6 Advanced Inventory Modelling

6.1 Modeling Transportation

Transportation is generally considered as an important element to include in Life Cycle Assessment. Transportation is often an important contributor to the total impacts in analysis of many types of systems. Therefore, simplifying your inventory by excluding transportation should only be done in cases where you have solid documentation to support this. There are various ways of modeling transportation. In this section we are going to explore some approaches and discuss the pros and cons of each of these.

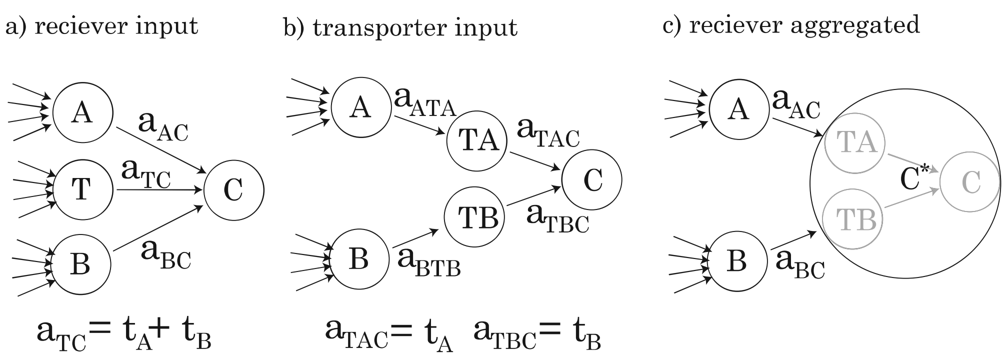

Figure 6.1: Alternative models of transport activities

In figure 6.1 we have illustrated three alternative ways of modeling transport. In all three cases we have two processes A and B which produce output that is required by process C. Transport, process T, is required to facilitate the delivery of the products from A and B to C. The amount of transport required to deliver products from A to C is t_A, and from B to C is t_B.

The first approach we refer to as *receiver input*. In this approach, process C requires input from A and B, a_{AC} and a_{BC}. In addition, a combined requirement of transport associated with the delivery of both A and B is placed upon process T, $a_{TC} = t_A + t_B$. This approach is simple to apply. The cooking recipe of process C is easy to understand: A given amount of A,B and transport, T, is required for the production of C. However, this approach does not enable us to distinguish between impacts associated with transportation of A and B individually.

In equation 6.1 it can be seen how process C requires input from both A, B and the transportation process T.

$$\begin{bmatrix} I - A_{ff} \\ -A_{bf} \end{bmatrix} = \begin{bmatrix} & \vdots & A[kg] & B[kg] & T[tkm] & C[kg] \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ A & \vdots & 1 & 0 & 0 & -a \\ B & \vdots & 0 & 1 & 0 & -b \\ T & \vdots & 0 & 0 & 1 & -(t_A + t_B) \\ C & \vdots & 0 & 0 & 0 & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ b & \vdots & -A_{bf,A} & -A_{bf,B} & -A_{bf,T} & -A_{bf,C} \end{bmatrix} \tag{6.1}$$

With the corresponding stressor matrix

$$S = \begin{bmatrix} & \vdots & A[kg] & B[kg] & T[tkm] & C[kg] & b \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ str. & \vdots & S_A & S_B & S_T & S_C & S_b \end{bmatrix} \tag{6.2}$$

The second approach, *transporter input* does reflect the actual events that takes place more accurately. Here, the two separate transportation processes, TA and TB, transport respectively products from A and B to C. In this approach, process C does not require outputs from process A and B directly, but indirectly through the transportation processes. The transportation processes, TA and TB require inputs of A and B, a_{ATA} and a_{BTB}. This is analogously to loading the goods onto the truck. Process C, in turn, require the transportation services which deliver the products, $a_{TAC} = t_A$ and $a_{TBC} = t_B$. This enable us to identify the transportation services associated with the individual transactions. However, we are required to model several individual transportation processes. The input structure (cooking recipe) of process C will also in this case appear somewhat misleading. It will only require transportation services and none of the goods required. Another aspect is that the constructed transportation processes are only valid for the distance specified. By inspection, we observe that the transportation processes require the good they are transporting in such a manner that for the specified distance, the required amount is transported. If the distance increases, more is loaded onto the truck. Therefore, the loading coefficients must be adjusted if the transportation distance is altered.

