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Aftermath & The Unseen Angles

Exploring the Impact of Game-Based Learning in Trigonometry - An Empirical Study

Master's thesis in Master of Science in Informatics
Supervisor: Alf Inge Wang
June 2023

Master's thesis

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Norwegian University of Science and Technology
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Abstract

Mathematics is often perceived as abstract and disconnected from reality, something which must be memorized rather than understood. This can lead to frustration, boredom, and disinterest. Meanwhile, students willingly spend several hours every day immersed in a virtual game world. How can this motivation be transferred into mathematics to facilitate a deeper and more comprehensible learning experience?

The research goal of this study is: *To develop an engaging and educational learning game and evaluate the effectiveness of game-based learning in high school mathematics.* Initially, a literature review was conducted to investigate relevant research, covering learning theories, mathematics, and enjoyment of video games. The literature review was used to develop Aftermath: a game-based learning platform about high school trigonometry, facilitating the exploration of mathematical concepts through an interactive notebook. The game was used in a practical experiment with 35 Norwegian high school students.

The study found that game-based learning significantly benefits students who view mathematics as something to be memorized, providing a more conceptual understanding of mathematics. General enjoyment of mathematics correlated with enjoyment in Aftermath. Enjoyment, learning outcomes, and motivation were found to be gender-independent. Video game players were less immersed but still benefit from game-based learning if the game is achievement and reward-focused, and overly difficult learning games reduce enjoyment and learning efficiency. A generalized model for estimating game enjoyment in mathematical learning games has been conceptualized, named the Game Enjoyment Factor (GEF). The research further suggests that game-based learning in trigonometry can effectively reinforce conceptual understanding and serve as a useful repetition tool, but its efficiency as an introduction tool was inconclusive.

These findings provide valuable insights for educators, game designers, and researchers about how personal attributes and game design can affect learning outcomes. The research emphasizes the potential of game-based learning to enhance student engagement and learning in mathematics.

Sammendrag

Matematikk blir ofte oppfattet som abstrakt og frakoblet fra virkeligheten, noe som må memoreres i stedet for å forstås. Dette kan føre til frustrasjon, kjedsomhet og manglende interesse. Samtidig tilbringer elever gjerne flere timer hver dag i en virtuell spillverden. Hvordan kan denne motivasjonen overføres til matematikk for å legge til rette for en dypere og mer forståelig læringsopplevelse?

Målet for denne studien er: *Å utvikle et engasjerende og pedagogisk læringspill og evaluere effektiviteten av spillbasert læring i matematikk på videregående skole.* Først ble det gjennomført et forstudie på eksisterende relevant litteratur innenfor læringsteorier, matematikk og glede av videospill. Forstudiet ble brukt til å utvikle Aftermath: en spillbasert læringsplattform om trigonometri i videregående skole, som tilrettelegger for utforskning av matematiske konsepter gjennom en interaktiv matematikkbok. Spillet ble brukt i et praktisk eksperiment med 35 elever ved norske videregående skoler.

Studien fant at spillbasert læring gir betydelige fordeler for studenter som ser på matematikk som noe som må memoreres, og gir en mer konseptuell forståelse av matematikk. De som likte matematikk hadde generelt sett en bedre spillopplevelse av Aftermath. Spillopplevelse, læringsresultater, og motivasjon viste seg å være uavhengig av kjønn. Elever som liker å spille var mindre engasjert, men drar fortsatt nytte av spillbasert læring hvis spillet er fokusert på prestasjon og belønning, og overdrevent vanskelige læringspill reduserer spillopplevelsen og læringseffektiviteten. En generell modell for å beregne spillopplevelse i matetmatiske læringsspill har blitt utviklet, kalt GEF (Game Enjoyment Factor). Forskningen antyder videre at spillbasert læring i trigonometri kan forsterke konseptuell forståelse og fungere som et nyttig repetisjonsverktøy, men også at effektiviteten som et introduksjonsverktøy ikke var entydig.

Disse funnene gir verdifull innsikt for pedagoger, spilldesignere og forskere om hvordan personlige attributter og spilldesign kan påvirke læringsresultater. Forskningen understreker potensialet til spillbasert læring for å øke studentenes engasjement og læring i matematikk.

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Part I

Introduction

This Part outlines the *why*, *what*, *how*, and *but* of the thesis. Firstly, *why* was this thesis made, describing the motivation that sparks the interest for this project. Secondly, *what* is this thesis about, detailing the research goal and the key questions the study seeks to answer. Thirdly, *how* was the research conducted, laying out the research methods used to answer the research questions. Lastly, *but*, what are the potential challenges and limitations concerning the validity of the research? With this introduction, the stage is set for a compelling voyage into the field of trigonometry and game-based learning.

Chapter 1

Motivation

There is something inherently fascinating about mathematics, a language that describes the intrinsic order of the universe. Yet, many perceive mathematics as abstract, distant, and difficult to understand. They see mathematics as an obscure web of symbols and equations that seems unrelated to our daily lives. Despite its pervasiveness and its undeniable importance in various fields throughout human history, mathematics often comes across as a discipline that is impersonal, difficult to grasp, and hence, daunting.

In traditional academic environments, mathematics is also often presented as a subject without real-world applications. Students are simply taught to follow the rules and are not given context. This detachment from reality often leads to students viewing mathematical concepts as something to be memorized rather than understood. However, mathematics is not a collection of static truths, but rather a dynamic system of interconnected concepts that reveal patterns, formulate theories, and solve practical problems.

Educational research has emphasized the need for context, engagement, and experiential learning in teaching complex subjects like mathematics [1][2]. The question then is how do we transform abstract symbols into comprehensible, relatable concepts? How do we move from passive learning and rote memorization techniques, to a deeper and more conceptual understanding? And how can learners take part in the discoveries of these concepts, making them feel like they could have invented them themselves? One answer may lie in game-based learning.

Game-based learning leverages the innate human desire for entertainment, transforming the educational process into an engaging and enjoyable experience. When mathematical concepts are presented through the lens of a game, they become less abstract and more a part of a player's reality. A concept such as trigonometry, often considered complex and abstract, can suddenly become a tool to solve a captivating puzzle or overcome a challenge in the game.

The true power of game-based learning, especially in mathematics, comes from its ability to visualize abstract ideas. The transformation of mathematical notions from abstract symbols into visual components can enhance conceptual understanding, promoting deeper and more meaningful learning. It allows students to *see* and *experience* the application of the principles they are learning, fostering a better comprehension of their significance and usage.

Moreover, game-based learning encourages trial and error, exploration, and problem-solving, skills that are critical not just to mathematics but also to broader cognitive and personal development. It can create a safe and interactive space where students can test their understanding, learn from their mistakes, and develop confidence in their abilities.

Apart from these benefits, game-based learning aligns with the psychological construct of *flow*, which is a mental state in which a person performing an activity is fully immersed and gets a sense of full involvement and enjoyment in the activity [3]. As video games inherently embody flow, game-based learning may turn mathematics, often seen as a tedious task, into a process of joyous discovery. Each new mathematical concept suddenly becomes a level-up and a win to celebrate.

To summarize, the motivation to embark on this project is driven by the idea that the right blend of technology, pedagogy, and gameplay can reshape the perception and understanding of mathematics. This Master's thesis is a journey towards building an educational learning game that would provide a platform for this shift – from the abstract, complicated and intimidating to the tangible, comprehensible, and enjoyable exploration of mathematical concepts.

> *"The essence of mathematics is not to make simple things complicated, but to make complicated things simple"*
>
> - Stanley Gudder

Chapter 2

Research Goal & Questions

Building on the motivation outlined in the previous Chapter, the primary objective of this particular study will be presented. This Chapter will describe the research goal of the project, which is then broken down into five research questions that the thesis will attempt to answer. An explanation and contextual description will be provided for each research question, highlighting why they are relevant and important to the thesis.

The research goal for this study is:

> *To develop an engaging and educational learning game and evaluate the effectiveness of game-based learning in high school mathematics.*

The research goal sets a broad task, which has multiple aspects that each pose its own set of unique challenges. To fully address these different aspects and challenges, five distinct research questions have been formulated, and a description of why they are important and how they relate to the goal is described.

- **RQ1:** ***How does attitude towards mathematics affect learning outcome and enjoyment in game-based mathematics learning?***

  RQ1 assesses the influence of students' attitudes towards mathematics on their learning outcomes and the enjoyment derived from a game-based learning approach. This directly addresses both the "*engaging*" and "*educational*" part of the research goal, by seeking to understand the role of students' predispositions in their engagement and success within the game-based learning environment.

- **RQ2:** ***Which personal attributes affect player enjoyment in game-based learning?***

  RQ2 investigates the personal attributes that might influence the level of enjoyment students experience in game-based learning. This question touches on the '*engaging*' aspects of the research goal, seeking to understand how personal attributes contribute to enjoyment, engagement, and the overall educational effectiveness of the game.

- **RQ3:** ***How effective is game-based learning as an introduction to trigonometry?***

  RQ3 explores the effectiveness of the developed game as an introductory tool for trigonometry. This addresses the "*educational*" aspect of the goal by assessing whether game-based learning can foster a solid foundational understanding and facilitate comprehension in this complex and abstract area of mathematics.

- **RQ4:** ***How effective is game-based learning as a repetition of trigonometry?***

  RQ4 examines the effectiveness of the game-based approach as a reinforcement tool for trigonometry. This also touches upon the "*educational*" aspect of the research goal by probing the role of game-based learning in facilitating repetition and consolidation, critical processes in mathematical learning. It aims to evaluate whether the developed game aids in improving conceptual understanding for students who use it for revising the subject matter.

- **RQ5:** ***How do the theories in game-based learning contribute to the players' enjoyment, motivation, and learning outcome in an educational mathematics game?***

  RQ5 investigates the impact of game-based learning theories on the players' enjoyment, motivation, and learning outcomes in the educational game. This addresses the fundamental aspects of the research goal to develop an engaging and effective learning tool. By interrogating the effectiveness of these theories, it hopes to understand their influence on player engagement, motivation, and learning outcomes in the educational game context.

With the research goal defined and research questions established, the study now transitions towards selecting appropriate research methods. These goals and questions will guide the choice of research methodologies, which will be discussed in the next Chapter.

Chapter 3

Research Methods

As described in Chapter 2, the purpose of this study is to explore the effectiveness of game-based learning through a prototype application aimed at high school mathematics students. Before conducting research, it is important to establish a clear research process. This chapter will describe the research model presented by Oates, B.J. in her book, *Researching Information Systems and Computing* [4]. Then, the methods that apply to this particular project will be described in further detail. Lastly, the specific research process for this project will be presented.

3.1 Research Model

Oates categorizes the different aspects of research as *the 6 Ps*: *purpose*, *products*, *process*, *participants*, *paradigm*, and *presentation* [4, pp. 11-13]. She further states that in academic research, it is not sufficient to just come up with an answer to a problem; the process followed must be put to the scrutiny of other academics [4, p. 32], and that a carefully planned research model is necessary for a research paper to be accepted.

To conduct reliable academic research and give a systematic overview of the research process, Oates proposes a generalized model [4, p. 33]. The model is shown in Figure 3.1, with the components used in this thesis highlighted in blue. It is composed of various components in different phases of the research project and helps provide an overview of the research process of a given research project. In the following sections, each of the highlighted components will be described.

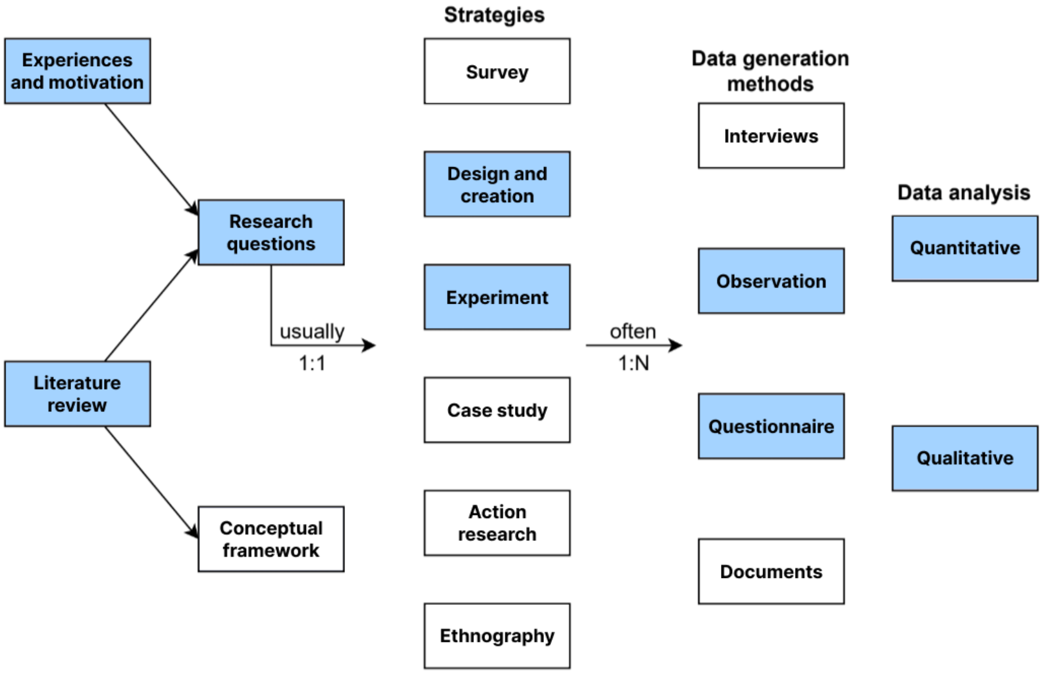


Figure 3.1: The research process model for this thesis, based on the generalized model by Oates [4, p. 33].

3.2 Experiences and Motivation

One of the aforementioned 6Ps of academic research is *purpose*. Determining the purpose of a research project (why the project is important or useful), is important for the quality of research. According to Oates, research without a purpose is unlikely to be good research [4, p. 11].

Before commencing a research project, it is important to think about your motivation, as well as your personal experiences, likes, dislikes, strengths, and weaknesses. This will help in thinking about possible research topics that may be addressed, and it is important for maintaining motivation through the project, particularly at boring or frustrating times [4, pp. 33-34].

3.3 Literature Review

Before a research project, it is also important to review previous research. Studying existing literature discovers what has been done before and what topics remain to be addressed. This is necessary when deciding upon a viable research question that has not already been fully addressed [4, p. 34], and lays the foundation for the research project [4, p. 73].

There is a wide variety of sources that may be used in a literature review, but books, journal articles, and conference papers are typically used [4, p. 34]. For research involving the design and/or creation of software, it is also important to study existing software that is related to the research project.

3.4 Research Questions

After figuring out the purpose and topics of the research, the next step is to concretize the topics by converting them into research questions. Oates does not explicitly define what a research question is or what constitutes a good research question, but a more clear definition is given by Mattick, K. et al. They define a research question as *"a question that a research project sets out to answer"* [5]. They also discuss what makes up a good research question, and suggest that a good research question is usually quite narrow or specific and focuses on an important topic. Research questions should aim to contribute a tiny step to the existing research knowledge, rather than making huge leaps, as broad questions are unlikely to be answered in a short time frame and can lack direction and impact [5]. Note that the article by Mattick, et al. is aimed at clinical studies, but should serve as a general definition and description of research questions across fields of study.

3.5 Design and Creation

With clearly defined research question(s), one or more research strategies can be selected. Design and creation is a research strategy that focuses on developing new IT products, such as a website, group support system or computer animation [4, pp. 108-109]. For such projects to be considered academic research, rather than only an illustration of technical skills, they should also demonstrate academic qualities and contribute to knowledge in some way [4, p. 109]. How such projects might contribute to knowledge depends on the role that the IT system plays in the project. The IT system can have one of three roles:

- **The main focus of the research:** In research projects where a new IT product is the main focus of the research, the IT product is itself a contribution to knowledge. For example, an IT application may be designed to automate a domain that has not previously been automated. An IT application can also incorporate a new theory as a way to test the theory, such as a web application testing a theory on the psychology of colors in different cultures.
- **A vehicle for something else:** An IT product may be developed not as the main focus of the research, but rather to supplement other research or visualize or demonstrate the results of other research. For example, conclusions from a literature review or field research may be illustrated via a prototype IT application, such as a website.

- **A tangible end-product where the focus is on the development process:** A research study analyzing development processes may develop an IT product simply to give test subjects something to develop. For example, two test groups may be asked to develop an application with either an agile or waterfall approach, with the purpose to compare the efficacy of the two approaches.

3.6 Experiment

The experiment research strategy focuses on investigating cause and effect, testing hypotheses and seeking to prove or disprove a causal link between a factor and an observed outcome [4, p. 35]. It is the central part of positivism, which is a philosophical paradigm where empirical evidence is used to prove or disprove hypotheses in an attempt to explain the world, and is a fundamental aspect of the scientific method [4, pp. 283-284]. In the experiment strategy, researchers start by developing a theory or hypothesis about a given topic and then designing and executing an experiment to test the hypothesis.

A common issue in experimental studies is pollution from other factors than the factor the research is meant to investigate. In other words, even if an experiment produces positive results, the research can not necessarily consider the experiment successful because the effect may have been caused by some unrecognized factor. Thus, firm conclusions can generally not be drawn from an experiment until the experiment has been repeated many times by themselves and other researchers [4, p. 127]. Minimizing the effects of other factors is important to ensure high internal and external validity, which will be explained in further detail in Chapter 4.

3.7 Observations

A data generation method is the means by which empirical field data or evidence is produced [4, p. 36]. One such data generation method is observations, where the researchers observe test subjects and their behavior. Oates distinguishes between two types of observations: overt and covert [4, pp. 203-204].

3.7.1 Overt and Covert Observations

In overt observations, the test subjects are told that they are part of a research project and their behaviour will be observed. One potential issue with this approach is the Hawthorne Effect, which is a change in the test subjects' behaviour due to test subjects being aware they are being observed [6, p. 1]. This effect may negatively affect the validity of the research and will be further described in Section 4.2.

The alternative approach to the observation method is covert observations, where the test subjects are not informed that they are being observed. This eliminates the

Hawthorne effect but introduces other issues. One issue is that the observers have to make sure no one realizes what they are doing, which means not asking too many questions, not criticizing anything that is seen as normal by those being observed, and not drawing attention to themselves [4, p. 204]. Furthermore, observing people without them knowing is an ethical issue as the test subjects have not given consent to the research.

3.7.2 Systematic and Participant Observation

Oates also distinguishes between systematic and participant observation. In a systematic observation, the researcher works with a pre-defined system of observations, which usually involves counting or timing, to generate quantitative data [4, pp. 204-205]. For example, researchers may want to observe the queue at a customer help desk to measure the time of arrival, time reaching the head of the queue, and time spent dealing with the customer query.

The alternative approach is participant observation, which is an observation method where the researcher takes part in the situation under study, so that it can be experienced from the point of view of the others in that setting [4, p. 208]. This typically leads to more qualitative results. Oates further divides participant observation into four sub-groups: complete observer, complete participant, participant-observer, and practitioner-researcher.

- A *complete observer* is present in the setting either overtly or covertly, observing everything that occurs, but takes no other part in the proceedings. For example, a researcher might sit in on a teacher's class to watch everything that occurs, but take no part in the lesson and class activities [4, p. 209].
- A *complete participant* uses covert observation and tries to become a member of the group being researched, to see the group's world from the inside.
- A *participant-observer* is similar to a complete participant, but can be used if you do not have the necessary credentials or for some other reason is unable to be a complete participant. A participant-observer shadows someone and follows people as they go about their lives or jobs, observing the activities and interactions, and taking part where possible.
- A *practitioner-researcher* is someone who is already part of the group and assumes the role of a researcher, either covertly or overtly.

3.8 Questionnaire

Another data generation method is questionnaire, which is a pre-defined set of questions assembled in a pre-determined order [4, p. 36]. Respondents are asked to answer the questions, often via multiple-choice options or Likert-scale questions, thus providing the researcher with data that can be analyzed and interpreted. Questionnaires are widely used in research because they provide an efficient way of collecting data from many people [4, p. 220].

Different question types may be used in a questionnaire, depending on what type of information the researchers want to gather, and a combination of different question types may be used. Questions are divided into two main categories: open questions and closed questions.

Open questions leave the respondent to decide what answer to give — you just leave a blank space for them to fill in as they see fit. Such questions are especially useful when there is a wide range of possible answers or it is difficult to assume how the respondents are likely to answer. On the other hand, open questions require more effort than closed questions from the respondents. The responses are also harder to code and analyze than responses to closed questions [4, pp. 222-223].

Closed questions force the respondent to choose from a range of pre-defined answers. Closed questions and their response formats take longer to design than open questions, as the researcher has to make sure all possible answers are provided. However, the responses can be more quickly analyzed, because they have been pre-coded. For closed questions with numerical values, statistical analysis of the responses is quick. Closed questions save the respondents time in answering, but can also cause them frustration if they can not find the pre-defined response that matches the answer they want to give. Closed questions can also be criticized for enabling respondents to answer quickly without thinking much about their responses, and for putting answers into the respondents' minds that they might not otherwise have come up with [4, p. 223].

3.9 Quantitative Analysis

After data has been generated with the chosen research strategies and data generation methods, the results must be analyzed. There are two types of results: quantitative and qualitative.

Quantitative data means data, or evidence, based on numbers [4, p. 245], and the goal is to find patterns in the data and draw conclusions. Tables, charts and graphs are typically used to detect patterns, but algorithms can also be used to automatically detect patterns in the data.

3.10 Qualitative Analysis

Qualitative data includes all non-numeric data (e.g. words, images, sounds, and so on), and qualitative analysis looks for themes and categories within this data [4, p. 38, 266]. Unfortunately, qualitative data analysis is not always a straightforward task. There are no hard and fast rules about how to do it. Whereas quantitative data analysis can draw upon well-established mathematical and statistical procedures qualitative analysis has fewer procedures and is more dependent on the skill of the researcher to see patterns and themes within the data [4, p. 267].

Note that there is a distinction between qualitative data and qualitative analysis. You can use quantitative analysis on qualitative data. For example, you may count the number of times a particular word or phrase occurs in some text [4, pp. 266-267].

3.11 Applied Research Process

With a description of the different research process components to be used in this project, the research process for the project may be presented.

3.11.1 Study Preparations

As mentioned in Chapter 1, game-based learning is a topic with growing interest within education, and a research project about game-based learning seemed exciting. As enthusiastic programmers and avid video game players, the previous experience and interest held by the writers of this thesis may be useful for such a project. However, the knowledge of previous research on learning principles is somewhat limited.

As a preparation for the research project, a literature review of the principles of learning was conducted, described in Chapter 5. Chapter 6 will give a more in-depth analysis of learning within the topic of mathematics, and Chapter 7 will give a brief overview of trigonometry, specifically the elements of trigonometry that have been used in the game-based learning platform developed for this research project. The literature review also studies previous work on the topic of game-based learning and enjoyment in video games, shown in Chapter 8. Based on this information, a few relevant existing applications have been analyzed, shown in Chapter 9.

Based on past experiences and motivation and the literature review, a research goal has been established and 4 research questions composed. The research goal and research questions are described in Chapter 2. With a clear objective for the research, suitable research strategies can be established.

3.11.2 Strategies

As shown in Figure 3.1, the research strategies chosen for this project are design and creation, and experiment. A game-based learning platform has been developed, aimed at teaching high school students about trigonometry. It is a prototype and the main focus of the research, as the research aims to study how such a learning platform may be further developed and used as a central part of high school education. Chapter 10 describes the final product, and Chapter 11 explains the reasoning for the development decisions, based on the previously conducted literature review.

Chapter 12 briefly describes the development process, and 13 briefly describes the technical implementation and technologies used in the application.

With the developed IT product, an empirical experiment has been conducted in three separate sessions. Chapter 14 explains the participant selection process and briefly describes the participant demographics and sampling. In Chapter 15, the execution of the experiment is described.

Two research strategies (design and creation, and experiment) have been used for the project. According to Oates, one research question typically has one research strategy [4, p. 35]. However, for this particular project it is reasonable to use two, as the design and creation of a prototype does not provide any academic results without an experiment, and an experiment can not be conducted without the prototype.

3.11.3 Data Generation

The experiments were conducted as participant observations, where the researchers acted as complete observers and observed the test subjects to take note of any behaviour of particular interest. For ethical reasons, the observations were made overtly. The results of the qualitative observations are shown in Chapter 21.

Another data generation method used in the project is a questionnaire. The test subjects were asked to fill out a digital questionnaire at the end of the experiment. A mix of closed questions (multiple-choice and Likert-scale questions) and open questions was used. All of the questions asked are listed in Chapter 16. The results from the multiple-choice questions are shown in Chapter 18, and the results from the text questions are shown in Chapter 20.

For this research project, both a qualitative (observation) and a quantitative (questionnaire) data generation methods were used. Using more than one data generation method is suggested by Oates to corroborate findings and enhance their validity, and is called method triangulation [4, p. 37]. Validity in empirical research will be further described in Chapter 4.

3.11.4 Data Analysis

The data from the closed questionnaire questions is quantitative and the data from the open text questions and qualitative observations are qualitative. The qualitative data has been studied in a statistical analysis, described in Chapter 19. In the analysis, correlations between different responses have been investigated. The validity of these correlations have been analyzed, using Mann-Whitney U test for correlations with two groups and Kruskal-Wallis test for other data. The qualitative data has been used to gain a basic understanding of the general perception of the game, and to support the statistical analysis.

3.12 Summary

Oates introduces *the 6 Ps* (purpose, products, process, participants, paradigm, and presentation) to categorize different aspects of research. She also describes a generalized research model to ensure reliable academic research with components outlining the research process. Determining the purpose and motivation for a project and reviewing previous research is an important first step to research, followed by the careful design of research questions which outline what the research aims to answer. The research process itself is divided into strategies, data generation and data analysis.

The strategies used in this thesis are design and creation, and experiment. The design and creation strategy aims to develop an IT product, which is then used in an empirical experiment. During this experiment, data is generated through observations and a questionnaire with a mix of open and closed questions. Lastly, the data is analyzed using both quantitative and qualitative methods. With an established research process, the next Chapter will explore some of the study's possible validity threats.

Chapter 4

Validity in Empirical Research

In experimental research, introduced in Section 3.6, validity generally refers to the extent that it measures what it is designed to measure and accurately performs the function(s) it is purported to perform [7, p. 129]. There are different categories of research validity. The two most common categories, introduced by Campbell in 1957 [8, p. 297], are *internal* and *external validity.* This chapter will describe these two in detail and describe the potential threats to each respective category.

4.1 Internal Validity

Internal validity relates to whether a study answers its research question in a manner free from bias [9, pp. 179-180]. High internal validity means that the results from the experiment are conclusive and not influenced by other factors. The threats to internal validity, as defined by Campbell, are history, maturation, testing, instrument decay, statistical regression, selection, and mortality [8, pp. 298-300].

History refers to events that occur during the experiment which are not directly related to the experiment itself. This may be an issue in studies that are conducted over an extended period (normally several days, months, or years). During this time, public news or other events may affect a test subject and their test results.

Maturation is similar to history. However, instead of external events, maturation relates to personal change. Depending on the duration of the test, a test subject may have grown older, hungrier, more tired, etc.

Testing is a threat to the internal validity of studies that modify the very thing they are meant to measure. For example, a test subject may be asked to do an IQ test at the beginning (pre-test) and end (post-test) of a study which is meant to measure intelligence over time. However, during the IQ test, the subject may learn to recognize certain patterns. At the end of the study, the test subject may score higher simply because they recognize the same patterns, not because the study made them smarter.

Instrument Decay (renamed to just *instrumentation* by Campbell and Stanley in 1966 [10, p. 9]), refers to changes in the measuring device during the test. This also applies to studies with humans as the measuring "device". For example, during an interview, an interviewer may become tired throughout a study. If an interviewer is interviewing several test subjects, the quality of the last interviews may not be as good as the first interviews due to the interviewer's fatigue.

Statistical Regression, also referred to as *regression toward the mean*, is the tendency for test subjects who score either very low or very high on an initial test to score closer to the mean the next time they are measured.

Selection is a potential bias related to the selection of test subjects. This also applies when a control group is used; who are placed in the test group and who are placed in the control group? Letting test subjects make this decision themselves or grouping test subjects based on some factor, may be a threat to internal validity, as the demographics and other properties of the control group no longer match the test group.

Mortality refers to the loss of test subjects during the study. Fortunately, mortality is not as morbid as the name may suggest. It is also referred to as *drop-out rate* or *attrition*, and refers to test subjects that are no longer members of the test group at the end of the test. For example, a study measuring students throughout their academic career may have a threat to its internal validity if students drop out of the university before they have received their degree.

4.2 External Validity

External validity asks the question of generalizability: To what populations, settings, treatment variables, and measurement variables can the effects of the experiment be generalized? [8, p. 297] High external validity means that the results from the experiments can be applied to the general population. Campbell introduces some threats to external validity in his original article [8, pp. 303-309], which are more clearly defined later by Campbell and Stanley as testing, selection bias, reactive effects of experimental arrangements, and multiple treatment interference [10, pp. 5-6]. Note that some of these threats overlap with the threats to internal validity described in Section 4.1. That is, some factors may be a threat to both internal and external validity.

Selection Bias is a commonly discussed term in the context of external validity, perhaps more so than for internal validity. To be able to confidently generalize the results of an empirical study, the test group must be a representation of the general population. Simply selecting random participants from the general population does indeed create a representative test group (assuming the sample size is large enough), but selecting randomly may often be difficult. Some participants may not want to participate, or researchers may (usually for the sake of simplicity) select one or more social groups within the general population.

Testing is a threat to external validity for a similar reason as to internal validity.

A pre-test might affect the test subject, which means that the results cannot be applied to a general population, as the rest of the population has not participated in that particular pre-test.

Reactive Effects of Experimental Arrangements is a general term for threats to generalization caused by the experiment itself influencing the test subjects [10, p. 6, 37]. There are several types of such reactive effects. One common experimental reactive effect is the **Hawthorne Effect**, which is a change in the test subjects' behavior or outcomes due to test subjects being aware they are being observed [6, p. 1]. For example, in an IQ test, particularly in a non-anonymous test, the test subjects may pay more attention because they know they are being tested. In studies where test subjects are introduced to a new technology, another type of such reactive effects is the **Novelty Effect**. The performance of test subjects may increase due to excitement and willingness to use the new technology, rather than because of any actual improvement. This performance increase is typically followed by a drop in performance later once its novelty has worn off [11]. The novelty effect is a common issue in educational empirical studies and studies on gamification and game-based learning [12][13].

Multiple Treatment Interference relates to studies that examine several factors in the same experiment. Even if the study gives positive results, the researchers cannot attribute the results back to one factor, because it is impossible to know which factor(s) contributed to the results.

4.3 Summary

The aforementioned 1957 study by Campbell and the 1966 study by Campbell and Stanley are considered pioneering studies on the topic of internal and external validity. More recent studies have introduced new threats and rephrased or removed some of the threats originally proposed. Furthermore, many of the threats depend largely on the field and type of study, and some threats may overlap others. In this chapter, the internal and external validity have been summarized, primarily based on the Campbell and the Campbell and Stanley studies, with a focus on the threats which relate to our particular field and type of study.

The validity threats described in this chapter will be referred to when describing the test subject sampling in Section 14.3, and in the discussion in Part VI, particularly in Chapter 26. In the next Part, the foundational introduction to the project will be used to conduct a literature review which will lay the foundation for the rest of the thesis.

Part II

Preliminary Research

Prior to the research and development of an educational game concept, preliminary research is necessary. This section comprises a summary of the studied material essential for this master's thesis. It begins by exploring the principles of learning and various learning theories, followed by the process of learning mathematics specifically, including its associated challenges. Next, the fundamental concepts of trigonometry are presented, providing insights into the learning material to be incorporated into the game developed for this thesis. Then, some of the theories related to game-based learning are presented, and the Part concludes with a review of existing applications relevant to this project.

Note that the principles of learning (Chapter 5), gamification and enjoyment in video games (Chapter 8) and analysis of existing applications (Chapter 9) are based on the pre-study conducted in the previous semester to prepare for this thesis [14].

Chapter 5

Principles of Learning

This chapter seeks to present some of the main theories within the field of learning. When developing a game-based learning platform, understanding the theory behind learning is as important as understanding the theory behind making a good game. The game is meant to not just be enjoyable, but also provide a learning outcome to its players. This Chapter is largely based on the pre-study conducted for this thesis [14].

5.1 What is Learning

Learning as a concept does not have a universally accepted definition. In the book *Learning Theories An Educational Persepctive* by Dale H. Schunk [15, p. 3], learning is defined as:

> *"an enduring change in behavior, or in the capacity to behave in a given fashion, which results from practice or other forms of experience"*

There are many theories on learning, and there seems to be no consensus on the categorization of these theories. Some view one theory as a subset of another theory, while others view a lot of them as standalone theories. However, there are some theories generally viewed as separate fields within learning theories: *behaviourism*, *cognitivism*, and *constructivism*. In the next sections, these learning theories will be defined and discussed. The theories are illustrated in Figure 5.1. The figure also includes subsets of the three learning theories. These are worth mentioning, but are beyond the scope of this project and will not be further discussed.

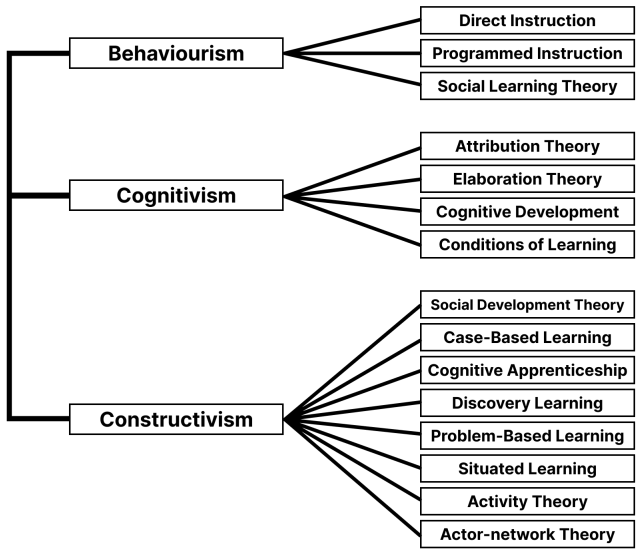


Figure 5.1: The three main types of learning theories [16]

5.2 Behaviourism

The behaviourism theory of learning is based on the work of Thorndikes, E.I., Watson, J.B., Pavlov, I., and Skinner, B.F. [15]. Within the field of behaviourism, Skinner's theory on *operant conditioning* is the most known. It views learning as something that happens through repetition of tasks, followed by reinforcement or punishment. Reinforcement is responsible for increasing the likelihood of a given response, while punishment seeks to decrease it. The rewards used for reinforcement are called reinforcers, and because they are defined by their effects, they can not be determined in advance [15, p. 90].

Skinner distinguishes two types of reinforcement: *positive* and *negative*. Positive reinforcement is providing something good to a situation, while negative reinforcement is removing something bad. Both reinforcements are used to increase the likelihood of a given response [15, p. 91]. An example of positive reinforcement is to receive praise on a correct answer, while an example of negative reinforcement is to stop criticism or remove homework. For punishment, the reinforcers are reversed to decrease the likelihood of a response, which simply means to take away something good or add something bad to a situation [15, p. 94]. In a classroom, a teacher models what they believe to be correct behaviour for the students, and then provides reinforcement for students that follow said behaviour. If the students do not follow this behaviour, they are punished in some form. For example, the teacher may lower students' grades, give out more homework, or in more extreme cases, punish students physically such as by slapping students' hands with a ruler. In video games, examples of positive reinforcement could be giving a player coins, special abilities or removing burdens that are slowing progress. A form of punishment would be to introduce burdens or remove coins and special abilities.

5.3 Cognitivism

The behaviourism theory in the previous section has later been criticized. Cognitivism is a general categorization of theories that emerged from dissatisfaction with the behaviourism theories.

In a study by Bandura, A., one of the challenges of behaviourism was presented. He concluded that humans could learn actions by simply observing others perform them. This suggests that reinforcement is not necessary for learning to occur [15, p. 118], which contradicts the behaviourist theories. Bandura's theory is referred to as *social cognitive theory* and is based on the assumption that people desire to control the events that affect their lives and to perceive themselves as agents [15, p. 122]. The theory views learning not only as a result of external impacts, but also through internal processes, or as Bandura explained it himself:

> *"People do not behave just to suit the preferences of others. Much of their behavior is motivated and regulated by internal standards and self-evaluative reactions to their own actions."*

The theory is known for the *triadic reciporcality* framework, illustrated in Figure 5.2, where interaction between personal factors, behaviour, and environment are the determinant factors for learning. Bandura coined the term *self-efficacy*, which simply put is the belief in one's ability to act in the ways necessary to reach specific goals. Self-efficacy is linked to the person within the triadic reciprocality, and the theory states that a person's perceived self-efficacy influences the behavior and vice versa (i.e. person <-> behavior) [15, p. 120]. An example would be to avoid doing tasks that are perceived to be difficult based on one's perceived capabilities, such as avoiding using your weaker foot when playing football (person → behaviour). In regards to video games, an example would be to choose the same player character or strategy one is comfortable with based on previous success (behavior → person). Further, the framework suggests that expectations from an environment also can affect the self-efficacy of students (environment → person). Hence, encouraging words could increase a student's self-efficacy. On the other hand, low self-efficacy could also affect the environment's expectations of them. As an example, teachers can judge students with learning disabilities as less capable and have a lower expectations for them than other students, even though they perform adequately (person → environment) [15, p. 120]. Lastly, the connections between behavior and environment can be exemplified with regards to teaching, where a teacher may reteach a subject if the students show confused behavior (behavior → environment). If students listen to the teacher by for example looking at an instructional video presented, the environment is affecting the behavior (environment → behavior).

Figure 5.2: Illustration of the triadic reciporcality [17]

5.4 Constructivism

Constructivism has a lot in common with cognitivism, and some would argue that it sprung out from the cognitivist field. Constructivism is mostly known from Piaget's theory of cognitive development [15, pp. 236-240], and Vygotsky's work on sociocultural theory [15, pp. 240-248]. Constructivism does, however, lack consistency in its definitions, and some would not consider constructivism alone to be a theory, but rather an epistemology or philosophy. Schunk argues that a theory needs to be testable, and constructivism does not provide a view of a world where principles of learning are to be discovered [15, p. 230]. This means that there are no scientific truths. A constructivist would argue that learners create their own learning, and that knowledge obtained is only true to that person. Knowledge is subjective and only a product of our cognition [15, p. 230-231]. Constructivists believe that learning is something that happens when new concepts are added to our existing structures in the brain. Thus, learning is something that would require active participants to connect new concepts, and teachers should therefore assume the role of facilitator rather than a source of knowledge the students should just listen to.

Both Piaget's and Vygotsky's theories have been important foundations for modern constructivism, but Piaget's theory is no longer a leading theory within the field of cognitive development. Thus, the rest of this paper will focus on Vygotsky's theory when discussing constructivism.

Vygotsky's theory stressed the importance of social, cultural, and individual factors as one of the most important factors for learning. Vygotsky rejected theories that viewed learning as something that happens in stages, which was the main theory of Piaget. He rather viewed learning as something that happened independently and as the result of social interactions. He stressed that the most critical component of psychological development was to think through symbols such as language, counting, and writing. Vygotsky's theory can therefore be viewed as a form of cognitive constructivism.

Throughout his work, one of his most influential concepts was Vygotsky's focus on the amount of learning a student can achieve under different conditions. He called one of these conditional situations the Zone of Proximal Development (ZPD), which he defined as:

"the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers"[18, p. 86].

The Zone of Proximal Development(ZPD) can be thought of as an environment where people are helped or guided by more competent and skilled individuals to reach their full potential. Vygotsky distinguished this environment from what you can do on your own, indicating that much more can be learned in such environments. This highlights the importance of knowledge sharing and collaboration to enhance learning. He also distinguished the Zone of Proximal development from the zone which can not be learned, indicating that humans have a limit on what they can achieve, even with the help of others. This relationship is illustrated in Figure 5.3

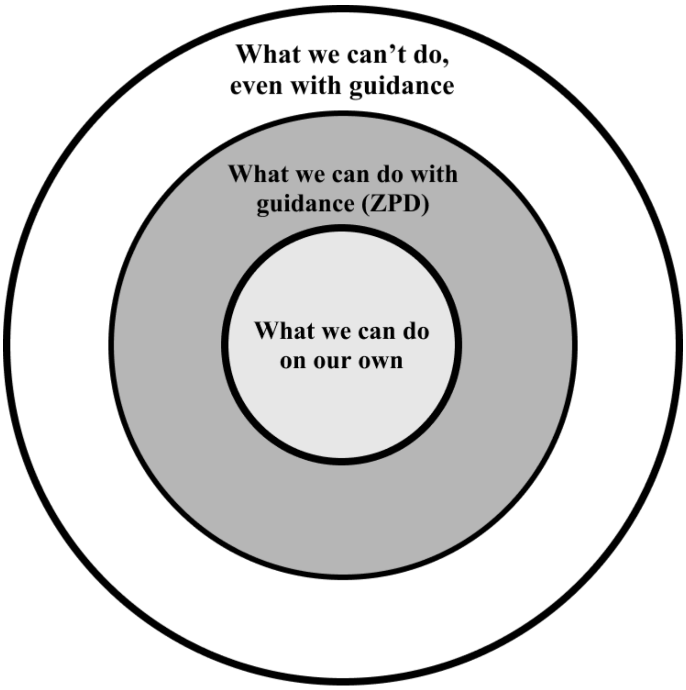


Figure 5.3: The Zone of Proximal Development [19]

5.5 Motivation

It is natural to assume that motivation is a key component to develop and master new skills. The more motivated a learner is, the more time and effort they will spend learning. However, there is a distinction in what motivates humans and the effects of these, categorized as *extrinsic* and *intrinsic motivation*.

