Anne-Sofie Johansen
Bendik Nag
Herborg Hermansen Tveit

Nurse Scheduling and Rescheduling: Combining Optimization with Machine Learning-Driven Demand Predictions

Master's thesis in Industrial Economics and Technology Management
Supervisor: Anders N. Gullhav, Helge Langseth
Co-supervisor: Thomas Bovim
June 2023

Master's thesis

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	Helge Langseth,	IDI
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Department of Industrial Economics and Technology Management

Preface

This master's thesis concludes our Master of Science degree at the Norwegian University of Science and Technology (NTNU). It is an interdisciplinary master's thesis that encompasses both the Department of Industrial Economics and Technology Management and the Department of Computer Science, with specialization in Applied Economics and Optimization and Artificial Intelligence. The work has been conducted during the Spring semester of 2023, building upon the foundation laid in our Specialization Report in the fall semester of 2022 (Johansen et al., 2022). This thesis is written in collaboration with the Clinic of Cardiology at St. Olavs Hospital.

We would like to thank our supervisors, Associate Professor Anders N. Gullhav and Professor Helge Langseth, and our co-supervisor Ph.D. Candidate Thomas Bovim, for valuable feedback and guidance throughout our work. We also want to thank the section managers at the Clinic of Cardiology, the clinic manager, Rune Wiseth, and the Head of the Department for Nursing Services, Gro Lillebø, for being available and providing insight and data to facilitate our work.

Anne-Sofie Johansen, Bendik Nag and Herborg Hermansen Tveit
Trondheim, June 2023

Abstract

Nurse scheduling is essential for all hospitals to ensure sufficient patient treatment and a balanced workload for the nurses. It is a complex problem, and the schedules are subject to uncertainties in both supply and demand for nurses. Despite this complexity, scheduling is often done manually, which is time-consuming for managers. A poorly constructed schedule leads to nurse rescheduling, resulting in additional time and cost burdens for the hospital and inconvenience for nurses. Nurse rescheduling refers to the reactive process of making adjustments to the schedule due to unexpected events such as nurse absences or changes in patient load. To combat disruptions, proactive strategies can be implemented to improve the robustness or flexibility of the schedule. Robustness refers to the schedule's ability to absorb disruptions, and flexibility is the ability to efficiently reestablish the schedule. It is interesting to identify how the interplay between proactive strategies in scheduling and reactive strategies in rescheduling can improve the overall quality of the schedules and facilitate reactive measures in rescheduling.

This thesis combines operations research and machine learning to improve the scheduling and rescheduling processes using real-life data from the Clinic of Cardiology at St. Olavs Hospital. We formulate a baseline multi-objective mixed-integer scheduling model. The objectives aim to create schedules that reflect the actual schedules at the clinic. We extend this model using a cross-section buffer strategy and flexible assignment strategy for increased robustness and flexibility. The generated schedules are used as input to the rescheduling problem, formulated as a baseline mixed integer model to minimize total rescheduling costs related to increased wage costs from reactive actions. To evaluate the schedule, the rescheduling model is embedded in a simulation framework using historical data from the clinic to imitate the supply of nurses and actual demand for the simulation period. We have developed two machine learning models for predicting future demand. The results from the most promising model are used in combination with the rescheduling model to improve the rescheduling costs.

Our results show that proactive strategies achieve similar objective values as the baseline schedule while outperforming the total costs of the baseline schedule during rescheduling. Cross-section buffer schedules efficiently distribute nurses by adapting to daily demand. The flexible assignments strategy demonstrates favorable cost outcomes, although with potential for improving the distribution of workload for the nurses. Machine learning predictions consistently decrease rescheduling costs.

Sammendrag

Turnusplanlegging er avgjørende for alle sykehus for å sikre tilstrekkelig pasientbehandling og balansert arbeidsbelastning for sykepleierne. Det er et komplekst problem, og turnusplanen er utsatt for usikkerhet både i tilbud og etterspørsel av sykepleiere. Til tross for denne kompleksiteten blir planleggingen ofte gjort manuelt, noe som er tidskrevende for lederne. En dårlig konstruert turnus fører til replanlegging, noe som er kostbart for sykehuset, tidkrevende for lederne og upraktisk for sykepleierne. Replanlegging refererer til den reaktive prosessen med å gjøre justeringer i planen på grunn av uforutsette hendelser som sykepleierfravær eller endringer i pasientbelastningen. For å motvirke behovet for replanlegging kan proaktive strategier implementeres for å forbedre robusthet og fleksibilitet i turnusen. Robusthet refererer til turnusplanens evne til å absorbere forstyrrelser, mens fleksibilitet er evnen til å gjenopprette turnusplanen effektivt. Det er interessant å identifisere hvordan samspillet mellom proaktive strategier i planlegging og reaktive strategier i replanlegging kan forbedre turnuskvaliteten, samt legge til rette for de reaktive tiltakene i replanlegging.

Denne masteroppgaven kombinerer optimering og maskinlæring for å forbedre prosessene for turnusplanlegging og replanlegging ved hjelp av reell data fra Klinikk for hjertemedisin ved St. Olavs sykehus. Vi utvikler en grunnleggende flerobjektiv, blandet heltallsmodell der målene er å skape turnusplaner som gjenspeiler de faktiske planene ved klinikken. Vi utvider denne modellen ved å implementere en bufferstrategi som går på tvers av seksjoner og en fleksibel strategi for økt robusthet og fleksibilitet. Modellen utvides ved å implementere strategier for å øke planenes robusthet og fleksibilitet. De genererte turnusene brukes som input til replanleggingsproblemet, som brukes for å evaluere turnusplanene. Replanleggingsproblemet er formulert som en grunnleggende blandet heltalls modell med objektiv om å minimere totale kostnader knyttet til økte lønnskostnader som følge av reaktive tiltak. Det integreres i et simuleringsrammeverk som benytter seg av historiske data fra Klinikk for hjertemedisin for å etterligne tilbudet etter sykepleiere og faktisk etterspørsel i simuleringsperioden. Vi har utviklet to maskinlæringsmodeller for å forutsi fremtidig etterspørsel. Resultatene fra den mest lovende modellen brukes i kombinasjon med replanleggingsmodellen for å forbedre replanleggingskostnadene.

Resultatene viser at de proaktive strategiene konstruerer turnuser med tilsvarende objektivverdier som de grunnleggende turnusene, samtidig som de reduserer de totale kostnadene for replanlegging. Bufferstrategien tilpasser seg den daglige etterspørselen for hver seksjon og fordeler tilgjengelige sykepleiere effektivt.Den fleksible strategien viser gunstige kostnadsresultater, selv om fordelingen av arbeidsbelastning for sykepleierne kan forbedres. Maskinlæringsprediksjoner reduserer konsekvent replanleggingskostnadene.

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Chapter 1

Introduction

The global nursing shortage results from a growing demand for healthcare with little resource increase. The aging population contributes to a growing need for healthcare services, resulting in a shortage of nurses (Helsedirektoratet, 2022). This shortage manifests in various ways, including a lack of nurses to meet fluctuating demand and disruptions due to nurse absences (Skjøstad, 2017). In addition, hospitals operate with limited budgets, and personnel cost is the main component of hospital operating costs. The combination of nurse shortage, unpredictable demand, and high hospital costs emphasizes the importance of efficiently utilizing existing nursing resources.

Hospitals usually provide around-the-clock services that require employees to work in shift rotations organized through a nurse schedule. Thus, scheduling the work for nurses is an essential task of hospital management. Creating high-quality nurse schedules is a complex and time-consuming process. The process involves complying with various laws and regulations and considering prohibited shift patterns to avoid fatigue. The nurses should work according to their contracted hours. There should also be sufficient staffing for each individual shift. In addition, nurses must be assigned weekend shifts in specific weekend rotations.

The nurse schedule is affected by uncertainties in nurse supply and patient load. Nurses may become unavailable for their scheduled shifts, or patient load may increase. These disruptions can lead to insufficient coverage for adequate patient care, requiring nurses to work unplanned shifts. This alteration of the schedule is known as nurse rescheduling.

The rescheduling process modifies the schedule to address understaffed shifts caused by disruptions while still adhering to critical constraints such as required nurse demand and workload limits. To ensure that the rescheduling is solved correctly, previously executed schedule rearrangements must also be considered when resolving the problem. Demand predictions can be a key factor in enhancing the performance of the rescheduling. Accurate predictions of nurse demand for upcoming days help decision-makers preemptively find replacements or additional nurses.

Managing schedule disruptions is both costly and time-consuming for decision-makers. It is desirable to have some protection against disruptions in the nurse schedules. This can be handled by utilizing proactive strategies when generating the schedule, which can improve the robustness and flexibility of the schedule. Robustness is defined as the ability to absorb disruptions, while flexibility is defined as the ability to reestablish the schedule efficiently. Utilizing such proactive strategies in nurse scheduling can facilitate more robust and flexible reactive strategies in the rescheduling problem.

The purpose of this thesis is to evaluate the effectiveness of proactive scheduling to facilitate cost-efficient reactive rescheduling with a focus on cross-section utilization of nurses. We aim to further enhance this evaluation by introducing machine learning models. These models are designed to predict nurse demand during the rescheduling phase and investigate its potential impact on decreasing rescheduling costs. The predictions are based on historical data from the Clinic of Cardiology at St. Olavs Hospital.

Our work is among the first to study the nurse scheduling problem and the nurse rescheduling problem together while accounting for uncertainties. To our knowledge, we are the first to make a rescheduling model that takes predicted demand from a machine learning model as input. Additionally, we introduce a novel proactive strategy combined with cross-section utilization. Studying the problems together enables a more realistic evaluation of proactive strategies. Moreover, all proposed models are based on real-world data from the Clinic of Cardiology, making them among the most realistic and applicable to date. Although the developed models are based on specific information, they are generic models that can be adapted to similar departments in other hospitals that wish to facilitate the utilization of resources across sections.

1.1 Outline

Chapter 2 provides background information on the Clinic of Cardiology and its nurse scheduling challenges. Chapter 3 provides the necessary background theory to give an understanding of machine learning methods and the field of operations research. Chapter 4 places our work within the existing literature. Chapter 5 provides a detailed description of the problem in this thesis, while Chapter 6 presents the corresponding optimization models. Chapter 7 outlines the simulation framework. Chapter 8 discusses real-world data and machine learning models used in this thesis. The results of the computational study are discussed in Chapter 9. Chapter 10 explores future research opportunities, and Chapter 11 concludes the thesis.

Chapter 2

The Clinic of Cardiology

This chapter presents the relevant background information connected to the Clinic of Cardiology (CC) at St. Olavs Hospital. Section 2.1 explains important terminology to understand the presented information. Section 2.2 gives an overview of St. Olavs Hospital and CC. Section 2.3 presents relevant laws and regulations before Section 2.4 describes the nurse scheduling and rescheduling processes at CC.

2.1 Terminology

- *Section.* A section is a unit within a hospital department.
- *Shift.* A shift is a defined working time on a given day.
- *Nurse schedule.* A nurse schedule is a complete shift plan for the nurses within a period. It shows which nurses are assigned to each shift.
- *Scheduling period.* The period a nurse's schedule is set for.
- *Competence.* Competence is a nurse's ability to perform specific tasks or cover specific types of demand. Competence is based on educational level.
- *Minimum staffing level.* The minimum staffing level describes the minimum number of nurses that must be present for a specific shift to deliver the required care level.
- *Staffing plan.* The staffing plan provides an overview of the minimum staffing level per shift. It lays the foundation for the creation of the nurse schedule.
- *Rescheduling.* Rescheduling is the process that ensures that the supply for a given shift meets demand in the occurrence of unforeseen absence or an increase in patient load.
- *Planning period.* The planning period is the period considered when rescheduling.
- *Additional hours.* Additional hours are defined as work beyond agreed working hours but within the statutory limit of a full-time equivalent.

- *Overtime payment.* Overtime payment is defined as all work hours that exceed a set number of working hours within a day or a week.
- *Bonus payment.* Bonus payments are additional payments for inconvenient shifts such as evening-, night- and weekend shifts. The main tariff agreement defines each type of bonus payment as a percentage of the nurse's base salary payment.
- *Compensation payment.* Compensation payment is given when a nurse is swapped from their original shift to work another shift. This payment takes effect if the swap is notified before the day of the original shift. If it is given on the day of the original shift, overtime payment is used instead.
- *The Working Environment Act.* The Working Environment Act is a Norwegian law that dictates regulations to ensure safe and fair employment conditions for all workplaces in Norway.
- *The main tariff agreement.* The main tariff agreement regulates wages, working conditions, pensions, and insurance schemes for nurses working in Norwegian hospitals. The agreement is made in collaboration with The Norwegian Nurses Organisation (NSF).
- *The Specialist Health Services Act.* The Specialist Health Services Act contains rules on responsibilities, duties, rights, and organization for anyone who offers or provides specialist health care services in Norway.
- *Local agreements.* The hospital departments can enter into local agreements that allow deviations from other central laws and agreements.

2.2 Description of the Clinic of Cardiology

St. Olavs Hospital in Trondheim is one of Norway's largest hospitals, with close to 11,000 employees, working about 9000 full-time equivalents (Helse Midt-Norge, 2021). The Clinic of Cardiology (CC) at this hospital specializes in cardiac disorders. The clinic's main goal is to provide the necessary help and treatment to patients with cardiac disorders. The organization structure in Figure 2.1 shows that CC is split into two departments, one for the medical services and the other for the nursing services. This thesis focuses on the Department for Nursing Services.

The Department for Nursing Services comprises seven sections, each specializing in different areas related to cardiac disorders and patient severity levels (St. Olavs Hospital, 2023). The Lab is responsible for examining patients within invasive cardiology and electrophysiology, while the Outpatient Clinic provides care for cardiac patients who are not hospitalized but have a referral from their doctor. The Intensive Care section is dedicated to treating patients with acute and critical cardiac disorders, while the 5-day section mainly deals with elective procedures. Patients are referred to this section by their doctors or local hospitals. Although the bed wards treat all types of cardiac disorders, each section has its specialization. Bed Ward 1 specializes in Transcatheter Aortic Valve Replacement (TARV), Bed Ward 2 focuses on patients with heart rhythm disorders, and Bed Ward 3 specializes in treating patients with cardiac failure and disorders in the heart valves. This thesis concentrates on the three bed wards mentioned: Bed Ward 1, Bed Ward 2, and Bed Ward 3.

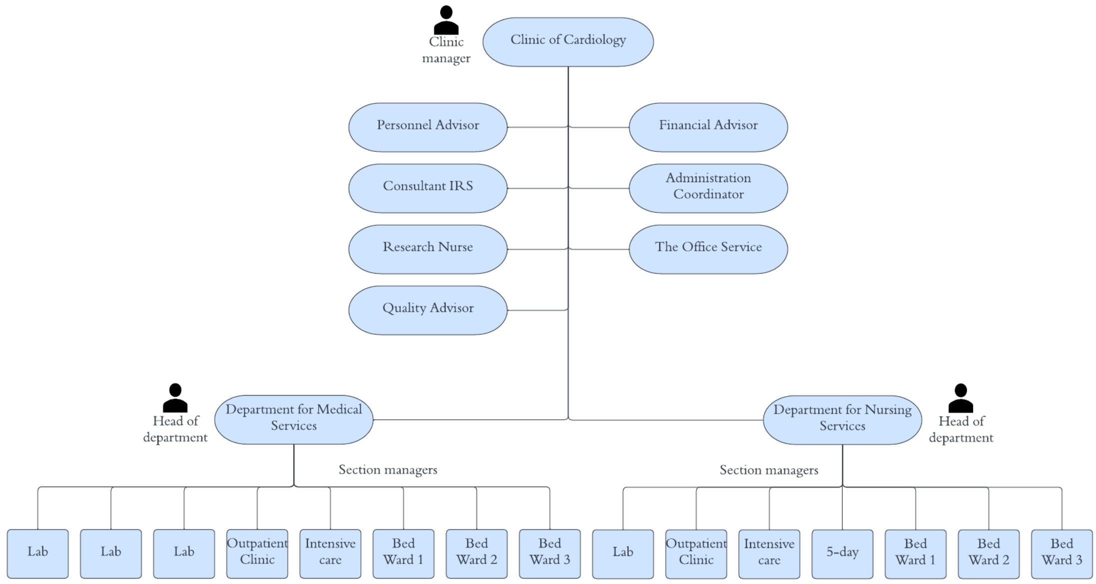


Figure 2.1: Structure of the Clinic of Cardiology

The three bed wards provide around-the-clock services all week and have a total of 104 employed nurses, with 32, 33, and 39 nurses assigned to Bed Ward 1, Bed Ward 2, and Bed Ward 3, respectively. While the nurses are primarily employed in one bed ward, they are also qualified to work in the other two bed wards. The employees in the bed wards have varying levels of competence and experience. The nurses' competence is categorized into three levels: assistant nurse, nurse, and specialized nurse. The experience level denotes the number of years of employment at CC.

The patients admitted to the bed wards all have a cardiac disorder, but the severity and how demanding a patient is can vary greatly. The primary factor is the type of cardiac disorder the patient has been admitted for, but other factors such as age and co-morbidities can also affect the patient's needs and level of care required. Different cardiac disorders require different treatment procedures, some more demanding than others.

2.3 Laws and regulations

Laws and regulations apply to nurses working in the hospital. The Working Environment Act and the main tariff agreement are the primary sources and form the foundation for hospital work laws (Arbeidsmiljøloven, 2022; NSF, 2022). Together, these regulate wages, working conditions, pensions, and insurance schemes for nurses in Norway. Most nurses work in a rotation and are obliged to work at inconvenient times, i.e., night shifts and weekend shifts. Nurses are scheduled to at least one weekend every three weeks. It is essential to follow the laws and regulations to take care of the nurses' health and well-being.

Due to the around-the-clock service and work at least every third Sunday, a standard workweek for a full-time nurse amounts to 35.5 hours. The contracted work hours for each nurse are the average work hours for the whole scheduling period. This means that the nurses can be scheduled to work more than 35.5 hours one week and less another week and still not get any overtime payments.

The local agreements for the bed wards have rules for rest time for working in rotational shifts. The minimum consecutive rest between shifts in any 24-hour window is specified to be at least 9 hours. However, the local agreements for CC specify that this number is set to 10 hours daily. Also, there is a minimum of 35 hours of rest between two consecutive shifts at least once every week.

Bonus payments are used for inconvenient shifts, such as evening, night, and weekend shifts. All bonus payments are illustrated in Table 2.1. For everyday work between 00:00 on Saturday and 24:00 on Sunday, the nurses get a weekend bonus of 23% of the hourly wage. For work between 17:00 and 23:00, there is an evening bonus of 28% of the hourly wage; for night work between 22:00 and 08:00, there is a night bonus of 28%.

Table 2.1: Bonus payments (Norsk Sykepleierforbund, 2022b).

Type of extra payment	**Percentage addition of hourly wage**
Weekend bonus	23%
Evening bonus	28%
Night bonus	28%
Overtime pay	100%
Swap shift before the day before the original shift	85%
Swap shift on the day of the original shift	100%

For nurses with full-time positions, all extra shifts worked outside of what is scheduled are considered overtime work and come with overtime payment. Furthermore, nurses with part-time positions are qualified for overtime pay if they work more than 35.5 hours during a week. Overtime pay is 100% of the hourly wage.

If a section manager needs to move a nurse from their scheduled shift to another, it is referred to as a swap. When a swap occurs, the nurse is entitled to a bonus payment. The amount of the bonus payment varies depending on the timing of the notification of the shift change. If the nurse is notified on the same day as their original shift, they are entitled to a percentage addition of 100% for the entire shift, which is the same rate as regular overtime pay. However, if the nurse is notified of the shift change prior to the day of their original shift, they are entitled to a compensation payment. This compensation payment is less than regular overtime pay and varies depending on the shift the nurse is swapped to. On average, the compensation payment is 85% of the regular overtime pay rate.

In addition to these laws and regulations, CC must comply with the Specialist Health Services Act, which deals with the hospital's duty of responsibility (Spesialisthelsetjenesteloven, 2022). This means that the hospital must also be organized so that the health personnel can provide professionally sound health care. The hospital should comply with its responsibilities to provide the individual patient with a comprehensive and coordinated health care service. This requirement dictates that the department's operations must be planned to ensure that the department has sufficient staff with the right expertise available for the patients at all times (Helsepersonelloven, 2022).

2.4 Scheduling

The around-the-clock service in the bed wards at CC requires a work distribution that covers all week hours. These coverage requirements result in a carefully planned nurse schedule. A nurse schedule outlines the assignment of nurses to work specified shifts over a predetermined scheduling period. The nurses are assigned to shifts to cover the demand for competencies and experience. The section managers are responsible for the creation and management of the schedules, with each section having its own schedule. The scheduling period for the bed wards at CC is one year, which presents challenges about the uncertainty in supply and demand over the long scheduling period.

Each section has a set of nurses, which should be distributed in the schedule. The nurses are only scheduled to the section they belong to, with the exception of weekend shifts. During the weekends, the three bed wards borrow nurses from the sections that only operate on weekdays, such as the Outpatient Clinic and the 5-day Clinic. The allocation of nurses to shifts in the bed wards is based on a staffing plan for each section. The section managers are responsible for scheduling the nurses for the whole scheduling period based on the staffing plan. Although the bed wards operate with different shift types, the schedule is generally simplified and divided into three main working shifts: day, evening, and night, denoted as D, E, and N, respectively.

The staffing plan specifies a minimum staffing level for each shift throughout the week, which typically remains constant weekly throughout the year. Ensuring that the minimum staffing level for each shift is fulfilled is crucial to ensuring adequate patient care. Weekend shifts tend to have a lower minimum staffing level. Table 2.2 provides an example of the minimum staffing levels for Bed Ward 2, which remain relatively unaltered throughout the scheduling period.

Table 2.2: One week in the staffing plan for Bed Ward 2.

Week X	Mon	Tue	Wed	Thu	Fri	Sat	Sun
D (day)	6	6	6	6	6	5	5
E (evening)	5	5	5	5	5	5	5
N (night)	3	3	3	3	3	3	3

The nurse scheduling process is complex for the section managers and requires much time and effort. The process starts with the section managers receiving the nurse's vacation and working preferences. Furthermore, they create the schedule based on these preferences, the nurse's contracted working hours, the minimum staffing level, and the laws and regulations described in 2.3. The section managers use the last year's resulting schedule as a base for creating the new schedule. According to the section managers at CC, generating nurse schedules for a scheduling period usually takes at least two weeks.

All nurses have competence and education in cardiac disorders and qualify to take any shifts at any of the bed wards at CC. A shift can have a set minimum or maximum number of nurses in a specific skill level. In addition, the section managers experience that a variety of experience and competence levels for the nurses for each shift are more efficient.

2.4.1 Rescheduling

When unforeseen events occur, for instance, when a nurse reports absence for their shift or the demand is higher than expected, the section managers must find nurses to step in and work a shift they are not initially assigned to. These unforeseen events happen daily at CC, and the work of finding a replacement is called rescheduling. Table 2.3 highlights the hierarchy of the different planning problems at CC.

Table 2.3: The hierarchy of staffing problems at CC.

Staffing Problem	**Description**	**Rate of change**
Staffing plan	Find the minimum staffing level per shift.	Rarely changed
Scheduling	Assign the nurses to work shifts in a scheduling period.	Once for every scheduling period, at least once a year.
Rescheduling	Finding replacements when adjustments to the schedule are required.	Every day

At the bed wards, the section managers first look towards their own employees to see if someone is available to step in. If not, they will look into the schedule of the other sections to see if there are some possibilities there. Each section's schedule is created independently, but it is possible to utilize nurses from the other sections in rescheduling. This is referred to as cross-section rescheduling.

Nurse absences can encompass both short-term and long-term absences. In both cases, reported absences lead to changes in the schedule, but different approaches are used to reschedule the vacant shifts. It is not known in advance how many schedule disruptions will occur for a given shift or for how many shifts a disruption will apply. The sick leave at CC mainly varied between 7% and 18% from 2020 to 2022, depending on the section and month. These percentages depict the many schedule disruptions and that some shifts have multiple disruptions simultaneously. Appendix B presents the numerical absence data.

In addition to nurse absences, the patient load varies at the bed wards. Some days and shifts are more demanding than others. This variation depends on many factors, but the main factors are the degree of severity of the patients and the turnover of patients. If multiple demanding patients are simultaneously hospitalized at the same section, this will increase the demand at the section. Also, if the patients are hospitalized for a short period, and new patients arrive frequently, this increases the demand. The number of nurses needed depends to some degree on the sum of patient load at each bed ward.

The rescheduling process can be complex, as multiple ways exist to fill a vacant shift. The rescheduling actions are described in Table 2.4.

Table 2.4: Actions for filling a vacant shift.

Action	**Description**
Swap	To swap shifts entail a nurse's scheduled shift being moved to another shift
Extra shift	Assign an extra shift to a nurse
Double shift	Assign a shift to a nurse already scheduled to a shift on the same day. This results in the nurse working two consecutive shifts

In rescheduling, various criteria are considered to determine which nurse should work a vacant shift. Even though the section managers strive to fulfill the laws and regulations for nurse scheduling, some are flexible when rescheduling. The priority is to meet the shift's demand and ensure that at least the minimum staffing level is met. Beyond this, the schedule planners need to balance several factors:

- **Cost** - The department runs on a strict budget and wishes to save money on salaries and the possibility of extra- and overtime pay.
- **Skills** - The nurses in the department have a wide variety of competence, qualifications, and experience.
- **Preferences** - The nurses plan their personal lives around the schedule. Therefore, the section managers strive to comply with the nurses' preferences and requests.

Chapter 3

Background Theory

This thesis takes an interdisciplinary approach; therefore, this chapter provides readers with the necessary background theory to give a basic understanding of machine learning and operations research. Section 3.1 provides an overview of the theory underlying the relevant machine learning paradigm. Section 3.2 describes the operations research theory necessary to comprehend the optimization methods utilized in this thesis. Finally, Section 3.3 highlights the interplay between these two fields and their prior applications.

3.1 Machine Learning

Machine learning is a subfield within artificial intelligence. Machine learning algorithms can enable machines to learn from data and make decisions or predictions based on what it learns. There are different machine learning algorithms based on the type of input data available and the desired outcome. It is common to consider three main types of learning; supervised, unsupervised, and reinforcement learning. Supervised learning uses labeled data, unsupervised learning uses unlabeled data, and with reinforcement learning, the machine learns from being rewarded or penalized for different decisions. This chapter mainly focuses on supervised learning, as this is the method used for the data we have been provided.

Supervised learning entails learning a mapping between a set of input variables, X, and output variables, Y (Cunningham et al., 2008). Using the input-output pairs, the algorithm can learn patterns and can use this knowledge to predict the output of unlabeled data with a matching set of features. Supervised learning is applicable to both classification and regression problems. In classification, the algorithm produces a discrete value from a predetermined set of classes, while in regression, the output is a float.

3.1.1 Machine Learning Methods

This section presents the theory behind the two supervised machine learning models explored in this thesis. Both these methods can handle classification and regression problems.

Artificial Neural Network

Artificial Neural Network is a type of machine learning based on the hypothesis that the mental activity in the human brain consists primarily of electrochemical activity in networks of brain cells called neurons (Russell & Norvig, 2010).

An artificial neural network consists of neurons, and these are connected by directed links (Nielsen, 2015). Figure 3.1 shows one type of an artificial neural network, the perceptron. This example shows that the perceptron takes the binary inputs, $x_0, x_1, and x_2$, and gives a single binary output. Each link is applied with a weight, w_i, which tells something about the strength of the connection between the neuron and its inputs. The neuron's output is determined by whether the weighted sum, $\sum_j w_j x_j$, is less or greater than a set threshold. We can also consider the neuron's threshold as the bias, b. The bias is the negative of the threshold and measures when the output should be 1. Equation (3.1.1) shows the activation function for the perceptron. The activation function decides whether or not a neuron should be activated. There are different activation functions, each with different behavior.

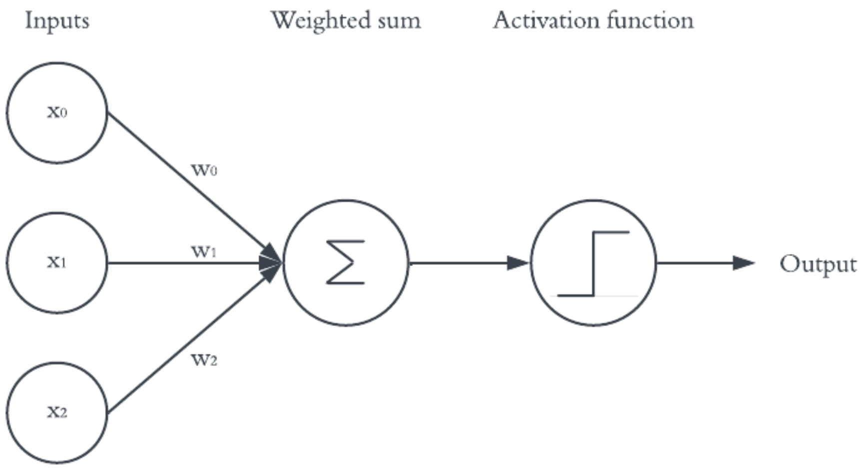


Figure 3.1: Perceptron

$$output = \begin{cases} 0 & \text{if } w \cdot x + b \leq 0 \\ 1 & \text{if } w \cdot x + b > 0 \end{cases} \tag{3.1.1}$$

Some of the most common activation functions are the Sigmoid, Rectified Linear Unit (ReLU), and Softmax activation functions. Equation (3.1.2) is the Sigmoid function, which outputs a value between 0 and 1 based on the input. It is often used for binary classification problems. Equation (3.1.3) is the ReLU function. If the input is positive, it outputs a value bigger than 0; otherwise, it outputs 0. The Softmax activation function is used to predict the probabilities of the different outputs when making a prediction. Equation (3.1.4) shows the mathematical formulation of the Softmax function. The task of the activation function is to add non-linearity to the input data.

$$f(x) = \frac{1}{1 + e^{-x}} \tag{3.1.2}$$

$$f(x) = \max(0, x) \tag{3.1.3}$$

$$f(x_i) = \frac{e^{x_i}}{\sum_{j=1}^{K} e^{x_j}} \tag{3.1.4}$$

There are two main types of neural networks; feed-forward neural networks and recurrent neural networks. These differ in the way they handle sequential data. In a feed-forward neural network, the data will only pass from input to output in that direction. In a recurrent network, the data moves in cycles between the layers. This is more complex than a feed-forward neural network due to its ability to retain and utilize sequential information.

A neural network can consist of several neurons in several layers. The input and output layers are essential components of a neural network, providing the necessary connections for data input and output. In addition to these layers, neural networks can incorporate hidden layers that perform nonlinear transformations in the neural network. Figure 3.2 illustrates a neural network with an input layer, one hidden layer, and an output layer. For a classification problem, the output layer will have the same number of neurons as classes to predict, while for a regression problem, the output layer will only contain one neuron, which produces one numeric value. In each layer, each neuron performs the steps shown in the perceptron in Figure 3.1, and the new value is then transferred to the next layer. The same procedure is performed until it reaches the output. This process is known as forward propagation.

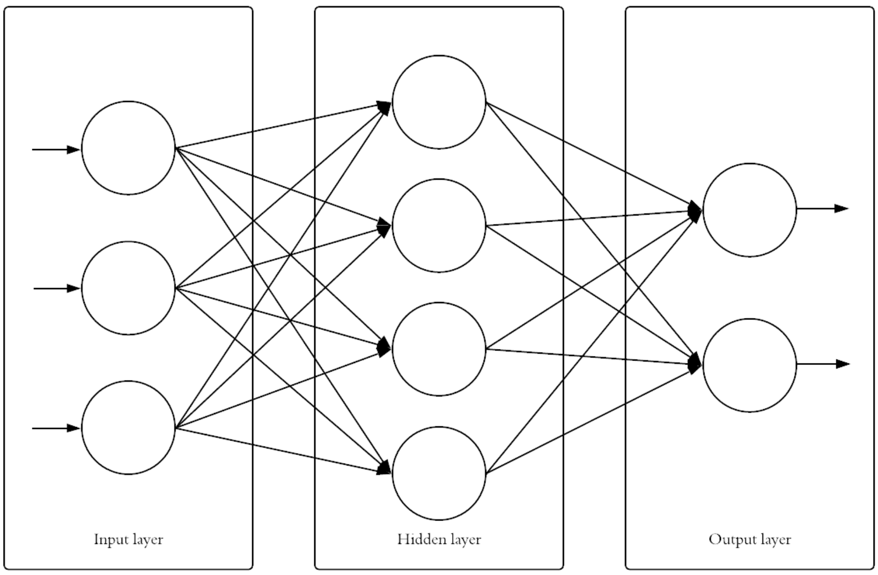


Figure 3.2: Architecture of a Feed-Forward Neural Network with input and output layer and a single hidden layers.

For the neural network to train and learn, the weights and biases of the neurons need to be adjusted to minimize the distance between the predicted output and the correct output.

For this to happen, forward and backward propagation is used. The error between the predicted output value and the correct target value is calculated when the neural network has done the forward propagation. The neural network utilizes a loss function to calculate the error. There are different loss functions for classification problems and regression problems. For classification problems, the most common loss function, presented in Equation 3.1.5, is the Categorical Cross-Entropy (CCE) which measures the dissimilarity between the predicted class probabilities, p_i, and the true class labels, y_i for each class C. For regression problems, the most commonly used loss functions are the mean squared error (MSE) and mean absolute error (MAE). Equations 3.1.7 and 3.1.6 present the mathematical formulas for calculating the MSE and MAE, respectively, where y_i is the true label and $\hat{y}_i$ is the predicted value for the label.

$$\mathrm{CCE} = -\sum_{i=1}^{C} y_i \log(p_i) \tag{3.1.5}$$

$$\mathrm{MAE} = \frac{1}{n}\sum_{i=1}^{n} |y_i - \hat{y}_i| \tag{3.1.6}$$

$$\mathrm{MSE} = \frac{1}{n}\sum_{i=1}^{n} (y_i - \hat{y}_i)^2 \tag{3.1.7}$$

In backpropagation, the neural network uses an optimization algorithm to adjust the weights and biases of the neural network. The most basic among the optimizers is the gradient descent, which is presented in Algorithm 1. In gradient descent, a gradient of the error is computed concerning the output weights. The gradient is then used to adjust the weights and biases in each layer by moving backward until the input layer. A learning rate, α, is used in the backpropagation process to determine the size of the weight adjustments. A learning rate too low may result in the network converging too slowly or being stuck in the local optima, and a learning rate too large may result in the network diverging. Each iteration containing both forward and backward propagation is called an epoch, and it is essential to choose a number of epochs such that the loss converges to a value. Each epoch considers all data points in the training set.

Algorithm 1 Gradient Descent Algorithm

1: $\mathbf{w} \leftarrow$any point in the parameter space
2: **for each** epoch **do**
3:     **for each** w_i in $\mathbf{w}$ **do**
4:         $w_i \leftarrow w_i - \alpha \frac{\partial}{\partial w_i}\mathrm{Loss}(\mathbf{w})$

To validate the performance of a trained neural network, the data is first split into two sets: a training and a test set. It is also common to create a validation set by randomly splitting the data between the training set and testing the performance of these random data points. The trained model should be able to predict the output of the test set, and the accuracy of the predictions can be computed by measuring the distance between the correct and predicted output. This is the score of how well the neural network can learn patterns in the data set.

Like many other machine learning methods, neural networks are sensitive to overfitting. Overfitting happens when the model becomes overly complex and excessively fits the training data, making it less capable of generalizing to new, unseen data. This phenomenon often occurs when the input data contains many features or the model has many parameters relative to the available training examples.

Decision Trees

Decision Trees are simpler supervised machine learning methods (Russell & Norvig, 2010). A decision tree consists of several nodes in a hierarchical structure, starting with the root node at the top and ending with leaf nodes at the end of each branch. Figure 3.3 shows a decision tree representation. Each decision node represents a specific condition or attribute, and based on the outcome of the decision, the tree branches out to different paths. Each decision node examines a feature of the data and determines which branch to follow based on a specific splitting criteria. The leaf nodes represent the final output of a combination of decisions made in the decision nodes.

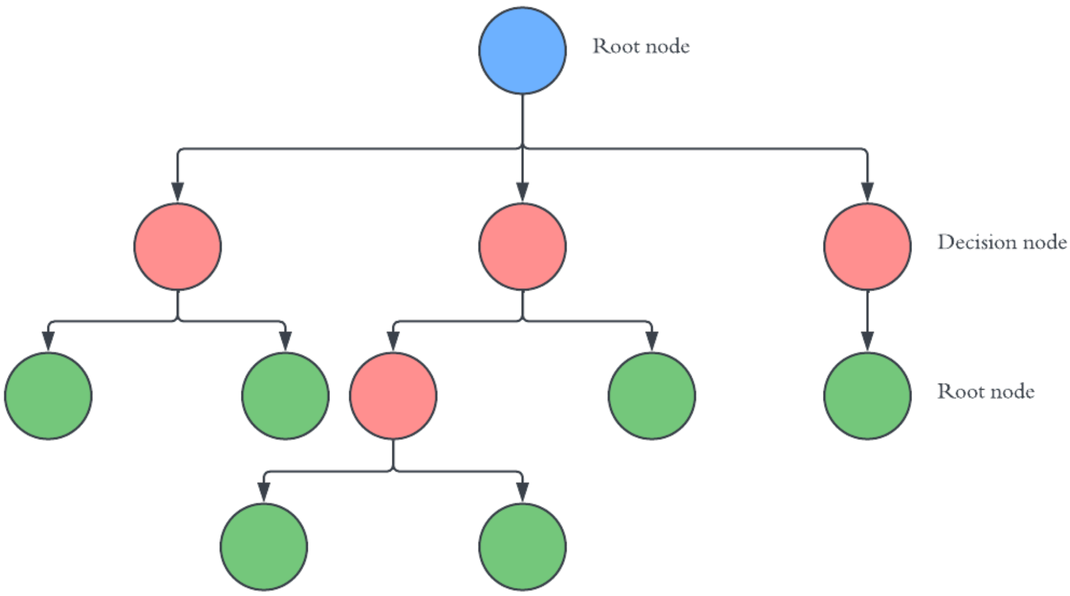


Figure 3.3: Decision tree with root, decision, and leaf nodes.

For classification problems, decision trees rely on computed Information Gain, and a splitting criteria is used to make each decision. Entropy and Gini Impurity are examples of splitting criteria represented as mathematical formulations. They aim to maximize the Information Gain at each split within the tree. The difference between the two is that the Entropy quantifies the randomness and uncertainty in the data. At the same time, the Gini Impurity calculates the probability of misclassification of a randomly selected element in the dataset. Equations (3.1.8) and (3.1.9) are the mathematical formulation of the Entropy and Gini Impurity for each class C, where p_i is the predicted probability for sample i. Features with the highest Information Gain are considered the most important since they provide more valuable information for decision-making. When dealing with a substantial number of features and the objective is to prioritize predictions on the most important features. Therefore, the Gini Impurity is often preferred over Entropy.

$$\text{Entropy} = -\sum_{i=1}^{C} p_i \log(p_i) \tag{3.1.8}$$

$$\text{Gini Impurity} = 1 - \sum_{i=1}^{C} p_i^2 \tag{3.1.9}$$

For regression problems, the splitting criteria most commonly used is the MSE or MAE. Both these criterion calculates the distance between the predicted and actual values. The MSE is more sensitive to outliers in the dataset than the MAE.

