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Norwegian and Swedish post-compulsory school students' perspectives on the purpose of school mathematics: An exploratory study

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Abstract

Many countries' educational aspirations are reflected in government-produced national curricula that specify, with varying degrees of prescription, what should be taught, when it should be taught, and even how it should be taught. These systemic ambitions are typically located in culturally constructed images of the ideal citizen dependent on, for example, whether cultures privilege the individual or the collective, or whether knowledge, particularly mathematics, is construed as practical or abstract. Despite such differences, little research has examined students' awareness of the systemic expectations that underpin their experiences of school mathematics. In this paper, drawing on data from 35 group interviews involving 92 students from post-compulsory schools, we present an exploratory study of Norwegian and Swedish students' perspectives on the purpose of school mathematics. Constant comparison analyses yielded six themes, or purposes, concerning the role of mathematics in support of *everyday shopping*, *future employment*, *the learning of other subjects*, *logical thinking and problem-solving*, *the development of an esthetic appreciation*, and *the management of personal finances*. However, despite extensive emphases in both the Norwegian and the Swedish curricula on the role of mathematical knowledge in the creation of just and equitable societies, no students discussed such societal goals. Some implications are discussed.

Keywords

group interviews, Norway, post-compulsory school, purpose of school mathematics, student perspectives, Sweden

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1. Introduction

The governments of many countries specify, in the form of written documents, the mathematics children in compulsory school are expected to learn. These documents, hereafter national curricula, effectively form the "contract" between the state and the teacher (Andrews et al., 2021) and highlight, inter alia, the extent to which a system's objectives are loosely or strongly framed (Bernstein, 1975). For example, the national curriculum for Cyprus not only specifies precisely the mathematics children should learn each year but also, leading to little teacher autonomy, mandates the use of government-produced textbooks (Xenofontos, 2019). In Singapore, the curriculum specifies precisely the content for each school year although, with textbooks only regulated and not mandated (Kaur, 2014), teachers are afforded some autonomy. In England, not only is the national curriculum less strongly framed but textbook production is effectively unregulated (Jones & Fujita, 2013), leaving teachers with considerable autonomy. Importantly, the learning objectives specified in these documents typically refer to assessable mathematical knowledge, which here we describe as tangible goals. However, where nations specify such tangible goals, they tend also to justify the subject's curricular inclusion on the basis of intangible goals like, for example, mathematics being a unique form of knowledge, a powerful tool for understanding and explaining the world, a means of personal empowerment, and a facilitator of social justice. Overall, though, a culture's curricular specification of mathematics, whether tangible or intangible, reflects not only that culture's construal of the "ideal person" (Cummings, 2003) but also its perspective on the purpose of school mathematics. This leads us to ask, to what extent are the recipients of that teaching aware of these tangible and intangible goals?

Interestingly, Niss (1996) observed that

> it is very difficult indeed to identify and locate the real goals of mathematics education in any given society. Firstly, it is often the case that these goals are not made explicit. What we can observe is the presence and the reality of mathematics education in its various formats and shapes, whereas the goals, like the underlying reasons and driving forces, are not directly observable. (p. 17)

In many ways the invisibility of these goals is unsurprising. For more than a century, scholars have acknowledged that education policy typically draws on "the secret workings of national life" (Sadler, 1900, as cited in Bereday, 1964, p. 310) and implicit rather than explicit rules (Williams, 1958). It draws on "folklore uncritically accepted from our ancestors" (Lauwerys, 1959, p. 294) and didactical folkways "warranted by their existence and taken-for-granted effectiveness" (Buchmann, 1987, p. 154). In this way, school mathematics should "be understood as a kind of cultural knowledge, which all cultures generate but which need not necessarily 'look' the same from one cultural group to another" (Bishop, 1988, p. 180). For example, the purpose of school mathematics in Spain has traditionally been to facilitate students' solving problems related to daily life and the workplace (García et al., 2013), while in Japan it has been unrelated to any sense of the real world (Yanagimoto & Yoshimura, 2001). Even reasoning, a core process competence of mathematics, is conceptualized in vague and sometimes contradictory ways in different curricula (Jeannotte & Kieran, 2017), while problem-solving, another core process competence, is manifested differently in different cultural contexts (Andrews, 2009).

In light of such matters, it is not unreasonable to assume that students leave school with experientially constructed views, informed by a range of hidden cultural imperatives (Wong et al., 2001), as to the purpose of school mathematics. Yet, despite evidence that some "philosophical work has been carried out on the aims of education in general… very little sustained work of this kind appears to have been carried out in mathematics education" (Huckstep, 2000, p. 8). This omission, particularly from the perspective of the student, reflects a surprisingly under-elaborated field (Samuelsson, 2008), and is something we address in this paper.

The research presented in this paper is located in two national education systems, Norway and Sweden. Both countries have long-established but regularly revised national curricula that specify, albeit in different ways, the tangible goals children are expected to have achieved by the end of different time points. The Norwegian mathematics content is presented as a series of statements for each school year (Utdanningsdirektoratet, 2019), while the Swedish curriculum presents them as objectives to be achieved by the end of grades three, six, and nine, respectively (Skolverket, 2018). In addition, both curricula include generic mathematical process goals relating to, for example, problem-solving, reasoning, and communication. In both countries, textbooks are unregulated (Ahl, 2016; Lepik et al., 2015) and, free to choose within the specified time periods when and how they teach expected content, teachers are afforded considerable autonomy. However, despite such autonomy, mathematics lessons in both countries, irrespective of a student's age, are structurally similar and typically comprise two phases. Teachers undertake a whole-class going-through of the lesson's new material, known as gjennomgang in Norway and genomgång in Sweden (Andrews & Nosrati, 2018), before inviting students to complete exercises from their textbooks (Andrews & Larson, 2017; Lepik et al., 2015).