The $(I - A)$ matrix of the foreground system and link to the background is shown equation 6.3. The hieratical structure of the interactions can be observed.

$$\begin{bmatrix} I - A_{ff} \\ -A_{bf} \end{bmatrix} = \begin{array}{cc} & \begin{array}{ccccc} A[kg] & B[kg] & TA[tkm_{w/A}] & TB[tkm_{w/B}] & C[kg] \end{array} \\ \begin{array}{c} \cdots \\ A \\ B \\ TA \\ TB \\ C \\ \cdots \\ b \end{array} & \left[\begin{array}{ccccc} \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 0 & -(a/t_A) & 0 & 0 \\ 0 & 1 & 0 & -(b/t_B) & 0 \\ 0 & 0 & 1 & 1 & -t_A \\ 0 & 0 & 0 & 0 & -t_B \\ 0 & 0 & 0 & 0 & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ -A_{bf,A} & -A_{bf,B} & -A_{bf,TA} & -A_{bf,TB} & -A_{bf,C} \end{array}\right] \end{array} \tag{6.3}$$

With the corresponding stressor matrix

$$S = \begin{array}{cc} & \begin{array}{cccccc} A[kg] & B[kg] & TA[tkm_{w/A}] & TB[tkm_{w/B}] & C[kg] & b \end{array} \\ \begin{array}{c} \cdots \\ str. \end{array} & \left[\begin{array}{cccccc} \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ S_A & S_B & S_T & S_T & S_C & S_b \end{array}\right] \end{array} \tag{6.4}$$

The third approach, *receiver aggregated*, principally aggregates the two transportation processes, TA and TB, plus C into a new process C^*. In doing this the transportation technology becomes a part of the requirement structure (technology description) of process C. By doing this, the number of processes to model are fewer than in the previous methods. However, the requirements structure is somewhat disturbed by the integration of the transportation services and one can therefore not distinguish between the production and transportation requirements. Following this it is not possible to identify the individual stressors associated with transport and production. The compactness of this formulation can be seen in equation 6.15. It should be noted that this approach involves aggregation of two processes using input coefficients. There are limitations to the applicability of this. Therefore, only apply this approach in cases where you have a unidirectional foreground system with no feedbacks.

$$\begin{bmatrix} I - A_{ff} \\ -A_{bf} \end{bmatrix} = \begin{bmatrix} & \vdots & A[kg] & B[kg] & C[kg] \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ A & \vdots & 1 & 0 & -a \\ B & \vdots & 0 & 1 & -b \\ C & \vdots & 0 & 0 & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ b & \vdots & -A_{bf,A} & -A_{bf,B} & -(A_{bf,C} + (t_a + t_b)A_{bf,T}) \end{bmatrix} \tag{6.5}$$

The stressor matrix takes on the following form.

$$S = \begin{bmatrix} & \vdots & A[kg] & B[kg] & C[kg] & b \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ str. & \vdots & S_A & S_B & S_C + (t_a + t_b)S_T & S_b \end{bmatrix} \tag{6.6}$$

6.1.1 Example

Assume the production of Product A and B, respectively, requires 0.3 kg and 0.2 kg of fuel and emits 0.8 and 0.3 kg of stressor j per kg produced. The production of 1 kg of product C requires 3.8 kg of product A and 4.2 kg of product and 0.1 kg of fuel. Per kg of C produced 0.2 kg of stressor j is emitted. Further, 1.6 tkm and 2.7 tkm of transport is required for the delivery of respectively, A and B to C. The transportation process requires 1.4 kg of fuel pr tkm and emits 0.7 kg of stressor j.

Apply the three transportation models to model the system and calculate output and total emissions and compare.

1) Receiver input

$$[I - A] = \begin{bmatrix} & \vdots & A[kg] & B[kg] & T[tkm] & C[kg] & Fuel[kg] \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ A & \vdots & 1 & 0 & 0 & -3.8 & 0 \\ B & \vdots & 0 & 1 & 0 & -4.2 & 0 \\ T & \vdots & 0 & 0 & 1 & -(1.6 + 2.7) & 0 \\ C & \vdots & 0 & 0 & 0 & 1 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\ Fuel & \vdots & -0.3 & -0.2 & -1.4 & -0.1 & 1 \end{bmatrix} \tag{6.7}$$

With the corresponding stressor vector

$$
S = \begin{bmatrix} & \vdots & A[kg] & B[kg] & T[tkm] & C[kg] & Fuel[kg] \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ str. & \vdots & 0.8 & 0.3 & 0.7 & 0.2 & 0.6 \end{bmatrix} \tag{6.8}
$$

For one unit demand of product C we obtain the following output vector.

