Two different methods of geoidal height determinations using a spherical harmonic representation of the geopotential, topographic corrections and the height anomaly–geoidal height difference

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Abstract. It is suggested that a spherical harmonic representation of the geoidal heights using global Earth gravity models (EGM) might be accurate enough for many applications, although we know that some short-wavelength signals are missing in a potential coefficient model. A 'direct' method of geoidal height determination from a global Earth gravity model coefficient alone and an 'indirect' approach of geoidal height determination through height anomaly computed from a global gravity model are investigated. In both methods, suitable correction terms are applied. The results of computations in two test areas show that the direct and indirect approaches of geoid height determination yield good agreement with the classical gravimetric geoidal heights which are determined from Stokes' formula. Surprisingly, the results of the indirect method of geoidal height determination yield better agreement with the global positioning system (GPS)-levelling derived geoid heights, which are used to demonstrate such improvements, than the results of gravimetric geoid heights at to the same GPS stations. It has been demonstrated that the application of correction terms in both methods improves the agreement of geoidal heights at GPS–levelling stations. It is also found that the correction terms in the direct method of geoidal height determination are mostly similar to the correction terms used for the indirect determination of geoidal heights from height anomalies.

Keywords: Geoidal height – Geopotential coefficients – Height anomaly – Topographic corrections

1 Introduction

An accurate solution of the boundary-value problem in physical geodesy has usually been found using Stokes' well-known formula for the anomalous gravity potential, with the geoidal height calculated through Bruns' formula. The geoid represents a vertical datum for orthometric heights used in many countries. An accurate geoid is also of interest in many other geophysical applications. However, with the increasing accuracy of the geopotential coefficients and the maximum degree of expansion to higher degree, the computation of geoidal heights from global gravity models has been an issue of increasing importance in the geodetic community. Rapp (1971, 1994a, 1994b) has examined different procedures for geoidal height computations using spherical harmonic coefficients of the global Earth gravity models. Rapp (1994a, 1994b, 1997) noted the use of height anomaly–geoidal height $(N-\zeta)$ difference and a height anomaly gradient correction term, which will be called the 'indirect' method for geoidal height determination. The 'direct' method of geoidal height determination referred to in this study involves determining the geoid undulations from the geopotential coefficients model alone. Smith (1998) and Smith and Small (1999) have investigated the use of direct and indirect geoid height determinations using EGM96 (Lemoine et al. 1997) geopotential coefficients.

In determining the geoid undulations from geopotential coefficients with the direct method for geoid height computation, we must expect a bias from the external harmonic series when applied at the geoid within the topographic masses. Sjöberg (1977, 1994) pointed out this bias and Sjöberg (1994, 1995), Vanicek et al. (1995), and Nahavandchi and Sjöberg (1998) derived different terms to handle this bias, which is here called the topographic correction for potential coefficients.

The purpose of this paper is to demonstrate the efficiency of geoidal height determinations from geopotential coefficient models using a set of GPS–levelling stations in two test areas in Iran. It will be suggested that the simple computation of topographical corrections and geoid heights with a set of spherical harmonics might be useful in practice, instead of the use of the very arduous procedure of classical gravimetric methods (Stokes' integral) and topographic effects. However, computations in different test areas are suggested. Rapp (1997) compared the geoidal heights derived from 960 GPS–levelling stations with the geoid undulations derived with the indirect approach through height anomaly and the OSU91A model (Rapp et al. 1991) over the USA. The root-mean-square (RMS) difference was ± 56 cm. Smith and Small (1999) also computed the geoidal heights by applying high-frequency corrections to the EGM96 geopotential model in a remove–restore technique. An RMS difference of ± 62 cm was found at the 31 GPS–levelling stations.

2 Direct geoidal height determination from a geopotential coefficients model alone

In modern methods of determining the geoidal undulations, the long-to-medium-wavelength components are frequently obtained from a global geopotential model in the modified Stokes formula. The short-wavelength information is then computed from Stokes' integral. In this study, these short-wavelength signals are disregarded and the geoidal heights (N) are determined from EGM96 geopotential coefficients using spherical harmonic representations by the following expansion that is complete to degree M (=360 in this study)(Heiskanen and Moritz 1967, Chaps. 1 and 2):

