development work. Furthermore, scholars report on successful results of teacher PD efforts when they are made together *with* teachers, rather than being designed with a top-down approach (e.g., Clarke & Hollingsworth, 2002; Nilsson, 2014). A similar distinction is suggested by Berry and Loughran (2010), who discuss traditional and newer forms of teacher PD. They argue that traditional forms, such as one-off courses and lectures, are ineffective for developing teachers' pedagogical content knowledge because they do not offer opportunities for sustained, collaborative learning. Instead, these forms of PD can be experienced as prescriptive and limiting, providing teachers with predefined solutions rather than empowering them to find their own (Berry & Loughran, 2010). In contrast, the authors argue that newer forms of PD, such as teacher networks, lesson study, and long-term practice-based programs,

are better suited to support teachers' learning and development. These approaches emphasize ongoing, collaborative learning opportunities that are situated in teachers' own context, allowing them to actively engage in the learning process (Berry & Loughran, 2010). These findings are supported by Roseler and Dentzau (2013), who argue that teachers might experience top-down approaches to teacher PD as de-professionalizing. Kennedy (2016) also argues that the effects of any PD program depend heavily on teachers' motivation to learn and to change their practice. She states that mandatory assignments may not influence teachers' learning as they most likely will forget about the program when they return to their classrooms.

When PD efforts are made together with teachers, they are invited to take part in decision-making processes. Knowles et al. (2005) claim that incorporating teachers in decision-making processes and seeing them as the heart of the decision-making around change constitutes a key principle in understanding, engaging, and developing ownership in adult learning. This principle is supported by Tan and Caleon (2016), who claim it is vital that teachers participate in the development work from the very beginning by contributing to defining the problem to work on, and within this process, learning will emerge. Feeney (2016) has also identified shared decision-making among practitioners as a key factor in supporting professional learning. He argues that a lack of communication and a lack of shared vision act as constraints on professional learning. Research has also shown that these processes are also important if teachers are to develop ownership of the PD. Al-Mahdi and Al-Wadi (2015) point to the importance of ownership in teacher PD and how it can lead to meaningful and relevant PD experiences. They argue that teacher ownership of PD is crucial for its success and sustainability as it allows teachers to take on an active role in their own development and promotes a sense of responsibility and accountability. Research also shows that the context in which the development effort unfolds matters. Scholars suggest that development programs contextualized out of school are limited in their connection between teacher learning and their actual practices (Desimone, 2009; Villegas-Reimers, 2003). An extensive body of research supports this idea, arguing that development efforts should enable teachers to develop their knowledge and ability to use new ideas and that the development effort, for this reason, should be connected to and contextualized within their practice (e.g., Ball & Bass, 2003; Ball & Even, 2009; Kennedy, 2016).

Bearing in mind the knowledge that research on teacher PD has provided, scholars have attempted to identify characteristics of PD to help principals and teacher educators (TEs) to

design and provide effective PD. Even though the research conducted after Corcoran (1995) devoted much attention to determining which characteristics of PD improve pupil attainment, there was little agreement on this among researchers in the following years (Guskey, 2003). However, as mentioned in the introduction, researchers have more recently found that PD is more likely to be more effective if it incorporates some key characteristics (e.g., Cordingley et al., 2015; Desimone, 2009; King & Stevenson, 2017; Putman & Borko, 2000; Timperley et al., 2007). It has been found that PD is more likely to improve pupil attainment if it is sustained over time, collaborative, has a high degree of teacher buy-in, is subject-specific, coherent, involves outside expertise, and involves opportunities to apply what has been learned in practice. These characteristics are often referred to as a consensus in the research field (e.g., Darling-Hammond et al., 2017; Desimone 2009; Wei et al., 2009), and have obtained a prominent impact on policymakers when it comes to how PD should be designed.

2.1.1 Teachers' prior knowledge and current practice as the starting point for their learning

Findings from several studies of teachers' PD show that teachers' current practice and knowledge need to be considered when new ideas about teaching and knowledge of teaching are to be implemented. For example, Pokhrei and Behara (2016) see teacher development as an ongoing process where teachers continue to grow through their own voluntary efforts, and they suggest that PD programs must address teachers' expectations and challenges. In their review of existing professional learning programs, Smith and Lindsay (2016) found that providers of external support should examine the current practice in school before providing learning opportunities for teachers. This is to ensure that programs provide opportunities where the teacher is always positioned as an active and empowered learner.

Exploring teachers' current practice and knowledge also contributes to identifying their development wants and needs. According to Watson (2015), the starting point for teachers' PD should be an assessment of their needs within their own school and classrooms. Moreover, this process is crucial for fostering teachers' sense of ownership of and engagement in their professional growth (Engeström & Sannino, 2010). Research suggests that a needs assessment has to be conducted before the start of the PD to ensure coherence with teachers' needs (Lindvall & Ryve, 2019). Moreover, when examining literature that focuses on teacher development, teacher learning, PD, and PD reform, Matherson and Windle (2017) found that teachers want PD opportunities that are interactive, engaging, and relevant for their students, that show them a more practical way to deliver content, are teacher-driven, and sustained over

time. Liljedahl (2018) argues that teachers approach professional learning opportunities with a complex collection of wants and needs, both conscious and unconscious, which they attempt to satisfy using PD opportunities as resources. Teachers' wants act as an agenda that guides their participation and determines the extent to which they seek and allow the development effort to influence their practice (Liljedahl, 2018). Similarly, Timperley et al. (2007) argue that teachers have different professional learning needs, and which needs are to be learned depends on their prior learning, skills, and dispositions. Teachers' PD should therefore commence with an understanding of teachers' needs at their own schools and in their classrooms to ensure that the development work is based on these needs. Scholars have also found that local education authorities, researchers, and facilitators should take leaders' and teachers' needs into consideration by inviting them to offer their opinions (Postholm, 2020). Postholm (2020) argues that these processes need to be carried out at the very beginning of the development work and serve as the foundation for ongoing and sustainable development. Therefore, more attention should be devoted to the start-up phase of development work so the teachers are supported in developing an understanding of its goal and why they should act on it (Postholm, 2008, 2020).

In addition to identifying teachers' development wants and needs, exploring and revealing their prior knowledge and practice can uncover their theories of action, which influence their learning outcomes from PD efforts. Timperley et al. (2007) argue that experienced teachers approach professional learning situations with rich theories about good teaching and students' learning in tow, and that these theories have a powerful effect on how they understand new learning experiences and how they are integrated into practice. "The extent to which new information is used is strongly influenced by the extent to which conceptual understandings and practical resources offered through the learning experience make sense to the recipients in terms of their existing understandings and practice contexts" (p. 7). As such, the set of theories and understandings the teachers bring with them to professional learning situations can be an advantage in terms of acquiring and integrating new knowledge, but only when the new information fits with their theories and practice. In the opposite case, Timperley et al. (2007) point out that the perspectives, theories, and understandings concerning teaching practice that the teachers currently have must be challenged in order to be reconstructed, but also that these are difficult to detect as they are usually tacit and difficult to articulate. As such, cueing and retrieving teachers' current knowledge and practice can both serve as a point of departure and lay the foundation for teachers' learning.

In the same way that teachers have theories of action that influence their learning in development work, external PD programs that teachers attend also have underlying theories of action that inform their actions and approaches. In a review study, Kennedy (2016) sorted PD programs according to their underlying theories of action when examining how these programs improve teaching. These theories of action included two central aspects: "a main idea that the teachers should learn, and a strategy for helping teachers enact the idea within their own ongoing systems of practice" (p. 945). This understanding of a PD program's theory of action comprises both the problem of practice that it aims to inform and the pedagogy used to help teachers to enact new ideas and translate them into their own practice. Kennedy (2016) argues that PD facilitators who work with practicing teachers are not merely offering a new idea but rather a different idea from the one that has guided teachers in the past. Timperley et al. (2007) take this one step further by arguing that professional learning experiences that seek to change practice need to help teachers understand the theories of action underpinning them. If not, the new learning might fail to be integrated with existing theories and as such be rejected as it does not correspond with their existing theories. Desimone and Garet (2015) found similar results, indicating that PD programs are more effective and have better implementation when they are explicitly linked to teachers' classroom lessons, attributing the failure of PD programs to the absence of this link. According to the authors, the effectiveness of PD is reduced when it fails to encourage teachers to incorporate new knowledge and strategies into their daily instructional routines and lessons. Therefore, they propose that PD should be designed with a focus on the ease of integrating it into teachers' lessons "and include support, guidance, and practice for teachers to integrate the knowledge or pedagogy into their daily instruction, rather than leaving that burden to them when they return to the classroom." (p. 256).

2.2 From knowledge to practice

Mathematics teacher education research emphasizes the importance of teacher content knowledge as reflected in the notion of pedagogical content knowledge introduced by Shulman (1986). Various models of knowledge for teaching in mathematics have been developed according to this concept. Scholars have investigated the characteristics of this knowledge, its development in teacher education, and its relationship to teaching quality and student learning (Hoover et al., 2016). This research shows that professional teacher knowledge has taken an important position in research in mathematics education in recent decades. While highlighting this importance, Charalambous and Pitta-Pantazi (2015) point to the long-recognized complexity of linking this knowledge with teaching practices and students' learning. Da Ponte

and Chapman (2016) echo this stance when discussing the complexity of mathematics teacher education. They point out that the development of mathematics teaching and knowledge of mathematics, key elements in this discussion are distinct but inherently connected. When trying to link practice and knowledge, researchers have found practice-based pedagogies of teacher PD to be important.

2.2.1 Practice-based pedagogy

It is now almost two and a half decades since Ball and Cohen (1999) suggested that mathematics teacher education should focus on how teachers learn "in and from practice" (p. 10), rather than focusing on what they should learn in preparation for practice. Much has happened since then and in a review of the literature on mathematics teaching practices and their teaching and learning since 2000, Charalambous and Delaney (2020) found that research on practice-based pedagogy in teacher education has increased in popularity. Even though practice-based pedagogy has been used to describe a wide range of approaches in teacher education (Forzani, 2014), it underscores the importance of engaging teachers to enact practice rather than only discussing its theoretical aspects. Thus, research on practice-based pedagogy in education has led to an understanding that teaching is a key part of the process of learning how to teach (e.g., Ball & Forzani, 2009; Grossman et al., 2009; Lampert, 2010). To this end, practice-based pedagogies as an approach to teacher education have contributed to a shift in focus away from the knowledge needed for teaching and to practices that require this knowledge (e.g., McDonald et al., 2013). Lesson Study (see Huang & Shimizu, 2016 for a systematic review), the LTP project (e.g., Ghousseini, 2017; Lampert et al., 2013; Kazemi et al., 2016), and the MAM program (e.g., Fauskanger & Bjuland, 2019; Wæge & Fauskanger, 2021, 2022) are examples of this shift. In recent years, there have also been several efforts that try to revamp or restructure existing teacher education programs or courses to better reflect practice-based pedagogies. Charalambous and Delaney (2020) found that these attempts are characterized by a focus on particular core or high-leverage practices. Decomposing practice into identifiable teaching practices often serves as the starting point, and representations of practices are used to help teachers to focus on and analyze the work of teaching.

According to Anthony et al. (2015) and Charalambous and Delaney (2020), the shift toward practice-based teacher education has taken two significant directions. The first focuses on identifying teaching practices that relate to teaching in general, and practices of teaching and learning specific school subjects, such as core or high-leverage practices in mathematics. The second relates to the use of pedagogies of practice in a broad perspective, which according to

Grossman et al. (2009) involves the three elements: representations of practice (e.g., records of teaching practice, observations, or student work); decompositions of practice (e.g., identification of specific components of practices such as talk moves); and approximations of practice (e.g., rehearsal, which is a simulation of certain aspects of practice).

While using Grossman et al.'s (2009) framework, Charalambous and Delaney (2020) looked at the empirical evidence that practice-based pedagogies have contributed. They found that studies reporting on opportunities for teachers to enact and reflect upon their own practice largely focused on the opportunities the environment created for supporting teacher candidates to enact certain practices in which opportunities to reflect on their own and colleagues' teaching is a critical aspect. This focus relates to scholars' attempts to explore and explicitly describe how the approach can be utilized to support teacher learning, and one example they highlight is rehearsals (e.g., Averill et al., 2016; Ghousseini, 2017; Kavanagh et al., 2020; Wæge & Fauskanger, 2020, 2022), which they argue provide such opportunities for reflection through debriefing sessions with the actors involved. Furthermore, Charalambous and Delaney (2020) found that studies have mainly focused on representations of practice, for example, videos or observations of teaching, and reported on how these representations contributed to developing teachers' ideas and perceptions about teaching. They also found that studies considering teaching practices mainly focused on a single practice, which mostly concerned the practice of leading classroom discussions. The authors argue that the focus on this particular practice is due to its frequent appearance in lists of core or high-leverage practices and the extensive research on orchestrating classroom discussions. I will elaborate more on core and high-leverage practices in section 2.3.1. In addition to this, Charalambous and Delaney (2020) found that the participants in these studies are teacher candidates and that the studies reported a notable variation in what they reported concerning their learning. For example, some studies highlighted how the teacher candidates' participation in practice-based learning environments contributed to a change in their development work, while others focused on mechanisms facilitating their learning. However, the one-sided emphasis on teacher candidates leads to a lack of empirical evidence from studies with other actors, such as experienced teachers. Charalambous and Delaney (2020) therefore call for research that explores the role of such practice-based learning environments in supporting the learning of teachers with different work experiences. They also maintain that as the majority of research on practice-based pedagogy has been carried out in an American context there is a need for studies in other countries.

With a practice-based pedagogy in mind, research has in recent years shown a growing interest in exploring how to bring PD programs to scale. In a review of research on mathematics PD, Sztajn et al. (2017) examined 144 papers published after 2005 that focused on PD programs for in-service K-12 mathematics teachers. Their aim was to examine what the field knows from research on mathematics PD programs that were purposefully planned to achieve specified learning goals for teaching and learning. This included facilitation to foster such learning that opened for the examination, refinement, and replication of a model for PD in mathematics. In this sense, this review builds upon studies that support the previously presented research showing that teacher learning is active, situated, and social (Putnam & Borko, 2000), and closely interrelated with practice (e.g., Ball & Cohen, 1999; Webster-Wright, 2009). Even though Sztajn et al. (2017) mostly found studies that examined PD offered at a single site, the review also shows a growing number of studies focusing on how to bring PD programs to scale. These studies emphasize the importance of the context where teacher learning experiences are designed and the roles of the school and district leaders as supporters of teachers' learning. The latter implies supporting both a vision of mathematics learning improvement and PD activities at school. Santagata et al. (2020) argue that PD programs that do not reach the desired outcomes can be the result of a disconnect between researchers and practitioners, conflicting visions, and a PD program design that does not take the systemic nature of teacher learning into account.

The importance of context is also underscored by Givvin and Santagata (2011) who in their study of evaluating a mathematics PD program for a middle school in the US found that there is a need to consider the institutional context in which PD is implemented and its role in shaping teacher participation. This finding is supported by Kazemi and Resnick (2020) who argue that researchers and practitioners need to work together to find more distinct ways of thinking about the processes of implementation of programs at scale because what works at one school cannot simply be copied and expected to work in another. Research on bringing PD efforts to scale has also pointed out the importance of ensuring that ownership is situated within the school. Coburn (2003) argues that bringing PD efforts to scale requires that the four important dimensions depth, sustainability, shift in reform ownership, and spread are addressed. According to Coburn, the aim must be to make reform efforts an internal rather than an external reform. She maintains that schools that take part in external reform efforts do not have ownership of the reform, as the knowledge and authority are mostly situated with the providers. Thus, conditions must be created that support the actors at the local schools to take ownership of the development work by shifting knowledge and authority from external stakeholders to stakeholders at school.

Research-practice partnership and boundary crossing

The above-mentioned research on mathematics PD related to approaches and artifacts for supporting teacher learning and change in teacher practice underscores the importance of collaboration and the context in which teacher learning experiences are designed, and the roles of the school and district leaders as supporters of teachers' learning. In the field of education, research has turned to research-practice partnerships to manage these key points. According to Coburn et al. (2013), research-practice partnerships can be distinguished from more traditional collaborations between researchers and practitioners in terms of four significant features: they are long term, focusing on problems of practice, committed to mutualism, use intentional strategies to foster partnership, and produce original analysis. In addition to the time span through which a working collaboration is maintained, the long-term feature allows for the development of trust between the partners involved where they can address and engage deeply in extensive questions. The focus on problems of practice shifts the researchers' attention from covering gaps in research with a pre-defined interventional approach to being more open-minded to the practitioners' needs and questions. The commitment to mutualism refers to the sustained interaction that helps to ensure that different perspectives are revealed and that a common goal is developed. This requires a careful process of relationship building and negotiation of priorities, which in turn contributes to the development of shared ownership and the possibility to learn from each other. This work of building partnerships is also organized according to intentional strategies using specific artifacts that guide the work. Producing an original analysis of data provides the basis for answering questions posed by the practitioners and also supports efforts to improve practice. As such, research-practice partnerships provide new insights into our understanding of the relationship between researchers or PD facilitators and practitioners engaged in PD efforts. Research-practice partnerships see context as an integral aspect of a complex system of improvement rather than only seeing contextual variables as factors that either restrain or support PD outcomes (Henrick et al., 2015). In this sense, this approach provides tools that contribute to integrating a research-practice relationship in the design of PD efforts.

Research-practice partnerships illustrate how researchers and practitioners can work together to advance knowledge on linking together teacher learning, leadership learning, and student learning. However, this requires relationship building between actors representing different practices in the context of the collaboration, especially because research-practice partnerships rely on effective sharing of knowledge and expertise across the systems these actors represent.

Santagata et al. (2020) argue that research-practice partnerships can benefit from the concept of boundary crossing (e.g., Akkerman & Bakker, 2011)[4]. They posit that boundary crossing provides opportunities for capacity building and professional growth for all actors involved in research-practice partnerships. The authors also point out that this type of work requires that the organizations the various actors represent create settings, devote time, and value the time and energy needed for individuals to engage in research-practice partnerships. Thus, they suggest that leaders play a fundamental role in crossing boundaries between teachers and researchers. However, they admit that institutionalizing this approach for educational improvement will take time as the culture within both PD and research often aspires for quick solutions and productivity.

Robutti et al. (2020) argue that partnerships between teachers and researchers can be viewed as a meeting between two professional communities, which can be further understood as joint work at boundaries. While focusing on the role of boundary objects in the context of collaboration between teachers and researchers within three PD programs for mathematics teachers, Robutti et al. (2020) characterize the idea of boundary objects in the context of these programs and highlight the learning mechanisms that can evolve from the interaction between different communities working on boundary objects. They point out that boundary objects are complex in structure but that actors' actions on specific structural components of the boundary object can foster a deeper understanding of it and extend the space within which they encounter each other. However, they underscore that a necessary condition is that the actors interact with the boundary object for it to exist and also evolve. Furthermore, they argue that translation actions on particular structural components of the boundary object make it possible for the different communities to agree on shared meanings for them. Through analysis of three PD programs, they found that the teachers and researchers developed different discourse levels when working on the boundary object that made it possible to have different levels of sharing between the communities. These levels of sharing concerned: transfer, through the development of a common vocabulary; translation, through a common interpretation of the vocabulary they use; and transformation, through referring to common new knowledge.

[4] A thorough description of boundary crossing and how it is understood in this thesis is found in section 3.2.2.

2.3 Ambitious mathematics teaching

Ambitious mathematics teaching is described as an approach to teaching that attends to the learning of all students and aims to deepen all students' understanding of complex mathematical ideas and performances (Lampert et al., 2010; Lampert et al., 2013; McDonald et al., 2013). This approach to teaching comprises the intellectually and socially ambitious goals of mathematical proficiency (Kilpatrick et al., 2001), which is the aim all reforms in mathematics education are striving to achieve. Ambitious mathematics teaching is underpinned by the notion that all students can "develop positive mathematical identities and become powerful mathematical learners" (Anthony & Walshaw, 2009, p. 6), and is recognized as challenging and yet vital for novice teachers to learn (Anthony & Hunter, 2012; Lampert at al., 2010). This involves finding skilled ways to elicit and respond to all students so they can learn meaningful mathematics and come to view themselves as competent mathematicians. Lampert et al. (2010) argue that this form of teaching supports learners to do mathematics competently, to make sense of what they do, and to be able to use their knowledge and skills to solve authentic problems. As such, it requires a teaching practice where the teacher engages deeply with all students' thinking and where their instructions are adjusted accordingly to promote students' learning. This also requires specialized knowledge for teaching and teaching mathematics, together with skills in orchestrating instructional activities and creating learning communities (Averill, 2012; Hunter & Anthony, 2011). Thus, ambitious mathematics teaching is an inherently interactive approach that supports meaningful participation for a wide range of students so they become powerful learners. This can then help to reverse the longstanding assumptions about who can and who cannot do mathematics (Averill, 2012).

2.3.1 Principles and practices

Ambitious mathematics teaching consists of a set of principles relating to student and teacher learning that guide the teachers in the use of classroom practices and mathematical knowledge (Kazemi, 2017). Some of these principles are: treating students as sense-makers; engaging deeply with students' thinking; designing instruction so that all students have equitable access to learning; having a clear instructional goal; and considering teaching as both intellectual work and a craft (Ghousseini et al., 2015; Kazemi, 2017; Lampert, 2013). Furthermore, Ghousseini et al. (2015) argue that these principles are a shift away from the traditional approach of transmitting information to students, also known as the teacher-centered teaching approach (Boaler, 2002), and toward a more student-centered approach. The development of these principles is the result of a partnership between researchers, teacher educators, and successful mathematics teachers to support the development of students' conceptual understanding (Ghousseini et al., 2015). These principles further rely on an ambitious view of teaching where a classroom community is developed and where discussion is valued. Within such a community, the teachers strive to elicit and respond to student reasoning, orienting students' ideas to one another as well as the mathematical goal, and positioning students as competent (Kazemi et al., 2009). This requires that the teacher focuses on the way the students make sense of mathematics, as well as the way they relate to each other, both socially and mathematically. It also requires that the teacher creates an inclusive learning environment that is intellectually rigorous and socioemotionally supportive, where students' experiences are taken into consideration, and where meaningful participation in mathematics is supported for all (Ghousseini et al., 2015). To help teachers develop this view of teaching, scholars have identified a set of core or high-leverage practices that are identifiable components of instruction that teachers enact to support learning and consist of "strategies, routines, and moves that can be unpacked and learned by teachers" (Grossman et al., 2018, p. 4).

Core practices can be defined as: "Identifiable components of teaching that teachers enact to support learning. These components include instructional strategies, and the subcomponents of routines and moves. Core practices can include both general and content-specific practices" (Grossman, 2018, p. 184). These practices are identified as vital components in supporting students' learning (Grossman et al., 2009; McDonald et al., 2013). They can be characterized as practices that: occur with high frequency in teaching; can be enacted by teachers in classrooms across different curricula or instructional approaches; allow for teachers to learn more about students and about teaching; preserve the integrity and complexity of teaching; and

are research-based and have the potential to improve student achievement (Grossman et al., 2018). A core practice model for teacher education has the defining feature of focusing on specific pedagogical practices, which in turn are associated with ambitious mathematics teaching (Forzani, 2014; McDonald et al., 2013). Core practices are also referred to as high-leverage practices (Forzani, 2014). These practices are considered to be an approach to teaching that is a vital part of helping the student to learn important content (Hatch & Grossman, 2009; Lampert, 2010; TeachingWorks, n.d.), and developed with a view to supporting teacher students in learning to teach. Ball and Forzani (2010) describe high-leverage practices as "the heart of the work of teaching that are most likely to affect student learning" (p. 43). They go on to explain that these practices include the most fundamental activities of teaching, and that a competent enactment of these practices serves as a foundation for new teachers to develop into highly effective professionals. Using TeachingWorks together with teachers and students, researchers at the University of Michigan have identified 19 high-leverage practices that include leading a group discussion and eliciting and interpreting student thinking (Grossman, 2018).

There is, however, disagreement on the practices identified within the two concepts, for example when it comes to how they are classified and for what reason, and how the practices relate to one another (Charlambous & Delaney, 2020). According to Charlambous and Delaney (2020), there are differences that need to be examined to achieve further consensus in order to develop a common language. Despite the lack of clarity, both core practices and high-leverage practices are identified as important for supporting novices so they can develop ambitious teaching and high-quality teaching (Forzani, 2014; Lampert et al., 2013; McDonald et al., 2013). There is also some agreement on identifying such important practices as eliciting, responding to students' ideas, and leading classroom discussions (Charlambous & Delaney, 2020; Forzani, 2014). In this study, I will draw on Grossman et al. (2018) who refer to the set of high-leverage practices identified by TeachingWorks as one of the better-known sets of core practices of teaching, and thus treat "core" and "high-leverage" as terms describing the same practices. The specific practices the MAM program draws on will be presented in section 3.3.

2.3.2 MAM-related research

The literature on the MAM program relates first and foremost to the studies conducted in Norway. However, the MAM program is based on a teacher education course aimed at helping pre-service mathematics teachers (LTP project) to enact ambitious mathematics teaching practices (Lampert et al., 2010; Lampert et al., 2013). Because research on ambitious mathematics teaching especially related to the LTP project has had an impact on the MAM program and its related research, I will include some of this research in this review.

Gradually more and more research has been conducted on practice-based teacher education in mathematics where the aim is to facilitate the development and enactment of core practices for ambitious teaching for novice teachers, teacher candidates, and prospective teachers. This research has mainly focused on the structural features of particular approaches, such as rehearsals, and the nature of the interactions between teacher educators and the aforementioned teachers (see for example, Averill et al., 2016; Kazemi et al., 2016). Much of this work is related to the LTP project, both in terms of designing and studying rehearsal with novice teachers (e.g., Ghousseini, 2017; Kazemi et al., 2016; Lampert et al., 2013). Findings from these studies show that rehearsals provide opportunities for novice teachers to learn an adaptive form of teaching. The researchers argue that rehearsal allows for shared decision-making and the development of knowledge, skills, and identities as an ambitious mathematics teacher. It is also found that rehearsals allow for question sequences that support novice teachers in learning to elicit and respond to students' thinking that builds on the principles of ambitious mathematics teaching (Ghousseini et al., 2015). Lately, scholars have also studied rehearsals in PD. For example, Kavanagh et al. (2020) found that reducing choices in the rehearsal setting makes it possible for teachers to increase their focus on how best to give full attention to, understand, and respond to student ideas.

The interest in research on structural features also applies to the literature related to the MAM program. For example, Fauskanger and Bjuland (2019) explored how the cycle of enactment and investigation provides opportunities for teachers to develop ambitious teaching of multiplicative properties. They found that the provided opportunities concerned mathematical language, strategies, and several important ambitious practices. Wæge and Fauskanger (2020, 2022) have investigated how teacher time-outs are used in rehearsals, and how they support and enable teachers to collectively learn core practices of ambitious teaching, and their development of pedagogical judgment. They found that teacher time-outs enable teachers to learn and work

on multiple practices together simultaneously and support their pedagogical reasoning and decision-making in the moment of teaching. Other researchers have also investigated the opportunities teachers have to learn professional noticing through the following stages: co-planning; rehearsing; co-enacting; reflection on learning cycles of enactment and investigation (Fauskanger & Bjuland, 2022); the opportunities teacher time-outs provide for teachers to develop ambitious teaching practices (Fauskanger, 2019); the components of ambitious teaching practices the teachers can learn through their work on instructional activities within the cycle of enactment and investigation (Fauskanger & Bjuland, 2019); and how the MAM program aligns with the shift in focus from the development of knowledge for teaching in mathematics to core practices in the guidelines of the mathematics curriculum in Norway since 2010 (Mosvold et al., 2018). This research on teacher learning or the possibilities to learn mathematics teaching practices report on either the positive impact or the promising opportunities the MAM program can provide for teachers to develop ambitious teaching practices.

Recently, there has been an increasing interest in studying the nature of the interactions between teacher educators and teachers within the ambitious teaching literature. A prominent part of this research is related to how teacher educators, as researchers or program facilitators, support teachers in their enactment of ambitious teaching practices through coaching (e.g., Averill et al., 2016; Gibbons & Cobb, 2016; Gibbons et al., 2017). Rawlins et al. (2020) explored novice teachers' perceptions on the introduction of rehearsal activities combined with in-the-moment coaching as they participated in a practice-based project designed to support them to become ambitious mathematics teachers. This project, entitled *Learning the work of ambitious mathematics teaching*, draws on the work from the LTP project and has adapted the cycle of enactment and investigation into its own context. While positioning themselves as learners in their role as coaches, Rawlins et al. (2020) found that the whole rehearsal process was highly valued by the novice teachers who claimed that this process contributed to their development of a more responsive way of teaching in terms of advanced levels of noticing and responding to student thinking. The authors claim that the rehearsal process expanded the opportunity these novice teachers had to reflect in, on, and for action.

Because research has shown that practice-embedded contexts, such as the one the MAM program provides, can support teachers' learning and development of ambitious teaching, scholars have slowly begun to pay attention to the facilitators' and principals' roles, and how

schools can be organized to support such learning. During a practice-embedded professional learning setting for a group of teachers, Gibbons et al. (2021) explored the complex work of experienced facilitators and how they foster professional learning environments to support teachers in exposing their practice to collective inquiry. They identified facilitation practices that included promoting collaborative learning, supporting teachers' experimentation with and analysis of instructional practice, and shaping the emotional space. The study shows that facilitators play a critical role in enacting these practices to develop professional learning environments that are supportive, nurturing, and collaborative, which then contributes to teachers' willingness to take risks that promote their own and each other's learning. Gibbons et al. (2019) investigated how principals and coaches coordinate their individual and collective work to support teachers' improvement of instructional practices. They found that coordination between different types of leadership support, such as coaching, PD, and evaluation, that are aligned with each other and the teachers' needs can promote instructional improvement in schools. This implies involving teachers in the coordination process, that the coordination is ongoing, and that leadership support is responsive to changing needs and contexts. Because district education-authority leaders are charged with shaping and supporting the work of the principal, Gibbons et al. (2019) also maintain that district leaders can support and help principals and coaches to develop a plan for how to coordinate their work at school. In their study, Kazemi et al. (2022) explore how principals' conceptions of teacher learning and PD can either facilitate or impede the implementation of a new mathematics curriculum. They argue that the leadership of principals plays a critical role in promoting school-wide transformation of mathematics teaching, which involves shaping the school's culture and promoting a shared vision for mathematics teaching. They also found that the principals' view of teacher learning and their ability to support teachers in ongoing PD are important factors in successful implementation of a new mathematics curriculum, and highlight the importance of aligning principals' conceptions of teacher learning with effective practices that promote school-wide transformation. Kazemi and Resnick (2020) have studied how schools are organized as workplaces for school leaders' learning and how this supports teachers' collective learning. They point out that it is important for schools to create a learning organization that prioritizes the ongoing learning and development of both teachers and leaders, which implies creating a culture of continuous improvement by using data to inform practice and provide opportunities for collaboration and shared decision-making. To accomplish this, Kazemi and Resnick (2020) argue that schools must invest in leadership development and create structures that support distributive leadership.

2.4 Leadership for teachers' learning

Scholars have long recognized that leadership influences teachers' learning and can be practiced in a way that might have a significant impact on promoting and sustaining change (Desimone, 2023; Fullan et al., 2005). Several studies point to the importance of the role the principal has in teachers' PD. Thoonen et al. (2011) found that by exercising a school-leadership practice that helps teachers to identify their development needs and enhances the implementation of new learning, the principal can contribute to creating a positive learning environment. These findings are supported by several scholars who argue that school leadership can create learning environments where teachers are supported in identifying their development needs, are encouraged to experiment, resources are provided to support teachers' learning, and implementation of new learning is enhanced (e.g., Thoonen et al., 2011; Vanblaere & Devos, 2016). Furthermore, research has revealed a positive association between learning-centered leadership that builds trust and the establishment of productive learning environments for teachers (Emstad & Birkeland, 2020; Pang et al., 2016; Piyaman et al., 2017; Talebizadeh et al., 2021). Scholars have found that teachers have the highest interest in collaboration when they perceive PD support and encouragement from their leaders (Silva et al., 2017; King & Stevenson, 2017). King and Stevenson's (2017) study presented a bottom-up approach supported from above (i.e., the school principals trusted the teachers). The teachers in this study were also given the time to plan and reflect together and cultivated an openness to working together. However, enough time to plan, observe, and reflect is often not set aside for development processes (Postholm, 2020).

While school leadership is argued to have great impact on fostering learning conditions for teachers' learning, research on teacher PD shows that the development work must be structured, facilitated, and coherently supported. According to Desimone and Garet (2015), leadership plays a vital role in supporting and encouraging teachers to implement the ideas and strategies they learned in PD sessions in the classroom. They maintain that leaders can provide teachers with valuable time and opportunities to participate in, practice, and apply what they have learned in PD. Research also shows that ideas and messages that are consistent and communication with colleagues about their practice are more robust when teachers experience a coherent system of support (e.g., Cobb et al., 2018; Gamoran, 2003; Knapp, 2003). Moreover, research on teacher PD also shows that there needs to be a plan for the teachers' development work in which the teachers are invited to take part in the planning (Darling-Hammond &

Richardson, 2009). As Earley and Bubb (2004, p. 8) state, "professional development does not just happen – it has to be managed and led".

Leithwood et al. (2020) argue that school leadership can have an especially positive influence on school and student outcomes when it is distributed. They found that distributed forms of leadership had a direct positive effect on teachers' capacity, motivation, and commitment, and their perceived working conditions, which in turn had a positive effect on students' learning and achievement. They also found that there is a correlation between a high level of student achievement and a high level of influence from all leadership sources, and that the principal's role had the greatest influence in all schools. According to Grootenboer and Hardy (2017), teacher PD leadership should be a collaborative effort, as the responsibility can be overwhelming for one person alone. This view is shared by Postholm (2019), who suggests that the task of managing developmental processes should be shared among multiple leaders or between leaders and teachers. While emphasizing distributed modes of leadership, Leithwood et al. (2020) also refer to the term "instructional leadership" when considering school leaders' contribution to building staff capacity. Instructional leadership in schools involves principals orienting their practices directly toward supporting teachers' instructional practice and students' learning (e.g., Hallinger, 2005; Leithwood et al., 2008; Murphy et al., 2007; Rigby, 2016). In their study, Kazemi and Resnick (2020) investigated how teachers' collective learning was supported by the way schools as workplaces are organized when making efforts to improve mathematics teaching, and how such an organization requires that leaders learn instructional leadership. They found that leadership practice must be considered across different learning structures, such as team meetings and classroom support, in order to understand how coherence can be effectively created by leaders, and how they support learning goals over time. They also maintain that in order to foster the desired leadership aspects within the district context, such as district policies, the expectations for leadership roles, and instructional materials, must be adjusted. They state: "Just as teachers need coherent systems that support intended changes in practice, so do leaders" (p. 414).

Because PD efforts for teachers often are led or facilitated by external actors, such as researchers or teacher educators, leadership of teacher learning also refers to PD facilitators. In the literature review presented in section 2.2., Sztajn, Borko, and Smith (2017) pointed to several studies that found that skillful facilitators have a great impact on ensuring the effectiveness of a PD program. They maintain that research on the knowledge and practices of

PD leaders has been given much more attention in recent years, and found several papers addressing what PD facilitators need to know and be able to do. In relation to facilitating practices, the review showed that successful facilitators create relationships with the teachers based on trust and promote discussions that are both challenging and supportive. Moreover, they found that such practices as orchestrating discussions during PD sessions that encouraged teacher participation, and being responsive to their contributions while maintaining focus on the goals of the PD activities, were successful. Sztajn and colleagues (2017) also found that the facilitators' role differs depending on whether the approach to PD is highly adaptive or specific. On the one hand, highly adaptive approaches characterized by attention to local contexts and general guidelines involve facilitation in identifying goals for teacher learning, anchoring PD activities, and preparing for guiding discussions. In specific approaches to PD, where PD is predetermined with specified goals, resources, and facilitation materials, facilitators need, in the planning phase, to obtain a thorough understanding of the core goals of the PD and be familiar with the materials they are going to use. However, despite these findings of examples of practices (and others) from studies of facilitation, Sztajn and colleagues (2017) claim that more research on what facilitators do and on general features of what it takes to lead PD is needed. They state that we need to know more about what facilitators must know and need to do to effectively lead PD for mathematics teachers if we are to prepare and support them.

2.5 The literature review as the knowledge base for the study

The presented literature review provides an overview of the research conducted on teacher PD in general and its specific application in the field of mathematics education. In brief, the review points out that teacher PD has undergone a shift from short-term programs situated outside the school context, such as one-off courses, toward more sustainable, practice-based initiatives. This shift has been driven by the realization of the limitations of traditional, one-shot PD programs and the need for more effective and impactful approaches to supporting teacher growth and development. The review suggests that short-term, one-shot PD programs often fail to make a lasting impact on teacher practice and student learning. In contrast, sustained, practice-based programs that involve ongoing collaboration and coaching have been found to be more effective in promoting teacher growth and improving student outcomes. Consequently, the research recommends that PD programs should be designed to be more ongoing, job-embedded, and collaborative, with a focus on supporting teachers in implementing new strategies and practices in their classrooms over an extended period of time.

The review further indicates that scholars have been paying increasing attention to the circumstances and contexts in which teacher PD efforts are situated, as well as the interplay between the actors involved. This reflects the growing recognition that PD efforts cannot be viewed in isolation from the broader social, cultural, and institutional contexts in which they occur, and that understanding these contexts is essential for designing effective and sustainable PD programs. Current research shows that the success of PD efforts depends not only on the quality and content of the PD itself, but also on such factors as school leadership, organizational culture, teacher collaboration, and the broader policy environment. Moreover, research has highlighted the importance of comprehending the perspectives and experiences of the various actors involved in PD, including teachers, administrators, coaches, and external facilitators. It is essential to understand the ways in which they interact and collaborate with one another to support teacher learning. Therefore, research suggests that PD programs should be designed to be contextually responsive, taking into account the unique needs, challenges, and opportunities of the schools and districts in which they are situated. They should also strive to ensure that the program is satisfying the needs of all stakeholders and promoting sustained improvement in teacher practice and student learning.

In my Ph.D. project I investigate the contribution of the MAM program to mathematics teachers' PD. This program complies with this shift toward practice-based PD efforts (see section 4.0). Furthermore, the thesis has a particular focus on the actions and interactions between the actors (i.e., the teachers, the principal, and the TEs) and their roles in supporting teachers' PD during the first year of the program. This review will therefore contribute to comparing and contrasting the findings of this study with the existing literature, identifying similarities and differences, and exploring the implications of the findings for theory and practice. Therefore, this literature review provides a knowledge base that can inform the meta-synthesis[5] by providing insights for interpreting and discussing the findings obtained from the study in relation to the research question.

[5] To answer the primary research question, the findings from three substudies were synthesized using a meta-synthesis approach. See section 5.5.3.

3.0 Theoretical framing

The theoretical foundation and the direction of qualitative research are shaped by assumptions about the nature of reality and the nature of knowledge. Therefore, understanding the theoretical foundation of a research study is important for evaluating its assumptions and perspectives, and for assessing the trustworthiness of its findings. In this section, I will elaborate on the theoretical framework that guides this study.

All qualitative research is value-laden (Creswell, 2013), meaning that the research is affected by the subjective, individual theories of the researcher (Postholm, 2010). Denzin and Lincoln (2011) argue that all research is conducted from a subjective and interpretive framework. They argue that researchers bring their own personal and professional experiences, assumptions, and perspectives to the research process, and that these factors inevitably influence the ways in which they interpret and understand the research data. According to Denzin and Lincoln, researchers' interpretive frameworks are shaped by a range of social and cultural factors, including their upbringing, education, social class, gender, and ethnicity. My interpretive framework as a researcher (e.g., Denzin & Lincoln, 2011) is also influenced by experiences from my education for teacher qualification and work as a lower secondary school teacher, as a TE in continuing education for experienced teachers, and as a facilitator in a national project called "Developing lower secondary school" (see for example, Postholm et al., 2017) under the auspices of the Norwegian education authority. These experiences, as well as the trends within the field of teachers' PD, form the basis of my interpretive framework and will thus to a certain extent be reflected in the choices I have made in this study.

Qualitative research, as described by Denzin and Lincoln (2011), is a situated activity that localizes the researcher in the real world. They argue that qualitative researchers focus their research on natural settings in their attempt to understand and interpret phenomena based on opinions ascribed to them by individuals in these settings. As such, the qualitative researcher is interested in understanding how people interpret their experiences, how they construct their worlds, and what meaning they attribute to their experiences. Walcott (2008) argues that the aim of qualitative studies is to elicit the emic perspective, which centers around the participants' points of view and emphasizes their specific interpretations within a context. According to Merriam (2002), the key to understanding qualitative research lies in the idea that meaning is socially constructed by individuals in their living world. Such an approach to qualitative studies

leads to an epistemological stance that posits that knowledge and understanding are actively constructed in the dynamic and social interaction between the researcher and the participants (Lincoln & Guba, 1985), a perspective that is rooted in the social constructivist paradigm (Postholm, 2010; Prawat, 1996).