Decision trees are subject to overfitting but can be handled with a technique called decision tree pruning (Russell & Norvig, 2010). This works by eliminating nodes that are not relevant to the decisions made. For decision trees, different parameters can be chosen to cause the predictions to be more accurate. For instance, it is possible to set a maximum depth or number of splits for the tree to perform. This allows the tree's complexity to be controlled, preventing it from overfitting the training data.

3.2 Operations Research

Optimization is the science of making the best possible decision for some goal given a set of restrictions (Lundgren et al., 2010). The decisions are made using a defined objective and restrictions on the decisions that can be made. The field of optimization can be viewed as part of operations research. Operations research encompasses a variety of quantitative methods for improving decision-making in complex systems. Mathematical programming is one of this field's most important and widely used techniques. Other operations research fields include statistics, queuing theory, simulation, control theory, and game theory (Lundgren et al., 2010). This section focuses on the theory in operations research relevant to this thesis.

3.2.1 Mathematical Programming

Mathematical programming is a subfield within operations research that focuses on optimizing mathematical models while adhering to a set of constraints (Lundgren et al., 2010). A mathematical model attempts to describe some part of the real world in mathematical terms for a particular purpose (Meyer, 2004).

When a decision problem is identified, the relevant aspects of the problem are formulated as a mathematical model. Equations (3.2.1) - (3.2.3) illustrate how a linear programming problem can be expressed in a general form.

$$\min \quad z = \sum_{j=1}^{n} c_j x_j \tag{3.2.1}$$

$$\text{s.t.} \quad \sum_{j=1}^{n} a_{ij} x_j \leq b_i \quad i = 1, ..., m \tag{3.2.2}$$

$$x_j \geq 0 \quad j = 1, ..., n \tag{3.2.3}$$

The objective of a problem is expressed through an objective function that is to be maximized or minimized and depends on decision variables. Equation (3.2.1) represents the objective function of a general minimization problem. The restrictions on the values of the decision variables are expressed through a set of constraints, illustrated in Equations (3.2.2)-(3.2.3) in the example. The constraints define a set of feasible solutions for the problem. This set of feasible solutions creates the solution space for the problem, whose size and structure are affected by the problem's complexity and the number of constraints. Mathematical programming uses techniques to identify the optimal solution to the problem among this set of feasible solutions.

Mathematical models can be formulated with one or more objectives. In single-objective optimization problems, it is always possible to say that one solution is better or worse than another. However, a wide variety of problems involve the simultaneous optimization of several objectives that present some conflict among them. These are referred to as multi-objective optimization problems (Jaimes et al., 2009). These problems usually seek a set of good alternative solutions with different trade-offs between the objectives. As there is no straightforward method to determine if a solution is better than another, multi-objective optimization methods rely heavily on the problem's decision-maker to provide information to find solutions that better fit their preferences.

There are several mathematical programming techniques for solving multi-objective optimization problems. These can be classified into three categories: a priori approaches, interactive approaches, or a posteriori approaches, depending on when preferences from the decision maker are incorporated into the search process (Jaimes et al., 2009). The lexicographic method is an example of an a priori approach. The problem is solved iteratively as a series of single-objective optimization problems, where the most important objective is solved first. The optimal value found for each objective is added as a constraint for subsequent iterations.

Mathematical programming can be classified into two main categories: deterministic and stochastic modeling (Kall et al., 1994). The main difference between them is how one should describe a problem in terms of uncertainty. Deterministic modeling is an approach where all input parameters are assumed to be precisely known. In other words, it is a model with no random component. As a result, a deterministic model has an unambiguous solution, and the result can often be obtained by direct calculation.

In contrast to deterministic modeling, stochastic modeling studies how to incorporate uncertainty in optimization problems into the model through probability distributions (King & Wallace, 2012). Stochastic programming requires assumptions about the probability distributions for uncertain parameters. Making correct assumptions about the distribution can lead to reliable results. However, with multiple possibilities for uncertainty realization, the estimations may be difficult, and finding a solution can be computationally demanding.

Simulation is a method that is used to assist in decision-making under uncertainty. In the context of operations research, simulation refers to the imitation of the behavior of a real-world process or system. Simulation is often used when there is a need to describe and model uncertainty in optimization and decision-making, particularly in dynamic systems that change over time (Figueira & Almada-Lobo, 2014). Simulation can be used in both deterministic and stochastic programming. In the approach of combining simulation and deterministic programming, uncertainty is not explicitly modeled as random variables but rather by running multiple scenarios or replications to capture the range of possible outcomes. If the problem can be accurately represented by a deterministic model but with some uncertainty in the input data, then simulation with deterministic modeling may be a more practical and efficient approach.

3.2.2 Solution Methods

Solution methods for optimization problems can be roughly divided into two categories: exact algorithms and heuristics (Lundgren et al., 2010; Sörensen, 2015). Exact algorithms are guaranteed to find the optimal solution within a finite amount of time, while heuristics aim to find a good solution quickly without such a guarantee. The choice of solution method depends on the type of problem, the available resources, and the desired level of accuracy. This section provides an overview of the solution methods and presents their strengths and shortcomings.

Exact Methods

One solution method that is widely used in mathematical programming is exact methods. They involve the use of algorithms to search the solution space and guarantee to find the optimal solution. However, they can be computationally expensive and may not be practical for large-scale problems.

There is a large number of commercial solvers for solving different optimization problems. The Simplex method is a commonly used method for solving linear programming (LP) problems, implemented in many commercial solvers. An LP problem involves optimizing a linear objective function under a set of linear constraints where all the variables are continuous. In $\mathbb{R}^2$, LP problems can be graphically solved and illustrated. Figure 3.4 shows an example of a graphical illustration of an LP problem, where the feasible region is denoted by X. The Simplex method systematically explores the feasible region of the problem. The algorithm starts at one of the extreme points of the feasible region. At each iteration, the algorithm moves from one extreme point to another in a systematic way, improving the objective function value. The algorithm terminates when no further improvement is possible and the current solution is optimal.

The Simplex method is designed to handle LP problems with continuous variables. When all the variables in an LP problem are required to be integers, the optimal solution may not be found at an extreme point. The branch and bound methods are better suited for

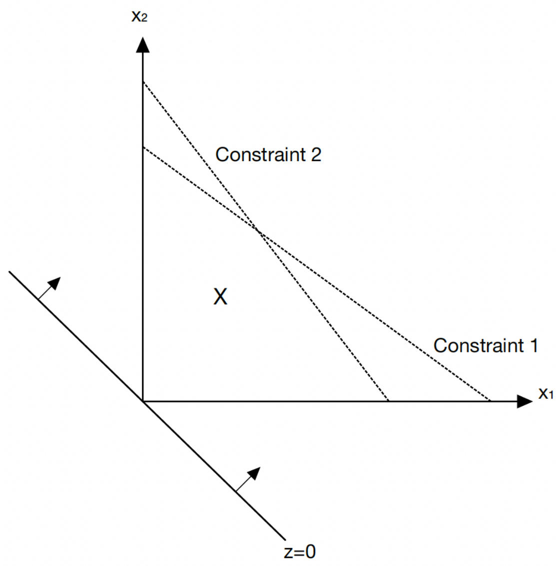


Figure 3.4: Graphical illustration of a LP problem. Figure based on illustration from Lundgren et al. (2010).

handling integer LP problems (Lundgren et al., 2010). In the branch and bound method, the problem is divided into subproblems and solved recursively by exploring a tree-like structure of potential solutions. The subproblems are then evaluated systematically until the best solution is found.

Figure 3.5 illustrates an example of a branch and bound tree for solving a maximization problem. At each node in the tree, a linear relaxation of the problem is solved using the Simplex method, giving a partial solution to the problem. This relaxation involves allowing the variables to take on fractional values, relaxing the integrality constraint. The edges represent the new constraints that define the new subproblems. The algorithms use bounds to help guide the search for the optimal solution and serve as a measure of the quality of the current solution. Bounds provide an estimation of the objective function value for each subproblem. For maximization problems, the lower bound represents the best feasible solution found so far, while the upper bound provides an upper limit on the optimal solution. The result from the subproblem is discarded if it cannot produce a better solution than the lower bound. The optimality gap is the difference between the upper bound and the lower bound, indicating the potential improvement in the solution. It is common to represent the optimality gap as the percentage distance from the current best solution to the current best bound. Optimality is proven when the gap reaches zero.

By iteratively solving subproblems and updating the bounds, the branch and bound method aim to minimize the optimality gap and prune subproblems that cannot yield better solutions. The algorithm terminates when the optimality gap becomes sufficiently small or when all subproblems have been explored. The branch and bound method guarantees to find an optimal solution to an integer LP problem, although it can be slow and require effort that grows exponentially with the problem size.

The Gurobi solver is an example of a commercial solver used to solve complex optimization problems efficiently and effectively. It is a widely used mathematical optimization solver

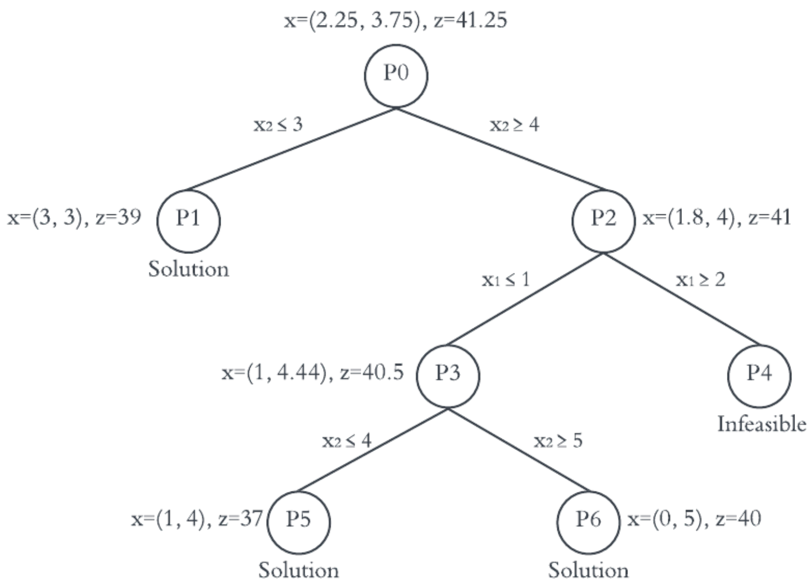


Figure 3.5: Example of a branch and bound tree for a maximization problem. The subproblem is computed at every node. The optimal solution is found in node P6.

developed by Gurobi Optimization, LLC. The Gurobi interface allows users to build an optimization model, pass the model to Gurobi, and obtain the optimization result. The solver utilizes a combination of state-of-the-art algorithms and techniques to find optimal or near-optimal solutions for linear programming (LP), mixed-integer programming (MIP), and other types of mathematical optimization problems. It automatically selects the most appropriate algorithm based on the problem characteristics. The solver's implementation and algorithms are proprietary to Gurobi Optimization. The point of the solver is that the users generally do not need to worry about the details of how the different techniques work. However, the solver is based on the branch and bound method, in addition to strategies like simplex-based methods, primal-dual interior-point methods, heuristics, cutting-plane techniques, and parallelism (Gurobi Optimization, n.d). Gurobi is known for its speed, reliability, and robustness, making it a popular choice among researchers, practitioners, and organizations across various industries for tackling optimization challenges.

Heuristics

Realistic formulations of the natural world are likely to lead to mathematical problems which are very difficult, if not impossible, to solve with exact methods in a reasonable amount of time and using reasonable amounts of computational resources (Juan et al., 2023). This is usually due to the combinatorial nature of practical problems. As a result, the search space of problems of high complexity is often too large or of too high dimensionality to be effectively solved to optimality, making mathematical programming impractical or too time-consuming to use. Using approximate heuristic solution methods is a common approach in complex computational problems instead of exact methods (Juan et al., 2023; Lundgren et al., 2010).

There are several definitions of heuristics. Silver et al. (1980) use the definition "heuristic methods are used for solving problems by an intuitive approach in which the structure of the problem can be interpreted and exploited intelligently to obtain a reasonable solution." They are often denoted as "rules of thumb" (Lundgren et al., 2010). Heuristics use previous experience and intuition to solve a problem (Petropoulos et al., 2023). A heuristic algorithm is designed to solve a problem in a shorter time than exact methods by using different techniques ranging from simple greedy rules to complex structures, which could be dependent on the problem's characteristics. However, their performance can vary depending on the problem being solved. In contrast to exact methods, heuristics do not guarantee to find the optimal solution and, therefore, generally return solutions that are worse than optimal (Sörensen, 2015).

Greedy algorithms are common examples of heuristics (Lundgren et al., 2010). They belong to the group of heuristics called construction heuristics, where one component is added to the solution in each iteration until a feasible solution is reached. Using a greedy heuristic entails that the element which provides the best contribution to the objective function is that which is added to the solution set. Greedy heuristics, like most heuristics, do not guarantee any solution quality (Lundgren et al., 2010).

3.3 Machine Learning and Operations Research

Optimization is a multidisciplinary field (Lundgren et al., 2010). For a successful result in a practical application, one often needs skills and competence in mathematics and computer science combined with technical or economic competence. Burke et al. (2004) believe that interdisciplinary collaborations are essential to make serious scientific advances in nurse scheduling research and to increase the utilization of that research in the real world. This will mainly require expertise from operations research and artificial intelligence.

There has been a significant increase in exploring machine learning in other scientific disciplines in the last decade, mainly using machine learning for optimization (Petropoulos et al., 2023). They are two closely related fields with many overlapping application areas, making machine learning a natural candidate for further research on optimization methods (Bengio et al., 2021). Machine learning approaches may exploit the patterns or specific characteristics that often occur for problems in practical situations of interest. The aim is that one can develop faster algorithms for practical cases by exploiting common patterns in the given instances.

There are several ways for interdisciplinary collaborations between operations research and machine learning. Bengio et al. (2021) have surveyed the existing literature by grouping the main contributions of machine learning for optimization into families of approaches. They classify three main groups: end-to-end learning, learning to configure algorithms, and machine learning alongside optimization algorithms.

End-to-end learning describes approaches where machine learning acts alone to provide a solution to the optimization problem. The machine learning model is trained to output solutions directly from the input instance. One example highlighted by Bengio et al. (2021) trains a neural network to predict the solution of a problem with uncertainty. The problem is formulated as a deterministic mixed-integer linear programming. The application needs to make decisions under incomplete information, so machine learning is used to predict solutions for the uncertainty aspect of the problem based on the available information. Figure 3.6 illustrates the process in end-to-end approaches.

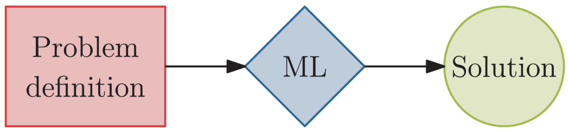


Figure 3.6: End-to-end learning. Figure from Bengio et al. (2021).

In cases where it is not sufficient to only use machine learning to tackle a problem, machine learning can be applied to provide additional information to an optimization algorithm. These approaches fall into the second group for combining machine learning and optimization, learning to configure algorithms, as defined by Bengio et al. (2021). Figure 3.7 illustrates the process for these approaches. An example approach in this group is when machine learning models are used to select the input parameters of an optimization method. Another example, highlighted by Bengio et al. (2021), is using machine learning on mixed-integer linear programming problems to estimate whether applying a Dantzig-Wolfe decomposition will reduce the solving time.

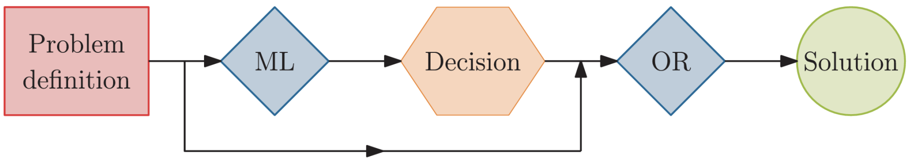


Figure 3.7: Using machine learning to augment an optimization algorithm. Figure from Bengio et al. (2021).

In the approaches where machine learning is used alongside optimization algorithms, the two methods interact during the solving process, as defined by Bengio et al. (2021). This approach differs from the other two groups in that the optimization algorithm repeatedly queries the same machine learning model to make decisions, using the current state of the algorithm as input. By integrating machine learning with optimization algorithms, the machine learning model is leveraged to help solve complex problems more efficiently. Bengio et al. (2021) highlight an example where a branch and bound method is used to solve a mixed-integer linear programming problem. Selecting the branching variable is either too heuristic or too slow, so machine learning is used for the branching decisions made at every node. Figure 3.8 illustrates the process for these approaches.

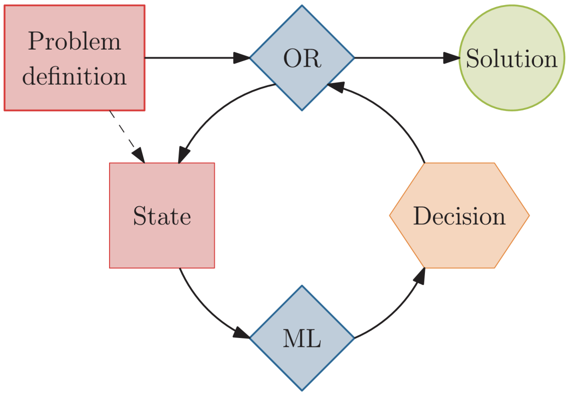


Figure 3.8: Using machine learning alongside optimization algorithms. Figure from Bengio et al. (2021).

Chapter 4

Literature Review

This chapter comprehensively reviews the literature on scheduling and rescheduling problems in hospital management. Section 4.1 outlines our search strategy, explaining how we identified relevant literature for this review. In Section 4.2, we position our research problem within the literature. Section 4.3 describes the nurse scheduling and rescheduling problems and compares different approaches to addressing these problems. Section 4.4 reviews aspects for handling demand uncertainty in nurse scheduling and includes reviewing the use of machine learning in personnel scheduling and rescheduling. Finally, Section 4.5 concludes the literature review and motivates this thesis.

4.1 Search Strategy

The literature reviews on personnel planning by Van den Bergh et al. (2013) and Mutingi and Mbohwa (2017) were important to discover the relevant literature presented in this chapter. Other parts of the literature were found using the search engine Google Scholar. The keywords used include nurse/ personnel scheduling/ rostering, nurse/ personnel rescheduling/ rerostering, shift scheduling, scheduling problem, rescheduling problem, operations research, demand prediction, and machine learning.

To find the most relevant literature for our study, we narrowed our literature review to focus solely on personnel scheduling, which has a wide range of applications in various industries. As reported by Van den Bergh et al. (2013), nurse scheduling is the most commonly studied area within personnel scheduling. Although other industries such as transportation, military, manufacturing, retail, and service personnel have also explored personnel scheduling in operations research, we prioritized literature on nurse scheduling as it is directly relevant to our research problem. However, the literature on nurse rescheduling was primarily retrieved from the literature review in our Specialization Report (Johansen et al., 2022). The machine learning literature was narrowed down to focus on personnel demand predictions within hospital scheduling.

4.2 Positioning in the literature

During recent decades, the interest in studying the healthcare industry in the context of operations research has increased (Van den Bergh et al., 2013). There is a growing demand for healthcare with little increase in resources, leading to a capacity problem and a staff shortage (Helsepersonellkommisjonen, 2023). This ever-increasing pressure leads to the desire for better use of available resources. There is much potential to organize hospital processes more efficiently and effectively (Hans et al., 2012).

Hulshof et al. (2012) provide a comprehensive overview of the typical decisions to be made within the managerial area of resource capacity planning and control in healthcare. To position and structure all healthcare planning and control decisions, they present a taxonomy that provides a method to identify, break down and classify these decisions. The taxonomy aims to support healthcare professionals in improved decision-making, resulting in improved performance in healthcare delivery. The taxonomy is divided into four hierarchical levels of control and six vertical areas of healthcare services. We use this taxonomy to position our work in the relevant literature and to put the scheduling problem at CC into a broader context of healthcare services. The taxonomy is illustrated in Figure 4.1.

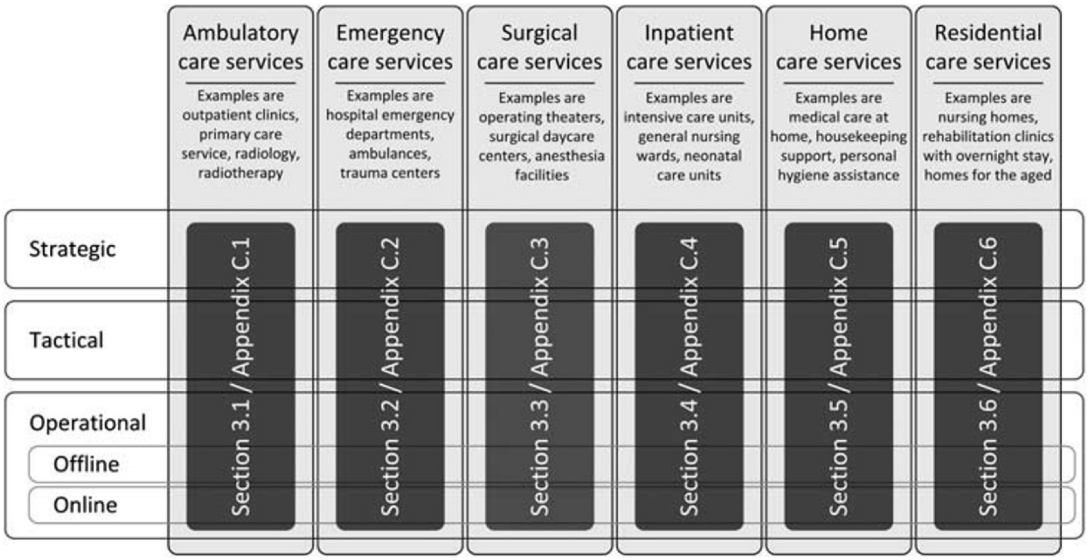


Figure 4.1: The taxonomy by Hulshof et al. (2012) for resource capacity planning and control decisions in healthcare.

The taxonomy's columns represent the different healthcare industry services. This aims to illustrate the context in which resource capacity planning and control decisions are made. We position our work within the inpatient care service. Inpatient care services provide care to hospitalized patients that are admitted overnight. This typically concerns 24-hour-a-day staffing levels, where the personnel works shifts. General nursing wards are examples of inpatient care services relevant to our nurse scheduling focus.

The hierarchical levels represent the hierarchical nature of decision-making in healthcare organizations and are divided into strategic, tactical, and operational. The operational level is then subdivided into offline and online operational planning. Problems related to the lower hierarchical levels can have less uncertainty as more information is known and the planning horizon is shorter.

The first hierarchical level is known as the strategic planning level. This level addresses the structural decision-making processes within hospital resource planning. Within the inpatient care service, the strategic level includes capacity dimensioning. This encompasses deciding on an appropriate number of employed nurses to meet demand variability.

Tactical planning is the second level within the hierarchy and addresses the organization and execution of the healthcare delivery process operations. Within the inpatient care service, tactical planning involves staff shift scheduling. This concerns deciding the exact staffing level to meet demand, where demand is based on forecasted assumptions.

The taxonomy's lowest level of the vertical axis represents the operational planning level. This involves the short-term decisions related to the healthcare delivery process execution. The operational planning level is separated into online and offline operational planning. Offline operational decisions concern capacity resource planning, where the decisions are made before executing the schedule. Within the inpatient care service, offline operational planning involves the nurse scheduling problem. It includes deciding what staff should be assigned to which shifts. The objective is to satisfy a complex set of restrictions involving work regulations and employee preferences while creating a schedule that can cope with future supply and demand uncertainties.

Online operational decisions regard time-critical decisions such as emergency coordination, rush orders, and decisions involving patient complications. Within the inpatient care service of Hulshof et al. (2012) 's taxonomy, online operational planning involves staff rescheduling. At this level, uncertainties in demand and supply are realized with the current demand fluctuations and nurse absences. Decisions on this level can therefore involve the reassignment of dedicated nurses. In the related literature, this problem of reestablishing a feasible schedule as a reaction to uncertainty is known as the nurse rescheduling problem.

This thesis focuses on personnel planning at the operational level. Decisions made at the strategic and tactical levels are considered out of scope. However, they affect the decisions made at the lower levels. The workforce size determined at the strategic level regulates the supply level for the scheduling and rescheduling problem. Decisions at the tactical level form a basis for the long-term forecasted demand.

At the operational level, both online and offline operational planning are studied. The problem at the offline level is creating a schedule for a fixed workforce. The problem at the online operational level is solved by a realization of the uncertainties in evaluating the performance of different schedules.

4.3 The Nurse Scheduling and Rescheduling Problem

The nurse scheduling problem has been widely studied in recent decades (Abdalkareem et al., 2021). The aim is to create an optimal schedule for nurses that meets patient demand while considering factors such as nurse preferences and labor regulations (Van den Bergh et al., 2013). The nurse scheduling problem is particularly challenging due to the complexity of nurse shift patterns and the need to balance the workload and preferences of nurses with varying patient loads.

The nurse rescheduling problem has attracted less attention in the literature than the nurse scheduling problem but is becoming increasingly popular. Moz and Pato (2003, 2007) were the first to formally define the nurse rescheduling problem using optimization methods (Mutingi & Mbohwa, 2017). As described in their study, nurse rescheduling problems occur when there is a disruption to the schedule. This happens due to unforeseen events such as nurse absences or increased demand. These disruptions lead to an imbalance between the supply and demand for nurses, which means the schedule for the given shift becomes infeasible. An efficient method to rebuild the schedule is required to meet the demand, which may result in multiple alterations.

The nurse scheduling problem and the nurse rescheduling problem are closely related problems that arise in healthcare operations management. Although the nurse scheduling and rescheduling problems are related, they are often studied separately in the literature. The strategies used in nurse scheduling will significantly impact the performance of the rescheduling strategies. Studying combinations of strategies in the two problems together is important to develop a more comprehensive scheduling model that can handle the complex and dynamic nature of nurse scheduling.

This section provides a comprehensive overview of the nurse scheduling and rescheduling problems and the associated uncertainty aspects. Section 4.3.1 first explores the key aspects of the nurse scheduling problem. Subsequently, Section 4.3.2 delves into the uncertainty aspects that affect nurse scheduling. Section 4.3.3 looks at how these uncertainty aspects are handled in the related literature. Building on this, Section 4.3.4 presents the key aspects of the nurse rescheduling problem.

4.3.1 Key Aspects in The Nurse Scheduling Problem

This section aims to provide an overview of the key aspects of the nurse scheduling problem to provide a better understanding of the problem and the various approaches that have been proposed to tackle it.

Demand

Hospitals must ensure that they have an adequate number of nurses with the right skill sets available to meet patient needs. Demand constraints may involve specifying a minimum number of nurses needed for each shift and the skills required to provide adequate care. Hard constraints are commonly used to guarantee that the minimum demand for each shift is fulfilled (Van den Bergh et al., 2013). Alternatively, understaffing and overstaffing are penalized in the objective (Ingels & Maenhout, 2015). Recently, it has become common to model demand as a soft constraint by including desired demand, where deviations from the desired demand penalize the objective (Bard & Purnomo, 2005b; Beckmann & Klyve, 2016; Fügener et al., 2018; Turhan & Bilgen, 2020).

With the demand for specific skills, it is common to include competence requirements within the demand requirements. Van den Bergh et al. (2013) present user-definable and hierarchical skills. In the case of user-definable skills, the scheduler has the freedom to define skills for every personnel member. Hassani and Behnamian (2021) and Schoenfelder et al. (2020) use definable skills. For hierarchical skills, the personnel with a higher skill classification can carry out the tasks of a lower-ranked employee but not vice versa (Beckmann & Klyve, 2016; Cowling et al., 2002; Lim & Mobasher, 2011).

Hours and shifts

The decision process in nurse scheduling is impacted by governing laws and regulations. Many studies limit schedules to assign a maximum of one shift per day per nurse (Bard & Purnomo, 2005b; Fügener et al., 2018; Hassani & Behnamian, 2021; Ingels & Maenhout, 2015; Schoenfelder et al., 2020; Turhan & Bilgen, 2020). Lim and Mobasher (2011) use an upper limit for the number of shifts assigned to a full-time nurse within the scheduling period. Some studies include an upper bound for weekly working hours (Beckmann & Klyve, 2016; Løyning & Melby, 2018).

Consecutiveness

A central aspect of scheduling is the distribution of work. It is common to include limits on the number of consecutive work shifts. Beckmann and Klyve (2016), Fügener et al. (2018), Ingels and Maenhout (2015) and Turhan and Bilgen (2020) include a maximum limit for consecutive working days. Turhan and Bilgen (2020) also include a minimum limit for consecutive working days in their model. Some studies penalize undesirable distribution of off-days in the objective, and some extend to include isolated off-days. Turhan and Bilgen (2020) do not allow isolated work days, as they are considered undesirable.

Illegal combinations

Due to laws and regulations, some shift combinations are illegal to schedule. There are various approaches to handling illegal shift assignments. One approach is to use minimum rest regulations as hard constraints (Bäumelt et al., 2016; Clark & Walker, 2011; Ingels & Maenhout, 2015, 2017; Kitada & Morizawa, 2013; Maenhout & Vanhoucke, 2011; Moz & Pato, 2003, 2004, 2007; Pato & Moz, 2008). Daily rest is the most common to include. Another approach is to prohibit certain shift combinations. Lim and Mobasher (2011) prohibit scheduling an early shift to follow a night shift. Cowling et al. (2002) assign shift patterns instead of individual shifts and connect a penalty value for each pattern assigned. Similarly, Beckmann and Klyve (2016) and Løyning and Melby (2018) combine hard constraints for daily rest with a reward for assigning desirable shift patterns.

Contracted Work

In most cases, the employees have a defined number of contracted hours in the scheduling period. Hassani and Behnamian (2021), Ingels and Maenhout (2015) and Schoenfelder et al. (2020) solve this by defining a minimum and a maximum number of assigned shifts per nurse during the scheduling period. Fügener et al. (2018) set the contracted hours in the scheduling period as an upper bound for the number of assigned hours per nurse. Maenhout and Vanhoucke (2013) use soft constraints for the minimum and maximum working hours during a week and penalize any violations of these limits in the objective. Bard and Purnomo (2005c) limit the number of overtime hours that a single nurse can be scheduled for. The scheduling rules and requirements regarding skill mix can make it challenging to create schedules with the same working hours every week. To facilitate more flexibility in the scheduling, some utilize the possibility of scheduling fewer hours during some weeks for some nurses and more hours in others where the average number of hours corresponds to the nurses' contracted hours (Beckmann & Klyve, 2016; Løyning & Melby, 2018).

Weekends

Weekend shifts are generally considered undesirable among nurses. Lim and Mobasher (2011) and Turhan and Bilgen (2020) set an upper limit for the number of weekend shifts assigned to each nurse. Maenhout and Vanhoucke (2013) handle varying policies across different departments, including partial and complete weekends, and penalize any policy violations or limits regarding weekend staffing. Beckmann and Klyve (2016) and Løyning and Melby (2018) state that nurses should work an entire weekend or no weekend shifts in a given week. Løyning and Melby (2018) use predefined weekend groups for the nurses to simplify the solution process.

Preferences

Nurses have different needs and obligations outside of work that affect their preferences to work certain shifts. Several studies in the literature have proposed different methods to handle nurse preferences in the scheduling models. Bard and Purnomo (2005b), Hassani and Behnamian (2021), Ingels and Maenhout (2015), Løyning and Melby (2018), Maenhout and Vanhoucke (2013) and Turhan and Bilgen (2020) incorporate preferences in the objective function to reward preferred shift assignments and penalize unpreferred shift assignments. Others, such as the model developed by Lim and Mobasher (2011), have assigned grades to employee preferences. Beckmann and Klyve (2016) maximize the number of respected requests in combination with preferable patterns, while the model by Schoenfelder et al. (2020) incorporates preferences through constraints by limiting the total number of unpreferred shift assignments for a given nurse.

Hassani and Behnamian (2021) and Turhan and Bilgen (2020) have extended their models to include planned vacations and restrict scheduling shifts within preplanned vacation periods for specific nurses. However, a focus on maximizing the number of fulfilled requests can sometimes lead to an imbalance in the perceived schedule for employees. In the worst case, one employee will be the victim of all the violations. For instance, the study by Wolbeck (2019) proposes a measure of fairness to maximize the quality of the worst individual schedule, while others have focused on minimizing the perceived difference between the best and worst schedules.

Cross-Section Scheduling

It is common to use a decentralized scheduling practice where each section within a hospital schedules its assigned nurses independently. However, due to the rise in personnel costs and shortage of full-time nurses, the cross-utilization of nurses is an approach that has gained attention (Brusco, 2008).

Cross-section scheduling entails utilizing resources across hospital sections to improve the efficiency of nurse scheduling. It refers to the process of assigning nurses to work in different sections within a hospital. Several articles have studied the effect of the cross-utilization of resources within healthcare and other industries. These studies found that coordination across sections in scheduling could be an effective strategy for hospitals. It improves the flexibility in responding to changes in demand and reduces staffing costs and undesirable staff schedules (Campbell, 1999; Fügener et al., 2018; Wright & Mahar, 2013).

Cross-section scheduling can be utilized to resolve structural personnel shortages (Maenhout & Vanhoucke, 2013). Studies that consider cross-section scheduling at the offline operational level often look at pools of cross-trained nurses who can work in different sections (Gnanlet & Gilland, 2014; Wright & Mahar, 2013). Both studies address the decision of which budget of float nurse hours should be available during a given period.

Kortbeek et al. (2015) highlight the importance of cross-utilization when studying the impact of demand variability and supply buffers in nurse scheduling. With cross-utilization, the variability in demand balances out due to economies of scale, so less buffer capacity is required. Without cross-utilization, the buffer capacity required to protect against uncertainty in demand can lead to regular overstaffing. Similarly, Maass et al. (2017) aims to determine the optimal number of cross-trained nurses to account for supply variability and how these should be scheduled.

4.3.2 Uncertainty in Nurse Scheduling

Most completed research on the nurse scheduling problem focus on the problem in a deterministic setting, while fewer studies include the variability and uncertainty aspects that occur (Burke et al., 2004; Hassani & Behnamian, 2021; Lim & Mobasher, 2011; Van den Bergh et al., 2013). In reality, the nurse scheduling problem faces several uncertain factors that impact the problem. Van den Bergh et al. (2013) define uncertainty of demand and uncertainty of capacity as two categories of uncertainty in personnel scheduling. The demand uncertainty for the nurse scheduling problem includes variability related to the number of patients and the patient load (Bard & Purnomo, 2005a; Lim & Mobasher, 2011). Uncertainty of capacity results from nurse absenteeism, sick leave, and other factors that affect nurse availability.

Uncertainty of demand may cause the expected demand determined in the nurse scheduling problem to differ from the actual demand for nurses realized in the nurse rescheduling problem. This deviation from expected demand may result from fluctuations in patient arrivals and departures, resulting in unstable numbers of patients. In addition, the workload related to each patient is subject to uncertainty (Bard & Purnomo, 2005a). This unpredictable fluctuation in patient workload results in nurse demand variability.

As for the uncertainty of demand, most literature on the nurse scheduling problem ignores uncertainty of supply (Van den Bergh et al., 2013). The supply of nurses is assumed to be deterministic and is only treated as an input parameter for creating the schedules. This approach overlooks the real-world uncertainties that affect the supply of nurses and may result in suboptimal schedules that require frequent modifications in the online operational phase.

4.3.3 Handling Uncertainty

Studies that focus on the nurse scheduling problem in a deterministic setting make assumptions concerning the demand and availability of nurses. During the execution of the schedule, these assumptions may prove to be insufficient representations of reality. To handle the uncertainty, it is crucial to generate schedules better equipped to handle disruptions caused by unexpected events (Hazır et al., 2010). A schedule should be able to deal with or absorb uncertainties by having predefined solutions for addressing those uncertainties (Ingels & Maenhout, 2015; Lim & Mobasher, 2011).

Mismatches between supply and demand can prove to be very costly. Therefore, it has become increasingly popular to identify ways to react to short-term supply and demand variations (Bard & Purnomo, 2005a; Schoenfelder et al., 2020). Labor costs represent a large portion of hospitals operating costs and can significantly increase with reactive schedule changes due to deviations between supply and demand (Ingels & Maenhout, 2015). In addition to increased costs, schedules with low robustness may lead to lower satisfaction among the nurses as it results in less predictability concerning their schedule (Ingels & Maenhout, 2015).

To deal with uncertainty, most studies propose various reactive measures (Ingels & Maenhout, 2015). Reactive scheduling means that the schedule is adjusted in response to changes in patient demand or nurse availability (Bard & Purnomo, 2005a; Ingels & Maenhout, 2015). This includes using overtime work, reallocating resources, making schedule changes, or accepting that demand cannot be fully met.

Ingels and Maenhout (2015) introduce the concept of proactive strategies. Proactive strategies are used at the tactical and offline operational levels to build robustness and flexibility into the schedules so that unexpected events will have less impact during schedule execution. Reserve duties, or buffers, are a common proactive strategy for incorporating robustness into nurse schedules. Ingels and Maenhout (2015) introduce several variations for utilizing reserve duties to construct stable shift schedules and evaluate the performance of each strategy. Hazır et al. (2010) evaluate if buffers are able to handle the uncertainty and conclude that schedules with larger buffers are preferred regarding robustness but may also deteriorate the cost. Their study addresses the important issue of determining the best trade-off between cost and robustness.

Several studies account for uncertainty using robust optimization defined by Soyster (1973). Even though Hassani and Behnamian (2021) argues that robust optimization in scheduling leads to a realistic presentation due to the importance of the problem and the need for sustainable schedules against uncertainty, this form of robust optimization should not be confused with robust strategies in nurse scheduling. Robust strategies are looser in definition than robust optimization.

4.3.4 Key Aspects in The Nurse Rescheduling Problem

The nurse rescheduling problem handles realized uncertainty by using reactive strategies to address the schedule disruptions. As with the scheduling problem, there are many important aspects to consider in the rescheduling problem. This section discusses the key aspects of rescheduling, including the similarities and differences from the scheduling problem.

Demand

The rescheduling problem occurs as a result of unmet demand. Once the original schedule is modified, the new schedule should guarantee that the minimum demand threshold is met (Bäumelt et al., 2016). Minimum demand is often classified in terms of the number of nurses needed. Some studies also include minimum demand for specific skills (Beckmann & Klyve, 2016; Lim & Mobasher, 2011). Similar to the nurse scheduling problem, demand constraints are often modeled as hard constraints to ensure no understaffed shifts (Bäumelt et al., 2016; Clark & Walker, 2011; Kitada & Morizawa, 2013; Lilleby et al., 2012; Moz & Pato, 2003, 2004, 2007; Pato & Moz, 2008). However, some studies use soft constraints for demand with penalization for understaffing in the objective (Bard & Purnomo, 2006; Ingels & Maenhout, 2017, 2018; Maenhout & Vanhoucke, 2011). Other approaches include minimizing the deviations from desired demand in the rescheduling, thereby minimizing both under- and overstaffing (Maenhout & Vanhoucke, 2011).

There are several variations in the relevant literature regarding how uncertainty in demand is treated. Ingels and Maenhout (2018) run a discrete-event simulation of the demand for each period during the day, meaning demand between periods is independent. Still, upper and lower bounds for staffing requirements are imposed. Long et al. (2022) handle uncertain demand with a distributionally robust model by minimizing the expected costs for the scheduling period using distributions to model the uncertainty.

Supply

In contrast to the nurse scheduling problem, most relevant literature regarding the rescheduling problem assumes the nurse supply is uncertain. Uncertainty in supply is treated in several ways within nurse rescheduling. Bard and Purnomo (2005c, 2006), Lilleby et al.

