NTNU
Norwegian University of Science and Technology
Faculty of Economics and Management
Dept. of Industrial Economics and Technology Management

Master's thesis

Helle Backer
Maria Fjelltun Dalvik
Sølvi Herabakka

Real Option Analysis of Hydro Turbine Replacements

Master's thesis in Industrial Economics and Technology Management
Supervisor: Stein-Erik Fleten
Co-supervisor: Erik Jacques Wiborg
June 2022

NTNU
Kunnskap for en bedre verden

Helle Backer
Maria Fjelltun Dalvik
Sølvi Herabakka

Real Option Analysis of Hydro Turbine Replacements

Master's thesis in Industrial Economics and Technology Management
Supervisor: Stein-Erik Fleten
Co-supervisor: Erik Jacques Wiborg
June 2022

Norwegian University of Science and Technology
Faculty of Economics and Management
Dept. of Industrial Economics and Technology Management

NTNU
Kunnskap for en bedre verden

Preface

This master's thesis has been conducted at the Norwegian University of Science and Technology as part of the Master's Programme Industrial Economics and Technology Management during the spring of 2022. The purpose of the thesis was to examine the optimal investment timing of a hydro turbine replacement with the use of real option analysis. In cooperation with Statkraft, we have aimed to make an investment decision model which further can be extended and used in the industry.

We would like to give our sincere gratitude to our supervisor Stein-Erik Fleten (NTNU). His knowledge of the field and willingness to review our work have been an essential contribution to the final result. Further, we want to extend our appreciation to our co-supervisor, Erik Jacques Wiborg (Statkraft). His enthusiasm, technical knowledge, and contacts in the industry have been extremely valuable to us during this research process.

Trondheim, June 11, 2022
Helle Backer, Maria Fjelltun Dalvik, Sølvi Herabakka

Abstract

In this master's thesis we investigate the optimal timing of a major hydro turbine replacement under two uncertainty factors; electricity price and risk of failure. We aim to develop an investment decision model applicable to the hydropower producer. As opposed to the commonly used net present value method, we use a real option framework, which accounts for the value of waiting for more information in an uncertain future. We simulate electricity prices and turbine lifetime, and solve the real option problem with the use of Least Squares Monte Carlo. Further, we use a base case to test our model, where the values are obtained from conversations with Statkraft and from previous literature. The results show that the variations in investment timing are highly dependent on uncertainty in both electricity prices and time in condition states. Additionally, the sensitivity analysis indicates that the lifetime estimations have the most significant impact on the results. Possible improvements to our model are dependent on both precise estimation models and industry practice.

Sammendrag

I denne masteroppgaven undersøker vi det optimale tidspunktet for en turbinutskifting i et vannkraftverk med to usikkerhetsfaktorer; strømpris og risiko for svikt. Målet vårt er å utvikle en investeringsmodell for en kraftprodusent. I motsetning til nettonåverdimetoden som vanligvis anvendes, benytter vi en realopsjonsmetode som inkluderer verdien av å vente på mer informasjon i en usikker fremtid. Vi simulerer strømpriser og turbinens levetid, og løser realopsjonsproblemet ved bruk av Least Squares Monte Carlo. Videre bruker vi et base case for å teste modellen vår, hvor vi har fått realistiske verdier gjennom samtaler med Statkraft og fra tidligere litteratur. Resultatene viser at variasjonene i investeringstidspunkt er svært avhengige av usikkerhet i både strømpriser og levetid. I tillegg indikerer sensitivitetsanalysen at levetidsestimatene har den største innvirkningen på resultatene. Mulige forbedringer av modellen vår er avhengige av både presise estimeringsmetoder og bransjepraksis.

Table of Contents

Nomenclature

Indices

i Condition state

j Turbine expert

s Simulation path

t Time (years)

Input data

α Annual growth rate of electricity prices (%)

η_0 Efficiency of turbine (when new) (%)

$\eta_{0,new}$ Efficiency of new turbine (when new) (%)

$\eta_{0,old}$ Efficiency of old turbine (when new) (%)

γ Annual efficiency reduction rate (%)

γ_{new} Annual efficiency reduction rate for new turbine (%)

γ_{old} Annual efficiency reduction rate for old turbine (%)

$\lambda(0)$ Condition state at $t = 0$

ω Age of the old turbine at $t = 0$ (years)

ϕ Time already spent in given condition state at $t = 0$ (years)

ρ Annual discount rate (%)

σ Annual volatility of electricity prices (%)

τ_{ij} Expected time spent in condition state i by turbine expert j (years)

C_{fail} Cost of a failure (MNOK)

C_{turb} Investment cost of a turbine replacement (MNOK)

M Number of simulation paths

N_i Number of expert judgements for condition state i

$P(0)$ Long-term electricity price at $t = 0$ (NOK/kWh)

Q Annual inflow in terms of energy (GWh)

T_{new} Expected lifetime of new turbine (years)

Endogenous variables and constants

β_i Scale parameter for gamma distribution of time spent in condition state i

$\eta(t)$ Turbine efficiency at time t (%)

$\lambda(t)$ Condition state at time t

θ_i Time in condition state i as a random variable (years)

A_i Intermediate value to calculate estimators of shape and scale parameters

$f(\theta_i)$ Probability density function of the gamma distribution for θ_i

k_i Shape parameter for gamma distribution of time spent in condition state i

$P(t)$ Long-term electricity price at time $t = 0$ (NOK/kWh)

V_{inv} Value of investing in a new turbine (MNOK)

V_{prod} Value of one year power production (MNOK)

V_{sys} Value of turbine system (MNOK)

1 Introduction

In this thesis, we aim to determine the optimal timing of a major hydro turbine replacement under failure risk and electricity price uncertainty. With the use of real option analysis (ROA), we examine how the uncertainty factors influence such an investment and identify the investment decision rules. We develop an applicable investment model for a hydropower producer. Further, we test the investment framework on a base case with realistic turbine values, developed through conversations with Statkraft. The terms investment and replacement will be used interchangeably about this decision throughout the thesis.

Today, hydropower accounts for around 90% of the power production in Norway, approximately 141 TWh (SSB 2021). With the increasing demand for electricity, as well as for sources of flexibility, hydropower will have an essential role in the green transition (IEA 2021). However, as most of the hydropower plants in Norway were built in the time period 1960-80, there is a great potential for upgrades and replacements of different components in order to meet this demand. According to Henriksen et al. (2020), replacements of hydro turbines installed before 1980 will give an increase in production of 3.1 TWh. For comparison, this is approximately one third of all wind production in Norway (SSB 2021).

Due to wear and tear, hydropower producers running old turbines will at some point need to replace the operating turbine. A hydropower producer should aim to find the optimal timing of the replacement. Today, most hydropower companies use net present value (NPV) for such an investment analysis (Horn et al. 2015). The method is considered adequate when cash flows are relatively safe and certain (Horn et al. 2015). However, the investment decision becomes more complex when facing uncertainty in cash flows, such as evolving electricity prices and the risk of failure. To incorporate the irreversibility of a turbine investment in addition to the uncertainty factors, we have chosen to use a real option framework for our investment analysis. A real option refers to a tangible asset, in which a company can decide when and whether to invest in or not (Dixit and Pindyck 1993). In our case, the tangible asset is a hydro turbine.

There are mainly two reasons why one would replace a hydro turbine; either to decrease the risk of failure or to increase the efficiency. A hydro turbine deteriorates naturally over time due to erosion, material fatigue, and/or cavitation. These factors can occur depending on location, type, size, water flow, and operation pattern, but can also be intensified due to flexible operation, including over-load, part-load, and start/stops (Eggen 2021; Trivedi et al. 2013). A replacement will reduce the risk of an unforeseen failure. In this thesis, a failure is defined as a major breakdown, i.e. the end of the hydro turbine's lifetime, where the hydropower producer is obligated to replace the turbine. We will ignore small breakdowns as they account for an insignificant cost compared to a full replacement. The efficiency of a turbine will also decrease with time, and a replacement will consequently increase the revenue as the efficiency is improved. Due to technological development, a new turbine will have higher initial efficiency than the turbine currently operating. Thus, installing a new turbine will set the efficiency to a higher level and reduce the risk of failure. The turbine which is to be replaced will be referred to as the old turbine throughout this thesis.

