CHAPTER 1

Introduction to CiviMatics

Bastian Vajen, Lara Gildehaus, and Heidi Strømskag

with *Yael Fleischmann, Timon Foss-Jähn, Michael Liebendörfer, Nicola Nagy, Frode Rønning, and Jakob Steinbachner*

Aims and Frameworks of CiviMatics

This handbook contains the results of the Erasmus+ project CiviMatics and offers an approach to linking mathematics education and civic education in the field of teacher education and training at the university level. The goal of CiviMatics is to offer educational tools to enhance the competences of future teachers to address complex societal challenges in their classrooms and to combine socio-scientific and mathematical perspectives to help their students understand various aspects connected to these issues. To achieve this, the project focuses on normative modelling, which is the way mathematical modelling and applications of mathematical models shape our reality and influence societal discourses as well as individual and collective behaviour. To exemplify these dimensions of mathematical modelling, various aspects connected to climate change and the human activities furthering it will be used as examples.

The teaching and learning approaches presented in this book as well as the competences connected to them can be linked to the OECD's PISA mathematics framework, aspects of the Sustainable Development Goal 4 and the OECD Learning Framework 2030. The PISA mathematics framework states that mathematical literacy of citizens must take priority over reproducing mathematical techniques or routines (OECD, 2018b, p. 43). In this regard, the PISA mathematics framework defines mathematical literacy as "an individual's capacity to reason mathematically and to formulate, employ, and interpret mathematics to solve problems in a variety of real-world contexts" (OECD, 2018b, p. 7). In particular, it is important for citizens in the modern world "to reason mathematically and to solve problems and interpret situations in personal, occupational, societal and scientific contexts" as well as "to draw upon certain mathematical knowledge and understanding" (OECD, 2018b, p. 22).

However, part of this mathematical literacy is not to be understood as the subject domain of mathematics, but as a cross-sectional task for education (Weber-Stein & Engel, 2021, p. 166). As mathematical literacy becomes more and more relevant to understand various societal processes due to the progressive mathematisation of society, it becomes more and more connected to other subjects, such as civic education (Mau, 2017, p. 24; Straehler-Pohl, 2017, p. 37; Weber-Stein & Engel, 2021). Consequently, sustainable development goals, such as to ensure "that all learners acquire the knowledge and skills needed to promote sustainable development, including, among others, through education for sustainable development", are intrinsically connected to mathematical competences, as they are essential for describing, understanding, predicting, and communicating various issues connected to climate change (Barwell, 2013, p. 2). Therefore, learning about climate change provides an ideal topic for linking civic education and mathematics, since the topic offers students a field of application for their skills that authentically reaches into the students’ lifeworld through the actual effects of mathematical models on political processes (OECD, 2018a, p. 5).

However, before students are able to grasp the connections between the mathematical and societal or political world, it is necessary to also offer teachers insights into these aspects. Thus, this handbook offers various tools to give prospective teachers of mathematics and civic education insight into the workings and principles of the respective subjects and to connect both subjects as part of teacher education. Hence, this handbook will offer an introduction to mathematics education, civic education and mathematical modelling as well as a number of different examples for lectures, seminars or other teaching and learning activities to familiarise future teachers with the links between the two subjects.

In the first chapter, the conceptual principles of the CiviMatics project will be presented. This will start with a brief overview of normative mathematical modelling, which is the cornerstone of this project, negotiating questions about what a model is, what types of models there are, and what relationships exist between classical mathematical modelling and civic education. Based on this, the framework for normative modelling developed in the project and its connection to different subjects is presented. After this theoretical foundation, a detailed description of different examples for a practical implementation as part of teacher education will follow, which include the various approaches in different educational settings from Norway (Chapters 2 & 3), Austria (Chapter 4), Germany (Chapters 5 & 6), and Romania (Chapter 7). These chapters will also provide a didactic commentary with experiences from practice and suggestions for adapting these courses for different educational settings. Additional materials, such as PowerPoint presentations or worksheets used in the seminars can be found in the appendix of the respective chapters as well as on the homepage of the project (https://www.civimatics.eu). The handbook concludes with a summary of its contents and a brief outlook on further research (Chapter 8), which also considers possible applications of the CiviMatics approach outside teacher education at the university level.

Introduction to Mathematical Modelling

Mathematical Modelling

The notion of a model relies on the notion of a system, that is, a reality subject to its own laws. A model is the result of a transformation of a system, usually a simplification, which is supposed to help in generating knowledge about the studied system. In practice, to answer a question relating to a system, one tries to build up a model which is easier, safer, and quicker to study than the system itself. Models are thus used for answering questions or exploring phenomena, possibly guided by research questions. Models always have a descriptive function, but they can also contain statements about what an individual should do, and be used to analyse the system it models. If this is the case, models are referred to as prescriptive, or normative. Depending on the case, a model can thus have descriptive, normative or prescriptive uses. For example, prognostic models (e.g., models of greenhouse gas emissions) are classified as descriptive, but they can be applied in a normative way.

