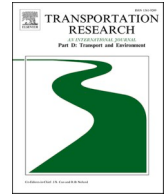


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Integrated optimization of charger deployment and fleet scheduling for battery electric buses

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ABSTRACT

Battery electric buses (BEBs) are considered effective options for sustainable mobility. Their operational efficiency is significantly influenced by the deployment of charging infrastructure and the scheduling of the bus fleet. In this research, we develop an integrated optimization model for charger deployment and fleet scheduling for BEBs under opportunity charging. The model jointly optimizes the battery nominal capacity, bus fleet size, and charger deployment at bus stops and the central terminal to minimize the total annualized costs. The time-varying characteristics of ridership, dwelling time, and travel time are considered to investigate realistic solutions with opportunity charging supplied by pantograph chargers. A real-life case study conducted in Oslo, Norway is presented to demonstrate the proposed model, and accordingly sensitivity analysis is further carried out to evaluate the system responses to a group of impact factors. The results indicate that BEB purchase costs and energy consumption rate have significant impacts on total annualized costs, and also imply that opportunity charging has a cost-effective advantage, which can save up to 13.38% of total annualized costs as compared to end station charging using the same type of pantograph charger.

1. Introduction

Electrification of city bus systems is becoming a widespread policy choice to mitigate climate change and promote sustainable mobility in the field of urban transportation (Qu et al., 2020). As one of the major types of alternative-fueled buses, battery electric buses (BEBs) are powered solely by rechargeable batteries and thus followed with significant attention owing to their environmental benefits, for example, zero tailpipe emission. Therefore, the promotion of BEB adoption has significance in the near future of public transit systems. During the past decades, thanks to the ground-shaking technological improvements and rapid market-share growths for BEBs, replacing conventional buses with BEBs is an increasing trend in many cities (Chen et al., 2019). However, unlike conventional buses, BEBs have a limited driving range and often need a long charging duration. To alleviate the negative effects resulting from these features, some transport agencies around the world intend to establish the opportunity charging system by using

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pantograph chargers to realize an effective transition from conventional buses to BEBs in the public transit system (Wang et al., 2017; Carrilero et al., 2018; Pelletier et al., 2019). Under such an opportunity charging system, the BEBs can be recharged in operation employing high power during the dwelling time at bus stops or operational intervals at terminals. In this way, the BEBs can be raised to a similar level of capability as their diesel counterparts with respect to driving range and operating time. Moreover, the primary investment costs for the BEB system include the BEB purchase costs and the costs for charger installation. In practice, these cost components usually are interdependent and thus should be integrated to plan the total investment costs in the BEB system. For example, the charger deployment is significantly affected by the battery nominal capacity due to limited driving range; the fleet schedule is influenced by the charger locations and battery nominal capacity, and further determines the fleet size; the battery nominal capacity is affected by the charger deployment and energy requirements in bus operation. In view of the characteristics of the BEB system with opportunity charging operation, special attention must be given to the integrated optimization of charger deployment and fleet scheduling.

Both charger deployment and fleet scheduling are critical problems for the efficiency of the electric transit system (Wei et al., 2018). Over the years, efforts toward the construction and management in terms of electrified transit systems have been made by both the industrial and academic communities (Li et al., 2018). Since BEBs have a limited driving range, unlike conventional buses, charging infrastructure deployment is essential to the efficient operation of the electric transit system. In recent years, as many cities tend to replace their conventional buses with BEBs, several studies have focused on the planning for charging facilities. Considering the energy consumption profile of BEBs, Liu et al. (2018) proposed a robust optimization model to deploy the charging infrastructure for a BEB system with the objective of minimizing the total investment costs. Liu et al. (2021b) further integrated seasonality and power matching into the charger deployment problem and developed a mixed-integer nonlinear model to optimize the charging station location for a BEB system. The results pointed out that the energy consumption characteristics of BEBs have significant effects on the charger deployment for an electric transit system. An (2020) jointly considered charging facility planning and fleet construction and developed a stochastic integer optimization model considering the charging demand uncertainty. The results show a close relationship between charger deployment and battery size. Uslu and Kaya (2021) proposed a mixed-integer linear mathematical model to determine the optimal location and capacity of charging stations in a BEB system, assuming that the waiting time at the charging stations is limited. Moreover, Lin et al. (2019) and Li et al. (2022) regarded the charger deployment for a BEB system as a multi-stage planning problem and proposed respective optimization models to determine the optimal locations of charging stations during each stage from a long-term perspective. However, in the previous studies, the charger deployment techniques were focused on the problem scenario that the buses are charged at the specific terminals, but rarely discussed the charging events at bus stops under the opportunity charging system, especially using the pantograph chargers. Though some studies pointed out that the dynamic wireless charging can make the BEBs be charged while in motion and thus proposed corresponding mathematical models to deploy the dynamic wireless charging infrastructure (Liu and Song, 2017; Alwesabi et al., 2022), dynamic wireless charger and pantograph chargers differ significantly in terms of charger deployment techniques, investment costs, and application environment.

Besides the charger deployment, how to schedule the bus fleet in an efficient way is also a critical issue for the BEB system. It is demonstrated that the reasonable scheduling of the bus fleet has great potential to reduce the fleet size and corresponding investment costs (Ibarra-Rojas et al., 2015). Therefore, it is of importance from a cost perspective to incorporate fleet scheduling into the BEB system management. For the scheduling of the BEB fleet, the problem is more complicated than that for the conventional bus fleet, because the driving range limitation and charging events during BEB operation should be considered. In view of this, Niekerk et al. (2017) introduced BEBs in the classical vehicle scheduling problem (VSP), and the limited driving range and linear charging process are involved in the proposed model. Wen et al. (2016) developed a mixed-integer programming formulation for electric VSP to minimize the BEB fleet size and employed an adaptive large neighborhood search heuristic to solve the model. Yıldırım and Yıldız (2021) proposed an integer programming model to determine the BEB fleet composition and vehicle scheduling, which aims to minimize the procurement costs of the buses and operating costs. Xiong et al. (2022) investigated the VSP of electric buses by considering the constraints of charging event time to determine the optimal fleet size coupled with the charging strategy. Bie et al. (2021) further explored the BEB fleet scheduling problem considering the uncertainty in travel time and energy consumption based on the stochastic optimization method. In addition, He et al. (2020) and Liu et al. (2021a) focused on the charging strategy for BEBs and developed optimization models to guarantee a stable operational bus fleet schedule. Zhang et al. (2021) incorporated the battery degradation and non-linear charging profile into the BEB scheduling problem based on a set partitioning model. However, in the aforementioned optimization methods related to BEB fleet scheduling, the locations of charging facilities are often assumed to be predetermined and their impacts on the investment costs are not discussed. As a matter of fact, the charger deployment has a close interaction with the fleet scheduling in the BEB system. Therefore, only the integration of charger deployment and fleet scheduling can optimize the investment costs from the system level. To this end, Liu and Ceder (2020) provided a bi-objective integer programming model to examine the BEB fleet scheduling problem with the optimal number of chargers installed at terminal stations, while the battery nominal capacity is assumed to be given in advance. Alwesabi et al. (2021) discussed the integrated optimization of charger deployment and fleet scheduling for the BEB system with dynamic wireless charging infrastructure to minimize the total costs of the system. Nonetheless, less attention has been focused on the integrated optimization of charger deployment and BEB fleet scheduling under opportunity charging. On the other hand, the previous studies overlooked the changes in the route information over different time periods during the day. In practice, the bus line information, such as the ridership, dwelling time, and travel time, may vary during different time periods, which has impacts on the decision-making process for both charger deployment and fleet scheduling.

In summary, though the previous studies have made progress in charger deployment and BEB fleet scheduling, there are still some limitations, as mentioned above. To further clarify the differences with existing studies, we compare this study with the aforementioned references in terms of considerations, as listed in Table 1 (considered aspects are marked as “√”; otherwise, unconsidered ones

are marked as “×”). To bridge the existing research gaps, we attempt to develop a comprehensive optimization model considering both charger deployment and BEB fleet scheduling. The major objective of this work is to optimize the total annualized costs of the BEB system considering the operating characteristics of the opportunity charging system using pantograph chargers and variations of bus line information during different time periods. To achieve this objective, an integrated optimization model is developed by considering the battery nominal capacity, vehicle scheduling with optimal fleet size, and the deployment of pantograph chargers at bus stops and the central terminal. Several managerial insights are discussed in detail based on the optimization results and sensitivity analysis. The results imply that the opportunity charging system has a cost-effective advantage by installing more chargers at bus stops, and indicate that the BEB purchase costs and energy consumption rate have significant impacts on the total investment costs. The proposed method can be used by public transit operators and related stakeholders to construct and manage the BEB-based urban transit system.

To be specific, the contributions of this study are summarized as follows. First, our work extends the literature in the realm of BEB system operation. To our best knowledge, it is the first time that the integrated optimization problem of charger deployment and fleet scheduling is formulated based on the opportunity charging system. In addition, the changes in the bus line information over different time periods and their impacts on BEB operation are considered, such as the ridership, dwelling time, and travel time, to improve the performance and applicability of the proposed model in real-world scenarios. Second, a case study conducted in Oslo, Norway is presented to demonstrate the effectiveness of the proposed model, and the energy consumption patterns during different time periods are further analyzed based on the optimal results. Finally, we conduct a group of sensitivity analysis to investigate the system response to the battery costs, BEB purchase costs, charger installation costs, charging power, energy consumption rate, and related upper bounds in the model, and thus evaluate their contribution to the optimal results. Several insights stemming from the numerical experiments are discussed.

