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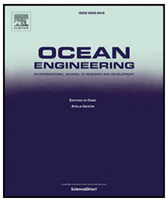

Intention modeling and inference for autonomous collision avoidance at sea ☆

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ABSTRACT

The open wording of the traffic rules of the sea, COLREGS, and the existence of unwritten rules, make it essential for an autonomous ship to understand the intentions of other ships. This article uses a dynamic Bayesian network (DBN) to model and infer the intentions of other ships in open waters based on their observed real-time behavior. Multiple intention nodes are included to describe the different ways a ship can interpret and conflict with the behavioral rules outlined in COLREGS. The prior probability distributions of the intention nodes are adapted to the current situation based on observable characteristics such as location and relative ship size. The resulting model is able to identify situations that are prone to cause misunderstandings and infer the state of multiple intention variables that describe how the ship is likely to behave. Different collision avoidance algorithms can use the resulting intention information to better know if, when, and how to act.

1. Introduction

When navigating at sea, understanding the intentions of other ships can be crucial for avoiding accidents (Chauvin, 2011). Blindly assuming that the other ship will follow the traffic rules put forth by the Convention on the International Regulations for Preventing Collisions at Sea (COLREGS) (IMO, 1972) is insufficient as shown in Chauvin and Lardjane (2008). They demonstrated the existence of local unwritten rules and agreements between captains that went contrary to the rules specified by COLREGS. Furthermore, COLREGS is open to disagreements making it unsafe to act only based on your own interpretation of the situation (Clawson, 2013; Woerner et al., 2019). For an autonomous ship to safely operate in these conditions, it is essential that the ship can pick up on the intentions of other ships.

A large variety of ship collision avoidance algorithms exists in the literature (Huang et al., 2020; Vagale et al., 2021). Most algorithms that consider COLREGS handle ships that do not fulfill the traffic rules by executing reactive evasive actions when the ships get close enough. In Eriksen et al. (2020) this is handled by having a separate short-term controller, in addition to their COLREGS compliant controller, which disregards COLREGS when the ships are close enough. A different approach is taken in Johansen et al. (2016) where they have a separate collision risk and COLREGS compliance penalties. The collision risk penalty increases when the ships get closer, ensuring that an evasive action will be taken even if it conflicts with the main COLREGS rules.

A different approach is taken in Tengesdal et al. (2020) where they instead simulate multiple possible future trajectories the other ships can follow. The probabilities of the different trajectories are based upon the likelihood of the other ships having different intentions, such as being COLREGS compliant. This enables the collision avoidance algorithm to take early and substantial actions if the intentions are uncertain or if it becomes apparent that the other ship does not act according to the rules. However, Tengesdal et al. (2020) does not consider how these intentions can be identified.

Different methods exist for identifying the intentions of other ships (Du et al., 2020; Woerner et al., 2019; Cho et al., 2021). Du et al. (2020) presents a method to identify whether the give-way ship is doing an evasive action or not. This enables the ship to comply with CORLEGS rule 17, which states that stand-on ships should act if the give-way ship is not taking appropriate action. Cho et al. (2021) presents a Bayesian model that evaluates the probability that the other ship follows its obligations as specified by COLREGS rules 14 to 17 based on its observed motion. Woerner et al. (2019) develop a scoring system to evaluate to what degree ships follow COLREGS rules 7, 8, and 13–17. This method is designed to evaluate different collision avoidance algorithms but can also be used online to evaluate how well other ships are acting in accordance with the rules.

These articles (Du et al., 2020; Woerner et al., 2019; Cho et al., 2021) evaluate whether the other ship is acting as expected based on

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the own-ships interpretation of the situation. They do not model the underlying causes making the ship not act as expected. These underlying causes could, for example, be a disagreement of the situation or one of the ships having priority over the other.

Works on intention modeling exist for air traffic (Krozel and Andrisani, 2006; Yepes et al., 2007), road traffic (Hardy and Campbell, 2013), and for robot pedestrian interactions (Chen et al., 2021; Hashimoto et al., 2015). These works show different ways of inferring the goal, behavior, or trajectories of the other agents in the encounter. Only Hashimoto et al. (2015) consider underlying causes that affect how an agent acts. They use information on whether a pedestrian is alone or in a group to affect the prior probability that it will hurry over at a flashing green light.

The present article uses a dynamic Bayesian network (DBN) to model and infer the intentions of other ships in open waters. Different intention variables are defined based on the different ways ships can interpret and conflict with the behavioral rules specified by COLREGS. The DBN combines these intention variables with a model based on COLREGS Rule 7, 8, 11, and 13 to 18 to define the possible ways the ship can act. A ship's intentions are gradually inferred by ruling out all possible combinations of intention states that contradict the observed course and speed. This way of modeling ensures that the intention probabilities are independent of how often the model is updated with new observations.

The contribution of this article and the novelty compared to earlier literature is a modeling framework that considers how underlying causes affect a ship's behavior and which can infer the state of multiple different intention variables based on measured properties. Modeling the underlying causes enables the model to identify situations that can cause misunderstandings, making it possible to take early actions to avoid a potentially dangerous situation. Furthermore, it enables the model to adapt to the current situation by letting additional information, such as relative ship size and location, affect the intentions. Being able to infer the state of multiple intention nodes enables the model to describe the future motion of other ships with higher fidelity than simply being COLREGS compliant or not. The resulting intention probabilities can be used for collision avoidance with algorithms that explicitly consider the intentions (Tengesdal et al., 2020) or as decision criteria replacing the static distance used to decide when to always act to avoid collision (Eriksen et al., 2020).

The rest of the article is structured as follows. Section 2 give background information on Bayesian networks. Section 3 presents the proposed DBN which is demonstrated in Section 4. The results are discussed in Section 5 and a conclusion is given in Section 6.

2. Background

Bayesian belief networks (BBN) are directed acyclic graphs (DAG) that model probabilistic relations. These networks consist of nodes that can be in a discrete set of states and arcs that define dependencies between nodes. An arc points from a parent node to a child node. Conditional probability tables (CPT) are supplied for all nodes and define the probability of the node being in a particular state as a function of the states of its parent nodes. If the nodes do not have a parent node, then the CPT defines the prior distribution of that node.

The Bayesian probability law is used to evaluate a node's probability distribution, given some evidence. Evidence is the set of information about the state of one or more nodes. If this information is uncertain, then virtual evidence can be used. Virtual evidence specifies the probability of observing this particular observation, given the state of the node. A state unlikely to result in the observation will be given a low probability, while one likely to result in the observation will be given a high probability. A thorough explanation of virtual evidence can be found in Ben Mrad et al. (2012).

BBNs can be made dynamic by repeating some or all of the nodes for each time step. Fig. 1 shows an example of the resulting DBN. DBNs make it possible to model how a system develops over time. The DBN can consist of time-independent nodes as well as time-dependent nodes.

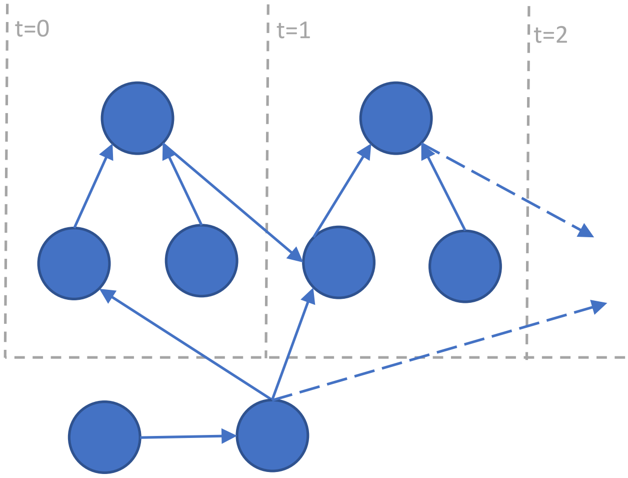


Fig. 1. Example of dynamic Bayesian network (DBN) consisting of three time-dependent nodes and two time-independent nodes. Two time steps are shown.

Software libraries such as Bayesfusion LLC (2021a) include different general solvers for evaluating DBNs and natively support the use of virtual evidence. More information on BBNs and DBNs can be found in Fenton and Neil (2018) and Russell and Norvig (2014).

3. Method

This section presents a DBN used to model and infer the intention of meeting ships. The term intentions will be used for a ship's internal states that we wish to infer such that we can understand how the ship will act. Examples of different intention variables are what the ship considers to be a safe distance, what priority it thinks it has relative to the other ships, and what it thinks that the COLREGS situation is.

The DBN model takes the perspective of a single ship, which will be called the reference ship, and models its relation to all other ships in the area. The index i will be used to identify the other ships in the area. To model multiple ships, the model must be repeated for each ship. How to make inference using the model is described in Section 3.1.

Each of the intention variables are modeled as nodes in the DBN. These nodes are stochastic variables as the intention is unknown. The intention nodes are modeled as time-independent nodes as it is assumed that the intentions do not change within one encounter. The prior distribution of the intention nodes describes how often the different intentions are encountered in situations similar to this one. How these priors are designed is described in more detail in Section 3.4.

The intentions are updated based on different measured properties that can be evaluated based on the relative position between the ships, theirs course, and their speed. The different measured properties are given in Table 3. A tracking system is assumed to be used to evaluate the ships course, speed, and position. The tracking system is assumed to give high quality tracks, such that the intention module does not need to account for measurement uncertainty.

The DBN evaluates the probability that a particular combination of measurements and intention node states are compatible. Which combinations that are compatible are defined by COLREGS and are described in Section 3.2 using logic statements. How these can be translated into CPTs is described in Section 3.3. The resulting DBN is shown in Fig. 5.

When a new observation is made, the different measured properties are inserted as evidence on the measurement nodes in a new time-step of the DBN. These measurement nodes are time-dependent, thereby enabling the system to combine information over time. The network can

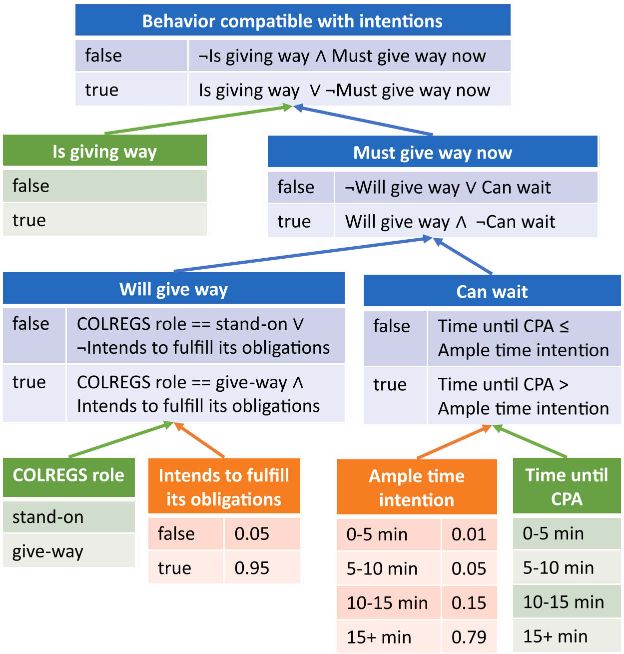


Fig. 2. A simplified example network used to illustrate the proposed inference method. Measurement nodes are shown in green, intention nodes in orange, and modeling nodes in blue. The initial probability distribution is shown for the intention nodes. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

be used to evaluate the probability that the observation is compatible with the prior distribution of the intention nodes. The distribution of the intention nodes can be updated by eliminating all combinations of intentions that contradict the observation. This is achieved by inserting evidence in the network stating that intentions and observed measurements are, in fact, compatible. The updated posterior distribution of the intention nodes can be used to give an updated prediction on how the reference ship will act. Two different ways of using the updated intention probability distributions for collision avoidance are outlined in Section 3.5.