Modelling is understood as the very process of building a model of a system and using it to answer questions about the system at stake. There are different tools for modelling, such as the modelling cycle in the framework of Blum and Leiß (2005) and the Herbartian schema in the Anthropological Theory of the Didactic (ATD, Chevallard, 2019). Insofar as modelling involves valuations, we speak of normative modelling. Models can be prescriptive but non-normative (e.g., a cake recipe), but very often prescriptive models will be normative, because certain actions or outcomes are treated as desirable. It is important to recognise that descriptive models can also be normative if, for example, the descriptive categories used are judgmental ("normal weight") or implicitly value certain actions or outcomes. For example, a model that relates CO_2 emissions in a country to people suggests different consequences than a model that relates CO_2 emissions to economic output. In shorter terms: While a model can be used in a purely descriptive way, that is, as a model of something, as is often the case in physics, for example, it can also be used in a normative way, being a model for something.

Mathematical Modelling in (Higher) Education

Mathematical modelling has become a fixed part of school curricula in numerous countries due to its advantages compared to classical mathematics teaching. Modelling tasks provide an opportunity to connect different types of mathematical knowledge, which classical mathematics instruction often does not accomplish sufficiently, and at the same time combine different competences such as reasoning, modelling, and problem solving that do not have their own place in content-oriented mathematics instruction (Bruder & Krüger, 2018). In particular, modelling tasks also train basic mathematical knowledge and skills that are often not available to students to the desired extent. Furthermore, modelling tasks cannot be solved schematically and can provide a remedy for the widespread problem that students often treat tasks without considering the content, i.e. they only try to extract the numbers and fit them into the currently typical calculation schemes (Bruder & Krüger, 2018).

Despite this great advantage of modelling tasks, modelling processes are mostly given little attention in teacher education and "(basic) competences for teaching mathematical modelling [are] [...] not sufficiently taught" (Borromeo Ferri & Blum, 2018, p. V). This can also be demonstrated in the research of Blum and Leiß (2006), who, when investigating how teachers deal with modelling tasks, found that these involve problems that do not occur in the same way in "classical" mathematics classes. Central to this is the difficulty balancing the independence of students and the intervention of teachers as well as the demanding comparison of results, as often different results to solve a task must be dealt with and different approaches must be discussed and reflected upon (Blum & Leiß, 2006). To be able to successfully incorporate modelling tasks in the mathematics classroom, it is thus important to highlight modelling competences in teacher education and to enable future and current teachers to incorporate relevant tasks in their classrooms.

In the context of structuring the didactic approaches to mathematical modelling, Blum and Borromeo Ferri (2010) described different competences that mathematics teachers need to enable a meaningful handling of modelling tasks in the classroom. In particular, the competence to prepare and conduct reality-based lessons plays a role in the teaching dimension, something that is also demanded outside of modelling processes as a central element of mathematics education to promote the experience of mathematics in everyday life and an opening up of the world through mathematics. Such realistic modelling tasks can also address learner types that are normally less enthusiastic about mathematics (Greefrath et al., 2013). Interdisciplinary approaches offer a useful starting point when seeking a connection to real-world phenomena. Social and economic challenges present an abundance of real-world topics, all of which are well-suited for modelling tasks. These are the kinds of issues typically explored in subjects like civic education or economics.

Link Between Mathematical Modelling and Civic Education

As a central principle, societal problems and their treatment are, as the subject of politics, also the subject of civic education (Goll, 2014, p. 258). Using real problems as the starting point for educational processes enables learners to better understand how democracies deal with challenges and how different viewpoints are negotiated to create possible solutions (Reinhardt, 2018, p. 100). Even if concrete problems and conflicts are constantly changing, their existence represents a fundamental component of democratic societies. Therefore, using societal problems for civic learning processes retains a constant relevance for civic education. However, the increasing complexity of modern societies leads on the one hand to an increasing complexity of problems and possible solutions (Triantafillou, 2020, p. 4). One consequence is a stronger mathematisation of society, which not only leads to new forms of information which influence various decision-making processes, but also reorganises the fundamental conditions of political and societal action (Straehler-Pohl, 2017, p. 37).