Importantly, both curricula (Skolverket, 2018; Utdanningsdirektoratet, n.d.) specify a range of, arguably, intangible goals. Both encourage students to adopt a critically outward perspective on the planet and, in so doing, highlight the importance of sustainability. Both emphasize the role of mathematics in preparing students for their everyday and professional lives and, rooted in a Western Humanist tradition, discuss the fostering of a sense of justice and tolerance. Both, rooted in respect for human rights, emphasize the importance of cultural diversity, the dignity of others, intellectual freedom, and equality in all its forms. Both stress the promotion of democratic values as the underpinning of a participative cultural life based on individual responsibility and the ability to interpret data and make reasoned decisions that will not impact negatively on others. In short, in great detail, both curricula stress the values on which the educational foundations of their systems are built.

2. Defining the purpose of school mathematics

A literature search on the "purpose of school mathematics" and variants of the phrase with *purpose* replaced by *goals*, *objectives*, *aims*, or *nature*, confirmed Samuelsson's (2008) perception of an underdeveloped and poorly defined field. For example, in their study of English primary teacher education students' emergent professionalism, Brown and McNamara's (2011, p. 70) found many students exhibiting "bafflement about the purpose of school mathematics". However, as the authors offered no indication as to their interpretation of the phrase, the purpose of school mathematics is clearly an unproblematic given with no need for definition. Others, studies, typically referring to systemic goals, have suggested that the *objective* of school mathematics is, somewhat vaguely, "to prepare all students for… their future career and bring some of them up to become professional mathematicians" (Sigurdsson, 2006, p. 49) or "the acquisition of mathematical knowledge and skills… the enhancement of mathematical thinking ability, and… the cultivation of problem-solving ability and attitude" (Pang, 2014, p. 265).

Broadly speaking, our reading of the literature has yielded, albeit tentatively, three broad purposes of school mathematics, which, in the interests of communicative convenience, we have labeled specialized mathematical knowledge, pragmatic mathematical knowledge, and critical mathematical knowledge respectively. In identifying the characteristics of these purposes, our aim has been less about critiquing their underlying ambitions, but uncovering how the purpose of school mathematics has been construed. Importantly, our identification of the three purposes has been largely based on material with only implicit connections to the theme. For example, while Ernest's (2016) paper is explicitly focused on values in mathematics, its content has allowed us to infer various connections to how the purpose of school mathematics has been historically construed. In other words, much of what follows is less a summary of colleagues' attempts to clarify the purpose of school mathematics, because these attempts do not exist, but a synthesis of inferences. That said, our hope is that there is nothing in the three purposes that will surprise the reader.

Specialized mathematical knowledge underpins both higher education and the numerate professions (Ernest, 2016; Greer & Mukhopadhyay, 2005; Noyes, 2007; Sigurdsson, 2006; Watson, 2004) and, more generally and depending on context, society's advancement (Fitzsimons, 2001; Wright, 2017). Specialized mathematical knowledge acts as a gatekeeper to life's opportunities, offers its possessors cultural capital and, importantly, the tools to liberate the oppressed (Gutstein, 2012). Before the 20th century, access to specialized mathematical knowledge was limited to an elite minority, with most children internationally experiencing little more than basic arithmetic (Clements et al., 2013). The social and political upheavals of the 20th century initiated considerable change and specialized mathematical knowledge became an explicit objective internationally. For example, focused on equity and a perceived need for national economic competitiveness, specialized goals were evident in the former Soviet Union (Ailes & Rushing, 1991; Karp, 2006; Romberg, 1984) and many of its satellites (Andrews, 2003; Bodovski et al., 2014). Elsewhere, following the end of the Second World War, the elite imperative could be found in the introduction of Bourbakian mathematics into many Western curricula (Andrews, 2015). However, the systemic presentation of such specialized goals for all students has not gone unchallenged. In the former Soviet Union, they failed to deliver equity (Silova, 2009) and elsewhere proved inaccessible to many students (Freudenthal, 1979; Gregg, 1995). Moreover, they problematically presented mathematics as morally neutral (Warnick & Stemhagen, 2007).

Pragmatic mathematical knowledge, typically comprising a nonspecialist set of "useful skills and procedures" (Watson, 2004, p. 361), incorporates some sense of functional numeracy, elements of mathematical modeling, and appropriate work-related knowledge (Ernest, 2016; Fitzsimons, 2001). It is increasingly construed as important by the mathematics education community (García et al., 2006; Geiger et al., 2018; Maass et al., 2018) and has prompted various educational systems to promote, almost as alternatives to formal mathematics, the teaching of numeracy, otherwise known as quantitative literacy, mathematical literacy or quantitative reasoning (Callingham et al., 2015), as a set of tools to facilitate learners' successful engagement with the demands of everyday life (Askew, 2015; Bennison, 2015; Venkat & Winter, 2015). Pragmatic mathematical knowledge resonates closely with the goals of international studies like the OECD's programme of international student assessment (PISA), which examines the extent to which 15-year-old students "are prepared to meet the challenges of today's societies" (OECD, 2003, p. 9). In particular, PISA moves "beyond the kinds of situations and problems typically encountered in school classrooms" toward situations people face when going about their daily lives and which necessitate the application of mathematical skills in less structured contexts (OECD, 2003, p. 24). That is, PISA aims to focus "on the capacity of students to put mathematical knowledge into functional use in a multitude of different situations in varied, reflective and insight-based ways" (Schleicher, 2007, p. 351).