Modeling whether a particular combination of observations and intention node states are compatible enables the system to gradually infer the reference ship's intentions without considering how often observations are given to the system. Giving the exact same observation multiple times to the system will not affect the probability distribution of the intention nodes, as the first observation has already eliminated all combinations of intentions that would be eliminated by the second observation.

A simplified example can illustrate this procedure. Fig. 2 shows a simplified network that only considers when the ship will act. COLREGS rule 8(a) states that a ship should act in "ample time". Two intention nodes are then needed, one modeling the reference ship's definition of ample time and the other modeling whether the reference ship intends to follow this rule. When an observation is made, the following evidence is inserted: time until closest point of approach (CPA), which role the reference ship has according to COLREGS, and whether the reference ship is giving way. In this example, the observation is only compatible with the intention of the reference ship if either of the following is true: it is giving way, it has a stand-on role, if it does not intend to follow the rules, or if the time until CPA is longer than the reference ships definition of ample time.

The intention probabilities can be updated to reflect the observation by inserting evidence on the "Behavior compatible with intentions" node stating that it must be in the "true" state. If it, for example, is observed that the time until CPA is 10 min, the reference ship has a give-way role, and it is not giving way, then the model can exclude the possibility that the ship intends to follow its obligation to give way while at the same time considers ample time to be more than 10 min. It is left with the possibility that it will not follow its obligations at all or that it considers ample time to be shorter than 10 min. For this example, the updated probability that the reference ship does not intend to fulfill its obligations evaluates to 47%. This is due to the prior likelihood that the reference ship will give way at a short distance (0.01+0.05 = 0.06) is similar to the prior likelihood that it will not fulfill its obligations (0.05). This simplified example is unable to model the underlying causes that influence how a ship will act. The rest of this section handles this by considering many more of the COLREGS rules.

Table 1
Abbreviations.

Abbreviation	Description
CPA	Closest point of approach
SO	Stand-on
GW	Give-way
HO	Head-on
OT_ing	Overtaking
OT_en	Overtaken
CR_SS	Crossing with other ship on starboard side
CR_PS	Crossing with other ship on port side

3.1. *Basic procedure*

For every new observation:

1. Insert information from observed position, course, and speed as evidence on the measurement nodes
2. Insert evidence stating that the compatible to all node (C) is in the state true.
3. Evaluate the updated probabilities for the different intention states
4. Expand the network with a new time-step

3.2. *Intention model logic*

This section presents a series of logic statements that define which combinations of intentions and observations that are compatible. These statements are based on the behavioral rules specified by COLREGS Rule 7, 8, 11, and 13 to 18. Rules regarding traffic separation schemes (Rule 10), narrow channels (Rule 9), and sailing vessels (Rule 12) are not considered in this article.

The section is structured following a top-down approach where the statement describing the most general model variable is presented first. Model variables that are used in more general statements are then gradually introduced. The different model variables are given in Table 4, intention variables in Table 2, measurement variables in Table 3, and parameters in Table 5. See Table 1 for abbreviations used in this model.

3.2.1. $C[t]$ - *Compatible to all*

An observation is compatible with the intention states of the reference ship at time step t if it is compatible towards all ships in the area at that time step. The area considered must be large enough to encompass all ships that potentially affect how the reference ship acts. All observations are also considered compatible if the ship intends to act in an unmodeled manner ($\mathcal{I}_U$). This state works as a catch-all for behavior that does not fit the behavioral rules described in this section. Mathematically, this is expressed through the following logical clause:

$$C[t] = \left(\wedge_{i=1}^{n} C_i[t]\right) \vee \mathcal{I}_U \tag{1}$$

Table 2
Intention variables.

Symbol	Description	States
$\mathcal{I}_{AT}$	What time until CPA the reference ship considers ample time	Real valued
$\mathcal{I}_{C}$	Whether the reference ship intends to be COLREGS-compliant when performing evasive maneuvers	Binary
$\mathcal{I}_{CS_i}$	What COLREGS situation the reference ship thinks it has towards ship i	“OT_ing”/“OT_en”/“HO”/“CR_PS”/“CR_SS”
$\mathcal{I}_{GS}$	Whether the reference ship acts according to good seamanship	Binary
$\mathcal{I}_{P_i}$	Whether the reference ship acts as if it has a lower or higher priority towards ship i	“higher”/“similar”/“lower”
$\mathcal{I}_{RC}$	What distance at CPA the reference ship considers a risk of collision	Real valued
$\mathcal{I}_{RCF}$	How far in front of a ship the reference ship considers a crossing as risky	Real valued
$\mathcal{I}_{SD}$	What the reference ship considers a safe distance at CPA	Real valued
$\mathcal{I}_{SDF}$	How far in front of a ship the reference ship considers a crossing as safe	Real valued
$\mathcal{I}_{SDM}$	What the reference ship considers a safe distance at CPA to the current midpoint (See 3.2.12).	Real valued
$\mathcal{I}_{SS}$	At what distance the reference ship consider that the situation starts	Real valued
$\mathcal{I}_{U}$	Whether the reference ship acts in an unmodeled way	Binary

Table 3
Measurement variables. The values are evaluated based on the measured position, speed, and course of the different ships in an encounter. Measurements that cannot be directly evaluated based on the position, speed, or course are described when the measurement is first used in Section 3.2.

Symbol	Description	States
$\mathcal{M}_{C}[t]$	Current course of the reference ship	Real valued
$\mathcal{M}_{S}[t]$	Current speed of the reference ship	Real valued
$\mathcal{M}_{CS_i}[t]$	Current COLREGS situation reference ship has towards ship i (See 3.2.15)	“OT_ing”/“OT_en”/“HO”/“CR_PS”/“CR_SS”
$\mathcal{M}_{D_i}[t]$	Current distance between the reference ship and ship i	Real valued
$\mathcal{M}_{DCPA_i}[t]$	Distance between reference ship and ship i at CPA assuming both will keep their current course and speed	Real valued
$\mathcal{M}_{DF_i}[t]$	How far the reference ship crosses in front on ship i assuming both keep their current course and speed. This value is set to ∞ if the ship does not cross in front of ship i	Real valued
$\mathcal{M}_{DM_i}[t]$	Distance at CPA to the current midpoint between the reference ship and ship i, assuming constant course and speed for the reference ship. (See 3.2.12)	Real valued
$\mathcal{M}_{P_i}[t]$	Whether reference ship has passed ship i. (See 3.2.6)	Binary
$\mathcal{M}_{TCPA_i}[t]$	Time until CPA between reference ships and ship i assuming both will keep their current course and speed	Real valued
$\mathcal{M}_{AF_i}[t]$	Whether the reference ship will pass aft or in front of ship i assuming both keep their current course and speed. (See 3.2.13)	“Aft”/“Front”

3.2.2. $C_i[t]$ - Compatible towards ship i

An observation is compatible with the intention states of the reference ship towards ship i if either of the following is true:

- The collision avoidance situation has not started yet (SS_i).
- The ships have passed each other safely (P_i)
- The ships will pass each other in such a manner that it is not a risky situation (RS_i).
- If the reference ship has a give-way role (R_i) and gives way correctly (GWC_i) towards ship i.
- If the reference ship has a stand-on role (R_i) and stands on correctly (SOC_i) towards ship i.

$$\begin{aligned} C_i[t] =& \neg SS_i[t] \vee P_i[t] \vee \neg RS_i[t] \\ & \vee \Big(\big(R_i == \text{“GW”}\big) \wedge GWC_i[t] \Big) \\ & \vee \Big(\big(R_i == \text{“SO”}\big) \wedge SOC_i[t] \Big) \end{aligned} \tag{2}$$

3.2.3. $SS_i[t]$ - Situation started

According to COLREGS Rule 11, the behavioral rules only apply for ships in sight of each other. COLREGS Rule 3 specifies that a ship is in sight if it can be seen visually. At what distance the reference ship sees ship i is unknown and modeled with the intention variable $\mathcal{I}_{SS}$. The situation starts whenever the distance between the ships ($\mathcal{M}_{D_i}$) is shorter than the situation start intention. Map data can be used to evaluate at which distance the ships are likely to see each other.

$$SS_i[t] = SS_i[t-1] \vee \big(\mathcal{M}_{D_i}[t] < \mathcal{I}_{SS}\big) \tag{3}$$

3.2.4. $RS_i[t]$ - Risky situation

If there is a risk of collision (RC_i) at one point of time after the situation starts (SS_i), then the situation should be considered as risky.

$$RS_i[t] = \begin{cases} \text{“false”} & if \neg SS_i[t] \\ RC_i[t] \vee RS_i[t-1] & otherwise \end{cases} \tag{4}$$

3.2.5. $RC_i[t]$ - Risk of collision

Actions to avoid collision are only needed if the reference ship considers that there is a risk of collision (COLREGS Rule 7, 12, and 14). According to COLREGS Rule 7(i), a risk of collision exists if the compass bearing from the reference ship to ship i “does not appreciably change” (IMO, 1972). How much change that is sufficient would depend on the distance between the ships, as one would experience a quicker bearing change once the ships get closer. To simplify this requirement, the expected crossing distance is used to evaluate whether there is a risk of collision. The acceptable distance when crossing in front can be larger than what is acceptable to the side of the ship. This is handled by defining two different intention variables, one specifying how far in front of a ship the reference ship considers it risky to cross ($\mathcal{I}_{RCF}$) and one specifying the distance at CPA that is considered risky ($\mathcal{I}_{RC}$). These are compared to the expected crossing distance in front ($\mathcal{M}_{DF_i}$) and at CPA ($\mathcal{M}_{DCPA_i}$) assuming both ships keep their current course and speed.

$$RC_i[t] = \big(\mathcal{M}_{DCPA_i}[t] < \mathcal{I}_{RC}\big) \vee \big(\mathcal{M}_{DF_i}[t] < \mathcal{I}_{RCF}\big) \tag{5}$$

3.2.6. $P_i[t]$ - Safely passed

If the reference ship has passed ship i ($\mathcal{M}_{P_i}$) and is at a safe distance (SD_i), then the reference ship does not need to consider the ship any longer. A ship is considered as passed if the time until closest point of approach, assuming constant course and speed for all ships, is negative.

$$P_i[t] = \mathcal{M}_{P_i}[t] \wedge SD_i[t] \tag{6}$$

Table 4
Model variables.

Symbol	Description	States
$C[t]$	Observation compatible with the intentions of the reference ship	Binary
$C_i[t]$	Observations and intentions compatible towards ship i	Binary
$CEM_i[t]$	Correct evasive maneuver towards ship i	Binary
$C\_CR\_SS_i[t]$	Correct crossing evasive maneuver with ship i on the starboard side	Binary
$C\_CR\_PS_i[t]$	Correct crossing evasive maneuver with ship i on the port side	Binary
$C\_HO_i[t]$	Correct head-on evasive maneuver towards ship i	Binary
$C\_OT_i[t]$	Correct overtaking evasive maneuver towards ship i	Binary
$CIC_i[t]$	Change in course towards ship i	“starboard”/“straight”/“port”
$CIS_i[t]$	Change in speed towards ship i	“higher”/“none”/“lower”
$GS_i[t]$	Good seamanship towards ship i	Binary
$GWC_i[t]$	Gives way correctly towards ship i	Binary
$IC_i[t]$	Initial course when the situation started towards ship i. Course is given in the NED frame.	Real valued
$IS_i[t]$	Initial speed when the situation started towards ship i	Real valued
$P_i[t]$	Has passed ship i safely	Binary
$PA_i[t]$	There has been a port action towards ship i	Binary
R_i	Role towards ship i	“GW”/“SO”
$RC_i[t]$	There is currently a risk of collision with ship i	Binary
$RS_i[t]$	It is a risky situation towards ship i	Binary
$SOC_i[t]$	Stands on correctly towards ship i	Binary
$SD_i[t]$	The reference ship will cross at a safe distance towards ship i	Binary
$SS_i[t]$	Situation has started towards ship i	Binary
$SA_i[t]$	There has been a starboard action towards ship i	Binary
$WGW_i[t]$	Will give way towards ship i	binary

Table 5
Example parameters chosen for demonstrative purposes. The parameters can be modified based on properties of the current situation, such as ship size, speed, and weather. The minimal acceptable definition of ample time (AT_{min}), safe distance at CPA (SD_{min}), safe distance front (SDF_{min}) and safe distance to the current midpoint (SDM_{min}), should be based on the maneuverability of the own-ship and how risk averse the operation should be.