(2012), Long et al. (2022) and Maenhout and Vanhoucke (2011) allow the pool of nurses to include external nurses, which consists of nurses employed externally to the section studied. On the other hand, Bäumelt et al. (2016), Clark and Walker (2011), Ingels and Maenhout (2017, 2018), Kitada and Morizawa (2013), Maenhout and Vanhoucke (2011), Moz and Pato (2003, 2004, 2007), Pato and Moz (2008) and Wolbeck et al. (2020) keep the pool of nurses fixed with only internal nurses.

Workload

There are rules and regulations for how the schedule should be set up. Most of the literature on the rescheduling problem applies similar rules and regulations in the original and updated schedules. However, in contrast to the scheduling problem, the constraints for the rescheduling problem can be modeled less strict. Bäumelt et al. (2016) and Wolbeck et al. (2020) argue that many of the important aspects of the workload restrictions in scheduling are less critical in rescheduling and that these violations are less severe than inadequate care. How these aspects are modeled depends on the problem focus, the labor rules, and the institutional constraints for the given problem.

Ingels and Maenhout (2015) list extra shifts and overtime work as examples of reactive strategies. Assignment of overtime is less used in the scheduling but serves as a strategy to avoid understaffing in the rescheduling problem. Ingels and Maenhout (2018) investigate reactively assigning extra shifts leading to overtime to improve flexibility.

An extension of regular extra shifts is to assign double shifts. Daily and weekly minimum rest requirements are often still modeled as hard constraints (Bäumelt et al., 2016; Clark & Walker, 2011; Ingels & Maenhout, 2017; Kitada & Morizawa, 2013; Maenhout & Vanhoucke, 2011; Moz & Pato, 2003, 2004, 2007; Pato & Moz, 2008; Wolbeck et al., 2020). This is, in part, to ensure that the available staff can provide high-quality care. If these constraints are not explicitly included in the model, the use of prohibited patterns ensures that employees get sufficient rest.

Planning Period

The planning period for the rescheduling problem refers to the time interval during which the nurses must be rescheduled to address schedule disruptions. The related literature has many variations for approaching the planning period. As described by Kitada and Morizawa (2013), many studies focus on the nurse rescheduling problem with a one-day absence of one nurse, even though absences sometimes continue for several consecutive days and can occur for several nurses simultaneously.

The nurse rescheduling problem can be explored by rescheduling the entire scheduling period or only a limited period (Wickert et al., 2019). Maenhout and Vanhoucke (2011) find it unnecessary to consider the entire scheduling period to obtain reasonable solutions. They determine that it is sufficient to consider a period before and after an absence, including the absence itself. Wolbeck et al. (2020) consider previous periods to get a feasible schedule and distribute the shift changes fairly among nurses over time.

Fairness and Preferences

As in the scheduling problem, fairness and preferences are often included as important aspects in the nurse rescheduling problem. Including fairness measures in the rescheduling problem has become increasingly popular and is often considered an important criterion for ensuring acceptance of the new schedule (Wolbeck et al., 2020). However, there are many variations on how to approach the aspect of fairness. Within rescheduling, it is common to consider fairness as an even distribution of shifts among nurses (Clark & Walker, 2011; Maenhout & Vanhoucke, 2011; Wolbeck et al., 2020). Bard and Purnomo (2005c) describe fairness as ensuring that each nurse has a reasonable number of satisfied requests.

In rescheduling, ensuring nurse preferences and infeasibilities due to schedule disruptions consist primarily of retaining the individual nurse's current shift assignments as much as possible (Maenhout & Vanhoucke, 2011). Thus, minimizing schedule changes is part of the objective in nearly all the related literature on personnel and nurse rescheduling to ensure that the preferences are maintained after rescheduling. This entails keeping the new schedule similar to the original one for all the nurses to avoid unnecessarily disturbing the nurses' private life. Many works focusing on minimizing changes also include nurse preferences in the objective function (Bäumelt et al., 2016; Ingels & Maenhout, 2017, 2018; Kitada & Morizawa, 2013; Maenhout & Vanhoucke, 2011; Moz & Pato, 2003, 2004, 2007; Pato & Moz, 2008; Wolbeck et al., 2020). These works try to ensure the quality of the schedules as perceived by the nurses themselves. Some also use this to ensure a fair or even workload among the nurses.

Costs

Few studies in the related literature consider monetary costs related to rescheduling. However, personnel costs significantly contribute to organizations' operating costs (Ernst et al., 2004; Van den Bergh et al., 2013). In addition, reactive changes in response to schedule disruptions may be costly and at the expense of the personnel (Ingels & Maenhout, 2017). Ingels and Maenhout (2018) emphasize how an appropriate personnel planning process is indispensable to managing these costs related to the personnel.

Ingels and Maenhout (2017, 2018) are some of the few works on rescheduling where the general objective is to minimize costs. This includes the personnel assignment costs and the costs related to understaffing. Lilleby et al. (2012) focus on balancing the extra long-term costs corresponding to a higher competence profile with reduced operational costs. The objective is to minimize the total costs related to different competence requirements. Long et al. (2022) aim to minimize the total operating cost over several periods under uncertain demand.

Cross-Section Rescheduling

While cross-section utilization can be viewed as a proactive strategy, as explained in Section 4.3.1, several studies also utilize it as a reactive strategy. Schoenfelder et al. (2020) introduce several quick response decisions that allow for day-to-day adjustments to supply and demand, where one of these includes cross-section rescheduling. Similarly, Bard and Purnomo (2005a) demonstrate that cross-section rescheduling could be an effective reactive strategy to limit over- and understaffing, with nurses being assigned to other hospital units than their home unit as a corrective measure in the online phase.

In addition to these studies, other researchers have explored the benefits of cross-section rescheduling in the online phase. Studies by Brusco (2008), Campbell (1999) and Inman et al. (2005) found that even a small amount of cross-section utilization can substantially aid in real-time staff adjustments, with under-utilized staff being rescheduled to areas with shortages. As a result, Inman et al. (2005) and Easton (2011) suggest that increased cross-section utilization may alleviate the need for absence anticipation strategies. Overall, these studies highlight the potential benefits of cross-section utilization as a reactive strategy in nurse scheduling to improve staff utilization and respond to daily fluctuations in supply and demand.

4.4 Estimation of Demand

Uncertainty in demand is a significant challenge in providing efficient schedules in nurse scheduling and rescheduling problems. Some studies in the operations research literature have tried to tackle this challenge through statistical methods. One approach that has emerged in the literature to address this challenge focuses on estimating demand based on historical data. For instance, Schoenfelder et al. (2020) model patient demand parameters after historically observed demand. The study illustrates that using a limited number of well-chosen demand scenarios as the basis of the nurse scheduling and rescheduling model results in high-performing schedules.

Similarly, Long et al. (2022) use demand estimations based on historical data. However, the study finds that the corresponding ambiguity set may be inaccurate when the data variance is significant. Therefore, they use a scenario-based approach, where uncertainty can be represented as part of a scenario, and each scenario represents a possible realization of the demand. Even though Fügener et al. (2018) assume deterministic demand, they use a discrete uniform distribution for each unit and day to adjust the deterministic expected demand to simulate the effect of stochastic demand per period.

Other studies utilize forecasting methods to obtain more accurate values for demand. For instance, Ordu et al. (2021) develop a decision support system to identify better forecasting methods. These forecasting methods include an ARIMA model, linear regression, and exponential smoothing.

It is not always accurate to only predict the number of patients for whom care must be provided but also the level of care that each will require. The nurse-to-patient ratio is a common metric for determining the number of required nurses to cover demand and is used to improve the determination of nurse demand. To address the issue of demand forecasting, Kortbeek et al. (2015) uses a model to predict the hourly workload at a care facility that consists of several care units. The predicted workload is then used to ensure the nurse-to-patient ratio is satisfied. They conclude that their model predictions closely follow historical data, resulting in an improved consistency in the delivered quality of nursing care.

4.4.1 Machine Learning for Improving Demand Prediction

An alternative method to address uncertain demand in scheduling and rescheduling is to utilize machine learning. As mentioned in Section 3.3, Bengio et al. (2021) propose combining machine learning as a tool to affect the uncertain input parameters of an optimization method. For the nurse scheduling problem, bed demand is uncertain because the number of beds occupied differs over time, and the severity of patients, and thus the demand, varies greatly. Tello et al. (2022) state that there is a relatively small number of previous research developed in the context of demand forecasting, particularly using machine learning models. In their work, they develop a prediction model using K-means clustering and Support Vector Regression (SVR) to predict weekly forecasts of the inpatient demand to assist in nurse scheduling.

One of the most common methodologies for estimating hospital demand is based on the valuation of the patient's length of stay (LOS) at the hospital (Tello et al., 2022). The patient's LOS often says something about the severity or complexity of the patient and is, therefore, a number worth valuing when predicting bed demand. Gül and Güneri (2015) train an artificial neural network to predict the patients' LOS at an emergency department.

4.5 Our Contribution

This thesis introduces a problem that builds upon existing literature while presenting new ideas to solve our particular problem. A significant difference from the existing literature is that the problem addressed in this report is based on Norwegian laws and regulations about nurse scheduling and rescheduling. As a result, the problem is tailored to the context of a Norwegian hospital, which may have distinct requirements, organizational structures, and approaches to scheduling and rescheduling.

In this thesis, the scheduling problem is modeled to reflect the scheduling process at CC realistically. As a result, our problem is formulated as a multi-objective model. Although many studies formulate multi-objective scheduling models, our specific objectives deviate from the literature. Our rescheduling model is formulated with a single objective focused on minimizing costs. As highlighted in the literature review, minimizing rescheduling costs has been studied in the rescheduling literature. However, to our knowledge, the objective in our work is significantly more detailed and captures a broader range of rescheduling costs than similar works in the literature.

While there is a significant body of relevant literature on nurse scheduling and rescheduling problems, few have examined the relationship between the two, despite their tight connection. Additionally, while some studies have addressed the uncertainty aspects of these problems, they have typically only used reactive or proactive strategies. However, given the impact of uncertainty handling on both problems, exploring the relationship between nurse scheduling and rescheduling can provide valuable insights for developing effective solutions for handling uncertainty. To our knowledge, our work is among the first to study the two problems together to evaluate uncertainty handling. This approach enables proactive scheduling to facilitate better utilization of reactive strategies, which can result in more efficient rescheduling.

We utilize a buffer strategy for handling uncertainty. Buffer strategies have been investigated in the literature. However, we extend this strategy by applying it to both the scheduling and rescheduling problems in combination with cross-section utilization. Cross-section buffers make it possible to schedule buffer shifts without utilizing reserve duties in the form of overstaffing. In contrast to similar studies in the literature, we are able to exploit the robustness provided by buffers without deteriorating the costs. Additionally, we contribute a novel flexible strategy that has, to our knowledge, not been explored in previous studies. Our use of these strategies, in combination with scheduling and rescheduling, is a unique aspect of our work.

The uncertainty of demand is an important aspect of rescheduling. Several studies have used methods for estimating demand based on real data. However, to our knowledge, no studies have utilized machine learning to gain better demand predictions. Our work contributes to the literature by combining the optimization approaches to solve the nurse scheduling and rescheduling problem with machine learning predictions based on historical data from the related clinic. This novel approach can potentially reduce the rescheduling frequency and enhance the overall cost results.

Some related works are selected to compare the main aspects of the scheduling and rescheduling problems with our approach. Table 4.1 displays the chosen studies, while Table 4.3 compares critical aspects of these articles with this thesis. The works have been selected to illustrate a broad specter of the presented aspects as best as possible. These have been selected based on their focus, model, and solutions.

Table 4.1: Relevant literature

Article number	**Article**
(1)	Ingels and Maenhout (2015)
(2)	Ingels and Maenhout (2018)
(3)	Løyning and Melby (2018)
(4)	Bard and Purnomo (2005a)
(5)	Kitada and Morizawa (2013)
(6)	Long et al. (2022)
(7)	Lim and Mobasher (2011)
(8)	Fügener et al. (2018)
(9)	Schoenfelder et al. (2020)
(10)	Kortbeek et al. (2015)
X	This thesis

The first column in Table 4.3 lists the most important aspects mentioned in this chapter. A cell is marked green when there is a similarity between an article and this report. The reader should be aware that this table only represents how we best interpreted the contents of each article. Table 4.2 displays the abbreviations used to describe the decisions for the articles in Table 4.3.

Table 4.2: Abbriviations

P	Personnel
N	Nurse
HC	Hard constraint
SC	Soft constraint
R	Rescheduling article
S	Scheduling article
B	Both scheduling and rescheduling in article

Table 4.3: Comparison of relevant literature.

	Article number										
Decision	**(1)**	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(8)**	**(9)**	**(10)**	**X**
Focus area	B	B	B	R	R	R	S	S	S	S	B
Employee group	P	P	N	N	N	N	N	N	N	N	N
Number of employees	20	10	117	20-200	24	20-44	16	85	-	-	104
Cross-section	No	No	No	No	No	Yes	No	Yes	No	Yes	Yes
Buffer	Yes	Yes	Yes	No	No	No	No	Yes	No	Yes	Yes
Flexible nurses	No	No	No	No	No	No	No	No	No	No	Yes
Uncertain demand	Yes	Yes	Yes	Yes	No	Yes	Yes	Yes	Yes	Yes	Yes
Uncertain supply	Yes	Yes	Yes	Yes	Yes	No	No	No	No	-	Yes
Minimum demand constraint	HC	SC	HC	SC	HC	HC	SC	HC	SC	HC	HC
Uses desired demand	No	No	No	No	No	No	No	Yes	No	Yes	Yes
ML for demand predictions	No	No	No	No	Yes	No	No	No	No	Yes	Yes
Skills	No	No	Yes	Yes	Yes	Yes	Yes	No	No	No	Yes
Multi-objective scheduling	Yes	No	Yes	-	-	No	Yes	Yes	No	No	Yes
Minimize rescheduling costs	Yes	Yes	No	Yes	No	Yes	-	-	-	-	Yes
Rolling horizon rescheduling	Yes	Yes	Yes	Yes	No	No	-	-	-	-	Yes

Chapter 5

Problem description

This thesis focuses on handling uncertainty in nurse schedules across three sections in the Clinic of Cardiology at St. Olavs Hospital. The problem is considered twofold, consisting of both the nurse scheduling problem and the nurse rescheduling problem. The nurse scheduling problem is the task of creating nurse schedules, where all nurses are assigned to working shifts. This complex optimization problem requires nurses to work in shifts to ensure around-the-clock patient care. The nurse rescheduling problem handles the consequences of the uncertainty in the nurse scheduling problem by modifying the schedule to ensure that demand is met. This thesis aims to produce schedules that can easily adapt to several uncertain factors, thus minimizing rescheduling costs.

Section 5.1 elaborates on the nurse scheduling problem. Section 5.2 describes the uncertainty aspects related to the problem. The uncertainty factors are tightly connected with the nurse rescheduling problem. Section 5.3 elaborates on the nurse rescheduling problem. The content of Section 5.3 is based on the related specialization report's content.

5.1 The Nurse Scheduling Problem

The nurse scheduling problem consists of creating feasible nurse schedules, which are created prior to the schedules' execution. A nurse schedule is a plan that assigns nurses to work specific shifts over a given scheduling period.

Shifts are categorized into work shifts and off shifts. Every section has three types of work shifts; day, evening, and night, each with a set duration. A nurse can only work full shifts. The night shift relates to the day when the shift starts; e.g., a night shift that begins at 23:00 on a Monday is considered a Monday shift. All shifts have a minimum staffing level that specifies the number of nurses that must be scheduled to ensure enough nurses are available to meet patient needs. However, the schedule should aim to fulfill the historical average demand for all shifts.

Each section has a set of employed nurses. The nurses are only scheduled to the section they belong to. The employees in each section consist of assistant nurses, nurses, and specialized nurses. These competence levels are hierarchical, meaning that a nurse can do the tasks of an assistant nurse, and a specialized nurse can do the tasks performed by a nurse. However, a nurse cannot do all tasks a specialized nurse is qualified for. The nurses are also categorized by their experience level, meaning the number of years of employment. All shifts have a desired demand related to experience. In addition, there is a desired demand for specialized nurses for each shift, while for nurse assistants, a maximum demand threshold is defined.

Nurse preferences are taken into account when generating the schedule. The preferences specify which shifts a nurse wishes to avoid during the week. Preferences for specific shifts during an arbitrary week are collected prior to the schedule generation. Each nurse has a set of contracted hours over the scheduling period corresponding to the nurse's employment percentage. A deviation from the total contracted hours over the scheduling period is allowed to ensure a feasible schedule generation.

Laws and regulations play a critical role in the nurse scheduling problem. Each nurse can be scheduled to work a maximum of one shift per day and cannot be scheduled to work more than a maximum number of hours per week. Nurses can be scheduled to work more or less than the exact number of contracted hours one week if the total number of hours over the scheduling period does not exceed the acceptable deviation from the total number of contracted hours. Nurses should be scheduled to work at most a specified maximum number of consecutive days. Due to the inconvenience of night shifts, a maximum number of consecutive night shifts is also included.

Regulations specify weekly and daily rest requirements. The rest requirements are that each nurse should have a minimum number of hours of consecutive rest within each 24-hour period and within a week. The daily and weekly requirements entail that some shift combinations are deemed illegal.

Weekend assignments are structured to ensure that a nurse scheduled for a weekend will work both a Saturday and Sunday shift. A nurse can only be scheduled for a weekend assignment with a specified interval. Moreover, predefined patterns must be followed for a nurse's weekend assignment. These patterns involve working a day shift and an evening shift or working two consecutive night shifts.

The nurse scheduling problem aims to minimize the deficit from the historical average demand for nurses while also achieving an even distribution of weekly workload. This objective should also ensure that the number of overstaffed shifts is evenly distributed. Finally, the objective aims to minimize the number of preference violations while achieving a balanced distribution of competence and experience for each shift.

5.2 Proactive Strategies for Handling Uncertainty

The nurse schedule is created before its execution and is based on expectations about demand and supply. Assumptions regarding patient load and nurse absences made during the creation of the schedule may be inaccurate at the time of the execution of the schedule. This results in an imbalance between supply and demand, causing the need for schedule adjustments. These reactive adjustments are known as the nurse rescheduling problem.

Changes to the schedule often lead to higher costs and dissatisfaction among the nurses.

Proactive strategies are employed during schedule generation to minimize the need for reactive rescheduling. These strategies aim to create more robust and flexible schedules that can withstand uncertainty. Robustness refers to a schedule's ability to absorb disruptions, while flexibility pertains to how easily the schedule can adapt to unexpected events causing imbalances in demand and supply.

To reduce the need for rescheduling, uncertainty is taken into account through proactive strategies when creating the nurse's schedule. These strategies are illustrated in Table 5.1.

Table 5.1: Proactive strategies.

Strategy	**Description**
Cross-section buffers	Resources exceeding the minimum demand can be scheduled to buffer shifts, which are not scheduled to a specific section. The sum of buffer assignments and section-specific assignments should cover the sum of the historical average demand across all sections for a given shift.
Flexible Assignments	Full-time nurses have a working ratio of X/ 100-X, where X represents a percentage of flexibility in their schedule. Only 100-X% are scheduled in the scheduling model.

5.3 The Nurse Rescheduling Problem

In the nurse rescheduling problem, reactive strategies are employed to adjust the nurse schedules due to schedule disruptions. In contrast to the nurse scheduling problem, the rescheduling problem covers a shorter period. The time window handled in the rescheduling problem consists of a defined planning period, which spans a specific number of days from the current day. In addition, days before and after the planning period are considered to ensure compliance with laws and regulations and allow greater flexibility in decision-making. The problem assesses new information regarding absence and demands daily and must handle future understaffed shifts each morning before the start of the day shift. Absences are notified before the day shift begins each day, and if an absence occurs, the duration of the absence is treated deterministically.

The rescheduling problem considers the historical average demand for nurses within the planning period, which must be fulfilled for all shifts within that period. However, the actual demand for nurses is revealed each morning before the day shift. This actual demand must be covered for all shifts on the current day while ensuring that the average demand for the remaining days in the planning period and post-period is still met. This requires a dynamic adjustment of the nurse schedule to accommodate the fluctuating actual demand while maintaining coverage for the average demand.

The laws and regulations that apply to the scheduling problem can be violated to avoid understaffing and to ensure adequate patient care. These violations entail that the requirements for skill composition on each shift are disregarded. In addition, the nurses are allowed to work overtime, a nurse can work more than one shift per day, and the weekly consecutive rest requirement can be overruled. This entails that the nurses can work extra shifts in addition to the ones they are scheduled to work. There are three types of reactive actions in rescheduling. These are displayed in Table 5.2.

Table 5.2: Reactive actions for the rescheduling model.

Actions	**Description**
Swap	An employee can be asked to work a different shift than planned in the schedule. This action can only happen when there is an understaffed shift, and an alternative shift has excess coverage in the following period.
Extra shift	Nurses can take extra shifts in addition to the assigned shifts in the nurse schedule. This action is allowed if the nurse does not breach the maximum weekly work hours.
Double shift	Nurses can take an extra shift on a day they are already scheduled to work. This is possible as long as the minimum hours of rest during 24 hours are maintained.

A nurse with a part-time position can work extra shifts to acquire additional hours. Additional hours are defined as work beyond agreed working hours but within the statutory limit for a full-time equivalent position. The wage for additional hours is equal to the nurse's regular wage. On the other hand, full-time nurses are required to work an average of the statutory limit, although the weekly total may vary. If a full-time nurse exceeds the workload they were originally scheduled to, overtime payment is triggered. Table 5.3 outlines the specific payments associated with different scenarios and actions. It is important to note that a nurse can only receive one type of extra payment at a time.

The focus of the rescheduling problem in this thesis is to minimize overall costs. The presented proactive strategies for handling uncertainty facilitate more effective reactive measures in the nurse rescheduling problem, thus contributing to limiting the rescheduling costs. Each day, in the rescheduling phase, the buffer shifts scheduled as a result of the cross-section buffer strategy are activated to a specific section based on the actual demand levels. Similarly, the flexible shifts corresponding to the flexible assignment strategy are activated in the rescheduling model to compensate for schedule disruptions. The flexible shifts can be activated to any of the sections, making flexible assignments a cross-section reactive strategy.

Table 5.3: Costs of overruling laws and agreements.

Triggers for extra payment	**Type of extra payment**
All extra hours for full-time nurses beyond their originally scheduled shifts	Overtime payment
Weekly hours of a part-time nurse exceed the statutory average hours of a full-time equivalent position	Overtime payment
More hours of work in a 24-hour time period than the statutory limit in the local agreements	Overtime payment
Work outside of regular working hours such as evening, night, and weekend	Evening/ night/ weekend bonus payment
Evening- or night shifts on the weekend	Evening or night bonus + weekend bonus
If one shift triggers both bonus payment and overtime payment, the bonus payment is removed, and only overtime payment is considered	Overtime payment
If a nurse swapped from a scheduled shift to an understaffed shift and is notified *before* the day of the original shift	Compensation payment for swaps
If a nurse swapped from a scheduled shift to an understaffed shift and is notified *on* the day of the original shift	Overtime payment

Chapter 6

Optimization Models

This chapter presents the mixed integer mathematical formulations of the two related problems that are used in this thesis: the nurse scheduling problem and the nurse rescheduling problem. The solution to the scheduling model provides a schedule, which serves as input to the rescheduling model. Various model extensions for the scheduling and rescheduling models are presented. These extensions include the strategies presented in Chapter 5, which aim at creating schedules that are better equipped to handle disruptions.

First, the nurse scheduling model for generating nurse schedules is presented in Section 6.1. Next, a rescheduling model for minimizing the overall rescheduling costs is presented in Section 6.2. All model extensions are presented in Section 6.3. Finally, Section 6.4 explains the models using illustrations and examples. Compressed versions of the scheduling model, the rescheduling model, and the model extensions are provided in Appendix A.

6.1 Nurse Scheduling Model

This section presents the optimization model for the nurse scheduling model. The mixed integer scheduling model in this section uses a lexicographic approach. This entails that the problem consists of several objectives that are solved iteratively as a set of single-objective optimization problems. Table 6.1 illustrates the order of the five objective functions. The inputs to the model are based on data provided by CC.

First, the indices, sets, parameters, and variables are defined. Next, the first step in the model, including the first objective function and corresponding constraints, is presented. The next steps of the model with additional parameters and constraints are then presented. Finally, the variables are declared.

Table 6.1: Overview of the lexicographic order of objectives in the scheduling model.

Objective one	Minimize the number of understaffed shifts
Objective two	Minimize total weekly deviation from the contracted hours for all nurses
Objective three	Maximize the distribution of overstaffed shifts
Objective four	Minimize the number of preference violations
Objective five	Minimize deviations from desired demand for experience levels and desired number specialized nurses per shift

6.1.1 Definitions

This section defines indices, sets, parameters, and variables. Sets are described with calligraphic letters. Parameters are written in capital letters, while variables are written using lower-case letters. Within the sets, parameters, and variables, subscripts indicate indices. Capital letter superscripts specify the meaning of the set, parameter, or variable. Overlines and underlines represent upper and lower bounds, respectively.

Indices

n nurse
c competence
e experience
b section
s shift
t day
k week

Sets

$\mathcal{B}$	set of sections, $\mathcal{B} = \{1, 2, 3\}$
$\mathcal{C}$	set of competence levels, $\mathcal{C} = \{AN, N, SN\}$
$\mathcal{E}$	set of experience levels
$\mathcal{N}$	set of nurses
$\mathcal{N}_c$	set of nurses with competence level c, $\mathcal{N}_c \subset \mathcal{N}$, $c \in \mathcal{C}$
$\mathcal{N}_e$	set of nurses with experience level e, $\mathcal{N}_e \subset \mathcal{N}$, $e \in \mathcal{E}$
$\mathcal{N}_b$	set of nurses in section b, $\mathcal{N}_b \subset \mathcal{N}$, $b \in \mathcal{B}$
$\mathcal{K}$	set of weeks included in scheduling period
$\mathcal{T}$	set of days in the scheduling period
$\mathcal{T}_k$	set of days in week k
$\mathcal{T}^{SUN}$	set of Sundays in scheduling period
$\mathcal{S}$	set of shifts, $\mathcal{S} = \{D, E, N, F, F_1\}$
$\mathcal{S}^{\mathcal{W}}$	set of work-shifts, $\mathcal{S}^W = \{D, E, N\}$, $\mathcal{S}^W \subset \mathcal{S}$
$\mathcal{S}^F$	set of off-shifts, $\mathcal{S}^F = \{F, F_1\}$. $\mathcal{S}^F \subset \mathcal{S}$

Scheduling Parameters

$\underline{D}_{bst}$	minimum demand in section b for shift s on day t
$\underline{D}_{ebst}$	desired demand for experience e in section b for shift s on day t
$\overline{D}_{bst}^{AN}$	maximum demand for assistant nurses in section b for shift s on day t
D_{bst}^{SN}	desired demand for specialized nurses in section b for shift s on day t
D_{bst}	historical average demand in section b for shift s on day t
$\overline{M}^D$	maximum number of consecutive work days
$\overline{M}^N$	maximum number of consecutive work nights
$\overline{L}$	maximum work hours in a week
H	hours in a full time work week
H_s	duration of shift s in hours
W	working weekend recurrence
C_n	contracted employment percentage for nurse n
$\overline{F}$	upper bound for allowed deviation from contracted hours
$\underline{F}$	lower bound for allowed deviation from contracted hours
K	number of weeks in scheduling period

Decision Variables

$$x_{nbst} = \begin{cases} 1, & \text{if nurse } n \text{ in section } b \text{ is scheduled for shift } s \text{ on day } t \\ 0, & \text{otherwise} \end{cases}$$

Auxiliary Variables

δ^{H-}_{nk}	weekly deficit of work hours from contract for nurse n in week k
δ^{H+}_{nk}	weekly surplus of work hours from contract for nurse n in week k
δ^{SN-}_{bst}	unsatisfied demand of specialized nurses in section b for shift s on day t
δ^{E}_{ebst}	unsatisfied demand for nurses with a defined experience level
δ^{D-}_{bst}	deficit from average demand in section b on shift s on day t
δ^{D+}_{bst}	surplus from average demand in section b on shift s on day t

6.1.2 Multi-objective Model

$$\min z^1 = \sum_{b\in\mathcal{B}} \sum_{s\in\mathcal{S}^W} \sum_{t\in\mathcal{T}} \delta^{D-}_{bst} \tag{6.1.1}$$

Objective (6.1.1) minimizes the number of understaffed shifts in the schedule. After solving this first objective, the model will return a value for the minimum number of shifts that were understaffed in the schedule. This value will be used as a constraint in the second step of the model.

Demand Coverage

$$\sum_{n\in\mathcal{N}} x_{nbst} \geq \underline{D}_{bst} \qquad b\in\mathcal{B}, s\in\mathcal{S}^W, t\in\mathcal{T} \tag{6.1.2}$$

$$\sum_{n\in\mathcal{N}} x_{nbst} \geq D_{bst} - \delta^{D-}_{bst} \qquad b\in\mathcal{B}, s\in\mathcal{S}^W, t\in\mathcal{T} \tag{6.1.3}$$

$$\sum_{n\in\mathcal{N}_{AN}} x_{nbst} \leq \overline{D}^{AN}_{cbst} \qquad b\in\mathcal{B}, s\in\mathcal{S}^W, t\in\mathcal{T} \tag{6.1.4}$$

Constraints (6.1.2) ensure minimum demand is covered for every shift. Constraints (6.1.3) measures the deficits from average demand. Constraints (6.1.4) limit the number of assistant nurses assigned to each shift.

Section Specific Assignments

$$x_{n1st} = 0 \qquad n\in\mathcal{N}/\{\mathcal{N}_{b=1}\}, s\in\mathcal{S}, t\in\mathcal{T} \tag{6.1.5}$$

$$x_{n2st} = 0 \qquad n\in\mathcal{N}/\{\mathcal{N}_{b=2}\}, s\in\mathcal{S}, t\in\mathcal{T} \tag{6.1.6}$$

$$x_{n3st} = 0 \qquad n\in\mathcal{N}/\{\mathcal{N}_{b=3}\}, s\in\mathcal{S}, t\in\mathcal{T} \tag{6.1.7}$$

Constraints (6.1.5), (6.1.6), and (6.1.7) ensure that no nurses can be scheduled to another section than their own. This encompasses the nurses for all three sections.

Legislative Constraints

$$\sum_{b\in\mathcal{B}}\sum_{s\in\mathcal{S}} x_{nbst} = 1 \qquad n\in\mathcal{N}, t\in\mathcal{T} \tag{6.1.8}$$

$$\sum_{b\in\mathcal{B}}\sum_{s\in\mathcal{S}^W}\sum_{t\in\mathcal{T}_k} H_s x_{nbst} \leq \overline{L} \qquad n\in\mathcal{N}, k\in\mathcal{K} \tag{6.1.9}$$

$$\underline{F}\sum_{k\in\mathcal{K}} C_n H \leq \sum_{b\in\mathcal{B}}\sum_{s\in\mathcal{S}^W}\sum_{t\in\mathcal{T}} H_s x_{nbst} \leq \overline{F}\sum_{k\in\mathcal{K}} C_n H \qquad n\in\mathcal{N} \tag{6.1.10}$$

$$\sum_{t'=t}^{T=t+\overline{M}^D}\sum_{b\in\mathcal{B}}\sum_{s\in\mathcal{S}^W} x_{nbst'} \leq \overline{M}^D \qquad n\in\mathcal{N}, t\in\{1,2,...,\mathcal{T}-\overline{M}^D\} \tag{6.1.11}$$

$$\sum_{t'=t}^{T=t+\overline{M}^N}\sum_{b\in\mathcal{B}} x_{nbNt'} \leq \overline{M}^N \qquad n\in\mathcal{N}, t\in\{1,2,...,\mathcal{T}-\overline{M}^N\} \tag{6.1.12}$$

Constraints (6.1.8) control that a nurse is only scheduled to one shift per day. Constraints (6.1.9) limit the total working hours for each nurse during one week. Constraints (6.1.10) sets an interval for how many hours each nurse should be assigned to work in the scheduling period. Constraints (6.1.11) sets an upper bound for consecutive days scheduled for each nurse. Constraints (6.1.12) sets an upper bound for consecutive nights scheduled for each nurse.

Weekend Assignments

$$\sum_{t'=0}^{T=W-1}\sum_{b\in\mathcal{B}}\sum_{s\in\mathcal{S}^W} x_{nbs(t+t')} = 1 \qquad t\in\mathcal{T}^{SUN}, n\in\mathcal{N} \tag{6.1.13}$$

$$\sum_{b\in\mathcal{B}}\left(x_{nbDt} - x_{nbE(t-1)}\right) = 0 \qquad n\in\mathcal{N}, t\in\mathcal{T}^{SUN} \tag{6.1.14}$$

$$\sum_{b\in\mathcal{B}}\left(x_{nbEt} - x_{nbD(t-1)}\right) = 0 \qquad n\in\mathcal{N}, t\in\mathcal{T}^{SUN} \tag{6.1.15}$$

$$\sum_{b\in\mathcal{B}}\left(x_{nbNt} - x_{nbN(t-1)}\right) = 0 \qquad n\in\mathcal{N}, t\in\mathcal{T}^{SUN} \tag{6.1.16}$$

Constraints (6.1.13) specify that each nurse should work exactly one weekend every W weekends. Constraints (6.1.14), (6.1.15), and (6.1.16) ensure that legal weekend patterns are assigned.

Rest Regulations

$$\sum_{b\in\mathcal{B}}\big(x_{nbNt}+x_{nbD(t+1)}\big)\leq 1 \qquad n\in\mathcal{N}, t\in\mathcal{T} \tag{6.1.17}$$

$$\sum_{b\in\mathcal{B}}\big(x_{nbNt}+x_{nbE(t+1)}\big)\leq 1 \qquad n\in\mathcal{N}, t\in\mathcal{T} \tag{6.1.18}$$

$$\sum_{b\in\mathcal{B}}\big(x_{nbN(t-1)}+x_{nbF_1t}+x_{nbD(t+1)}\big)\leq 2 \qquad n\in\mathcal{N}, t\in\mathcal{T} \tag{6.1.19}$$

$$\sum_{b\in\mathcal{B}}\big(x_{nbN(t-1)}+x_{nbF_1t}+x_{nbE(t+1)}\big)\leq 2 \qquad n\in\mathcal{N}, t\in\mathcal{T} \tag{6.1.20}$$

$$\sum_{b\in\mathcal{B}}\big(x_{nbE(t-1)}+x_{nbF_1t}+x_{nbD(t+1)}\big)\leq 2 \qquad n\in\mathcal{N}, t\in\mathcal{T} \tag{6.1.21}$$

$$\sum_{n\in\mathcal{N}}\sum_{b\in\mathcal{B}}\sum_{t\in\mathcal{T}_k} x_{nbF_1t}=1 \qquad k\in\mathcal{K} \tag{6.1.22}$$

Constraints (6.1.17) and (6.1.18) specify shift patterns that should not be assigned. Constraints (6.1.19), (6.1.20) and (6.1.21) specify the illegal patterns with regards to the required weekly rest day F1. Constraints (6.1.22) ensure that the required rest day is scheduled once a week.

Minimize hours deviations

$$\min z^2=\sum_{n\in\mathcal{N}}\sum_{k\in\mathcal{K}}\big(\delta_{nk}^{H^-}+\delta_{nk}^{H^+}\big) \tag{6.1.23}$$

Objective (6.1.23) is the second objective function in the lexicographic order. It minimizes the total weekly deviation from the contracted number of hours for all nurses. This is to ensure an even distribution of workload per week during the whole scheduling period.

$$\sum_{b\in\mathcal{B}}\sum_{s\in\mathcal{S}}\sum_{t\in\mathcal{T}_k} H_s x_{nbst}=C_nH+\delta_{nk}^{H^-}-\delta_{nk}^{H^+} \qquad n\in\mathcal{N}, k\in\mathcal{K} \tag{6.1.24}$$

Constraints (6.1.24) measure the weekly deviations from contracted hours.

$$\sum_{b\in\mathcal{B}}\sum_{s\in\mathcal{S}^W}\sum_{t\in\mathcal{T}}\delta_{bst}^{D^-}\leq z^1 \tag{6.1.25}$$

Constraint (6.1.25) ensures that the optimal solution from the first objective function is upheld when solving the second step of the lexicographic order.

Distribute overstaffing

Additional Parameter

$\overline{\delta}$ largest value of either the deficit or surplus of nurses working hours

Additional Variable

$$\alpha_{bst} = \begin{cases} 1, & \text{if there is overstaffing in section } b \text{ on shift } s \text{ on day } t \\ 0, & \text{otherwise} \end{cases}$$

$$\max z^3 = \sum_{b \in \mathcal{B}} \sum_{s \in \mathcal{S}^W} \sum_{t \in \mathcal{T}} \alpha_{bst} \tag{6.1.26}$$

Objective (6.1.26) is the third objective function in the lexicographic order. It maximizes the number of overstaffed shifts. This is to ensure the overstaffing is evenly distributed over shifts, preventing that overstaffing occurs in large quantities on only a select few shifts.

$$\sum_{n \in \mathcal{N}} x_{nbst} = D_{bst} - \delta_{bst}^{D-} + \delta_{bst}^{D+} \qquad b \in \mathcal{B}, s \in \mathcal{S}^W, t \in \mathcal{T} \tag{6.1.27}$$

Constraints (6.1.27) measure the deviations from average demand. These replace Constraints (6.1.3).

$$\alpha_{bst} \leq \delta_{bst}^{D+} \qquad b \in \mathcal{B}, s \in \mathcal{S}^W, t \in \mathcal{T} \tag{6.1.28}$$

Constraints (6.1.28) ensure that α can only take a value when there is overstaffing.

$$\delta_{nk}^{H+} + \delta_{nk}^{H-} \leq \overline{\delta} \qquad n \in \mathcal{N}, k \in \mathcal{K} \tag{6.1.29}$$

Constraints (6.1.29) ensure that a nurse cannot have a deviation from working hours that is larger than the largest value from step two in the lexicographic order.

$$\sum_{n \in \mathcal{N}} \sum_{k \in \mathcal{K}} \delta_{nk}^{H+} + \delta_{nk}^{H-} \leq z^2 \tag{6.1.30}$$

Constraint (6.1.30) ensures that the optimal solution from the second objective function is upheld within an allowed deviation when solving the third step of the lexicographic order.

Preference Violation

Additional Parameter

I_{nst} nurse n wants to avoid working shift s on day t

$$\min z^4 = \sum_{n \in \mathcal{N}} \sum_{b \in \mathcal{B}} \sum_{s \in \mathcal{S}^W} \sum_{t \in \mathcal{T}} I_{nst} x_{nbst} \tag{6.1.31}$$

Objective (6.1.31) is the fourth objective function in the lexicographic order. It minimizes the number of preference violations in the schedule.

$$\sum_{b \in \mathcal{B}} \sum_{s \in \mathcal{S}^W} \sum_{t \in \mathcal{T}} \alpha_{bst} \geq z^3 \tag{6.1.32}$$

Constraint (6.1.32) ensures that the optimal solution from the third objective function is upheld when solving the fourth step of the lexicographic order.