$$N(R,\varphi,\lambda) = \frac{GM_3}{R\gamma}\sum_{n=0}^{M}\left(\frac{a_1}{R}\right)^n \sum_{m=0}^{n}\left[\left(\frac{GM_1}{GM_3}C_{nm} - \frac{GM_2}{GM_3}\left(\frac{a_2}{a_1}\right)^n C'_{nm}\right)\cos m\lambda + \right.$$
$$\left. + \frac{GM_1}{GM_3}S_{nm}\sin m\lambda\right]P_{nm}(\sin\varphi) - \frac{1}{\gamma_0}(W_0 - U_0) \qquad (1)$$

where

R	= mean geoid radius
(φ, λ)	= spherical latitude and longitude of the computation point
γ_0	= normal gravity at the ellipsoid to which the geoidal height N will refer
a_1	= equipotential scale factor of EGM96 (6378.1363 km)
a_2	= equipotential radius of GRS-80 (6378.137 km)
GM_1	= gravity-mass constant of EGM96 ($3.986004415\times 10^{14}$ m^3/s^2)
GM_2	= gravity-mass constant of GRS-80 ($3.986005000\times 10^{14}$ m^3/s^2)
GM_3	= best estimate of gravity-mass constant for the Earth ($3.986004418\times 10^{14}$ m^3/s^2)
W_0	= adopted gravity potential on the geoid (62636856.88 m^2/s^2)
U_0	= defined normal gravity potential on the ellipsoid (62636860.85 m^2/s^2)
C_{nm} and S_{nm}	= fully normalized geopotential coefficients of degree n and order m of EGM96 in non-tidal system (C_{00}=1.0; S_{00}=0.0; C_{1m}=S_{1m}=0.0; C_{20}=non-tidal; S_{20}=0.0)

C'_{nm} and S'_{nm} = fully normalized normal potential coefficients of degree n and order m of GRS-80 in non-tidal system (C'_{00}=1.0; S'_{00}=0.0; C'_{1m}=S'_{1m}=0.0; C'_{20}=non-tidal; S'_{20}=0.0). We have also made use of the fact that S'_{nm}=0.0 for all n and m.

P_{nm} = fully normalized Legendre functions.

The geoid undulation given by Eq. (1) defines the absolute level of the geoid through the value of W0 (Bursa 1995). It also adopts the GRS-80 ellipsoid (Moritz 1988) as the reference ellipsoid. A best global value of gravity-mass constant (GM3) (Bursa 1995) is also adopted. Definition of these parameters allows us to determine the geoid undulation with Eq. (1) in $30' \times 30'$ grids relative to the GRS-80 ellipsoid. The tide system in which the geoid height model is to be given is a non-tidal system to be more consistent with EGM96.

In employing the geopotential coefficients for the geoidal height computations (direct method), the assumption is that the external harmonic series expansion is convergent on the Brillouin sphere. The downward continuation of this series inside the sphere presents some problems of convergence, and Sjöberg (1977) emphasized this point. On the other hand, Jekeli (1981, 1982) pointed out that the convergence problem is non-existent for a finite series. Even if the convergence problem is negligible we must expect a bias for external harmonic series when applied at the geoid within the topographic masses. This bias can be estimated by removing the topographic masses (such that we can now continue the external harmonic series of the geopotential downwards to the geoid – they are now harmonic between the geoid and the topography), and then restore the masses. The removal and restoration of topography implies a direct and indirect topographic correction on the geopotential. Nahavandchi and Sjöberg (1998) used the second Helmert condensation method for handling the topographic masses and derived the following formula for the direct topographical correction on the geoid to the third power of elevation:

$$\delta N_{\mathrm{Dir}} = -\frac{2\pi\mu}{\gamma}\sum_{n=0}^{M'}\sum_{m=-n}^{n}\frac{n+2}{2n+1}(H^2)_{nm}Y_{nm}(P) - \frac{2\pi\mu}{R\gamma}\sum_{n=0}^{M'}\sum_{m=-n}^{n}\frac{(n+2)(n+1)}{3(2n+1)}(H^3)_{nm}Y_{nm}(P) \tag{2}$$

and for the indirect topographical correction on the geoid to the third power of elevation (Nahavandchi and Sjöberg 1998)

$$\delta N_{\mathrm{Ind}} = -\frac{2\pi\mu}{\gamma}\sum_{n=0}^{M'}\sum_{m=-n}^{n}\frac{n-1}{2n+1}(H^2)_{nm}Y_{nm}(P) + \frac{2\pi\mu}{3R\gamma}\sum_{n=0}^{M'}\sum_{m=-n}^{n}\frac{n(n-1)}{(2n+1)}(H^3)_{nm}Y_{nm}(P) \tag{3}$$