Critical mathematical knowledge presents school mathematics as a source of social empowerment (Aguirre et al., 2017; Ernest, 2016; Gonzalez, 2009; Stemhagen, 2016) that underpins citizenship (Noyes, 2007; Watson, 2004) and democratic participation (Gutstein, 2012; Stemhagen, 2011). It has become the goal of those engaged in what is known as critical mathematics education (Agudelo-Valderrama, 2009; Brantlinger, 2013; Frankenstein, 1983; Gonzalez, 2009), whereby mathematics, focusing on equity, diversity, and social justice, helps students to question, critique and take action concerning important social and political issues (Aguirre et al., 2017; Felton-Koestler, 2017; Gutstein, 2012; Stemhagen, 2016). It challenges the view that mathematics lies outside the moral and ethical imperatives that underpin cultural life (Davis, 2001; de Freitas, 2008; Warnick & Stemhagen, 2007) and facilitates students finding an answer to their oft-asked question, "what good is all this stuff?" (Brelias, 2015). Implicit in such a view of mathematics is the sense that the purpose of school mathematics must be more than just the provision of a "credit note" for students (Gregg, 1995). However, even within mathematics education itself, researchers tend to polarize "between those who think of their work as primarily critical in nature versus those who see their work as more math content-focused" (Stemhagen, 2016, p. 96), while others, despite ambitions to counter them, reinforce deficit narratives (Adiredja, 2019).

3. Researching students' perspectives on the purpose of school mathematics

As indicated earlier, research on students' perspectives on the purpose of school mathematics is an under-researched field (Samuelsson, 2008). A small number of qualitative studies, undertaken subsequent to interventions focused on introducing students to the role of mathematics in creating more equitable and just societies, have evaluated teacher education (Felton-Koestler, 2017) and high school students' (Brelias, 2015) perspectives on the purpose of school mathematics. However, such studies tend to be methodologically problematic in relation to the exploratory study presented below. For example, with respect to preservice teachers, Felton-Koestler (2017) analyzed data from a variety of coursework assignments designed to induct participants into an awareness of the sociopolitical dimensions of mathematics teaching and learning. In so doing, his data reflected a bias toward a predetermined set of desirable outcomes rather than an authentic attempt to elicit students' uninfluenced views. With respect to school students, Brelias (2015) reports on how statistical and mathematical modeling activities can facilitate high school students' awareness of the use of mathematics as a tool for social inquiry. She found, albeit by means of leading questions, that students not only began to see mathematics as a valuable support to their emergent awareness and understanding of social issues but recognized its role in supporting their critical engagement with evidence. Such studies, driven by ambitions explicitly relating to the sociopolitical dimensions of school mathematics, tend to produce somewhat one-dimensional outcomes of limited relevance to an authentic exploratory study.

Similar problems emerge from the small number of quantitative studies alluding to the student voice (Sayers & Andrews, 2018; OECD, 2013; Samuelsson, 2008). Here, researchers have typically adopted top-down approaches, whereby predetermined categorizations have been operationalized for quantitative analysis. For example, as part of its PISA 2012 assessment of mathematical knowledge, the OECD (2013) evaluated *students' instrumental motivation to learn mathematics* against four broad statements set against a Likert-like scale. These were (OECD, 2013, p. 79):

> Making an effort in mathematics is worth it because it will help me in the work that I want to do later on.
>
> Learning mathematics is worthwhile for me because it will improve my career prospects and chances.
>
> Mathematics is an important subject for me because I need it for what I want to study later on.
>
> I will learn many things in mathematics that will help me get a job.

While the OECD's scale could be construed as an indicator of students' perspectives on the purpose of school mathematics, it seems both limited and uncertain in its conceptualization. Indeed, with three statements addressing later employment and one later education, a two-dimensional scale of this nature would be unlikely to survive the rigors of a factor analysis. In a similar vein, Samuelsson (2008), drawing on government data from 120 compulsory schools, identified a construct he labeled *importance of mathematics*. However, of the six statements implicated in his scale, two focused on some generic sense of importance, one on personal interest, one on mathematics' importance in future education, another on its importance in future employment and, finally, a vague allusion to the use mathematics. Similar criticisms can be made of Sayers and Andrews' (2018) study, where, drawing on a factor analysis of survey data from Swedish teacher education students, a factor based on just two items were identified. This factor, which they labeled *mathematics as an important life tool*, comprised just two essentially unrelated items, namely:

> School mathematics is taught to develop the ability to think logically.
>
> School mathematics is taught because it will be useful in everyday life.

In short, such scales are methodologically problematic. As with other problematic mathematics education studies (Andrews et al., 2022), the aggregation of different constructs yields single measures of limited value (Hardesty & Bearden, 2004). In this paper, therefore, our aim is to avoid such problems by undertaking a bottom-up investigation of Norwegian and Swedish upper secondary students' perspectives on the purpose of school mathematics. Moreover, while reference to each of the three forms of mathematical knowledge discussed above can be found in the curricular goals of both countries (Skolverket, 2018; Utdanningsdirektoratet, 2019), no study has investigated the resonance between such goals and students' perspectives on what they have been taught. Importantly, the decision to focus on upper secondary students was based on the rationale that they would have completed compulsory school and be well placed to comment on why they think they have been learning mathematics for 10 years. This exploratory study is framed by the question:

> How do Swedish and Norwegian upper secondary students construe the purpose of school mathematics?

4. The study and its methods

The data on which the following is based derives from 35 semi-structured group interviews involving 50 Swedish and 42 Norwegian upper secondary school students. In both countries, upper secondary school refers to a 3-year program that follows the completion of compulsory school at the end of year 9 in Sweden and year 10 in Norway. In Sweden, the upper secondary school comprises 18 tracks, of which 12 are vocational and 6, permitting access to higher education, are academic. In a similar vein, Norwegian upper secondary school comprises 15 tracks, 10 vocational and, also permitting access to higher education, 5 academic tracks.