Distribution of Competence and Experience

$$\min z^5 = \sum_{e \in \mathcal{E}} \sum_{b \in \mathcal{B}} \sum_{s \in \mathcal{S}^W} \sum_{t \in \mathcal{T}} \delta^E_{ebst} + \sum_{b \in \mathcal{B}} \sum_{s \in \mathcal{S}^W} \sum_{t \in \mathcal{T}} \delta^{SN-}_{bst} \tag{6.1.33}$$

Objective (6.1.33) is the fifth objective function in the lexicographic order. It minimizes the deviations from desired demand for experience levels and the desired number of specialized nurses per shift. This is to ensure an even distribution of competence and experience levels per shift.

$$\sum_{n \in \mathcal{N}_e} x_{nbst} \geq \underline{D}_{ebst} - \delta^E_{ebst} \qquad e \in \mathcal{E}, b \in \mathcal{B}, s \in \mathcal{S}^W, t \in \mathcal{T} \tag{6.1.34}$$

Constraints (6.1.34) measure the deficit from desired demand for experience.

$$\sum_{n \in \mathcal{N}_{SN}} x_{nbst} \geq D^{SN}_{bst} - \delta^{SN-}_{bst} \qquad b \in \mathcal{B}, s \in \mathcal{S}^W, t \in \mathcal{T} \tag{6.1.35}$$

Constraints (6.1.35) measure the deficit from desired demand for competence.

$$\sum_{n \in \mathcal{N}} \sum_{b \in \mathcal{B}} \sum_{s \in \mathcal{S}^W} \sum_{t \in \mathcal{T}} I_{nst} x_{nbst} \leq z^4 \tag{6.1.36}$$

Constraint (6.1.36) ensures that the optimal solution from the fourth objective function is upheld when solving the fifth step of the lexicographic order.

Variable Declarations

$$x_{nbst} \in \{0,1\} \qquad n \in \mathcal{N}, b \in \mathcal{B}, s \in \mathcal{S}, t \in \mathcal{T} \tag{6.1.37}$$

$$\alpha_{bst} \in \{0,1\} \qquad b \in \mathcal{B}, s \in \mathcal{S}^W, t \in \mathcal{T} \tag{6.1.38}$$

$$\delta_{nk}^{H^-} \geq 0 \qquad n \in \mathcal{N}, k \in \mathcal{K} \tag{6.1.39}$$

$$\delta_{nk}^{H^+} \geq 0 \qquad n \in \mathcal{N}, k \in \mathcal{K} \tag{6.1.40}$$

$$\delta_{bst}^{SN^-} \geq 0 \qquad b \in \mathcal{B}, s \in \mathcal{S}^W, t \in \mathcal{T} \tag{6.1.41}$$

$$\delta_{bst}^{E} \geq 0 \qquad b \in \mathcal{B}, s \in \mathcal{S}^W, t \in \mathcal{T} \tag{6.1.42}$$

$$\delta_{bst}^{D^-} \geq 0 \qquad b \in \mathcal{B}, s \in \mathcal{S}^W, t \in \mathcal{T} \tag{6.1.43}$$

$$\delta_{bst}^{D^+} \geq 0 \qquad b \in \mathcal{B}, s \in \mathcal{S}^W, t \in \mathcal{T} \tag{6.1.44}$$

6.2 Nurse Rescheduling Model

This section presents the optimization model for the nurse rescheduling model. The rescheduling model is based on the rescheduling model from our Specialization Report (Johansen et al., 2022). The model takes a schedule with reported absences as input. The inputs are based on the schedule generated by the scheduling model. The day of rescheduling is always denoted as t_0. In this model, actual demand is revealed daily on day t_0. For all upcoming days after t_0, average demand is used.

This section is structured similarly to Section 6.1. First, indices, sets, parameters, and variables are defined. Next, the objective function and corresponding constraints are presented. Finally, the variables are declared.

6.2.1 Definitions

This section defines indices, sets, parameters, and variables. Sets are described with calligraphic letters. Parameters are written in capital letters, while variables are written using lower-case letters. Within the sets, parameters, and variables, subscripts indicate indices. Capital letter superscripts specify the meaning of the set, parameter, or variable. Overlines and underlines represent a maximum or minimum, respectively.

Indices

n	nurse
b	section
s	shift
t	day
c	competence
k	week

Sets

$\mathcal{B}$	set of sections, $\mathcal{B} = \{1, 2, 3\}$
$\mathcal{C}$	set of competence levels, $\mathcal{C} = \{AN, N, SN\}$
$\mathcal{N}$	set of nurses
$\mathcal{N}^{100}$	set of full time nurses, $\mathcal{N}^{100} \subset \mathcal{N}$
$\mathcal{N}_c$	set of nurses with competence level c, $\mathcal{N}_c \subset \mathcal{N}$, $c \in \mathcal{C}$
$\mathcal{T}^R$	set of days in planning period
$\mathcal{T}^{PRE}$	set of days before planning period
$\mathcal{T}^{POST}$	set of days after planning period
$\mathcal{T}$	set of all days, $\mathcal{T} = \left\{\mathcal{T}^{PRE} \cup \mathcal{T}^R \cup \mathcal{T}^{POST}\right\}$
$\mathcal{T}^A$	set of days in planning period or post period, $\mathcal{T}^A = \left\{\mathcal{T}^R \cup \mathcal{T}^{POST}\right\}$
$\mathcal{K}$	set of weeks included in planning period
$\mathcal{T}_k$	set of days in week k
$\mathcal{T}_k^R$	set of days in week k included in planning period, $\mathcal{T}_k^R = \left\{\mathcal{T}^R \cap \mathcal{T}_k\right\}$
$\mathcal{T}^W$	set of weekend days in planning period
$\mathcal{T}^{WA}$	set of weekend days in planning period and post period
$\mathcal{S}^W$	set of shifts, $\mathcal{S}^W = \left\{D, E, N\right\}$

Rescheduling Parameters

D_{bst}	historical average demand in section b for shift s on day t
$D^{A}_{bst_0}$	actual demand in section b for shift s on the day of rescheduling
W_n	hourly wage for nurse n
$\overline{L}$	maximum work hours in a week
P^{O}	penalty percentage for overtime hours
P^{OS}	penalty for overtime hours of swap
P^{W}	penalty payment for weekend hours
P^{N}	penalty payment for night hours
P^{E}	penalty payment for evening hours
H	hours in a full-time work week
$\overline{H}^{D}$	maximum hours during a day before overtime is triggered
H_s	duration of shift s in hours
H^{M}	duration of double shift, $H^{M} = H_D + H_E$
H^{PRE}_{nk}	scheduled hours for nurse n during a week k in the original schedule
X_{nbst}	scheduled value for nurse n in section b working shift s on day t
A_{nt}	nurse n is available to work on day t
Δ_{nt}	nurse n has been scheduled to work or has already worked a double shift on day t
Ω_{nk}	total overtime hours planned for nurse n during week k
Λ_{nst}	overtime hours caused by nurse n working shift s on day t

X_{nbst}, A_{nt}, Δ_{nt}, Ω_{nt} and Λ_{nst} are time-dependent dynamic parameters. Their values depend on when the model is solved. All the other parameters have set values.

Decision Variables

$$x^{+}_{nbst} = \begin{cases} 1, & \text{if nurse } n \text{ in section } b \text{ is rescheduled to work shift } s \text{ on day } t \\ 0, & \text{otherwise} \end{cases}$$

$$x^{-}_{nbst} = \begin{cases} 1, & \text{if nurse } n \text{ in section } b \text{ is removed from shift } s \text{ on day } t \\ 0, & \text{otherwise} \end{cases}$$

$$x'_{nbst} = \begin{cases} 1, & \text{if nurse } n \text{ in section } b \text{ works shift } s \text{ on day } t \\ 0, & \text{otherwise} \end{cases}$$

$$u_{ns_1t_1t_2} = \begin{cases} 1, & \text{if nurse } n \text{ swapped a shift from day } t_2 \text{ to work shift } s_1 \text{ on day } t_1 \\ 0, & \text{otherwise} \end{cases}$$

$$\epsilon_{nst} = \begin{cases} 1, & \text{if nurse } n \text{ is assigned to work an extra shift } s \text{ on day } t \\ 0, & \text{otherwise} \end{cases}$$

$$d_{nt} = \begin{cases} 1, & \text{if nurse } n \text{ is assigned to work double shifts on day } t \\ 0, & \text{otherwise} \end{cases}$$

Auxillary variables

ω_{nk} overtime hours for nurse n in week k
λ_{nst} overtime hours for nurse n caused by working shift s on day t

6.2.2 Minimum Cost Objective

The objective function in this model concerns minimizing the total costs of rescheduling.

$$z^1 = \sum_{n \in \mathcal{N}} \sum_{b \in \mathcal{B}} \sum_{s \in \mathcal{S}^W} \sum_{t \in \mathcal{T}^R} H_s W_n x^+_{nbst} \tag{6.2.1}$$

The first term of the minimum cost objective, term (6.2.1), covers the base pay of all rescheduled shifts.

$$z^2 = \sum_{n \in \mathcal{N}} \sum_{k \in \mathcal{K}} P^O W_n \omega_{nk} - \sum_{n \in \mathcal{N}} \sum_{s \in \mathcal{S}^W} P^O W_n \big(1 - A_{nt_0}\big) \Lambda_{nst_0} \tag{6.2.2}$$

Term (6.2.2) sums the weekly overtime payments triggered by the rescheduling and subtracts the costs for shifts scheduled with overtime where the nurse subsequently has called in absent. t_0 represents the current day.

$$z^3 = \sum_{n \in \mathcal{N}} \sum_{t \in \mathcal{T}^R} P^O W_n \big(H^M - \overline{H}^D\big) d_{nt} \tag{6.2.3}$$

Term (6.2.3) sums the daily overtime payments caused by working a double shift.

$$z^4 = \sum_{n \in \mathcal{N}} \sum_{s_1 \in \mathcal{S}^W} \sum_{t_1 \in \mathcal{T}^R} \sum_{t_2 \in \mathcal{T}^A} P^{OS} H_{s_1} W_n u_{ns_1t_1t_2} \tag{6.2.4}$$

$$z^5 = \sum_{n \in \mathcal{N}} \sum_{s_1 \in \mathcal{S}^W} \sum_{t_1 \in \mathcal{T}^R} P^O H_{s_1} W_n u_{ns_1t_1t_0} \tag{6.2.5}$$

Term (6.2.4) accounts for the penalty payments associated with a swapped shift, where the swap was done before the day of the original shift. Term (6.2.5) computes the overtime cost for swapped shifts where the original shift was on the same day as the rescheduling.

$$z^6 = \sum_{n \in \mathcal{N}} \sum_{t \in \mathcal{T}^R} P^E H_E W_n \epsilon_{nEt} - \sum_{n \in \mathcal{N}} \sum_{t \in \mathcal{T}^R} P^E W_n \lambda_{nEt} \tag{6.2.6}$$

$$z^7 = \sum_{n \in \mathcal{N}} \sum_{t \in \mathcal{T}^R} P^N H_N W_n \epsilon_{nNt} - \sum_{n \in \mathcal{N}} \sum_{t \in \mathcal{T}^R} P^N W_n \lambda_{nNt} \tag{6.2.7}$$

$$z^8 = \sum_{n \in \mathcal{N}} \sum_{s \in \mathcal{S}^W} \sum_{t \in \mathcal{T}^W} P^W H_s W_n \epsilon_{nst} - \sum_{n \in \mathcal{N}} \sum_{s \in \mathcal{S}^W} \sum_{t \in \mathcal{T}^W} P^W W_n \lambda_{nst} \tag{6.2.8}$$

Terms (6.2.6), (6.2.7) and (6.2.8) sum all the extra payments connected to working an evening shift, a night shift, or a weekend shift, respectively. During weekends these payments are stacked. If a shift triggers overtime, it should not count both overtime hours and extra payments. Therefore, the amount of overtime hours connected to the specific shifts, λ_{nst}, is subtracted for each equation.

$$z^9 = \sum_{n\in\mathcal{N}}\sum_{b\in\mathcal{B}}\sum_{s\in\mathcal{S}^W}\sum_{t\in\mathcal{T}^A} A_{nt}H_sW_nx^-_{nbst} \tag{6.2.9}$$

Term (6.2.9) removes the base payment from a shift that an available nurse is no longer scheduled to work.

$$z^{10} = \sum_{n\in\mathcal{N}}\sum_{b\in\mathcal{B}}\sum_{t\in\mathcal{T}^A} A_{nt}P^EH_EW_nx^-_{nbEt} \tag{6.2.10}$$

$$z^{11} = \sum_{n\in\mathcal{N}}\sum_{b\in\mathcal{B}}\sum_{t\in\mathcal{T}^A} A_{nt}P^NH_NW_nx^-_{nbNt} \tag{6.2.11}$$

$$z^{12} = \sum_{n\in\mathcal{N}}\sum_{b\in\mathcal{B}}\sum_{s\in\mathcal{S}^W}\sum_{t\in\mathcal{T}^{WA}} A_{nt}P^WH_sW_nx^-_{nbst} \tag{6.2.12}$$

Terms (6.2.10), (6.2.11), and (6.2.12) remove the bonus payments from a shift that an available nurse is no longer scheduled to work.

$$\min z^T = \sum_{i=1}^{8} z^i - \sum_{j=9}^{12} z^j \tag{6.2.13}$$

The total minimum cost objective function (6.2.13) summarizes all the different terms represented in Equations (6.2.1)-(6.2.12).

Dependency in Variables

For better readability, we have defined variables x'_{nbst} and ϵ_{nst}. x'_{nbst} represents the final output schedule, while ϵ_{nst} is defined to keep track of which shift assignments are extra shifts.

$$x'_{nbst} = X_{nbst} + x^+_{nbst} - x^-_{nbst} \qquad n\in\mathcal{N}, b\in\mathcal{B}, s\in\mathcal{S}^W, t\in\mathcal{T}^R \tag{6.2.14}$$

$$x'_{nbst} = X_{nbst} - x^-_{nbst} \qquad n\in\mathcal{N}, b\in\mathcal{B}, s\in\mathcal{S}^W, t\in\mathcal{T}^{POST} \tag{6.2.15}$$

$$\sum_{b\in\mathcal{B}} x^+_{nbst_1} = \epsilon_{nst_1} + \sum_{t_2\in\mathcal{T}^A} u_{nst_1t_2} \qquad n\in\mathcal{N}, s\in\mathcal{S}^W, t_1\in\mathcal{T}^R \tag{6.2.16}$$

$$\sum_{s\in\mathcal{S}^W}\sum_{t\in\mathcal{T}_k^R}\lambda_{nst}+\sum_{s\in\mathcal{S}^W}\sum_{t\in\mathcal{T}_k}\Lambda_{nst}=\omega_{nk}+\Omega_{nk}\qquad n\in\mathcal{N},k\in\mathcal{K}\tag{6.2.17}$$

$$\lambda_{nst}\leq\sum_{b\in\mathcal{B}}H_s x^+_{nbst}\qquad n\in\mathcal{N},s\in\mathcal{S}^W,t\in\mathcal{T}^R\tag{6.2.18}$$

$$x^-_{nbst}\leq X_{nbst}\qquad n\in\mathcal{N},b\in\mathcal{B},s\in\mathcal{S}^W,t\in\mathcal{T}^{\mathcal{A}}\tag{6.2.19}$$

Constraints (6.2.14) and (6.2.15) ensure that variable x'_{nbst} sums the input schedule with the rescheduled shift changes found by the model. Constraints (6.2.16) track if a newly assigned shift is an extra shift. Constraints (6.2.17) ensure the balance between the variables counting overtime hours per shift and total overtime hours. Constraints (6.2.18) secure that new overtime hours can only occur for a newly assigned shift. Constraints (6.2.19) guarantee that only scheduled shifts can be removed.

Demand Coverage

$$\sum_{n\in\mathcal{N}}x'_{nbst}\geq D_{bst}\qquad b\in\mathcal{B},s\in\mathcal{S}^W,t\in\mathcal{T}^A/\{t_0\}\tag{6.2.20}$$

$$\sum_{n\in\mathcal{N}}x'_{nbst_0}\geq D^A_{bst_0}\qquad b\in\mathcal{B},s\in\mathcal{S}^W\tag{6.2.21}$$

Constraints (6.2.20) ensure average demand is covered for every shift. Constraints (6.2.21) ensure actual demand is covered for every shift on the day of rescheduling.

Legislative Constraints

$$\sum_{b\in\mathcal{B}}\sum_{s\in\mathcal{S}^W}\sum_{t\in\mathcal{T}_k}H_s x'_{nbst}\leq\overline{L}\qquad n\in\mathcal{N},k\in\mathcal{K}\tag{6.2.22}$$

$$\begin{aligned}\sum_{b\in\mathcal{B}}\sum_{s\in\mathcal{S}^W}\sum_{t\in\mathcal{T}_k}H_s x'_{nbst}-\big(\omega_{nk}+\Omega_{nk}\big)\leq{}&\max(H,H^{PRE}_{nk})\\&+\sum_{t\in\mathcal{T}_k^R}\big(H^M-\overline{H}^D\big)d_{nt}\\&+\sum_{t\in\mathcal{T}_k}\big(H^M-\overline{H}^D\big)\Delta_{nt}\\&n\in\mathcal{N}/\{\mathcal{N}^{100}\},k\in\mathcal{K}\end{aligned}\tag{6.2.23}$$

$$\sum_{b\in\mathcal{B}}\sum_{s\in\mathcal{S}^W}\sum_{t\in\mathcal{T}_k} H_s x'_{nbst} - \big(\omega_{nk} + \Omega_{nk}\big) \leq H^{PRE}_{nk} + \sum_{t\in\mathcal{T}_k^R}\big(H^M - \overline{H}^D\big) d_{nt} + \sum_{t\in\mathcal{T}_k}\big(H^M - \overline{H}^D\big)\Delta_{nt} \qquad n\in\mathcal{N}^{100}, k\in\mathcal{K} \tag{6.2.24}$$

Constraints (6.2.22) limit the total working hours for each nurse during one week. Constraints (6.2.23) register overtime hours for all part-time nurses that work more hours than a full-time work week during a given week. Constraints (6.2.24) register overtime hours for all full-time nurses that work more hours than originally scheduled.

Technical Constraints for the Output Schedule

$$x'_{nbst} \leq A_{nt} \qquad n\in\mathcal{N}, b\in\mathcal{B}, s\in\mathcal{S}^W, t\in\mathcal{T}^R \tag{6.2.25}$$

$$\sum_{b\in\mathcal{B}} x'_{nbst} \leq 1 \qquad n\in\mathcal{N}, s\in\mathcal{S}^W, t\in\mathcal{T} \tag{6.2.26}$$

$$\sum_{b\in\mathcal{B}}\sum_{s\in\mathcal{S}^W} x'_{nbst} \leq 1 + \big(d_{nt} + \Delta_{nt}\big) \qquad n\in\mathcal{N}, t\in\mathcal{T}^R \tag{6.2.27}$$

$$\sum_{b\in\mathcal{B}}\sum_{s\in\mathcal{S}^W}\sum_{t\in\mathcal{T}^A} A_{nt}H_s X_{nbst} \leq \sum_{b\in\mathcal{B}}\sum_{s\in\mathcal{S}^W}\sum_{t\in\mathcal{T}^A} H_s x'_{nbst} \qquad n\in\mathcal{N} \tag{6.2.28}$$

Constraints (6.2.25) secure that only available nurses can work shifts in the planning period. Constraints (6.2.26) check that no one is scheduled to work more than one shift at the same time. Constraints (6.2.27) control that a nurse can only work two shifts if the nurse works one of the specified double shifts. Constraints (6.2.28) secure that no nurse works fewer hours in the new plan compared to the input schedule, except in the cases where nurses have been absent from the original schedule.

Technical Constraints for Actions

$$\sum_{b\in\mathcal{B}}\sum_{s\in\mathcal{S}^W} A_{nt_2} x^-_{nbst_2} = \sum_{s_1\in\mathcal{S}^W}\sum_{t_1\in\mathcal{T}^R} u_{ns_1t_1t_2} \qquad n\in\mathcal{N}, t_2\in\mathcal{T}^A \tag{6.2.29}$$

$$\sum_{b\in\mathcal{B}} x^+_{ns_1bt_1} \geq \sum_{t_2\in\mathcal{T}^A} u_{ns_1t_1t_2} \qquad n\in\mathcal{N}, s_1\in\mathcal{S}^W, t_1\in\mathcal{T}^R \tag{6.2.30}$$

$$\sum_{b\in\mathcal{B}}\big(x'_{nbDt} + x'_{nbNt}\big) \leq 1 \qquad n\in\mathcal{N}, t\in\mathcal{T} \tag{6.2.31}$$

$$\sum_{b\in\mathcal{B}}\left(x'_{nbNt}+x'_{nbD(t+1)}\right)\leq 1 \qquad n\in\mathcal{N}, t\in\mathcal{T} \tag{6.2.32}$$

$$\sum_{b\in\mathcal{B}}\left(x'_{nbEt}+x'_{nbNt}\right)\leq 1 \qquad n\in\mathcal{N}, t\in\mathcal{T} \tag{6.2.33}$$

$$\sum_{b\in\mathcal{B}}\left(x'_{nbNt}+x'_{nbE(t+1)}\right)\leq 1 \qquad n\in\mathcal{N}, t\in\mathcal{T} \tag{6.2.34}$$

Constraints (6.2.29) and (6.2.30) secure that if an available nurse is moved from one shift to another, it is a valid swap and is registered in the variable $u_{nt_1t_2}$. Constraints (6.2.31), (6.2.32), (6.2.33), and (6.2.34) specify illegal shift patterns, where illegal combinations of double shifts are handled.

Variable Declarations

$$x^+_{nbst}\in\{0,1\} \qquad n\in\mathcal{N}, b\in\mathcal{B}, s\in\mathcal{S}^W, t\in\mathcal{T}^R \tag{6.2.35}$$

$$x^-_{nbst}\in\{0,1\} \qquad n\in\mathcal{N}, b\in\mathcal{B}, s\in\mathcal{S}^W, t\in\mathcal{T}^A \tag{6.2.36}$$

$$x'_{nbst}=X_{nbst} \qquad n\in\mathcal{N}, b\in\mathcal{B}, s\in\mathcal{S}^W, t\in\mathcal{T}^{PRE} \tag{6.2.37}$$

$$x'_{nbst}\in\{0,1\} \qquad n\in\mathcal{N}, b\in\mathcal{B}, s\in\mathcal{S}^W, t\in\mathcal{T}^A \tag{6.2.38}$$

$$u_{ns_1t_1t_2}\in\{0,1\} \qquad n\in\mathcal{N}, s_1\in\mathcal{S}^W, t_1\in\mathcal{T}^R, t_2\in\mathcal{T}^A \tag{6.2.39}$$

$$d_{nt}\in\{0,1\} \qquad n\in\mathcal{N}, t\in\mathcal{T}^R \tag{6.2.40}$$

$$\omega_{nk}\geq 0 \qquad n\in\mathcal{N}, k\in\mathcal{K} \tag{6.2.41}$$

$$\lambda_{nst}\geq 0 \qquad n\in\mathcal{N}, s\in\mathcal{S}^W, t\in\mathcal{T}^R \tag{6.2.42}$$

6.3 Model Extensions

To improve the effectiveness of the nurse scheduling and rescheduling models in handling uncertainty, this section presents two model extensions. These extensions affect both models, as the proactive strategies apply to the scheduling model and the corresponding reactive strategies apply to the rescheduling model. The extensions build upon the models presented in Sections 6.1 and 6.2. Rather than providing a complete model description, this section focuses on presenting the deviations from the original models. For each extension, only the additional sets, parameters, variables, and constraints are presented. Section 6.3.1 first introduces an extension based on the cross-section buffer strategy. Next, Section 6.3.2 presents the strategy that utilizes flexible assignments to handle unforeseen absences in the rescheduling model.

6.3.1 Buffer Strategy

The buffer strategy is applied to ensure optimal utilization of surplus nurses across the three sections. The proactive strategy is applied to the scheduling model to generate schedules with additional robustness for handling unexpected events. In the rescheduling model, the buffer shifts are activated for the sections with an increase in demand or decrease in available nurses. This section presents the details of the buffer strategy, including how it is implemented in both models.

Scheduling Extensions

In the nurse scheduling model, the proactive buffer strategy schedules nurses to buffer shifts. These shifts are not defined for a specific section, allowing nurses to work in any of the three sections as needed. The buffer shifts are activated for each nurse on the day of rescheduling, t_0. Until then, nurses are unaware of which section they will be working in, only knowing what shift they will work.

Sets

$\mathcal{B}$	set of sections, $\mathcal{B} = \{1, 2, 3, 0\}$
$\mathcal{B}^W$	set of working sections, $\mathcal{B}^W = \{1, 2, 3\}$, $\mathcal{B}^W \subset \mathcal{B}$

Buffer shifts do not apply to a specific section. Instead, a fourth section is defined. This fourth section represents all nurses who are scheduled for a buffer shift and have not yet been assigned to any particular section.

Parameters

β_{st}	upper bound for how many buffer nurses can be scheduled each shift

Auxiliary Variables

δ_{st}^{D-}	deficit from total average demand on shift s on day t
δ_{st}^{D+}	surplus from total average demand on shift s on day t

While the nurses scheduled to buffer shifts contribute to cover the total average demand across all sections, they cannot be counted towards fulfilling the minimum demand of any specific section. Furthermore, since these nurses are not assigned to a particular section until the day of rescheduling, they cannot contribute to meeting the average demand of any specific section. Consequently, the deviations from the average demand can no longer be calculated for each specific section, but instead, consider the total demand. As a result, the auxiliary variables change from δ_{bst}^{D-} and δ_{bst}^{D+} to δ_{st}^{D-} and δ_{st}^{D+}.

Maximize buffer

$$\max z^6 = \sum_{n \in \mathcal{N}} \sum_{s \in \mathcal{S}^W} \sum_{t \in \mathcal{T}} x_{n0st} \tag{6.3.1}$$

Objective (6.3.1) is the sixth step in the lexicographic order. This objective aims to maximize the number of nurses scheduled for a buffer shift.

Constraints

$$\sum_{n \in \mathcal{N}} x_{nbst} \geq \underline{D}_{bst} \qquad b \in \mathcal{B}^W, s \in \mathcal{S}^W, t \in \mathcal{T} \tag{6.3.2}$$

Constraints (6.3.2) replace Constraints (6.1.2) and ensure that only the non-buffer shifts can cover the minimum demand.

$$\sum_{n \in \mathcal{N}} \sum_{b \in \mathcal{B}} x_{nbst} = \Big(\sum_{b \in \mathcal{B}^W} D_{bst} \Big) - \delta_{st}^{D-} + \delta_{st}^{D+} \qquad s \in \mathcal{S}^W, t \in \mathcal{T} \tag{6.3.3}$$

Constraints (6.3.3) replace Constraints (6.1.27) and measure the deviations from average total demand.

$$\sum_{n \in \mathcal{N}} \sum_{t \in \mathcal{T}} x_{n0st} = 0 \qquad s \in \mathcal{S}^F \tag{6.3.4}$$

Constraints (6.3.4) ensure that no nurses can have their off-shifts scheduled as buffer shifts.

$$\sum_{n \in \mathcal{N}} x_{n0st} \leq \beta_{st} \qquad s \in \mathcal{S}^W, t \in \mathcal{T} \tag{6.3.5}$$

Constraints (6.3.5) limit the number of nurses that can be scheduled to the buffer section per shift.

$$\sum_{e \in \mathcal{E}} \sum_{b \in \mathcal{B}} \sum_{s \in \mathcal{S}^W} \sum_{t \in \mathcal{T}} \delta_{ebst}^{E} + \sum_{b \in \mathcal{B}} \sum_{s \in \mathcal{S}^W} \sum_{t \in \mathcal{T}} \delta_{bst}^{SN-} \leq z^5 \tag{6.3.6}$$

Constraint (6.3.6) ensures that the optimal solution from the fifth objective function is upheld when solving the sixth step of the lexicographic order.

Rescheduling Extensions

In the nurse rescheduling model, the buffer shifts are activated for specific sections when there is an increase in demand or a decrease in available nurses. This reactive strategy ensures that the available nurses are optimally utilized to cover unexpected events.

Sets

$\mathcal{B}$	set of sections, $\mathcal{B} = \{1, 2, 3, 0\}$
$\mathcal{B}^W$	set of working sections, $\mathcal{B} = \{1, 2, 3\}$, $\mathcal{B}^W \subset \mathcal{B}$

Similarly, as for the nurse scheduling problem in the buffer strategy, a fourth section is defined for all nurses that are scheduled for a buffer shift. All original constraints in Equations (6.2.1)-(6.2.42) use the set $\mathcal{B}^W$.

Variables

$$a_{nbst} = \begin{cases} 1, & \text{if buffer shift for nurse } n \text{ in section } b \text{ is activated on shift } s \text{ on day } t \\ 0, & \text{otherwise} \end{cases}$$

Constraints

$$\sum_{n \in \mathcal{N}} \sum_{b \in B} x'_{nbst} \geq \sum_{b \in B^W} D_{bst} \qquad s \in \mathcal{S}^W, t \in \mathcal{T}^R / \{t_0\} \tag{6.3.7}$$

Constraints (6.3.7) replace Constraints (6.2.20) and ensure that nurses scheduled to buffer shifts are included when ensuring that demand is covered for the whole planning period.

$$x'_{n0st_0} = 0 \qquad n \in \mathcal{N}, s \in \mathcal{S}^W \tag{6.3.8}$$

Constraints (6.3.8) ensure that, after rescheduling, no nurses can remain scheduled to a buffer shift on the current day.

$$x'_{n0st} = X_{n0st} - \sum_{b \in \mathcal{B}^W} a_{nbst} - x^-_{n0st} \qquad n \in \mathcal{N}, s \in \mathcal{S}^W, t \in \mathcal{T}^R \tag{6.3.9}$$

Constraints (6.3.9) ensure that activated nurses are removed from the buffer and moved to a section.

$$x'_{nbst} = X_{nbst} + x^+_{nbst} + a_{nbst} - x^-_{nbst} \qquad n \in \mathcal{N}, b \in \mathcal{B}^W, s \in \mathcal{S}^W, t \in \mathcal{T}^R \tag{6.3.10}$$

Constraints (6.3.10) replace Constraints (6.2.14) and ensure that the schedule is updated for all nurses.

Variable Declaration

$$a_{nbst} \in \{0,1\} \qquad n \in \mathcal{N}, b \in \mathcal{B}^{\mathcal{W}}, s \in \mathcal{S}^{W}, t \in \mathcal{T}^{R} \tag{6.3.11}$$

6.3.2 Flexible Assignments

The flexible assignments strategy is a proactive approach that enhances the scheduling model's flexibility to handle unforeseen absences. This approach allows for the scheduling of flexible nurses who can be assigned to cover unexpected events. The flexible assignments strategy aims to optimize the utilization of nurses while providing additional flexibility to the rescheduling model in handling unexpected events.

Scheduling Extensions

All full-time nurses are scheduled with an employment percentage of 100-X%. Therefore, there are no changes to the nurse scheduling model except for adjustments to the scheduled hours for full-time nurses.

Rescheduling Extensions

The nurse rescheduling model activates the flexible shifts for all full-time nurses in the flexible assignment strategy. However, there are some restrictions on how these shifts can be activated. For instance, they cannot be activated on shifts that a nurse prefers to avoid working. Moreover, the flexible shifts adhere to many of the same laws and regulations as the regular shifts scheduled by the nurse scheduling model.

Sets

$\mathcal{N}^F$	set of flexible nurses $\mathcal{N}^F \subset \mathcal{N}$

Parameters

I_{nst}	nurse n wants to avoid working shift s on day t
R_n	remaining number of flexible shifts for nurse n

R_n is a time-dependent parameter and is updated for each iteration if the model utilizes a flexible shift.

Variables

$$f_{nbst} = \begin{cases} 1, & \text{if flexible shift for nurse } n \text{ in section } b \text{ is scheduled to shift } s \text{ on day } t \\ 0, & \text{otherwise} \end{cases}$$

Constraints

Some of the scheduling constraints apply to flexible shifts, f_{nbst}, in the rescheduling model. A flexible nurse can not be scheduled to double shifts with their flexible shifts. Thus, Constraints (6.1.8) applies to f_{nbst}. In addition, the constraints concerning the number of consecutive shifts and consecutive nights apply. This refers to Constraints (6.1.11) and (6.1.12), respectively.

$$I_{nst} f_{nbst} = 0 \qquad n \in \mathcal{N}^F, b \in \mathcal{B}, s \in \mathcal{S}^W, t \in \mathcal{T}^R \tag{6.3.12}$$

Constraints (6.3.12) ensure that flexible shifts can not be scheduled for undesired shifts.

$$\sum_{b \in \mathcal{B}} \sum_{s \in \mathcal{S}^W} \sum_{t \in \mathcal{T}^R} f_{nbst} \leq R_n \qquad n \in \mathcal{N}^F \tag{6.3.13}$$

Constraints (6.3.13) ensure that the number of scheduled flexible shifts for a nurse never exceeds the remaining number of flexible shifts.

$$x'_{nbst} = X_{nbst} + x^+_{nbst} + f_{nbst} - x^-_{nbst} \qquad n \in \mathcal{N}^F, s \in \mathcal{S}^W, b \in \mathcal{B}^W, t \in \mathcal{T}^R \tag{6.3.14}$$

$$x'_{nbst} = X_{nbst} + x^+_{nbst} - x^-_{nbst} \qquad n \in \mathcal{N}/\{\mathcal{N}^F\}, s \in \mathcal{S}^W, b \in \mathcal{B}^W, t \in \mathcal{T}^R \tag{6.3.15}$$

Constraints (6.3.14) ensure that the schedule is updated for flexible nurses. Constraints (6.3.15) ensure that the schedule is updated for all nurses except flexible nurses. These constraints replace Constraints (6.2.14).

Variable Declaration

$$f_{nbst} \in \{0,1\} \qquad n \in \mathcal{N}, b \in \mathcal{B}, s \in \mathcal{S}^W, t \in \mathcal{T}^R \tag{6.3.16}$$

6.4 Illustrations

This section displays several illustrations that are created as supplements for further understanding the presented optimization model. Section 6.4.1 illustrates the connection between the various models, while Section 6.4.2 presents illegal patterns. Section 6.4.3 illustrates how the rescheduling process works.

6.4.1 Connection between the Models

The nurse scheduling model and the nurse rescheduling model share many commonalities in terms of indices, sets, and parameters. Most shared elements are described identically in both models to ensure seamless integration. The set of nurses used in both models remains the same, maintaining consistency throughout the scheduling and rescheduling processes. Although nurse competence is not explicitly utilized in the nurse rescheduling model, it is relevant for determining nurse wages in the rescheduling costs.

In the rescheduling model, the parameter X_{nbst} represents the scheduled values for all nurses in all sections on each day within the planning period. In both models, the set of all days denoted $\mathcal{T}$ corresponds to all days the model considers. In the scheduling model, this value is dependent on the number of weeks to schedule. However, in the rescheduling model, the set of all days is dependent only on sets $\mathcal{T}^{PRE}$, $\mathcal{T}^{R}$, and $\mathcal{T}^{POST}$. On the first day of the rescheduling model, X_{nbst} reflects the output schedule from the scheduling model, where each value corresponds to the decision variable x_{nbst} from the scheduling model. As the rescheduling process progresses through iterations, the values of X_{nbst} are updated to incorporate the completed rescheduling actions. This ensures that the rescheduled values accurately reflect the modifications made to the original schedule.

Figure 6.1 illustrates the connectivity of all models, offering a preview of the fully integrated system. To further improve uncertainty handling, a machine learning model can be included. The machine learning model aims to enhance the observed demand information using predictions. When the machine learning model provides predictions on the nurse demand to the rescheduling model, these predictions replace the historical average demand parameter with a predicted demand parameter. The machine learning method is described in detail in Section 8.2.

As illustrated in Figure 6.1, the rescheduling model and the machine learning model are embedded in a rolling horizon simulation framework which solves each day with updated predictions from the machine learning model. The simulation framework will be explained more in-depth in Chapter 7.

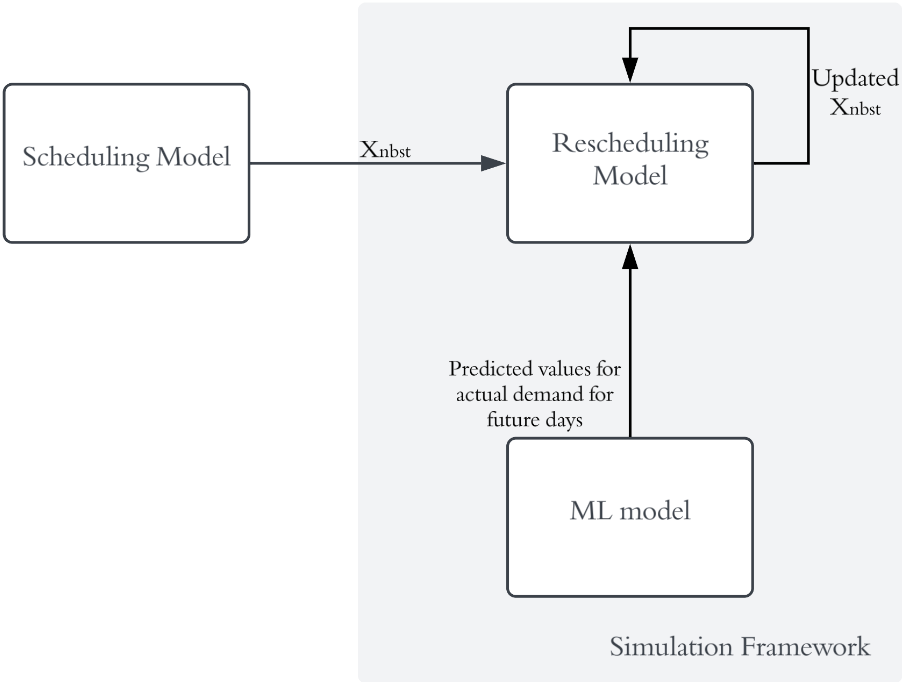


Figure 6.1: Connection between the Nurse Scheduling Problem and the Nurse Rescheduling Problem.

6.4.2 Illegal Patterns

Minimum Rest in a Week

Constraints (6.1.22) specify that all nurses must be scheduled to an F1 shift every week, which entails a weekly minimum consecutive rest. Constraints (6.1.19)-(6.1.21) in the presented optimization model represent illegal shift patterns for this minimum consecutive rest during a week. As presented in Chapter 2, the duration of an F1 shift amounts to 35 hours. Figure 6.2 illustrates the illegal shift patterns in the schedule. The red sections in the figure indicate free periods for the scheduled nurse, while the blue sections represent scheduled shifts. The combinations are illegal because the duration of the F1 shift is less than 35 hours.

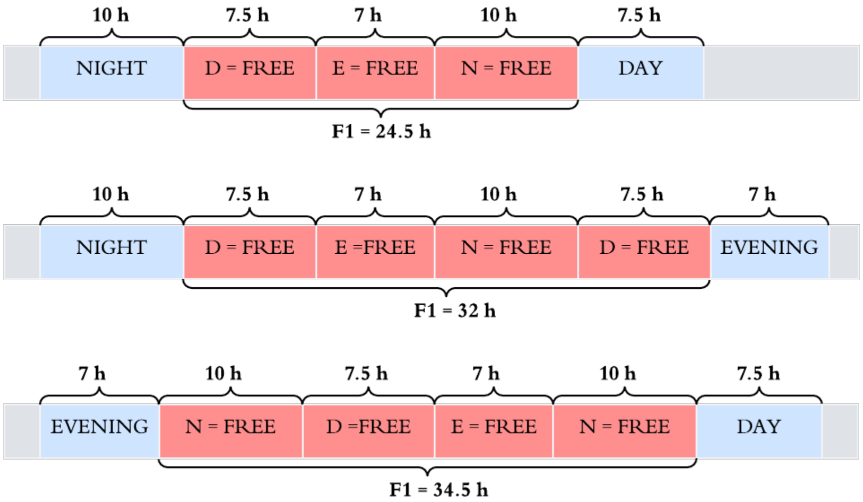


Figure 6.2: Illegal shift patterns for ensuring F1 day.