Looking back to the Behaviourist view of learning described in Section 5.2, learning can be interpreted as something that occurs from the stimulus provided by the environment in the form of punishments or rewards. A person would for example be motivated to do a certain task in a certain way, just because they would be rewarded for it. This type of motivation is called *extrinsic motivation*, and it is defined as *"the performance of an activity in order to attain some separable outcome"* [20, p. 71]. With regard to education, an example could be a learner that wants to achieve higher grades to please their parents or to receive a teacher's approval.

A different type of motivation comes from a desire to engage in an activity not to receive a specific reward or to seek a specific outcome, but rather for no obvious reward than the task engagement itself [15, p. 386]. This type of motivation is what is commonly known as *intrinsic motivation*, and its importance on learning is shown by numerous research [15, p. 386]. In regards to education, a student may work harder on their mathematics homework, not because they want a higher grade, but because they think mathematics is interesting and want to learn more.

With both extrinsic and intrinsic motivation defined, motivation can generally be seen as a reinforcement coming from the environment or from ourselves. However, this does not mean that the two types are mutually exclusive and unable to affect each other. A study conducted by Edward Daci investigated external reinforcement on intrinsic motivation and found that external reinforcement such as money would have a negative impact on intrinsic motivation, while verbal reinforcements from social interactions tend to enhance it [21]. Extrinsic and intrinsic motivation may also be used in conjunction. A teacher may begin by offering a rewards such as verbal praise (extrinsic motivator), and work toward building student pride in their accomplishments (intrinsic motivator).

5.6 Flow

Another theory related to motivation, and particularly related to Intrinsic motivation, is the Flow theory derived by Mihaly Csikszentmihalyi [3]. The theory is based on the mental state of a person when they are engaging in a task that is intrinsically motivating. From his research Csikszentmihalyi found that people experienced a state of full immersion, a state which he coined the state of *flow*. This state can be understood as being so engaged in an activity that nothing else matters. Being in a flow state is often associated with peak performance, as people in this state can perform at their highest level. According to Csikszentmihalyi, the key to achieving a state of flow is to find a balance between the challenge of a task and the individual's skill level, which is illustrated in Figure 5.4. The graph shows that a task whose challenge is way below the skill level of the person performing it, can lead to boredom. And on the flip-side, a challenge that is to challenging can lead to anxiety. Csikszentmihalyi's flow theory is also the foundation of GameFlow and Kiili's experiential model, which will be described in more detail in Section 8.2 and 8.3.2 respectively.

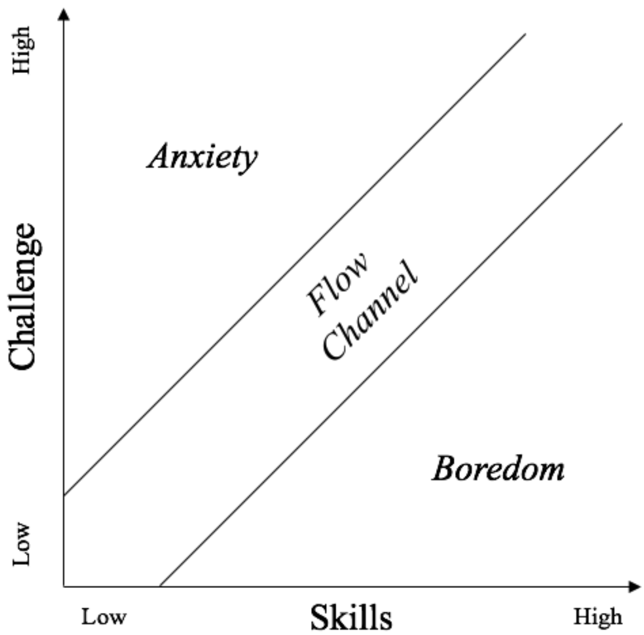


Figure 5.4: Flow-channel model adapted from Csikszentmihalyi [3]

5.7 Summary

This Chapter has covered some of the fundamental learning theories. The Behaviorism theory of learning involves the repetition of tasks followed by reinforcement or punishment to increase or decrease the likelihood of a response. Cognitivism emerged as a critique of Behaviorism and suggests that learning can occur through observation without the need for reinforcement, emphasizing internal processes and self-evaluation. Constructivism, stemming from cognitivism, emphasizes learners creating their own subjective knowledge through active participation. Motivation also plays a crucial role in learning, with extrinsic motivation driven by external rewards or punishments and intrinsic motivation stemming from an inherent desire to engage in an activity for its own sake. Furthermore, the Flow theory states that when individuals are fully immersed in intrinsically motivating tasks, they experience a state of flow characterized by complete engagement. With this foundational understanding of general learning theories, the next Chapter will explore learning specifically related to mathematics.

Chapter 6

Learning Mathematics

With an understanding of general learning concepts, this chapter delves into important aspects specifically regarding learning mathematics. Firstly, Math Anxiety, an emotional response that often hinders math performance, is explained. Then, the two concepts Procedural and Conceptual Understanding are discussed, emphasizing their critical role in grasping mathematics. Lastly, the relation between Concept Image and Concept Definition is explored, highlighting how personal interpretations and official definitions of concepts influence mathematical understanding.

6.1 Math Anxiety

One of the most researched topics within the domains of both learning and mathematics is *Math Anxiety*, which can be defined as [22, p. 176]:

> *"A feeling of tension, apprehension, or fear that interferes with maths performance."*

6.1.1 Performance Implications

Numerous studies have found that *math anxiety* is negatively correlated with math performance. One of the most prominent research within this field is the study *"The Relationships Among Working Memory, Math Anxiety, and Performance"* by Ashcraft, M. H. and Kirk, E. P. [23]. The authors wanted to understand the connection between math anxiety, working memory, and math performance. They found a strong negative correlation between math anxiety and math performance, meaning that students who reported higher levels of math anxiety tended to perform worse on math tests [23, pp. 235]. This is also supported by previous research [22, pp. 195 - 196]. Additionally, and most importantly, Ashcraft and Kirk discovered that math anxiety negatively impacted working memory, which is essential for complex problem-solving tasks [23, p. 227]. They hypothesized that the cognitive load induced by anxiety consumes working memory resources, leading to decreased performance [23, pp. 235-236]. In essence, math anxiety not only affects a person's emotional state but can also hinder the cognitive processes needed to perform mathematical tasks effectively.

However, academic anxiety is not limited to mathematics alone, and phenomena such as *test anxiety* and *general anxiety* have been shown to be more closely related to math anxiety than academic performance and ability [24, pp. 44-45]. In a 2016 review, Downker, A. et al. investigated the research on math anxiety over the past 60 years. The authors concluded that math anxiety can not be reduced to either general anxiety or test anxiety [25, p. 2], thus it should be viewed as an entity on its own. The authors reference the research conducted by Punaro and Reeve [26], which found mathematics to elicit stronger emotional reactions, and especially anxiety, than most other academic subjects. However, they also suggest this topic needs further research.

Other prominent findings within math anxiety research include the findings of Aschraft et al., which looked at learners' attitudes towards math. They discovered that low confidence and low motivation in math are strongly correlated to math anxiety [22, p. 178].

6.1.2 Strategies to reduce Math Anxiety

In the pursuit of effective treatments for math anxiety, researchers have identified several strategies that could potentially be incorporated into the design of a math game. In a literature review by Blazer, C., some of the most prominent treatments are highlighted. The treatments that apply to game-based learning are listed and summarized below:

- **Relating math to real life** - an emphasis on relating math to real life can counteract the abstract and intimidating nature of mathematics [27, p. 3]. Designing game scenarios that echo real-world applications can help students perceive mathematics as a practical and relevant tool.
- **Encourage active learning** - a second strategy is to encourage active learning [27, p. 3]. Games by their nature are engaging, and a well-designed math game can allow students to explore, practice, and actively apply mathematical knowledge, thereby mitigating anxiety.
- **Shift focus** - researchers suggest placing less emphasis on correct answers and computational speed [27, p. 3]. This aligns with the shift in math instruction towards a more understanding-based, process-oriented approach. A math game can therefore focus on the journey, rather than the end result, and provide feedback on the methodology, rather than the answer alone. Examples of practical implementations can be to avoid time constraints.
- **Manipulatives** - The use of manipulatives is also a noteworthy strategy [27, p. 4]. By including digital manipulatives within the game, abstract mathematical

concepts can be made more concrete and accessible, helping to lower barriers to understanding and thereby reducing anxiety.

- **Use of technology** - incorporating technology in the classroom can serve as a supportive tool for mitigating math anxiety [27, p. 4]. In this context, a math game can utilize technology to provide a friendly, engaging, and interactive platform for learning mathematics.

In summary, these strategies offer considerations when designing a math game aimed at reducing math anxiety, focusing on real-life relevance, active learning, process-oriented thinking, the use of digital manipulatives, and the application of technology.

6.2 Procedural vs Conceptual Understanding

One important aspect of learning mathematics involves the distancing between procedural and conceptual understanding. Procedural understanding refers to the knowledge of skills, algorithms, or procedures, while conceptual understanding refers to the comprehension of mathematical concepts, operations, and relations [28, p. 1119]. In short, and with relation to mathematics, procedural understanding can be thought of as knowing *how* to do something, while conceptual understanding is knowing both *how* to do it and *why*.

A significant contributor to the field of procedural versus conceptual understanding in mathematics was Richard Skemp, a British mathematician and educator. In his influential work *"Relational Understanding and Instrumental Understanding"*, he discussed these two forms of understanding under different terms [29]. Instrumental understanding, which parallels procedural understanding, was described as *"rules without reasons"* [29, p. 2], while relational understanding, akin to conceptual understanding, involved knowing both what to do and why. Skemp's work emphasized the importance of relational understanding, suggesting that this approach provides learners with the ability to adapt and apply their knowledge to new problems and contexts. This suggests conceptual learning can improve students learning outcomes and problem solving abilities.

Subsequent research has built on Skemp's foundational work, illustrating the intertwined relationship between procedural and conceptual understanding. One influential study conducted by Rittle-Johnson and Alibali found that the relationship between the two is iterative and reciprocal [30]. Learning procedures can lead to conceptual understanding, and conversely, understanding concepts can also support the learning of new procedures. However, their findings suggest that the influence of conceptual knowledge on procedural knowledge seems stronger than the reverse [30, p. 188].

In recent years, the exploration of these concepts has extended to digital environments. The growing interest in game-based learning has opened up new avenues for understanding how learners develop both procedural and conceptual knowledge in mathematics. For instance, a study conducted by Kiili et al. demonstrated the potential of digital games in developing mathematical understanding [31]. In their research, they used a game named Semideus [1] to evaluate and assess students' conceptual rational number knowledge. Their findings indicated that Semideus could effectively assess students' knowledge in this area and identify specific misconceptions related to whole number bias [31, p. 51]. They also conducted an intervention study using the Wuzzit Trouble game[2], which focuses on whole number arithmetic. Surprisingly, even though the game's primary focus is not rational numbers, they found that playing the game significantly improved students' rational number understanding [31, p. 51]. Their results highlighted the integrated nature of numerical development. Despite the differences between whole and rational numbers, understanding whole number magnitudes and achieving fluency in whole number arithmetic create a foundation for understanding rational numbers. This finding reinforces the idea that conceptual and procedural understanding are not isolated, where learning in one area can strengthen the other.

6.3 Grasping Mathematical Concepts

Another critical facet in the process of understanding mathematics is encapsulated in the theory of concept image and concept definition, proposed by Tall and Vinner [32]. Their work emphasizes the intricate relationship between these two aspects and their role in the comprehension and application of mathematical concepts.

The concept definition represents the official mathematical description of a concept, as provided in textbooks or formal teachings [32, p. 152]. For instance, a circle is defined as a shape wherein all points are at an equal distance from a central point. Conversely, the concept image embodies our personal visualizations and experiences associated with a particular concept. It is built up over the years through experiences and includes mental images, examples, and counterexamples [32, p. 152]. Continuing with the circle example, our concept image could include a pizza, a wheel, or the sun.

These two elements do not always perfectly align, which can lead to confusion or misconceptions, potentially hindering mathematical understanding. For example, a common divergence occurs with the concepts of square and rectangle. Formally, a square is a type of rectangle since they both have four right angles. However, our concept image typically differentiates these two shapes, based on our experiences that often classify squares and rectangles as distinct entities.

In the field of educational game design, it's vital to consider the two aspects of mathematical understanding: the formal *concept definition* and the personal *concept image*. These elements shape how learners perceive and interact with mathematical concepts. By incorporating strategies that address both aspects, game designers can create experiences that foster a more complete and nuanced

[1] https://seriousgamessociety.org/2016/09/22/semideus-a-game-for-mastering-rational-numbers/
[2] https://www.youcubed.org/resources/wuzzit-trouble/

understanding of mathematics. This can be particularly effective in games, which can provide diverse and engaging contexts that help link abstract mathematical ideas to concrete experiences, enhancing the formation of accurate and robust concept images.

6.4 Summary

This chapter has covered the important aspects of learning mathematics: Math Anxiety, Procedural vs Conceptual Understanding, and Grasping Mathematical Concepts. Addressing Math Anxiety in the design of educational games can create an environment that helps students overcome emotional barriers and perform better. The study of procedural and conceptual understanding opens pathways for game-based strategies that encourage a conceptual understanding, providing players with tools to comprehend the *how* and the *why* of mathematical procedures and concepts. The alignment between concept image and definition is also crucial. Designing games that help to form accurate mental images of mathematical concepts can prevent misconceptions and foster a deeper understanding. Overall, these theories intertwine to inform the development of educational video games, creating an environment conducive to learning, exploration, and mastery of mathematics. Following this exploration of learning mathematics, the next Chapter will delve into trigonometry, a specific branch of mathematics, and provide an overview of the mathematical knowledge necessary to create the game-based learning this thesis is built upon.

Chapter 7

An Overview of Trigonometry

This project aims to research how gamification and interactive visualizations may affect the quality of mathematics education in Norwegian high schools. High school mathematics is a broad topic, and to appropriately narrow the scope of the experiment the project focuses specifically on trigonometry. This chapter will provide an overview of the trigonometry taught in Norwegian high schools. However, the reader is assumed to be familiar with high school educational material, and the chapter will only provide a brief overview of the topic. If necessary, the reader is encouraged to explore *Trigonometry* by Sundstrom, T. and Schlicker, S. [33], which this chapter is based on. Other educational material on trigonometry may suffice, but note that the formulas and variables in this chapter are obtained from that book and may be different in other books or articles.

7.1 Unit Circle

Trigonometry is a branch of mathematics that can be used to model periodic phenomena such as sound and light waves, the tides, the number of hours of daylight per day at a particular location on Earth, and many other phenomena that repeat values in specified time intervals [33, p. 1]. Trigonometric functions are special because they are periodic; they repeat themselves in regular patterns. Consequently, to model periodic phenomena it is usually enough to just look at one of these repeatable patterns, rather than the whole graph of the function [33, p. 3]. One complete repeatable pattern is called a period, which will be further explained later.

To model and understand trigonometric functions, the x-value of one of these patterns may be wrapped around a circle, called a unit circle. It's called a unit circle simply because its radius is always equal to 1. The circumference of a circle is $2 * r * \pi$, which for the unit circle (where $r = 1$) is 2π. In other words, at $x = 2\pi$, one revolution (one period, one of the repeatable patterns) has been completed. The y-value of our periodic function corresponds to the y-value in the unit circle, and the x-value of the periodic function corresponds to the distance traveled around the circumference of the unit circle [33, pp. 2-5].

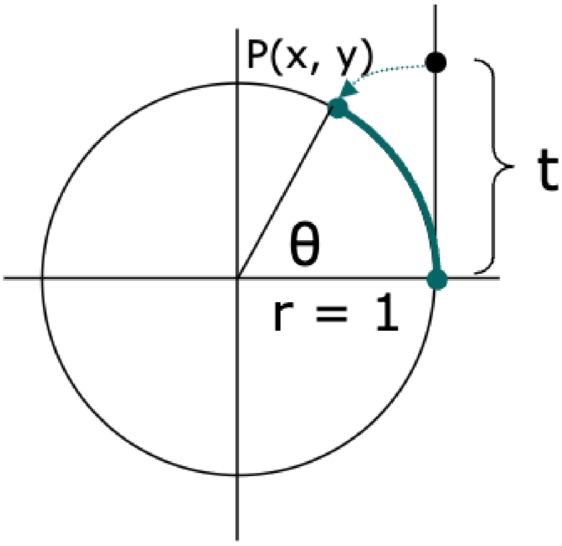


Figure 7.1: A trigonometric function being wrapped around the unit circle

7.2 Sine and Cosine

As explained in the section above, the y-value of the unit circle corresponds to the y-value in the periodic function. This y-value can be calculated with the sine function, often shortened to *sin*. That is, the y-value in the unit circle is given by $y = sin(t)$, where t is the x-value of the original periodic function (or the distance traveled around the circumference of the circle). Similarly, the cosine function, often shorted to *cos*, is used to calculate the x-value in the unit circle. The x-value is given by $x = cos(t)$. Thus, the point reached after traveling t distance around the circle is given by: $(x, y) = (cos(t), sin(t))$ [33, pp. 13-15].

Note that in the functions $y = sin(t)$ and $x = cos(t)$, the variable t is often given in radians. A radian is the angle that gives a corresponding arc with length 1. See Figure 7.2. As mentioned above, when one full revolution around the circle is completed, the length of the arc is 2π. In other words, $360° = 2\pi \approx 6.28$ [33, pp. 24-32].

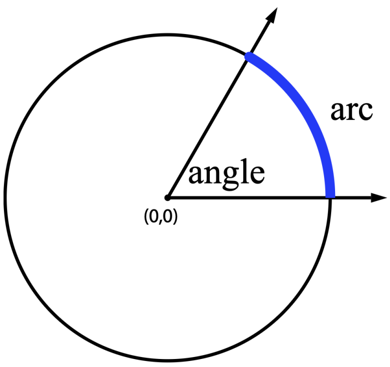


Figure 7.2: An arc and its corresponding angle

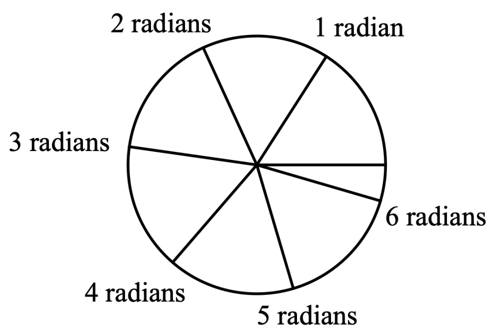


Figure 7.3: The unit circle with radians 1, 2, 3, 4, 5, and 6

7.3 Right Triangles & Tangent

Another way to think about sine and cosine is with right triangles, i.e. triangles with one 90° angle. As previously mentioned in Section 7.2, $y = sin(t)$. In Figure 7.4, sin can also be seen as the length of the vertical line opposite to the angle. Similarly, cos is the length of the horizontal, adjacent line. Formally, sine is defined as the length of the opposite line divided by the hypotenuse, $sin = \frac{opposite}{hypothenus}$. But, as the figure shows and as explained in Section 7.1, the hypotenuse (or radius) in a unit circle is always equal to 1, which effectively means $sin = opposite$. A similar deduction can be made with cosine, giving $cos = adjacent$ [33, pp. 178-181].

The last trigonometric function to mention is the tangent, or tan. The tangent function is given by the length of the opposite line divided by the length of the adjacent line, $tan = \frac{opposite}{adjacent}$. It follows that tan can also be expressed as $tan = \frac{sin}{cos}$ [33, pp. 63-64]. Note that tan is not defined for all real numbers. For example, $t = \frac{\pi}{2}$ is not valid, as this would give $tan(\frac{\pi}{2}) = \frac{sin(\frac{\pi}{2})}{cos(\frac{\pi}{2})} = \frac{1}{0}$, which gives an undefined result. As explained in Section 7.1 and Section 7.2, trigonometric functions are periodic, so $tan(t)$ is also not defined for for $t = \frac{3\pi}{2}$, $t = \frac{5\pi}{2}$, $t = \frac{7\pi}{2}$, and so on. Generally, $tan(t)$ is undefined when $t = \frac{\pi}{2} + k\pi$ for every integer k [33, p. 64].

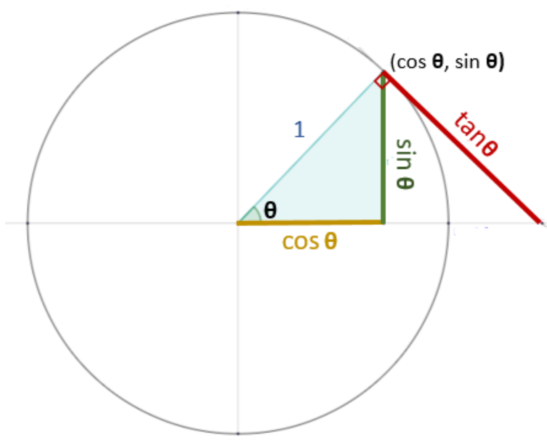


Figure 7.4: Sin, cos, and tan, expressed with right triangles

7.4 Graphs of Trigonometric Functions

Wrapping trigonometric functions around the unit circles gives a good understanding of sine, cosine, tangent, and the relationship between them. By unwrapping the functions, the actual graphs of the functions can be investigated. The unwrapped function $y = sin(t)$ is shown in Figure 7.5. Note the repeatable pattern for every 2π on the x-axis, as explained previously in Section 7.1 and in Section 7.2. Similarly, the unwrapped function $y = cos(t)$ is shown in Figure 7.6. The graphs of $y = sin(t)$ and $y = cos(t)$ are called sinusoidal waves, and the sine and cosine functions are called sinusoidal functions [33, p. 78].

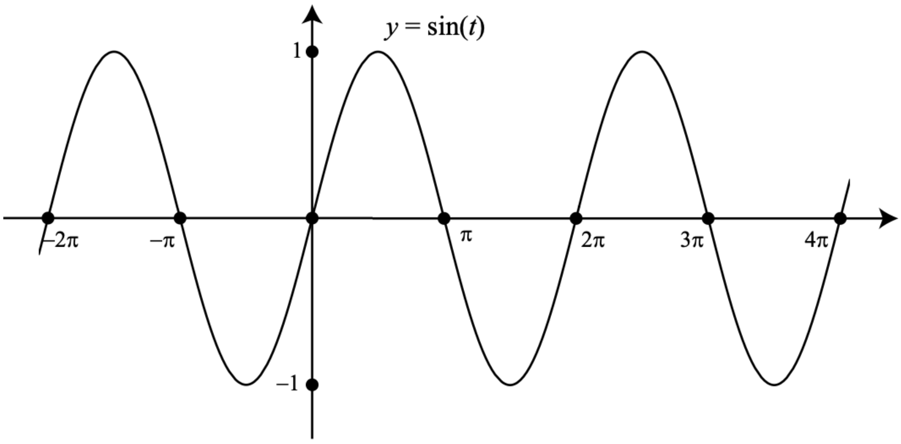


Figure 7.5: The graph of $y = sin(t)$

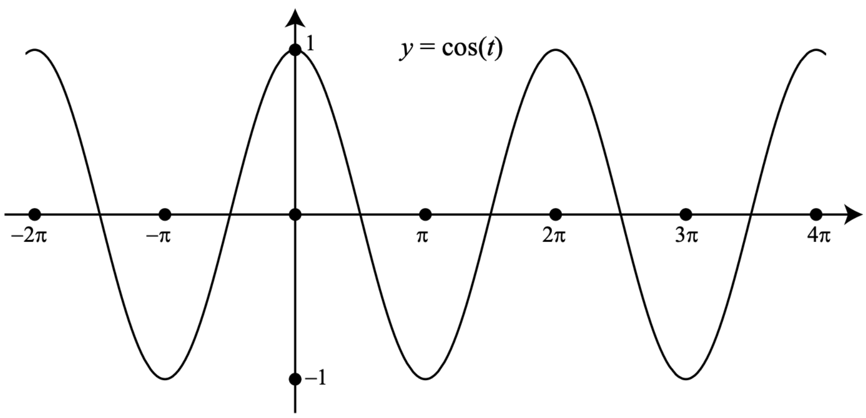


Figure 7.6: The graph of $y = cos(t)$

Sinusoidal functions have different properties which affect the shape of the graph. The general sine function is defined as

$$y = Asin(B(t - C)) + D \tag{7.1}$$

as provided by [33, p. 90]. The variables A, B, C, and D will be explained in the following sections.

7.5 Amplitude

As explained in Section 7.1 and Section 7.2, trigonometric functions are periodic and thus have repeatable patterns. In fact, these patterns repeat indefinitely. Formally, for the functions $y = sin(t)$ and $y = cos(t)$, the domain (the possible x-values) is $\langle -\infty, \infty \rangle$ [33, pp. 4-5]. In other words, any real number is a valid x-value. For the y-values, the range of valid values is $[-1, 1]$. This can be seen in Figure 7.5 and Figure 7.6, and follows from the fact that the radius of the unit circle is equal to 1, as explained in Section 7.1.

By multiplying the trigonometric function by a constant, A, the range becomes $[-A, A]$. The constant A is called the amplitude of the function and is formally defined as one-half the distance between the maximum and minimum functional values [33, p. 79]:

$$\text{Amplitude} = \frac{1}{2}|(\text{max y-coordinate}) - (\text{min y-coordinate})| \tag{7.2}$$

Figure 7.7 shows a sine function with the amplitude marked. As the figure shows, the amplitude can also be considered as the length between the midpoint (the equilibrium line) and the top/bottom y-values of the graph.

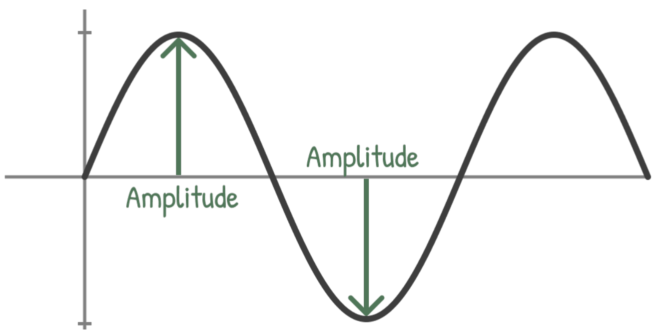


Figure 7.7: A generic sinusoidal function with the amplitude marked

7.6 Period & Frequency

When discussing an expression such as $sin(t)$ or $cos(t)$, the expression inside the parentheses is often referred to as the argument of the function [33, p. 92]. As explained in Section 7.1 and Section 7.2, the period of a trigonometric function is 2π By multiplying t with some constant B, the period of the function changes. Generally, the period of a trigonometric function is defined as abs($\frac{2\pi}{B}$). This can be seen in Figure 7.8. As B grows, the period becomes smaller, i.e. the distance on the x-axis between each repeatable pattern shrinks.

The period of a trigonometric function is also closely related to the frequency of the function. Formally, the frequency of a sinusoidal function is the number of periods (or cycles) per unit time [33, p. 111]. Since frequency is the number of cycles per

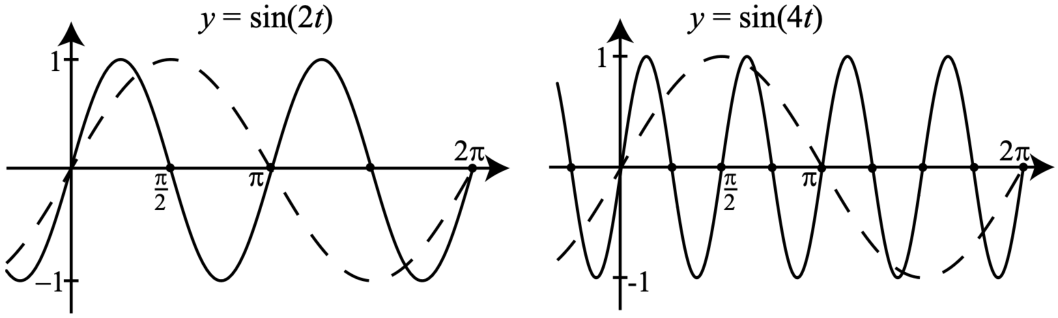


Figure 7.8: Graphs $y = sin(2t)$ and $y = sin(4t)$. The graph $y = sin(t)$ is also shown as dashed lines.

unit of time, and the period is the amount of time to complete one cycle, frequency and period are related as follows [33, p 111]:

$$\text{Frequency} = \frac{1}{\text{period}} = \frac{B}{2\pi} \tag{7.3}$$

7.7 Phase Shift

In the previous section, t from the function $y = sin(t)$ was multiplied with a constant B. In this section, a constant C will instead be subtracted, i.e. the function $y = sin(t - C)$ will be investigated. Figure 7.9 shows the graphs $y = sin(t - 1)$ and $y = sin(t - \pi)$.

Note that when $C > 0$, the sinusoidal function is horizontally translated to the right by C units. Similarly, when $C < 0$, the function is translated to the left by C units. When working with a sinusoidal graph, such a horizontal translation is called a phase shift [33, pp. 96-97]. Figure 7.10 shows the graphs $y = cos(t+1)$ and $y = cos(t + \pi)$.

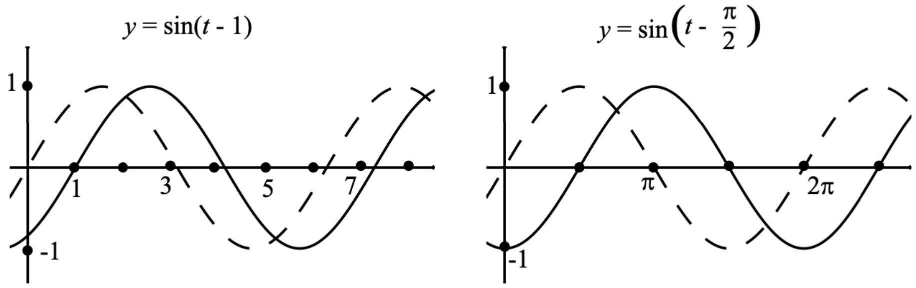


Figure 7.9: Graphs $y = sin(t - 1)$ and $y = sin(t - \pi)$. The graph $y = sin(t)$ is also shown as dashed lines.

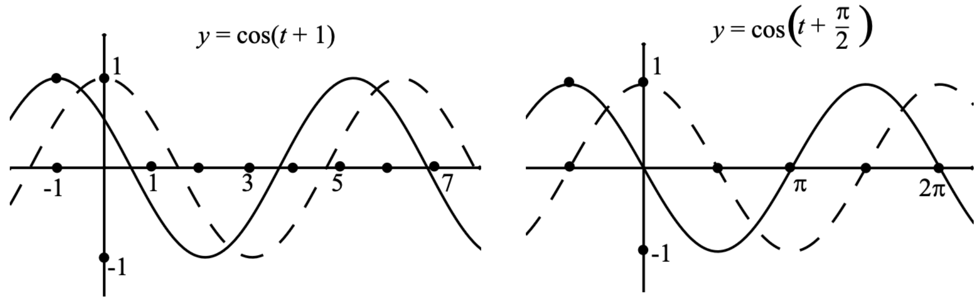


Figure 7.10: Graphs $y = cos(t + 1)$ and $y = cos(t + \pi)$. The graph $y = cos(t)$ is also shown as dashed lines.

7.8 Vertical Shift

Similar to how phase shift is a horizontal translation of the sinusoidal function, the function may be translated vertically by adding a constant D to the function. Figure 7.11 shows the graph $y = sin(t) + D$. When $D > 0$, the function $y = sin(t) + D$ is shifted upwards D units relative to $y = sin(t)$. Similarly, when $D < 0$, the function is shifted downwards D units [33, p. 100].

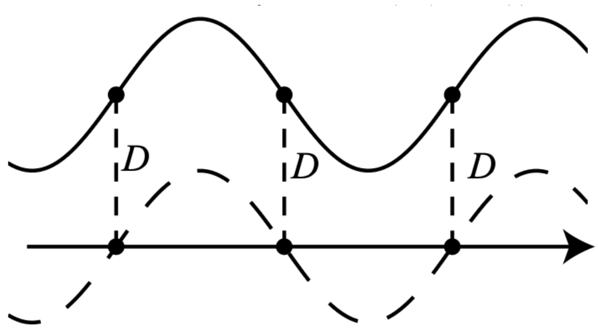


Figure 7.11: Graphs of $y = sin(t) + D$. The graph $y = sin(t)$ is also shown as dashed lines.

7.9 Simplifying Trigonometric Functions

Consider the general sine formula in Equation 7.1. An alternative expression can be found by removing the parentheses. This gives the formula

$$y = Asin(Bt - BC) + D \tag{7.4}$$

The element BC is just two constants multiplied together, and can naturally be combined into a single constant to simplify the expression. The constant can also be multiplied by (-1) to avoid the subtraction, which gives the formula

$$y = Asin(Bt + C) + D \tag{7.5}$$

which in some sources is used as the general sine formula [34, p. 1], as opposed to the general formula defined by Sundstrom and Schlicker [33, p. 90] shown in Equation 7.1. Note that the C in Equation 7.5 is not the same as the C in Equation 7.1, but rather $C_1 = -\frac{C_2}{B}$ where C_1 is the C in Equation 7.5 and C_2 is the C in Equation 7.1. Furthermore, although Equation 7.5 is simpler than Equation 7.1 due to fewer parentheses, note that the phase shift now becomes $-\frac{C}{B}$. That is, in the function $y = sin(Bt + C)$, the function is translated $-\frac{C}{B}$ units to the right.

Both Equation 7.1 and Equation 7.5 are valid representations of the general sine formula. But for the sake of consistency, Equation 7.5 will be used for the rest of this report. This is also the equation used in the game designed for the empirical experiment that is the basis of this report. The game will be described in Part III, and the methodology of the experiment will be described in Part IV.

7.10 Summary

This chapter has summarized the majority of trigonometry taught in Norwegian high schools. The unit circle has been explained and used to describe the fundamental trigonometric functions sine, cosine, and tangent. With these functions defined, the corresponding graphs of the functions have been explored. Different properties of the graphs have been described, namely amplitude, period, frequency, phase shift, and vertical shift. The general trigonometric formula has also been reordered to simplify the expression. This knowledge will be useful later in the creation of a game-based learning platform. To create a game-based learning platform about trigonometry, it is also important to understand gamification and what makes games fun. This will be explained in the next chapter.

Chapter 8

Gamification & Enjoyment in Video Games

Video games are becoming more and more popular. According to a 2021 study by Newzoo led by Wijman, T., more than 3 billion people in the world play video games regularly, and the games market is estimated to grow to more than $ 200 billion by 2024 [35]. This Chapter will investigate what makes video games fun and how elements from games can be used to enhance learning. Firstly, certain elements of what makes games fun will be analyzed in detail, followed by the GameFlow theory and an experiential gaming model. Then, the topic of gamification will be introduced, followed by an overview of various game reward systems.

8.1 What Makes Things Fun to Learn?

In the paper *What makes things fun to learn? Heuristics for Designing Instructional Computer Games* [36], Thomas W. Malone provides a set of guidelines for designers of instructional computer games. The foundation on which the guidelines have been created is based on theory and experiments presented in some of Malone's other work [37]. Malone's focus is towards educational games, but he emphasises that the focus is based on what makes games fun and not what makes them educational [36, p. 162]. Malone highlights three characteristics of such games: *Challenge*, *Fantasy*, and *Curiosity*.

8.1.1 Challenge

Malone suggests that a game's challenge is based on whether it provides a **goal** whose attainment is uncertain [36, p. 162]. Three key concepts related to the challenge of a game, namely Goals, Uncertain Outcome and Self-Esteem, will be explained.

Goal - Based on his own study [37], Malone found that video games that had a goal, also had a correlation with preference between games of the players. However, not all goals are equally good, and Malone highlights certain aspects of good goals. One of them is that the skill being taught (in a game) should be a means to achieve a goal, and not the goal itself. In other words, learning should not be the main goal in a game but rather a side effect which arises from problem-solving when trying to achieve the goal of the game. Furthermore, Malone provides a list of four concerns regarding good goals:

1. Simple games should provide an obvious goal.
2. Complex environments without built-in goals should be structured so that users can generate goals of appropriate difficulty.
3. The best goals are often practical or fantasy goals.
4. Players need to know if they are getting closer to the goal through performance feedback.

Uncertain Outcome - When games become predictable, they also become less fun because you already know what is going to happen. Malone suggests four ways to ensure Uncertain outcomes:

1. **Variable Difficulty level** - The difficulty is either (a) automatically determined, (b) chosen by the player, or (c) determined by the opponent's skill.
2. **Multiple Level goals** - By dividing the goals into multiple levels, the players can reach new goals and climb up the ”goal-achievement ladder”. Malone mentions two levels of goals one can include: a basic goal, which can be to answer right on a quiz question, and a meta-goal like scorekeeping and speeded response, where the goal is to get as close to the absolute limit, maximizing their score. Some examples can be in quiz games like Kahoot! [1], where players want to answer as fast as possible, while still answering right, to get more points on the scoreboard.
3. **Hidden information** - by hiding information from players, their curiosity can increase and which contributes to the challenge of the game.
4. **Randomness** - Randomness can be a tool to heighten interest as it provides uncertain outcomes. However, Malone does not specify the type of randomness. In this thesis, randomness is interpreted as the player's **perceived randomness**, and not actual randomness. This is important both in video games and outside of the game domain. For example, music services commonly do not use actual randomness when you shuffle a playlist. A truly random list could have similar artists and songs appear sequentially. Although this is in fact true randomness, it does not *feel* random for the user.

[1]https://kahoot.com

Self-esteem - As with all goals and challenges, a player succeeding in a computer game can boost their self-esteem. However, the opposite effect of this is that players might feel worse if they have to many failures in the game. This highlights the importance of variable difficulty levels, as it gives players the opportunity to play at appropriate levels based on their ability, which also can be related to the Flow state illustrated in Figure 5.4. Furthermore, the performance feedback should be presented in a way that minimizes self-esteem damage. An example could be to reduce the feedback when the players' performance is bad and amplify the feedback on success.

8.1.2 Fantasy

Fantasy is, by Malone's definition, *to show or evoke images of physical objects or social situations not actually present* [36, p. 164]. Malone makes a distinction between Extrinsic Fantasy and Intrinsic Fantasy as illustrated in Figure 8.1.

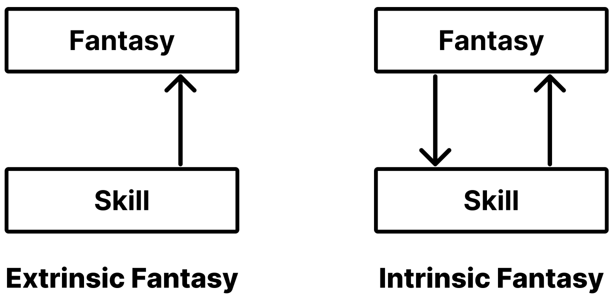


Figure 8.1: Illustration of the dependencies in Malones Extrinsic and Intrinsic fantasy [36, p. 164]

Extrinsic Fantasy - Extrinsic Fantasy is when the fantasy in a game directly depends on the use of a skill. An example of this could be a quiz game with two teams. Both teams have a rocket that moves towards the other team for each right answer they get. If they get close enough, the rocket explode, and the team win. Examples likes this can easily be applied in an educational game with different curriculum, as it is domain independent. Malone also points out that extrinsic fantasy is a relatively easy way to increase the fun of learning, and provides seven examples of both goals to reach and catastrophes to avoid that can be used in learning games.

Reaching a goal

- A train on a track is approaching a city.
- A rocket is passing the other planets of the solar system on its way to earth.
- A complicated building is being built, piece by piece.

- A fleet of space invaders is being destroyed, one by one.

Avoiding a Catastrophe

- A man is hung, one body part at a time.
- A person advances toward the edge of a cliff, one step at a time.
- A time bomb is ticking toward an explosion.

Intrinsic Fantasy - Intrinsic fantasy is when the fantasy is dependent on the skill, but the skill is also dependent on the fantasy. An example of this is the game Tetris illustrated in Figure 8.2, where the player should stack different shapes of boxes onto a map and fill as many horizontal lines as possible. The players placement choices will affect the fantasy, as it directly affects the map they need to work with for the next rounds. Malone suggests that Intrinsic Fantasy is both more interesting and instructional than extrinsic fantasy, and that it shows *"how the skill could be used to accomplish some real world goal"*.

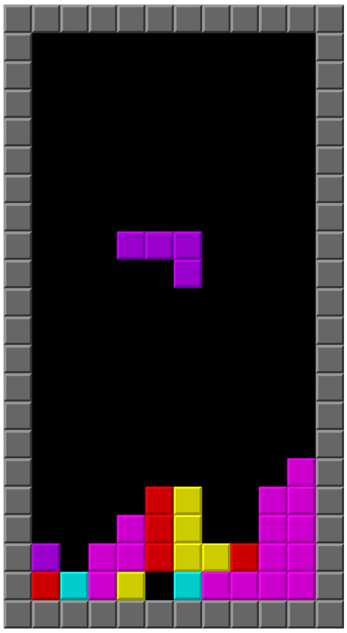


Figure 8.2: Illustration of the game Tetris [2]

Emotional Aspects - Malone makes an assumption that games with strong emotions such as war, destruction and competition, are more likely to be popular than those with less emotional fantasies. However, Malone also states that *"different people will find different fantasies appealing"*, pointing out that game designers can have a broader appeal if they design games with multiple fantasy choices. He also points out that the deciding factors of the appeal is dependent on much more than just the sex of players. However, he does not mention any examples of such factors.