Symbol	Description	Value
$\mathcal{P}_{CIC}$	Max change in course that is considered as keeping the course	10°
$\mathcal{P}_{CIS}$	Max change in speed that is considered as keeping the speed	$2\,\mathrm{m\,s^{-1}}$
AT_{min}	Ownships minimal accepted definition of ample time	60 s
SD_{min}	Ownships minimal accepted definition of safe distance at CPA	75 m
SDF_{min}	Ownships Minimal accepted definition of safe distance to cross in front	100 m
SDM_{min}	Ownships minimal accepted definition of safe distance to midpoint	75 m

3.2.7. $SOC_i[t]$ - *Stands on correctly*

The reference ship stands on correctly towards ship i if it does not change its course (CIC_i) or speed (CIS_i), or if it does a correct evasive maneuver (CEM_j) towards another ship (j) it has a give-way role (R_j) for (Rule 17).

$$\begin{aligned} SOC_i[t] = &\Big(\big(CIC_i[t] == \text{“straight”}\big) \wedge \big(CIS_i[t] == \text{“none”}\big)\Big) \\ &\vee_{j=1}^{n} \Big(\big(R_j == \text{“GW”}\big) \wedge CEM_j[t]\Big) \end{aligned} \tag{7}$$

3.2.8. $GWC_i[t]$ - *Gives way correctly*

The reference ship gives way correctly towards ship i if it is executing a correct evasive maneuver CEM_i. According to COLREGS Rule 8, the ship must take evasive actions in what it considers “ample time” ($\mathcal{I}_{AT}$). The “time” in ample time is measured as the time until CPA assuming both ships keep their current course and speed ($\mathcal{M}_{TCPA_i}$). How long before CPA the reference ship consider as “ample time” is modeled with the intention variable $\mathcal{I}_{AT}$. The ship is allowed to stand on correct (SOC_i) before what it considers “ample time”.

$$GWC_i[t] = CEM_i[t] \vee \Big(\big(\mathcal{M}_{TCPA_i}[t] > \mathcal{I}_{AT}\big) \wedge SOC_i[t]\Big) \tag{8}$$

3.2.9. $CEM_i[t]$ - *Correct evasive maneuver*

For an evasive maneuver to be correct, it must comply with “good seamanship” (GS_i) (COLREGS Rule 8) if the reference ship has an intention to act with “good seamanship” ($\mathcal{I}_{GS}$). Additionally, the maneuver must fulfill the requirements specified by COLREGS if the reference ship has an intention to be COLREGS-compliant when performing evasive maneuvers ($\mathcal{I}_C$). COLREGS specify a set of situations and how to act in each scenario. These consist of overtaking (OT_ing, Rule 13) another vessel, being overtaken (OT_en, Rule 17), head-on (HO, Rule 14), crossing with the other ship on the starboard side (CR_SS, Rule 15), and crossing with the other ship on the port side (CR_PS, Rule 17). What COLREGS situation the reference ship believes it has towards ship i is denoted as $\mathcal{I}_{CS_i}$.

$$\begin{aligned} CEM_i[t] = &\big(\neg\mathcal{I}_{GS} \vee GS_i[t]\big) \wedge \Bigg(\neg\mathcal{I}_C \vee \Big(\big((\mathcal{I}_{CS_i} == \text{“OT\_ing”}) \\ &\quad \vee (\mathcal{I}_{CS_i} == \text{“OT\_en”})\big) \wedge C\_OT_i[t]\Big) \\ &\quad \vee \Big(\big(\mathcal{I}_{CS_i} == \text{“HO”}\big) \wedge C\_HO_i[t]\Big) \\ &\quad \vee \Big(\big(\mathcal{I}_{CS_i} == \text{“CR\_SS”}\big) \wedge C\_CR\_SS_i[t]\Big) \\ &\quad \vee \Big(\big(\mathcal{I}_{CS_i} == \text{“CR\_PS”}\big) \wedge C\_CR\_PS_i[t]\Big)\Bigg) \end{aligned} \tag{9}$$

3.2.10. $SD_i[t]$ - *Safe distance*

According to COLREGS Rule 8, actions to avoid collision shall result in the ships passing at a safe distance. Whether the reference ship and ship i will pass at a safe distance is evaluated by assuming that both ships will keep their current course and speed. This assumption holds for ship i if it has a stand-on role, as stand-on ships are required to keep their course and speed (COLREGS Rule 17). If the reference ship has a give-way role, then it is expected to mark its intent by substantially changing its course or speed (COLREGS Rule 8) before returning to the initial course. Assuming that it will keep its course and speed should result in passing at a safe distance if the ship has started to act to avoid a collision. As with risk of collision ($RC_i[t]$), different intention and measurement nodes are included for a safe crossing distance in front ($\mathcal{M}_{DF_i}$, $\mathcal{I}_{SDF}$) and at CPA ($\mathcal{M}_{DCPA_i}$, $\mathcal{I}_{SD}$).

$$SD_i[t] = \big(\mathcal{M}_{DCPA_i}[t] > \mathcal{I}_{SD}\big) \wedge \big(\mathcal{M}_{DF_i}[t] > \mathcal{I}_{SDF}\big) \tag{10}$$

3.2.11. $C\_OT_i[t]$ - *Correct overtaking evasive maneuver*

COLREGS Rule 13 specifies that the overtaking vessel shall keep out of the way of the vessel being overtaken. Checking that the ships are crossing at a safe distance (SD_i) is therefore sufficient.

$$C\_OT_i[t] = SD_i[t] \tag{11}$$

3.2.12. $C\_HO_i[t]$ - *Correct head-on evasive maneuver*

For head-on situations, COLREGS Rule 14 specifies that the ships must make a starboard turn such that they pass each other port to port. As both ships have to give way in this situation, assuming that ship i will keep its current course is unrealistic. Instead, a new measurement is used that considers the distance at CPA to the current midpoint between the ships ($\mathcal{M}_{DM_i}$). As the current midpoint does not change when the ships courses change, considering a safe distance to the current midpoint thereby requires that the reference ship has to do an evasive maneuver even though ship i has already changed its course. The distance at CPA to the current midpoint is evaluated assuming the reference ship will keep its current course and speed. The distance to the midpoint is set to 0 if the ship passes with the midpoint on the starboard side. This ensures that the ship has to pass on the correct side. Which distance to the midpoint the reference ship considers as safe is denoted as $\mathcal{I}_{SDM}$.

$$C\_HO_i[t] = \left(\mathcal{M}_{DM_i}[t] > \mathcal{I}_{SDM}\right) \tag{12}$$

3.2.13. $C\_CR\_SS_i[t]$ - *Correct crossing starboard-side evasive maneuver*

In a crossing situation, Rule 15 of COLREGS specifies that a ship should, in addition to cross at a safe distance (SD_i), avoid crossing in front of another ship it has on its starboard side. Whether the reference ship crosses aft or front of ship i ($\mathcal{M}_{AF_i}$) is evaluated by first finding the intersection point of the paths followed by the ships assuming that they keep their current course. Which ship that first arrives at this point crosses in front of the other.

$$C\_CR\_SS_i[t] = \left(\mathcal{M}_{AF_i}[t] == \text{“aft”}\right) \wedge SD_i[t] \tag{13}$$

3.2.14. $C\_CR\_PS_i[t]$ - *Correct crossing port-side evasive maneuver*

If a ship with the other on its port side is forced to take action, then COLREGS Rule 17(c) specifies that it, in addition to cross at a safe distance (SD_i), should avoid changing its course (CIC_i) towards port.

$$C\_CR\_PS_i[t] = \left(CIC_i[t] \neq \text{“port”}\right) \wedge SD_i[t] \tag{14}$$

3.2.15. $\mathcal{M}_{CS_i}[t]$ - *COLREGS situation*

According to COLREGS Rule 13(b), a ship is overtaking another when it is coming up on the ship “from a direction more than 22.5 degrees abaft her beam” (IMO, 1972). Uncertainty in the heading of the other ship can lead to different interpretations of the situation. Uncertainty in whether it is an overtaking situation is modeled by using the classifier as shown in Fig. 3. The size of the uncertainty region can be based on a combination of historical data and expert opinion. Different situations could be presented to different experienced captains where they could express their trust that other ships would identify this situation correctly. The values used in this article are chosen for demonstrative purposes.

A head-on situation is defined by COLREGS Rule 14(a) to be when two vessels are meeting on “nearly reciprocal courses”, while Rule 14 (b) specifies when a head-on situation exists based on the visibility of different lights of the other ship. This opens up for disagreements from different definitions of “nearly reciprocal” and how the ships observe each other. With the presence of current and winds, a ship observing the course of the other by radar or AIS might come to a different conclusion than one observing the heading of the other ship based on the visibility of lights (Woerner et al., 2019). Furthermore, measurement uncertainties in the course of the other ship can lead to misunderstandings. The classifier shown in Fig. 4 is used to accommodate this uncertainty. Identifying the uncertainty and mean of which angle a head-on situation starts can be evaluated similarly to the overtaking case. In addition, the mean can be chosen based on case law and certifying agency requirements as proposed in Woerner et al. (2019).

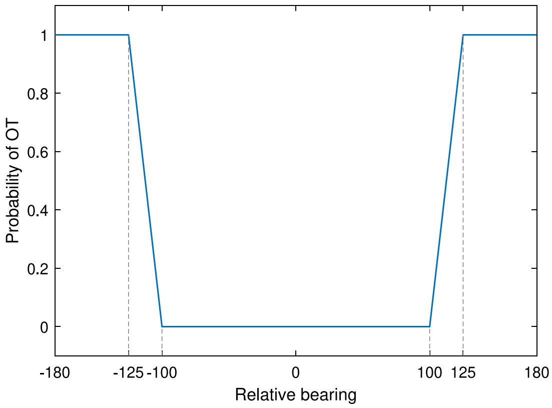


Fig. 3. Classifier giving the probability that it is an overtaking situation. Relative bearing is defined as the bearing from the ship being overtaken to the overtaking ship relative to the heading of the ship being overtaken. 22.5° abaft the beam as specified in COLREGS Rule 13 is the same as ±112.5° relative to the heading. This classifier considers a 15° uncertainty in the situation.

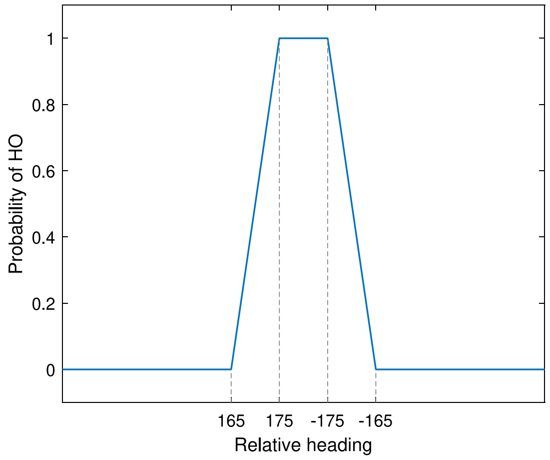


Fig. 4. Classifier giving the probability that it is a head-on situation. The relative heading between the two ships defined the probability. This classifier considers a 10° uncertainty in the situation.