3.1 The social constructivist paradigm and socio-cultural theory

The social constructivist paradigm highlights the role of social and cultural context in shaping individual and collective learning. Rather than being objective and neutral, knowledge is constructed through social interaction and interpretation in this paradigm. Prawat (1996) puts socio-cultural theory within the social constructivist paradigm, arguing that this paradigm offers a more comprehensive and accurate explanation of the learning process than traditional individualist or behaviorist approaches. It acknowledges the importance of social and cultural context in shaping learning, as well as the active role of learners in constructing knowledge through social interaction. The work of Lev Vygotsky is framed by this view of knowledge, and his thoughts and ideas are known as socio-cultural theory in Western countries. Vygotsky argued that learning is a social and cultural process that occurs through interaction with more knowledgeable others (Vygotsky, 1978). According to him, learning occurs through the zone of proximal development, which is the difference between what a learner can do independently and what they can do with the help of a more knowledgeable other. This means that learning takes place in collaboration between two or more people. Furthermore, Vygotsky (1978) argues that society plays a central role in the process of meaning-making because the environment in which children grow up will influence how they think and what they think about. He claimed:

> Every function in the child's cultural development appears twice: first, on the social level and, later on the individual level; first, between people (interpsychological) and then inside the child (intrapsychological) (p. 57).

This means that the origin of an individual's knowledge is found in their interactions with their surroundings and other people before their knowledge is internalized. It also means that learning is a social process where individuals and their social environment are dialectically related to each other (Postholm, 2010; Prawat, 1996). In the socio-cultural perspective, individuals use mediating artifacts to interact with their surroundings and other people (Vygotsky, 1978). Mediation is the most important principle in socio-cultural theory, describing the process through which individuals use external tools, symbols, and cultural artifacts to

understand and interact with the world (Vygotsky, 1978). Vygotsky (1978) also argues that language plays a crucial role in transforming an individual's thinking and enhancing their cognitive abilities, making it the primary mediating tool and a key component in interpersonal interactions.

Merriam (1998) points out the importance of reflexivity in the social constructivist paradigm, arguing that researchers must acknowledge their own role in the construction of knowledge. She states that researchers are not objective observers, but active participants in the construction of knowledge. Therefore, it is important for researchers to be reflexive and to acknowledge their own social and cultural background and experiences.

Socio-cultural theory is particularly relevant for the study of teacher PD, as it emphasizes the importance of social and cultural context in shaping learning. In my study, the socio-cultural theory is useful in understanding the ways in which teachers interact with one another, with TEs, and with a principal in the context of the program. The interactions between these actors can shape the ways in which teachers construct knowledge about mathematics teaching and learning, and can ultimately influence the effectiveness of the PD program.

3.2 Cultural Historical Activity Theory

Cultural Historical Activity Theory (CHAT) was initially developed by Leont´ev (1978, 1981), who based this theory on Vygotsky's work (Wertsch, 1981). Thus rooted in socio-cultural theory (Wertsch, 1981), CHAT can be described according to several features. The first main feature of CHAT comprises the three levels of analysis of *activity.* According to Leont´ev, activity breaks down the distinction between the external world and the world of internal phenomena (Wertsch, 1981), and it has a predominate place in CHAT. The first level of analysis is that activities are distinguished according to their motive and the object toward which they are oriented. It is important to note that within CHAT, "object" is understood as the overall goal of the activity. The second is that actions are distinguished according to their goals, and third that operations are distinguished according to the conditions under which they are carried out (Wertsch, 1981). The second feature of CHAT is the involvement of the notion of "goal" and that the work or actions performed within the activity are goal-directed. These actions are carried out to move the practice toward the object, which means that each action is focused on specific goals. The third feature is that activity is mediated, where mediation is extended to not only include tools, but also signs and the use of language (Cole, 1996). CHAT emphasizes

development, which is the fourth feature, where both culture and history are taken into account in the understanding of development. The fourth feature also indicates that human activity, the mediating artifacts, tools, and signs have emerged and are developed through social interaction. Internalization (Wertsch, 1981), the fifth feature, refers to Vygotsky's (1978) general genetic law of cultural development, cited above.

It is important to mention that Vygotsky (1981) and his colleagues did not see the individual as passive in this process of transformation, even though they saw social reality as having a primary role. According to Vygotsky, the consciousness is produced in the interactions between individuals and society and is therefore not a product of society. External and internal activities must thus be seen as having a developmental relationship where external processes are transformed to create internal processes, which leads to the notion that the individual is active in both transforming the process and also changing its structure. This process of externalization also has a central place in CHAT theory (Leont´ev, 1981; Engeström, 1999), where internalization refers to the reproduction of the culture and externalization to the process of creating new processes or new ways to use them. The above-mentioned term "object" has a predominant place in CHAT. Leont´ev (1978) argued that the object is material or ideal:

> The object of the activity is twofold: first, in its independent existence as subordinating to itself and transforming the activity of the subject, second; as an image of the object, as [a] product of its property of psychological reflection that is realized as an activity of the subject (p. 52).

This means that the object can be either material (such as a physical object) or ideal (such as a concept or idea), has an independent existence, and can be something that people are working toward. In CHAT, the object is not a fixed or achievable endpoint but rather a dynamic concept that propels activity and encourages people to engage in it. The object is not simply something to be attained, but rather something that shapes and directs the activity, providing a sense of purpose and direction. In CHAT, the object is considered to be an ideal that guides activity, and while it may never be fully attained, it serves as a driving force for ongoing activity and development. This perspective emphasizes the importance of the process of activity, rather than just the achievement of a particular outcome. Furthermore, Leont´ev (1981) stated that "the object is the true motive" (p. 59) for people's actions, which refers to the first feature of CHAT. The object is in this sense the reason for an activity's existence, which leads to the notion that

one activity can be determined from another only by looking at the object. In other words, subjects who perform similar actions but have different objects are involved in different activities. This means that in CHAT, the unit of analysis is not just the actions themselves, but the activity as a whole, which includes the reason for the actions, the tools used, and the people involved. Therefore, in CHAT the activity is the unit of analysis, where it is composed of subject, object, actions, and operations (Leont´ev, 1978). In the context of teacher PD, it is necessary for individuals aiming to develop their practice toward a specific goal to know about, or ideally share a collective motive to act upon. As Sannino, Engeström, and Lemos (2016) note, "A thing out there in the environment becomes the object of an activity when it meets a need and is invested with meaning and motivating power" (p. 602). Teachers' motivation should be integrated into the object since their practice and requirements serve as the basis for imbuing them "with initiative and commitment" (Sannino & Engeström, 2017, p. 81).

3.2.1 The activity system

Engeström (1987) extended Leont´ev's ideas of activity by including the concepts of community and rules that support it, and developed the concept of an "activity system". The activity system, a graphic rendition of the activity theory, is considered to be a unit of analysis of both collective and individual human activity (Engeström, 1987, 1999, 2001; Engeström & Miettinen, 1999). The system consists of seven factors or nodes: subject, mediating artifacts, object, outcome, rules, community, and division of labor (see Figure 1.). A change in one factor will influence another one in the system, and furthermore the system as a whole, making the factors related and the system dynamic. The acting subject in the activity system refers to a person or a group of people using cultural mediating artifacts to move the practice toward the object. The outcome represents how the subject has moved toward the object, the desired aim of the activity. The "rules", "community" and "division of labor" represent the context in which the activity is carried out. The context is not, however, just a surrounding element but interwoven in the actions. It may, according to Engeström (1987), determine the premises and possible restrictions for the subject's goal-directed actions toward the object. "Community" refers to the people in the activity system who share the same object. As the activity is object oriented, a collective activity system is driven by a deeply shared motive embedded in the object of the activity (Engeström, 2001). This means that the people involved in a community must conduct their work or goal-directed actions toward the same object, guided by a set of rules, such as norms and conventions. The work or goal-directed actions are divided between the people in the community and referred to as a division of labor, which makes it possible to

distinguish between collective activity and individual action (Cole, 1996; Engeström, 1987, 2001; Engeström & Miettinen, 1999). A graphic development of the activity system is visualised in Figure 1 below.

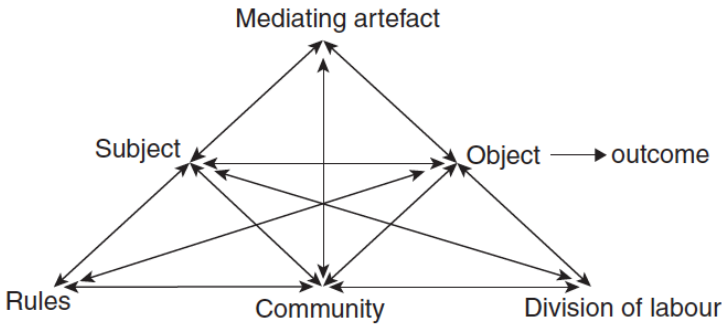

Figure 1. *The complete activity system (Engeström, 1987, 1999, 2001).*

Engeström (1999) describes the development of the activity system as a form of expansive learning with a cycle of development that begins with an emphasis on internalization. Internalization refers to the reproduction of the topical culture, whereas externalization is the innovative process of finding new ways of using old artifacts or creating new ones that in the end develop into internalization. The activity system can therefore be a useful tool when a development activity is analyzed to pinpoint tensions and contradictions in the implementing process. Engeström (1987, 2001) further expanded the activity system to include two or more systems into a third generation of CHAT. This third generation forms a network of interacting systems where the focus is directed on the collaboration between them. This means that the subjects in the various systems act on their own objects, and at the same time, in their network they act on an object that is partially shared between the systems. A graphic development of the third generation of CHAT is visualized in Figure 2 below.

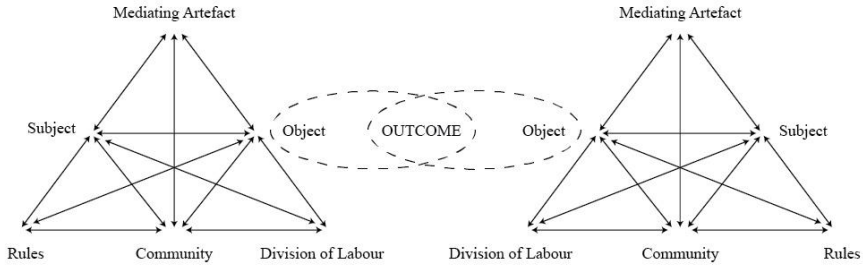

Figure 2. *Two interacting activity systems representing the third generation of CHAT (Engeström, 2001).*

In activity theory, tensions and contradictions constitute the foundation for development and change (Engeström & Miettinen, 1999). Engeström (2001) argues that tensions and contradictions are not just temporary or superficial problems that can be easily resolved, but are deeply embedded in the structure of the activity systems themselves. Over time, tensions and contradictions can accumulate within an activity system or between different activity systems and can arise from conflicts between different goals, values, or practices (Engeström, 2001). The concept of contradictions highlights the idea that change and development in activity systems do not happen smoothly or linearly but are often propelled by tensions and conflicts that emerge from contradictions within and between activity systems. By recognizing and addressing these contradictions activity systems can evolve and transform (Engeström, 2015).

3.2.2 Boundary object, boundary crossing, and transformation

Since an activity system is what Engeström et al. (1995) refer to as a complex community of practice, a network of activity systems can be seen as a network of complex communities of practice. As a network of systems brings attention to the interaction between them, it necessitates communication and collaboration between resource persons across different communities of practice, such as teachers in a school and TEs in a university. To do this, practitioners need to be able to move between activity systems. This movement is described as "boundary crossing" which Engeström et al. (1995) characterize as "horizontal expertise where practitioners must move across boundaries to seek and give help, to find information and tools wherever they happen to be available" (p. 332).

However, in their work, practitioners operate in and move between multiple parallel activity systems. These activity systems demand and afford different, complementary but also conflicting cognitive tools, such as language and concepts, theories, and specific rules and patterns of social interaction to be followed with students, colleagues, and administrators (Engeström et al., 1995). The standards or expectations for constituting knowledge and skills may differ across various contexts (Engeström & Sannino, 2010). Practitioners face challenges when negotiating because words naturally mean different things to different people (Wertsch & Toma, 1995), and ingredients are combined from different activity systems to achieve hybrid solutions (Engeström, 2001). Therefore, boundary crossing must be facilitated if practitioners are to effectively navigate and integrate the various cognitive tools, rules, and patterns of social interaction that are demanded and afforded by the multiple parallel activity systems they operate in and move between (Engeström, 2001). Without effective boundary crossing, practitioners may struggle to understand or effectively communicate with others from different activity systems, which can lead to misunderstandings, conflicts, and solutions that do not address the needs or goals of all involved parties (Engeström et al., 1995). In this facilitation of boundary crossing, boundary objects are important (Engeström et al., 1995). Star and Griesemer (1989) describe boundary objects as passive artifacts that can fulfill a specific function in bridging intersecting communities of practice. They maintain that boundary objects "both inhabit several intersecting worlds and satisfy the informational requirements of each of them" (p. 393). Within CHAT, we have already seen that these artifacts can be physical but also take the shape of signs and tools. Engeström et al. (1995) argue that a boundary object is a concept or artifact that can be shared and understood by multiple parties within different contexts, thus facilitating communication and coordination between those parties. Engeström et al. also point out that boundary objects are not fixed or static, but rather are flexible and can be adapted or reinterpreted by the various parties involved. As such, boundary objects can help to bridge the gap between different cultures, disciplines, or professions, and enable the creation of new knowledge or practices through the collaborative efforts of those involved in boundary-crossing actions.

It is important to note Akkerman and Bakker's (2011) findings that a boundary object in CHAT can have a twofold meaning in understanding its function and nature. On the one hand, it can be understood as artifacts, in line with the above-mentioned conceptualization by Star and Griesemer (1989), that are similar or identical across different activity systems, and can be used to facilitate communication and collaboration between them. On the other hand, a boundary

object within the third generation of CHAT (Engeström, 2001) can be understood as "the potentially shared or jointly constructed object" between two or more activity systems (Engeström, 2001, p. 136). In this sense, boundary objects are not necessarily identical across activity systems, but rather are objects or concepts that are jointly constructed or negotiated between them. They are shared motives or goals that can help to align the practitioners' work from different activity systems toward a common purpose (Engeström, 2001).

Akkerman and Bakker (2011) suggest that boundary objects should be confined within Star's original conceptualization to prevent confusion. However, such a notion could create confusion in the CHAT framework as artifacts within an activity system are understood as tools or instruments that are used to accomplish goal-directed actions toward a specific object. In this sense, the artifacts are not just passive objects but are actively used by practitioners to accomplish specific goals. For the purpose of this study, I draw on Engeström's (2001) conceptualization of a boundary object as a partially shared object toward which the subjects of various activity systems accomplish goal-directed actions by using mediating artifacts. By engaging in the construction and negotiation of a partially shared object in a "shared meeting ground" (Engeström & Toivainen, 2011, p. 35), the subjects can share their insights and perspectives to facilitate knowledge sharing and collaboration between individuals or groups from different activity systems.

Furthermore, Engeström and Toivainen (2011) argue that the shared meeting ground, which is the space where practitioners can engage in boundary crossing, can be both physical and virtual. In this space, ideas can be adopted from one another and lead to developmental transfer (Engeström & Sannino, 2010). According to Engeström (2001), transformation occurs when practitioners work together on the boundary object and engage in boundary crossing. They may encounter new perspectives and ways of thinking that challenge their assumptions and ways of doing things and broaden their understanding of the object and its context. Encountering new perspectives and being open to learning from others can lead to a process of reflection and re-evaluation understood as transformative learning, which is a process of fundamental change in a person's perceptions, values, and worldview that can lead to profound changes in practice (Engeström & Toivainen, 2011). As such, the dialog between different activity systems becomes the key focus in the process of boundary crossing and transformation.

4.0 The MAM program and the accomplished project

In this chapter, I will present the practice-based program (MAM program) that has been explored in this study. As mentioned above in the introduction, the MAM program has been refined and evolved throughout its existence. Thus, the accomplished project in this study will be referred to as MAM 2019.

4.1 Description of the conducted MAM 2019

MAM 2019 commenced with two start-up sessions held in the spring of 2019. During these sessions, the program's content was introduced to the participating teachers and principals, which included practices of ambitious mathematics teaching and instructional activities. Moreover, participants were informed of the model *cycle of enactment and investigation* for PD (see Figure 4). Over the next two school years, the program consisted of ten sessions that featured a complete cycle of enactment and investigation. These sessions were held at one of the participating schools, which is referred to as the host school, with five sessions taking place each year. The host school made classrooms and students available to carry out the cycles. Additionally, there were two reflective sessions conducted at the end of each school year. In the sessions where a full cycle of enactment and investigation was conducted, the teachers were divided into three groups, each guided by a TE. The teacher groups and the associated TE were fixed for all sessions, and this group structure enabled them to work together throughout the entire cycle. Figure 3 provides an overview of the sessions during the first year of the program period.

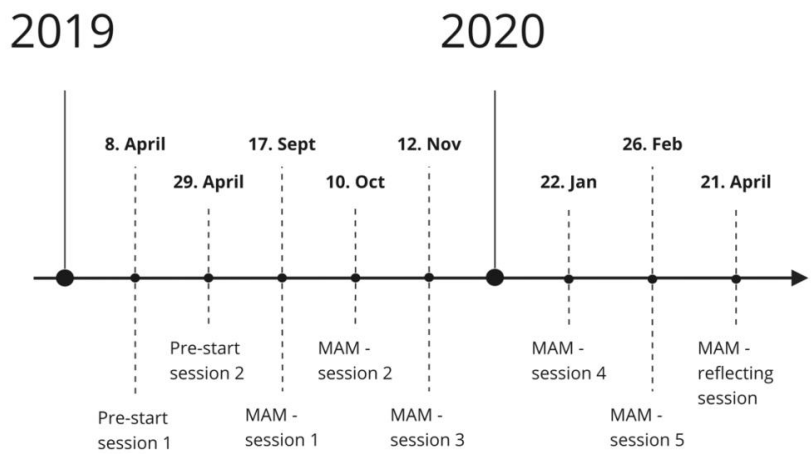

Figure 3. *Overview of the sessions during the first year of the program period.*

The cycle of enactment and investigation invites teachers to engage in collective exploration, observation, and reflection by focusing on one instructional activity each time as a common tool, guided by TEs (Wæge & Fauskanger, 2020, 2022). The cycle consists of six phases which are repeated routines that provide opportunities for teachers to both practice teaching and reflect on that practice:

1. *Preparation*. Prior to each session, the teachers read an article related to the theme of the session. These articles typically involved the relation between the instructional activity and the principles and practices emphasized in the session. They also read a description of the activity that formed the foundation for the session, which they also were encouraged to try out with their own students in their own classroom.

2. *Collective analysis*. In this stage, all participating teachers jointly analyze the article they have read, guided by the TEs. An important part of this process is to analyze the principles and practices important to the instructional activity and discuss how they are used or can be used in a responsive manner in orienting the students toward the learning goal.

3. *Co-planning*. Groups of teachers and TEs co-plan to design an instructional activity, that includes the principles and practices they analyzed and discussed during the collective analysis, to be taught to a group of students.

4. *Rehearsal.* In a fixed setting, a selected teacher or pair rehearse to enact the activity planned by the group. The remaining teachers and the TE act as students. In this stage, all the members of the group can initiate a *teacher time-out* (TTO) which allows them to pause instruction and engage in collective in-the-moment thinking. TTOs allow members to discuss how the teacher should respond to student contributions and determine the direction of further instruction.

5. *Classroom co-enactment.* The teachers co-enact the planned activity with a group of real students in a real classroom. Because all the members of the group are equally responsible for the instruction, the selected teacher or pair led the activity, whereas the others took TTOs whenever necessary.

6. *Collective analysis.* The groups of teachers and TEs engage in an analysis of the co-enactment and reflected on their own learning, with a focus on how the important principles and practices of the instructional activity were manifested with the students. Subsequently, the entire group engages in collective analysis and reflection, followed by preparation for the subsequent cycle.

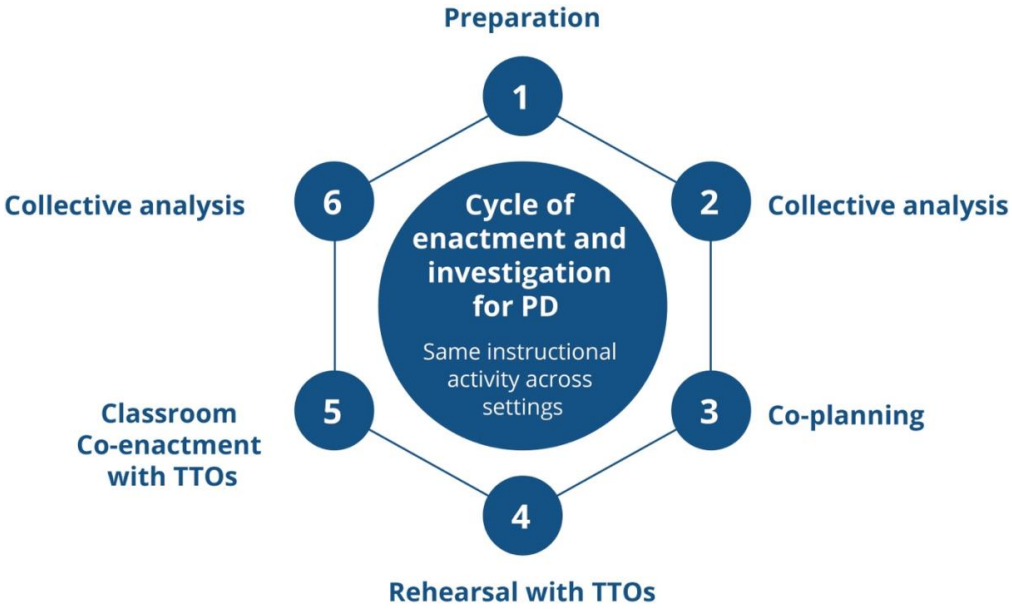

Figure 4. *The cycle of enactment and investigation for PD (Norwegian Centre for Mathematics Education, n.d.).*

One of the defining features of a core practice model for PD, such as the cycle of enactment and investigation, is its emphasis on specific pedagogical practices that are closely linked to the goal of ambitious mathematics teaching (Forzani, 2014; McDonald et al., 2013). Furthermore, the MAM program is modelled on research on effective forms of PD[6], and informed by theory on teachers' collective learning in a community of practice (Wenger, 1998). The circle of enactment and investigation provides opportunities for the teachers to actively take part in mutual processes of negotiation of meaning to create a joint enterprise (Wenger, 1998). Moreover, the teachers are invited to engage in collective exploration, observation, and reflection by using the instructional activities as a common tool, guided by TEs (Wæge &

[6] See literature review section 2.1 for research on effective forms of PD.

Fauskanger, 2020). Thus, in addition to promoting opportunities to learn to enact the principles, practices, and mathematical knowledge entailed in ambitious mathematics teaching, the MAM program can be seen as offering a model for teachers' PD.

4.2 The principles and practices in the MAM program

The cycle of enactment and investigation is structured around a number of particular pedagogical practices and principles[7] aimed at fostering student learning and supporting teachers' development in teaching. These principles and practices have been identified as crucial for supporting students' learning (Ghousseini et al., 2015; Grossman et al., 2018; Kazemi 2017; Lampert, 2013). The MAM 2019 program focuses on the following principles and core practices:

Principles:

- *Students are sense-makers*: Involves understanding how students make sense of mathematical concepts, valuing and leveraging their prior knowledge and experiences, and providing opportunities for students to reason, explain, and justify their thinking, engage in mathematical discussions, and work collaboratively with their peers.
- *Designing instruction so that all students have equitable access to learning*: Involves providing all students with equal opportunities to engage in challenging and advanced mathematics that fosters equitable access to learning. This principle also requires acknowledging and addressing the diverse needs, backgrounds, and experiences of learners, recognizing each student as a capable and valued learner with the potential to succeed in mathematics.
- *Having a clear instructional goal*: Involves the teacher planning and conducting activities with a clear learning goal in mind, often related to the main ideas of mathematics. It requires a deep understanding of the mathematical concept being taught, as well as a clear vision of what students should comprehend and be able to accomplish as a result of the instruction. This understanding is necessary to effectively support student learning and achievement.

[7] See literature review section 2.3.1 for research on practices and principles of ambitious mathematics teaching.

- *Knowing the students*: Involves getting to know the students as individuals and as learners. This includes recognizing their unique strengths, habits, prior knowledge, learning preferences, interests, and challenges. Additionally, it involves developing an awareness of how students perceive themselves as learners.

(Ghousseini et al., 2015; Kazemi, 2017)

All these principles underpin an ambitious view of classroom instruction that entails creating a classroom environment that is culturally responsive, inclusive, and supportive, and where students feel safe and comfortable expressing themselves while engaging in learning activities (Ghousseini et al., 2015).

Core practices:

- *Launching problem* refers to the teacher's initial introduction of a problem to the students in a way that motivates and engages them in the problem-solving process. This involves setting the task in a clear and accessible manner that provides an entry point for all students, while also offering opportunities for students to engage in deeper and more complex mathematical thinking.
- *Using mathematical representations* involves teachers utilizing various representations to facilitate student learning of mathematical concepts. Through purposeful use, teachers can support students' sense-making, reasoning, and engagement with mathematical ideas, leading to a deeper understanding of concepts. Therefore, students must have diverse opportunities to work with different types of representations and develop their ability to use them effectively to communicate mathematical ideas.
- *Facilitating student talk and eliciting and responding to students' mathematical ideas* involves eliciting and responding to student thinking in a way that promotes mathematical discourse and supports student reasoning, which is crucial for promoting deeper mathematical understanding and engaging students in meaningful mathematics learning experiences. It necessitates the need for teachers to create a classroom culture that values student thinking and encourages students to share and justify their mathematical ideas.
- *Aiming toward a mathematical goal* involves setting clear mathematical goals for lessons, as well as designing instructional activities and tasks that support the attainment of those goals. Teachers need to have a deep understanding of mathematical concepts and be able to connect them to their goals for student learning. This involves careful

planning, flexibility, and responsiveness to student thinking and needs, where the aim is to create a cohesive and meaningful learning experience for students that builds on their prior knowledge and supports the development of new mathematical understandings.

(Ghousseini et al., 2015; Kazemi, 2017; Kazemi & Wæge, 2015; Lampert et al., 2013)

4.3 Instructional activities

The MAM program uses particular instructional activities that serve as *containers* (Fauskanger & Bjuland, 2019: Kazemi & Wæge, 2015) that provide a structured and organized way for teachers to learn and train in teaching practices, and use principles and knowledge that are involved in ambitious mathematics teaching. These instructional activities support collaborative learning, critical reflection, and a focus on ambitious teaching practices that improve mathematics teaching and learning (Lampert et al., 2013). The teachers engage in these instructional activities through the cycle of enactment and investigation, allowing them to learn how to implement the principles and practices effectively. Additionally, this process provides opportunities for teachers to reflect on their own teaching practice, make necessary adjustments, and develop their skills. In this sense, instructional activities serve as a vehicle for teachers to develop their professional knowledge and expertise.

Furthermore, these instructional activities are intentionally selected and designed to help teachers develop a deeper understanding of the content and pedagogical strategies that they need to effectively teach their students. By reducing the complexity of teachers' learning (Lampert et al., 2013), the activities provide scaffolding that enables them to make informed judgments and decisions in the classroom (Kazemi & Wæge, 2015). Thus, the selection of these particular activities is not intended to signal the importance of these activities in themselves or that these are the "correct" ones. Rather the choice of an instructional activity is intended to indicate that learning in and from practice needs an instructional vehicle to focus on practice in ways that enable the teachers to enact and learn the complex and demanding endeavor of ambitious teaching (e.g., Lampert et al., 2010; Lampert et al., 2013). The following instructional activities were used (Kazemi & Wæge, 2015):

- *Choral counting* is a classroom activity in which students count together in unison, typically starting from a given number and counting by a specific interval (e.g., counting by 19 starting from 19), either forwards or backwards. The activity is

designed to support students' understanding of number patterns, place value, and other mathematical concepts. During the activity, the teacher stops the count at strategic points to ask questions or make comments that prompt students to think about the counting sequence and to make connections to other mathematical concepts.

- *Quick images* is an activity where a group of students is shown a visual image or a small set of objects for a brief moment. Students are then asked to reconstruct the image in their minds and use mathematical reasoning to describe what they saw. The activity is designed to support students' visual and spatial reasoning skills, as well as their ability to describe and explain mathematical concepts.
- *Problem solving* is a process in which students are presented with a mathematical problem or situation that requires them to use problem-solving strategies to find a solution. The activity is typically open-ended and designed to promote students' mathematical thinking, sense-making, and reasoning skills, as well as to deepen students' understanding of mathematical concepts. The activity involves group work, with students working collaboratively to develop and share their ideas, and during this work, teachers support students in verbalizing and responding to each other's thinking and accordingly facilitate mathematical discussions.

Additionally, *Strings* (Kazemi & Wæge, 2015) were also planned to be used as an instructional activity but were not included during the first year of the program period.

4.4 Guidelines for the school management and expectations for the participants

The MAM program includes guidelines for school management in participating schools and sets expectations for teachers who participate in the program. The guidelines for the school management involve that they actively engage with and understand the requirements of the MAM model. They should prepare teachers for this type of PD, find the "right" teachers to participate, ensure stability within the group, and either provide external support to lead the group's sessions or delegate responsibility to a dedicated teacher within the school. School management should also attend some of the sessions and support the TEs by expressing expectations for the teachers' collective development work. Moreover, school management should create an organizational structure that enables progress in implementing new teaching methods, and allocate sufficient time in the timetable for teachers to engage in collaborative development work at school.

The MAM program also recommends that school management should establish a steering group responsible for ensuring progress in the development work. The group can consist of one or two school leaders responsible for the development work, the expert who will lead the sessions (either internal or external), and one teacher. The school should consider which teachers will participate in the development work, engage teachers in an evaluation of the characteristics of mathematics education in the school, and agree on what they want to achieve through the MAM modules. They should create a goal formulation for evaluating their development work and that should be subject to regular review and adjustment. The school management should also consider the resources available and the conditions under which teachers will collaborate, and create an organizational structure that allows them to incorporate the development work into the daily operations of the school. Finally, they should create a progress plan with scheduled meeting times for the steering group and teachers to ensure their commitment, and allocate time to evaluate the development work at least once every six months.

The expectations for the participating teachers involve norms for collaboration and a positive attitude to the development work.[8] The purpose of these norms and expectations is to establish a positive and productive learning community among the participants taking part in the PD program. The norms are designed to create a safe and respectful environment where teachers can take risks, share their ideas, and learn from each other. The expectations are intended to ensure that all participants are fully engaged and committed to the learning process, and that they actively contribute to the development of a collaborative and supportive learning community. These norms and expectations aim to foster a culture of continuous learning and improvement among the participating teachers.

[8] For a more comprehensive account of the norms and the expectations for the attending participants in the MAM program, the interested reader is directed to the following website: https://www.matematikksenteret.no/kompetanseutvikling/mam/mam-modellen

5.0 Methodology and methods

This chapter presents the design of the study and the methodological considerations and methods used. I will describe how the data collection was planned and carried out, and how the empirical data has been analyzed in accordance with the substudies. An explanation of how CHAT was used to identify tensions and contradictions based on the findings from the substudies will then be presented. Thereafter, I will present how a meta-synthesis approach was used to integrate the findings from the three substudies, followed by a discussion on ethical considerations and the quality of the study.

5.1 Qualitative research design

The point of departure for this research project was to explore how the MAM program can contribute to the PD of mathematics teachers, and the chosen method for accomplishing this was a qualitative research approach. A qualitative study is appropriate for exploring and gaining a deeper understanding of complex phenomena in their social and cultural context, while taking into account such human aspects as social interaction. Bearing this in mind, I developed a qualitative research design. Even though researchers emphasize different characteristics of qualitative research so that it does not represent a definite set of components, Creswell and Poth (2018) argue that the field still has some common aspects. First the design of this study will be presented, and then an explanation of how a selection of the characteristics contributed to the development of the study will follow.

Qualitative research is *context-dependent* and the data are often collected in their *natural setting* (e.g., Creswell & Poth, 2018). This means that the study is a situated activity within the participants' context, enabling the researcher to gather close-up information about the participants and the features of the context in which they act. With the problem statement in mind, I needed to find a development project that included the MAM program and participants who were willing to let me conduct research based on their development work. In 2018, the NSMO was approached by a school owner who asked for assistance in finding a PD project for mathematics teachers in several primary and lower secondary schools from one district in Norway. The school owner wanted the NSMO to contribute to their development project by supplying the MAM program, a request the NSMO was happy to take on. It was decided to run two parallel programs, one for primary schools and one for lower secondary schools, and both programs were planned to start at the beginning of the 2019 school year. As this teacher PD

project suited my Ph.D. study well, both in terms of time and research topic, I decided to ask the actors involved if they were willing to join my research, to which they replied in the positive. The program intended for the lower secondary school was selected for my research. In addition to the objectives outlined in the introduction, this selection was based on the assumption that my background as a lower secondary mathematics teacher could help me to better understand the teachers' context and experiences. *Context-dependent* research conducted in a *natural setting* (e.g., Creswell & Poth, 2018) further implies that the researchers are present in the research field when collecting the data material, rendering the reality visible (Denzin & Lincoln, 2011). To further comply with the characteristics, I wanted one of the participating schools to be the context of the study. By following one of the schools throughout the first year of the program period and being present for a longer period of time when collecting the data material, I could gain a thorough "understanding of how events, actions, and meaning are shaped by the unique circumstances in which they occur" (Maxwell, 2013, p. 30). The context of the study and how it was selected are described in sub-section 4.3.

Addressing the purpose of a research project can be approached in different ways. However, the focus is framed by the problem statement, which is followed by a set of research questions that guide the inquiry in that they "explain specifically what your study will attempt to learn or understand" (Maxwell, 2005, p. 67). This also means that the research questions determine how and where data are to be collected. As presented in the introduction, the focus of this study was on the roles the teachers, the school leader, and the TEs as the program leaders have as different actors in the teachers' PD work, and how these roles act and interact to contribute to the teachers' PD. Such a focus led to an investigation into the interactions between these actors, which can be studied through their lived experiences and social processes (Denzin & Lincoln, 2011) concerning the development work in relation to the program. Moreover, people's interaction in development processes is influenced by such conditions such as their motivation, knowledge, perceptions, and expectations (e.g., Kennedy, 2016; Sannino et al., 2016; Timperley et al., 2007). Thus, an examination of these conditions in connection with MAM 2019 was important for understanding the actions that were taken. This means that the data material was intended to provide insight into the actors' expectations of the program as well as their experiences from the development work. Based on these considerations, the three subsidiary research questions were developed.

The first subsidiary research question concerns the teachers' perception related to the content of the program. In addition to providing insight into conditions that can influence their actions concerning their development work, the intention of this substudy was to gather information about the teachers' current perspectives, theories, and understandings of teaching practices and students' learning. As the development of teaching practices is the very core of what the MAM program is offering, the teachers' perceptions of teaching and students' learning before they enter a practice-based development program can indicate what the MAM program can contribute. The second subsidiary research question pertains to school management and their support for teachers' development work. This substudy specifically focuses on the principal's role as a facilitator for teacher PD. The intention has been to gather information about how the principal planned to support teachers' development work, and how this plan was implemented in practice as the program progressed. The aim was to gain insight into the organizational development structures at the school and the interaction between the principal and teachers. The third and final subsidiary research question concerns the TEs, as the program leaders, and how they supported the teachers' development of ambitious mathematics teaching. The program leader's role can be seen as offering support to the teachers to develop ambitious mathematics teaching, which is the purpose of the program. The intention of this substudy was to collect information about how the TEs experienced the first year of the program period, particularly in relation to their collaboration with the teachers and the principal. The purpose was to gain insights into how they experienced the results of their supporting actions and interactions with other actors.

These three subsidiary research questions attend to all the actors involved in the teachers' PD work, which opens for the development of a complex picture of the research issue, as multiple perspectives are taken into account. Creswell (2013) argues that such a holistic account requires that the researcher describes the complex interactions of factors in the study, which in turn paves the way for exploiting the use of CHAT as an analytical framework. One of the strengths of CHAT as an analytical framework is to identify tensions and contradictions for change and development (Engeström, 2001).

5.2 A case study approach

A case study approach is a frequently used qualitative research methodology, particularly due to its contribution of varied views on the world by providing knowledge that is context-dependent (Flyvbjerg, 2006). Like other forms of qualitative research, a qualitative case study searches for meaning and understanding where the researcher is the primary instrument for data collection and analysis. The researcher has an abductive strategy for investigation, where the end product is thickly described (Merriam & Tisdell, 2015). Several scholars have contributed to a general understanding of the nature of case studies. For example, Yin (2013) defines case study in terms of the research process as "a contemporary phenomenon within its real-life context, especially when the boundaries between a phenomenon and context are not clear and the researcher has little control over the phenomenon and context" (p. 13). Stake (1995) focuses on trying to pinpoint the case as the unit of analysis, thus viewing the case "as an object rather than a process" (p. 2). According to Creswell (2013), a case study is a bounded system, bounded in time and space. Such a view is also supported by Thomas (2015, p. 21), who argues that the case study is defined by the "edges you put around the case", meaning that the researcher can delimit what he or she is going to study. Merriam (1998) defines case study as an in-depth description and analysis of a bounded system, and defines "the case as a thing, a single entity, a unit around which there are boundaries" (p. 27). The case could then be a single person, a group of people, a program, and so on.

The methodology should be selected because of the nature of the research problem and the questions being asked. Merriam (1998) points out:

> A case study design is employed to gain an in-depth understanding of the situation and meaning for those involved. The interest is in process rather than outcomes, in context rather than a specific variable, in discovery rather than confirmation (p. 19).

As such, case studies can be particularly useful for studying a process, program, or individual in an in-depth, holistic way that allows for deep understanding (Merriam, 1998). The intention of this research study is to explore how a practice-based PD program, and more specifically the MAM 2019, can contribute to mathematics teachers' PD. As mentioned in section 3.3, MAM 2019 takes place in a specific setting and real-life context. By following the program period's first year, it also has a defined start and end. Therefore, it is a bounded system, bounded in time and space (Creswell, 2013), that can provide an in-depth understanding of the development of

the mathematics teachers' PD. Moreover, the investigation of MAM 2019 can lead to an in-depth understanding of the different perspectives and experiences of the various actors involved, meaning that the end product can be thickly described (Merriam & Tisdell, 2015).

5.3 Participants and context of the study

Four lower secondary schools signed up for MAM 2019. As the focus of this study was on the interactions between the actors, including the teachers and the principal, I had to follow both the school management and the participating teachers from the same school. Selecting which school or the number of schools to follow required careful consideration. Focusing on multiple cases has several advantages. First, it enables comparability across different contexts (Stake, 1995). Second, it provides richness and complexity the opens for a balanced understanding of the phenomenon and helps to identify contextual factors that influence its manifestation and outcome (Merriam, 1998; Stake, 1995). Third, it enables triangulation of data from different sources that can enhance the rigor and trustworthiness of the study's findings (Creswell, 2013). Additionally, including multiple schools can ensure data flow, particularly if one of the schools decides to withdraw from the study. However, there are several reasons for only focusing on a single school. First, it allows the researcher to observe and analyze the details of the PD project more thoroughly, including the interactions between the participants, the context, and the outcomes of the project. This in-depth analysis provides a rich and balanced understanding of the main phenomenon that would be difficult to achieve with a larger sample (Stake, 1995). Second, by focusing on a single school, the researcher can establish a deeper relationship with the participants and gain their trust, which can lead to more honest and open communication (Creswell, 2013). This, in turn, can lead to a more complete understanding of the phenomenon being studied, as the participants may be more likely to share their thoughts and experiences. Third, by studying a single school in depth, the researcher can generate more detailed and context-specific findings that can be particularly useful for informing future PD programs or interventions. These findings can provide a rich description of the complexities and challenges involved in carrying out PD programs in a specific context, which can help other educators and researchers to better understand the phenomenon and develop more effective interventions, and can be used to inform practice and policy (Merriam, 1998). Bearing these reasons in mind, I chose to focus on a single school.

