The direct effect and downward continuation correction of atmospheric masses in gravimetric geoid determination

Hossein Nahavandchi
Norwegian University of Science and Technology, Department of Geomatics, Trondheim, Norway
hossein.nahavandchi@ntnu.no
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Summary. – A new procedure to the direct atmospheric gravity effect, in connection with Stokes's integral formula, was presented by Nahavandchi (2002). This formula was derived by combining the local contributions of the effect of atmospheric masses, computed from a detailed Digital Terrain Model (DTM) and a set of spherical harmonics to account for the global contributions. This formula is valid for a point on the geoid, which indicates this fact that the direct effect and the effect of analytical continuation of gravity anomalies to sea level (due to the atmospheric masses) are combined into the so-called direct atmospheric effect. These two terms are studied here and a new strategy for the effect of atmospheric masses in geoid computations is presented. This strategy has been investigated over a test area.

Keywords: atmospheric effect, downward continuation, geoid, Stokes's formula.

1. – INTRODUCTION

Geoid determination by Stokes's well-known formula requires that there are no masses outside the geoid. Stokes's formula also requires that gravity anomalies Δg must refer to the geoid. Consequently, the effect of the atmospheric masses must be removed or reduced, which corresponds to the so-called direct effect on gravity anomaly. Stokes's formula applied to these corrected gravity anomalies results to the co-geoid. The restoration of the atmosphere corresponds to the indirect effect, i.e., the correction from the co-geoid to the geoid. Ecker and Mittermayer (1969) derived a formula for the direct atmospheric effect on the gravity, which later named the IAG approach. This method is described in Moritz (1980). Sjöberg (1993) emphasized that there could be additional significant direct and indirect atmospheric effects stemming from a more detailed treatment of Earth's topography than is made in the IAG approach. Sjöberg (1999) and Sjöberg and Nahavandchi (2000) investigated the direct and indirect atmospheric gravity and geoid effects in the original and modified Stokes's formula. They derived a formula for the direct atmospheric gravity and geoid effects using a spherical harmonic representation of the topography. Later, Nahavandchi (2002) derived another formula for the direct atmospheric effect, combining a local contribution term with a set of spherical harmonics. This effect was derived at a point on the geoid. In this paper, we will improve this formula for a point on the topography (direct atmospheric effect on the topography), and then, the effect of downward continuation of gravity anomalies to the geoid (due to the atmospheric masses) will be investigated. This means that the direct atmospheric effect, computed at the geoid, will be split to two terms. These formulas will also be investigated numerically in a test area.

2. – DIRECT ATMOSPHERIC EFFECT COMPUTED AT A POINT ON THE GEOID

Following the approach presented in Sjöberg and Nahavandchi (2000), the atmospheric potential at an arbitrary point P can be written:

$$V^a(P) = \iint_\sigma \int_{r_S}^{\infty} \frac{\rho_a r^2 dr}{\ell_P} \tag{1}$$

where ρ_a is the density of the air scaled by the gravitational constant G, $\ell_P = \sqrt{r_P^2 + r^2 - 2r_P r \cos\psi}$, r_P, r, r_S are the geocentric radii of P, the running point under the integral and Earth's surface, respectively, ψ is the geocentric angle and σ is the unit sphere. Finally, the atmospheric potential to the first power of topographic height H can be written as (Sjöberg and Nahavandchi 2000):

$$V^a(P) = 4\pi\rho_0 R^2 \left\{\frac{1}{\upsilon-2} - \sum_{n=0}^{\infty} \frac{1}{2n+1}\left(\frac{r_P}{r}\right)^n \frac{H_n(P)}{R}\right\} \tag{2}$$

where

$$H_n(P) = \sum_{m=-n}^{n} H_{nm} Y_{nm}(P) \tag{3}$$

and

$$H_{nm} = \frac{1}{4\pi}\iint_\sigma H\, Y_{nm} d\sigma \tag{4}$$

and

$$H_P = \sum_{n,m} H_{nm} Y_{nm}(P) = \sum_{n=0}^{\infty} H_n(P) \tag{5}$$

Here the harmonics Y_{nm} obey

$$\frac{1}{4\pi}\iint_\sigma Y_{nm}\, Y_{n'm'} d\sigma = \begin{cases} 1 & \text{if n=n' and m=m'} \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

Also, ρ_0 is the density of the atmosphere at the radius of sea-level (ρ^0) multiplied by the gravitational constant (G), $\upsilon > 2$ is a constant and R is the mean Earth radius.