where M' is the maximum degree of height coefficients in a spherical harmonic representation, $\mu = G\rho$, ρ being the density of crust considered constant, and Y_{nm} are fully normalized spherical harmonics obeying the following rule:

$$\frac{1}{4\pi}\int\int_{\sigma} Y_{nm}Y_{n'm'}\,d\sigma = \begin{cases}1 & \text{if } n = n' \text{ and } m = m' \\ 0 & \text{Otherwise}\end{cases} \tag{4}$$

and

$$(H^{\upsilon})_{nm} = \frac{1}{4\pi}\int\int_{\sigma} H_P^{\upsilon} Y_{nm}\, d\sigma; \qquad \upsilon = 2,3 \tag{5}$$

$$H_P^{\upsilon} = \sum_{n,m} (H^{\upsilon})_{nm}\, Y_{nm} \tag{6}$$

3 Indirect geoidal height determination through height anomaly

Geoidal height can also be determined from the height anomaly (ζ) by the well-known approximate formula (Heiskanen and Moritz 1967, p. 327) and an additional term dependent on H^2 (Sjöberg 1995)

$$N_P(\varphi,\lambda) = \zeta_P(r,\varphi,\lambda) + \frac{(\Delta g_B)_P}{\bar{\gamma}} H_P + \frac{H_P^2}{2\bar{\gamma}}\left(\frac{\partial \Delta g^F}{\partial H}\right)_P \tag{7}$$

where Δg_B and Δg^F are the Bouguer and free-air anomalies, respectively, and (Heiskanen and Moritz 1967)

$$\left(\frac{\partial \Delta g^F}{\partial H}\right)_P = \frac{R^2}{2\pi}\int\int_{\sigma} \frac{\Delta g^F - \Delta g_P^F}{\ell_0^3} d\sigma - \frac{2}{R}\Delta g_P^F; \qquad \upsilon = 2,3 \tag{8}$$

where ℓ_0 is the spatial distance between the computation point P and the running point, σ is the unit sphere, H_P is the orthometric height at P, and $\bar{\gamma}$ is an average value of normal gravity between Q' (corresponding to P) on the ellipsoid and the point Q'' (corresponding to P) on the telluroid. The geoidal height given by Eq. (7) is the separation between a reference ellipsoid and Earth's gravity equipotential surface (geoid). Rapp (1997) rewrote Eq. (7) in terms of computer efficiency in the following form (although he neglected the term dependent on H^2 which is given below):

$$N(\varphi,\lambda) = \zeta_0(r,\varphi,\lambda) + C_1(\varphi,\lambda) + C_2(\varphi,\lambda) \tag{9}$$

where (Rapp 1997)

$$C_1(\varphi,\lambda) = \frac{\partial \zeta}{\partial r} H + \frac{\partial \zeta}{\partial \gamma}\frac{\partial \gamma}{\partial h} H \tag{10}$$

$$C_2(\varphi,\lambda) = \frac{\Delta g_B}{\bar{\gamma}} H + \frac{H^2}{2\bar{\gamma}}\left(\frac{\partial \Delta g^F}{\partial H}\right) \tag{11}$$

The ζ_0 value can be computed from (Heiskanen and Moritz 1967)

$$\zeta_0(r_e,\varphi,\lambda) = \frac{GM_3}{r_e\gamma}\sum_{n=0}^{M}\left(\frac{a_1}{r_e}\right)^n \sum_{m=0}^{n}\left[\left(\frac{GM_1}{GM_3}C_{nm} - \frac{GM_2}{GM_3}\left(\frac{a_2}{a_1}\right)^n C'_{nm}\right)\cos m\lambda + \frac{GM_1}{GM_3}S_{nm}\sin m\lambda\right] P_{nm}(\sin\varphi) - \frac{1}{\gamma}(W_0 - U_0) \tag{12}$$

where r_e is the ellipsoid radius, γ is the normal gravity at the telluroid (i.e. normal height), and the same notations and explanations as for Eq. (1) are used.