Swedish participants attended one of four Stockholm schools, whose available programs reflected the expectations and aspirations of their locality. One offered only academic tracks, one offered only vocational tracks and two offered both vocational and academic. Norwegian participants derived from two schools offering only academic tracks in Oslo and Trondheim respectively and an Oslo school offering only vocational tracks. Procedurally, a diverse selection of upper secondary schools in both countries was contacted informally and, having obtained appropriate permissions, classes were visited and the project explained. From this process volunteers emerged, typically in self-selected friendship groups ranging in size from two to five. In both countries, once the agreed number of group interviews had been confirmed, a decision that we discuss below, no further schools were contacted. All participants, who were fully aware of their rights to withdrawal and anonymity, signed participation agreements. Students were interviewed during the early semesters of their upper secondary program, so would have typically been aged either 16 or 17 at the time. The sample, reflecting the willingness of students to participate, comprised 31 academic track students and 19 vocational track students from Sweden, and 24 academic track and 18 vocational track students from Norway. At the time of their interviews, all students had completed the first mathematics course of one semester's duration.

When undertaking an exploratory study, deciding how many interviews to conduct is important for at least two, somewhat conflicting, reasons. The pragmatic reason concerns ensuring thematic saturation, or the point after which no new ideas are generated by the analysis (O'Reilly & Parker, 2013). The ethical decision concerns ensuring that the goodwill of informants is not abused by the undertaking of more interviews than is necessary. Prior research has indicated that where participants are drawn from relatively homogeneous groups, few interviews are necessary. For example, in the context of group interviews, a recent study of parents' views on school-home communication was achieved with just six interviews (Campbell et al., 2016). Of even greater relevance to this study, 2 studies focused on upper secondary students' perspectives on different aspects of mathematical learning found 9 interviews sufficient for investigating Mexican students' emotional responses to mathematics (Martínez-Sierra & García-González, 2017) and 12 for an in-depth analysis of Swedish students' understanding of linear equations (Andrews & Öhman, 2019). Consequently, acknowledging that exploratory studies aim to exhaust possible themes rather than offer generalities and in light of prior research, it was concluded that 20 interviews in each country should be sufficient for achieving thematic saturation without undermining participants' goodwill. In practice, while 2 interviews failed to materialize in Sweden and 3 failed to materialize in Norway, 18 interviews in the former and 17 in the latter proved more than sufficient for uncovering students' perspectives on the purpose of school mathematics.

Interviews, which typically lasted around 40 min, were structured around four broad questions, one of which invited students to reflect on why they were expected to study mathematics throughout their school careers. Group interviews were undertaken because they facilitate exploratory research by "allowing opinions to bounce back and forth and be modified by the group, rather than being the definitive statement of a single respondent (Frey & Fontana, 1991, p. 178). Interviews were undertaken at a time and place determined by the students, video recorded on laptop computers, and transcribed. In both countries, some interviews were conducted in English according to the linguistic competence and wishes of participants.

To avoid masking culturally nuanced differences, data from each country were analyzed separately and results were compared cross-nationally. In each country, acknowledging the exploratory nature of the study, data were subjected to the following data-driven constant comparison analysis (Glaser & Strauss, 1967; Strauss & Corbin, 1998). A transcript was read and reread and categories of responses related to the purpose of school mathematics were identified and recorded. The second transcript was then read and reread with two explicit objectives. First, to identify further evidence of any categories yielded by the first transcript to facilitate both thematic saturation and clarity of definition. Second, to identify further categories, in which case previous transcripts were reread to see if the new category had been missed earlier. This process was repeated until all transcripts had been read and coded. To minimize the loss of contextual meaning, transcripts were analyzed in the language used by informants before quotes selected for inclusion in this paper were translated into English, a process entailing the transformation of Norwegian and Swedish idioms into forms recognizable to an English-speaker without losing any intended meaning. Finally, due to written Swedish being intelligible to a Norwegian and vice versa, any uncertainty with either analysis was discussed by both authors and an agreement was reached.

5. Results

The analytical process described above yielded six broad themes, hereafter purposes. These present the purpose of school mathematics as enabling or supporting

- everyday shopping;
- future employment;
- the learning of other subjects;
- logical thinking and problem-solving;
- one's appreciation of mathematics; and
- the management of one's personal finances.

Despite thematic similarity, some of the purposes distinguished the two cohorts, while others were manifested similarly. In the following, the presentation of each purpose, exemplified by selected student responses, reflects these similarities and differences. To facilitate the reader's understanding, each student's self-selected pseudonym is appended by one of the four suffixes, (NA), (NV), (SA), and (SV), denoting Norwegian academic, Norwegian vocational, Swedish academic, and Swedish vocational tracks, respectively.

5.1 Supporting everyday shopping

Few students did not speak about the everyday world of shopping and, broadly speaking, the two cohorts offered similar perspectives. At a general level, typical Norwegian perspectives were found in the comments of Tania (NA), who noted that "you need it in daily life and… you're in the shop, you want to buy something, and then you must add… eh… sum the price of the things you buy and stuff…" and Robin (NV), who commented that "when you are going to buy food and stuff, you need to know how much things cost and stuff… and how much you can spend and stuff…". Similar perspectives emerged from the Swedish data, as seen in Alice's (SA) observation that "it's something you use in everyday life, it's typical when you go shopping, it's just the basics", and Markus' (SV) view that "it's always good to have a base in maths. So, if you go shopping and have a hundred, then you cannot buy for more than a hundred".