The probability that the reference ship evaluates the current situation as an overtaking or head-on situation is based on the two classifiers given in Figs. 3 and 4. The remaining probability gives the probability that the reference ship evaluates the situation to be a crossing situation. Whether the reference ship is in front or back of the other ship when the situation starts defines whether it is overtaking (“OT_ing) or being overtaken (“OT_en”). Whether the other ship is on the port or starboard side defines whether it is a crossing port side (“CR_PS”) or crossing starboard side (“CR_SS”) situation. This information is inserted as virtual evidence on the measured COLREGS situation node, $\mathcal{M}_{CS_i}$.

According to COLREGS Rule 13(d), subsequent alterations in bearing do not change the situation. The situation is therefore defined when the situation starts, which can lead to misunderstandings as the different ships may define that the situation starts at different time points (Clawson, 2013). To model the uncertainty caused by when the reference ship thinks that the situation starts, a situation measurement node ($\mathcal{M}_{CS_i}$) is introduced. The state of this node is equal to the state of the situation intention node ($\mathcal{I}_{CS_i}$) only at the time-step where the reference ship thinks that the situation starts. At all other time-steps, the probability of measuring the different states of the measurement node is unaffected by the state of the intention node. Which time-step the reference ship thinks that the situation starts is uncertain, making it uncertain which measurement that defines the intention state. There should be an equal probability of measuring all states when the measurement node is independent of the intention node.

$$\mathcal{M}_{CS_i}[t] = \begin{cases} \mathcal{I}_{CS_i} & if\, SS_i[t] \wedge \neg SS_i[t-1] \\ [0.2, 0.2, 0.2, 0.2, 0.2] & otherwise \end{cases} \tag{15}$$

3.2.16. R_i - Role

A ship must give way if it has lower priority ($\mathcal{I}_{P_i}$), either as specified in COLREGS Rule 18 or due to unwritten rules (Chauvin and Lardjane, 2008). If the ship has higher priority, it must stand on. If the priority is similar, then the role is given by Rule 13 to 15. In a head-on situation, both ships must give way (Rule 14). In an overtaking situation, the overtaking vessel must give way (Rule 13). In a crossing situation, the one with the other ship on its starboard side must give way (Rule 15).

$$R_i = \begin{cases} & if\, \left(\mathcal{I}_{P_i} == \text{“lower”}\right) \vee \Big(\left(\mathcal{I}_{P_i} == \text{“similar”}\right) \\ \text{“GW”} & \quad \wedge \big(\left(\mathcal{I}_{CS_i} == \text{“HO”}\right) \vee \left(\mathcal{I}_{CS_i} == \text{“CR\_SS”}\right) \\ & \quad \vee \left(\mathcal{I}_{CS_i} == \text{“OT\_ing”}\right) \big) \Big) \\ \text{“SO”} & otherwise \end{cases} \tag{16}$$

3.2.17. $GS_i[t]$ - Good seamanship

Good seamanship is difficult to define and can contain many different behaviors. In this article, good seamanship restricts the ship from changing which side it turns towards to avoid collision. The ship is not allowed to have made both a starboard action (SA) and a port action (PA) during a collision encounter.

$$GS_i[t] = \neg\left(SA_i[t] \wedge PA_i[t]\right) \tag{17}$$

$$SA_i[t] = \begin{cases} \text{“false”} & if \neg SS_i[t] \\ \left(CIC_i[t] == \text{“starboard”}\right) \vee SA_i[t-1] & otherwise \end{cases} \tag{18}$$

$$PA_i[t] = \begin{cases} \text{“false”} & if \neg SS_i[t] \\ \left(CIC_i[t] == \text{“port”}\right) \vee PA_i[t-1] & otherwise \end{cases} \tag{19}$$

3.2.18. $CIC_i[t]$ - Change in course

A change in course is evaluated by comparing the initial course (IC_i) with the measured course ($\mathcal{M}_C$). The initial course is saved when the situation starts (SS_i). If the change in course is less than $\mathcal{P}_{CIC}$ then it is considered as keeping the course. $\mathcal{P}_{CIC}$ should be chosen small enough to ensure that all intended course changes are marked as such, while being large enough to ensure that measurement uncertainty and small oscillations due to waves are not marked as a course change.

$$IC_i[t] = \begin{cases} \mathcal{M}_C[t] & if \neg SS_i[t] \\ IC_i[t-1] & otherwise \end{cases} \tag{20}$$

$$CIC_i[t] = \begin{cases} \text{“starboard”} & if\, \mathcal{M}_C[t] > \left(IC_i[t] + \mathcal{P}_{CIC}\right) \\ \text{“port”} & if\, \mathcal{M}_C[t] < \left(IC_i[t] - \mathcal{P}_{CIC}\right) \\ \text{“straight”} & otherwise \end{cases} \tag{21}$$

3.2.19. $CIS_i[t]$ - Change in speed

The initial speed (IS_i) and change in speed are evaluated in the same manner as for the course. The same considerations should be made when choosing $\mathcal{P}_{CIS}$.

$$IS_i[t] = \begin{cases} \mathcal{M}_S[t] & if \neg SS_i[t] \\ IS_i[t-1] & otherwise \end{cases} \tag{22}$$

$$CIS_i[t] = \begin{cases} \text{“higher”} & if\, \mathcal{M}_S[t] > \left(IS_i[t] + \mathcal{P}_{CIS}\right) \\ \text{“lower”} & if\, \mathcal{M}_S[t] < \left(IS_i[t] - \mathcal{P}_{CIS}\right) \\ \text{“none”} & otherwise \end{cases} \tag{23}$$

3.3. Translation into DBN

A DBN is made from the logic statements given in Section 3.2. A node is introduced for each intention variable, measurement variable, and model variable. Arcs are introduced based on the dependencies given by the equations in Section 3.2. The resulting topology can be seen in Fig. 5.

The logical statements given in Section 3.2 need to be translated into CPTs to be used by the DBN. This can be done by evaluating whether the output is “true” or “false” for all combinations of inputs. This results in CPTs consisting of 0/1 probabilities. Nodes that according to Tables 2, 3, and 4 are real-valued must be discretized. A suitable range and discretization step must be defined. The software GeNIe (Bayesfusion LLC, 2021b) allows the user to specify equations and to use real-valued nodes. It can then automatically discretize and translate these equations into CPTs.

3.4. Priors

Information from the current situation, such as ship types and the type of environment, can improve the prior distributions of the intention nodes. Examples of different factors that could be considered are shown in Table 6. These influencing factors can be included as time-independent nodes that affect the intention states.

Different approaches can be followed to identify factors that affect the intentions. One way is to have a workshop with experts in the field, such as experienced captains. This workshop can be similar to risk analysis workshops such as Hegde et al. (2018) and Rokseth et al. (2017). Another option is to study captains during operation as done in Chauvin and Lardjane (2008). This has the advantage of being more correct than a workshop, but some factors might not show up during the study. A last option is to analyze historical data logged with the automatic identification system (AIS) that larger vessels are required to be equipped with IMO (2021). This method would be more general as much more data from different ships and situations could be analyzed. It will, however, be limited to the information that is logged with the AIS, which does not necessarily include all factors that could be of interest. A combination of the three approaches is preferable to maximize correctness and completeness.

The same methods can be used for quantifying how the intention nodes are affected by the identified factors. AIS data could be used to build prior distributions on, among others, how far before CPA different types of ships tend to give way and how close they tend to be at CPA. This information could be supplemented with data from operation studies and expert judgment to model how factors not included in the AIS affect the distribution. Different methods for building CPTs based on expert information are analyzed in Mkrtchyan et al. (2016).

Performing a thorough identification and quantification is outside the scope of this article. Table 7 shows the quantification used to produce the results presented in Section 4.

3.5. Using the intentions

This section presents two different ways of using the evaluated intention probabilities for collision avoidance.

3.5.1. Decision criteria

The first approach considers whether the own-ship should consider the reference-ship in the collision avoidance algorithm. Collision avoidance algorithms similar to Eriksen et al. (2020) do not need to consider the reference ship if the own-ship has a stand-on role, and the reference ship is planning to give way. A new node can be introduced into the network to evaluate whether the reference ship is planning to give way or not. A threshold can be proposed that defines how likely it must be that the reference ship will give way for it to be safely ignored by the collision avoidance algorithm.

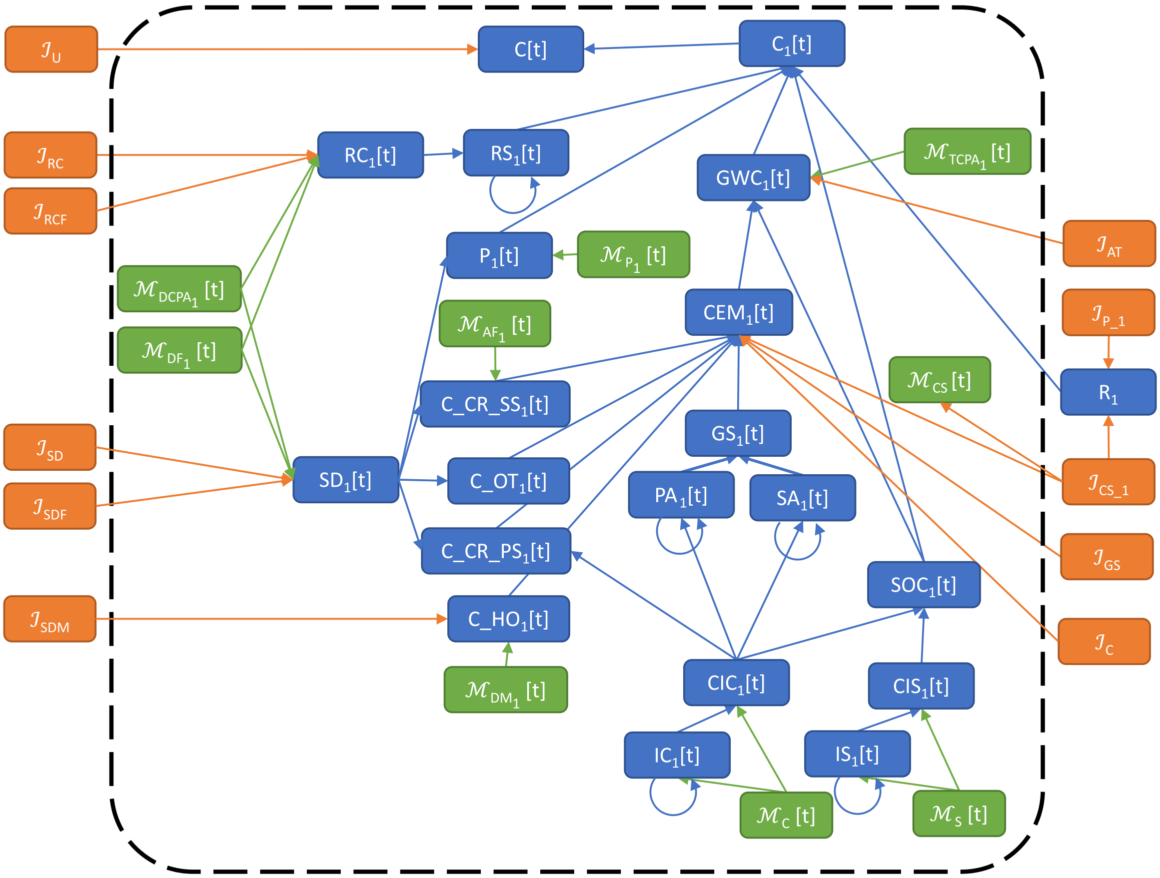


Fig. 5. Figure showing the topography of the resulting DBN for a single ship encounter. Nodes related to situation start ($\mathcal{I}_{SS}$, $SS_i[t]$) are omitted to reduce complexity. See Tables 2, 3, and 4 for abbreviations. Subscript 1 indicates that this model considers the relation between the reference ship and ship with index 1. In a multi-ship encounter, all nodes with index subscript would be repeated for any additional ship in the encounter. Green nodes represent measurements, orange node intentions, and blue nodes model variables. All nodes inside the box are time-dependent and are repeated for each time step. Circular arrows indicate connections between subsequent time steps.