The next step is to evaluate the C_1 term. Using Eq. (12) we can evaluate the first term of C_1 from

$$\frac{\partial \zeta}{\partial r} H(r,\varphi,\lambda) = \frac{-GM_3}{\gamma r^2} H \sum_{n=0}^{M} (n+1) \left(\frac{a_1}{r}\right)^n \sum_{m=0}^{n} \left[\left(\frac{GM_1}{GM_3} C_{nm} - \frac{GM_2}{GM_3} \left(\frac{a_2}{a_1}\right)^n C'_{nm}\right) \cos m\lambda + \frac{GM_1}{GM_3} S_{nm} \sin m\lambda\right] P_{nm}(\sin\varphi) \tag{13}$$

The second term of Eq. (10) can obviously be rewritten as

$$\frac{\partial \zeta}{\partial \gamma} \frac{\partial \gamma}{\partial h} H\ (r,\varphi,\lambda) = 0.3086 H \left[\frac{GM_3}{\gamma^2 r} \sum_{n=0}^{M} \left(\frac{a_1}{r}\right)^n \sum_{m=0}^{n} \left[\left(\frac{GM_1}{GM_3} C_{nm} - \frac{GM_2}{GM_3} \left(\frac{a_2}{a_1}\right)^n C'_{nm}\right) \cos m\lambda + \frac{GM_1}{GM_3} S_{nm} \sin m\lambda\right] P_{nm}(\sin\varphi) - \frac{1}{\gamma^2}(W_0 - U_0\right] \tag{14}$$

We next consider the evaluation of the C_2 term. The Bouguer anomalies can be found with the following formula from free-air anomalies and a digital terrain model (DTM) (Heiskanen and Moritz 1967):

$$\Delta g_B(\varphi,\lambda) = \Delta g^F(\varphi,\lambda) - 0.1119 H(\varphi,\lambda) \tag{15}$$

where it is assumed that the density of crust ρ is a constant value equal to 2760 kg m^{-3}. The free-air gravity anomaly in Eq. (15) can be computed from EGM96 potential coefficients using (Heiskanen and Moritz 1967)

$$\Delta g^F(r,\varphi,\lambda) = \frac{GM_3}{r^2} \sum_{n=0}^{M} (n-1) \left(\frac{a_1}{r}\right)^n \sum_{m=0}^{n} \left[\left(\frac{GM_1}{GM_3} C_{nm} - \frac{GM_2}{GM_3} \left(\frac{a_2}{a_1}\right)^n C'_{nm}\right) \cos m\lambda + \frac{GM_1}{GM_3} S_{nm} \sin m\lambda\right] P_{nm}(\sin\varphi) - \frac{2}{r}(W_0 - U_0 \tag{16}$$

with the same notations and explanations as for Eq. (1).

The digital elevation model in this study is available in a global grid. The evaluation of Bouguer anomaly can also be done in a global grid of mean values whose size is compatible with the maximum degree of expansion (in this study 360). The free-air anomalies are determined at the surface of the Earth. Thereafter the C_1 and C_2 values are computed on a grid with mean values in 30×30 cells. These two corrections can also be computed at each desired point and then added to the ζ_0 value calculated with Eq. (12), resulting in the geoidal height.

It should also be mentioned that the direct method gives a spherical approximation of the geoid while the indirect method gives an ellipsoidal approximation of the geoid. Some differences between the two methods are caused by these different approximations, although we have used the two approximations locally in two test areas.

4 Numerical investigations

4.1 Data sources

The first area of study is limited by latitudes 54º N and 55º N and longitudes 30º E and 31º E. The elevation in this area varies from 1400 to 2400 m. GPS–levelling stations are used to demonstrate the efficiency of different procedures of geoidal height determination as an independent data set. There are 33 GPS–levelling stations in this area. The orthometric heights of GPS–levelling stations vary from 1431 to 2289 m. Although there is not enough information for assessing the accuracy of these orthometric heights, Hamesh (1991) has estimated that the accuracy of the orthometric heights is about 70 cm. He has also mentioned that the terrestrial gravity data have not been used in the determination of the orthometric heights. The accuracy of the ellipsoidal height computed from GPS is estimated to about 25 cm (Nilforoshan 1995). The gravimetric geoidal heights of these 33 stations are also known (Hamesh 1991). The gravimetric geoid heights are computed using a modification of Stokes' formula combining the short-wavelength contributions from the terrestrial gravity (11 000 grid observations in 110″ × 160″ cells) and height data (1 km × 1 km) with the long-wavelength contributions from a global geopotential model [OSU89B (Rapp and Pavlis 1990), to degree and order 360]. Topographic and atmospheric corrections are also applied. The intention of this study is to determine the geoidal heights with the direct and indirect methods from a global geopotential model and then to compare the results of these two procedures with the above-mentioned gravimetric geoid heights at 33 GPS–levelling stations. These comparisons will show us the efficiency of using very simple computations of the geopotential coefficient models compared with the classical formula of Stokes with the arduous computations of integral formulae including the topographic corrections.

