

# Evaluating Modelling Approaches for State-Dependent Environmental Constraints in Medium-Term Hydropower Scheduling

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**Abstract**—This paper addresses the inclusion of state - dependent environmental constraints in medium-term scheduling of hydropower plants with reservoirs. The reservoir handling and release plans of hydropower production can lead to dry river sections and lakes, create barriers for fish mitigation and impact terrestrial ecosystems. In Norway, many reservoirs are also used for recreational purposes. Environmental constraints are imposed to facilitate synergies in reservoir usage and ensure high enough water levels. Some environmental constraints are challenging to mathematically include in hydropower scheduling models, due to nonconvex characteristics or binary logic. State-dependent constraints can make the problem formulation nonconvex, and are therefore not included in existing scheduling tools. This paper compare different approaches for representing such constraints in medium to long-term scheduling models and evaluate the difference in optimality.

**Index Terms**—hydropower scheduling, environmental constraints, state-dependent constraints

## NOMENCLATURE

### Index Sets

$\mathcal{T}$  Set of stages (weeks)  
 $\mathcal{S}$  Set of scenarios (inflow and price)

### Parameters

$\lambda_t^s$  Power price in stage  $t$ , given scenario  $s$ , in  $\frac{\text{€}}{\text{MWh}}$   
 $i_t^s$  Inflow in stage  $t$ , given scenario  $s$ , in  $\text{Mm}^3$   
 $v_t^0$  Initial reservoir level in stage  $t$ , in  $\text{Mm}^3$   
 $E$  Energy conversion factor, in  $\frac{\text{MWh}}{\text{Mm}^3}$   
 $\bar{V}$  Maximum reservoir volume, in  $\text{Mm}^3$   
 $\bar{U}$  Maximum discharge, in  $\text{MW}$   
 $\tilde{V}_t$  Environmental limit on reservoir volume in stage  $t$ , in  $\text{Mm}^3$   
 $F_{v_{t+1}}$  Future profit function of  $v_t$

### Variables

$q_t$  Plant outflow in stage  $t$ , in  $\text{Mm}^3$   
 $s_t$  Spilled outflow in stage  $t$ , in  $\text{Mm}^3$   
 $u_t$  Plant output in stage  $t$ , in  $\text{MWh}$   
 $v_t$  Reservoir level at end of stage  $t$ , in  $\text{Mm}^3$   
 $\alpha_{t+1}$  Future expected profit in stage  $t$ , in  $\text{€}$   
 $\alpha_t$  Expected profit in stage  $t$ , in  $\text{€}$   
 $\gamma_t$  Environmental binary variable for stopping production in stage  $t$

## I. INTRODUCTION

Climate change and environmental degradation are existential threats to Europe and the world. To overcome these challenges, power producers are encouraged to operate in an environmentally sustainable way. The dominating energy source in Norway is hydropower, and to limit the ecological burden of hydropower production, the Norwegian government has imposed rules and regulations on the reservoir volume and release plans [1].

In Norway, many large reservoirs are also used for recreational activities such as fishing, boat trips, and swimming for the locals. There are strict regulations at the minimum reservoir level to keep these activities alive [1]. The goal of the environmental constraint is to avoid low water levels in the reservoirs, which leads to great dissatisfaction among the local population.



Fig. 1. Example of low water level in a recreational area.

State-dependent environmental constraints are imposed on operation of several Norwegian hydropower plants, and may be imposed on more hydropower plants in near future as a result of revision of the concession terms of existing plants [2]. State-dependent restrictions are often more economically efficient and can be better targeted in terms of environmental

gains but have the disadvantage of being mathematically challenging to model [3].

Medium-term hydropower scheduling models currently used in the Nordic hydropower industry do not include accurate representations of state-dependent constraints as they often lead to nonconvexities and the need for logical conditions. State-of-the-art solution methods for medium- to long-term hydropower scheduling in the Nordic are based on stochastic dual dynamic programming (SDDP) [4], which require a convex model formulation. These models therefore rely on linear approximations of state-dependent and nonlinear constraints [5] [6]. Using stochastic dynamic programming (SDP) [7] will enable the possibility to include nonconvexities and logical conditions but is suitable only for small systems. Previous research considers an accurate representation of state-dependent environmental constraints using SDDiP [8] and SDP [9], and linear approximations in SDDP. This work differs by using an SDP algorithm to compare an exact formulation to linear approximations in an industrial case with uncertainty in price and inflow.