[2] https://upload.wikimedia.org/wikipedia/commons/7/7c/Emacs_Tetris_vector_based_detail.svg

8.1.3 Curiosity

Curiosity is, by Malone's definition *"The motivation to learn, independent of any goal seeking or fantasy-fulfilment"* [36, p. 165]. Malone points out that the environments of a game should be balanced with an *optimal level of informational complexity.* This means that a game should provide surprises without being totally incomprehensible. Optimally, games should provide some expectations to players, giving them some sense of what will happen, but not always meet their expectations. Malone further divides Curiosity into two types, Sensory Curiosity and Cognitive Curiosity.

Sensory Curiosity involves attracting our attention to changes in the environment that can be picked up by our senses. This can for example be a change in sounds, light and other sensory stimuli. The most common way to appeal to sensory curiosity in video games is through audio and visual effects. These effects can also be used to appeal to other motivations such as:

1. Decoration of the game, to make it more interesting.
2. Enhance the fantasy, which is a type of decoration.
3. As Reward to highlight good performance.
4. As a representation system of information other than words and numbers.

Cognitive Curiosity is about the learner's urge to fulfill their own knowledge structures. Malone claims that learners want their cognitive structures to be complete, consistent and parsimonious, which relates to theories within cognitivism described in Section 5.3. To trigger a learner's cognitive curiosity he suggests that information should be presented such that the learner feels their knowledge is incomplete, inconsistent or unparsimonious. Or, in other words, the learners should feel like they do not see the full picture yet.

8.1.4 Summary

Malone's theory provides guidelines for designing video games and important aspects to pay attention to. The theory can be used for analysis of existing learning games and the development of game concepts, which will be explained in Chapter 9 and 10, respectively.

8.2 GameFlow

As discussed in Section 5.5, motivation is one of the key factors the learning outcome in education and intrinsic motivating task can lead to flow and enjoyment. In other words, the more someone enjoys a lecture or another educational task, the higher the learning outcome. However, this may be somewhat vague, as enjoyment is difficult to quantify. Sweetser, P. and Peta, W. attempted to quantify enjoyment in computer games using various heuristic models [38]. They coined this model as GameFlow, which is based on Csikszentmihalyi's flow theory [3], described in Section 5.6. The GameFlow model consists of eight elements: concentration, challenge, skills, control, clear goals, feedback, immersion, and social interaction. The model can be used to review games and distinguish between high and low-rated games and identify why some succeed and the others fail. Although this study is aimed specifically at actual video games rather than applications with gamification elements integrated, it should serve as a basis for analysing the enjoyment, and thereby the learning quality, of gamified education.

8.2.1 Concentration

To induce flow, Sweetser & Wyeth state that a game should quickly capture the player's concentration, and provide enough stimuli from different sources to make the player feel it is worth attending to. When a game requires all of a player's relevant skills to meet its challenges, it fully absorbs the player's attention, and no excess energy is left over to process anything other than the activity [39]. Games should be designed to quickly capture and sustain the player's attention, whether it's their first few seconds or their hundredth hour of play. Engaging game worlds, tasks that feel meaningful, and minimization of non-game-related interactions are all strategies to maintain concentration. However, the player's workload should be carefully calibrated to ensure that tasks are challenging but not overwhelming [38, pp. 4-6].

8.2.2 Challenge

Games should offer a suitable degree of challenge that aligns with the player's skill level, adapts to their progress, and is paced accordingly. The balance between challenge and skill levels is an essential to achieve flow, and Sweetser and Wyeth consider challenge the most important aspect of good game design [38, p. 6]. If the challenge outweighs the skills, the player might feel anxious, and if the challenge is too easy, they may experience apathy [38, p. 6].

Games generate enjoyment by challenging players, pushing their memory and performance boundaries [38, p. 6]. These challenges can vary from accomplishing difficult tasks, overcoming opponents, testing and mastering skills, to achieving goals and coping with suspenseful danger. The rewards of facing such challenges are intrinsic, and the process itself is its own reward [38, p. 6].

Furthermore, games should also have variable levels of difficulty to meet all players at the correct level of challenge [38, p. 7]. There are different ways to achieve this. Some games allow players to select a difficulty level that corresponds to their skill level, and games with levels should have a gradually increasing difficulty for each level. This helps maintain player interest wile providing a more substantial challenge as they learn the game and increase their skills [38, pp. 6-7]. Pacing is also crucial, maintaining appropriate challenge levels and tension throughout the game, applying pressure without causing frustration. Player fatigue can be minimized by varying activities and pacing during gameplay [38, p. 7].

8.2.3 Skills

Closely related to the challenge element are player skills. To provide an engaging gaming experience, it's vital to encourage the development and mastery of player skills [38, p. 7]. This is crucial for achieving a state of flow in games, where the players' perceived skills align with the challenges presented by the game. Instructional methods such as comprehensive and interesting tutorials facilitate quick involvement and progression in learning game mechanics [38, p. 7]. Additionally, the concept of learning through gameplay is important, allowing players to acquire and practice skills in a contextual and paced manner [38, p. 7].

Starting a game should be simple and not require manuals or lengthy explanations [38, p. 7]. In-game assistance, such as hints or context-sensitive help, can be provided to support players without breaking immersion [38, p. 8]. Furthermore, game design should adhere to platform conventions and industry standards to shorten the player's learning curve and create intuitive, easy-to-use interfaces [38, p. 8]. This includes consistent game controls and interfaces, learnable input mechanisms, and the use of real-world metaphors and analogies to assist player navigation and interaction within the game [38, p. 8].

8.2.4 Control

A player's sense of control in a game is fundamental for achieving flow, allowing their intentions to translate into in-game actions effectively [38, p. 8]. This includes control over character movements, exploration of the environment, manipulation of in-game objects, and mastery over the game interface and controls. Furthermore, the player should be able to easily start the desired type of game, turn the game on and off, and save the game in different states [38, p. 8].

Moreover, players should feel that their decisions impact the game world [38, p. 8-9]. Games should also allow a level of freedom for players to experiment and to take on the game in their preferred way, rather than being bound by a linear path set by the game designer [38, p. 9]. This includes offering multiple paths through the game or multiple ways to win, thereby avoiding a single optimal strategy. Ultimately, the player should feel like they are playing the game, rather than being played by it [38, p. 9].

8.2.5 Clear goals

Games should establish clear goals for players at the right times to promote engagement and flow [38, p. 9]. There are generally two types of goals: overriding goals and intermediate goals. Having a clear, overriding goal, typically presented early in the game, is important to give the player a purpose. This is often done through an introductory cinematic that establishes the background story [38, p. 9]. However, each level should also have multiple goals, and games often use briefings to describe a mission that outline the immediate goals of the current part of the game and suggest some of the obstacles that the players might face [38, p. 9].

8.2.6 Feedback

Feedback is a critical component of engaging gameplay and achieving flow. Games must provide players with frequent and appropriate feedback to maintain concentration and measure progress towards objectives [38, p. 9-10].

This feedback can be offered in various forms, such as scores, in-game interfaces, and sounds, which help players understand their current status and how well they are progressing [38, p. 9-10]. Games should also give immediate responses to player actions, reinforcing their sense of engagement and agency [38, p. 10]. Moreover, feedback upon a player's loss is essential to inform them if they are moving in the right direction and to encourage mastery of the game [38, p. 9-10].

Essentially, rewarding players with consistent, immediate feedback on progress and success enhances the overall gaming experience, promoting a sense of achievement and motivation [38, p. 10].

8.2.7 Immersion

Immersion in a game refers to the deep and effortless involvement of the players, often leading to a loss of self-awareness and a diminished concern for everyday life [38, p. 10]. Immersion can result in high emotional investment due to the time, effort, and attention devoted to gameplay. The game becomes the primary focus of the player's attention, and their emotions are directly influenced by the game [38, p. 10].

For a game to be immersive, it should draw players in emotionally and viscerally, and make the interface seem invisible. Elements like audio, narrative, and other sensory details play a crucial role in achieving this effect. Sound effects and soundtracks can help to maintain immersion, while a compelling narrative can make players feel like they're part of the story [38, p. 10]

8.2.8 Social Interaction

Sweetser & Wyeth suggest that games should support and create opportunities for social interaction [38, p. 10]. Although social interaction is not an element of flow, it is a significant factor in enhancing enjoyment. Players often engage in games for the social interaction they provide, even if they may not particularly like the game itself [38, p. 10].

To encourage social interaction, games should promote player competition, cooperation, and connection [38, pp. 10-11]. Games can support these interactions through features like chat functions and online boards. Online games, in particular, offer the appeal of a virtual community where players can spend hours interacting with friends, forming groups, and gradually improving their characters [38, p. 11]. The game experience should be designed to encourage player-to-player interaction and generate enjoyment when playing with others both inside and outside the game [38, p. 10-11]. Social competition is another crucial aspect of social interaction in games, as players often derive satisfaction from competing against and defeating others [38, p. 11].

8.2.9 Summary

This section explored the GameFlow model, a tool based on Csikszentmihalyi's flow theory [3] and crafted by Sweetser and Peta for quantifying enjoyment in video games [38]. The model consists of the eight components: concentration, challenge, skills, control, clear goals, feedback, immersion, and social interaction.GameFlow serves as a useful tool for evaluating games, differentiating between top and lower-rated ones, and determining why some become successful while others do not. In Section 18.3, we will introduce a metric that draws on both GameFlow and Malone's theory (mentioned in the previous section) to calculate game enjoyment.

8.3 Experiential Gaming Model

Based on various theories, including the flow theory of Csikszentmihalyi [3] described in section 5.6, Kristian Kiili [40, p. 13] developed a model for experiential gaming. He observed a lack of models that integrated both educational learning theory and game design aspects. In summary, experiential learning can be explained as learning by doing, and that you learn from your experiences. This theory will be explained in further detail below.

8.3.1 Gameplay and learning

Although flow is very important, Kiili also highlights the importance of gameplay, problem-solving, and experiential learning.

Gameplay is about keeping the players motivated and engaged throughout an entire game. Gameplay is important, and lack of good gameplay will likely make a game disappear from the market quickly [40, p. 16]. Kiili suggests that games with excessive educational aspects or technology can ultimately sacrifice quality of gameplay. Here, connections can be drawn to Malone's theory described in section 8.1. For educational games, this is sensible as they focus too much on trying to make the game seem like a learning game, where the learning itself is the goal of the game rather than a side effect of playing.

Problem-Solving is among the most important skills humans need to master, and games are, according to Kiili, generally a provider of a meaningful environment for problem-based learning. Generally within a game, a problem is *"anything that somehow restricts a player's progress in the game world"* [40, p. 17]. And they may be divided into two types, Well-structured- and ill-structured problems. Well-structured problems have a definitive answer, while ill-structured problems have no clear goals and incomplete information regarding the problem. Thus, ill-structured problems do not have a definitive solution, and the problem solver needs to prioritize what they want to achieve to come up with the best solution. This is important, as it gives the problem solver opportunities to use different strategies. Problem-solving is also associated with discovery learning, where the learner discovers new rules instead of memorizing them.

Experiential Learning encompass learning through direct experiences and reflective observation. A central model used in the field of experiential learning Kolb's model, which includes four stages [2, p. 21]:

1. The learning begins with a learner getting a concrete experience.
2. They get a collection of data and reflective observations about the experience.
3. The learner makes generalisations, draws conclusions, and forms hypothesis about the experience.
4. The learner tests the hypothesis in new circumstances through active experimentation.

8.3.2 Kiili's model

Kiili's model is constructed to link gameplay with experiential learning in order to facilitate the flow experience and is illustrated in Figure 8.3. The model is similar to the human blood-vascular system and the heart of the model is the challenges that are derived from learning objectives. The heart is responsible for keeping the motivation and engagement by "pumping" challenges to an ideation loop. Here the player comes up with ideas to solve challenges. Then, the learner tests the ideas in an experience loop to find a solution. The core takeaway from the model is that the learner needs to use both the idea generation and experience loop to keep up the circulation that fuels the heart, which is the motivation and engagement of the learner and produces flow. Imagine if a player only tries and retries random strategies without coming up with ideas themselves. This would not contribute to idea generation, which metaphorically means less circulation and decreased feeling of flow.

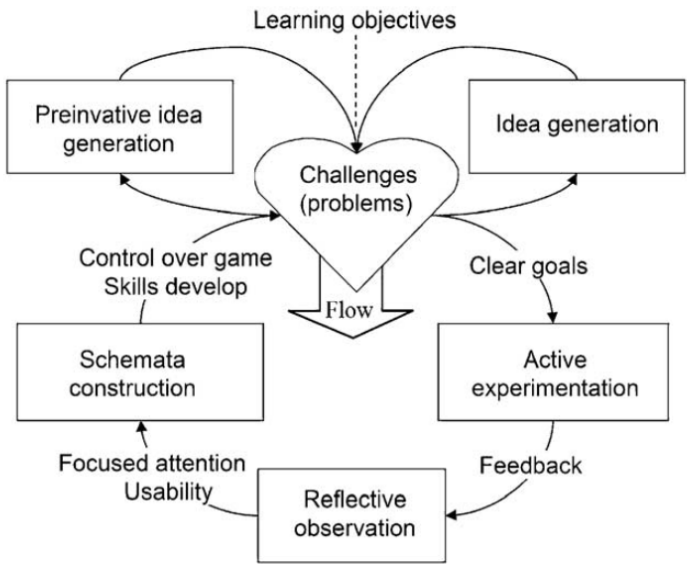


Figure 8.3: Kiili's experiential gaming model [40, p. 18]

One thing the model is not concerned with is social interaction in learning, and therefore it only explains the motivation and engagement from a single player's perspective. Kiili's model can be used to design and analyze educational games, but it only works as a link between educational theory and game design. It does not provide a "how-to" on designing a game.

8.3.3 Summary

This section has delved into the Experiential Gaming Model by Kristian Kiili. His model seeks to integrate educational learning theories with game design principles, placing a strong emphasis on gameplay, problem-solving, and experiential learning. Kiili's model highlights the need for sustaining player engagement, and high-quality gameplay, while simultaneously offering a meaningful environment for learning through problem-solving. Drawing parallels with the human cardiovascular system, Kiili's model works on the circulation of challenges to stimulate player motivation and engagement, using ideation and experiential loops for problem-solving. As will be revealed in Chapter 11, the solid foundation provided by Kiili's model will shape the design of an experiential learning game about trigonometry.

8.4 Gamification

As described in the previous sections, video games can certainly exhibit flow and be motivating and entertaining. A common criticism of video games is their purpose, or rather lack thereof. However, studies show that elements from video games can be included in other areas such as education to enhance the learning outcome.

This phenomenon is commonly referred to as gamification. Deterding, S. et al. attempted to give a definition to this term. They defined Gamification as *"the use of game design elements in non-game contexts"* [41]. Gamification typically involves the use of positive reinforcements such as XP and achievements, as well as negative reinforcements such as obstacles or removing upgrades. This relates to and is a practical example of Behaviourists theories described in Section 5.2.

In another study by Hamari, J. et al. various peer-reviewed empirical studies on gamification were analyzed [12]. The literature review attempts to answer whether gamification actually works, and presents a framework for examining the effects of gamification by drawing from definitions of gamification and discussions of motivational affordances. The review indicates that gamification has positive effects, but also that these effects are highly dependent on context and the users involved [12, p. 3028]. However, the study does point out that the result of gamification might be caused by the *novelty effect*, explained in Section 4.2 as a form of reactive effect of experimental arrangements.

In summary, Gamification is the application of game design elements in non-game contexts. Although generally positive, its effects vary based on context and the users involved. The *novelty effect* (see Section 4.2) might also influence its perceived success, which is highly related to the validity of this research project.

8.5 Game Reward Systems

Game reward systems are systems that are implemented in games to enhance both the motivation and enjoyment of players in various ways. This section will cover different types of reward systems, how they relate to *flow* and *motivation*, and the considerations proposed when implementing such systems in games. The theories are mainly based on the study conducted by Sun T. and Wang H. [42], which discusses reward systems from several viewpoints.

8.5.1 Forms of Rewards

Reward systems play a crucial role in player motivation and engagement, and can greatly influence the overall player experience. Based on multiple surveys and analysis of video games, Wang H. and Sun T. identified eight forms of rewards in video games [42, pp. 3-5]:

1. **Score Systems** mark player performance using numbers, and are typically used for self-assessment and comparison with other players. The score system introduces competition as well as virtual identities and player status that can be accumulated over a longer period rather than the performance of a single play.

2. **Experience Point Reward Systems** reward players in games where players control avatars that can develop and "level up". This usually happens through experience points. These points represent a facility type of reward as they enhance the avatar's abilities, including new skills or increased attributes.

3. **Item Granting System** rewards consist of virtual items that can be used by the player's avatar. Such systems encourage exploration and maintain player interest during slower parts of the game. They are widely used in RPGs and MMORPGs.

4. **Resources** are valuables that can be collected and used to affect gameplay. Examples include virtual wood in Age of Empires III and life counts in Super Mario Bros. These are primarily sustenance rewards as they are practical and usable within the game.

5. **Achievement Systems** reward players with titles that are bound to avatars or player accounts, which are earned by fulfilling specified conditions. They encourage players to complete specific tasks and explore the game worlds.

6. **Feedback Messages** are used to provide instant rewards in response to successful actions. They create positive emotions in players and serve as a form of computer-generated praise.

7. **Plot Animations and Pictures** are rewards given after important events such as defeating a major enemy or clearing a new level. They are visually attractive and serve as milestones marking the player achievement.

8. **Unlocking Mechanisms** rewards give players access to new game content once certain requirements are met. These mechanisms reward players as games progress by gradually exposing hidden parts of the game world. This keeps players curious about what might be available in future play.

8.5.2 Rewards, Flow and Motivation

One of the most important aspects when designing Reward Systems is to keep players excited and motivated [42, p. 8].

The Flow theory described in Section 5.6 is widely used in gaming research, and it states that optimal experiences are characterized by a balance between challenge and skill, clear goals, and immediate feedback, among other factors. One problem associated with Flow is boredom, illustrated in Figure 5.4. Wang and Son argue that this problem can be solved with reward systems that either modify the players' emotions or help players establish higher challenges. However, based on their analysis of previous studies, Wang and Son still caution against the overuse of rewards, as too many extrinsic rewards may diminish *intrinsic motivation* [42, p. 9]. At last Son and Wang highlights the importance of immediate feedback in clarifying short-term goals, using examples of games that break down complex quests into shorter tasks. Immediate feedback is especially important in single-player games.

8.5.3 Considerations

To summarize their analysis, Wang and Sun propose several considerations for designing effective game reward systems [42, pp. 12-13], these are:

1. **Life constraint:** Target casual players with rewards that can be accessed during short play sessions to accommodate their flexible time commitments.
2. **Autotelic experiences:** Employ multi-level goals to create intrinsically rewarding experiences and facilitate immersion.
3. **Balance:** Ensure rewards align with effort and time investment. This includes balancing rewards between hardcore and casual players in multiplayer games, and those buying or not buying virtual items.
4. **Uncertainty and secrecy:** Utilize uncertainty to create excitement, while maintaining predictable critical resources. Secret elements can encourage deeper engagement and community contribution.
5. **Accumulated vs. instant feedback:** Accumulated rewards can create a sense of long-term achievement and enable comparison, while instant feedback enhances game responsiveness and helps maintain player engagement.
6. **Social purposes:** Rewards can serve to differentiate players, establish status, and facilitate sharing and comparison. They can also strengthen group identity among advanced players.
7. **Physical world activities:** Using mobile technology to link rewards with real-world activities can encourage engagement and health-conscious behaviors, transforming the perception of game-playing.

8.5.4 Summary

This section discusses the various reward mechanisms in games to boost player motivation and enjoyment. The key to designing effective reward systems is to maintain player excitement and motivation. However, overuse of rewards could potentially diminish intrinsic motivation. When designing game reward systems, considerations include accommodating casual players, creating intrinsically rewarding experiences, balancing rewards, utilizing uncertainty and secrecy, and linking rewards to real-world activities, among others. This analysis will be relevant for the creation of a new video game, aimed to motivate and immerse students to foster a more conceptual understanding of mathematics. This foundational understanding of video games and GameFlow will be used in the next Chapter to analyze existing applications.

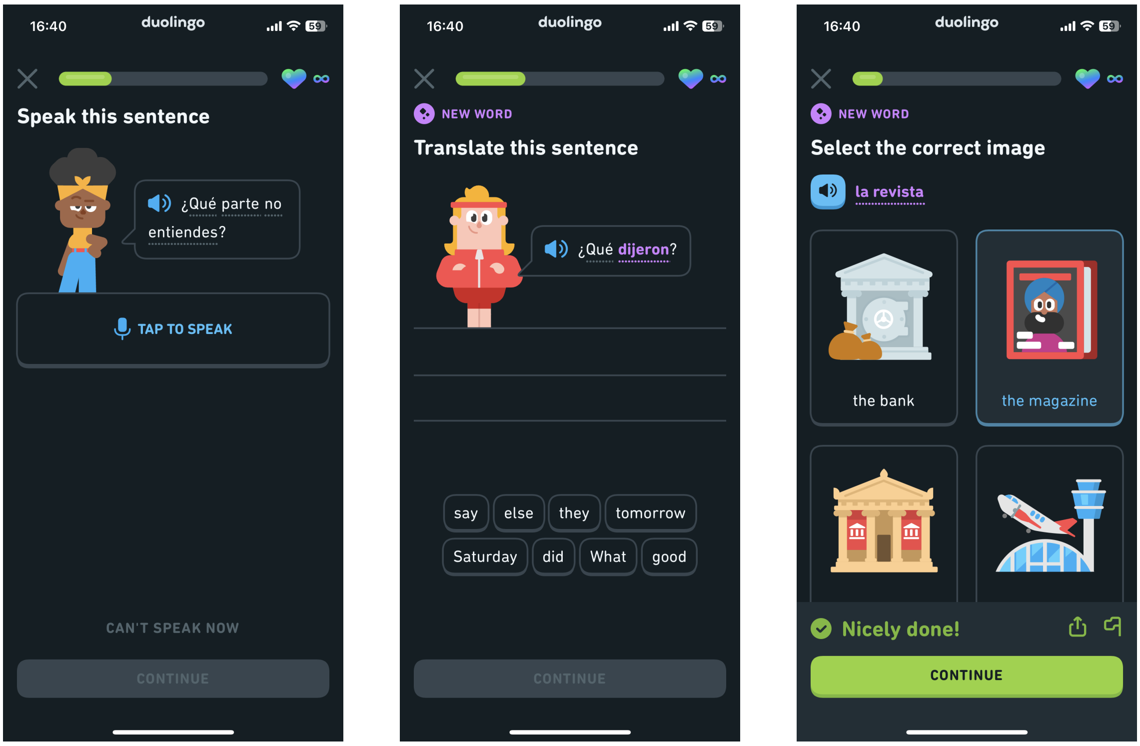
Chapter 9

Analysis of Existing Applications

As discussed in the previous Chapter, gamification of learning can be beneficial because it can make learning more engaging and enjoyable for students. By incorporating game-like elements such as points, rewards, and challenges, gamification can motivate students to stay focused and engaged with the material. This section will describe some learning applications which use gamification elements to enhance users' motivation and thereby learning outcomes. Firstly, Duolingo will be described, followed by Brilliant.org, and Dragon Box.

9.1 Duolingo

Duolingo is a language learning platform that offers courses in various languages. It uses a gamified approach to help users learn and retain new vocabulary and grammar skills through a variety of lessons and activities. Duolingo is available as a website and as a mobile app, and it is free to use. The platform uses a variety of methods, including multiple-choice quizzes, fill-in-the-blank exercises, and listening and speaking exercises, to help users improve their language skills. Figure 9.1 shows some of the types of exercises in the English to Spanish course.



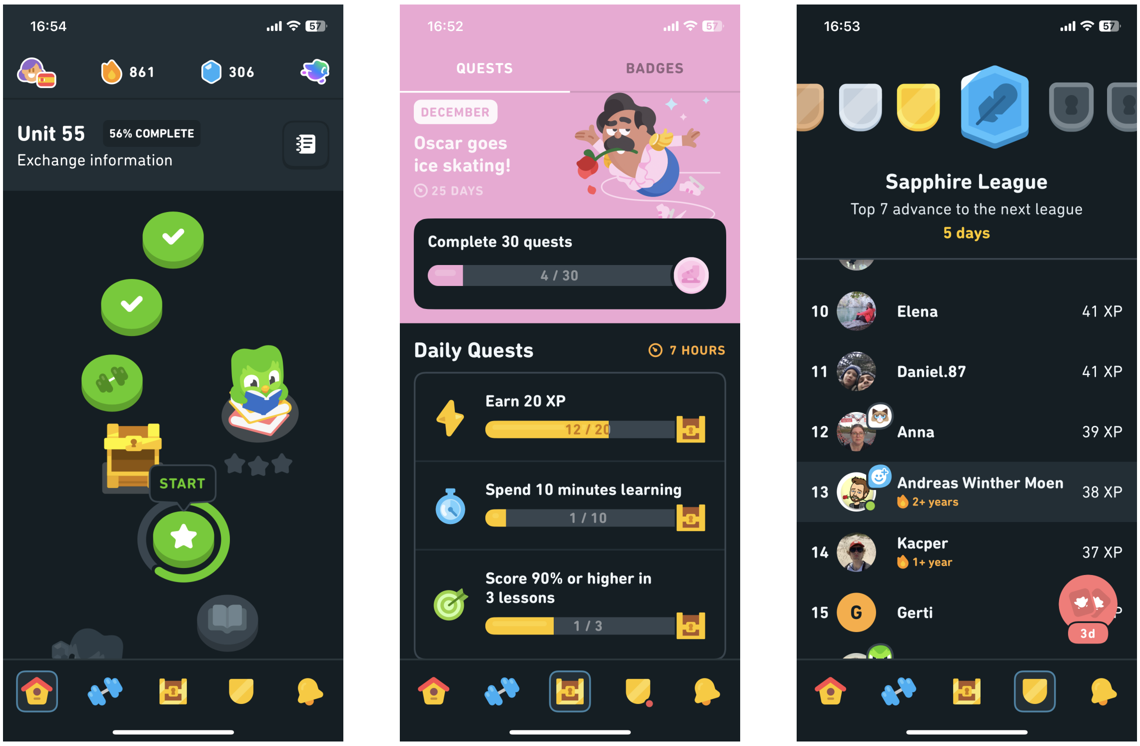
(a) Speech (b) Sentence translation (c) Image selection

Figure 9.1: Different language exercises in the Duolingo application [1]

According to Duolingo's own statistics, there are more than 500 million registered users as of December 2022[2]. There is no doubt that Duolingo is a popular and efficient tool to learn new languages. One reason is that the exercises are easily accessible and simple with a gradual increase in difficulty. But, arguably more importantly, the application features many gamification elements, which are generally based around extrinsic motivation as mentioned in Section 5.5.

Duolingo is heavily based on Rewards Systems as described in Section 8.5. Particularly, they use *Experience Point (XP) systems*, where the users receive XP based on how well they did. There are also daily and monthly quests to give more XP, to maintain user retention. The application also has *Unlocking Mechanisms*, such as a weekly division-based leaderboard. The users with the most XP are promoted to the next division, and the users with the least XP are demoted, encouraging competition. Furthermore, if you do at least one exercise per day, you get a streak. Streaks, scores and achievements can be shared and compared with your friends. Some of the gamification elements of Duolingo are shown in Figure 9.2.

[1] https://www.duolingo.com
[2] https://blog.duolingo.com/2022-duolingo-language-report/



(a) Learning path (b) Bonus XP quests (c) Weekly leaderboard

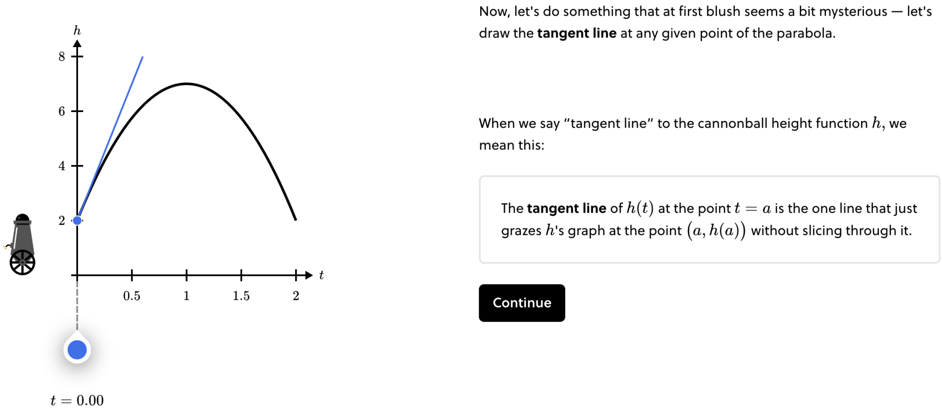
Figure 9.2: Various gamification elements in the Duolingo application [3]

9.2 Brilliant

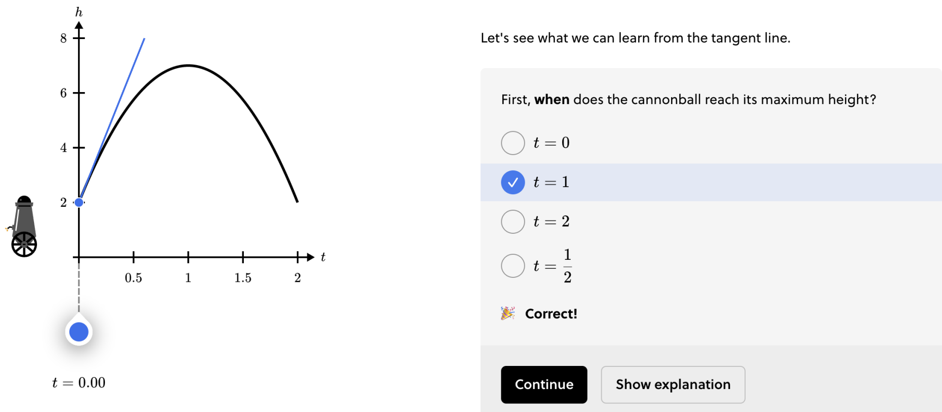
Similar to Duolingo, Brilliant is a learning platform with gamification elements to enhance motivation and learning. However, Brilliant focuses on STEM (Science, Technology, Engineering, and Mathematics) education. Brilliant provides a structured approach to learning, with each subject divided into courses and further into small, digestible modules. Courses are designed to be engaging and interactive, often requiring the learner to solve problems or answer questions as they progress. This approach aims to promote understanding through active problem-solving rather than passive reading or memorization.

As this is a more intricate field of study than language learning and requires more in-depth explanations, it appears more like an educational tool and less like a game. However, traditionally STEM topics are taught in a classroom and with books, and Brilliant provides interactive visualizations which may make it easier to understand core concepts. Screenshots of a calculus exercise on Brilliant are shown in Figure 9.3.

[3] https://www.duolingo.com



(a) Description of tangent lines

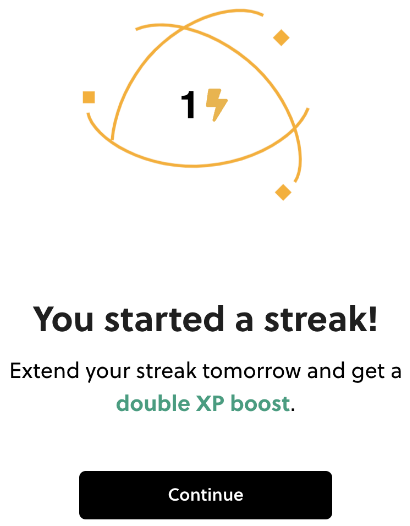


(b) Task related to tangent lines

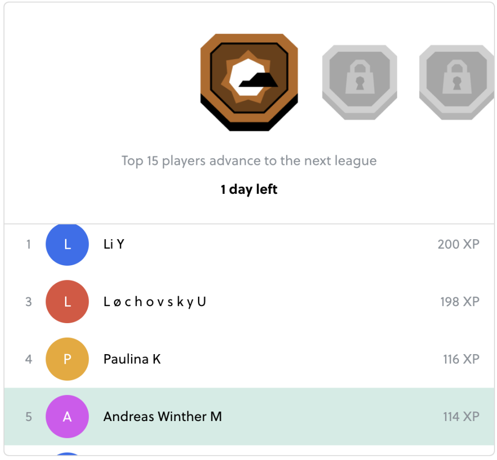
Figure 9.3: A calculus exercise on tangents and derivatives from Brilliant.org [4]

Brilliant uses various gamification elements to motivate its users. Similar to Duolingo, Brilliant has a streak mechanism to make users come back to the application for consecutive days (see Figure 9.4a). By completing tasks you also get XP which counts towards a league system (see Figure 9.4b). The league system in particular appears to be heavily inspired by Duolingo. However, Brilliant's gamification elements are not as extensive. Brilliant is also created to be used with a computer on a larger screen, while Duolingo is primarily a mobile application. Brilliant's tasks are also more complex.

After using both applications for a while, Brilliant feels more like traditional studying and requires more focus and time where the goal is the learning itself, while Duolingo feels more like a leisure application you use when you have some minutes to spare. Brilliant's approach to learning seems to be more focused on conceptual understanding as mentioned in 6.2. While Duolingo uses cartoon-like characters as visual elements, Brilliant uses graphics mostly as a visualization tool for illustrating mathematical equations to give a deeper understanding of the topic. It is clear that Duolingo aims toward a wide age group, while Brilliant aims more toward high school and university students, which is also the target audience of this research.



(a) Daily streaks



(b) League system

Figure 9.4: Gamification elements at Brilliant.org [5]

9.3 Dragon Box

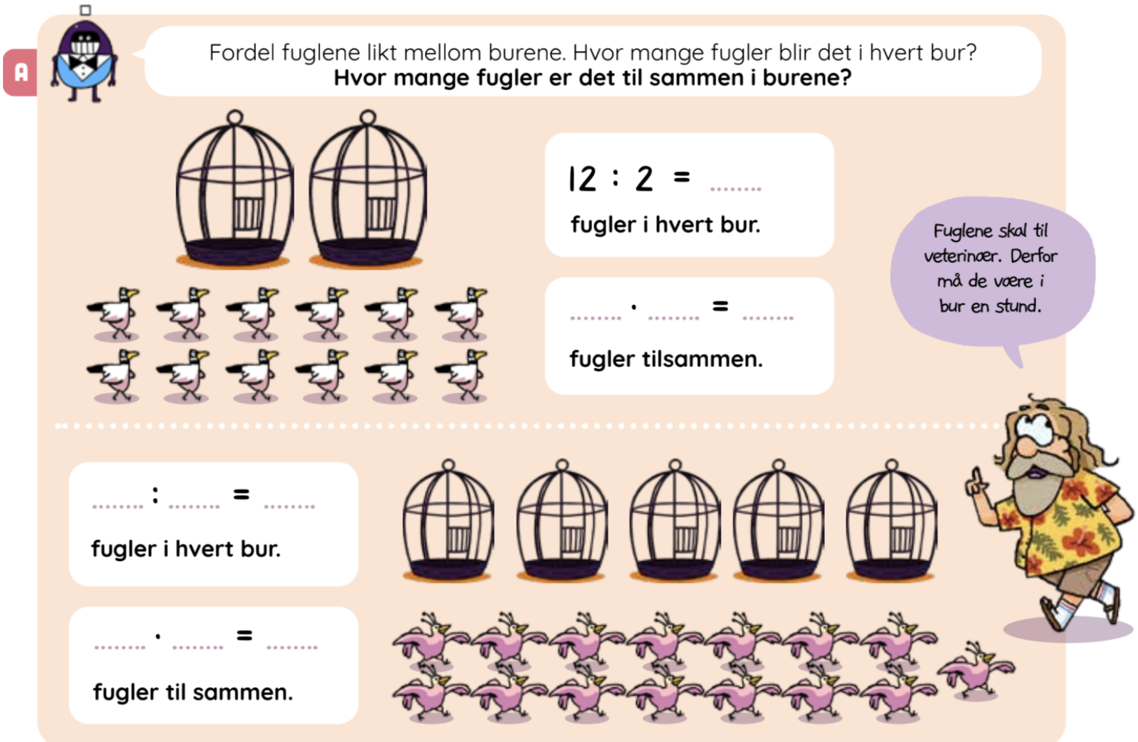
Another educational tool used for STEM topics, particularly within mathematics, is Dragon Box. It is less known than the two aforementioned applications, and is in only Norwegian and aimed at Norwegian grade schools. While Duolingo and Brilliant are tools aimed at individuals where the users choose to sign up individually to use the service, the paying customers of Dragon Box are educational institutions. Dragon Box is meant to be used by teachers as a supplement to traditional education. Being focused on grade school pupils aged 6-10, the application features a very playful design (see figure 9.5a). As Dragon Box is meant to be used in class, it doesn't have retention gamification elements as seen in Duolingo and Brilliant, such as daily streaks and public leaderboards.

The tasks in Dragon Box are small math questions, but instead of just using plain numbers and equations, the goal is to visualize the problem to make them easier to comprehend. Figure 9.5b shows one task which teaches division and multiplication by dividing a certain number of birds among a certain number of cages.

In a 2021 randomized controlled trial study, the efficacy of Dragon Box was measured [43]. The study shows an increase in motivation and some improvements in performance among all demographics.

[5] https://brilliant.org/

(a) Illustration of levels overview



(b) Illustration of a division and multiplication task in DragonBox

Figure 9.5: Illustrations of DragonBox[6]

[6]https://www.dragonbox.no/

9.4 Summary

Three relevant applications have been analyzed. Duolingo is a language learning platform that relies on several gamification and reward systems to enhance user retention and engagement. Brilliant.org is an interactive learning platform that is focused on active learning through problem-solving and includes topics for mathematics. It uses gamification elements and reward systems such as XP Systems, but not as extensively as Duolingo. Both applications are learning platforms with added gamification elements, rather than video games with learning elements. Dragon Box appears more as a video game and is aimed at mathematics, but the target audience is younger than the intended target audience for this project. With an analysis of some related existing applications, the next Part describes the creation of a new learning game which will be the basis of this project.

Part III

Product

Following the preliminary research and a newly acquired understanding of learning theories, trigonometry, and game-based learning, this Part will introduce the mathematical learning game developed for this thesis. The first chapter will provide a surface-level description of the game, highlighting its key features and how it facilitates experiential learning. The following Chapter will delve more into the design decisions, which are grounded in the theory described in the preliminary research. Then, the development process will be described, followed by an overview of the game's technical implementation and the tools used during development.

Chapter 10

Game Description

In this Chapter, the game-based learning platform developed for this thesis will be described. The game is named *Aftermath*, and will lay the foundation for the rest of the project. The description in this Chapter will describe Aftermath from a user's perspective, starting with a brief description of the game and its objective, followed by an explanation of the game mechanics. Then, the game environment and art style will be described, before the game progression is introduced. Subsequently, the game's educational content is presented, including how it relates to the curriculum whilst fostering experiential learning. Then, the game's user interface and controls are presented, followed by a brief consideration of the game's technical requirements.

Figure 10.1: Landing page of the Aftermath game[1]

[1] https://aftermath-game.vercel.app

10.1 Game Objective

Aftermath is a single-player 2D game developed as a game-based learning platform for trigonometric functions, specifically sine, cosine and tangent functions. The goal of Aftermath is to let players explore a sandbox environment with visual and interactive elements, and thereby provide a more experiential approach to learning. The game is not meant to replace traditional education, but rather serve as a supplement which provides a deeper comprehension of the educational material.

The game's objective is to collect coins that are distributed across a coordinate system. The player must to craft a sinusoidal function in a control panel by choosing a function and adjusting the function's various input fields. To craft the correct function, the player needs to understand the concepts of the sinusoidal function variables: Amplitude, Frequency, Phase shift and Vertical Shift. These concepts are all described in Chapter 7, specifically in Section 7.5-7.8, respectively. How the player interacts with these through the control panel will be further described in Section 10.2.

