

Planning a maritime supply chain for liquefied natural gas under uncertainty

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ABSTRACT

This paper studies the design of a mid-scale maritime supply chain for distribution of liquefied natural gas (LNG) from overseas sourcing locations, via a storage located at the coast, before transporting the LNG on land to industrial customers. The case company has signed contracts with a number of initial customers and expect that there will be more customers and increased demand in the years to come. However, it is currently uncertain whether and when new contracts will be signed. To capture this uncertainty with regard to which and how many future customers there will be, which directly affects the demand, we propose a multi-stage stochastic programming model, which maximizes the expected profits of the supply chain. The model aims at aiding decisions concerning the import of LNG, investments in floating storage units and customer distribution systems, and it has been applied on a real case study for distributing LNG to customers in a Brazilian state. It is shown that explicitly considering uncertainty in the modeling of this problem is very important, with a Value of Stochastic Solution of 13.2%, and that there are significant economies of scale in this supply chain. Most importantly, the multi-stage stochastic programming model and the analysis presented in this paper provided valuable decision support and managerial insights for the case company in its process of setting up the LNG supply chain.

1. Introduction

In the challenge of meeting the world's rising energy demand, the consumption of natural gas is predicted to increase by almost 45% by 2040 compared to the level in 2017 (IEA, 2018). Natural gas is traditionally transported through pipelines, but as a share of the expected increase in demand is projected to originate from developing countries, new markets without extensive pipeline connections are likely to emerge. Therefore, there is a need for other ways of transporting, storing, and handling the gas. By cooling natural gas down to about $-162\text{ }^{\circ}\text{C}$, the gas condensates and takes the form of Liquefied Natural Gas (LNG). LNG takes up only about 1/600th of the volume compared to its gaseous form (Mokhatab et al., 2013). This significant volume reduction enables other means of transportation (e.g., specialized LNG vessels) and storage, thus reducing the need for high investments in pipeline connections.

A typical LNG supply chain is illustrated in Fig. 1. It consists of five main elements and starts with (1) the extraction of the natural gas, which is then cleaned and purified before (2) being liquefied. Due to its cryogenic nature, LNG is kept in insulated pressure tanks designed to maintain its low temperature. However, as the tanks are not perfectly insulated, a slightly higher surface

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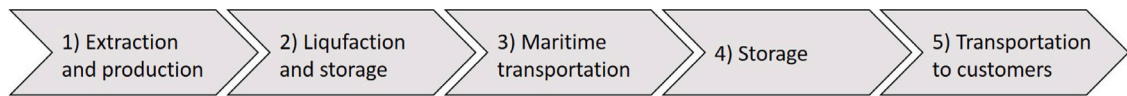


Fig. 1. A typical LNG supply chain.

temperature makes for a constant boiling and evaporation of LNG. This process is called boil-off and is an essential attribute when storing and transporting LNG. The boil-off rate is constant and typically in the range of 0.10–0.15% of the tank's total capacity per day (Mokhatab et al., 2013). After being liquefied and stored at the liquefaction plant, the LNG is (3) transported to its destination terminal(s). The great volume reduction gained from liquefying the gas makes several means of transport economically viable. While LNG can be transported through pipelines over shorter distances, the most common transportation method over longer distances is maritime transportation using specialized LNG vessels. Land-based transportation is also an alternative for shorter distances and smaller volumes and is done by filling smaller insulated pressure containers transported by trucks. At the destination terminal, the LNG is again (4) temporarily stored before being (5) transported to the customers.

In this paper, we consider the planning of a new mid-scale maritime supply chain for distribution of LNG consisting of elements (3)–(5) of the supply chain illustrated in Fig. 1. This planning problem comes from a real case for a company which is in the planning process of setting up a maritime supply chain for the distribution of LNG to a Brazilian coastal state. Due to reasons of confidentiality, we refer to the company only as the *Company*. The actual supply chain is restricted to a specific structure, where LNG will be imported to a single destination terminal in Brazil from overseas locations with LNG vessels. The destination terminal will consist of one or more *Floating Storage Units* (FSUs) where the LNG is temporary stored, and from which it will be distributed to inland regasification stations near the customers. Due to lack of available pipeline connections in the market's state, the LNG must be distributed in containers by trucks to regasification stations serving the customers.

Setting up this supply chain is a strategic planning problem and the Company has to make the following inter-connected decisions: (1) with how many and which industrial customers to form contractual delivery contracts; (2) the size and number of FSUs to employ (i.e., the total storage capacity at the destination terminal); (3) plan the shipments from the overseas destinations (i.e., determine the size and number of shipments from overseas locations). The customers, which are the end users of the LNG, are typically major production facilities in need of energy. The Company has already established delivery contracts or letters of intent with some customers with known demand, while there is a set of other potential customers, with given demand, with whom the Company might sign contracts in the future. However, the decisions above must be made now, which means they have to be made under uncertainty with regard to which future customers, and hence demand, that will appear. Due to the supply chain's complexity and high capital intensity, as well as the uncertainty faced, the Company seeks to use optimization-based decision support in the process of designing it.

There exist several studies on optimizing the distribution of LNG at different planning levels. At the operational planning level, there are for example several studies on the LNG maritime inventory routing problem, e.g., Grønhaug and Christiansen (2009), Fodstad et al. (2010) and Andersson et al. (2010). The planning of an Annual Delivery Program (ADP) is an important planning problem at the tactical level and consists of scheduling the LNG deliveries to a set of customers for the next year, e.g., Rakke et al. (2011), Stålhane et al. (2012), Al-Haidous et al. (2016), Mutlu et al. (2016) and Andersson et al. (2017). However, in what follows we focus on studies for the strategic design of an LNG supply chain, which is most relevant for the planning problem we consider in this paper.

Several studies for the strategic planning of small-scale LNG supply chains use mixed-integer programming (MIP) to model and solve their decision problem. Jokinen et al. (2015) consider the strategic planning of a small-scale LNG supply chain. Its components consist of a large distribution terminal, smaller satellite terminals, and a customer network. LNG is distributed directly by trucks to customers in close proximity to the main terminal. If customers are located farther away, LNG vessels transport LNG to the satellite terminals from which further distribution is done by trucks. Bittante et al. (2017, 2018) also present MIP models for the optimal design of small-scale supply chains where LNG is delivered from supply-side terminals to demand-side terminals by ship transportation and, subsequently, by land-based truck transportation to the customers. The models are used on case studies from the region around the Gulf of Bothnia and the Caribbean, respectively. In a following-up work, Bittante and Saxén (2020) extend the single-period MIP models from the previous studies into a multi-period model. The use of the model is demonstrated on two case studies for a set of islands in Indonesia and in the north region of the Baltic Sea, respectively. Koza et al. (2017) consider a planning problem for a future scenario in which a liner shipping company may use LNG fueled container vessels on some of their services. Decisions are made with regard to the capacities of the onshore storages and the fleet of LNG vessels to supply the container vessels, with the objective of minimizing the sum of investment costs and operational costs. A similar problem as considered in the above studies, though for a liquid helium supply chain, is studied by Malinowski et al. (2018).

