Uncertainty and sensitivity analysis for the performance evaluation of ship dynamical model

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ABSTRACT

Model uncertainty is pervasive and inherent in the engineering field. It could bring potential risks in real applications, especially for ship behaviour prediction under environmental disturbances. The evaluation and quantification of model uncertainty are of importance for accurate ship motion prediction. This study applies model uncertainty analysis and sensitivity analysis methods to evaluate the ship motion model's level of uncertainty against environmental disturbances and ship manoeuvres. Firstly, three models are created based on the a dynamical model (Mariner) in Marine Systems Simulator. After that, models are tested on various predefined scenarios. The similarity of predicted trajectories and the reference is evaluated by Euclidean distance and used to quantify the uncertainty of models. Next, statistical analysis is used to analyze the uncertainty of models. Sensitivity analysis (SA) method called 'PAWN' and 'UnivariateSpline' interpolation technology are combined to identify which factors contribute the most to model's performance. The results suggest that the uncertainty caused by external factors varies from different models under different manoeuvres. SA can tell us which factors (wind angle, wind velocity, and surge speed) have a large influence to the model uncertainty given a ship maneuver. Such analyses, on the one hand, contribute operators to choosing the optimum model according to the current conditions for better ship motion prediction. On the other hand, they can pick up the most important factors for fast uncertainty modelling.

KEY WORDS: Ship motion prediction; Model uncertainty analysis; Statistical analysis; Sensitivity analysis; UnivariateSpline interpolation.

INTRODUCTION

Digital technology has become an enabler for making ship motion more intelligent and safe. A large variety of advanced ship motion prediction models have been developed and present a good performance. However, the model's robustness and stability have been a huge challenge that might lead to serious accidents. This problem results from model uncertainty. Uncertainty is ubiquitous in modelling-related engineering fields, especially for ship motion prediction [Arendt et al., 2012]. Therefore, it is of importance to evaluate model uncertainty for the support of model improvement and accurate ship motion prediction.

Model uncertainty comes from two sources: model discrepancies and input parameters [Smith, 2013]. Model discrepancies are caused by an imprecise representation of real physics. Input uncertainty refers to uncertain model parameters. These two factors are inherent in ship motion models. In addition, the collected sensor data probably worsens this case as it is mixed with noise from the environment and measured errors. Researchers proposed a large variety of models for future ship position prediction. For example, the extended Kalman filter was used to estimate ship motion states and predict the trajectory of a vessel by means of a kinematic model [Perera et al., 2012; Sutulo et al., 2002; Perera, 2017]. Li et al. leveraged the Support Vector Machine to predict the heave motion under the impact of waves [Li et al., 2016]. Skulstad et al. offer ship position predictions based on an integration of a supervised Machine Learning model of the ship into the ship dynamic model [Skulstad et al., 2020]. Although these models could achieve a good performance in terms of ship motion prediction, their robustness can not be guaranteed due to the lack of model uncertainty evaluation and quantification, which would bring risks in real applications. This problem highlights the necessity of uncertainty analysis (UA) of ship models.

The uncertainty mainly contains two types: aleatoric uncertainty and epistemic uncertainty [Smith, 2013]. Aleatoric uncertainty is also called data uncertainty which is caused by errors in the dataset. Epistemic uncertainty refers to model uncertainty that comes from model structure and model parameters. That means different methods applied to the same set of data often obtain different conclusions [Young and Holsteen, 2017]. That would bring a huge risk for decision making in the engineering field, especially for accurate motion control. This problem implies the necessity of model uncertainty evaluation. A novel data-driven (named SDMU) was proposed to conduct hydrologic model uncertainty evaluation [Pathiraja et al., 2018]. It is critical to provide improved predictions of system outputs. Mathevet et al. assessed the performance of two conceptual rainfall-runoff models based on Kling-Gupta Efficiency metric [Mathevet et al., 2020]. Simulations present the limitation of both models when they were applied in different watersheds. Hu et al. carried out time-dependent reliability analysis by integrating the fast integration method and surrogate model method [Hu et al., 2016]. This method is