The rest of this paper is organized as follows. Section 2 presents the problem description. The integrated optimization model of charger deployment and fleet scheduling is presented in Section 3. In Section 4, the numerical case study and sensitivity analysis are furnished. Section 5 discusses the conclusions and future studies.

2. Problem description

In this study, we considered a public transport system that intends to replace conventional buses with BEBs under opportunity charging. To this end, the operator optimizes the charger deployment and fleet scheduling to minimize the total investment costs while satisfying the predetermined timetabled schedule. Specifically, charger deployment involves the tasks that determine the optimal locations and numbers of chargers installed at bus stops and the central terminal, respectively; fleet scheduling is performed to determine the optimal matches between BEBs and timetabled trips, where the required battery nominal capacity and fleet size for optimal scheduling are obtained. Note that a single-terminal transit network is considered in this study, where all the timetabled round-trips need to depart and return to the central terminal. Such a transit network is widely used in medium-sized cities and has been studied in a number of related works (Kim and Schonfeld, 2013; Rinaldi et al., 2020; Zhang et al., 2021). For simplicity, we refer to the round-trip as a trip for short below. The trips may derive from different bus lines and each bus line comprises several bus stops, where some bus stops may be shared by different bus lines, i.e., the transfer stops. To ensure the efficiency of the bus operations, some pantograph chargers should be installed at bus stops and the central terminal. The pantograph charger is an automatic connection fast charger that has been widely used in several countries such as Norway, the Netherlands, and Poland. Fig. 1 provides a real-world example of a BEB using a pantograph charger in Drammen, Norway. As shown, the BEB is automatically connected to the charger through the pantograph on the top of the vehicle for charging while it is at the bus stop. The main advantages of the pantograph charger lie in the following two points. First, the pantograph charger allows high charging power, and thus the bus can be charged within a short time when onboarding and/or offboarding passengers at bus stops. Second, the operational process of the pantograph charger does not require additional actions on the part of the driver in the charging process. During the charging process, the pantograph automatically rises and connects to the charger within a few seconds. Given these advantages, the pantograph charger is suitable for applications in the opportunity charging system, where there is a possibility of frequent charging on routes, i.e., at bus stops. Note that since the connecting time to the charger often accounts for a negligible proportion of the dwelling time, the charging time at the bus stop is generally considered to be equivalent to the dwelling time when onboarding and/or offboarding passengers (Bi et al., 2018; Lajunen, 2018; Zeng et al., 2022).

Without loss of generality, we consider that each bus stop can accept at most one charger while the central terminal can

Table 1
Comparison with previous studies.

References	Considerations		
	Charger deployment	Fleet scheduling	Opportunity charging
(Liu et al., 2018; Liu et al., 2021b; An, 2020; Uslu and Kaya, 2021; Lin et al., 2019; Li et al., 2022; Liu and Song 2017; Alwesabi et al., 2022)	√	×	×
(Niekerk et al. 2017; Wen et al., 2016; Yildirim and Yildiz, 2021; Xiong et al., 2022; Bie et al., 2021; He et al., 2020; Liu et al., 2021a; Zhang et al., 2021)	×	√	×
(Liu and Ceder, 2020; Alwesabi et al., 2021)	√	√	×
This study	√	√	√

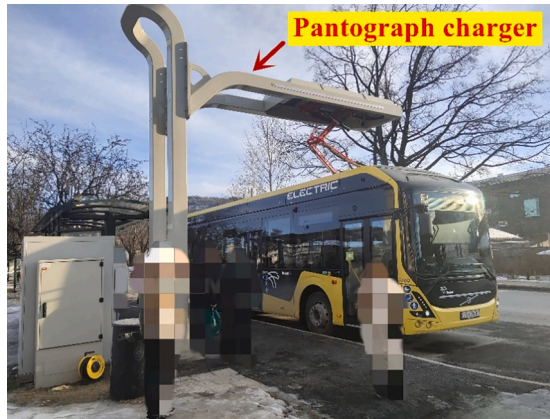


Fig. 1. A BEB using a pantograph charger for charging.

accommodate more than one charger under a given upper bound due to the land resource limitation. During the BEB operation under opportunity charging, the vehicle can be recharged at the bus stop during dwelling times, if a pantograph charger is installed at the bus stop. The BEB can also be charged when finished a trip and returned to the central terminal. Under this operational characteristic, there is a significant interaction between charger deployment and battery nominal capacity. That is, the locations of chargers installed at bus stops determine the variation trend of battery energy and thus affect the required nominal capacity of the batteries equipped in the BEBs. Meanwhile, the battery nominal capacity also influences the energy consumption rate due to its contribution to the vehicle mass and thus affects the charger deployment (Basma et al., 2022). Both the charger deployment and battery nominal capacity have impacts on the charging time at the central terminal, and thus affect the fleet scheduling to satisfy the predetermined timetabled schedule, which further determines the BEB fleet size. The outline of the integration optimization of charger deployment and fleet scheduling for the BEB system is illustrated in Fig. 2. As shown, there is an interaction between charger deployment (top part of the figure) and fleet scheduling (bottom part of the figure), where the battery nominal capacity bridges an important link due to its effects on both charger deployment and fleet scheduling, as mentioned above.

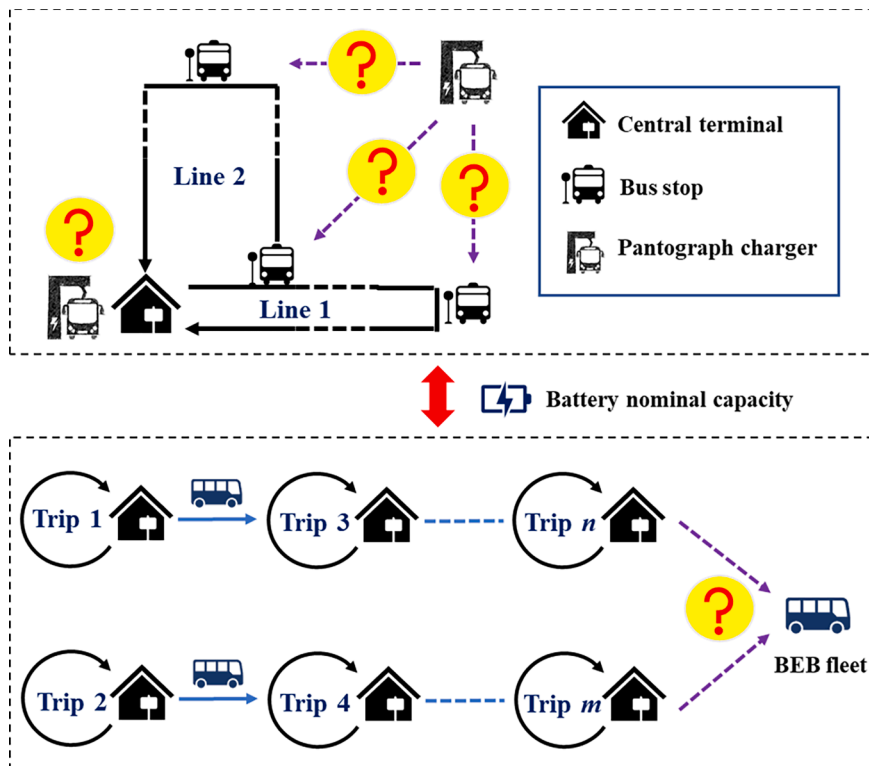


Fig. 2. Outline of the integration optimization of charger deployment and fleet scheduling.

The overarching research question in this study is how to determine an optimal subset of bus stops for installing pantograph chargers, the optimal number of pantograph chargers installed at the central terminal, and the optimal scheduling of the BEB fleet. The studied problem can be regarded as the combination of the strategic issue for charger deployment and the operation issue for fleet scheduling. Furthermore, this study considers that the chargers are dedicated to charging the BEBs that serve the related bus lines, while other vehicles or buses are not allowed to use them, which is commonly seen in public transit systems and BEB scheduling models (Bi et al., 2018; An, 2020; Liu and Ceder, 2020). The objective of the proposed model is to minimize the total annualized costs, including BEB purchase costs and the costs for charger installation at both bus stops and the central terminal. In particular, we further consider the variations of bus line information over different time periods in the problem formulation, including the ridership, dwelling time on bus stops, and travel time for a trip. Specifically, the ridership takes significant effects on the vehicle mass and further influences the energy consumption rate; the dwelling time on bus stops has an influence on the amount of charged energy during BEB operation; the travel time has an impact on the required number of BEBs to serve all the trips from the predetermined timetabled schedule. In practice, these three realistic factors have time-varying characteristics and may vary among different time periods, i.e., peak, moderate, and off-peak times. Therefore, their variations over different time periods should be considered to improve the performance and applicability of the proposed model in real-world scenarios.

To further clarify the problem and facilitate model development, we make some assumptions related to chargers, BEBs, and bus line information as follows:

Assumption 1. All BEBs are homogeneous with the same battery size.

Assumption 2. All chargers are homogeneous pantograph chargers, and each charger is equipped with one outlet.

Assumption 3. The bus line information during different time periods is known, which involves the travel time for each trip, dwelling time on each bus stop, and ridership on each bus line.

Assumption 4. The energy consumption is linearly proportional to the driving distance, and the energy consumption rate is also partly related to the vehicle mass.

Assumption 5. The energy recharged is linearly proportional to the charging duration within the predetermined battery state-of-charge (SOC) range.

Assumption 6. The BEB must be recharged during dwelling time when reaching the bus stops with chargers. Meanwhile, when the bus completes its current trip and returns to the central terminal, it must be charged to the upper limit of the predetermined SOC range.