The GPS–levelling geoidal heights are computed by the following well-known formula with the combination of the ellipsoidal height h, computed from GPS, and the orthometric height H, computed from precise levelling:

$$N = h - H \tag{17}$$

The statistics of differences between the gravimetric and GPS–levelling geoidal height models at the 33 GPS stations are shown in Table 1. A maximum difference between these two geoid models of 1.238 m is computed. The main reasons for these differences between gravimetric and GPS–levelling geoid heights might be the systematic errors in orthometric heights and the terrestrial gravity observation errors (Hamesh 1991). Another other reason could be the fact that the terrestrial gravity data are not used to determine the orthometric heights.

Table 1. The statistics of differences between GPS-levelling-derived and gravimetric geoid height models for 33 GPS stations [units in m]

Min	Max	Mean	SD	RMS
−0.119	1.238	0.519	0.469	0.700

Table 2. The statistics of differences between GPS-levelling-derived and EGM96 geoid height models for 33 GPS stations excluding the topographic corrections [units in m]

Min	Max	Mean	SD	RMS
0.408	1.692	1.011	0.481	1.124

4.2 Direct geoidal height determination from the EGM96 geopotential model alone

Equation (1) is used to compute the geoidal heights in the 33 GPS–levelling stations. The global EGM96 to degree and order 360 is used in these computations. The statistics of the differences between the geoidal heights computed from EGM96 and from GPS–levelling data are shown in Table 2. A maximum difference of 1.692 m is observed between these two geoidal height models. The short-wavelength information missing from EGM96 can account for some of the high-frequency differences with the GPS–levelling geoidal heights. However, the large bias comes mainly from the fact that the computation points lie inside the topographic masses. This large bias will be treated below.

In the next step the direct topographic correction [Eq. (2)] and indirect topographic correction [Eq. (3)] are evaluated in the first test area. To do this, the height coefficients $(H^2)_{nm}$ and $(H^3)_{nm}$ are determined from Eqs. (5) and (6). For this, a 30′ × 30′ DTM is generated using the Geophysical Exploration Technology (GETECH) 5′ × 5′ DTM (GETECH 1995). This 30′ × 30′ DTM is averaged using area weighting. Since the interest is in continental elevation coefficients and we are trying to evaluate the effect of the masses above the geoid, the heights below sea level are all set to zero. The spherical harmonic coefficients of topographic heights are computed to degree and order 360. Parametric definitions are as follows: $\mu = G\rho$, where $G = 6.673 \times 10^{-11} m^3 kg^{-1} s^{-2}$ and $\rho = 2670$ kg/m^3, R=6371 km, and $\gamma = 981$ Gal. The topographic corrections are computed to degree and order 360 so that the corresponding cell size is 30′ × 30′. Figures 1 and 2 show respectively the direct and indirect topographic corrections in the first test area. The statistics of the direct and indirect topographic corrections on the geoid are shown in Tables 3 and 4. An absolute maximum value of 0.367 m for the direct effect and an absolute maximum value of 0.139 m for the indirect effects on the geoid are found. Thereafter, the geoidal heights computed with Eq. (1) are corrected for the effect of topographic masses and compared with the 33 GPS–levelling geoidal heights. Table 5 shows the statistics of differences between these two geoidal height models. As can be seen from the values given in Table 5, better results are surprisingly obtained with the geoid derived from the geopotential coefficients model and corrected for topographic effects, compared with the results for the gravimetric geoidal heights at 33 GPS–levelling stations. The RMS of the difference between gravimetric geoidal height and the 33 GPS–levelling geoidal heights is ±0.7 m, while it is computed as ±0.718 m for EGM96 geoidal heights, which is a good result considering that the short-wavelength contributions must be missing in the EGM96 geoid. This interesting result was also reported by Hamesh (1991). He compared the gravimetric geoid at 55 GPS–levelling stations in Iran. The RMS error was computed to ±1.5 m. The RMS of differences reduced to ±0.95 m at the same GPS stations when the gravimetric geoidal heights were computed only by the OSU89B model and topographic corrections (the 'direct' method in this study). Finally, the values given in Tables 2 and 5 demonstrate that the application of the direct and indirect topographical corrections to geoidal heights computed from EGM96 coefficients yield a better fit of geoidal heights to GPS–levelling data than if no topographical corrections were applied.