The research presented in this paper aims to enlighten how different modelling approaches of state-dependent environmental constraints in water value calculations affect water values and production plans. Our contribution includes a description of two different modelling approaches, an exact representation and a linear approximation of the environmental constraint. The two approaches are compared towards the current situation, where these constraints are not considered in the planning. The comparison is conducted for a case study of the Driva hydropower plant in mid-Norway, using data provided by the operator of the plant, TrønderEnergi. Both formulations can be used in methods that do not require a convex model formulation, such as SDP. In models based on SDDP, a convex model formulation is required and a linear approximation is necessary.

## II. MODEL DESCRIPTION

A medium-term hydropower scheduling model based on SDP is used to investigate the solution quality of the different representations of the constraint. An SDP model framework is chosen due to its straightforward implementation and good opportunities for formulation flexibility, including nonconvexities and hereby state-dependent constraints. The same model framework is used for both implementations to avoid additional noise in the comparison.

The developed SDP-model takes the perspective of a power producer, and the objective is to maximize revenue while complying with all physical and regulatory constraints, including environmental constraints. The dynamic structure in the hydropower scheduling problem enables the ability to solve smaller scheduling problems for each weekly stage independently and use the connection between the weekly steps to establish the optimal solution for the whole scheduling problem. The connection between each weekly stage is the reservoir level and, due to strong autocorrelation, the level of the power price. The operational decisions in one step

determines the reservoir level, affecting the decisions in the next step.

### A. Modeling Uncertainties

Inflow and power price are considered uncertain and are represented in the model as stochastic variables. The uncertainty is represented using a Markov model with weighted probabilities. In addition, autocorrelation in price is considered by modeling price as a state variable. The correlation from last week is represented by using the power price in the previous week  $t - 1$  as a state variable in week  $t$ .

### B. Weekly Stage Problem

The SDP-algorithm solves the decision problem for each weekly stage  $t = 1, \dots, \mathcal{T}$ , for all discrete reservoir states and all stochastic states, see e.g. [9] for a description of a similar SDP-algorithm. The weekly decision problem is formulated with (1a)-(1i).

$$\max \alpha_t = \lambda_t^s \cdot u_t + \alpha_{t+1} \quad (1a)$$

$$v_t = v_t^0 - u_t - s_t + i_t^s \quad (1b)$$

$$\alpha_{t+1} \leq F_{\alpha_{t+1}}(v_t) \quad (1c)$$

$$u_t = E \cdot q_t \quad (1d)$$

$$v_t \geq \gamma_t \cdot \tilde{V}_t \quad (1e)$$

$$u_t \leq \gamma_t \cdot \bar{U} \quad (1f)$$

$$v_t \leq \bar{V} \quad (1g)$$

$$u_t, v_t, \alpha_{t+1}, q_t, s_t \geq 0 \quad (1h)$$

$$\gamma_t \in \{0, 1\} \quad (1i)$$

The objective of the weekly stage problem (1a) is to maximize revenue from the current week, as well as the future revenue of remaining reservoir volume. The resulting reservoir level of each week is determined by (1b) and the future revenue is set by (1c). The energy conversion is described in (1d) and is modeled as a constant relation. Equations (1e) and (1f) ensures that the environmental constraint is being complied with. If the reservoir level is lower than the environmental threshold,  $\tilde{V}_t$ , the binary variable  $\gamma_t$  is set to zero and the production has to stop.  $\gamma_t$  can be set to one when the reservoir level is higher than the threshold. Then the hydropower plant can produce power, but the resulting reservoir level has to be above the threshold. Equation (1g) ensures reservoir level within the physical boundaries.

After solving all the decision problems in each stage, the expected future profit is calculated and used when solving the previous stage ( $t - 1$ ). When the problem has been solved for all weeks, the algorithm re-solves the entire planning horizon, using the water values from the first stage as end-value setting in the last stage. To avoid unwanted end of horizon-effects, this continues until the algorithm converges, i.e. when the water values in the first step equals the water values in the last step. When the SDP algorithm has converged, the calculated water values can be used for a final forward simulation in order to obtain production plans.

### C. Solution Method

In order to compare the different approaches of modeling the state-dependent environmental constraint, production plans are simulated using three different sets of calculated water values. The three different approaches for calculating water values are presented below:

- 1) Without Environmental Constraint: To calculate water values without inclusion of environmental constraints is often the currently used method in commercial Nordic hydropower scheduling. In this case, the weekly stage problem presented in II-B is modified by excluding constraint (1e) and set  $\gamma_t$  to 1 for all stages.
- 2) Near Exact Formulation: To include the environmental constraint with a near exact formulation, the weekly stage problem presented in II-B is used to calculate water values.
- 3) Linear Approximation: The formulation in II-B is non-convex and uses binary logic. To avoid this, a linear relaxation is imposed, setting  $\gamma_t$  to a continuous variable between 0 and 1 for all stages.