For **Assumption 1**, we refer to the existing research results that the fleet scheduling with uniform battery nominal capacity is more cost-effective than route-specific battery sizes (Alwesabi et al., 2020). For **Assumption 2**, the pantograph charger with one outlet is a popular type of automatic connection fast charger, and this assumption has also been used in BEB modeling literature (Wang et al., 2017). Moreover, this assumption can be relaxed as it does not change the structure of the optimization model. **Assumption 3** shows that the bus line information is deterministic, which is acceptable for a tactical decision-making problem targeted by this study. For **Assumption 4**, several studies clarify the significant linear relationship between energy consumption and driving distance, and such an assumption is commonly used in the existing literature (Wang et al., 2018, 2021). On this basis, we further consider the relationship between vehicle mass and energy consumption rate for a BEB, which would affect the battery capacity decision involved in the model (Redelbach et al., 2014). **Assumption 5** is realistic because the charging rate is constant within the specific SOC range, i.e., usually between 20% and 80% (Hwang et al., 2017). In addition, to extend the battery life, deep charging and discharging are often avoided in practical applications. **Assumption 6** is realistic for the opportunity charging system, where the BEBs often need to be frequently charged during operation, because their batteries usually have a relatively small nominal capacity, as discussed in the literature (Qin et al., 2016; Xylia et al., 2017; Bi et al., 2018). This assumption also ensures the deterministic initial SOC of BEBs at the beginning of each trip, which is commonly seen in BEB scheduling models and is also beneficial to reduce the computational burden (Zhang et al., 2021).

3. Methodology

In this section, the mathematical formulations of the optimization model are developed based on the problem description in the previous section. To solve the complex optimization problem, we apply model linearization so that the optimal solutions can be obtained using a standard optimization solver. The notations used throughout this study are summarized in the Appendix.

3.1. Model formulation

The proposed integrated optimization framework involves three primary decision-making tasks. Firstly, we need to select the optimal locations to install pantograph chargers at bus stops and determine the optimal number of pantograph chargers installed at the central terminal. Secondly, we determine the optimal battery nominal capacity for the BEBs by considering the bus line information and the charger locations. Finally, we optimize the fleet scheduling to satisfy the predetermined timetabled schedule. Let S and I denote the sets of bus stops and trips, respectively. Accordingly, the optimization model has four decision variables, as follows. Firstly, the decision variable involved in the charger deployment at bus stops is a binary variable x_s , which represents whether to install a pantograph charger at the bus stop $s \in S$. Secondly, the decision variable involved in the charger deployment at the central terminal is

the integer variable z , which represents the number of pantograph chargers installed at the central terminal. Thirdly, the positive integer variable g is defined as the decision variable to represent the battery nominal capacity. Finally, the decision variable involved in the BEB fleet scheduling is denoted by a binary variable φ_{ij} , which represents whether a BEB serves trips $i \in IUO$ and $j \in IUD$ consecutively. Thereinto, we use O and D to represent the original and destination depots respectively for notational convenience. This is a commonly-used notational definition in bus scheduling models to capture the start and end of a schedule for a specific bus (Zhang et al., 2021). In this way, the optimal number of BEBs from the fleet scheduling scheme can be captured by summing φ_{Oi} , which can be used to represent whether a BEB is dispatched to serve its first trip i during the daily operation. Similarly, φ_{iD} can represent whether a BEB finishes its last trip i . Based on these definitions, the integrated optimization model is formulated as

$$\min \sum_{i \in I} \varphi_{Oi} \cdot c_{bat} \cdot g + \sum_{i \in I} \varphi_{Oi} \cdot c_{bus} + z \cdot c_{pc} + \sum_{s \in S} x_s \cdot c_{pc} \tag{1}$$

s.t.

$$e_{O'r\hat{t}} = 0, \forall r \in R, \hat{t} \in \hat{T} \tag{2}$$

$$e_{sr\hat{t}} = e_{O'r\hat{t}} + q_{r\hat{t}} \cdot d_{O'r}, \forall s \in S_r, r \in R, (O', s) \in L_r, \hat{t} \in \hat{T} \tag{3}$$

$$e_{s'r\hat{t}} = e_{sr\hat{t}} + q_{r\hat{t}} \cdot d_{sr} - \rho \cdot x_s \cdot \tilde{\tau}_{r\hat{t}}, \forall s' \in S_r \cup D', s \in S_r, r \in R, (s, s') \in L_r, \hat{t} \in \hat{T} \tag{4}$$

$$q_{r\hat{t}} = q' \cdot \left(1 - \frac{w_b^{bat} - \frac{g}{\delta} + \Delta w_{r\hat{t}}^{adj}}{w_b^{bus}} \times 4.5\% \right), \forall r \in R \tag{5}$$

$$e_{sr\hat{t}} \leq g \cdot (e^{ub} - e^{ul}), \forall s \in S_r \cup D', r \in R, \hat{t} \in \hat{T} \tag{6}$$

$$g^{\min} \leq g \leq g^{\max} \tag{7}$$

$$\sum_{j \in IUD} \varphi_{ij} = 1, \forall i \in I \tag{8}$$

$$\sum_{i \in IUO} \varphi_{ij} = 1, \forall j \in I \tag{9}$$

$$\sum_{i \in T} \mu_{it} \cdot t \geq t_i + \tau_{r\hat{t}}, \forall i \in I_r, \hat{t} \in \hat{T}, t_i \in \hat{t} \tag{10}$$

$$\sum_{i \in T} \eta_{it} \cdot t = \sum_{i \in T} \mu_{it} \cdot t + \frac{e_{O'r\hat{t}}}{\rho}, \forall i \in I_r, r \in R, \hat{t} \in \hat{T}, t_i \in \hat{t} \tag{11}$$

$$\sum_{i \in T} \mu_{it} = 1, \forall i \in I \tag{12}$$

$$\sum_{i \in T} \eta_{it} = 1, \forall i \in I \tag{13}$$

$$\theta_{it} = \sum_{i \leq t} \mu_{it} - \sum_{i \leq t} \eta_{it}, \forall i \in I, t \in T \tag{14}$$

$$\sum_{i \in I} \theta_{it} \leq z, \forall t \in T \tag{15}$$

$$1 \leq z \leq k \tag{16}$$

$$t_j \geq \left(\sum_{i \in T} \eta_{it} \cdot t \right) \cdot \varphi_{ij}, \forall i, j \in I, t \in T \tag{17}$$

$$g \in Z^+ \tag{18}$$

$$z \in Z \tag{19}$$

$$x_s \in \{0, 1\}, \forall s \in S \tag{20}$$

$$\varphi_{ij} \in \{0, 1\}, \forall i, j \in I \cup O \cup D \quad (21)$$

$$\eta_{it} \in \{0, 1\}, \forall i \in I, t \in T \quad (22)$$

$$\mu_{it} \in \{0, 1\}, \forall i \in I, t \in T \quad (23)$$

Objective (1) minimizes the total annualized costs considering the trade-off between the fleet size, battery nominal capacity, and the number of chargers installed at bus stops and the central terminal, where c_{bat} , c_{bus} , c_{pc} respectively represent the annualized costs of the battery per unit, BEB, and pantograph charger, which can be obtained by dividing their acquisition costs into their respective life spans (Liu et al., 2018). The first term calculates the annualized costs of batteries used in the BEB fleet, which depend on the optimal results for both battery nominal capacity and feet size; the second term determines the annualized costs for the BEB fleet composition depending on the number of BEBs required for optimal scheduling scheme; the third and final terms calculate the annualized costs for the charger deployment, which depend on the optimal number of chargers installed at bus stops and the central terminal, respectively. Compared to the formulations in previous studies, the objective function in the proposed model jointly considers the battery nominal capacity, bus fleet size, locations of pantograph chargers at bus stops, and the number of pantograph chargers at the central terminal, and thus can optimize the total investment costs from the system level.

Constraints (2)-(7) present the restrictive conditions regarding charger deployment at bus stops coupled with the battery nominal capacity. As Bi et al. (2018) pointed out, energy consumption rate and dwelling time have a critical influence on the selection of bus stops to install chargers. However, existing charger deployment works have rarely considered the time-varying characteristics of bus line information. In this study, we consider the variations of ridership and dwelling time during different time periods in the model formulation to explore a more realistic solution. To be specific, constraints (2)-(4) calculate the cumulative energy demand for each bus line $r \in R$ until stop s , where R is the set of bus lines, and $e_{sr\hat{t}}$ represents the cumulative energy demand for route r until stop s during the time period $\hat{t} \subseteq \hat{T}$, where \hat{T} denotes the set of time periods; O' and D' are the dummy nodes for central terminal representing the departure point and arrival point, respectively, for every trip; d_{sr} represents the driving distance from bus stop s to the subsequent one on bus line r ; ρ is the charging power of the pantograph charger; $\bar{\tau}_{r\hat{t}}$ represents the dwelling time at each bus stop on bus line r during time period \hat{t} ; $q_{r\hat{t}}$ represents the energy consumption rate for bus line r during time period \hat{t} , which can be obtained using constraint (5). Moreover, constraints (2)-(4) present the variation trend of battery energy with the charger deployment decision, and also reflect the operational characteristics of the opportunity charging system, which attempts to improve the system efficiency by introducing more chargers and allowing charging at bus stops. The constraints also imply that installing chargers at transfer stops has greater potential to reduce the charger deployment costs. For constraint (5), we refer to the research results from Bi et al. (2015), which have demonstrated that 10% vehicle mass reduction contributes to about 4.5% energy consumption reduction for a BEB. On this basis, constraint (5) considers the effects of battery weight and ridership weight on the energy consumption rate, where q' is the base energy consumption rate of the reference BEB; w_b^{bat} and w_b^{bus} represent the base battery weight and base bus weight of the reference BEB, respectively; δ is the battery specific energy, which associates the battery nominal capacity with the battery weight; $\Delta w_{r\hat{t}}^{adj}$ is the ridership weight adjustment for bus line r during period \hat{t} . Constraint (6) indicates the ideal battery SOC range and ensures that the cumulative energy demand must be between the upper and lower SOC limits, where e^{ub} and e^{ll} are the upper and lower bounds of the battery SOC, respectively. Constraint (7) indicates that the battery nominal capacity could be within the reasonable range depending on the market availability, where g^{\min} and g^{\max} are respectively the lower and upper bounds for the battery nominal capacity.