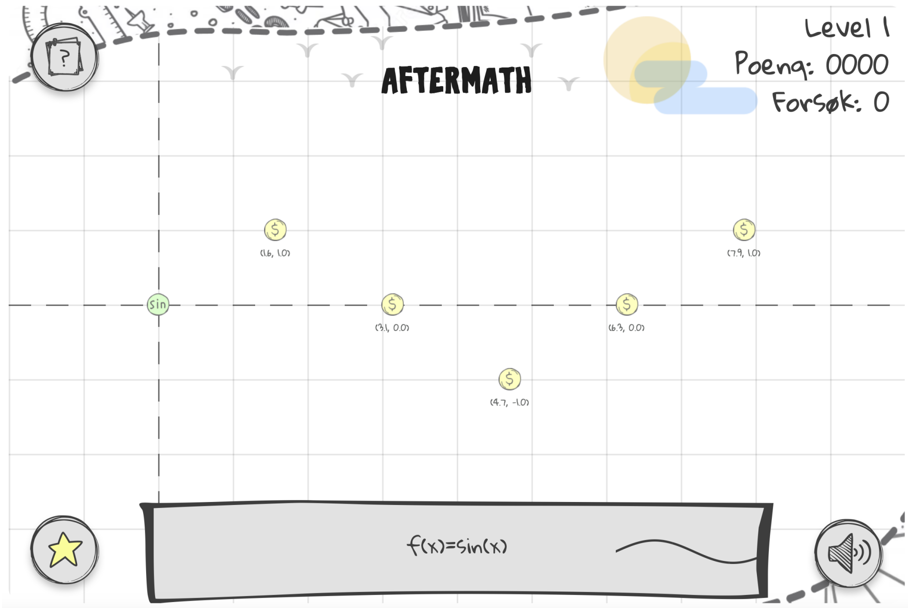


Figure 10.2: Level one of Aftermath

Figure 10.2 illustrates the first level of Aftermath and shows how coins are distributed throughout the coordinate system. The first level has no input fields for the sine function, and the user only needs to press the spacebar to shoot the function from the origin. A firing attempt from a player is illustrated in Figure 10.3. The user receives a score based on a scoring algorithm for each level and is awarded 0-3 stars for each level based on their score. The goal of the game is to complete as many levels as possible and to get as many stars as possible. The scoring algorithm and scoring system will be described more in-depth in Section 10.2.3.

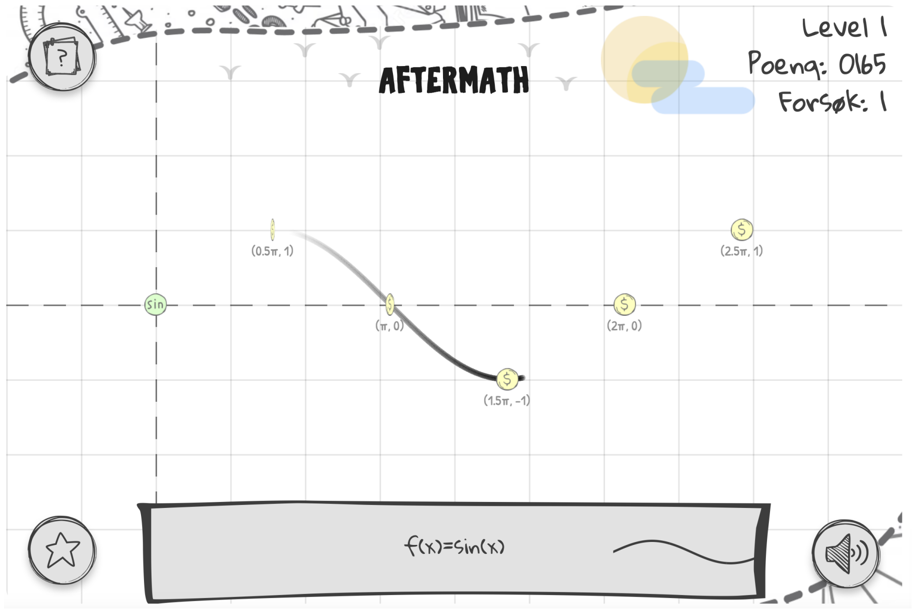


Figure 10.3: Firing the sine function to collect coins

10.2 Game Mechanics

Game Mechanics describe the rules that govern the gameplay, how players interact with the game, how actions impact the game state, and the scoring system that is used to evaluate the players' performance. This section will give a thorough introduction to the different game mechanics in Aftermath.

10.2.1 Function Control Panel

Figure 10.2 and Figure 10.3 show the most basic control panel that is introduced to the player. As mentioned, in the first level the player only has to press the spacebar to complete the level. Throughout the game, the control panel is gradually extended with new concepts. The control panel includes all the possible choices and combinations the player has available to craft a function to complete each level. On the right side of the control panel, a simple preview of the sinusoidal graph is shown. This preview immediately updates when the player changes the function. Different examples of the control panel are illustrated in Figure 10.4. They include choices for setting the Amplitude, Vertical Shift, Frequency, Phase Shift, and switching the core function between sine and cosine.

The different variables are individually capped at a certain number in both directions. This is illustrated to the user by removing the up and down button in the capped direction. Capping the variables reduces the number of combinations available to the player, thereby reducing the span of dominating strategies. For example, if it were possible to set the Angular frequency to an arbitrarily large number, the function would cover the whole map and hit all the coins no matter where they are placed.

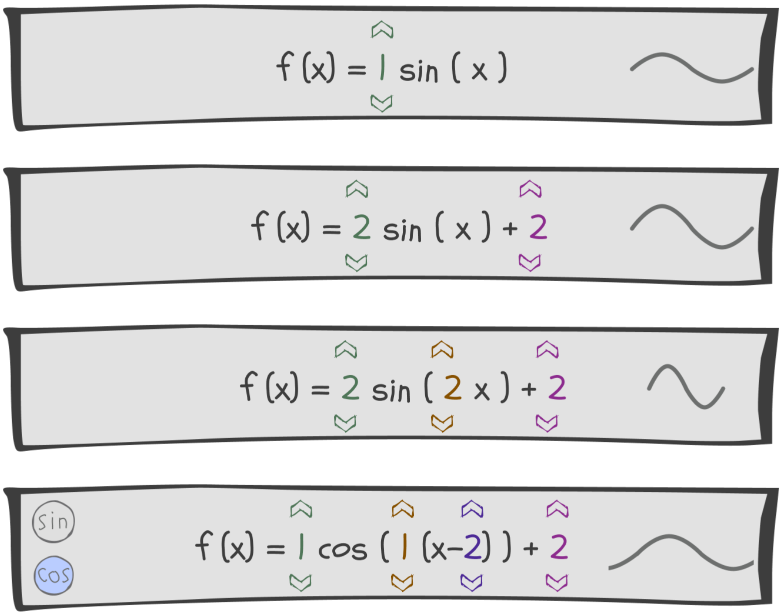


Figure 10.4: Different variations of the function-control panel with gradually increasing complexity.

10.2.2 Power Bar and Bombs

As players progress, the game introduces a Power Bar which indicates the power the user wants to shoot in the function domain. A higher value will cause the function to go further, and the maximum value corresponds to the right end of the visible domain. Alongside the Power Bar, bombs are also introduced. If the player fully charges the power bar, they risk hitting the bombs that can be located at the end of the visible function domain. Bombs will affect the users' score if they are hit and usually cause so much point deduction that the users will not be able to proceed to the next level. These concepts are illustrated in Figure 10.5.

10.2.3 Score System

As explained in Section 10.1, the goal of the game is to collect coins in the coordinate system to reach the highest score possible. For each level, the player's performance is ranked on a scale from zero to three stars, where zero means the level is incomplete, and three stars indicate that the player has found a perfect solution. The star score thresholds are set individually for each level, which means if a score gives 3 stars on one level, it is not guaranteed to be enough for 3 stars on another level. The score reached on the completion of a level is communicated through a scorecard illustrated in Figure 10.6.

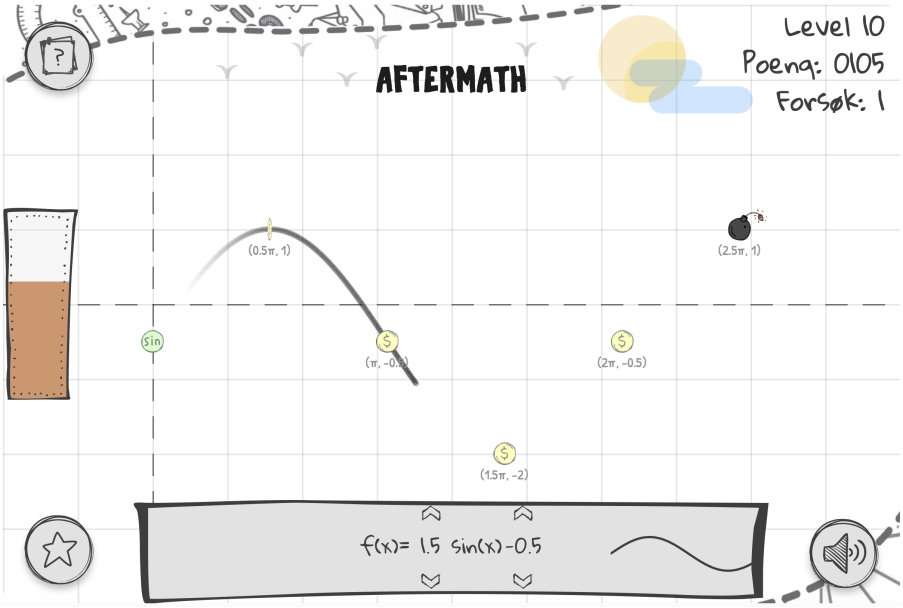


Figure 10.5: Introduction of the Power Bar and Bombs

Figure 10.6: Score Card for an arbitrary level

For each level, the players receive a score based on their performance. This is meant to improve the players' motivation and will be further discussed in Section 11.4. The score, S_i, for a given shot i is given by the formula:

$$S_i = \sum_{j=1}^{c} C_j, \tag{10.1}$$

where C_j is the score for a collected coin j. C_j is defined as

$$C_j = 100 * n_j * m_j, \tag{10.2}$$

where n_j is a multiplier given by the number of coins collected in this shot, defined as

$$n_j = 1 + c_j * 0.1, \tag{10.3}$$

where c_j is the number of coins collected in this shot before collecting coin j. Lastly, m_j in formula 10.2 is a multiplier based on the current shot index, given by

$$m_j = \frac{1}{(k_j - 1) * 0.2 + 1}, \tag{10.4}$$

where k_j is the total number of shots attempted when hitting coin j. m_j is inversely proportional to k_j, which means that the negative effects of using many attempts diminish. For example, $m_1 = 1$, $m_2 \approx 0.83$, $m_3 \approx 0.71$, $m_4 \approx 0.63$. This is meant to both encourage good players to complete the level in one attempt, but also not unreasonably punish struggling players for using many attempts. The reasoning for this experiential approach will be discussed further in Section 11.1.

Based on the score for each shot, S_i, defined in Equation 10.1, the total score, S, for a level is a simple summation of all shots:

$$S = \sum_{i=1}^{n} S_i \tag{10.5}$$

where n is the total number of shots used at this particular level.

10.3 Game Environment and Settings

As shown in Figure 10.1, the game environment in Aftermath is inspired by a grid notebook. Such notebooks are commonly used in Math courses, and this design decision was made to give players the feeling of a paper-like textbook in a digital environment. The hand-drawn art style is consistently used throughout the game, providing a unique and cohesive aesthetic to all elements. Furthermore, sound effects were added for coin hits, star collection, and bomb explosions to enhance the player's experience. To further enhance the gameplay, the sine function is animated to fade while moving, creating a snake-like effect as it traverses the map. Overall, the game is designed to evoke the feeling of a digital notebook, combining a familiar setting with innovative gameplay elements to facilitate the learning of trigonometric functions.

10.4 Game Progression

The game's progression follows a fixed ascending order, with 25 levels in total. The player begins at level 1 and progresses through the game sequentially, tackling one level at a time. Players need to complete a level to unlock the next. The overview of the different levels is shown in Figure 10.7.

The initial levels are designed to be relatively easy, serving to introduce the players to the core concepts. A common issue in game design is balancing the difficulty, as the game developers naturally have more experience playing the game. Thus, they may consider a certain level easier than new players, because the developers understand the game mechanics more thoroughly. This is one of many reasons why extensive user-testing is important when developing games or other applications.

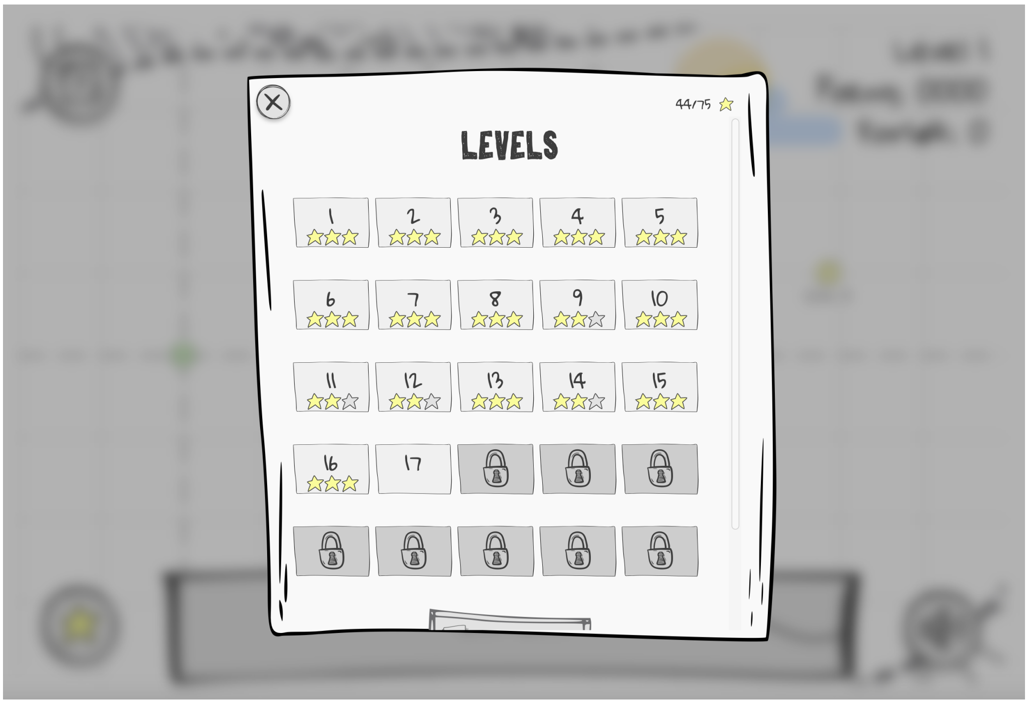


Figure 10.7: Level Modal in Aftermath

As the player progresses through the levels and new concepts are introduced, the difficulty increases. In a high school class, there is typically a difference in students' skills. It is important to both ensure low-skilled players do not lose motivation, and to ensure high-skilled players do not finish the entire game before the end of the experiment. Thus, the later levels are significantly more difficult. Figure 10.8 shows the last level of the game. To solve the level, the player must use all the different game mechanics introduced throughout the game.

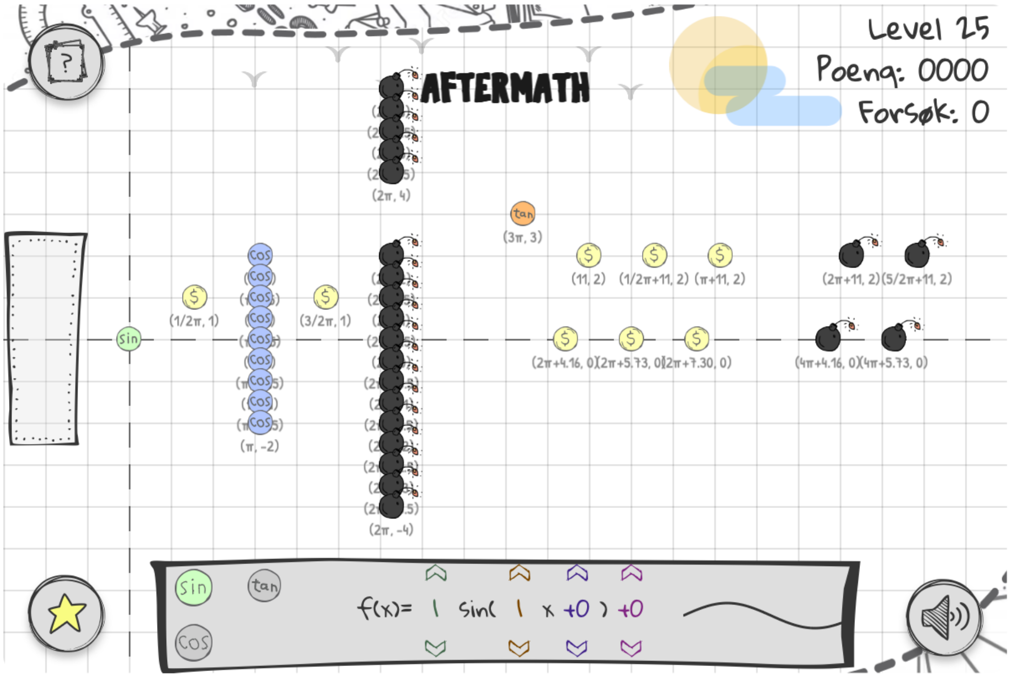


Figure 10.8: Complex level

10.5 Educational Content Integration

In addition to letting the player explore trigonometric functions in a sandbox environment, the game also briefly describes the related material from the students' curriculum. The educational material is based on Chapter 7 and is aligned with the trigonometry taught in Norwegian high schools. However, the game uses animations in an attempt to explain the material better than the books which are used in traditional education. The aim is to help players gain a visual understanding of trigonometric functions, hoping that computer graphics will serve as a more effective tool for illustration compared to traditional textbooks. However, the educational content in Aftermath is rather brief, as the main purpose of the game is to allow players to explore at their own pace. This makes students learn through experiential learning, rather than through reading and memorization.

The players are introduced to the educational content through Information Cards that appear before a level which introduces a new concept. The content presented in the information cards are inspired by Brilliant.org[2] and the theory presentation of Khan Academy[3]. The content for each concept includes a title and a combination of text, images and animations. Figure 10.9 illustrates an Information Card introducing the concept of vertically shifting a function.

[2]https://brilliant.org/
[3]https://www.khanacademy.org/

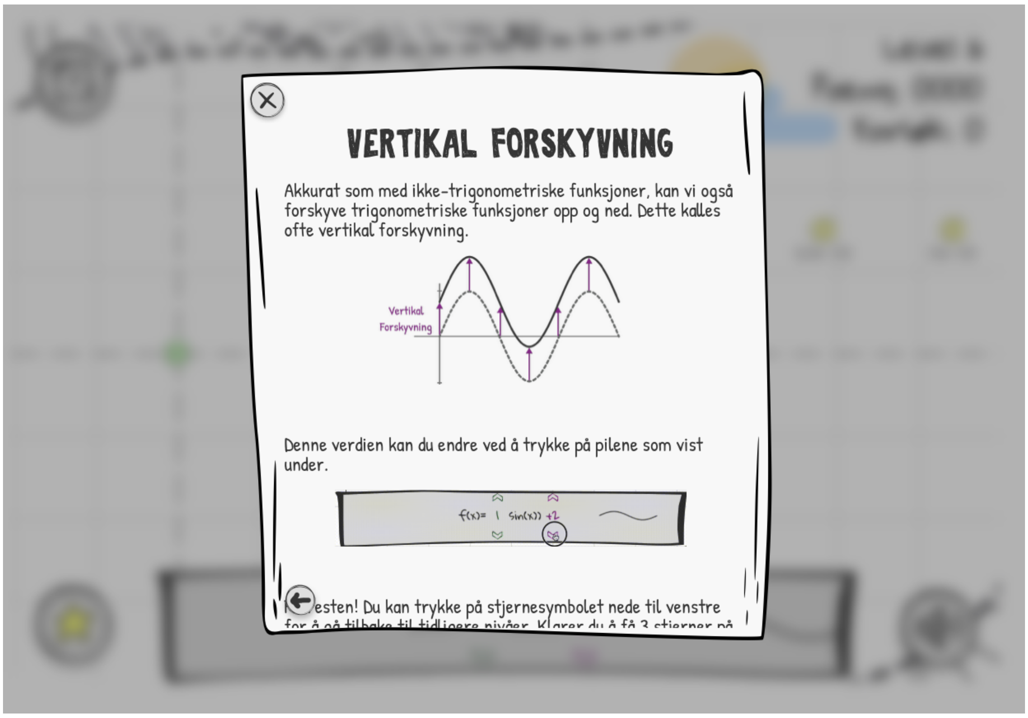


Figure 10.9: A Information Card introducing the concept of Vertical Shifting

As mentioned previously, the educational material in the Information Cards are mostly introductory. The text is usually quite brief and tries to explain the material using simpler and more informal language than the language typically used in school books. This is intentional, as the goal is to give students a basic understanding of the various mathematical concepts, rather than strictly following the mathematical language and descriptions from the curriculum. This decision will be further discussed in Chapter 11.

10.6 User Interface and Controls

The user interface and controls are designed to be both engaging and easy to learn. The coordinate system, which is the primary focus of the game, is displayed at the center of the screen, with target coins that players aim to hit and bombs they need to avoid. Positioning the coordinate system at the center helps to draw the player's attention to the core gameplay.

Around the borders of the game screen, there are various control buttons. Placing the buttons at the border of the screen prevents any obstruction to the gameplay. One button lets the player access the Information Card if they want to review the educational material. Another button lets the player open the levels overview, seen in Figure 10.7. There is also a button to toggle sound effects. Lastly, the control panel is provided, as described earlier in Section 10.2.2. All of these interactive elements can be viewed in Figure 10.10.

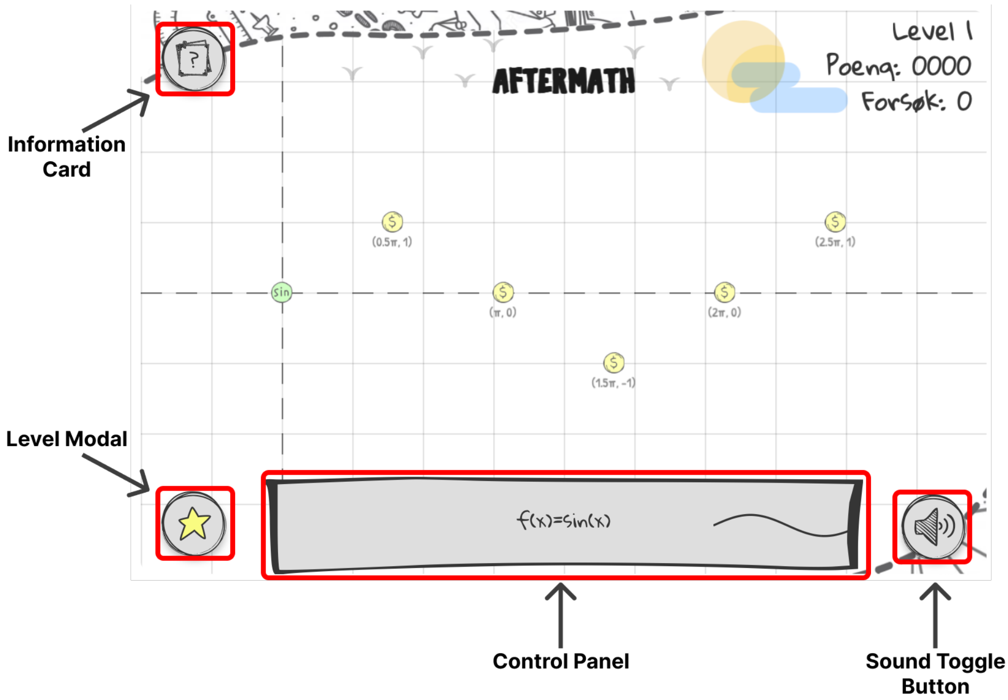


Figure 10.10: UI-layout with intractable elements highlighted

Aftermath also includes various feedback systems to help users understand the consequences of their actions, confirm their input and track their progress. Visual feedback is provided with the use of colors for each of the trigonometric concepts. An example can be seen in Figure 10.9, where the vertical shift illustration uses the same color as the vertical shift selector in the control panel. Using a consistent color scheme for the different concepts can help in users understanding. Another visual feedback is the use of animations for all clickable elements when the user hovers over them. Interactive elements have drop shadow to indicate that they are clickable. If an interactive element is disabled, such as the levels overview being disabled while the sinusoidal wave is firing, the drop shadow is removed to indicate that the element is no longer clickable.

The control panel includes an abstract illustration of the trigonometric function currently selected. A change in the current variables in the control panel would cause the illustration to animate to its new state, which in turn gives the users visual feedback of how their actions are affecting the function they are creating.

To track progress, the user can view their current score for each level in the levels overview, as illustrated in Figure 10.11. The overall progress can be viewed by the amount of stars achieved in the top right corner.

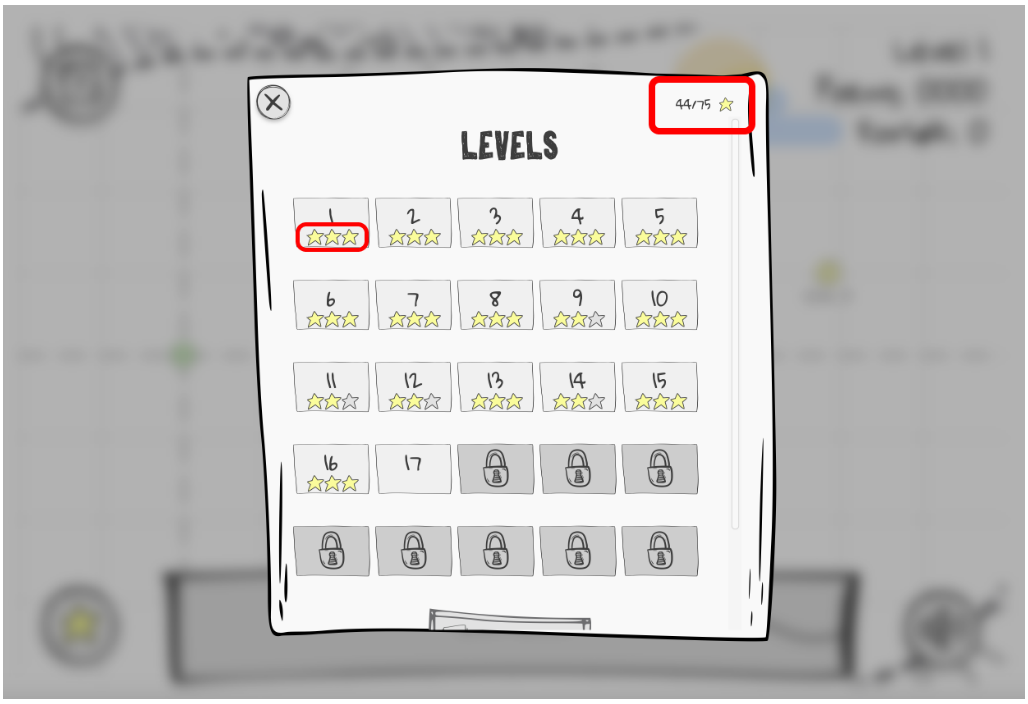


Figure 10.11: The levels modal with player progress indicators highlighted.

10.7 Technical Requirements

Aftermath is a browser-based game for desktop and laptop computers. The game is rendered on a canvas with a resolution of 1200x800 pixels, and the game runs well on any computers with a matching or higher resolution. As a browser-based game, Aftermath is compatible with virtually any modern computers. The game has been tested and confirmed to work well with Chrome and Safari browsers, while compatibility with other browsers may vary and has not been extensively verified.

There is no specific hardware requirements, but through manual testing the game was found to work fine on devices with any modern CPUs. Initially, there were some issues with hit detection on older or low-quality devices. In this previous version of the game, the coins, bombs and other collectibles had a box collider which was used to detect collisions with the trigonometric wave. The wave had a similar box collider at the front of the wave. On devices with poor performance, the number of frames per second (fps) was lower, which meant that the trigonometric wave sometimes appeared to phase through the collectibles. In one frame, the wave was behind the collectible, and in the next it was ahead. This is a common issue in many physics-based games [44], as movement and collisions are calculated discretely. In Aftermath, this was solved by pre-calculating the path of the trigonometric function when the user shoots, and detecting whether or not a given collectible would be hit. Then, when the wave reaches the x-value of the collectible, it registers the hit regardless of the current y-value of the drawn wave. This is possible because the path is deterministic, and ensures the hit detection will work on any computer, regardless of its technical specifications.

10.8 Summary

This Chapter introduced the game *Aftermath*, a 2D single-player game created as an interactive learning platform for trigonometric functions. It uses a sandbox environment to allow players to explore the trigonometric functions through the collection of coins in a coordinate system. The game serves as a supplementary tool for traditional education and aims to enhance conceptual understanding. The game includes educational content inspired by Brilliant.org and Khan Academy and is modeled on the curriculum of Norwegian high schools. Through its game mechanics, carefully planned game progression, integrated educational content, and an aesthetically pleasing user interface, *Aftermath* combines game-based learning with immersive gameplay, providing a novel approach to learning trigonometric functions. With this thorough description, the next Chapter will explore and discuss the design decisions made during the development process.

Chapter 11

Game Design

This Chapter will cover how the game is designed based on the theories of gamification and game enjoyment presented in Part II. The design decisions made in this Chapter will lay the foundation for the Chapter 25 which will discuss RQ5: *How do the theories in game-based learning contribute to the players' enjoyment, motivation, and learning outcome in an educational mathematics game?*. This Chapter begins by exploring the experiential learning model and how it relates to Aftermath. Then, the game will be discussed in regard to Malone's theory, including the three aspects: *challenge*, *curiosity* and *fantasy*. Then, a discussion of how Aftermath relates to GameFlow is presented, followed by a similar discussion about reward systems.

11.1 Experiential Learning

Aftermath is built with a learning-by-doing design, rewarding players for giving things a try, even if they do not get it right the first time. Instead of having to work out everything perfectly from the start, players learn from the process of trial and error. This idea conforms with Experiential Learning described in Section 8.3 and the game takes great inspiration from Brilliant's approach through active problem-solving rather than passive reading or memorization. One could also argue that the approach falls under the umbrella term of a Behaviouristic approach to learning, which is about learning from actions and consequences, as described in Section 5.2. However, it is important to note that the game does not strictly enforce right or wrong actions. It sees failure not as a setback, but as a learning opportunity. The game pushes the players to take a closer look at where they went wrong and to understand the game's principles from these experiences.

The game takes most of its theoretical inspiration from Kiili's model [40], described in Section 8.3.2. First, the game conveys knowledge to the player via Information Cards. It then challenges players to figure out the correct function to collect all the coins, thereby encouraging problem-solving skills. The players are prompted to apply the information given, form a hypothesis, evaluate their results, and iterate their actions until all coins are collected and the level is completed. When players decide the parameters and launch the function, they successfully navigate the initial phase of Kiili's model - idea generation. Following the execution, the players must assess the function they have created, and consider any necessary adjustments.

Summarized, Aftermath is inspired by the theoretical models of Kiili[40] while taking some inspiration from Brilliant's presentation of the course material before they introduce problem-solving tasks. Both of these sources of inspiration are grounded in the direction of experiential learning, which has heavily influenced the development of Aftermath.

11.2 Making a Fun Game

Malone's theory of what makes things fun to learn lists three overall aspects that a game design should consider: Challenge, Curiosity and Fantasy [36]. This section will look at all these aspects individually and discuss how they are applied in the game.

11.2.1 Challenge

Malone highlights three core concepts that relate to the challenge of a game: *Goals*, *Uncertain Outcomes* and *Self-Esteem*.

Goals

Aftermath is a game where the players try to collect as many coins as possible, and by doing so they unconsciously learn trigonometry. This conforms with Malone's theory, which suggests that the skill being taught in a game should be a means to achieve a goal, rather than the goal itself. In the case of Aftermath, the trigonometric concepts are tools the player can use to collect coins and achieve a higher score. Consequently, a robust understanding of these principles naturally translates to higher scores.

In the creation of Aftermath, some of the crucial aspects outlined by Malone (Section 8.1.1) were taken into account. Firstly, the game's premise and objectives were designed to be both simple and obvious, making them easy for players to understand. The objective of collecting coins can be perceived as realistic and to some extent, driven by fantasy. Moreover, the game provides the player with feedback in the form of sounds and visuals whenever a coin is collected, along with a scoring system that reflects the player's performance. These features all align with Malone's recommendations for creating goals.

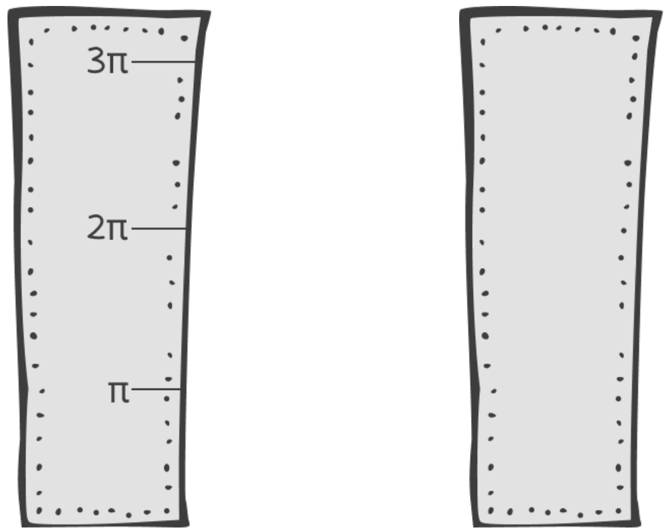
Uncertain outcomes

Introducing uncertainty in a game based on mathematics may be difficult, as mathematical functions are inherently deterministic. To make the gameplay less predictable, bombs and a power bar were introduced. These are shown in Figure 10.5 and described in Section 10.2. If a player hits a bomb, the wave immediately stops (which means no more coins can be collected) and the player's score is reduced. This makes the levels more uncertain, as the player does not know whether they will hit a bomb or not, even with a mathematically correct solution. Thus the player does not only need to craft the correct function but they also need to estimate the shooting distance. The following list highlights other aspects of the game that contribute to the challenge and uncertain outcomes.

- **Hidden Information:** Two clear instances of hidden information has been used in Aftermath. Firstly, only the first level is available to the player, and they must complete the level to unlock subsequent levels. This may increase players' curiosity about what comes next. The locked levels can be viewed in Figure 10.7. Secondly, to increase the challenge, the power bar does not clearly show how far the wave will go, as just described under uncertain outcomes. In an earlier version of the game, the power bar had markers indicating the length the wave would travel in the x-axis, given in values of π. A prototype of this version of the power bar is shown in Figure 11.1a. A typical trigonometry exercise is to calculate the period of a function, and this version of the power bar would be a gamified version of such an exercise. However, during user-testing (see Section 12.4) it was suggested to remove this to make the game less predictable and thus more fun to play. Ultimately, it was a trade-off between potential learning outcomes and enjoyment, and the period markers were removed from the power bar in the final version of the game. The final version of the power bar is shown in Figure 11.1b.

- **Multiple Level Goals:** In Aftermath, each level has at least one set solution, and the star system rewards players with one to three stars. For players who strive for perfection, achieving three stars requires crafting the optimal solution in the control panel. This allows players to set their personal mastery level, thereby creating multiple layers of goals. The primary objective is to advance to the next level by collecting all coins, while the higher-level goal is to do so in the most efficient manner, by formulating optimal solutions. If a player finds satisfaction in merely completing a level, they can aim for one star, whereas those who want to perfect a level can strive for three stars on the first try. This design enables players to self-regulate their game flow, deciding for themselves whether to progress to subsequent levels.

- **Variable Difficulty Level:** Malone suggests that games should have variable difficulty levels, either adjusted based on player performance or by letting the player manually select a difficulty level. This is not the case in Aftermath, as it is important for the empirical study to collect objective data about the



(a) Period Info (b) Info Hidden

Figure 11.1: Power Bar with and without period info

players' performances. However, if the game was developed further and used as part of a high school mathematics curriculum, one could consider adding variable difficulty levels. For example, the player could choose a difficulty level at the beginning (or this could be automatically set based on their grade in the course). The levels could also become easier or harder depending on the player's performance by spawning fewer or more bombs on the level. Another way to vary the difficulty level could be to show level-specific hints to players who are struggling. For a more in-depth discussion on the topic of variable difficulty levels, see Part VI.

Self-esteem

Self-esteem was an important factor during the design and development of Aftermath. Several design decisions were made based on this element of Malone's theory. It is also reasonable to assume a correlation between *self-esteem* and *math anxiety*, described in Chapter 6. Thus, the causes of this was also taken into consideration when developing the game.

Firstly, as described in Section 10.2.3, the scoring algorithm has diminishing negative effects. That is, the points reduction for completing a level in two shots as opposed to one is more severe than the points reduction for completing a level in three shots as opposed to two. This decision is meant to reward good performance whilst not unreasonably punishing poor performance.

Secondly, the game uses sounds and visual effects as feedback when the player does something positive, such as collecting coins or achieving stars. A similar effect is used when a player hits a bomb, but other than this particular example, the audio and visual feedback are focused on emphasizing good performance rather than poor performance.

Thirdly, the game levels were designed with a gradually increasing difficulty level. The first few levels are very easy, and the last few levels are very difficult. With this design, even those who struggle to understand trigonometry should complete a certain number of levels. One could argue that such a *regression towards the mean* design does not even negatively affect good players' sense of achievement. An over-achieving player will likely focus mostly on the fact that they are doing *better* than their peers, while an under-achieving player will likely focus on *how much* worse they perform. For example, if you are on level 17 while your friend is on level 15, you feel proud that you are in the lead, but you would not necessarily feel much better about your own performance if your friend was on level 5. However, your friend will feel that he is right behind you and doing almost as well, even though you know that levels 16 and 17 are likely too difficult for your friend to complete.

11.2.2 Fantasy

The fantasy of Aftermath tries to evoke the feeling of working with mathematics problems in a grid notebook, as a student would do if they were using traditional pen and paper. The concepts of math functions such as Sine, Cosine and Tangent are represented in the same manner as the coins and bombs the users try to hit and avoid.

Based on Malone's two types of fantasy, Aftermath is designed with intrinsic fantasy, where the players' skill is affecting the grid notebook and its elements and the fantasy also affects the players' skills. An example of intrinsic fantasy is when the player uses their skills to create a function to launch into the grid book. The function will traverse the grid notebook and collect all the coins along its path. If the function is optimal it would collect all the coins. However, if the function were sub-optimal, only collecting a subset of the coins, the players can now choose a different strategy to collect the remaining coins, rather than crafting the function that was initially optimal. Also, when creating functions and launching them, the player can see graphically whether their function is too high or low, and by how much. If the players hit bombs on the map, they will understand how long was too long in the power indicator, which means the skill is somewhat dependent on the fantasy.

11.2.3 Curiosity

Curiosity is a central element of Aftermath through the exploration and discovery of new game mechanics and levels. Both Sensory Curiosity and Cognitive Curiosity have been important throughout the design and development of Aftermath.

Sensory Curiosity

Sensory curiosity in Aftermath is primarily realized through audio and visual effects, adhering to Malone's principles. Sensor curiosity could also be related to the GameFlow element of feedback, described in 8.2.6.

Firstly, Aftermath contains some decorative graphics, such as the sun, clouds, and birds visible at the top of the screen, shown in Figure 10.3. These are purely decorative and do not directly influence the gameplay. The decorative graphics were added to make an otherwise plain design a bit more interesting, and these particular graphics are suitable because some students tend to fill their notebooks with various doodles.

Secondly, the sine wave is designed to mimic a pencil sketching a line across a notebook. This is achieved by using a color similar to a regular pencil, and by using a fading trail animation. This amplifies the fantasy element of a grid notebook.

Thirdly, Aftermath includes several systems for representing information. For example, the stars on the Score Card in Figure 10.6 are a visual representation of player performance. If the player did not find an optimal solution, one or more of the stars are greyed out. Another example of a representation system can be found in the function indicator seen in the control panel in Figure 10.10. It provides a preliminary visualization of the shape of the expected graph without displaying all information. This is also important for cognitive curiosity which will be described below, and for player control which will be described in Section 11.3.

Lastly, as covered previously, sound effects were added to reward good performance, such as hitting coins and collecting stars. Additionally, on the Score Card the three stars appear with a slight delay for each consecutive star. This is meant to increase the tension and excitement when a player receives their score. Furthermore, the volume for a star's sound effects increases for each one, which is meant to further emphasize a three stars achievement.

Cognitive Curiosity

To evoke players' Cognitive Curiosity, several features were added. Firstly, the graph indicator was added to give the player feedback on how their actions affect the function. Because the graph is not based around a coordinate system, it can make the player feel like they do not see the full picture, and thus trigger their cognitive curiosity.

Secondly, by adding sine, cosine and tangent coins, it can be challenging for a player to immediately see how the whole map should play out, as these coins change how the function behave by transforming it. A transformation from a sine function to a cosine function when hitting a cosine coin is illustrated in Figure 11.2. Enhancing levels with multiple of these coins could seem challenging at first, but also cause an increase in cognitive curiosity, as the users want to figure out how it all comes together in the perfect solution for each level.