The four schools were similar in terms of the number of students and teachers, and no other factors that potentially could affect the aim of the study were discovered. Therefore, all four

schools, including the principals and the mathematics teachers participating in MAM 2019, were asked if they wanted to take part in my research study, to which three of the four volunteered. The subsequent opportunity to choose between three participating schools made it possible to use purposeful sampling (Creswell, 2013). The chosen school, with its participating mathematics teachers, was selected for two reasons. First, the selected school and its participating mathematics teachers were the first to show interest. I assumed that informants who were particularly interested would make an extra effort to facilitate my work as a researcher in conducting the necessary data collection for the research study. Second, the selected school was asked and agreed to be the hosting school for the cycles. I wanted to be present at the plenary sessions throughout the first year of the program period to observe how the development work evolved. By selecting the hosting school as the context for my research, I had the opportunity to spend additional time at the research site, in addition to the data-collection periods.[9] During these sessions, I could gain further insight into the contextual conditions that characterized the selected school. This gave me the opportunity to maximize what I could learn from the participants I had selected and the context in which they acted (Stake, 1995). The selected school had seven mathematics teachers, all of whom attended the program. It had 330 pupils divided into 8th to 10th grades (ages 13–16) and it was multicultural with students who had different ethnicity backgrounds.

In the process of selecting informants for interviews and observation, all the participating mathematics teachers were asked if they wanted to contribute to my study by being interviewees and observation subjects. Almost all the teachers volunteered, which again made it possible to use purposeful sampling (Creswell, 2013). The aim was to select participating teachers who could satisfy the purpose of the study (Postholm, 2010), and who were willing to share their knowledge and experiences. I also considered selecting teachers with varying teaching experiences who worked with students at different grade levels as a criterion. This is because their teaching experience and membership in a working team could potentially influence their perceptions and experiences of the development work. One of the teachers immediately answered my call. This particular teacher was teaching 9th grade mathematics in the current year, which is considered to be the grade least interrupted by activities that interfere with the "normal" teaching, such as further-education exhibitions, exam preparations, and various kinds

[9] See section 5.4 for a thorough description of planning and data collection.

of excursions. Thus, selecting a 9th grade teacher would reduce the chances of events in the field disrupting the data collection. As this teacher volunteered both to be an interviewee and allow me to observe her classroom practice, she became my main informant. I assigned her the pseudonym Sofie. She had 10 years of teaching experience and formal teacher education of 30 credits according to the European Credit Transfer and Accumulation System (ECTS) (one year of full-time studies is 60 ECTS). To comply with the aforementioned criteria, I selected two additional teachers who taught mathematics in grades 8 and 10, both of whom immediately volunteered to be interviewed as part of my research. These teachers were given the pseudonyms Harald and Stig. Harald had five years of teaching experience and formal teacher education of 180 ECTS, while Stig had 13 years of teaching experience and formal teacher education of 60 ECTS. Additionally, three other teachers volunteered to participate in focus-group interviews (Kamberelis & Dimitriadis, 2011).

The school management volunteered to participate in the study while also agreeing to serve as the hosting school and the research context. It consisted of the principal and vice-principal. The three TEs responsible for leading MAM 2019 throughout the entire period were asked to contribute by being interviewees in focus-group interviews, to which all three agreed.

5.4 Planning and data collection

For this research project I planned two data-collection periods, each with a duration of three weeks. The first was carried out in the fall of 2019 before the start of MAM 2019, and the second in the spring of 2020 after the first year of the program period. The data-collection schedule was carefully planned in advance to ensure that I managed to collect the data I wanted during the three weeks in question. In the following I will elaborate on my data-collection methods and describe how the two data-collection periods were planned and conducted.

5.4.1 Interviews

Interviews constitute the main data-collection method, which is a commonly used strategy and primary source of data in qualitative research. According to Merriam and Tisdell (2015), interviewing is sometimes the only way to collect data when the object of study cannot be directly observed. This refers, for instance, to people's feelings, interpretations, or perceptions, or to past events. From the social constructivist perspective, reality is constructed during the interview within the frame of a social, historical, and cultural context (Wardekker, 2000). Brinkmann and Kvale (2015) claim that "[K]nowledge is constructed in the interaction between the interviewer and the interviewee" (p. 4), and furthermore that the researcher attempts to unfold the meaning of the interviewees' experiences to understand their world from their point of view.

There are different types of interviews, structured, semi-structured, or unstructured, which can be conducted in various ways, and can be directed toward single individuals or a group of individuals. In this study, I have conducted face-to-face individual semi-structured interviews, focus-group interviews, and online interviews.

Semi-structured interviews

In semi-structured interviews, both the interviewer and the interviewee engage in a formal interview where the intent is to understand the subjects' perspectives (Brinkmann & Kvale, 2015). This type of interview is positioned between the completely structured interview, where the interviewer strictly follows a prepared interview schedule with questions that are usually closed in character, and unstructured interviews, which are more free-flowing and are characterized by emergent conversations related to a general topic rather than specific questions. In semi-structured interviews, the interviewer prepares a list of topics to be explored and open-ended questions to be asked during the interview. The interviewer also ensures that the questions elicit open responses from the participants, which allows for unplanned conversations to be developed in ways or about themes that could not have been anticipated when the interview was planned (e.g., McIntosh & Morse, 2015). Open-ended questions also minimize the influence of the theories and attitudes the researcher might have beforehand (Creswell, 2013). As such, the semi-structured interview allows the participants to express narratives about their personal experiences and thus enables the researcher to learn and understand the participants' opinions, statements, and convictions. According to McIntosh and

Morse (2015), the semi-structured interview can be conducted both in person and online using social media platforms.

Focus-group interviews

In contrast to individual interviews, data collection in focus-group interviews occurs in and is facilitated by a group setting (Stewart & Shamdasani, 2015). These interviews can be used as a method of collecting data in qualitative research when a group of people has knowledge of the topic being studied. As focus-group interviews give the participants the security of sitting together as a group where opinions are discussed, the method might seem less threatening to them compared to individual interviews (Krueger & Casey, 2015). A social constructivist perspective underlies the data-collection procedure of focus-group interviews as the data obtained is socially constructed within the interaction of the group. Hennink (2014) points out that the interactive discussion through which the data are generated is one of the most unique characteristics of focus-group research. She argues that during the group discussion the participants both share and listen to the views of others and might also refine their views based on what they hear. Therefore, data generated from focus-group interviews are different and not accessible through individual interviews.

Online interviews

The internet and the development of communication technologies have given researchers the possibility to collect data through online venues, for instance by using computer-mediated communication (Salmons, 2014). Depending on the availability, these tools have given the researcher the opportunity to talk directly with participants anywhere and at any time. Salmons (2014) states that online interviews emancipate the researcher from geographical constraints when considering participants and the context in which the interview is to take place. Furthermore, she argues that most computer-mediated communication allows for video or audio recordings to be made which can be useful if the researcher wants to relive the interview or look for nonverbal cues. However, like all other data-collection methods there are also weaknesses related to conducting online interviews. For example, Merriam and Tisdell (2015) point out that access to and knowledge of how to use computer-mediated communication systems can vary, and also that technology is always subject to breakdowns. Moreover, Salmons (2014) argues that online interviews may be more challenging when trying to establish a personal connection between the interviewer and interviewee, particularly when it comes to establishing trust. Thunberg and Arnell (2022) also claim that a lack of physical presence can

make it more difficult to establish a sense of shared understanding than in face-to-face interviews.

5.4.2 Observations

Like interviews, observations are also considered a primary source of data, and this data-collection method has been used in a variety of disciplines for collecting data about people, processes, and cultures in qualitative research (Creswell, 2013). According to Marshall and Rossmann (1995), observation is defined as "as the systematic description of events, behaviours, and artefacts in the social setting chosen for study" (p. 79). In qualitative research, observations take place in the setting where the phenomenon of interest naturally occurs and are therefore also called naturalistic (Angrosino & Pérez, 2000). In this sense, the observational data represent a firsthand encounter with the phenomenon of interest, which is therefore favorable when studying actual behavior. Although observation as a method is most commonly used to collect data in relation to the research question, Merriam and Tisdell (2015) maintain that observations can also be conducted to provide knowledge or information that can be used as common reference points for subsequent interviews. They also argue that an outside observer can notice things that may have become routine for the participants themselves, and for that reason would not have been highlighted in an interview. Such information enables the researcher to ask more detailed questions in the following interview, for instance, what the participant was thinking when performing a specific action and thus contributing to gaining a more thorough understanding of the phenomenon. Observing the natural context in which the participant is acting can also provide valuable information about the context (Merriam & Tisdell, 2015) that in turn can contribute to understanding what is said in the following interview.

There is no distinct role the observer can take that is suitable for all situations. The role of the observer normally depends on the extent to which he or she is to be involved in the research setting. Gold (1958) states that observation is an activity that moves along a continuum: from *complete participant, participant as observer*, *observer as participant,* and *complete observer*. When researchers assume the role of a complete participant, they take on an insider role and are a full part of the setting. Participant as observer means that the researcher engages in the research setting and there is full interaction with the participants, but the participants are aware of the researcher. In the role of observer as participant, the researcher/observer has some connection to the setting being studied, but the involvement is minimal. When the researcher

assumes the role of a complete observer, he or she is present in the setting being studied but does not take any active part at all. It is important to note that Gold's classification implies that the observation is participatory, meaning that the observer is always present at the research site, and that the researcher is not a member of the society being studied.

5.4.3 The first data-collection period

For the first data collection period I planned three individual semi-structured interviews (Brinkmann & Kvale, 2015) with three mathematics teachers, and two focus-group interviews (Kamberelis & Dimitriadis, 2011), one with five mathematics teachers and another with school management. The group of five mathematics teachers included two of the teachers who had participated in the individual interviews, as well as three others who were also involved in the MAM 2019 program, as mentioned in section 5.3. Moreover, I planned to observe all the mathematics teaching lessons Sofie had during the entire period. The length of the data-collection period was determined so there was enough time to become acquainted with the school and get a grasp of its daily life. Such an acquaintance is important to give a thick description of the context where the research is conducted (Stake, 1995), and can furthermore be important to better understand what is really going on in the analysis (Corbin & Strauss, 2008). Figure 5 provides an overview of the first data-collection period.

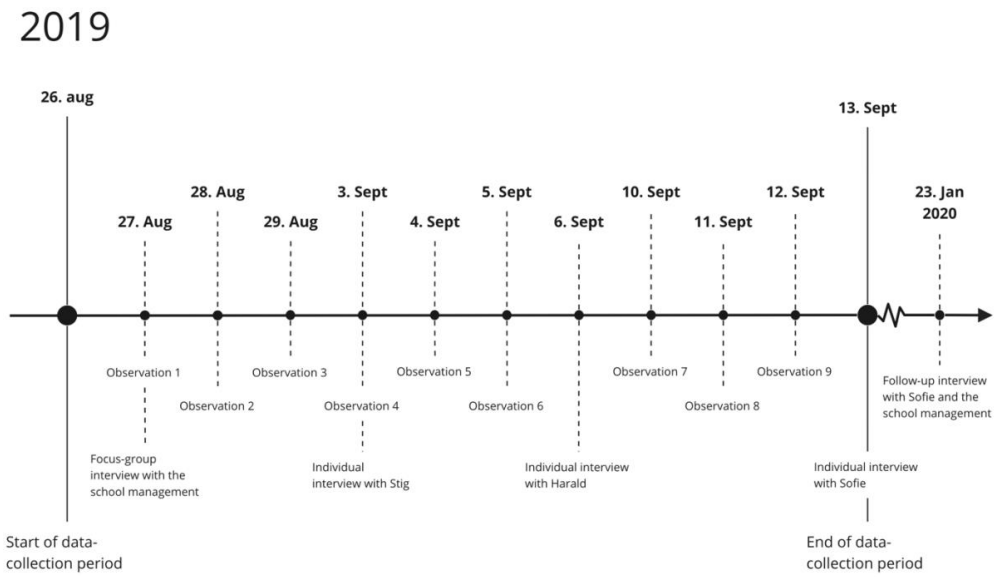

Figure 5. *Overview of the first data-collection period.*

The first data-collection period started by getting acquainted with the teachers, school management, students, and the context of the selected school. As a novice researcher, my lack of experience made it challenging to know what to look for and what to pay attention to in order to achieve a significant understanding and recognition of good sources of data (Stake, 1995). I also had to be careful not to be predisposed and I had to be aware of prejudices or other assumptions I might have, including theoretical perspectives. Such a position, known as sensitivity and skepticism (Postholm, 2019; Stake, 1995), is important to better investigate the meaning of what is being studied, which includes the entire data-collection process. As such, I tried to write down my daily observations and impressions with as little value-laden impressions as possible before attempting to understand what the observations were all about. These observations and impressions from conversations were written down on a daily basis in a logbook and then interpreted at the end of the day. The continuous work with my logbook constantly led me to new interesting questions and anticipations concerning the organization and school management of the teachers' PD, the mathematics teachers' perceptions related to teaching and student learning, and the contextual conditions that characterized the school. These written descriptions, interpretations, questions, and anticipations became useful perspectives in my work on the interview guide, and helped me to understand what the participants were saying during the interviews and in the process of analysis.

Observations

The observations were related to Sofie's classroom teaching of her 9th grade students. This group had three mathematics lessons a week, each on a different day, which made it possible to schedule a total amount of nine observations. The aim of the observations was mainly to provide information that could be used in the construction of the interview guide, and about situations that could be used as reference points for discussing teaching practices in the following interview (Merriam & Tisdell, 2015). I was also aware that observing the natural context in which the teacher was acting could provide valuable information that in turn could contribute to an understanding of what was mentioned in the following interview, such as actions that were difficult to explain or actions that have become routine for the teacher herself (Merriam & Tisdell, 2015). Therefore, I was also looking for situations that gave me the opportunity to ask what she was thinking when conducting her teaching practices and to find follow-up questions for the coming interview that could get her to explain the different choices she made.

Since I wanted to observe Sofie in her natural setting (Angrosino & Pérez, 2000), I asked if the observations could be conducted in her regular classroom with her regular group of students, to which she agreed. This means that the observations contain lessons or teaching episodes that were part of her regular classroom routine, and according to the school's learning program. The work of teaching often appears in several forms, such as writing on the board, talking to students, modelling, or using concretes, thus making it more or less impossible for the researcher to write everything down. Therefore, I planned to video-record the observations. A video recording also makes it possible for the researcher to relive the situation and in turn continue the observation and the analysis (Postholm, 2010). Even though I was not interested in video-recording the students, I could not guarantee that the camera would not capture some of them, on screen or verbally. Thus, I obtained informed consent from both the students and their superiors in accordance with Norwegian Centre for Research Data (NSD)[10][11] (NSD, 2019) regulations.

Even though I had carefully planned the observation and Sofie had informed the students about the observations I would be making, including handing out the consent form I had e-mailed several days in advance, the observations were not completely conducted according to the plan. Several students had forgotten about the consent form, and even with repeated reminders from Sofie, only half of the students had remembered to return a signed consent form. Thus, the two first observations were carried out without video recording, and field notes were therefore the only data from these observations. The rest of the observations were conducted as planned. The immediate impressions from the observations were written down in my logbook immediately after the end of each observation while they were still fresh in mind. How the first collected data material is understood is important for continuing the data collection process (Postholm, 2019). The immediate impressions were thoughts and descriptions of Sofie's teaching practice that I wanted to look into in more detail in the video-recordings, and which were further used in my preparation for the upcoming observations. The impressions also led me to pay more attention to similar situations and to questions that could serve as the initial interview guide. The extent to which the observer is engaged in the situation may vary depending on what the

[10] The Norwegian Centre for Research Data (NSD) became a part of the Norwegian Agency for Shared Services in Education and Research (Sikt) on January 1, 2022.
[11] See Appendix 2 for the study information and signature form for the research participants' informed consent.

observation aims to serve. As my aim was to observe Sofie's regular classroom practices before taking part in a professional learning setting, I did not want to intervene in or influence her lessons in any way. Thus, I assumed the role of *complete observer* (Gold ,1958), meaning that I was present to take field notes and record data without getting directly involved with the activity or the students.

Interviews

The purpose of the individual interviews with the three teachers was to learn about the mathematics teachers' current perspectives, theories, and understandings of teaching practices and students' learning. Therefore, my intention was to ask questions that could uncover the teachers' perceptions and knowledge regarding teaching and students' learning, which were grounded in their classroom practices, as well as the knowledge required to effectively teach school mathematics. The purpose of the focus-group interview with the teachers was to develop knowledge about their experiences of the school management's support and facilitation of their PD. I also wanted the group to express their thoughts on what kind of principles are or should be grounded in their classroom teaching practices. The purpose of the focus-group interview with school management was to gain data about how they prepared for their participation in MAM 2019, how they planned to organize the development work at school, and how they planned to support the teachers' development work. I also asked questions aimed at eliciting their expectations for the program and how they assumed the teachers perceived their leadership practice.

Before the interviews started, I explained their purpose and the aims of my research project. Even though I had already explained both the purpose of the interviews and participants' rights carefully in advance, and in accordance with the NESH (NESH, 2021) regulations, it was important for me that the interviewees were totally aware of the situation. All the interviews were conducted face-to-face and took place at the school. They were conducted as a conversation, but with a clear focus and a few pre-prepared questions[12] that were used as a guide throughout the interviews without any specific order. The same interview guide was used in all three individual interviews. The conversation gave the teachers and school management the opportunity to raise interesting aspects or themes that I had not considered before the

[12] See Appendix 3 for all the interview guides.

interview. I acted as a moderator (Chrzanowska, 2002) throughout the interviews by asking questions to encourage dialog between the participants. During the interview, I wrote down small remarks, possible follow-up questions, and comments that could help me in the further analysis. I also wrote down thoughts and remarks right after the interviews that were related to what I had experienced. The individual interviews lasted for approximately 40 minutes, whereas the group interview with the teachers lasted for 50 minutes, and the group interview with the school management for one hour. All the interviews were audio-recorded so I could give my undivided attention to the conversation. For the same reason as for the observations, this was also done to make it possible for the researcher to relive the situation and in turn continue the analysis (Postholm, 2010). I decided, moreover, to video-record the group interview as it might retrospectively be difficult to distinguish the interviewees from one another.

5.4.4 The second data-collection period

The second data-collection period was initially planned to follow the same schedule as the first. However, the pandemic situation that arose in early 2020 made it impossible for me to conduct the second data-collection period as planned, as the restrictive national health regulations did not allow me to visit the school. Accordingly, the second data-collection period consisted of three focus-group interviews, one with the same teachers as in the first data-collection period, and one interview with school management, which also included a follow-up interview to clarify concepts. They were all conducted online through the communication platform Microsoft Teams. The second data-collection period also included a focus-group interview and a follow-up interview with the same three TEs. An overview of the second data-collection period is presented in Figure 6.

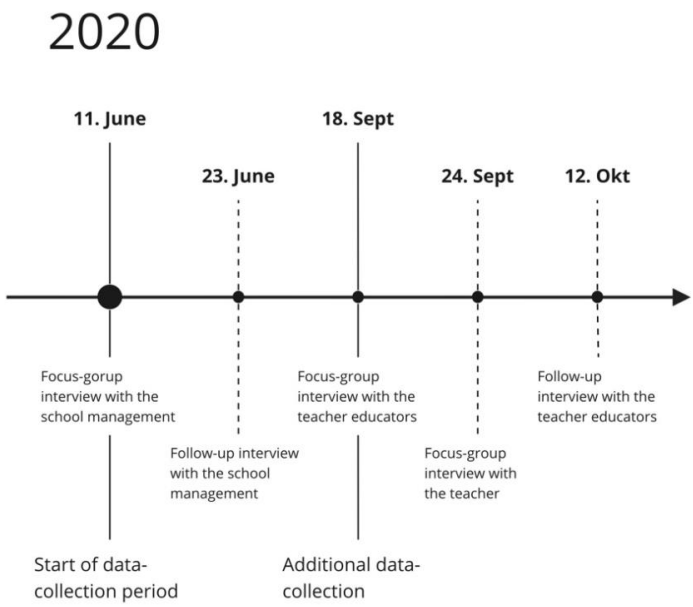

Figure 6. *Overview of the second data-collection period.*

The purpose of all the group interviews was to elicit the interviewees' experiences of the first year of the program. For the teachers, this involved experiences related to their development work within the program and at the school, and how they experienced the principal's facilitation and support. For school management, this involved experiences related to their organization of the development work at school, and their facilitation and support of the teachers' development work. Furthermore, I wanted both the teachers and school management to reflect on their prior expectations and goals for the program, to consider the extent to which these expectations and goals were fulfilled, and reflect on which factors might have promoted or restrained this work. For the TEs, this involved experiences related to the participating teachers and how these experiences reflected their expectations concerning foreknowledge and development. I also wanted them to reflect on the principal's involvement, their own role, and their interaction with the other actors.

All the interviewees were once again made aware of the purpose of the interviews in accordance with the NESH (NESH, 2021) regulations to ensure their awareness of the conditions. This time around I also acted as moderator (Chrzanowska, 2002) throughout the interviews by asking questions to encourage dialog between the participants, and the interviews were conducted as a conversation with a clear focus and a few pre-prepared questions. I was aware that online

interviews could entail possible challenges in terms of reduced personal connection between me and the teachers (Salmons, 2014), difficulties in establishing shared understandings (Thunberg & Arnell, 2022), or technological breakdown issues (Merriam & Tisdell, 2015). However, the first data-collection period and the continuing meetings during the program sessions contributed to establishing a trustworthy relationship with the participants. I also tried to avoid any misunderstandings by asking follow-up questions and requesting the participants to repeat utterances. Small remarks, possible follow-up questions, and comments that could help me in the further analysis were noted during the interviews, and interesting thoughts and remarks about what I had experienced were written down right after the interviews. The group interview with the teachers lasted for one hour, while the group interview with the school management lasted for 50 minutes. The group interview and the follow-up interview with the TEs lasted, respectively, for two hours and two and a half hours. All interviews were audio-recorded by me with a small recorder placed on the table.

5.5 Data analysis

In this thesis I have used the constant comparative analysis method (Corbin & Strauss, 2008; Strauss & Corbin, 1990, 1998) in all three substudies. Even though the method was originally developed to generate theory from empirical data within grounded theory, Corbin and Strauss (2008) claim that the method can be used in all studies, including case studies, and is widely used in qualitative research. While the constant comparative analysis was the main method used in the first substudy, it was used to conduct initial analysis for organizing the data material to create a scale for further analysis (Charmaz, 2014) in the second and third substudies. Based on these initial analyses, I have used CHAT and the activity system in the second and third substudy (Engeström, 1987, 1999, 2001) to identify tensions and contradictions that have the potential for development and change (Engeström & Miettinen, 1999). Finally, to integrate the findings from the three substudies into a comprehensive whole, I have used a meta-synthesis approach (Sandelowski & Barroso, 2007). Below, I will elaborate on the two methods and how they were applied to analyze the data of the three substudies, as well as how the findings from the three substudies were integrated using a meta-synthesis.

5.5.1 Constant comparative analysis method

I decided to use the constant comparative analysis method to analyze the data material (Corbin & Strauss, 2008; Strauss & Corbin, 1990, 1998). This method helps the researcher to make sense of the data in a systematic and rigorous way. I found this method useful for the analysis in my study, particularly because of its systematic approach (Postholm, 2019) that can be applied to various types of data, including interviews, focus groups, and observations within case studies. Although Strauss and Corbin (1990, 1998) state that the analysis process is not linear, this method provides procedures and tools that are useful for structuring the data material, making it easier to process and simultaneously ensuring that the researcher thoroughly examines and interprets his findings (Postholm, 2019). Alvesson and Sköldberg (2009) argue that in this form of data analysis, the researcher engages in a process of moving between the data and the field to collect information about a phenomenon. This information is then generated into codes and categories that are continually tested and refined throughout the analysis. According to Strauss and Corbin (1990, 1998), this process helps to ensure that the analysis is grounded in the data and that the findings are robust and trustworthy (Lincoln & Guba, 1985). As such, the constant comparative analysis method allows researchers to generate rich and balanced insights from their data and to develop a more robust understanding of the research context.

According to Postholm (2019), the analysis process in qualitative research begins as soon as the researcher enters the research field and constantly continues throughout the study to gain a better understanding of the phenomenon being studied. This is because the researcher's initial understanding affects the focus of the data collection. For this study, my initial understanding as a researcher was based on the written impressions in my logbook from observations and conversations during the first data-collection period, on questions and remarks noted during the interviews, and on descriptions of my initial experiences noted immediately after classroom observations and the interviews. This initial understanding is then the starting point for the analysis. Table 1 presents an example of reflections related to the interviews, observations, and logbook.

Table 1. *Example of reflections from the logbook.*

Situation	Reflections
Log day 1	…In a longer conversation with Sofie today she told me about a mathematics teacher course they have recently attended. She talked about what they learned from the course, and she specifically highlights mathematical conversation, problem solving, and that students must be allowed to explore and develop their own solutions in the work with a mathematical problem. I wonder how she understand these concepts and how they affect her classroom practices…
Observation 1	…Considering what we talked about yesterday, this teaching lesson was a bit of surprise to me. As the class begins she has a very strong focus on mathematical notation, and she is determined that the students have to solve the equation tasks in a special way. She argues that she, as well as the other mathematics teachers, knows what it takes to learn equations and that they know this is important according to what will be emphasized on the upcoming exam. How does she think this is related to the concepts mentioned the day before? Why does she do things differently? I have to ask more about this in the interview. This lesson has some aspects reminiscent of a behavioristic approach to learning...
Individual interview - Sofie	…She is using much of her time to talk about the importance of teacher explanation. It seems like the power of good explanations is an important aspect for her in relation to teaching in mathematics. However, she also seems to vacillate a little bit as, on the one hand she highlights how she can transfer her knowledge from herself to the students with the use of good explanations or "show the students", and on the other hand she talks about facilitating for problem solving, classroom discussions, and inquiry. Further insight is needed to fully understand what she really means when she talks about teacher explanations and "showing the students".

The interviews were transcribed verbatim shortly after they were conducted. During the transcription process, I took notes that included initial interpretations relating to what the interviewees said, research that could support my interpretations, and questions. I also color-coded bigger chunks of the data that could be developed as categories. The coloring made it easier to see which parts of data were related to a specific code, as shown in Table 2. To gain

an overview of the transcriptions and my notes, I organized everything into a four-column matrix: transcription text, initial analysis, related research, and codes and questions. An example of how the transcriptions were organized is presented in Table 2. I then divided the transcriptions into smaller sections to make the raw data easier to work with. Each section contained one to three statements depending on how the reasoning or argumentation was connected.

Table 2. *An example of how the transcriptions were organized from substudy 2.*

Transcription	Initial analysis	Related research	Codes and questions
Researcher: Do your expectations align with your experiences of this first year of the program? Principal: …It's always easy to be wise after the fact, but what I now see as a big challenge with the whole program is that it has become "one-size-fits-all" approach, and the implementation phase went very quickly. There was a short period of time from when we received the information to when we made a decision and started. I, as the principal, feel like I don't have enough control over the project. I don't feel I have any real impact on either the content or the organization because it has to go this one way. Researcher: What do you mean by "one size fits all"? Principal: Well, now it's the same series of courses for all the participating secondary schools, but I think we would have succeeded better if there were opportunities to make adjustments in the organization and content in relation to each school's needs. And it's a bit unclear to me who defines the content and organization, whether it's the NSMO or the school owner, or us principals. I've asked about this and I haven't been given a clear answer. The school owner is involved with professional support and is coordinating the project, but when, I ask it doesn't seem like they have the mandate to make changes if necessary. Researcher: In what way have you supported the mathematics teachers in their work with the MAM project?	"one size fits all". Reflecting on the implementation period and how the program is organized in relation to their own development needs. This is interesting because he says he is experiencing a lack of control. What is this need for control about? Especially concerning the content. Also, his job is to facilitate teachers' development work by supporting a collaborative learning environment at school (see first interview). Organizing the development work at school by devoting time. Did not participate in the sessions in the program or in the meetings with the teachers at school,	Leithwood et al. 2020 King & Stevenson, 2017 Kazemi & Resnick, 2019	What does he mean when he says: "I don't have enough control over the course of the project?" or "I don't feel I have any real influence on either the content or the organization." Implementation and organization

Principal: Well, we have tried to follow the plan we originally made, which was to set aside time between the meetings for the math teachers. One of the math teachers has reduced teaching time so there is room to coordinate the collaboration between the math teachers. So they've had a session of about two hours between each meeting to work here at the school, both to expand their knowledge and work on the "homework" they have been given at the sessions. So...yes, but of course, our participation is lacking to a great extent, both at the sessions and between them. So, the development work has not been led by us other than organizing the use of time.	and did not lead the development work. This is way off from all the aspects of the plan they described in the first interview! Could be possible reasons for not having an overview of the teachers' development work.		Facilitating and supporting teachers' learning

Strauss and Corbin (1990, 1998) describe this type of analysis as a three-step coding process with: "open coding", "axial coding" and "selective coding" phases, and with constant movement between them. Although Strauss and Corbin point out that there is no sharp dividing line between the coding processes, I will describe how these stages were used to analyze the data material in this study.

Open coding

During the open coding phase the researcher reads through the data material several times to identify patterns, concepts, and themes (Strauss & Corbin, 1990, 1998). He looks for similarities and differences in the data and codes it by labeling the concepts and themes that emerge to develop main categories. This process helps the researcher to organize and make sense of the data, and to identify the most relevant and significant themes and concepts for further analysis (Postholm, 2019). With the research questions in mind, I started to scrutinize the data material by using the codes as the starting point for identifying patterns, concepts, and themes (Strauss & Corbin, 1998). In this process, I identified initial categories that captured the essence of the codes, which became the categories used to code the rest of the data material. This was done by going through the data material systematically and applying codes to each piece of data relevant to the category. Instead of analyzing the transcription line by line, it was analyzed sentence by sentence. This approach allows for larger units to be covered by each code, making it easier to work with the data material (Postholm, 2019). Throughout the open coding process I constantly compared the data to identify similarities and differences, and the codes and categories were continually tested and refined. This interactive process of going back and forth between data collection and analysis is referred to as an abductive approach (Alvesson

& Sköldberg, 2009). I asked questions such as: "what does this mean?" or "what can this be an expression of?" and noted immediate thoughts about what I believed the communication and utterances represented. The process was repeated several times as new categories were developed, refined, and revised as new patterns and themes emerged. The codes were grouped into categories across the interviews, generating the main categories for the three substudies.[13]

Axial coding

Strauss and Corbin (1990, 1998) state that during the axial coding phase the researcher examines the relationships between the codes and categories developed in the open coding phase and looks for connections and patterns between them. Corbin and Strauss (2008) maintain that the researcher can then use this information to develop a more detailed understanding of the data and to refine the categories and codes developed in the open coding phase. Strauss and Corbin (1990, 1998) propose a paradigm model as the starting point for this phase that involves the following factors: causal circumstances, the phenomenon, context, intervening circumstances, action/interaction strategies, and consequences. They argue that these factors can guide the axial coding phase in developing a comprehensive and systematic understanding of the dynamic nature of the phenomenon being studied and the different factors that shape it. By analyzing the data through the lens of the paradigm model, researchers can identify the relationships between the different factors and how they contribute to the phenomenon (Strauss & Corbin, 1990, 1998).

For my study, the causal circumstances refer to factors or reasons that led to the conduction of MAM 2019, which can be understood as reasons for the schools' participation, as the study would never have been started without their inclusion. The phenomenon is the MAM program itself, which is the focus of the study, and the specific school I followed where the program was carried out represents the context. Furthermore, the intervening circumstances refer to the unexpected events or factors that may have influenced how the program was carried out. Action/interaction strategies relate to the different actions taken by the actors involved, meaning the teachers, principal, and TEs for this study, and how these actions impact the teachers' PD. The consequences are related to the impact the teachers, the school, and the TEs had on the outcome of the program, both positive and negative.

[13] See Figures 7, 8 and 9 for an overview of the main categories of the three substudies.

To develop sub-categories related to the research questions in the study, I used the different factors of the paradigm model to further analyze and refine the main categories. In substudy 1, I used the phenomenon factor to explore the specific aspects of *engaging in students and their thinking* and *mathematical discussion*.- These were the most relevant factors for the research question. I also used the action/interaction strategies factor to analyze the specific strategies and techniques that the teachers used to engage in students and their thinking and to facilitate mathematical discussion. Both of these factors were helpful in developing sub-categories related to the teachers' perceptions of teaching and students' learning in mathematics, as well as how these prior perceptions might influence their participation in the development program. In substudy 2, I used the context and intervening circumstances factors to examine the specific contextual and situational strategies the school management used to support and facilitate the teachers' PD, and to examine the process of developing ownership of the development work. The same strategies were also used to explore factors that influence the teachers' experiences of lack of ownership, motive, and school management support. I also used the intervening circumstances factor to explore the school leader's ability to lead the development work at school.

In substudy 3, the phenomenon and context factors were used to explore the specific instructional activities and their purpose in the development program, as well as to examine how they were related to the broader educational context in which the program was carried out. These factors were also used to examine the specific contextual properties and the communication with the actors, and how these properties and interactions influenced the school's participation. Action and interaction were examined to determine which aspects underpinned the expectations of the teachers and the school, and how they experienced the influence these factors had on the teachers' participation. Additionally, in substudies 2 and 3, the causal circumstances factor was prominent. The reason for the school's participation was claimed to be the school's and the mathematics teachers' perceived need for PD. However, when analyzing the interviews, it became evident that this need was perceived by the school owner and not by the school and the mathematics teachers, as shown in substudies 2 and 3. Even though the principal stated that the school was given a choice as to whether or not they wanted to participate, the school's participation was initiated by the school owner. The causal circumstances factor was used to analyze the specific barriers and challenges that the teachers, principal, and program leaders faced in addressing this issue of the school owner's perceived need for PD. Throughout the entire process of developing sub-categories, I asked questions

such as when, why, and under which circumstances did this category materialize, how and what does it lead to (Strauss & Corbin, 1998) when identifying the different factors that contribute to the emergence of a particular code or category, and how these factors are related to each other.

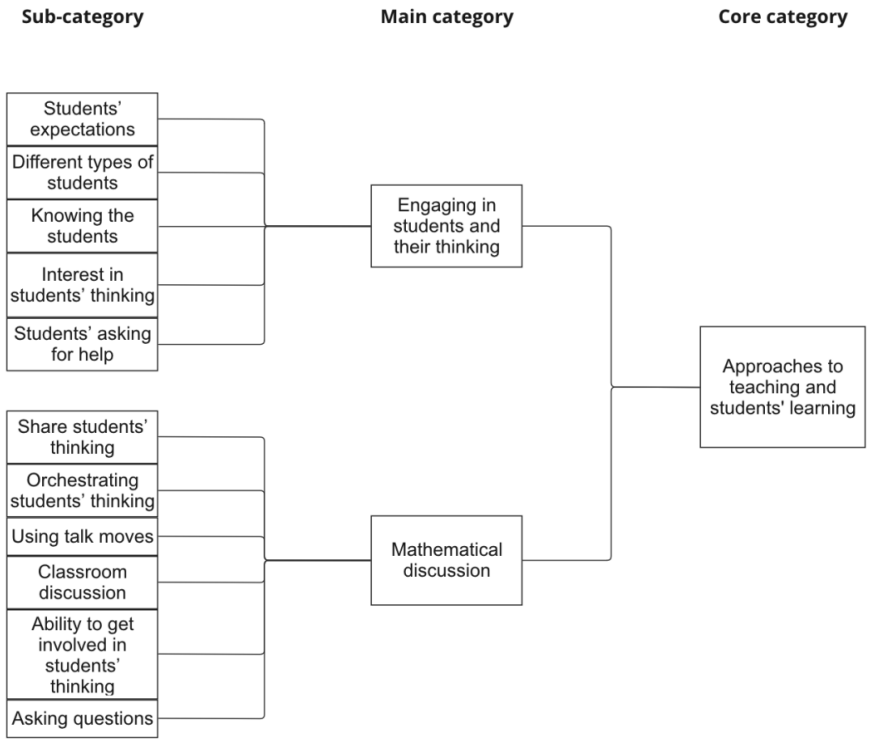

Figure 7. *Overview of the main categories, sub-categories, and core categories in substudy 1.*

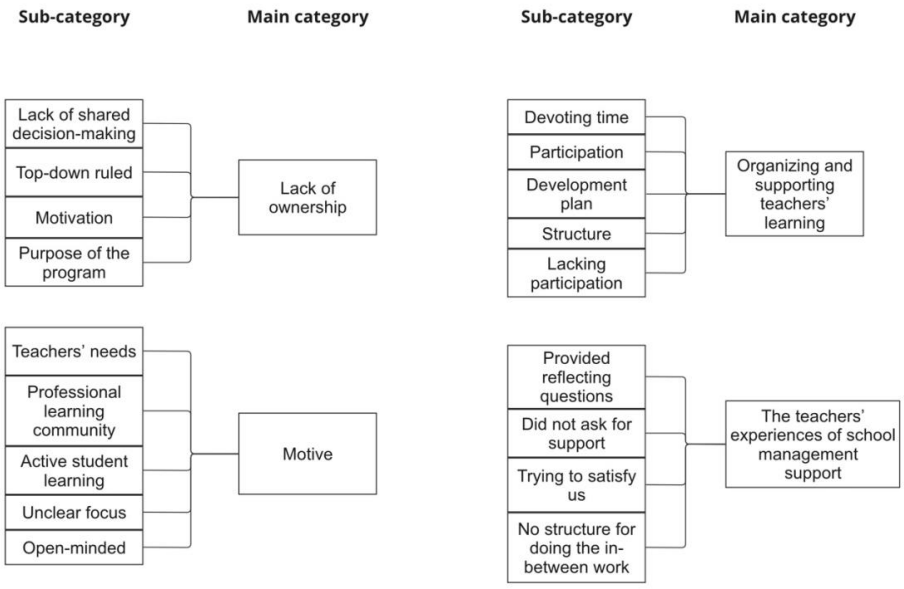

Figure 8. *Overview of the main categories and sub-categories in substudy 2.*

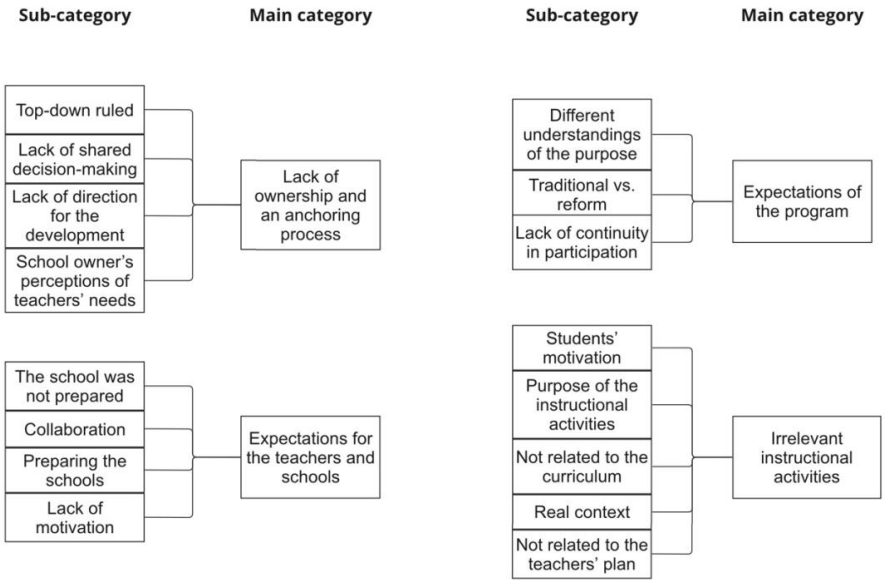

Figure 9. *Overview of the main categories and sub-categories in substudy 3.*

Selective coding

The final coding phase, selective coding, is described as the phase in which the researcher identifies the central category that is central for answering the research question and connects all the other categories and organizes the data around this category (Strauss & Corbin, 1990, 1998). The most significant categories are selected, refined, and modified, and organized into a coherent structure that explains the data (Strauss & Corbin, 1990, 1998). As mentioned above, constant comparative analysis constituted the main method for analyzing the data in substudy 1, whereas it was used to organize the data material to create a scale for further analysis (Charmaz, 2014) in substudies 2 and 3. After careful consideration of the data material in substudies 2 and 3, I decided not to develop a core category during the selective coding phase. This was due to the fact that the data material in these substudies did not lend itself to a single, overarching category that could adequately capture the complexity and richness of the material. Instead, I decided to focus on refining and modifying the most significant categories that emerged during the open and axial coding phases. This allowed me to maintain the integrity and complexity of the data while still identifying patterns and relationships between categories. Furthermore, using the analysis from the open and axial coding phases as the point of departure for further analysis with the use of CHAT and the activity system enabled me to consider all relevant aspects of the activity system and ensure that important elements were not overlooked. This approach also opened for a more nuanced understanding of the data and provided a strong foundation for further analysis. Therefore, I did not develop a core category for substudies 2 and 3, but rather used the analysis from the open and axial coding phase as the point of departure for further analysis using CHAT and the activity system. This process is described in section 5.5.2.