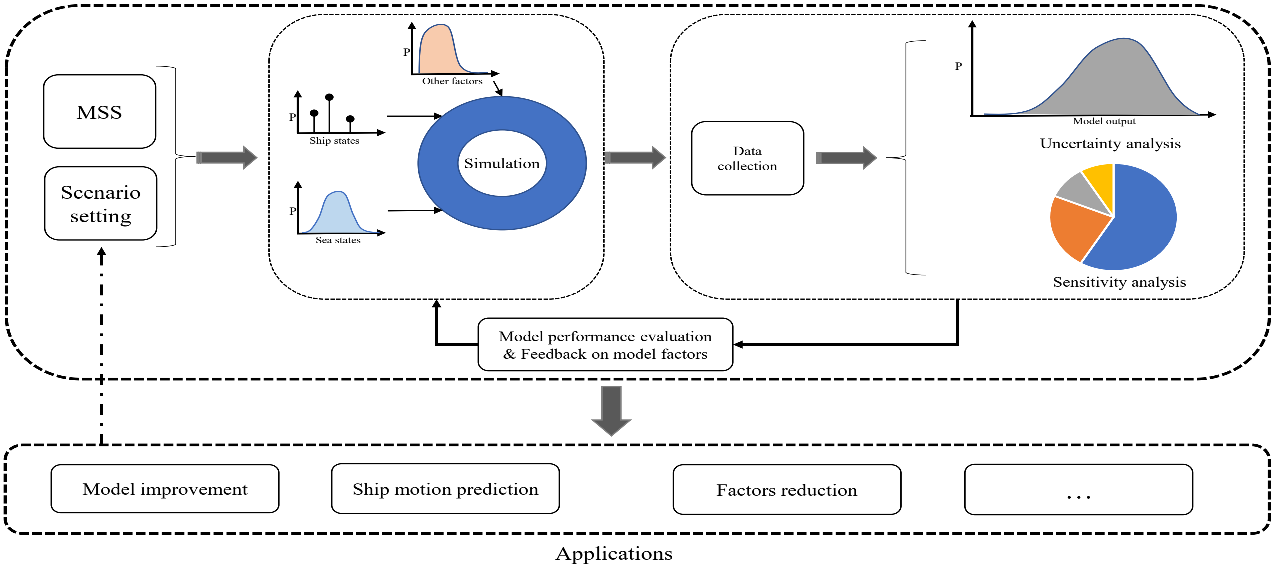


Fig. 1 The framework of model UA on the ship model.

efficient to conduct model uncertainty quantification. From the point of methodology, Xi et al. proposed reliability analysis strategies given three representative scenarios [Xi, 2019]. It is found that the confidence-based reliability analysis can make estimation errors controllable with less conservativeness. Young et al. proposed a computational framework for multimodel analysis, which can allow the researcher to unbundle their model specifications and observe the influence of each model ingredient [Young and Holsteen, 2017]. Dash et al. applied a stochastic response surface approach to investigate the parameters uncertainty of ship model [Dash and Nagarajan, 2014]. It aims to investigate how parameters uncertainty is propagated to the engine torque and propeller revolution in different manoeuvres.

Inspired by the model UA in other fields and [Dash and Nagarajan, 2014], this study is to explore how to assess the uncertainty of ship models under the influence of various factors such as sea states and ship states. Recognizing the pros and cons of each model is beneficial to combine models to construct an optimum predictor according to the current conditions for better ship motion prediction. For the purpose of model UA, we need a model as the benchmark and other models as the test objectives. Therefore, the ship model in Marine Systems Simulator (MSS) is employed as the referenced one in this study [Perez et al., 2006]. The main contributions are as follows: 1) a model UA framework is proposed to evaluate the uncertainty of models; 2) statistical analysis is applied to quantify the uncertainty of ship models in different manoeuvres; 3) sensitivity analysis (SA) method called 'PAWN' and interpolation technology ('UnivariateSpline') are used to pick up important factors which cause impacts to model uncertainty.