Constraints (8)-(17) give the restrictive conditions with respect to the fleet scheduling coupled with the charger deployment at the central terminal. Several studies recently investigated the BEB fleet scheduling problems, whereas little research has incorporated the impacts of charger deployment into BEB fleet scheduling under opportunity charging (Liu and Ceder, 2020; Zhang et al., 2021; Xiong et al., 2022). In the proposed model, the battery energy variations during BEB operation are influenced by both the energy consumption rate and the locations of chargers at bus stops. To be specific, constraints (8) and (9) ensure that all the trips must be implemented once to satisfy the predetermined timetabled schedule. To formulate the charging events during the fleet scheduling process, the scheduling horizon is discretized into finite time slots of a certain interval. On this basis, constraints (10)-(15) restrict and calculate the charging start time and end time as the BEB finishes its current trip and returns to the central terminal. Constraint (10) ensures that the charging start time after trip i must be later than the arrival time of the trip considering the variations of driving time during different time periods, where μ_{it} is a binary variable represents whether a BEB begins to charge in time slot t after trip i at the central terminal; t_i is the departure time of trip i ; $\tau_{r\hat{t}}$ represents the travel time for bus line i during the time period \hat{t} . Constraint (11) calculates the charging end time after trip i as the charging start time is determined, where η_{it} is a binary variable representing whether a BEB ends charging in time slot t after trip i at the central terminal. More importantly, this constraint presents the impacts of charger locations at bus stops on the fleet scheduling, where $e_{D'r\hat{t}}$, representing the amount of charged energy, is determined based on the decisions regarding charger deployment at bus stops. Constraints (12) and (13) ensure that a BEB must be charged when it finished trip i and returned to the central terminal. Constraint (14) links variables μ_{it} and η_{it} , where θ_{it} is a binary variable representing whether a BEB is charged in time slot t after trip i . Constraint (15) ensures that the number of BEBs being charged in each time slot i cannot be larger than the number of chargers installed at the central terminal. Constraint (16) indicates that the number of chargers installed at the central terminal needs to be limited within a given range, in view of the foundational charging requirements and restricted construction land space. Constraint (17) is to make sure that the departure time of trip j must be no earlier than the charging end time

after trip i , if a BEB serves trips i and j consecutively. In addition, Eqs. (18)-(23) present the definitional constraints of the variables.

3.2. Model linearization

Note that the integrated optimization model is a mixed-integer nonlinear optimization problem due to the nonlinearity in both the objective function and constraints. Specifically, for the objective function, it can be observed that the first term ($\sum_{i \in I} \varphi_{O_i} \cdot c_{bat} \cdot g$) is nonlinear; for the constraints, constraint (17) is nonlinear. The nonlinearity of the model results in significant-high complexity to solve the problem. This is because conventional exact algorithms or commercial solvers have limited ability to solve complex nonlinear models. To deal with the nonlinearity in the objective function, an auxiliary variable ω_{O_i} is defined, and a large enough constant M is further introduced with the following relationship between g and ω_{O_i} :

$$\omega_{O_i} \geq g + (\varphi_{O_i} - 1) \cdot M, \forall i \in I \tag{24}$$

$$\omega_{O_i} \geq 0, \forall i \in I \tag{25}$$

Thus, the nonlinear term ($\sum_{i \in I} \varphi_{O_i} \cdot c_{bat} \cdot g$) can be replaced by ($\sum_{i \in I} \omega_{O_i} \cdot c_{bat}$), and the objective function is equivalently transformed to Eq. (26).

$$\min \sum_{i \in I} \omega_{O_i} \cdot c_{bat} + \sum_{i \in I} \varphi_{O_i} \cdot c_{bus} + z \cdot c_{pc} + \sum_{s \in S} x_s \cdot c_{pc} \tag{26}$$

In addition, the nonlinear constraint (17) can be replaced by the linear constraint (27):

$$t_j \geq \sum_{i \in T} \eta_{ij} \cdot t + (\varphi_{ij} - 1) \cdot M, \forall i, j \in I \tag{27}$$

After model linearization, the model can be regarded as a mix-integer linear programming problem, and thus can be resolved by using a standard optimization solver, for example, the Gurobi (Gurobi Optimization, 2021).

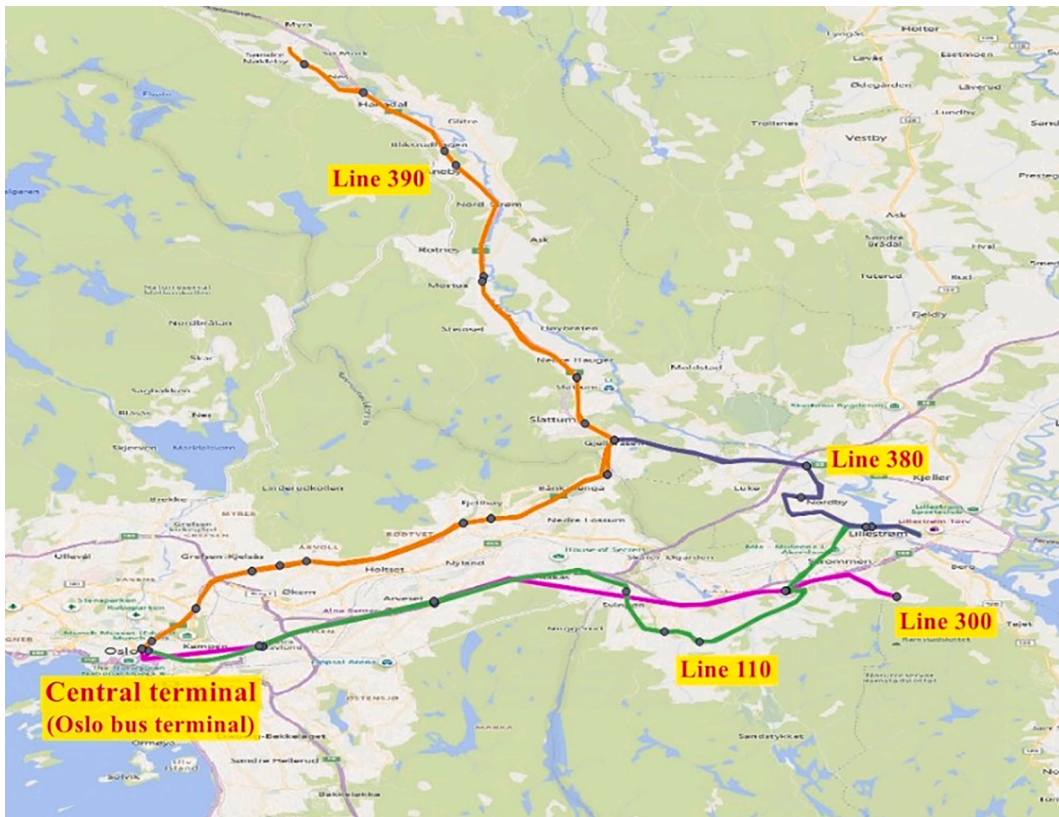


Fig. 3. Bus lines in the case study.

4. Case study

4.1. Example scenario description

In this section, a case study is presented to demonstrate the integrated optimization model for charger deployment and fleet scheduling under opportunity charging. Referring to the real-world scenario from Oslo, Norway, we apply the proposed model to a public transit system with four bus lines, including line 110, line 300, line 380, and line 390, as shown in Fig. 3. All these four bus lines depart from and return to the same central terminal, namely the Oslo bus terminal, and the trip lengths of the bus lines are 50.44 km, 41.32 km, 57.3 km, and 79.32 km, respectively. There are 228 bus stops of the four bus lines in the road network, and the distances between two adjacent bus stops are often less than 1 km. However, it is unnecessary to install chargers at two bus stops within a very short distance (Sebastiani et al., 2016). On the other hand, considering too many bus stops would bring a significant challenge for computing. Therefore, we apply the K-means clustering method to reduce the scale of bus stops in the case study, which is a widely used approach to effectively simplify the network of public transit systems (De Bona et al., 2021). Finally, 31 candidate stops are considered for installing pantograph chargers, and some of them are shared by more than one bus line.

In the city of Oslo, the urban bus system operates throughout the day. To reduce the problem scale, we select the time frame 6:00–17:00 as the scheduling horizon covering the three periods. Specifically, the peak duration is the time frame 6:00–9:00; the moderate duration covers the time frames 9:00–11:00 and 15:00–17:00; the off-peak duration is the time frame 11:00–15:00. During each time period, the information of each bus line, including the ridership weight adjustment Δw_{rt}^{adj} , dwelling time on each candidate stop $\tilde{\tau}_{rt}$, and travel time for trip τ_{rt} , is listed in Table 2. The ridership weight adjustment for each bus line refers to the deviation from the base ridership weight. This case study refers to the related literature and considers that the number of passengers for the base ridership is 27 and the average weight per passenger is 68 kg (Hjelkrem et al., 2021). For example, if the number of passengers is 28, the ridership weight adjustment is the weight of one passenger, i.e., 68 kg.