Others, from both cohorts and typically academic, offered more nuanced perspectives. For example, Emma (NA) noted that "if you are going to the shop to buy clothes that are half price, or 25% off, then there are things you learnt at school that you have to use…", while Hanna (SA) commented that "if you've gone shopping and there is a discount and you know how to calculate that… It is like useful to use maths like this".

Overall, the above has highlighted a not unexpected and commonly held perception that learning mathematics allows one to manage day-to-day economic transactions. This ability to check not only that one has sufficient money for what one wants to buy but also to check one's bill recurred throughout the interviews.

However, despite such broad similarities, there was a key difference between the two cohorts. Half of all Swedish interviews, but not one Norwegian, included some reference to comparing the price of items across different sellers, as seen in André's comment:

> Yeah, especially groceries. So, you have to calculate how much money you have and what you can buy and how much that will cost and (…) you always look at the prices compare them to the prices in another store…

That said, there was also evidence of some skepticism, including Kevin's (SV) rather tongue-in-cheek suggestion that people typically buy what they want without thinking about the financial consequences.

5.2 Supporting future employment

A second, future-oriented, purpose emerged from all but one Norwegian interview and concerned how students saw mathematical competence as contributing to future employability. At the most prosaic end of the spectrum, dominated by Swedish vocational students, were comments focused on unskilled occupations. For example, Leo (SV) spoke of the basic mathematical competence necessary for being a shopkeeper, while Sheldon (SV) offered similar comments with respect to becoming a bus driver.

Moving beyond such unskilled employment, vocational students from both countries alluded to the supportive role of mathematics in various trades. For example, Dennis (SV) commented that to be "an electrician you need…, current, voltage, and everything like that; it requires maths", while Matheus (NV) noted:

> I plan to be a fire technician, and you need a bit of maths there… so if I am good at maths, it will help me… like… measure and stuff… you don't need to check on the calculator all the time if you know it.

The most commonly occurring comments, representing around half of all students' utterances across both countries and levels of education, concerned the professions. For example, Sheldon (SV), who had much to say on the matter, added that mathematics was a necessary prerequisite if one wanted to work in astronomy or be a medical doctor. In a similar vein, Tomas (SV) commented that

> if you want to be an architect for example you know, it's very important to know specific stuff in maths or if you want to be a designer, you know whatever you do, it's good to know like specific parts of maths.

Similar views were expressed by Norwegian students, as in Kristine's (NA) argument that:

> … it depends a lot on what you plan to do after high school… if you have planned to take a fairly high education then it is clear that there is a lot of maths, if, for example, you are going to be a chemist or a physicist or… medical studies too… but if you plan to work at McDonalds, for example, then it is not so important…

Others suggested that even if one knows what employment route one expects to take, it is wise to continue to study mathematics even if its relevance seems uncertain. For example, Kenneth (SV) commented that "even when you think you have found out what you want to be, you might want to switch careers later and then maybe you need more knowledge in maths", while Sarah (NA) noted that "you can't know in advance, so everyone has to learn everything… so that… you can be what you want…". In other words, many students seemed clear that taking a flexible view with respect to studying school mathematics may be wise. In this respect, Göte's (SA) comment seemed particularly far-sighted. He said, of some hypothetical person, that

> they know it could always change in life. They could be in an accident and do not get to keep their job, or do not have the right conditions to be able to keep that job and have to change, and then maybe they have too little maths skills.

In conclusion, as summarized by Alice (SA), a common theme throughout both sets of interviews was that "when one has studied maths, one can study further and get… more opportunities for the future… more career choices". Moreover, alongside the consensus that mathematical competence helps one realize one's life choices was an awareness that such competence gives students an advantage over their less competent peers, as seen in Leo's (SV) comment that "when you search for a job… it's like you are higher status than everybody else and you say, 'I know more than you, I am smarter'".

5.3 Supporting the learning of other subjects

Few students did not speak about the relationship between mathematics and the learning of other school subjects. Moreover, students' perspectives were indistinguishable across the two national contexts. Broadly speaking, utterances typically focused on the natural sciences, as in the comments of Lise (SA), who commented that "we will be reading chemistry or physics next year, maths is fundamental to all these scientific topics" and Malin (NA), who noted that

> you need to use maths a lot in other subjects, so you have it in chemistry and physics and stuff… so maths is a very important tool… so it is not… if you look at the maths as a subject… just a subject, then it can be a lot of numbers and a bit like… but if you put it in the context of other science subjects, then it is the most important tool you have.

Others' comments, both Norwegian and Swedish, were more general, as in Werner's (SV) observation that mathematics "works well with other subjects that also need maths, like, they complement each other", while Mukhtar (NV) simply noted that "it is a tool really… to be able to use maths".

In sum, students across the majority of interviews were clear that learning mathematics supports the learning of other subjects, particularly science. Embedded in their views was a strongly articulated awareness, as reflected in Oskar's (SA) comment, that "maths is connected to pretty much everything, every subject. It is, you know, the background to everything".

5.4 Supporting the development of logical thinking and problem-solving

Despite a strong collective view that the purpose of school mathematics is essentially utilitarian, the subject's intrinsic rather than extrinsic properties were frequently visible. In particular, the development of a person's logical thinking, analytical capacity, and problem-solving competence were mentioned frequently by academic students from both countries.

From the perspective of logical thinking, the comments of Oskar (SA) and Emily (NA) summarized the perspectives of such students from both countries. Oskar (SA) commented that,

> A big part of it is, you know, logic… The brain develops as you get older, but I think developing your thinking… requires something that helps it and I think… maths is one of the things where you think… and you connect different parts, where maths is like one of the logical… one of the logical things where the brain has to really work hard.