This article is organized as follows. In Section 2, the UA framework is proposed. In Section 3, experiment settings and results are elaborated and analyzed. Conclusion is given finally.

UNCERTAINTY ANALYSIS PROCEDURE

This section mainly introduces the framework of model UA on ship models. As shown in Fig. 1, the whole analysis procedure is divided into three parts. The first part is to set up scenarios based on the corresponding applications. The second part is to carry out a model evaluation based on experiment settings to generate ship trajectories. The last one is to use statistical analysis and SA to evaluate and quantify the uncertainty of the ship model. Analysis results can give support for applications of interest, such as model improvement, ship motion prediction, and factors reduction.

Ship motion model

In this study, the 3-DOF Abkowitz model of a Mariner class cargo vessel is taken as a ship model. The model structure in non-dimensional form is expressed as

$$\begin{bmatrix} m' - X'_{\dot{u}} & 0 & 0 \\ 0 & m' - Y'_{\dot{v}} & m' x'_g - Y'_{\dot{r}} \\ 0 & m' x'_g - N'_{\dot{v}} & I'_{zz} - N'_{\dot{r}} \end{bmatrix} \begin{bmatrix} \dot{u}' \\ \dot{v}' \\ \dot{r}' \end{bmatrix} = \begin{bmatrix} X' + \tau_x \\ Y' + \tau_y \\ N' + \tau_n \end{bmatrix} \tag{1}$$

where the superscript denotes non-dimensional variables. m' is the mass of ship, x'_g is the position of gravity center. $\dot{u}'$, $\dot{v}'$, $\dot{r}'$ are the accelerations in surge, sway, and yaw direction. X', Y', and N' are forces along the ship logitudinal and lateral directions, and the moment in the vertical axis, respectively. $X'_{\dot{u}}$, $Y'_{\dot{v}}$, $Y'_{\dot{r}}$, $N'_{\dot{v}}$, and $N'_{\dot{r}}$ are dimensionless added mass coefficients. I'_{zz} is the inertia moment. The definition of non-dimensional variables can be referred to [Wang et al., 2021b]. τ_x, τ_y, and τ_n are the wind forces and moment, respectively. Their definition can be found in [Fossen, 2011].

The non-dimensional forms of hydrodynamic forces/moments are expressed by Eq. 2.

$$\begin{aligned}
X' &= X'_u \Delta u' + X'_{uu} \Delta u'^2 + X_{uuu} \Delta u'^3 + X_{vv} v'^2 \\
&+ X_{rr} r'^2 + X_{rv} r' v' + X_{\delta\delta} \delta^2 + X_{u\delta\delta} \Delta u' \delta^2 \\
&+ X_{v\delta} v' \delta + X_{uv\delta} \Delta u' v' \delta \\
Y' &= Y_v v' + Y_r r' + Y_{vvv} v'^3 + Y_{vvr} v'^2 r' + Y_{vu} v' \Delta u \\
&+ Y_{ru} r' \Delta u' + Y_\delta \delta + Y_{\delta\delta\delta} \delta^3 + Y_{u\delta} \Delta u' \delta + Y_{uu\delta} \Delta u'^2 \delta \\
&+ Y_{v\delta\delta} v' \delta^2 + Y_{vv\delta} v'^2 \delta + Y_0 + Y_{0u} u' + Y_{0uu} u'^2 \\
N' &= N_v v' + N_r r' + N_{vvv} v'^3 + N_{vvr} v'^2 r' + N_{vu} v' \Delta u' \\
&+ N_{ru} r' \Delta u' + N_\delta \delta + N_{\delta\delta\delta} \delta^3 + N_{u\delta} \Delta u' \delta + N_{uu\delta} \Delta u'^2 \delta \\
&+ N_{v\delta\delta} v' \delta^2 + N_{vv\delta} v'^2 \delta + N_0 + N_{0u} u' + N_{0uu} u'^2
\end{aligned} \tag{2}$$

where ρ is the density of water, L is the ship length; $U = \sqrt{(U_0 + \Delta u)^2 + v^2}$ is the instantaneous ship speed; Δu is perturbed surge velocity about nominal speed U_0; δ is the rudder angle. The hydrodynamic derivatives $(X'(), Y'(), N'())$ are the parameters that need identifying.