One example of this is the problem of climate change. Mathematical processes are central both in describing the problem and in predicting its further development and weighing possible solutions (Barwell, 2013, p. 3). Although the complexity of the problem and the competencies required to understand it exceed the scope of civic education, its analysis and use in the classroom are nevertheless indispensable for civic education. In this respect, it is important for civic education to strive for a stronger interdisciplinary cooperation with other subjects when dealing with complex societal and political problems, such as climate change, and, in this context, to remain open to the contents and didactic principles of other disciplines. An interdisciplinary connection between subjects—such as mathematics and civic education—can be advantageous for both disciplines. For example, mathematical modelling can be used to develop a better understanding of the generation of knowledge about societal problems and the cause-effect relationships of political decisions, while taking a civic perspective on mathematical models can help to better grasp the use of modelling processes and its applicability to the real world.

Normative Modelling and Civic Education

Principles of Civic Education

Although models are used on a regular basis in civic education, didactic discourses about the structure of models as well as their uses for educational processes are lacking. When using models in civic education, the goal is mainly to exemplify political processes and help students to understand their underlying principles and structure. One prevalent model used to exemplify the process of political decision making is the policy cycle. This model describes the policy process as evolving through a sequence of distinct stages. Initially introduced as a normative model in political science aiming to provide an ideal framework for planning and decision making, it has developed into a widely applied framework to organise research on public policy (Jann & Wegrich, 2007). It was also introduced into civic education to help teachers and students grasp real political situations in their complexity, interdependence, and formative elements. The phases of the political cycle, and the categories that influence it, are reformulated into key questions guiding the process of understanding political decision making (Massing, 1995, p. 86). Such categories are for example the division of the political domain into polity (form), policy (process) and politics (content) (Oberle, 2016, p. 25). The policy cycle can be seen as a tool to exemplify the processual structure of political decision-making, which may be influenced by a variety of different variables, but consists of a distinct pattern (Massing, 1995, p. 88). Thus, the use of models such as the policy cycle is twofold: On the one hand, the models should provide students with a simplified, and thus somewhat flawed, picture of reality. On the other hand, they can be used as a tool for analysis, by comparing real political processes with elements of the model and offering a basis for inquiry.

Connected to the use of models such as the policy cycle are often other principles of civic education, such as the problem orientation (Ackermann et al., 2018, pp. 31–33). Problem orientation states that (political) problems and their treatment are, as objects of politics, at the same time objects of civic education (Goll, 2014, p. 258). By taking up political problems (e.g., climate change) and making them the focus of learning about politics, decisions are made about the methodological form of the teaching-learning process. The teaching of problems aims at problem-solving thinking and, if successful, promotes a high degree of judgement competence and political maturity in the learners. To achieve this, Goll (2014) proposes that most approaches have three methodical steps: The analysis of the situation, the discussion of possibilities and the formation of a judgement (p. 263). Such principles of civic education are aimed at helping students acquire the ability to form a political judgement, which represents the core of political education processes (Juchler, 2012, p. 24). The importance of independent political judgement arises from the close connection between the ability to judge and the concept of political maturity. The goal of civic education is to contribute to the development of "political, moral, and ethical autonomy" through political maturity, which as part of self-determination always requires the ability to make independent judgements (Henkenborg, 2012, pp. 28–29). In this context, learners should be enabled to make independent assessments of political, economic, or social issues while weighing different criteria (Reinhardt, 2018, p. 24).

However, the ability to judge cannot be regarded as a stand-alone competency, but has to be integrated into the subject-didactic "triad" of political analysis competence, judgement competence, and action competence (Henkenborg, 2012, pp. 32–34). On the one hand, judgement is therefore dependent on a well-developed analytical competence, since a well-founded judgement appears impossible without penetrating social and political facts and structures; on the other hand, it is also linked to the competence to act, since the rational and independent judgement represents the basis of the political action of democratic citizens. Using societal problems as a topic for civic education processes can help learners understand the causes of such issues and enable them to analyse the political processes that are involved in solving them. Models can be a useful tool to facilitate learning in this context, both regarding the analysis of an issue as well as the political steps that can be taken to solve it. For an issue such as climate change, the understanding of which is dependent on mathematical and political competences, it can be beneficial to combine principles of civic education and mathematics education to enable learners to grasp the issue, analyse possible solutions and act in accordance with their own judgement.

Normative models, which serve as a framework for the interdisciplinary approach to modelling, are approached in two different ways in the context of this project: On the one hand, with the help of a normative modelling cycle developed in the project and, on the other hand, with the help of Study and Research Paths, using the Herbartian schema. These approaches will be briefly presented in the following.