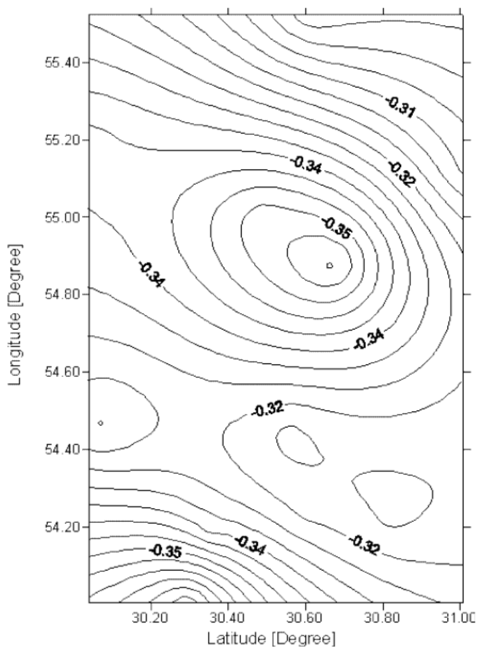

Fig. 1. The direct topographic correction to degree and order 360 [units in m]

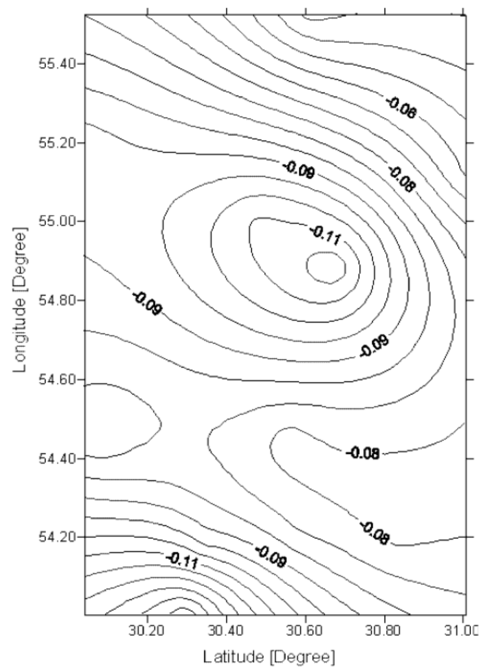

Fig. 2. The indirect topographic correction to degree and order 360 [units in m]

4.3 Indirect geoidal height determination through the height anomaly

The height anomaly is firstly computed with Eq. (12) from EGM96 coefficients at the 33 GPS–levelling stations. The degree and order of expansion is 360. The height anomalies are then compared with the GPS–levelling–derived geoidal heights. The statistics of the differences are shown in Table 6. A maximum difference of 1.571 m is observed between these two geoid height models. The main reason for the large differences is the height anomaly–geoidal height difference.

The next step is the calculation of the C_1 and C_2 correction terms. Equation (10) is used to compute the C_1 term in the test area. The 30′ × 30′ height information (mentioned above) and global EGM96 model to degree and order 360 are used to estimate this term. This means that the C_1 term will be computed in a 30′ × 30′ cell size. Figure 3 shows the C_1 term in the first test area. An absolute maximum value of 0.16 m is found for this term. To compute the C_2 term, the Bouguer anomalies are computed from free-air anomalies and the DTM using Eqs. (15) and (16). The EGM96 model to degree and order 360 and the 30′ × 30′ height information are used to evaluate this term. The C_2 term is computed in a 30′ × 30′ cell size. In order to compute the second term of C_2 in Eq. (11), i.e. the term dependent on H^2, we have employed Eq. (8). To do this the integration area is extended to 20 from the computation points (see Nahavandchi 1998) and the free-air gravity anomalies are computed in this extended area from EGM96 to degree and order 360. The results of computations of this second term in C_2 (depending on H^2) show that this term has at most 21% of the magnitude of the C_2 term, resulting in 0.092 m in this study. Figure 4 depicts the C_2 term in the test area. An absolute maximum value of 0.44 m is computed. The statistics of C_1 and C_2 terms are also given in Table 7.