The Abkowitz model can be viewed as a linear model with respect to the hydrodynamic coefficients. The motion equations are discretized by Euler's stepping method and the regression model is expressed by

$$
\begin{aligned}
&\frac{(u(k+1)-u(k))m_{11}}{\Delta t U_0^2} - \tau'_x && = AX(k)\\
&\frac{(v(k+1)-v(k))m_{22}L}{\Delta t U_0^2} + \frac{(r(k+1)-r(k))m_{23}L^2}{\Delta t U_0^2} - \tau'_y && = BY((k)\\
&\frac{(v(k+1)-v(k))m_{32}L}{\Delta t U_0^2} + \frac{(r(k+1)-r(k))m_{33}L^2}{\Delta t U_0^2} m_{33} - \tau'_n && = CN(k)
\end{aligned}
\tag{3}
$$

where Δt is the time interval, and τ'_x, τ'_y and τ'_n are the non-dimensional wind forces and moment; $m_{11} = m' - X'_{\dot{u}}$, $m_{22} = m' - Y'_{\dot{v}}$, $m_{23} = m'x'_G - Y'_{\dot{r}}$, $m_{32} = m'x'_G - N'_{\dot{v}}$, $m_{33} = I'_z - N'_{\dot{r}}$; A, B, and C are the vectors of hydrodynamic coefficients to be identified, and $X(k)$, $Y(k)$ and $N(k)$ are the input vectors as shown in Eq. 2.

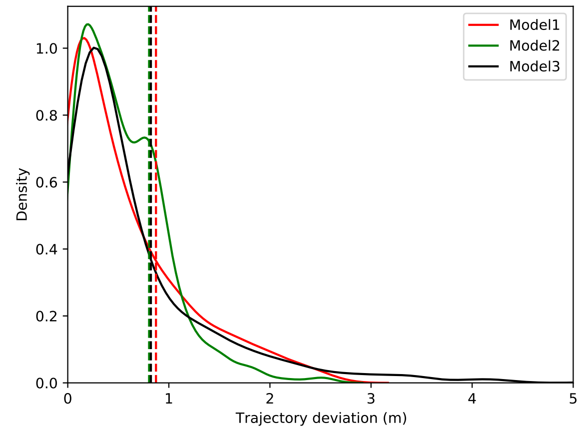
The parameters uncertainty in three DOFs of X, Y, N could lead to the performance of constructed mathematical model varying in different scenarios. The evaluation and quantification of model uncertainty is of importance for model improvement and accurate ship motion prediction.

Uncertainty analysis and Sensitivity analysis

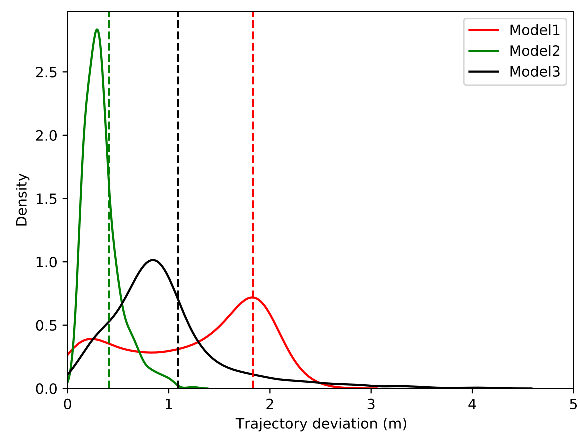
The trajectory deviation of models and Mariner is used as an indicator of model performance. It is quantified by the Euclidean distance method [Cha, 2007]. The larger the Euclidean distance, the worse the model performance is. The following analysis is carried out using obtained deviations in different predefined conditions. The statistical analysis aims to mine informative knowledge from the collected data by descriptive statistics such as Probability Density Function (PDF), maximum value, and mean value. It is regarded as a simple but efficient approach for UA. Therefore, in this study, common descriptive statistics are used for the uncertainty evaluation of the constructed models. They can present knowledge about the advantages and limitations of each model and contribute to the optimum model selection according to the current manoeuvre and environmental factors.