Thereafter, the atmospheric gravity and gravity anomaly can be derived as (Sjöberg and Nahavandchi 2000):

$$g^a(P) = -\frac{\partial}{\partial r_P} V^a(P) \doteq 2\pi\rho_0 H(P) \tag{7}$$

and

$$\Delta g^a(P) = -\left(\frac{\partial}{\partial r_P} + \frac{2}{r_P}\right) V^a(P) \doteq 4\pi\rho_0 \sum_{n=0}^{\infty} \frac{n+2}{2n+1} H_n(P) \tag{8}$$

Nahavandchi (2002) improved eq. (8) at a point on the geoid to the first power of elevation H as:

$$\Delta g^{a}(P) = 2\pi\rho_0[H_P + 3\bar{H}_P] \tag{9}$$

where

$$\bar{H}_P = \sum_{n=0}^{\infty} \frac{1}{2n+1} H_n(P) \tag{10}$$

To derive the above equations $\upsilon = 3$ and $r_P = R$ are used. Equation (9) shows that there are both local and global contributions to the direct atmospheric gravity effect.

3. – THE DIRECT ATMOSPHERIC EFFECT AT A POINT ON THE TOPOGRAPHY

A more detailed treatment of the formula for the direct atmospheric effect may lead to additional significant effects. Equation (8) is derived based on the assumption that $r_P = R$, i.e., neglecting the topographical height H. Rewriting this formula for a point P' at the topographical surface, one obtains:

$$\Delta g^{a}(P') = 4\pi\rho_0 \sum_{n=0}^{\infty} \left(\frac{r_{p'}}{R}\right)^{n-1} \frac{n+2}{2n+1} H_n(P) \tag{11}$$

Equation (11) can also be written similar to eq. (9). It is

$$\Delta g^{a}(P') = 2\pi\rho_0\left[H_{P'} + 3\overline{H'}_{P'}\right] \tag{12}$$

where

$$\overline{H'}_{P'} = \sum_{n=0}^{\infty} \left(\frac{r_{p'}}{R}\right)^{n-1} \frac{1}{2n+1} H_n(P') \tag{13}$$

The difference between eq. (8) and eq. (11) can be regarded as the downward continuation effect of the atmospheric masses on the gravity anomalies and can be written as:

$$\delta\Delta g^{a}(P') = 4\pi\rho_0 \sum_{n=0}^{\infty} \left[\left(\frac{r_{p'}}{R}\right)^{n-1} - 1\right] \frac{n+2}{2n+1} H_n(P') \tag{14}$$

This effect can be computed with a spherical approximation of $r_{P'} = R + H_{P'}$. A more detailed treatment of eq. (14) may lead to additional significant atmospheric effects. It is obvious that the spherical harmonics will only provide global information and some short-wavelength contributions will be missing (considering the degree and order of expansions in this study). A strategy is, if possible, to include the local contributions in the formula. This may be achieved if one can split Eq. (14) to two different terms. To do this, we first rewrite eq. (14) as follows:

$$\delta\Delta g^a(P') = 4\pi\rho_0 \sum_{n=0}^{\infty} \left[\left(\frac{r_{p'}}{R}\right)^{n-1} - 1\right] \frac{1}{2}\left(\frac{3}{2n+1} + 1\right) H_n(P') \qquad (15)$$

or

$$\delta\Delta g^a(P') = 4\pi\rho_0 \left\{\frac{-H_{P'}}{2} + \sum_{n=0}^{\infty} \left[\left(\frac{r_{p'}}{R}\right)^{n-1} - 1\right] \frac{1}{2}\left(\frac{3}{2n+1}\right) H_n(P') + \frac{1}{2}\sum_{n=0}^{\infty} \left(\frac{r_{p'}}{R}\right)^{n-1} H_n(P')\right\} \qquad (16)$$

where eq. (5) is used.