In our model, electricity prices and turbine lifetime are considered uncertainty factors. In this thesis, uncertainty in turbine lifetime will be equivalent to risk of failure. We simulate electricity prices ignoring short-term effects with the use of geometric Brownian motion (GBM) and simulate the turbine lifetime with the use of gamma distributed time in condition states. A condition state represents the condition of a deteriorating component and is a well-incorporated grading system in the Norwegian hydropower industry today (Solvang et al. 2011). To our knowledge, we are the first to incorporate these condition states as uncertainty factors within the field of ROA on hydropower.

A base case with realistic values is constructed to test the model. The optimal investment timing is then found with the use of Least Squares Monte Carlo (LSM). This approach is compared to a benchmark, which only considers the technical condition of the turbine when deciding investment timing. Our LSM model yields significantly higher investment values for the turbine system compared to the benchmark.

In the following sections, we present our contributions in light of existing literature. We describe today's best practice on turbine replacements in the Norwegian hydropower industry. Further, Section 2 gives a more thorough description of the problem and how we have approached it by developing an investment decision model. Section 3 presents base case values used to test our modeling framework. A benchmark representing a simplified investment rule is also introduced in the section and used for comparison purposes. The results and a discussion regarding our model are given in Section 4, before we give concluding remarks in Section 5.

1.1 Relevant Literature

We start by defining real option problems and present methods to solve them. An overview of previous work done on ROA within investment decisions is given before we elaborate on the electricity price and hydro turbine lifetime as uncertainty factors in a hydro turbine investment decision.

1.1.1 Introduction to Real Options

The term real option was first introduced by Myers (1977) and refers to option pricing theory applied to the valuation of real (non-financial) assets where the investor has the opportunity to delay, expand, switch, or suspend an investment (Lambrecht 2017). Similar, Dixit and Pindyck (1993) define a real option as the opportunity to acquire real assets. Compared to a financial option which considers trading an underlying asset at a predetermined price, a real option is the right to act on an investment opportunity. This is an opportunity and not an obligation similar to the financial option. Further, three properties must be present in a project in order to call it a real option (Dixit and Pindyck 1993):

1. Irreversibility of investment decision
2. Uncertainty in future rewards from the investment
3. Possibility to delay the investment to obtain more information about the future

All these properties are present in a hydro turbine investment, which will be more thoroughly argued for in Section 2.2. One main advantage of the real option approach is the possibility of valuing investments with flexibility and uncertainty in cash flows, which today's managers face with rapid changes in the business environment (Horn et al. 2015). Thus, a real option approach is an appropriate method when considering a hydro turbine replacement.

Early studies on the usage of ROA for investment decisions are mainly divided into two categories: upgrades and replacements (Mauer and Ott 1995). The literature on replacement decisions is of most interest to us, as this thesis concerns a hydro turbine replacement. Mauer and Ott (1995) is one of the first studies on replacements and marks the beginning of the use of real options in the industry. Mauer and Ott (1995) analyze the determinants of replacement decisions with maintenance and operation cost uncertainty, as well as realistic tax effects. With the use of real options, optimal investment timing of a deteriorating asset with a replacement producing the same product is determined. Factors influencing this timing are the cost volatility, purchase price, corporate tax rate, depreciation rate, technological development uncertainty, and tax law changes (Mauer and Ott 1995). Our work will be based on many of the same principles as Mauer and Ott (1995), as we want to replace a deteriorating turbine with a new equally sized turbine.

Another study looking at replacement models is Richardson et al. (2013), which includes lead time as an uncertainty factor in their real option model. Lead time is here defined as the time from an investment decision is made until the equipment is installed. It is considered an uncertainty factor as their paper is about heavy mobile equipment in the mining industry, where there are significant variations in lead time. For simplicity, lead time is excluded from our thesis but is a possible extension that will be further discussed in Section 4.4.

Three relevant studies using ROA within the hydropower industry are Andersson et al. (2014), Bøckman et al. (2007) and Dønnestad et al. (2022). These also use ROA for optimal investment timing, but not necessarily on a component level. However, they are subject to many of the same uncertainty factors as we discuss for a turbine replacement. Andersson et al. (2014) aim to find the optimal investment timing for an upgrade of the production capacity in an already existing hydropower plant. Hydropower scheduling is included in the ROA, which gives more realistic cash flows but requires numerical solutions (Andersson et al. 2014). Similarly, Bøckman et al. (2007) include the capacity choice in the ROA on investment timing of small hydropower projects. Both studies focus on the uncertainty in electricity prices and find the required electricity price level as a decision rule for the optimal investment timing. Dønnestad et al. (2022) examine the switch from a refurbishment to a replacement of a hydro turbine and include electricity prices as an uncertainty factor. Our study is most similar to Dønnestad et al. (2022) as we both examine the optimal timing of a hydro turbine replacement, including electricity prices as an uncertainty factor. In addition, we include turbine lifetime as an uncertainty factor. Our study stands out as we do a more thorough analysis of a specific component and include its condition state, making the ROA more interpretable and applicable for the hydropower producer.

1.1.2 Methods to Solve Real Option Problems

In order to solve real option problems, there are mainly two methods: dynamic programming and contingent claims analysis. In contingent claims analysis, a replicating portfolio is constructed. The portfolio consists of financially traded assets, which are supposed to have the same risk and return characteristics as the underlying real option. The value of the real option is then determined based on no-arbitrage arguments. However, a replicating portfolio that spans the risk of the project can be challenging to find (Dixit and Pindyck 1993). Kern et al. (2015) tried to find a replicating portfolio for hydropower revenues but concluded that the method performed relatively poorly. When a replicating portfolio is difficult to find, the dynamic programming approach is useful. This is a more general approach, which can be used for more than real option problems. Moghaddam and Usher (2011) and Mo and Hågenvik (2020) are two examples of studies applying dynamic programming on replacement problems. Moghaddam and Usher (2011) used dynamic programming to solve an optimization problem for maintenance or replacements of a multiple component system. Moghaddam and Usher (2011) tried to maximize the reliability of the system and minimize the costs, which can be compared to our problem of maximizing the investment value of a replacement. Further, Mo and Hågenvik (2020) used dynamic programming as a part of an investment decision simulator made for the hydropower industry to calculate the optimal time of an investment. Our model tries to solve a similar problem, however, we utilize dynamic programming with a real option framework.

The dynamic programming approach assumes an exogenous discount rate. The discount rate can be interpreted as the opportunity cost of capital (Dixit and Pindyck 1993). From conversations with Statkraft, it becomes clear that most hydropower companies operate with a fixed required rate of return when considering investment decisions. As an exogenous discount rate for the turbine investment exists, we will use the dynamic programming approach.

LSM is a state-of-the-art approximate dynamic programming approach introduced by Longstaff and Schwartz (2001). The main advantage of LSM is that it can value and manage options with early or multiple exercise opportunities and can thus be used in both financial engineering and for real option analysis. The concept behind the LSM approach is that the option holder must compare the immediate payoff value with the expected value of continuation, and then choose the exercise value with the highest expected payoff. With LSM, the expected payoff from a later exercise is computed by regressing the later payoff values as a function of the current level or state of the uncertainty factors (Longstaff and Schwartz 2001). Therefore, the value of continuation is conditional on these factors, which in this thesis are electricity price and turbine condition state.