Table 6
Factors that can influence the intentions of the reference ship. Table 7 specifies and quantifies the dependencies.

Factor	Reason	States
Maneuver-ability	A poor maneuverability requires earlier actions and larger margins	Low/medium/high
Location	Ships tend to act earlier and have larger margin in open seas than inland waterways	Open sea/innland
Ship type	A leisure craft is less likely to know and follow rules and best practice	Commercial/leisure
Relative ship size	Larger ships tends to have priority over smaller ships (Chauvin and Lardjane, 2008)	Smaller/similar/larger
Speed	Ships require larger safety margins when going at a fast speed	Slow/fast

The node representing whether the reference ship is planning to give way depends on whether the reference ship has a give-way role (R_i), considers it a risky situation (RS_i), and if its definitions of ample time ($\mathcal{I}_{AT}$), safe-distance at CPA ($\mathcal{I}_{SD}$), safe distance when crossing in front ($\mathcal{I}_{SDF}$), and safe distance to the current midpoint ($\mathcal{I}_{SDM}$) are acceptable. Additionally, the reference ship is assumed not to give way if it acts in an unmodeled manner ($\mathcal{I}_U$). Eq. (24) shows the logic statement that defines whether the ship will give way towards ship i (WGW_i).

$$\begin{aligned} WGW_i[t] = & \left(R_i == \text{“GW”}\right) \wedge \left(\mathcal{I}_{AT} > AT_{min}\right) \\ & \wedge \left(\mathcal{I}_{SD} > SD_{min}\right) \wedge \left(\mathcal{I}_{SDF} > SDF_{min}\right) \\ & \wedge \left(\mathcal{I}_{SDM} > SDM_{min}\right) \wedge RS_i[t] \wedge \neg \mathcal{I}_U \end{aligned} \tag{24}$$

3.5.2. *Candidate trajectories*

The second approach evaluates whether a candidate trajectory for the reference ship is compatible with the estimated intentions. Measurements can be evaluated based on the candidate trajectory and inserted into the network. The network can then be used to evaluate the probability that this trajectory is compatible with the reference ship's intentions ($C[t]$). These candidate trajectories with corresponding probability can be used as scenarios in scenario-based collision avoidance algorithms similar to Tengesdal et al. (2020)

Minor alterations are needed to evaluate the measurements based on trajectories. All measurements that consider that the reference ship is keeping its course and speed are instead evaluated using the candidate trajectory of the reference ship while only assuming that all other ships in the encounter will keep their course and speed. The current course ($\mathcal{M}_C$) and speed ($\mathcal{M}_S$) must be evaluated a bit into the candidate trajectory so that the ship has time to execute the potential evasive action. If the situation has not started, then a trajectory keeping the course and speed will be wrongly given a high probability. This is avoided by setting the current distance ($\mathcal{M}_{D_i}[t]$) to zero. The time until CPA ($\mathcal{M}_{TCPA_i}$) is not relevant for the candidate trajectories as the entire future motion of the ship is considered as known. Instead, this measurement is set to the minimum acceptable time (AT_{min}). An intention to give way at a shorter time than acceptable will evaluate a high probability for trajectories that keep the course and speed. This makes the collision avoidance algorithm take evasive actions if it is likely that the reference ship will give way at an unacceptable short

Table 7
Prior probability distribution used in the simulation study for the different intention states as a function of the influencing factors. To keep the list short, factors are only included that were of relevance to the scenarios presented in Section 4. States marked in bold are used unless otherwise specified. $\mathcal{N}(\mu,\sigma)$ indicates a truncated normal distribution with expected value μ, standard deviation σ, and limited to be larger than 0. For binary states the probability of "true" is given. Discrete states are given in the order specified in Table 2.

Intention	Influencing factor	Prior distribution
$\mathcal{I}_{AT}$	Maneuverability: low **Maneuverability: medium**	$\mathcal{N}(480\,\mathrm{s}, 80\,\mathrm{s})$ $\mathcal{N}(360\,\mathrm{s}, 75\,\mathrm{s})$
$\mathcal{I}_{C}$	Ship type: commercial	0.99
$\mathcal{I}_{CS_i}$	None	[0.2, 0.2, 0.2, 0.2, 0.2]
$\mathcal{I}_{GS}$	Ship type: commercial	0.995
$\mathcal{I}_{P_i}$	**Relative ship size: similar** Relative ship size: larger	[0.05, 0.90, 0.05] [0.01, 0.59, 0.4]
$\mathcal{I}_{RC}$	Maneuverability: medium, Location: open sea	$\mathcal{N}(1\,\mathrm{km}, 175\,\mathrm{m})$
$\mathcal{I}_{RCF}$	Maneuverability: medium, Location: open sea	$\mathcal{N}(1.5\,\mathrm{km}, 250\,\mathrm{m})$
$\mathcal{I}_{SD}$	**Maneuverability: medium,** **Location: open sea,** **Speed: slow**	$\mathcal{N}(300\,\mathrm{m}, 75\,\mathrm{m})$
	Maneuverability: low, Location: open sea, Speed: slow	$\mathcal{N}(700\,\mathrm{m}, 100\,\mathrm{m})$
$\mathcal{I}_{SDF}$	Maneuverability: medium, Location: open sea, Speed: slow	$\mathcal{N}(500\,\mathrm{m}, 120\,\mathrm{m})$
$\mathcal{I}_{SDM}$	Maneuverability: medium, Location: open sea, Speed: slow	$\mathcal{N}(300\,\mathrm{m}, 75\,\mathrm{m})$
$\mathcal{I}_{SS}$	Maneuverability: medium, Location: open sea	$\mathcal{N}(7\,\mathrm{km}, 1.7\,\mathrm{km})$
$\mathcal{I}_{U}$	None	0.9999

time before CPA. The rest of the measurements can be evaluated as usual.

There are many different ways of generating candidate trajectories. This article generates trajectories based on line-of-sight guidance, as proposed in Johansen et al. (2016). These trajectories are generated by simulating a simple ship model that uses a line-of-sight guidance rule to evaluate a reference course that gradually converges towards the nominal path (Fossen, 2011). The nominal path is assumed to go in a straight line going through the position where the ship was first observed, pointing in the same direction as the ship's course at this point. Adding different constant offsets to the reference course generates different trajectories that quickly move away from and then align parallel to the original course. Figs. 6 to 14 shows the resulting trajectories with a constant offset in speed or course. All the trajectories assume that evasive actions are done at the current time-step and not at future time-steps. This assumption can be acceptable for collision avoidance, as it is enough to know if the other ship will give way in time and to what side it will give way.

4. Results

This section presents different simulation scenarios that demonstrate the capabilities of the intention model. Scenario 1 to Scenario 7 go into detail on specific situations to highlight how the intention model works. The specific scenarios focus on trajectories the ships can take in the future. The probabilities of different candidate trajectories being compatible with the reference ship's intentions ($C[t]$) are presented. Note that probabilities for all trajectory candidates do not need to sum to 1 as there can be multiple trajectory candidates that are compatible with the intentions of the reference ship.

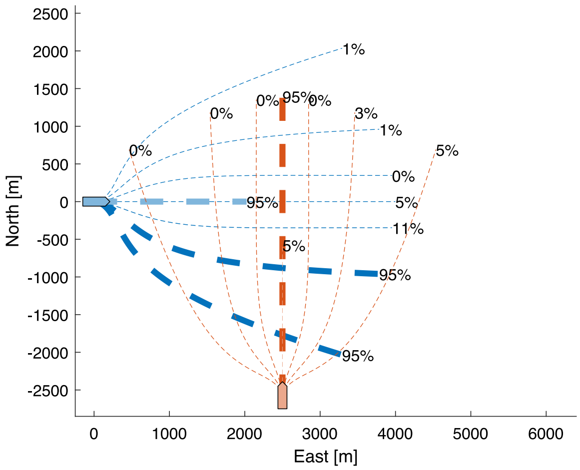


Fig. 6. Scenario 1. Two ships are meeting on a collision course in a clear crossing situation. The figure shows the different candidate trajectories (dashed lines). The probability at the end of each trajectory and the thickness of the line shows the probability that the trajectory is compatible with the ship's intentions ($C[t]$). Trajectories with reduced speed are shown with a lighter color. The ship symbols are scaled for visualization purposes and do not represent the true ship size. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

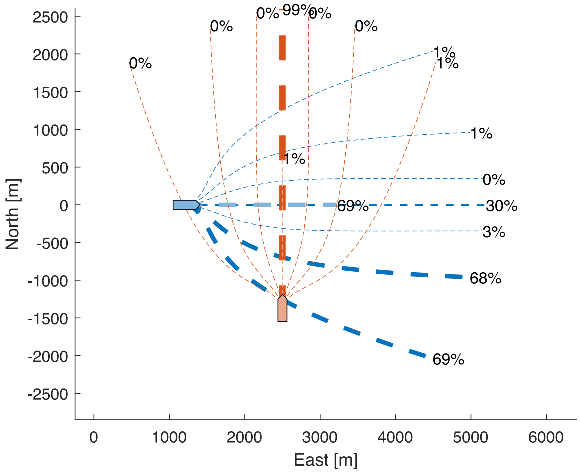


Fig. 7. Scenario 1. Shows the same encounter as Fig. 6 at a later time-point. The figure shows the different candidate trajectories (dashed lines). The probability at the end of each trajectory and the thickness of the line shows the probability that the trajectory is compatible with the ship's intentions ($C[t]$). Trajectories with reduced speed are shown with a lighter color. The ship symbols are scaled for visualization purposes and do not represent the true ship size. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Scenario 8 to Scenario 11 show sets of many different similar situations to demonstrate the sensitivity of the intention model to changes in the situation. In these scenarios the focus is on the underlying intentions and whether the ship will give way (WGW_i). How these states develop as the ships approach each other is shown.

The DBN is in each scenario evaluated using the SMILE (Bayesfusion LLC, 2021a) library for C++. A separate instance of the model is run for all ships in the encounter.

Scenario 1 - gradual inference

This scenario demonstrates an ability to identify the intentions based on observations. Fig. 6 shows two ships meeting on a collision course. The situation is a clear crossing situation where, according to

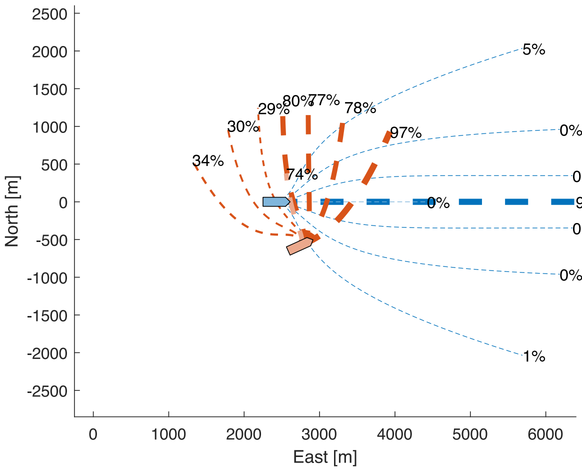


Fig. 8. Scenario 1. Shows the same encounter as Figs. 6 and 7 at a later time-point. The red ship has changed its course 45° to starboard and halved its speed. The figure shows the different candidate trajectories (dashed lines). The probability at the end of each trajectory and the thickness of the line shows the probability that the trajectory is compatible with the ship's intentions ($C[t]$). Trajectories with reduced speed are shown with a lighter color. The ship symbols are scaled for visualization purposes and do not represent the true ship size. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

COLREGS Rule 15, the blue ship is responsible for giving way while the red should stand on. The model evaluates a 93% chance that the blue ship will give way (WGW) and a 6% chance that the red ship will give way. The blue ship can give way either by reducing its speed or making a starboard turn.