There are also a few studies that explicitly consider uncertainty in future demand, which is an important aspect of our problem, though using rather different modeling and solution approaches than we do in this paper. Zhang et al. (2017) develop a multi-scenario MIP model, which is solved by a heuristic combining a MIP-solver with ant colony optimization. The methodology is tested on a case study concerning an LNG supply chain along the Yangtze River in China. In contrast to our stochastic programming approach, they perform sensitivity analyses to see the effect of different LNG prices instead of optimizing across the different scenarios. Cardin et al. (2015) look at the importance of flexibility in large-scale and capital intensive projects, with flexibility defined as the ability to adapt to changes in the market due to realizations of uncertainties. A case of an onshore LNG production

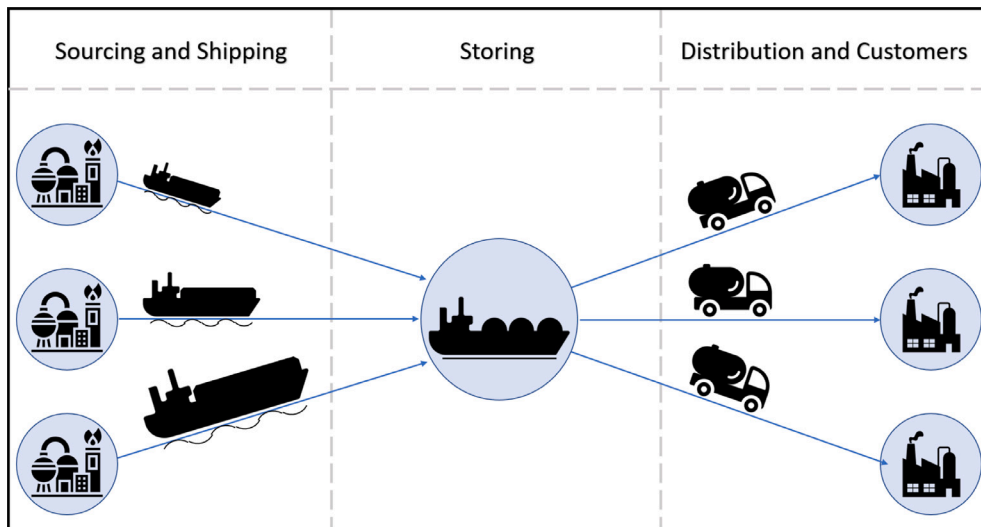


Fig. 2. Overview of the LNG supply chain. LNG is shipped from overseas sourcing locations to a destination terminal where the LNG is stored in one or more Floating Storage Units (FSUs), before being further distributed by trucks to customers.

project is presented. In contrast to what we do in this paper, they analyze the different scenarios one at a time. Through both statistical and economical analysis of different scenarios, they show that the economical performance can be considerably improved by designing a supply chain with sufficient flexibility to adapt to market changes. Finally, we can mention the study by [Sangaiah et al. \(2020\)](#) where the authors propose a robust optimization approach to handle the demand uncertainty in an LNG supply chain.

In this paper, we propose a MIP model to support the Company in setting up its mid-scale maritime LNG supply chain for LNG distribution. We explicitly consider uncertainty with regard to future customers in a way that differs from the previous studies. Particularly, in our case the uncertain element to consider is whether individual customers will sign a contract with the Company or not. The demand associated with each customer is however known and stated in the contract. Our contributions are as follows: (1) We propose a novel and efficient node-based multi-stage stochastic programming model for the planning of a mid-scale LNG maritime supply chain with uncertain customers, and (2) we test the model on a real case for the Company. Our tests show that explicitly considering uncertainty in the modeling of this problem leads to higher expected profits and that the supply chain is characterized by significant economies of scale. The paper also serves as an example of how using Operations Research can provide valuable decision support and managerial insights for solving a real planning problem.

The remainder of this paper is organized as follows: Section 2 provides some background information for the real case study considered in this paper, while Section 3 gives a problem definition. The multi-stage stochastic programming model is provided in Section 4.3. Section 5 presents the input data for the case study, before the computational analyses and results are shown in Section 6. Finally, we provide a summary and draw concluding remarks in Section 7.

2. Background for the case study

The Company plans to set up a mid-scale supply chain for the distribution of LNG to a Brazilian coastal state. The state is currently served with LNG through the GASBOL pipeline, connecting the Bolivian gas resources to the Brazilian market. GASBOL's capacity is limited to about two million m^3 of natural gas per year. However, the Company estimates the total demand in the state to be around 15 million m^3 per year, thus currently yielding a substantial deficit. By importing LNG from overseas locations, the Company aims to fill some of this current deficit. The scope of the planned supply chain operations is illustrated in Fig. 2. This includes the parts (3)–(5) of the whole LNG supply chain illustrated in Fig. 1, i.e., the maritime transportation from the liquefaction plants (overseas import of the LNG), temporary storage in FSUs at the coast of the state, and further distribution and regasification of the LNG to their customers.

The considered LNG production plants to source from are all located in the Atlantic basin. The Company plans to purchase LNG from one or several of these sources and then pay to have it shipped by specialized LNG vessels to their destination terminal in Brazil. The purchasing price of LNG is given by the market price of natural gas with an added surcharge covering the cost of the liquefaction process. The Company intends to order shipments on a voyage charter basis, where they pay a freight rate for a specified volume of LNG to be delivered in Brazil. To secure steady freight rates and due to the logistics of their suppliers, they plan to set their shipment schedule on a yearly basis and order their shipments a year in advance of arrival (similar to the ADP).

The Company wants to apply for a permanent license to dock one or more Floating Storage Units (FSU) in a given port along the coastline of the Brazilian state, and use these as temporary storage of the LNG before distributing it to the customers. Due to the lack of accessible pipeline connections combined with a goal of having operations up and running in the near future, FSUs represent

the only viable storage option. This can also be a flexible solution in the case of changing (i.e., increased) demand. The potential customers are mostly production plants in need of energy which are located at different places in the Brazilian state. To serve the customers, the Company plans to build and operate regasification stations in close proximity to them and distribute the LNG from the FSU to the regasification stations in insulated pressure containers using trucks. To reduce the consequences of downtime and delays in the distribution system, some LNG can be temporarily stored in containers at the regasification stations.

Prior to the start of operations, the Company has received interest from several potential customers, and some of these have already signed contracts binding them as customers when operation begins. Others have signed letters of intent for when the supply chain is up and running. Even though the future customers are uncertain, the Company predicts more potential customers to appear along the planning horizon, and estimates doubled demand within a few years. The use of *take-or-pay contracts* is common in the energy supply industry. A take-or-pay contract is a contract where the customer is bound to pay for the contractually predetermined amount of LNG, whether they can receive the entire amount or not. Furthermore, the Company's selling price is simply an add-on to the current market and purchase price, meant to cover their supply chain costs and potential profits. In this way, price uncertainty is born by the customers and the Company ensures stable and predictable revenues from each customer once a contract is signed. However, exactly which customers that will become available in the future is still uncertain.