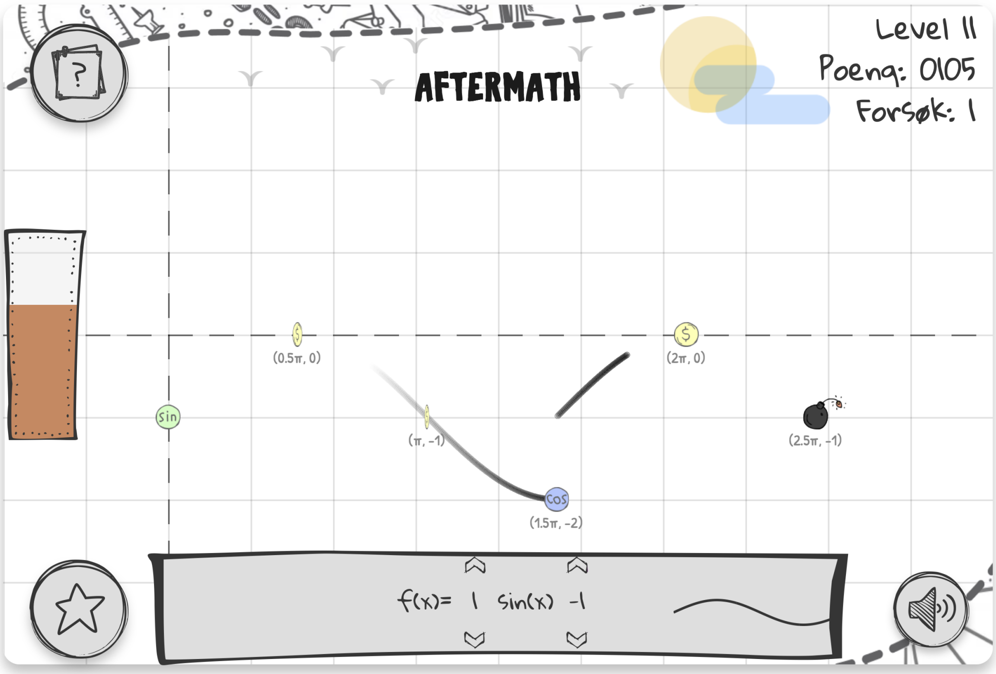


Figure 11.2: Cosinus coin affecting the sine wave

11.3 GameFlow

This section will cover how the game is designed to produce flow for the players. The design decisions will be discussed with consideration of the GameFlow theory described in Section 8.2. Previous sections have covered other theories in depth, and due to some overlap between the theories, this section seeks to cover aspects specific to the GameFlow theory. Therefore, this section has deliberately chosen to omit the parts about Goals and Feedback, as this is covered in previous sections. Also, Social Interaction is not relevant within Aftermath because it is a single-player game.

11.3.1 Concentration

Keeping players concentration is vital when achieving *Flow*. Aftermath is designed with a level-based system that presents users with different challenges as they progress through the game. This can make the player have to rethink each level, and thus provide enough new stimuli to keep their focus. The game captures the players' attention quickly as the first level only requires the user to press space to start and watch how the game is played. This will be further elaborated in Section 11.3.3.

11.3.2 Challenge

As mentioned in Section 8.2.2 Challenge is one of the most important aspects in the game design, where the challenge should be aligned with the players' skill to avoid players being bored or overwhelmed. Given the static design of the game, Aftermath introduces concepts gradually. This is both intentional and was part of our user-testing during development. The gradual introduction of concepts helps players get comfortable with each concept before moving on. This can also mitigate the pitfall of creating a game that is too challenging, which will also have an effect on the users' concentration, as challenging tasks can make users frustrated and make them give up. However, as the game is static, it is harder to balance each level because the differences in player skills can vary. To conquer this, the star system can broaden the range of the perceived challenge, where the highly skilled players can try to achieve 3 stars, while the less skilled players can focus on completing the level. Thus, the players choose the level of challenge themselves.

11.3.3 Player Skill

As mentioned in Section 11.3.2, the players are only required to press the spacebar to complete the first level. This action works as a very minimalistic tutorial and gives the player a feel of how the controls work, what the consequences of their actions are, and thus, how the game is played. The design of the first level was introduced to make it as easy as possible for the player to get started, and also make it part of the game. This decision was based on the findings of Sweetser And Wyeth, that good game designs rarely should include lengthy explanations [38, p. 7].

11.3.4 Control

During the design of the user interface, the most important aspects were to reduce the number of options available and highlight only the vital controls that are used the most. The most important task in the game is to create a function to complete each level. The function picker is thus located in the middle and bottom of the screen. As mentioned before, controlling the function and receiving feedback can make the player feel the impact of how their actions affect the game world. Other important control features include being able to change levels, read the information cards, and set the game volume. These three actions are all located on the front page. Also, as explained in Section 11.1, the game has a sandbox environment approach, where players can explore the game levels and its controls without being guided in a linear path of how to play. Even though levels have a unique mathematically correct solution, the levels can be completed in numerous ways. This is consistent with the Sweetser And Wyeth's suggestion of having multiple ways to win [38, p. 9].

11.3.5 Immersion

Immersion was arguably one of the most difficult elements to design for in Aftermath. To create immersion, a lot of the other aspects need to work together, and players need to be emotionally attached to the game. This is usually done through audio effects, a story-driven narrative, and good visual effects [38, p. 10]. Aftermath does not follow a storyline, and creating a narrative thus seemed challenging. Also, when learning, and concentrating on math problems, audio and background music could potentially be harmful to learning outcomes. Immersion should come from the solving of the levels themselves, where the players focus on solving the task. The task of puzzle-solving itself requires players' full concentration, and immersion can be accomplished through this approach.

11.4 Reward systems

The design of Aftermath incorporates different types of reward systems. Most notably, players are awarded coins and stars upon the completion of each level. The accumulation of coins and stars serves as a measure of the player's progress and experience. Other than unlocking new levels, the coins and stars do not add any new features, but act as a form of player achievement.

A gradual unlocking mechanism has also been implemented in the game, enabling players to access new mathematical concepts and levels as they progress. This approach helps avoid information overload and provides players with an opportunity to familiarize themselves with new concepts before proceeding. The gradual unlocking of new functions is illustrated in Figure 10.4.

The game responds with auditory and visual cues when players collect coins or encounter obstacles. As players progress, they can unlock various game abilities, such as the ability to change the functions amplitude, frequency, vertical shift and phase shift, as well as changing the function between sine, cosine and tangent. This adds another layer to the reward system and promotes gradual learning.

Certain reward systems were first considered but not ultimately included in Aftermath. For instance, no rewards were set for daily engagement, since Aftermath is designed for 30-minute gameplay sessions, rather than daily usage. A high score list was also omitted from the design, as the game is meant to encourage players to explore the game at their own pace and not be driven by competition. Furthermore, based on the research regarding learning mathematics and math anxiety from Chapter 6, a high score list might potentially lead to increased *math anxiety*, or prompt players to rush through the game without sufficiently learning the concepts.

If Aftermath, or a similar game, is further developed and designed for long-term gameplay spanning over several months, additional reward systems might be considered. For example, stars earned could be used to purchase in-game items such as costumes or hints, and the player could be rewarded for completing a level in consecutive days. Regardless, the design principle would remain the same, ensuring that the most enjoyable way of progressing through the game is also the most educational. It's critical to prevent a scenario where the most efficient strategy is also the most tedious, as this could limit the game's learning outcomes and enjoyment factor.

11.5 Summary

The chapter discussed the design principles that went into the creation of Aftermath, focusing on experiential learning, what makes the game fun, GameFlow elements, and reward systems. The game encourages trial-and-error learning, incorporates challenge, curiosity, and fantasy elements, and aims to maintain players' concentration. It provides gradual difficulty progression, player control, and immersion through visual and auditory effects. The reward systems include coins, stars, and unlocking mechanisms. Overall, Aftermath combines educational and enjoyable elements while addressing *math anxiety* and emphasizing the development of conceptual understanding in mathematics. With a clear description and discussion of Aftermath, the next chapter will focus on *how* the application was developed, from the initial literature review and ideation to the finalized product.

Chapter 12

Development Process

This chapter will give a brief overview of the research process. First, the overall timeline of the project will be presented. Then, the ideation phase with an initial prototype will be shown and discussed, followed by a description of the project and team management. Lastly, the user testing process will be presented.

12.1 Project Timeline

This project started with a literature review of learning principles and gamification and enjoyment in video games, conducted as a specialization project in the fall of 2022 to prepare for the master's thesis. The thesis project itself started on January 23rd, 2023, and lasted 21 weeks, ending June 19th. During the first 14 weeks of the project, the primary focus was developing Aftermath, followed by two weeks of practical experiments. The last five weeks were solely spent writing and finalizing the thesis, although some writing, mostly notes and drafts, were written throughout the whole semester. Figure 12.1 shows the project timeline graphically.

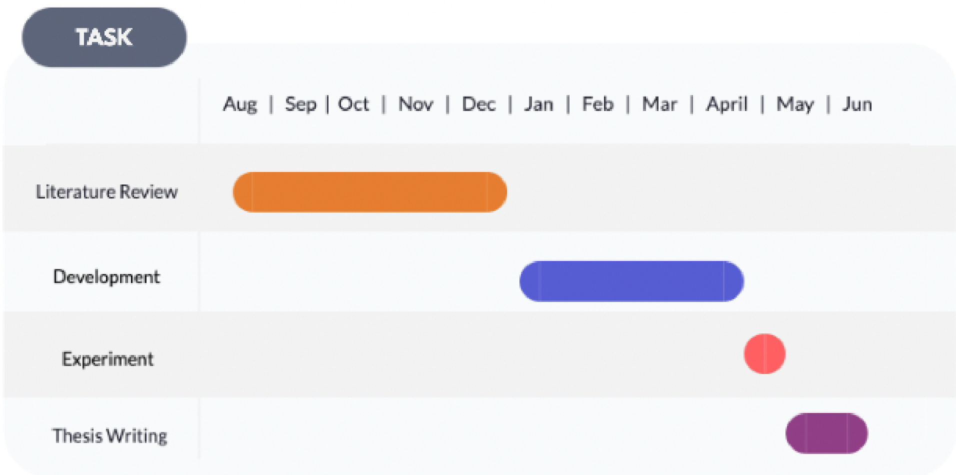


Figure 12.1: Project timeline

12.2 Ideation

The first part of the development phase was to come up with concept ideas and create initial design sketches in Figma[1]. The initial design is shown in Figure 12.2. Note the name, *Triggerd!*, which is a homonym that refers both to *trigonometry* and pulling the *trigger* of a gun. At this point of the development phase, the concept was an action game where the player controlled a soldier and shot trigonometric rockets at enemies. However, after consideration of theories related to learning mathematics, specifically math anxiety and conceptual understanding, it was hypothesized based on the theory that a competitive game with high-stress levels would not be ideal to facilitate conceptual understanding [27][29]. Instead, the choice was made to focus more on a sandbox-like experience where students could experiment at their own pace.

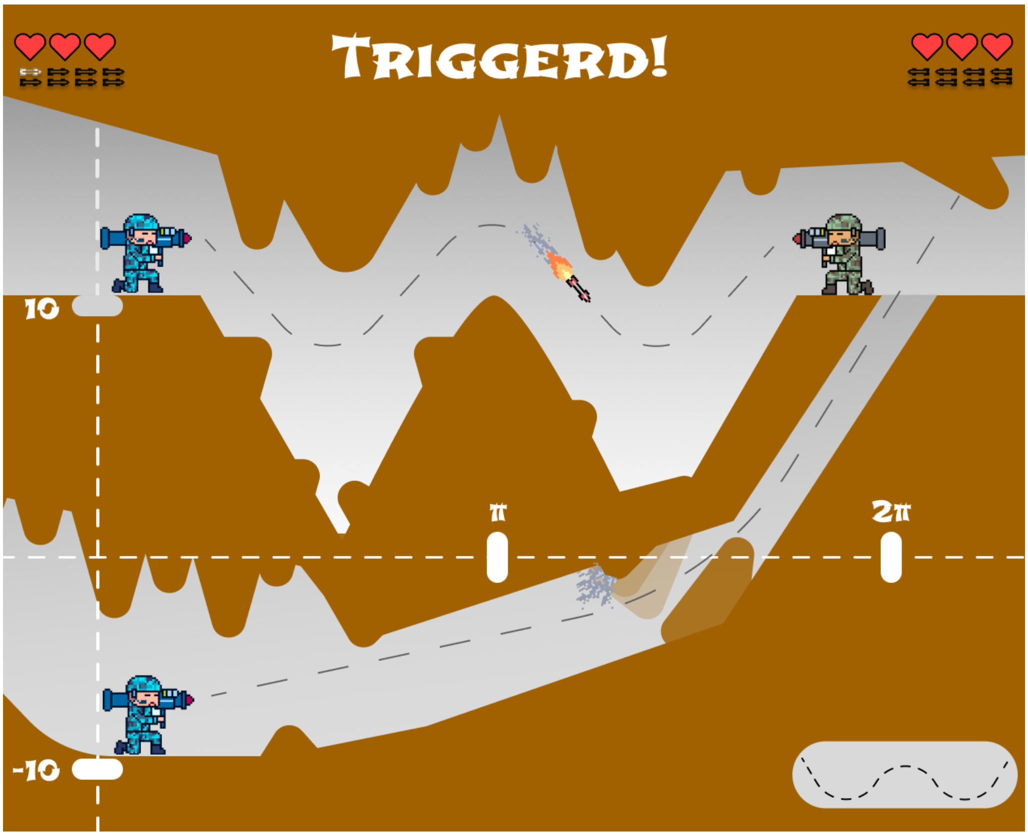


Figure 12.2: An initial design sketch of the game concept Triggerd!

12.3 Project Management

The project, particularly the development phase, followed an agile process with small and gradual increments and quick adjustments. The process was loosely based on Scrum[45], although some elements, such as daily standups and sprint planning meetings, were deemed unnecessary due to the small team size (2 team

[1] https://www.figma.com/

members). Throughout the entire project, including the specialization project, supervisor meetings were organized weekly. The time between each supervisor meeting corresponds to a Scrum sprint of one-week [45].

The two team members worked side by side most of the time, and they could easily discuss ideas and issues. Git was used as a versioning tool to more efficiently work together on the same code base, and GitHub[2] was used to upload and share code. However, due to the team size and work procedures, extensive planning tools, such as Kanban boards, were considered unnecessary. Instead, GitHub's issue page was used to report bugs and add planned features. When working on an issue, the team member would first assign themselves to the issue and submit a pull request on completion, to be reviewed by the other team member. Figure 12.3 shows a screenshot from GitHub's issue page.

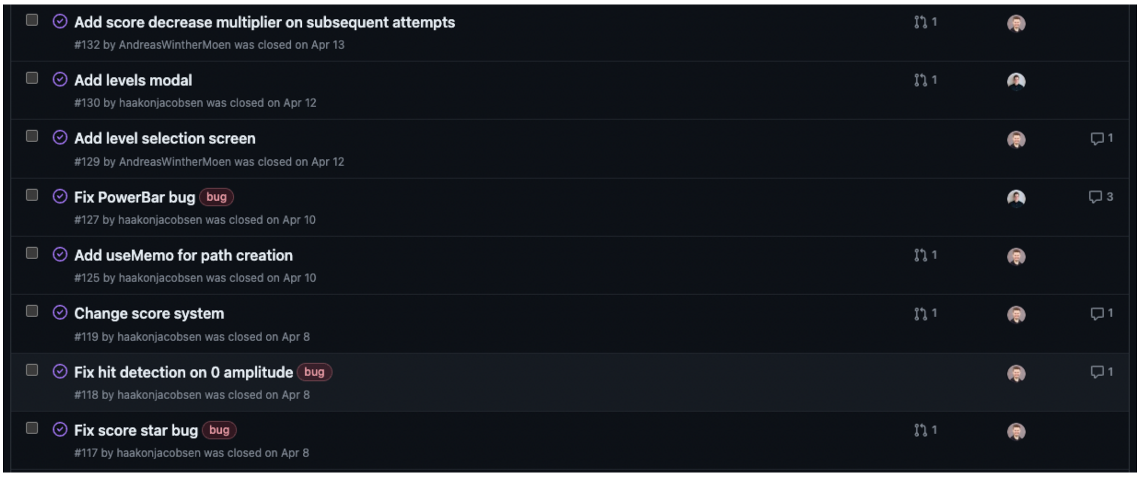


Figure 12.3: Project issues on GitHub

12.4 User Testing

During the development phase, some informal and non-systematic user testing was conducted. The tests were meant to balance the game difficulty and the usability of core game mechanics and controls. In the game balance tests, the test subjects were given a level and asked to complete it. The test subject would normally get more than one level for each test session. These tests were mostly performed by people who had seen and tried the game earlier. The usability tests were performed by people who had never seen the game before. All of the user tests were performed by fellow students and friends. As mentioned, the tests were conducted in a non-systematic way, which means the tests were not planned in advance but rather conducted arbitrarily when a new feature needed testing and there were potential test subjects nearby.

[2]https://github.com

12.5 Summary

In this chapter, an overview of the development process for the research project has been provided. The project started with a literature review in the fall of 2022, and the experimental research project lasted from January 23rd, 2023 until June 19th, 2023. During the ideation phase, initial design sketches were created and discussed, and the game concept changed from an action game involving shooting to a design based on a mathematics notebook. The chapter has also covered project management, following an agile process based loosely on Scrum, but with some adjustments due to a small team size. The user testing process is briefly described, mentioning game balance and usability tests. With the development process established, the next chapter will delve into the technical details of Aftermath and the tools used during development.

Chapter 13

Technical Implementation

This chapter will describe the technology stack used to develop Aftermath. First, the quality attributes and other implementation considerations will be described. Based on these considerations, the chosen technology stack will be presented, starting with the platform, followed by graphics libraries and developer tools. While describing the chosen technology stack, some of the promising technologies that were ultimately not chosen will also be described.

13.1 Implementation Considerations

Before selecting a technology stack and planning the development of a new application, it is important to consider factors such as what type of users the application will have, how many concurrent users there will be, the developers' previous knowledge and experiences, the application size and its performance requirements and potential security issues. The most important factors for this particular application are listed below and will influence the choices made in the following sections.

- **Development Time:** As shown in Section 12.1, Aftermath was developed over 14 weeks. The exact time frame was not known at the start of the project, but it was certainly a relatively short duration for developing a full-fledged application. Thus, development time, i.e. the time spent by the developers to complete a certain feature, is an important factor when selecting the technology stack. Using familiar and popular technologies will help reduce development time. Choosing technologies with a high level of abstraction will also generally reduce development time.
- **Code Quality:** As described in Section 3.11, Aftermath is created to conduct an empirical experiment, and the application is not meant to be further developed, neither by the researchers of this thesis, nor by other developers. Thus, development time is more important than code quality. That is, the code quality should only be prioritized if it does not significantly affect the development time in a negative manner.

- **Usability:** As will be explained in Section 15.1, only 30 minutes are allocated to the execution of the experiment. Thus, ease of use and the time required to access and start the application is important.

- **Performance:** Video games typically involve some sort of animation and movement, which makes performance important. If the computer is unable to render the frames quickly enough, the game will stutter, which may negatively affect the user's perception of the game. For a game such as Aftermath which is part of an academic experiment, it is particularly important, as bad performance may affect the test results. The test subjects may use different computers with different specifications, which means bad performance may introduce a bias in the data. However, the animations and calculations in Aftermath are fairly simple and should not significantly influence the technology stack. Note that there were some performance issues rendering the trigonometric wave (see Section 10.7), but these issues were due to an inefficient rendering algorithm and unrelated to the technology stack and were resolved before the experiment.

13.2 Platform

One of the first decisions before developing an application is which platform to use. Video games are typically made in a game engine, with two well-known and free-to-use alternatives being Unity[1] and Unreal Engine[2]. The former was a possible platform option for Aftermath, as one of the developers and researchers for this thesis has past experience with it and it is a fairly simple game engine to learn. However, a problem with applications developed in Unity is that they are native binary applications that must be compiled and uploaded, then downloaded and installed on the user's device. This would increase the time required to start the application, and would likely make test subjects start playing at slightly different times. Furthermore, the applications developed in Unity are platform-specific, which means a separate version must be compiled and uploaded for Linux, Mac, and Windows to ensure everyone is able to play. It is possible to export WebGL applications that can run in a browser, but they are still large-scale applications that require significantly longer load times than native web applications.

For a small-scale game such as Aftermath, a simple web application written in JavaScript is more than sufficient. As JavaScript is an interpreted language that does not require ahead-of-time compilation, it allows for hot reloading, i.e. making code changes and immediately seeing the results without losing the current state of the app. This reduces the development time and improves the developer experience. Both of the researchers also have extensive experience building web applications, which makes it a suitable choice to maximize development efficiency.

Web applications are traditionally static HTML pages with CSS styling and

[1]https://unity.com/
[2]https://www.unrealengine.com/

dynamic functionality through JavaScript. However, a more modern approach is to use single-page applications where the entire application is written in JavaScript and then rendered to an empty HTML page. This allows for much better global state management and enhances code reusability through reusable components. Aftermath is a game with many dynamic and interactive elements that affect several parts of the application, which makes it suitable as a single-page application. The most commonly used framework for single-page applications, both by the researchers of this thesis and the overall population of web developers, is React. Thus, it was a natural choice for Aftermath as it makes it easy to find solutions to problems and reduces development time.

13.3 Graphics Library

A web-based video game typically renders graphics to a canvas, which is an HTML element that makes it possible to draw simple graphics, such as circles, rectangles, etc., to a web page. The canvas is cleared and re-drawn at a certain time interval (e.g. 30 times per second) to create the illusion of moving objects. This is a low-level implementation that requires the developers to manually control all the aspects of the rendering. There are more high-level solutions that offer abstractions on top of the native canvas implementation and handle low-level rendering, frame-rate control, layer optimizations, etc. These reduce the development time without significantly affecting performance and are good choices for Aftermath.

There are several canvas-based rendering frameworks for web applications. Some are designed for large games with 3D graphics, for example Babylon.js[3]. However, as explained previously, 3D rendering is not suitable for Aftermath as it only relies on 2D graphics.

A seemingly interesting option is Construct[4], which is a no-code game engine and a visual tool with a drag-and-drop interface. Although this may seem intriguing, it has somewhat limited functionality. Generally, the more abstractions the easier it is to implement, but the range of development options is also limited. At the start of the project, the vision for the game was relatively unclear, and since the researchers of this thesis are experienced programmers and enjoy writing code, it was natural to prioritize feature flexibility over simplicity.

Two game frameworks with suitable abstraction levels are Phaser[5] and Pixi.js[6]. They are both optimized for making 2D games rendered to a canvas and have a wide variety of features. However, Pixi.js is a rendering engine more reminiscent of a traditional HTML canvas with some abstractions, while Phaser is more reminiscent of a game engine like Unity or Unreal Engine. Ultimately, Pixi.js was chosen, as it seemed to be a good balance between abstraction and feature flexibility, and is a much more commonly used framework than Phaser. React also integrates better

[3]https://www.babylonjs.com/
[4]https://www.construct.net/
[5]https://phaser.io/
[6]https://pixijs.com/

with Pixi.js than with Phaser through Pixi's solution PixiReact[7].

13.4 Developer Tools

As an application grows, it is important to properly organize the project and consistently follow a coding standard. This is particularly important for larger teams. However, strictly following rules and not approving pieces of code that do not follow the rules will also increase development time. As Aftermath is not meant to be a large application and is only developed by two people, it is not as important to be very strict in regard to coding standards, although some standards were agreed upon at the start of the project.

13.4.1 Coding Standards

The IDEs were configured to automatically format code according to AirBnB's JavaScript style guide[8]. This is a commonly used style guide and one of the default formats provided in the IDE. For large projects, linting tools are typically configured to ensure that uploaded code follows the coding standards. It is also possible to configure continuous integration which automatically reviews pull requests and rejects code that does not follow the standards. However, this was considered unnecessary, as there were only two developers and both developers' IDEs were configured to automatically format the code.

13.4.2 Static Types

JavaScript is a dynamically typed language, which means it automatically assumes the type of a variable. This makes it easier and quicker to write JavaScript code, particularly small code segments. However, errors are more likely to occur and it is often more difficult to comprehend code written by someone else. A solution is to use TypeScript[9], which is a statically typed language built on JavaScript. It provides easier debugging due to fewer bugs associated with incorrect type errors. TypeScript is sometimes difficult for inexperienced developers, as it requires the code to be written in a more strict manner. However, the developers of Aftermath have extensive experience with TypeScript, and although Aftermath is a relatively small project, it was decided to use TypeScript for the whole project.

[7]https://pixijs.io/pixi-react/
[8]https://airbnb.io/javascript/
[9]https://www.typescriptlang.org

13.5 Summary

This chapter has presented the technical implementation of Aftermath. The primary factors considered in the selection of technologies and third-party applications were development time, code quality, usability and performance. Aftermath was built as a web application with React, and Pixi.js was used to render a dynamic game screen using an HTML canvas. Automatic code formatting and static typing through TypeScript were used to ensure high code quality. This Chapter concludes Part III and the description of the developed product. The next Part will describe the related practical experiments.

Part IV

Experimental Design

This Part describes the approach taken for the experiment. First, it explains how the participants were chosen, including details about the pool of students, the sampling, and the experiment session. Then, the experiment itself is described, providing a clear overview of its different parts. The Part ends with a look at the questionnaire and its components, giving a complete picture of how the research was carried out.

Chapter 14

Participant Selection

This Chapter will present the selection of test subjects for the experiment. The Chapter will first describe the different mathematics programs in Norwegian high schools, followed by an overview of the three experiment sessions conducted. Lastly, the sampling strategy and its potential validity issues will be briefly discussed.

14.1 Math Programs in Norwegian High Schools

The experiment has been performed on mathematics students in 2nd and 3rd grade. In Norwegian high schools, students choose from a variety of mathematical study programs. In 1st grade, students can choose either 1T or 1P, with the former being more theoretical and the latter being more practical. Then, in 2nd grade, the 1P students normally choose 2P, and the 1T students can choose either R1 or S1, followed by R2 or S2 respectively in 3rd grade. R-math is more advanced and teaches subjects such as geometry, trigonometry, algebra, combinatorics, and differential equations. R-math is recommended for students who want to persue higher education in STEM fields (Science, Technology, Engineering & Mathematics). S-math is more focused on social studies and teaches algebra, functions, probability, linear optimization, and statistics. A visual overview of the mathematical study programs is shown in Figure 14.1. R1-students who are particularly interested in mathematics may also choose X-math as an additional subject. X-math explores number theory, complex numbers, probability, and statistics on a deeper, more fundamental level.

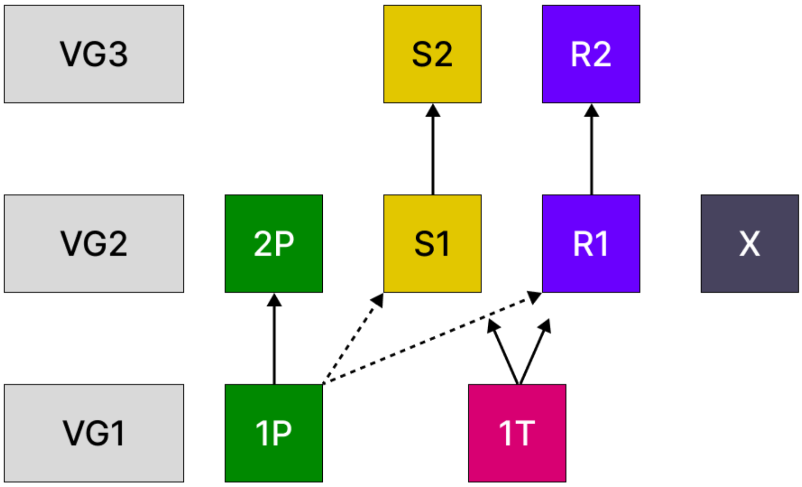


Figure 14.1: The study program paths in Norwegian high schools[1], with dashed lines indicating paths that are not recommended.

14.2 Experiment Sessions

The experiment was conducted over three separate sessions, respectively on 28th April, 3rd May, and 12th May 2023. The first two studies were performed at Strinda Videregående Skole and the third study at Kristen Videregående Skole Trøndelag. In the first and third studies, the test subjects were in second grade, i.e. 17 and 18 years old, while the test subjects in the second study were 3rd-grade students (18 and 19 years old). The demographics of the test subjects are shown in Table 14.1.

Session	Date	Grade	Study Program	No. Students
1	28th Apr	2nd	X	7
2	3rd May	3rd	R2	14
3	12th May	2nd	R1	14

Table 14.1: Experiment Sessions

Table 14.1 also shows the math study program of the test subjects. As the Table shows, each session was performed on a class from a different mathematical study program. In the first session, the test subjects were X-math students, the second session R2-students, and third session R1-students. The game levels and theory in Aftermath are mostly based on the curriculum taught in R2-mathematics, although some elements are introduced in 1T and R1. The motivation for conducting the experiment on different groups was to measure Aftermath's efficacy in two manners. Game-based learning may be an effective tool to rehearse and practice material already taught in traditional manners, as it gives the students a new perspective

[1] www.data.utdanning.no/sites/default/files/styles/test/public/matematikk_pa_videregaende_0.png

and deeper understanding of mathematical theories. However, game-based learning may also be useful during the introductory part of a course, as it may make it easier to grasp mathematical theories in the future. See Chapter 24 for a discussion of the results and Aftermath as such an educational tool.

14.3 Sampling

As described in Chapter 4, selection bias is a potential threat to both the internal and external validity of the results of the study. Ideally, the test group should be a randomized selection of the general population (which in this study would be all high school students in Norway). However, this would require students from many different schools across the whole country and is far beyond the funding and scope of this project. A common sampling technique to reduce costs without significantly introducing selection bias is called *cluster sampling*. Clusters are natural groupings of people, and cluster sampling involves obtaining a random sample of clusters from the population, with all members of each selected cluster invited to participate [46]. The technique of selecting classes from randomly selected schools is considered cluster sampling. Note that the selection of schools is not entirely random, but rather based on the geographical location of the researchers, i.e. the city of Trondheim. However, this should not introduce a bias assuming the personal attributes such as intelligence, skill level, and personality of high school students are similar across different geographical regions of Norway.

14.4 Summary

The experiment was conducted over three sessions at two different schools. A mix of X, R1 and R2 students comprised the test group. The selection of test subjects has followed the cluster sampling technique to simplify the process without significantly introducing a selection bias. With this brief overview of the experiment, the next section will delve into details about the practical execution of the experiment.

Chapter 15

The Experiment

This Chapter will describe the practical execution of the experiments. First, the overall schedule will be presented. The schedule is divided into three sections, introduction, practical experiment, and questionnaire, which will be described separately.

15.1 Schedule

All three experiment sessions listed in Table 14.1 were conducted similarly. One lecture (45 minutes) was set aside for each experiment, which was divided into three sections (see Table 15.1). Both researchers (the authors of this report) were present for the experiments.

Time	**Section**
5 min	Introduction (describing the study experiment and background)
30 min	Practical experiment (students playing the game)
10 min	Questionnaire

Table 15.1: The timetable for the experiment

15.2 Introductory Segment

The study was briefly described during the introduction, including what the test subjects would do during the experiment. As explained in Chapter 10, the game levels may be easy to complete, but getting a full score (3 stars) on each level may be more difficult. The introduction emphasized that the game may be challenging to complete fully and that everyone is not expected to complete everything. This was done to reduce the potential of math anxiety and the chance of test subjects getting frustrated, thereby losing concentration and/or motivation. Other than that, the experiment was not described in detail to not give test subjects a pre-defined opinion of the game, which minimizes bias in the results.

15.3 Practical Experiment

At the start of the practical experiment, the students were given a URL to the game[1] and asked to open the website in a browser. The students used their own computers, and everyone had a computer available. During the experiment, qualitative observations were made of the test subjects, including their performance and general mood. To reduce the Hawthorne Effect (see Section 4.2), the observations were made as anonymously as possible, with researchers observing from a distance and trying to avoid eye contact. The results of the qualitative observations will be shown in Chapter 21. Generally, students did not receive any help unless they asked specifically. This decision was made to improve the validity of the test results. Helping some students and not others would be a case of testing being a threat to both the internal and external validity of the results, as explained in Section 4.1 and Section 4.2, respectively.

Some students experienced minor technical issues during the experiment. The problems were related to canvas scaling on computers with low-resolution screens but were resolved by zooming out and refreshing the screen. Although a minor inconvenience, these issues should not drastically affect the experiment results, as they were resolved quickly.

15.4 Questionnaire

After the practical experiment, the students were asked to complete a questionnaire, which will be described in the next Chapter. The questionnaire was easily accessible through a button in the game. During the questionnaire, the researchers moved to the front of the classroom to be unable to see the test subjects' screens. This was done for anonymity and to reduce the Hawthorne Effect (see reactive effect of experimental learning in Section 4.2). At the start of the questionnaire, it was also emphasized that the results were anonymous, and the students were encouraged to be honest with their results.

15.5 Summary

The experiment was divided into three sections: an introduction, a practical experiment, and a questionnaire. The introduction lasted approximately 5 minutes, during which the researchers briefly presented the research and described the experiment. During the 30-minute experiment, the students played Aftermath while the researchers made qualitative observations and helped students fix any technical issues. At the end of the experiment, the students answered a questionnaire, which lasted approximately 10 minutes. This questionnaire, including all the questions asked, will be described in more detail in the following Chapter.

[1] https://aftermath-game.vercel.app/

Chapter 16

Questionnaire Form

This chapter will present the questions and statements asked in the questionnaire answered by the test participants after the experiment. First, some multiple-choice questions will be presented, followed by text questions, and a set of Likert scale statements.

Note that this grouping of questionnaire elements was done for the report, but does not match the order they were asked in the questionnaire. The students were first asked demographic questions (age and gender), followed by some questions about their interest and knowledge in math, including their perceived knowledge benefit from playing Aftermath. Then, students were asked about their perception of the game. These questions were based on the theory from Malone [36] and GameFlow [38], which is covered in Section 8.1 and Section 8.2 respectively). Lastly, the students were asked various other questions, including video game interest and whether the student had any other feedback. Grouping questions based on the topic rather than the type of question is more logical for a test subject.

All of the questions were mandatory to answer, excluding the free-text questions. Note that the answers are anonymous, and the students were not asked for information that can be used to identify them.

A total of four multiple-choice questions and four open questions were asked, shown in Table 16.1 and Table 16.2, respectively. 22 Likert scale statements were asked, shown in Table 16.3. For all the statements, a 1-5 Likert scale was used. The bottom of the scale (1) was annotated with *Disagree*, and the top (5) was annotated with *Agree*.

ID	Question	Options
Q1	What is your age?	• <17 • 17 • 18 • 19 • >20
Q2	What is your gender?	• Female • Male • Other
Q3	Which math grade do you think you will get this year?	• 1 • 2 • 3 • 4 • 5 • 6
Q4	How would you describe the game's difficulty?	• Too easy • Easy • Ideal • Hard • Too hard

Table 16.1: Multiple Choice Questions

ID	Question
Q5	What did you like about the game?
Q6	What did you dislike about the game?
Q7	Is there anything you struggled with in the game?
Q8	Do you have any other feedback?

Table 16.2: Open Questions

ID	Question
S1	Math is fun
S2	Math is mostly about memorization
S3	I was familiar with trigonometric functions before playing
S4	I discovered something new about trigonometric functions after playing
S5	I understand trigonometry better after playing
S6	I got a new perspective on trigonometric functions after playing
S7	Game-based learning is more motivating
S8	Game-based learning as a supplement can yield better learning outcomes than traditional teaching
S9	I often tried to calculate the optimal solution
S10	It was easy to understand how to start playing
S11	The game mechanics were comprehensible
S12	It was difficult to understand the goal of the game
S13	I understood if I did something right or wrong in the game
S14	Time went fast whilst playing
S15	I thought little about other things than the game itself and solving the tasks
S16	I was curious about the next levels in the game
S17	I liked the design of the game
S18	I kept my concentration whilst playing
S19	I was tired whilst playing
S20	I like playing games in my spare time
S21	I prioritized level progression over stars
S22	I have used game-based learning platforms before

Table 16.3: Likert Statements

16.1 Summary

The questionnaire that the students were asked to fill out at the end of the experiment consists of four multiple-choice questions, four open questions and 22 Likert-scale statements. The questions aim to answer and systematize the students' demographics and their opinions of the game and of game-based learning and mathematics in general. In the next Part, the results from the questionnaire and the rest of the experiment will be presented.

Part V

Results

This section presents the findings from the experiment explained in Part IV. It starts by describing the test group's demographics, including students' grades, age, and gender. Next, it covers the results from the questionnaire and uses statistical analysis to discover any interesting links between different groups. Then, it shares the participants' open-ended responses, followed by the qualitative observations made by the researchers during the experiment.

Chapter 17

Participant Demographics

This chapter will provide an overview of the demographics of the test subjects from the experiments. Firstly, the number of students in each mathematics class will be presented, followed by an overview of the age and gender of the students.

17.1 Mathematics Class

As explained in Section 14.2, the experiment was conducted over three separate sessions, with a total of 35 test participants. The first and third sessions were conducted on 2nd-grade students (X-math and R1-math, respectively), and the second session on 3rd-grade students (R2-math).

There were seven students in the X-math class and 14 students in both the R2-class and R1-class. Figure 17.1 shows the grade distribution of the test subjects. As the Figure shows, the number of R1 and R2 students was twice as high as the number of X students. This is reasonable and the data should apply to the rest of the population, as X is an optional course and typically has fewer students than the mandatory mathematics courses.

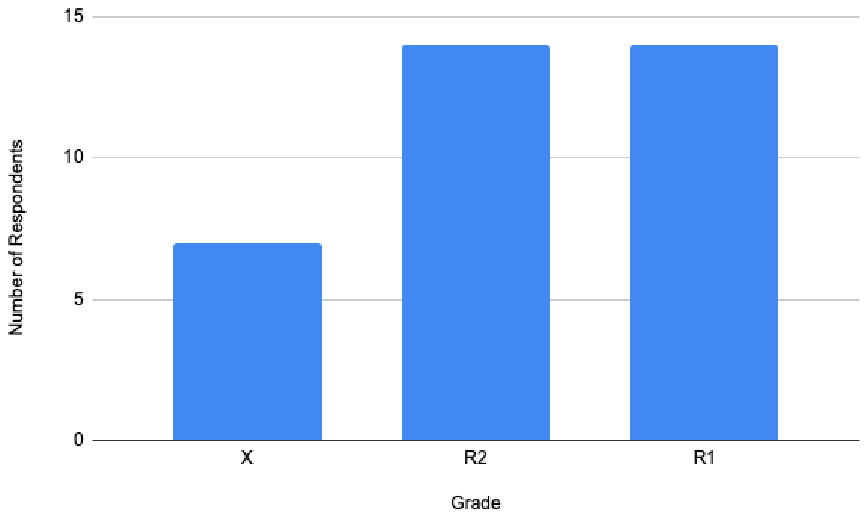


Figure 17.1: Grade distribution of test subjects

17.2 Age

Students in 2nd grade are typically 17 or 18 years old, and 3rd-grade students are typically 18 or 19. In the experiments, there were 17 students aged 17, 12 students aged 18, and 6 of the students were 19 years old. Figure 17.2 shows the age distribution of all the test subjects. Note that the experiments were conducted in late April and early May, which may explain the tendency towards low age that the figure suggests.

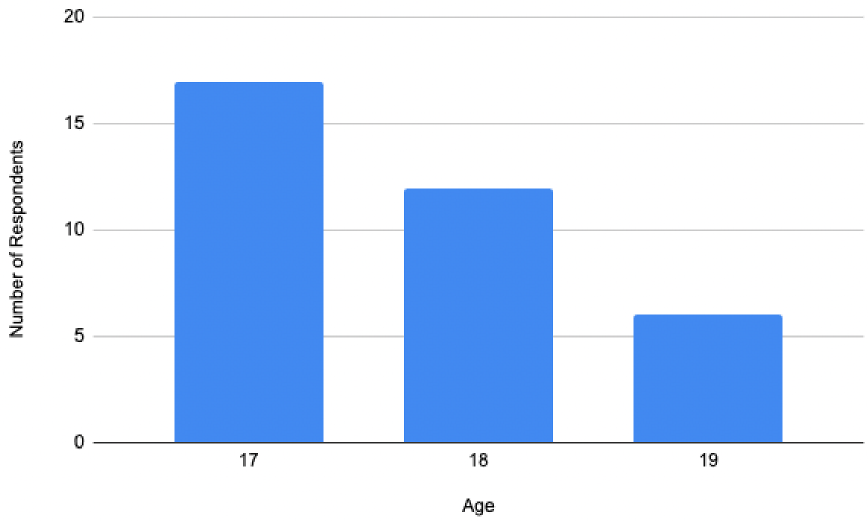


Figure 17.2: Age distribution of test subjects

17.3 Gender

In total, there were 11 females and 24 males in the study. Table 17.1 shows the gender distribution in the different classes, and Figure 17.3 shows the total gender distribution of the test subjects. The data shows a somewhat skewed gender ratio, with approximately two-thirds male and one-third female. This is a slightly less even distribution than the general population. According to *udir.no*[1], approximately 59% of students in the mathematics courses X, R1, and R2 are male.

Gender	X	R2	R1
Male	86%	86%	43%
Female	14%	14%	57%

Table 17.1: Gender distribution by math class in the experiment

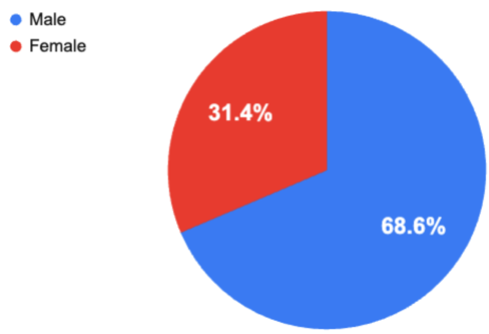


Figure 17.3: Gender distribution of test subjects

17.4 Summary

In total, there were 35 test participants, divided into three separate sessions. There were seven X-math students in the first session, 14 R2-students in the second session, and 14 R1-students in the third session. Most of the students were 17 or 18 years old, and some of the R2-students were 19. Approximately two-thirds of the test subjects were male, indicating a slightly uneven distribution compared to the national average. This chapter has presented the questionnaire results related to demographics, and the next Chapter will present the rest of the questionnaire data.