Other parameters in the case study are given in Table 3, such as the parameters related to the annualized costs, energy consumption, and specific coefficients in the proposed model. Observably, most of the parameters are assigned with reference to the related literature (Bi et al., 2018; An, 2020; Hjelkrem et al., 2021). The maximum number of chargers that can be built at the central terminal is empirically given due to the lack of related data.

Moreover, for the fleet scheduling, we discretize the scheduling horizon, i.e., 660 min, into 132 time divisions, where the measurement interval of each time division is 5 min. Such a setting for the scheduling horizon discretization is also used in the existing bus scheduling strategy to reasonably reduce the computational burden (Liu et al., 2021). During the scheduling horizon, the timetables for the bus lines are listed in Table 4. In total, 113 trips are considered in the case study.

4.2. Optimal results

We use Gurobi optimizer 9.1.2 to solve the optimization model. The case study is conducted on a computer with Windows 10 operating system equipped with an Intel (R) Core (TM) i7-10610U CPU at 1.80 GHz 2.30 GHz, 32 GB of RAM. The model consists of 293 continuous variables and 58,129 integer variables, including 58,115 binary variables. The model also has 29,521 constraints. The results obtained from the optimal solution show that the overall annualized costs of the BEB system under opportunity charging are €1,204,028. Note that as the proposed mixed-integer linear programming model is NP-hard, the solver is unable to guarantee to find the optimal solution within an acceptable amount of time. Nonetheless, the Gurobi optimizer can obtain the solution with high quality measured by the optimality gap. Specifically, the relative optimality gap is lower than 4%, and the absolute gap is lower than € 48,000 after the solver operates within 30 min. In view of the problem scale and large annualized costs invested for the BEB system, it is reasonable to believe that the solution got from the solver is sufficiently accurate and acceptable for the transit agencies, as pointed out in related studies (Huang et al., 2021; Li et al., 2022). The total annualized costs include BEB purchase costs, battery costs, and charger deployment costs. It is found that 28 BEBs are required to implement a total of 113 trips, and the nominal capacity of the battery for each BEB is 152 kWh. The optimal scheduling of the BEB fleet for the case study is illustrated in Fig. 4. This figure provides the chain of connected trips assigned to each BEB to ensure that all the timetabled trips are served once; the ordinate and abscissa respectively present the bus lines and timetables, i.e., departure times, for the trips; the line with specific color and marker illustrates the chain of connected trips for each BEB. Note that the number of binary variables has a dominant influence on the model efficiency; most binary variables come from the scheduling part of the model; the number of variables mainly depends on the number of trips in the bus network. A simulation test shows that the proposed model can handle the problem with up to 321 trips within five hours, where the

Table 2
Bus line information during different time periods.

	Off-peak duration			Moderate duration			Peak duration		
	Δw_{rt}^{adj} (kg)	$\tilde{\tau}_{rt}$ (s)	τ_{rt} (min)	Δw_{rt}^{adj} (kg)	$\tilde{\tau}_{rt}$ (s)	τ_{rt} (min)	Δw_{rt}^{adj} (kg)	$\tilde{\tau}_{rt}$ (s)	τ_{rt} (min)
Line 110	68	30	95	1564	45	100	4284	60	105
Line 300	680	30	43	2516	45	46	4080	60	50
Line 380	340	30	118	1972	45	122	4420	60	128
Line 390	884	30	158	2040	45	164	3944	60	170

Table 3
Scenario parameters of the case study.

Parameters	Symbols	Values	References
Annualized purchase costs of a BEB (£/year)	c_{bus}	24,625	(An, 2020; Yildirim and Yildiz, 2021)
Annualized costs of a pantograph charger (£/year)	c_{pc}	20,000	(An, 2020; Pelletier, et al., 2019)
Annualized battery costs per unit (£/kWh/year)	c_{bat}	88	(An, 2020; Yildirim and Yildiz, 2021)
Lower bound for battery nominal capacity (kWh)	g^{min}	60	(Gao et al., 2017)
Upper bound for battery nominal capacity (kWh)	g^{max}	200	(Gao et al., 2017)
Base consumption rate (kWh/km)	q'	1.24	(Bi et al., 2018)
Base battery weight (kg)	w_b^{bat}	2,492	(Bi et al., 2018)
Battery specific energy (kWh/kg)	δ	13	(Bi et al., 2018)
Base bus weight (kg)	w_b^{bus}	15,000	(Bi et al., 2018)
Charging power of the pantograph charger (kW)	ρ	300	(Pelletier, et al., 2019)
The upper bound of the usable SOC range	ϵ^{ub}	0.8	(Hwang et al., 2017)
The lower bound of the usable SOC range	ϵ^{ll}	0.2	(Hwang et al., 2017)
Maximum number of chargers at the central terminal	k	5	/

Table 4
Timetables for the bus lines.

Line 110	Line 300	Line 380	Line 390
6:10:00	6:05:00	6:05:00	6:10:00
Every 15 min	Every 15 min	Every 20 min	Every 20 min
8:55:00	9:05:00	8:05:00	8:50:00
Every 20 min	Every 20 min	Every 30 min	Every 30 min
10:55:00	11:05:00	16:35:00	16:50:00
Every 30 min	Every 30 min		
14:55:00	15:05:00		
Every 20 min	Every 20 min		
16:55:00	16:45:00		

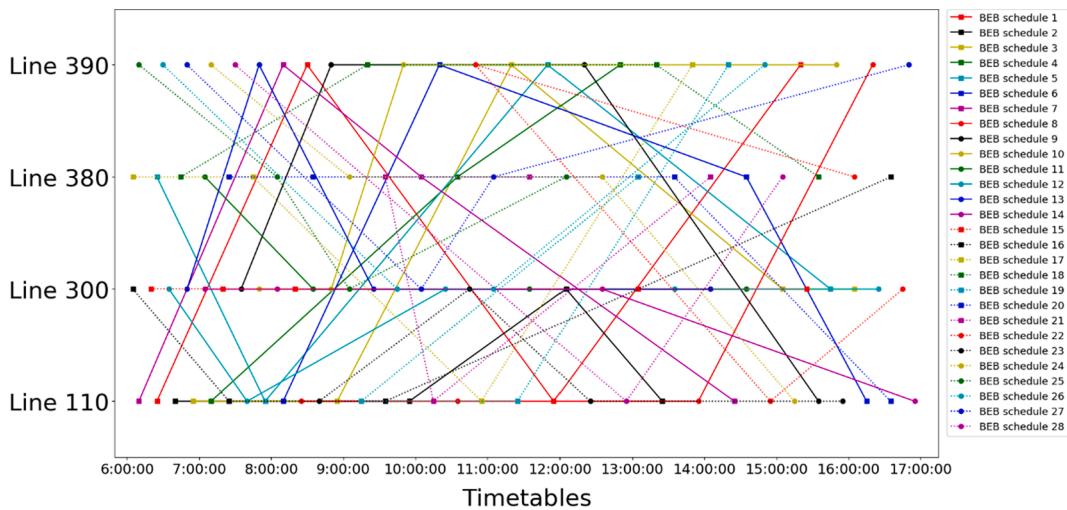


Fig. 4. Optimal scheduling of BEB fleet for the case study.

model has 200,423 binary variables and the relative optimality gap is lower than 4%.

Furthermore, it is found that the model selects four bus stops to install the pantograph chargers, and their locations are presented in Fig. 5. The underlying reasons for such a distribution of the selected bus stops primarily lie in the following two points. First, the locations of the pantograph chargers need to ensure that a BEB can successfully finish a trip from any bus line without violating the usable battery SOC. Second, the proposed model attempts to select the bus stops that are shared by more than one bus line, i.e., transfer stops, to install the charger. In this way, the charger deployment costs can be reduced, and the utilization efficiency of the charger is improved. Therefore, as shown in Fig. 5, two chargers are installed at the transfer stops. Meanwhile, the solution shows that three pantograph chargers are required to be installed at the central terminal.

In the proposed model, the impacts of vehicle mass on energy consumption rate are considered, and thus the energy consumption