While Emily (NA) noted that mathematics

> is a very particular understanding that you also copy in other subjects… to see systems for example, to see… oh, here you have a system, that, for example, you can use in other subjects too… it helps you with the logical… to understand that there are systems in things, that there is a logic in this…

In addition to discussing mathematics' role in the development of logical thinking, other academic students from both countries spoke about the ways in which mathematics offers particular ways of looking at things. For example, Karin (SA) commented that

> the person who knows maths has different ways of looking at things… He has seen a lot of problems from a lot of different angles and that person knows that you have to, like, look for different angles to maybe find the best way to solve the problem.

The sense of cognitive flexibility and self-belief implicit in Karin's comment found resonance in Magnus' (NA) comment that

> maths has developed me a lot when it comes to like problem-solving, generally, not just in like special maths knowledge, but also more general problem-solving in all situations in life… that is for *me* though.

In broad terms, however, academic students from both countries were clear as to the importance of mathematics in the development of problem-solving competence and logical thinking. Interestingly, while such views were typically the domain of academic students, a small number of vocational students from both countries offered similar, but less sophisticated, perspectives. For example, from the Swedish perspective, Andreas (SV) described mathematics as something "to stimulate our brains or so, making it easier for us to learn new things", while, from the Norwegian perspective, Dennis (NV) suggested that the role of "maths is… to train… you to think quicker, you maybe calculate in your head if you're smart". Overall, a significant proportion of students from both countries, typically but not exclusively, academic, were clear that learning mathematics provides a set of problem-solving skills that enhance one's logical thinking and analytical capacity.

5.5 Supporting the development of one's appreciation of mathematics

While many students commented on the role of mathematics in the development of logical thinking and problem-solving, many also spoke about the satisfaction they derived from mathematics in general and problem-solving in particular. Broadly speaking, however, this fifth purpose tended to distinguish the two groups, with the consequence that the two cohorts' responses are presented separately.

From the Swedish perspective, Hanna and Sam, who were interviewed together, were the only students to discuss explicitly the enjoyment they derive from their engagement with mathematics. For example, Hanna (SA) noted that

> all these problems that you solve are interesting and they're at least fun and you can have fun with them. And I think that's what makes math kind of interesting and fun because you can do so much with a simple problem.

For Sam (SA), however, his relationship with mathematical problems was ambivalent. He commented that mathematical problem-solving gives him

> A lot of (…) head-scratching and (laughter) a lot satisfaction when you actually solve the hard problems that you couldn't solve earlier. But it's… it's kind of tough. Sometimes I hate mathematics but sometimes I love it. I usually love it… And I love it because it's hard and because it makes me think… outside of the box. But I also hate it because it's hard and… it makes me have to work.

In sum, both saw solving mathematics problems as intrinsically satisfying, even if tinged with ambivalence. Such enthusiasm, however, was occasionally met with what seemed like low-level derision aimed at those who derive pleasure from mathematics. For example, Ivan (SA) referred to “a guy in the class… the only thing he does is maths. He thinks it’s great fun to study”. Outside these extremes, nothing was said, particularly by vocational students.

By way of contrast, notions of fun permeated many Norwegian academic students’ comments. For example, Kristine (NA) noted that

> … it is a lot of fun when you crack the code for something you have been working on for a while… and then suddenly it opens up… and then it is great fun. When you get (understand) stuff then most things are fun really… but maths is very special in that way…

Others, such as Charlotte (NA), commented that “I think maths is a fun subject too… at least compared to many other subjects”, while Kristine (NA) formulated it as a form of confession, “I like maths a lot, I can be completely open about that… maths is fun”. Others alluded to an excitement that emerges from working with mathematics, as seen in Emma’s (NA) comment that “it is exciting to sit there, and… and find out how things are related” and Thomas’ (NA) view that

> I think it is fun to get challenges and sit and work with things and in a way get the feeling that I can do this, and this is challenging, and… that I do something that is exciting… I just think it is exciting.

However, the sense of fun frequently mentioned by Norwegian academic students rarely appeared in the utterances of Norwegian vocational students, as seen in Ruben’s (NV) “no, for me, when I sit there and do it… I don’t feel that I get anything out of it” and Simen’s (NV) “no, I have to say, I don’t feel I get anything out of it”. However, unlike some of their Swedish counterparts, these students did not disparage those peers who are excited by mathematics.

In sum, while pleasure seemed an intrinsic part of mathematical learning for academic track students, many vocational students articulated something different from Sigrid’s (NA) positive view that “when you are about to understand something” not only do “you get an ‘Aha! -experience’” but “something else in life too”.

5.6 Supporting the management of one’s personal finances

Finally, a future-oriented purpose evident in all Swedish and most Norwegian interviews, concerned students’ personal finance and the management of money. Due to substantially differing emphases in their responses, the two cohorts are reported separately.

Almost all Swedish students, academic and vocational, spoke of understanding interest, as with Jacob’s (SV) comment that “percentages in terms of interest and loans and things like… that’s very good to know because you might not really figure out in your head how much you can spend”. In a similar vein, Omar (SA) commented that mathematics

> simplifies everyday life. One sees things differently when you get to them in everyday life, particularly with discounts and interest rates. It helps of course when one becomes an adult and begins to save and borrow, so you know roughly how much you will need to pay.