Fig. 7 shows the same situation at a later time-point. As the blue ship has not yet done any action to avoid collision, it becomes more likely that it believes it has a higher priority making it not give way at all. The model, therefore, evaluates a 68% chance that the blue ship will give way. As the red ship has not changed its course or speed, it becomes less likely that it thinks it has lower priority, which results in a 1% chance that it will give way.

When the red ship starts to make an evasive maneuver, as shown in Fig. 8, it becomes more likely that the red ship acts to avoid collision. Note that the candidate trajectories are generated relative to the nominal path of the ship, which is assumed to continue northwards. As the time until CPA is very short, it is unlikely that the red ship has such a short definition of ample time. The model, therefore, evaluates a 32% chance that the red ship acts in an unmodeled manner. The probability that the red ship will give way is evaluated to be 29%.

Scenario 2 - COLREGS incompliant action.

This scenario demonstrates the modeling of incompliant behavior. Fig. 9 shows two ships meeting on a collision course where the blue ship has turned its course to port to cross in front of the red ship. The model predicts that the blue ship will continue to cross in front even though this is not compliant with COLREGS.

Scenarios 3 - uncertain COLREGS situation

This scenario demonstrates how uncertainty in the COLREGS situation affects the model. Fig. 10 shows a scenario where the red ship is approaching the blue ship from an angle that is close to the border between an overtaking and a crossing situation. The situation metric evaluates a 54% chance of it being an overtaking situation, in which case the red ship should give way to either side. The remaining 46% is evaluated as a crossing situation, in which case the blue ship should give way behind the red ship. This results in a substantial probability

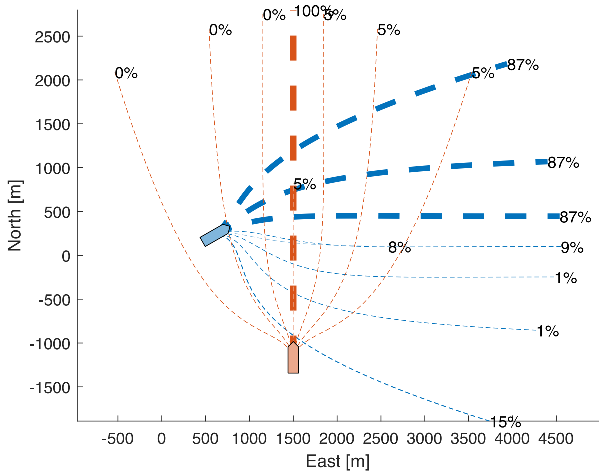


Fig. 9. Scenario 2. The ships approached in the same manner as shown in Fig. 6. The blue ship performed a COLREGS incompliant maneuver by changing course to port to avoid collision. The figure shows the different candidate trajectories (dashed lines). The probability at the end of each trajectory and the thickness of the line shows the probability that the trajectory is compatible with the ship's intentions ($C[t]$). Trajectories with reduced speed are shown with a lighter color. The ship symbols are scaled for visualization purposes and do not represent the true ship size. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

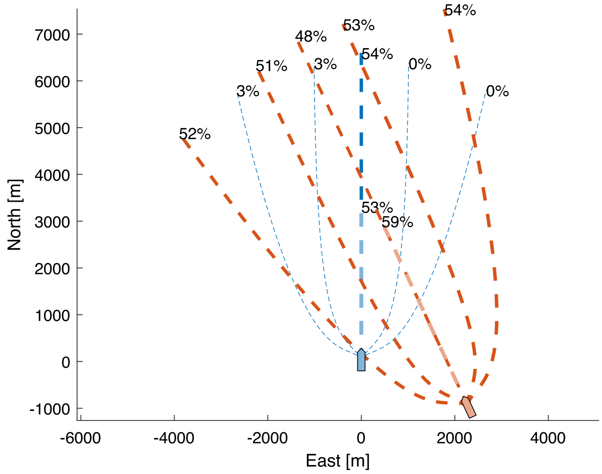


Fig. 10. Scenario 3. The red ship is approaching the blue ship with a higher speed and a relative bearing of 113 degrees relative to the heading of the blue ship. The bearing is close to the limit between overtaking and crossing, which can cause uncertainty. The figure shows the different candidate trajectories (dashed lines). The probability at the end of each trajectory and the thickness of the line shows the probability that the trajectory is compatible with the ship's intentions ($C[t]$). Trajectories with reduced speed are shown with a lighter color. The ship symbols are scaled for visualization purposes and do not represent the true ship size. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

for both keeping the course and speed and taking evasive actions. For the blue ship, none of the candidate trajectories where course alone was changed made the blue ship cross behind the red ship at a safe distance. The only option among the candidate trajectories that gave way behind the red ship was for the blue ship to reduce its speed.

Scenario 4 - risk of collision

This scenario demonstrates uncertainties that arise from whether there is a risk of collision (RC_i). Fig. 11 shows two ships meeting in a head-on situation. The model evaluates a 61% chance that there is a risk of collision, and a 86% chance that either ship will give way. If

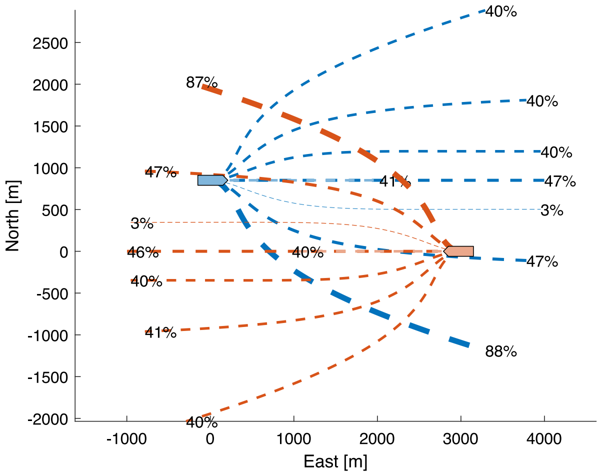


Fig. 11. Scenario 4. Two ships are approaching in a head-on situation where it is uncertain whether there is a risk of collision (RC_i). The figure shows the different candidate trajectories (dashed lines). The probability at the end of each trajectory and the thickness of the line shows the probability that the trajectory is compatible with the ship's intentions ($C[t]$). Trajectories with reduced speed are shown with a lighter color. The ship symbols are scaled for visualization purposes and do not represent the true ship size. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

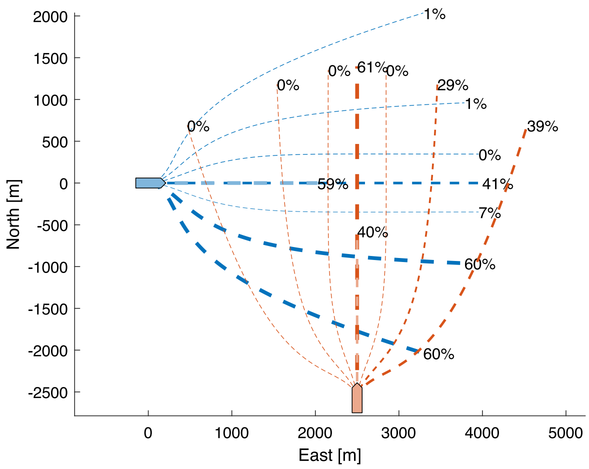


Fig. 12. Scenario 5. Same situation as Scenario 1. Information that the blue ship is substantially larger than the red ship is inserted as prior information. The figure shows the different candidate trajectories (dashed lines). The probability at the end of each trajectory and the thickness of the line shows the probability that the trajectory is compatible with the ship's intentions ($C[t]$). Trajectories with reduced speed are shown with a lighter color. The ship symbols are scaled for visualization purposes and do not represent the true ship size. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

there is no risk of collision, then all actions that keep the ships at a risk-free distance are acceptable. Either way, making a large starboard turn is acceptable as it results in crossing as specified in COLREGS Rule 14.

Scenarios 5 and 6 - effect of priors

These scenarios demonstrate how utilizing prior information to modify the prior probability distributions affects the model. Fig. 12 shows the same scenario as Scenario 1 but utilizes information that the blue ship is significantly larger than the red ship. The model, therefore, evaluates a substantially larger probability that the blue ship has priority over the red, which results in a 58% chance that the blue ship will give way and a 40% chance that the red ship will give way.

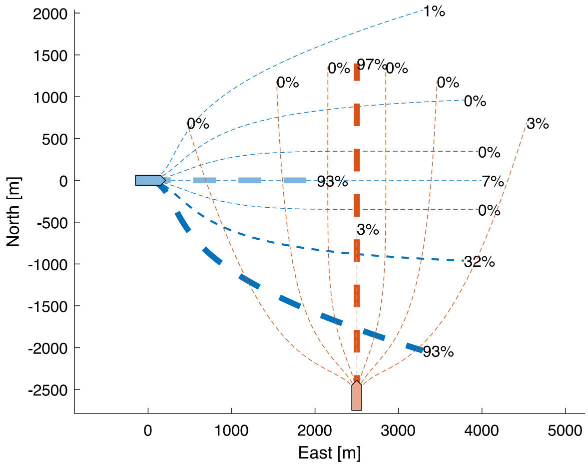


Fig. 13. Scenario 6. Same situation as Scenario 1. Information that both ships have a low maneuverability is inserted as prior information. The figure shows the different candidate trajectories (dashed lines). The probability at the end of each trajectory and the thickness of the line shows the probability that the trajectory is compatible with the ship's intentions ($C[t]$). Trajectories with reduced speed are shown with a lighter color. The ship symbols are scaled for visualization purposes and do not represent the true ship size. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

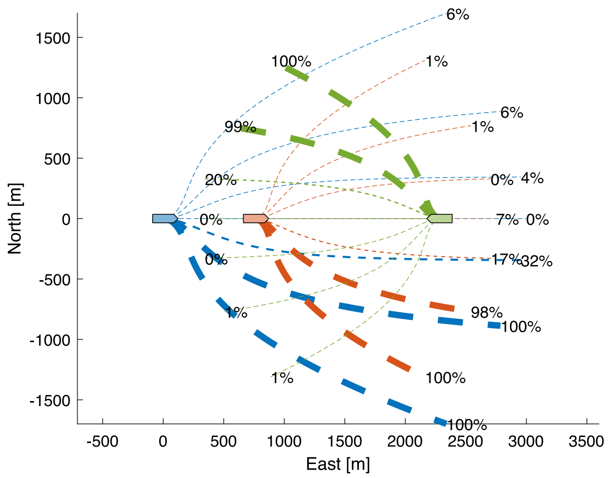
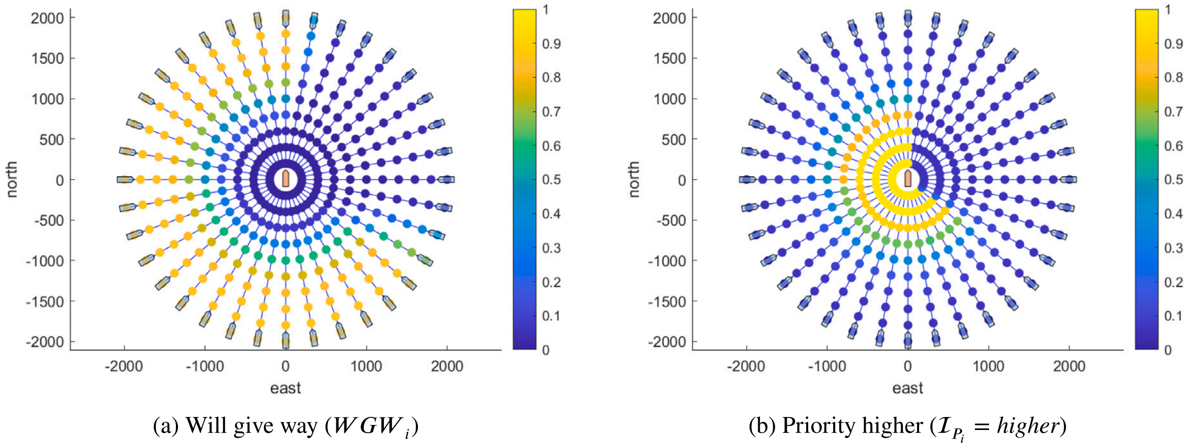


Fig. 14. Scenario 7. A collision encounter consisting of three ships. The figure shows the different candidate trajectories (dashed lines) with their respective probability of being compatible with the ship's intentions ($C[t]$). The thickness of the line is proportional to the probability of that trajectory being compatible with the ship's intentions. The ship symbols are scaled for visualization purposes and do not represent the true ship size. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Similarly, Fig. 13 shows the same scenario as Scenario 1 but with the maneuverability of both ships set to low. This makes it more likely that the blue ship will try to cross with a larger distance between the ships.