When identifying a core category in substudy 1, I first turned to the literature in search of a unifying concept that could combine all categories that were related to the teachers' perception of teaching and students' learning in mathematics. Using literature related to the categories, I found that they were all related to the practices and principles of ambitious mathematics teaching (e.g., Kazemi, 2017). While searching for a unifying concept in the literature can be useful in terms of connecting the data with theory, Corbin and Strauss (2008) also claim that the researcher must be aware of the risk that the concept may not fully fit the data. Thus, I returned to the data material to ascertain the relationship amongst the categories and between the categories and the theoretical concept. In this process, I found that the sub-categories and therefore also the categories contained other aspects of teaching and students' learning that

could be related to the practices and principles of ambitious mathematics teaching, such as "teacher explanation", and "showing strategy". To unify the categories into a combined core category, I rejected the theoretical concept and rather asked myself "what is all this really about"? I realized that all the categories involved different approaches to teaching and students' learning, and the core category was therefore entitled *Approaches to teaching and students' learning*.

5.5.2 CHAT and the activity system

CHAT[14] is a framework for analyzing human activity and understanding how people interact with each other and their environment to accomplish their goals (Engeström, 1999). The second and third generation of CHAT views human activity as a system and a network of systems that is situated within a larger social and cultural context. They acknowledge the importance of understanding the interplay between various elements of the system and between systems in order to gain a holistic understanding of how they function (Engeström, 2001, 2015). Furthermore, Engeström (2001) argues that one of the key strengths of using the activity system to analyze the data material is its ability to identify and address tensions and contradictions within and between activity systems.[15] As such, CHAT as a framework for further analysis enabled me to gain insights into how the various nodes of the activity system in substudy 2 and between systems in substudy 3 interacted with one another and contributed to their functioning. This approach further enabled me to consider the social and cultural factors that might have influenced the activity systems, provided a deeper understanding of the broader context in which the activity systems operated, and allowed for identifying tensions and contradictions within and between the systems that could then be a starting point for change and development.

In substudy 2, my aim was to explore how the school management supports and facilitates mathematics teachers' PD as they participate in MAM 2019. I began the process of using CHAT and the activity system to further analyze the data by identifying the activity and mapping out the activity system by identifying the different nodes, including the rules, division of labor, tools, community, and outcomes.[16] I considered the activity in this study to be the teachers' PD, with the teachers as the acting subject in a related activity system. I further considered MAM

[14] See section 3.2 for a thorough description of CHAT.
[15] See section 3.2.1 for a thorough description of the activity system.
[16] The nodes in the activity system are described in section 3.2.1.

2019 as the mediating artifact that helped the teachers to move toward the object, defined as developing their teaching practice. The school management was seen as a part of the community, working to support and facilitate the development process. Because the initial analysis revealed that the school owner initiated the school's participation in the program, I also placed the school owner as a member of the community (see Figure 10).

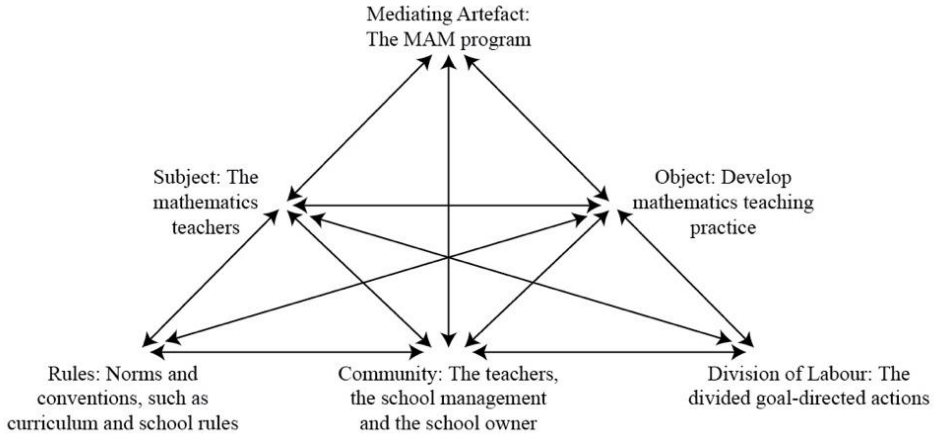

Figure 10. *The activity system for teacher PD with the teachers as the subject.*

With the main categories and their sub-categories developed from the initial analysis as the starting point, I further analyzed the data material to identify interactions within or between the nodes in the activity system. I also looked for data that could reveal tensions and contradictions within and between the nodes of the activity system, which could potentially serve as the starting point for change and development. This process led to three prominent tensions and contradictions. The first was a tension within the community between the teachers and the school leader as to how the MAM program should serve as a mediating artifact. The data from the three categories, *Lack of ownership*, *Motive*, and *The teachers' experiences of school management support*, showed that the motive for participating in MAM 2019 was not shared between the teachers and the school management. Second, in addition to a lack of shared motive, the data from all four main categories showed that neither the teachers nor the school management had a clear understanding of the purpose of the MAM program. This leads to a contradiction between the community and the MAM program as the mediating artifact because the lack of clear understanding of the purpose of the program will impede their goal-directed

actions toward the object. Thirdly, the data from the two categories, *Lack of ownership*, and *Organizing and supporting teachers' learning*, showed that the school owner and the school management were not fully aware of what their roles involved. This led to a tension in the division of labor because goal-directed actions had not been divided between the school leader and the school owner.

In substudy 3 my aim was to explore how the TEs experienced a practice-based development program for mathematics teachers from their point of view. The school management and the school owner were considered part of the community in the activity system where the mathematics teachers were the acting subject because they played an important role in facilitating and supporting the teachers' PD. As actors who are leading MAM 2019, the TEs were a crucial part of the development project as they played a role in facilitating and supporting the teachers' PD by using MAM 2019 as the mediating artifact. This means they can be regarded as members of the same community. However, Engeström (2001) argues that the activity system is flexible and adaptable to different contexts and research questions. This means that researchers can modify and adjust the classification of actors and components within an activity system to suit their specific research needs. He also argues that the activity system is context-dependent on the specific details of the context in which the activity system is being studied and that the context can shape the roles and relationships of the actors involved. The focus in substudy 3 was on the role of the TEs, who are not a part of the immediate context of the mathematics teachers' work. Although they play an important role in mediating the development program and facilitating the teachers' work, they are external to this context and therefore indirectly involved in the activity system through their mediating role and their interactions with the teachers. Thus, the TEs, the program leaders, were regarded as the acting subject in their own activity system (see Figure 11), and the practice-based development program for mathematics teachers defined the activity. The object of this activity system is the teachers' development of ambitious mathematics teaching, and the mediating artifact is MAM 2019 that helps the TEs move toward their object. Similar to substudy 2, I used the categories and their sub-categories from the initial analysis as the starting point to further analyze the data material to identify interactions within or between the nodes in the activity system.

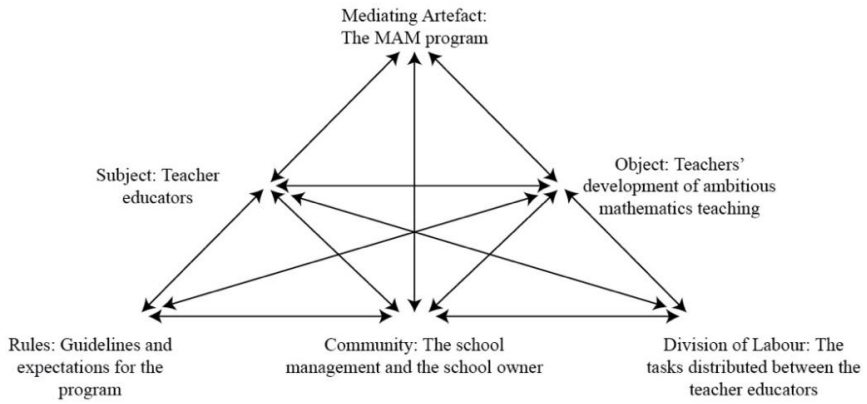

Figure 11. *The activity system for teacher PD with the TEs as the subject.*

In this process I quickly realized that the data material did not describe any interactions, neither within or between the nodes in the activity system, that indicated any tension or contradictions. However, the data revealed a mismatch between what the TEs considered to be the object of their work and what they experienced as the object of the school's participation in the program. The data from the *expectations for the program* category revealed that MAM 2019 had a predetermined aim, but the data from the *expectations for the program* and *the relevance of the instruction activities* categories showed that the TEs experienced that this aim did not align with the teachers' motive for participating in the program. To obtain a better understanding of what this mismatch was all about, I used the third generation of CHAT (Engeström, 2015) (see Figure 12). Then I observed that the TEs experienced that the school and its teachers lacked a clear and common direction for the development work. In other words, the TEs experienced that the school had not developed a common motive for attending the program. This leads to an understanding of a contradiction between the two activity systems when it comes to how the MAM program is to function as a mediating artifact.

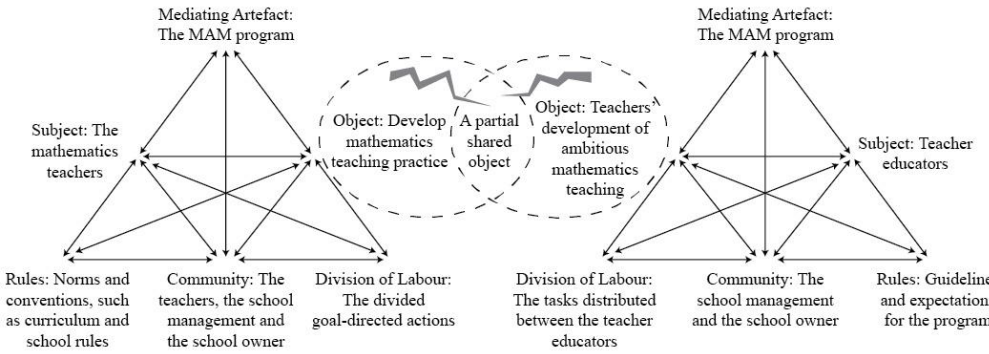

Figure 12. *A network of activity systems for teacher PD.*

5.5.3 Meta-synthesis

To understand how the findings from the three substudies can contribute to answering the main research question I needed to integrate the findings into a comprehensive whole. For this purpose I chose to synthesize the findings across the three substudies using a meta-synthesis approach (e.g., Sandelowski & Barroso, 2007). Meta-synthesis research involves synthesizing qualitative research studies on a particular topic to develop new insights and understandings (Bondas & Hall, 2007). McCormick et al. (2003) describe this as a process of interpreting interpretations that goes beyond mere summarization of the findings; it entails a reconceptualization of the findings and their interpretation, leading to the generation of new insights that surpass the scope of individual studies (Campbell et al., 2003). Thus, the primary goal of meta-synthesis is to combine individual studies at a higher level of abstraction and to utilize the conclusions drawn from these studies as the primary data. Herber and Barroso (2020) state that a variety of methodological approaches to qualitative synthesis have emerged in recent years, one of which includes the qualitative meta-summary/meta-synthesis developed by Sandelowski and Barroso (2007).

According to Sandelowski and Barroso (2007), synthesizing qualitative research involves a six-step procedure: conceiving the qualitative synthesis study, searching and retrieving qualitative literature, appraising reports of qualitative studies, classifying the findings in qualitative reports, synthesizing qualitative findings into meta-summaries, and/or synthesizing qualitative research findings into meta-synthesis. They add that meta-summary and meta-synthesis are two different approaches to synthesizing qualitative research. Meta-summary involves summarizing

and organizing the findings of multiple studies into a single report where a deeper level of analysis, interpretation, or integration of the findings from the studies is not required (Sandelowski & Barroso, 2007). In contrast, the authors posit that meta-synthesis requires a more in-depth analysis and integration of the findings from multiple studies to generate new insights or theories. According to Sandelowski and Barroso (2007), a qualitative meta-synthesis refers to an interpretive combination of qualitative research findings obtained from primary research reports. They point out that the outcome of such a meta-synthesis involves integrating research findings and going beyond mere summarization. Thorne et al. (2004) concur with this view and suggest that a meta-synthesis should offer fresh interpretations of the findings or serve as the platform from which to develop conceptual descriptions, models, or theories, thereby presenting "integrations that are more than the sum of parts" (p. 1358).

Conducting a meta-synthesis in qualitative research is a challenging and demanding task. For example, Bondas and Hall (2007) point out that meta-synthesis research is a complex and demanding process that requires researchers to carefully consider the methods used in the original studies, as well as the broader social, cultural, and political context in which they are conducted. They note that the diversity of qualitative research approaches, combined with the lack of standardization in reporting methods, can make it difficult to compare and combine findings across studies. Moreover, Thorne et al. (2004) caution novice researchers against conducting meta-analysis in qualitative research due to its complexity in transforming new conceptualizations. However, in this study, the constant comparative analysis method was used in all three substudies. Furthermore, as the researcher, I was responsible for the data collection, analysis, and findings from the substudies, which were situated in the same research context. These conditions enabled me to overcome the challenges pointed out by Bondas and Hall (2007) and Thorne et al. (2004) and made it easier to ensure that the particular contribution of each substudy was maintained. Additionally, since I already had the findings from the three substudies in hand, Sandelowski and Barroso's (2007) first four steps had already been conducted.

Sandelowski and Barroso (2007) suggest several analytical methods that can be used to approach meta-synthesis, including the constant comparative analysis method (Strauss & Corbin, 1998). Using the core category from substudy 1 and the main categories, tensions, and contradictions from substudies 2 and 3 as the starting point, herein referred to as "codes", I synthesized them into a meta-synthesis using the constant comparative analysis method.

Following the three-step coding process described in section 5.5.1, I started the open coding phase by looking for patterns to group the codes into categories. Similar codes were grouped together into categories, and during this process, three prominent categories emerged: *The MAM program as a mediating artifact*, *A lack of foundation and leadership practice for PD work at school,* and *A lack of a partially shared object*.

As I examined how the codes and the three categories were connected in the axial coding phase I faced the challenge that several codes could be connected to more than one main category of the meta-synthesis. For example, the code "Expectations of the program" from substudy 3 could be connected to all three main categories in the meta-synthesis. These different relations between the codes and the three categories made it difficult to develop distinctive sub-categories. To address this challenge I turned to the tensions and contradictions identified in substudies 2 and 3. As mentioned above, these tensions and contradictions were identified by using CHAT and activity systems, based on the sub-categories and main categories that emerged in the initial analysis. Moreover, these tensions and contradictions were already related to the sub-categories and main categories within their respective studies, which made it easier to see how the findings were connected to the main categories in the meta-synthesis. Therefore, I used the four tensions and contradictions, (1) A tension within the community between the teachers and the school leader as to how the MAM program should serve as a mediating artifact, (2) A contradiction between the community and the MAM program as the mediating artifact, (3) A tension in the division of labor because goal-directed actions have not been divided between the school leader and the school owner, (4) A contradiction between the two activity systems regarding how the MAM program is to function as a mediating artifact, to group the codes into higher-level concepts. Then I was able to separate one category from another. This approach also helped me to concentrate on the focus of the study – the actors' actions and interactions. Furthermore, the relationship between the findings across the three substudies and the tensions and contradictions was explored.

While connecting the categories with the four tensions and contradictions, I posed questions such as when, why, and under which circumstances the categories emerged, as well as how and what outcomes they yielded. This process led to the development of five sub-categories:

Contradictions in theories of action, *A lack of foundation for PD work at school*, *Leadership for teachers' learning*, *A need for boundary crossing and transformation learning*, and *Challenges in the nature of the MAM program*. The purpose of this meta-synthesis was to integrate and consolidate the findings from the three substudies, thereby achieving a higher level of abstraction that would enable the main research question to be answered. According to Strauss and Corbin (1990, 1998), the core category is a higher-order concept that synthesizes the categories and sub-categories to explain what the data is all about. *Interaction and dialog as a foundation for practice-based PD programs* was developed as the core category in this meta-synthesis. This overarching theme connects and explains the findings from the three substudies. Figure 13 below provides an overview of the main categories, sub-categories, and the core category. This meta-synthesis and the categories that emerged from it will structure the discussion of the findings.

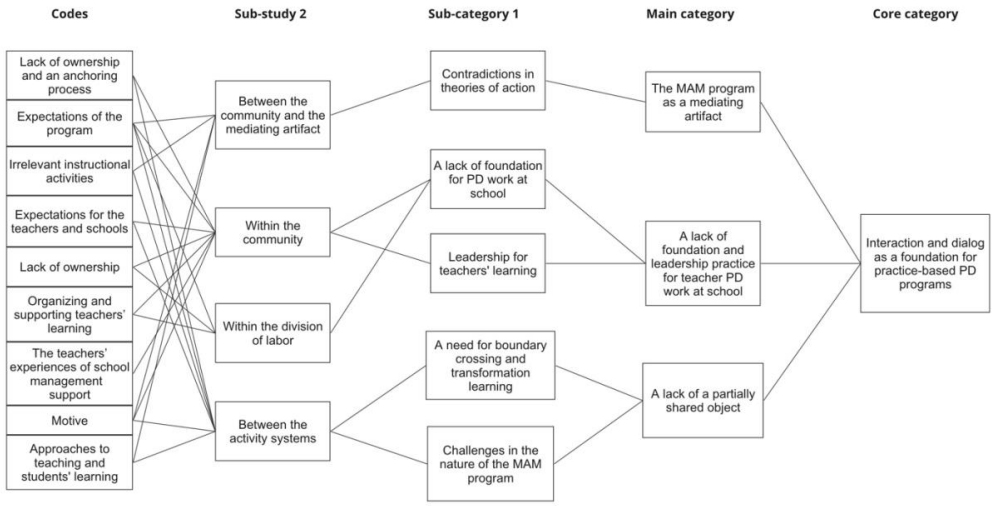

Figure 13. *Overview of the main categories, sub-categories, and core categories in the meta-synthesis.*

5.6 Ethical considerations and trustworthiness

Ethical considerations are crucial in ensuring that qualitative research is conducted in a manner that is respectful, responsible, and sensitive to the needs and experiences of research participants. By adhering to ethical principles, researchers can ensure that their work is credible, trustworthy, and contributes to the advancement of knowledge in a responsible and respectful manner. For this study, the guidelines from the National Research Ethics Committee in Social

Sciences and the Humanities (NESH, 2016) were followed throughout the entire research process. There are five parts to the guidelines, each addressing distinct ethical obligations and comprising multiple aspects. While some aspects of ethical considerations and trustworthiness relevant to the current research study have been briefly discussed in Chapter 5.0, a more focused discussion on them in relation to this study will be provided in the following sections.

According to NESH (2016):

> Researchers have responsibilities towards all persons involved in or affected by research. Researchers should respect the participants' human dignity and consider their personal integrity, safety, and well-being. Participation in research should, as a rule, be based on information and consent.

To comply with this guideline, all research participants were provided with sufficient information to ensure their comprehensive understanding of the research project and what they were agreeing to before project start-up and again before data collection. The provided information included the research project's aim and procedures, and the participants were informed that their participation was voluntary and that they could withdraw at any time without further explanation. Anonymity is a crucial aspect of research ethics. Researchers should strive to guarantee anonymity whenever possible to protect the privacy, confidentiality, and welfare of research participants (NESH, 2016). Therefore, the participants were informed that the collected data material would be anonymized to protect their identity and workplace. They were also informed that data collection, storage, and the research were all approved by NSD[17] (2019) and that the collected data would be stored on a secure server accessible only to the researcher and his two supervisors. No data was stored locally. Participants were also informed that they could access their specific data if they wished, but none of them expressed an interest in doing so. A destruction date was set to ensure the secure disposal of the data. All written and transcribed material was also anonymized as quickly as possible and stored on the same secure server. These measures were taken to minimize the risk of interfering in the participants' private lives (Boeije, 2009). All this information was provided both orally and in writing so the

[17] See Appendix 1 for NSD approval.

participants could give their informed consent, as described in section 5.4.3. They all signed the informed-consent form to indicate their agreement to participate in the research project.

Even though different data-collection methods were used, the ethical considerations are the same in qualitative research. To this end, creating safe and ethical data-collection situations is essential, which can be done by establishing trust between the researcher and the participants. In addition to informing the participants about the guarantee of anonymity, the purpose of the interviews, as well as the observations, was once again clearly explained. For all the interviews, the booked meeting rooms were private and safe from intrusion. It was also made clear and agreed that what was said in the interview would not go beyond the participating parties. Researchers must also be mindful of power imbalances that might exist between themselves and participants (Dodgson, 2019; NESH, 2016), and take steps to minimize any potential harm or coercion. Thus, the designed questions were open and in accordance with the purpose of the study, and I strived to not ask leading follow-up questions. Coffee and cookies were also supplied in an attempt to make the situation comfortable for the participants, and the interviews also started with a loose conversation that was not related to the data-collection situation. From the outset, the teachers responded positively to me, which helped establish safe and secure interview situations with clearly defined boundaries.

Ethical problems may arise when analyzing data. Merriam (1998) states that there is a risk of losing some information if it conflicts with the researcher's own perceptions. Even though I was aware that mistakes might occur subconsciously, I focused on being true to the data. Throughout the transcription process it was crucial to accurately represent the research participants' statements and to quote them verbatim when necessary. The transcription was first done in the official Norwegian language form, Bokmaal, and then translated into English. Translating quotations into English has an advantage as it can increase anonymity by neutralizing the impact of specific Norwegian dialects and structural language variations on the statements. Quotations were used to present the data from substudies 2 and 3 (articles 2 and 3). In substudy 1, the participants provided rich and complex statements in the first substudy, so I chose to use a narrative format to present the data. Riessman (2008) suggests that presenting the findings as narratives can make them more accessible to the reader, especially considering the rich and complex statements provided by the participants during the interviews. The process of writing the narratives (e.g., Polkinghorne, 1995; Riessman, 2008) involved putting together all relevant pieces of data from the individual interviews of the three teachers. The creation of

the narratives involved a process of gradually condensing or configuring the text (e.g., Polkinghorne, 1995), resulting in a story that incorporates the researcher's voice. Once the stories were completed, they were shared with the two teachers in question for proofreading and comments. Two of the teachers fully accepted the narratives as they were written, whereas the third teacher requested a minor change, which was made and thereafter accepted. This form of member-checking (Lincoln and Guba, 1985) was also conducted with the quotations in the other two articles, which strengthened the quality of the study.

5.6.1 Trustworthiness

While the ultimate goal of any researcher is to produce reliable and trustworthy findings, there has been an ongoing debate on the scientific validity of qualitative and interpretive research. The measures and procedures used to ensure trustworthiness, as well as the broader concepts that underpin the approach, have been questioned (e.g., Angen, 2000). While the quality of quantitative research is commonly discussed in terms of reliability, validity, and generalizability, scholars argue that these concepts should be replaced by more appropriate or relevant concepts for evaluating the quality of qualitative research. For example, Maxwell (2002) suggests using such concepts as authenticity and understanding in place of validity, which he sees as more connected to accounts than data or methods. Miles and Huberman (1994) put reliability on the same level as dependability and auditability, while internal validity is comparable to credibility and authenticity, and external validity to transferability and fittingness. In a similar vein, Denzin and Lincoln (2000) advocate replacing internal and external validity with such concepts as trustworthiness and authenticity within the interpretive paradigm. They propose an alternative framework for evaluating the quality of qualitative research that is based on naturalistic axioms, arguing that traditional criteria of validity and reliability may not fully capture the complex and subjective nature of qualitative research. Instead, they propose four criteria that point to the importance of the researcher's interpretation and understanding of the research context: *credibility*, *transferability*, *dependability*, and *confirmability*. These alternative criteria provide a means of assessing the quality and trustworthiness of qualitative research, which I will use to evaluate the trustworthiness of the current study.

Credibility in qualitative research means ensuring that the findings accurately represent participants' perspectives and experiences. Techniques such as prolonged engagement, triangulation, and member-checking are used to achieve credibility. The purpose of these

techniques is to confirm that the researcher's interpretation of the data aligns with the participants' perspectives. Only the participants can judge the credibility of the findings, as qualitative research aims to understand phenomena from their point of view (Lincoln & Guba, 1985). Ultimately, credibility is established when the research findings are credible or believable from the participant's perspective.

Transferability in qualitative research, means that the researcher provides a detailed description of the research context, participants, setting, data-collection methods, and assumptions. This will make it possible for readers to assess the extent to which the findings can be applied or generalized to other contexts or settings. Transferability is primarily the responsibility of the person generalizing the findings, and the qualitative researcher's role is to provide the necessary information. Therefore, the reader's role is to evaluate the transferability of the findings to their own context, and to judge the sensibility of the transfer.

Dependability is a critical aspect of qualitative research, referring to the consistency and stability of findings across contexts and over time. Achieving dependability can be challenging since the context in which research occurs is constantly changing. To address this, researchers need to describe contextual changes and how they affect the research process. Techniques such as audit trails help to improve dependability by documenting the decision-making process and providing a record of the research. However, researchers cannot measure the same thing twice, which makes the process of achieving dependability more complex.

Confirmability is a crucial criterion in qualitative research, referring to the extent to which the research findings are grounded in the data and not influenced by the researcher's subjective viewpoints. It also pertains to the degree to which the findings can be verified or corroborated by others. Techniques such as reflexivity and diligent data checks are used to promote confirmability, ensuring that the findings are shaped by the participants' perspectives rather than the researcher's subjective viewpoints or motivations. The ultimate aim is to establish impartiality in the findings, enabling readers to evaluate how much they are based on the data rather than the researcher's assumptions or prejudices.

Triangulation was used in this study to establish credibility and confirmability (Lincoln & Guba, 1985). This is a widely used technique in qualitative research that involves using multiple data sources to raise the degree of credibility and dependability of the findings. However, it is

important to note that triangulation is not a method for verifying findings or testing for credibility and dependability. As Angen (2000) argues, triangulation assumes that weaknesses in one method can be compensated for by another method, and that it is always possible to reconcile different accounts obtained from different methods. This is not always the case in practice. Despite this limitation, triangulation remains a powerful tool for gaining a deeper understanding of a phenomenon. It is used to ensure that each category under study is rich, robust, comprehensive, and well-developed, and can facilitate a more comprehensive and nuanced understanding of the research question. Danzin (1978) and Patton (1999) identified different types of triangulation. The one used in this study is within their definition of *data-source triangulation*, which involves using multiple data sources to investigate the same research question. In this study, I have collected data from different participants to gain a more comprehensive understanding of the phenomenon of mathematics teachers' PD.

Another technique used to establish credibility, as well as dependability, is member-checking (Lincoln & Guba, 1985). As mentioned above, member checking was used in all three substudies in accordance with the ethical guidelines outlined by NESH (2021). This method allows participants to assess the researcher's intentions through their actions, providing the opportunity to correct misunderstandings and challenge the researcher's interpretations. Member checking also makes it possible to collect additional information and summarize preliminary findings. Moreover, it offers participants the opportunity to evaluate the adequacy of the data and preliminary findings, and approve specific aspects of the data. However, using member checking to establish the credibility and transferability of qualitative research has been criticized by some authors, such as Morse (1994) and Angen (2000). They have pointed out specific problems, such as the assumption of a fixed truth or reality that can be confirmed by a respondent. This assumption is problematic when using an interpretive perspective, where understanding is co-created and there is no objective truth to which the findings from a study can be compared. Furthermore, relying solely on member checking to establish the dependability of research may result in misunderstandings instead of confirmation for a variety of reasons, such as participants changing their perspectives, the interview influencing their original perceptions, and new experiences occurring since the time of the interview. Even though I sent the narratives and quotations to the research participants before publication to verify accuracy and received no negative feedback, with only one minor change requested, it is impossible for the researcher to control for all factors. Therefore, as a researcher, I recognize

that the research participants' possible varying perspectives may affect the findings, resulting in distinct responses to the data.

A final aspect that I want to discuss, which is raised as important for producing trustworthy findings in qualitative research, is reflexivity. According to Berger (2015), reflexivity refers to describing the contextual intersecting relationship between the researcher and the research participants. She maintains that such a description will increase the credibility of the findings and deepen the understanding of the work. To this end, the position of the researcher as an insider or outsider, and their shared experiences with the research participants, are particularly significant when examining the similarities and differences between the researcher and the participants (Berger, 2015; Teh & Lek, 2018). It is important for the researcher to be aware of these resemblances and variances and to disclose them to the readers (Dodgson, 2019). Although my background as a mathematics teacher allowed me to relate to some extent to the teachers' experiences, as a researcher, I did not share any experiences with the teachers or school management. Therefore, I consider myself an outsider in relation to the teachers and school management.

While I positioned myself as an outsider in relation to the teachers and school management, my situation was different regarding the TEs. I had been working with them as colleagues for the past seven years, and we had established a positive working relationship. Because we had shared experiences, I consider myself an insider in relation to the TEs. It is important to note that in this study, I was only a researcher and not part of the group that led the MAM program or made any program-related decisions. During the sessions, I observed their teaching practices. Nonetheless, it is possible that my relationship with the TEs inadvertently impacted the research, as researching one's own colleagues can potentially result in biased perspectives and errors in judgment (Ry Nielsen & Repstad, 2006). As Ry Nielsen and Repstad (2006) suggest, the primary challenge for the researcher is to maintain objectivity and distance themselves from the situation and their own role. To achieve this, I engaged in discussions with other scholars, which included non-involved colleagues and researchers in the field. These discussions aimed to help me focus on the research question and minimize any preconceived notions that I may have had during the data collection and analysis process. As Ry Nielsen and Repstad (2006) point out, it is crucial for the researcher to become "an eagle that hovers high with a large overview and at the same time with a sense of the relevant details" (p. 274) to ensure accurate and reliable findings.

Dodgson (2019) defines reflexivity as the process of self-awareness and self-reflection that researchers engage in to examine their own values, assumptions, perspectives, and subjectivity that may influence the research process and findings. Despite my efforts to be critical of my own interpretations (Thagaard, 2009), I recognize that my assumptions may have had an impact on my understanding of what I wanted to study. Previous studies related to the MAM program, as described in section 2.3.2, have reported either positive impacts or promising opportunities for teachers to develop ambitious teaching practices. Based on these studies and my own experiences of previous MAM projects, I assumed that the PD program would have a positive impact on the teachers. Furthermore, I also assumed that the study would provide new knowledge and insight into how these actors should act and interact to contribute to and support the teachers' development of ambitious mathematics teaching during the program period. Hence, the negative experiences shared by the research participants were unexpected, and this may have led me to seek out reasons or explanations for these negative experiences, instead of remaining open to what the data material means during the analysis process.

While conducting this qualitative study, my theoretical background as a researcher has inevitably impacted my perspective and interpretation of the data, as acknowledged by Postholm (2010). However, my intention has not been to strive for complete objectivity. Through my analysis I have made interpretations of the data that have shaped my own frame of reference, including my sensations, experiences, and opinions, to derive meaning from the data. As Thagaard (2009) notes, the researcher's influence on the data analysis is greater than in the actual data collection. Therefore, I critically reflected on my subjectivity and made a conscious effort to maintain transparency throughout the data-analysis process.

6.0 Findings

This section presents the findings from the case study, including the three substudies reported in the three articles.

6.1 Substudy 1: Teachers' perceptions of teaching and students' learning

The aim of the first study was to investigate the participating teachers' perceptions on teaching and students' learning before they entered MAM 2019. The study focused on the teachers' role, where the purpose was to explore their current understanding of the core of what they were to develop through participation in the program. These understandings can thus represent the starting point of the teachers' development journey and then, in correlation with the aim and purpose of the development program, indicate what might contribute to promoting or limiting their development work. The research question was: *What perceptions do three Norwegian lower secondary mathematics teachers have about classroom practice and students' learning?* The constant comparative analysis method (Corbin & Strauss, 2008; Straus & Corbin, 1990, 1998) used to structure and analyze the data material to answer the research question resulted in the following two main categories: *Engaged with students and their thinking* and *Mathematical discussion*, and the core category: *Approach to teaching and student learning*. The analysis and discussion in this study arrived at two key points that structure the presentation of the findings from interview study 1.

6.1.1 A need for a common understanding of key concepts

The study revealed that the concepts the teachers used when describing their perceptions were similar to those used in ambitious mathematics teaching. They used such concepts as "engaging in students' thinking" and "mathematics discussion" when describing features of good classroom practice. However, the teachers' perceptions of these key concepts did not align with the descriptions in ambitious mathematics teaching (e.g., Kazemi, 2017), even when they initially appeared to correspond. For instance, Sofie used the concepts of "engaging in students' thinking" and "facilitating mathematical discussions," which are important principles of ambitious mathematics teaching (e.g., Gibbons et al., 2017), but her underlying purpose in acting on these practices did not align. Sofie believed that engaging in students' thinking and facilitating mathematics discussions were important for accessing what the students did or did not understand, which in turn helped her explain mathematics to them.

Bearing this finding in mind, the article argues that it may be useful to examine teachers' understanding of concepts that are important for the development program, either in the start-up phase or before the project begins. This helps the teachers to develop an understanding of the purpose of the program and why they should act upon it (Postholm, 2008, 2020). Even though it might seem that we are using a common language with the same concepts in a natural context, our understanding of the concepts can differ from one person to the next. For these reasons, it would be expedient to have a better idea of how the participants understand key concepts to avoid unnecessary misunderstandings that might undermine the teachers' work with their PD.

6.1.2 A need to understand teachers' perceptions

The study shows that the teachers' described perceptions of teaching and students' learning may have many similarities with reform-based mathematics teaching, such as ambitious mathematics teaching, in terms of the principles and so-called core practices. However, when focusing on the teachers' underlying purposes of their perceptions it was found that these purposes reflect a teacher-centered approach to teaching. Even though the teachers' perceptions showed traces of a reform-based approach and the teachers maintained that their teaching practice could not be characterized as traditional, their purpose for acting on practices supporting the key concepts proved otherwise. Sofie argued that the reason for engaging in students' thinking and facilitating mathematical discussions was to gain access to their understandings, which might indicate a teacher-centered teaching approach. However, her purpose in accessing students' understanding through these practices was to gather the information that would better equip her to explain mathematics to them. Furthermore, two of the teachers said that they believed teacher explanation and solving many tasks could be a very good and necessary approach to helping students learn. These purposes appeared to be important principles that guide teachers in their classroom practice. The study shows that the teachers' perceptions are rooted in underlying purposes for acting on these perceptions, which seem to reflect their views on teaching and students' learning. Therefore, it is these purposes that are their true perceptions of teaching and students' learning.

Bearing this finding in mind, the article suggests that the program providers need to invest in exposing the teachers' underlying purposes for acting on their described perceptions to understand what their perceptions are really about. This is particularly important as there appeared to be no direct correlation between the teachers' described perceptions and their

underlying purposes. Thus, the underlying purpose of the perceptions, in terms of their view on teaching and students' learning, is what represents their true perceptions and are what the teachers relate to when assessing whether the PD makes sense in terms of their existing understandings and knowledge (e.g., Desimone, 2009; Timperley et al., 2007). Moreover, the article argues that the teachers need to be challenged to reflect on their current perceptions of what they are to develop, which for this study means the teachers' perceptions of key concepts. This will support their development of the purpose of the program and why they should act upon it (Postholm, 2008, 2020), and will help to avoid unnecessary misunderstandings that might undermine their development work.

6.2 Substudy 2: Principals' leadership of teachers' professional development

The aim of the second study was to explore how a Norwegian school management team facilitates and supports its mathematics teacher's PD in their first year of participation in a particular practice-based development program (MAM 2019). The study focuses on the school management's role in terms of how they support and facilitate the mathematics teacher's development work, especially the relations between the principal's leadership and the mathematics teacher's participation in the program. The research question was: *How does school management support and facilitate mathematics teachers' professional development as they participate in a practice-based development program?* The constant comparative analysis method (Corbin & Strauss, 2008; Straus & Corbin, 1990, 1998) was used to structure and conduct an initial analysis of the data material, which resulted in the four main categories: *Lack of ownership*, *Motive*, *Organizing and supporting teachers' learning*, and *The teachers' experiences of school management support*. This initial analysis created a scale for further analysis (Charmaz, 2014), and based on this, the activity system was used to identify tensions and contradictions that can serve as the point of departure for change and development. The study found two key points related to the research question, and using the activity system, three tensions or contradictions were identified. The two key points, tensions, and contradictions structure the presentation of the findings from interview study 2.

6.2.1 Lack of ownership and joint motive

The first key point concerns teachers' ownership of and motive for the development work. The findings revealed that the school was omitted from at least two important processes related to the teachers' attendance in MAM 2019 and their development work. First, the school was not invited to take part in the process of finding a suitable development program to attend. Second,

the school management did not have a dialog with the teachers concerning their development needs and motivation. The data material indicates that the school's participation in MAM 2019 was a top-down decision made by the school owner who first found and presented the MAM program to the schools, and thereafter made the decision alone that the schools would participate. Without consulting the schools as to the relevance of the program leads to the notion that the school owner made the decision of participation *for* and not *together* with the schools. Such an approach to teachers' PD is contradictory to what researchers have found to underpin successful teacher PD (Clarke & Hollingsworth, 2002). The choice of PD program and the decision to participate was based on what the school owner believed complied with the teachers' development needs and not with what they actually wanted or felt they needed to improve their classroom practice. Thus, the process of developing ownership of the teachers' learning was not present as they were not acknowledged as the heart of decision-making (Knowles et al., 2005), and the school owner was therefore the true stakeholder in the development work with MAM 2019.

The study also revealed a mismatch in motives between the principal and the teachers. The principal's goal was aimed at the learning process itself as he wanted the teachers to learn how to learn together to develop a learning community, which he hoped would have a knock-on effect for the entire school. The teachers, on the other hand, had a content focus as they believed the goal of the program was to explore new teaching methods and learn to activate students in mathematics conversations. However, the study indicates that the teachers did not have a clear and common understanding of the overriding goal and what they were to develop through their participation in the program. As one of the teachers said, "we didn't quite see where we were going with the project". Furthermore, the study shows that the teachers' motivation for participating in the program was low from the beginning and even declined during the program period. The teachers' lack of motivation can be seen in relation to their unclear interpretation of the purpose of the program, and that they did not see how the program contributed to their PD. The study suggests that the mismatch in motives between the principal and the teachers, and the teachers' lack of a clear and common understanding of the aim of the program are a consequence of not allotting enough time in the start-up phase to build the teachers' sense of ownership and develop a shared overarching goal or object.

6.2.2 Lack of structure and practice for development work at school

The study shows that the school leader had a plan for organizing and supporting teachers' PD, both within and alongside their participation in MAM 2019. For example, he
treated leadership as a shared enterprise (Grootenboer & Hardy, 2017) by delegating responsibility to the vice-principal and to a teacher working as a mathematics-subject coordinator. The vice-principal was assigned the task of managing organizational issues and supervising the teachers' participation in the program. The mathematics-subject coordinator was assigned the task of coordinating the teachers' day-to-day job with the development work. The principal himself planned to be updated on the teachers' development process by obtaining an overview of what they were doing and engaging directly in their learning process. Moreover, time was set aside in the timetable for facilitating the teachers' development work, which gave them the opportunity to collaborate between the sessions. However, this plan was only partially followed in practice. The study shows that only the organizational part of the plan was implemented.

The school leader and the teachers did not establish a common understanding of what the allocated time was intended for. The data material indicates that the teachers did not perceive that the intention behind meetings between the sessions was that they should immerse themselves in the content of the program, for example by reading and discussing the given articles. The study also showed that both the school leader and the vice-principle did not provide sufficient support for the teachers' ongoing PD work, as they were not present nor engaged in the program sessions or the collaboration meetings. The study suggests that a structure and practice for development work must be preestablished at the school if a development plan is to be successfully implemented and fulfill teachers' need for continuous developmental support. The school leader is important for teachers' learning, and can have a substantial influence on teachers' learning (Leithwood et al., 2020). The study shows what a lack of management and active leadership of teachers' PD can lead to when new learning is to be developed and implemented. The conditions were not in place for the teachers to create a learning community where they could develop their understanding of the core practices and principles of ambitious teaching. Without sufficient support from school management, the teachers did not manage to create a "historical new form of societal activity that was collectively generated" (Engeström, 1987, p. 174).

6.2.3 The CHAT analysis

With the use of CHAT and the activity system, three tensions or contradictions were identified that can be the point of departure for change and development. First, due to the mismatch in motives for participating in MAM 2019, there is a contradiction within the community between the teachers and the school leader in the activity system with respect to how MAM 2019 should serve as a mediating artifact. The study thus finds that the development of a shared motive to act on the object could be the starting point for change and development for this school. This aligns with Leont´ev's (1981) statement that "the object is the true motive" (p. 59) for people's actions. Such a development process should be led and facilitated by the school leader, for example by allocating time and resources in the start-up phase for the teachers to identify with the topic for the PD work (Postholm, 2008, 2020), and by helping the teachers to identify their development needs and enhance the implementation of new learning (Thoonen et al., 2011; Vanblaere & Devos, 2016). In this way, the teachers' and the school leaders' motives are built into the object because their practice and actual needs constitute the starting point of the development work (Sannino & Engeström, 2017). As such, the PD work is "owned" by the practitioners (Engeström & Sannino, 2010), and their collective motive for acting on the object is "invested with meaning and motivating power" (Sannino et al., 2016, p. 602).