The final simulations are conducted as parallel simulation, i.e. assuming a fixed start-reservoir level in week 1 for all simulated weather and price scenarios. This is the selected simulation approach because this resembles the industrial process of production planning in TrønderEnergi.

### III. CASE STUDY

Finally, the model is applied to a single-reservoir hydropower plant case study. The case study described in this section is the production planning of the Driva power plant (150MW), with Gjeviltvatnet as the main reservoir (280Mm<sup>3</sup>), located in Norway. TrønderEnergi, the Norwegian energy company that operates the power plant, initiated the study by request due to existing challenges regarding the inclusion of environmental constraints in their production planning process. Gjeviltvatnet is, in addition to being a hydropower reservoir, an assembly point for recreational activities. Every summer, many people come from surrounding cities to this area to spend their vacation fishing, swimming, and boating in Gjeviltvatnet. Therefore, it is of great interest that the reservoir level is kept high enough to ensure that visitors can do these activities. The case study is a compelling case as there is a lot of pressure from the local population and the authorities that the reservoir level must be high in the summer. A main motivation of the work has been the close industry collaboration and the access to actual data from TrønderEnergi.

#### A. Price and Inflow Data Inputs

All input data is provided from TrønderEnergi. The inflow scenarios are based on historical data, and the price scenarios are simulated from a fundamental model (EMPS) that uses historical weather years as the stochastic input. The price and inflow scenarios are presented in Figures 3 and 2, respectively.

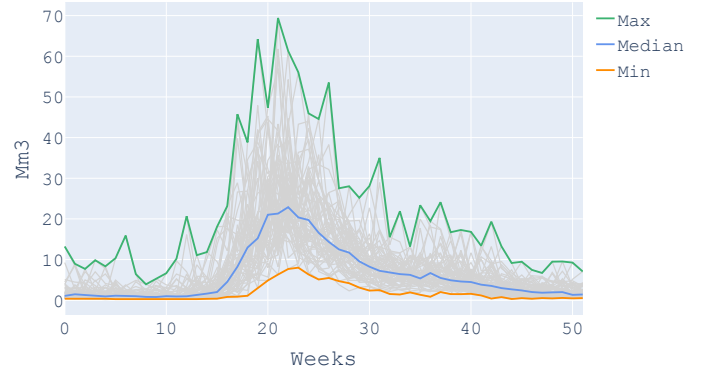


Fig. 2. Input data for inflow scenarios.

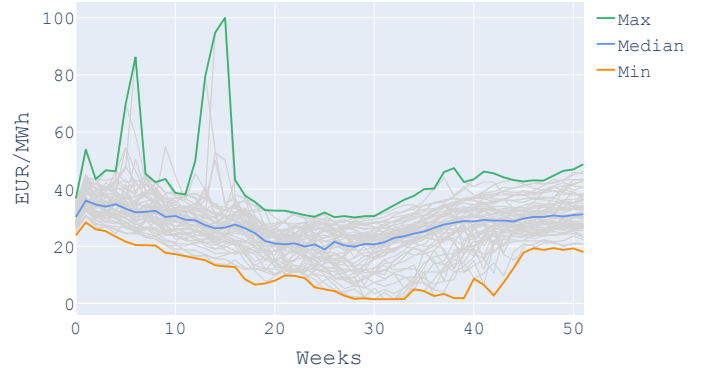


Fig. 3. Input data for price scenarios.

#### B. Results and Discussion

The three approaches described in II-C are used to calculate water values and simulate production plans using parallel simulation. The results of the simulations are presented in the following section.

To simulate production plans for the hydropower plant, historical inflow and price prognosis from 57 years are used together with calculated water values from each of the three approaches described in II-C. The resulting production plans follow a traditional, seasonal curve for reservoir management. This is reasonable considering that the assumed power price has a characteristic curve, with high prices in the winter and low in the summer. Comparing the results from simulations with different water values, we see that the production plans are equal between the three approaches for most of the simulated scenarios. With water values from the near exact formulation, 65% of the simulated production plans were identical to the plans without consideration of the restriction. With water values from the linearly approximated formulation, 93% of the simulated production plans were equal to the plans simulated with water values that did not include the restriction. The relatively low impact of considering the constraint can be

explained by the assumed power price. Because of low prices during summer, power production within the restriction period is already less beneficial than the rest of the year, dampening the effect of the constraint.