Scenario 7 - multi-ship encounters

Fig. 14 shows an encounter with three ships, where the red and green ship have a head-on encounter, while the blue ship has an overtaking encounter with the red ship and a head-on encounter with the green ship. If the blue ship had only considered the red ship, then it would be allowed to cross on either side of the ship. As the evasive maneuver has to be correct towards both ships, it can only change its course towards starboard.



(a) Will give way (WGW_i) (b) Priority higher ($\mathcal{I}_{P_i} = higher$)

Fig. 15. Multiple different simulations of two ships approaching each other from different angles. The red ship is standing still and the blue ship is keeping its course and speed constant. The figure demonstrates how the intentions develop as the ships approach. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Scenario 8 - entry angle

Fig. 15 shows how the intentions develop as two ships approach from different angles. Fig. 15(a) shows how the blue ship is initially assumed to give way except if it is approaching in a crossing situation with the red ship on its port side. The gradual transition in whether the blue ship will initially give way demonstrates the gradual transitions between the different COLREGS situations. Fig. 15(b) shows how the belief that the ship thinks it has higher priority gradually increases as it approaches.

Scenario 9 - maneuver angle

Fig. 16 shows how the intentions develop as two ships approach in a crossing situation with different angles on the avoidance action. Fig. 16(a) shows that as long as the avoidance action is large enough it will be assumed that the blue ship will give way, if not then Fig. 16(b) shows that it is assumed that the blue ship acts as if it has higher priority. In the cases where the blue ship has changed course to port, Fig. 16(c) shows a low probability that the blue ship has an intention be COLREGS-compliant when performing evasive maneuvers. In the cases where the port maneuver is small enough to be marked as the ship acting as if it had a higher priority the COLREGS-compliant evasive maneuver will not fall as the model already has an explanation for the observed behavior.

Scenario 10 - maneuver times

Fig. 17 shows how the intentions develop as two ships approach in a crossing situation with port and starboard maneuvers at different times. Fig. 17(a) shows that for sufficiently early actions the blue ship will be marked as giving way. In the cases where these actions are to the port side, the COLREGS-compliant evasive maneuver state will drop as shown in Fig. 17(c). In the cases the actions are too late, Fig. 17(d) shows that the ship is marked as showing unmodeled behavior as the ship changes course into a collision.

Scenario 11 - head-on offset

Fig. 18 shows how the intentions develop as two ships approach in a head-on situation with different sideways offsets. Fig. 18(a) shows how it was initially assumed that the blue ship will give way as long as the sideways offset is not too large, when it is too large it is assumed that there is no risk of collision as shown in Fig. 18(d). In the cases where the ships get too close, the behavior is explained with the ship acting as if it has a higher priority, shown in Fig. 18(b), or that there was no risk of collision, after all, shown in Fig. 18(d). Fig. 18(c) does not have any significant changes as the state is only affected by non-compliant evasive maneuvers. As no maneuver is made the behavior is instead explained with a higher priority.

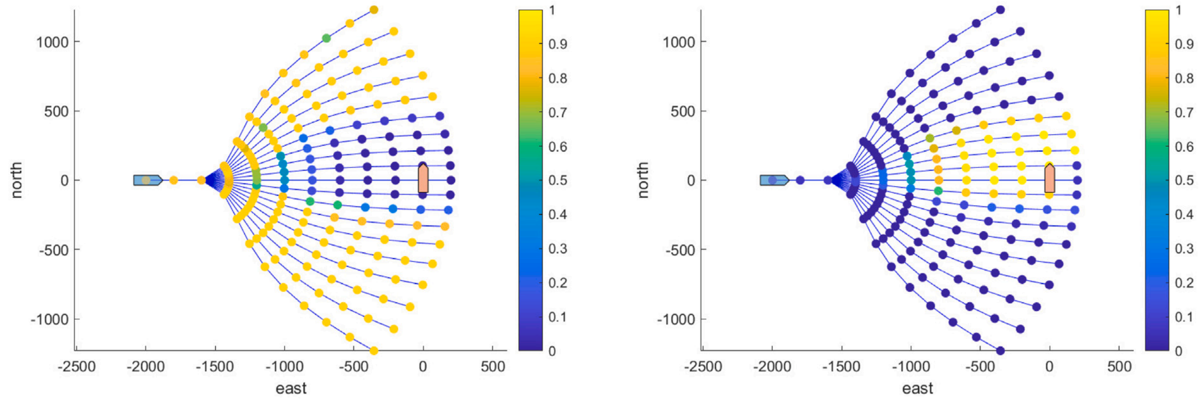
5. Discussion

Scenario 1 demonstrates that the model is able to infer the intentions of a ship based on its observed position, course, and speed. The blue ship did not change its course as it approached. This behavior could be explained by the blue ship having high priority or by having a short ample time. Once the ships came closer, the probability that the blue ship had a definition of ample time that was lower than the remaining time until CPA decreased. This increased the probability that the blue ship had higher priority. The red ship changed its course and speed shortly before CPA to avoid collision. Before this point in time, the model did not increase the chance that the red ship would give way as it did not give any indications of giving way. When the red ship finally changed course, the time until CPA was very short, making it quite unlikely that the red ship had such a short definition of ample time. As this behavior does not fit very well with the model, a high chance was evaluated that the red ship acts in an unmodeled way. A collision avoidance algorithm using this intention inference module should display conservative behavior when unmodeled behavior is observed. This will be the case when evaluating candidate trajectories, as all trajectories will have an increased probability of being compatible. When using the intentions as decision criteria, unmodeled behavior will count as not giving way, thereby making the own-ship give way.

Scenario 1 and Scenario 2 show that having multiple different intention variables that can explain a ship's behavior increases the fidelity of the model. In both scenarios, the blue ship acted in a COLREGS incompliant manner. Modeling how the ships are incompliant enables the model to distinguish between Scenario 1 where the blue ship will stand on and the red ship must give way, and Scenario 2 where the blue ship does an evasive maneuver, although to the wrong side.

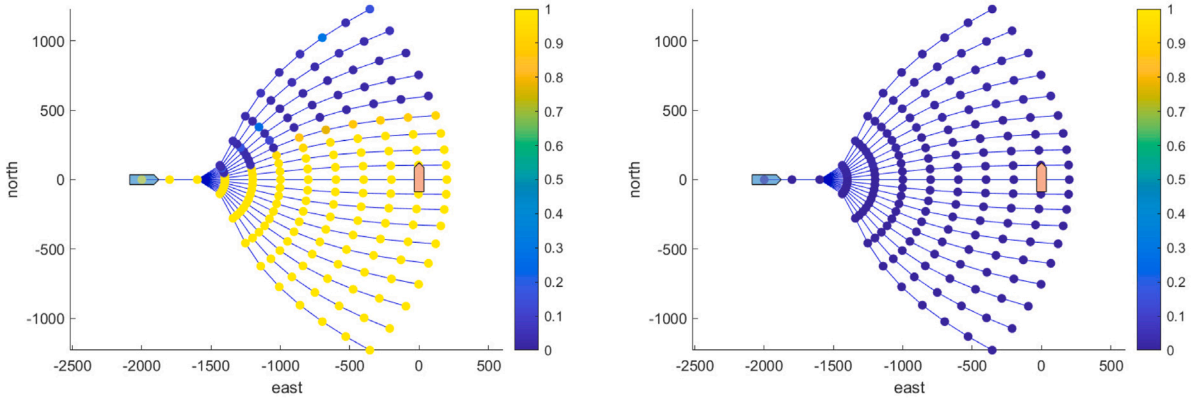
Modeling of the underlying causes that can cause misunderstandings is demonstrated in Scenario 3. Having a clear distinction between the different COLREGS situations is prone to cause misunderstandings, as it is unlikely that the ships will evaluate borderline situations exactly the same. By modeling this uncertainty, it becomes clear that its insufficient to blindly trust the own-ships interpretation of the situation.

In Scenario 4 the uncertainty stems from whether there is a risk of collision. This scenario gives an example where it is insufficient to consider a single parameter for collision avoidance, such as if the ship



(a) Will give way (WGW_i)

(b) Priority higher ($\mathcal{I}_{P_i} = higher$)



(c) COLREGS-compliant evasive maneuver ($\mathcal{I}_C$)

(d) Unmodeled behaviour ($\mathcal{I}_U$)

Fig. 16. Multiple different simulations of two ships approaching in a crossing situation where the blue ship performs an avoidance maneuver with different angles. The red ship is standing still. The figure demonstrates how the intentions develop as the ships approach. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

will give way. In most other situations, the own-ship must give way if the other ship does not fulfill its obligation. In this situation, the opposite is true; if the other ship fulfills its obligations, then both ships must give way. If the other ship keeps its course, then the own-ship can turn a safe situation into a potentially dangerous one by giving way with a significant starboard maneuver, which is required by COLREGS rule 14.

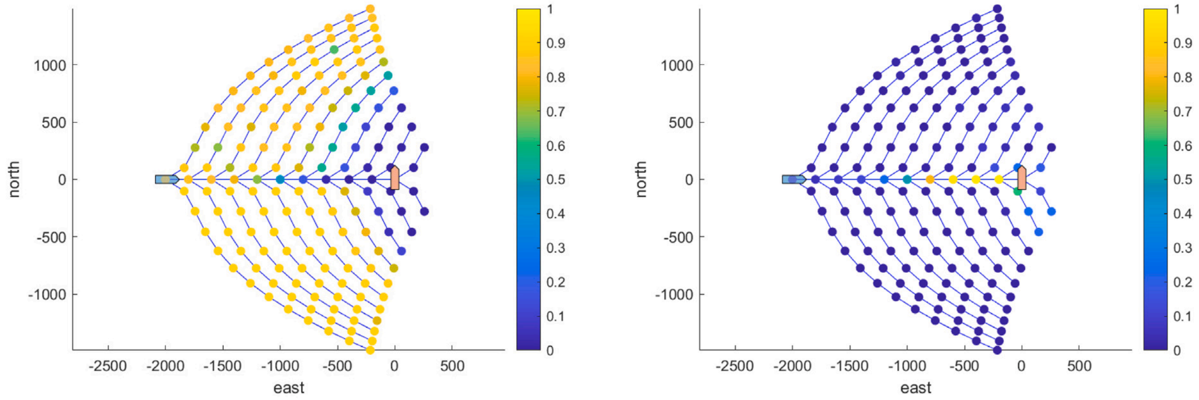
Scenario 5 and Scenario 6 show that additional information, such as the relative ship size or ship maneuverability, can be used to affect the intention probabilities. Having a collision avoidance algorithm that adapts to the current situation is crucial as ships act in very different manners in different situations, such as open waters and inland waterways. The proposed intention model presented in this article is a step towards this ability as it gives the collision avoidance algorithm an understanding of how the other ship will act in the current situation.