Second, both the teachers and the school leader struggled to understand how to benefit from their participation in the program, which can be understood as a tension between the community and MAM 2019 as a mediating artifact. What makes this tension even more complex is the aforementioned mismatch in motives between the principal and the teachers. Their work or goal-directed actions when using MAM 2019 are therefore not aimed toward the same object. For the MAM program to function as a meditating artifact the teachers and the school leader need to understand what the program can contribute and assess how this contribution can help to achieve their goals in the ongoing development work. The MAM program aims to provide teachers with the opportunity to develop ambitious mathematics teaching, an aim that relates to the teachers' motive. Additionally, the program draws on effective forms of PD (e.g., Desimone, 2009; Putman & Borko, 2000), and is informed by theory on teachers' collective learning in a community of practice (Wenger, 1998). Thus, both motives could be built into the object of this activity system and the MAM program could serve them both.

Third, the conducted work or goal-directed actions have not been divided between the teachers, the school leader, and the school owner who constitute the community as a whole, which leads to a tension in the division of labor (Engeström, 1987). The data material shows that the teachers lack a collective motive, and initiative and commitment (Sannino & Engeström, 2017), as mentioned above. Moreover, the data material shows that the school leader called for more defined roles as he expressed a need to clarify who is in control and making the decisions for the development work. As argued above, that the school leader has an important role as a facilitator for teachers' learning. However, the study also shows that the school leader needs to be supported in this work to manage the responsibility that lies within his role, which in this case should be supplied by the school owner. As the stakeholder of MAM 2019, the school owner needs to be aware of his role as a facilitator for the school leader's work, for example, providing resources such as internal or external support. According to the data from the study, it appears that the school owner did not fulfill his responsibility as a stakeholder in MAM 2019. The study suggests that the school leader felt a lack of control over the teachers' education in the development program, was uncertain of his role, and was not included in decision-making processes.

6.3 Substudy 3: Teacher educators' experiences of a failed practice-based development program

The aim of the third and final study was to explore the TEs' experiences in enacting the first year of MAM 2019. The study focuses on the three TEs' role as program leaders and their experiences of leading MAM 2019, which the school at this stage had decided to terminate. The research question was: *How do teacher educators experience a practice-based development program for mathematics teachers?* Similar to the second interview study, the constant comparative analysis method (Corbin & Strauss, 2008; Straus & Corbin, 1990, 1998) was used to structure and conduct an initial analysis of the data material. The initial analysis resulted in four main categories: *Lack of ownership and an anchoring process*, *Expectations for the teachers and schools*, *Expectations for the program*, and *Irrelevant instructional activities*. The activity system (Engeström, 1987, 1999, 2001) was then used to analyze and discuss the findings, and to identify tensions and contradictions that can be potential starting points for change and development. The study resulted in three key points relating to the research question and the identification of one important contradiction. The key points and the contradiction structure the presentation of the findings from interview study 3.

6.3.1 Lack of ownership and an anchoring process

The first key point is related to how the TEs experienced the initiating process for choosing the MAM program as an aid to the teachers' PD work, and their impression of how the school and the teachers were prepared for participation. The findings show that the TEs experienced that the school's participation in MAM 2019 was a top-down decision made solely by the school owner. From meetings with the school owner and the school leaders, and plenary sessions with the teachers, the TEs had the impression that there was a lack of communication between the parties. The TEs experienced that the teachers were omitted from several important decision-making processes concerning participation and how the program's object aligns with their ongoing PD work. According to the TEs' experiences, this indicates that the teachers' actual development needs and voices were not used to create the foundation for the decision to participate in the program. Instead, the decision was based on the school owner's assumptions of what the teachers needed. In other words, the TEs' experiences indicate that the school lacked a shared vision for participating in the program, which can impede the teachers' professional learning (Feeney, 2016). Furthermore, the lack of a common vision reveals a lacking, or at best, a rushed process of anchoring the MAM program to the teachers' PD work, which may have led to the teachers taking part in a development work they did not feel they owned (Engeström & Sannino, 2010).

6.3.2 Expectations for the school and their teachers

The TEs had expected that the school and the teachers were more thoroughly prepared in relation to the expectations and premises of the program, and thus were ready to play an actively engaged role in their own and others' learning. However, the TEs' experiences showed that this was not the case. Based on a meeting with the principal, the TEs argued that the teachers were not used to the kind of collaborative development work that MAM 2019 required, and that the school was undermined by its lack of a collaborative learning culture. Active collaborative learning is a very important part of the MAM program, which is based on research on effective forms of PD (e.g., Desimone, 2009; King & Stevenson, 2017), and provides teachers with the opportunity to create a joint enterprise by actively taking part in mutual processes of negotiation of meaning in a community of practice (Wenger, 1998). According to the TEs, the teachers' lack of experience of working in collaborative learning processes made it challenging for them to establish a community of practice.

The TEs also experienced a distinction between the aim of the program and the teachers' perception of this aim and how it is to be achieved. According to the TEs, the teachers expected to be given teaching resources and to be told how their teaching practice could be improved, and this aligned with a traditional form of teacher PD (Berry & Loughran, 2010). Such an approach is in contrast with the approach in the MAM program, which is based on research on effective forms of PD grounded in socio-cultural views of learning and development and aligns with a newer form of teacher PD (Berry & Loughran, 2010). Furthermore, the analyses of the study indicate that the teachers' expectations for the program might have affected their motivation negatively. The TEs experienced that the teachers did not prioritize the participation in the sessions as they did not really want to be there, which strongly indicates a lack of motivation.

The article argues that the school's lack of a collaborative learning culture and their poorly conducted preparatory work could be detrimental to the teachers' development work in MAM 2019. It also argues that the teachers' different expectations, together with their lack of motivation, might indicate that the teachers' motivation was not embedded in the overriding goal or object of the development work (Leont´ev, 1981). For these reasons, the findings suggest that more attention should be devoted to the start-up phase in an attempt to address these issues. The TEs themselves argue they could have supported the principal in preparing the teachers for the program by offering reflective questions they could discuss to better comply with the expectations and premises of the program. However, the article points to research arguing that only reflecting on these issues might not be enough. Structure and culture must interact if teachers are to learn together (Postholm, 2018), which is a process that tends to be demanding and long-lasting. The article also argues that reflecting on these issues can uncover the need to create structures and develop a collaborative culture for learning that can lay the foundation for successful PD (Postholm, 2018). In this sense, the reflective questions can be the first step toward such a development.

6.3.3 Instructional activities

The instructional activities used in the MAM program are designed so the teachers can learn to enact the teaching practices in interaction with students (e.g., Fauskanger & Bjuland, 2019). Therefore, the TEs naturally expected the instructional activities to fulfill their purpose when used in the sessions with the teachers. However, the TEs experienced that the teachers did not find the instructional activities to be relevant. The TEs argued that the teachers assessed the

relevance of the instructional activities according to the extent to which they believed the students found the context interesting, the extent to which they could be related to the curriculum, and how likely it was the activities would appear on the exam. Therefore, the TEs believed the instructional activities were detrimental to the teachers' learning process, and also a reason why the program failed to be completed. Some research supports the TEs' assumptions, arguing that teachers' learning is strongly influenced by the extent to which the practical resources offered through learning experiences make sense to the participants (Timperley et al., 2007). However, the article suggests that the relevance of the instructional activities needs to be discussed in more depth. For the MAM program, the instructional activities are mediating artifacts that are intentionally selected to help the teachers develop ambitious mathematics teaching. The article suggests that the discussion on the relevance of the instructional activities should be based on this purpose and that the TEs should rather question the teachers' understanding of this purpose instead of considering alternative instructional activities that comply with the teachers' assessment of their relevance.

6.3.4 The CHAT analysis

With the use of CHAT to further analyzing the findings, one contradiction was identified that can be the point of departure for change and development for the TEs, the PD program, and the school. As outlined above, the TEs experienced a distinction between the aim of the program and the teachers' perception of this aim and how it could be achieved. The activity system analysis (Engeström, 2001) pointed out that elaborated distinctions in the aim can be understood as a contradiction in objects between two activity systems where the TEs are the acting subject in the one activity and the participating teachers are the subject in the other. According to the activity system, the TEs, as the program providers, can be considered to be the subject of one activity. The MAM program is the mediating artifact that the TEs use as an aid to facilitate the teachers in developing their teaching practice toward the object, which is to promote opportunities for teachers to learn to enact principles and practices of ambitious mathematics teaching. Furthermore, the teachers, as the learners of the program, act as the subject in the other activity where the MAM program is the mediating artifact that they use as an aid in their ongoing PD work. If the MAM program is to be a genuine aid for the teachers' ongoing PD work, the aim of the program, and therefore also the aim of the TEs, must be in accordance with the object of the activity in which the teachers are the subject. In other words, the objects of the two activities need to be partially shared (Engeström, 2015).

When analyzing the TEs' experiences, it becomes clear that the two objects in the two activity systems hardly can be considered partially shared. The TEs experiences show that MAM 2019 was brought into the teachers' PD work with a predetermined aim. Considering the TEs' experiences of a lack of communication among the participants, the teachers' perceived lack of relevance of the instructional activities, and the teachers' lack of motivation, it is reasonable to assume that the teachers' and TEs' expectations and objects developed independently from one another. Thus, the article suggests that the development work for MAM 2019 was undermined by an insufficient anchoring process and, moreover, argues that a partially shared object could have been developed through such a process. Furthermore, the article also suggests that there are several reasons to believe that the issues elaborated on here could have been addressed if a thorough anchoring process had been conducted. First, the teachers could have been invited into the process and have been given the opportunity to voice their opinions (Postholm, 2020) and thus be treated as the decision-making core in their own development work (Knowles et al., 2005). In doing so, the development work could have been based on the practitioners' development needs and expectations (Watson, 2015), and in this sense constitute the foundation for deciding to participate in the program. Second, if the TEs were invited to take part in the process, they would be able to ascertain the teachers' needs and expectations and take them into consideration before starting the program. This would then have enabled the TEs to see the teachers' needs and expectations in relation to the MAM program in terms of its purpose, content, and view on learning and a foundation for the learning conditions would have been established. The TEs could also help the school and teachers to relate the purpose of the MAM program to the teachers' development needs and in this sense contribute to building the teachers' engagement into the object so that it could become their true motive (Leont´ev, 1981). A thorough anchoring process can in this way coordinate the expectations and goals of the people involved in the development work and thus facilitate the development of an object that is partially shared.

7.0 Discussion and conclusions

In this final section I will discuss how the findings presented in the previous section contribute to answering the main research question of my study: *How can a practice-based development program contribute to mathematics teachers' professional development?* The aim of this research project was to explore how the MAM program contributes to teachers' PD by examining the roles of teachers, principals, and TEs (program leaders) and how they acted and interacted to contribute to and support teacher PD during the first year of the program. The discussion will therefore focus on the actions and interactions between the actors. The main categories developed from the integration process in the meta-synthesis, namely *The MAM program as a mediating artifact*, *A lack of a foundation and leadership practice for teacher PD work at school*, and *A lack of a partially shared object*, and their subcategories, will structure the discussion in the following three sections (7.1, 7.2, and 7.3). Moreover, as tensions and contradictions form the basis for development and change (Engeström & Miettinen, 1999), I will discuss how the four tensions and contradictions identified in the three substudies can serve as a starting point for change and development in separate sub-sections in sections 7.1 and 7.2, and throughout section 7.3. Finally, the potential limitations of this study will be discussed, after which conclusions will be drawn and recommendations for further research provided.

7.1 The MAM program as a mediating artifact

PD programs of any kind are a form of supplementary teacher training that need to be treated as an aid to teachers' ongoing PD. In relation to CHAT, teacher PD programs such as the MAM program can be understood as mediating artifacts aiding teachers to accomplish goal-directed actions toward their specific object. However, the findings from substudies 2 and 3 reveal a contradiction between the teachers as the acting subject and MAM 2019 as the mediating artifact that can be understood as a difference in theories of action. Research shows that both teachers and PD programs possess a set of theories that inform their actions and approaches (Kennedy, 2016; Timperley et al., 2007). This means that both parties bring a set of pre-developed theories to their encounters. The findings of this study show that the teachers and the MAM program have different theories of teaching and learning, which leads to a contradiction in both what and how the program can contribute to the teachers' PD.

7.1.1 Contradictions in theories of action

Teachers who have been in the profession for several years and have acquired knowledge and skills through training, further education, and practicing the profession are often referred to as "experienced" teachers. Timperley et al. (2007) argue that these teachers have developed robust theories on good teaching and students' learning that they turn to when approaching professional learning situations. These theories have also significantly influenced their interpretation of new learning experiences and how they are integrated into their practice (Timperley et al., 2007). Findings from the three studies indicate that the teachers in this study are no exception, where two particular aspects stand out that can be understood as a contradiction between the teachers' and the MAM programs' theories of action. The first aspect, found in substudy 1, is related to the theoretical view on teaching and students' learning. The findings show that the teachers have a teacher-centered view on teaching and students' learning in mathematics. This does not align with the student-centered view of ambitious mathematics teaching (Ghousseini et al., 2015), which the MAM program aims for. The teachers' perceptions of such key concepts as "engaging in students' thinking" and "mathematics discussion" differed significantly from the principles of ambitious mathematics teaching. These perceptions were grounded in their understanding of how to facilitate students' learning and appeared to work as principles that guided them in their work as mathematics teachers. In this sense, the teachers' perceptions of the concepts were informed by their view on teaching and students' learning and accordingly were used in alignment with their teacher-centered approach to teaching.

This aspect of the contradiction might not be problematic per se, as an ambitious view of teaching and students' learning is at the core of what the teachers are to develop through participating in the program (Wæge & Fauskanger, 2020, 2022). However, viewed through the lens of CHAT, it is important to understand that the MAM program is a mediating artifact that is intended to aid the teachers' PD work. Therefore, it is crucial that teachers and facilitators involved in the development work are aware of the differences in their views on teaching and students' learning if they are to be able to work toward a common object. The findings show that how the teachers perceived these key concepts was not obvious, nor were the differences easy to detect as they initially appeared to correspond with those described in ambitious mathematics teaching. Thus, this aspect of the contradiction between the teachers' and the MAM program's theories of action can potentially lead to implications for the program leaders or others involved in the PD work as they might be left with the impression that the teachers

are talking about key principles for ambitious mathematics teaching (Gibbons et al., 2017) when they in fact are not. These implications can further limit the teachers' ability to develop an understanding of the practices and principles of ambitious mathematics teaching that the MAM program provides.

The second aspect of the contradiction in theories of action is related to the purpose of the instructional activities used in MAM 2019. Findings from substudy 3 show that there were differences in how teachers and TEs understood the purpose of these activities. The TEs experienced that the teachers did not consider the instructional activities to be relevant, as they maintained that the teachers assessed the relevance of these activities according to other parameters than the purpose they were intended to serve in the program. As outlined in section 5.2.1, the instructional activities used in the MAM program are designed for teachers to learn and enact the teaching practices of ambitious teaching in interaction with the students (Lampert et al., 2013). Therefore, the purpose of the instructional activities was never to be tasks that should accommodate the parameters the teachers value as relevant but rather to scaffold their learning of ambitious teaching practices. Their teacher-centered approach to teaching and the lack of a shared understanding of the program's goals and overall object, as revealed by the findings from substudies 1 and 2, suggest that this aspect of the contradiction in theories of action could stem from the teachers' perception of what the instructional activities are intended to achieve. This notion is supported by another finding from substudy 3, which shows that the teachers expected to be given teaching resources and to be told how to improve their teaching practice, aligning with a traditional form of teacher PD (Berry & Loughran, 2010). Given that teachers' theories of action significantly impact how they interpret new learning experiences in their own practice (Timperley et al., 2007), it is reasonable to assume that the instructional activities were difficult for teachers to make sense of within their existing understandings and practice contexts, and as a result, they found them irrelevant to their learning.

7.1.2 The contradictions as the point of departure for change and development

The aim of the MAM program is to promote opportunities for learning to enact the principles and practices, and mathematical knowledge entailed in ambitious mathematics teaching (Fauskanger & Bjuland, 2019; Wæge & Fauskanger, 2020). If the teachers are to make use of the MAM program as a mediating artifact, it is crucial for them to understand how central concepts that describe the principles and practices they are to learn are defined. However, it is equally important that the program leaders understand the teachers' point of departure if their

actions are to contribute to the development of a new form of practice (Engeström & Sannino, 2010). Examining the teachers' perceptions of these concepts and, if necessary, clarifying the conceptual understanding of them seems to be an obvious action to take to resolve this contradiction. Substudy 1 shows that it might be beneficial for PD leaders to focus on exposing teachers' perceptions of key concepts to determine whether it is necessary to clarify how these concepts are to be understood in the context of ambitious mathematics teaching. In doing this, unnecessary misunderstandings that might impede the teachers' development work can be avoided.

Substudy 1 suggests that a discussion on the relevance of the instructional activities should be based on the extent to which they fulfill their purpose in the program. The instructional activities are designed so the teachers will learn to enact the teaching practices in interaction with the students (e.g., Lampert et al., 2010). Therefore, the focus should not shift to providing teachers with instructional activities with the aim of aligning with their expected relevance. However, researchers suggest that PD programs must address teachers' expectations and challenges (Pokhrei & Behara, 2016), and the degree to which the teachers make sense of the resources offered has a strong impact on the extent to which new information is used (Timperley et al., 2007). Therefore, the program providers should consider developing new instructional activities that align with the relevance parameters the teachers value, while ensuring that the main focus remains on the purpose of the activities. This is also suggested by the TEs themselves as they state that when the teachers experience that the instructional activities are irrelevant this is detrimental to their learning process. For this reason they claimed they should have found other activities that the teachers could have perceived as more relevant. Instructional activities that are more aligned with the material the teachers are teaching can lead to significant positive impact on the implementation of the PD effort (Desimone & Garet, 2015). This could also contribute to complying with the teachers' wants (Matherson & Windle, 2017), which in turn can build motivating power into the object of the development work (Sannino et al., 2016).

Using what the teachers see as relevant instructional might not be sufficient, as this contradiction really is about how the teachers perceive the purpose of the instructional activities. The study does not explicitly reveal how the teachers perceive this purpose, but, considering the teachers' traditional approach to this PD effort (Berry & Loughran, 2010), it is likely that they do not see the instructional activities as a means for learning to enact the practices and principles of ambitious teaching (e.g., Lampert et al., 2010). As the teachers were

looking for resources and guidance on how their teaching practice could be improved, it is likely that they considered the instructional activities to be self-contained. Thus, more effort needs to be put into clarifying the purpose of the instructional activities in relation to the overriding aim of the program.

In spite of the fact that clarifying both the way key concepts are to be understood and the purpose of the instructional activities seems to be an obvious action to take to develop common ground, a deeper analysis of the teachers' perceptions indicates that this contradiction is of a more theoretical nature than the practices and principals and instructional activities themselves. Substudy 1 shows that the teachers' perceptions of the key concepts were grounded in their underlying view on teaching and students' learning, which reflected a traditional teacher-centred approach to teaching (Boaler, 2002), and which even appeared to imply a behavioristic view of learning. Bearing in mind that teachers' existing theories influence how they understand new learning experiences and how they are integrated into practice (Timperley et al., 2007), this leads to the finding that the teachers' perceptions of the key concepts are only a consequence of their existing theories and ideas about teaching and students' learning. This is supported by other findings from substudy 1 showing that the principles, grounded in a theoretical view on teaching and students' learning, are what guide the teachers in their classroom practice.

Due to the theoretical nature of the contradiction presented above, the following question arises: is there a need to have a stronger focus on the theoretical underpinning of the practices and principles of ambitious teaching? Considering the MAM programs' theory of action, the obvious answer is no. The principles embedded in ambitious teaching underpin an ambitious view of classroom instruction that emphasizes the classroom community where discussion and shared meaning-making are valued, and where the students are placed at the center of their own learning (e.g., Ghousseini et al., 2015; Kazemi, 2017). The meanings and entailments of this ambitious view are constantly negotiated through the use of instructional activities in the cycle of enactment and investigation, guided by the TEs throughout the sessions (Wæge & Fauskanger, 2020). Although the teachers had engaged in processes of negotiating the meaning of this student-centered focus for a year, the substudies indicate that they were not able to make sense of what MAM 2019 offered. For example, they expected something different than what the program provided, did not find the instructional activities relevant, and did not have a clear understanding of the overriding goal of the program. Kennedy (2016) argues that PD facilitators

are not just introducing a new idea, but rather one that is quite different from what has traditionally guided teachers in the past. Because the teachers' perceptions of these key concepts were grounded in their underlying view on teaching and students' learning, this new idea MAM 2019 is offering them is not only an ambitious mathematics teaching practice but a radically new perspective on how learning occurs. If the MAM program aims for the teachers' classroom practices to be guided by the principles of ambitious mathematics teaching, the findings from this study show that the program should not only focus on the practices and principles, but also pay more attention to the theoretical underpinnings of this ambitious teaching approach. This is because the perspectives, theories, and understandings the teachers have about teaching practice must be challenged so they can be reconstructed (Timperley et al., 2007). Without this emphasis on the theoretical underpinnings, the practices offered by the program may only expand teachers' toolkit of practices for their teacher-centered teaching approach.

7.2 A lack of foundation and leadership practice for teacher PD work at school

A collective activity system is, according to Engeström (2001), driven by a collectively shared motive that is embedded in the object of the activity. According to Leont´ev (1981, p. 59), the true motive for people's action is the object, meaning that the individuals within a community who aspire to enhance their practice toward a specific object should be aware of, or ideally, share a collective motive to act on the object. The findings from substudy 2 reveal a contradiction within the community between the principal and the teachers when it comes to how MAM 2019 should function as a mediating artifact. The findings show that the principals' goal for participation was for the teachers to develop a learning community that could be implemented schoolwide, whereas the teachers believed the object of the program was to explore new teaching methods and learn to activate students in mathematics. In other words, the teachers' motive concerned *what* to learn, and the principals' motive concerned *how* to learn it, which represents a focus on content and process, respectively. The mismatch in motives between the principal and the teachers indicates that a common understanding of how the MAM program can aid them in their ongoing development work was not developed before the decision to participate was made, thus a collectively shared motive for the development work was not established. This contradiction was also identified in the findings of substudy 3, as the TEs described that the school lacked a shared vision for participating in the program.

The findings from substudy 2 also revealed a second contradiction that concerned the community, but this time between the community and the division of labor. The principal called for more defined roles and stated that he did not know who was making the decisions for the development work. Moreover, the school owner did not invite the principal into the decision-making process when selecting a suitable PD program to attend and also stepped away from his responsibility to support the principal's work. Findings from substudy 3 also show that the TEs experienced a lack of communication between the school owner and the school leader. These findings indicate that the goal-directed actions were not divided between the school leader and the school owner; their responsibilities were not clarified or assigned.

While identifying contradictions can provide opportunities for change and development (Engeström & Miettien, 1999), the findings from substudies 2 and 3 can also explain why these contradictions occurred. With insights from this understanding, it will be possible to refine and nuance how to address them. These findings suggest significant issues with the foundation for PD, as well as the leadership of teachers' learning at the school. These issues were likely the primary source of these contradictions. Moreover, the findings indicate that these issues may have had an equal or greater impact on teachers' learning than the contradictions themselves.

7.2.1 A lack of foundation for PD work at school

Findings from substudies 2 and 3 show that the principal and teachers were insufficiently prepared for participating in MAM 2019. It is evident through the findings from these two substudies that the school's participation in MAM 2019 was a top-down decision from the school owner. The PD program was selected by the school owner alone, without consulting the schools and the teachers as to whether or not the program was relevant to their development needs. The findings from substudy 2 show that the school management had the final decision concerning the school's participation, but did not consult the teachers when making the decision. Thus, the teachers were omitted from both the process of finding a suitable PD program adapted to their development needs, and from the decision-making process that led to their participation in MAM 2019. Al-Mahdi and Al-Wadi (2015) argue that developing ownership of the development work is crucial for ensuring PD experiences that are meaningful and relevant. Shared decision-making, which is strongly associated with the development of ownership (e.g., Knowles et al., 2005; Feeney, 2016; Tan & Caleon, 2016), implies that teachers are invited to take part in decision-making processes from the very beginning of the development work. This was not the case for teachers in this study, which made it challenging

for them to take ownership of the development work. Furthermore, disregarding the teachers from decision-making processes around change by making decisions *for* them and not *together with* them is in contrast to what research argues to be the underpinning of successful teacher PD (Clarke & Hollingsworth, 2002). Moreover, when the school management makes their decision without consulting the teachers, the project can be understood as a mandatory assignment. This can have a detrimental effect on teachers' motives and learning (Kennedy, 2016). Indeed, the school owner's decision to initiate the teachers' PD work without consulting them, as an outside member of the community, can be construed as a de-professionalization of the teacher role (Roseler & Dentzau, 2013), where the outsider implies that identifying development needs is beyond the capability of the teachers.

Another important aspect in developing ownership is related to the process of identifying teachers' development needs at their school and in their classroom as a basis for their learning and development (Engeström & Sannino, 2010; Timperley et al., 2007: Watson, 2015). Teachers approach professional learning opportunities with a complex collection of wants and needs that influence their participation (Liljedahl, 2018). It is important to invite teachers to offer their opinions so that their wants and needs are taken into consideration (Postholm, 2020); this invitation was not extended to the teachers in this study. Findings from substudy 3 also show that the decision to engage in the MAM program was not based on the teachers' actual needs but on the school owner's perception of their needs. Furthermore, findings from substudy 2 showed that the teachers lacked motivation when entering the PD program and their motivation even declined during the program period. Considering that the teachers were omitted from several decision-making processes, this finding is not surprising. Their motivation was never built into the object of the development work, which, as pointed out above, is critically important (Sannino et al., 2016). Failure to take teachers' motivation to learn and change their practice seriously can significantly undermine the success of any PD program (Kennedy, 2016). What is needed is open communication with the principal, shared decision-making, learning structures, autonomy in decision-making, and a shared vision. If these factors are lacking, the development will be constrained (Feeney, 2016). Therefore, if these factors are not properly addressed, teachers' work in MAM 2019 may not lead to a successful outcome.

While the lack of developing ownership was prominent, the school's lack of preparation before starting in MAM 2019 also highlighted the absence of a culture and practice for development work at the school. Findings from substudy 2 show that time was set aside in the timetable so

the teachers could both participate in the program and work on the related development work between the sessions. However, the findings also show that the principal and the teachers had not established a common understanding of what the allocated time was intended for. The principal argued that it was devoted to giving the teachers the opportunity to immerse themselves in the content of the program, which involved discussing how it could be incorporated into their teaching practices, and reading and discussing the article assigned between each session. This intention was not perceived by the teachers. They argued that there was no facilitation for work related to MAM 2019 outside of the sessions. The findings also show that the teachers did not use the allocated time for reading and discussing the articles, rather they read the night before the session—if at all—which indicates that this task was not given priority. A culture for development work at school is argued to be an important factor for teachers' learning and the development of collaborative learning processes (Postholm, 2018). When the teachers did not use the allocated time for the related PD work, this suggests that the school did not have a culture for collaborative learning, and that the allotted time, for this reason, did not lead to development.

Findings from substudy 3 show that the TEs experienced that the teachers were not used to practicing collaborative learning processes and went on to argue that this lack of experience of such learning processes made it challenging to establish a community of practice (Wenger, 1998). The collective perspective on learning and development is of major importance in the MAM program as the teachers' learning of ambitious mathematics teaching relies on their participation in the cycle of enactment and investigation (Wæge & Fauskanger, 2020, 2022). This demonstrates that if amenable conditions for PD are not present at the school, such as a culture and structure that is open to development work, the teachers' learning and growth in the program can be impeded.

7.2.2 Leadership for teachers' learning

The findings from substudy 2 show that the school leader had developed a plan to organize and support the teachers' development work. His development plan aligned with the notion that leadership needs to be treated as a shared enterprise (Grootenboer & Hardy, 2017). This was reflected in the fact that the vice-principal and a mathematics coordinator were involved in the shared task of leadership. Additionally, the development plan included devoting time in the timetable for teacher collaboration and reflection, which is an essential aspect when facilitating continuous development for teachers (King & Stevenson, 2017). The principal also planned to engage directly in the teachers' learning process. Therefore, the principal created a plan to structure and support the teachers' development work, which can create favorable conditions for teacher learning (Postholm, 2018). A good development plan together with good leadership can create a learning environment at schools that enhances the implementation of new learning (e.g., Thoonen et al., 2011; Vanblaere & Devos, 2016). However, "professional development does not just happen – it has to be managed and led" (Earley & Bubb, 2004, p. 8). The findings from substudy 2 indicate that the principal did not fully implement the development plan, as he was not present nor engaged in the program sessions or the collaboration meetings. The teachers did not receive the necessary support and encouragement to incorporate the new ideas and strategies they learned in the PD program (Desimone & Garet, 2015). Furthermore, the lack of a shared understanding between the teachers and the principal with respect to the purpose of the allocated time suggests that the development plan was not collaboratively created, where the teachers should have been invited to take part in the planning process (Darling-Hammond & Richardson, 2009). This lack of collaboration may have contributed to the failure to establish a shared understanding.

The findings also point out that the principal's absence in supporting the teachers' learning was not due to him neglecting his responsibility, but rather to a lack of understanding of how to provide such support. The findings from substudy 2 indicate that MAM 2019 was perceived by the principal as challenging, given its organizational complexity and the level of support required for the teachers' PD. Practice-based PD programs like the MAM program require that the participating schools have a flexible organization, especially the host school. The principal needs to allocate rooms and time for plenary sessions, group work, teaching sessions, and collaboration between the teachers at school. Students must also be made available for enactment. In this case the principal experienced that the time spent on these organizational issues limited his participation in the teachers' development work, both during and between

sessions. The level of organization required for such a program is higher than that of a traditional course where teachers are simply gathered in a meeting room all day. However, the findings also show that the principal was more concerned about how his lack of control over MAM 2019 prevented him from making organizational and subject-matter adjustments in line with the school's needs. While research on school-based development emphasizes the importance of contextualizing PD work (e.g., Kazemi & Resnick, 2020; Postholm, 2020; Santagata, 2020), it is crucial to note that the MAM program, although job-embedded, is not explicitly designed as a school-based project. Rather, it is carried out in practice at one of the participating schools. The findings show that the principal's primary concern appeared to be focused on modifying or adapting MAM 2019 rather than facilitating and supporting teachers' involvement in the program to enhance their classroom practices.

7.2.3 The contradiction as the point of departure for change and development

Because "the object is the true motive" for people's actions (Leont´ev, 1981, p. 59), the contradiction regarding the lack of a collectively shared motive for participating in the MAM program between the principal and the teachers must be addressed if their goal-directed actions are to lead toward the joint object. It is important to note that the difference between the two motives in terms of content and process is not contradictory but rather different understandings of how to benefit from the program. The two motives can, however, lead to contradictory actions if they are not shared. Similar to Leont´ev's (1981) example of hunting directed to the object of obtaining food, the teachers and the principal will focus their goal-directed actions on what they believe will contribute to moving their practice toward the object. Because their motives are not shared, as members of the community they lack a foundation for distributing work or actions that are collectively focused on a common need (Engeström, 1987, 2001). Based on the presented findings, one obvious action to take is for the school owner and the school management to invite the teachers into decision-making processes related to their PD work. In this way, they will participate in the development work from the very beginning and be given the opportunity to contribute to defining the problem to work on (Tan & Caleon, 2016). Postholm (2020) argues that these processes should be included in the start-up phase of the development work. This will then serve as the foundation for ongoing improvement and sustainability. She maintains that this phase involves engaging stakeholders, securing funding, and setting clear goals and shared motives. These processes can create a space where both teachers and the principal can offer their opinions (Postholm, 2020), which in turn can elicit

possible motives for participation. Negotiation over these motives can then be developed into a shared vision for the development work (Feeney, 2016; Sztajn et al., 2017).

Developing ownership of the PD work, where motives are aligned, requires that leaders understand the importance of the role they have in facilitating teachers' PD. For example, the principal can establish a learning environment where teachers are given time to reflect together (King & Stevenson, 2017), are supported and encouraged to collaborate (Silva et al., 2017), and receive help to identify their development needs (Thoonen et al., 2011). These processes of developing ownership can contribute to ensuring that the development effort is based on the teachers' actual needs, rather than the school owner's assumption of their needs. As such, the teachers' practice and needs can serve as the starting point for their development work, which in turn will contribute to building teachers' motivation into the object that loads them "with initiative and commitment" (Sannino & Engeström, 2017, p. 81). However, the findings from substudy 2 showed that the principal struggled to fulfil the development strategy that had been planned before the program period started. As district leaders, such as the school owner, have the responsibility to shape and support the work of principals, Gibbons et al. (2019) argue that they can do so by assisting them in developing a plan for coordinating their work at school. Thus, the school owner must acknowledge its role of supporting school leaders in managing their responsibilities and become involved in the contextual challenges the principal faces at school. As such, responsibility can be negotiated, and roles can be clarified so that their goal-directed actions together contribute to the teachers' development work (Engeström, 2001). This could then lead to a more coherent system of support (Cobb et al., 2018) and facilitate the principal's leadership growth that further contributes to promoting school-wide development of mathematics teaching (Kazemi et al., 2022).

Developing a shared motive is important to ensure that the goal-directed actions are well aligned and satisfy teachers' needs, which for the informants in this study implies a thorough process of developing ownership. However, the findings from substudy 2 also identified several factors that can impede teachers' development work, including a lack of a culture and practice for development work at school and inadequate leadership of teachers' learning. Thus, schools that participate in practice-based professional learning programs, such as the MAM program, should also prioritize building a strong foundation for PD within the school. Such a dual focus is also supported by research indicating that both a focus on content and process should be integrated and complement each other in the development work (Postholm et al., 2013). As the

MAM program can be viewed as a model for teachers' PD,[18] this dual focus not only reflects the two motives of the teachers and school management but also the opportunities for development that exist within the MAM program. These opportunities can contribute to shifting the principals' approach to the MAM program from a school-based project toward a mediating artifact for developing leadership practice for teachers' learning, which in turn can lead to the development of structure, culture, and practice for development work at the school.

The cycle of enactment and investigation provides a systematic model for PD where the teachers are invited to engage in collaborative learning processes in an active and sustained way (Wæge & Fauskanger, 2020). This is done by teachers taking part in mutual processes of negotiation of meaning to create a joint enterprise in a community of practice (Wenger, 1998). These processes are also guided by the TEs and involve collective exploration, observation, and reflection by using the instructional activities as a common tool, guided by the TEs (Wæge & Fauskanger, 2020). The way this development program is designed offers a model for PD that the principal can use as an aid or thinking tool for creating developmental structures and practices contextualized to the school. Furthermore, the processes led by the TEs can function as examples of instructional leadership practices that demonstrate how teachers are supported in collaborative learning and where meanings concerning teaching instruction and students' learning are negotiated (e.g., Leithwood et al., 2020; Rigby, 2016). Through their participation in the program, the teachers can gain valuable experiences of this PD model that can be leveraged by the principal to practice distributed forms of leadership, which in turn can have a positive impact on teachers' capacity, motivation, commitment, and perception of working conditions (Leithwood et al., 2020). By taking part in the program, the principal can take advantage of this opportunity to improve his leadership practice so he can manage to lead the development processes after the program period is over. This means that both motives can be built into the object, which can then expand the school's possibilities for learning that the program provides. In this way, the MAM program can not only help to enhance teachers' PD in ambitious mathematics teaching, but it can also serve as a means for schools to invest in leadership development. This is an investment that will pay dividends for schools as structures will be created that support distributive leadership (Kazemi & Resnick, 2020).

[18] See section 4.1 for a thorough description of the MAM program as a model for teachers' PD.

While the guidelines[19] for school management's participation in the MAM program were intended to help them avoid the above-mentioned challenges relating to structuring and supporting teachers' PD work, the findings show that it is not enough to simply provide information. It cannot be assumed that participating schools have the necessary development structures in place, or that they have followed the guidelines completely. Furthermore, the findings also show that schools may not recognize the opportunities for leadership development that exist within the program. Schools cannot be expected to address their organizational and leadership learning needs independently, and instead, they require support, as elaborated on in section 7.3.

7.3 A lack of a partially shared object

The activity system analysis (Engeström, 2001, 2015) used to examine the dynamics of human activity suggests that the teachers in this study can be regarded as active subjects in the process of teacher PD, with the school management and the school owner as members of the community. Additionally, the analysis shows that the TEs are regarded as the acting subject in another activity, and together the activities constitute a network of systems. While a collective activity system is driven by a shared motive embedded in the object of the activity (Engeström, 2001), a network of activity systems is driven by an object that is partially shared (Engeström, 2015). According to Engeström (2015), the objects constructed by the activity systems are moved toward a potentially shared or jointly constructed object that constitutes the rationale for the network's existence. Therefore, according to CHAT, all development efforts should begin with a discussion on and negotiation over the purpose and creation of goals and an overarching object so a partially shared object can be developed. Then, the involved parties use mediating artifacts to move toward this object. In this study, the two activity systems use the MAM program as a shared mediating artifact, which the school has chosen to participate in and the TEs have chosen to lead, connecting the two activity systems together. The purpose of the MAM program is to aid the actors in conducting goal-directed actions that move the activities toward a partially shared object so they can reach the desired outcome of developing their teaching practices. To do so, a partially shared object must be developed, and this requires that the actors engage in boundary crossing (Engeström et al., 1995).

[19] See section 4.4 for a thorough description of the guidelines.

According to Engeström et al. (1995), boundary crossing occurs when individuals move cognitively across different activity systems to navigate the differences between them. They argue that effective boundary crossing is important as it can promote effective communication and interaction. Thus, it is crucial for the actors in this study to engage in boundary crossing to create the condition for the development of a partially shared object. Even though there were instances when the actors could have engaged in boundary crossing, the findings presented in section 7.2 show that they did not actively do so. For example, even though the cycle of enactment and investigation provides opportunities for collaboration and negotiation between the teachers and the TEs on the content of the program, the teachers did not have a common and clear understanding of the overriding goal of the program, even after one year of participation. Moreover, the two start-up sessions that were used to provide information about the program could have been utilized as a shared meeting ground (Engeström & Toivainen, 2011) for the actors to engage in boundary crossing. Other than the two start-up sessions, the design[20] of MAM 2019 did not facilitate expectations and motives to be revealed and negotiated. Therefore, the actors did not engage in boundary crossing and a partially shared object was never developed.

The lack of a constructed partially shared object leads to the notion that the teachers and the TEs were using the MAM program as a mediating artifact without a common agreement on the direction toward which the program should aid to improve their practice. Section 7.1 revealed a misalignment between the theories of action held by the teachers and the program. The discussion highlighted that the teachers' perceptions of key concepts and instructional activities show that they interpreted important aspects of the program differently than its intended purpose. This was also pointed out by the TEs, who experienced that the teachers expected something substantially different from what the MAM program aimed to contribute, both in terms of content and view on learning and development. Furthermore, in section 6.2, it is evident that the teachers and school management had different motives for participating in the program. This divergence arose from their contrasting understandings of what the program should contribute. These findings show that the MAM program is interpreted and understood differently by the two activity systems, and further demonstrate that there is not only a contradiction within the teachers' activity system but also between the two activity systems

[20] See section 4.1 for a thorough description of the design of the MAM 2019.

when it comes to how MAM 2019 is to function as a mediating artifact. Thus, these contradictions reinforce the need for the actors to engage in boundary crossing to gain insight into each other's expectations, interpretations, and comprehension of the mediating artifact they share. This is necessary, not only to enable them to engage in effective communication and negotiation (Engeström et al., 1995), but also to ensure effective program implementation and achievement of the intended goals.

7.3.1 A need for boundary crossing and transformation learning

Despite the actors' lack of leadership and understanding of boundary-crossing processes, the three substudies reveal compelling reasons why such processes should have taken place. The findings also indicate that specific actions could have been taken not only to promote the creation of a partially shared object but also to overcome the contradictions presented in sections 7.1 and 7.2. The presented findings in the previous sections have shown a complex situation for the actors in terms of contradictory motives, and accordingly, there were different expectations for how the MAM program should function as a mediating artifact. As mentioned above, there were contradictions both within the teachers' activity system, as well as between the two activity systems that require boundary crossing to ensure effective program implementation and achievement of the intended goals. One boundary-crossing action that could have been taken to overcome the contradiction between the TEs and the teachers would have been to examine their theories of action so they could have been revealed and negotiated, and in turn better understood across the two parties. Dialog is the primary focus in the process of boundary crossing (Engeström & Toivainen, 2011). Thus, revealing these theories of action involves creating conditions for communication where the actors gain insight into each other's expectations, interpretations, and understandings of their teaching practice for the students' learning and the MAM program. This is particularly related to the key concepts of ambitious mathematics teaching and the purpose of the instructional activities. However, the findings in section 7.1 indicate that revealing the teachers' perceptions might not be straightforward as these perceptions were not obvious nor easy to detect. Additionally, they were grounded in an underlying view of teaching and students' learning that was substantially different from the student-centered view embedded in ambitious mathematics teaching (Ghousseini et al., 2015). In other words, the teachers' perception of key concepts represents a conflicting cognitive tool (Engeström, 1999) that can challenge negotiations. As stated in section 6.1, the TE might be left with the impression that the teachers are talking about key principles for ambitious mathematics teaching when in fact they are not.