Comparing Figs. 1 and 2 with Figs. 3 and 4, we find that Fig. 1 (the direct topographic correction) is mostly similar with Fig. 4 (the C_2 term) in shape and magnitude. Also, Fig. 2 (the indirect topographic correction) is mostly similar in shape and magnitude with Fig. 3 (the C_1 term). This means that the correction terms in either of the two direct and indirect methods of geoidal height determination mostly have the same shape and magnitude in this study. However, these results should be tested in other areas.

Table 3. The statistics of direct topographic corrections computed with EGM96 to degree and order 360 for 33 GPS stations [units in m]

Min	Max	Mean	SD
−0.367	−0.306	−0.349	0.016

Table 4. The statistics of indirect topographic corrections computed with EGM96 to degree and order 360 for 33 GPS stations [units in m]

Min	Max	Mean	SD
−0.139	−0.048	−0.092	0.024

Table 5. The statistics of differences between GPS-levelling derived and EGM96 geoid height models for 33 GPS stations including the topographic corrections [units in m]

Min	Max	Mean	SD	RMS
−0.051	1.271	0.582	0.421	0.718

Table 6. The statistics of differences between GPS-levelling derived geoidal height and geoidal height computed through height anomaly for 33 GPS stations excluding the correction terms [units in m]

Min	Max	Mean	SD	RMS
0.196	1.571	0.902	0.501	1.031

Finally, the geoidal heights are computed using Eq. (9), including the C_1 and C_2 correction terms, and compared with the GPS–levelling–derived geoidal heights. Table 8 shows the statistics of differences between these two geoidal height models. The values given in Tables 6 and 8 show that the application of the two-correction term (C_1 and C_2) to height anomalies computed from a geopotential model yields geoidal heights that fit better to GPS–levelling data than if no correction terms were applied. A maximum difference of 1.114 m is computed between the geoidal height derived through height anomalies and GPS–levelling geoid heights. Table 8 also shows very good results at GPS–levelling stations compared with the results of gravimetric geoidal heights fitting at the same stations. Surprisingly, the RMS difference between geoidal heights determined through height anomaly (including two correction terms) and the GPS–levelling geoid is computed to be ±0.612 m, while it was ±0.7 m with the gravimetric geoid heights compared at the same stations.

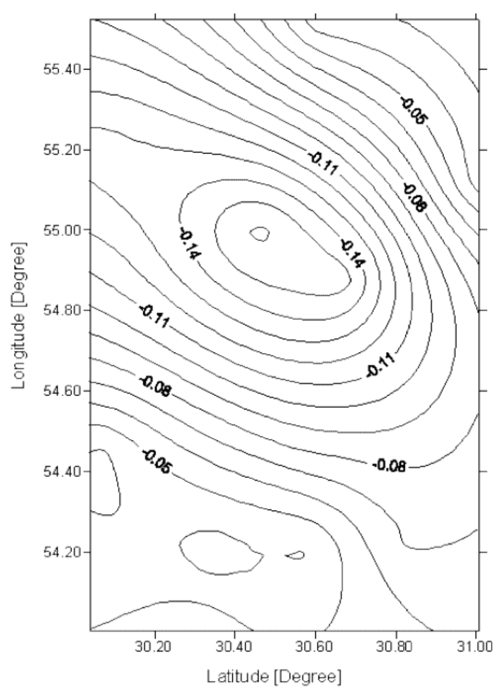

Fig. 3. C_1 correction term in the test area [units in m]

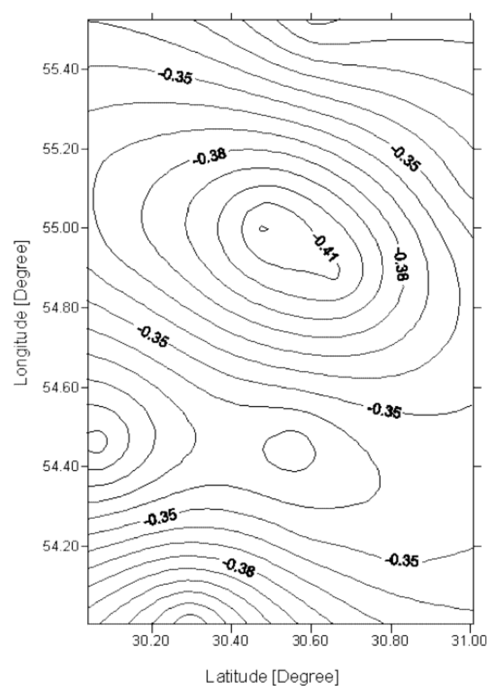

Fig. 4. C_2 correction term in the test area [units in m]