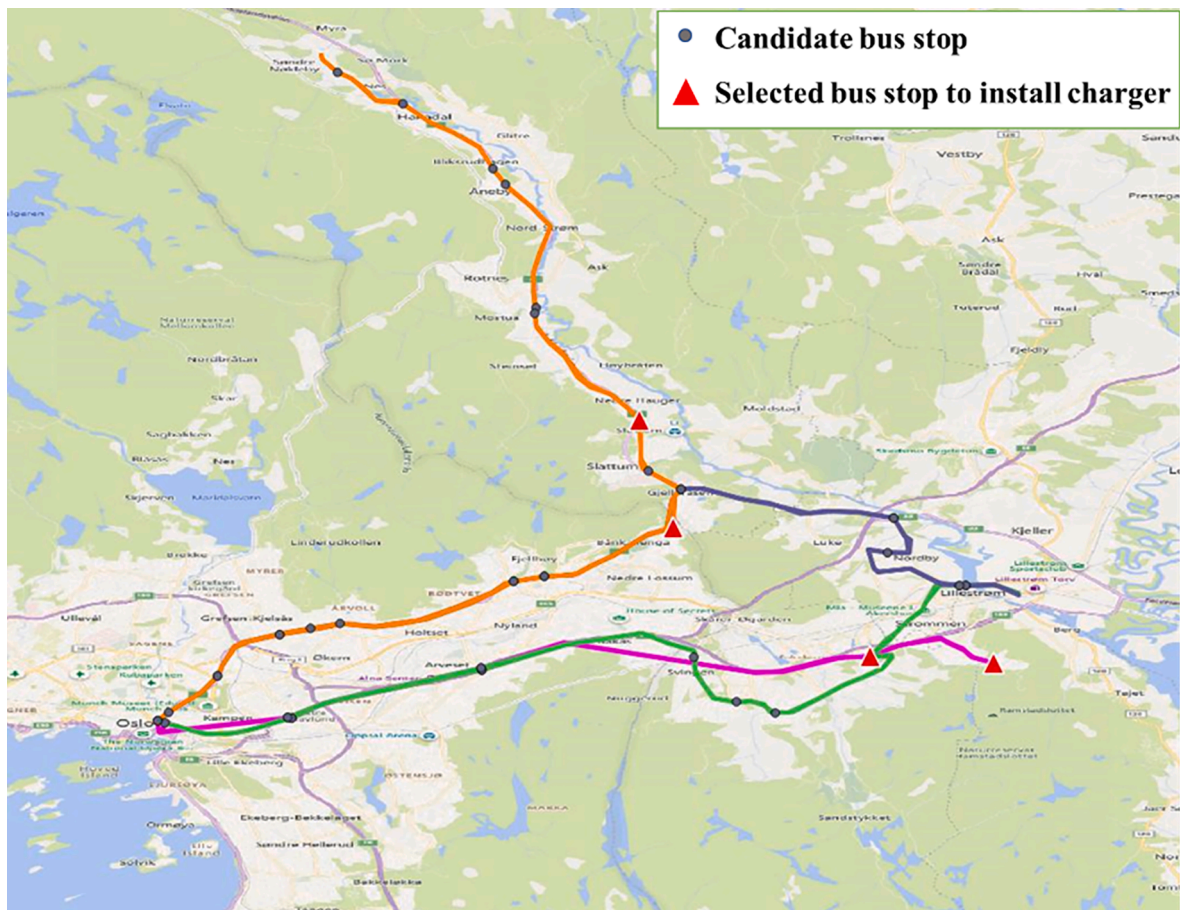


Fig. 5. The bus stops that are selected to install the pantograph chargers.

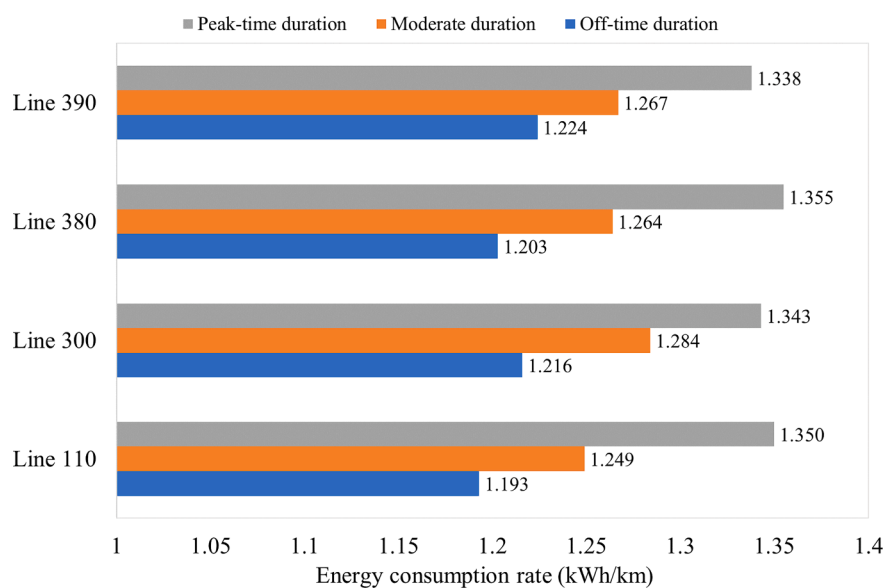


Fig. 6. The energy consumption rate for each bus line during different time periods.

rate varies during different time periods because the ridership weights are time-variant. Fig. 6 shows the energy consumption rate for each bus line during different time periods. It is found that the energy consumption rate for each bus line has significant differences among different time periods, where the peak and off-peak hours respectively contribute to the highest and lowest energy consumption rates, making the maximum difference in energy consumption rate 0.157 kWh/km.

Once the locations of chargers at candidate bus stops are determined, the variation trend of battery SOC, i.e., SOC pattern, for each bus line can be derived, which has significant effects on battery nominal capacity and fleet scheduling. Specifically, line 110, line 300, line 380, and line 390 respectively have 12, 9, 14 and 17 candidate bus stops in the case study. Fig. 7 shows the SOC patterns for the bus lines during different time periods and presents the battery SOC when the BEB departs from the bus stops. The numbers on the horizontal axis represent the serial numbers of the bus stops following the sequence of the round-trip along the corresponding bus lines. For example, the bus stops with the first and last serial numbers are both the spatially nearest bus stops to the central terminal, while they imply two opposite directions for a round-trip. The results present the differences in the SOC patterns among different time periods, which are mainly caused by the differences in energy consumption rate and dwelling time on each candidate bus stop. The peak-time duration results in a higher energy consumption rate while longer dwelling time on the candidate bus stops, compared to other time periods. Therefore, the battery SOC exhibits a faster descending rate in motion but shows a more dramatic rise after charging at bus stops during the peak-time duration.

In this study, we focus on the opportunity charging system, where the BEBs can be charged at the bus stops during the dwelling time. In a real-world case, some of the public transit agencies only use the pantograph chargers at the central terminal (Lajunen, 2018). To evaluate the effectiveness of the proposed model, we further solve the optimization model for end station charging and then compare its optimal results with the proposed model for opportunity charging. In the optimization model for end station charging, the only difference with the proposed model for opportunity charging is that the decision variables regarding the locations of pantograph chargers at bus stops are not involved. The results indicate that the opportunity charging can reduce the total annualized costs by 3.55% compared to the end station charging. Specifically, the end station charging system requires 29 BEBs equipped with 178 kWh batteries and four pantograph chargers installed at the central terminal to implement the 113 trips in the case study. Fig. 8 (a) and (b) provide the component proportions of the total annualized costs of the opportunity charging and end station charging systems, respectively. It is found that installing pantograph chargers at bus stops under the opportunity charging system can reduce the battery nominal capacity and BEB fleet size, and thus saves the total annualized costs, even though the number of chargers under opportunity charging is increased compared to the end station charging. Note that the battery degradation resulting from intermittent charging with a short time under opportunity charging is not considered in this study. It is because the battery used in the BEB, i.e., Lithium iron

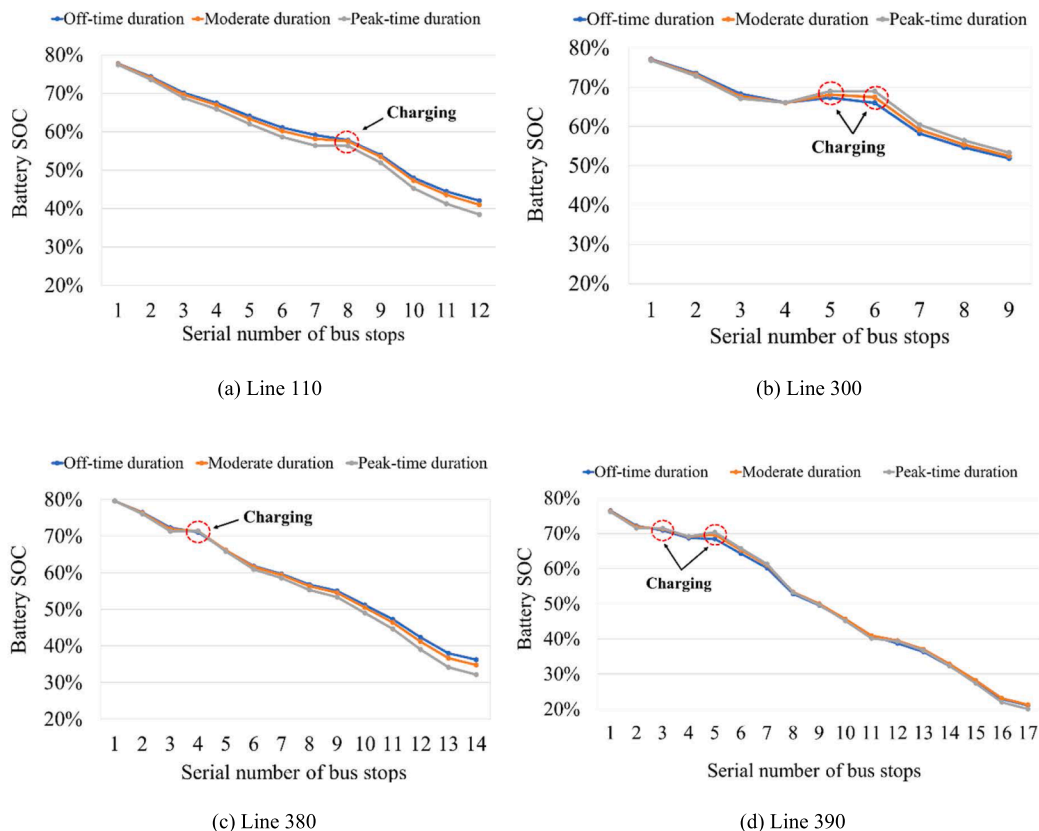


Fig. 7. SOC patterns for the bus lines during different time periods.