Scenario 7 demonstrates that the model can consider encounters with multiple ships. The model considers whether an observed position, course, and speed are compatible with the intention towards all vehicles. The model does not consider that the reference ship has an idea of what the other ships plan to do. This could, for example, be that the blue ship in Fig. 14 predicts that the red ship will make a starboard turn and therefore chooses to take an even larger starboard turn.

Scenario 8 to Scenario 11 demonstrates the sensitivity of the intention models to how the ships meet and act. Additionally, it shows the effect of the initial distribution of the different intention variables. Scenario 8 shows the effect of the situation classifiers given in Figs. 3 and 4 on the initial probability that the ship will give way. Furthermore, it shows the effect of ample times ($\mathcal{I}_{AT}$) initial distribution on when it becomes likely that the ship has a higher priority. Scenario 9 shows the effect of safe distance ($\mathcal{I}_{SD}$) and safe distance fronts ($\mathcal{I}_{SDF}$) initial distributions on what is considered a valid avoidance action. Scenario 11 shows the effect of safe-distance midpoints ($\mathcal{I}_{SDM}$) initial distribution on how far to the side the blue ship has to pass the red ship for it to be considered as giving way. It also shows the effect of risk of collisions ($\mathcal{I}_{RC}$) initial distribution.

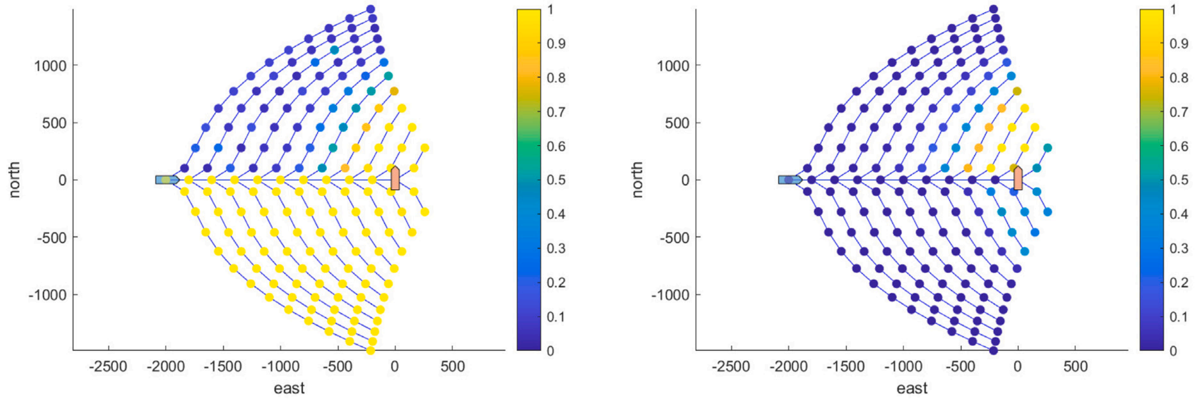
Evaluating different candidate trajectories has some advantages, such as being able to better portray situations such as the one shown in Scenario 4. For the trajectories to realistically portray how the reference ship will act, there must be a candidate trajectory that adequately describes the other ship's trajectory. The candidate trajectory and actual trajectory must be close enough to result in the correct collision avoidance behavior for a collision avoidance algorithm utilizing these intentions. Choosing suitable candidate trajectories is not a trivial task. The ones used in this article cannot handle more complicated situations, such as those where the ship is unable to act at the initial time-step but can act at a later one and where the reference ships make more drastic or sequential changes in course or speed.

The probabilities associated with each candidate trajectory do not represent the probability that the reference ship will follow this trajectory. Instead, it represents the probability that this trajectory is something the reference ship would consider acceptable when only considering properties related to COLREGS. If it is known that the ship will follow COLREGS and how it defines the different ambiguities such as ample time and safe distance, then all trajectories that adhere to this definition of the rules will be given a 100% probability of being compatible with the intentions.



(a) Will give way (WGW_i)

(b) Priority higher ($\mathcal{I}_{P_i} = higher$)



(c) COLREGS-compliant evasive maneuver ($\mathcal{I}_C$)

(d) Unmodeled behaviour ($\mathcal{I}_U$)

Fig. 17. Multiple different simulations of two ships approaching in a crossing situation where the blue ship performs either a starboard or port avoidance action at different times. The red ship is standing still. The figure demonstrates how the intentions develop as the ships approach. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

This article has not considered grounding risk or the COLREGS rules regarding traffic separation schemes (Rule 10), narrow channels (Rule 9), and sailing vessels (Rule 12). Regarding traffic separation schemes and grounding risk, generating candidate trajectories will be more challenging as the trajectories must cover the ship's different options, such as following and leaving the traffic separation scheme correctly. In these situations, it might be necessary to dynamically generate the trajectories based on the current circumstances. An additional challenge arises in narrow channels due to stand-on vessels being allowed to change their course to follow the channel (Woerner et al., 2019).

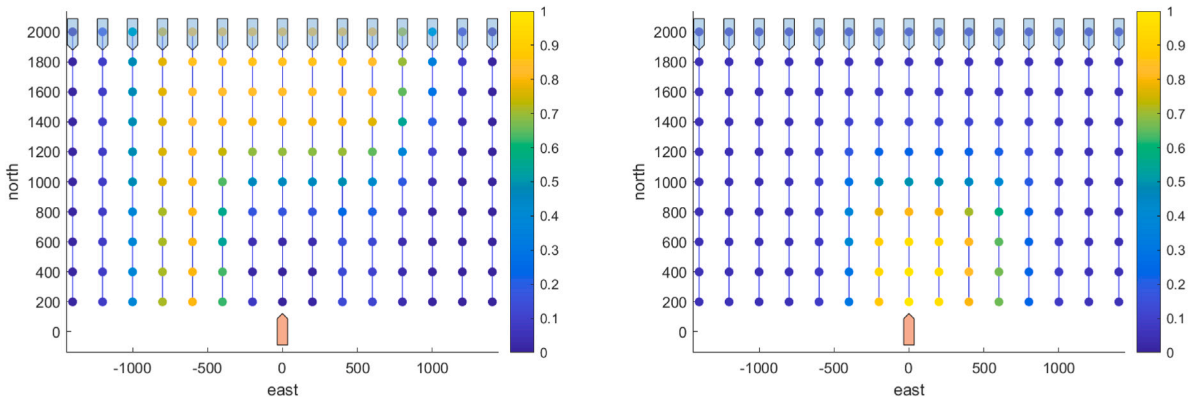
Furthermore, the model does not explicitly consider measurement uncertainties. This should not be a problem as long as the noise is less than $\mathcal{P}_{CIC}$ and $\mathcal{P}_{CIS}$. If the noise is substantial, then measurement uncertainty should be modeled as well. This can be achieved by having separate nodes representing the measured state and the measurement itself. The measurements themselves should be child nodes of the measured state, and their CPTs should describe the measurement uncertainty. This way of modeling is called the measurement idiom (Fenton and Neil, 2018).

The model assumes all initial changes in course are large enough to avoid collision without requiring additional course changes. This assumption does not hold if the model is fed an observation in the middle of a course change. The model can then evaluate that the ship is not standing on correct (as it changed its course), nor is it giving way correct (as the course change is too small to avoid collision). This can be handled by introducing a node indicating whether the other ship is currently changing its course.

To have acceptable computational time, the number of time-steps in the DBN must be limited. This can be achieved with a sliding window approach where only the last couple of observations are considered. The priors for the intention nodes must be updated to represent the information that is no longer inside the window. This is done by setting the intention priors equal to what the posterior was at the last time-step that is no longer in the window. With a limited window, the frequency of new observations inserted into the model must be considered. Feeding information more often makes the window consider a shorter time span which will contain more similar observations. This will reduce the inference capabilities of the model. Feeding information less often makes the model respond to changes slower. Not all measurements need to be saved as a time-step in the DBN. The newest time-step of the DBN could be updated at a quick frequency and then only saved as a new time-step if it contained substantial new information relative to the previously saved time-steps. This should make the DBN respond quickly and keep a high inference quality with a limited window.

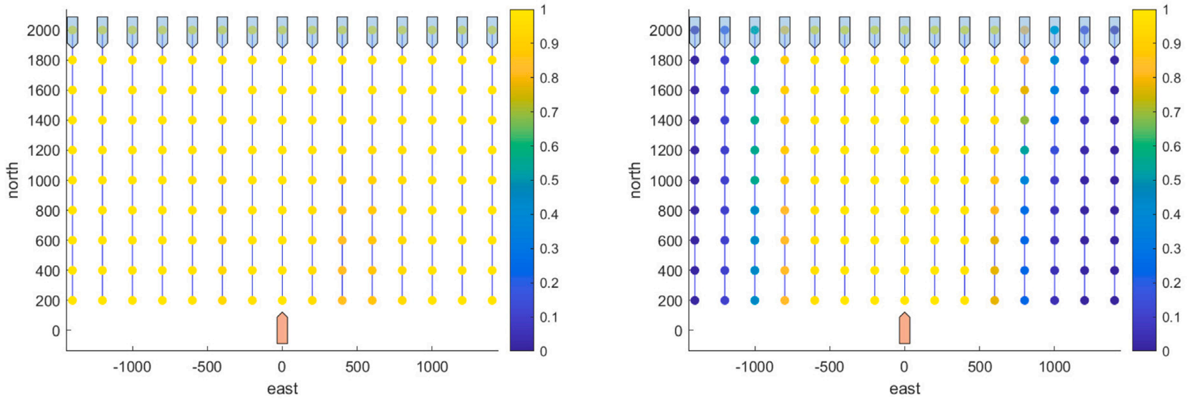
6. Conclusion

This article presents a novel approach for modeling and inferring the intentions of other ships in a potential collision encounter at sea. The simulation study shows that the method is able to infer the state of different intention nodes, identify situations that are likely to lead to misunderstanding, and adapt the intention probabilities to the current situation. This opens up for new possibilities for collision avoidance algorithms. It could enable collision avoidance algorithms to act more safely and predictably as they will better understand the future motion



(a) Will give way (WGW_i)

(b) Priority higher ($\mathcal{I}_{P_i} = higher$)



(c) COLREGS-compliant evasive maneuver ($\mathcal{I}_C$)

(d) Risk of collision ($\mathcal{I}_{RC}$)

Fig. 18. Multiple different simulations of two ships approaching in a head-on situation with different sideways offsets. The red ship is standing still and the blue ship is keeping its course and speed constant. The figure demonstrates how the intentions develop as the ships approach. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

of meeting traffic. They could become able to take early proactive actions to turn a situation prone to misunderstandings into a clear situation where all ships agree on how to act. Lastly, it opens up for collision avoidance algorithms to adapt to the current situation, such as relative ship size and locations. This is an essential feature for collision avoidance algorithms working in multiple different situations where different tuning parameters are needed.

The focus of this article is the enhanced modeling and inference capabilities achieved with the proposed framework. Future work is needed on expanding the model to include the parts of COLREGS that were not considered, to consider grounding, to consider factors outside of COLREGS that affect how ships behave, and to validate the model with historical data. Furthermore, work is needed on gathering the statistics that work as priors for the different intention states and on identifying how they are affected by available information on the current situation. Lastly, collision avoidance algorithms must be developed that can utilize the increased situational awareness provided by this model.

CRediT authorship contribution statement

Sverre Velten Rothmund: Conceptualization, Methodology, Software, Validation, Investigation, Writing – original draft. **Trym Tengesdal:** Conceptualization, Software, Writing – review & editing. **Edmund Førland Brekke:** Conceptualization, Writing – review & editing, Supervision, Project administration, Funding acquisition. **Tor Arne Johansen:** Conceptualization, Writing – review & editing, Supervision, Project administration, Funding acquisition.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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