One finally obtains

$$\delta\Delta g^a(P') = 2\pi\rho_0 \left[-H_{P'} + 3\overline{H''}_{P'} + \sum_{n=0}^{\infty} \left(\frac{r_{p'}}{R}\right)^{n-1} H_n(P')\right] \qquad (17)$$

where

$$\overline{H''}_{P'} = \sum_{n=0}^{\infty} \left[\left(\frac{r_{p'}}{R}\right)^{n-1} - 1\right] \frac{1}{2n+1} H_n(P') \qquad (18)$$

Equation (17) shows that there are both local and global contribution terms to the gravity anomalies from the downward continuation effect of the atmospheric masses. The derived formula is the difference between the direct atmospheric effect computed at the topography and at the geoid. It is believed this new strategy, i.e. combination of the direct atmospheric effect at the topography [eq. (12)] with the downward continuation correction of eq. (17) treats the atmospheric effects in a more precise way, as the local contribution terms are present in both formulas. Only using the correction terms of eqs. (12) and (17), for removing the effect of atmospheric masses, the corrected gravity anomalies are allowed to be used in Stokes's formula (other correction terms to the gravity anomaly, the most important one the topographical corrections, must also be accounted. They are not studied here.)

The effect of atmospheric masses on the geoid can also be computed. The Stokes formula must be used. The effect on geoid becomes:

$$N_{\mathrm{dir}}^a(P') = \frac{R}{4\pi\gamma} \iint_\sigma S(\psi)\, \Delta g^a(P') d\sigma \qquad (19)$$

or

$$N_{\mathrm{dow}}^a(P') = \frac{R}{4\pi\gamma} \iint_\sigma S(\psi)\, \delta\Delta g^a(P') d\sigma \qquad (20)$$

where $S(\psi)$ is the Stokes function and γ is the mean normal gravity at sea-level.

4. – NUMERICAL INVESTIGATIONS

The direct atmospheric effect at a point on the topography will be computed from eq. (12) in a test area in Iran. This formula includes two terms representing the local contributions and global information. To investigate the effect of downward continuation of the atmospheric masses on the gravity anomalies, the differences between the direct atmospheric effects at the geoid and topography [eq. (14)] as well the new expression for these differences [eq. (17)] are computed in a test area in Iran. The test area is limited by latitudes 51° N and 54° N and longitudes 33° E and 36° E. The topography in this test area, depicted in fig. 1, varies from 50 to 5671 m. The height coefficients H_{nm} are determined from eqs. (4) and (5). For this, a 30' × 30' DTM is generated using the Geophysical Exploration Technology (GETECH) 5' ×5' DTM (GETECH, 1995). This 30' × 30' DTM is averaged using area weighting. Since the interest is in continental elevation coefficients and we are trying to evaluate the effect of the masses above the geoid, the heights below sea level are all set to zero. The spherical harmonic coefficients of topographic heights are computed to degree and order 360. Parameter definitions are as follows: $\rho_0 = G\rho^0$, where G = 6.673 × 10^{-11} $m^3kg^{-1}s^{-2}$ and $\rho^0 = 1.23\, kg/m^3$, R=6371 km, and γ = 981 Gal. The effects are computed to degree and order 360 so that the corresponding cell size is 30' × 30'. To compute the local contributions in the new formulas, a 1 km by 1 km DTM over Iran is used. The computation points are the same as the location of 1 km by 1 km DTM. It means that the local contributions are computed using the heights of computation points (H) (coming from 1 km by 1 km DTM) and the global contributions are determined from the spherical harmonic representation of topographical heights (H_n).

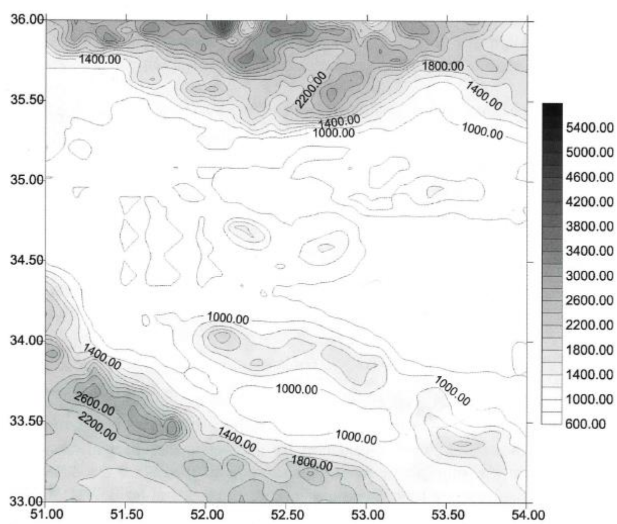

Fig. 1 – Presentation of topography in test area. Units in m.