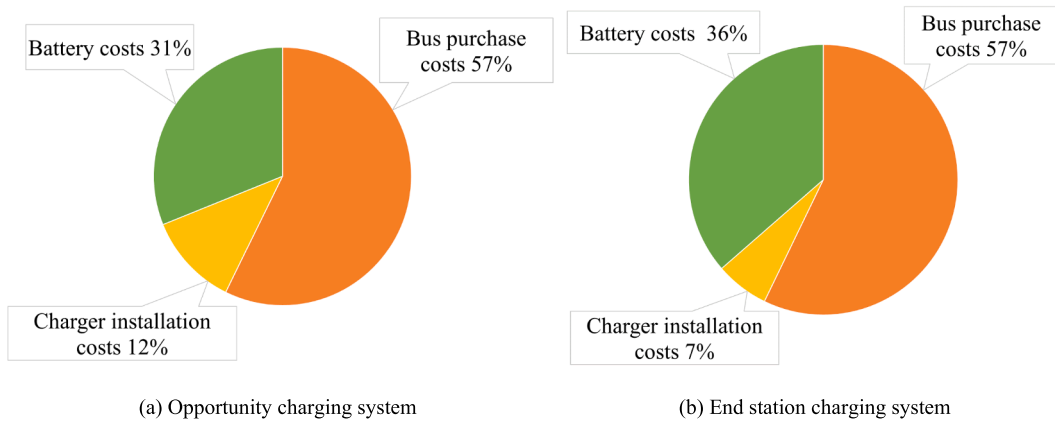


Fig. 8. Component proportion of the total annualized costs for opportunity and end station charging systems.

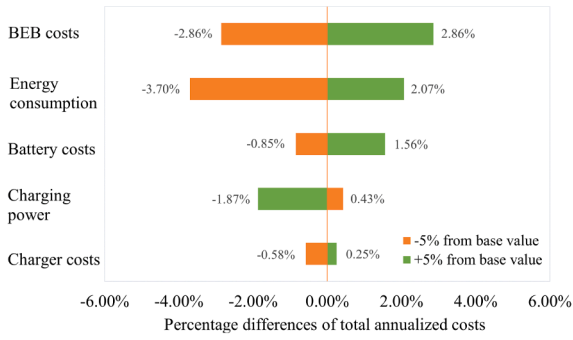
phosphate-based battery, often has a low memory effect, and thus the intermittent charging without overcharge has a limited impact on the battery health (Lukic et al., 2008; Han et al., 2019; Zhang et al., 2021). If necessary, it is straightforward to integrate battery degradation costs into the objective function by adjusting the battery costs. We will investigate the influence of battery cost variation on the optimal results in sensitivity analysis, as seen in Section 4.3. In addition, overnight charging is not discussed in this section because it often utilizes the slow charger and BEBs cannot be charged during the service operation. The installation costs and charging power of the slow charger are not involved in this study, which is significantly different from that of the pantograph charger.

4.3. Sensitivity analysis

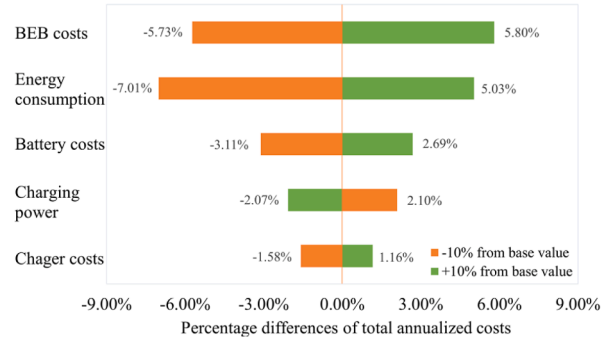
In the case study, several key parameters of the example scenario are referenced in existing studies, as mentioned in Section 4.1. On this basis, we further carry out a series of experiments for sensitivity analysis to explore the impact of the key parameters on the total annualized costs. The parameters considered in the sensitivity analysis include the annualized battery costs per unit c_{bat} , annualized purchase costs of a BEB c_{bus} , annualized installation costs of a pantograph charger c_{pc} , charging power of the pantograph charger ρ , and base energy consumption q' . The sensitivity analysis evaluates these parameters by changing their values individually. Specifically, the values of each parameter in sensitivity analysis are determined by respectively increasing and decreasing 5%, 10%, 20%, and 40% from the values presented in the case study, and then the corresponding optimal total annualized costs are compared with the baseline results as mentioned in Section 4.2. The results of the sensitivity analysis are summarized in Fig. 9, which presents the percentage differences of total annualized costs with a variation of each parameter under different changing ranges. As seen, the optimal total annualized costs are significantly affected by the annualized purchase costs of BEB and the base energy consumption, while the results are less affected by the charging power and annualized installation costs of the charger. Meanwhile, the annualized battery costs per unit have a moderate impact on the results as compared to other parameters.

Besides the aforementioned key parameters, the optimization model also gives the upper bounds of the battery nominal capacity g^{\max} and the number of chargers that can be built at the central terminal k . Here, we further consider other conditions to explore their impacts on the optimal results. Fig. 10 and Fig. 11 respectively illustrate the optimal solutions regarding the battery nominal capacity and the number of chargers at the central terminal, as their upper bounds change, coupled with corresponding total annualized costs. It is found from Fig. 10 that the minimum requirement for the battery nominal capacity is 98 kWh, which contributes to larger total annualized costs compared to the baseline results because more chargers are required as the battery nominal capacity decreases. Note that the model is infeasible as the upper bound g^{\max} is less than 98 kWh due to the limitations in terms of the usable SOC range and locations of candidate bus stops in the case study. With an increase g^{\max} , the optimal battery nominal capacity increases, and the total annualized costs decrease accordingly. However, when g^{\max} increases to a certain value and beyond, the optimal battery nominal capacity no longer changes. We find that the optimal battery nominal capacity is 152 kWh when g^{\max} is larger than 200 kWh, similar to the baseline results.

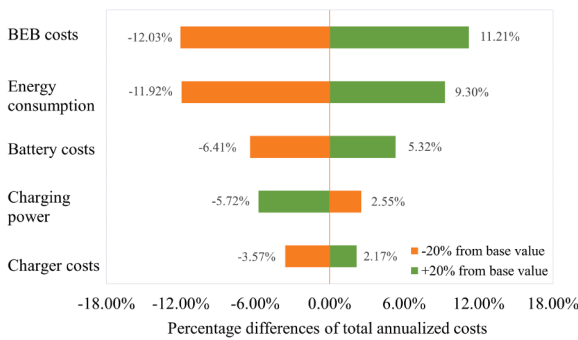
In Fig. 11, the maximum number of chargers (k) that can be built at the central terminal is adjusted to analyze its effects on the solution. It can be seen from the figure that the minimum requirement for the number of chargers installed at the central terminal is two, while the total annualized costs are higher than the baseline results. This is because more chargers are required to be installed at candidate bus stops as the number of chargers at the central terminal decreases. Meanwhile, it is expected that the number of chargers affects the fleet size required to finish the predetermined trip service because the charging duration time of BEBs at the central terminal would increase with fewer available chargers. Note that the model is infeasible if k equal to one due to the limitations regarding the charging duration time and locations of candidate bus stops. With the increase k , the optimal number of chargers installed at the central terminal increases, and the total annualized costs decrease. However, when k increases to a certain value and beyond, the optimal number of chargers installed at the central terminal remains unchanged. We find that the optimal number of chargers installed at the central terminal is three when k is larger than five, similar to the baseline results. This finding also suggests that there is a trade-



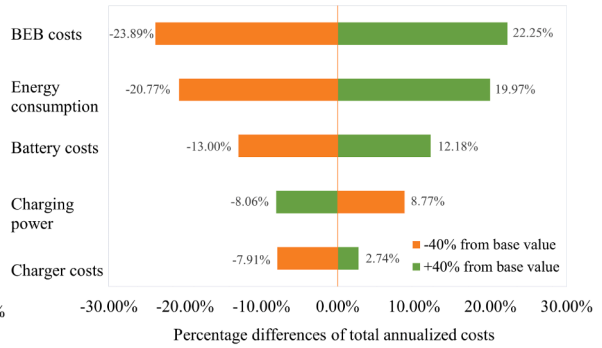
(a) 5% changing range of each parameter



(b) 10% changing range of each parameter



(c) 20% changing range of each parameter



(d) 40% changing range of each parameter

Fig. 9. Percentage differences of total annualized costs with a variation of each parameter.

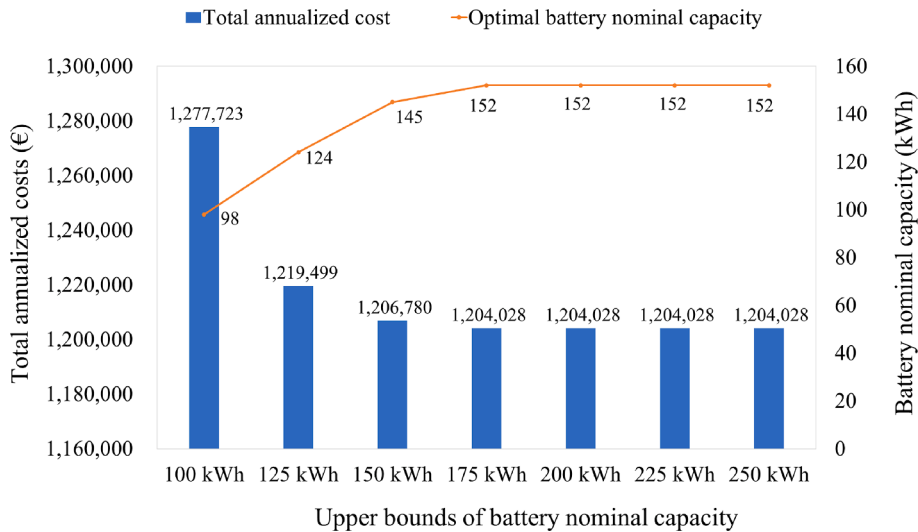


Fig. 10. Optimal results as the upper bound of battery nominal capacity changes.

off between the charging duration time of BEBs and the number of chargers at the central terminal, from the perspective of total investment costs.