Considering real options within the hydropower industry, techniques similar to LSM have been developed for production management, but also for capacity investment settings (Nadarajah et al. 2017). One example of applying LSM for hydropower production management is the work of Denault et al. (2013). Similar to our model, the paper sets the random electricity price as exogenous and defines a finite investment horizon with a discrete number of time periods. Denault et al. (2013) include the available amount of energy in the water reservoir as an endogenous factor dependent on the production decisions. This is however different from our model, as we set the available water energy as deterministic.

To our knowledge, the LSM approach within hydropower real options is not widely used. The application of LSM within electricity distribution management seems to be however more prominent (Blanco, Olsina et al. 2011; Blanco, Waniek et al. 2011; Pringles et al. 2015; Tian et al. 2012). An example is Pringles et al. (2015), who considers transmission network expansions in power markets, where the LSM is applied to determine optimal decision regions to execute, postpone or reject the investment projects. The real option model includes two stochastic processes: the development of electricity demand and generation costs (Pringles et al. 2015). The work of Pringles et al. (2015) has similarities with our work, as we also aim to find optimal decision rules in order to execute or postpone an investment. However, we do not consider the option to reject a project, as we assume that there will always be a turbine replacement within our investment horizon. Another similarity is the inclusion of two stochastic processes in the analysis, though we consider different processes as stochastic compared to Pringles et al. (2015).

There are several other industries that can be mentioned where the LSM method has been applied to solve energy real options. Assereto and Byrne (2021) assess the economic feasibility of utility-scale solar in Ireland using a real option framework. Further, Abadie and Chamorro (2009) determine the value of doubling the capacity of a gas-fired plant, while Zhu (2012) investigates nuclear power investments in China, both using the LSM approach. Another case study from China also applying LSM, is the carbon capture and storage investment evaluation model developed by Zhu and Fan (2011). Common for many studies on energy real options is the identification of electricity prices (or electricity demand) as the main source of uncertainty (Kozlova 2017). The electricity price level is one out of two uncertainty factors determining the optimal time of a turbine investment in our model as well. Modeling electricity price as a stochastic process makes the LSM an applicable method for numerical and complex problem-solving.

1.1.3 Electricity Prices

Electricity price is a crucial factor when considering a turbine investment. The future price levels will largely determine the power plant revenue and be important in estimating the cost of lost production if a failure with a long downtime occurs. Hydropower producers selling electricity in the power market face uncertainty in electricity prices. Through conversations with Statkraft, it became clear that the hydropower companies spend much time considering future price scenarios.

Literature on the topic shows that it is common to differentiate between the short-term and the long-term development of commodity prices, such as electricity prices. One example from the literature is Schwartz and Smith (2000), which develop a two-factor model for commodity prices that accounts for both short-term and long-term effects. Short-term deviations are expected to follow a mean-reverting Ornstein-Uhlenbeck (OU) process, meaning that the short-term deviations will revert to zero. The long-term equilibrium price level is assumed to follow a geometric Brownian motion (GBM) with drift (Schwartz and Smith 2000). Considering the development of electricity prices, the short-term and long-term variations can be explained by different external factors. The weather, availability of production capacity, and transmission bottlenecks can explain short-term variations. Long-term variations are on the other hand subject to uncertainties in electricity demand, substitute energy prices, inflation, technological development, as well as climate politics and regulations (Bøckman et al. 2007; Haukeli et al. 2021; Schwartz and Smith 2000).

There are multiple proposed models found in the literature trying to catch the dynamics of electricity prices. Lucia and Schwartz (2001) investigate several one and two-factor models and build on the work of Schwartz and Smith (2000) when presenting the two-factor models. Lucia and Schwartz (2001) also apply a mean-reverting OU process to represent short-term variations but use arithmetic Brownian motion to model long-term effects instead of a GBM in the two-factor models. The paper concludes that two-factor models perform better than one-factor models when including both long-term and short-term effects in electricity prices (Lucia and Schwartz 2001).

For a hydropower producer considering a long-term power generating project, only the uncertainty in time average electricity price over the project period is relevant. Short-term mean reversion has a minor influence on investment values and decisions (Bøckman et al. 2007). Thus, only the long-term electricity price levels will be of interest when considering whether and when to invest in a new hydro turbine. Schwartz and Smith (2000) argue that a one-factor model using a GBM process, only concerning uncertainty in equilibrium prices, performs well for many long-term investments and that short-term variations safely can be ignored. The work of Pindyck (2001) also concludes that applying a GBM to catch long-term commodity price evolution will lead to minor errors for long-term energy-related investments.

The work of Bøckman et al. (2007) is one example of a paper considering long-term electricity price dynamics modeled as a GBM. Both the problem and approach discussed in Bøckman et al. (2007) have similarities with what we consider in this thesis. Bøckman et al. (2007) aim to find the optimal investment timing for small hydropower projects but differs from our problem as it is not on component level and also discuss the optimal capacity choice. However, both Bøckman et al. (2007) and this thesis apply a real option framework with dynamic programming to approach the problems.

As a hydro turbine replacement can be considered a long-term power generating project, we are only interested in catching the dynamics of the time-averaged electricity price over the project period. Thus, modeling the electricity price levels as a GBM is suitable for our purpose, as we ignore short-term deviations. The GBM model for electricity price development will be described in detail in Section 2.3.

1.1.4 Lifetime of Hydro Turbine

The lifetime of a hydro turbine is limited due to deterioration caused by erosion, cavitation, fatigue, and material defects (Kumar and Singal 2015; Trivedi et al. 2013). The distribution of these deterioration types varies depending on the type, location, water flow, and operating pattern. Deterioration models can be divided into three groups (Nicolai et al. 2007):

- Black-box models
- Grey-box models
- White-box models

Black-box models use lifetime distributions to describe the random time to failure. Such models will only determine if a component is functioning or not, and are not modeling deterioration as a function of time. On the other hand, white-box models require knowledge about the deterioration process and model through simulations of measurable deterioration and failure physics. In order to develop a white-box model, the condition of the system must be monitored continuously.

To overcome the simplicity of black-box models and deal with the lack of continuous monitoring required in white-box models, a grey-box model is the most appropriate for this paper. Grey-box models are typically stochastic processes with time-dependent deterioration and failure. In contrast to white-box, grey-box models only require periodic monitoring of the system's condition (Nicolai et al. 2007; Welte 2008).

Welte (2008) is a great contribution to the field of deterioration models. In his model, he incorporated condition states, which is a set of states describing a component's condition. The conditions are frequently used in the Norwegian hydropower industry and will be further described in Section 1.2.1. The simulation of the turbine's lifetime in our model will be based on the deterioration model developed by Welte (2008). To our knowledge, this is the most prominent research in the field today.

1.2 Today's Industry Practice

In order to develop a transparent real option model for a hydro turbine replacement, it is essential to understand today's industry practice. A description of condition states used for condition monitoring in the Norwegian hydropower industry is given in the following. Then, the process of replacing a hydro turbine in Statkraft is described.

1.2.1 Condition Monitoring in the Norwegian Hydropower Industry

Visual inspections are regularly assessed to determine component deterioration in the Norwegian hydropower industry. The inspections are qualitative assessments done by maintenance personnel which follow the same guidelines in order to standardize and quantify the evaluations. The classification system, which is widely applied in the industry today, was introduced by the Norwegian Electricity Industry Association (EBL) in the 1990s (Welte 2008). Five condition states describe the deterioration of hydropower plant components, where state 1 defines a new component and state 4 indicates the most critical condition (Welte and Eliasson 2011). An imaginary state 5 implies that the component has failed. A general description of the five states is presented in Table 1, while Table 2 lists the states for a Francis turbine runner with fatigue as a failure mechanism, which is the turbine type we study in the base case described in Section 3.

Table 1: General description of condition states according to EBL. Adapted from Welte, Heggset et al. (2011).

Condition state	Description
1	No indication of weakening.
2	Some indication of deterioration. The condition has noticeably worsened.
3	Serious deterioration. The condition has considerably worsened.
4	The condition is critical.
5	Failure.