[1] https://www.udir.no/tall-og-forskning/statistikk/statistikk-videregaende-skole/karakterer-vgs/

Chapter 18

Questionnaire Results

This chapter will present the results of the questionnaire. Before the data is presented, the presentation of the data is briefly discussed, outlining the decisions made to enhance the readability of the results. Then, the questionnaire results are presented, starting with questions related to learning outcomes, followed by questions related to game enjoyment. Lastly, the rest of the questions are presented, including interest in video games and game-based learning in general, and the students' moods and strategies during the experiment.

18.1 Presentation of Data

The complete data set (see Appendix A) presents the raw data from the questionnaire. In this Chapter, the data is presented with some alterations to make the data more clear and comprehensible.

Firstly, the data is displayed in percentage values as opposed to absolute values. This makes the data easier to read and compare, especially in a table format.

Secondly, the majority of the questions from the questionnaire are 1-5 Likert-scale statements, where the students answer to what degree they disagree or agree with the statement. To further improve the readability of the tabled data, answers 1 and 2 were grouped and renamed as Disagree (**D**), 3 as Neutral (**N**), and 4 and 5 as Agree (**A**). A similar grouping was done for question Q4: *How would you describe the game's difficulty?*. The answers to this question were presented as a 1-5 Likert-scale question, but the ends of the answer were denoted with **Too Easy** and **Too Hard**, as opposed to **Disagree** and **Agree**. Thus, answers 1 and 2 are grouped and renamed as **Too Easy**, 3 as **Ideal**, and 4 and 5 as **Too Hard**.

18.2 Learning Outcome

Table 18.1 shows the other questions (i.e. Likert-scale statements) from the questionnaire that are related to mathematics, and Table 18.2 shows the math grade the test subjects assume they will get this year. The data seems to suggest that the test group is better at math than the general population, as the majority seems to think math is fun. Furthermore, the average expected grade is 4.6, which is higher than the national average of 4.2 in both R1 and R2, according to *udir.no*[1]. The average grade for X students is reportedly 5.1, but the data from Udir lacks information for this particular study program, with only 56 students reported nationally. According to the same statistics, the average grade for R2 students at Strinda Videregående Skole is also 4.2. The data from Kristen Videregående Skole is unavailable.

ID	**Question**	**D**	**N**	**A**
S1	Math is fun	6%	43%	51%
S2	Math is mostly about memorization	26%	37%	37%
S3	I was familiar with trigonometric functions before playing	9%	29%	63%
S4	I discovered something new about trigonometric functions after playing	17%	29%	54%
S5	I understand trigonometry better after playing	26%	37%	37%
S6	I got a new perspective on trigonometric functions after playing	11%	31%	57%
S7	Game-based learning is more motivating	26%	34%	40%
S8	Game-based learning as a supplement can yield better learning outcomes than traditional teaching	37%	17%	46%
S9	I often tried to calculate the optimal solution	71%	7%	21%

Table 18.1: Questions related to students' knowledge and opinion of math

Most of the students also claim to know some trigonometry before playing, and the majority of the test group seemed to discover something new or get a new perspective on trigonometry. However, they seemed to prefer attempting several times with different solutions rather than trying to calculate the solution first. The rest of the results are more balanced and non-conclusive and will be discussed in Part VI.

[1] https://www.udir.no/tall-og-forskning/statistikk/statistikk-videregaende-skole/karakterer-vgs/

ID	Question	1	2	3	4	5	6
Q3	Which math grade do you think you will get this year?	0%	6%	14%	20%	34%	26%

Table 18.2: The grade the test subjects assume they will get this year

18.3 Game Enjoyment Factor (GEF)

Table 18.3 shows the test subjects' opinion of the game's difficulty, and suggests that the game was somewhat difficult and that the vast majority did not consider the game too easy. The other questions related to the students' perception of the game are shown in Table 18.4. Most of the students seemed to understand the game's goal and how to play. Furthermore, most of the students also seemed to be immersed and they liked the game design. For a discussion of these results, see Chapter 22.

In total, 10 questions were asked about game enjoyment. Discussing the correlation between game enjoyment and other factors is difficult with many questions, as the number of correlations would grow exponentially. To easier discuss the correlation between game enjoyment and other factors, the responses were aggregated to one combined score, named Game Enjoyment Factor (GEF). The GEF score is based on the preliminary research conducted in Chapter 8. Three questions were asked based on the challenge, fantasy, and curiosity elements of Malone's theory [36], described in Section 8.1. The GEF score also encompasses seven elements from GameFlow [38] which do not overlap with Malone's theory (see Section 8.2). Question Q4 and statements S16 and S17 are based on Malone's theory, and statements S10-S15 and S18 are based on the GameFlow theory. Two of the questions contributing to the GEF score come from the *immersion* category of GameFlow, as this category was considered important with its strong connection to the *flow theory*. Also, it was challenging to encapsulate all facets of immersion within a single question, which is why the question was divided into two separate ones.

The GEF score ranges from 1 to 5, where 1 is worst and 5 is best. For questions with a regular 1-5 Likert scale, the GEF is simply the average Likert score. However, S12 is inverted (which means 1 is best and 5 is worst), and for Q4, 3 is best and 1 and 5 are worst. A general formula for the GEF score of a given question i is found by taking the average distance between test subjects' answer and the *ideal* answer:

$$\text{GEF}_i = 5 - \frac{1}{n}\sum_{j=1}^{n}(B_i - R_{i,j}) * m_i \tag{18.1}$$

where B_i is the best answer for question i (for most questions $B = 5$) and $R_{i,j}$ is the response given by student j to question i. m_i is a multiplier to correct for non-Likert answers. For Likert answers, $m = 1$, and for Q4, $m = 2$. This is necessary to make Q4's GEF score range from 1 to 5. The **total average GEF** score is found by averaging the GEF score of each question:

$$\text{Total Average GEF} = \frac{1}{n}\sum_{i=1}^{n}\text{GEF}_i \qquad (18.2)$$

The last column in Table 18.3 and Table 18.4 shows the GEF score for each question, and Figure 18.1 graphically visualizes the GEF scores for each question and statement. Figure 18.2 shows the average GEF score for each test subject. **The total average GEF score for all students and questions was 3.70**, which suggests that the overall enjoyment was positive. 91% of students (32 out of 35) reported a GEF score above or equal to 3, indicating a neutral or above enjoyment of the game. 34% of the participants received a GEF score of 4 or above, which indicates a high enjoyment of the game.

ID	**Question**	**Easy**	**Ideal**	**Hard**	**Category**	**GEF**
Q4	The game's difficulty was	3%	54%	43%	Challenge	3.51

Table 18.3: Game Difficulty

ID	**Question**	**D**	**N**	**A**	**Category**	**GEF**
S10	It was easy to understand how to start playing	9%	14%	77%	Skills	4.06
S11	The game mechanics were comprehensible	9%	23%	69%	Control	3.94
S12	It was difficult to understand the goal of the game	57%	6%	37%	Clear Goals	3.23
S13	I understood if I did something right or wrong in the game	11%	14%	74%	Feedback	4.00
S14	Time went fast whilst playing	14%	20%	66%	Immersion 1	3.77
S15	I thought little about other things than the game itself and solving the tasks	20%	20%	60%	Immersion 2	3.69
S16	I was curious about the next levels in the game	29%	23%	49%	Curiosity	3.29
S17	I liked the design of the game	11%	26%	63%	Fantasy	3.86
S18	I kept my concentration whilst playing	29%	23%	49%	Concentration	3.34

Table 18.4: Game Enjoyment Questions

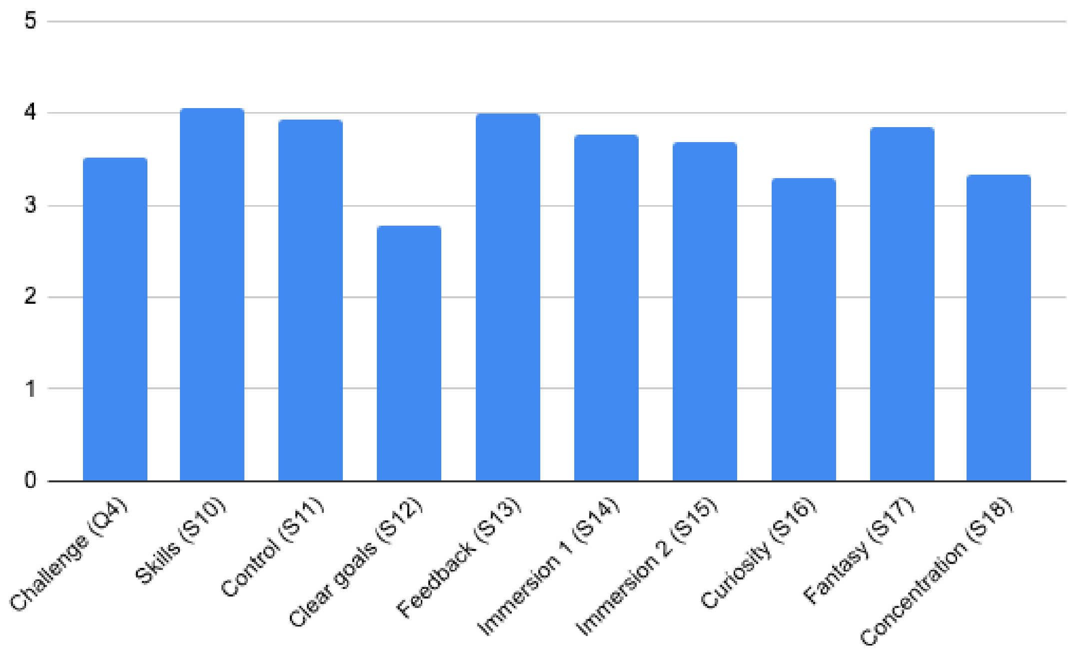


Figure 18.1: Game Enjoyment Factor score for each question and statement

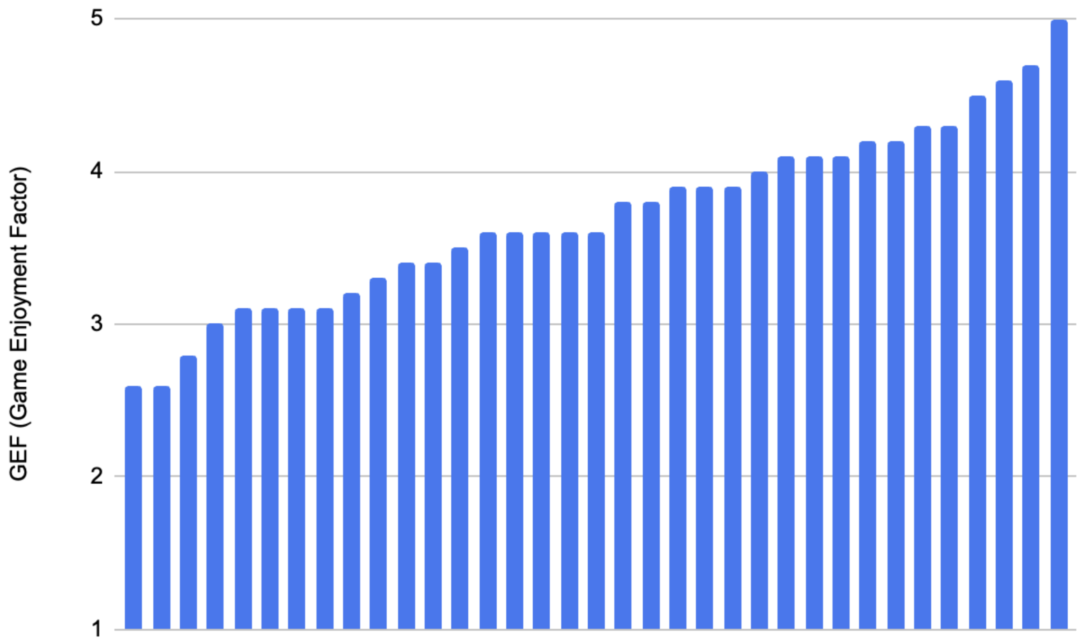


Figure 18.2: Game Enjoyment Factor score for each test subject

18.4 Other Questionnaire Results

After the first experiment, some questions were added to the questionnaire. The second experiment occurred towards the end of the day, from 3 pm to 3:30 pm.

Furthermore, the students in the second experiment were in 3rd grade, which means they were partaking in the traditional Norwegian *russefeiring*, where high school students celebrate the end of compulsory school. Thus, the test subjects in the second and third experiments were asked in which regard they felt tired whilst participating in the experiment. This will be discussed in Part VI.

Additionally, based on qualitative observations during the first experiment, there seemed to be some correlation between video game interest and performance in Aftermath. Therefore, the students in the second and third experiment were asked about their past experiences with game-based learning and video games in general, as well as how they approached Aftermath's scoring system. Question S9 (see Table 18.1) was also added after the first experiment.

Although some changes were made to the questionnaire between the first and second experiments, the data is still reliable, as there were only seven participants in the first experiment.

The questions described above are shown in Table 18.5, and show that the majority of students like playing video games, but the majority also haven't used game-based learning platforms earlier. The data also shows that most students preferred to focus on getting a high score for each level, rather than unlocking new levels. Lastly, the table shows that some students were tired and some were not. For a further discussion on this data and its correlations, see Chapter 23.

ID	**Question**	**D**	**N**	**A**
S19	I was tired whilst playing	32%	36%	32%
S20	I like playing games in my spare time	18%	21%	61%
S21	I prioritized level progression over stars	54%	21%	25%
S22	I have used game-based learning platforms before	61%	18%	21%

Table 18.5: Questions regarding attitudes towards games and exhaustion

18.5 Summary

According to the data from the questionnaire, the test group seems to have higher enjoyment and grades in mathematics than the national average. The GEF score was introduced to measure game enjoyment, based on the theory behind game-based learning covered in Chapter 8. According to the GEF scores, the students seemed to overall enjoy the game and be immersed. In the following Chapter, the results and the correlations between the data will be further analyzed.

Chapter 19

Statistical Analysis

This chapter will provide insight into the most prominent correlations in the questionnaire data presented in the previous chapter. The correlations have been selected based on their statistical significance and relevance to the research questions. First, the strategies used to group and analyze the data will be explained and briefly discussed. Then, the most prominent correlations will be presented, with one section for each question from the questionnaire. Each section will contain a set of graphs of the correlations and a corresponding table with the statistical significance of the correlations, with statistically significant results highlighted.

19.1 Grouping and Analysis Strategies

The 1-5 Likert-scale statements are grouped equally to the description in Section 18.1. Answers 1-2 are grouped as *Disagree*, 3 as *Neutral* and 4-5 as *Agree*.

To better answer the research questions, some groups have been further divided. The students who particularly liked math have been compared to those who dislike or are neutral about math. That is, those who answered 4 or 5 to S1: *Math is fun*, have been grouped together as *Fun* and compared with those who answered either 1, 2 or 3, grouped as *Boring/Neutral.* Similarly, those who showed no interest in calculating the answers in S9: *I often tried to calculate the optimal solution* have been compared against those who attempted to calculate the answer in some regard. In other words, those who answered 3, 4 or 5 have been grouped together as *Calculate*, and those who answered 1 or 2 have been grouped together as *No calculate.*

With regards to RQ3 (*How effective is game-based learning as an introduction to trigonometry?*) and RQ4 (*How effective is game-based learning as a repetition of trigonometry?*), students from the X-math class and students from the R1-class have been grouped together as *2nd-grade*, as neither of these have learned much about trigonometric functions in school. The students from the R2-class have been categorized as 3rd-grade students to follow the same nomenclature.

The answers to question Q3: *Which math grade do you think you will get this year?*, has been grouped as *1-3*, *4*, and *5-6* to align with the national average mathematics grades[1] and provide more comprehensible correlation graphs. The strategy will be further discussed in Chapter 26, and the original questionnaire data can be found in Appendix A.

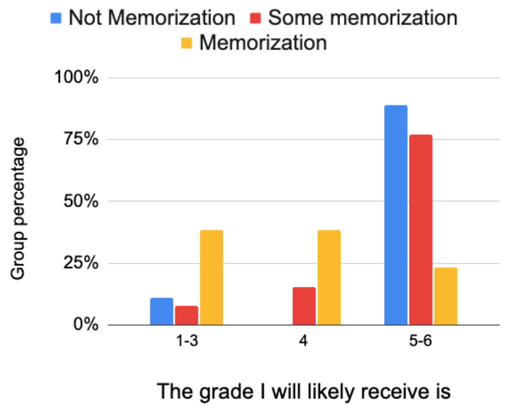
19.2 Math & Memorization

In S2, students were asked whether they perceive mathematics as a subject about memorization. Figure 19.1 shows some of the correlations between this question and other factors. Those who agreed to the statement seem get lower grades, as shown in Figure 19.1a, which is a statistically significant result, as shown in Table 19.1.

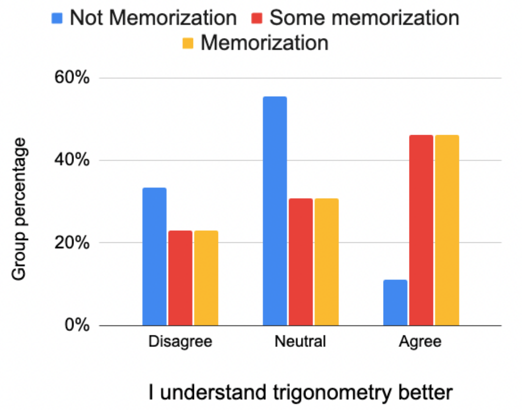
According to Figure 19.1b, the same students seem to have learned more about trigonometry from playing Aftermath than those who think mathematics is not just about memorization. However, the results are not statistically significant.

The students who perceive mathematics as a subject about memorization also seemed to enjoy Aftermath more. According to Figure 19.1c, they were more curious about the next level, and according to Figure 19.1d, they also got a higher GEF score. The former is not statistically significant but the latter is.

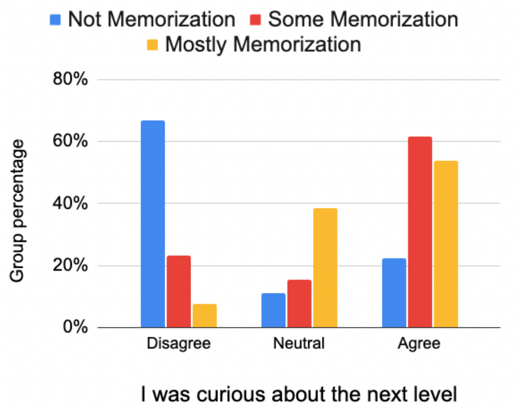
[1] https://www.udir.no/tall-og-forskning/statistikk/statistikk-videregaende-skole/karakterer-vgs/



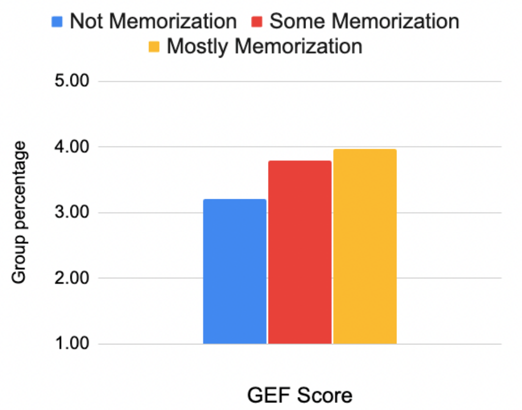
(a) Q3: *Which math grade do you think you will get this year?*



(b) S5: *I understand trigonometry better after playing*



(c) S16: *I was curious about the next levels in the game*



(d) GEF Score

Figure 19.1: Correlation between groups from S2: *Math is mostly about memorization* and other factors

Grouped by S2: ***Math is mostly about memorization***					
Question	**Group**	**1-3**	**4**	**5-6**	**p**
Q3: *Which math grade do you think you will get this year?*	Not Memorization	11%	0%	89%	**0.0085**
	Some Memorization	8%	15%	77%	
	Mostly Memorization	38%	38%	23%	
Statement	**Group**	**D**	**N**	**A**	**p**
S5: *I understand trigonometry better after playing*	Not Memorization	33%	56%	11%	0.3361
	Some Memorization	23%	31%	46%	
	Mostly Memorization	23%	31%	46%	
S16: *I was curious about the next levels in the game*	Not Memorization	67%	11%	22%	0.1131
	Some Memorization	23%	15%	62%	
	Mostly Memorization	8%	38%	54%	

Table 19.1: Kruskal-Wallis analysis on responses grouped by attitude towards S2: *Math is mostly about memorization*

Grouped by S2: ***Math is mostly about memorization***			
Measurement	**Group**	**Score**	**p**
GEF Score	Not Memorization	3.21	**0.0139**
	Some Memorization	3.78	
	Mostly Memorization	3.96	

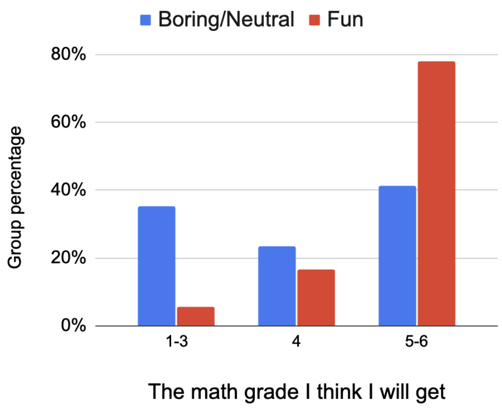
Table 19.2: Kruskal-Wallis analysis of GEF Scores grouped by attitude towards S2: *Math is mostly about memorization*

19.3 Math Enjoyment

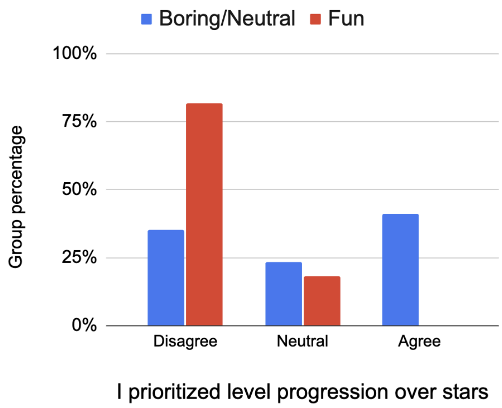
Figure 19.2 shows some prominent correlations between S1: *Math is fun* and other factors. According to Figure 19.2a, those who enjoy mathematics seem to get better grades. This result is statistically significant, according to Table 19.3.

There also seem to be some correlations in regard to Aftermath. According to Figure 19.2b, students who consider mathematics fun preferred to fully complete each level and collect as many stars as possible, instead of trying to unlock as many levels as possible. This result is also statistically significant.

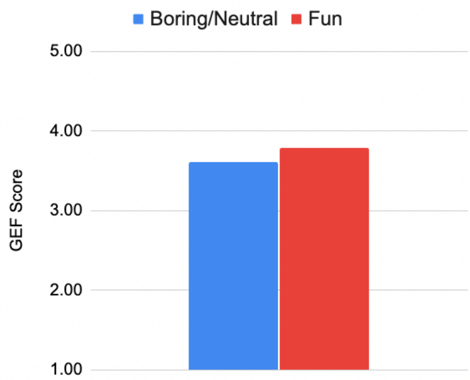
According to Figure 19.2c, those who enjoy mathematics also seemed to enjoy Aftermath, based on their respective GEF scores. However, this result is not statistically significant.



(a) Q3: *Which math grade do you think you will get this year?*



(b) S21: *I prioritized level progression over stars*



(c) S1: *GEF Score*

Figure 19.2: Correlation between groups from S1: *Math is fun* and other factors

Grouped by S1: ***Math is fun***					
Question	**Group**	**1-3**	**4**	**5-6**	**P**
Q3: *Which math grade do you think you will get this year?*	Boring/Neutral	35%	24%	41%	**0.0056**
	Fun	6%	17%	77%	
Statement	**Group**	**D**	**N**	**A**	**p**
S21: *I prioritized level progression over stars*	Boring/Neutral	35%	24%	41%	**0.0149**
	Fun	82%	18%	0%	

Table 19.3: Mann-Whitney U analysis on responses grouped by attitude towards S1: *Math is fun*

Grouped by S1: ***Math is fun***			
Measurement	**Group**	**Score**	**p**
GEF Score	Boring/Neutral	3.61	0.3714
	Fun	3.79	

Table 19.4: Mann-Whitney U analysis of GEF Scores grouped by attitude towards S1: *Math is fun*

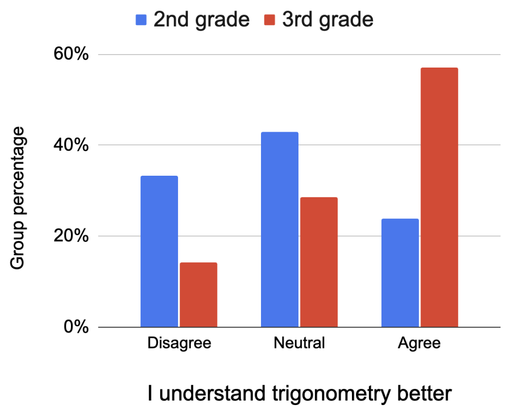
19.4 Study Year

The first and third experiments were performed on 2nd-grade students, and the second experiment on 3rd-grade students. Grouping the experiments based on the study year shows some correlations to other factors, shown in Figure 19.3.

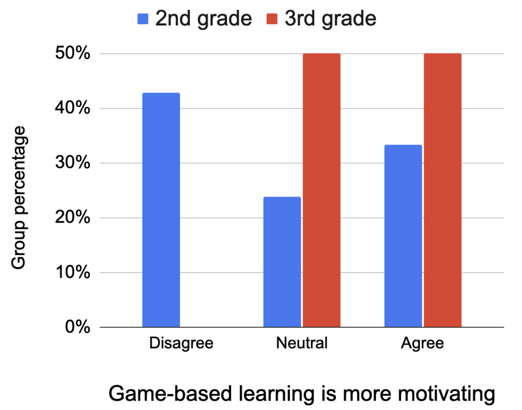
The learning outcomes from playing Aftermath seemed to be higher for the 3rd-grade students than the 2nd-grade students, according to Figure 19.3a. This is a statistically significant result, according to Table 19.5.

According to Figure 19.3b, 3rd-grade students consider game-based learning as more motivating than traditional education in a higher regard than 2nd-grade students. Figure 19.3c shows that 3rd-grade students also believe game-based learning as a supplement to traditional education may yield better learning outcomes. The result regarding motivation is statistically significant, but the result regarding learning outcomes is not.

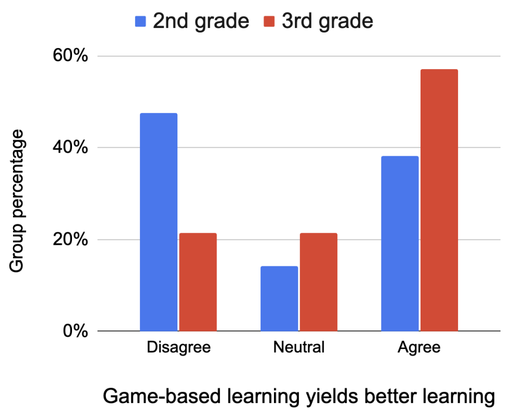
2nd-grade students did not seem to try to calculate the answer before attempting a shot, as shown in Figure 19.3d. The answers by the 3rd-grade students are more varied. According to Table 19.5, this result is statistically significant.



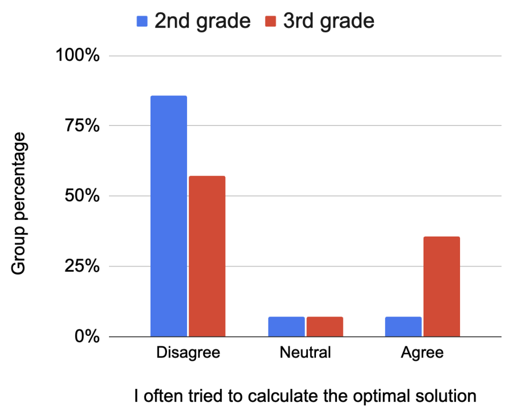
(a) S5: *I understand trigonometry better after playing*



(b) S7: *Game-based learning is more motivating*



(c) S8: *Game-based learning as a supplement can yield better learning outcomes than traditional teaching*



(d) S9: *I often tried to calculate the optimal solution*

Figure 19.3: Correlation between groups of students' study year and other factors

Grouped by study year					
Statement	**Group**	**D**	**N**	**A**	**p**
S5: *I understand trigonometry better after playing*	2nd-grade	33%	43%	24%	**0.0216**
	3rd-grade	14%	29%	57%	
S7: *Game-based learning is more motivating*	2nd-grade	43%	24%	33%	**0.0298**
	3rd-grade	0%	50%	50%	
S8: *Game-based learning as a supplement can yield better learning outcomes than traditional teaching*	2nd-grade	48%	14%	38%	0.0776
	3rd-grade	21%	21%	57%	
S9: *I often tried to calculate the optimal solution*	2nd-grade	86%	7%	7%	**0.0148**
	3rd-grade	57%	7%	36%	

Table 19.5: Mann-Whitney U analysis on responses grouped by study year

19.5 Preference to Calculate Solution

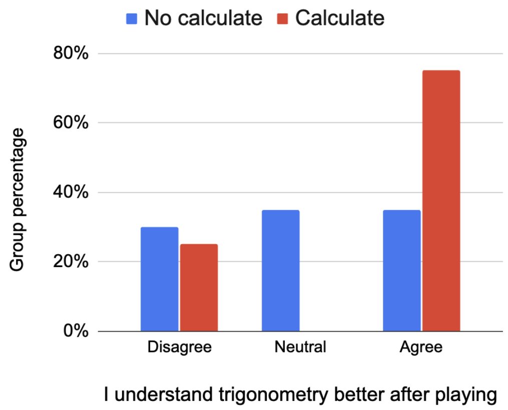
One of the Likert statements from the questionnaire was S9: *I often tried to calculate the optimal solution*. The data suggest some correlations with this statement and other factors, shown in Figure 19.4.

According to Figure 19.4b, the students who attempted to calculate the optimal solution before shooting were more immersed in the game. This is a statistically significant result, according to Table 19.6.

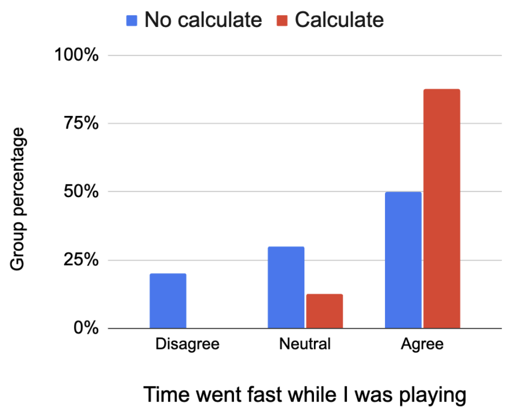
Figure 19.4a shows that the students who tried to calculate the optimal solution understand trigonometry better after playing, but the result is not statistically significant.

Figure 19.4c shows a correlation between students who tried to calculate the optimal solution and a lower perceived difficulty of the game. However, the data is not statistically significant.

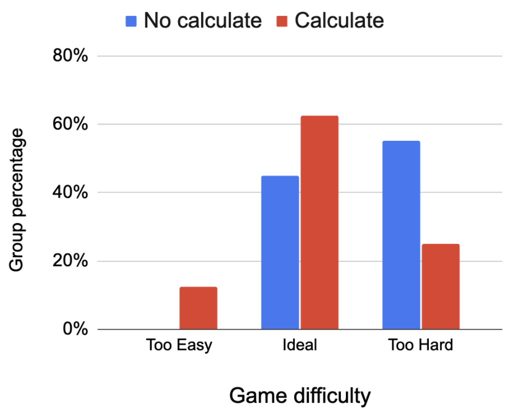
The correlation with the GEF score is shown in Figure 19.4d, and indicates that those who tried to calculate the optimal solution enjoyed the game more. The result is statistically significant.



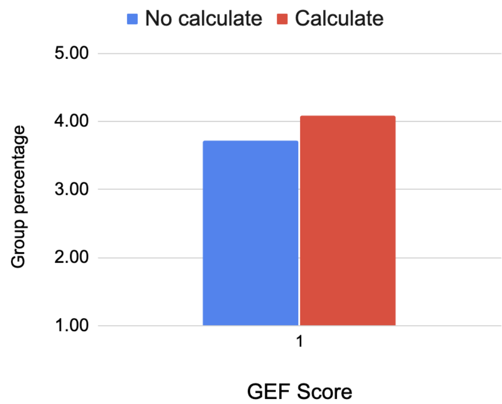
(a) S5: *I understand trigonometry better after playing*



(b) S14: *Time went fast whilst playing*



(c) Q4: *How would you describe the game's difficulty?*



(d) GEF Score

Figure 19.4: Correlation between groups from S9: *I often tried to calculate the optimal solution* and other factors

Grouped by S9: *I often tried to calculate the optimal solution*					
Statement	**Group**	**D**	**N**	**A**	**p**
S5: *I understand trigonometry better after playing*	No calculate	30%	35%	35%	0.2175
	Calculate	25%	0%	75%	
S14: *Time went fast whilst playing*	No calculate	20%	30%	50%	**0.0071**
	Calculate	0%	13%	88%	
Question	**Group**	**Easy**	**Ideal**	**Hard**	**p**
Q3: *Which math grade do you think you will get this year?*	No calculate	0%	45%	55%	0.2193
	Calculate	13%	63%	25%	

Table 19.6: Mann-Whitney U analysis on responses grouped by S9: *I often tried to calculate the optimal solution*

Grouped by S9: *I often tried to calculate the optimal solution*			
Measurement	**Group**	**Score**	**p**
GEF Score	No calculate	3.72	**0.0424**
	Calculate	4.09	

Table 19.7: Mann-Whitney U analysis of GEF Scores grouped by attitude towards S9: *I often tried to calculate the optimal solution*

19.6 Gamer

The students' interest in video games was investigated in the Likert-statement S20: *I like playing games in my spare time*. Figure 19.5 shows some correlations with this statement.

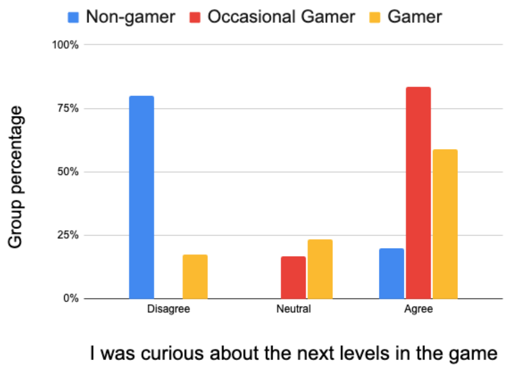
Figure 19.5a shows that the students who like to play video games in their spare time were more curious to reach new levels. This is a statistically significant result.

The students who like to play video games also seemed to prioritize collecting stars over level progression, as shown in Figure 19.5b. The graph also indicates that Non-gamers were more concerned with progression than to collect stars. However, this result is not statistically significant.

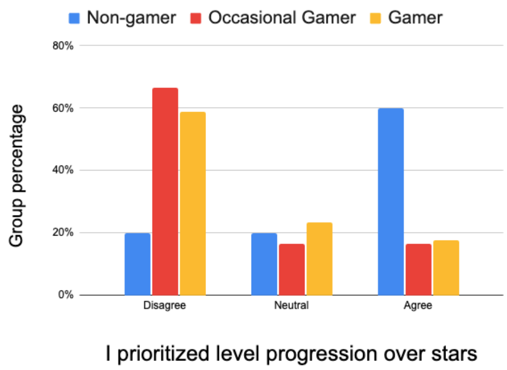
According to 19.5c, gamers are the only group that was not immersed during the gameplay. All non-gamers, and a majority of occasional gamers, thought little about other things than the game itself and solving the task.

Gamers were generally less concentrated than occasional and non-gamers. 50% of gamers were not concentrated whilst playing.

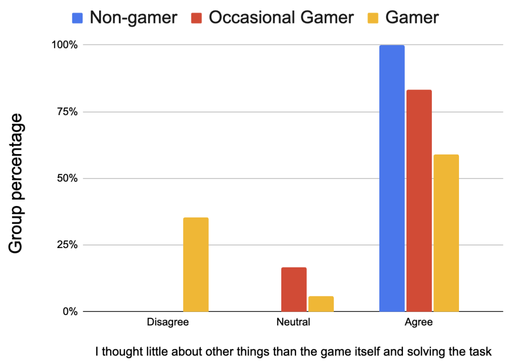
According to Figure 19.6, occasional gamers and gamers got a higher GEF score than non-gamers. However, this result is (slightly) not statistically significant, according to Table 19.9.



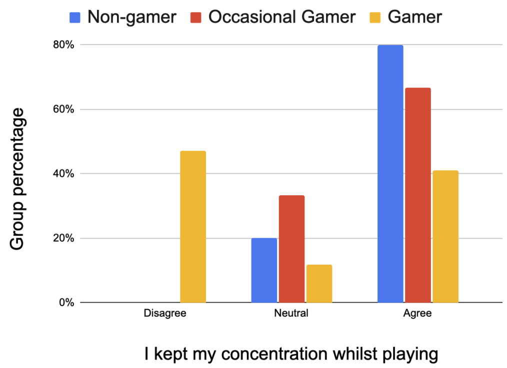
(a) S16 *I was curious about the next levels in the game*



(b) S21: *I prioritized level progression over stars*



(c) S15: *I thought little about other things than the game itself and solving the tasks*



(d) S18: *I kept my concentration whilst playing*

Figure 19.5: Correlation between S20: *I like playing games in my spare time* and other factors

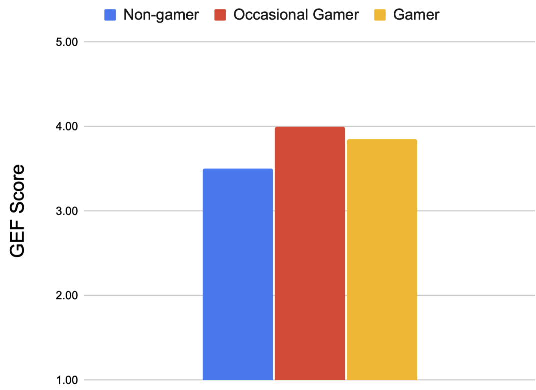


Figure 19.6: Correlation between S20: *I like playing games in my spare time* and GEF

Grouped by S20: *I like playing games in my spare time*					
Statement	**Group**	**D**	**N**	**A**	**p**
S16: *I was curious about the next levels in the game*	Non-Gamer	80%	0%	20%	**0.0415**
	Occasional Gamer	0%	17%	83%	
	Gamer	18%	24%	59%	
S21: *I prioritized level progression over stars*	Non-Gamer	20%	20%	60%	0.0730
	Occasional Gamer	67%	17%	17%	
	Gamer	59%	24%	18%	
S15: *I thought little about other things than the game itself and solving the tasks*	Non-Gamer	0%	0%	100%	0.3819
	Occasional Gamer	0%	17%	83%	
	Gamer	35%	6%	59%	
S18: *I kept my concentration whilst playing*	Non-Gamer	0%	20%	80%	0.1823
	Occasional Gamer	0%	33%	67%	
	Gamer	47%	12%	41%	

Table 19.8: Kruskal-Wallis analysis on responses grouped by attitude towards S20: *I like playing games in my spare time*

Grouped by S2: *Math is mostly about memorization*			
Measurement	**Group**	**Score**	**p**
GEF Score	Not Memorization	3.50	0.0577
	Some Memorization	4.00	
	Mostly Memorization	3.85	

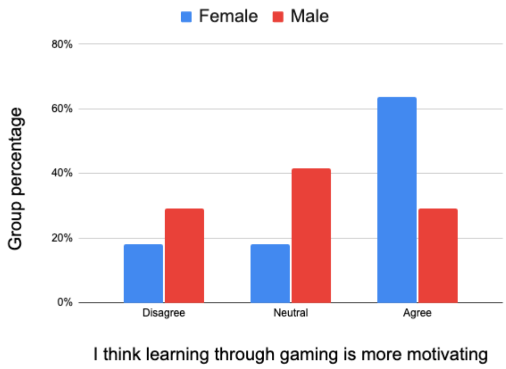
Table 19.9: Kruskal-Wallis analysis of GEF Scores grouped by attitude towards S20: *I like playing games in my spare time*

19.7 Gender

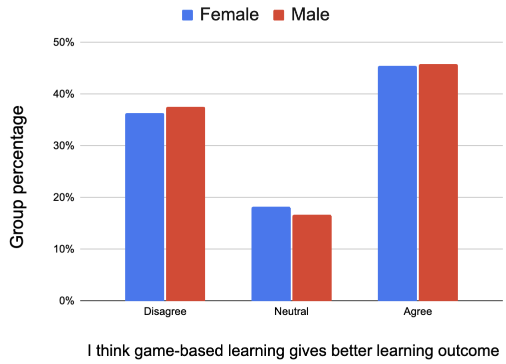
Students were asked what their gender is. Figure 19.7 shows some correlations with students' gender and other factors. Note that the gender option *Other* was available in the questionnaire, but is omitted in the following correlations because no students selected this answer.

Figure 19.7c shows that male students easier understood how to start playing, and Figure 19.7d shows that more male students like to play games in their spare time. Both results are statistically significant, according to Table 19.10.