UA aims to analyze how much the uncertainty of model output is caused by the variation of input parameters. In contrast, SA is to investigate how much the variation of model output is proportioned to each input parameter. The impact of uncertain factors is different over different ship manoeuvres. In order to gain insights into the relationship between model uncertainty and external factors, it is of importance to identify the factors that contribute the most to the model capability. Therefore, 'PAWN' is used to compute the sensitivity of factors based on the obtained trajectory difference [Wang et al., 2021a]. 'PAWN' method is to quantify the discrepancy between unconditional and conditional cumulative distribution functions (CDFs). The longer the discrepancy is, the larger the sensitivity index of x_i is. The sensitivity index 'S_i' is computed by the Eq. 4.

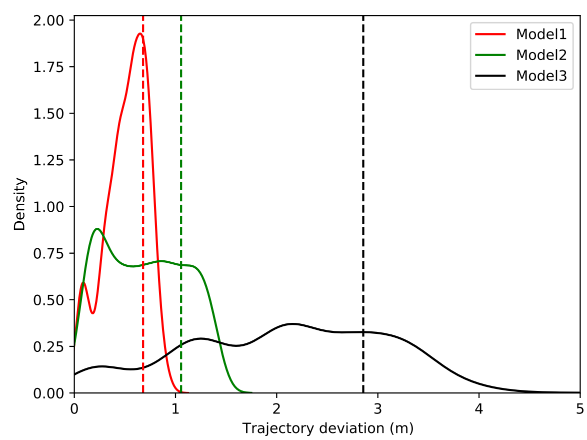
$$
\begin{cases}
\hat{S}_i = \underset{k=1,\ldots,l}{max} KS(I_k)\\
KS(I_k) = \max\limits_{y} |F_y(y) - F_{y|\tilde{x}_i}(y|\tilde{x}_i \in I_k)|
\end{cases}
\tag{4}
$$



(a) Linear motion



(b) Turning



(c) Zigzag

Fig. 2 The comparison of models' performance under the maneuvers of linear, turning, and zigzag motion.

where KS is Kolmogorov-Smirnov statistic; $F_y(y)$ is unconditional CDF

Table 1 The Kolmogorov-Smirnov statistic of wind angle, wind velocity, and surge velocity under different maneuvers for three models, separately.

Maneuver	Ship model	Wind angle	Wind velocity	Surge velocity
Linear	Model1	**0.50**	0.34	0.27
	Model2	0.37	0.28	**0.90**
	Model3	0.28	**0.74**	0.24
Turning	Model1	0.03	0.05	**0.97**
	Model2	0.15	**0.62**	0.41
	Model3	0.32	0.19	**0.47**
Zigzag	Model1	0.30	0.17	**0.64**
	Model2	0.21	0.11	**0.98**
	Model3	0.05	0.02	**1.00**

where $y \subseteq Y$ and $F_{y|\tilde{x}_i}(y|\tilde{x}_i \in I_k)$ is conditional CDF where $\tilde{x}_i$ is fixed.