Table 2: Description of condition states. Component: Turbine runner. Failure mechanism: Fatigue. Inspection method: Visual inspection. Adapted from Welte, Heggset et al. (2011).

Condition state	Description
1	Surface is plane and bright. No sign of damage.
2	Minor areas of the runner have a dull surface.
3	Surface is rough. Pitting. Small cracks evaluated as uncritical.
4	Critical cracks in the turbine runner.
5	Failure.

Regular visual inspections and classification of deterioration through the EBL system are well incorporated into the hydropower industry. All hydropower companies that participated in a survey from 2006 to 2007 confirmed that they used condition state inspections (Solvang et al. 2011). In a survey conducted by Eggen and Solvang (2002), condition indicators that were registered or measured by the Norwegian hydropower industry were identified. The results showed that approximately 50% of the indicators were judged utilizing the EBL classification system for mechanical equipment in a hydropower plant, such as the turbine. A deterioration model that incorporates this system will have the advantage of the necessary data already obtained.

Visual inspections are performed at regular intervals and are done more frequently as the component is observed to be in one of the last states. As the monitoring is not continuous, the exact time for a transition between two condition states can not be accurately determined. Despite this, the inspections are the most accurate data available to tell how the condition of a hydropower component has developed over time.

1.2.2 Investment Decisions for Hydro Turbines in Statkraft

The replacement of a hydro turbine is a long process, as it involves several departments, as well as potential turbine suppliers. From conversations with Statkraft, the process can be divided into several steps illustrated in Figure 1. It is emphasized that this is a simplification of the process. Each replacement will be different, and thus time spent in each step are approximations. A brief description of the process will be given in the following.

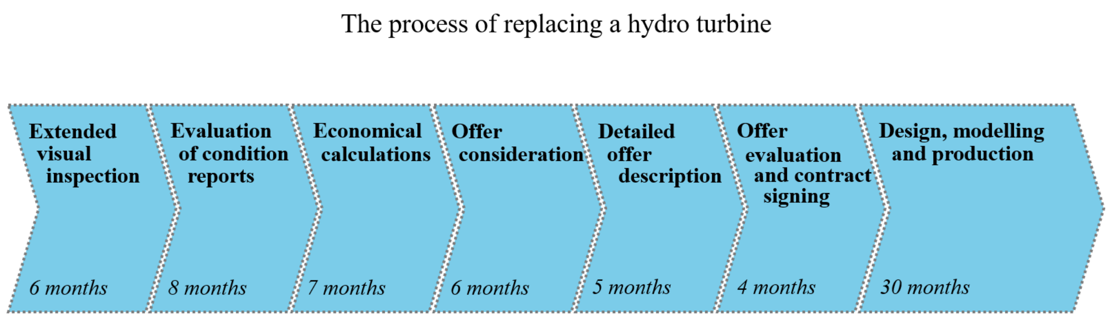

Figure 1: Steps in the process of replacing a hydro turbine, based on conversations with Statkraft.

The replacement of a hydro turbine is usually triggered by an on-site extended condition assessment which is performed when the turbine has reached a certain age. If the technical condition is considered unsatisfactory, or the efficiency is substandard compared to that of a new turbine, the process of replacing the turbine proceeds. The first step after the condition monitoring is the evaluation of the condition reports by a manager, which may conclude that the turbine needs an upgrade or replacement. Specialists will then consider whether other parts of the hydropower plant should be replaced or maintained at the same time to utilize the expected down period maximally.

Further, economic calculations are required to determine the value of the replacement. Both the expected costs and the expected increase in income from the replacement are considered. The costs include the purchase of the turbine and required equipment, as well as the time spent on the project. A replacement could also lead to a loss in production during the project period, although a well-planned project should minimize these costs. The increase in income can be caused by increased turbine efficiency, changes in operation pattern, and reduced maintenance costs.

Through an NPV analysis, the value of a replacement is compared to the value of not investing. A project application then has to be prepared. If approved, determining a procurement strategy begins, and inquiries are sent to relevant suppliers. Offers from suppliers must then be considered before the company decides on a supplier. Before a final contract can be signed, the chosen supplier writes a more detailed offer description. Then designing, modeling and production of the new turbine are carried out, before the new turbine is installed and tested. The entire process can take as long as six years. This is highly dependent on the size and type of turbine, and some turbines can be delivered in as little as 14-25 months.

2 Hydro Turbine Replacement Model

Section 2 gives a thorough description of the hydro turbine replacement problem and how we have approached it by developing an investment decision model. We first scope the problem before arguing that a real option framework is appropriate when analyzing a hydro turbine replacement. Then, we give mathematical descriptions of how we have modeled electricity prices, turbine lifetime, turbine efficiency changes, and the value of the turbine system. At last, the LSM-based algorithm for solving the problem is described.

2.1 Scope and Assumptions

There are several factors determining the value of the turbine system. In this thesis, we include the factors regarded as the most influential for an investment decision. We model the electricity prices and turbine lifetime as stochastic processes. Changes in efficiency due to technological development and wear and tear are assumed to be deterministic. Further, annual inflow is assumed to be the same for all years, as seasonal variations are irrelevant for long-term investment decisions. For simplicity, we ignore the possible effects that climate change has on the annual inflow over the investment horizon. We also assume that the old and the new turbine have the same installed capacity and will be operating for the same amount of hours per year. Thus, differences in energy output between the old and the new turbine will be caused by differences in efficiency. The investment cost related to a runner replacement is independent of which year the replacement happens.

The total turbine lifetime we simulate, as will be described in Section 2.4, is the mechanical lifetime of the hydro turbine. Time spent in each condition state 1, 2, 3, and 4 is random, and the mechanical lifetime is defined as the sum of time spent in all states. The turbine can only fail at the end of condition state 4, after running through condition states 1, 2, and 3 in ascending order. Thus, a failure will be unforeseen as the total lifetime is random and unknown in advance of the failure for the hydropower producer. We assume that the turbine *must* be replaced as the total lifetime expires because it can not be used for its intended purpose after this. However, a hydro turbine *should* be replaced when its economic lifetime expires. This is the point in time when it is profitable to replace in terms of economics. The economic lifetime is shorter than the mechanical lifetime, and any hydropower producer should strive to find this economical optimal time of replacement. Further, we also assume that a failure will result in a long stop in production, incurring the hydropower producer extremely high costs. Similar to the investment cost related to a runner replacement, the cost of a failure is independent of which year the failure occurs. Turbine failures resulting in short downtime with no need for a replacement are excluded from the study, as the related costs are negligible when considering a long-term investment decision.

The problem is scoped to only consider one turbine replacement, meaning that replacing the new turbine at a later point in time is excluded from the study. During the investment horizon, the hydropower producer will once a year make a decision whether to invest in the current year or to wait. In real life, the hydropower producer will not always be free to choose the optimal time for a turbine replacement. The replacement must be coordinated with the available workforce, power production, and the corporate budget. As those factors are company-specific and we aim to develop a general model, we ignore this and assume it is possible to freely choose the time of a replacement. The latest possible time to invest is at the end of the mechanical lifetime of the old turbine, and thus the longest simulated turbine lifetime determines the finite investment horizon. Expanding the problem to consider more than one turbine replacement would have required modeling of electricity prices and condition states very far into the future, which will result in larger uncertainties. Due to the robustness of the turbine, there is several decades between two replacements. With a newly installed turbine, the next replacement will not be of concern for a few decades. Thus, scoping the problem to only consider one replacement will not influence our decision rules.