In the first attempt, the direct atmospheric effects at the point on the topography are computed and depicted in fig. 2. It reaches to 0.358 mGal in the test area. This amount of direct atmospheric effect agrees with the results of Andersson et al. (1975) investigation. They noted that the real atmosphere is neither laterally homogeneous nor regular in shape. In particular, its lower boundary is very irregular and, indeed, takes on the shape of Earth's topography. Because of this, the atmospheric gravity effect is not the same even for points of equal altitude. They came up with a second-order atmospheric effect to gravity and the geoid. The magnitude of this correction on the gravity reaches to a few hundreds micro gal on gravity and more than 40 cm on the geoid.

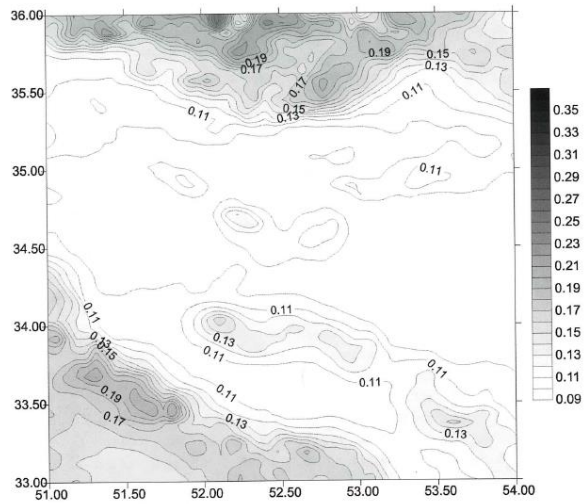

Fig. 2 – The direct atmospheric effect on the gravity anomalies computed on the topography. Units in mGal.

Fig. 3 shows the differences on gravity between the direct atmospheric effect computed at the geoid and at the topography (e.g. downward continuation correction). Spherical harmonics [eq. (14)] are used to present the differences. It reaches to 0.015 mGal over Damavand Mountain in the test area with a maximum elevation of 5671. It is obvious that this effect is not significant and can be neglected in a precise geoid determination. This effect in its present form, however, is computed through a set of spherical harmonics, which only presents the long-wavelength contributions. The new formula, on the other hand, i.e. eq. (17), presents the same effect in a more precise way. It includes the local information. The downward continuation effects, computed with eq. (17), are depicted in Fig. 4. It reaches to 0.354 mGal. It is obvious that the local contributions are the reason for the differences between the two eqs. (14) and (17). The differences between these two approaches are numerically realized in Fig. 5. It reaches to 0.35 mGal. The differences between two methods seem to be large enough to be considered in precise geoid computations. These amounts of differences were expected as eq. (14) only uses a spherical harmonic representation of the topography, but eq. (17) uses both spherical harmonics and a dense DTM in this study. It is obvious that denser DTM than the one used in this study (1 km by 1 km) will show better the local irregularities.

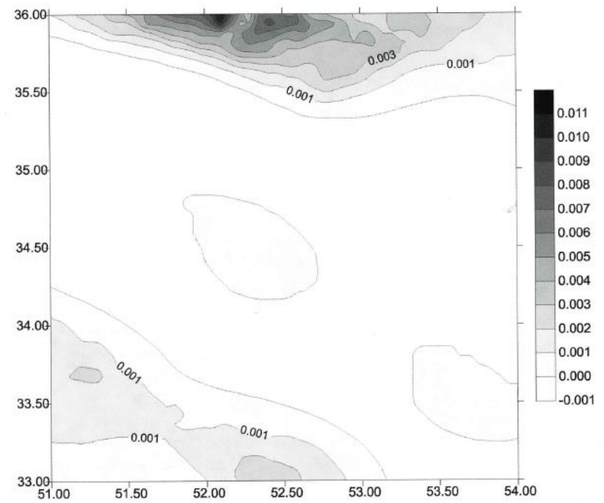

Fig. 3 – The downward continuation correction to the atmospheric gravity effect computed using spherical harmonics. Units in mGal.