Interestingly, in two interviews, students spoke of the need to understand the process of amortization, as seen in Pedram’s (SV) observation that “amortisation, it is important to be able, interest rates, interest expenses, to be able to figure it out, it is important” and Göte’s (SA) view that mathematics is necessary "if you buy a tenancy and stuff and amortisation and things like that". Others spoke more generally about the management of personal finances. For example, Markus (SV) commented about the need to manage "larger sums, as well as your salary", while Kenneth, in the same interview, added that "it becomes much easier to make financial plans… if one has several years of mathematics". In addition, a theme raised in just one interview, addressed the need to understand the Swedish pension system. Here, Sheldon (SV) commented that

> You have to do a lot of calculations here, because if you put your money in automatically, it stays there (in the state-run low risk account). So, to keep your money from decreasing, you have to constantly check what's going up what's going down.

Finally, several students spoke of the need to avoid overextension, as in Alice's (SA) concerns with respect to "SMS loans and stuff, there are many who do not know how much you lose there", comments reflected in Omar's (SA) summary that, "there are too many adults today who do not really understand interest and how it works".

From the above several issues can be inferred. First, Swedish students were clearly aware of the problems of overextension and how this can create serious financial difficulties. In so doing, their comments indicated a concern for others beyond themselves. Second, they spoke with authority and some sophistication, whether in their use of words like amortization or Sheldon's (SV) understanding of the Swedish pension system. Third, they spoke in ways indicative of a sound understanding of interest and its impact on matters financial. Overall, students' utterances, including an unexpected use of the word amortization, indicated a clear understanding of how the financial world works and that knowing mathematics should facilitate their successful participation in that world.

By way of contrast, Norwegian students did not mention loans or interest rates. Their comments about personal economics focused on more basic concerns. For example, Tania (NA), alluding to the need to check that tax and social deductions have been calculated accurately, commented that

> … if you work, and if you want to know how much you earn, and you want to know your net income… if you know it yourself you can double check if the person who does it has done it right.

Anna (NA) argued that "maths is something you meet again, in… in daily life… paying tax and… like practical maths… you kinda have to use maths in daily life, and then it is OK to have a bit of control on it", while Dennis (NV) simply noted that "when you get out in grown up life then you have to pay for… you have to calculate how much you spend". In sum, in comparison with their Swedish peers, Norwegian students articulated a considerably less sophisticated understanding of the issues surrounding personal finance.

6. Discussion

In this paper, motivated by the argument that bottom-up studies of the form presented here have the potential to uncover how well systemic aims have been realized, we have presented an exploratory study of Swedish and Norwegian post-compulsory school students' perspectives on the purpose of school mathematics. Such students, having completed compulsory school, should be well placed to comment on why they think they have been learning mathematics for 10 years. As found with studies of upper secondary students' mathematics cognition (Andrews & Öhman, 2019) and affect (Martínez-Sierra & García-González, 2017), the number of interviews undertaken in each country proved sufficient for ensuring thematic saturation.

The analyses yielded six purposes of school mathematics, common across the two cohorts, which we discuss below. While most were not unexpected, their resonance with the three forms of mathematical knowledge yielded by the literature was not unproblematic, confirming an inadequately conceptualized field. Also, although there were occasional allusions to the tangible goals of the two curricula, the six purposes resonated closely with various intangible goals concerning students' preparation for participation in their respective communities. In other words, when considering why they learn mathematics, students typically distinguished between curricular content and the broad principles underlying what they learn.

The first purpose, *mathematics supports everyday shopping*, occurred frequently across both sets of interviews. It resonates unproblematically with the pragmatic mathematical knowledge discussed above, in its drawing on those nonspecialist skills and procedures (Watson, 2004) related to the functional numeracy that facilitates an individual's successful and critical engagement with the demands of everyday life (Askew, 2015; Bennison, 2015; Callingham et al., 2015; Venkat & Winter, 2015). The purpose also seems to resonate with PISA's goals concerning the situations people face when going about their daily lives and which necessitate the application of mathematical skills in less structured contexts (OECD, 2003). However, this particular purpose distinguished the two cohorts, with Swedish students frequently discussing a need to compare prices, something their Norwegian peers did not.

The second purpose, *mathematics as a support for future employment*, yielded a broad spectrum of expectations. At one end, the mathematical needs of shopkeepers and bus drivers, occupations discussed uniquely by Swedish vocational students, draw on the functional numeracy (Ernest, 2016; Fitzsimons, 2001; Watson, 2004) of pragmatic mathematical knowledge. At the other end, employment in science, engineering, medicine or architecture requires the advanced specialist knowledge (Ernest, 2016) that supports higher education and the numerate professions (Greer & Mukhopadhyay, 2005; Sigurdsson, 2006; Watson, 2004). Between the two, the mathematical needs of, say, plumbers and electricians seem to go beyond pragmatic mathematical knowledge but not as far as specialized mathematical knowledge. That said, both Norwegian and Swedish students, whether vocational or academic, seemed clear that being mathematically competent not only facilitates the realization of one's employment ambitions but gives one an advantage over others in the manner of Gutstein's (2007) mathematical knowledge as gatekeeper to life's opportunities. However, neither pragmatic mathematical knowledge nor specialized mathematical knowledge seems sufficient to account for the variation in students' employment-related expectations, thus challenging the adequacy of such knowledge conceptualizations.

Third, *mathematics as a support for the learning of other subjects* typically emerged from academic track students' statements concerning the importance of mathematical knowledge as the foundation on which other subjects' knowledge is constructed, particularly chemistry and physics. Interestingly, vocational students seemed less aware of such relationships, prompting the question, is their lack of awareness a consequence of the ways in which they have been taught or, perhaps, a more general failure on their part to reflect on their educational experiences? In this context, while the purpose of school mathematics could be construed as implicitly pragmatic, it was, for these Norwegian and Swedish academic students, resonant with the specialized mathematical knowledge discussed earlier (Ernest, 2016; Greer & Mukhopadhyay, 2005; Noyes, 2007; Sigurdsson, 2006; Watson, 2004).