2.2 Hydro Turbine Replacement as a Real Option

We define the opportunity to replace a hydro turbine as a real option, defined by the three properties mentioned in Section 1.1.1:

1. The investment decision is irreversible. Once the hydropower producer has decided to replace the old turbine, the new turbine is uniquely designed for the hydropower plant and does not have the flexibility to be used at another location.
2. Uncertainty in electricity prices, as well as the risk of failure, lead to uncertainty in future rewards from the investment. If future electricity prices are low, the hydropower producer might not recover the investment cost of a replacement in a short-term perspective. On the other hand, a turbine failure will result in extremely high costs in terms of lost production as well as additional costs caused by inconveniences with extended downtime.
3. It is possible to delay the turbine investment to obtain more information about the future electricity prices and the condition of the old turbine. Thus, the value of waiting for more information is accounted for in a real option analysis.

From this, we see that a real option framework is appropriate when analyzing a hydro turbine replacement.

2.3 Modeling Electricity Prices

Long-term electricity prices are crucial for hydropower producers considering a turbine replacement, as the price development largely impacts the value of the investment. As stated in Section 1.1.3, we will model long-term time-averaged electricity prices as a GBM process. The prices will represent the annual unit revenue for the hydropower producer.

The Brownian motion, also called a Wiener process, is a continuous time-stochastic process. One of the most important characteristics of such a process is the Markov property, which says that only the current value is useful when forecasting the future path of the process. Past values of the process or any other existing information will not affect the future path (Dixit and Pindyck 1993). Using GBM, it is assumed that the prices are expected to grow at a constant rate, also referred to as the annual drift. Further, it is assumed that the variance increases in proportion to time. If the electricity price level decreases (or increases) more than expected in one time period, electricity price forecasts for all times in the future will be decreased (or increased) in the same proportion (Schwartz and Smith 2000). For the hydropower producer, this means that only the current electricity price level is valuable, and past electricity price values will not affect the future price development.

Mathematically, the GBM is described by the following stochastic differential equation (SDE)

$$dP(t) = \alpha P(t)dt + \sigma P(t)dZ(t), \tag{1}$$

where $P(t)$ is a measure of the long-term time averaged electricity price level at time t. The parameter α is the annual growth rate and σ is the annual volatility. The term $dZ(t)$ is an increment of a standard Wiener process with mean zero and unit variance (Dixit and Pindyck 1993).

The analytic solution of the SDE, which is has been applied for simulation, is given as follows

$$P(t) = P(0)e^{(\alpha - 0.5\sigma^2)t + \sigma\sqrt{t}Z}, \tag{2}$$

where $P(0)$ is the current long-term electricity price level and $Z \sim \mathcal{N}(0,1)$. We assume that the mean electricity price is at a given level within a year, therefore the Monte Carlo simulations have an annual resolution for the electricity price dynamics.

As we aim to catch the dynamics of a long-term time-averaged electricity price, we have to assume that α and σ are constant in order to use GBM. This assumption is suitable for our problem because we do not consider seasonal and other short-term variations, making the volatility time-dependent.

Given that the long-term electricity price follows (1), the expected price is (Dixit and Pindyck 1993)

$$E[P(t)] = P(0)e^{\alpha t}. \tag{3}$$

This is one of the results of the Markov property, saying that the future given the present state, is independent of the past.

2.4 Modeling Hydro Turbine Lifetime

We base the modeling of hydro turbine lifetime on the work of Welte (2008). The model follows a continuous-time semi-Markov process with discrete state space, which Welte (2008) suggests as a general deterioration model for the Norwegian hydropower industry. The sojourn time θ_i in a condition state i is assumed to be a random variable. The θ_i's are modeled using a gamma distribution and suggested to be statistically independent (Welte 2008). The distributions and turbine lifetime development are illustrated in Figure 2.

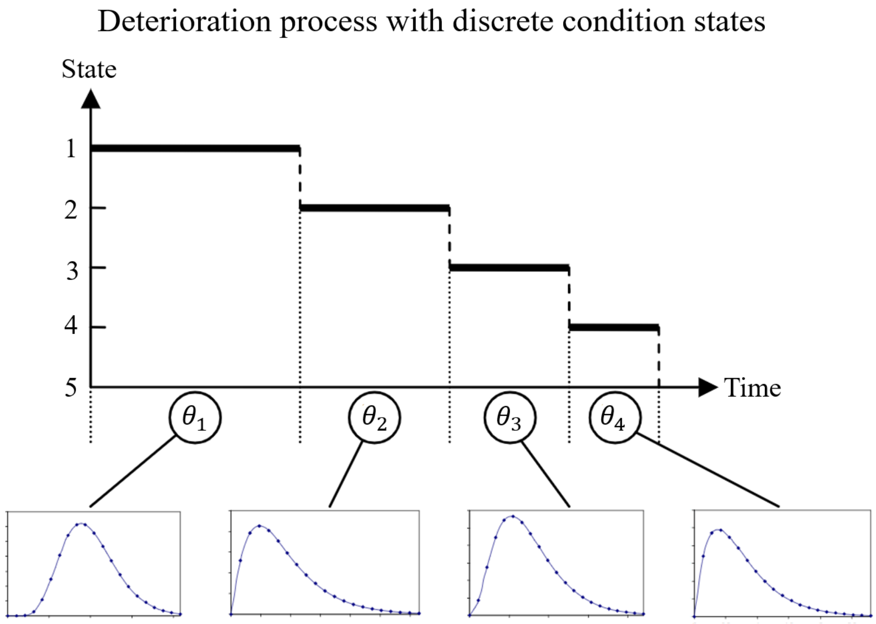

Figure 2: Modeling of deterioration by a semi-Markov process with discrete condition states. Adapted from Welte (2008).

The probability density function of the gamma distribution is

$$f(\theta_i) = \frac{1}{\beta_i^{k_i}\Gamma(k_i)}\theta_i^{k_i-1}e^{-\frac{\theta_i}{\beta_i}} \quad \text{for } \theta_i \geq 0, \tag{4}$$

where $\Gamma(.)$ is the gamma function (Gertsbakh and Kordonskiy 1969). k_i is the shape parameter and β_i is the scale parameter for condition state $i \in \{1, 2, 3, 4\}$. Condition state 5 will not have any distribution, as this is an imaginary state, indicating a failure.

In this study, the shape and scale parameters have been estimated through maximum likelihood estimation (MLE). For each condition state i, we have a number of N_i expert judgments. τ_{ij} is the time one expert expects the turbine to spend in condition state i. To calculate the MLEs, an intermediate value A_i is first calculated from (5). Then, the shape and scale estimators are respectively calculated from (6) and (7) (Husak et al. 2007).

$$A_i = \ln \overline{\tau}_i - \frac{\sum_{j=1}^{N_i} \ln \tau_{ij}}{N_i} \tag{5}$$

$$\widehat{k}_i = \frac{1}{4A_i}\left(1 + \sqrt{1 + \frac{4A_i}{3}}\right) \tag{6}$$

$$\widehat{\beta}_i = \frac{\overline{\tau}_i}{\widehat{k}_i} \tag{7}$$

Applying this deterioration model, we can simulate transitions between condition states. We do this by randomly picking sojourn times θ_i from each of the four distributions. By applying the random times, we simulate how long one hydro turbine is operating in a given condition state. Time already spent in the starting condition state at the beginning of the investment horizon, referred to as ϕ, is subtracted from the simulated total time in this state. If the time already spent in this condition state exceeds the simulated time, the turbine will transition to the next condition state in the succeeding year.

$\lambda(t)$ represents the condition state at time t in the investment horizon. Time in each condition state θ_i is summarized to determine the total lifetime of that turbine. The condition states and turbine lifetime are simulated for each turbine path.

2.5 Modeling Efficiency Changes

The efficiency of a hydro turbine is expected to decrease during its lifetime. Models to determine efficiency reduction can be classified as stochastic or deterministic (Welte 2008). By applying a stochastic process, the complexity would have increased significantly, as the electricity prices and turbine lifetime are modeled as stochastic processes in this thesis. Thus, a deterministic efficiency model will be used in this paper to keep the complexity at a reasonable level.