In summary, the results of sensitivity analysis indicate that the BEB purchase costs and energy consumption rate are important impact factors in determining the total investment costs for the BEB system under opportunity charging. The reasons behind these results are that the BEB purchase costs account for the largest proportion of the total investment costs, and meanwhile, the energy

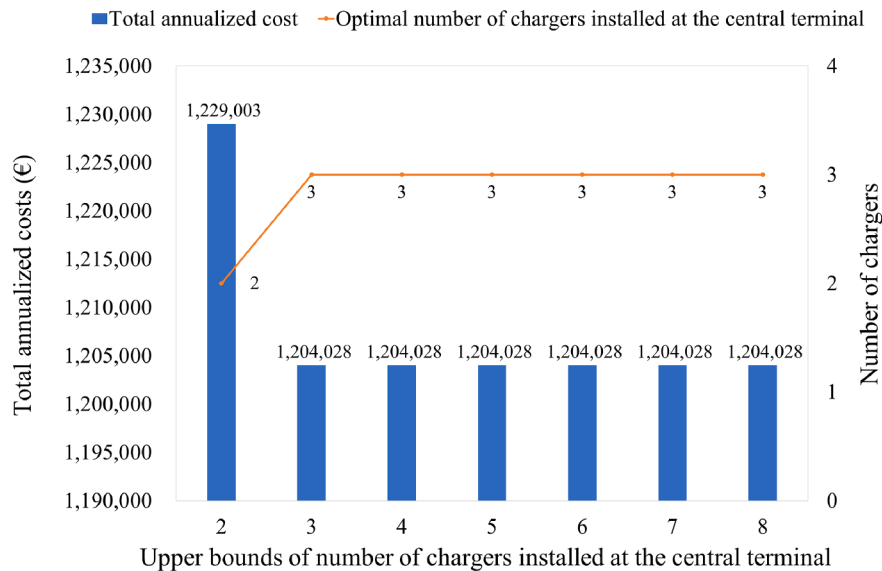


Fig. 11. Optimal results as the upper bound of the number of chargers installed at the central terminal changes.

consumption rate affects both the battery nominal capacity and charger deployment. The battery costs per unit are also identified as an important factor, and public transit operators should pay attention to their downward trends due to the rapid development of the automotive battery industry in recent decades (Nykvist and Nilsson, 2015). In addition, the results provide public transit operators with the implication that it is unnecessary to pursue the extremely high charging power for the BEB system. It is because the improvement of charging power leads to the limited reduction of total annualized costs and may exert adverse effects on battery health (Wang et al., 2021). The results also imply that the opportunity charging system has a cost-effective advantage by introducing more chargers at bus stops while reducing the BEB fleet size coupled with the battery nominal capacity because the total annualized costs are least sensitive to the annualized installation costs of the pantograph charger. Moreover, since the optimal results regarding the battery nominal capacity and the number of chargers installed at the central terminal no longer change as their upper bounds reach certain values and beyond, public transit operators should determine the suitable ranges of choice when constructing the BEB system.

As mentioned above, the sensitivity analysis evaluates the system responses to the key parameters by changing their values individually. In the foreseeable future, it is expected that some parameters will change simultaneously based on technological advancements, such as battery costs, energy consumption rate, and charger installation costs (Lotfi et al., 2020). If the key parameters are changed simultaneously, the opportunity charging system may require considerably less total annualized costs than the baseline results. For example, the total annualized costs can be reduced by 28.10% when the following conditions are satisfied simultaneously: annualized costs of battery per unit and charger installation are reduced by 10%, annualized BEB purchase costs and base energy consumption rate are reduced by 20%, and charging power of pantograph charger is increased by 20%. Besides, we further consider the cases in which the opportunity charging is replaced by the end station charging while key parameters remain the same. The results indicate that the opportunity charging can save 1.23–13.45% of total annualized costs compared with the end station charging over different conditions, where the annualized installation costs of the pantograph charger play a critical role in widening the cost gap.

5. Conclusions

In this study, we present a methodology for the integrated optimization of charger deployment and fleet scheduling for BEBs under opportunity charging, by taking the variations of the bus line information over different time periods into account. The efficiency and applicability of the proposed model are evaluated by extensive numerical experiments with a real-world case study conducted in Oslo, Norway. The optimization results provide the optimal battery nominal capacity, bus fleet size, and charger deployment at both bus stops and the central terminal. Besides, sensitivity analysis reveals several useful findings based on the system behavior in response to a group influencing factors in the model. For example, the operators are suggested to pay more attention to the BEB purchase costs that have significant impacts on the total investment costs, where the total annualized costs can be saved up to 23.89% as the BEB purchase costs reduce by 5–40%. By contrast, the installation costs of pantograph chargers have a limited effect on the total investment costs. Inspired by the case study and sensitivity analysis, the main finding of this study is that the opportunity charging system has a cost-effective advantage by introducing more chargers at bus stops. The results indicate that the total annualized costs can be saved up to 13.38% by using opportunity charging, as compared to the end station charging that also utilizes the same type of pantograph charger, which provides operators with guidance to consider opportunity charging in the BEB system.

Notably, this study can be extended in a few directions. Firstly, the energy consumption of BEBs may be influenced by several uncertain factors in a real-world scenario, such as weather conditions, road conditions, and driving patterns (Chen et al., 2021).

Therefore, the more realistic energy consumption models will be applied in extending the optimization model. Secondly, this research considers the problem scenario with a single-terminal transit network. Our future work will investigate the more complicated problem of public transit networks with multiple central terminals, which allows chargers to be deployed at several central terminals and thus affects the temporal utilization rate of chargers. Thirdly, incorporating different charging methods and partial charging options has the potential to further improve the operational efficiency of the BEB system. Thus, another future work will discuss the hybrid strategy with a combination of different charging methods, and further investigate the problem that allows partial charging at the central terminal. Fourthly, the proposed model introduces the ridership weight to adjust the energy consumption rate, while does not discuss the passenger flow distribution based on transit assignment models (Jiang and Szeto, 2016). To further improve the performance of the optimization model, our future work will incorporate a transit assignment model to accurately predict passenger ridership by considering realistic features, such as personalization and bounded rationality (Jiang and Ceder, 2021). Finally, we will further explore the solution algorithms based on advanced optimization techniques, such as the hybrid heuristics, to efficiently solve the integrated optimization problems for the BEB system, especially for large-scale instances.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix. List of notations

Sets	
R	Set of bus lines, indexed by r
I	Set of trips, indexed by i and j
S	Set of bus stops, indexed by s and s'
L	Set of directed road segments connecting sequential bus stops, indexed by (s, s')
T	Set of time slots, indexed by t
\widehat{T}	Set of time periods, including peak, moderate and off-peak durations, indexed by $\widehat{t}, \widehat{T} \subseteq T$
I_r	Set of trips coming from bus line $r, I_r \subseteq I$
S_r	Set of bus stops that form bus line $r, S_r \subseteq S$
L_r	Set of directed road segments that form bus line $r, L_r \subseteq L$
Parameters	
c_{bat}	Annualized battery costs per unit
c_{bus}	Annualized purchase costs of a BEB
c_{pc}	Annualized installation costs of a pantograph charger
O	Original depot
D	Destination depot
O'	Dummy node for central terminal representing departure point of every trip
D'	Dummy node for central terminal representing arrival point of every trip
d_{sr}	Driving distance from bus stop s to subsequent one on bus line $r, s \in S_r, r \in R$
q'	Base energy consumption rate of the reference BEB
ρ	Charging power of the pantograph charger
w_b^{bat}	Base battery weight of the reference BEB
w_b^{bus}	Base bus weight of the reference BEB
δ	Battery specific energy
$\Delta w_{r\widehat{t}}^{adj}$	Ridership weight adjustment for bus line r during time period $\widehat{t}, r \in R, \widehat{t} \in \widehat{T}$
ϵ^{ub}	Upper bound of the usable battery SOC
ϵ^{il}	Lower bound of the usable battery SOC
t_i	Departure time of trip $i, i \in I$
$\tau_{r\widehat{t}}$	Travel time for route r during time period \widehat{t}
$\widetilde{\tau}_{r\widehat{t}}$	Dwelling time at each bus stop for charging for route r during time period \widehat{t}
k	Maximum number of chargers that can be installed at central terminal
g^{\min}	Lower bound for battery nominal capacity
g^{\max}	Upper bound for battery nominal capacity
M	A large enough constant used for model linearization

(continued on next page)

(continued)

Sets	
Variables	
g	Uniform nominal capacity of the battery equipped in BEBs
φ_{ij}	A binary variable, representing whether a BEB serves trips i and j consecutively, where trip i begins earlier than trip j , $i \in I \cup O$, $j \in I \cup D$, $i \neq j$
x_s	A binary variable, representing whether to install a pantograph charger at bus stop s , $s \in S$
z	An integer variable, representing the number of chargers installed at central terminal
$e_{sr\hat{t}}$	Cumulative energy demand for route r until stop s during time period \hat{t} , $s \in S_r \cup O' \cup D'$, $r \in R$, $\hat{t} \in \hat{T}$
$q_{r\hat{t}}$	Adjusted energy consumption rate for route r during time period \hat{t} , $r \in R$, $\hat{t} \in \hat{T}$
μ_{it}	A binary variable, representing whether a BEB begins to charge in time slot t after trip i at central terminal and before serving the subsequent trip, $i \in I$, $t \in T$
η_{it}	A binary variable, representing whether a BEB ends charging in time slot t after trip i at central terminal and before serving the subsequent trip, $i \in I$, $t \in T$
θ_{it}	A binary variable, representing whether a BEB is being charged in time slot t after trip i at central terminal and before serving the subsequent trip, $i \in I$, $t \in T$
ω_{Oi}	Auxiliary variable used for model linearization, $i \in I$

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