Normative Modelling Cycle

The combination of civic education and mathematics education requires an adaptations of established modelling cycles in order to make political analysis and judgement explicitly visible. Previous representations aim at a mathematical result, which is often checked for its correctness at the end (e.g., by a measurement or an experiment). This unambiguity and verifiability is not given in normative modelling. Our proposal on the methodological level therefore consists of a combination of the steps of political didactic problem-orientation and mathematical modelling, which are made visible in a common model. The basis for this is a modelling cycle that already contains a so-called situation model, that is, a mental representation of the situation (Blum & Leiß, 2005; Borromeo Ferri, 2006, p. 92). Normative modelling, however, requires more, namely a political analysis of the situation (in addition to a mathematical analysis), a discussion of political possibilities, and a judgement formation as subsequent steps. At a minimum, the discussion of political possibilities requires that different possibilities emerge from the mathematical models or that they can be considered from the very beginning. Therefore, the question of selecting models or families of models arises. Policy analysis also requires identification of the interests of involved stakeholders. Neither alternative models nor affected interests emerge on their own. To incorporate these steps, we suggest a new modelling cycle, based on established approaches but adding additional steps for the modelling processes (see Figure 1).

The first step, constructing (1.) does not involve conscious steps, but suggests that in normative modelling we may need to reconcile different conceptions of reality if we are to negotiate solutions in our societies. Simplifying (2.) is one of the most relevant steps. Which parts of the situation model are included at all and how interrelation-ships are simplified essentially determines the result. Here, alternatives have to be considered, their consequences for the model have to be estimated and they have to be classified with regard to political interests. Mathematization (3.) is in itself a technical step, provided that the real model is specified precisely enough. In practice, however, the real model is specified more concretely in this step, so that simplifications similar to those in (2.) are to be expected here as well. The mathematical work (4.) will rarely provide starting points for the political discussion.

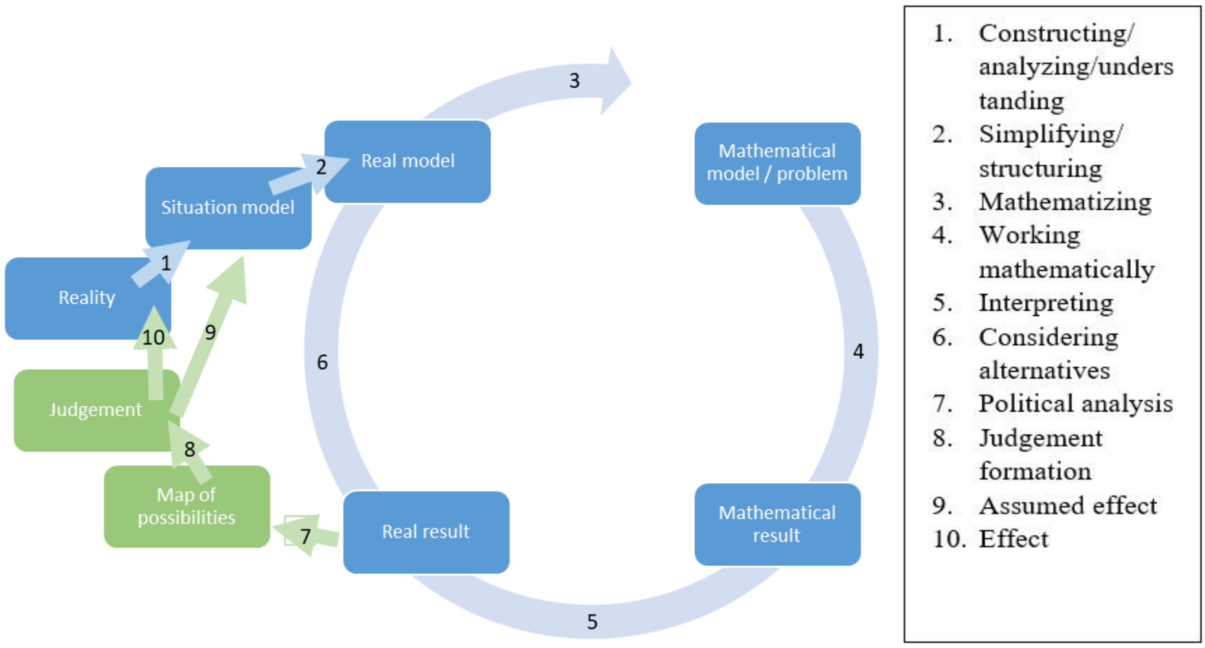
Although alternative actions exist here (e.g., obtaining solutions algebraically or numerically), the differences, if any, should be irrelevant. Interpreting (5.) should also be more of a technical step because it initially involves only the translation of mathematical variables, functions, etc. into reality. However, generalizations could be made at this step, concerning e.g. model assumptions or restrictions of variable ranges, etc. Moreover, the presentation of the results will very often suggest actions, at least implicitly. Such (normative) statements can never be the result of a mathematical calculation and should therefore always be outsourced to the further steps. First, the different possibilities and the different implications related to the stakeholders' interests should be noted through the reflection and critique of the modelling just described. We named that to build a "map of possibilities". After that, everyone

Co-funded by
the European Union

should form their own judgment (8.) by weighing the interests. Finally, it should be acknowledged that decision taken in the classroom might have an impact on the world as we assume it to be at that moment (in terms of our situation model; 9) and as it is (reality; 10).