The regulator, the Norwegian Water Resources and Energy Directorate (NVE), suggests that the Norwegian hydropower producers can use a linear model, saying that the annual efficiency reduction is constant over the turbine's lifetime (NVE 2017). Thus, we will model the efficiency reduction due to wear and tear as a linear function

$$\eta(t) = \eta_0 - \gamma t, \tag{8}$$

where η_0 is the turbine efficiency when the turbine is new. γ is the annual efficiency reduction rate, and $\eta(t)$ is the turbine's efficiency when it has been operating for t years.

However, when considering a replacement of an old turbine with a new one, we are not only interested in how much the efficiency of the old turbine has decayed. The value of interest is the total increase in efficiency by setting a new turbine into operation. The total increase in efficiency includes both the recovery from wear and tear but also technological development. This is illustrated in Figure 3.

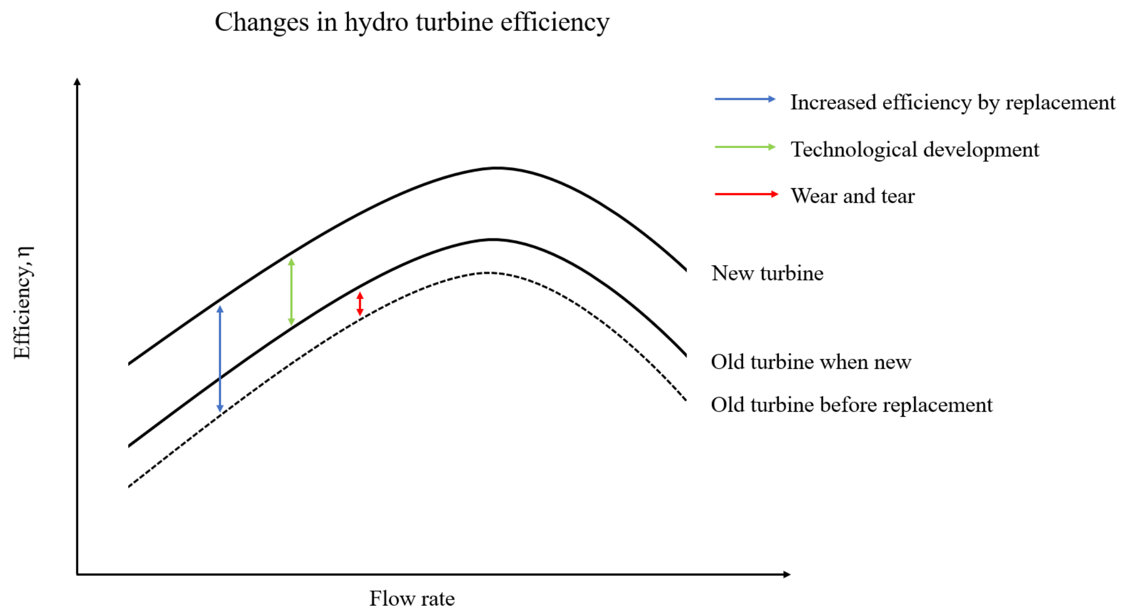

Figure 3: Changes in turbine efficiency due to both wear and tear and technological development. Adapted from NVE (2017).

We model the technological development by setting the starting efficiency of the new turbine $\eta_{0,new}$ at a higher level than the starting efficiency of the turbine currently operating $\eta_{0,old}$. In this thesis, it is assumed that the currently operating turbine has not experienced this technological development and is thus described as the old turbine.

Both the initial efficiency and the annual rate of efficiency reduction are assumed to be improved by technological development. Thus, the speed of efficiency reduction will be slower for the new turbine, and $\gamma_{old} > \gamma_{new}$. The difference between the old and the new turbine in terms of efficiency is shown in Figure 4.

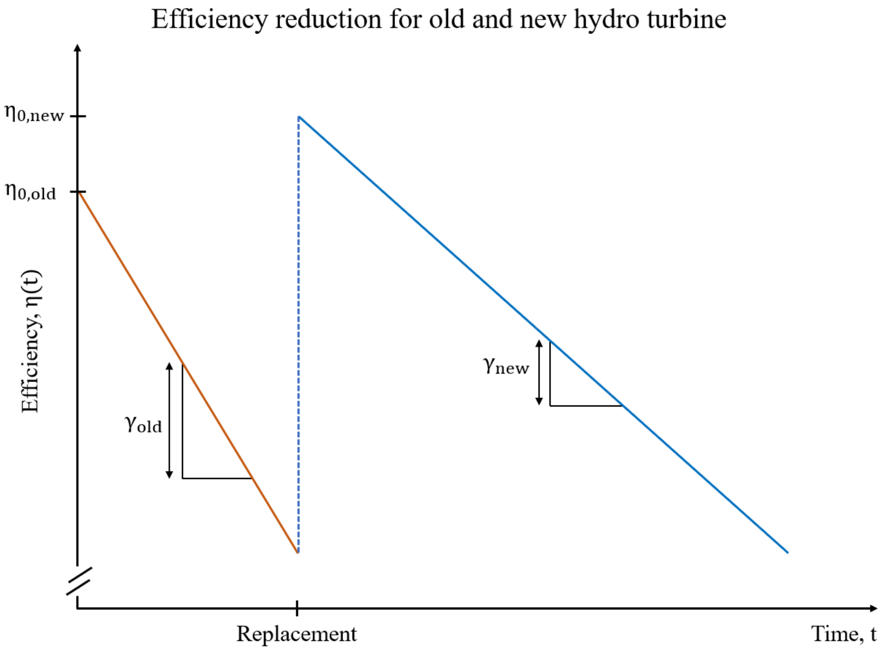

Figure 4: Difference between the old and the new turbine in terms of initial efficiency and the annual rate of efficiency reduction.

2.6 Defining the Value of the Turbine System

In order to determine the optimal time of a turbine replacement, we consider the value of the turbine system, which is referred to as V_{sys} and expressed in (9). The value of the turbine system is defined as the value of having an old turbine in operation while also having the option of investing in a new turbine to replace the old turbine. V_{sys} is a piecewise function divided into two parts. The first part expresses the assumption introduced in Section 2.1, which says that we are forced to invest in a new turbine if the old turbine reaches condition state 5. The value of the turbine system will then be V_{inv}. The second part expresses the maximal value of whether to invest immediately or postpone the replacement if the old turbine has not failed.

$$V_{sys}\big(t, P(t), \lambda(t)\big) = \begin{cases} V_{inv}\big(t, P(t), \lambda(t)\big) & \text{if } \lambda(t) = 5 \\ max \begin{cases} V_{inv}\big(t, P(t), \lambda(t)\big) \\ E\big[V_{sys}\big(t+1, P(t+1), \lambda(t+1)\big)\big]e^{-\rho} + V_{prod}\big(t, P(t)\big) \end{cases} & \text{if } \lambda(t) \neq 5 \end{cases} \tag{9}$$

The value of replacing the old turbine and investing in a new one is given in (10). Similar to V_{sys}, V_{inv} is dependent on the current condition state $\lambda(t)$ of the old turbine. If the old turbine has failed, the hydropower producer is forced to both pay a cost related to the failure C_{fail}, but also to pay the cost of investing in a new turbine runner C_{turb}. C_{turb} includes the cost of materials, transport, engineering, modeling, testing, assembly, and so on. C_{fail} includes both costs easy to measure in terms of money, such as lost production due to long downtime, but also costs hard to measure. Such costs include potential environmental damage and loss of reputation. Human life and health can also be put in danger if personnel or other individuals are present at severe failure events (Welte 2008). To include all the disadvantages of a failure, C_{fail} will be a significant cost.

$$V_{inv}\big(t, P(t), \lambda(t)\big) = \begin{cases} \sum_{m=0}^{T_{new}} \Big(P(t)e^{\alpha m}\big(\eta_{0,new} - m\gamma_{new}\big)Qe^{-\rho m}\Big) - C_{fail} - C_{turb} & \text{if } \lambda(t) = 5 \\ \sum_{m=0}^{T_{new}} \Big(P(t)e^{\alpha m}\big(\eta_{0,new} - m\gamma_{new}\big)Qe^{-\rho m}\Big) - C_{turb} & \text{if } \lambda(t) \neq 5 \end{cases} \tag{10}$$

The investment will create value in terms of production over the expected lifetime of the new turbine T_{new}. Q is the annual inflow in terms of energy, and $P(t)$ is the electricity price level at time t. Given $P(t)$, the expected future price levels can be calculated from (3) with an annual drift α. The initial efficiency of the new turbine $\eta_{0,new}$ will decrease with an annual speed γ_{new}.