Minimum Rest in a 24-hour period

Constraints (6.2.31)-(6.2.34) in the presented optimization model represent illegal shift patterns for rescheduling. These patterns are based on the regulations presented in Chapter 2, which specify that a nurse must have a minimum of 10 hours of rest within a 24-hour timeframe. Figure 6.3 illustrates all illegal shift pattern combinations. The red sections in the figure indicate free periods for the scheduled nurse, while the blue sections represent scheduled shifts. The combinations are illegal because they violate the minimum rest requirement.

Chapter 2 specifies the shift durations. Consecutive shifts overlap for 15 minutes, resulting in the total shift durations summarizing to 24.5 hours.

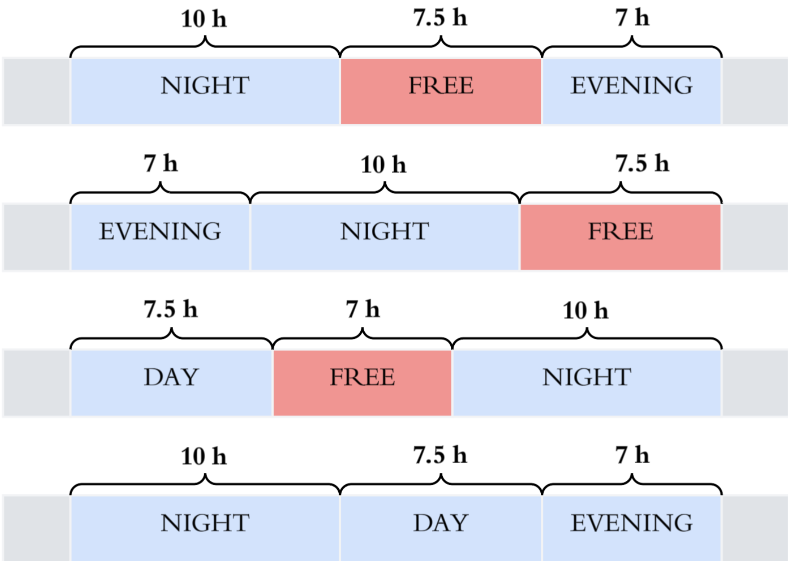


Figure 6.3: Illegal shift patterns for ensuring enough rest in a 24-hour period.

6.4.3 Rescheduling Example

Table 6.2 illustrates an example of a 7-day schedule for six different nurses, where day 6 and 7 represent the weekend. The nurses belong to three different sections within the same department. The abbreviations D, E, N, and F denote a day, an evening, a night, and an off shift, respectively.

The schedules in Table 6.2 are subject to uncertainty. During the execution of the schedules, several absences occur. Available nurses then fill these absences across all three sections. The output is a modified schedule where the supply in all understaffed shifts has been adjusted to meet the demand. The modified schedule is illustrated in Table 6.3, where the absences are marked in red, and the rescheduled shifts as a result of these absences are marked in green.

Table 6.2: Input schedule before rescheduling occurs.

		Days						
		1	**2**	**3**	**4**	**5**	**6**	**7**
Section 1	Nurse 1	D	D	F	D	N	F	F
	Nurse 2	E	E	D	F	F	E	D
Section 2	Nurse 3	N	N	F	F	D	F	F
	Nurse 4	F	D	E	F	E	D	E
Section 3	Nurse 5	D	D	N	N	F	F	F
	Nurse 6	F	E	F	F	D	N	N

Table 6.3: Modified schedule after rescheduling.

		Days						
		1	**2**	**3**	**4**	**5**	**6**	**7**
Section 1	Nurse 1	D	D	D	D	N	F	F
	Nurse 2	-	-	-	F	F	E	D
Section 2	Nurse 3	N	N	F	F	D	N	N
	Nurse 4	F	D+E	F	E	E	-	E
Section 3	Nurse 5	D	D	N	N	F	D	F
	Nurse 6	E	E	F	F	D	-	-

Chapter 7

Simulation Framework

A simulation framework is developed to implement the rescheduling model as a rolling horizon. We use this framework to review how the rescheduling model performs over time and evaluate the quality of the generated schedules. Section 7.1 presents a general overview of the simulation framework. Next, Section 7.2 describes how the framework generates instances. Finally, Section 7.3 discusses the limitations of the results.

7.1 General Overview

The simulation framework generates nurse absences and demand requirements for the nurse rescheduling model. Absence and future demand are assumed to be stochastic. New absences with their corresponding durations are notified each morning, and the actual daily demand is revealed. Demand requirements for upcoming days are based on average values from historical data as a simple estimate for the upcoming demand. With the inclusion of the machine learning model, the demands for the upcoming days are based on the model's predictions. As described in Chapter 5, the rescheduling model runs each morning, updating the schedule provided by the nurse scheduling model.

The rolling horizon simulation begins with generated absences and actual demand for the first iteration and then solves the rescheduling problem. The output from the model updates the input schedule for the following day. The dynamic parameters, described in Section 6.2.1, are corrected using the updated input schedule. Next, new absences and actual demand for the following day are provided. This step represents the move from day t to day $t+1$. Figure 7.1 illustrates how the system develops throughout the simulation period.

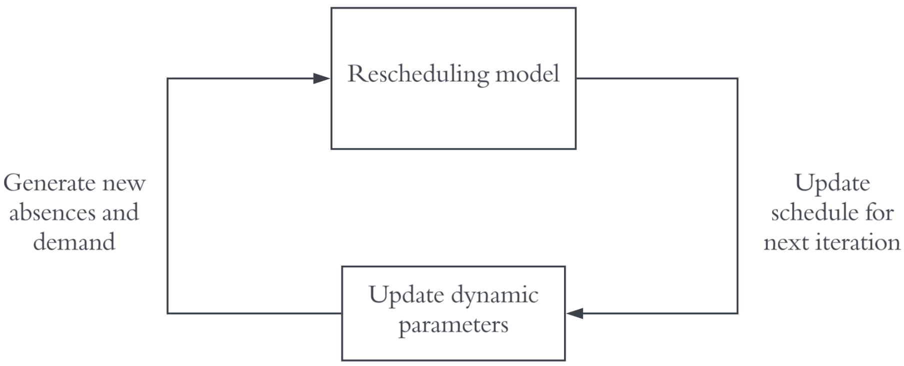


Figure 7.1: Overview of the simulation framework.

Figure 7.2 displays the time development in the rolling horizon and which parts of the total timespan the rescheduling model considers in each iteration. The displayed scheduling period corresponds to the period covered in the scheduling model. The schedule represents a defined scheduling period over a given number of weeks.

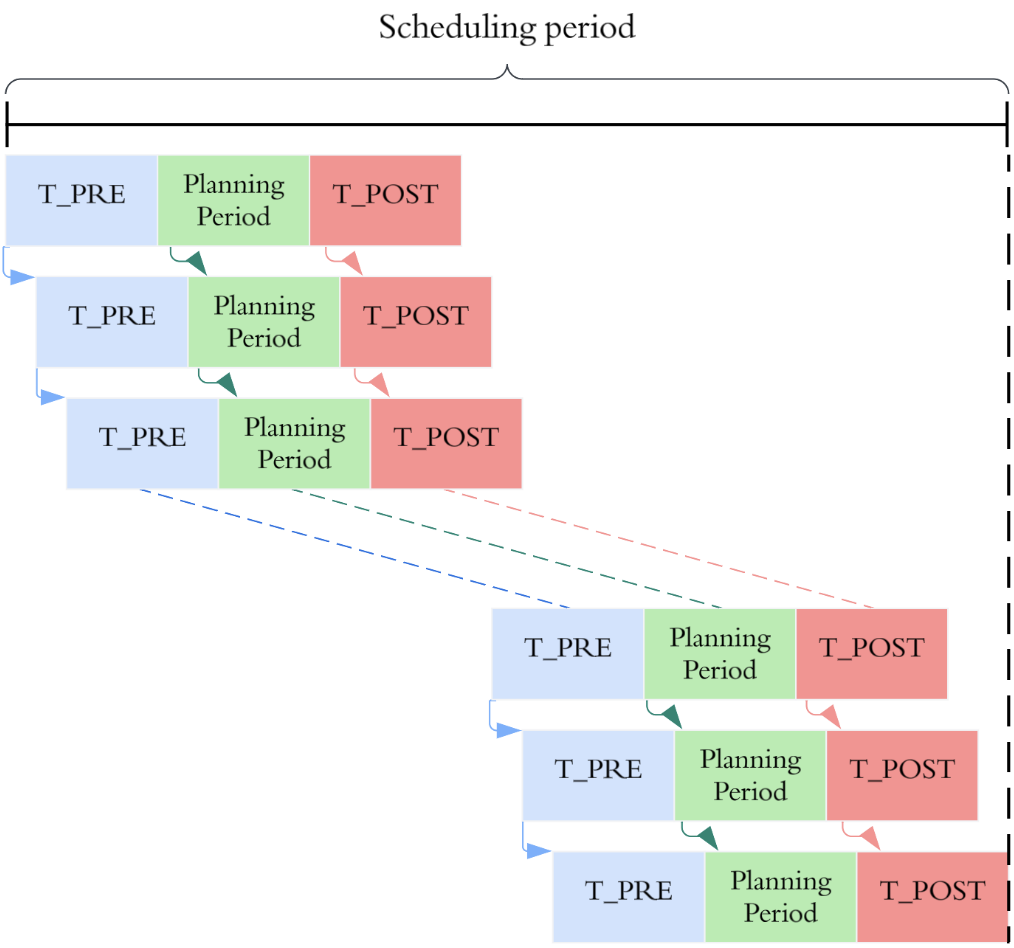


Figure 7.2: Time development in the framework

7.2 Random Number Generation

Each simulation iteration generates new values for the stochastic variables. The absence generation is based on numerical absence data provided by CC, while the actual demand values are based on actual staffing levels at CC.

7.2.1 Generation of Absences

A Markov model has been developed based on the possible states each nurse can be in and the transitions between these states. Using this Markov model entails that the transition probabilities apply to all nurses in the system and that probabilities are constant for the duration of the simulation. Figure 7.3 shows the possible states and how the nurses can move between states.

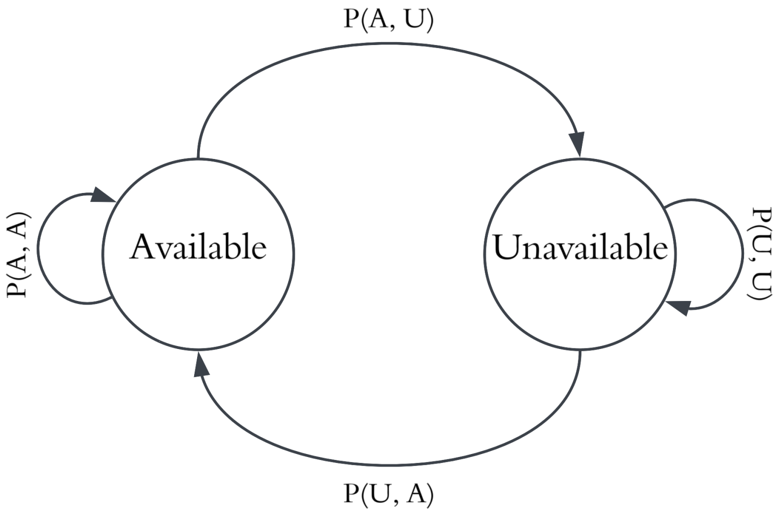


Figure 7.3: Transitions between the states Available (A) and Unavailable (U) with corresponding transition probabilities.

As explained in Chapter 5, the absence duration is assumed deterministic at the point of rescheduling. Consequently, the absences for the whole scheduling period are generated in advance using the Markov model. These values are used to update the A_{nt} matrix during one simulation. The simulation framework ensures that today's absences are revealed to the rescheduling model for each iteration in the rolling horizon. The rescheduling model also holds information about previously registered absences.

Table 7.1 illustrates what absence information is available to the rescheduling model at the time of the rescheduling. The green and red cells represent known information about the nurses' availability for the model today, t_0. If a cell takes the value 1, it means a nurse is available, while 0 represents an absence. The grey cells represent unknown values and are all assumed to have the value 1. The A_{nt} matrix is updated with only the available information.

Table 7.1: Illustration of the model's absence information.

	Days			
	t_{-1}	t_0	t_1	t_2
Nurse 1	1	0	0	1
Nurse 2	0	0	0	1
Nurse 3	1	1	1	0
Nurse 4	1	1	1	1
Nurse 5	1	1	0	0

7.2.2 Demand Sampling

Demand values are sampled in periods corresponding to the duration of the schedule. Each morning the actual demand for that day is revealed to the model to act as the minimum required staffing per shift this day. The demand for the remainder of the period is set by the historical average of the demand levels.

The actual demand serves as today's demand requirement, while the average demand is the required demand for the remaining days included in the planning period and post period. Table 7.2 shows the model knowledge for each step in the simulation.

Table 7.2: Illustration of the model's demand information.

		Planning period				
Days	t_{-1}	t_0	t_1	t_2	t_3	t_4
Average	9	9	8	4	4	8
Actual	11	12	9	3	3	10
Knowledge	11	12	8	4	4	8

7.3 Limitations

The absence generation does not consider the scheduled workload. There could be a significant correlation between workload and absence, but this is not handled in the absence generation process.

The Markov model generates the absences using estimated parameters. The actual transition probabilities are unknown, and different transition probabilities yield different results. The Markov model does not handle potential seasonal variations.

The historical values for average demand are not necessarily an accurate measure of upcoming demand. As an example, the COVID-19 pandemic could skew the numbers in one direction or the other and create an imbalance in the average values. However, the average values would provide a pointer for the expected supply.

Chapter 8

Data Analysis and Machine Learning Method

This chapter presents the data that has been retrieved for this thesis and the implemented machine learning methods. Section 8.1 introduces and analyzes each dataset. Next, Section 8.2 describes the implementation of the machine learning methods and illustrates the implementation of machine learning with optimization.

8.1 Data Analysis and Prepossessing

In this section, three datasets are analyzed. First, the preference data is presented and discussed. Next, the nurse and activity data are presented and analyzed to provide a better understanding of the datasets.

8.1.1 Preference Data

The preference data consists of expressed disinterest in specific shifts from the current employees at CC. We created an employee survey for the nurses at the three bed wards and gathered preference data from their answers. A total of 39 answers were collected from the survey. The data presented in this section is used in the preference generation process, which is explained in Appendix D.

Figure 8.1 presents the expressed disinterest in the shifts during the weekdays. The general trend is that night shifts are unpopular, and day shifts are preferred. Not surprisingly, Friday evening is not popular as it is the evening shift leading up to the weekend. Other fluctuations can result from which days the clinic schedules surgeries or other procedures.

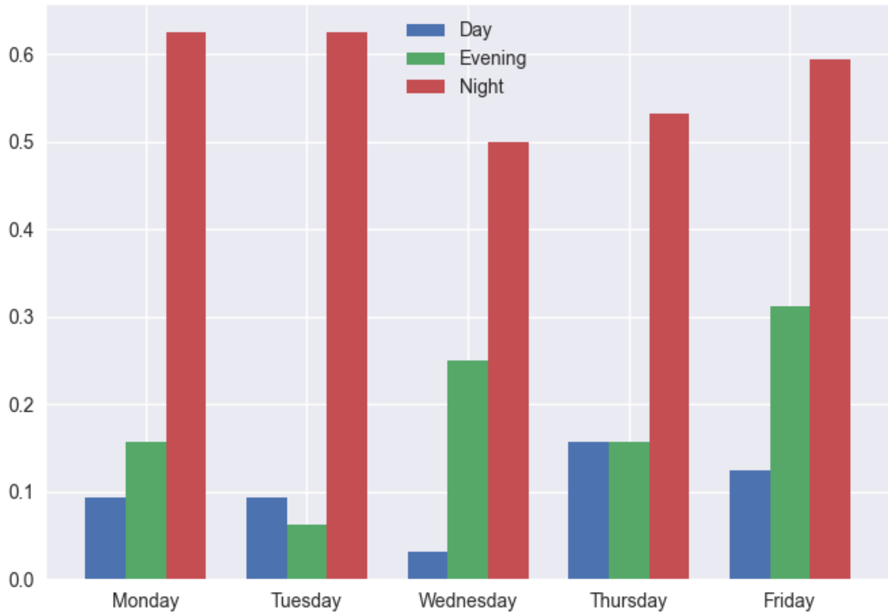


Figure 8.1: The fraction of nurses who have expressed disinterest for specific weekday shifts.

As described in Chapter 5, nurses work in specific weekend rotations. A nurse works one day and one evening shift or two night shifts during a work weekend. Figure 8.2 illustrates the expressed disinterest in weekend shifts. Since the shifts are assigned dependent on each other, the values displayed in the figure represent the average values for day, evening, and night shifts on Saturday and Sunday.

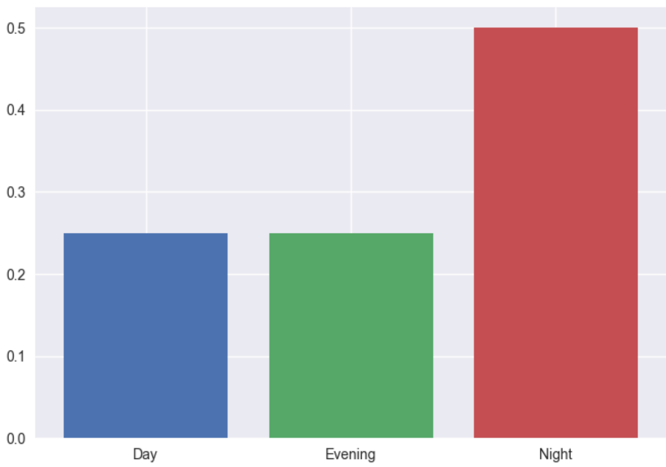
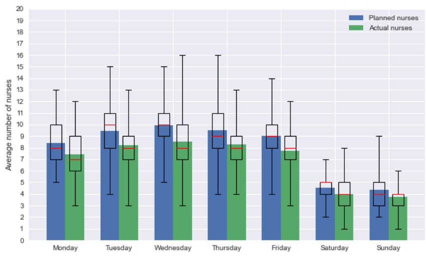


Figure 8.2: The fraction of nurses who have expressed disinterest for specific shifts on the weekend.

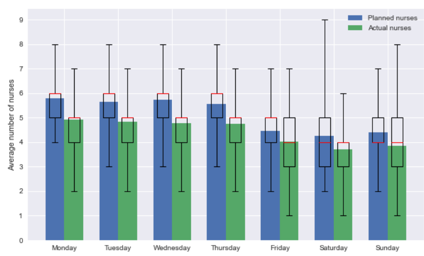
8.1.2 Nurse Data

The planned nurse schedule can differ significantly from the actual nurse coverage due to unforeseen events such as absences. The retrieved nurse data includes the planned and actual nurse coverage for all shifts from the three bed wards from January 2018 to November 2022.

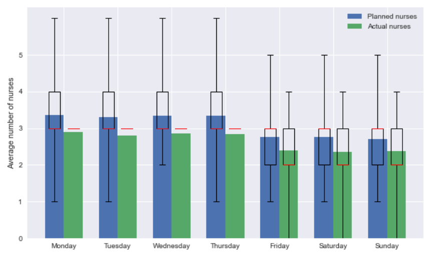
As described in Chapter 2, the standard shifts are day shift, D, evening shift, E, and night shift, N. In the schedule, these shifts have various shift codes connected to each. To generalize the data, all shift codes have been aggregated based on their corresponding shift types to provide an overview of all the shifts associated with either day, evening, or night shifts. Figure 8.3 shows the average number of nurses working in Bed Ward 1 for each shift type, D, E, and N, respectively. The blue bars represent the planned number of nurses, while the green bars indicate the actual number of nurses who worked. It is worth noting that the values on the y-axis are different for the three plots. The variations in the day shifts are more significant than in the evening and night shifts. The analysis reveals consistent trends across all three bed wards. On average, the planned number of nurses consistently exceeds the actual number of nurses for all shift types. Moreover, the figures demonstrate that nurse demand tends to be higher on weekdays than on weekends.



(a) Day shift



(b) Evening shift



(c) Night shift

Figure 8.3: Average planned and actual staffing of nurses in Bed Ward 1 for different shift types. The blue bar is the planned staffing for nurses, and the green bar is the actual nurse staffing.

The box plots in Figure 8.3 provide insights into the variability of the data. The boxes represent the interquartile range (IQR), capturing the middle 50% of the data for each shift. In a box plot, the whiskers are lines set to extend at most two times the IQR size from the box's edges. Data points lying outside the whiskers are considered outliers within the dataset. However, these outliers are not visually represented in these plots. The red horizontal line within the box represents the median value for each bar. The values in the dataset are all integers. As a result, the edges of the box, the median, and the ends of the whiskers coincide with integer values in the box plots.

For some of the night shifts in Figure 8.3c, the box plot does not display any boxes or whiskers. This is because the IQR for these days only contains the number 3. Consequently, since the whiskers' length depends on the IQR's size, they are not visible in this case since they will start and end in 3. The box plots show a considerable variation in the nurse demand for Saturday and Sunday day and evening shifts. As presented in Chapter 2, the three bed wards borrow nurses from other sections on weekend shifts. Therefore, weekend staffing varies depending on how many nurses the bed wards utilize from other sections.

Figure 8.4 illustrates the distribution of the number of nurses working the day shifts on Mondays in Bed Ward 1 in the dataset. Most Mondays have between 6 and 9 nurses working, although there is some variation. The same dataset is the basis for determining the average nurse scheduling and rescheduling demand, specifically for day shifts on Mondays. Similarly, the average weekly demand for all other shifts is determined based on the corresponding shift data for the other shift types and bed wards.

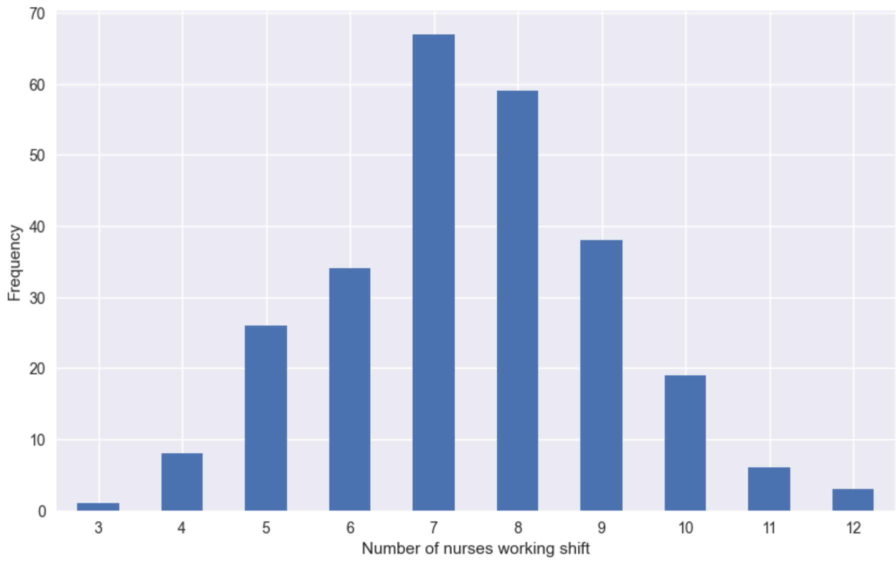


Figure 8.4: Number of nurses for day shift in Bed Ward 1 on Mondays in the dataset.

Figure 8.5 displays the monthly average of the planned and actual number of nurses. The figure shows no clear indication of any significant seasonal variations. This could imply that seasonality does not have a large impact when determining nurse demand.

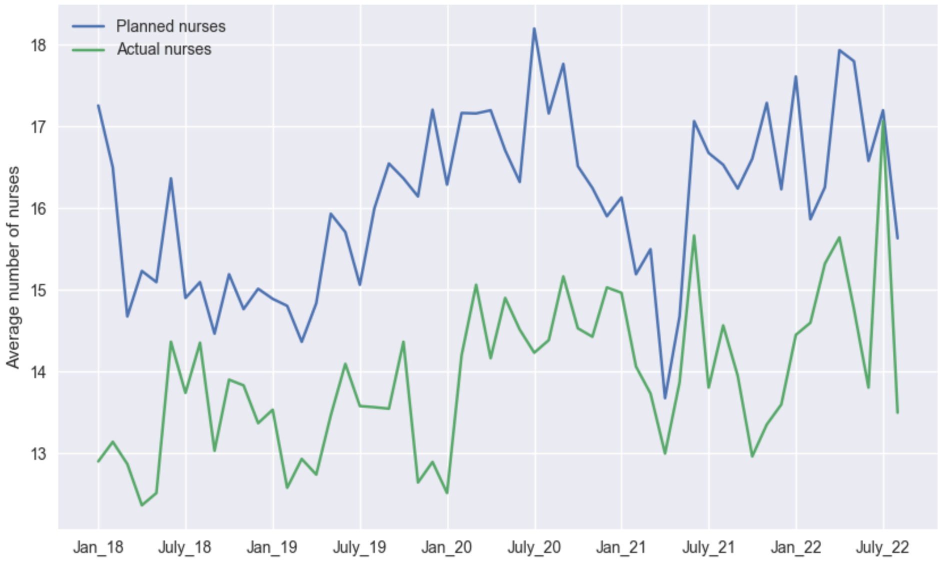


Figure 8.5: Average number of working nurses for all shifts in Bed Ward 1 for each month in the dataset.

8.1.3 Activity Data

The activity data primarily focuses on the patient-related activities within each section. The provided data for the three bed wards spans from January 2018 to November 2022 and includes information about the number of patient arrivals and departures and patient demographics such as age and diagnoses.

Patient load is a significant factor contributing to the variation in the need for nurses. It is, therefore, interesting to analyze potential factors that can influence patient workload and, consequently, the demand for nurses. Various factors can contribute to an increase in the patient load. Examples are the number of patients in need of nursing care, the presence of patients with severe and complex diseases, patients with multiple comorbidities, and age. To better understand the dataset, we got input from the section managers and the clinic manager at CC regarding patient load. Their expert knowledge helped us identify some factors that consistently influence the nurse demand at different levels of patient load.

The clinic manager explained that the patient load often increases with the patient's age and that the total length of stay (LOS) for the patients could indicate the patient's need for nursing care. Figure 8.6 presents the distribution of the patients' ages. The figure shows that most patients in the bed wards are between 60 and 80 years. Diagnosis for older patients is often more severe compared to younger patients with the same diagnosis. An older patient group can therefore be an indication of higher nurse demand. Figure 8.7 displays the average LOS for patients in each age group. The figure demonstrates a positive correlation between age and LOS. In addition, the box plot shows that the variation in the LOS for patients increases with age. As described earlier, in Section 8.1.2, the box plots are set to not show the outliers, but the LOS data contains many outliers.

Ages groups from 15 to 40 place the top edge of the box lower than the average LOS. This indicates that these age groups include large outliers that greatly influence the average value.

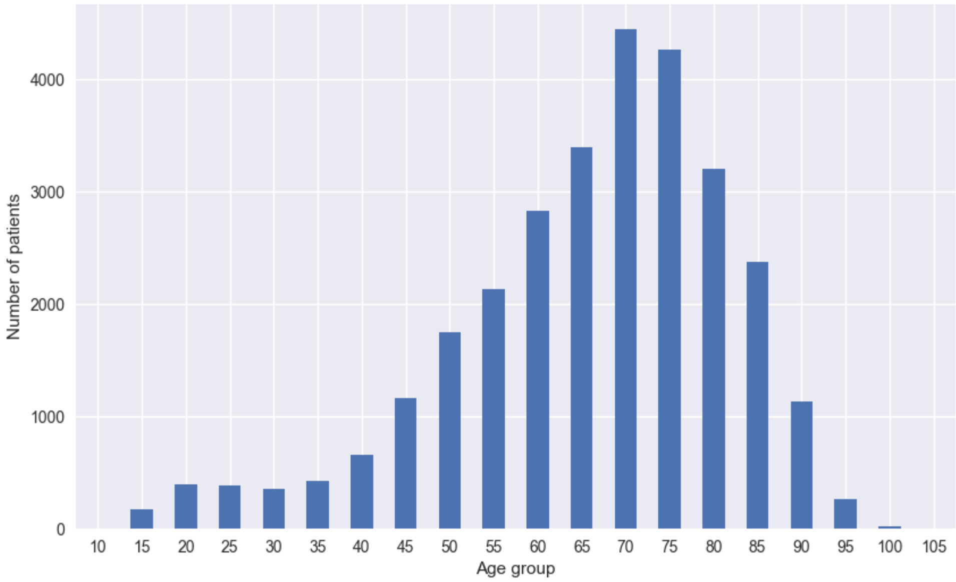


Figure 8.6: Number of patients in each age group.

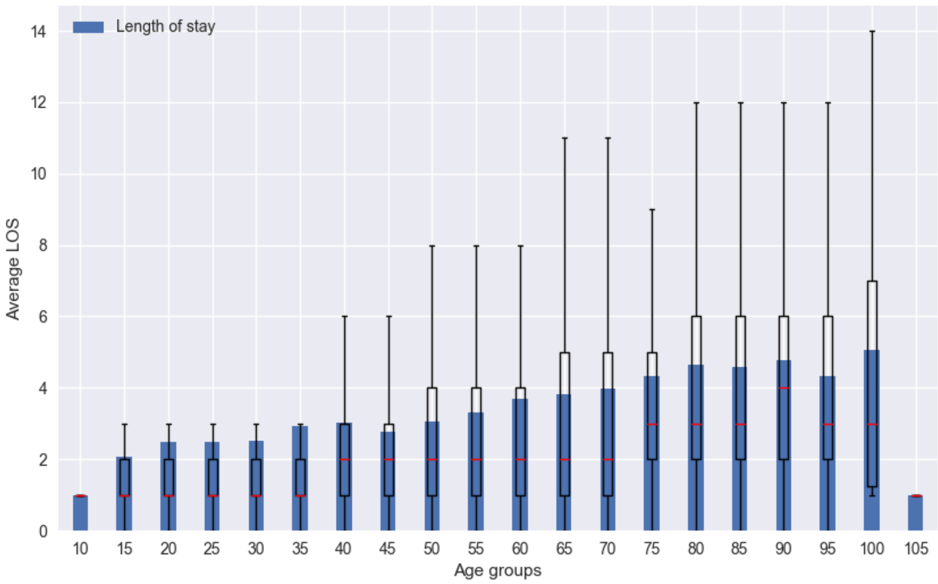


Figure 8.7: Average length of stay (LOS) for patients in each age group.

The clinic manager also informed us that cardiac failure is among the most challenging medical conditions encountered in the bed wards. The patients arriving with cardiac failure are always acute and critical. Cardiac failure typically requires a significant amount of nursing care regardless of the patient's age. Figure 8.8 shows the fraction of patients with cardiac failure within each age group. It shows that the occurrences of cardiac failure generally increase with age. Figure 8.9 shows the average LOS for patients with and without cardiac failure. As with age, cardiac failure shows a clear positive correlation with the total LOS.

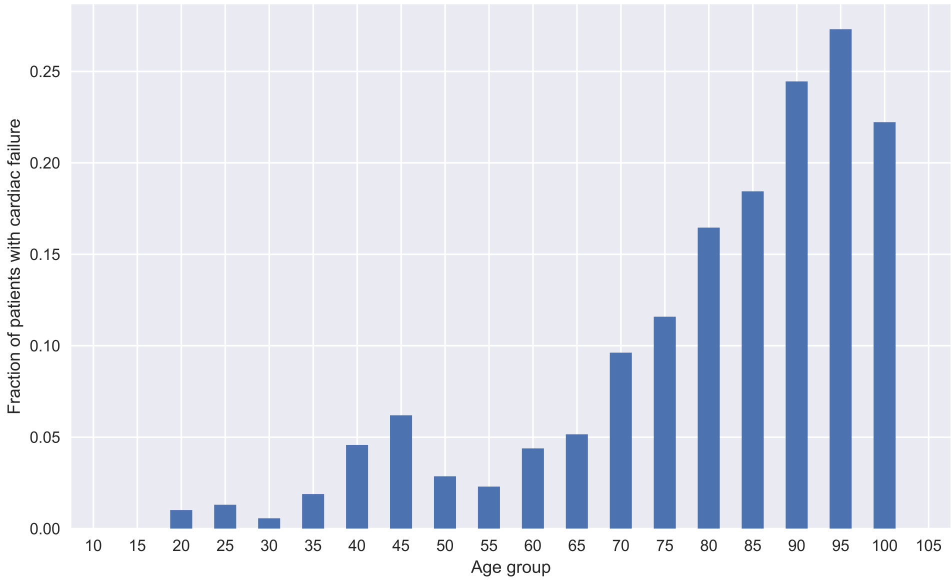


Figure 8.8: The fraction of patients in each age group with cardiac failure.

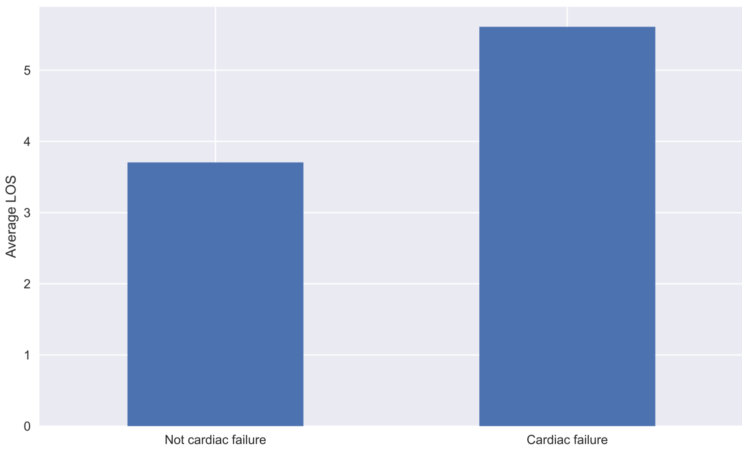


Figure 8.9: Average length of stay (LOS) for the patients with and without cardiac failure.

8.2 Machine Learning Methods

This section provides an overview of the machine learning methods employed in this thesis. These methods aim to evaluate whether they can yield more accurate predictions of nurse demand compared to the historical average demand and, as a result, reduce rescheduling costs in the nurse rescheduling model. The machine learning models should be capable of predicting the nurse demand based on the input features, considering a specific number of future days from the time the predictions are generated.

Figure 8.10 illustrates how the input data has been created. The dataset used to create machine learning models is a combination of the nurse data and activity data presented in Section 8.1. The dataset is combined on the date and only includes numerical data. The input data comprises sparse information, primarily due to incorporating features like age groups and weekdays. The prediction models' target feature is the actual demand for nurses for future days. This feature represents the integer count of nurses required for each shift on a specific day, making it suitable as both a target value for classification and regression problems.

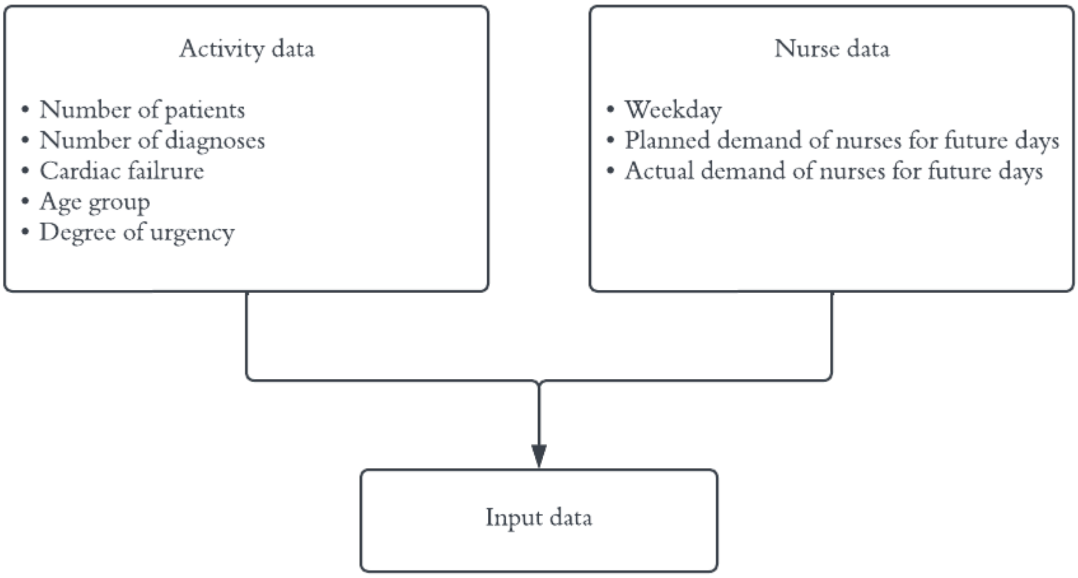


Figure 8.10: The merge of activity and nurse data into input data for the machine learning models.

Given the substantial number of features in the input data, it becomes critical to employ a method capable of figuring out which features contribute to patterns within the dataset. The models chosen and described in this section are a neural network model and a decision tree model. By implementing both models, we can evaluate their performance and determine the one that achieves the best results for further analysis. This comparison enables us to decide on the most effective machine learning model.

The following sections explain the architecture, hyperparameters, features, and the training procedure for the machine learning models. In addition, the implementation of the machine learning models and the optimization models is described and visualized.

8.2.1 Artificial Neural Network

The implemented neural network architecture comprises one input layer and one output layer. The neural network can be initialized with a number of hidden layers, enabling the model to find non-linear patterns in the data. It is essential to keep the number of hidden layers small enough not to make the model too complex. For classification cases, the output layer consists of a number of neurons equal to the unique number of historic numbers of actual demand. For regression problems, the output layer contains one single neuron.

Table 8.1 describes the hyperparameters of the artificial neural network, which are parameters that are set before the learning process begins and determine the architecture and behavior of the network. These parameters are not learned from the data but are manually chosen to ensure that the neural network performs well. The process of finding the optimal hyperparameters is often done through experimentation and testing to find the values with the best performance for the demand predictions.

Table 8.1: Hyperparameters for the neural network.

Parameter	**Description**
Neurons in hidden layer	Choosing the number of neurons in hidden layers involves balancing model complexity for capturing patterns without overfitting or underfitting.
Activation function	The activation function determines the non-linearity and learning capacity of the model. Activation functions are utilized in the hidden layers, and for classification models, an activation function is also utilized in the output layer.
Loss Function	The loss function quantifies the difference between predicted and true values. Regression and classification models generally use different loss functions.
Optimizer	The optimizer impacts training speed and quality.
Epochs	The number of epochs should be large enough for the loss to converge.
Learning Rate	The learning rate is based on finding a value where the loss converges rather than diverges during the training process.

8.2.2 Decision Tree

A decision tree was implemented as an alternative machine-learning method to assess its performance compared to the neural network. The decision tree is also implemented as both a regression problem and a classification problem. Table 8.2 describes the hyperparameters used for the decision tree. Same as for the neural network, these hyperparameters are set before the training of the model.

Table 8.2: Hyperparameters for decision tree.

Parameter	Description
Splitting criteria	The selection of the splitting criteria for the decision tree is important as it directly impacts the quality of the splits made at the decision nodes. The choice of splitting criteria is based on the numerous features and sparse data.
Maximum depth	The maximum depth is determined by trying different values and analyzing what performs best. A larger maximum depth could result in an overly complex decision tree, especially when dealing with many features. This can make it more difficult to retrieve patterns and potentially lead to overfitting, where the tree memorizes the training data instead of learning general patterns.