From the TEs' perspective, the examination of the teachers' perceptions must involve a more thorough process than just inviting the teachers to explain their perceptions of these concepts. They need to study their current practice to gain insight into what their perceptions really mean. In this way, they can overcome the challenge of negotiating in a language where important concepts mean different things to each other (Wertsch & Toma, 1995). If they succeed in this, they can take part in a negotiation where their understandings can be shared more between them. From the teachers' perspective, they must engage in boundary crossing with the TEs to challenge their own theories and assumptions and be open to expanding their understanding of the purpose of the MAM program and its content. This is essential, as the purpose of the MAM program is to aid them in developing a view on teaching and students' learning that is embedded in ambitious mathematics teaching, which is substantially different from their current position. The negotiation also means that the teachers will be helped by the TEs to understand the theories of action underpinning the MAM program so they can be integrated within their own existing theories (Timperley et al., 2007). These boundary-crossing actions can avoid language from being a conflicting cognitive tool (Engeström, 1999), which can lead to potential conflicts and solutions that do not address the needs or goals of all involved parties (Engeström et al., 1995). Furthermore, the boundary-crossing actions can lay the foundations for teachers' extant perspectives, theories, and understandings of teaching and students' learning to be challenged and reconstructed (Timperlay et al., 2007) toward an ambitious view of mathematics teaching.

Another reason why the actors should work together on a partially shared object and engage in boundary crossing is to allow transformational learning to occur (Engeström, 2001). Although the cycle of enactment and investigation situates teachers' learning and development in real classrooms with real students that are very close to their own reality, this reality does not represent the participating teachers' *actual* practice, as it is not their classroom or their students. With such an interpretation, the practice where the cycle is conducted must be understood as a constructed reality. Despite its pervading practice-based approach, this shows that the MAM program is not explicitly linked to the teachers' actual classroom lesson, which according to Desimone and Garat (2015) can be a reason for failure of PD programs. Therefore, the teachers must undergo a transformation when they return to their own classrooms that further needs to be supported and guided for them to "integrate the knowledge or pedagogy into their daily instruction, rather than leaving that burden to them when they return to the classroom" (Desimone & Garat, 2015, p. 256).

It may be difficult for the TEs to support the teachers in this transformation as it will demand a great deal of effort to follow up all the participating teachers from the different schools on a daily basis. But, this daily support can be provided by the principal who can play a critical role in promoting school-wide development of mathematics teaching (Kazemi et al., 2022). As suggested in section 7.2, the MAM program provides a model for PD that the principal can utilize as a thinking tool to design developmental structures and practices tailored to the specific needs of the school and to invest in leadership development. Nonetheless, the discussion in section 7.2 also states that recognizing these opportunities requires support for the principal. Therefore, these opportunities cannot only be informed through general guidelines but also negotiated through boundary crossing by the principal and the TEs to become a viable cognitive tool (Engeström, 2001) that takes the context of the school into consideration. Together with an active school owner who is committed to his responsibility of shaping and supporting the principal's work (Gibbons et al., 2019), such negotiation can also contribute to dividing their goal-directed actions so teachers experience a coherent system of support (e.g., Cobb et al., 2018; Gamoran, 2003; Knapp, 2003). As such, these opportunities can become an integrated component of a partially shared object that the principal can act on to enhance his leadership practice. This component can be crucial, not only to enable the principal to manage to lead the development processes after the program period is over but also to support the teachers' work in transforming ambitious mathematics teaching, as a new idea, into the context of their own practice (Kennedy, 2016).

7.3.2 Challenges in the nature of the MAM program

While boundary crossing is argued to be crucial for the actors in this study to develop a partially shared object, there are challenges concerning the nature of the MAM program that could potentially impede the possibility of boundary crossing to occur. Firstly, the MAM program expects schools participating in the program to have anchored the development work according to the mathematics teachers' development needs before they attend. Even though the MAM program takes place in one of the participating schools and involves job-embedded teacher development, it should not be regarded as a school-based project. Therefore, the design does not include the process of needs assessment to identify areas of improvement tailored to the unique needs, interests, and goals of the particular school and its teachers. As, such, the design makes it challenging to conduct the above-mentioned boundary-crossing actions. Secondly, it is important to remember that the MAM program provides the opportunity for more than one school to participate simultaneously. This means that it is a program at scale where participants

represent different schools that might have different expectations, interpretations, and understandings of the different aspects of the MAM program. Based on the findings from this study, these challenges can lead to PD situations where the teachers have as many different understandings of the MAM program as the number of participants.

The concept of boundary crossing is not new in the field of practice-based PD for mathematics teachers. It has been highlighted as an essential notion by scholars who adopt a research-practice approach, including those who study how to scale up PD programs (e.g., Robutti et al., 2020; Santagata et al., 2020). Even though MAM 2019 was not a research-practice partnership, as the TEs did not plan to conduct research, the development work required a collaborative partnership between the TEs and practitioners. Collaborative partnerships are crucial aspects in research-practice approaches (Coburn et al., 2013). Henrick et al. (2015) argue that research-practice partnerships view context as an essential element of a multifaceted improvement system, rather than regarding contextual variables solely as factors that either impede or facilitate PD outcomes. Therefore, to address the aforementioned challenges, the MAM program designers could benefit from examining what can be learned about establishing such partnerships from this research. For example, Coburn (2003) argues that opportunities must be created for collaboration and shared decision-making among all stakeholders, including teachers, administrators, and external providers, to support those at the local school to assume ownership of the development work. Kazemi and Resnick (2020) caution against just copying what works in one school at another, as PD programs should be tailored to the specific context through the collaboration process between the researcher, teachers, and school leaders. Coburn et al. (2013) stress the importance of "commitment to mutualism", which involves engaging in reciprocal and mutually beneficial interactions where the partners are willing to invest time and put effort into the partnership based on shared goals and joint decision-making. As this research recommends breaking down boundaries between different stakeholders, tailoring the program to the specific context, and fostering a sense of shared ownership and responsibility, it is evident that these findings clearly support the boundary-crossing actions suggested in section 7.3.1. Based on this research, it is recommended that the MAM program designers consider changing the program's design to make it more flexible and adaptable to the local context. By doing so, assistance can be provided to identify goals and motives for teachers' learning, anchor the PD activity, and prepare for guiding discussions (Sztajn et al., 2017). This, in turn, will enable the actors to "move across boundaries to seek and give help, to find information and tools wherever they happen to be available" (Engeström et al., 1995, p. 332). Because no matter how well

prepared the schools are prior to their participation, in their attempt to combine ingredients from different activity systems to achieve hybrid solutions (Engeström, 2001) the actors will always face the challenge of negotiation because words naturally mean different things to different people (Wertsch & Toma, 1995).

7.4 Limitations

This section will reflect on modifications that could have possibly been made to the research study if it were to be conducted again. While the presented study has proven its strengths through the characteristics of trustworthiness, the way the research was conducted and the choices I have made have influenced and limited the findings. Two prominent limitations have been identified in this research.

The first concerns the purpose behind the data collection, especially related to the individual interviews with the teachers. In retrospect, it is clear that the actors' motives should have been a primary focus of the data collection, as "the object is the true motive" (Leont'ev, 1981, p. 59) for people's actions, as were a prominent aspect in the findings from this study. Although the study as a whole provided enough information to suggest that the teachers' motives for participating in the MAM program were not clear and shared by all of them, and furthermore, differed from the principals' motives and the program's object, the teachers' motives were first explored after the first year of the program period and after they decided to withdraw from the program. Therefore, what I was really exploring was their perceptions of the motives they had when the program started one year earlier. It is possible that these perceptions could have been influenced by the negative experiences they had and emotions they felt from attending the program. Moreover, teachers may be less likely to share their true motives with me as a researcher after the program is over, especially if they believe that their criticism might reflect poorly on themselves. This could lead to a distorted or incomplete understanding of the teachers' motives for participating in the program. Thus, the limitation of not exploring the teachers' motives for participating in the program in depth may have affected the validity of the findings. I should therefore have placed a greater emphasis on exploring the teachers' motives for participating in the program in substudy 1.

The second limitation concerns the choice of informants for this research. While the study explored the roles of the teacher, school management, and TEs, providing an extensive, rich, and comprehensive data foundation, the school owner's voice was not included. Instead, he was

represented through the perspectives of the TEs and the principal. Therefore, the opinions and actions of the school owner are portrayed according to how the TEs and the principal perceived them. As pointed out in the study, the school owner was a crucial stakeholder in the teachers' PD program and had his own unique perspective on the goals, priorities, and expectations for the school and its teachers that were not shared with the others. Thus, it must be acknowledged that the omission of the school owner's involvement in the study could have affected the credibility of the findings.

Even if taking these limitations into account may have resulted in a more balanced picture of the studied phenomenon, it is important to note that a Ph.D. project is inherently limited in scope. There will always be voices or elements that could have been included, and the researcher may face several dilemmas due to such factors as time, available resources, and sample size. While acknowledging the limitations of a research project is essential for responsible and credible research, it is equally important to justify the choices made, such as the choice of selecting one school instead of multiple schools.[21]

[21] See section 5.3 for an account of the criteria used to select participants and context of the study.

7.5 Conclusion

This thesis aimed to explore how practice-based PD programs can contribute to the PD of mathematics teachers. This was done by investigating how the first year of the MAM program was carried out in a lower secondary school in Norway. The study focused on the roles of the teachers, the principal, and the teacher educators (TEs) as program leaders, and how these roles acted and interacted to contribute to and support the teachers' PD. As mentioned in section 5.6, my assumption was that the MAM program would have a positive impact on the teachers, and therefore I assumed that the study would provide new knowledge and insight into how these actors could act and interact to contribute to and support successful teachers' PD. This study has shown that the interaction between the actors is indeed important to contribute to and support teachers' PD, but especially at another stage of the development work and with respect to other areas than assumed.

This study found that interaction and dialog between actors are crucial, particularly in the start-up phase of practice-based PD programs. Through CHAT and the activity system analysis, several tensions and contradictions were identified that were caused by conditions and factors impeding the participating teachers' PD. These included a misalignment between the theories of action held by the teachers and the program, a lack of ownership for the development work, a lack of shared motives for participating, and an absence of a partially shared object of development work. By positioning tensions and contradictions as the foundation for change and development (Engeström & Miettinen, 1999), this thesis argues that interaction and dialog are crucial for resolving these issues and for establishing a foundation for the MAM program to contribute effectively to the PD of mathematics teachers.

The thesis points out the importance of integrating boundary-crossing actions into the MAM program. By negotiating these processes, perceptions and expectations can be clarified, motives can be aligned, and a partially shared object for the development work can be developed. In particular, the thesis argues that boundary-crossing processes can reveal the need for the principals to develop leadership practices alongside the teachers' development of ambitious mathematics teaching. Since the MAM program is not explicitly linked to teachers' actual classroom lessons, teachers must undergo a transformation when they return to their own classrooms. Focusing on developing leadership practices can enable the principal to lead the development processes after the program period ends and support teachers' work in transforming ambitious mathematics teaching within the context of their own practice. As such,

the MAM program can contribute not only to teachers' PD of ambitious mathematics teaching but also build a foundation for sustainable PD work at school.

The study revealed that the actors did not engage in boundary-crossing actions, as their perceptions and expectations were not clarified, motives were not shared and negotiated, and a partially shared object was not developed. However, negotiating on these issues is critical if a foundation for practice-based programs is to be established and succeed in contributing to mathematics teachers' PD. This process requires the actors to position themselves where the diversities within them are exposed. Achieving this requires interaction and dialog between them. Therefore, it can be concluded that interaction and dialog serve as the foundation for practice-based PD programs.

In light of the literature review presented in Chapter 2.0, this thesis makes several contributions to the research field, and also confirms previous findings. These concluding remarks reinforce the idea that all development efforts should begin with a discussion on and negotiation over the intention and creation of goals and a comprehensive object, which enables the development of a partially shared object (Engeström, 2015). The thesis provides additional evidence to support the idea that effective leadership by the principal plays a crucial role in promoting comprehensive improvements in mathematics teaching throughout a school (Kazemi et al., 2022). Additionally, it points to the importance of schools prioritizing the continuous learning and development of both teachers and leaders (Kazemi & Resnick, 2020). This thesis also enhances our understanding of the essential qualities and actions of facilitators who lead PD for mathematics teachers and provides insights into the knowledge and competencies TEs need as PD leaders and facilitators (Kennedy, 2016; Sztajn et al., 2017). The thesis also points out that boundary-crossing actions are an integral and essential part of the program to contribute to teachers' PD, and that such inclusion requires knowledge and understanding of these processes and how to lead them. With interaction and dialog as the point of departure, this study shows that leadership and comprehension of boundary-crossing processes should be included as part of the knowledge and competencies required by facilitators who lead PD for mathematics teachers.

7.6 Further research

The present study has pointed out that certain factors, such as the circumstances and context in which a practice-based PD program is carried out, shared motives, and the development of a common object for the development work, as well as the support provided, have a significant impact on teachers' PD. In this thesis I have proposed and discussed several changes and actions that can be considered to cope with the challenges these factors raise. For example, I have recommended several actions that can facilitate boundary crossing and discussed how this can be achieved. However, there is no guarantee that these suggestions will yield positive results. To better understand the implication of these findings, future studies could explore whether and how the proposed opportunities for boundary crossing can be integrated into the design of the MAM program and how they can contribute to the development of a partially shared object. Furthermore, it would be worthwhile to investigate how a dual focus in the MAM program could support both the development of principals' leadership learning and mathematics teachers' development of ambitious mathematics teaching. It also would be worthwhile examining whether incorporating a focus on principals' development of leadership learning into the aim of the MAM program can increase the likelihood of ensuring teachers' learning of ambitious mathematics teaching.

While most existing research on practice-based learning environments has focused on teacher candidates in the United States, the present study contributes to answering Charalambous and Delaney's (2020) call for more research on how these environments can enhance the learning of teachers with diverse work experiences in other countries by studying a practice-based PD program for lower secondary mathematics teachers in Norway. This study has also responded to Sztajn et al.'s (2017) call for research that can provide further insights into the knowledge and competencies required for effective PD facilitation. The thesis demonstrates that the development of a partially shared object between teachers and teacher educators (TEs) can significantly impact the outcome of a practice-based PD program. Nonetheless, the present study yields ambiguous results compared to the few existing studies related to the MAM program that have demonstrated promising results on teachers' PD without implementing these actions (e.g., Fauskanger & Bjuland, 2019; Wæge & Fauskanger, 2020, 2022). Thus, in accordance with Cohen and Mehta's (2017) recommendation, I advocate for further exploration in the field to understand why some educational reforms succeed while others do not.

The third and final recommendation for further research is of a theoretical nature. The MAM program has been designed in accordance with Wenger's (1998) theory of teachers' collective learning in a community of practice. The circle of enactment and investigation in the program provides teachers with opportunities to engage in mutual processes of negotiating meaning, leading to the creation of a joint enterprise guided by teacher educators (Wæge & Fauskanger, 2020). When the teachers and TEs consult, the consulting individuals will have varying levels of experience, expertise, age, personality, and authority, which will be unique to each group and program. This diversity can create challenges in terms of power dynamics, as TEs naturally hold more power or authority than practitioners. This power differential can erect barriers to effective communication and collaboration that is essential, not only for mutual engagement and the negotiation of meanings within the community of practice, but also for engaging in boundary-crossing practices. Therefore, I will encourage future research to explore and determine the challenges to collaboration and negotiation that may arise from power dynamics, and how these challenges can be addressed. Such knowledge can contribute to developing successful boundary-crossing practices, and to understanding how these practices can constitute an important part of the facilitation practices that are effective in promoting practice-embedded teacher learning (Gibbons et al., 2021).

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Appendices

Appendix 1: Assessment of processing of personal data from the Norwegian Social Science Data Service (NSD)

Sikt

Vurdering av behandling av personopplysninger

Referansenummer	Vurderingstype	Dato
934742	Standard	07.08.2019

Prosjekttittel
Mathematics teachers' professional development in a practice-based development program

Behandlingsansvarlig institusjon
Norges teknisk-naturvitenskapelige universitet / Fakultet for samfunns- og utdanningsvitenskap (SU) / Institutt for lærerutdanning

Prosjektansvarlig
Eskil Braseth

Prosjektperiode
01.08.2018 - 31.12.2022

Kategorier personopplysninger
Alminnelige

Lovlig grunnlag
Samtykke (Personvernforordningen art. 6 nr. 1 bokstav a)

Behandlingen av personopplysningene er lovlig så fremt den gjennomføres som oppgitt i meldeskjemaet. Det lovlige grunnlaget gjelder til 31.12.2024.

Meldeskjema

Kommentar
Det er vår vurdering at behandlingen av personopplysninger i prosjektet vil være i samsvar med personvernlovgivningen så fremt den gjennomføres i tråd med det som er dokumentert i meldeskjemaet med vedlegg den 07.08.19, samt i meldingsdialogen mellom innmelder og NSD. Behandlingen kan starte.

MELD VESENTLIGE ENDRINGER
Dersom det skjer vesentlige endringer i behandlingen av personopplysninger, kan det være nødvendig å melde dette til NSD ved å oppdatere meldeskjemaet. Før du melder inn en endring, oppfordrer vi deg til å lese om hvilke type endringer det er nødvendig å melde:

https://nsd.no/personvernombud/meld_prosjekt/meld_endringer.html

Du må vente på svar fra NSD før endringen gjennomføres.

TYPE OPPLYSNINGER OG VARIGHET
Prosjektet vil behandle alminnelige kategorier av personopplysninger frem til 31.12.22. Datamaterialet med personopplysninger lagres videre til 31.12.24. Datamaterialet vil oppbevares internt ved behandlingsansvarlig institusjon til videre forskning.

LOVLIG GRUNNLAG
Prosjektet vil innhente samtykke fra de registrerte til behandlingen av personopplysninger. Vår vurdering er at prosjektet legger opp til et samtykke i samsvar med kravene i art. 4 og 7, ved at det er en frivillig, spesifikk, informert og utvetydig bekreftelse som kan dokumenteres, og som den registrerte kan trekke tilbake. Lovlig grunnlag for behandlingen vil dermed være den registrertes samtykke, jf. personvernforordningen art. 6 nr. 1 bokstav a.

PERSONVERNPRINSIPPER
NSD vurderer at den planlagte behandlingen av personopplysninger vil følge prinsippene i personvernforordningen om:

- lovlighet, rettferdighet og åpenhet (art. 5.1 a), ved at de registrerte får tilfredsstillende informasjon om og samtykker til behandlingen
- formålsbegrensning (art. 5.1 b), ved at personopplysninger samles inn for spesifikke, uttrykkelig angitte og berettigede formål, og ikke behandles til nye, uforenlige formål
- dataminimering (art. 5.1 c), ved at det kun behandles opplysninger som er adekvate, relevante og nødvendige for formålet med prosjektet
- lagringsbegrensning (art. 5.1 e), ved at personopplysningene ikke lagres lengre enn nødvendig for å oppfylle formålet

DE REGISTRERTES RETTIGHETER
Så lenge de registrerte kan identifiseres i datamaterialet vil de ha følgende rettigheter: åpenhet (art. 12), informasjon (art. 13), innsyn (art. 15), retting (art. 16), sletting (art. 17), begrensning (art. 18), underretning (art. 19), dataportabilitet (art. 20).

NSD vurderer at informasjonen om behandlingen som de registrerte vil motta oppfyller lovens krav til form og innhold, jf. art. 12.1 og art. 13.

Vi minner om at hvis en registrert tar kontakt om sine rettigheter, har behandlingsansvarlig institusjon plikt til å svare innen en måned.

FØLG DIN INSTITUSJONS RETNINGSLINJER
NSD legger til grunn at behandlingen oppfyller kravene i personvernforordningen om riktighet (art. 5.1 d), integritet og konfidensialitet (art. 5.1. f) og sikkerhet (art. 32).

For å forsikre dere om at kravene oppfylles, må dere følge interne retningslinjer og/eller rådføre dere med behandlingsansvarlig institusjon.

OPPFØLGING AV PROSJEKTET
NSD vil følge opp underveis (hvert annet år) og ved planlagt avslutning for å avklare om behandlingen av personopplysningene er avsluttet/ pågår i tråd med den behandlingen som er dokumentert.

Lykke til med prosjektet!

Kontaktperson hos NSD: Silje Fjelberg Opsvik
Tlf. Personverntjenester: 55 58 21 17 (tast 1)

Appendix 2: Study information and signature form for the research participants' informed consent

For the students and their guardians:

Matematikksenteret
Nasjonalt senter for matematikk i opplæringen

Samtykkeerklæring om filming av matematikkundervisning i forbindelse med forskning på kompetansehevingstilbudet «Mestre Ambisiøs Matematikkundervisning»

NTNU, Matematikksenteret har etter forespørsel av utdanningsetaten i Oslo (UDE) tilbudt et kompetansehevingsprosjekt «Mestre Ambisiøs Mateamtikkundervisning» (MAM) for fire ungdomsskoler og ni barneskoler i Osloområdet. Prosjektet er en del av en større satsning U22 (Ungdomsskoler mot 2022). Matematikksenteret skal over fire semestre, høst 2019-vår 2020-høst 2020-vår 2021, gjennomføre et etterutdanningskurs med matematikklærerne fra de deltakende skolene.

I forbindelse med gjennomføringen av kompetansehevingstilbudet skal jeg (Eskil Braseth ved NTNU, Matematikksenteret) forske på hvilken måte MAM programmet kan bidra til matematikklærernes læring. Videre skal jeg se på hvordan skolens ledelse og lærerutdannerne som leder utviklingsprogrammet kan legge til rette for og bidra til en slik utvikling. Forskningen inngår i mitt doktorgradsprosjekt ved NTNU.

Matematikklæreren til ditt barn er en av deltakerne i mitt forskningsprosjekt. Som en del av studien skal jeg observere matematikklæreren i egen undervisning, og i den forbindelse har jeg behov for å ta videoopptak av noen matematikktimer i klassen til barnet ditt. Videoopptakene vil foregå i matematikktimene i en sammenhengende treukers periode på høsten 2019 og våren 2020. Formålet med observasjonen og videoopptakene er å observere *lærerens* praksis, men et videoopptak i et klasserom kan i noen tilfeller fange opp stemmer og/eller bilder av elevene. Jeg ønsker derfor deres samtykke om at deres barn kan filmes i matematikktimene i observasjonsperioden.

Videoopptakene skal brukes til forskning knyttet til læreres læring og utvikling av matematikkundervisning. Noen utdrag av videoopptakene vil bli transkribert og publisert i forsknings- og populærvitenskapelige artikler. Elever/lærere/skoler vil være anonymisert i publikasjonene, og personidentifiserbare data vil kun være tilgjengelig for Eskil Braseth, de to veilederne May Britt Postholm (Professor ved Institutt for lærerutdanning, NTNU) og Reidar Mosvold (Professor i matematikkdidaktikk ved UiS), og involverte personer. Data med personopplysninger vil bli oppbevart fram til utgangen av kalenderåret 2024, to år utover

avtalt prosjektperiode for videre forskning, og deretter slettet. Forskningen som skal gjennomføres er meldt inn og godkjent av Norsk Senter for Forskningsdata (www.nsd.uib.no). Videoopptakene vil bli oppbevart sikkert, i tråd med retningslinjer fra NSD.

Deltakelse er frivillig og alle deltakere står fritt til å trekke seg når de måtte ønske i løpet av prosjektperioden. Deltakerne har også rett til å be om innsyn, retting, sletting, begrensning og dataportabilitet. De har også rett til å klage til Datatilsynet. Vedlagt svarslipp skal fylles inn og sendes på epost til undertegnede innen fredag 23. august 2019. Ta gjerne kontakt ved eventuelle spørsmål.

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Samtykkeerklæring, videopptak av matematikkundervisning

Elevens navn:

Foresatte

☐ **Ja,** jeg/vi samtykker i at barnet vårt kan filmes i matematikktimene, og at opptakene brukes slik det er skissert i skrivet.

☐ **Nei,** jeg/vi samtykker IKKE i at barnet vårt kan filmes i matematikktimene, og at opptakene brukes slik det er skissert i skrivet.

Dato:_____ Sted:_____

Signatur foresatte:_____

Elev

☐ **Ja,** jeg samtykker i at jeg kan filmes i matematikktimene, og at opptakene kan brukes slik det er skissert i skrivet.

☐ **Nei,** jeg samtykker IKKE i at jeg kan filmes i matematikktimene, og at opptakene brukes slik det er skissert i skrivet.

Dato:_____ Sted:_____

Signatur elev:_____

For the participating teachers, school management, and teacher educators:

Matematikksenteret
Nasjonalt senter for matematikk i opplæringen

Samtykkeerklæring om innsamling og bruk av datamateriale i forbindelse med forskning på kompetansehevingstilbudet "Mestre Ambisiøs Matematikkundervisning"

NTNU, Matematikksenteret har etter forespørsel av utdanningsetaten i Oslo (UDE) tilbudt et kompetansehevingsprosjekt «Mestre Ambisiøs Mateamtikkundervisning» (MAM) for fire ungdomsskoler og ni barneskoler i Osloområdet. Prosjektet er en del av en større satsning U22 (Ungdomsskoler mot 2022). Matematikksenteret skal over fire semestre, høst 2019-vår 2020-høst 2020-vår 2021, gjennomføre et etterutdanningskurs med matematikklærerne fra de deltakende skolene.

I forbindelse med gjennomføringen av kompetansehevingstilbudet skal jeg (Eskil Braseth ved NTNU, Matematikksenteret) forske på hvilken måte MAM programmet kan bidra til matematikklærernes læring. Videre skal jeg se på hvordan skolens ledelse og lærerutdannerne som leder utviklingsprogrammet kan legge til rette for og bidra til en slik utvikling. Forskningen inngår i mitt doktorgradsprosjekt ved NTNU.

Du er en av de som skal delta i forskningsprosjektet og vi ber om ditt samtykke til innsamling og bruk av følgende type datamateriale:

Data av type A

- Videoopptak av gruppeintervjuer som skal gjennomføres høsten 2019 og våren 2020
- Bakgrunnsinformasjon om alder, kjønn, utdanning og arbeidserfaring

Datamaterialet vil brukes til:

- Forskning knyttet til matematikklæreres læring og skoleledelsenes tilrettelegging av en slik læring. Forskningen vil bli gjort av Eskil Braseth i forbindelse med vedkommende sitt doktorgradsprosjekt. Noen utdrag av videoopptakene vil bli transkribert og publisert i forsknings- og populærvitenskapelige artikler. Involverte personer og skolen vil være anonymisert i publikasjonene på en slik måte at de ikke kan spores tilbake, og personidentifiserbare data vil kun være tilgjengelig for Eskil Braseth, de to veilederne May Britt Postholm (Professor ved Institutt for lærerutdanning, NTNU) og Reidar Mosvold (Professor i matematikkdidaktikk ved UiS), og involverte personer. Data med personopplysninger vil bli oppbevart fram til utgangen av kalenderåret 2024, to år utover avtalt prosjektperiode for videre forskning, og deretter slettet.

Forskningen som skal gjennomføres er meldt inn og godkjent av Norsk Senter for Forskningsdata (www.nsd.uib.no). Videoopptakene vil bli oppbevart sikkert, i tråd med retningslinjer fra NSD.

Data av type B

- Lydopptak av to individuelle intervjuer med deg. Intervjuene skal gjennomføres i løpet av høsten 2019 og våren 2020

Datamaterialet vil brukes til:

- Forskning knyttet til matematikklæreres læring og skoleledelsenes tilrettelegging av en slik læring. Forskningen vil bli gjort av Eskil Braseth i forbindelse med vedkommende sitt doktorgradsprosjekt. Noen utdrag av lydopptakene vil bli transkribert og publisert i forsknings- og populærvitenskapelige artikler. Involverte personer og skolen vil være anonymisert i publikasjonene på en slik måte at de ikke kan spores tilbake, og personidentifiserbare data vil kun være tilgjengelig for Eskil Braseth, de to veilederne May Britt Postholm (Prosfessor ved Institutt for lærerutdanning, NTNU) og Reidar Mosvold (Professor i matematikkdidaktikk ved UiS), og involverte personer. Data med personopplysninger vil bli oppbevart fram til utgangen av kalenderåret 2024, to år utover avtalt prosjektperiode for videre forskning, og deretter slettet. Forskningen som skal gjennomføres er meldt inn og godkjent av Norsk Senter for Forskningsdata (www.nsd.uib.no). Lydopptakene vil bli oppbevart sikkert, i tråd med retningslinjer fra NSD.

Data av type C

- Videoopptak av din matematikkundervisning i egen klasse i to perioder med en varighet på 3 uker. Den første perioden gjennomføres høsten 2019 og den andre perioden gjennomføres våren 2020
- Videoopptak av to intervju med deg. Intervjuet skal gjennomføres i løpet av høsten 2017

Datamaterialet vil brukes til:

- Forskning knyttet til matematikklæreres læring og skoleledelsenes tilrettelegging av en slik læring. Forskningen vil bli gjort av Eskil Braseth i forbindelse med vedkommende sitt doktorgradsprosjekt. Noen utdrag av videoopptakene vil bli transkribert og publisert i forsknings- og populærvitenskapelige artikler. Involverte personer og skolen vil være anonymisert i publikasjonene på en slik måte at de ikke kan spores tilbake, og personidentifiserbare data vil kun være tilgjengelig for Eskil Braseth, de to veilederne May Britt Postholm (Prosfessor ved Institutt for lærerutdanning, NTNU) og Reidar Mosvold (Professor i matematikkdidaktikk ved UiS), og involverte personer. Data med personopplysninger vil bli oppbevart fram til utgangen av kalenderåret 2024, to år utover avtalt prosjektperiode for videre forskning, og deretter slettet. Forskningen som skal gjennomføres er meldt inn og godkjent av Norsk Senter for Forskningsdata (www.nsd.uib.no). Videoopptakene vil bli oppbevart sikkert, i tråd med retningslinjer fra NSD.

Data av type D

- Videoopptak av ett gruppeintervju. Intervjuene skal gjennomføres i løpet av høsten 2020

Datamaterialet vil brukes til:

- Forskning knyttet til matematikklæreres læring og skoleledelsenes tilrettelegging av en slik læring. Forskningen vil bli gjort av Eskil Braseth i forbindelse med vedkommende sitt doktorgradsprosjekt. Noen utdrag av videoopptakene vil bli transkribert og publisert i forsknings- og populærvitenskapelige artikler. Involverte personer og skolen vil være anonymisert i publikasjonene på en slik måte at de ikke kan spores tilbake, og personidentifiserbare data vil kun

være tilgjengelig for Eskil Braseth, de to veilederne May Britt Postholm (Prosfessor ved Institutt for lærerutdanning, NTNU) og Reidar Mosvold (Professor i matematikkdidaktikk ved UiS), og involverte personer. Data med personopplysninger vil bli oppbevart fram til utgangen av kalenderåret 2024, to år utover avtalt prosjektperiode for videre forskning, og deretter slettet. Forskningen som skal gjennomføres er meldt inn og godkjent av Norsk Senter for Forskningsdata (www.nsd.uib.no). Videoopptakene vil bli oppbevart sikkert, i tråd med retningslinjer fra NSD.

Deltakelse er frivillig og alle deltakere står fritt til å trekke seg når de måtte ønske i løpet av prosjektperioden. Deltakerne har også rett til å be om innsyn, retting, sletting, begrensning og dataportabilitet. De har også rett til å klage til Datatilsynet. Vedlagt svarslipp skal fylles inn og sendes på epost til undertegnede innen torsdag 11. september 2020. Ta gjerne kontakt ved eventuelle spørsmål.

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Samtykkeerklæring, data av type A

Navn:

□ **Ja,** jeg samtykker i at jeg kan filmes i forbindelse med forskningsprosjektet (data av type A), og at opptakene og min bakgrunnsinformasjon kan brukes slik det er skissert i skrivet.

□ **Nei,** jeg samtykker IKKE i at jeg kan filmes i forbindelse med forskningsprosjektet (data av type A), og at opptakene og min bakgrunnsinformasjon kan brukes slik det er skissert i skrivet.

Dato:_____ Sted:_____

Signatur:_____

Samtykkeerklæring, data av type B

Navn:

□ **Ja,** jeg samtykker at det kan gjøres lydopptak i forbindelse med forskningsprosjektet (data av type B), og at opptakene kan brukes slik det er skissert i skrivet.

□ **Nei,** jeg samtykker IKKE i at det kan gjøres lydopptak med forskningsprosjektet (data av type B), og at opptakene kan brukes slik det er skissert i skrivet.

Dato:_____ Sted:_____

Signatur:_____

Samtykkeerklæring, data av type C

Navn:

□ **Ja,** jeg samtykker i at jeg kan filmes i forbindelse med forskningsprosjektet (data av type C), og at opptakene kan brukes slik det er skissert i skrivet.

□ **Nei,** jeg samtykker IKKE i at jeg kan filmes i forbindelse med forskningsprosjektet (data av type C), og at opptakene kan brukes slik det er skissert i skrivet.

Dato:_____ Sted:_____

Signatur:_____

Samtykkeerklæring, data av type D

Navn:

□ **Ja,** jeg samtykker i at jeg kan filmes i forbindelse med forskningsprosjektet (data av type D), og at opptakene kan brukes slik det er skissert i skrivet.

□ **Nei,** jeg samtykker IKKE i at jeg kan filmes i forbindelse med forskningsprosjektet (data av type D), og at opptakene kan brukes slik det er skissert i skrivet.

Dato:_____ Sted:_____

Signatur:_____

Appendix 3: Interview guides

Structure of the individual semi-structured interview guide for the mathematics teachers, fall 2019.

	Questions. All questions start with: In your opinion…
1	What kind of knowledge is needed by mathematics teachers to teach mathematics?
2	What type of requirements are there for teaching mathematics?
3	Is there a shared perception among mathematics teachers at your school as to what type of knowledge mathematics teachers need for teaching mathematics in school?
4	Are the mathematics teachers at your school aware of each other's teaching practice? If yes, how?
5	Can you briefly describe a teaching lesson where your classroom practice is visible?
6	What are your expectations for the MAM project?

Structure of the focus-group interview guide for the school management, fall 2019.

	Questions
1	How is the school management organized in relation to the MAM project?
2	What support do you find the mathematics teachers' need to be able to learn together?
3	How would you describe school management's role as a facilitator for mathematics teachers' professional development?
4	Can you give a specific description of how you facilitate for mathematics teachers' professional development?
5	How is time for development work structured at your school?
6	How would you describe the mathematics teachers' need to develop their knowledge about teaching and teaching practice?
7	How do you assume the mathematics teachers perceive the school's leadership of their development work?
8	How do you understand the MAM program, and how do you assume the program will influence the participating mathematics teachers?
9	What do you think the mathematics teachers will learn through their participation in the MAM program?

Structure of the focus-group interview guide for the school management, spring 2020.

	Questions
1	How has the school management been organized in relation to the MAM program?
2	In what way have you supported the mathematics teachers in their work with the MAM program?
3	Can you describe your role as facilitators for the mathematics teachers' professional development this last year?
4	How was the time for development work structured this last school year?
5	How would you describe the mathematics teachers' need to develop their knowledge about teaching and teaching practice?
6	How do you assume the mathematics teachers perceive the school's leadership of their development work?
7	What are your perceptions or experiences of the MAM program halfway through the period?
8	How do you assume that the MAM program has influenced the participating mathematics teachers?
9	What do you think the mathematics teachers have learned through their first year of participation in the MAM program?

Structure of the focus-group interview guide for the mathematics teachers, fall 2019.

	Questions
1	In your opinion, how does the principal, or school management, facilitate for your development of knowledge about teaching and teaching practice in mathematics?
2	How do you perceive the development work in mathematics at your own school?
3	What are your opportunities for collaboration in the school, and what occurs during these meetings?
4	Do you find that the collaboration contributes to your development of knowledge about teaching and teaching practice in mathematics?

Structure of the focus-group interview guide for the mathematics teachers, fall 2020.

	Questions
1	What is, or was, your perception of the overall goal of the MAM program?
2	What is your experience of school managements' facilitation of your development work with the MAM program?
3	How motivated were you for participating in the MAM program?
4	How did you find the start-up phase of the MAM program?
5	What were your opportunities for collaboration in connection with the MAM project, and what did you do during these meetings?
6	Do you find that the collaboration contributed to your development of knowledge about teaching and teaching practice in mathematics?

Structure of the focus-group interview guide for the TEs, fall 2020.

	Questions
1	How do you experience the teachers' perception of the purpose of the MAM project?
2	What foreknowledge do you expect the teachers to have to participate in the MAM project?
3	How have you experienced the teachers' development of knowledge during the first year?
4	How have you experienced the communication with the participants in the project?
5	Would you do anything differently if you could start over? If so, what?
6	How do you experience the principal's involvement in the MAM program?
7	In what way were you involved in the initiation phase of the project?
8	Are there any special experiences you have had after the first year of the program period that you would like to share?

The articles

Article 1

Mathematics Teachers' Perceptions of Teaching Practices Alignment with Ambitious Teaching

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Norwegian University of Science and Technology

Received: 25 November 2020 Accepted: 16 August 2022

Research on practice-based mathematics teacher education has identified core practices, principles and design features that lead to effective programs. Yet, some teachers do not perceive such practice-based development programs as relevant or useful. In response to this, the study reported in this article investigated three Norwegian teachers' current perceptions on mathematics teaching and student learning. Findings from the study indicate that teachers' perceptions on classroom practices can be described by using key concepts similar to those that are used to describe practices in ambitious teaching. However, the teachers' perceptions are guided by underlying purposes that differ from the view of teaching and student learning grounded in the characteristics of ambitious mathematics teaching. The implications of the findings are discussed in relation to the potential benefits of studying teachers' current understandings of concepts related to a professional development program before entry to or during the start-up phase of the program.

Keywords · practice-based · craft knowledge · ambitious teaching · teacher professional development

Introduction

Ever since Ball and Cohen (1999) called for a practice-based theory of teacher education the focus on practice-based pedagogy in research on mathematics teacher education has increased (e.g., Charalambous & Delaney, 2020). The need for a focus on practice-based teacher education is argued by McDonald et al. (2013), who described it as "a major shift—a turn away from a predominant focus on specifying the necessary knowledge for teaching toward specifying teaching practices that entail knowledge and doing" (p. 378). Hence, there is a need to bridge the gap between knowledge for teaching and knowledge of teaching, between theory and practice, and between university courses and field work (Ball & Forzani, 2009; Grossman et al., 2009; Zeichner, 2012). According to Forzani (2014), there is little consensus as to what practice-based teacher education means, as the term has been used to describe a wide range of programs that differ from the academic model of teacher education. Practice-based teacher education has nevertheless led to understanding that teaching is a key part of the process of learning to teach (Ball & Forzani, 2009; Grossman at al., 2009; Lampert, 2010; McDonald et al., 2014). Bailey and Taylor (2015) argued that practice-based teacher education can be identified as having two directions. The first focuses on core practices of ambitious teaching and the second deals with a range of pedagogical practices relating to novice teachers engaging with representations, decompositions, and approximations of practice. In this article, the focus is on the direction that centres around core practices and ambitious teaching.

Ambitious teaching is described as teaching that attends to the learning of all students and aims to deepen all students' understanding of complex mathematical ideas and performances (Lampert et al., 2010; Lampert et al., 2013; McDonald et al., 2013). A large body of research on practice-based teacher education aims to facilitate novice teachers', teacher candidates' and prospective teachers' development and enactment of core practices of ambitious teaching. For example, Bailey and Taylor (2015) focused on novice teachers' learning of core high-leverage teaching practices through engaging in a problem-solving approach to explore learning and teaching mathematics. Having reviewed key findings from research on teaching practices and practice-based pedagogy undertaken since 2000, Charalambous and

Delaney (2020) discussed the progress in making practice a key aspect of understanding and improving teaching and teacher education. Kazemi and Wæge (2015) investigated prospective teachers' learning experiences of participating in a practice-based-methods course focusing on a set of core practices of classroom teaching. Less research, however, has been dedicated to investigating in-service mathematics teachers' learning of ambitious mathematics teaching. Most of the research found in this area centred on coaching that supports teachers in their enactment of ambitious teaching practices, and in understanding what in-service teachers learn from participating in such professional development (PD) (e.g., Fauskanger & Bjuland, 2019; Gibbons & Cobb, 2016; Gibbons et al., 2017), or centred on organising schools to support teachers' PD (e.g., Gibbons et al., 2019; Kazemi & Resnick, 2020). In this article, the focus is on in-service mathematics teachers, referred to hereon as "teachers."