In this study, Eq. 4 is modified to adapt to the current study by Eq. 5. It can compare CDFs pairwise. The mean of KS values is regarded as the sensitivity index of the variable of interest.

$$\begin{cases} \hat{S}_i = mean(KS(I_k)) \\ KS(I_k) = \max_{j \neq i} |F_{y|\tilde{x}_j}(y|\tilde{x}_j \in I_k) - F_{y|\tilde{x}_i}(y|\tilde{x}_i \in I_k)| \end{cases} \tag{5}$$

KS value is estimated based on the same 'y'. However, the outputs 'y' are not consistent for different CDFs. As a consequence, it leads to the difficulty of KS computation. Therefore, this study introduces interpolation technology called 'UnivariateSpline' which can fit all points to a one-piece function [Geng and Wu, 2021]. It can achieve the estimation of probability given a 'y'. The detailed elaboration can be referred to Python library (Scipy).

UA could gain the knowledge of each model's performance under various conditions, which could benefit for accurate ship motion prediction via combine different models together. The important factors can be identified by means of SA. That is useful for factors reduction in order to implement fast uncertainty modelling.

EXPERIMENT

Experiment setting

This study aims to explore how to apply uncertainty analysis and sensitivity analysis in ship model performance evaluation. Such analyses can be transferred to evaluate any data-driven or mathematical models. Based on uncertainty analysis results, different models could be combined to achieve more accurate ship motion prediction in real world. Owing to the lack of different ship models, therefore, three dynamic models are constructed as research objectives based on Mariner model in this study. As model construction is not the main concern of this paper, this can be done by adjusting randomly the parameters of three DOFs (X,Y,N) in Eq. 2, separately. The parameters of three DOFs (X,Y,N) are added by random noise separately to make three different ship models. After that, the reference model and three models are used to make a comparison under different conditions. Wind velocity varies from 1 to 10 m/s, wind angle ranges from 0^o to 180^o with an interval of 30^o; The initial surge speed of the ship changes from 1 to 10 m/s. The simulator is run for 600 seconds with sampling frequency 10 (HZ) to render the trajectories of the referenced model and three models. The manoeuvers are set as 20^o turning and $20^o/20^o$ zigzag. The initial rudder angle is set as 0 and it varies within $[-40^o, +40^o]$.

Uncertainty analysis

This section is to analyze the model uncertainty by average trajectory deviations. Due to different experiment settings, the model evaluation needs to be done 600 times to obtain 600 Euclidean distance points when each of the models is simulated for each manoeuvre. Fig. 2 shows three models are applied for different manoeuvres including linear motion, turning, zigzag. Therein, curves represent PDFs and vertial dashed lines are the third quartile which is used to describe data distribution.

From Fig. 2(a), the third quartile of the three models is similar around 1 m. That means the stability of the three models are close to each other when they are used for ship linear motion prediction. The tail of the PDF of model3 is quite longer than the others. It shows that model3 performs worse for certain cases. The large difference between the three models occurs when they are applied for turning prediction in Fig. 2(b). The three dashed lines are distinct from each other and show model2 outperforms the others. In addition, the distribution of data points demonstrates that the performance changes over different sea states and ship surge speeds. For zigzag prediction, model1 and model2 have a better performance than model3 as shown in Fig. 2(c).