$$
x' = \begin{bmatrix} & \vdots & A[kg] & B[kg] & T[tkm] & C[kg] & Fuel[kg] \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ & \vdots & 3.8 & 4.2 & 4.3 & 1 & 8.1 \end{bmatrix} \tag{6.9}
$$

and the following vector of stressor by processes and total emissions of the given stressor.

$$
e = \sum_{pro} E_{pro} = \begin{bmatrix} & \vdots & A[kg] & B[kg] & T[tkm] & C[kg] & Fuel[kg] \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ str. & \vdots & 3.04 & 1.26 & 3.01 & 0.2 & 4.86 \end{bmatrix} = 12.37 \tag{6.10}
$$

2) Transporter input

$$
I - A = \begin{bmatrix} & \vdots & A[kg] & B[kg] & TA[tkm_{w/A}] & TB[tkm_{w/B}] & C[kg] & Fuel[kg] \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & \cdots \\ A & \vdots & 1 & 0 & -(3.8/1.6) & 0 & 0 & 0 \\ B & \vdots & 0 & 1 & 0 & -(4.2/2.7) & 0 & 0 \\ TA & \vdots & 0 & 0 & 1 & 1 & -1.6 & 0 \\ TB & \vdots & 0 & 0 & 0 & 0 & -2.7 & 0 \\ C & \vdots & 0 & 0 & 0 & 0 & 1 & \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ b & \vdots & -0.3 & -0.2 & -1.4 & -1.4 & -0.1 & 1 \end{bmatrix} \tag{6.11}
$$

The stressor vector is also expanded to include both TA and TB.

$$
S = \begin{bmatrix} & \vdots & A[kg] & B[kg] & TA[tkm_{w/A}] & TB[tkm_{w/B}] & C[kg] & Fuel[kg] \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ str. & \vdots & 0.8 & 0.3 & 0.7 & 0.7 & 0.2 & 0.6 \end{bmatrix} \tag{6.12}
$$

For one unit demand of product C we obtain the following output vector.

$$x' = \begin{bmatrix} & \vdots & A[kg] & B[kg] & TA[tkm_{w/A}] & TB[tkm_{w/B}] & C[kg] & b \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ & \vdots & 3.8 & 4.2 & 1.6 & 2.7 & 1 & 8.1 \end{bmatrix} \tag{6.13}$$

We here see that the output of the two transportation processes, TA and TB sum up to the output of the single transport process, T, in the receiver input case. Consequently, this is also the case for the vector of stressor by processes. That is, the emissions from TA andTB sums up to that of T. We also observe that the total emissions sum up to the same value as in the previous case.

$$e = \sum_{pro} E_{pro} = \begin{bmatrix} & \vdots & A[kg] & B[kg] & TA[tkm_{w/A}] & TB[tkm_{w/B}] & C[kg] & b \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ str. & \vdots & 3.04 & 1.26 & 1.12 & 1.89 & 0.2 & 4.86 \end{bmatrix} = 12.37 \tag{6.14}$$

3) Receiver Aggregated

$$I - A = \begin{bmatrix} & \vdots & A[kg] & B[kg] & C[kg] & Fuel[kg] \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ A & \vdots & 1 & 0 & -3.8 & 0 \\ B & \vdots & 0 & 1 & -4.2 & 0 \\ C & \vdots & 0 & 0 & 1 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ b & \vdots & -0.3 & -0.2 & -(0.1 + (1.6 + 2.7) * 1.4) & 1 \end{bmatrix} \tag{6.15}$$

Here the stressor vector takes on the following form.

$$S = \begin{bmatrix} & \vdots & A[kg] & B[kg] & C[kg] & Fuel[kg] \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ str. & \vdots & 0.8 & 0.3 & 0.2 + (1.6 + 2.7) * 0.7 & 0.6 \end{bmatrix} \tag{6.16}$$

For one unit demand of product C we obtain the following output vector.

$$x' = \begin{bmatrix} & \vdots & A[kg] & B[kg] & C[kg] & Fuel[kg] \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ & \vdots & 3.8 & 4.2 & 1 & 8.1 \end{bmatrix} \tag{6.17}$$

and the following vector of stressor by processes and total emissions of the given stressor.

$$e = \sum_{pro} E_{pro} = \begin{bmatrix} & \vdots & A[kg] & B[kg] & C[kg] & Fuel[kg] \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ str. & \vdots & 3.04 & 1.26 & 3.21 & 4.86 \end{bmatrix} = 12.37 \quad (6.18)$$

6.1.2 Problem

Assume the production of Product A and B, respectively, requires 0.4 kg and 0.6 kg of fuel and emits 0.5 and 0.2 kg of stressor j per kg produced. The production of 1 kg of product C requires 2.1 kg of product A and 0.3 kg of product and 0.4 kg of fuel. Per kg of C produced 0.5 kg of stressor j is emitted. Further, 0.6 tkm and 0.7 tkm of transport is required for the delivery of respectively A and B to C. The transportation process requires 2.3 kg of fuel pr tkm and emits 2.8 kg of stressor j pr tkm. Each kg of fuel used emits 0.6 kg of stressor.