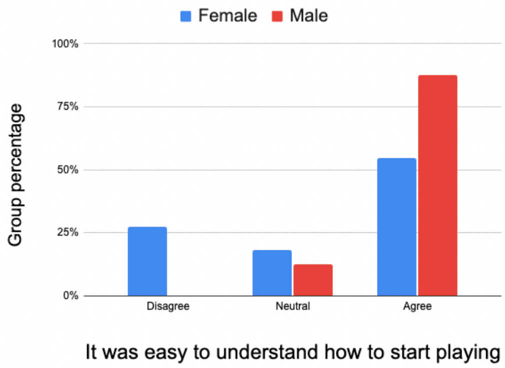
According to Figure 19.7a, more female students think game-based learning is more motivating than traditional education, but there is no significant difference between the genders genders can not be determined. Also, Figure 19.7b shows that there is no significant difference between the genders in the perceived learning outcome of game-based learning. Furthermore, Figure 19.8 shows that the GEF score is also independent of gender.



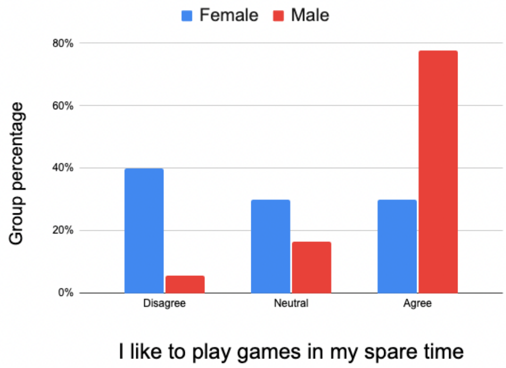
(a) S7: *Game-based learning is more motivating*



(b) S8: *Game-based learning as a supplement can yield better learning outcomes than traditional teaching*



(c) S10: *It was easy to understand how to start playing*



(d) S20: *I like playing games in my spare time*

Figure 19.7: Correlation between gender and other factors

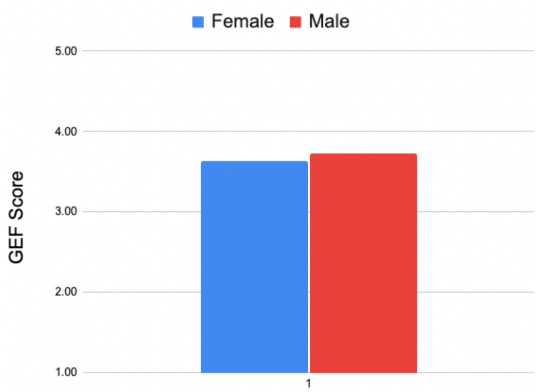


Figure 19.8: Correlation between gender and GEF Score

Grouped by gender					
Statement	**Group**	**D**	**N**	**A**	**p**
S7: *Game-based learning is more motivating*	Female	18%	18%	64%	0.3990
	Male	29%	42%	29%	
S8: *Game-based learning as a supplement can yield better learning outcomes than traditional teaching*	Female	36%	18%	45%	1.0000
	Male	38%	17%	46%	
S10: *It was easy to understand how to start playing*	Female	27%	18%	55%	**0.0243**
	Male	0%	13%	88%	
S20: *I like playing games in my spare time*	Female	40%	30%	30%	**0.0366**
	Male	6%	17%	78%	

Table 19.10: Mann-Whitney U analysis on responses grouped by gender

Grouped by gender			
Measurement	**Group**	**Score**	**p**
GEF Score	Female	3.64	0.5328
	Male	3.73	

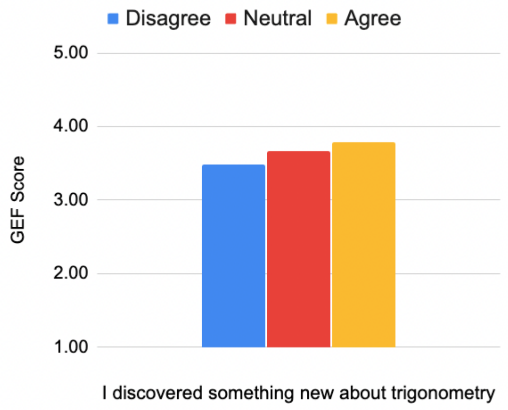
Table 19.11: Mann-Whitney U analysis of GEF Scores grouped by gender

19.8 GEF

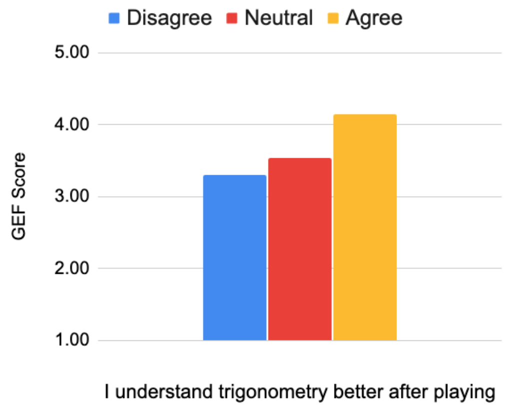
Some correlations with GEF scores have been shown in previous sections. The rest of the noteworthy GEF score correlations are shown in Figure 19.9 and 19.10.

According to Figure 19.9a, students who discovered something new about trigonometric function got a higher GEF score. Similarly, the students who reportedly understood trigonometry better after playing also got a higher GEF score, as shown in Figure 19.9b. The former is not statistically significant, but the latter is.

Figure 19.9c shows the correlation between game difficulty and GEF score, and indicates that higher difficulty leads to lower game enjoyment. However, the data is not statistically significant.

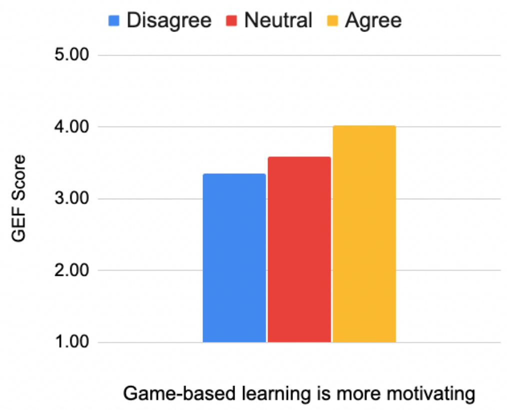


(a) S4: *I discovered something new about trigonometric functions after playing*



(b) S5: *I understand trigonometry better after playing*

(c) Q4: *How would you describe the game's difficulty?*



(d) S7: *Game-based learning is more motivating*

Figure 19.9: Correlation between various factors and GEF score

The students who consider game-based learning as more motivating than traditional education got a higher GEF score, as shown in Figure 19.9d. This result is statistically significant. Similarly, the students who believe game-based learning can yield better learning outcomes got a higher GEF score, according to Figure 19.10a. However, this data is not statistically significant.

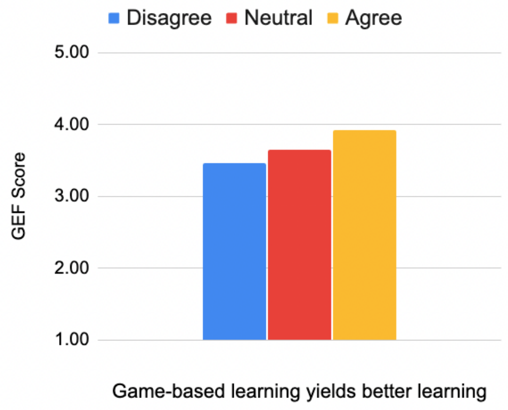
Prioritizing level progression over collecting stars seems to give lower game enjoyment, as shown in Figure 19.10b, but the results are (slightly) not statistically significant.

According to Figure 19.10c, the students who easily understood the game mechanics got a higher GEF score, and this data is statistically significant.

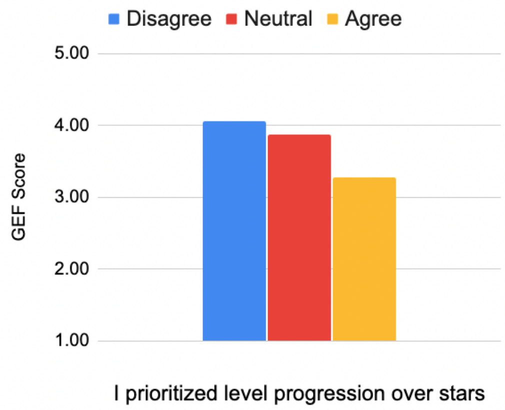
Lastly, the students who get good grades generally got a lower GEF score, according to Figure 19.10d, but the result is not statistically significant.

GEF Score				
Question/Statement	**D**	**N**	**A**	**p**
S4: *I discovered something new about trigonometric functions after playing*	3.48	3.67	3.79	0.4049
S5: *I understand trigonometry better after playing*	3.50	3.54	4.15	**0.0016**
S7: *Game-based learning is more motivating*	3.36	3.59	4.02	**0.0333**
S8: *Game-based learning as a supplement can yield better learning outcomes than traditional teaching*	3.46	3.65	3.92	0.1251
S11: *The game mechanics were comprehensible*	3.03	3.35	3.90	**0.0084**
S21: *I prioritized level progression over stars*	4.06	3.87	3.27	0.0580
Question/Statement	**1-3**	**4**	**5-6**	**p**
Q3: *Which math grade do you think you will get this year?*	3.90	3.94	3.56	0.1800
Question/Statement	**Easy**	**Ideal**	**Hard**	**p**
Q4: *How would you describe the game's difficulty?*	4.00	3.85	3.49	0.1909

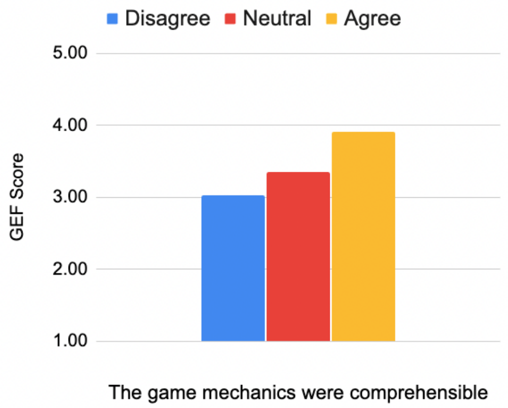
Table 19.12: Kruskal-Wallis analysis of GEF Scores grouped by various questions and statements.



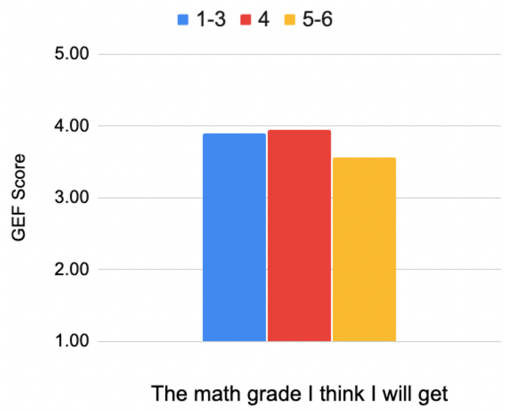
(a) S8: *Game-based learning as a supplement can yield better learning outcomes than traditional teaching*



(b) S21: *I prioritized level progression over stars*



(c) S11: *The game mechanics were comprehensible*



(d) Q3: *Which math grade do you think you will get this year?*

Figure 19.10: Correlation between various factors and GEF score

19.9 Summary

According to the data, students who enjoy mathematics and perceive mathematics as a subject about memorization enjoyed Aftermath more. Performance, learning outcomes and enjoyment of Aftermath also depend on study year and previous knowledge, with 3rd-grade students generally scoring higher than 2nd-grade students. The students who worked to calculate the optimal solution rather than experiments with various solutions generally learned more, were more immersed and considered the game less difficult. Furthermore, the analysis shows that students who like to play video games in their spare time were less immersed and more achievement-oriented. Various gender differences have also been analyzed, showing that girls generally spend less time playing video games but have the same opinion of Aftermath and game-based learning. Some other correlations with game enjoyment have been analyzed, suggesting that a primary focus during the design and development of game-based learning platforms should be to make the games not too difficult, but rather entertaining and engaging and with a focus on achievements and rewards. This Chapter concludes the analysis of the closed questions from the questionnaire, and in the next Chapter the open questions will be presented.

Chapter 20

Text Answers

Four open-ended text questions were asked. The responses to these questions are summarized in the following sections. Note that the answers are sometimes paraphrased and categorized with similar responses to convey the information in a more organized manner. The original data is shown in Appendix D.

20.1 What Students Liked

Question Q5 asked *What did you like about the game?* The responses are summarized in the list below, and the raw data is shown in Table 8. Overall, the students seemed to like several parts of the game. They considered the game engaging and fun, and thought it was easy to understand due to clear instructions and quick feedback on player inputs. The students also seemed to think that the game has educational value. One student suggested developing the game further and thought it had the potential to become a useful tool in high school education.

- **Engaging and fun gameplay:** Students found the game to be enjoyable and a good break from traditional learning. The game's concept and setup were appreciated, and the levels and scoring system added an engaging element.
- **Easy to understand:** The game was generally found to be easy to understand. The clear instructions at the beginning and the quick feedback on the suggested functions were useful. Also, the ability to easily change functions was appreciated.
- **Educational value:** The game was seen as informative and educational. It was mentioned that the game provided a more foundational understanding of the trigonometric concepts. The game also gave a practical explanation of functions, which was appreciated.
- **Good visual design and animations:** The visual aspect of the game, including its animations and user interface, was well received. The game was described as visually pleasing, and the website was referred to as *"clean"*.

- **Potential for further development:** There were positive comments about the potential of the game, especially if multiplayer functionality was added. The game was seen as potentially beneficial for younger children growing up with little concentration.

20.2 What Students Disliked

Question Q6 asked *What did you dislike about the game?* The responses are summarized in the list below, and the raw data is shown in Table 9. Although most of the students considered the game easy to understand at the beginning, students seemed to struggle at later levels. There were also some reports of various minor bugs in the game related to bombs exploding without being hit and zoom levels not working properly. Some students also said they didn't learn much from the game. This contradicts the positive educational value in the previous section, which shows that some students learned something while other students did not.

- **Difficulty understanding advanced game mechanics:** Many students found it difficult to understand the later levels, especially those without much prior knowledge of trigonometry. A few players suggested that the game should have more detailed explanations between levels to help understand what is going on. It was also mentioned that it would be helpful to have a "tip" button or the option to get hints if a player is stuck.
- **Issues with game elements and interaction:** Some students reported that the game wasn't always intuitive. There were mentions of bugs, such as bombs exploding without being hit, or the function not registering when it passed through an object. Some found the full-screen mode problematic.
- **Lack of learning:** A few students mentioned that they didn't learn much from the game. There were comments that the game was mostly trial and error, and they rarely calculated or thought about what to do. Some didn't understand the point of the game and felt the connection between the game and the learning material (trigonometry) wasn't clear.

20.3 What Students Struggled With

Question Q7 asked *What did you struggle to understand (if any)?* The responses are summarized in the list below, and the raw data is shown in Table 10. Most prominently, the students seemed to struggle to understand the advanced levels, as mentioned in the responses to Q6 shown in the section above. These responses were mostly from students in the first and third experiments, i.e. those who were in 2nd grade. There was also some feedback that the information cards didn't sufficiently explain how to complete the level.

- **Knowledge of Trigonometric Functions:** Several players expressed that they had little prior knowledge of trigonometric functions, which affected their understanding of the game.
- **Understanding the Purpose of the Game:** Some players mentioned they didn't understand the objective of the game or the purpose of certain game mechanics, such as the sine and cosine buttons.
- **Need for Preliminary Instruction:** A few players wished for some preliminary instruction or more familiarity with the functions before playing the game. The information cards before each task were perceived as inadequate by some.

20.4 Other Feedback

Question Q8 asked *Do you have any other feedback?*, and was asked to ensure test subjects could add any feedback not covered by other questions. Most of the students did not answer this question, but the received responses are summarized in the list below, and the raw data is shown in Table 11.

- **Use of Stars for Enhancements:** Some players suggested that it would be enjoyable if the stars collected in the game could be used to purchase enhancements, such as different backgrounds or sound effect packs. This addition could encourage players to accumulate as many stars as possible.
- **Game Enjoyment and Change of Pace:** One respondent appreciated the game's entertainment value, expressing that it was a positive change from the usual learning routine.
- **Incorporation of Engaging Music:** One respondent suggested integrating exciting music in addition to the sound effects.

20.5 Summary

Based on the answers to the open questions from the questionnaire, the students seemed to generally like the gameplay and found it easy to understand. They believe the game has an educational value and that there is potential for further development. However, some students expressed difficulty understanding some of the more advanced game mechanics and reported a lack of learning because of it. Some students struggled because they had not learned much about trigonometric functions in school and the educational material taught in the game was inadequate for someone with no previous references to the topic. The students suggested it should be possible to use stars for enhancements, and that the game should have some engaging music. With the results from the open questions, the next Chapter will finalize the results by presenting the qualitative observations made by the researchers during the experiment.

Chapter 21

Qualitative Observations

During the experiments, observations were made and noted. The observations were related to the general mood of the group, progression, and any unusual behaviour by test subjects. This chapter will present the notes made during each experiment.

21.1 First Experiment

The first experiment was conducted on 28th April with seven X-math students. The observations made by the researchers were noted down on paper and later categorized and summarized in the list below.

- **Level Progression:** After the first 10 minutes, most students had completed at least 10 levels. After that, progress slowed down. The highest level someone reached was 18.
- **Collaboration:** Students sat at tables with two seats. Three students sat alone, but four students sat in groups of two. The students who sat together started to ask each other questions about the levels when they were struggling.
- **Calculations:** At the beginning, students seemed to try to calculate the optimal solution before attempting a shot. Later, they gradually started quickly trying new solutions without calculating. This was a trend for all students.
- **Sound:** Initially, all of the students had muted their computers. After a few minutes, one student noticed the mute button in the game and turned on their computer audio. After shooting and hearing coin sounds the student exclaimed "Oh, cool!". However, they seemed somewhat uncomfortable to bother their fellow students, and lowered the computer volume and later muted the sound entirely.
- **Frustration:** Towards the end of the experiment, some students expressed frustration by sighing or exclaiming "No!" after shooting. This was most

prominent at difficult levels or if they barely missed a coin. However, everyone was playing throughout the whole experiment without giving up.

21.2 Second Experiment

The second experiment was conducted on 3rd May with 14 R2-math students. In total, there were 24 students in the class, but not all of them showed up for the experiment. This, including its impact on the results, will be further discussed in Section 26.2. The observations made by the researchers are categorized and summarized in the list below.

- **Level Progression:** Similar to the previous experiment, the students progressed quickly at the beginning but slowed down after approximately 10 minutes. However, the students in this experiment progressed a bit further. The highest level someone reached was 20, and the average was higher than in the previous experiment.
- **Technical Issues:** Several students had technical issues related to their computers' screen resolution. Some time was spent figuring out the solution, which was to zoom out and refresh the page. After approximately four minutes, the technical issues were resolved for all students.
- **Sound:** Similar to the previous experiment, most students did not use sound or started with sound but turned it off eventually. However, one student used headphones whilst playing, and some students played with sound at low volume.
- **Collaboration:** Most students sat in groups of two. Some of them collaborated. Two students who seemed to be friends started working together on one computer and gave up the progress on the other.
- **Bombs:** One student exclaimed "No! Of course I hit the bomb..." when hitting a bomb, and seemed to be quite engaged in the game.
- **Concentration:** The students were generally less concentrated than in the previous experiment, particularly towards the end of the session. Two students, who sat together, picked up their phones after approximately 15 minutes and did not pay much attention after that.
- **Shortcut:** Some students learned they could shoot a horizontal line by setting the amplitude to 0, and then shot one horizontal line for every coin to clear the level. One student used this strategy for several levels.
- **Replaying:** Some students replayed the level after completing it to get 3 stars, while other students seemed to prefer to move to the next level.
- **Calculations:** Similar to the previous experiment, students seemed to attempt to calculate the optimal solution at the beginning of the experiment but gradually started trying several solutions without calculating.

- **Return to Game:** One student completed the questionnaire fast and opened the game again whilst waiting for the rest of the class to finish the questionnaire.

21.3 Third Experiment

The third experiment was conducted on 12th May with 14 R1-math students. Similar to the two previous experiments, the researchers made qualitative observations during the experiment. The observations are categorized and summarized in the list below.

- **Level Progression:** The level progression followed the same pattern as the previous experiments. The highest level someone reached was 19, and the average was similar to the first experiment.
- **Technical Issues:** Some students experienced similar technical issues as the second experiment. These were quickly resolved as the solution to the problem was already discovered in the previous experiment. After approximately two minutes, all of the technical issues were resolved.
- **Sound:** Three students used headphones, and some students played with computer speakers at low volume. Most students had turned off the sound at the end of the game.
- **Collaboration:** All of the students sat at tables with two or three people. However, there was little to no collaboration or talking throughout the experiment.
- **Calculations:** There was more trial and error than in previous experiments. The students seemed to not try to calculate the answers before attempting a shot, particularly after the first few levels.
- **Shortcut:** Some students discovered the 0 amplitude described in the section above, but none of them used the strategy for many levels.
- **Distractions:** The teacher opened the window after approximately 13 minutes. There was some noise from the playground outside.
- **Frustration:** There was generally more frustration than in previous experiments. Students seemed to struggle to complete levels and understand game concepts.
- **Concentration:** All of the students were paying attention to the game. However, there was some yawning towards the end.

21.4 Summary

This chapter has presented the qualitative observations from each experiment session. In each experiment, students initially progressed quickly but slowed down later, and the test subjects from the second experiment generally performed better. Some collaboration was observed among some students sitting together, particularly in the first and second experiments. The students often used a trial-and-error strategy instead of trying to calculate the correct answer, especially towards the end of the experiment when the levels were more difficult. A few students discovered a strategy which would give a low score but also let them complete the level without any calculation, but this strategy was not used much. Generally, some frustration was expressed, increasing with the difficulty level. Some technical issues arose but were quickly resolved. With all of the results from the questionnaire and the qualitative observations, the next Part will discuss the results and their implications.

Part VI

Discussion & Conclusion

This Part examines the results of the study in relation to the research questions. It delves into a detailed discussion of the results from Part. Each chapter addresses a specific research question, interpreting the corresponding results and providing possible explanations for the findings. Following the discussion of the research questions, other potential validity threats not covered in the earlier discussion will be examined. The section concludes with a summary of the key findings and a glance towards possible further work, mapping out the trajectory for future research.

Chapter 22

Research Question 1

This chapter will discuss the results related to RQ1: *How does attitude towards mathematics affect learning outcome and enjoyment in game-based mathematics learning?* First, the results related to students' perception of mathematics will be discussed, followed by a similar discussion of students' enjoyment of mathematics. Lastly, we will delve into the methods used by students, examining findings related to their preferences for solving problems through calculations."

22.1 Math & Memorization

Generally, the data suggests that game-based learning may be useful in high school mathematics education. A majority positive answer to S4 (*I discovered something new about trigonometry*) and S6 (*I got a new perspective on trigonometry*) (see Table 18.1) suggests that game-based learning may be an effective tool. However, the benefits of game-based learning seem to depend on students' perception of mathematics.

The data shows that the students who perceive mathematics as a subject about memorization, rather than understanding, get lower grades (see Figure 19.1a). These findings conform with Skempt's theory about procedural and conceptual understanding [29].

However, interestingly, a more conceptual understanding does not seem to correlate with higher enjoyment of game-based learning. In fact, there appears to be a negative correlation. Students who perceive mathematics as a subject about memorization were more curious about the next level in the game (see Figure 19.1c). The correlation suggests those who have a less conceptual understanding of mathematics enjoyed the game more. Similarly, the GEF (Game Enjoyment Factor) score was also higher for students who perceive mathematics as a subject about memorization (see Figure 19.1d). The data also indicates that students who perceive mathematics as a subject about memorization learned more about trigonometry from playing the game (see Figure 19.1b).

Aftermath is a different approach to learning than traditional education, and one of the goals is to give students a more conceptual understanding. The notion that students who perceive mathematics as a subject not about memorization benefit less from game-based learning may be a reasonable assumption, as they may already have a conceptual understanding of mathematics (see Section 6.2) and correct concept images (see Section 6.3). In summary, this indicates that Aftermath can assist those who view math as a subject to be memorized, as it can enhance their conceptual understanding.

22.2 Math Enjoyment

Similar to the perception of memorization in math, there seems to be some correlation between enjoyment and performance in mathematics and Aftermath. The students who enjoyed mathematics got a higher GEF score, although with a somewhat small difference and not a statistically significant result (see Figure 19.2c). Students who enjoy mathematics also seem to get better grades (see Figure 19.2a). The notion that motivation leads to better performance may not be very surprising, but it is worth confirming pre-existing assumptions. An interesting observation is that higher grades lead to lower game enjoyment (see Figure 19.10d). This may indicate that those who are good at math enjoyed the game less, perhaps because they were bored. However, the results are not statistically significant.

Another interesting math enjoyment correlation is the tendency to prioritize collecting stars over higher level progression. The students who consider math fun prioritized collecting stars and getting a higher score over level progression (see Figure 19.2b). This does not necessarily suggest that those who enjoy mathematics are more score-oriented, but it may suggest they want to fully understand the level before moving on to the next level. It is reasonable to assume that the students who replay a level to get a full score also learn more than those who just frantically attempt different solutions and go straight to the next level.

22.3 Preference to Calculate Solutions

Students who meticulously calculate their answers seem to perform better and learn more (see Figure 19.4). The students who tried to calculate the correct formula before attempting a shot were more immersed in the game. This also corresponds with a higher GEF score. The correlation between preference to calculate solutions and GEF score is statistically significant, which indicates that calculating solutions in the game yields higher overall enjoyment.

This correlation may be reasonable, as the shots by those who carefully craft their functions will more likely be successful. This can yield a better sense of accomplishment for the player, and they might benefit more from the game's reward systems, which were discussed in Section 11.4. Also, calculating, evaluating, and then carefully adjusting the parameters based on the observation enforce experiential learning better than just attempting randomly.

However, when analyzing game enjoyment and a player's tendency to calculate solutions, it is also worth discussing which factor is the cause and which is the effect. Did they enjoy the game because they carefully calculated the answers, or did they carefully calculate the answers because they were immersed and engaged in the game? The reality may also be a combination of both, which may explain the relatively high statistical significance of the correlations. There may also be a significant difference in the type of students. Those who are not particularly good at mathematics may spend more time and effort solving the problem if it means they will perform better in a game they are immersed in. Meanwhile, those who enjoy and are proficient in mathematics may enjoy a game that plays to their ego and reinforces their notion that they are good in mathematics.

In addition to higher game enjoyment, there also seems to be a correlation with high learning outcomes. The students who preferred to calculate the optimal solution seemed to learn more about trigonometry (see Figure 19.4a). This may not be a surprise, as those who spend more time solving mathematical problems will naturally learn more. But it is an interesting observation in conjunction with the discussion above. The overarching goal of a learning game is to improve learning. If more calculations lead to better learning and higher game enjoyment leads to more calculations, then it follows that higher game enjoyment leads to better learning. This confirms the basic assumption that game-based learning must be fun to be an effective learning tool.

Another interesting correlation with a player's tendency to calculate solutions is that the students who just attempted different answers until they found the correct one perceived the game as more difficult than those who calculated the correct answer (see Figure 19.4c). In fact, those who preferred the calculation strategy perceived the game difficulty as very balanced, with almost two-thirds answering 3 on the 1-5 Likert-scale. However, similar to the correlation with game enjoyment discussed above, the cause and effect may be reversed. Did players struggle more with the game because they did not try to calculate the answer before shooting, or did they not calculate because they were struggling? Perhaps some students were mostly calculating initially but gave up and stopped trying to calculate as the levels became more difficult. Regardless, the correlation is interesting in the design and development of game-based learning platforms. A game that is too difficult leads to less motivation and less effective learning. As only one person responded that the game was too easy, no inference about the effects of too easy learning games can be concluded from the data. However, based on the data and the flow theory (see Section 5.6), it can be assumed that the enjoyment and learning outcomes are best when the game difficulty is balanced.

Aftermath was developed with this assumption in mind. As explained in Section 11.3.2 under *Self-esteem*, the levels were designed with a gradually increasing difficulty level to both motivate the players who are bad at math and also give the players who are good at math a challenge. However, the data may suggest that the game was slightly too difficult. This is a common issue in game design. The game developers have spent perhaps hundreds of hours developing the game and obviously understand the game concepts and mechanics much better than someone who has never seen the game before. A level that a developer perceives as easy is likely perceived as more difficult by a new player. Extensive user testing, particularly with people who have not tried earlier versions of the game, is necessary to find a good game balance. As explained in Section 12.4, some user testing was performed. But the tests were performed non-systematically and somewhat informally and were sometimes performed on people who had played the game earlier. The test subjects were also mostly friends from the university, who have likely taken more trigonometry classes than the intended target audience. More extensive user testing may have helped to balance the levels.

22.4 Summary

This section discussed how students' attitudes towards mathematics impact their learning outcomes and enjoyment of game-based mathematics learning. It reveals that students who perceive math as a memorization-oriented subject tend to derive more enjoyment and learning from the game, evident by higher GEF scores and an increased understanding of trigonometry. The section also discussed how enjoyment and performance in mathematics are interconnected, with motivated students generally performing better and showing a propensity towards a complete understanding of game levels. The game's effect also varies based on students' grade and previous knowledge, with 3rd-grade students demonstrating a higher understanding and motivation compared to 2nd-grade students. The students who preferred to calculate solutions showed higher game immersion, learning outcomes, and enjoyment, confirming the fundamental assumption that game-based learning must be enjoyable to be effective. However, the game was perceived as more difficult by those who did not calculate solutions, suggesting an optimal balance in game difficulty is key for maximum learning efficacy.

Chapter 23

Research Question 2

RQ2 asked *Which personal attributes affect player enjoyment in game-based learning?* The GEF score was conceptualized to be able to answer this question. In the following sections, the GEF score for different types of people will be analyzed and discussed, based on results from the questionnaire.

23.1 Video Games

Going into the project, there were some initial hypotheses related to RQ2. As Aftermath is a video game, it is natural to assume there may be some correlations between video game preferences and enjoyment of Aftermath. The hypothesis was that students who like to play video games in their spare time would enjoy game-based learning more than those who do not. However, the data suggests the truth is more nuanced, and perhaps even opposite from the hypothesis.

The students who like to play video games were generally less immersed in the game (see Figure 19.5c and Figure 19.5d). This seems to disprove the original hypothesis. One reason may be that those who like to play video games have a higher standard for what qualifies as a good and enjoyable video game, and thus did not experience the same level of flow as someone with less video game experience.

However, this explanation seems to be refuted by the GEF scores. According to the data, those who like to play video games, either occasionally or often, got a slightly higher GEF score than those who do not (see Figure 19.6). But this may be due to how the GEF system attempts to estimate flow and enjoyment. Statements such as S11: *The game mechanics were comprehensible* and S13: *I understood if I did something right or wrong in the game* were also elements of GEF, and video game players generally scored higher on these statements.

Another correlation with S20: *I like playing games in my spare time* is that those who do not play video games were far less curious to see the next level (see Figure 19.5a). This suggests a personality difference between gamers and non-gamers. Even though the video game players were less immersed, they still wanted to complete the levels and see what comes next. This correlation is further confirmed by Figure 19.5b, which suggests that video game players prioritized collecting as many stars as possible as opposed to just going to the next level. The correlation suggests gamers are more completionists than non-gamers. Another personal attribute worth investigating further is different *player types* and what types of games they like to play. This will be elaborated further in future work in Chapter 28.

These findings may be interesting for the design of future game-based learning platforms if the platform is aimed at a specific target audience in regard to video game interest. The data suggests that video game players are more motivated by level-based games where they get some sort of an award or achievement for completing each level. This does not necessarily mean game-based learning is ineffective for non-gamers, though, but perhaps a game-based learning platform aimed specifically at non-gamers should be less achievement-oriented and more focused on carefree exploration.

23.2 Gender

The majority of video game players are male (see Figure 19.7d). Based on pre-existing bias and the consensus in society, it was hypothesized that the game would resonate more with males. However, interestingly, male and female students got a very similar GEF score (see Figure 19.8), which indicates that females enjoyed the game as much as male students. Furthermore, female students seem to think equally positively about game-based learning for motivation and learning outcomes (see Figure 19.7a and Figure 19.7b).

These findings are interesting, as they seem to suggest that female students play video games less frequently than male students but still enjoy video games just as much as male students. Several studies suggest that the gender difference in video games is not as severe as the general consensus in society suggests. According to Romrell, D., there has been a declining difference between genders, with approximately 55% of video game players being male [47]. A more recent report from the Entertainment Software Association suggests that 48% of video game players are female [48]. However, the statistics are based on the American population and the data does not consider the frequency and duration of play, as they classify a video game player as someone who plays more than 1 hour a week [48, p. 2].

Based on the data from these reports and our research, game-based learning seems to be just as effective for male and female students. Gaming is socially more acceptable to men than women, even though they have the same learning outcome and the same attitudes towards it.

23.3 Game Difficulty

Another correlation with game enjoyment is the game's difficulty. Those who perceived the game as too hard got a lower GEF score (see Figure 19.9c). This is also confirmed by Figure 19.10c, which suggests that those who easily understood the game mechanics got a significantly higher GEF score. Note that S11: *The game mechanics were comprehensible* was also a part of the GEF calculations, which amplifies the correlation.

The notion that game difficulty is related to game enjoyment is not too surprising, as you will experience more frustration if you are struggling with a game. There is a correlation between the GEF score and students' learning outcomes from Aftermath (see Figure 19.9b). Those who got a high GEF score also generally think game-based learning yields better motivation (see Figure 19.9d) and better learning outcomes (see Figure 19.10a). This suggests it is important to avoid making learning games too difficult, as it will make players less motivated and the learning outcomes will be reduced. This conforms with the *flow* theory [3]. According to Figure 5.4, too much challenge may lead to anxiety. This is particularly problematic in a mathematical educational game, as it leads to math anxiety, which may further reduce math performance, as explained in Section 6.1. According to Figure 5.4, too easy games are also problematic because they may lead to boredom. This can not be confirmed by the data from this study, as only one student answered that the game was too easy.

23.4 Summary

This chapter has investigated personal attributes affecting player enjoyment in game-based learning, in an attempt to answer RQ2: *Which personal attributes affect player enjoyment in game-based learning?*. The results highlighted three main areas: video game preferences, gender, and game difficulty. Video game enthusiasts showed an interesting relationship. They were less immersed in the game but achieved higher GEF scores, indicating they may have higher standards for game enjoyment. Moreover, they showed a higher desire to complete levels and achieve rewards, indicating a more completionist mindset. Concerning gender, males and females achieved similar GEF scores, suggesting both genders enjoy game-based learning similarly, regardless of the societal perception that gaming is more common among males. Finally, game difficulty showed a significant impact. Players perceiving the game as too hard had lower GEF scores. This underlines the importance of balanced game difficulty to avoid frustration or boredom and maintain motivation and learning outcomes. Future research could further explore these factors to optimize game-based learning environments.

Chapter 24

Research Question 3 & 4

RQ3 and RQ4 asked: *How effective is game-based learning as an introduction to trigonometry?* (RQ3) and *How effective is game-based learning as a repetition of trigonometry?*(RQ4). This section will look at RQ3 and RQ4 together, as they are closely related and based on the same data. This chapter will present the relevant data provided in the results, starting with a discussion of Aftermath as a repetition tool and followed by a discussion of Aftermath as an introduction tool.

24.1 Aftermath as a Repetition Tool

The benefits of game-based learning seem to depend on the students' study year and previous knowledge. According to the qualitative observations, the students seemed to struggle more in the first and third experiments (2nd-grade students) than in the second experiment (3rd-grade students). They completed fewer levels and generally expressed more frustration throughout the experiments. Similarly, the students who reported a lack of learning in the open text questions were primarily students from the third experiment, which also suggests a correlation between study year and learning outcomes. Furthermore, 3rd-grade students reported a significantly higher understanding of trigonometry after playing than 2nd-grade student (see Figure 19.3a). 3rd-grade students also seemed to be much more motivated (Figure 19.3b).

Conventional wisdom might suggest that those who already understand trigonometric concepts would have less new information to learn from a game like Aftermath. On the contrary, the results suggest that students who are already familiar with trigonometry felt they enhanced their understanding through gameplay.

These findings indicate that Aftermath is an effective tool for the reinforcement and enhancement of pre-existing knowledge. The results underscore the game's capacity to deepen students' conceptual understanding of trigonometry, by providing a more interactive, engaging mode of revisiting the subject matter. This could potentially support a broader application of game-based learning in education as a means of reinforcing and enhancing the understanding of previously studied concepts.

One could argue that traditional education is more focused on procedural learning, especially the grading system with written exams. Based on the author's personal experiences, exam questions are typically similar from year to year, following the same procedures but with different numbers and values. If a student only does rote memorization on previous exams to achieve a better grade, they might learn how to solve the exam questions better by following certain procedures, rather than building a deeper understanding of mathematical concepts. Aftermath facilitates a more conceptual understanding for students with a procedural understanding, thereby suggesting that game-based learning is an effective tool to build a more conceptual understanding. In regards to RQ4, this suggests that game-based learning may be good as a repetition tool.

24.2 Aftermath as an Introduction Tool

RQ3, on the other hand, asks how effective game-based learning is as an introduction to a new topic, rather than as a repetition of the topic. According to Rittle-Johnson and Alibali, it is easier to learn procedurally with a pre-existing conceptual understanding rather than inversely [30, p. 188]. Based on the reasoning above, which considers Aftermath a tool for conceptual understanding and traditional education a tool for procedural understanding, Rittle-Johnson and Alibali's findings could suggest that Aftermath is more effective as an introduction to a new mathematical concept than as a repetition.

The data mentioned in the previous section seems to contradict this theory though, as the 2nd-grade students expressed more frustration and completed fewer levels than the 3rd-grade students. However, this type of experiment may not be optimal to measure the effectiveness of Aftermath as an introduction tool because the experiment was only conducted over one 45-minute long session. To measure the effectiveness of Aftermath as an introduction to a new topic, the students would have to be evaluated next year after they have learned trigonometry in school. This experiment would attempt to evaluate whether the students who have played Aftermath are more capable of gaining a procedural understanding than those who only follow traditional education.

Another reason why Aftermath shows poor performance as an introduction tool in the experiment may be that the game was arguably made more as a repetition tool than an introduction tool. The practical experiments only lasted 30 minutes, and the game tries to summarize and explain most of the trigonometry curriculum in these 30 minutes. The information cards do not go much in-depth about each new concept but rather introduce the concepts with a brief description and let the player explore the concepts themselves through trial and error. If the game were designed to be used throughout an entire semester, there could be more levels for each new concept. One could argue that a game like Aftermath may be more useful as an introduction tool if it had more levels and information cards for each mathematical concept. The player would also need more time to carefully investigate and explore the concepts by letting the players use Aftermath on their own, rather than in a controlled experiment environment.

24.3 Summary

This chapter has explored Aftermath both as a repetition tool and as an introduction tool. The findings suggest that Aftermath may be highly effective for reinforcing and enhancing pre-existing knowledge in trigonometry. The game proved beneficial in deepening students' understanding of the subject matter through an interactive, engaging platform, transforming procedural knowledge into a more conceptual understanding.

However, the effectiveness of Aftermath as an introduction tool is less evident. The 2nd-grade students expressed more frustration and struggled more to complete the levels. But the current study design, based on single-session experiments, does not necessarily provide a solid basis to assess its effectiveness as an introductory tool. To assess the game's effectiveness as an introductory tool, it should be evaluated over a full academic year, after the students have learned trigonometry in a traditional setting.

Chapter 25

Research Question 5

RQ5 states: *How do the theories in game-based learning contribute to the players' enjoyment, motivation, and learning outcome in an educational mathematics game?* This section will discuss whether the design decisions made in Chapter 11 have contributed to enhancing the enjoyment, motivation, and learning outcome of Aftermath. The main theories included is: Kiili's Experiential Gaming Model [40], What Makes Things Fun to Learn? by Thomas Malone [36] and GameFlow by Sweetser and Wyeth [38]. The evaluation of these theories will be based on the empirical evidence conducted in the questionnaire, where questions were specifically mapped to each of the contributing factors from all theories. The chapter ends with a discussion on actual tradeoffs between enjoyment, motivation, and learning outcomes in the development of Aftermath.

25.1 Experiential Learning

As Aftermath is designed with an experiential approach, the learning outcomes can generally be attributed to this approach. The data shows that Aftermath leads to higher learning outcomes, which suggests an experiential approach is good for learning in this type of game (see S4, S5 and S6 in Table 18.1). The majority of students reported that they discovered something new about trigonometric functions (S4) and that they understood trigonometry better (S5). The vast majority of students also reported that they got a new perspective on trigonometric functions (S6). However, it is worth mentioning that the results only measure subjective and perceived learning, as no objective measurements were made. This issue will be further discussed in Chapter 26.

Based on the overall GEF score, the players seemed to generally enjoy the experiential approach to education (see Section 18.3). Students also responded generally positively to S7: *Game-based learning is more motivating* and S8: *Game-based learning as a supplement can yield better learning outcomes than traditional teaching.* Furthermore, the majority of the students disagreed with S9: *I often tried to calculate the optimal solution.* This suggests players preferred a trial-and-error approach.