Yearly revenue for the base case, without the restriction in water value calculations, was 17.1M€. The average change in profit of all simulations is presented in Table I. The economic results in Table I are calculated considering the change in yearly revenue from power production and the difference in the value of the reservoir level at the end of the analysis period.

TABLE I  
ECONOMICAL IMPROVEMENT FROM BASE CASE

	Formulation Method	
	<i>Linear Approximation</i>	<i>Exact Formulation</i>
Absolute average	2333 EUR/yr	40 993 EUR/yr
Relative average	0.01%	0.24 %

The average change in profit from Table I shows a slight variance between the different approaches. The average difference is considerably more prominent for the exact formulation than for the linear approximated formulation but less than 0.5% in both cases. The low economic gains could be due to many weather years resulting in equal production plans for each approach.

The linear approximation approach did not change the production plans of any economic significance, indicating that it is not the most suitable method. Therefore, further studying results from the approximated constraint was seen as less valuable than the exact formulation to analyze how the new water values affect the production plans.

The following observations and discussions are comparing the exact constraint formulation to the base case method. The identical scenarios are filtered out to see what differences occur by including the restriction in the water value calculations. In other words, we only look at the 20 weather years that changed the production plan after introducing the exact restriction formulation. The average economic results in which the exact formulation deviates from the base method are presented in Table II. The scenario with the best improvement resulted in an economical gain of 1.6 M€, while the worst scenario gave a loss of 1.3M€. The average profit gain of all years with improvement was 460 000 € and the average profit loss of all years with deterioration was 302 000 €.

TABLE II  
ECONOMICAL IMPROVEMENT FROM BASE CASE  
OF FILTERED SIMULATIONS<sup>a</sup>

Absolute average	117 000 EUR/yr
Relative average	0.67 %

<sup>a</sup>Simulations that resulted in unequal production plans.

The average reservoir levels for each week are presented in Figure 4. An important remark is that the average values do not fully reflect the spread in the curves, and for some scenarios, there are greater differences. The production plans

appear similar, but there is an interesting difference during the restriction period at two particular points. At the beginning of the restriction period, the reservoir level is lower for the water values that consider the constraint. After a few weeks, this reverses, and during the last weeks of the restriction period, the reservoir level is higher for the water values that consider the constraint.

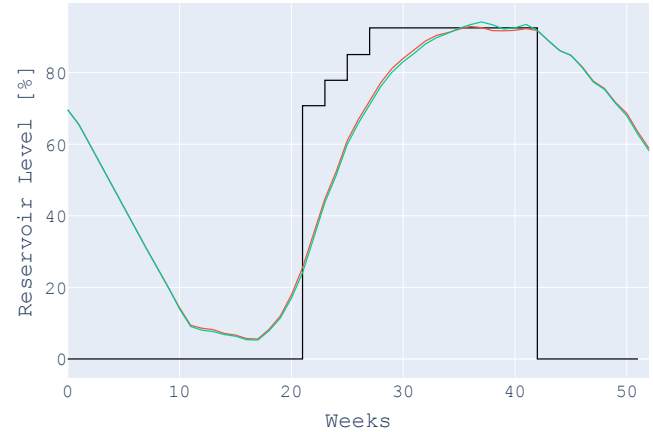


Fig. 4. Average reservoir level of simulations that resulted in unequal production plans. Red line is the base case simulations and green line is the exact formulation simulations.

The turning point, where the average reservoir levels of the exact formulation exceed the base case, is further illustrated in Figure 5. This means that the water values considering the environmental constraint are lower right before the restriction period and become higher during the following weeks. The producer has no chance to govern differently until the reservoir level reaches the threshold in the restriction period, i.e., the turning point comes from how the individual scenarios that have already reached the limit are handled. The same reasoning also explains why the average reservoir level crosses the boundary in weeks 37 and 39.

Despite the changes discussed previously, the reservoir levels in the case study do not change of any practical significance. The small changes in the reservoir levels may point to the case's price distribution, with lower prices in the summer and higher in the winter. The model does not see an incentive to save water in the winter to reach a high enough level to be allowed to produce water earlier in the summer.

The model is economically driven and therefore governed by the earning potential in the period with the restriction. Even though the case study resulted in a financial gain from including the constraint in the water value calculations, the threshold is not reached any earlier in the restriction period. The purpose behind the restriction is not better achieved, but the operation does not violate the terms of the restriction. A different price distribution will likely affect how the policy's purpose, which is to get more water, is met.