3. Problem definition

This section formally defines the problem in which a mid-scale maritime LNG supply chain is to be designed with the objective of maximizing expected profits, while accounting for the uncertainty regarding which customers become available in the future. As explained in Section 2 and illustrated in Fig. 2, LNG is imported from overseas locations to a given destination terminal in Brazil with LNG carriers. The destination terminal consists of one or more FSUs, from which the LNG is distributed to inland regasification stations near the customers. The decisions to be made concern the following:

- Customers
There is a pool of potential customers to supply. Customers may manifest their interest in signing a supply contract at any point in time. We call an *available customer* a customer that has manifested their interest in signing a contract. The Company is to decide with which available customers to form contractual agreements. Once a contract is formed, it is binding for the remaining planning horizon.
- FSUs
There are several FSU types, varying in storage capacities and costs. The Company must decide which FSU(s) to acquire and at what point in time to do so. At least one FSU must be acquired at the start of operations to temporarily store the LNG for the initial and known customers, but more FSUs can be added later in the planning horizon if needed to serve new customers. Once an FSU is acquired, it must be kept for the remaining planning horizon.
- Shipments
Shipments must be planned from sourcing locations with LNG vessels varying in size. It is assumed that shipments are ordered periodically, and that choices of sourcing location and vessels can vary between the different periods.

The decisions to be made in this strategic planning problem are all interconnected. As there is uncertainty regarding which customers become available, the future rise in demand is uncertain. The resulting decisions in a solution to this problem should therefore account for the possibility of an increase in the number of customers and answer the question: If a customer becomes available, should it be signed? The answer lies in the trade-off between the potential revenues generated from the customer and the additional costs it inflicts on the supply chain.

In the following, we provide more details about each of the three relevant parts of the supply chain and the corresponding decisions that must be made.

3.1. Customers

Based on the information provided by the Company, a set of customers is defined. This set includes the customers with which contracts have already been signed, as well as numerous other potential customers that might appear along the planning horizon. Each customer has a known periodic (yearly) demand and a given startup period (year). Customers that already have signed contract are bound as customers for the remainder of the planning horizon, during which they are referred to as *active customers*. An assumption is made that a customer can only be signed from its first period of demand, that is, its startup period, and if signed, take-or-pay contracts are used. If a contract is not formed in this period, the customer is assumed to have its demand met by other energy suppliers. The mentioned customers who have signed contractual agreements prior to the start of the operations are active for the entire planning horizon. These are referred to as *initial customers*.

Each active customer requires an individual distribution and regasification system consisting of specialized containers, trucks and regasification stations. As illustrated in Fig. 3, the size of this system depends on the customer's given demand. Consequently, the costs of acquiring, operating, and maintaining the system for the land distribution of LNG from the FSU(s) vary from customer to customer, but can be pre-calculated. Capital expenses of acquiring the systems' components are depreciated over the remainder of the planning horizon. These costs are directly connected to the specific customers and referred to as the *direct costs*. In contrast, as the FSU and overseas shipping costs are not directly connected to specific customers, these are shared among all active customers and are referred to as *indirect costs*.

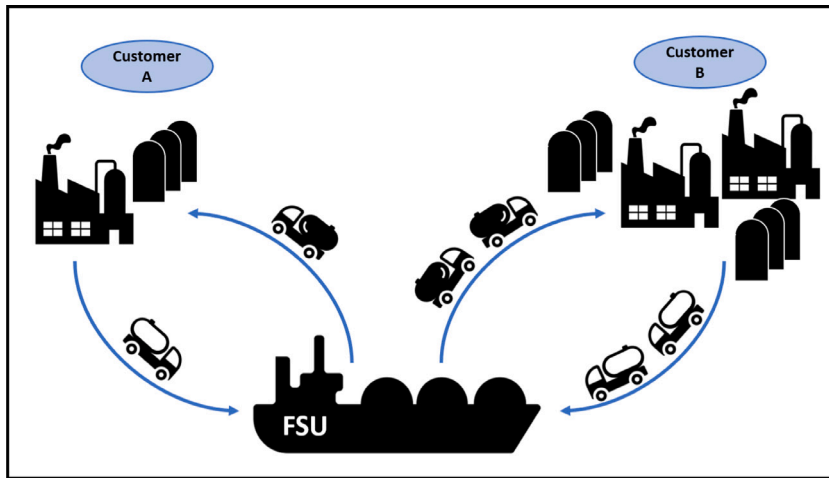


Fig. 3. To serve customers with different levels of demand, different numbers of containers, trucks, drivers and regasification stations are needed.

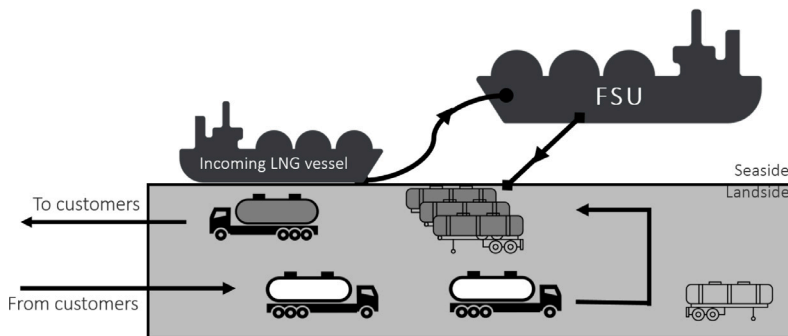


Fig. 4. Illustration of the terminal infrastructure.

As stated in Section 2, the Company does not take any risk concerning fluctuations in the LNG market prices. Its purchasing price of LNG is fully recovered in the selling price to their customers, and therefore, the purchasing price has no impact on the supply chain profits. By separating the selling price into two parts, where the first covers the purchasing price and the second covers the remaining supply chain costs and profits, one can determine the contribution a customer makes towards the profits without considering the current market price of LNG. The selling price's second part is referred to as an *add-on price*, and for the reasons mentioned, only the add-on price is considered further. Therefore, by multiplying the add-on price with a customer's demand, the revenues generated from the customer are found. Furthermore, by subtracting the customer's direct costs from the revenues, we find the customer's *gross margin*. The gross margin is the part of the revenues meant to cover the indirect costs and profits. It is assumed that the add-on price is the same for all customers.

3.2. Storage

Another important decision to be made is which FSU(s) to acquire. We assume there is a given set of FSUs to choose from, differentiated by their storage capacity and cost. More than one FSU can be acquired, but once acquired, they are kept for the remainder of the planning horizon. Due to already signed contracts with the initial customers, there is demand from the start of the planning horizon, and at least one FSU must be acquired to handle this demand. There is a periodic cost of operating an FSU, which includes both operating and charter costs. It is assumed that the boil-off gas originating from the FSU is used to operate the FSU, and thus, the value loss of boil-off is covered by the operating cost. There is also a cost for setting up a new FSU. This cost will only be imposed the first time a new FSU is being used. Since the Company argue that they will never enter into an FSU contract for very short time periods due to the cost for setting it up, we assume that once an FSU is acquired, the Company will have it until the end of the planning horizon. This assumption is also justified by the expected increase in demand over time in this case study. However, we assume that we can add more FSUs at later decision stages (time periods) to facilitate for this increased demand.

Fig. 4 illustrates the terminal infrastructure, where the FSU(s) will receive LNG from incoming LNG vessels. Specialized LNG containers are then filled up from the FSU(s) and brought to the customers' tanks or regasification stations by trucks.