In order to obtain further insight into the differences between eqs. (14) and (17), the geoid atmospheric effect in downward continuation of gravity anomalies is computed using eq. (20). This means that the results of eqs. (14) and (17) on the gravity anomalies are transferred on the geoid using eq. (20). The same test area is chosen. The geoid atmospheric effect reaches to 0.81 cm for the results of eq. (14) while it reaches to 4.2 cm using eq. (17). This means that the local contributions of the downward continuation effect to the geoid (due to the atmosphere) are significant and must be considered in a precise geoid computation.

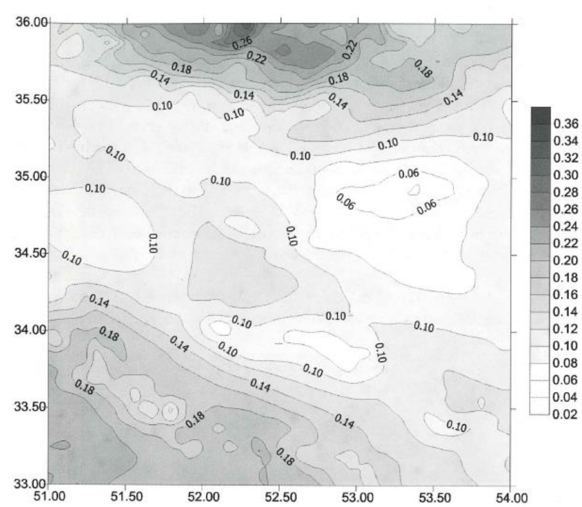

Fig. 4 – The downward continuation correction to the atmospheric gravity effect with new formula. Units in mGal.

Preliminary results show that the use of this new procedure in a gravimetric geoidal height computation (i.e. the use of eqs. (12) and (17) for removing the effect of atmospheric masses on the gravity anomalies) yields better agreement with the Global Positioning System (GPS)-levelling derived geoidal heights, which are used to demonstrate such improvements, than the results of gravimetric geoidal heights at to the same GPS stations but using the old formula.

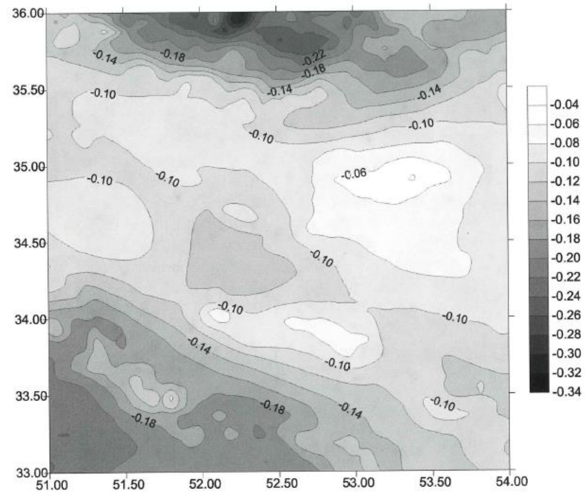

Fig. 5 – The differences between the downward continuation effect of atmospheric masses computed from the new and old formulas. Units in mGal.

5. – CONCLUSIONS

A strategy to compute the direct atmospheric effect presented by Nahavandchi (2002) used the geoid surface for the computation points. To treat this effect in a more precise way, the direct atmospheric effect computed on the geoid is split to two terms: i) a direct effect at a point on the topography and ii) a downward continuation correction term from the topography to the geoid. It is shown that both these terms include local contributions and long-wavelength information. This implies that the previous formula may have some numerical problems in representing of all significant contributions. This new strategy and the previous one are then realized in a test area with the maximum elevation of 5671 m. It is shown that the differences can be significant. A maximum difference of 0.35 mGal is achieved, which is significant and must be included in a precise geoid determination. On the geoid, the differences reach to more than 4 cm.

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