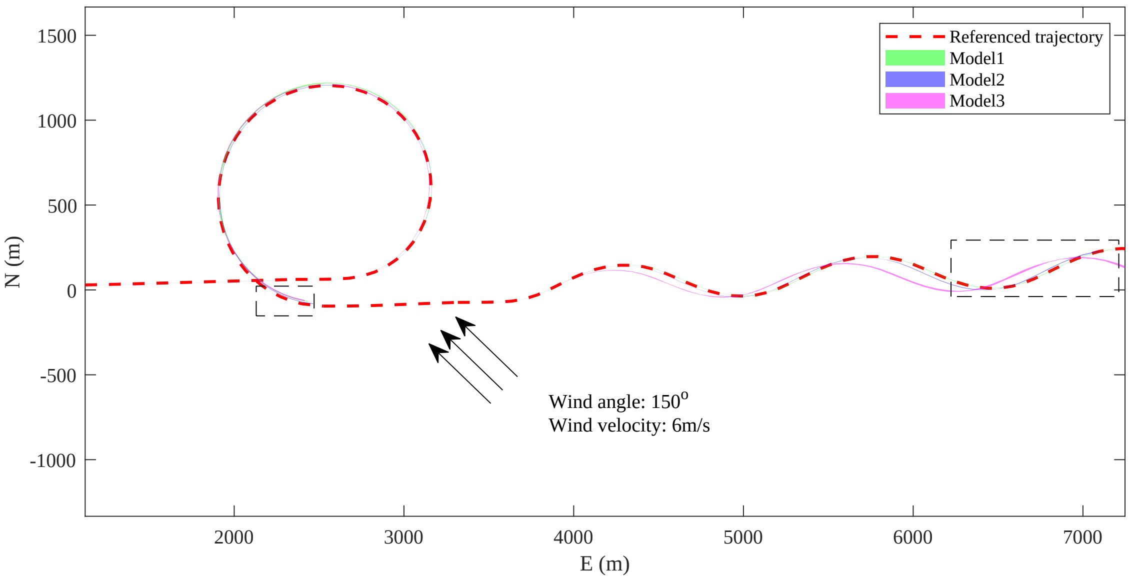
The knowledge is summarized as follows:

- The performance of three models are highly dependent on ship maneuvers.
- For linear motion prediction, parameters' errors in three DOFs do not cause much differences as three models have similar performance and stability.
- Y-DOF parameters are not sensitive to turning motion as model2 remains the high accuracy of prediction. In contrast, the errors in X-DOF cause a very large influence to the predicting accuracy.
- For zigzag, model3 has the worst performance. That shows N-DOF parameters are quite sensitive to zigzag.
- All models' performance are affected by sea states and ship surge speeds to some extent.

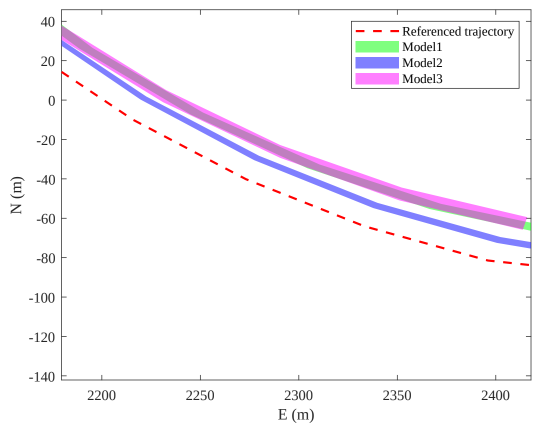
Based on the insights of the pros and cons of different models, they can be assembled to make ship motion prediction under different maneuvers to improve holistic accuracy.

Sensitivity analysis

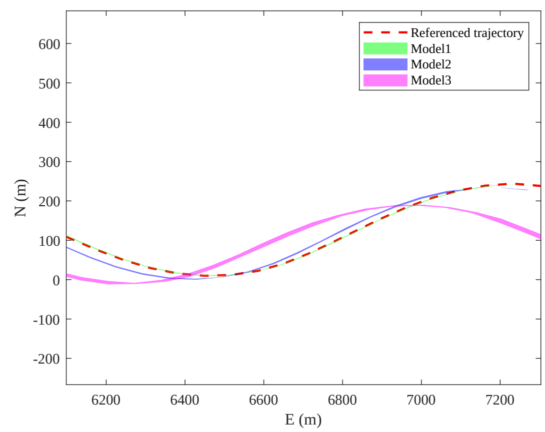
The previous section investigates how the uncertainty of input parameters propagates to the model output. This section focuses on how much the variation of the output can be proportioned to input factors. That could provide supports for researchers to do factor prioritisation and factor fixing.