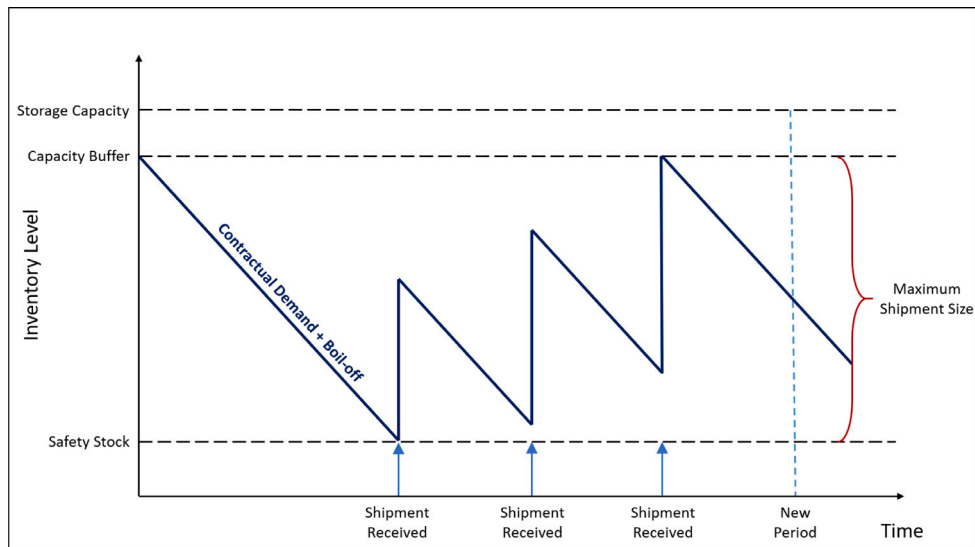


Fig. 5. Illustration of the inventory level as a function of time.

Each FSU has a limited storage capacity. The Company might want to have some storage capacity designated for a *safety stock* and a *capacity buffer* to introduce more robustness in the solutions. The safety stock is retained LNG in the FSU(s) to account for potential delays in shipments. The capacity buffer is vacant storage space to account for potential delays in distribution, e.g., in the case where some customers are not able to receive the LNG over a short period of time. Fig. 5 shows how the safety stock and capacity buffer affect the usable storage capacity and the size of the incoming shipments. It should be noted that the size of the capacity buffer is the difference between the storage capacity and the capacity buffer, as shown in the figure.

3.3. Sourcing and shipping

Regarding the import of LNG, decisions on sourcing locations, LNG vessels to use, and the number of deliveries must be made. A set of possible sourcing locations is given, where each location has a known annual availability of LNG and a distance to the destination terminal in Brazil. The periodic availability of LNG at each sourcing location is assumed to be constant throughout the planning horizon. This assumption can be justified that the status of sourcing locations, which are major LNG liquefaction plants, is not expected to change much.

We assume there is a set of given LNG vessels differing in capacities, sailing speeds and costs and that these can be chartered on voyage charters, where a price is paid for using the vessel to transport LNG from the chosen sourcing location to the given destination terminal. The price or freight rate includes the costs of fuel, vessel crew and port fees, and varies with the LNG carrier's size and sailing time. Due to the economies of scale in the shipping industry, freight rates per volume of LNG decrease with increasing vessel capacities. It is assumed that the LNG vessels are loaded to their maximum capacity prior to leaving the sourcing location, meaning that shipment sizes are decided by the chartered LNG vessel's capacity. However, due to boil-off during sailing and the LNG heel needed for the return voyage, the volume loaded differs from the volume delivered. It is assumed that shipment schedules are determined on a yearly basis, which is common in the LNG business.

3.4. Uncertainty

Which customers will become available to sign a contract in the future is currently uncertain. As stated in Section 3.1, each of the potential customers has a defined startup period, and prior to this period, it is uncertain whether the customer will appear or not. The only customers that are certain to have a demand are the initial customers, which are already bound as customers through signed contractual agreements. In other words, there is a given initial demand, but the future increase in demand due to new customers is uncertain. Strategic decisions must account for the uncertainty to design a supply chain flexible enough to handle potential increases in demand. Each customer has a given probability of appearing, and all initial customers are guaranteed to remain active customers for the entire planning horizon. The probability of a customer signing a contract is assumed to be known and independent of the decisions made by other customers.

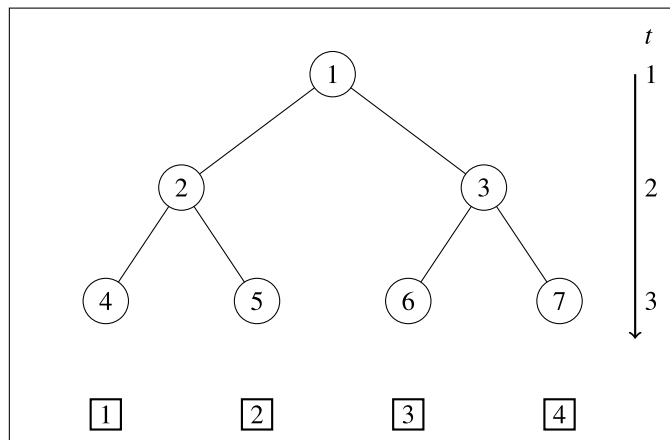


Fig. 6. Illustration of a qualitative three-stage scenario tree. The numbers inscribed in the nodes are arbitrary node indices. The numbers inscribed in squares at the bottom of the scenario tree are arbitrary scenario indices.

4. Multi-stage stochastic programming model

In this section we propose a multi-stage stochastic programming model for the planning problem presented in Section 3. We adopt a so-called *node formulation*, which is general with respect to the scenario tree describing the uncertainty as well as the number of decision stages included. Vitali et al. (2020) compare the node formulation with the more common scenario formulation and show that the former is more efficient, though perhaps not as intuitive. We also refer to Kall et al. (1994) and Birge and Louveaux (2011) for a general introduction to stochastic programming, to Pantuso et al. (2016) and Skålnes et al. (2020) for stochastic programming models of maritime transportation problems and, particularly, and finally to Ormevik et al. (2020) and Bakkehaug et al. (2014) for two examples where node formulations have been used. In the remainder of this section we start by briefly introducing the concept of a scenario tree in Section 4.1. Subsequently, in Section 4.2 we present the notation and in the mathematical model.

4.1. Scenario trees

This section provides a short introduction about scenario trees. We do this using Fig. 6, which describes a small example of a three-stage scenario tree. A scenario tree is a utility that allows us to describe and encode a discrete stochastic process. It is made of a set of nodes. Each node represents a “state of the world”, i.e., a collection of the uncertain information realized until the decision stage where the node belongs. Looking at Fig. 6 we see that the scenario tree has seven nodes. At the first decision stage which corresponds to the first time period, i.e., $t = 1$, we have only one node (node 1). This is to say that the information we have today is not uncertain. At the second decision stage, either node 2 or node 3 will materialize, each representing different realizations of the uncertain parameters related to $t = 2$. In the focal problem, nodes 2 and 3 will represent two different sets of customers signing a contract with the company in that particular period. Further on, at decision stage 3, four nodes may materialize, representing as many (possibly conditional) realizations of the uncertainty. That is, if node 2 materialized in stage 2, one might end up either in node 4 or in node 5. Likewise, after node 3, one might end up either in node 6 or 7. Each node has a probability of materializing, and the sum of the probabilities of the nodes at the same stage sums to 1. Each node, except for the root node, has a parent node. As an example, node 2 is the parent of nodes 4 and 5. We call the nodes at the last decision stage as *leaf nodes*. Finally, a scenario tree contains a finite set of scenarios, representing complete realizations of the entire stochastic process. In Fig. 6 we find four scenarios, identified by squares beneath the scenario tree. As an example, scenario 1 represents the realization of the uncertainty made of the nodes 1 (in time period 1), 2 (in time period 2) and 4 (in time period 3). Also scenarios have probabilities, and their probabilities sum to one. In this simple example, the number of time periods and stages are the same, i.e., they are both three. It should be noted that it is possible to have more than one time period in each stage. In the scenario for our case study, described in Section 5.2, we have for example two time periods in each stage.