The fourth purpose, which was manifested similarly across the two cohorts, alluded to students' awareness of the role of mathematics in the development of *logical thinking and problem-solving*. A superficial interpretation of their comments, as with the previous purposes, indicated a construal of mathematics as utilitarian. However, both academic and vocational students' comments, albeit in different ways, seemed to go beyond mere acknowledgment of logical thinking and problem-solving to indicate a role for mathematics in the development of one's cognitive flexibility. In so doing, students' perspectives appear resonant with the intangible goals of both countries relating to the development of critical thinking (Skolverket, 2018; Utdanningsdirektoratet, 2019). Importantly, beyond an implicit sense that such a purpose may draw on some sense of specialized mathematical knowledge, there appears to be little explicit connection between this particular purpose and the three broad purposes yielded by the literature. More importantly, perhaps, is the likelihood that construals of mathematics as a support to the development of logical thinking are culturally conditioned by teaching influenced by Western perspectives relating to the "theory of formal discipline" (Bass & Ball, 2018).

Fifth, *supporting one's appreciation of mathematics* highlighted two related but essentially opposing perspectives on mathematics, although some have suggested that such goals do not constitute a purpose of school mathematics, in that drawing "pupils' attention to the esthetics of mathematics may be an aim - something we are trying to do within mathematical education - but this remains distinct from the question of what such a mathematical education is for" (Huckstep, 2000, p. 8). On the one hand, Norwegian academic students, together with a couple of Swedish, spoke positively of the satisfaction they derive from mathematical problem-solving (Dreyfus & Eisenberg, 1986; Ernest, 2016). However, despite their clearly articulated pleasure, not one student explicitly mentioned the esthetic properties of mathematics, notions of proof (Lange, 2009; Weber, 2008), or mathematical modeling (Ferrando & Albarracín, 2021; García et al., 2006). On the other hand, Norwegian vocational students rarely spoke in such positive terms, tending to find little pleasure in mathematics, an emotional response likely to prevent further voluntary engagement with the subject (Hoffman, 2010; Li et al., 2021; Nardi & Steward, 2003). In short, not only does mathematics seem to invoke widely differing emotions (Eberle, 2014), particularly among Norwegian students, but those who spoke positively had a very one-dimensional view of the subject.

Sixth, *supporting the management of one's personal finances* was the most discriminative of the six purposes. The manner in which Swedish students spoke of amortization and pensions indicated a well-developed understanding of sophisticated personal finance, which contrasted with the prosaic views of their Norwegian peers. Indeed, while the mathematics implicated in Swedish students' utterances resonated with various forms of specialized mathematical knowledge (Ellerton & Clements, 1988; Greer & Mukhopadhyay, 2005; Sigurdsson, 2006), the mathematics implicated in the Norwegian students' utterances tended to draw on those nonspecialist skills and procedures of pragmatic mathematical knowledge (Ernest, 2016; Fitzsimons, 2001; Watson, 2004). Moreover, highlighting the potential uniqueness of their utterances, the extent to which Swedish students were able to discuss matters pertaining to amortization and pensions indicated a well-developed understanding of matters of serious personal finance, and contradicts research from the United States indicating high school and college students' limited understanding of such matters (Avard et al., 2005; Chen & Volpe, 1998).

Finally, with respect to the six purposes, only two, mathematics as a support in the learning of other subjects and mathematics as a support in the development of logical thinking and problem-solving, seemed to emerge comparably from the two cohorts. The remaining four distinguished the two cohorts in two ways. First, Swedish students seemed to present a less innocent or more street-wise perspective on their future lives than their Norwegian peers. For example, they spoke about comparing prices when shopping, they spoke about the employment advantage that knowing mathematics imparts, they were more aware of the role mathematics plays in supporting the learning of other subjects, and they were considerably more sophisticated in their understanding of the role mathematics plays in the management of one's personal finances. Second, Norwegian students seemed more likely than their Swedish peers to comment positively on the emotional impact that learning mathematics evokes.

Importantly, while students' utterances resonated in various ways with both pragmatic and specialized mathematical knowledge, the typical Scandinavian "emphasis on providing individuals with prerequisites for competent, active, concerned and critical citizenship" (Niss, 1996, p. 24) was missing. Indeed, acknowledging the emphases found in both curricula on sustainability, social justice, tolerance, equality, the rights and dignity of others, cultural diversity, and democracy (Skolverket, 2018; Utdanningsdirektoratet, n.d.), the lack of any reference to critical mathematical knowledge and its role in ensuring social justice was particularly surprising. In other words, while the intangible goals of both curricula emphasize both individual empowerment and collective responsibility, only the former was present in students' utterances. They were clearly conscious of individually focused intangible goals but unaware of, or chose not to comment on, collectively focused intangible goals. It is possible that students had been introduced to such intangible collectively focused goals in ways other than through mathematics. If so, then not only is more research on Scandinavian students' engagement with critical mathematics necessary, but the curricular authorities in both countries may need to re-examine how such matters are reified. Alternatively, if students from both countries typically experience mathematics as teacher exposition followed by exercises from the book (Andrews & Larson, 2017; Lepik et al., 2015), then it may be reasonable to infer that teacher educators and textbook authors may need to focus greater attention on the incorporation of these collectively focused intangible goals. That said, irrespective of how one tries to explain the problem, the evidence of this study suggests that many of the collectively focused intangible goals of both countries seem not to have permeated the mathematics-related consciousness of either Norwegian or Swedish upper secondary students.

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