Research of teachers' PD of ambitious mathematics teaching through practice-based development programs has largely focused on the knowledge the teachers should develop and what they learned through participation, or the specific goals of practice-based development programs. Little attention has been devoted to teachers' existing knowledge and understandings and how these might influence their participation in a practice-based development program. In response to this, the research reported in this article investigated teachers' current perceptions on teaching and students' learning before they entered a practice-based development program in mathematics and explored how these perceptions might influence their participation. Researchers have identified several key characteristics that are of major importance in the work of teachers' PD (Borko, 2004; Desimone, 2009; Timperley et al., 2007). One of these is related to the extent to which the teachers' learning is consistent with their knowledge (Desimone, 2009), which implies that what the teachers experience as taught in PD is in accordance with their existing knowledge. Timperley et al. (2007) argued that the extent to which conceptual understandings and practical resources offered through the learning experience make sense to the recipients in terms of their existing understandings and practice contexts strongly influences the degree to which new information is used. According to Elmore (2002), teachers' PD must be of high quality and relevant to their needs if it is to be effective and successful. Teachers' PD is an ongoing process in which their continuous growth depends on their effort (Pokhrel & Behera, 2016). Research indicated that more attention should be devoted to the start-up phase of development work so the teachers are supported in developing an understanding of the goal and why they should act on it (Postholm, 2008; 2021). Thus, it may be helpful to examine teachers' perceptions on what they are to develop and what impact these perceptions might have on their own learning process when participating in a practice-based development program in mathematics.

One way of investigating teachers' existing knowledge and understandings about teaching and students' learning in mathematics is through identifying their craft knowledge. Craft knowledge is described as the professional qualities, formal knowledge and set of competencies developed through practice and experience (Ruthven & Goodchild, 2008). The purpose of this article, however, is not to determine teachers' craft knowledge, rather it is to attempt to understand teachers' current perceptions on teaching and students' learning before entering a practice-based development program in mathematics, here defined as a part of their craft knowledge. This stance is based on research on teachers' PD that claimed teachers' knowledge has an impact on their engagement in PD programs (e.g., Desimone, 2009; Timperley et al., 2007). In this study, three Norwegian lower secondary teachers' current perceptions on teaching and students' learning before they enter a practice-based development program in mathematics are investigated. The research aims to answer the research question: What perceptions do three lower secondary mathematics teachers have about classroom practice and students' learning?

Background of the Study

The aim of the study reported in this article was to investigate teachers' perceptions of teaching and students' learning in mathematics before they begin a practice-based development program: *Mastering Ambitious Mathematics Teaching* (MAM project) for in-service mathematics teachers in Norway (e.g., Fauskanger & Bjuland, 2019; Wæge & Fauskanger, 2021). The MAM project was developed and

contextualised to the Norwegian situation from the *Learning Teaching in, from, and for Practice* project (e.g., Ghousseini, 2017; Kazemi et al., 2016; Lampert et al., 2013), which aimed to promote opportunities for novice teachers to learn to enact ambitious teaching in practice (e.g., Kazemi & Wæge, 2015; Lampert et al., 2013). The MAM project adapted the pedagogy of ambitious teaching to mathematics in the Norwegian context. The work has led to the development of a model and related resources for school-based PD for teachers in Norway (Fauskanger & Bjuland, 2019), for which the aim was to support teachers in learning to enact the complex and demanding endeavour of ambitious teaching (e.g., Lampert et al., 2010; McDonald et al., 2013). The study reported in this article focused on teachers' perceptions of their classroom practice and students' learning before participating in a PD program based on promoting the use of ambitious mathematics teaching practices.

Theoretical Framework and Related Research

The study was grounded in social-constructivist theory, meaning that individuals and their social environment are dialectically related to each other (Postholm, 2010; Prawat, 1996). In school this means that the teaching context is decisive for the pupils' learning. A key element in the view of ambitious teaching is that the emerging ideas in the classroom are built on and extended directly from student thinking and reasoning. To both elicit and respond to students' thinking and reasoning, the teacher needs to create a discussion-based classroom community (Kazemi et al., 2009). The work of ambitious teaching is also about orienting the students to each other's ideas and the mathematical goal, which means that the teacher must attend to the way students make sense of mathematics and relate to one another, both socially and mathematically (Ghousseini et al., 2015; Gibbons et al., 2017). The view that ambitious teaching aims to enhance the learning outcome for all students requires the creation of an inclusive learning environment that takes the students' experiences into account and supports meaningful participation.

Ambitious mathematics teaching is defined by Lampert et al. (2010) as the work of teaching that entails the intellectually and socially ambitious goals of mathematical proficiency (Kilpatrick et al., 2001). The work attends to improve the learning of all students and aims to deepen their understanding of complex mathematical ideas and performances (e.g., Forzani, 2014; Lampert et al., 2010). Ambitious teaching is built upon a set of principles relating to student and teacher learning that is pivotal in the demanding endeavour of ambitious teaching (Kazemi, 2017). These principles guide teachers in the use of classroom practices and mathematical knowledge and aim to maximise students' ability to learn important mathematics with meaning (Lampert et al., 2013). The principles involve treating all students as sense-makers, knowing the students as individuals and learners, learning with and from students and designing instruction with clear instructional goals (Ghousseini et al., 2015; Gibbons et al., 2017). Ambitious teaching involves an approach to teaching that, together with similar approaches such as realistic mathematics education (Van den Heuvel–Panhuizen, 2003; Freudenthal, 1991), problem-based learning (Lampert, 2001), inquiry-based pedagogy (Artigue & Blomhøj, 2013) and thinking classrooms (Liljedahl, 2016), is considered *reform-based* (Boaler, 2002), where the learning process is student centred. In contrast to this reform-based teaching approach, a *traditional* teaching approach is teacher-centred.

Teachers need to make a large number of choices during their day-to-day work. Their decisions are based on their set of competencies that has been developed throughout their careers within the practice of teaching, which can be referred to as *craft knowledge* (Ruthven & Goodchild, 2008). According to Cooper and McIntyre (1996), "craft knowledge describes the knowledge that arises from and, in turn, informs what teachers do" (p. 76). They further maintained that teachers develop professional craft knowledge through their involvement in processes of reflection and practical problem solving. Cooper and McIntyre (1996) also stated that craft knowledge is not the knowledge the teachers draw on when explaining their thinking that underlies their teaching practice, and that it thus must be distinguished from the knowledge that is not linked directly to practice. Craft knowledge in this sense is more directly linked to practice than other forms of knowledge and is of a practical nature. Teachers' current classroom practices can in this way be considered as elements of craft knowledge which are to be

developed towards ambitious teaching through enacting core learning and teaching practices. Teachers' perceptions on teaching and students' learning in mathematics will therefore be informed by their craft knowledge and will come to light through their classroom practice. In this article, Cooper and McIntyre's (1996) description of the concept is adopted.

Inferring teachers' perceptions and understandings of teaching and students' thinking is far from straight forward, Ruthven and Goodchild (2015) argued that craft knowledge is action-oriented and not generally made explicit by teachers. It may be that teachers find the ideas difficult to articulate or may be unaware of using craft knowledge. This research takes the approach that a teacher's perceptions make sense to the individual who has the perception. The focus is therefore on what the teachers perceive, rather than what they do not perceive. Such an approach also means that teachers may not consider their perceptions to be contradictory, even if an external observer might see them as being so.

The Study

The study, which took place in a lower secondary school in Norway, aimed to investigate mathematics teachers' perceptions related to their classroom practices and students' learning before the start of a PD program. Hence, the research reported in this article was part of the MAM project outlined above. By acknowledging teachers' perceptions as parts of their craft knowledge, it was necessary to conduct a qualitative study to reach an in-depth understanding of what these perceptions are all about. The data were collected over a period of three weeks, which provided enough time to become acquainted with the school and get a grasp of its daily life.

Methodology

The research reported in this article was a qualitative interview study (Brinkmann & Kvale, 2015) of three lower secondary teachers at the same school. Qualitative research is a situated activity that localises the researcher in the real world, and qualitative researchers thus focus their research on natural settings and attempt to understand and interpret phenomena based on opinions ascribed to them by individuals in these settings (Denzin & Lincoln, 2011). The aim of qualitative studies is to bring forward the emic perspective, which centres around the participants' points of view and emphasises their specific interpretations within a context (Wolcott, 2008).

The epistemological stance in qualitative studies is that knowledge and understanding are constructed in the encounter between the researcher and the participants (Lincoln & Guba, 1985), following a social-constructivist paradigm (Postholm, 2010; Prawat, 1996). Based on analyses of the data, narrative texts were constructed to present the findings (Polkinghorne, 1989; Riessman, 2008).

Participants and Data Collection

The research participants were three teachers working at the same lower secondary school in Norway. They were selected through purposeful sampling (Creswell, 2013). Four lower secondary schools planned to participate in the MAM project, all of whom were asked to contribute to this study. As the four schools were similar in terms of the number of students and teachers, and because no other factors that potentially could affect the aim of the study were discovered, the first school that volunteered to participate was selected. It was important that the teachers worked at the same school as their perceptions were to form the basis of the discussion on how these perceptions might influence their participation as individuals and as a school in the MAM project. The teachers at the school were asked if they were interested in contributing to the study, and many volunteered. The aim was to select teachers at the school that could satisfy the purpose of the study (Postholm, 2010), and who were willing to share their knowledge and experiences. As their teaching experience and membership in a working team might be possible factors influencing their perceptions, three teachers were chosen who had different teaching experiences, were working with students at different year levels, and appeared to be the most interested. The first participant, with 10 years of teaching experience and formal teacher

education of 30 credits according to the European Credit Transfer and Accumulation System (ECTS) (one year of full-time study is 60 ECTS), was given the pseudonym, Sofie. The second participant, with five years of teaching experience and formal teacher education of 180 ECTS, was given the pseudonym, Harald. The third participant, with 13 years of teaching experience and a formal teacher education of 60 ECTS, was given the pseudonym, Stig. Of these three, Sofie expressed the most interest in contributing to the study and thus became the main participant.

The data in this study were collected from three individual semi-structured interviews (Brinkmann & Kvale, 2015) with the three participants, and from classroom observations of one interviewee. The individual semi-structured interviews were conducted as a conversation because Brinkmann and Kvale (2015) claimed that "Knowledge is constructed in the interaction between the interviewer and the interviewee" (p. 4), but with a clear focus on six prepared questions. This type of interview conversation provided the teachers with the opportunity to refer to interesting aspects or themes the researcher did not think of before the interview. The interviews were audio-recorded, and the same interview guide (see Table 1) was used in all three individual interviews. A follow-up interview with Sofie was conducted to clarify concepts and ideas identified. The initial interviews took approximately 40 minutes while the second interview with Sofie lasted 35 minutes.

The aim of the observations was to provide information that was used to construct the interview guide and to provide contextual information that was used in the dialogue during the interviews. Observations of Sofie's classroom were focused on her classroom teaching of one group of 19 students in Year 9 (14–15 years of age). In all, nine classroom observations were conducted in the natural classroom setting (Angrosino & Pérez, 2000), which means lessons or teaching episodes that were part of the regular classroom routine in line with the learning program ascribed by the school were observed.

Table 1
Structure of the individual semi-structured interview guide

	Questions. All questions start with: In your opinion…	Purpose
1	What kind of knowledge is needed by mathematics teachers to teach mathematics?	Knowledge about teaching
2	What type of requirements are there for teaching mathematics?	Framework (e.g., given by the school management or education authority)
3	Is there a shared perception among mathematics teachers at your school as to what type of knowledge mathematics teachers need for teaching mathematics in school?	Knowledge about teaching
4	Are the mathematics teachers at your school aware of each other's teaching practice? If yes, how?	Teaching practices
5	Can you briefly describe a teaching lesson where your classroom practice is visible?	Teaching practices
6	What are your expectations for the MAM project?	Prepared for the project

The immediate impressions from the observations were written down in a logbook after each lesson. The observations were also video-recorded and studied several times, together with the written impressions in the logbook, in search of teaching actions that could serve as starting points for discussions during the interviews. The teacher informed all the students about the observation activity. The researcher did not interfere in the teaching and did not take part in the discussions during the lessons. The researcher assumed the role of *complete observer* (Gold, 1958), meaning that the researcher was present but did not take any active part in the lessons.

Data Analysis and the Construction of Narratives

For this study, the constant comparative analysis method (Corbin & Strauss, 2008; Strauss & Corbin, 1998) was used to analyse the data. Corbin and Strauss (2008) argued that this method can be used to analyse data in all qualitative studies. The transcription of the interviews started shortly after they were conducted, and they were transcribed in their entirety. To acquire an overview of the data collected in the interviews, the transcriptions were organised into a matrix with columns where initial analysis, related research, and labels and questions were entered. An example of one sequence is shown in Table 2. The transcriptions were further divided into smaller sections, one to three statements, in an attempt to understand the essence of what was expressed in the raw data (Corbin & Strauss, 2008). Related research and previous experiences from working with ambitious mathematics teaching were used as reflective tools to understand what was said from the participants' point of view. This way of interacting between inductive and deductive approaches, from theory to data and vice versa, can be considered an *abductive* approach (Alvesson & Sköldberg, 2009).

Excerpts of labelled data were labelled, and colour coded. These were compared and given codes, each covering whole sentences and sometimes even whole paragraphs. In this way, the codes covered larger units and as such made working with the data easier (Postholm, 2019). Then, the codes that could be related to teaching and students' learning in mathematics were grouped into categories. For instance, codes involving a particular teaching practice, such as "talking with students", were grouped into one category, and codes involving a particular characteristic of teaching and students' learning, such as "students' thinking", were grouped into another. Throughout this process, it appeared the data were either about mathematical discussions or the teachers' interest in students and their thinking. Thus, "Mathematical discussion", and "Engage with students and their thinking" became the main categories for this study. Furthermore, the categories were structured and specified by asking questions, such as when, why and under which circumstances did the categories materialise, and how and what did this lead to? This process identified eleven sub-categories, which are presented together with their related main categories in Table 3.

Table 2
An example of how the transcriptions were organised

Transcription	Initial analysis	Related research	Labels and questions
Researcher: Can you say more about how you collect information from your students? Sofie: Well, that's not easy…In my opinion, I collect the most valuable information when I wander around in the classroom while the students are working on tasks. Then I can talk with the students and observe how they solve the task. Because not everyone raises their hand and asks for help. And also, through small tests or small "checkouts" as we call them, where I give them just a few minutes to solve one or two tasks on a piece of paper at the end of a lesson, and then I collect them and see how it went. Then I get a very good overview, I think, as long as they don't peek at their neighbour's work. Eh, but we try not to have so many big tests, but I think small tasks like that work very well…and that's it. And walking around, trying to talk with everyone during a lesson when they're working to collect information. And of course, there are a lot, or not a lot, but some students also contribute verbally. And then you get information in that way from those who ask about things and say "Oh, I don't understand that" or "I don't get it from here." But often they just say, "I don't understand" or "I don't get it", so they don't really know what they don't understand.	Sofie says she talks with the students, but she does not elaborate on how this conversation takes place. Sofie says some students are asking questions if they do not understand something. This can be an invitation into a mathematical conversation.	Mathematical conversation. See principles of ambitious teaching (e.g., Kazemi et al., 2009; Lampert et al., 2013)	Ask Sofie if she can elaborate on what "talking with students" implies Ask Sofie what she does when a student says: "I don't understand" Observing and talking with students Checkouts Students inviting into a mathematical discussion.

Table 3
Main categories and sub-categories

Main categories	Engage with students and their thinking	Mathematical discussion
Sub-categories	Students' expectations Different types of students Knowing the students Interest in students' thinking Students' asking for help	Share students' thinking Orchestrating students' thinking Using talk moves Classroom discussion Ability to get involved in students' thinking Asking questions

In the process of developing a core category a question such as, "What is this all about?" (Corbin & Strauss, 2008) was asked, and the data were analysed in light of the literature relating to teaching and students' learning in mathematics. The teachers' perceptions were approached with the notion that they were related to each other and made sense for the person who had the perceptions. This process revealed that the teachers' utterances within the categories did not always cohere, as their purpose for acting on similar perceptions within their classroom practices and students' learning in mathematics varied. In the search for a unifying concept (Corbin & Strauss, 2008), it was discovered that differences in the teachers' purposes for acting on similar perceptions were informed by an incoherence in their approach to teaching and student learning. "Approaches to teaching and student learning" thus became the core category and will be explored further in the analysis and discussion together with the main categories. A narrative format has been used to present the data. As the participants provided rich and complex statements throughout the interviews, according to Riessman (2008), the findings will be more accessible to the reader if they are presented as narratives. A narrative text (Polkinghorne, 1989; Riessman, 2008) for each teacher based on their responses and the developed categories was constructed. The categories form the structure of the narratives.

Ethical Considerations and Quality of the Study

The study, approved by the Norwegian Centre for Research Data (NSD), complies with the ethical principles laid down by the Norwegian Ethical Research Committee (NESH, 2021). Informed consent was obtained from all the interviewees in accordance with the NSD guidelines, and participants were given pseudonyms for reporting purposes to ensure their anonymity. The teachers were also informed that all information in the study was completely confidential and that they could withdraw at any time without needing to provide further explanation (NESH, 2021). No participants withdrew from the study.

The quality of this study was improved by using member-checking (Lincoln & Guba, 1985). The three teachers received their respective narratives by email and were asked to approve or comment if they thought they were misrepresented in the narrative. All three approved the narratives without any data being either added or excluded. The findings from this study may have importance beyond the immediate data collection context. Even though the presented descriptions and the analysis were connected to three teachers from one specific school, the findings can contribute knowledge to understanding similar situations and contexts. This means that the reader can use the presented descriptions as a thinking tool if they perceive processes as parallel experiences and adapt them to their own situation, thus conducting naturalistic generalisations (Stake & Trumbull, 1982).

Findings

This section presents the three teacher narratives based on the responses from the interviews.

Sofie

Sofie had worked at the same school for ten years and described herself as well informed about her colleagues' teaching practice in mathematics. She did not consider the school's teaching practice in mathematics to be traditional. "The students are always allowed to collaborate with a partner or in a group ... but the teacher standing in front of the board and explaining, and things like that, is something we do. That's perhaps because we feel it's necessary." Sofie added that the students' work involved solving many tasks, which she considered to be a prerequisite for their mathematical learning.

Sofie believed that teachers should be good at explaining mathematical concepts and processes. "I want the students to understand what we're doing. And that idea is something that permeates everything in a way. If they are to understand the material, they need to have it explained in a way that makes them able to understand." She further expressed that teachers' explanations are mainly aimed at helping students understand how to solve tasks and why a strategy works. "If I'm going to explain how to solve a task by using an example on the board, then I have to be able to explain it in a way that helps them understand why we do what we do." Sofie further explained that she had tried to start the lessons

by giving the students a mathematical problem with the aim of motivating them, creating curiosity and encouraging them to develop their own strategies and share their thinking. However, she claimed that she had experienced this way of teaching as being too time-consuming. She said that it was not possible to do this in every lesson because it took time away from getting through the lesson plan.

Sofie maintained that the teacher must be interested in the students and curious about what they do not understand and why to determine which explanation to use in different situations. She said the explanations must be adapted to the students' level, and the teacher needs the ability to understand which explanation to choose and when. For example, she stated that "... the ability to explain is very much related to the ability to understand why students do not understand. You can't just learn a lot of different ways to explain." Sofie stated that she finds out what students do not understand when she walks around the classroom talking to them and observing how they solve tasks, and by asking her students to explain to her what they do not understand.

Sofie described that she must show and explain things to the students. For instance, she explained the standard algorithm for solving equations with different types of numbers, parentheses, and so on. Sofie further explained that the mathematical discussions that took place between her and her students where when she asks what to do next and the students suggest different approaches, such as moving all the unknowns to one side of the equal sign or solving fractions. Sofie maintained that group work provides opportunities for the students to ask each other for help if they need it while working on a set of tasks individually.

Stig

Contrary to Sofie, Stig maintained that the school's teaching practice in mathematics is traditional, where the teachers often stand in front of the board providing examples while the students solve the given tasks. "If you look into a mathematics classroom, then you either see a teacher in front of the board explaining and showing examples, or students sitting and solving tasks." Later, Stig added that he acknowledged this way of teaching can be beneficial, "But both the instruction and the tasks can be really good, so I don't claim it's wrong to do it this way."

Stig argued that the teacher must understand how students think and how they experience mathematics when it is taught, and that teachers not always considering the students' point of view might be the reason why the students fail to understand. "What do the students experience when we show them math? It's not always what we think." He argued that finding out how students work and think mathematically are important if teachers are to help the students at their level of understanding, and he believed that doing this makes him better equipped to explain the mathematical content to his students.

Stig stated that he would have liked to know more about misconceptions and why students do not understand mathematics. He also wanted to know more about different types of students, and which types have difficulty understanding various things in mathematics. Stig claimed that teachers need to know where and in what situations mathematics is needed so they can explain to the students why they need to learn mathematics.

Stig also claimed that facilitating conversations about mathematics with students and between students is something he does more and more often. The scope of the conversations may vary, he added, and this variation mostly depends on the teacher's preparation. "The better prepared you are able to be, the more beneficial questions you might have prepared," he said and added, "If the plan for the lesson is really good, more or less everything is possible." Stig maintained that the teacher also must consider along the way if the students are interested and eager, and then adapt accordingly, and identify students who do not like to talk as much so they can be put together in pairs or groups.

Harald

Harald suggested that "We math teachers need a 'toolbox' with suggestions about how to teach or change teaching to something more modern." He elaborated that the point was to determine how to get students to think differently than they do when the teacher is in front of the board and they are

solving tasks, as in traditional teaching. Harald argued that traditional teaching is out of date, but that some teachers, including himself, still persist with the practice to some extent, especially in relation to giving the students a lot of similar tasks. "I think math teachers would benefit from being challenged to teach the subject in other ways." Harald believed that mathematics teaching should be adjusted to all students, both the strongest and the weakest, which is important for student motivation.

Harald said he usually begins lessons with discussions, especially when introducing a new topic. Following such discussions, he said he often organises the students into predetermined groups based on how they normally work; he tried to allow the students to work in a way that suits them best. "I divide them into groups based on their personality and not based on what mathematical level they might be on. Based on what needs they have in a way. Some are quiet and some like to discuss, so I put them together." Harald explained that he often ends a lesson by letting students discuss in pairs what they think other students may perceive as difficult about the task, and how other students might have solved it. These conversations typically differ from one class to the next, he said, and they depend on the students' mathematical level.

> One class I teach really likes to talk out loud. They're on a more equal level mathematically and are better at engaging in a plenary conversation. The students in the other class are on a very different level so if one speaks out loud the others may lose interest … so it's different from class to class.

Analysis and Discussion

In this section, the teachers' perceptions relating to teaching and students' learning in mathematics, and how their perceptions make sense to them are analysed and discussed. The main categories, "engaged with students and their thinking" and "mathematical discussion", and the core category, "approach to teaching and student learning", are used to structure the discussion. The perceptions are discussed across the interviewees.

Engaging with Students and Their Thinking

The perception of engaging with students' thinking is particularly emphasised by Sofie and Stig, who both repeatedly pointed this out as an important practice in mathematics teaching. Their descriptions of engaging with students' thinking share many similarities with the principles of ambitious mathematics teaching, which focus on treating all students as sense-makers, knowing the students as individuals and learners, and learning with and from students (Ghousseini et al., 2015; Gibbons et al., 2017). For instance, Sofie and Stig argued that the teacher must show interest in their students, be curious about what they do and do not understand, and how they experience mathematics when it is taught. Both Sofie and Harald pointed out the importance of enabling students to develop their own strategies. Stig added that students' thinking must not be taken for granted, as it might not always be what one expects. With these perceptions, it can be assumed that both Sofie and Stig were acting on these principles of ambitious mathematics teaching in their classroom practice, or at least similar principles that focus on a student-centred teaching approach (Boaler, 2002).

Although Sofie's and Stig's perception of engaging with students' thinking seems in many ways to resemble some of the principles of ambitious teaching, their understanding of this concept is rooted in a different underlying purpose when it comes to acting on this perception. The aim of the principles of ambitious mathematics teaching is to deepen all students' understanding of complex mathematical ideas and performances by eliciting and responding to the students' thinking and reasoning as they emerge in discussion-based classroom communities (e.g., Forzani, 2014; Lampert et al., 2010). For Sofie and Stig, however, their purpose behind engaging with students' thinking was to obtain information that they could use when explaining mathematics to them. They claimed that engaging with students' thinking provides them with important information about how students learn, which in turn enables them to be on the students' wavelength and to determine their learning trajectory. Bearing this in mind, the teachers would then decide which explanation to use in each particular situation. Sofie and Stig

perceived that engaging with students' thinking can therefore be considered to be a method that better equips them to explain mathematics to students.

Mathematical Discussion

Although the three teachers emphasised different aspects of the mathematical discussion and the facilitation of it, they all agreed that it is particularly important in their teaching and the students' learning of mathematics. Again, their perceptions appear to be in accordance with the principles of ambitious teaching (Kazemi et al., 2009), and therefore associated with a student-centred teaching approach (Boaler, 2002). Sofie emphasised the importance of facilitating students to share their thinking by asking them questions. Stig pointed to the conditions for having classroom discussions and mathematics discussions between students, which he believed rely on the teacher's prior preparation of good questions. This way of preparing a lesson is an important aspect that enables teachers to enact key practices that ensure the principles of ambitious teaching (Lampert et al., 2013). Harald claimed that organising the students into groups enabled them to discuss each other's strategies and reflect on how other students might think. Orienting the students to each other's ideas is one of the key features in productive mathematics discussions (Ghousseini et al., 2015; Gibbons et al., 2017). However, Harald did not specify how the students' ideas were shared and oriented to other students' ideas. This makes it difficult to determine the extent of the similarities in his teaching practice with the principles of ambitious teaching. Nonetheless, it seems that some of the ideas were shared.

In the same way as for the perception "engaging with students' thinking", Sofie's and Stig's understanding of mathematics discussions also appeared to have some differences compared to ambitious teaching (e.g., Kazemi, 2017). Although Sofie and Stig maintained that they use mathematical discussions as a means for gaining access to the students' thinking, which partly aligns with the description of ambitious teaching (e.g., Kazemi, 2017), they also stated that the intention behind including mathematical discussions in teaching is to be better equipped to explain mathematics to students. This is an understanding that differs from the idea of creating a discussion-based classroom community where the discussions are based on the students' emerging ideas (Kazemi et al., 2009).

Approach to Teaching and Student Learning

The three teachers seemed to be in a developmental process in their approach to teaching and student learning. Although they did not agree on the extent to which traditional teaching characterises the school's teaching practice, the findings reveal traces of both a traditional and a reform-based teaching approach in their perceptions (Boaler, 2002). These traces are most evident in Sofie's and Stig's descriptions. On the one hand, they claimed that a traditional teaching approach is outdated. On the other hand, they believed that teacher explanation and solving many tasks can be a very good and necessary approach to helping students learn. Both Stig and Sofie seemed to be focused on teachers' explanations in their approach to teaching and maintained that this was very important for facilitating students' learning in mathematics. Such a view of teaching and students' learning in mathematics fits with how Boaler (2002) described traditional teacher-centred mathematics teaching, and also appears to imply a behaviouristic view on learning.

Teacher explanation was important in Sofie's perception on teaching and students' learning, and Stig also appeared to agree with such a position. As mentioned above, the perception on "engaging with students' thinking" sees it is a method that better equips teachers to explain mathematics to students, which also applied to the perception of "mathematical discussion". These two perceptions can therefore be considered as practices they act on to support their view on students' learning. Teacher explanation thus appeared to be a principle that guided Sofie and Stig in their classroom practice in the same way as the principles in ambitious teaching guide teachers in the use of classroom practices and mathematical knowledge (e.g., Forzani, 2014; Lampert et al., 2013; McDonald et al., 2013). Another principle that appears to guide Stig and Harald in their classroom practice is the perception that there are different types of students who can be categorised by the way they learn. This perception affected both the composition of student groups and the number of mathematical discussions. This perception

also seemed to be an important reason for engaging with students' thinking, as this could inform the teacher both about how students learn and their learning trajectories. The teachers' intention behind engaging with students' thinking and facilitating mathematical conversations may therefore appear to be of a different nature than described for ambitious mathematics teaching (e.g., Kazemi & Wæge, 2015; Lampert et al., 2013).

Conclusion and Potential Implications

This study aimed to explore teachers' perceptions of their classroom practice and students' learning before participating in a PD program based on promoting the use of ambitious mathematics teaching practices. Bearing the analysis and discussion of the findings in mind, this last section will conclude and point out possible implications that the teachers' perceptions can have when participating in a practice-based PD program that promotes the use of ambitious mathematics teaching.

A Need for Common Understanding of Key Concepts

The findings reported in this paper show that the teachers' perceptions on key concepts related to classroom practice and students' learning in mathematics may differ, even when they initially appear to correspond. When analysing the data, the researcher found that the teachers used concepts similar to ambitious teaching when describing their perceptions, which they also believed were important features of good classroom practice. These perceptions were grounded in their understanding of how to facilitate students' learning and appeared to work as principles that guided them in their work as mathematics teachers. Sofie's situation is a good example because her first description relating to engaging with students' thinking and facilitating for mathematical discussions aligned with the principles in ambitious teaching (Gibbons et al., 2017), while her underlying purpose for acting on practices supporting this principle was not. Therefore, the difference in the teachers' perceptions on these key concepts is not found in their description of the perceptions but in the underlying purpose for acting on these perceptions. This underlying purpose is based on the ideas embedded in the teachers' craft knowledge, which guided their classroom practice (Ruthven & Goodchild, 2008). Both Sofie and Stig maintained that mathematics must be explained to the students in the process of helping them to learn. The underlying purpose can therefore be considered as their view on how students learn mathematics and can in this sense be understood as an important part of their craft knowledge (e.g., Cooper & McIntyre, 1996; Ruthven & Goodchild, 2008; 2015).

This study has shown that the teachers used some of the same concepts as in ambitious mathematics teaching (Gibbons et al., 2017) in their description of their perceptions of teaching and students' learning. However, their understandings did not always align, which was evident when the underlying purposes behind these perceptions were revealed. Working with teachers who have perceptions on key concepts that differ from those described in the development program might lead to possible implications for the providers or others involved. For example, teacher educators or other actors contributing to the teachers' PD work might be left with the impression that teachers are talking about key principles for ambitious mathematics teaching (Gibbons et al., 2017) when they describe how they perceive teaching and students' learning, rather than what really reflects their intentions behind their perceptions and their actual classroom practice. Thus, the program providers might benefit from clarifying the conceptual understanding of important key concepts used in the development program at an early stage to ensure that the PD leaders and teachers involved are using the same language. As such, clarifications might also contribute to aligning the information and practical resources provided in the program with the teachers' existing understandings and practice, which strongly influences the extent to which they are willing to use them (Timperley et al., 2007).

A Need to Understand Teachers' Perceptions

Differences in how teachers understand concepts may also concern teacher educators when conducting a teacher PD program. Timperley et al. (2007) argued that the conceptual understandings and practical resources offered through the learning experience within teacher PD must make sense to the recipients in terms of their existing understandings and practice contexts. The findings in this study show that it is important to take the teachers' view on teaching and learning into account as this view appeared to affect their perception of the aspects to be developed in the PD programme. In this way, the teachers will be better able to experience the content as relevant to their needs (Elmore, 2002). Moreover, the findings in this study show that the teachers have a different set of perceptions than what is seen on the surface. Bearing in mind Cooper and McIntyre's (1996) finding that teachers develop their professional craft knowledge through their involvement in processes of reflection and practical problem solving, PD programs therefore need to challenge the teachers' underlying perceptions and encourage them to reflect on their view on students' learning so they can develop their classroom practices and further expand their vision of what is possible. It seems essential that a practice-based development program, like the MAM project, must therefore aim to develop teachers' existing perceptions on what they are to develop by first mapping the teachers' craft knowledge in relation to the topic. It is this knowledge that the teachers relate to when assessing whether the teacher PD makes sense in relation to their existing understandings and knowledge (e.g., Desimone, 2009; Timperley et al., 2007). In this particular case, it may be necessary to identify the teachers' understanding of key concepts, either before starting a practice-based development program or in the start-up phase. Identifying and challenging the teachers' understanding of key concepts could support their development of the purpose of the program and why they should act upon it (Postholm, 2008, 2021). Additionally, the differences in understanding may be addressed, challenged, or promoted according to the goals and aims of the PD program, thus avoiding unnecessary misunderstanding that might undermine the teachers' development.

Mapping this terrain of teachers' perceptions is nevertheless easier said than done. A person's perception may not have the same meaning as what an observer might think it means, as the teachers' perceptions do not necessarily reveal the underlying purpose for acting on them. One unfortunate drawback could be to assume that there is a one-to-one correspondence between what is stated and the concept that is used by the teacher, and how those statements and concepts are understood by another person. However, in this study, the underlying purpose determined which practices made sense to the teachers and, therefore, also which practices they were willing to use. Hence, teachers must be challenged cognitively to reflect on these underlying purposes (Avalos, 2011), for example through questions such as, "What kind of classroom practices do these perceptions on teaching lead to? What impact do they have on students' learning?" and more importantly, "How do the teachers' perceptions align with (or not) the coterminous nature of teacher development programs? Examining teachers' craft knowledge to ensure consistency between teachers' existing knowledge and the content of a practice-based development program might therefore not be enough if "craft knowledge describes the knowledge that arises from and, in turn, informs what teachers do" (Cooper & McIntyre, 1996, p. 76).

The findings in this study show that teachers' actual perceptions on teaching and students' learning in mathematics are the ones related to their underlying purposes for acting on them. These perceptions are based on the knowledge they draw on when explaining their thinking that underlies these purposes, and they first become visible through these explanations. Moreover, the findings show that this knowledge is not directly linked to their practice, but largely informs the choices they make in their practice. The teachers' underlying purposes for what they do is therefore not accessible by only observing their practice or discussing it superficially. Therefore, if the teachers' existing understandings and knowledge are to be taken into account, there must be an investigation into their perceptions of the underlying purpose of their teaching and classroom actions. As has been seen here, the underlying purposes for what they do might be hidden and remain hidden, not only to the project management but also to the teachers themselves. The purpose is what needs to be understood and further challenged if their classroom practices are to be developed. Such an investigation into the teachers' perceptions

can form an important starting point for their PD and further contribute to understanding how a PD program, in which they participate, can function as an aid to broaden each teacher's understanding of ambitious teaching and how it promotes student learning.

Acknowledgement

The study presented in this article is part of the *Mastering Ambitious Mathematics Teaching* (MAM) project organised by the Norwegian Centre for Mathematics Education. I would like to thank the organisation, the people who ran the project, and the teachers who participated in the research.

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Article 2

frontiers in Education

ORIGINAL RESEARCH
published: 10 September 2021
doi: 10.3389/feduc.2021.697231

Principals' Leadership of Mathematics Teachers' Professional Development

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Leadership has long been recognized for having a significant impact on teacher learning. While research on development programs for mathematics teachers has suggested a change in focus from teachers being passive participants to becoming active learners in practice-based development programs, little is said about how this change in focus affects the principals' role as leaders of teachers' professional development (PD). In response to this, the presented study investigates how a Norwegian school management team facilitates and supports its mathematics teacher's PD in their first year of participation in a particular practice-based development program. Findings from the study show that supporting teachers' PD is easier said than done. The study highlights the importance of building teachers' sense of ownership and having a shared overarching goal for participating in a practice-based development program. Moreover, there must be a structure and a practice for development work at school if a plan for development in practice is to be successfully implemented and fulfill teachers' need for continuous development support. Based on the findings from this study and the use of cultural historical activity theory (CHAT) and the activity system, the article suggests that at least two prerequisites must be present for practice-based development programs to serve as mediating artifacts for teachers' PD. First, the roles involved in the development work must be defined so that the work or goal-directed actions divided between the people in the shared community act towards the same object. Second, the school leader needs support in his work as a leader of teachers' PD.

Keywords: school leaders' role, practice-based, mathematics teachers' professional development, teacher leadership, supporting school leader, cultural historical activity theory

OPEN ACCESS

Edited by:
Carlos Francisco De Sousa Reis, University of Coimbra, Portugal

Reviewed by:
Agus Mutohar, Walisongo State Islamic University, Indonesia
Arne Jakobsen, University of Stavanger, Norway

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Specialty section:
This article was submitted to Leadership in Education, a section of the journal Frontiers in Education

Received: *19 April 2021*
Accepted: *30 August 2021*
Published: *10 September 2021*

Citation:
Braseth EA (2021) Principals' Leadership of Mathematics Teachers' Professional Development. Front. Educ. 6:697231. doi: 10.3389/feduc.2021.697231

INTRODUCTION

In a two-year-long practice-based development program in mathematics, teacher educators and a group of teachers from a number of schools come together five times a year for a daylong, job-embedded professional learning event, driven by teacher educators. The events, called a cycle of enactment and investigation, take place in a genuine school context. Each cycle starts with a discussion based on a pre-read article or a short video vignette of a teaching sequence related to ambitious mathematics teaching. Then the teachers are divided into arranged groups where they plan to carry out an instructional activity together with their supervisor, focusing on how to enact particular practices for ambitious mathematics teaching. One or two of the teachers are responsible for carrying out the instructional activity with a group of real students in an actual classroom context. At the end of the planning sessions, they have a rehearsal where the other teachers act as "students", asking questions that real students might ask. The rehearsal gives the teachers the opportunity to try

out and discuss the teaching strategies and moves they have planned. During the rehearsal and the conduction of the instructional activity, the teachers, as well as the supervisor, can pause the instruction by initiating a teacher time-out that instantly freezes the situation and enables the group to think out loud together in the moment and determine the direction of the further instruction. The cycle ends with a group discussion where the teachers reflect on the conducted instructional activity and the planning process together with the supervisor before briefly preparing for the next upcoming event.

The practice-based development program briefly described above is called the *Mastering ambitious mathematics teaching (MAM) program,* aimed for in-service mathematics teachers in Norway (e.g., Fauskanger & Bjuland, 2019; Wæge & Fauskanger, 2020). The program has been developed and contextualized for the Norwegian situation from the *Learning Teaching in, from, and for Practice* (LTP) project (e.g., Ghouessini, 2017; Kazemi et al., 2016; Lampert et al., 2013). Research on practice-based pedagogy has become increasingly popular within mathematics teacher education and teacher learning over the past 2 decades (e.g., Charalambous & Delaney, 2020). One approach to teachers PD that has become particularly popular and also given an important direction of practice-based PD, is Lesson Study (see Huang & Shimizu, 2016 for a systematic review). Scholars have shown that Lesson Study can improve teachers' knowledge and build productive professional learning communities (e.g., Lewis et al., 2009). Research on practice-based pedagogies has led to an understanding that teaching is a key part of the process of learning to teach (e.g., Ball & Forzani, 2009; Grossman et al., 2009; Lampert, 2010).

Teachers' professional development (PD) is essential if classroom practice is to be changed (Borko, 2004), and is an ongoing process in which teachers' continuous growth depends on their own effort (Pokhrel & Behera, 2016). Research has also indicated that meaningful support from school principals is crucial in promoting teacher learning through PD (Akiba et al., 2015; King & Stevenson, 2017; Silva, Amante & Margoda, 2017). The principal can support teacher learning by creating a learning culture, shaping learning opportunities and providing resources, time, encouragement, and monitoring (Desimone, 2009). School leaders need to acknowledge their role as facilitators for teachers' learning and ensure that proper learning conditions are established to create a culture of learning at the school (Walker, 2007).

Research on practice-based development and the school leaders' role in teachers' learning and development has received much attention. However, little of this attention has been devoted to the principals' role in leading mathematics teachers' learning as they participate in practice-based development programs that are job-embedded. The study presented in this article aimed to investigate the relations between a principal's leadership and mathematics teacher's participation in a PD by examining a Norwegian lower secondary school's first year of participation in the MAM program (see description below). This article focuses on school managements' role in terms of how they support and facilitate in-service mathematics teachers' PD when they participate in a job-embedded practice-based development program such as the MAM program. While focusing on the school leaders' role, I have used Cultural Historical Activity Theory (CHAT) and the activity system (Engeström, 1987, 1999, 2001) as the theoretical framework for analyzing and discussing the findings. The activity system contributes to describing and analyzing activities within an organization, such as teachers' PD in a school, and can thus be used as a tool for discovering aspects that have development potential (Postholm M. B., 2020). The study presented in this article is driven by the research question:

How does school management support and facilitate mathematics teachers' professional development as they participate in a practice-based development program?