If the turbine has not failed at the time we choose to invest, the value of the investment includes the value of the new turbine as well as the replacement costs. Production revenue from the new turbine is discounted with the factor ρ, which is the annual discount rate required from the hydropower producer.

As V_{inv} in (10) has now been explained, we once again consider V_{sys} in (9) in the situation where the old turbine has not failed, and we are not forced to invest immediately. Applying the dynamic programming approach, the sequence of possible decisions is broken down into just two components: the value of investing immediately and the continuation function that includes the consequences of all subsequent decisions (Dixit and Pindyck 1993). In this situation, the hydropower producer will compare the value of investing now with the value of postponing the investment. The second alternative includes the expected value of the turbine system the subsequent year and the value of one-year production revenue from the old turbine V_{prod}. V_{prod} is given as

$$V_{prod}\big(t, P(t)\big) = P(t)\big(\eta_{0,old} - \gamma_{old}(\omega + t)\big)Q. \tag{11}$$

The old turbine has an initial efficiency $\eta_{0,old}$ and decreases with an annual speed γ_{old}. ω is defined as the age of the old turbine at the beginning of the investment horizon $t = 0$. The expected value of the turbine system in the subsequent year is determined from regression in the LSM algorithm and will be further described in Section 2.7.

2.7 Algorithm to Determine Optimal Investment Timing

The algorithm for determining the optimal investment time for a new hydro turbine will be described in the following. The model is implemented in Python and the code is attached in Appendix A. Electricity prices and turbine lifetime are simulated as described in Section 2.3 and Section 2.4 M times over a given investment horizon, where M is a large number. As the electricity price $P(t)$ is modeled as a stochastic process, $P(t)$ has different values for each simulated path s, as well as for each time t. Also, the total turbine lifetime will have unique values for each path s, as each simulated turbine will differ in time spent in each of the four first condition states.

At the beginning of the investment horizon $t = 0$, values for the current long-term electricity price level $P(0)$, current condition state $\lambda(0)$ and time already spent in this state ϕ, as well as the age of the turbine in operation ω, should be known for the hydropower producer. Those values are set as a starting point for the simulation. The decrease in efficiency for the old turbine will develop over time but be the same for each path as it is deterministic.

The value of investing V_{inv}, as given in (10), is calculated for each time for each simulation path. Annual inflow in terms of energy Q, the expected lifetime of new turbine T_{new}, investment cost C_{turb} and failure cost C_{fail} are assumed to be constant for all paths and all times for simplicity.

After simulating M paths and calculating the value of investing in a new turbine at each time t, the LSM algorithm recursively determines if it is most profitable to invest now or wait. The length of the investment horizon is determined by the turbine with the longest simulated lifetime. The turbine has to be replaced at the time of failure, when λ=5, and the value of the turbine system is therefore defined to be the value of investment V_{inv}, at the time of failure for each path.

To compare the value of investing now to waiting, we do a non-linear least squares regression of all paths s to find the conditional expected value of waiting. Our model includes two state variables determining the system value at time t: electricity price level $P(t)$ and condition state $\lambda(t)$ for the old turbine. The function to determine the conditional expected value of the system should include both state variables, as well as the square terms of both $P(t)$ and $\lambda(t)$ and the cross-products (Jafarizadeh and Bratvold 2015; Longstaff and Schwartz 2001). There are a number of different basis functions that can be used for the regression, for example, Hermite, Legendre, Chebyshev, Gegenbauer, and Jacobi polynomial. Numerical tests have indicated that even the basic power of state functions give accurate results (Longstaff and Schwartz 2001). Thus, our regression function in the LSM-based algorithm is a second-order polynomial, given in (12).

$$\begin{aligned} E[V_{sys}(t+1, P(t+1), \lambda(t+1))] &= g(P(t), \lambda(t)) \\ &= F_t P(t)^2 + G_t P(t) + H_t \lambda(t)^2 + I_t \lambda(t) + J_t P(t)\lambda(t) + K_t \end{aligned} \tag{12}$$

F_t, G_t, H_t, I_t, J_t and K_t are the regression coefficients determined for each time t.

The value of the turbine system in the subsequent year is discounted in order to be comparable to the value of investing now. If the immediate investment value exceeds the conditional expected value of waiting, the value of the turbine system is set to be the investment value, and we continue to work recursively. The assessment of whether to invest now or wait was executed recursively for all paths s and for all times t, i.e., once a year.

3 Base Case

In order to test our model, we define a base case with reasonable values for the option of replacing an old Francis turbine runner with a new one. This is the most common hydro turbine installed in Norway (Fladen et al. 2010). We consider a turbine in condition state 2 at the beginning of the investment horizon and assume it has been operating for 20 years. Further, we assume that it was 15 years since the hydropower producer registered the transition from condition state 1 to 2. Thus, the hydropower producer will be interested in considering a replacement in the relatively near future. The assumptions for the base case are the same as described in Section 2.1, where the main objective is to determine the optimal investment timing.

Many of the parameter values required in our model will have large variations due to factors such as installed capacity, operational pattern, and location of the plant. As our purpose is to test the model, it is appropriate to define a base case with imaginary yet reasonable magnitudes. However, it is possible to apply our model for real-world problems with values directly referring to a specific plant.

3.1 Parameter Values

The parameter values used in the base case are summarized in Table 3 and will be justified in the following. Most of the parameter values have been chosen based on previous work on similar topics. Some of the data required for the model have not been available from open sources, and a group of experts from the hydropower industry were asked to list values within a realistic range.

Estimating the annual growth rate of the long-term electricity price level requires extensive in-depth analysis. As a detailed study of electricity price development has not been in focus in this thesis, we use previous work to set a value for the parameter. A long-term power market analysis for 2021 to 2040 predicts Norwegian electricity price levels from 30 future weather scenarios (Haukeli et al. 2021). With 0.30 kr/kWh as the five-year moving electricity price average in 2021, the expected electricity price level in 2040 is 0.50 kr/kWh (Haukeli et al. 2021). From this, we expect an annual growth rate of 2.6%. We choose to apply a conservative value and set the annual growth rate α equal to 2%. We also base our value for the current long-term electricity price on the work of Haukeli et al. (2021) and set $P(0)$ equal to 0.30 kr/kWh.

Similar to α and $P(0)$, the annual volatility of the electricity price level σ is chosen based on previous work. We set a value based on the work of Bøckman et al. (2007), which estimated the annual volatility of electricity prices from five one-year forward contracts in the Nordic electricity market to be 13.35%. As mentioned in Section 1.1.1, the work of Bøckman et al. (2007) shares many similarities with our problem. We choose to round σ to 13% in our case study.

When determining the value of replacing an old turbine, the expected lifetime of the new turbine is an important parameter. Compared to the mechanical lifetime modeled in Section 2.4, the expected lifetime of the new turbine will be a proxy of the economic lifetime. Thus, we only want to have the new turbine in operation as long as it is profitable. There are examples of successful operation of hydraulic turbines running for almost 100 years, but the economic lifetime of such turbines is usually 30-40 years (Georgievskaia 2019). Ruud (2017) used an expected lifetime of 30 years when studying the optimal time of replacing a Francis turbine runner in a large Norwegian hydropower plant. According to the turbine experts, the turbine lifetime is in the range of 30-40 years. We choose to apply a conservative value and set the expected lifetime of the new turbine T_{new} to 30 years.