8.2.3 Feature Selection, Training, and Comparison

Feature Selection

The actual demand for nurses in each of the sections for each shift type is based on the input data. The actual demand is revealed on day t_0, corresponding to today. The planned staffing and the actual demand are used as inputs for the models. The models predict the nurse demand for future days in the planning period, aiming to predict future demand accurately. The planned staffing for the day to predict is also included as a feature.

The selection of the features in the input data for the machine learning models was based on an analysis of the dataset, aiming to identify existing patterns within the data. Additionally, input from the clinic manager played a significant role in determining which features should be included in the models. According to the clinic manager, certain factors are known to contribute to an increased nurse demand, and these factors were considered during the feature selection process.

The clinic manager specifically highlighted two key features that were believed to be critical factors influencing nurse demand: the occurrence of cardiac failure and the age of the patients. The input data also includes other features, such as the current day, the number of patients in one section on a specific day, the number of comorbidities, and if the patients are acutely or planned hospitalized.

The features in the input dataset are date-specific and connected to specific shifts (D, E, N) and to the bed wards. To utilize the activity data linked to specific patients, it was necessary to merge the data based on the dates in a reasonable way. One prediction model for each shift type (D, E, and N) has been created for each bed ward. The input features in Table 8.3 are for a specific date for a specific bed ward.

Table 8.3: The format of the input data for the models.

Feature	**Values**
Number of patients	Sum
Cardiac failure	Sum
Degree of urgency	Sum
Number of diseases	Mean
Planned number of nurses day 0	Sum for specific shift
Planned number of nurses day 1	Sum for specific shift
Planned number of nurses day 2	Sum for specific shift
Actual number of nurses day 0	Sum for specific shift
Age groups	Sum for each age group
Weekdays	Mon, Tue, Wed, Thu, Fri, Sat, Sun

Training

The input data was divided into two subsets: a training set and a test set to facilitate effective training and evaluation of the neural network. The training set was comprised of 80% of the original dataset, while the remaining 20% was reserved for the test set, ensuring an unbiased assessment of the model's performance.

For the neural network, to further enhance the training process and prevent overfitting, a validation set was extracted from the training set to evaluate the model's performance during training. This validation set accounted for 20% of the training data and was randomly chosen.

The models have been trained on data spanning the years 2018 to 2021. However, the models were not trained on the data from 2022. The 2022 data is the data simulated in the rescheduling model.

Comparison

The mean absolute error (MAE) is used as a metric to compare two regression models. Further, for comparison between a classification and a regression model, accuracy is the comparison metric used. The accuracy is a percentage of how often the prediction is correct, and the MAE is the average deviation from the actual demand. These metrics indicate how well the models perform.

8.2.4 Implementation of Machine Learning in the Optimization Model

Figure 8.11 is an extension of Figure 6.1. It shows the connectivity of the optimization models and the machine learning model, with a particular focus on the machine learning model. The machine learning models have been structured into two distinct stages. The decision to employ a two-stage process is motivated by the recognition that the predicted value of nurse demand from the initial step can potentially influence subsequent predictions. The prediction for day 1 serves as an additional input variable to the next prediction model, which predicts the value for day 2. These predicted values are sent to the rescheduling model and used as input parameters.

In Figure 8.11, the features known on day 0 are highlighted in blue. The planned demand for the specific day of prediction is highlighted in yellow. In stage 2, the predicted value is displayed with a green color.

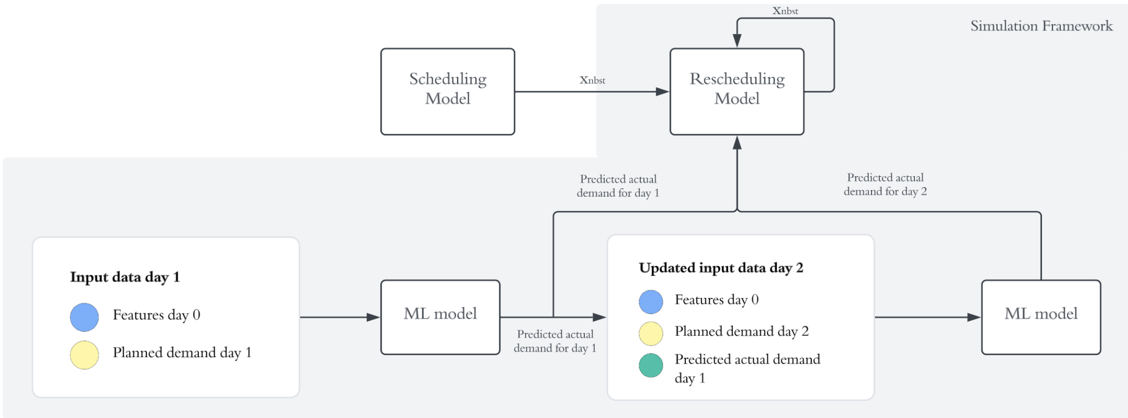


Figure 8.11: Implementation of all the models, both optimization and machine learning models.

New Parameter

D^P_{bst} Predicted demand in section b for shift s on day t

New Demand Constraints

$$\sum_{n \in \mathcal{N}} x'_{nbst} \geq D^P_{bst} \qquad b \in \mathcal{B}, s \in \mathcal{S}^W, t \in \mathcal{T}^R / \{t_0\} \tag{8.2.1}$$

$$\sum_{n \in \mathcal{N}} \sum_{b \in B} x'_{nbst} \geq \sum_{b \in B^W} D^P_{bst} \qquad s \in \mathcal{S}^W, t \in \mathcal{T}^R / \{t_0\} \tag{8.2.2}$$

Constraints (8.2.1) and (8.2.2) show the updated demand constraint in the rescheduling model. These constraints incorporate machine learning-based predicted demand, D^P_{bst}, replacing the historical average demand D_{bst} in the original demand Constraints (6.2.20) and (6.3.7) for the days included in the planning period.

Table 8.4 shows the rescheduling model's knowledge when incorporating the machine learning model's predictions. As the table displays, the demand in the planning period is decided by predictions and not average demand as in Table 7.2.

Table 8.4: Illustration of the model's demand information with machine learning predictions.

		Planning period				
Days	t_{-1}	t_0	t_1	t_2	t_3	t_4
Average	9	9	8	4	4	8
Predicted	12	10	10	3	-	-
Actual	11	12	9	3	3	10
Knowledge	11	12	10	3	4	8

Chapter 9

Computational Study

This chapter introduces a series of test cases designed to evaluate the effectiveness of the models proposed in this thesis. The objectives are to assess whether proactive scheduling improves efficiency and reduces the costs of rescheduling and to analyze if machine learning-driven demand predictions improve the rescheduling process.

In Section 8.2, we presented machine learning models to improve the results of the nurse rescheduling model. First, these methods must be evaluated and analyzed in order to determine which methods should be utilized in combination with the rescheduling models.

Furthermore, several test cases are developed to enable us to comprehensively assess the performance of the formulated optimization models in addressing the nurse scheduling and rescheduling problems outlined in Chapter 5. The baseline models formulated in Chapter 6 are created to solve the nurse scheduling and nurse rescheduling problems. To further enhance these baseline models and better handle uncertainties, we have introduced model extensions that incorporate proactive measures in the scheduling model and corresponding reactive measures in the rescheduling model.

The computational outcomes of the scheduling models are analyzed by comparing them to predefined criteria, ensuring that schedules employing proactive strategies yield at least comparable results to the baseline schedule. Subsequently, the corresponding rescheduling models are evaluated by their ability to address uncertainties and reduce total costs, with a comparative analysis between the baseline rescheduling model and the models utilizing the strategies.

The final step in this chapter aims to combine the results from the analysis of the machine learning methods with the optimization models and evaluate whether incorporating the demand predictions yield better results than using demand based solely on historical averages. Improved results should limit the need for rescheduling, thus decreasing the total costs.

Outline

Firstly, Section 9.1 presents the hardware and software specifications for running the models. To ensure the comparability of results, all models are executed in the same test environment. Section 9.2 describes the parameter values for all static parameters in the optimization models, which remain the same for all test cases. To ensure the models provide as realistic results as possible, all parameter values are based on real-world data from CC. Section 9.3 provides the experimental plan for all test cases related to the optimization models. This includes test cases for evaluating the baseline models and the model extensions, along with various test cases featuring different absence and demand information levels.

In Section 9.4, the results from the machine learning models implemented are examined, and results in the decision of which model to utilize for the rescheduling model. Continuing, all the test cases from Section 9.3 are comprehensively analyzed and compared. Finally, Section 9.5 concludes the computational study by discussing the limitations of the results.

9.1 Test Environment

The optimization models and machine learning methods are implemented in Python. The optimization models are solved using the commercial optimization software Gurobi Optimizer. All tests are completed using a Dell Inc. OptiPlex 9020 computer. Table 9.1 includes further descriptions of the software- and hardware-specific information used to implement and solve the models.

Table 9.1: Details of software and hardware specifications.

Processor	Intel(R) Core(TM) i7-4770
Cores / Frequency	4 / 3.4GHz
Operating System	Windows 10 Education
RAM	16GB
Python version	3.10.9
Gurobi version	10.0.1
Tensorflow version	2.10.0

9.2 Initialization of Parameters

The schedule generated by the nurse scheduling model should replicate the situation at the three bed wards as realistically as possible. Table 9.2 gives an overview of the current staffing in each section and some demand-related parameter values used in the scheduling model. Table 9.3 presents the desired demand values for experience for the various shifts, which is the same across all sections and days. More detailed information about the nurse information is illustrated in Appendix C.

Table 9.2: The number of nurses and demand-related values for the competence.

	BW1	BW2	BW3
Total number of nurses	32	33	39
Number of specialized nurses	10	6	7
Desired demand for specialized nurses, D^{SN}_{bst}	1	1	1
Maximum demand for assistant nurses, D^{AN}_{bst}	1	1	1

Table 9.3: Desired demand values for the number of years of experience for all three sections, $\underline{D}_{ebst}$.

	Day	Evening	Night
Under 1 year	0	0	0
1 - 5 years	1	1	0
Over 5 years	2	2	1

Table 9.4 presents the minimum and average demand for each shift for all three sections. The staffing numbers in Table 9.2 and Table 9.3 combined with the daily average demand values in Table 9.4 lay the foundation for the output schedule. As explained in Chapter 2, the values for the minimum demand are based on staffing plans supplied by CC, while the values for historical average demand are the results of the data analysis presented in Chapter 8.

Table 9.4: Minimum and historical average demand for each bed ward (BW) for the nurse scheduling model in the format ($\underline{D}_{bst}$, D_{bst}).

		Mon	Tue	Wed	Thu	Fri	Sat	Sun
	Day	(6, 8)	(6, 9)	(6, 9)	(6, 9)	(6, 8)	(3, 4)	(3, 4)
BW1	Eve	(5, 5)	(5, 5)	(5, 5)	(5, 5)	(5, 5)	(3, 4)	(3, 4)
	Night	(3, 3)	(3, 3)	(3, 3)	(3, 3)	(3, 3)	(2, 2)	(2, 2)
	Day	(6, 8)	(6, 10)	(6, 10)	(6, 9)	(6, 9)	(3, 4)	(3, 4)
BW2	Eve	(5, 5)	(5, 5)	(5, 5)	(5, 5)	(5, 5)	(3, 4)	(3, 4)
	Night	(3, 3)	(3, 3)	(3, 3)	(3, 3)	(3, 3)	(2, 3)	(2, 3)
	Day	(7, 11)	(7, 12)	(7, 12)	(7, 12)	(7, 10)	(3, 5)	(3, 5)
BW3	Eve	(5, 5)	(5, 5)	(5, 5)	(5, 5)	(5, 5)	(3, 4)	(3, 4)
	Night	(3, 3)	(3, 3)	(3, 3)	(3, 3)	(3, 3)	(2, 2)	(2, 2)

Table 9.5 displays the values of the scheduling parameters presented in Section 6.1. The shift durations are based on a generalization of the shift durations at CC. The maximum

number of work hours in seven days and required rest during a week is determined by the Norwegian Working Environments Act (Arbeidsmiljøloven, 2022). As described in Chapter 2, the main tariff agreement determines the number of hours in a full-time work week. This number is specified to be 35.5 hours.

As outlined in Chapter 2, the nurses follow a shift rotation system where they work every third weekend. To accommodate this weekend rotation within the scheduling period, the number of weeks in the scheduling period should be in increments of three. Moreover, the scheduling period should be sufficiently long to thoroughly evaluate the rescheduling model. Consequently, the scheduling period is established for a 6 week period.

Table 9.5: Initialization value of scheduling parameters.

Parameter	**Description**	**Value**
$\overline{M}^D$	maximum number of consecutive work days	5
$\overline{M}^N$	maximum number of consecutive work nights	3
$\overline{L}$	maximum work hours in a week	48
H	hours in a full time work week	35.5
H_D	duration of day shift in hours	7.5
H_E	duration of evening shift in hours	7
H_N	duration of night shift in hours	10
W	working weekend recurrence	3
$\overline{F}$	upper bound for allowed deviation from contracted hours	105%
$\underline{F}$	lower bound for allowed deviation from contracted hours	95%
K	number of weeks in scheduling period	6

In our Specialization Report, we evaluated the impact of the planning period duration on the rescheduling model (Johansen et al., 2022). Our findings indicate that while the planning period duration does influence the results, the observed differences are relatively minor and do not significantly affect the outcomes. Based on discussions with the section managers at CC, it was concluded that a three-day planning period is realistic in real-world scenarios, and extending beyond three days introduces additional uncertainties related to nurse availability and patient load. In addition, it is most urgent to find replacements for the shifts in the near future, and the urgency decreases each day from the day of the reported absence. Consequently, all rescheduling tests will employ $\mathcal{T}^R = \{t_0, t_1, t_2\}$.

Table 9.6 displays the values of the rescheduling parameters presented in Section 6.2. In addition, values for parameter C_n are listed in Appendix C. The nurse wages and extra payments for the weekend, evening, and night shifts are based on information from Norsk Sykepleierforbund (2022b) on the minimum yearly wages for nurses with and without specialization in Norwegian hospitals. The wage calculator provided by The Norwegian Nurses Organisation finds the corresponding hourly wage (Norsk Sykepleierforbund, 2022a).

Table 9.6: Initialization value of rescheduling parameters.

Parameter	**Description**	**Value**
$\lvert\mathcal{T}^R\rvert$	number of days in the planning period	3
W_n	hourly wage in NOK for assistant nurse n where $n \in \mathcal{N}_{c=AN}$	210
W_n	hourly wage in NOK for regular nurse n where $n \in \mathcal{N}_{c=N}$	250
W_n	hourly wage in NOK for specialized nurse n where $n \in \mathcal{N}_{c=SN}$	280
P^O	penalty percentage for overtime hours	100%
P^{OS}	penalty for hours in a swapped shift	85%
P^W	penalty payment for weekend hours	23%
P^N	penalty payment for night hours	28%
P^E	penalty payment for evening hours	28%
$\overline{H}^D$	maximum hours during a day before overtime is triggered	10
H^M	duration of double shift, $H^M = H_D + H_E$	14.5

The parameters H, $\overline{L}$, and H_s are present in both models and have the same value for rescheduling as presented in Table 9.5. H_{nk}^{PRE} is calculated based on the schedule provided by the scheduling model. The values of the dynamic parameters handling overtime and double shifts, namely Ω_{nk}, Λ_{nst}, and Δ_{nt}, are initialized with values of 0. These dynamic parameters are updated between each step in the rolling horizon, while the parameters explained in Table 9.6 remain constant through each step. A_{nt} is initialized and updated as explained in Chapter 7. I_{nst} is generated as explained in Appendix D.

9.3 Case Definitions

Multiple test cases have been developed to sufficiently evaluate and analyze the optimization models presented in Chapter 6 and the machine learning models presented in Section 8.2. The evaluation process includes examining both the baseline nurse scheduling and rescheduling models, as well as the model extensions.

Section 9.3.1 presents the test cases designed explicitly for the baseline models. These test cases include evaluating the nurse scheduling and rescheduling models to determine their effectiveness in generating appropriate and realistic schedules. Section 9.3.2 focuses on the evaluation of the model extensions presented in Section 6.3. These extensions incorporate proactive strategies for the scheduling model and corresponding reactive strategies for the rescheduling model. The test cases aim to assess whether implementing these strategies improves results compared to the baseline models. To further assess the performance and capabilities of the various models under different scenarios, Section 9.3.3 presents several uncertainty levels in terms of absence and demand levels. These uncertainty levels provide valuable insights for evaluating and comparing all model variations.

9.3.1 Baseline Models

This section presents the test cases for the baseline models. The baseline models encompass both the scheduling and rescheduling models, which were introduced in Sections 6.1 and 6.2, respectively. These test cases are designed to assess the performance of the models. The test cases provide a baseline for comparing the results of the extended models.

In order to evaluate the impact of the lexicographic objective constraints in the baseline scheduling model, the test cases presented in Table 9.7 are formulated. The test cases have variations of the slack when solving the lexicographic objectives. The test cases enable us to evaluate the difference in the generated schedules and determine what provides the best baseline schedule.

Table 9.7: Test cases for objective function setup.

Test Case	**Description**
Base 1	Objective function as defined in Chapter 6.1 *with slack* for objective constraints.
Base 2	Objective function as defined in Chapter 6.1 with *no slack* for objective constraints.

9.3.2 Model Extensions

This section provides test cases for the model extension, which aims to gain insights into the performance of the proactive strategies and their ability to handle uncertainties. The results of these test cases will contribute to a more thorough understanding of the effectiveness and practicality of the model extensions in addressing the challenges of nurse scheduling and rescheduling.

The model extensions encompass both proactive and reactive strategies. The proactive strategies influence the schedule generation process, while the reactive strategies involve strategy activation during the rescheduling phase. However, it is important to note that the test cases are specifically formulated for the scheduling model, as the characteristics of each test instance influence the corresponding rescheduling models.

Buffer

For the buffer strategy, evaluating the impact of the parameter β_{st} is interesting. As described in Section 6.3, it sets an upper limit on the number of buffer nurses scheduled for each shift. Evaluating different values of this parameter provides insights into its effect on the generated schedule and, consequently, on the corresponding rescheduling model. In these cases, all nurses can be scheduled to buffer shifts. Table 9.8 presents the test cases that explore varying buffer levels.

Table 9.8: Test cases for the buffer strategy.

Test Case	Description	Values
Buffer 1	No limit on buffer levels	β_{st} equals an arbitrarily large number, thus not limiting the number of buffer nurses scheduled to a shift.
Buffer 2	$\beta_{st} = 3$	The buffer is limited to 3 nurses per shift.
Buffer 3	$\beta_{Dt} = 6$ $\beta_{Et} = 3$ $\beta_{Nt} = 3$	The buffer is limited to 6 nurses per day shift, 3 nurses per evening shift, and 3 nurses per night shift.

Flexible Assignments

The flexible assignments strategy involves the utilization of flexible nurses, as described in further detail in Table 5.1. In this strategy, it is interesting to evaluate how varying values for the flexible percentage X impact the results. Table 9.9 presents three separate test cases with different X levels to explore these variations. Flexible shifts are only assigned to full-time nurses, which remains equal to a total of 32 nurses in all cases. These test cases enable an evaluation of the flexible assignment strategy and its impact on the scheduling and rescheduling models.

Table 9.9: Test cases for the flexible assignments strategy.

Test Case	Description	Values
Flex 10	X = 10%	Gives a distribution of 90% scheduled work and 10% flexible work.
Flex 20	X = 20%	Gives a distribution of 80% scheduled work and 20% flexible work.
Flex 30	X = 30%	Gives a distribution of 70% scheduled work and 30% flexible work.

9.3.3 Uncertainty Levels

This section provides several uncertainty levels to further assess and compare the performance of the baseline models and model extensions. These levels focus on absence and demand. Since absence and demand levels can vary significantly in real-world scenarios, it is interesting to assess how the models perform under different levels of uncertainty. The uncertainty levels are used in the evaluation of the different schedules. This allows us to identify which model variations are better suited to address the challenges of uncertainty in nurse rescheduling.

Cross-Section Rescheduling

To assess the impact of cross-section rescheduling, it is necessary to compare it with in-section rescheduling. With in-section rescheduling, there are no shared resources between the sections. To evaluate the effect of cross-section rescheduling, a comparison is made between the total costs of cross-section and in-section rescheduling by gradually increasing the absence levels for one section. By computing the total costs at each 5% increment, we can compare the cost development for increasing absence levels within one section. Furthermore, the difference in total costs can be analyzed when rescheduling is performed using only in-section rescheduling versus incorporating cross-section rescheduling.

Varying Absence Levels

Nurse availability is an uncertain factor that can significantly impact the rescheduling results. High levels of nurse absences often increase the need for reactive actions, while lower levels of absences reduce the need for rescheduling. Table 9.10 presents the absence levels that have been developed to properly evaluate and analyze the rescheduling model's performance.

Table 9.10: Absence levels for rescheduling.

Absence level	**Description**	**Values**
Level 0	No absence	0% absence in all sections.
Level 1	Low absence level in all sections	5% absence in all sections.
Level 2	Representative absence levels in all sections	10% absence in all sections. based on data in Appendix B.
Level 3	High absence in one section	30% absence in one section and 10% absence in other sections.
Level 4	High absence level in all sections	30% absence in all sections.

The simulation framework presented in Section 7.2.1 generates the absences used in the model. Table B.1 in Appendix B displays the absence data provided by CC, where the monthly percentages of absences are given from 2020 to 2022. These percentages show that absences vary over time and between sections. Section B.0.1 provides a detailed overview of how the absences are generated using the data from CC and the simulation framework in Section 7.2.1.

Demand Information

Patient load is another source of uncertainty that affects the need for rescheduling. Varying demand levels can lead to increased total rescheduling costs. While the actual demand for nurses is known before the morning shift each day, the demand for the rest of the planning period remains unknown. This was illustrated in Table 8.4.

Table 9.11 presents different levels of demand information. A comparison between using simple average estimations, machine learning predictions, and revealing perfect information can provide insight into the value of accurate demand information. The actual demand is known every morning at all levels. However, with perfect information, the actual demand for the remainder of the planning period is also revealed.

Table 9.11: Test case for evaluating the value of perfect information of demand.

Information of demand	**Description**	**Values**
Average	Historical average demand	Demand for the upcoming two days in the planning period are based on historical averages, while today is using actual demand.
Perfect	Perfect information	The demand in the planning period is based on perfect information, where the values are given in a similar manner as actual demand.
Predicted	Machine Learning predictions	Demand for the upcoming two days in the planning period are based on machine learning predictions, while today is using actual demand.

The information level with demand predictions is included in the rescheduling model using the demand constraints presented in Section 8.2.4. Utilizing the different levels of demand information will provide insights into whether utilizing machine learning predictions can add value to the rescheduling and perform with a total cost closer to the cost when having perfect information.

9.4 Results

This section presents the results of the different test cases. The analysis aims to evaluate the performance of the different test cases and compare the effectiveness of the various strategies in both the scheduling and the rescheduling models. First, the machine learning results are presented and evaluated in Section 9.4.1. Section 9.4.2 presents the key metrics used to discuss and compare the scheduling results. Building on this, the scheduling results for the baseline model and the model extensions are presented in Section 9.4.3. Next, the key metrics for the rescheduling model are presented in Section 9.4.4. Section 9.4.5 present the rescheduling results for the baseline model and the corresponding model extensions.

Finally, Section 9.4.6 analyze the results for combining the best machine learning method with the various rescheduling strategies.

9.4.1 Machine Learning Results

This section presents the implemented machine learning models and compares them to each other. The initialization of the hyperparameters is presented and discussed before the results from the models are presented.

The analysis includes two regression models, one neural network, and one decision tree model. The regression models output a float, but since we can only have integer nurse demand, the output number will be rounded to the nearest integer for both models. As described in Section 8.2.4, the models are first set up in a two-stage process and trained on data spanning from 2018 to 2021. After the regression analyses, the decision tree will also be evaluated with classification.

Table 9.12 presents the initialization of hyperparameters for the neural network regression problem, while Table 9.13 presents the initialization of the hyperparameters for the decision tree models for both the regression problem and classification problem.

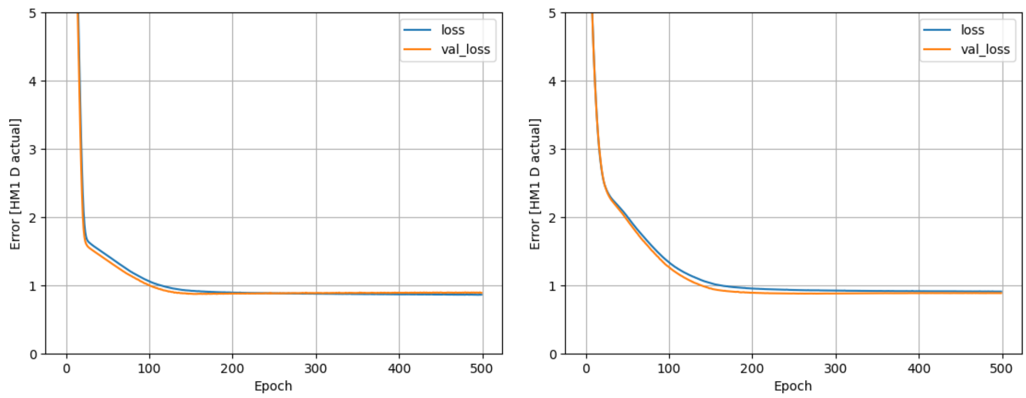
Table 9.12: Initialization of hyperparameters for the Neural Network regression.

Parameter	**Description**
Neurons in hidden layer	The neural network comprises a single hidden layer with 64 neurons.
Activation function	The ReLU is chosen as the activation function for the hidden layer chosen due to its capability to encourage sparse representations. By only activating for positive input values and outputting zero for negative inputs, ReLU promotes sparsity in the network, allowing it to focus on the important features for discovering patterns.
Loss Function	The MAE is used as the loss function, measuring the average absolute difference between predicted and actual values.
Optimizer	The Adam optimizer is employed due to its adaptive learning rate mechanism.
Epochs	The model is trained for 500 epochs.
Learning Rate	The model uses a learning rate of 0.00003.

Table 9.13: Initialization of hyperparameters values for Decision Tree regression.

Parameter	Description
Splitting criteria	For the regression problem, the MAE is chosen as the splitting criteria for evaluating splits in the decision tree because it is not that sensitive to outliers in the dataset. The Gini Impurity is selected as the splitting criteria for the classification problem as it is suitable for datasets with multiple features.
Maximum depth	Considering the significant number of features, the tree is initialized with a maximum depth. After analysis, a maximum depth of 3 was determined to be best.

Figure 9.1 displays the error loss observed during the training of the neural networks on days 1 and 2, where day 0 is the day the rescheduling is done. This figure presents the MAE during training of the neural network on the day shift in Bed Ward 1 for the predictions. As the epochs progress, the model learns, and the loss steadily decreases.



(a) Loss graph for day 1 prediction

(b) Loss graph for day 2 predictions

Figure 9.1: Loss graphs for days 1 and 2 of training with the neural network for Bed Ward 1 on day shift.

Surprisingly, we found that the accuracy and MAE for predictions for both the machine learning models on day 1 compared to day 2 did not exhibit significant differences. This suggests that the activity data may not strongly influence the discovered pattern within the neural network. Instead, it is likely that the primary factors shaping the pattern stem from other features like the planned staffing.

Table 9.14 presents the MAE values for the predictions on the 2022 data. It is observed that the decision tree model generally achieves smaller MAE values compared to the neural network model. In cases where the neural network outperforms the decision tree in terms of MAE, the difference between the errors is not significant. Given the objective to minimize the deviation of predictions from the historical actual demand, the decision tree model is prioritized.

Table 9.14: Comparison of MAE for neural network regression (NN) and decision tree regression (DT) for each section and shift type.

		D		E		N	
Day number		1	2	1	2	1	2
BW1	NN	1.04	0.97	0.65	0.69	0.28	0.33
	DT	0.91	0.92	0.67	0.67	0.31	0.30
BW2	NN	0.95	0.95	0.66	0.61	0.41	0.48
	DT	0.97	0.95	0.61	0.56	0.38	0.39
BW3	NN	1.05	1.02	0.69	0.74	0.54	0.53
	DT	0.98	0.97	0.72	0.75	0.53	0.53

The reason why the decision tree provides predictions with a smaller error from the target value than the neural network could be attributed to the significant number of features provided. Both prediction models demonstrate the ability to detect patterns within the collected features from the nurse data. However, the decision tree's maximum depth restricts the number of features it considers, allowing it to focus on the most influential ones. On the other hand, the neural network might discover patterns in features not considered by the decision tree and lead to overfitting for the neural network. To enhance the value of the activity data and uncover clearer patterns, it could be beneficial if the nurses could record the perceived patient load for each shift by utilizing a scale from less to more patient demand than usual. Analyzing this additional data might reveal more distinct patterns in the patient data.

The actual demand is an integer. However, the solution space is limited and can be divided into classes. Therefore, both classification and regression methods can be suitable for addressing this problem. The comparison metrics reveal that decision tree regression outperforms neural network regression. Next, a comparison between the decision tree regression and decision tree classification is presented.

Since we compare two models with different ways to provide the output, the comparison metric used is accuracy. Table 9.15 shows the accuracy of the two decision tree models compared to each other. This table shows that the regression problem is more accurate in most of the predictions than the classification problem. Notably, the decision tree regression utilizes MAE as its splitting criteria, while the classifier disregards the magnitude of incorrect predictions. This, coupled with the higher accuracy achieved by the regression approach, suggests that it is sensible to continue analyzing the nurse rescheduling problem using predictions generated by the decision tree regression model.

Table 9.15: Comparison of prediction accuracy between decision tree regression and classification for each section and shift type.

		D		E		N	
Day number		1	2	1	2	1	2
BW1	Regression	0.36	0.34	0.43	0.42	0.71	0.71
	Classification	0.34	0.34	0.40	0.43	0.71	0.72
BW2	Regression	0.33	0.35	0.49	0.51	0.64	0.63
	Classification	0.33	0.35	0.48	0.49	0.64	0.63
BW3	Regression	0.37	0.37	0.41	0.40	0.52	0.52
	Classification	0.36	0.36	0.41	0.41	0.52	0.51

9.4.2 Key Metrics Scheduling

Several metrics are defined to evaluate the effectiveness of the scheduling model. These are used to assess and compare the performance of the different scenarios and test cases. Table 9.16 presents the metrics used to measure the results of the scheduling model. The table also displays how an ideal schedule should evaluate all the metrics. It is worth noting that the overstaffing metric refers to the distribution of overstaffed shifts. The aim is to distribute the overstaffing over as many shifts as possible, limiting the amount of overstaffing per shift.

Table 9.16: Key metrics for the scheduling model.

Metric	**Values**	**Aim**
Understaffing	Number of shifts	Minimize
Hours deviation	Total hours deviated	Minimize
Max deviation	Highest hour deviation for a nurse	Minimize
Overstaffing	Number of shifts	Maximize
Preference	Percentage of preference compliance	Maximize
Experience	Number of shifts violating requirements	Minimize
Competence	Number of shifts violating requirements	Minimize
Buffer shifts	Number of buffer shifts assigned	Neutral
Flexible shifts	Number of flexible shifts assigned	Neutral
Shifts scheduled	Number of shifts	Neutral
Runtime (s)	Seconds	Minimize

9.4.3 Scheduling Results

This section analyzes the results obtained from the nurse scheduling model. Both the results from the baseline scheduling model and the proactive strategies in the model extensions are presented, enabling for comparison of the performance of the model variations.

Baseline Nurse Scheduling Model

Table 9.17 presents the results for the test cases presented in Table 9.7. These test cases aim to determine how slack in the objective function constraints impacts the various metrics, thus determining a tradeoff between runtime and slack in the lexicographic constraints.

Table 9.17: Results for solving the cases with and without slack in the objective function in the scheduling model.

Metric	**Test case**	
	Base 1	**Base 2**
Understaffing	0	0
Hours deviation	351.9	337.9
Max deviation	3.25	3.25
Overstaffing	213	205
Preference	93.0%	91.3%
Experience	2	4
Competence	12	8
Shifts scheduled	2287	2289
Runtime (s)	17 350	109 154

The schedule with slack in the constraint of previously solved objectives, Base 1, ran with a time limit of 7 200 seconds per objective, while the schedule with no slack, Base 2, ran with a time limit of 36 000 seconds per objective. Both schedules found solutions within 1.5% of the optimality gap for understaffing, hours deviation, overstaffing, and preferences. The objective related to an even distribution of competence and experience ran until the time limit with a lower bound equal to 0 in both cases.

Base 1 and Base 2 show similar results in the measures. For hours deviation, Base 1 utilizes the available slack and performs slightly worse. However, Base 1 is able to better allocate the overstaffing across more shifts, which leads to increased robustness in the schedule. Base 1 also achieves a higher percentage of shifts without violating the preferences of the nurses. The distribution of experience and competence is similar, but Base 2 performs slightly better than Base 1.

The model in the Base 1 case permits a 5% increase in the objective concerning deviations from contracted hours, a 10% decrease in the number of overstaffed shifts, and a 10% increase in the number of preference violations. Despite this, the model still produces comparative results for all measures while significantly outperforming Base 2 for runtime. Therefore, the remaining scheduling test cases allow slack in the constraints.

Strategies for the Nurse Scheduling Model

Table 9.18 presents all results for the test instances presented in Tables 9.8 and 9.9, which are related to the scheduling model in both the buffer strategy and the flexible assignment strategy.

Table 9.18: Results for solving each test instance on proactive strategies. This includes buffer scheduling and scheduling using the flexible assignment strategy.

Metric	**Test case**					
	Buffer 1	Buffer 2	Buffer 3	Flex 10	Flex 20	Flex 30
Understaffing	0	0	0	0	0	0
Hours deviation	351.3	351.1	352.35	300.7	371.2	500.9
Max deviation	3.25	3.25	3.25	3.75	3.25	4.65
Overstaffing	101	103	103	162	108	110
Preference	91.6%	90.9%	91.3%	92.5%	92.8%	93.6%
Experience	8	4	4	12	0	14
Competence	14	8	12	4	4	20
Buffer shifts	720	357	462	-	-	-
Flexible shifts	-	-	-	96	192	288
Shifts scheduled	2291	2288	2288	2192	2157	2074
Runtime (s)	47 725	48 227	49 643	26 590	22 570	28 610

In contrast to Base 1 and the cases with flexible assignments in scheduling, the buffer test cases have a higher time limit of 10 800 seconds. This extension is implemented due to the inclusion of an additional section in the model that tracks scheduled buffer shifts. The incorporation of this extra section adds complexity to the solving process, resulting in longer runtimes for these particular test cases. All test cases related to the strategies have a notably longer runtime than Base 1. Though, the runtime still represents a significant improvement over the actual time spent by section managers in practice.

Both strategies are able to avoid understaffing. Across all cases, preferences yield good results with only small differences. There are some variations in the results connected to the deviations from the number of contracted hours for all nurses. The buffer test cases provide similar results to Base 1, while the flexible assignment test cases exhibit some noticeable differences. Flex 10 shows improved results compared to all other test cases. This could be attributed to shift combinations in the model, where some align better with a 90% nurse employment percentage than full-time employment. In contrast, Flex 30 performs significantly worse than all other test cases in terms of deviations from contracted hours.

Overstaffing provides similar results in all test cases, with only Flex 10 standing out with better results. However, all test cases perform significantly worse on the overstaffing distribution than Base 1. This is somewhat expected in comparison with the flexible assignments, as Base 1 has more available hours to distribute for covering shifts. For the buffer schedules, however, it is not relevant to directly compare the results from overstaffing with other test cases. The values for overstaffing in the buffer instances reflect the total number of nurses scheduled for a specific shift across all three sections. The other cases look at overstaffing for a specific shift in a given section. This means the buffer schedules have fewer possible shifts to overstaff in total.

For the experience and competence values, it is noteworthy that Flex 20 stands out positively, indicating a lower number of deviations, while Flex 30 stands out negatively.

9.4.4 Key Metrics Rescheduling

Similarly to the scheduling model, several metrics are defined to evaluate the effectiveness of the rescheduling model. These are used to assess and compare the performance of the different scenarios and test cases. Table 9.19 presents these metrics and how they are measured. The model's performance is measured by evaluating the total costs, which should be minimized as much as possible. This implies that the number of nurses ordered to take swap, double or extra shifts should be as low as possible. When comparing the results, the aim is to explore if the model extensions provide a cost reduction compared to the Base 1 schedule, where a positive percentage for cost reduction corresponds to cost improvement.

Table 9.19: Key metrics for the rescheduling model.

Metric	**Values**
Swaps	Number of shifts
Extra shifts	Number of shifts
Double shifts	Number of shifts
Overtime	Number of hours
Total cost	Amount in NOK during the whole scheduling period
Flex usage	Number of flexible shifts utilized
Cost reduction	Percentage cost reduction compared to Base 1 instances

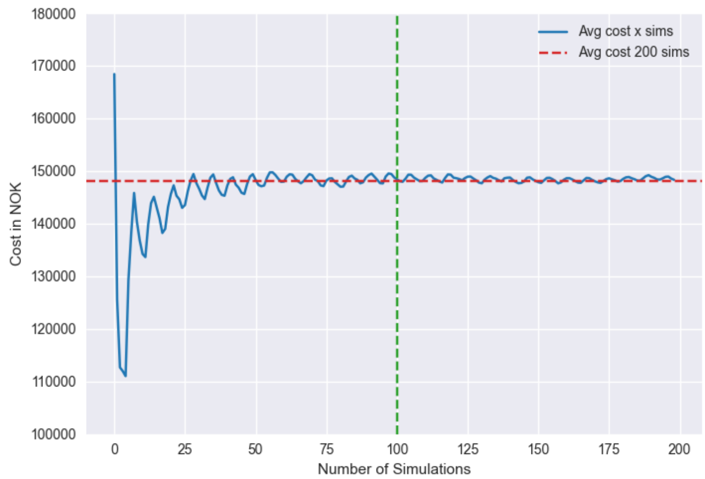
9.4.5 Rescheduling Results

This section analyzes the results obtained from the nurse rescheduling model without the use of machine learning predictions. The results are derived from running the rescheduling model on the output for some of the scheduling test cases presented in Section 9.3. To ensure comparable results, all tests are run with the same random seeds. The results specify which scheduling case is used as input to generate the presented rescheduling results.

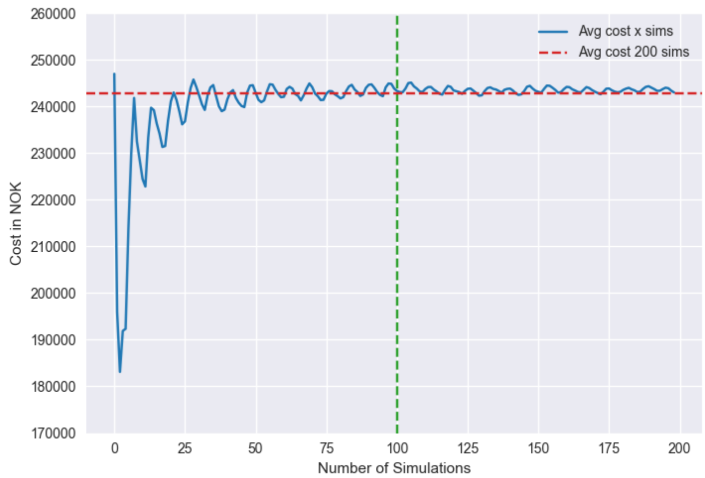
To ensure accurate and reliable test results, we first analyze the number of simulations required for the subsequent analysis. We present the test results from the baseline rescheduling model considering varying absence levels and average demand. These results provide insights into the model's performance under different circumstances. Next, the results of cross-section or in-section rescheduling are examined. Further, we analyze the results obtained from the model extensions of the rescheduling model. The extensions incorporate the reactive part of the strategies, which involve activating the proactive measures from the schedules. This enables a comparison of results between the baseline model and the model extensions. Finally, the rescheduling results with perfect information on demand levels are presented.