4.2. Notation

Given a scenario tree, a node formulation can be derived by connecting parameters and decisions to nodes in the scenario tree. As an example, in Fig. 6 one will make decisions at node 1, then at nodes 2 and 3, and so on. Likewise, uncertain parameters will take values which depend on the specific node. Before we provide a formulation, we introduce some notation which creates the necessary structure (e.g., set of nodes and all auxiliary elements).

Sets

| | |
|---|--|
| \mathcal{N} | Set of nodes in the scenario tree |
| $\mathcal{N}^L \subseteq \mathcal{N}$ | Subset of leaf-nodes, i.e., the nodes at the last decision stage |
| $\mathcal{N}_n^P \subseteq \mathcal{N}$ | Sequence of nodes from the root node to node n |
| \mathcal{L} | Set of available sourcing locations |
| \mathcal{J} | Set of potential customers |
| $\mathcal{J}^I \subseteq \mathcal{J}$ | Sub-set of initial customers |
| \mathcal{V} | Set of available LNG vessels |
| \mathcal{F} | Set of available FSU types |

It should be noted that the scenario tree, represented by the nodes in the sets \mathcal{N} , \mathcal{N}^L and \mathcal{N}_n^P , also includes the information about the periods (e.g., years) in the considered planning horizon.

Parameters

| | |
|------------|--|
| p_n | Probability of node n in the scenario tree occurring |
| $a(n)$ | Parent node of node n |
| A_i | Availability of LNG at sourcing location i given in m^3 |
| Q_v^V | Carrying capacity of vessel v |
| Q_{iv}^I | Received volume when shipping from location i using vessel v (after boil-off) |
| C_{iv}^S | Shipment costs for sourcing from location i with vessel v |
| Q_f^F | Storage capacity of an FSU of type f |
| C_f^{FO} | Cost of chartering and operating an FSU of type f |
| C_f^{FS} | Cost of setting up an FSU of type f |
| B^S | Boil-off rate |
| B^{CB} | Capacity buffer as percentage of contractual demand at the given time |
| B^{SS} | Safety stock as percentage of contractual demand at the given |
| D_{jn} | Contractual demand from customer j in scenario tree node n |
| P_{jn} | Gross margin from customer j in scenario tree node n |
| K_{jn} | 1 if customer j becomes available to sign a contract in node n , 0 otherwise, i.e., it is 1 for all nodes in the scenario tree corresponding to the customer's first time period |

Variables

| | |
|------------|---|
| d_n | Total contractual demand from active customers in scenario tree node n |
| s_n | Inventory level in FSU after the decision made in node n |
| x_{ivn} | Number of shipments ordered from sourcing location i with vessel v in node n |
| q_{ivn} | Binary variable which is equal to 1 if LNG is shipped from location i with vessel v in node n , 0 otherwise |
| y_f | Binary variable which is equal to 1 if an FSU of type f is chartered, 0 otherwise |
| z_{jn} | Binary variable which is equal to 1 if contract with customer j is active in node n , 0 otherwise |
| y_{fn}^R | Number of extra FSUs of type f chartered in scenario tree node n |

4.3. Mathematical model

Objective function

$$\max \sum_{n \in \mathcal{N}} p_n \left(\sum_{j \in \mathcal{J}} P_{jn} z_{jn} - \sum_{v \in \mathcal{V}} \sum_{i \in \mathcal{L}} C_{iv}^S x_{ivn} \right) \tag{1}$$

$$- \sum_{f \in \mathcal{F}} C_f^{FO} \left(y_f + y_{fn}^R \right) \tag{2}$$

$$- \sum_{f \in \mathcal{F}} C_f^{FS} \left(y_f + \sum_{n \in \mathcal{N}^L} p_n y_{fn}^R \right) \tag{3}$$

The first part of the objective function calculates the expected value of the gross margins from active customers minus shipping costs. The second term considers the costs of chartering and operating the FSU(s). The last term calculates the expected FSU setup costs.

Constraints

$$\sum_{v \in \mathcal{V}} Q_v^V x_{ivn} \leq A_i, \quad i \in \mathcal{L}, n \in \mathcal{N} \quad (4)$$

$$Q_{iv}^I q_{ivn} \leq \sum_{f \in \mathcal{F}} Q_f^F (y_f + y_{fn}^R) - (B^{CB} + B^{SS})d_n, \quad i \in \mathcal{L}, v \in \mathcal{V}, n \in \mathcal{N} \quad (5)$$

Constraints (4) ensure that the total volume shipped from sourcing location i for each scenario tree node n does not exceed the availability of LNG at the given sourcing location. Constraints (5) restrict the incoming shipment sizes to be lower than the storage capacity minus the safety stock and capacity buffer.

$$\sum_{f \in \mathcal{F}} y_f \geq 1, \quad (6)$$

$$y_{fa(n)}^R \leq y_{fn}^R, \quad f \in \mathcal{F}, n \in \mathcal{N} \setminus \{0\} \quad (7)$$

$$\sum_{v \in \mathcal{V}} \sum_{i \in \mathcal{L}} Q_{iv}^I x_{ivn} + s_{a(n)} = d_n + \sum_{f \in \mathcal{F}} B^S Q_f^F (y_f + y_{fn}^R) + s_n, \quad n \in \mathcal{N} \setminus \{0\} \quad (8)$$

$$s_n \leq \sum_{f \in \mathcal{F}} Q_f^F (y_f + y_{fn}^R) - B^{CB} d_n, \quad n \in \mathcal{N} \quad (9)$$

$$s_n \geq B^{SS} d_n, \quad n \in \mathcal{N} \quad (10)$$

Constraints (6) and (7) ensure that at least one FSU must be acquired at the beginning of the planning horizon, and if an FSU is acquired, it must be kept for the entire planning horizon. That is, if an FSU is available at a given scenario node it is also available at the nodes emanating from it. It should be noted that these constraints can easily handle the decision of whether to acquire multiple FSUs of the same type. In that case, it is sufficient to add to the set multiple copies of the same FSU. Constraints (8) make sure that the inventory is managed correctly between all consecutive nodes in the scenario tree. Constraints (9) ensure that the capacity buffer remains unfilled. The constraint is necessary as the capacity buffer is meant to be vacant storage space, only used in the case of uncertainty on the operational level. Constraints (10) ensure that the inventory level is always greater than the desired safety stock.