Apply the three transportation models to model the system and calculate output and total emissions and compare.

6.2 Modeling Recycling

Modeling recycling is not difficult, but it requires that you keep your tongue straight in your mouth. A lot of literature has been devoted to various ways of modeling recycling. We are going to explore a condensed representation of recycling systems in order to get to the core of the issue at hand. In short terms, the challenge is how to model and allocate impacts from series, or cascades, of loops.

We will start with a generalized model of a system with recycling. Figure 6.2 has five nodes (processes) labeled with capital letters. Starting from left we have a process producing a virgin material, denoted V. This material goes into the production process, denoted P. This product is required for use by a consumer, U. The collection of waste from the use is modeled as a separate process and is denoted, W. The collected waste is used in recycling, R, which in turn provides input to the production process.

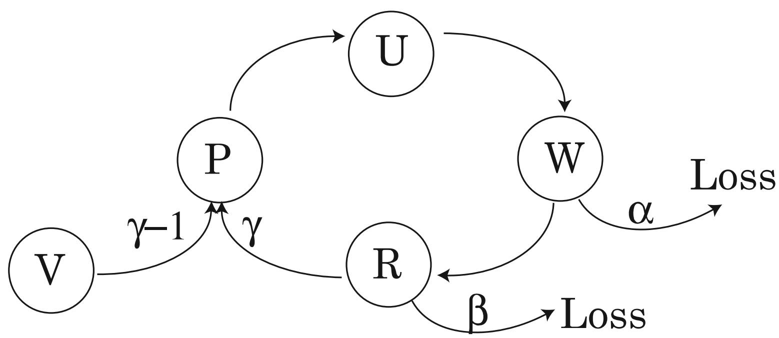

Figure 6.2: Generalized representation of recycling

We have introduced three parameters, α, β and γ. The latter represents the fraction of recycled material that is used in the production process. So, $\gamma = 0.4$ means that 40% of the material used in production in process P, is recycled material and 60% is virgin material. The parameters α and β represents losses in the two processes waste and recycling. The first, α, indicates the amount of material is lost due to incorrect sorting of the waste. Analogously, β, represents fraction of the amount that is received to recycling that is polluted in some manner so that it cannot be recycled. Based on this we can set up the following A matrix.

$$A = \begin{array}{cc} & \begin{array}{ccccc} P & U & W & R & V \end{array} \\ \begin{array}{c} P \\ U \\ W \\ R \\ V \end{array} & \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1+\alpha & 0 & 0 \\ 0 & 0 & 0 & 1+\beta & 0 \\ \gamma & 0 & 0 & 0 & 0 \\ (1-\gamma) & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \tag{6.19}$$

We shall now investigate this system using geometric series expansion. First, we are going to investigate the output in the first three tiers for a demand of 1 placed on the use process, U, see below.

$$x_t = \begin{array}{cc} & \begin{array}{ccccc} 0 & 1 & 2 & 3 & 3_{acc} \end{array} \\ \begin{array}{c} tier \\ P \\ U \\ W \\ R \\ V \end{array} & \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & \gamma(1+\beta) & \gamma(1+\beta) \\ 0 & 0 & \gamma & 0 & \gamma \\ 0 & 0 & (1-\gamma) & 0 & (1-\gamma) \end{bmatrix} \end{array} \tag{6.20}$$

What we see here is the following: In the zeroth tier, output is required of the use process. This initiates output in the production process in tier 1. In the next turn, this requires supply of virgin and recycled material in tier 2. In tier three, the recycled material initiates output from the waste process. We see the amount of output from the waste collection process required, is $\gamma(1+\beta)$. The accumulated activity in tier 3 shows that our demand has now initiated activity in production, virgin and recycled materials production as well as in activity in the waste collection process. This seems fine, however as we proceed with our series expansion throughout tier 4 to 7, some questions arise that we will get back to.