Number of Simulations

The simulations in the test cases are run several times to cover the variations that occur and provide more representative results. To improve the credibility of the rescheduling analyses, two test cases are run with 200 simulations with the goal of finding a reasonable threshold for the number of simulations further in the study. Figures 9.2a and 9.2b illustrate the results from running the simulations of nurse rescheduling on the Base 1 schedule with absence level 2 and level 3, respectively, as presented in Table 9.10.



(a) Average cost with absence level 2 and average demand.



(b) Average cost with absence level 3 and average demand.

Figure 9.2: Average cost from the objective function for different numbers of simulations.

Figure 9.2 shows how the average cost stabilizes with the increasing amount of simulations for both simulation test cases. In both cases, the average cost has stabilized at 100 simulations, as indicated by the vertical dotted line. Hence, the rest of the rescheduling cases will use 100 simulations to find the estimation for the key metrics. The horizontal dotted line in both figures shows the average total cost after 200 simulations.

Cross-Section vs In-Section Rescheduling

Figure 9.3 compares total costs between cross-section and in-section rescheduling. The simulations are run with increasing absence percentage in Bed Ward 1, while the absence percentages in the other two sections remain constant with 10% absence. The total cost is computed for every five percent increase in absence levels in Bed Ward 1 in the interval from 5% to 30%.

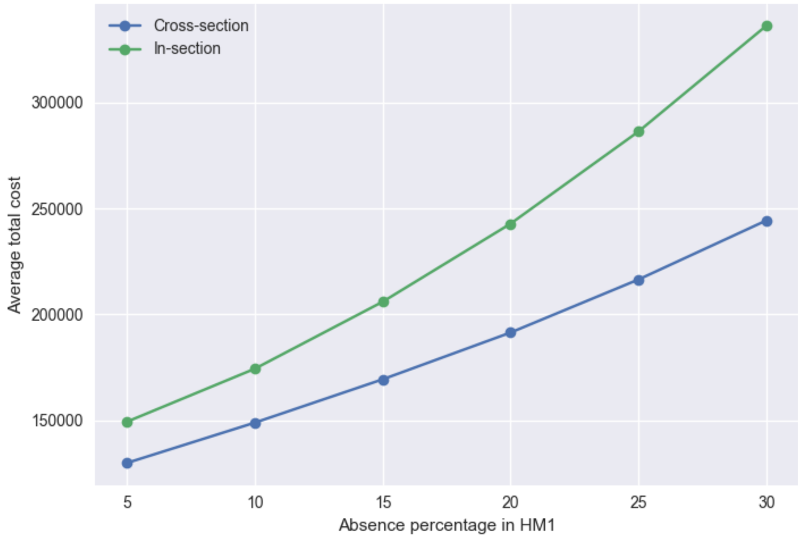


Figure 9.3: Cost development for cross-section vs. in-section rescheduling with increasing absence in Bed Ward 1 for 100 simulations on each absence level.

The cost increase is notably higher with in-section rescheduling. The utilization of cross-section rescheduling demonstrates a significant improvement to in-section rescheduling when the absence variations between sections are higher, while also performing better when the percentages are equal across all sections. This aligns with the findings of our specialization report, which highlighted the positive effects of cross-section rescheduling (Johansen et al., 2022). All subsequent test cases will exclusively focus on cross-section rescheduling based on these conclusive positive effects.

Absence Variations

Table 9.20 outlines the average results for the test cases considering varying absence levels, as presented in Table 9.10. The results illustrate how the baseline rescheduling model performs on varying absence levels. The Base 1 schedule is utilized as input for the baseline rescheduling model.

Table 9.20: Average results for solving each test instance on varying absence levels for 100 simulations.

Test instance		**Metrics**				
		Swap shifts	Extra shifts	Double shifts	Over-time	Total cost
Strategy: Base 1 Absence: Level 0 Demand: Average		61.6	5.9	0.1	3.0	53 504
Strategy: Base 1 Absence: Level 1 Demand: Average		97.7	16.5	0.6	6.2	95 451
Strategy: Base 1 Absence: Level 2 Demand: Average		142.2	30.8	1.4	10.6	148 754
Strategy: Base 1 Absence: Level 3 Demand: Average		207.8	53.3	2.8	24.6	243 801
Strategy: Base 1 Absence: Level 4 Demand: Average		301.8	163.0	14.3	89.0	584 762

Table 9.20 illustrates that higher absence levels lead to higher total cost as more reactive measures must be performed. In all instances, the number of swaps is notably high compared to the number of extra and double shifts. This difference is not entirely unexpected, given that swap-related costs are lower than those associated with additional hours or overtime hours for extra and double shifts. Swaps enable the reassignment of nurses from shifts with excess capacity to those with higher demand, effectively reducing total wage costs by optimizing resource allocation and minimizing unnecessary overstaffing. The utilization of double shifts is expected to be limited, given that assigning a double shift always induce overtime payments. Generally, as absence levels increase, there is a notable increase in the total number of swaps, extra shifts, and double shifts.

Figure 9.4 compares the total costs for absence levels 0, 1, 2, and 4. The figure illustrates that there is close to a linear relationship in costs between level 0, level 1, and level 2. This indicates that for lower absence levels, the actual demand provokes rescheduling actions and which leads to higher rescheduling costs. In contrast, it is clear that the increased absence in level 4 has a considerable effect on the total cost. Using absence level 4, rescheduling actions will likely have to occur regardless of the actual demand on the given day. Therefore, the rescheduling outcome with absence level 4 is less dependent on the actual demand. It will also lead to more expensive actions. As shown in Table 9.20, both overtime and double shifts are more frequently used in this test instance.

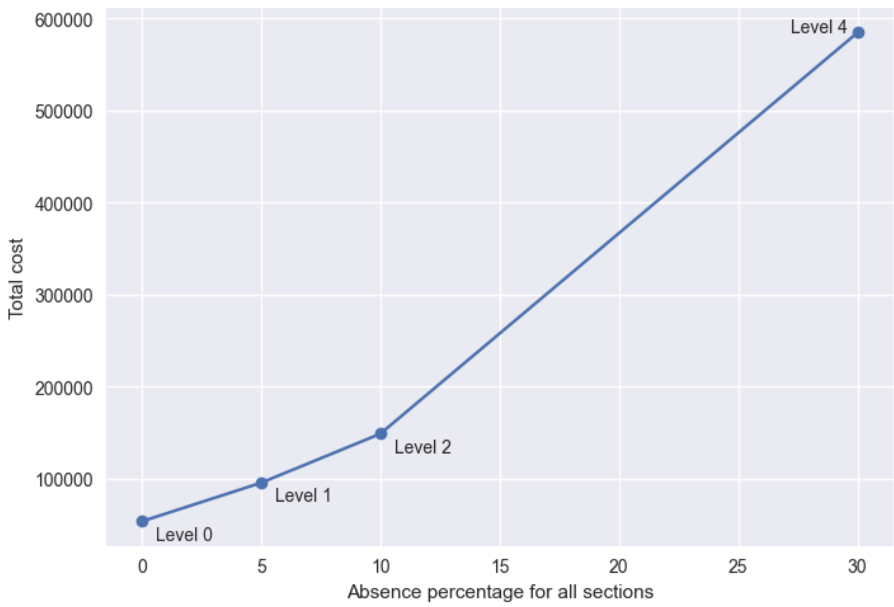


Figure 9.4: Cost comparison for test instances from Table 9.20 with the specified absence levels.

Subsequent test cases will exclusively utilize absence level 2 and level 3. Absence level 2 is particularly valuable as it closely mirrors the real-world context at CC, making it interesting to study with various strategies. On the other hand, absence level 3 exhibits differences in absence levels across the sections, with a high absence percentage in one section. This variation highlights the notable contrast between cross-section and in-section rescheduling approaches. Given the importance of cross-section utilization in this thesis, analyzing absence level 3 offers valuable insights into how the different strategies utilize cross-section rescheduling.

Buffer Activation

Table 9.21 provides an overview of the results of running the rescheduling model with the buffer strategy. In this model, all scheduled buffer shifts are activated in response to the actual demand. The results are derived from running the rescheduling model on the three output schedules generated from the test cases outlined in Table 9.8. To assess the model's performance under varying absence levels, each buffer schedule is tested with two different absence levels.

The buffer strategy shows an overall improvement over the baseline models. Comparing the results with those of the baseline model, as presented in Table 9.20, Buffer 1 and Buffer 3 reduce comparable total costs by over 20%. The cost improvement shows the value of the model's ability to distribute the buffer nurses among the sections based on the observed actual demand. For both absence levels, lower costs are observed due to a decrease in swaps and extra shifts. For all instances, the number of overtime hours and double shifts show minimal variations compared to the baseline.

Buffer 1 and Buffer 3 demonstrate higher performances compared to Buffer 2. Even though

Table 9.21: Average results for solving each test instance on varying buffer levels for 100 simulations.

Test instance		Metrics					
		Swap shifts	Extra shifts	Double shifts	Over-time	Total cost	CR
Strategy: Absence: Demand:	Buffer 1 Level 2 Average	109.1	26.3	1.1	11.1	114 919	23%
Strategy: Absence: Demand:	Buffer 1 Level 3 Average	161.0	47.7	3.3	24.5	192 858	21%
Strategy: Absence: Demand:	Buffer 2 Level 2 Average	112.3	24.7	1.1	7.6	127 959	14%
Strategy: Absence: Demand:	Buffer 2 Level 3 Average	183.4	52.3	3.6	21.8	216 829	11%
Strategy: Absence: Demand:	Buffer 3 Level 2 Average	100.1	23.2	1.3	6.4	112 843	24%
Strategy: Absence: Demand:	Buffer 3 Level 3 Average	166.5	47.7	3.0	17.8	194 569	20%

the Buffer 1 schedule facilitates more buffer shifts than the Buffer 3 schedule, as illustrated in Table 9.18, Buffer 1 and Buffer 3 deliver very similar results with minimal variations across all metrics. This suggests that the additional robustness provided by Buffer 1 is unnecessary, as the extra buffer shifts it introduces are likely distributed evenly across the sections to meet the real demand, which typically exceeds the minimum demand. The nearly identical outcomes suggest that the limitations imposed in Buffer 3 on the number of buffer shifts are reasonable. However, Buffer 3 performs slightly worse than Buffer 1 when absence levels increase, indicating that the extra robustness of Buffer 1 may be better suited for handling high absence levels.

Figure 8.3 highlights that variations in actual demand are highest during day shifts and lowest during night shifts. This suggests that having no limits on the number of buffer shifts during evening and night shifts may not be advantageous, which could explain why Buffer 1 did not exhibit improvement over Buffer 3. As the only distinction between Buffer 2 and Buffer 3 is the limit for buffer shifts on the day shifts, it implies that the number of buffer shifts allocated to day shifts significantly impacts the total costs.

Flexible Assignment Activation

Table 9.22 presents the results for running the rescheduling model using the flexible assignment strategy. The results are obtained from running the rescheduling model on the output schedules from the test cases presented in Table 9.9. In addition, varying absence levels are tested for all test cases.

Table 9.22: Average results for solving each test instance with flexible assignments for 100 simulations.

Test instance		**Metrics**						
		Swap shifts	Extra shifts	Double shifts	Over-time	Flex usage	Total cost	CR
Strategy: Absence: Demand:	Flex 10 Level 2 Average	122.8	30.3	23.4	38.2	53.0	141 072	5%
Strategy: Absence: Demand:	Flex 10 Level 3 Average	172.6	51.8	67.8	36.7	72.6	225 683	7%
Strategy: Absence: Demand:	Flex 20 Level 2 Average	103.3	8.8	20.9	23.4	91.0	106 233	29%
Strategy: Absence: Demand:	Flex 20 Level 3 Average	165.9	18.4	35.1	39.1	123.3	187 918	23%
Strategy: Absence: Demand:	Flex 30 Level 2 Average	66.0	7.2	7.2	10.6	135.4	59 149	60%
Strategy: Absence: Demand:	Flex 30 Level 3 Average	124.3	16.4	22.3	33.2	178.6	141 936	42%

Similarly to the buffer strategy, all test cases utilizing flexible nurses yield improved results compared to the baseline model. The improvements become more pronounced as the level of flexibility increases. As expected, the test case with the least flexibility, Flex 10, exhibits the lowest cost reduction of under 10% for both absence levels.

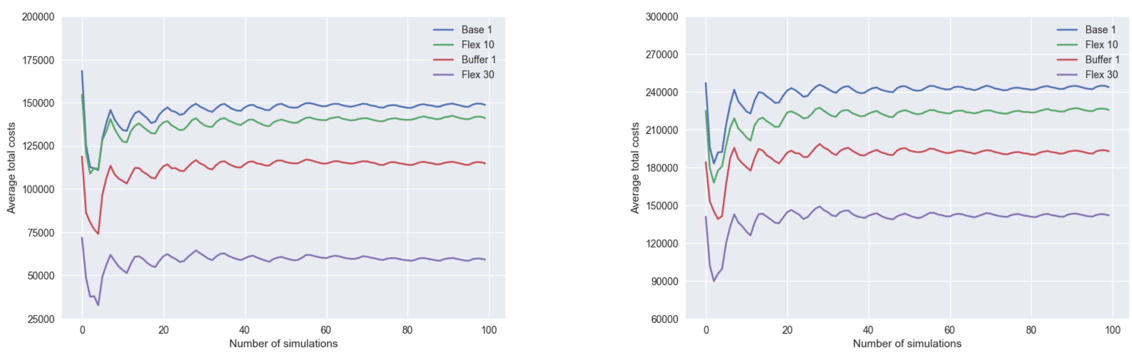
In addition to reduced total costs, the number of swaps and extra shifts also decreases as the level of flexibility increases. However, double shifts remain significantly higher in all test instances compared to the results from the buffer strategy and the baseline models. This difference is due to the stricter scheduling constraints imposed on flexible shifts, which limit the available shift patterns. Consequently, many flexible shifts are scheduled as double shifts. Although the number of double shifts is notably high, it decreases as the level of flexibility increases. With higher flexibility, the nurses are scheduled for a lower percentage of the total shifts. This makes it easier to utilize flexible shifts sensibly during rescheduling without resorting to double shifts.

The number of overtime hours in rescheduling results of the flexible strategy is higher compared to the results from the buffer strategy and the baseline model. The difference is most pronounced in Flex 10, which is natural considering that it yielded the best results in terms of deviations from an even distribution of working hours, as shown in Table 9.18. With an even distribution and the flexible nurses scheduled with 90% employment, most flexible shifts will trigger overtime payments.

The comparison between the results in Table 9.21 and Table 9.22 reveals that Flex 20 and Flex 30 consistently outperform the buffer strategy on total costs. This indicates that the flexibility provided in Flex 20 and Flex 30 is more cost-efficient compared to the robustness provided by the buffer strategy. The most important factor contributing to this improvement is that the flexible shifts are not bound to a specific shift and can be utilized on days with high demand. Another factor that may account for this difference in results is that the buffer shifts must be activated. In contrast, flexible shifts are only activated if it reduces the total costs.

The flex usage column in Table 9.22 indicates the average number of activated flex shifts across 100 simulations. Unsurprisingly, higher absence levels correspond to higher flex usage in all three test cases. However, as the flexible percentage increases, the number of available flexible shifts to utilize also increases. Consequently, while the usage of flexible shifts increases with greater flexibility, the percentage utilization decreases due to the scheduling constraints that limit the activation of all available flexible shifts. This entails that Flex 10 with high absence levels has the highest utilization with an average of 76%, while Flex 30 with low absence levels has the lowest utilization with an average of 47%.

Figure 9.5 compares the total cost results for instances using strategies Base 1, Buffer 1, Flex 10, and Flex 30 for absence levels 2 and 3, where all use average demand. These test cases are selected for further comparisons of the performance of the baseline model, the buffer strategy, and the flexible strategy. The selection is based on the findings discussed earlier in the analysis of the respective cases, where these cases provided interesting results. As a result, these test cases will be used in subsequent test instances. The figure shows the performance in comparison and displays the cost gap between the different strategies.



(a) Total costs comparison for selected instances with absence level 2.

(b) Total costs comparison for selected instances with absence level 3.

Figure 9.5: Comparison of total cost results from selected instances.

Perfect Information of Demand

Table 9.23 presents the results obtained using perfect information for demand, as described in Table 9.11. To analyze the effect of perfect information and compare the results across different model variations, the test is conducted on the baseline rescheduling model, the rescheduling model utilizing the buffer strategy, and the rescheduling model utilizing the flexible assignments strategy.

Table 9.23: Average results for solving each test instance with perfect information of demand for 100 simulations.

Test instance		Metrics						
		Swap shifts	Extra shifts	Double shifts	Over-time	Flex usage	Total cost	CR
Strategy: Absence: Demand:	Base 1 Level 2 Perfect	128.9	28.0	0.8	9.5	-	128 144	14%
Strategy: Absence: Demand:	Base 1 Level 3 Perfect	195.0	47.9	1.9	21.6	-	217 873	11%
Strategy: Absence: Demand:	Buffer 1 Level 2 Perfect	92.3	20.6	0.7	8.7	-	96 210	35%
Strategy: Absence: Demand:	Buffer 1 Level 3 Perfect	144.0	37.1	2.6	19.8	-	167 809	31%
Strategy: Absence: Demand:	Flex 10 Level 2 Perfect	109.4	29.3	15.7	34.5	44.5	115 148	23%
Strategy: Absence: Demand:	Flex 10 Level 3 Perfect	164.1	47.6	26.6	64.0	64.8	190 953	22%
Strategy: Absence: Demand:	Flex 30 Level 2 Perfect	43.4	6.3	4.0	8.4	125.9	40 007	73%
Strategy: Absence: Demand:	Flex 30 Level 3 Perfect	93.5	14.6	14.9	22.6	167.6	102 713	58%

Perfect demand information in the planning period yields the expected result of improving the total rescheduling costs. Once the demand with perfect information is revealed, the need for further rescheduling actions due to demand variations is eliminated. The only remaining factor necessitating rescheduling in the planning period is nurse absenteeism. This reduced requirement for rescheduling is also evident in other metrics, where the number of swaps, extra shifts, double shifts, and overtime hours are reduced compared to results based on historical average demand.

The flexible assignment strategy with perfect information on demand demonstrates a decrease in the utilization of flexible shifts compared to the results with historical average demand, as shown in Table 9.22. This decrease can be attributed to the reduced necessity for rescheduling actions due to limited demand uncertainty. Consequently, less than half of the available flexible shifts are utilized.

Among the presented results in the computational study, the Flex 30 strategy stands out with the lowest total costs. By utilizing perfect information on demand, this strategy achieves a remarkable decrease of 73% in total costs compared to the baseline rescheduling model that relies on historical average demand. This highlights the benefits of employing proactive strategies and having accurate demand information. Moreover, even when comparing only the baseline models without proactive strategies, the advantage of having precise demand information becomes evident as it eliminates the uncertainty associated with demand fluctuations, enabling more effective adjustments to patient demand.

9.4.6 Results from Combining Machine Learning and Rescheduling

Table 9.23 highlights the advantage of incorporating precise demand information in the rescheduling models. As discussed in Section 9.4.1, machine learning techniques can generate demand predictions based on historical data, offering more reliable demand information than historical average demand. Integrating demand predictions from the decision tree regression with the rescheduling models aims to minimize the need for excess rescheduling and improve the overall model performance. The same test cases presented in Table 9.23, now using demand predictions, are used to evaluate the outcomes of combining the machine learning model with the rescheduling model.

Machine Learning without Upper Bound

Table 9.24 illustrates the results from using the machine learning-based demand predictions in combination with the various rescheduling models. The demand predictions are based on the results presented in Section 9.4.1.

Table 9.24: Average results for solving each test instance with demand predictions for 100 simulations.

Test instance		**Metrics**						
		Swap shifts	Extra shifts	Double shifts	Over-time	Flex usage	Total cost	CR
Strategy: Absence: Demand:	Base 1 Level 2 Prediction	158.3	38.0	1.1	13.4	-	167 456	-13%
Strategy: Absence: Demand:	Base 1 Level 3 Prediction	227.0	57.1	2.7	31.0	-	266 152	-9%
Strategy: Absence: Demand:	Buffer 1 Level 2 Prediction	118.1	28.3	0.8	10.7	-	125 746	15%
Strategy: Absence: Demand:	Buffer 1 Level 3 Prediction	170.4	46.6	3.1	24.2	-	203 225	17%
Strategy: Absence: Demand:	Flex 10 Level 2 Prediction	136.1	39.8	19.7	48.2	31.6	154 527	-4%
Strategy: Absence: Demand:	Flex 10 Level 3 Prediction	188.1	57.6	34.4	74.2	72.2	239 603	2%
Strategy: Absence: Demand:	Flex 30 Level 2 Prediction	69.6	9.9	7.4	20.2	143.3	67 225	55%
Strategy: Absence: Demand:	Flex 30 Level 3 Prediction	127.0	20.7	21.5	42.8	181.4	147 694	39%

Even though several of the test instances provide positive cost reductions compared to the baseline results, the results illustrate a significant increase in the average total costs for all cases compared to the corresponding test cases without demand predictions. This increase is mainly due to an increase in the average number of swaps. For Flex 30 instances, there is also a significant increase in overtime hours. This could be a result of the simulations where the predictions overestimate demand compared to the actual demand, leading to unnecessary actions. The monetary punishment of overestimating future demand raises the average total costs across all 100 simulations.

Machine Learning with Upper Bound

As discussed, using machine learning predictions in the rescheduling model poses challenges when the predictions overestimate the demand, resulting in unnecessary actions and increased total cost. To address this issue, a variation is introduced to mitigate the drawbacks of overestimation while leveraging the benefits of predictions when both predicted and actual demand is lower than the historical average demand. This updated demand constraint is incorporated into the rescheduling models to improve effectiveness.

$$\sum_{n\in\mathcal{N}} x'_{nbst} \geq \min(D_{bst}, D^P_{bst}) \qquad b\in\mathcal{B}, s\in\mathcal{S}^W, t\in\mathcal{T}^R/\{t_0\} \tag{9.4.1}$$

$$\sum_{n\in\mathcal{N}}\sum_{b\in B} x'_{nbst} \geq \min\Big(\big(\sum_{b\in B^W} D_{bst}\big), \big(\sum_{b\in B^W} D^P_{bst}\big)\Big) \qquad s\in\mathcal{S}^W, t\in\mathcal{T}^R/\{t_0\} \tag{9.4.2}$$

Constraints (9.4.1) and (9.4.2) show the updated demand constraint in the model variation, replacing constraints (8.2.1) and (8.2.2).

Table 9.25 provides an overview of the results obtained by incorporating the model variation with an upper bound equal to the average demand when solving the same test instances as shown in Table 9.24.

Table 9.25: Average results for solving each test instance with demand predictions for 100 simulations.

Test instance		**Metrics**						
		Swap shifts	Extra shifts	Double shifts	Over-time	Flex usage	Total cost	CR
Strategy: Absence: Demand:	Base 1 Level 2 Prediction	136.9	23.7	1.3	8.3	-	136 955	8%
Strategy: Absence: Demand:	Base 1 Level 3 Prediction	208.0	43.7	2.9	19.2	-	231 123	5%
Strategy: Absence: Demand:	Buffer 1 Level 2 Prediction	105.0	21.0	1.2	9.3	-	105 671	29%
Strategy: Absence: Demand:	Buffer 1 Level 3 Prediction	157.3	38.8	3.1	19.8	-	179 542	26%
Strategy: Absence: Demand:	Flex 10 Level 2 Prediction	120.9	25.1	23.0	27.1	50.0	130 171	13%
Strategy: Absence: Demand:	Flex 10 Level 3 Prediction	171.2	42.0	36.6	51.3	71.1	210 129	14%
Strategy: Absence: Demand:	Flex 30 Level 2 Prediction	56.6	5.8	5.9	7.7	125.8	50 492	66%
Strategy: Absence: Demand:	Flex 30 Level 3 Prediction	118.9	13.4	19.2	20.7	168.3	126 614	48%

The results demonstrate that the model variation leads to a reduction in total cost for all cases compared to their corresponding cases without predictions. Moreover, it significantly decreases the costs when compared to the original machine learning-guided rescheduling model. These findings suggest that while the machine learning model may occasionally overestimate the demand, its predictions for lower demand compared to the historical average yield superior outcomes compared to the cases without predictions.

Figure 9.6 shows that rescheduling with predictions from the machine learning model provides solutions that are closer to the results with perfect information compared to instances using the historical average demand. While there is a slight decrease observed in all metrics, the improvement from the test instances without machine learning primarily stems from a reduction in the number of extra shifts and overtime hours. In the Flex 10 test case, the improvement in total costs is also a result of a large decrease in the number of double shifts.

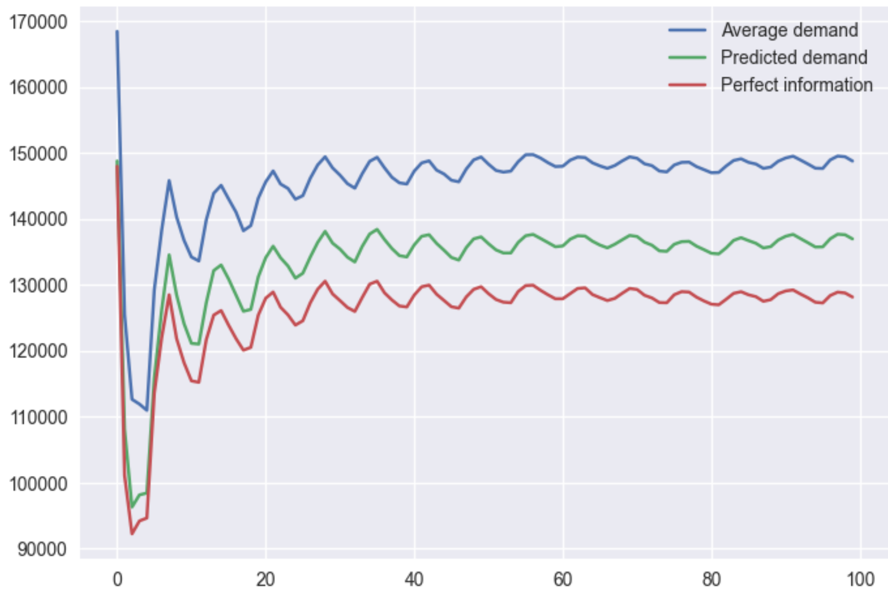


Figure 9.6: The total cost for all 100 simulations for average demand, predicted demand, and demand with perfect information.

Although the machine learning-guided test cases exhibit only a marginal reduction in the number of swaps compared to the cases without machine learning, the model appears to allocate the available swaps more efficiently. With more accurate demand information, there is a lower probability of allocating excessive resources to a shift, resulting in a decreased need for swapping away from shifts. This efficient use of swaps also reduces the reliance on more expensive actions like extra shifts and assigning overtime hours, as the shifts are adequately staffed early in the planning phase through effective swapping.

9.5 Limitations

In this chapter, we have used the rescheduling model within the simulation framework outlined in Chapter 7 to evaluate the scheduling strategies with various absence and demand combinations. Our results show that the use of proactive strategies in the scheduling model facilitates more cost-efficient reactive measures in the rescheduling model. However, there are several limitations related to the results and the strategies that are worth considering.

The results for the buffer strategy, the flexible assignments strategy, and the use of demand predictions show promise in comparison to the baseline model. However, the lack of information regarding actual rescheduling costs at CC poses a challenge in directly comparing our results to the current practice. Consequently, it becomes difficult to precisely evaluate the performance of our models in relation to the actual rescheduling process. Nevertheless, our models utilize real-life data and incorporate multiple components that closely reflect the circumstances at CC. While further research and comparisons with actual rescheduling costs are necessary for a comprehensive assessment, our findings provide valuable insights into the potential benefits of improving the rescheduling process at CC.

Despite the promising results obtained from the strategies, a key challenge lies in gaining acceptance and willingness from nurses to be scheduled for buffer shifts or assigned flexible shifts. The flexible strategy, which yields the best total cost outcomes, introduces less predictability for the nurses. Convincing nurses to embrace a flexibility range of 10% to 30% may prove challenging. On the other hand, the buffer strategy offers a more predictable schedule for nurses but provides a lower cost improvement. However, it should be acknowledged that implementing buffer shifts involves scheduling all nurses, including those who are not full-time. This may pose difficulties in obtaining approval from a larger number of nurses. To incentivize nurses, the cost savings achieved through the utilization of these strategies open up the possibility of offering bonus payments to those working buffer shifts or flexible shifts. Among the strategies, the flexible assignments strategy holds the greatest potential for such incentives, given its higher cost savings and the lower number of flexible nurses required.

The feasibility of implementing the buffer shifts or flexible shifts strategies goes beyond the nurses' willingness to accept them, requiring consideration of their practicality within the existing operational framework at CC. The cross-sectional nature of these strategies necessitates coordination and communication among section managers during both the scheduling and rescheduling process. Moreover, section managers are accustomed to their individualized schedule generation and rescheduling processes, which can exhibit significant variations across sections.

The general trend in all rescheduling cases is that swaps are performed more frequently than extra shifts. The high utilization of swaps differs from current practice at CC, where extra shifts are often used for schedule disruptions. With higher absence levels, more frequent extra shifts are necessary to cover the demand. Swaps can be problematic as it reduces the protection from future schedule disruptions. If the model had no limitations, a scenario could emerge where all nurses were swapped from a shift, and multiple rescheduling actions would have to be performed to cover the demand. Our model employs hard constraints on future demand levels to mitigate this effect.

The models developed in this thesis are based on real-world data obtained from CC. There are, however, several aspects where real and sufficient data is unavailable, which makes it challenging to make accurate assumptions. For instance, the simulations of nurse preferences rely solely on a survey conducted among nurses working in the bed wards at CC. The responses from the survey are generalized to represent weekly preferences, but in reality, individual nurses' preferences can vary significantly depending on various factors, such as the individual nurses' personal life, workload, and seasonal variations.

In addition to data regarding nurse preferences, the results obtained in this study rely on general absence data from CC. Obtaining more comprehensive absence data that includes nurse-specific absence information for each shift would enhance the accuracy of simulations regarding nurse availability. Incorporating such data into the prediction models can improve the nurse rescheduling results and handle unexpected absences more effectively.

The utilization of actual demand data from CC in our simulations has revealed certain days with unusual staffing levels, making it challenging to accurately predict demand for these atypical days based on the available data. While these anomalies are likely attributed to specific events, the underlying causes remain unexplained. In addition, our scheduling models do not currently account for holidays, which have staffing levels equal to weekends in the actual demand data. As a result, the omission of holiday considerations in the models leads to lower rescheduling costs when the actual demand is lower than expected.

The target feature that machine learning models aim to predict is the actual demand. However, it has been observed that there is a correlation between the actual demand and the planned demand. It is important to note that the planned demand label is determined well in advance before the schedule is executed. Due to this correlation, relying only on the actual demand as the label may not generate highly accurate predictions. Still, considering the available data, the actual demand remains the preferred label to utilize in the predictions.

Chapter 10

Future Research

The flexible assignment strategy has shown promising results in terms of total costs, but it has limitations in terms of high utilization of double shifts and overtime hours, as well as low percentage utilization of flexible shifts. These limitations suggest potential areas for improvement in proactive strategies related to flexible nurses. Future research should explore alternative approaches to address these limitations. One possible approach is to give lower flexibility per nurse while including other nurses beyond those working full-time, giving a better distribution of flexibility. Optimizing the distribution of hours among flexible nurses would enable easier activation and utilization of flexible shifts, reducing reliance on double shifts and overtime hours. Flexible nurses receive full pay regardless of their flex usage. An improved distribution may give a higher flexible shift usage, resulting in better utilization of the contracted hours. Future research should also focus on developing a variant of the strategy that limits the number of flexible shifts utilized per day, promoting an even distribution of flexible shift usage. Efficiently arranging shifts for flexible nurses in the schedule would facilitate better utilization of available hours and improve rescheduling outcomes.

An interesting direction for future research would involve investigating the potential benefits of combining the buffer strategy and the flexible strategy, leveraging the strengths of both. By integrating the robustness of the buffer strategy with the flexibility of the flexible assignment strategy, it is possible that even more favorable outcomes can be achieved, surpassing the obtained results from using each strategy individually. This could be further extended by looking at the higher strategical levels, like the tactical level or strategic level, in combination with the proactive strategies to facilitate an improved utilization of the available nurses.

Future extensions of our work should look at utilizing better practices for workload registrations. Although the predicted demand utilized in this thesis is based on almost five years of patient data, it is important to note that predicting future workload based on this data relies on assumptions and discussions with managers at CC. The lack of specific information regarding the connections between patients and workload levels hinders our ability to more accurately capture patterns in the data. Obtaining better information on the relationship between patients and workload levels would be valuable and could potentially provide the machine learning models with even more insightful observations and patterns. This could yield more precise predictions, resulting in improved efficiency and cost savings.

The successful application of machine learning-based demand predictions in the rescheduling model highlights the potential for enhancing nurse scheduling processes using similar predictions. The computational study revealed that even in instances with zero nurse absences, costs of rescheduling occurred due to incorrect assumptions about demand in the scheduling model. To address this issue, future research should explore the integration of machine learning-driven demand predictions into the nurse scheduling model itself. This integration would enhance the accuracy of demand estimation from the nurse scheduling process, leading to improved robustness and effectiveness of the schedules, reducing disruptions, and further minimizing the need for rescheduling.

The rescheduling model and machine learning model operate on sequential data. Introducing a recurrent neural network that incorporates the demand from the previous days could be done in future work. It has the potential to enhance the accuracy of demand predictions compared to the feed-forward neural network and decision tree employed in this thesis. By considering the demand patterns from recent days, an RNN can better capture temporal dynamics and potentially provide more precise demand predictions.

Chapter 11

Concluding Remarks

The purpose of this Master's thesis was to assess the effectiveness of proactive and reactive cross-section strategies when solving the nurse scheduling and rescheduling problem. In addition, the aim was to evaluate the value of incorporating machine learning-driven demand predictions based on real-life data. To achieve these goals, we developed a MIP scheduling model and a MIP rescheduling model guided by demand predictions. The rescheduling model was run within a simulation framework where new absences, demand information, and the updated schedule were provided for each iteration.

The findings of this thesis demonstrate the importance of considering cross-section rescheduling as a valuable approach to optimizing resource allocation. The implementation of cross-section rescheduling exhibited substantial improvements in total rescheduling costs across various absence levels. By sharing resources and covering imbalances in supply across bed wards, cross-section utilization proves beneficial for departments with overlapping competence between sections, enhancing the efficiency of the reactive rescheduling process.

Our results show that proactive scheduling facilitating cross-section rescheduling can further improve the benefits of shared resources. The introduced strategies, namely buffers and flexible assignments, were introduced with the intention of mitigating the need for rescheduling. Both strategies significantly reduced the total costs of the rescheduling process while producing similar metrics in the scheduling objectives. The cross-section utilization of the buffer strategy allows for the scheduling of buffer shifts without resorting to overstaffing, ensuring robustness without increasing costs, distinguishing our study from existing literature, and distinguishing our study from existing literature. The flexible assignments strategy is a novel approach to tackling schedule flexibility. These strategies offer promising solutions to effectively manage workforce allocation and mitigate the challenges posed by unpredictable absences and demand variations. However, determining the best strategy requires a trade-off between minimizing total costs and increasing the number of double shifts and overtime hours. The flexible assignments strategy provided very promising results in terms of total costs, while the buffer strategy provided better utilization of rescheduling actions and offered increased predictability for nurses.

One key aspect that sets this thesis apart from previous research is the integration of machine learning-driven demand predictions. The integration of machine learning predictions in the rescheduling phase has proven instrumental in bridging the gap between perfect demand information and simple average estimates of upcoming demand. It is clear from the rescheduling results that there is great cost-saving potential in accurate demand predictions, with the demand prediction significantly closing the gap between results using historical average demand and perfect demand information. Having demand predictions for multiple days in the planning period facilitates appropriately staffing shifts early on, resulting in more efficient execution of reactive actions. Moreover, our analysis has highlighted the potential for improved predictions through improved data quality and by refining machine learning models. In addition to the visible advantages reflected by the key rescheduling metrics, the demand predictions enhance predictability for section managers and nurses. More reliable demand information facilitates rescheduling actions conducted in advance rather than on the morning of each shift.

In summary, this thesis contributes to the existing body of knowledge by providing novel insights into nurse scheduling and rescheduling. The combination of proactive and reactive cross-section strategies, along with the integration of machine learning-driven demand predictions, offers a comprehensive approach to optimize resource allocation in healthcare settings. The findings emphasize the practicality and effectiveness of these strategies in reducing costs, enhancing predictability, and improving decision-making in nurse scheduling and rescheduling processes.