The analysis and discussion of the findings related to the research question will lead to further discussions of possible opportunities for change and development in the frame of CHAT and the activity system. By using the activity system as the unit of analysis, I will identify tensions and contradictions that can be a starting point for change and development. In the following, I will start by presenting related research before elaborating on CHAT and the activity system, and the context of the study. Then I will present a description of the method, and explain how the data was collected and analyzed. Finally, I will present and discuss the findings prior to making my concluding remarks.

RELATED RESEARCH

Teacher Professional Development

Several researchers claim that Teacher PD is the key to successful school reform and student learning (Desimone, 2009). It is understood as activities that improve teachers' knowledge, skills, and attitudes towards teaching practices (OECD, 2014). These activities can have various forms and have traditionally been identified as official events, such as conferences, workshops, and degree programs (Burns & Darling-Hammond 2014). However, researchers have suggested that out-of-school programs are limited in their connection between teacher learning and the actual practices in school (Villegas-Reimers, 2003; Desimone, 2009). This idea is supported by an extensive body of research arguing that teacher's PD should be connected to and contextualized within practice, and in that sense it should enable teachers to develop their knowledge and ability to use new ideas (e.g., Ball & Bass, 2003; Ball & Even, 2009; Kennedy, 2016). Teacher PD should also treat teachers as active students, be maintained over time, and open for collaborative participation (Desimone, 2009) which further facilitates teacher collaboration that is assumed by researchers to contribute to PD and instructional improvement (DuFour and Fullan, 2012).

Watson (2015) argues that teachers' PD usually begins with an understanding of teachers' needs at their own school and in their classroom, and the effects of any PD program depend heavily on teachers' motivation to learn and to change their practice (Kennedy, 2016). Furthermore, Engeström and Sannino (2010) have found that the development work must be "owned" by the practitioners and therefore based on their development needs.

This means that the teachers must take part in the development work right from the beginning and be acknowledged as the heart of the decision-making around change, which is a key principle in understanding, engaging, and developing ownership in adult learning (Knowles et al., 2005). The development effort is, in this way, made together with the teachers instead of being designed as doing things to teachers, an approach that aligns with what researchers have found in successful teacher PD (Clarke & Hollingsworth, 2002). From a contrary point of view, it is argued that PD initiated by an outside member of the community suggests that problems identified externally are beyond the capability of the teachers within a given community to solve and can further promote a de-professionalization of the teacher (Roseler & Dentzau, 2013).

Furthermore, Timperley et al. (2007) found that teachers should at least understand the purpose of the work and why they should attempt to move their practice in that direction. Time must therefore be allotted for the teachers to develop an understanding of the goal and why they should act upon it in the start-up phase of the development work (Postholm, 2008; Postholm, M. B. 2020). Moreover, the work of teachers' PD is a matter of what to develop and how to develop it, and research shows that the focus on content and the process must go hand-in-hand and be integrated in the development work (Postholm et al., 2013).

Leadership for Teachers' Learning

It is widely acknowledged that leadership can be practiced in a way that might have a significant impact on promoting and sustaining change (Fullan et al., 2005). Research on educational management and leadership concludes that school principals possess an important position that can have substantial influence on teachers' learning (Leithwood et al., 2020). For instance, findings from a study in England indicated that PD for school improvement can result in real change if the school leader understands its potential (Opfer et al., 2011). The principal can contribute to creating a learning environment by exercising a school leadership practice that helps teachers to identify their development needs and enhances the implementation of new learning (Thoonen et al., 2016). A leadership practice can involve several leaders who interact with each other and the actual learning situations (Spillane, 2005). Spillane (2005) argues that "structures, routines and tools are the means through which people act" (p. 147). Furthermore, Darling-Hammond and Richardson (2009) argue that there needs to be a plan for teacher PD, and Earley and Bubb (2004, p. 80) state that "professional development does not just happen—it has to be managed and led," and has to be supported and encouraged by the leaders (Silva et al., 2017). Research shows that teachers need continuous development support in their PD work (King & Stevenson, 2017), and that it is the school leaders' task to arrange for the teachers' learning in schools (Elmore, 2000).

Although leading teachers' PD is often considered to lie within the school leaders' role, research shows that teachers can be development leaders in their own schools. Grootenboer and Hardy (2017) claim that the leading of teachers' PD needs to be a shared enterprise as the task is often too much to handle for one person alone, a notion that is supported by Postholm (2019), who argues that the work with developmental processes should be distributed between different leaders, or between leaders and teachers. To do this, the principals must have the courage to let go of leadership and be willing to place their trust in their teachers' beliefs, values, and judgements, which is considered to be the challenge for leadership (European Commission, 2010). Building professional trust is important when establishing a productive learning environment for the teachers (Liu et al., 2016), and can furthermore allow teacher leadership to flourish (Smylie et al., 2007). However, certain conditions must be taken into account if teacher leadership is to be fruitful. For instance, Birky et al. (2006) argue that school administrators must encourage and motivate their teachers to be effective leaders through their words and actions. The principal can therefore influence teacher leaders' motivation to exercise their leadership role effectively through his or her style and actions.

CULTURAL HISTORICAL ACTIVITY THEORY

CHAT was developed by Leontèv (1978, 1981) on the basis of Vygotsky's work, which implies that learning and development are rooted in socio-cultural theory (Wertsch, 1981). Leont´ev (1981) says that "the object is the true motive" (p. 59) for people's actions. Teachers, school leaders, or other educators should therefore share a collective motive to act on the object, or at least know about the object they aim to develop their practice towards. The object can in this way become "invested with meaning and motivating power" (Sannino et al., 2016, p. 602), and the teachers' motivation should therefore be built into the object because it is their practice and needs that serve as the starting point. Engeström (1987) expanded on Vygotsky's individual definition of the zone of proximal development to include a collectivist and social perspective, seeing how the activity can develop a collective, such as a team of teachers and a school as a whole, into a new form of social activity. He defines this as follows: "It is the distance between the present everyday actions of the individuals and the historically new form of the societal activity that can be collectively generated" (p. 174).

The Activity System

As explained above, CHAT is the result of Leont´ev's expansion on Vygotsky's work. The activity system (shown in **Figure 1** below) is a graphic development of the activity theory (Engeström, 1987, 1999, 2001; Engestrøm and Miettinen, 1999) and is therefore formed on the basis of CHAT. This system, considered to be a unit of analysis of human activity, consists of the seven factors: subject, mediating artifacts, object, outcome, rules, community, and division of labour (Engeström, 1987). These factors are related and thus have a mutual impact on each other, thus forming a dynamic system where a change in one factor will influence another in the system and also the system as a whole (see **Figure 1** below).

The acting subject refers to a person or a group of people from whose viewpoint the analysis of the activity system is conducted

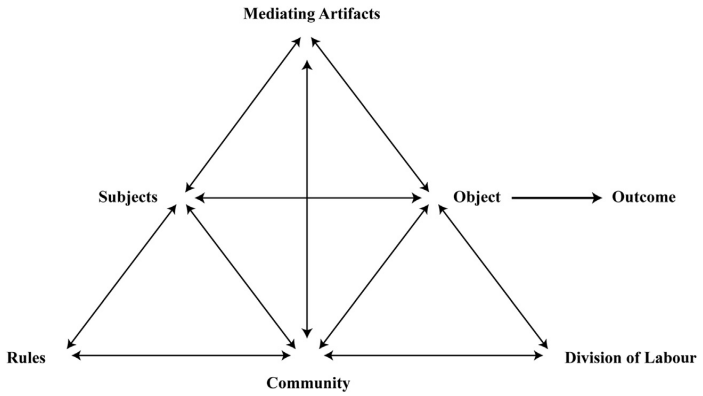

FIGURE 1 | The complete activity system (Engeström, 1987,1999,2001).

(Engeström, 1999). A team of teachers can thus be an active subject in the system that utilizes cultural mediating artifacts to move practice towards the object, here defined as an overall goal. Mediating artifacts can comprise such physical artifacts as smartboards, tablets, books, or pencils, but also language and even a development program which a group of teachers participates in, as it can be defined as an aid or a thinking tool (Postholm M. B., 2020) that aims to support the development of their teaching. How the subject has moved towards the object and the desired result is shown as the outcome in the system. In the context of teachers' PD, the outcome might also include students' learning if the attention is on the teacher's classroom practice. The three remaining factors represent the context in which the activity is carried out and may determine the premises and possible restrictions for the subject's goal-directed actions towards the object (Engeström, 1987). The context is therefore not just a surrounding element but rather interwoven in the actions. The community refers to the people who share the same object, as in relation to how the teachers' PD can refer to the actual teachers and leaders who aim to facilitate and support the development process. The people in the shared community act within a set of rules such as norms and conventions that guide the actions in the activity system. The conducted work or goal-directed actions are divided between the people in the shared community and are described as the division of labour. Tensions or contradictions between the various factors in this activity system may occur and are, according to Engeström and Miettinen (1999), the basis and thus the starting point for change and development.

THE MAM PROGRAM AND THE CONTEXT OF THE STUDY

MAM is a PD program for in-service mathematics teachers that aims to promote opportunities for learning to enact the principles, practices, and mathematical knowledge entailed in ambitious mathematics teaching in an adaptive manner (Fauskanger & Bjuland, 2019; Wæge & Fauskanger, 2020). As stated above, the MAM program has been developed and contextualized on the basis of the LTP project (e.g., Ghouessini, 2017; Kazemi et al., 2016; Lampert et al., 2013) to fit the Norwegian situation. Whereas the LTP project originally was developed to support teacher students to enact ambitious mathematics teaching practices (Lampert 2010; Lampert et al., 2013), the MAM program attempts to adapt this pedagogy of ambitious mathematics teaching for in-service mathematics teachers' PD (Fauskanger & Bjuland, 2019; Wæge & Fauskanger, 2020). The core of the MAM program is to engage teachers through the daylong job-embedded PD events called *cycles of enactment and investigation* for PD (e.g., Lampert et al., 2013), earlier introduced in a vignette. Each cycle focuses on one instructional activity and through their work in these cycles the participating teachers will engage with a set of instructional activities during the program. The cycles include the six stages: preparation, collective analysis, co-planning, rehearsal, classroom co-enactment, and collective analysis. The teachers work together in groups, and are planning, rehearsing, enacting, and debriefing instruction throughout these six stages[1].

The MAM program is modelled on research of effective forms of PD that are argued to be sustainable over time, and that build systematic support and provide opportunities for active learning (e.g., Putnam and Borko, 2000; Desimone, 2009). Furthermore, the MAM program is informed by theory on teachers' collective learning in a community of practice (Wenger, 1998). The circle of enactment and investigation provides opportunities for the teachers to actively take part in mutual processes of negotiation of meaning to create a joint enterprise (Wegner, 1998). Moreover, the teachers are invited to engage in collective exploration, observation, and reflection by using the instructional activities as a common tool, guided by teacher educators (Wæge & Fauskanger, 2020). Thus, in addition to promoting opportunities to learn to enact the principles, practices, and mathematical knowledge entailed in ambitious mathematics teaching, the MAM program can be considered to offer a model for teachers' PD.

In addition to two informative start-up sessions, this program was planned to have a duration of 2 years, starting in the fall of

2019 and consisting of 12 sessions held at one of the participating schools. A full cycle of enactment and investigation was planned for ten of the sessions, five each year. The last two sessions were reserved for reflection, one at the end of each school year. The two start-up sessions were held prior to the end of the previous school year and were used to inform the participating teachers and school leaders about the program. Teachers from eleven primary schools in the same district also participated in the start-up sessions as they were attending a MAM program for primary mathematics teachers. The teachers were introduced to the practices and principles of ambitious teaching and the instructional activities.

The context of this study is a Norwegian lower secondary school with 330 students, seven mathematics teachers and a school leader team consisting of the school principal and a vice-principle. The school is multicultural with students of different ethnicities. The school participated in the MAM program together with three other lower secondary schools in the same district. This study followed the school's first year of participation.

METHODOLOGY

To address the research question presented in this article a qualitative interview study (Kvale and Brinkmann, 2015) was conducted at one of the lower secondary schools that was participating in the MAM program in Norway. All the participating lower secondary schools were asked to take part in the research study, and three of the four schools volunteered. To answer the research question and determine what the school managements' decisions concerning the support and facilitation of the mathematics teachers' PD were all about, I found it necessary to conduct a thorough investigation and cultivate this within the research context (Walcott, 2008). The three schools that volunteered were relatively similar in the number of mathematics teachers and number of students. However, two of the schools had been through several changes in school management in recent years. As I did not want to risk a major change in the school management during the period of the study, I selected the school that had had the most stable school management in recent years. Structures and practices for leadership and developmental work are usually created prior to or at the beginning of a development process, and the start-up phase is argued to have an impact on learning and enduring change (Postholm, 2008, 2020). Thus, the study was conducted during the school's first year of participation. The informants in the study have been selected through purposeful sampling (Creswell, 2013) and are: five mathematics teachers, the school principal and the vice-principal, working at a lower secondary school.

Data Collection

The data material in this study has mainly been collected from four focus-group interviews (Kamberelis and Dimitriadis, 2011). The five teachers participated in two of the interviews, the first conducted before the start of the project in the fall of 2019, and the second in the fall of 2020. The other two focus-group interviews were conducted with the principal and vice-principal, the first conducted before the start of the project, fall 2019, and the second in the spring of 2020. Data material was also collected from three follow-up interviews with the principal and vice-principal to clarify concepts. All the four focus-group interviews were conducted as a conversation to comply with Brinkmann and Kvale's (2015) claim that "knowledge is constructed in the interaction between the interviewer and the interviewee" (p. 4), but with a clear focus on pre-prepared questions related to the research question. The interview guides are presented in **Tables 1–4** below. This type of conversation also provides the interviewees with the opportunity to bring forward interesting aspects or themes the researcher did not think of before the interview. I acted as a moderator (Chrzanowska, 2002) throughout the interviews by asking questions to encourage dialog between the participants. All the focus-group interviews were audio-recorded and conducted with the use of a digital communication program due to COVID-19 restrictions.

Data Analysis

The constant comparative analysis method (Corbin & Strauss, 2008; Straus and Corbin, 1990, 1998) was used to structure and analyze the data material in this study. The transcription work

TABLE 1 | Structure of the focus-group interview guide for the school management, fall 2019.

	Questions
1	How is the school management organized in relation to the MAM project?
2	What support do you find the mathematics teachers' need to be able to learn together?
3	How would you describe school management's role as a facilitator for mathematics teachers' professional development?
4	Can you give a specific description of how you facilitate for mathematics teachers' professional development?
5	How is time for development work structured at your school?
6	How would you describe the mathematics teachers' need to develop their knowledge about teaching and teaching practice?
7	How do you assume the mathematics teachers perceive the school's leadership of their development work?
8	How do you understand the MAM program, and how do you assume the program will influence the participating mathematics teachers?
9	What do you think the mathematics teachers will learn through their participation in the MAM program?

TABLE 2 | Structure of the focus-group interview guide for the school management, spring 2020.

	Questions
1	How has the school management been organized in relation to the MAM program?
2	In what way have you supported the mathematics teachers in their work with the MAM program?
3	Can you describe your role as facilitators for the mathematics teachers' professional development this last year?
4	How was the time for development work structured this last school year?
5	How would you describe the mathematics teachers' need to develop their knowledge about teaching and teaching practice?
6	How do you assume the mathematics teachers perceive the school's leadership of their development work?
7	What are your perceptions or experiences of the MAM program halfway through the period?
8	How do you assume that the MAM program has influenced the participating mathematics teachers?
9	What do you think the mathematics teachers have learned through their first year of participation in the MAM program?

TABLE 3 | Structure of the focus-group interview guide for the mathematics teachers, fall 2019.

	Questions
1	In your opinion, how does the principal, or school management, facilitate for your development of knowledge about teaching and teaching practice in mathematics?
2	How do you perceive the development work in mathematics at your own school?
3	What are your opportunities for collaboration in the school, and what occurs during these meetings?
4	Do you find that the collaboration contributes to your development of knowledge about teaching and teaching practice in mathematics?

TABLE 4 | Structure of the focus-group interview guide for the mathematics teachers, fall 2020.

	Questions
1	What is, or was, your perception of the overall goal of the MAM program?
2	What is your experience of school managements' facilitation of your development work with the MAM program?
3	How motivated were you for participating in the MAM program?
4	How did you find the start-up phase of the MAM program?
5	What were your opportunities for collaboration in connection with the MAM project, and what did you do during these meetings?
6	Do you find that the collaboration contributed to your development of knowledge about teaching and teaching practice in mathematics?

commenced immediately after the interview ended. Then the transcriptions were carefully scrutinized and organized into smaller sections and given codes (Straus and Corbin, 1990, 1998). Using an abductive approach in this process (Alvesson & Sköldberg, 2009), I was looking for descriptions that could be related to the research question. The interviews were treated separately, but with an attention to look for connections between them. I used related theory and my own experiences as a reflecting tool when trying to understand the informant's utterances from their point of view. I thereafter examined the data material for differences and similarities to allow subtle discrimination and differentiation between the categories (Strauss and Corbin, 1998). As relevant categories emerged, the remaining sections were examined to see if the relevant categories were presented. Four key points that emerged during the open coding process (Straus and Corbin, 1990, 1998) became my main categories:

- Lack of ownership
- Motive
- Organizing and supporting teachers' learning
- The teachers' experiences of school management support

To define and specify the categories, the sub-categories were situated within the main categories by asking questions such as why, when, and under which conditions did the categories materialize (Straus and Corbin, 1990, 1998). The data material was also mirrored with the literature relating to the research question. This initial analysis used the constant comparative analysis method to create the scale for further analysis (Charmaz, 2014). Based on this initial analysis and discussion of these findings, I have used CHAT and the activity system to identify tensions and contradictions that can be the starting point for change and development.

Ethical Considerations and Quality Assurance

The study presented in this article has been approved by the Norwegian Centre for Research Data (NSD) and follows the ethical principles laid down by the Norwegian Ethical Research Committee (NESH, 2006). The participants in the study signed a

consent form based on informed consent in accordance with the NSD guidelines. They were also guaranteed full confidentiality and anonymity (NESH, 2006). Neither the school nor the participants are named, but rather referred to as teacher, principal, or vice-principal. The participants were also informed that they could withdraw from the study at any time without further explanation (NESH, 2006; Creswell, 2013).

The quality of this study was ensured through member-checking (Lincoln & Guba, 1985). Although the descriptions and analysis presented in this article are only connected to the teachers and school management from one specific school, the findings from this study may have importance beyond its immediate context if the reader is willing to have a creative and imaginative approach (Geertz, 1973), transforming it into a thinking tool (Gudmundsdottir 2001). Thus, hopefully, the findings from this study can contribute knowledge and considerations to similar situations and contexts.

FINDINGS

The findings are presented as extracts from the focus group-interviews and follow-up interviews. The developed main categories structure the presentation of the findings.

Lack of Ownership

The interviews with the school leader group revealed that the MAM program was initiated by the school owner (the local education authority) in the district. The principal stated:

> When we agreed to join the MAM program, one of the prerequisites was that we should be able to continue our work developing a more inquiry-based mathematics teaching. [. . .] on those grounds, a decision was made over our heads to create collective teacher development work in mathematics for several schools in the district, which was reached together between our leader and the Norwegian Centre for Mathematics Education. The development work was submitted to us, and at that point there was no actual choice about whether to participate or not. And then it was presented to us that this is how it's going to be, the program looked good, so we decided to participate. But at this point we did not have any dialog with our teachers about what they actually wanted or needed to improve in this process.

Later he added:

> The decision was taken over our heads [. . .] what I felt that we could choose was the number of participants.

Although the teachers expressed diminishing motivation, they claimed they were open-minded and entered the program with a positive attitude. The teachers said:

> Teacher 1: To sum up, we may not have been the most motivated people.
> Teacher 2: We were skeptical, but I don't feel that we were negative.
> Teacher 1: No.
> Teacher 2: It was more like "what is this?". No, we weren't the most top-motivated people, but we weren't at the bottom either.
> Teacher 3: Our motivation level sank during the period.

Motive

The school management team stated that they wanted the teachers to develop how they could learn together. The principal said:

> What I mainly hope the teachers learn is how to learn together. That means that the MAM program is first of all about how we can work with the professional learning community at school, more than the teachers learning a specific teaching method in the classroom. Because if we as a school learn how to best learn together, we can in some way use what we have learned in the MAM program to further develop other things we need to learn.

When the mathematics teachers were asked about their motive for attending the MAM program, they answered (the excerpt below starts after a 5-s pause):

> Teacher 1: Pass.
> Everyone laughs.
> Teacher 2: Well. . .
> Teacher 3: Well, indeed.
> Teacher 1: It became a bit vague, developing the quality of mathematics teaching. We're supposed to get better at teaching, but that's in a way the purpose of all courses.
> Teacher 2: I felt that we should get better at teaching in a way that activates the students more. That we should become better at having mathematical conversations in the classroom and using the kind of tasks that we could present in a different way than explaining from the blackboard, or not in a way what many would call traditional teaching. Exploring new methods that should activate the students. To improve these things and practice them throughout this project. At least that's how I interpreted it.
> Teacher 4: That was also probably what became decisive for us, that we didn't quite see where we were going with the project. Well, it's quite clear that the goal was in many ways as you say, "active student learning", and methods to achieve it, but I felt the course itself was not always characterized as being useful for that purpose. So, I didn't quite understand what the overall goal was here. I found that difficult to catch.

Organizing and Supporting Teachers' Learning

When school management was asked how they planned to organize the teachers' PD, the principal said:

> I found it natural to delegate the work of supervising the development work with the mathematics teachers to the vice-principal. He's a former mathematics teacher and thus has a special competence which will probably give him an advantage when working with the teachers. So, basically, the vice-principal is the one coordinating the development work, together with a teacher-coordinator in mathematics.

Although the principal delegated the main responsibility to the vice-principal, he stated that he was planning to participate in the sessions as much as he could, and that it was crucial that he was also involved in the teachers' development. He stated:

> I'm pretty sure that it's crucial that we're both equally up-to-date on the content of the project, and what's going on here at the school concerning the teaching of our teachers if we're going to succeed in this project.[. . .] It's crucial for all types of development projects that the school management team has good information about what's going on so that we can be part of the process together with the teachers. I think the leaders' participation is crucial. It might be easy to delegate or think that the teachers can do this on their own, but if it really is to mean anything, we also have to show the teachers that this is important to us. So, we have to prioritize our time, focus on this because we believe in it.

The school leader explained that they gave the mathematics teachers designated time to work on the development program. The principal said:

> . . .One of the mathematics teachers has been given earmarked time to coordinate the collaboration between the mathematics teachers. They have also been given time for a two-hour collaboration meeting between the sessions, both to immerse themselves in the content and to work with "homework" that is given at the sessions.. . .

The vice-principal added:

> . . .and facilitated for the teachers to have the opportunity to participate in the courses. There are teachers who are made available for the work, and the financial framework for the teachers to participate has been arranged, etc.

In the focus-group interview with school management after the first year of participation in the program, they reflected on the work they had done to support and facilitate the teachers' development. The principal said:

> Our participation is to a great extent lacking, both at the sessions and between them. So, the development work has not been led by us other than organizing the use of time [. . .] The plan was that the vice-principal should participate in the discussions at the sessions together with the teachers. But he had to spend the time organizing things like coffee, lunch, supplying teachers, making students available for the lessons, instead of participating.

They maintain that it would have been better if school management had participated in the development program together with the teachers as a part of the program's participation group. The principal added:

> The situation could clearly be different if we, school management, participated in the sessions. If so, we would have picked up some of the feedback the teachers are giving us now, and we could have done something at an earlier stage. Perhaps we also could have managed to increase their motivation and "seen" the teachers better.

Furthermore, the principal also reflected on challenges related directly to the MAM program, he stated:

> It's easy to be wise after the fact, but what I now see as a big challenge with the whole program is that it has become "one-size-fits-all" [. . .] I don't feel I have a real impact on either the content or the organization because it has to go this one way. [. . .] It's the same series of courses for all the participating secondary schools, but I think we would have succeeded better if there were opportunities to make adjustments in the organization and content in relation to each school's needs.

Later in the interview he added:

> I experience this as top-down governed. [. . .]I have no control over this education, and it's a difficult situation to experience for a leader. I don't know who the owner of the project is and who makes the decisions . . . I don't know.

The Teachers' Experiences of School Management Support

However, the teachers did not experience school management as being absent from their development process but were rather satisfied with the job it had done in supporting and facilitating their PD. One of the teachers said:

> Teacher 1: [. . .] They have not been negative to the course. I think they just really hoped that we would be satisfied. They have been very good at listening to us when we have provided input. So, in my opinion, I think they have been very accommodating and done what they can. [. . .]But they may not have planned for us to work a lot with this besides the sessions, but we have not asked for it either as we may not have really wanted to.

Another teacher adds:

> Yes, that is more or less true, and as you say said, if we had been super enthusiastic, they might also have become more engaged and given us even more time.

A third teacher adds:

> Well, when we have had such meetings and math meetings for the math teachers at the whole school, we have been told to discuss things such as "what are we going to do next with MAM", "how are we going to work with it?". That is, they have engaged in it and made sure that we don't forget. So, I have nothing to say about that.

The teachers reflected on the work they were supposed to do between the sessions, which was to read an article. One teacher said:

> Honestly, I don't know. It feels like it's homework, something I have to do, but is this really something I need? That's what I feel.

Another teacher adds:

> To me it was a little like that I forgot it a little, and then I remembered it, we got a reminder by e-mail 1 week before the session or something like that, and then it kind of sat there so I read it maybe the night before or in the morning before the session. I didn't prioritize it because as a teacher there are so many things to prioritize and remember, so many conversations to have, so it was never on the top of my priority list. So, it was only read right before the session.

ANALYSIS AND DISCUSSION

In this section I will first analyse and discuss how school management has supported and facilitated the mathematics teachers' PD in their first year of participation in the MAM program. Then I will use CHAT and the activity system to further analyze and discuss tensions and contradictions that can serve as a starting point for future PD. The presented data are analyzed and discussed across the main categories.

A Lack of Ownership and Joint Motive

The interviews with the informants revealed that the decision to work with the MAM program as a development project for the mathematics teachers was made solely by the school owner. The school was not invited to take part and therefore was not included in the process of finding a suitable development program for the mathematics teachers. Furthermore, school management did not have a dialog with the teachers as to what they actually wanted or needed to improve their classroom practice. Thus, the decision to participate in the MAM program was made on the basis of what was presented, which school management believed aligned with their conditions for participating, and not with the teachers' actual needs. Watson (2015) argues that teacher PD should be based on the teachers' needs at their school and in their classroom. As the school was not included in the process of deciding what development program to attend, there is also reason to believe that they were deprived of the opportunity to determine how the program fit with their development needs. Omitting mathematics teachers from such processes can restrain their learning and development, as acknowledging the teachers as the heart of decision-making around change is a key principle in understanding, engaging, and developing ownership in adult learning (Knowles et al., 2005). A development program chosen and decided by the school owner can of course be both relevant and based on the teacher's needs. However, this way of making decisions *for* the school and the teachers on the basis of what someone else thinks is best for them, instead of making decisions *together* with the teachers, is the opposite of what researchers have found to be the underpinning of successful teacher PD (Clarke & Hollingsworth, 2002). This can rather be characterized as a traditional top-down approach to teacher PD which can be argued as de-professionalizing the teacher (Roseler & Dentzau, 2013).

There appears to be a mismatch in motives between the principal and the teachers. The principal is more concerned about how the MAM program can contribute to developing the learning community where the teachers in the whole school can learn together, not just the mathematics teachers, than the mathematics teachers' development of teaching methods. There are good reasons to focus on such a goal. The MAM program draws on research on effective forms of PD, and has a collective perspective on learning where the teachers take part in mutual processes of negotiation of meaning to create a joint enterprise in a community of practice (Wenger, 1998). It is far from certain that the principal's statement is based on Wenger's ideas. Nevertheless, it shows that the MAM program can contribute to achieving the principal's main goal for participating, which is for the teachers to develop collective learning processes, and, furthermore, that such a focus on PD has the potential to improve the school, which can result in real change as the overarching goal (Opfer et al., 2011). The teachers seemed uncertain as to what the goal of the MAM program was and what they were supposed to learn through their participation. They hesitated to answer questions on this, and it did not seem that there was a clear and common understanding of what they were to develop through their participation in the program. According to Timperley et al. (2007), the participating teachers should at least have developed an understanding of the purpose of the development work, and moreover why they should attempt to move their practice towards the object of the work. The uncertainty the teachers show about the motive of the development work, and the fact that they and the principal have different motives, might be the consequence of not allotting enough time in the start-up phase to develop a shared overarching goal for participating in the MAM program.

The effects of any PD program depend heavily on teachers' motivation to learn and to change their practice (Kennedy, 2016). The presented data show that the teachers were not highly motivated to participate in the MAM program from the beginning, and that the motivation also decreased during the program period. The teachers' satisfaction with school management's support and facilitation, despite their absence, and the teachers' lack of initiative to spend time on the MAM program beyond the sessions can also be understood as a sign of a lack of motivation. This is not surprising if we see this in terms of

Leont´ev (1981) statement that "the object is the true motive" (p. 59) for people's actions. The teachers' have to share a collective motive for acting on the object if it is to be "invested with meaning and motivating power" (Sannino et al., 2016). As the teachers' needs did not serve as a starting point for the development work, and as there was a lack of a common understanding of the purpose, it appears that their motivation was not built into the object. Also, the teachers said they did not ask for more time and support to work with the MAM program outside the meetings, which indicates a lack of initiative and commitment (Sannino et al., 2016).

A Plan for Organizing and Supporting Teacher Learning—in Word but not in Deed

The principal had a clear strategy for how to lead and organize the teachers' PD as they participated in the MAM program, a strategy that included delegating responsibility to the vice-principal and a mathematics-subject coordinator. Leading teachers' PD can often be too much to handle for the principal alone, and the principal's way of treating leadership as a shared enterprise (Grootenboer & Hardy, 2017) might be useful in trying to avoid this challenge. Delegating the management of organizational issues and supervision of the teachers' participation to the vice-principal is also a way to build trust by acknowledging his competence as a former mathematics teacher. The same acknowledgment was given to one of the mathematics teachers who was assigned the task of coordinating the teachers' day-to-day job related to the development work. Building professional trust is important for establishing a productive learning environment for the teachers (Liu et al., 2016), and for giving teacher leadership the opportunity to flourish (Smylie et al., 2007). The principal's strategy also included a plan for facilitating the teachers' development work by providing them with designated time in the timetable, which enabled them to collaborate between the sessions. Teacher collaboration is assumed by researchers to contribute to PD and instructional improvement (DuFour and Fullan, 2012), and making sufficient time available for the participating teachers to collaborate is an important feature in teacher learning (Desimone, 2009). The principal furthermore planned to have an overview and engage directly in the teachers' learning process, not by controlling, but by keeping up-to-date on what the teachers could learn in the program and getting involved in their development process. Despite a messy start with this development work, it seems that the school, in accordance with what Darling-Hammond and Richardson (2009) maintain, had a plan for organizing and supporting the mathematics teachers' PD, both within and alongside their participation in the MAM program.

Nevertheless, a plan must be implemented in practice to fulfil its intention, and the data show that the plan school management produced was only partly followed in practice. The principal delegated the work and designated time for the teachers to collaborate and reflect on their learning, which indeed is a way to support teachers' PD (King & Stevenson, 2017). However, organization is not sufficient on its own, as teachers need continuous development support (King & Stevenson, 2017). Although the principal had planned for the teachers to use the designated time to immerse themselves in the content of the program and do the homework, which involved reading and discussing an article, the teachers perceived this in another way. They argued there were not facilitated any work related to the MAM program besides the sessions, and they did not perceive that the collaboration meetings between these sessions should be used to read the given article. Thus, a common understanding had not been established between the teachers and school management about what the allocated time was intended for. Furthermore, Birky et al. (2006) maintain that school administrators must encourage and motivate their teachers to be effective leaders through their words and actions if the teacher leadership is to be fruitful. As the principal and vice-principal were not present at the sessions or the collaboration meetings, the teachers' statements indicate that the teacher who was assigned the responsibility of coordinating the teachers' day-to-day work with their participation was left alone and did not receive sufficient support from school management. Spillane (2005, p. 147) argues that "structures, routines and tools are the means through which people act". School management could have interacted on aspects of the teachers' learning situations, including using a variety of tools, routines, and structures (Spillane, 2005), but such a practice did not seem to be established in this case.

The principal says he experienced a lack of control over the MAM program and described it as a "one-size-fits-all" project with no opportunities to make ongoing adjustments in line with the school's needs, both organizationally and in terms of subject matter. The importance of the development work being closely linked to the participants' context is well documented, especially when it comes to school-based development (e.g., Postholm, 2008, 2020; Smith & Landsay, 2016). Furthermore, Engeström and Sannino (2010) maintain that the development work must be owned by the practitioners, which means it must be based on their development needs. Although the MAM program is a job-embedded teacher development program that takes place in practice at one of the participating schools, it is not a school project. Teachers from several schools are participating in the program together, and the possibilities for making adjustments based on all the schools' needs are therefore limited. The MAM program is a PD program that aims to promote opportunities to develop ambitious mathematics teaching (e.g., Fauskanger & Bjuland, 2019; Wæge & Fauskanger, 2020), which the school participated in to further develop inquiry-based teaching. It is of course important to remember that the school was omitted from the process of finding a suitable development program for the mathematics teachers. Nevertheless, it seems that the principal was more concerned with how to adjust or change the MAM program rather than how to arrange and support the teachers' participation in the program so that it could contribute to improving their classroom practices.

As Earley and Bubb maintain (2004, p. 80), "professional development does not just happen—it has to be managed and led," and needs to be supported and encouraged by the leaders (Silva et al., 2017). The analysis in this study shows that the principal paid too little attention to leading and supporting the

teachers' PD in building a structure for the development work, as management ended up only organizing the teachers' PD in time and place. Moreover, the teachers did not manage to use the allocated collaboration time to create a learning community where they could develop their understanding of the core practices and principals of ambitious teaching. In other words, without more detailed arrangement and support from school management, the teachers did not manage to create a "historical new form of societal activity that was collectively generated" (Engeström, 1987, p. 174).

The MAM Program as a Mediating Artifact

As described above, the MAM program is here defined as a mediating artifact meant to function as an aid for the teachers to develop their classroom practice. However, the findings reveal contradictions within and tensions between the factors in the activity system (Engeström, 1987, 1999, 2001). I have already pointed out that the teachers do not feel that they "owned" the development work, and that there was a mismatch in the motive for participating in the MAM program between the principal and the teachers. In other words, there is a contradiction within the community in the activity system as to how the MAM program should serve as a mediating artifact. Developing a shared motive to act on the object can therefore be a reasonable starting point for this school's change and development (Engestrøm and Miettinen, 1999; Leont´ev, 1981). To accomplish this, time and resources must be allocated in the start-up phase for the teachers to identify with the topic for the PD work (Postholm, 2008; Postholm, M. B. 2020). This process must also be led and facilitated by the school leader. As the principal possess an important position that can have a substantial influence on teachers' learning (Leithwood et al., 2019), the responsibility lies within his role to create a learning environment that helps teachers to identify their development needs and enhance the implementation of new learning (Thoonen et al., 2011; Vanblaere & Devos, 2016). Through such a process, the teachers' motivation would be built into the object because their practice and needs serve as the starting point for the development work (Sannino et al., 2016). Also, restrained development work due to a lack of motivation on the part of the teachers can be avoided.

The findings reveal that the participants, both school management and the teachers, struggled to understand how to benefit from their participation in the MAM program in the mathematics teachers' development work. This could be understood as a tension between the community and the MAM program as a mediating artifact. Bearing in mind that the teachers' and school management's motive for participating in the MAM program differed when it came to content and process, they tried to understand how to benefit from the program from different points of views. However, as research shows that the focus on content and process must go hand-in-hand and be integrated into the development work (Postholm et al., 2013), I would argue that both motives could be built into the object in this activity system, and furthermore that the MAM program could serve them both. The program provides mathematics teachers with the opportunity to develop ambitious mathematics teaching through the cycle of enactment and investigation (Lampert et al., 2013), together with other teachers and a teacher educator in a community of practice (Wenger, 1998). In this way, the MAM program is modelling a form of PD which can be used as an aid or a thinking tool that contributes to create developmental structures adjusted to the school. As such, the MAM program can become a mediating artifact (Engeström, 1987), not only for improving mathematics teachers' classroom practices towards ambitious teaching (Lamper et al., 2010), but also for building a foundation for mathematics teachers' PD in general. To take advantage of this opportunity, the principal has to take part in the program so he can learn and manage to lead the development processes after the program period is over, as it is the school leaders' task to arrange and facilitate the teachers' learning in schools (Elmore, 2000).

Supporting the School Leader

The study presented in this article has shown that leading mathematics teachers' PD is easier said than done. With an explicit focus on teachers' PD and the knowledge that it has to be managed, led, encouraged, and supported (Earley & Bubb, 2004; Elmore, 2000:; King & Stevenson, 2017; Silva et al., 2017), one might quickly forget that the need for support also refers to the school leader. As the overall leader supplying the school with development resources, and as the initiator of the MAM program, the school owner has to be placed in the community of the activity system as well (Engeström, 1987, 1999, 2001). The school owner's role must therefore be to support the school leader in conducting development work at school by providing resources, for example, internal or external support, that enable the school leader to conduct his work in the best possible way. However, such goal-directed actions were not clear in this study. The school leader's uncertainty relating to who is in control and who is making the decisions strongly indicates a need for more clearly defined roles. The latter, in combination with a top-down initiated development program, rather indicates that the school owner is stepping away from liability. The conducted work or goal-directed actions have not been divided between the people in the shared community and this leads to a tension in the division of labour (Engeström, 1987).

Hulsbos et al. (2016) found that reflecting with colleagues and participating in networks are workspace learning activities that are highly appreciated by leaders. As several schools are participating in the MAM program, the program itself could serve as a starting point for creating a network for the leaders from the participating schools. The school leaders could in this way draw on each other's experiences, thus developing their own profession as school leaders. However, as Earley and Bubb (2004, p. 80) found, "professional development does not just happen—it has to be managed and led". While it may not be the school owners' responsibility to lead these kinds of networks, it is the school owners' responsibility, as the overriding leader, to support and enable these networks to blossom, which could be done by supplying external expertise. In the same way as the school leaders need to acknowledge their role as facilitators for teachers' learning (Walker, 2007), this study has shown that the school owners also need to acknowledge their role as facilitators for the school leaders' work as leaders of teachers' PD.

CONCLUSION

In exploring how school management facilitates and supports mathematics teachers' PD when they participate in a practice-based development program, this study has found that words do not necessarily become deed, or practice, on their own. The school needs to have a structure and a practice for development work if a development plan is to be successfully implemented and conducted. Furthermore, the study presented in this article has illuminated how difficult development work can be if those involved do not aim their actions in the same direction. Teacher ownership and a shared overarching goal must be the foundation and form the basis for participating in a practice-based development program. By using the activity system as the unit of analysis, I have identified tensions and contradictions relating to the mathematics teachers' development work that can be the point of departure for change and development. The MAM program was supposed to be an aid or a tool that could help the mathematics teachers to develop their classroom practice and can thus be considered to be a mediating artifact. The findings indicate that the roles involved in the development work were not defined so that the conducted work or goal-directed actions divided between the people in the shared community act towards the same object. Furthermore, the school leader needs support in his work as a leader of teachers' PD, and the school owner must acknowledge his role as the facilitator for this work.

Further research should aim to understand how school management should be included in such practice-based development programs for teachers. As such, school management can also develop their professionality and be better prepared to facilitate and support the teachers' development work during the program and after its completion. The development program can then serve as a mediating artifact for mathematics teachers' PD.

DATA AVAILABILITY STATEMENT

The raw data supporting the conclusions of this article will be made available by the authors, without undue reservation.

AUTHOR CONTRIBUTIONS

EAB conducted the research, analyzed the data and is the author of this manuscript.

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Conflict of Interest: The author declares that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Article 3

Teacher educators' experiences of a failed practice-based development program in mathematics

Running title: Teacher educators' experiences of a failed development program

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Acknowledgement(s) of people, grants, funding, etc.:

EB conducted the research, analysed the data and is the author of this manuscript. No grants or fundings were given for this research project.

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This paper is submitted for publication and is therefore not included.

ISBN 978-82-326-7558-6 (printed ver.)
ISBN 978-82-326-7557-9 (electronic ver.)
ISSN 1503-8181 (printed ver.)
ISSN 2703-8084 (online ver.)

NTNU
Norwegian University of
Science and Technology