Technically the waste collection process places a demand in tier 4 on the use process with $\gamma(1+\beta)(1+\alpha)$. Now, remember this is not "our" use process. This is the use of the material in the previous use round. That is, use by the person drinking coca-cola from an aluminium can before it was recycled and became your coca-cola can. Proceeding, we obtain the following for tier 4 through 7.

$$x_t = \begin{bmatrix} tier & \vdots & 4 & 5 & 6 & 7 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ P & \vdots & 0 & \gamma(1+\beta)(1+\alpha) & 0 & \\ U & \vdots & \gamma(1+\beta)(1+\alpha) & 0 & 0 & \\ W & \vdots & 0 & 0 & 0 & \gamma(1+\beta)(1+\alpha)\gamma(1+\beta) \\ R & \vdots & 0 & 0 & \gamma(1+\beta)(1+\alpha)\gamma & \\ V & \vdots & 0 & 0 & \gamma(1+\beta)(1+\alpha)(1-\gamma) & \end{bmatrix} \tag{6.21}$$

What we observe here, is that the section of our series, from tier 4 throughout 7, is identical to that of tier 0 to 3 except that the demand placed on the use phase is $\gamma(1+\beta)(1+\alpha)$ rather that 1. This is because our initial demand of 1 unit of use, causes a demand of $\gamma(1+\beta)(1+\alpha)$ onto use in the previous round. Remember, this is because our production activity uses recycled material.

The question that arises now, is how should one allocate responsibility for outputs and the associated emissions in the various tiers. Few would argue against attributing the impacts accumulated in tier 3, to the external demand of 1 placed upon the use process, U. The controversy arises when we move backwards into the previous life cycle. That is, tiers 4 through 7. Should the use of a good in one round, be held accountable for the virgin material outtake in a previous round ? Also, If we proceed further through more tiers towards infinity, we will have many more rounds.

In most cases we are not interested in an infinite amount of recycling rounds, rather when we are interested in the single round that we interfere with. That is, when we use aluminium as a input material to one of our processes, what are the interested in the impacts associated with this one time use of aluminium. These will vary depending on the level of recycling. To obtain this one generally models a broken recycling circle. So instead of a loop we model a unidirectional system that has a start and a stop. This principally means that we do not concern ourselves with what has occurred in previous rounds or in later rounds. The challenge is where to break the circle.

In figure 6.3 we have modeled a system where we attribute impacts from the recycling activities from the waste in the previous round to our service. In doing this we do not attribute any impacts downstream of our use. That is, the next round of recycling.

The matrix describing this system is shown below. It is identical to that in equation 6.19 except that here, a dummy process ,U^*, has been added to represent the use phase in the previous round. Activity in this node will however not be attributed with any stressors.

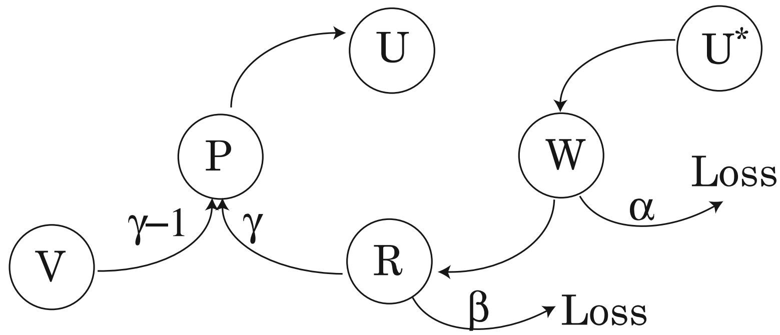

Figure 6.3: Single round recycling

$$A = \begin{array}{c|cccccc} & P & U & U^* & W & R & V \\ \hline P & 0 & 1 & 0 & 0 & 0 & 0 \\ U & 0 & 0 & 0 & 0 & 0 & 0 \\ U^* & 0 & 0 & 0 & (1+\alpha) & 0 & 0 \\ W & 0 & 0 & 0 & 0 & (1+\beta) & 0 \\ R & \gamma & 0 & 0 & 0 & 0 & 0 \\ V & (1-\gamma) & 0 & 0 & 0 & 0 & 0 \end{array} \tag{6.22}$$

Putting a final demand of 1 unit on the use process, U, gives the following x vector.

$$x_t = \begin{array}{c|c} P & 1 \\ U & 1 \\ U^* & \gamma(1+\beta)(1+\alpha) \\ W & \gamma(1+\beta) \\ R & \gamma \\ V & (1-\gamma) \end{array} \tag{6.23}$$

The output vector is identical to the accumulated output in tier 3 found in equation 6.20, except for the activity in process U^* which belongs to the previous round. From this we conclude that our unidirectional model now represents one round of the recycling loop.

There is not one optimal way of modeling recycling. However, understanding the main principles you should now be able to assess different approaches and construct models for various applications.