$$\sum_{n' \in \mathcal{N}_{a(n)}^P} K_{jn'} z_{jn'} \geq z_{jn} - K_{jn}, \quad j \in \mathcal{J}, n \in \mathcal{N} \setminus \{0\} \quad (11)$$

$$z_{ja(n)} \leq z_{jn}, \quad j \in \mathcal{J}, n \in \mathcal{N} \setminus \{0\} \quad (12)$$

$$z_{jn} = 1, \quad j \in \mathcal{J}^I, n \in \mathcal{N} \quad (13)$$

Constraints (11) state that, if some customer j is available at some decision node n and it is not the first year the customer has been available, i.e., z_{jn} and $K_{jn} = 0$, then the contract must have been signed up in the first year of appearance of the customer, that is, there must be some predecessor n' of node n , for which $K_{jn} = 1$ (indicating the customer's first year) and $z_{jn} = 1$ (indicating that the customer was made active in that year. Constraints (12) force customers that are signed and become active customers, to stay active for all succeeding nodes, i.e., in the remaining of the planning horizon. All initial customers must be accepted, which is enforced through Constraints (13).

$$d_n = \sum_{j \in \mathcal{J}} D_{jn} z_{jn}, \quad n \in \mathcal{N} \quad (14)$$

$$s_n \geq 0, \quad n \in \mathcal{N} \quad (15)$$

$$x_{ivn} \geq 0, \quad i \in \mathcal{L}, v \in \mathcal{V}, n \in \mathcal{N} \quad (16)$$

$$y_f \in \{0, 1\}, \quad f \in \mathcal{F} \quad (17)$$

$$y_{fn}^R \in \mathbb{N}^+, \quad f \in \mathcal{F}, n \in \mathcal{N} \quad (18)$$

$$z_{jn} \in \{0, 1\}, \quad j \in \mathcal{J}, n \in \mathcal{N} \quad (19)$$

$$q_{ivn} \in \{0, 1\}, \quad i \in \mathcal{L}, v \in \mathcal{V}, n \in \mathcal{N} \quad (20)$$

Constraints (14) define the d_n variables as the sum of the demands of the active customers, while the remaining constraints define the domain of the decision variables.

Table 1
Considered LNG vessels and their respective characteristics.

| | Capacity (m ³) | Charter rate (USD/day) | Average speed (knots) |
|----------|----------------------------|------------------------|-----------------------|
| Vessel 1 | 75 000 | 87 714 | 19.0 |
| Vessel 2 | 100 000 | 96 875 | 19.0 |
| Vessel 3 | 125 000 | 104 756 | 19.0 |
| Vessel 4 | 145 000 | 109 998 | 19.0 |
| Vessel 5 | 160 000 | 113 475 | 19.0 |
| Vessel 6 | 170 000 | 115 616 | 19.0 |

Table 2
FSUs considered and their respective characteristics. All costs are rounded to nearest 1000.

| | Capacity (m ³) | Annual charter cost (USD) | Setup cost (USD) |
|-------|----------------------------|---------------------------|------------------|
| FSU 1 | 125 000 | 18 624 000 | 10 000 000 |
| FSU 2 | 140 000 | 21 900 000 | 10 000 000 |
| FSU 3 | 170 000 | 25 676 000 | 10 000 000 |

Table 3
The considered customers' yearly demand of LNG in each period of the planning horizon, given in m³, and their probability of appearing. A probability of one indicates the customer is a initial customer.

| | Year 1 | Year 2 | Year 3 | Year 4 | Year 5 | Year 6 | Probability |
|-------------|---------|---------|---------|---------|---------|---------|-------------|
| Customer 1 | 91 000 | 91 000 | 182 000 | 182 000 | 273 000 | 273 000 | 1.00 |
| Customer 2 | 109 000 | 109 000 | 218 000 | 218 000 | 334 000 | 334 000 | 1.00 |
| Customer 3 | 121 000 | 121 000 | 182 000 | 182 000 | 243 000 | 243 000 | 1.00 |
| Customer 4 | 243 000 | 243 000 | 364 000 | 364 000 | 485 000 | 485 000 | 1.00 |
| Customer 5 | – | – | 243 000 | 243 000 | 364 000 | 364 000 | 0.75 |
| Customer 6 | – | – | 243 000 | 243 000 | 516 000 | 516 000 | 0.90 |
| Customer 7 | – | – | – | 197 000 | 212 000 | 229 000 | 0.70 |
| Customer 8 | – | – | 237 000 | 249 000 | 263 000 | 277 000 | 0.95 |
| Customer 9 | – | – | – | – | 70 000 | 77 000 | 0.83 |
| Customer 10 | – | – | – | – | 146 000 | 148 000 | 0.60 |
| Customer 11 | – | – | – | – | – | 59 000 | 0.50 |
| Customer 12 | – | – | – | – | – | 237 000 | 0.45 |
| Customer 13 | – | – | – | 254 000 | 262 000 | 270 000 | 0.66 |
| Customer 14 | – | – | 209 000 | 211 000 | 214 000 | 216 000 | 0.55 |

5. Input data and scenario tree generation

This section presents the input data gathered and used in our real-life case study (Section 5.1) and the generation of the scenario tree for modeling the uncertainty about which customers that become available in the future (Section 5.2).

5.1. Case study data

The Company is considering a total of 11 LNG sourcing locations in the Atlantic Basin. Each of these has a given annual supply capacity of LNG and sailing distance to the destination terminal in Brazil. The set of available vessels to choose among for the overseas sourcing of LNG has been selected in collaboration with the Company. This set gives a representation of the relevant vessel types available in the market. The candidate vessels and their respective capacities, time charter rates and average sailing speeds are given in Table 1. Time charter rates are either based the Company's own estimates, or estimated through logarithmic regression in such a way to ensure diminishing rates per unit for increasing vessel capacities, similarly to the method used by Koza et al. (2017). As the Company intends to have their LNG shipped on voyage charter, the time charter rates given in Table 1 are converted into a voyage charter cost for every combination of vessel and sourcing location. This is done by finding the total time charter cost given the vessel's average sailing speed and the location's distance from the destination terminal, and adding estimates of extra costs for fuel, crew, operations and other fees.

The candidate FSUs and their characteristics are given in Table 2. These are considered to give a representation of the available relevant FSUs in the market, and all figures are based on the Company's own estimates.

We consider a planning horizon of six years. The customers and their respective demands and probabilities of appearing are given in Table 3. The initial customers with which the Company has already signed contracts are numbered from 1 to 4, and these have a given demand already from the first year. In addition, customers 5 and 6 have both announced their interest of being supplied by the Company. Based on the Company's estimates the remaining customers have been generated with starting demands ranging from 30,000 to 275,000 m³ of LNG per year. The probabilities of the customers are also estimate by the Company.

A regasification and distribution system, as described in Section 3.1, gives both long- and short-term costs. All costs related to setting up, operating and maintaining the system, in addition to the capacities of the different components, are given in Table 4. Furthermore, assumptions are made regarding the number of each component needed to cover each customer's demand. Enough

Table 4
Costs and capacities of the elements in a distribution and regasification system.

| CAPEX | | |
|--------------------------|-----------|---------------------------|
| Truck | 50 000 | USD/Unit |
| Container | 85 000 | USD/Unit |
| Regasification station | 1 000 000 | USD/Unit |
| OPEX | | |
| Truck driver | 20 000 | USD/Year |
| Maintenance trucks | 5 000 | USD/Year/Unit |
| Insurance trucks | 2 000 | USD/Year/Unit |
| Insurance containers | 2 000 | USD/Year/Unit |
| Regasification station | 150 000 | USD/Year/Unit |
| Maintenance reg. station | 150 000 | USD/Year/Unit |
| Capacities | | |
| Container | 45 | m ³ of LNG |
| Regasification station | 667 | m ³ of LNG/day |

containers to cover four days of demand are needed, these include one day for transporting the daily demand, plus three days of safety stock stored at the customer's location. Due to differences in the customers' locations, an assumption is made stating that a driver can make only one roundtrip delivery to one customer per day. Two shifts of drivers are assumed to work each day so that there is only one truck for every two drivers needed. Lastly, the number of regasification stations needed per customer location is chosen so that the total daily regasification capacity exceeds the customer's daily demand.