Table 7. The statistics of C_1 and C_2 correction terms for 33 GPS stations [units in m]

	C_1	C_2
Min	−0.160	−0.440
Max	−0.031	−0.294
Mean	−0.092	−0.365
SD	0.054	0.045

Table 8. The statistics of differences between GPS-levelling-derived geoidal height and geoidal height computed through height anomaly for 33 GPS stations including the correction terms [units in m]

Min	Max	Mean	SD	RMS
−0.172	1.114	0.462	0.402	0.612

In order to obtain further insight into the comparison results, another test area in Iran has been chosen. This second area is limited by latitudes 49º N and 51º N and longitudes 33º E and 35º E. The elevation in this area varies from 1900 to 2450 m. Twenty-three GPS-levelling stations are used for the comparisons. GPS-levelling heights are determined with the same procedure and accuracy as for the first test area. The same results for the comparisons as for the first area are obtained, with minor differences. Comparisons of the gravimetric geoid height at these 23 GPS-levelling stations show an RMS of ±0.682 m, while the RMS difference was computed to ±0.772 m by the direct method and ±0.601 m by the indirect method of geoidal height determination at the same stations. However, we recommend these computations be carried out in other test areas with different gravimetric geoid models. These results show that, at least in the test areas of this study, the estimation of the geoidal heights with very simple computations of the height anomaly (including correction terms) from the spherical harmonic representations of the geopotential, topography, and height anomaly–geoidal height difference agrees better with the GPS-levelling geoid undulations than the very arduous computations of geoidal heights with Stokes' integral and topographic corrections.

It is shown that the direct and the indirect presentation of the geoidal height from the geopotential coefficient models differ from each other at the GPS stations. However, Figs. 1–4 depict that the correction terms in both methods are mostly similar in shape and magnitude (in this study). Therefore, one possible reason for the differences between these two methods is the procedure used for computation of the geoidal height itself. In fact, the indirect method of computation [Eq. (12)] gives better results than the direct method of computation with Eq. (1) in this study. The reason might be the convergency problem in Eq. (1), which is mentioned in Sect. 2, as the computation points lie inside the topographic masses. However, these computations must be tested in other areas.

5 Discussions and conclusions

Two different procedures to determine the geoidal heights from a geopotential coefficient model are presented. In the first method the geoidal heights are computed directly at the geoid inside the topographic masses from a geopotential model alone, and thereafter the effects of topographic masses (direct and indirect topographic effects) are corrected. In the second approach, the height anomalies are calculated first. Subsequently, the height anomalies are converted to the geoidal heights using two correction terms, one of them representing the height anomaly gradient term and the second one the height anomaly–geoidal heights difference. Both of these approaches use the spherical harmonics to estimate the geoidal heights and correction terms and are very suitable and simple to use from the computational point of view compared to the classical gravimetric approach of geoidal height determination using the Stokes formula, which requires very arduous integral computations including the topographical corrections.

The two methods of geoidal height determination from the geopotential coefficient model and the gravimetric geoid computed with Stokes' formula are investigated at 33 and 23 GPS-levelling stations respectively in two test areas. The results show very good agreement of the two methods of this study with the GPS-levelling data compared with the gravimetric method, in terms of a few centimetres. Surprisingly, better results are obtained with the indirect computations of geoidal heights through height anomaly at GPS stations than the gravimetric geoid heights at the same stations. This means that, at least in the areas of this study, the use of the geopotential model agrees better with GPS-levelling data than the gravimetric method for geoid height determination. However, the gravimetric geoid heights of this study might be not accurate enough and these computations should be done in other test areas too.

It is found that the correction terms in the direct approach of geoidal height determination (i.e. topographic corrections) are mostly similar in shape and magnitude to the correction terms in the indirect method of geoidal height determination (i.e. the height anomaly – geoidal height difference and the height anomaly gradient term). Therefore, the better result of the second method is caused by better treatment of the geoidal height computations in the indirect method with the computation points at the Earth's surface than in the direct method of geoidal height computations with the computation points at the geoid inside the topographic masses. The convergency problem is suggested out as the reasons for this.

Geoidal height determination through height anomaly (indirect method) has demonstrated good agreement with the GPS-levelling data of this study, although we know that some high-frequency information (local contributions) is missing in this approach. The results could be improved in the near future by increasing the accuracy of the potential coefficient models and the maximum degree of expansion.

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