Figure 1

An interdisciplinary modelling cycle



The presented cycle can be helpful for the creation of models as well as for the analysis of modelling, because it explicitly points out working steps. For example, arguments that one has "recalculated" certain effects are made discussable. Different real results are usually not based on different interpretations (5) or mathematical solutions (4), but partly on mathematisations (3) and especially simplifications (2) as well as perceptions of reality (1), which all have to be discussed explicitly. In the process, it may be possible to identify the interests of the actors concerned, which frame such assumptions.

The Anthropological Theory of the Didactic and Study and Research Paths

The Notion of Praxeology

The *anthropological theory of the didactic* (ATD) postulates that any activity related to the production, diffusion, or acquisition of knowledge should be interpreted as an ordinary human activity, and thus proposes a general model of human activities built on the notion of *praxeology*. This is a key notion in the ATD, explained like this:

> A praxeology is, in some way, the basic unit into which one can analyse human action at large. […] What exactly is a praxeology? We can rely on etymology to guide us here – one can analyse any

> human doing into two main, interrelated components: praxis, i.e. the practical part, on the one hand, and logos, on the other hand. “Logos” is a Greek word which, from pre-Socratic times, has been used steadily to refer to human thinking and reasoning – particularly about the cosmos. […] One fundamental principle of the ATD [states that] no human action can exist without being, at least partially, “explained”, made “intelligible”, “justified”, “accounted for”, in whatever style of “reasoning” such an explanation or justification may be cast. […] Of course, a praxeology may be a bad one, with its “praxis” part being made of an inefficient technique – “technique” is here the official word for a “way of doing” – and its “logos” component consisting almost entirely of sheer nonsense – at least from the praxeologist’s point of view! (Chevallard, 2006, p. 23).

A praxeology in the ATD is a unit composed of four components (Chevallard, 2019): T, τ, θ and Θ (sometimes referred to as “the four t-s”). T (Latin capital letter t) is a *type* of tasks, τ (Greek tau) is a technique (or a set of techniques) to solve the tasks, θ (Greek theta) is a *technology* (i.e., a discourse) to describe and explain each technique, and Θ (Greek capital theta) is a theory that justifies the technology. T and τ belong to the praxis block of a praxeology, whereas θ and Θ belong to the logos block. A praxeology $\wp$ is thus written: $\wp = [T / \tau / \theta / \Theta]$. *A priori* praxeological analyses are important for classroom experiments, where praxeological models of the knowledge at stake are instrumental in designing interventions to be implemented in the classroom. Praxeological analyses can also be done *a posteriori* to analyse how a praxeology has been built up during the solution of a problem (see e.g., Strømskag, 2021).

From Knowledge “Visits” to Dynamic World Inquiries: A Paradigm Shift

The prevailing didactic paradigm, which we may refer to as the paradigm of *visiting works*, is fundamentally rooted in the notion that there are specific bodies of knowledge, or curricula, which bear significant social relevance (Chevallard, 2015). Within this framework, learners engage primarily with selected praxeologies, and often do so without truly grasping the raisons d’être underpinning them. While this paradigm is not strictly synonymous with a teacher-centric approach, it tends to prioritise the foundational praxeologies over the activities and challenges that might otherwise contextualise them. This is particularly evident in how mathematical knowledge is presented—as a polished, final product. The intrinsic motivations or initial questions that paved the way for its evolution are frequently pushed to the sidelines. Such an approach culminates in what Chevallard (2015) describes as the “monumentalisation” of the curriculum. Here, mathematical entities are held in reverence, almost to the extent of being sacrosanct, with little room for inquiry or contestation. The potential pitfall of this paradigm is that it might render the curriculum as something distant, perhaps even intangible, to students.

On the other hand, the paradigm of *questioning the world* offers a more encompassing pedagogical strategy. The foundational methodological component underpinning the paradigm of questioning the world is the notion of *Study and Research Path* (SRP) based on the so-called *Herbartian schema* (Chevallard, 2019):

$$[S(X, Y, Q) \rightarrow M] \hookrightarrow A^{\heartsuit}$$

Here, the didactic system $S(X, Y, Q)$ is not formed around a given praxeology to be studied, but around a question Q to which X (the students), with the help of Y (the teacher/teachers), has to provide an answer $A^{\heartsuit}$. The study of Q generates an inquiry process involving a didactic milieu M made up of different types of objects or tools for the inquiry:

$$M = \{A_1^{\Diamond}, A_2^{\Diamond}, \ldots, A_m^{\Diamond}, W_1, W_2, \ldots, W_n, Q_1, Q_2, \ldots, Q_p, D_1, D_2, \ldots, D_q\},$$

where the components of M signify the following: $A_i^{\Diamond}$ are existing answers to Q found in the literature and elsewhere; W_j are all types of work that must be used in order to study and understand all the other components of M; Q_k are questions generated by the study of Q and the other components in M; and D_l are datasets that are collected through various types of research during the study of Q.