With regard to the customer demands given in Table 3 and the described cost elements of the regasification and distribution systems shown in Table 4, the direct periodical cost for each customer is calculated. These costs represent the cost of acquiring, operating and maintaining a regasification and distribution system with sufficient capacity to handle the customer's demand. It should be emphasized that due each customer's demand is relatively large compared to the capacity of the containers and trucks used for the land-side distribution, very little can be gained by pooling, i.e., using containers and trucks to serve different customers. We can therefore pre-calculate this distribution cost separately for each customer.

The gross margin of each customer is as mentioned calculated by multiplying the demand with the add-on price. As the add-on price is based on the Company's estimates and considered as confidential, this data is not shown here.

In Section 3.2, and as illustrated in Fig. 5, we discussed the safety stock and the capacity buffer as means to handle delays in incoming shipments and short-term fluctuations in customer demands, respectively. The size of the safety stock is, in collaboration with the Company, set to be three days of total customer demands. Since the Company expects only minor short-term fluctuations in customer demands and if this still happens, they assume these fluctuations can be handled through interaction with the spot market, the capacity buffer is set to zero.

5.2. Scenario tree generation

The model presented in Section 4.3 is formulated as a general multi-stage stochastic program, which can adapt to problems with any (finite) number of decision stages. In our case study we consider a six-year planning horizon and assume decisions are formalized every second year. This yields a three-stage stochastic program with decisions (subsequent nodes in the scenario tree) separated by a two-year period.

The uncertainty affecting the problem (i.e., which customers become available at a given decision stage) is of a binary type (i.e., a customer may become available or not) thus can be described by a finite, but combinatorial, number of scenarios. Particularly, considering that, in our case study, there are six potential customers that could appear between year three and four (second decision stage) and four that could appear between year five and six, we have a total of $2^6 \times 2^4 = 1024$ scenarios. Fig. 7 shows a qualitative description of this scenario tree, where a scenario is any path from the node in decision stage 1 to any node at decision stage 3. This number of scenarios is too large to obtain a solvable stochastic program. Therefore, we solve approximations of the problem obtained by means of a scenario tree with a smaller number of scenarios. The approximating scenario tree is obtained by means of Monte Carlo sampling techniques (Shapiro, 2003). In Section 6.1 we discuss how we choose the sample size.

6. Computational results

The problem presented in Section 4.3 was solved using the Python libraries of the commercial MIP-solver Gurobi (version 9.0.1) on a PC with a 3.60 GHz Intel i7 processor and 32 GB RAM. In what follows, Section 6.1 presents the analysis performed to determine a sufficient number of scenarios for providing a good trade-off between accuracy in the modeling of uncertainty and tractability of the stochastic program, as well as an evaluation of the benefit from explicitly considering uncertainty, i.e., the Value of the Stochastic Solution (VSS). Section 6.2 presents and discusses detailed results and provides managerial insights for the real case study.

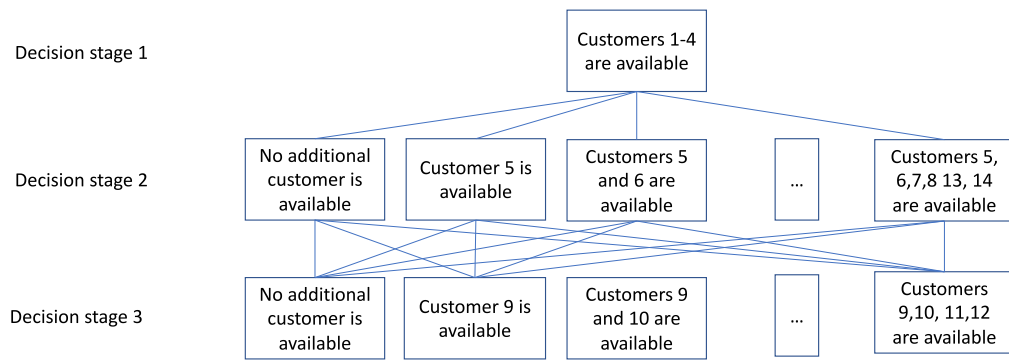


Fig. 7. Qualitative description of the scenario tree describing the uncertainty for the Company.

Table 5

Results from the in-sample analysis show that a sample size of 100 scenarios is considered to yield sufficiently stable solutions, while also maintaining solvability.

| Nr. of scenarios | Nr. of runs | Average runtime (s) | Relative SD | Avg. gap |
|------------------|-------------|---------------------|-------------|----------|
| 100 | 10 | 3973 | 1.61% | 0.15% |

SD = Standard Deviation.

6.1. Stability analysis, solution times and value of stochastic solution

As discussed in Section 5.2, solving the multi-stage stochastic programming model presented in Section 4.3 for the complete three-stage scenario tree (1024 scenarios) is computationally demanding. In our experiments, we obtain an optimality gap not smaller than 133% after 14,400 s. Therefore, we sample a smaller scenario tree. In general, the deviation between the objective value of a solution obtained using the complete scenario tree and that of a solution obtained from a sampled scenario tree decreases as the sample size increases. This decrease is a result of having a larger number of the whole scenarios population represented. However, this increase of accuracy comes at the expense of increased computational complexity of the resulting stochastic program.

In addition, it must be noted that the objective value of a sampling-based approximation is itself a random variable. That is, different sampled scenario trees having the same number of scenarios (sample size) lead, in general, to a different objective value. We conduct an in-sample stability analysis to determine a sample size that yields sufficiently stable solutions, while still maintaining tractability (Kaut and Wallace, 2007; King and Wallace, 2012). That is, we settle to a sample size that gives a sufficiently low variability of the objective value of the resulting problem. Our in-sample stability illustrates that a sample of 100 scenarios is sufficient for our purpose. The results of the analysis are reported in Table 5. The analysis is performed by solving the problem ten times, each time with a different scenario tree consisting of 100 scenarios that are randomly sampled from the total of 1024 scenarios. For each run, we limit the computational runtime to 7200 s or an optimality gap of 0.1%, whichever comes first. We then measure the relative standard deviation of the objective values obtained. The standard deviation is as low as 1.61%, with an average optimality gap of 0.15%. We deem these values sufficient for the purpose of our study and therefore settle on approximations obtained by means of 100 sample scenarios.

As stated by Birge (1982), the Value of Stochastic Solution (VSS) is a measurement of the potential benefit from solving a stochastic program over solving the deterministic Expected Value Problem (EVP), in which expected values replace random parameters. As many stochastic programs, and multi-stage models in particular, become significantly harder to model and solve than their corresponding EVPs, we want to calculate the VSS to check whether it is worth explicitly considering the uncertainty. Escudero et al. (2007) present a method for finding the expected value of a multi-stage stochastic model. Based on this, the VSS in our case study is found to be 13.2%, which is significantly higher than the standard deviation of the in-sample stability analysis. Considering this is a capital intensive industry, we conclude that there seems to be significant gains from explicitly considering the uncertainty through a stochastic programming model.