The new water values, which take into account the environmental constraint, reflects a production halt in the weeks

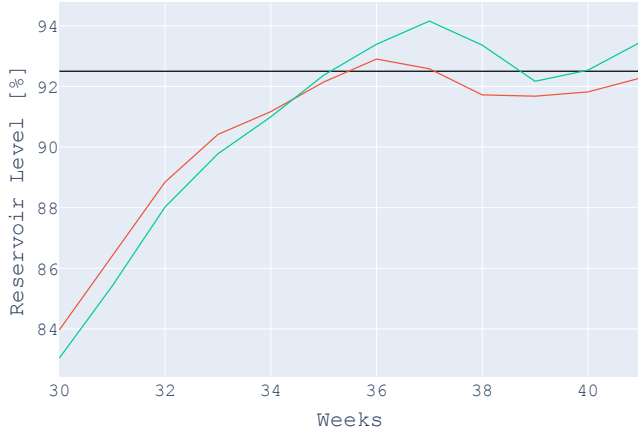


Fig. 5. Excerpt from Figure 4. Red line is the base case simulations and green line is the exact formulation simulations.

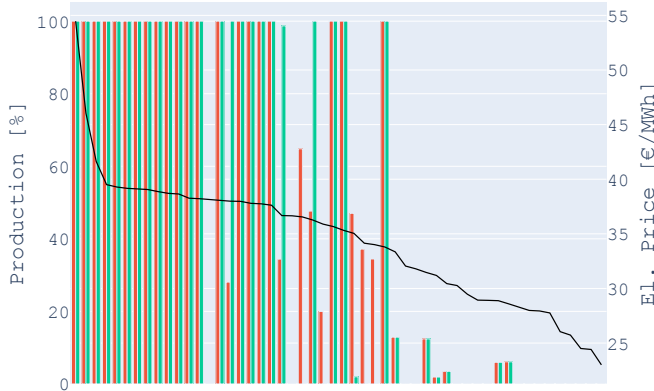


Fig. 6. Duration curve of a selected simulation. Red bars are the base case and green bars are the exact formulation approach.

following activation of the restriction. By anticipating the restriction in advance, power producers can move production to weeks with higher prices. The shift in production is illustrated with the duration curve in Figure 6. The duration curve shows weekly power production in a selected scenario for the water values with and without the restriction, sorted by descending price. Here, the model manages to move production to weeks with a higher price and thus get more profit from the water resource.

The production is forced to stop whenever the reservoir level is lower than the threshold during the restriction period. From Figure 4 it is clear that there, on average, are a lot of simulations resulting in a no-production directive. From the duration curve in Figure 6 most of the "no-production" weeks are further to the right, meaning that these weeks have a low price and it would not be beneficial to produce regardless of the restriction. This emphasizes what was seen from the

production plans, that the reservoir level often does not change when the restriction is included in the water value calculations. In addition, it also further substantiates the observation that including environmental constraints in water value calculations does not necessarily lead to higher fulfillment of the purpose behind the restriction. Improved modeling of the constraint in the medium-term scheduling was not found to improve the fulfillment of the underlying purpose of the constraint.

There may, however, be some years where this distribution does not occur. While the price distribution in this case study is typical, some years may be abnormal, with higher summer prices. Years with this atypical price distribution predictions could incentivize planning for the restriction. The model can weigh the benefit of producing in the winter against the disadvantage of experiencing stop requirements in the summer; hence, the price distribution influences the model. In addition to price sensitivity, the results from the case study are also case specific in terms of the characteristics of the hydropower plant where the constraint is imposed and the regulatory definition of the constraint.

#### IV. CONCLUSIONS

This research paper has investigated state-dependent environmental constraints in medium-term hydropower scheduling. The authors aimed to contribute to the research field by implementing and comparing suggested methods of including environmental constraints. A case study was performed to compare an exact formulation to a linear approximation. The two approaches were compared to the base case method, excluding the restriction in water value calculations.

The main findings from the case study showed performance improvement when including an exact formulation of the state-dependent constraints. The financial results indicate an earning potential, and the duration curve illustrated how planning ahead for the restriction could ensure production in higher priced weeks. On the other hand, the overall reservoir level did not increase substantially. Despite a financial gain, a higher fulfillment of the purpose behind the restriction, which is to get more water for recreational purposes, was not seen. Still, the model is very price sensitive, and it is expected that planning for the restriction could have a larger impact with a different seasonal price profile.

There was no significant difference between linear approximation and the base case method, indicating that a complete relaxation of the binary variables is not a suitable method. A possible extension of this study is to look at other tighter approximation methods.

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