(a) Random manoeuvres



(b) Zoomed turning trajectories



(c) Zoomed zigzag trajectories

Fig. 3 The comparison among different models under different maneuvers when wind angle, wind velocity, and surge speed are 150^o, 6 m/s, and 8 m/s.

Table. 1 shows the KS values of wind angle, wind velocity, and surge velocity for three models under three manoeuvres. From this table, the influence of factors varies over three models under different manoeuvres. The knowledge is shown as follows:

- For linear motion, wind angle is the most sensitive factor for Model1 as its KS value (0.5) is the largest; wind velocity and surge velocity are the influential factors for Model3 and Model2, separately. In other words, the parameters of X,Y,N-DOFs are sensitive to wind angle, surge velocity, and wind velocity, separately.
- For turning, Model2 is sensitive to wind velocity while Model1 and Model3 are more sensitive to surge velocity. That means the parameters of N-DOF is prone to the impact of wind velocity.
- For zigzag, the variation of surge velocity contributes the most to all models. The KS values of Model1 and Model2 are 0.98 and 1.00 separately. The changing surge velocity leads to the drastic variation of their performance.

To sum up, SA is to investigate how model performance is affected by the changes in other variables known as input variables (wind velocity, wind angle, and surge velocity). After SA, given a specific manoeuvre and a ship model, influential factors can be found for factors reduction and fast uncertainty modelling.

Case study

Besides factors analysis, UA can provide supports for better ship motion prediction as well. This section introduces a case regarding how to improve the accuracy of ship motion prediction through an assembled model when a ship performs random manoeuvres.

Three models are tested under random manoeuvres from linear motion to turning, linear, and zigzag) as shown in Fig. 3(a). Wind angle, wind velocity and initial ship surge speed are set as 150^o, 6m/s, and 8m/s, separately. Fig. 3(b) displays the zoom-in trajectories in the left dashed square area in Fig. 3(a). The coloured shaded areas represent the uncertainty interval of the predicted trajectories of the three models. From this figure, we can conclude that model2 has a better performance as its trajectory is quite close to the reference (red dashed line). Therefore, in this case, Model2 should be chosen for turning prediction. Fig. 3(c) displays the zoom-in trajectories in the right dashed square area in Fig. 3(a). For zigzag, Model1 appears to outperform other models. Its predicted trajectory is consistent with the referenced one.

Based on the analyses above, ship motion prediction should take advantage of the strength of each model and combine them together according to the current conditions. For example, Model2 is selected to predict ship turning movement and it is replaced by Model1 when the ship performs a zigzag manoeuvre.

CONCLUSION

This study proposed a model UA procedure to carry out performance evaluation of ship models. That is useful to get insights regarding the pros and cons of models for better ship motion prediction in specific scenarios. To verify the UA procedure, several case studies are designed and corresponding analyses are given. First, three models are constructed based on the Mariner model in MSS toolbox. For the convenience of UA, three models are built by adjusting the parameters of 3 DOFs randomly. All models are run under predefined experiment settings to obtain predicted trajectories and the referenced one. Model evaluation and statistical analysis are used to mine knowledge regarding the performance of models under different manoeuvres. Sensitivity analysis and interpolation technology (‘UnivariateSpline’) are employed to pick up the most important factors such as wind angle and surge velocity. Such analyses conduce to sharpening the stakeholders’ view of model uncertainty and provides insights regarding the optimum combination of models.

The results suggest that the uncertainty of external factors are propagated to trajectory prediction by a ship model. That leads to the changing ship performance over different ship manoeuvres, environmental factors, and ship states. Based on UA, different models could be combined for accurate ship motion prediction when a ship is executing demanding operations. SA shows different models’ uncertainties arise from different sources as shown in Table. 1. That highlights we should pay more attention to the most sensitive factors for further uncertainty modeling.

The performance of ship models is prone to the influence of various disturbances such as wind, wave, and current. In future work, these factors will be considered into model UA and construct an uncertainty evaluation model for onboard supports of accurate ship motion prediction.

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