6.2. Detailed results and economies of scale

In this section, we look more closely into the optimal solution. It can be noted that the smallest FSU type is chosen as an initial investment in the first stage. Thus, shipments sizes are restricted by the FSU's capacity, and we see that only the two smallest LNG vessels, with capacities of 75,000 and 100,000 m³, are used. With regard to which customers are accepted in the different scenarios, an interesting observation is that all customers that become available in the second stage are accepted. Only in the third and last stage, there are differences in customer acceptance among the scenarios. Here, the two customers with the least demand (i.e., customers 9 and 11 in Table 3) are accepted in most but not all the scenarios. This is most likely due to two reasons. Firstly, initial investments have to be made to set up the distribution and regasification system for each customer. The earlier a customer is

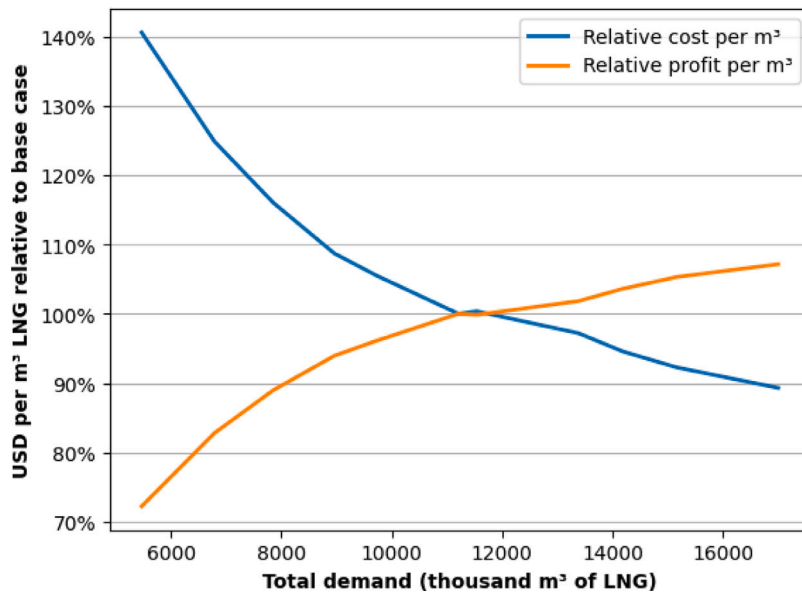


Fig. 8. The cost per unit decreases and the profit per unit increases as the total demand increases, indicating that the supply chain enjoys economies of scale. The costs and profits per unit are given relative to the those of the base case, which is set to 100%.

signed, the more periods these costs can be depreciated over. Secondly, the restrictions in shipment sizes due to the limited available storage capacity forces the use of smaller shipments with a greater shipping cost per unit. To be able to select all customers in the third stage, the Company must in some scenarios investment in additional storage capacity at the terminal (i.e., to charter an additional FSU) and the extra cost for this is higher than the extra revenue for these customers.

Economies of scale correspond to the economic benefits of having higher demand, as costs can then be distributed over a larger number of units (Pindyck and Rubinfeld, 2015). To estimate the economies of scale in this problem, we run the model with adjusted demand of each customer varying from 50 to 150% of the original demands. Fig. 8 shows how the profit per unit increases and cost per unit decreases with increased demand. In other words, a larger share of the add-on price, discussed in Section 3, contributes towards profits when demand increases. To a large extent, the unit cost reduction is due to the FSU cost making up a large share of the total costs. As mentioned in Section 3, this is an indirect cost, meaning it is not directly affected by the total demand. However, with increased demand, the cost per unit of LNG sold decreases. This all indicates the LNG supply chain studied in this paper has significant economies of scale, which further encourages the acceptance of new customers.

Even though it is shown that the mid-scale LNG supply chain enjoys economies of scale, the solutions to some of the adjusted demand instances indicate that not all customers should always be accepted. As previously explained, customers appearing in the last of the three stages, that is one of the last two years, might not be profitable to accept. While these customers are accepted for the majority of the scenarios they appear in, it is clear that they should not automatically be accepted. However, it should be mentioned that the solutions presented here are affected by the finite planning horizon of the long-term model. Even if a customer is defined to only have demand in one or two of the last years, in real-life a long-term contract might be signed, exceeding the planning horizon considered here. However, these results still indicate that short-term contracts might not be preferred.

While the Company has secured an initial demand by forming contractual agreements with certain customers prior to the start of their operations, i.e., the initial customers, this might not be the case when planning similar LNG supply chains elsewhere. It is therefore of interest to analyze how the solution changes when the number of initial customers, and thus the initial demand, decreases. Due to the industry's capital intensity, it is highly unlikely that investments in an LNG supply chain are made without the guarantee of a certain volume of demand. We therefore test varying the number of starting customers from one to three among the four first customers shown in Table 3. The decisions with regard to the shipping and FSU made in the solutions to the cases with fewer initial customers are similar to those of the base case with four initial customers. This is expected as a solution to the base case includes the smallest FSU, which again limits the shipment sizes. Therefore, lowering the initial demand cannot result in a smaller FSU being chosen as long as there is at least one initial customer which needs to be serviced. However, with regard to the customers chosen in later stages/periods, there are certain differences. As previously shown, the supply chain enjoys economies of scale and therefore seeks to increase the total demand to further utilize the diminishing costs per unit. Even so, we saw that certain customers appearing only in the third and last decision stage are not selected in the base case. However, for solutions to both the cases with one and two starting customers, all customers are accepted in all scenarios no matter what stage they appear in. In other words, the lowered initial demand encourages further acceptance of customers in the subsequent periods due to the observed economies of scale.

7. Concluding remarks

In this paper, we studied the design of a mid-scale maritime supply chain for distribution of liquefied natural gas (LNG). The planning problem includes decision about the sourcing and shipping of LNG, temporary storage in one or more FSUs at the given destination terminal, and subsequent road-based distribution to industrial customers. Even though the case company has signed contracts with some initial customers, they still expect there will be more customers and increased demand in the years to come. To capture this uncertainty with regard to which and how many future customers there will be, which directly affects the demand, we proposed a multi-stage stochastic programming model, which maximizes the expected profits of the supply chain. The model is a node formulation which is formulated through the use of nodes rather than the standard scenario formulation. This reduces the number of variables and constraints in the model and removes the need for explicit non-anticipativity constraints.

The multi-stage stochastic programming model was used on a real planning problem for a case company of designing a supply chain for the distribution of LNG from overseas sourcing locations to industrial customers in a Brazilian state. It was shown that explicitly considering uncertainty in the modeling of this problem is very important, as the Value of Stochastic Solution (VSS) was as high as 13.2%. The analysis also showed how the profit per unit increased and the cost per unit decreased with increased demand, emphasizing the importance of economies of scale in this supply chain. Finally and most importantly, the multi-stage stochastic programming model and the analysis presented in this paper provided valuable decision support and managerial insights for the case company in its process of setting up the LNG supply chain.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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