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Appendix A

Compressed Models

A.1 The Nurse Scheduling Model

A.1.1 Definitions

Indices

n	nurse
c	competence
e	experience
b	section
s	shift
t	day
k	week

Sets

$\mathcal{B}$	set of sections, $\mathcal{B} = \{1, 2, 3\}$
$\mathcal{C}$	set of competence levels, $\mathcal{C} = \{AN, N, SN\}$
$\mathcal{E}$	set of experience levels
$\mathcal{N}$	set of nurses
$\mathcal{N}_c$	set of nurses with competence level c, $\mathcal{N}_c \subset \mathcal{N}$, $c \in \mathcal{C}$
$\mathcal{N}_e$	set of nurses with experience level e, $\mathcal{N}_e \subset \mathcal{N}$, $e \in \mathcal{E}$
$\mathcal{N}_b$	set of nurses in section b, $\mathcal{N}_b \subset \mathcal{N}$, $b \in \mathcal{B}$
$\mathcal{K}$	set of weeks included in scheduling period
$\mathcal{T}$	set of days in the scheduling period
$\mathcal{T}_k$	set of days in week k
$\mathcal{T}^{SUN}$	set of Sundays in scheduling period
$\mathcal{S}$	set of shifts, $\mathcal{S} = \{D, E, N, F, F_1\}$
$\mathcal{S}^{\mathcal{W}}$	set of work-shifts, $\mathcal{S}^W = \{D, E, N\}$, $\mathcal{S}^W \subset \mathcal{S}$
$\mathcal{S}^F$	set of off-shifts, $\mathcal{S}^F = \{F, F_1\}$. $\mathcal{S}^F \subset \mathcal{S}$

General Parameters

$\underline{D}_{bst}$	minimum demand in section b for shift s on day t
$\underline{D}_{ebst}$	desired demand for experience e in section b for shift s on day t
$\overline{D}_{bst}^{AN}$	maximum demand for assistant nurses in section b for shift s on day t
D_{bst}^{SN}	desired demand for specialized nurses in section b for shift s on day t
D_{bst}	historical average demand in section b for shift s on day t
$\overline{M}^{D}$	maximum number of consecutive work days
$\overline{M}^{N}$	maximum number of consecutive work nights
$\overline{L}$	maximum work hours in a week
H	hours in a full time work week
H_s	duration of shift s in hours
W	working weekend recurrence
C_n	contracted employment percentage for nurse n
$\overline{F}$	upper bound for allowed deviation from contracted hours
$\underline{F}$	lower bound for allowed deviation from contracted hours
K	number of weeks in scheduling period

Decision Variables

$$x_{nbst} = \begin{cases} 1, & \text{if nurse } n \text{ in section } b \text{ is scheduled for shift } s \text{ on day } t \\ 0, & \text{otherwise} \end{cases}$$

Auxiliary Variables

δ_{nk}^{H-}	weekly deficit of work hours from contract for nurse n in week k
δ_{nk}^{H+}	weekly surplus of work hours from contract for nurse n in week k
δ_{bst}^{SN-}	unsatisfied demand of specialized nurses in section b for shift s on day t
δ_{ebst}^{E}	unsatisfied demand for nurses with a defined experience level
δ_{bst}^{D-}	deficit from average demand in section b on shift s on day t
δ_{bst}^{D+}	surplus from average demand in section b on shift s on day t

A.1.2 Multi-objective Model

$$\min z^1 = \sum_{b\in\mathcal{B}} \sum_{s\in\mathcal{S}^W} \sum_{t\in\mathcal{T}} \delta_{bst}^{D-} \tag{A.1.1}$$

Demand Coverage

$$\sum_{n\in\mathcal{N}} x_{nbst} \geq \underline{D}_{bst} \qquad b \in \mathcal{B}, s \in \mathcal{S}^W, t \in \mathcal{T} \tag{A.1.2}$$

$$\sum_{n\in\mathcal{N}} x_{nbst} \geq D_{bst} - \delta_{bst}^{D-} \qquad b \in \mathcal{B}, s \in \mathcal{S}^W, t \in \mathcal{T} \tag{A.1.3}$$

$$\sum_{n \in \mathcal{N}_{AN}} x_{nbst} \leq \overline{D}^{AN}_{cbst} \qquad b \in \mathcal{B}, s \in \mathcal{S}^W, t \in \mathcal{T} \tag{A.1.4}$$

Section Specific Assignments

$$x_{n1st} = 0 \qquad n \in \mathcal{N}/\{\mathcal{N}_{b=1}\}, s \in \mathcal{S}, t \in \mathcal{T} \tag{A.1.5}$$

$$x_{n2st} = 0 \qquad n \in \mathcal{N}/\{\mathcal{N}_{b=2}\}, s \in \mathcal{S}, t \in \mathcal{T} \tag{A.1.6}$$

$$x_{n3st} = 0 \qquad n \in \mathcal{N}/\{\mathcal{N}_{b=3}\}, s \in \mathcal{S}, t \in \mathcal{T} \tag{A.1.7}$$

Legislative Constraints

$$\sum_{b \in \mathcal{B}} \sum_{s \in \mathcal{S}} x_{nbst} = 1 \qquad n \in \mathcal{N}, t \in \mathcal{T} \tag{A.1.8}$$

$$\sum_{b \in \mathcal{B}} \sum_{s \in \mathcal{S}^W} \sum_{t \in \mathcal{T}_k} H_s x_{nbst} \leq \overline{L} \qquad n \in \mathcal{N}, k \in \mathcal{K} \tag{A.1.9}$$

$$\underline{F} \sum_{k \in \mathcal{K}} C_n H \leq \sum_{b \in \mathcal{B}} \sum_{s \in \mathcal{S}^W} \sum_{t \in \mathcal{T}} H_s x_{nbst} \leq \overline{F} \sum_{k \in \mathcal{K}} C_n H \qquad n \in \mathcal{N} \tag{A.1.10}$$

$$\sum_{t'=t}^{T=t+\overline{M}^D} \sum_{b \in \mathcal{B}} \sum_{s \in \mathcal{S}^W} x_{nbst'} \leq \overline{M}^D \qquad n \in \mathcal{N}, t \in \{1, 2, ..., \mathcal{T} - \overline{M}^D\} \tag{A.1.11}$$

$$\sum_{t'=t}^{T=t+\overline{M}^N} \sum_{b \in \mathcal{B}} x_{nbNt'} \leq \overline{M}^N \qquad n \in \mathcal{N}, t \in \{1, 2, ..., \mathcal{T} - \overline{M}^N\} \tag{A.1.12}$$

Weekend Assignments

$$\sum_{t'=0}^{T=W-1} \sum_{b \in \mathcal{B}} \sum_{s \in \mathcal{S}^W} x_{nbs(t+t')} = 1 \qquad t \in \mathcal{T}^{SUN}, n \in \mathcal{N} \tag{A.1.13}$$

$$\sum_{b \in \mathcal{B}} \left(x_{nbDt} - x_{nbE(t-1)}\right) = 0 \qquad n \in \mathcal{N}, t \in \mathcal{T}^{SUN} \tag{A.1.14}$$

$$\sum_{b \in \mathcal{B}} \left(x_{nbEt} - x_{nbD(t-1)}\right) = 0 \qquad n \in \mathcal{N}, t \in \mathcal{T}^{SUN} \tag{A.1.15}$$

$$\sum_{b \in \mathcal{B}} \left(x_{nbNt} - x_{nbN(t-1)}\right) = 0 \qquad n \in \mathcal{N}, t \in \mathcal{T}^{SUN} \tag{A.1.16}$$

Rest Regulations

$$\sum_{b\in\mathcal{B}}\big(x_{nbNt}+x_{nbD(t+1)}\big)\leq 1 \qquad n\in\mathcal{N}, t\in\mathcal{T} \tag{A.1.17}$$

$$\sum_{b\in\mathcal{B}}\big(x_{nbNt}+x_{nbE(t+1)}\big)\leq 1 \qquad n\in\mathcal{N}, t\in\mathcal{T} \tag{A.1.18}$$

$$\sum_{b\in\mathcal{B}}\big(x_{nbN(t-1)}+x_{nbF_1t}+x_{nbD(t+1)}\big)\leq 2 \qquad n\in\mathcal{N}, t\in\mathcal{T} \tag{A.1.19}$$

$$\sum_{b\in\mathcal{B}}\big(x_{nbN(t-1)}+x_{nbF_1t}+x_{nbE(t+1)}\big)\leq 2 \qquad n\in\mathcal{N}, t\in\mathcal{T} \tag{A.1.20}$$

$$\sum_{b\in\mathcal{B}}\big(x_{nbE(t-1)}+x_{nbF_1t}+x_{nbD(t+1)}\big)\leq 2 \qquad n\in\mathcal{N}, t\in\mathcal{T} \tag{A.1.21}$$

$$\sum_{n\in\mathcal{N}}\sum_{b\in\mathcal{B}}\sum_{t\in\mathcal{T}_k} x_{nbF_1t}=1 \qquad k\in\mathcal{K} \tag{A.1.22}$$

Minimize hours deviations

$$\min z^2=\sum_{n\in\mathcal{N}}\sum_{k\in\mathcal{K}}\big(\delta_{nk}^{H^-}+\delta_{nk}^{H^+}\big) \tag{A.1.23}$$

$$\sum_{b\in\mathcal{B}}\sum_{s\in\mathcal{S}}\sum_{t\in\mathcal{T}_k} H_s x_{nbst}=C_nH+\delta_{nk}^{H^-}-\delta_{nk}^{H^+} \qquad n\in\mathcal{N}, k\in\mathcal{K} \tag{A.1.24}$$

$$\sum_{b\in\mathcal{B}}\sum_{s\in\mathcal{S}^W}\sum_{t\in\mathcal{T}}\delta_{bst}^{D^-}\leq z^1 \tag{A.1.25}$$

Distribute overstaffing

Additional Parameter

$\overline{\delta}$ largest value of either the deficit or surplus of nurses working hours

Additional Variable

$$\alpha_{bst}=\begin{cases}1, & \text{if there is overstaffing in section } b \text{ on shift } s \text{ on day } t\\ 0, & \text{otherwise}\end{cases}$$

$$\max z^3 = \sum_{b \in \mathcal{B}} \sum_{s \in \mathcal{S}^W} \sum_{t \in \mathcal{T}} \alpha_{bst} \tag{A.1.26}$$

$$\sum_{n \in \mathcal{N}} x_{nbst} = D_{bst} - \delta_{bst}^{D-} + \delta_{bst}^{D+} \qquad b \in \mathcal{B}, s \in \mathcal{S}^W, t \in \mathcal{T} \tag{A.1.27}$$

$$\alpha_{bst} \leq \delta_{bst}^{D+} \qquad b \in \mathcal{B}, s \in \mathcal{S}^W, t \in \mathcal{T} \tag{A.1.28}$$

$$\delta_{nk}^{H+} + \delta_{nk}^{H-} \leq \overline{\delta} \qquad n \in \mathcal{N}, k \in \mathcal{K} \tag{A.1.29}$$

$$\sum_{n \in \mathcal{N}} \sum_{k \in \mathcal{K}} \delta_{nk}^{H+} + \delta_{nk}^{H-} \leq z^2 \tag{A.1.30}$$

Preference Violation

Additional Parameter

I_{nst} nurse n wants to avoid working shift s on day t

$$\min z^4 = \sum_{n \in \mathcal{N}} \sum_{b \in \mathcal{B}} \sum_{s \in \mathcal{S}^W} \sum_{t \in \mathcal{T}} I_{nst} x_{nbst} \tag{A.1.31}$$

$$\sum_{b \in \mathcal{B}} \sum_{s \in \mathcal{S}^W} \sum_{t \in \mathcal{T}} \alpha_{bst} \geq z^3 \tag{A.1.32}$$

Distribution of Competence and Experience

$$\min z^5 = \sum_{e \in \mathcal{E}} \sum_{b \in \mathcal{B}} \sum_{s \in \mathcal{S}^W} \sum_{t \in \mathcal{T}} \delta_{ebst}^{E} + \sum_{b \in \mathcal{B}} \sum_{s \in \mathcal{S}^W} \sum_{t \in \mathcal{T}} \delta_{bst}^{SN-} \tag{A.1.33}$$

$$\sum_{n \in \mathcal{N}_e} x_{nbst} \geq \underline{D}_{ebst} - \delta_{ebst}^{E} \qquad e \in \mathcal{E}, b \in \mathcal{B}, s \in \mathcal{S}^W, t \in \mathcal{T} \tag{A.1.34}$$

$$\sum_{n \in \mathcal{N}_{SN}} x_{nbst} \geq D_{bst}^{SN} - \delta_{bst}^{SN-} \qquad b \in \mathcal{B}, s \in \mathcal{S}^W, t \in \mathcal{T} \tag{A.1.35}$$

$$\sum_{n \in \mathcal{N}} \sum_{b \in \mathcal{B}} \sum_{s \in \mathcal{S}^W} \sum_{t \in \mathcal{T}} I_{nst} x_{nbst} \leq z^4 \tag{A.1.36}$$

Variable Declarations

$$x_{nbst} \in \{0,1\} \qquad n \in \mathcal{N}, b \in \mathcal{B}, s \in \mathcal{S}, t \in \mathcal{T} \tag{A.1.37}$$

$$\alpha_{bst} \in \{0,1\} \qquad b \in \mathcal{B}, s \in \mathcal{S}^W, t \in \mathcal{T} \tag{A.1.38}$$

$$\delta_{nk}^{H-} \geq 0 \qquad n \in \mathcal{N}, k \in \mathcal{K} \tag{A.1.39}$$

$$\delta_{nk}^{H+} \geq 0 \qquad n \in \mathcal{N}, k \in \mathcal{K} \tag{A.1.40}$$

$$\delta_{bst}^{SN-} \geq 0 \qquad b \in \mathcal{B}, s \in \mathcal{S}^W, t \in \mathcal{T} \tag{A.1.41}$$

$$\delta_{bst}^{E} \geq 0 \qquad b \in \mathcal{B}, s \in \mathcal{S}^W, t \in \mathcal{T} \tag{A.1.42}$$

$$\delta_{bst}^{D-} \geq 0 \qquad b \in \mathcal{B}, s \in \mathcal{S}^W, t \in \mathcal{T} \tag{A.1.43}$$

$$\delta_{bst}^{D+} \geq 0 \qquad b \in \mathcal{B}, s \in \mathcal{S}^W, t \in \mathcal{T} \tag{A.1.44}$$

A.2 The Nurse Rescheduling Model

A.2.1 Definitions

Indices

n	nurse
b	section
s	shift
t	day
c	competence
k	week

Sets

$\mathcal{B}$	set of sections, $\mathcal{B} = \{1, 2, 3\}$
$\mathcal{C}$	set of competence levels, $\mathcal{C} = \{AN, N, SN\}$
$\mathcal{N}$	set of nurses
$\mathcal{N}^{100}$	set of full time nurses, $\mathcal{N}^{100} \subset \mathcal{N}$
$\mathcal{N}_c$	set of nurses with competence level c, $\mathcal{N}_c \subset \mathcal{N}$, $c \in \mathcal{C}$
$\mathcal{T}^R$	set of days in planning period
$\mathcal{T}^{PRE}$	set of days before planning period
$\mathcal{T}^{POST}$	set of days after planning period
$\mathcal{T}$	set of all days, $\mathcal{T} = \left\{\mathcal{T}^{PRE} \cup \mathcal{T}^R \cup \mathcal{T}^{POST}\right\}$
$\mathcal{T}^A$	set of days in planning period or post period, $\mathcal{T}^A = \left\{\mathcal{T}^R \cup \mathcal{T}^{POST}\right\}$
$\mathcal{K}$	set of weeks included in planning period
$\mathcal{T}_k$	set of days in week k
$\mathcal{T}_k^R$	set of days in week k included in planning period, $\mathcal{T}_k^R = \left\{\mathcal{T}^R \cap \mathcal{T}_k\right\}$
$\mathcal{T}^W$	set of weekend days in planning period
$\mathcal{T}^{WA}$	set of weekend days in planning period and post period
$\mathcal{S}^W$	set of shifts, $\mathcal{S}^W = \left\{D, E, N\right\}$

Parameters

D_{bst}	historical average demand in section b for shift s on day t
$D^A_{bst_0}$	actual demand in section b for shift s on the day of rescheduling
W_n	hourly wage for nurse n
$\overline{L}$	maximum work hours in a week
P^O	penalty percentage for overtime hours
P^{OS}	penalty for overtime hours of swap
P^W	penalty payment for weekend hours
P^N	penalty payment for night hours
P^E	penalty payment for evening hours
H	hours in a full-time work week
$\overline{H}^D$	maximum hours during a day before overtime is triggered
H_s	duration of shift s in hours
H^M	duration of double shift, $H^M = H_D + H_E$
H^{PRE}_{nk}	scheduled hours for nurse n during a week k in the original schedule
X_{nbst}	scheduled value for nurse n in section b working shift s on day t
A_{nt}	nurse n is available to work on day t
Δ_{nt}	nurse n has been scheduled to work or has already worked a double shift on day t
Ω_{nk}	total overtime hours planned for nurse n during week k
Λ_{nst}	overtime hours caused by nurse n working shift s on day t

Decision Variables

$$x^{+}_{nbst} = \begin{cases} 1, & \text{if nurse } n \text{ in section } b \text{ is rescheduled to work shift } s \text{ on day } t \\ 0, & \text{otherwise} \end{cases}$$

$$x^{-}_{nbst} = \begin{cases} 1, & \text{if nurse } n \text{ in section } b \text{ is removed from shift } s \text{ on day } t \\ 0, & \text{otherwise} \end{cases}$$

$$x'_{nbst} = \begin{cases} 1, & \text{if nurse } n \text{ in section } b \text{ works shift } s \text{ on day } t \\ 0, & \text{otherwise} \end{cases}$$

$$u_{ns_1t_1t_2} = \begin{cases} 1, & \text{if nurse } n \text{ swapped a shift from day } t_2 \text{ to work shift } s_1 \text{ on day } t_1 \\ 0, & \text{otherwise} \end{cases}$$

$$\epsilon_{nst} = \begin{cases} 1, & \text{if nurse } n \text{ is assigned to work an extra shift } s \text{ on day } t \\ 0, & \text{otherwise} \end{cases}$$

$$d_{nt} = \begin{cases} 1, & \text{if nurse } n \text{ is assigned to work double shifts on day } t \\ 0, & \text{otherwise} \end{cases}$$

Auxillary variables

ω_{nk} overtime hours for nurse n in week k
λ_{nst} overtime hours for nurse n caused by working shift s on day t

A.2.2 Minimum Cost Objective

$$z^1 = \sum_{n\in\mathcal{N}} \sum_{b\in\mathcal{B}} \sum_{s\in\mathcal{S}^W} \sum_{t\in\mathcal{T}^R} H_s W_n x^{+}_{nbst} \tag{A.2.1}$$

$$z^2 = \sum_{n\in\mathcal{N}} \sum_{k\in\mathcal{K}} P^O W_n \omega_{nk} - \sum_{n\in\mathcal{N}} \sum_{s\in\mathcal{S}^W} P^O W_n \big(1 - A_{nt_0}\big) \Lambda_{nst_0} \tag{A.2.2}$$

$$z^3 = \sum_{n\in\mathcal{N}} \sum_{t\in\mathcal{T}^R} P^O W_n \big(H^M - \overline{H}^D\big) d_{nt} \tag{A.2.3}$$

$$z^4 = \sum_{n\in\mathcal{N}} \sum_{s_1\in\mathcal{S}^W} \sum_{t_1\in\mathcal{T}^R} \sum_{t_2\in\mathcal{T}^A} P^{OS} H_{s_1} W_n u_{ns_1t_1t_2} \tag{A.2.4}$$

$$z^5 = \sum_{n\in\mathcal{N}} \sum_{s_1\in\mathcal{S}^W} \sum_{t_1\in\mathcal{T}^R} P^O H_{s_1} W_n u_{ns_1t_1t_0} \tag{A.2.5}$$

$$z^6 = \sum_{n\in\mathcal{N}} \sum_{t\in\mathcal{T}^R} P^E H_E W_n \epsilon_{nEt} - \sum_{n\in\mathcal{N}} \sum_{t\in\mathcal{T}^R} P^E W_n \lambda_{nEt} \tag{A.2.6}$$

$$z^7 = \sum_{n\in\mathcal{N}} \sum_{t\in\mathcal{T}^R} P^N H_N W_n \epsilon_{nNt} - \sum_{n\in\mathcal{N}} \sum_{t\in\mathcal{T}^R} P^N W_n \lambda_{nNt} \tag{A.2.7}$$

$$z^8 = \sum_{n\in\mathcal{N}} \sum_{s\in\mathcal{S}^W} \sum_{t\in\mathcal{T}^W} P^W H_s W_n \epsilon_{nst} - \sum_{n\in\mathcal{N}} \sum_{s\in\mathcal{S}^W} \sum_{t\in\mathcal{T}^W} P^W W_n \lambda_{nst} \tag{A.2.8}$$

$$z^9 = \sum_{n\in\mathcal{N}} \sum_{b\in\mathcal{B}} \sum_{s\in\mathcal{S}^W} \sum_{t\in\mathcal{T}^A} A_{nt} H_s W_n x^-_{nbst} \tag{A.2.9}$$

$$z^{10} = \sum_{n\in\mathcal{N}} \sum_{b\in\mathcal{B}} \sum_{t\in\mathcal{T}^A} A_{nt} P^E H_E W_n x^-_{nbEt} \tag{A.2.10}$$

$$z^{11} = \sum_{n\in\mathcal{N}} \sum_{b\in\mathcal{B}} \sum_{t\in\mathcal{T}^A} A_{nt} P^N H_N W_n x^-_{nbNt} \tag{A.2.11}$$

$$z^{12} = \sum_{n\in\mathcal{N}} \sum_{b\in\mathcal{B}} \sum_{s\in\mathcal{S}^W} \sum_{t\in\mathcal{T}^{WA}} A_{nt} P^W H_s W_n x^-_{nbst} \tag{A.2.12}$$

$$\min z^T = \sum_{i=1}^{8} z^i - \sum_{j=9}^{12} z^j \tag{A.2.13}$$

Dependency in Variables

$$x'_{nbst} = X_{nbst} + x^+_{nbst} - x^-_{nbst} \qquad n\in\mathcal{N}, b\in\mathcal{B}, s\in\mathcal{S}^W, t\in\mathcal{T}^R \tag{A.2.14}$$

$$x'_{nbst} = X_{nbst} - x^-_{nbst} \qquad n\in\mathcal{N}, b\in\mathcal{B}, s\in\mathcal{S}^W, t\in\mathcal{T}^{POST} \tag{A.2.15}$$

$$\sum_{b\in\mathcal{B}} x^+_{nbst_1} = \epsilon_{nst_1} + \sum_{t_2\in\mathcal{T}^A} u_{nst_1t_2} \qquad n\in\mathcal{N}, s\in\mathcal{S}^W, t_1\in\mathcal{T}^R \tag{A.2.16}$$

$$\sum_{s\in\mathcal{S}^W} \sum_{t\in\mathcal{T}^R_k} \lambda_{nst} + \sum_{s\in\mathcal{S}^W} \sum_{t\in\mathcal{T}_k} \Lambda_{nst} = \omega_{nk} + \Omega_{nk} \qquad n\in\mathcal{N}, k\in\mathcal{K} \tag{A.2.17}$$

$$\lambda_{nst} \le \sum_{b\in\mathcal{B}} H_s x^+_{nbst} \qquad n\in\mathcal{N}, s\in\mathcal{S}^W, t\in\mathcal{T}^R \tag{A.2.18}$$

$$x^-_{nbst} \le X_{nbst} \qquad n\in\mathcal{N}, b\in\mathcal{B}, s\in\mathcal{S}^W, t\in\mathcal{T}^{\mathcal{A}} \tag{A.2.19}$$

Demand Coverage

$$\sum_{n\in\mathcal{N}} x'_{nbst} \geq D_{bst} \qquad b\in\mathcal{B}, s\in\mathcal{S}^W, t\in\mathcal{T}^A/\{t_0\} \tag{A.2.20}$$

$$\sum_{n\in\mathcal{N}} x'_{nbst_0} \geq D^A_{bst_0} \qquad b\in\mathcal{B}, s\in\mathcal{S}^W \tag{A.2.21}$$

Legislative Constraints

$$\sum_{b\in\mathcal{B}}\sum_{s\in\mathcal{S}^W}\sum_{t\in\mathcal{T}_k} H_s x'_{nbst} \leq \overline{L} \qquad n\in\mathcal{N}, k\in\mathcal{K} \tag{A.2.22}$$

$$\begin{aligned}\sum_{b\in\mathcal{B}}\sum_{s\in\mathcal{S}^W}\sum_{t\in\mathcal{T}_k} H_s x'_{nbst} - \big(\omega_{nk}+\Omega_{nk}\big) \leq & \max(H, H^{PRE}_{nk})\\ & + \sum_{t\in\mathcal{T}^R_k}\big(H^M - \overline{H}^D\big)d_{nt}\\ & + \sum_{t\in\mathcal{T}_k}\big(H^M - \overline{H}^D\big)\Delta_{nt}\\ & n\in\mathcal{N}/\{\mathcal{N}^{100}\}, k\in\mathcal{K}\end{aligned} \tag{A.2.23}$$

$$\begin{aligned}\sum_{b\in\mathcal{B}}\sum_{s\in\mathcal{S}^W}\sum_{t\in\mathcal{T}_k} H_s x'_{nbst} - \big(\omega_{nk}+\Omega_{nk}\big) \leq H^{PRE}_{nk} & + \sum_{t\in\mathcal{T}^R_k}\big(H^M - \overline{H}^D\big)d_{nt}\\ & + \sum_{t\in\mathcal{T}_k}\big(H^M - \overline{H}^D\big)\Delta_{nt}\\ & n\in\mathcal{N}^{100}, k\in\mathcal{K}\end{aligned} \tag{A.2.24}$$

Technical Constraints for the Output Schedule

$$x'_{nbst} \leq A_{nt} \qquad n\in\mathcal{N}, b\in\mathcal{B}, s\in\mathcal{S}^W, t\in\mathcal{T}^R \tag{A.2.25}$$

$$\sum_{b\in\mathcal{B}} x'_{nbst} \leq 1 \qquad n\in\mathcal{N}, s\in\mathcal{S}^W, t\in\mathcal{T} \tag{A.2.26}$$

$$\sum_{b\in\mathcal{B}}\sum_{s\in\mathcal{S}^W} x'_{nbst} \leq 1 + \big(d_{nt}+\Delta_{nt}\big) \qquad n\in\mathcal{N}, t\in\mathcal{T}^R \tag{A.2.27}$$

$$\sum_{b\in\mathcal{B}}\sum_{s\in\mathcal{S}^W}\sum_{t\in\mathcal{T}^A} A_{nt}H_s X_{nbst} \leq \sum_{b\in\mathcal{B}}\sum_{s\in\mathcal{S}^W}\sum_{t\in\mathcal{T}^A} H_s x'_{nbst} \qquad n\in\mathcal{N} \tag{A.2.28}$$

Technical Constraints for Actions

$$\sum_{b\in\mathcal{B}}\sum_{s\in\mathcal{S}^W} A_{nt_2}x^-_{nbst_2} = \sum_{s_1\in\mathcal{S}^W}\sum_{t_1\in\mathcal{T}^R} u_{ns_1t_1t_2} \qquad n\in\mathcal{N}, t_2\in\mathcal{T}^A \tag{A.2.29}$$

$$\sum_{b\in\mathcal{B}} x^+_{ns_1bt_1} \geq \sum_{t_2\in\mathcal{T}^A} u_{ns_1t_1t_2} \qquad n\in\mathcal{N}, s_1\in\mathcal{S}^W, t_1\in\mathcal{T}^R \tag{A.2.30}$$

$$\sum_{b\in\mathcal{B}}\left(x'_{nbDt}+x'_{nbNt}\right)\leq 1 \qquad n\in\mathcal{N}, t\in\mathcal{T} \tag{A.2.31}$$

$$\sum_{b\in\mathcal{B}}\left(x'_{nbNt}+x'_{nbD(t+1)}\right)\leq 1 \qquad n\in\mathcal{N}, t\in\mathcal{T} \tag{A.2.32}$$

$$\sum_{b\in\mathcal{B}}\left(x'_{nbEt}+x'_{nbNt}\right)\leq 1 \qquad n\in\mathcal{N}, t\in\mathcal{T} \tag{A.2.33}$$

$$\sum_{b\in\mathcal{B}}\left(x'_{nbNt}+x'_{nbE(t+1)}\right)\leq 1 \qquad n\in\mathcal{N}, t\in\mathcal{T} \tag{A.2.34}$$

Variable Declarations

$$x^+_{nbst}\in\{0,1\} \qquad n\in\mathcal{N}, b\in\mathcal{B}, s\in\mathcal{S}^W, t\in\mathcal{T}^R \tag{A.2.35}$$

$$x^-_{nbst}\in\{0,1\} \qquad n\in\mathcal{N}, b\in\mathcal{B}, s\in\mathcal{S}^W, t\in\mathcal{T}^A \tag{A.2.36}$$

$$x'_{nbst} = X_{nbst} \qquad n\in\mathcal{N}, b\in\mathcal{B}, s\in\mathcal{S}^W, t\in\mathcal{T}^{PRE} \tag{A.2.37}$$

$$x'_{nbst}\in\{0,1\} \qquad n\in\mathcal{N}, b\in\mathcal{B}, s\in\mathcal{S}^W, t\in\mathcal{T}^A \tag{A.2.38}$$

$$u_{ns_1t_1t_2}\in\{0,1\} \qquad n\in\mathcal{N}, s_1\in\mathcal{S}^W, t_1\in\mathcal{T}^R, t_2\in\mathcal{T}^A \tag{A.2.39}$$

$$d_{nt}\in\{0,1\} \qquad n\in\mathcal{N}, t\in\mathcal{T}^R \tag{A.2.40}$$

$$\omega_{nk}\geq 0 \qquad n\in\mathcal{N}, k\in\mathcal{K} \tag{A.2.41}$$

$$\lambda_{nst}\geq 0 \qquad n\in\mathcal{N}, s\in\mathcal{S}^W, t\in\mathcal{T}^R \tag{A.2.42}$$

A.3 Mathematical Model Extensions

A.3.1 Buffer

Scheduling Extensions

Sets

$\mathcal{B}$	set of sections, $\mathcal{B} = \{1, 2, 3, 0\}$
$\mathcal{B}^W$	set of working sections, $\mathcal{B}^W = \{1, 2, 3\}$, $\mathcal{B}^W \subset \mathcal{B}$

Parameters

β_{st}	upper bound for how many buffer nurses can be scheduled each shift

Auxiliary Variables

δ^{D-}_{st}	deficit from desired demand on shift s on day t
δ^{D+}_{st}	surplus from desired demand on shift s on day t

Maximize buffer

$$\max z^6 = \sum_{n \in \mathcal{N}} \sum_{s \in \mathcal{S}^W} \sum_{t \in \mathcal{T}} x_{n0st} \tag{A.3.1}$$

Constraints

$$\sum_{n \in \mathcal{N}} x_{nbst} \geq \underline{D}_{bst} \qquad b \in \mathcal{B}^W, s \in \mathcal{S}^W, t \in \mathcal{T} \tag{A.3.2}$$

$$\sum_{n \in \mathcal{N}} \sum_{b \in \mathcal{B}} x_{nbst} = \Big(\sum_{b \in \mathcal{B}^W} D_{bst} \Big) - \delta^{D-}_{st} + \delta^{D+}_{st} \qquad s \in \mathcal{S}^W, t \in \mathcal{T} \tag{A.3.3}$$

$$\sum_{n \in \mathcal{N}} \sum_{t \in \mathcal{T}} x_{n0st} = 0 \qquad s \in \mathcal{S}^F \tag{A.3.4}$$

$$\sum_{n \in \mathcal{N}} x_{n0st} \leq \beta_{st} \qquad s \in \mathcal{S}^W, t \in \mathcal{T} \tag{A.3.5}$$

$$\sum_{e \in \mathcal{E}} \sum_{b \in \mathcal{B}} \sum_{s \in \mathcal{S}^W} \sum_{t \in \mathcal{T}} \delta^{E}_{ebst} + \sum_{b \in \mathcal{B}} \sum_{s \in \mathcal{S}^W} \sum_{t \in \mathcal{T}} \delta^{SN-}_{bst} \leq z^5 \tag{A.3.6}$$

Rescheduling Extensions

Sets

$\mathcal{B}$ set of sections, $\mathcal{B} = \{1, 2, 3, 0\}$
$\mathcal{B}^W$ set of working sections, $\mathcal{B} = \{1, 2, 3\}$, $\mathcal{B}^W \subset \mathcal{B}$

Parameters

D^P_{bst} predicted demand for section b on shift s on day t

Variables

$$a_{nbst} = \begin{cases} 1, & \text{if buffer shift for nurse } n \text{ in section } b \text{ is activated on shift } s \text{ on day } t \\ 0, & \text{otherwise} \end{cases}$$

Constraints

$$\sum_{n\in\mathcal{N}} \sum_{b\in B} x'_{nbst} \geq \sum_{b\in B^W} D_{bst} \qquad s \in \mathcal{S}^W, t \in \mathcal{T}^R/\{t_0\} \tag{A.3.7}$$

$$x'_{n0st_0} = 0 \qquad n \in \mathcal{N}, s \in \mathcal{S}^W \tag{A.3.8}$$

$$x'_{n0st} = X_{n0st} - \sum_{b\in\mathcal{B}^W} a_{nbst} - x^-_{n0st} \qquad n \in \mathcal{N}, s \in \mathcal{S}^W, t \in \mathcal{T}^R \tag{A.3.9}$$

$$x'_{nbst} = X_{nbst} + x^+_{nbst} + a_{nbst} - x^-_{nbst} \qquad n \in \mathcal{N}, b \in \mathcal{B}^W, s \in \mathcal{S}^W, t \in \mathcal{T}^R \tag{A.3.10}$$

Variable Declaration

$$a_{nbst} \in \{0, 1\} \qquad n \in \mathcal{N}, b \in \mathcal{B}, s \in \mathcal{S}^W, t \in \mathcal{T}^R \tag{A.3.11}$$

A.3.2 Flexible Assignments

Rescheduling Extensions

Sets

$\mathcal{N}^F$ set of flexible nurses $\mathcal{N}^F \subset \mathcal{N}$

Parameters

I_{nst}	nurse n wants to avoid working shift s on day t
R_n	remaining number of flexible shifts for nurse n

Variables

$$f_{nbst} = \begin{cases} 1, & \text{if flexible shift for nurse } n \text{ in section } b \text{ is scheduled to shift } s \text{ on day } t \\ 0, & \text{otherwise} \end{cases}$$

Constraints

$$I_{nst} f_{nbst} = 0 \qquad n \in \mathcal{N}^F, b \in \mathcal{B}, s \in \mathcal{S}^W, t \in \mathcal{T}^R \tag{A.3.12}$$

$$\sum_{b \in \mathcal{B}} \sum_{s \in \mathcal{S}^W} \sum_{t \in \mathcal{T}^R} f_{nbst} \leq R_n \qquad n \in \mathcal{N}^F \tag{A.3.13}$$

$$x'_{nbst} = X_{nbst} + x^+_{nbst} + f_{nbst} - x^-_{nbst} \qquad n \in \mathcal{N}^F, s \in \mathcal{S}^W, b \in \mathcal{B}^W, t \in \mathcal{T}^R \tag{A.3.14}$$

$$x'_{nbst} = X_{nbst} + x^+_{nbst} - x^-_{nbst} \qquad n \in \mathcal{N}/\{\mathcal{N}^F\}, s \in \mathcal{S}^W, b \in \mathcal{B}^W, t \in \mathcal{T}^R \tag{A.3.15}$$

Variable Declaration

$$f_{nbst} \in \{0, 1\} \qquad n \in \mathcal{N}, b \in \mathcal{B}, s \in \mathcal{S}^W, t \in \mathcal{T}^R \tag{A.3.16}$$

Appendix B

Absence Data

Table B.1 displays the absence data provided by CC. The absences are given as a monthly percentage from January 2020 to September 2022. From our understanding, the percentages only display absences registered on scheduled nurses.

The data is given per bed ward (BW), and the corresponding percentages are related to the number of nurses for the given section. To find a total absence percentage across the sections, each section's individual percentages must first be linked to their respective number of nurses.

B.0.1 Generation of Absences

The generation of absences is based on section-specific data on aggregated absence percentages across CC from January 2020 through September 2022, as shown in Table B.1. The data is used to estimate the expected absence percentage, which is approximately 10%. The provided data reflects the absence of nurses with scheduled shifts, not the nurses' availability. To generate the probability of absences across sections, we assume that 10% of all nurses, both scheduled and unscheduled, are unavailable on average.

Table B.2 presents the values used in the Markov model from Chapter 7 to generate absences.

Table B.2: Parameter values for Markov model

P(A, A)	0.945
P(A, U)	0.055
P(U, U)	0.500
P(U, A)	0.500

$$\lim_{n\to\infty} \begin{bmatrix} A & U \end{bmatrix} \begin{bmatrix} 0.945 & 0.055 \\ 0.5 & 0.5 \end{bmatrix}^n \approx \begin{bmatrix} 0.9 & 0.1 \end{bmatrix}, \tag{B.0.1}$$

$$A + U = 1, \qquad A, U \geq 0 \tag{B.0.2}$$

Equations (B.0.1) and (B.0.2) show how the parameters in Table B.2 fit the estimated availability percentage. The equations show that the specified transition probabilities converge towards a 90% possibility of being available and a 10% possibility of being unavailable. When the system is initialized with a 10% random sample of unavailable nurses, the system should preserve a similar percentage throughout the whole simulation period. Since we do not have any information on the average duration of a short-term absence, it is assumed to be two days. The transition probabilities from the unavailable state entail an expected stay of two days in that state. The assumption that absence, on average, lasts two days is also used to avoid long-time absences in the simulation, as CC handles long-time absences using other methods than only rescheduling.

In other cases than 10% absence, the expected absence length of two days is still kept. This means that only the P(A, A) and P(A, U) values are altered to ensure convergence towards the wanted absence percentage.

Although the absence percentages are reported, this only covers the availability of nurses scheduled to work specific shifts. The exact availability percentages would improve the realism of the simulations. Additionally, there are likely seasonal variations in the provided data. Access to personal absence percentages would enhance the precision of our simulation.

Table B.1: Overview of absence data from the Clinic of Cardiology

Month	**Year**	**BW 1**	**BW 2**	**BW 3**
January	2020	16.0%	9.5%	15.8%
	2021	10.5%	7.8%	7.9%
	2022	15.2%	8.6%	7.2%
February	2020	16.5%	7.7%	8.1%
	2021	8.5%	9.1%	8.9%
	2022	23.0%	12.1%	10.5%
March	2020	14.1%	10.8%	12.0%
	2021	7.9%	12.5%	10.6%
	2022	24.7%	14.8%	9.6%
April	2020	11.4%	6.4%	4.8%
	2021	7.8%	11.8%	12.7%
	2022	16.7%	6.0%	5.7%
May	2020	7.4%	8.9%	4.0%
	2021	11.1%	15.0%	11.5%
	2022	18.3%	5.6%	12.1%
June	2020	11.4%	5.0%	5.6%
	2021	14.5%	12.9%	6.4%
	2022	13.8%	7.2%	10.7%
July	2020	5.2%	8.5%	8.9%
	2021	8.7%	5.3%	11.0%
	2022	11.5%	4.9%	7.0%
August	2020	7.0%	9.4%	7.2%
	2021	12.7%	4.6%	10.6%
	2022	12.9%	4.2%	7.2%
September	2020	15.1%	6.0%	6.7%
	2021	12.2%	7.9%	7.6%
	2022	12.8%	5.1%	6.7%
October	2020	12.6%	7.3%	6.7%
	2021	14.5%	4.3%	6.7%
November	2020	10.2%	3.7%	6.9%
	2021	18.4%	7.1%	8.9%
December	2020	10.6%	4.2%	8.8%
	2021	16.0%	7.2%	6.3%

Appendix C

Nurse Data

Table C.1 presents an overview of the information about the nurses employed at Bed Ward 1, Bed Ward 2, and Bed Ward 3. It includes details on the number of nurses categorized by their competence level, experience level, and their employment percentage C_n at each bed ward.

Table C.1: Overview of nurse information from the Clinic of Cardiology.

		Assistant Nurse			**Nurse**			**Specialized Nurse**		
	C_n	$\mathcal{E}$=1	$\mathcal{E}$=2	$\mathcal{E}$=3	$\mathcal{E}$=1	$\mathcal{E}$=2	$\mathcal{E}$=3	$\mathcal{E}$=1	$\mathcal{E}$=2	$\mathcal{E}$=3
BW1	50%	0	0	1	0	0	0	0	0	0
	60%	0	0	1	3	0	3	0	0	1
	75%	0	0	2	2	1	3	0	0	0
	90%	0	0	4	0	3	1	0	0	0
	100%	0	0	2	0	3	2	0	0	0
BW2	50%	0	0	0	0	1	1	0	0	0
	60%	0	0	2	0	1	1	0	0	1
	75%	0	0	1	2	1	2	0	0	1
	90%	0	0	0	0	7	3	0	0	0
	100%	0	0	3	0	1	4	0	0	1
BW3	50%	0	0	0	0	1	3	0	0	0
	60%	0	0	0	0	1	5	0	0	1
	75%	0	0	1	0	1	4	0	0	1
	90%	0	0	1	0	2	0	0	0	2
	100%	0	1	4	1	7	3	0	0	0

Appendix D

Preference Generation

Uniform random numbers are used to generate the I_{nst} matrix (Hillier & Lieberman, 2015). For each nurse n, shift s, and day t, a random number between 0 and 1 is generated. If the generated number is below the value threshold presented in Figure 8.1 and Figure 8.2 for the corresponding shift and day, I_{nst} sets the value to 1, representing disinterest in working that specific shift s on day t. If the random number is above the threshold, the matrix value is set to 0.

NTNU
Norwegian University of
Science and Technology