Even though there is no doubt Aftermath is an experiential approach to learning, the questions from the questionnaire do not sufficiently document players' approach to experiential learning. This makes it difficult to measure the actual effect of an experiential approach on learning outcomes, as it is difficult to make correlations in the data set. Asking a question such as "When you made an unsuccessful shot, did you stop and evaluate the results before attempting to shoot again?" could have been useful to determine whether Aftermath successfully utilized its experiential potential. Such questions could potentially discover whether students actually evaluated their actions, before attempting again, which corresponds with the last part of the iteration of Kiili's model [40], explained in Section 8.3.2.

25.2 GEF

GEF was introduced as a tool in this thesis to evaluate player enjoyment. The overall GEF was high amongst the students, where 91% of them reported a GEF score above or equal to 3, indicating a neutral or above enjoyment of the game. This indicates that a vast majority of the participants got some enjoyment from playing Aftermath. Also, 34% of the participants received a GEF score of 4 or above, indicating significantly high enjoyment of the game. This section will break down each element of the GEF score, comprised of the theory by Malone [36] and the GameFlow theory [38]. The GEF scores will be discussed, including their implications on RQ5.

25.2.1 Malone

Malone's theory was the basis for three of the questions and statements that comprised the GEF score (See Table 19.12). The data suggests that the game's *challenge* was generally decent, with a GEF score of 3.51 to Q4: *How would you describe the game's difficulty?*. However, some students perceived the game as slightly too difficult. The game used variable difficulty levels, where each level became progressively more difficult, but the data from the questionnaire and the qualitative observations suggest that the difficulty increased too fast, causing frustration for some students (Chapter 21).

Another element from Malone's theory was *fantasy*, which was asked through S17: *I liked the design of the game*. This statement had a high GEF score of 3.86. However, retrospectively, this question does not accurately measure Malone's fantasy element, as it does not ask specifically about extrinsic and intrinsic fantasy. With the exception of this it is still an important contribution to game enjoyment and the general fantasy of the game. The results from the open-ended questions also support this notion, suggesting that the students generally seemed to appreciate the design of the game (see Chapter 20). As an example, one student answered that "it felt satisfying to shoot sine and cosine functions" (Appendix 8). Another student also expressed that the game *"...gives a better understanding because you can see* *with your own eyes what happens when you change the different function attributes"* (Appendix 8). This highlights exactly the design decisions made for evoking intrinsic fantasy, as described in Section 11.2.2.

Curiosity was also measured in the questionnaire, namely in S16: *I was curious about the next levels in the game.* This statement got a GEF score of 3.29. It is one of the lower GEF scores overall, but the data also suggests a clear distinction in this data based on the player types, namely students' approach to video games. The data suggests that video game players were generally more curious, scoring high on curiosity, but also that non-gamers were not particularly curious (see Figure 19.5a). This distinction is interesting, as it suggests that game-based learning platforms should be created differently depending on the target audience, with games aimed at gamers being focused more on levels and achievements.

25.2.2 GameFlow

The GameFlow theory comprised seven of the ten elements of the total GEF score, namely: player skills, control, clear goals, feedback, immersion, and concentration. The *player skills*, covered by S10: *It was easy to understand how to start playing* got the highest GEF score of 4.06 (see Table 18.4). The design of the games introduction was very simplistic, where the user is only required to hit the spacebar to understand how the controls work, and what the consequences of their actions are.

The *control* was measured by S11: *The game mechanics were comprehensible*, and gave a GEF scores of 3.94 (Table 18.4) suggesting that although the game was a bit difficult, the game mechanics were easy to understand. The answers were also generally positive to S13: *I understood if I did something right or wrong in the game*, which measured the *feedback* in Gameflow with a GEF score of 4.00 and approximately 75% of students agreeing to the statement. Note that the feedback measure of GameFlow overlaps with the Sensor Curiosity mentioned in Section 11.2.3.

The findings indicate that the game provided visual stimuli to the users' actions, which helped guide the users throughout the game. Generally, the responses suggest that the game mechanics and explanations were easy to understand. This is an important element of GameFlow and game design in general, as it is simpler to create easier levels than to completely redesign the game mechanics and their intuitiveness. This suggests Aftermath may be further developed by simply adding some new levels and polishing some minor aspects of the game. Potential future development of the game will be discussed in Chapter 28.

The results from S12: *It was difficult to understand the goal of the game*, which measured if the game had *clear goals*, gave surprising results with a GEF score of 3.23 (Table 18.4). The answers were generally polarized; many students either disagreed or agreed, but very few answered neutrally. One explanation could be the negation of the question, as it asked if the students found it *"difficult"* to understand the goals. A further discussion on why negating questions in a questionnaire can be problematic is provided in the next Chapter.

Arguably the most important factor of GameFlow is *immersion*. In fact, due to its importance, two of the statements that comprise the GEF score are based on immersion, namely S14: *Time went fast whilst playing* and S15: *I thought little about other things than the game itself and solving the tasks*. The majority of students answered positively to both of these statements, and the GEF scores were 3.77 and 3.69, respectively (Table 18.4). This suggests that the students were generally immersed in the game and that tedious tasks such as trigonometry may become engaging and immersive with the right tools.

Interestingly, the answers were more mixed to S18: *I kept my concentration whilst playing*, which measured the *concnetration*. This is somewhat surprising, as concentration is necessary for immersion. However, retrospectively, this may be due to the phrasing of S18. *I kept my concentration whilst playing* talks about the player's concentration throughout the session, whilst S14 (*Time went fast whilst playing*) and S15 (*I thought little about other things than the game itself and solving the tasks*) do not mention duration. So, a student who was initially immersed but lost concentration and immersion throughout the session may answer more negatively to S18 than S14 and S15.

25.3 Tradeoffs

In the realm of game-based learning, the interplay between players' enjoyment, motivation, and learning outcomes often involves significant tradeoffs. For example, motivation, which is a critical driver of both enjoyment and learning, was a recurring focus when considering potential game features. One specific trade-off emerged with the proposal of a *divide coin*, which was intended to split the trigonometric wave into two separate waves. This element is showcased in Figure 25.1 and could have facilitated more diverse and challenging levels. However, the divide coin idea was eventually disregarded because a divide symbol may be confusing as dividing a trigonometric function does not actually produce such a function graph. This decision underscored a commitment to prioritize mathematical accuracy over additional game features, despite the potential for increased player motivation and enjoyment.

Other trade-offs were also discussed. During the early phase, when the name of the game was *Triggerd!* and the game was more themed as an action and shooter game, one suggested idea was to enable the player and enemies to interact with the coordinate system. For example, an enemy could blow up the y-axis, making it more difficult for opponents to calculate the correct solution. This could enhance *intrinsic fantasy* (see Section 8.1.2) as the players' actions directly affect the game environment. This idea was disregarded, but it is an interesting example of a trade-off between enjoyment/fantasy and educational value. Generally speaking, the more such elements are introduced, the more the game deviates from traditional education. This may be positive as it may motivate students more, particularly those who struggle with traditional learning methods, but it may also be negative because the learning outcome is not necessarily applicable to the curriculum and would therefore be less useful.

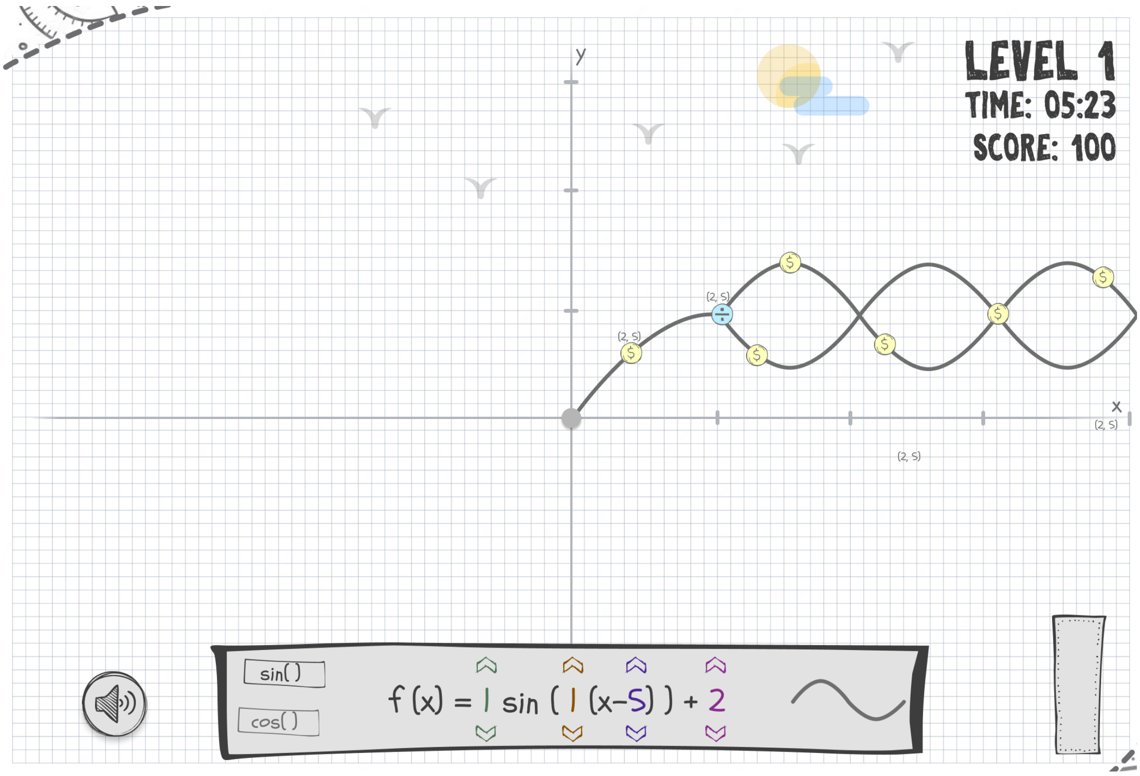


Figure 25.1: A design sketch for a potential divide coin

In examining these tradeoffs, it appears that Aftermath strikes a suitable balance between fostering learning outcomes and player enjoyment. This balance is crucial in maintaining the game's attractiveness and its educational value for a broad range of students. The application of game-based learning theories in Aftermath enhances the gameplay experience, while also contributing positively to its educational effectiveness. The empirical data collected suggests that the blend of enjoyment and education in Aftermath is well-received by students, indicative of the successful utilization of game-based learning theories.

25.4 Summary

This Chapter looked at Research Question 5, which explores the impact of game-based learning theories on enjoyment, motivation, and learning outcomes in Aftermath. The findings indicate that the game's experiential approach contributed to an enhanced conceptual understanding of trigonometric functions, a preference for a trial-and-error learning style, and high motivation and enjoyment levels among students, with 91% of students reporting a GEF score of 3 or above. Furthermore, the consideration of tradeoffs in Aftermath's design demonstrates a successful integration of game-based learning theories, striking a suitable balance between fostering learning outcomes and player enjoyment. This balance is reflected in the game's reception, suggesting that the blend of enjoyment and education in Aftermath effectively engages a wide range of students.

Chapter 26

Other Validity Concerns

In the previous chapters, some of the validity issues have been analyzed in accordance with the research questions. This chapter will discuss some of the other potential threats to the validity of the results. First, some of the issues with the questionnaire will be presented. Then, the students' lack of attendance and sample size will be discussed. Lastly, a discussion of the novelty effect and its impact on the results will be elaborated.

26.1 Questionaire

The questionnaire is arguably the most significant data source used in this thesis. Oates suggests there are many possible issues related to the phrasing of the questions in a questionnaire [4].

26.1.1 Ambiguity

A question may not necessarily be interpreted the same way it was intended by the researchers. Retrospectively, the phrasing of S5: *I understand trigonometry better after playing* may be problematic. If you did not feel that you learned anything, should you answer 1 or 3 to this question? If you believe it is impossible for a learning game to reduce your knowledge of a topic you may answer 1, but if you assume it's possible for a game to make you more confused about the topic you may answer 3.

26.1.2 Question Negation

Another possibly problematic question from the questionnaire is S12: *It was difficult to understand the goal of the game.* This question is negated, meaning 1 is the *optimal* answer. This is a technique used to prevent a type of response bias called "acquiescence bias", where respondents have a tendency to agree with statements or consistently respond at one end of the scale. Reverting the meaning of questions aims to ensure that participants are reading and thinking about each item rather than simply responding out of habit or without thought. However, this approach can possibly introduce another problem where respondents become confused by the inconsistent directionality of the items and answer incorrectly, thus inclusion of negatively worded items can result in less accurate responses [49]. This may have been a problem in the questionnaire, as some of the students who seemed to enjoy the game, learn a lot, and get a good GEF score agreed to the statement *It was difficult to understand the goal of the game.* This suggests that they misinterpreted the statement.

Retrospectively, it could have been better to not negate any questions. Alternatively, there should have been more emphasis added on the fact that the question was negated, for example by highlighting the word *difficult* with italic or bold text. Furthermore, more thorough pilot testing of the questionaire could have been helpful to ascertain that respondents comprehend the item as intended. Some pilot testing was done with the supervisor, and a university student during the user testing, but more pilot testing may have been beneficial.

26.1.3 Perceived Learning Outcome

Some other questions that may be a threat to the validity of the results are those questions related to learning outcomes:

- Q3: *Which math grade do you think you will get this year?*
- S4: *I discovered something new about trigonometric functions after playing*
- S5: *I understand trigonometry better after playing*

These questions and statements may not be an accurate representation of the learning outcomes, as they rely on subjective answers. Students may either overestimate or underestimate their own performance, giving a less accurate result than an objective test would. Furthermore, students may also *intentionally* overestimate their own performance as a result of the Hawthorne effect, described in Section 4.2. Because the students knew they were being observed, they may answer higher to the aforementioned questions because they want to be perceived in a certain way.

However, as mentioned in Section 15.4, it was emphasized at the start of the questionnaire that the results were anonymous, and the researchers moved to the front of the classroom to be unable to see the test subject's screens. This was done in an attempt to reduce the Hawthorne effect.

26.2 Lack of Attendance

Another potential bias in the results is selection bias, described in Section 4.1 and Section 4.2. The test subjects were selected using a cluster sampling method, which is intended to easier obtain test subjects without introducing significant biases. However, a problem related to selection bias is that some of the students did not show up. In the second experiment, 24 were expected but only 14 showed. In the first and third experiment, all students were present.

Students not being present for the experiment may be a threat to the validity of the results, as there may be a bias in which students choose to not show up to a class. Generally, students with low attendance may be those who get lower grades. This may be a reasonable assumption, as the test population's assumed grades were higher than the national and the respective school's average, indicating a bias due to poor-performing students not attending. However, it is worth noting the discussion in the previous section, which suggests that the assumed grade may be higher than the actual grade.

Another reason why students were not present may be attributed to *Russefeiring*, which is a traditional celebration in Norway where students celebrate the end of high school. It is not uncommon for students to have lower attendance during *Russefeiring*[1]. This may also introduce a bias, as some types of students may care more about this celebration than others.

26.3 Sample Size

According to Oates, for small-scale research projects a good rule-of-thumb is to have a final sample size of at least 30 [4, p. 100]. Originally, the plan was to only conduct experiments 1 and 2 (X-math and R2 math students, respectively), as the earliest date possible for the third experiment was 12th May. Because fewer than expected students showed up in the first experiments as mentioned in the previous section, the third experiment was conducted to give a sample size that conforms with Oates' rule-of-thumb.

Although the sample size is in accordance with Oates, it may not have been large enough to conduct such extensive correlational statistical analysis as done in Chapter 19. Oates does not suggest a similar rule for the statistical analyses, but retrospectively, the sample size should likely be larger when the data is divided into smaller groups to observe correlations with other data.

Another suggestion by Oates to enhance the validity of the results is method triangulation, which is defined as using two or more data generation methods [4, p. 37]. In this research project, data was collected through a questionnaire with both open and closed questions, and qualitative observations were made. Another possible way to collect data could be to record anonymous metrics during

[1] https://www.nrk.no/vestfoldogtelemark/videregaende-skoler-melder-om-hoyt-fravaer-blant-russen-1.15948478

gameplay to measure player progress. This would be a more systematic observation (see Section 3.7) and could add more information and validity to the results. However, this could be an ethical dilemma as the analytics would have to record player actions. It would also require more development time, as the data would need to be sent to a server and stored in a database.

26.4 Novelty effect

As described in Section 4.2, the Novelty effect is a common issue in empirical studies on gamification and game-based learning. When test subjects are exposed to a new technology, their performance may increase because they are excited about the technology, rather than because the technology actually has positive effects. This effect is amplified in shorter experiments and is a very likely bias in this particular study.

To mitigate the Novelty effect, the statistical analysis and interpretation of results have focused more on the correlations between the different types of test subjects, rather than the effectiveness of Aftermath as a definitive answer. For example, although the average GEF score of all test subjects is higher than 3, indicating a net positive perception of the game, this result has not been emphasized much as it is likely affected largely by the Novelty effect. However, analyzing and comparing the GEF score of two different groups of people, for example, those who enjoy mathematics and those who do not, is a more reliable analysis, as the impacts of the Novelty effect are likely more or less equal for both groups.

26.5 Summary

The research in the thesis presents several validity concerns, primarily related to the phrasing and interpretation of questionnaire items, including ambiguities in questions, negatively-worded questions, and possible divergence between students' perceived and actual grades. Selection bias may also have skewed the results, potentially favoring higher-performing students. The small sample size may have been insufficient for the statistical analysis. Lastly, the novelty effect was considered a probable bias, wherein students' excitement over new technology could have falsely inflated perceived effectiveness. Despite attempts to mitigate these concerns, their potential impact on the validity of the research findings remains a noteworthy issue.

Chapter 27

Conclusion

The goal of this thesis was *"To develop an engaging and educational learning game and evaluate the effectiveness of game-based learning in high school mathematics"*. Aftermath has been developed as a game-based learning platform for trigonometry, and an experiment has been conducted on three high school mathematics classes.

RQ1 asked *"How does attitude towards mathematics affect learning outcome and enjoyment in game-based mathematics learning?"* The results indicate that students who perceive mathematics as a subject about memorization benefit more from game-based learning, as it helps them gain a more conceptual understanding of mathematics. The benefits of game-based learning also depend on enjoyment of the relevant subject, as those who enjoy mathematics also enjoy mathematical learning games and thus learn more from them.

The enjoyment and effectiveness of game-based learning are affected by several attributes. **RQ2** asked *"Which personal attributes affect player enjoyment in game-based learning?"* Video game players are less immersed in game-based learning platforms but still benefit from game-based learning if the game is designed with a focus on achievements and rewards. Aftermath was perceived as difficult by certain students, and learning games that are too difficult seem to be less enjoyable and have a reduced learning efficiency. As very few students perceived the game as too easy, the findings can not be used to make conclusions about the effectiveness of learning games that are too easy. Societal perceptions indicate that gaming is more common among males and that male students benefit more from game-based learning. The findings of this study challenge these perceptions, suggesting that the enjoyment, learning outcomes and motivation towards game-based learning in mathematics are independent of gender.

RQ3 asked *"How effective is game-based learning as an introduction to trigonometry?"*, and **RQ4** asked *"How effective is game-based learning as a repetition of trigonometry?"* The findings of this study can not be used to determine the effectiveness of game-based learning as an introduction tool. However, the findings suggest that game-based learning helps build a more conceptual understanding for those who already have a procedural understanding and that game-based learning is an effective repetition tool.

RQ5 examined *How do the theories in game-based learning contribute to the players' enjoyment, motivation, and learning outcome in an educational mathematics game?*. The use of Kiili's experiential learning approach, Malone's theory, and the GameFlow theory was found to be effective in Aftermath, and thus in the creation of a mathematical learning games. The theories helped shape Aftermath's design, guiding important decisions and tradeoffs. A standout point was Aftermath's ability to find a balance between being educational and enjoyable at the same time. 91% of students measured a GEF score of 3 (neutral) or above, and more than 50% reported they discovered something new, and got a new perspective within trigonometry after 30 minutes of playing.

Chapter 28

Future Work

Aftermath was positively received, and the findings suggest that game-based learning in general may be effective to help students gain a more conceptual understanding of mathematics. There is certainly a potential for Aftermath or a similar game-based learning platform to be further developed and used as a supplement to traditional education.

As discussed in the previous Chapter, the findings suggest that Aftermath is an effective repetition tool, but also that the study can not conclusively determine its effectiveness as an introduction tool, partially due to the experiments only lasting 30 minutes. However, the study does suggest that Aftermath helps students gain a more conceptual understanding of mathematics, and according to existing theory, it is easier to gain a procedural understanding with an existing conceptual understanding [30, p. 188]. In conjunction, these suggest further research should be conducted to investigate the effectiveness of game-based learning as an introduction tool.

In a future study, the experiments should be conducted over a longer time period to test the hypothesis that it is easier for students to learn mathematical formulas and procedures after experimenting with a game such as Aftermath. Conducting an experiment over a longer time period could also make the data more reliable through objective measurements of learning outcomes. In this study, the effects on learning outcomes are only based on subjective feedback and perceived learning, which may skew the results. Future studies should also employ a control group to compare the results to traditional teaching methods. Furthermore, as this study has relied on one-session experiments, the data may be less reliable due to the *novelty effect*. In a future experiment conducted over a longer time period, the novelty effect would be reduced as the excitement over the new technology gradually wears off [11].

One interesting result investigated in this project is the correlation between gamers and non-gamers, including the gender difference in regard to video game usage and preferences. These correlations suggest some interesting results that may challenge societal perceptions and should be investigated further. In this study, no distinctions between the types of players were made, only how often they like to play games. The study could be elaborated in future research by investigating different types of players, based on the type of games they like to play, when they play, why they play games, whether they like to play alone or with friends, etc.

Several improvements can also be made to Aftermath and used in future studies. While the game is a prototype developed over only a few months by two students, the game shows potential for expansion beyond its current focus on trigonometry. Given its visually engaging approach, the platform could be further refined to teach other fields of mathematics that benefit from a visual learning environment, such as geometry, calculus, and applied mathematics.

New game mechanics could also be added to the existing trigonometry game, such as a multiplayer element to investigate the effects of competition and collaboration in education. For example, high scores could be added to facilitate competition, and a two-player mode where one player has to unlock trigonometric function features for their teammate who can use these features to solve tasks. Competition could potentially enhance student engagement, and collaboration could enhance learning outcomes when players collaborate with more knowledgeable peers, as stated by Vygotsky's Theory on the Zone of Proximal Development [18, p. 86]. It could be interesting to further investigate how competition and collaboration affect the enjoyment, motivation, and learning outcomes, and the effects they have on math anxiety.

If Aftermath is expanded to be used in an experiment conducted over a longer time period, it should also incorporate more gamification elements, such as those described in Duolingo. Features such as *Experience Point Reward Systems* and *Achievement Systems* with daily streaks and divisions could improve user retention. This was not a focus in the prototype of Aftermath, as the experiment was only conducted over one session, but a learning game should also attempt to engage players over longer periods of time.

This study has proved that game-based learning enhances conceptual learning of mathematics and gives a more foundational understanding for students. While the study suggests some promising avenues, it is merely scratching the surface of game-based learning's potential. By dedicating more time, research, and resources to refining these platforms and broadening their scope, the traditional approach to learning will gradually be reshaped. Through these steps, we may begin to uncover the rest of *the unseen angles* in mathematics, making it more engaging and accessible for everyone.

Bibliography

[1] S. Freeman, S. L. Eddy, M. McDonough *et al.*, 'Active learning increases student performance in science, engineering, and mathematics', *Proceedings of the National Academy of Sciences*, vol. 111, no. 23, pp. 8410–8415, 2014. DOI: 10 . 1073 / pnas . 1319030111. eprint: https://www.pnas.org/doi/pdf/10.1073/pnas.1319030111. [Online]. Available: https://www.pnas.org/doi/abs/10.1073/pnas.1319030111.

[2] D. Kolb, *Experiential Learning: Experience As The Source Of Learning And Development*. Jan. 1984, vol. 1, ISBN: 0132952610.

[3] M. Csikszentmihalyi, *Beyond boredom and anxiety: Experiencing flow in work and play.* San Fransisco: Jossey-Bass, 1975.

[4] B. J. Oates, *Researching Information Systems and Computing.* Sage Publications Ltd., 2006, ISBN: 1412902231.

[5] K. Mattick, J. Johnston and A. de la Croix, 'How to... write a good research question', *The Clinical Teacher*, vol. 15, no. 2, pp. 104–108, 2018. DOI: https://doi.org/10.1111/tct.12776. eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1111/tct.12776. [Online]. Available: https://onlinelibrary.wiley.com/doi/abs/10.1111/tct.12776.

[6] P. Sedgwick, 'The hawthorne effect', *BMJ*, vol. 344, 2012, ISSN: 0959-8138. DOI: 10.1136/bmj.d8262. eprint: https://www.bmj.com/content/344/bmj.d8262.full.pdf. [Online]. Available: https://www.bmj.com/content/344/bmj.d8262.

[7] M. L. Patten and M. Newhart, *Understanding Research Methods*, 10th ed. Routledge, 2017, ISBN: 9781315213033.

[8] D. T. Campbell, 'Factors relevant to the validity of experiments in social settings', *Psychological Bulletin*, vol. 54, 4 1957.

[9] T. Dybå and T. Dingsøyr, 'Strength of evidence in systematic reviews in software engineering', in *Proceedings of the Second ACM-IEEE International Symposium on Empirical Software Engineering and Measurement*, ser. ESEM '08, Kaiserslautern, Germany: Association for Computing Machinery, 2008, pp. 178–187, ISBN: 9781595939715. DOI: 10.1145/1414004.1414034. [Online]. Available: https://doi.org/10.1145/1414004.1414034.

[10] D. T. Campbell and J. C. Stanley, *Experimental and Quasi-Experimental Designs for Research*, 1st ed. Wadsworth Publishing, 1966.

[11] C. H.-H. Tsay, A. K. Kofinas, S. K. Trivedi and Y. Yang, 'Overcoming the novelty effect in online gamified learning systems: An empirical evaluation of student engagement and performance', *Journal of Computer Assisted Learning*, vol. 36, no. 2, pp. 128–146, 2020. DOI: https : / / doi . org / 10 . 1111 / jcal . 12385. eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1111/jcal.12385. [Online]. Available: https://onlinelibrary.wiley.com/doi/abs/10.1111/jcal.12385.

[12] J. Hamari, J. Koivisto and H. Sarsa, 'Does gamification work?–a literature review of empirical studies on gamification', in *2014 47th Hawaii international conference on system sciences*, Ieee, 2014, pp. 3025–3034.

[13] L. Rodrigues, F. D. Pereira, A. M. Toda *et al.*, 'Gamification suffers from the novelty effect but benefits from the familiarization effect: Findings from a longitudinal study', *Int J Educ Technol High Educ*, vol. 19, 13 2022.

[14] A. W. Moen and H. R. Jacobsen, 'Learning by gaming - a specialization project for the development of a collaborate learning game', 2022.

[15] D. H. Schunk, *Learning Theories: An Educational Perspective*. Pearson Education Limited, 2011, ISBN: 0-13-707195-7.

[16] M. Ahmad, L. Rahim and N. I. Arshad, 'A review of educational games design frameworks: An analysis from software engineering', Jun. 2014, pp. 1–6, ISBN: 978-1-4799-4390-6. DOI: 10.1109/ICCOINS.2014.6868452.

[17] A. Bandura, 'Social foundations of thought and action', *Englewood Cliffs, NJ*, vol. 1986, no. 23-28, 1986.

[18] L. S. Vygotsky, *Mind in Society - The Development of Higher Psychological Processes*. Harvard University Press, 1978, ISBN: 0-674-57628-4.

[19] R. Wass and C. Golding, 'Sharpening a tool for teaching: The zone of proximal development', *Teaching in Higher Education*, vol. 19, no. 6, pp. 671–684, 2014. DOI: 10.1080/13562517.2014.901958.

[20] R. M. Ryan and E. L. Deci, 'Self-determination theory and the facilitation of intrinsic motivation, social development, and well-being', 2000. DOI: "https://doi.org/10.1037/0003-066X.55.1.68".

[21] E. L. Deci, 'Intrinsic motivation, extrinsic reinforcement, and inequity.', *Journal of personality and social psychology*, vol. 22, no. 1, p. 113, 1972.

[22] M. H. Ashcraft, E. P. Kirk and D. Hopko, 'On the cognitive consequences of mathematics anxiety.', 1998.

[23] M. Ashcraft and E. Kirk, 'The relationships among working memory, math anxiety, and performance', *Journal of experimental psychology. General*, vol. 130, pp. 224–37, Jul. 2001. DOI: 10.1037//0096-3445.130.2.224.

[24] R. Hembree, 'The nature, effects, and relief of mathematics anxiety.', *Journal for Research in Mathematics Education*, vol. 21, pp. 33–46, 1990.

[25] A. Dowker, A. Sarkar and C. Y. Looi, 'Mathematics anxiety: What have we learned in 60 years?', *Frontiers in Psychology*, vol. 7, 2016. DOI: 10.3389/fpsyg.2016.00508.

[26] L. Punaro and R. Reeve, 'Relationships between 9-year-olds' math and literacy worries and academic abilities', *Child Development Research*, vol. 2012, Oct. 2012. DOI: 10.1155/2012/359089.

[27] C. Blazer, 'Strategies for reducing math anxiety. information capsule. volume 1102.', *Research Services, Miami-Dade County Public Schools*, 2011.

[28] B. Rittle-Johnson and M. Schneider, 'Developing conceptual and procedural knowledge of mathematics', in Jan. 2015, pp. 1102–1118. DOI: 10.1093/oxfordhb/9780199642342.013.014.

[29] R. R. Skemp, 'Relational understanding and instrumental understanding', *Mathematics teaching*, vol. 77, no. 1, pp. 20–26, 1976.

[30] B. Rittle-Johnson and M. Alibali, 'Conceptual and procedural knowledge of mathematics: Does one lead to the other?', *Journal of Educational Psychology*, vol. 91, pp. 175–189, Mar. 1999. DOI: 10.1037/0022-0663.91.1.175.

[31] K. J. M. Kiili, K. Devlin, A. Perttula, P. Tuomi and A. Lindstedt, 'Using video games to combine learning and assessment in mathematics education', *International Journal of Serious Games*, vol. 2, no. 4, Dec. 2015. DOI: 10.17083/ijsg.v2i4.98. [Online]. Available: https://journal.seriousgamessociety.org/index.php/IJSG/article/view/98.

[32] D. Tall and S. Vinner, 'Concept image and concept definition in mathematics with particular reference to limits and continuity', *Educational Studies in Mathematics*, vol. 12, May 1981. DOI: 10.1007/BF00305619.

[33] T. Sundstrom and S. Schlicker, *Trigonometry*. Grand Valley State University, 2022.

[34] N. Bornstein, 'Teaching transformations of trigonometric functions with technology', *Journal of Interactive Media in Education*, Jul. 2020. DOI: 10.5334/jime.503.

[35] T. Wijman, 'Newzoo global games market report 2021', 2021.

[36] T. W. Malone, 'What makes things fun to learn? heuristics for designing instructional computer games.', *Proceedings of the 3rd ACM SIGSMALL symposium and the first SIGPC symposium on Small systems*, pp. 162–196, 1980.

[37] T. Malone, 'What makes things fun to learn? a study of intrinsically motivating computer games', *Pipeline*, vol. 6, Jan. 1981.

[38] P. Sweetser and P. Wyeth, 'Gameflow: A model for evaluating player enjoyment in games', *Comput. Entertain.*, vol. 3, no. 3, p. 3, Jul. 2005. DOI: 10.1145/1077246.1077253. [Online]. Available: https://doi.org/10.1145/1077246.1077253.

[39] M. Csikszentmihalyi, 'Flow: The psychology of optimal experience', in Jan. 1990.

[40] K. Kiili, 'Digital game-based learning: Towards an experiential gaming model', *The Internet and Higher Education*, vol. 8, no. 1, pp. 13–24, 2005, ISSN: 1096-7516. DOI: https://doi.org/10.1016/j.iheduc.2004.12.001. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S1096751604000776.

[41] S. Deterding, D. Dixon, R. Khaled and L. Nacke, 'From game design elements to gamefulness: Defining" gamification"', in *Proceedings of the 15th international academic MindTrek conference: Envisioning future media environments*, 2011, pp. 9–15.

[42] H. Wang and C.-T. Sun, 'Game reward systems: Gaming experiences and social meanings', p. 15, May 2012.

[43] F. F. Vennerød-Diesen, F. Siddiq, J. Smedsrud, M. Bugge and S. Daus, 'Innovativ matematikkundervisning på barnetrinnet førte til positive resultater', 2021.

[44] C. Ericson, *Real-Time Collision Detection*. CRC Press, 2004.

[45] H. Kniberg, *Scrum and XP from the Trenches*. InfoQ, 2007. [Online]. Available: http://www.infoq.com/minibooks/scrum-xp-from-the-trenches.

[46] P. Sedgwick, 'Cluster sampling', *BMJ*, vol. 348, 2014. DOI: 10.1136/bmj.g1215. eprint: https://www.bmj.com/content/348/bmj.g1215.full.pdf. [Online]. Available: https://www.bmj.com/content/348/bmj.g1215.

[47] D. Romrell, 'Gender and gaming: A literature review', in *annual meeting of the AECT International Convention, Hyatt Regency Orange County, Anaheim, CA*, ERIC, 2014, pp. 170–182.

[48] E. S. Association, '2022 essential facts about the video game industry', 2022.

[49] C. A. Schriesheim and K. D. Hill, 'Controlling acquiescence response bias by item reversals: The effect on questionnaire validity', *Educational and psychological measurement*, vol. 41, no. 4, pp. 1101–1114, 1981.

Appendix

A Questionnaire

B Multiple-Choice Questions

ID	**Question**	**<17**	**17**	**18**	**19**	**>19**
Q1	What is your age?	0	17	12	6	0

Table 1: Question 1

ID	**Question**	**Female**	**Male**	**Other**
Q2	What is your gender?	11	24	0

Table 2: Question 2

ID	**Question**	**1**	**2**	**3**	**4**	**5**	**6**
Q3	Which math grade do you think you will get this year?	0	2	5	7	12	9

Table 3: Question 3

ID	Question	Too easy	Easy	Ideal	Hard	Too hard
Q4	How would you describe the game's difficulty?	0	1	19	11	4

Table 4: Question 4

C Likert-Statements

ID	Question	1	2	3	4	5
S1	Math is fun	0	2	15	7	11
S2	Math is mostly about memorization	2	7	13	8	5
S3	I was familiar with trigonometric functions before playing	0	3	10	13	9
S4	I discovered something new about trigonometric functions after playing	1	5	10	12	7
S5	I understand trigonometry better after playing	2	7	13	9	4
S6	I got a new perspective on trigonometric functions after playing	0	4	11	10	10
S7	Game-based learning is more motivating	4	5	12	6	8
S8	Game-based learning as a supplement can yield better learning outcomes than traditional teaching	5	8	6	11	5
S9	I often tried to calculate the optimal solution	10	10	2	4	2

Table 5: Questions related to students' knowledge and opinion of math

ID	Question	1	2	3	4	5
S10	It was easy to understand how to start playing	0	3	5	14	13
S11	The game mechanics were comprehensible	0	3	8	12	12
S12	It was difficult to understand the goal of the game	10	10	2	4	9
S13	I understood if I did something right or wrong in the game	1	3	5	12	14
S14	Time went fast whilst playing	2	3	7	12	11
S15	I thought little about other things than the game itself and solving the tasks	2	5	7	9	12
S16	I was curious about the next levels in the game	4	6	8	10	7
S17	I liked the design of the game	0	4	9	10	12
S18	I kept my concentration whilst playing	2	8	8	10	7

Table 6: GEF Questions

ID	Question	1	2	3	4	5
S19	I was tired whilst playing	4	5	10	7	2
S20	I like playing games in my spare time	3	2	6	10	7
S21	I prioritized level progression over stars	10	5	6	5	2
S22	I have used game-based learning platforms before	3	14	5	4	2

Table 7: Questions regarding attitudes towards games and exhaustion

D Open-Text Questions

Note that the responses are from the raw data conducted, and are in Norwegian. Also, note that all non-answer responses, such as "I don't know" or "Nothing", have been omitted from the tables with open-text questions.

bra animasjoner og visuelt tilfredstillende
funksjonene
Det var artig, men ble fort lei.
kjapp tilbakemelding på formen til funksjonen du har foreslått, gøy å lete etter løsninger
Selve oppsettet og konseptet var bra.
UI var bra. Bra ide. Jeg likte godt tankegangen.
Alt var helt greit til formålet.
Levler, og måten man får score
Alt
var lett å forstå hvordan spillet fungerte. Morsommere måte å lære på
Det meste fungerte som det skulle
Gir en grunnleggende forståelse.
God forklaring i starten
Lagd sånn at det er greit å forstå, samt et morsomt avbrekk fra regning
At det vær en annen måte å gjøre matte på som var kult
Engasjerende og lav terskel for læring
Lærerikt
Synes nettsiden så clean ut og det føltes godt å skyte sin/cos funksjoner
Spillet var lett og forstå og har høyt potensiale til å bli et verdenskjent spill dersom du gjør det multiplayer. Mattespill er bedre å gjøre for små barn som vokser opp med lite konsentrasjon.
Informativ
fange mynter
Enkelt å forstå
Gøy og lærerikt
Det var lett å se hva man skulle gjøre, hvis man forsto oppgaven.
Fin måte å gjøre matte litt mer interresant
Lett å holde fokus siden det var morsomt. Gir bedre forståelse siden man får se hva som skjer med egne øyne når man endrer de ulike tingene.
At det var enkelt å endre funksjonen
Det var lett å forsta hva man skulle gjøre.
Det var lett å spille og det er fint med visualisering av ulike funksjoner.
fint design, sikkert bra for de som skjønner sammenhengen mellom cosinus og sinus og sånn
Det hadde mye informasjon under veis og forklarte det praktiske ganske bra.
Fint med informasjonen som kom, og artig med utfordringen med kraft og bomber.

Table 8: Responses to Q5: *What did you like about the game?*

Fullskjerm
lite intuitivt noen ganger, kanskje det går at å bedre forklare de praktiske forandringene med grafen som skjer når du forandrer en nyintrodusert faktor.
Koordinatene kunne gjerne ha vært angitt som multiplum av pi (evt legge inn denne muligheten i k). Hvis man prøver flere ganger, men har truffet de første myntene, blir disse borte, kanskje de kunne ha blitt værende (men svakere/skygget ut) så man husker hvor man traff dem?
Var litt vanskelig å justere hvordan man justerte ting i spillet
Følte at jeg satt fast på enkelte nivå uten noen måte å gå videre på.
Synes det fungerte bra
Skaff multiplayer. Skyte andre sine romskip med alle slags type funksjoner og velge selv hvilke og hvordan de vil se ut.
vanskelig å forstå i starten
Kunne ha mer forklaring mellom de ulike nivåene, slik at man kan få mer forståelse. Vise hvordan man kan tenke på enhetssirkelen for å kunne treffe myntene.
skjønte ikke helt meningen med det
skjønte ikke helt greia med cosinus og når det kunne være mer praktisk enn cosinus. lyden var litt øreskjærende. "Chargen" til å sende ut en sinus eller cosinus bølge var litt rar og skjønte ikke hvorfor den trengte å være med
Noen ganger skjønte jeg ikke hva som var meningen. Litt lite forklaring.
var enkelte bugs som jeg forsto det
Hadde vært fint om det var noe som gjorde det mulig å få hint underveis om man skulle stå fast.
Det burde vært et fasit svar sånn at man ser hva som er riktig.
Starten fordi det var litt for stort og jeg kunne ikke krysse ut tutorial.
Vanskelig å skjønne hvordan å løse oppgavene uten å gjette seg fram til svaret. Dette kan ha med at vi ikke har hatt om trigonometri i sammenheng med funksjoner, og at det dermed var vanskelig å forstå.
Ingenting funket dårlig men jeg skjønte lite av teorien hehe.
litt utfordrende å skjønne hva som var poenget siden det er lenge siden vi har hatt om trigometri.
Var mest prøving å feiling for å komme seg frem til svaret, var sjeldent jeg regnet eller prøvde å tenke ut hva som var lurt å gjøre/velge.
Var mest prøving å feiling for å komme seg frem til svaret, var sjeldent jeg regnet eller prøvde å tenke ut hva som var lurt å gjøre/velge.

Table 9: Responses to Q6: *What did you dislike about the game?*

Vet ikke så mye om trigonometriske funksjoner fra før av
Det var ganske klart, men vanskelig når jeg ikke kan nok om slike funksjoner fra før.
Alt fungerte greit, skulle ønsket noe undervisning på forhånd, for eksempel kjenskap til funksjonene. De tekst blokkene med illustrasjoner før hver oppgave var litt lite.
ikke helt
Skreiv om dette ved forbedringsdelen
Nei, var veldig bra.
vanskelig å forstå i starten, men skjønte etter hvert
forsto ikke hva målet var
Cosinusbølgen og den relevanse
Forsto ikke målet med spillet.
forsto ikke helt poenget med de sinus og cosinus knappene
Nei, ingen problemer med dette.
tror det bare var at jeg ikke skjønte stoffet

Table 10: Responses to Q7: *Is there anything you struggled with in the game?*

Matematikk.org vibe. Kunne gjerne ha litt kul kahoot musikk.
Svært gøyalt spill, positiv forandring til en kjedelig læringsverdag
Ikke noe jeg kommer på
Hadde vært gøy om man kunne bruke myntene til noe. Til å kjøpe en annen bakgrunn eller en annen pakke med lydeffekter eller noen andre ting som gjør at man vil få flest mynter som mulig.

Table 11: Responses to Q8: *Do you have any other feedback?*

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