In the Herbartian schema, the concept of "visiting works" remains but is driven by the need to find productive answers, $A_i^{\Diamond}$, even if it means delving into vast knowledge domains with expert guidance. The motivation behind such visits is not the prestige of $A_i^{\Diamond}$ but its utility in forming $A^{\heartsuit}$. The Herbartian schema outlines the key components of the inquiry process. The dynamics of such a process are articulated through various dialectics, with Bosch (2018) highlighting three as especially significant:

- *Question-Answer Dialectic*: this embodies the iterative essence of research, where answers to initial questions spark further questions, necessitating a deeper exploration into the subject;
- *Media-Milieu Dialectic*: here, a distinction is drawn between the media, which propagates messages, and the milieu, a system devoid of any intention with respect to the question studied. For a message to gain credence, it must endure the scrutiny of the milieu, affirming its authenticity and relevance;
- *Individual and Collectivity Dialectic*: this emphasises the balance between individual contributions and collective aspirations in the research process.

The new paradigm is based on three principles related to curricula (Chevallard, 2018). Firstly, every human community has duties towards its members. An essential duty is that of defining and implementing a community curriculum to ensure that all members of the community are *enabled to think and act appropriately*, in a way beneficial to themselves and to others, in the different social worlds (in particular the worlds of family, profession, and citizenship) in which they are or will be led to live. This aligns with Lange's (2008) emphasis on the competence area of social learning. Similarly, Print (2013) identifies competences essential for democratic citizenship. These competences encompass beliefs in social justice, equality, and the equal treatment of all citizens. Furthermore, they also include the skills necessary for coalition building, cooperation, and the ability to thrive in a multicultural environment.

Secondly, the curriculum within the community should empower its members, either as individuals or in collective groups, to discern, articulate, and address the questions they encounter. This relates to democratic competences like the ability to critically assess information, evaluate stances or decisions, adopt a viewpoint, and substantiate that position, as outlined by Print (2013). Thirdly, to achieve this goal, the community shall define (and revise regularly) a curriculum *core made up of questions* that members of the community "have the right not to be allowed to avoid" (Gagnon, 1995, p. 72).

In summary, the paradigm of visiting works offers a structured approach, prioritising established bodies of knowledge, which can sometimes risk making the curriculum seem fixed or unrelatable to students. On the other hand, the paradigm of questioning the world leans towards fostering inquiry and critical reflection, placing emphasis on the journey of discovery rather than just the destination. This latter approach may present education as more interactive and dynamic.

Practical Applications

The ability to make autonomous political judgments, which is a central component in civic education, is dependent on extensive analytical competence, because without an understanding of social and political issues, a well-founded judgement seems impossible. Complex issues and their solutions, like climate change, require an under-standing of mathematics and civics to properly analyse them and form an informed judgement. Mathematics education and civic education offer a high potential for cooperation in the teaching of complex societal problems due to similar objectives and the reference to comparable concepts. Practical examples of such an interdisciplinary approach will be provided in the next chapter.

References

Ackermann, P., Breit, G., Cremer, W., Massing, P., & Weinbrenner, P. (2018). *Politikdidaktik kurzgefasst: 13 Planungsfragen für den Politikunterricht*. *Wochenschau Politik*. Schwalbach/Ts.: Wochenschau-Verl.

Barwell, R. (2013). The mathematical formatting of climate change: critical mathematics education and post-normal science. *Research in Mathematics Education*, *15*(1), 1–16.

Beck, U. (2015). Emancipatory catastrophism: What does it mean to climate change and risk society? *Current Sociology*, *61*(1), 75–88.

Blum, W., & Leiß, D. (2006). Filling up – In the problem of independence-preserving teacher interventions in lessons with demanding modelling tasks. In M. Bosch (Ed.), *Proceedings of the Fourth Conference of the European Society for Research in Mathematics Education* (pp. 1623–1633). Barcelona: Universitat Ramon Llull and ERME.

Blum, W., & Borromeo Ferri, R. (2010). Mathematical modelling in teacher educatio: Experiences from a modelling seminar. In V. Durand-Guerrier, S. Soury-Lavergne, & F. Arzarello (Eds.), *CERME 6: Proceedings of the sixth congress of the European Society for Research in Mathematics Education, January 28th-February 1st 2009 Lyon (France)* (pp. 2046–2055). Institut national de recherche pédagogique.

Borromeo Ferri, R. (2006). Theoretical and empirical differentiations of phases in the modelling process. *ZDM – Mathematics Education*, *38*, 86–95.

Borromeo Ferri, R., & Blum, W. (2018). Vorwort. In R. Borromeo Ferri & W. Blum (Eds.), *Lehrerkompetenzen zum Unterrichten mathematischer Modellierung* (pp. V–IX). Springer Fachmedien Wiesbaden.

Bosch, M. (2018). Study and research paths: A model for inquiry. In B. Sirakov, P. N. de Souza, & M. Viana (Eds.), *International Congress of Mathematicians* (Vol. 3, pp. 4001–4022). Singapore: World Scientific Publishing. https://doi.org/10.1142/9789813272880_0210

Bruder, R., & Krüger, U.-H. (2018). Lerngelegenheiten für Mathematisches Argumentieren, Modellieren und Problemlösen (LEMAMOP). In R. Biehler, T. Lange, T. Leuders, B. Rösken-

Winter, P. Scherer, & C. Selter (Eds.), *Mathematikfortbildungen professionalisieren* (pp. 225–248). Springer Fachmedien Wiesbaden. https://doi.org/10.1007/978-3-658-19028-6_12

Chevallard, Y. (2006). Steps towards a new epistemology in mathematics education. In M. Bosch (Ed.), *Proceedings of the Fourth Conference of the European Society for Research in Mathematics Education* (pp. 22–30). Barcelona: Universitat Ramon Llull and ERME.

Chevallard, Y. (2015). Teaching mathematics in tomorrow's society: A case for an oncoming counter paradigm. In S. J. Cho (Ed.), *Proceedings of the 12th International Congress on Mathematical Education* (pp. 173–187). Cham: Springer. https://doi.org/10.1007/978-3-319-12688-3_13

Chevallard, Y. (2018). The curricular conundrum in the light of the ATD. In Y. Shimizu & R. Vithal (Eds.), *The Twenty-fourth ICMI Study: School Mathematics Curriculum Reforms: Challenges, Changes and Opportunitie* (pp. 213–220). Tsukuba: University of Tsukuba.

Chevallard, Y. (2019). Introducing the Anthropological Theory of the Didactic: An attempt at a principled approach. *Hiroshima Journal of Mathematics Education*, *12*, 71–114. https://www.jasme.jp/hjme/download/05_Yves%20Chevallard.pdf

Douglas, K. M., & Sutton, R. M. (2015). Climate change: Why the conspiracy theories are dangerous. *Bulletin of the Atomic Scientists*, *71*(2), 98–106.

Gagnon, P. (1995, December). What should children learn? *The Atlantic*. Retrieved from https://www.theatlantic.com/magazine/archive/1995/12/what-should-children-learn/376491/

Goll, T. (2014). Problemorientierung. In W. Sander (Ed.), *Reihe Politik und Bildung: Band 69. Handbuch politische Bildung* (4th ed., pp. 258–266). Schwalbach/Ts.: Wochenschau Verlag.

Greefrath, G., Kaiser, G., Blum, W., & Borromeo Ferri, R. (2013). Mathematisches Modellieren – Eine Einführung in theoretische und didaktische Hintergründe. In R. Borromeo Ferri, G. Greefrath, & G. Kaiser (Eds.), *Mathematisches Modellieren für Schule und Hochschule* (pp. 11–37). Springer Fachmedien Wiesbaden. https://doi.org/10.1007/978-3-658-01580-0_1

Henkenborg, P. (2012). Politische Urteilsfähigkeit als politische Kompetenz in der Demokratie: Der Dreiklang von Analysieren, Urteilen Handeln. *Zeitschrift Für Didaktik Der Gesellschaftswissenschaften*, *3*(2), 28–50.

Jann, W., & Wegrich, K. (2007). Theories of the Policy Cycle. In F. Fischer & G. Miller (Eds.), *Public Administration and Public Policy. Handbook of Public Policy Analysis: Theory, Politics, and Methods* (pp. 43–62). London: Routledge.

Juchler, I. (2012). Politisches Urteilen. *Zeitschrift Für Didaktik Der Gesellschaftswissenschaften*, *3*(2), 10–27.

Kant, I. (2015). *Die drei Kritiken: Kritik der reinen Vernunft (1781/87) ; Kritik der praktischen Vernunft (1788) ; Kritik der Urteilskraft (1790)*. Köln: Anaconda.

Lange, D. (2008). Citizenship Education in Germany. In V. B. Georgi (Ed.), *Schriftenreihe / Bundeszentrale für Politische Bildung: Vol. 666. The making of citizens in Europe: New perspectives on citizenship education* (pp. 89–95). Bonn: Bundeszentrale für Politische Bildung.

Massing, P. (1995). Wege zum Politischen. In P. Massing & G. Weißeno (Eds.), *Schriften zur politischen Didaktik: Vol. 24. Politik als Kern der politischen Bildung: Wege zur Überwindung unpolitischen Politikunterrichts* (pp. 61–98). Wiesbaden: VS Verlag für Sozialwissenschaften.

Mau, S. (2017). *Das metrische Wir: Über die Quantifizierung des Sozialen*. Suhrkamp Verlag.

Michelsen, G., & Overwien, B. (2020). Nachhaltige Entwicklung und Bildung. In P. Bollweg, J. Buchna, T. Coelen, & H.-U. Otto (Eds.), *Handbuch Ganztagsbildung* (pp. 557–574). Wiesbaden: Springer Fachmedien Wiesbaden.

Niss, M. (1983). Considerations and experiences concerning integrated courses in mathematics and other subjects. In M. J. Zweng, T. Green, J. Kilpatrick, H. O. Pollak, & M. Suydam (Eds.), *Proceedings of the Fourth International Congress on Mathematical Education* (pp. 247–249). Boston: Birkhäuser.

Oberle, M. (2016). Politikwissenschaft als Bezugsdisziplin der Politischen Bildung. In M. Oberle & G. Weißeno (Eds.), *Politische Bildung. Politikwissenschaft und Politikdidaktik: Theorie und Empirie* (pp. 17–29). Wiesbaden: Springer Fachmedien Wiesbaden.

OECD. (2018b). *PISA 2022 mathematics framework (draft)*. Paris. https://pisa2022-maths.oecd.org/files/PISA%202022%20Mathematics%20Framework%20Draft.pdf

OECD. (2018a). *The future of education and skills: Education 203*. Paris. https://www.oecd.org/education/2030/E2030%20Position%20Paper%20(05.04.2018).pdf

Ojose, B. (2011). Mathematics Literacy: Are We Able To Put The Mathematics We Learn Into Everyday Use? *Journal of Mathematics Education*, *4*(1), 89–100.

O’Neill, J. (2002). The rhetoric of deliberation: Some problems in Kantian theories of deliberative democracy. *Res Publica*, *8*, 249–268.

Print, M. (2013). Competencies for democratic citizenship in Europe. In M. Print & D. Lange (Eds.), *Civic and political education: Vol. 3. Civic education and competences for engaging citizens in democracies* (pp. 37–51). Rotterdam: Sense Publishers.

Reinhardt, S. (2013). Teaching for democratic learning. In M. Print & D. Lange (Eds.), *Civic and political education: Vol. 3. Civic education and competences for engaging citizens in democracies* (pp. 99–109). Rotterdam: Sense Publishers.

Reinhardt, S. (2018). *Politik Didaktik: Handbuch für die Sekundarstufe I und II*. *Fachdidaktik*. Berlin: Cornelsen.

Straehler-Pohl, H. (2017). De|mathematisation and Ideology in Times of Capitalism: Recovering Critical Distance. In H. Straehler-Pohl, N. Bohlmann, & A. Pais (Eds.), *The Disorder of Mathematics Education* (pp. 35–52). Springer. https://doi.org/10.1007/978-3-319-34006-7_3

Strømskag, H. (2021). Prospective teachers solving a percentage problem: An analysis of the construction of a praxeology. In B. Barquero, I. Florensa, P. Nicolás, & N. Ruiz-Munzón (Eds.), *Extended Abstracts Spring 2019. Trends in Mathematics 13* (pp. 77–86). Cham: Birkhäuser. https://doi.org/10.1007/978-3-030-76413-5_9

Triantafillou, P. (2020). Trapped in the Complexity Bowl? Public Governance and the Liberal Art of Governing. *International Journal of Public Administration*, *43*(14), 1228–1236.

Weber-Stein, F., & Engel, J. (2021). Civic Statistical Literacy und politische Bildung im Informationszeitalter. Kooperative statistik- und politikdidaktische Erkundungen im Feld der Demokratiemessung. In C. Deichmann & M. Partetzke (Eds.), *Demokratie im Stresstest* (pp. 165–192). Springer Fachmedien Wiesbaden. https://doi.org/10.1007/978-3-658-33077-4_10

Zweng, M. J., Green, T., Kilpatrick, J., Pollak, H. O., & Suydam, M. (Eds.). (1983). *Proceedings of the Fourth International Congress on Mathematical Education*. Boston: Birkhäuser. https://doi.org/10.1007/978-1-4684-8223-2

Co-funded by
the European Union