The turbine experts were also asked to list expected times spent in each condition state, which were used to calculate the shape and scale parameters for the gamma distributions. We set the expected time in condition state 1, 2, 3, and 4 to, respectively 5, 18, 9, and 1 year, which is the average of the values listed by the experts. State 5 is a fictional state which indicates turbine failure, and the expected time in this state is therefore set to 0.

Based on industry experience, the experts were asked to give reasonable values for turbine efficiency, annual inflow in terms of energy, investment cost, and cost related to a major failure. Starting efficiency for an old turbine and a new turbine were thereby set to respectively 94.25% and 95.20%, where technological development explains the gap. Further, a turbine with an annual inflow of 150 GWh is used for this base case. We set the investment cost for a new turbine to 20 MNOK. Considering costs hydropower producers face from a failure, lost production, loss of reputation, as well as the potential damage to human life and the environment are included. Thus, C_{fail} will be a major cost and is set to 1,000 MNOK.

Machinery in the hydropower industry is in general robust, and for the turbine runner, this means both a long lifetime and low efficiency reduction. NVE recommends the Norwegian hydropower producers to use an efficiency reduction rate of 0.043% per year due to wear and tear (NVE 2017). As mentioned in Section 2.5, it is assumed that technological development has led to a lower speed of efficiency reduction. Thus, we have used the recommended efficiency reduction from NVE (2017) as an indicative value and assumed that it is lower than the annual efficiency reduction for the old turbine and higher than the annual efficiency reduction for the new turbine. In the base case, we respectively set γ_{old} and γ_{new} to 0.073% and 0.033%.

The discount rate should reflect the risk-return characteristics of the investment option. A low discount rate will be beneficial for technologies with high upfront investment costs and low running costs. This cost pattern is typical for renewable energy production and will be more characteristic for technologies with long lifetimes. Thus, investment projects within hydropower should generally be valued with a low discount rate (Edenhofer et al. 2012). The discount rate can be challenging to obtain, as this usually is sensitive information from the hydropower producer's perspective. Also, investors will often have to rely on both advice from valuation experts, as well as their own experience to decide on a discount rate. However, the consulting and accounting firm Grant Thornton did an extensive survey considering discount rates applied within renewable energy projects. For levered hydro projects in the Nordics, the survey concluded that an indicative discount rate should be 5.75% (GrantThornton 2018). Further, Edenhofer et al. (2012) used discount rates of respectively 3, 7 and 10% when calculating levelized cost of electricity in their analysis of cost trends within hydropower projects. From conversations with Statkraft, it became clear that a discount rate above 5.75%, but less than 10%, is reasonable. Thus, we set the discount rate ρ to be 6% in the base case.

Table 3: Parameter values used in the base case.

Parameter description	**Symbol**	**Value**	**Unit**
Condition state at $t = 0$	$\lambda(0)$	2	-
Age of old turbine at $t = 0$	ω	20	years
Time already spent in condition state at $t = 0$	ϕ	15	years
Long-term electricity price at $t = 0$	$P(0)$	0.30	NOK/kWh
Annual growth rate	α	2	%
Annual volatility	σ	13	%
Annual inflow in terms of energy	Q	150	GWh
Expected lifetime of new turbine	T_{new}	30	years
Efficiency of existing turbine (when new)	$\eta_{0,old}$	94.25	%
Efficiency of new turbine (when new)	$\eta_{0,new}$	95.20	%
Annual efficiency reduction rate for old turbine	γ_{old}	0.073	%
Annual efficiency reduction rate for new turbine	γ_{new}	0.033	%
Investment cost by a turbine replacement	C_{turb}	20	MNOK
Cost by a failure	C_{fail}	1,000	MNOK
Expected time in condition state 1, 2, 3, 4	$\overline{\tau_i}$	5, 18, 9, 1	years
Annual discount rate	ρ	6	%

3.2 Benchmark

The benchmark models a simplified view of a hydropower producer wanting to exploit most of the old turbine's lifetime. It is assumed that the producer regularly monitor the technical condition and that this is the only factor accounted for when considering a replacement. Based on industry experience for time spent in each condition state, we set the decision rule in the benchmark as follows: *replace the old turbine after five years of operating in condition state 3.* Though the decision rule differs from the LSM approach, the investment value is calculated as in (10) when the investment timing is known. All parameter values are equal to the base case. It is essential to notice that the cost of failure will be included if the simulated lifetime is shorter than the investment timing for the benchmark rule.

Compared to the LSM approach, the economical perspective is not included when determining the time of replacement in the benchmark. The decision rule is only dependent on the condition state of the turbine. With the use of the parameter values from the base case, it is possible to compare the results from the LSM to the benchmark. Thus, we obtain implications of the effect of the electricity price uncertainty on the investment timing. Section 4.2 gives a thorough discussion on this, including an analysis of both investment timing and turbine system value at the time of replacement.

4 Results and Discussion

In this section, we discuss the results obtained from testing our model with the base case values described in Section 3.1, and then compare it with the benchmark results. Furthermore, a sensitivity analysis examining the two uncertainty factors in our model is conducted, before a general discussion regarding our model is given.

4.1 Base Case Results and Discussion

We obtained convergence in the results by running the code with 10,000 simulated paths. Also, running the code with 10,000 simulations compared to 100,000 reduced the computational time significantly without compromising considerably on convergence.

4.1.1 Time of Investment

The time of replacing the base case turbine has been determined by our model and is illustrated in Figure 5 for all simulated paths. We defined that the turbine had been operating for 15 years in condition state 2 at $t = 0$. It must however be noticed that the turbine may have transitioned to a higher condition state by the time of investment. Figure 5 shows that more than 40% of the turbine replacements will be executed between five and eight years from the time we start to study the turbine. The first replacements happen after two years, while the latest replacements happen after 22 years.

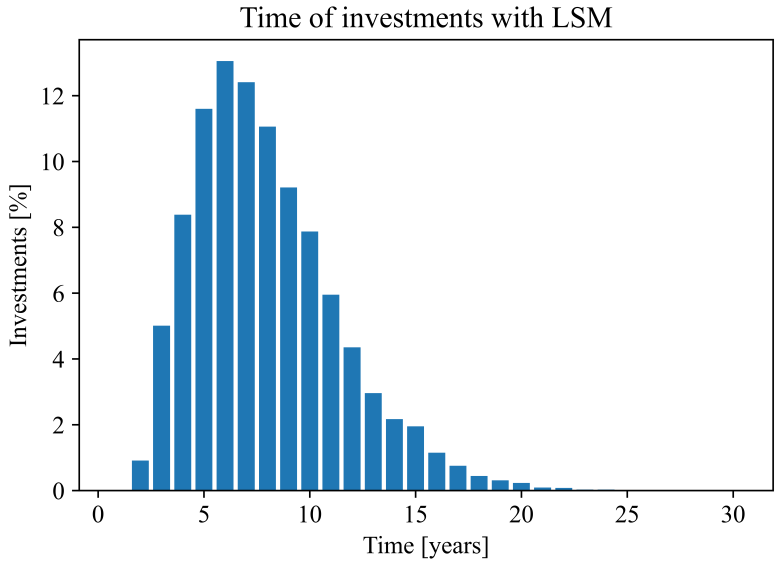

Figure 5: Time of replacing the base case turbine, determined by the LSM approach.

The base case turbine has an expected remaining lifetime of 13 years at $t = 0$, and there are few paths where the replacement happens after this. Though most replacements are executed in the earliest years, there is a great range of 20 years in investment timing. This can be explained by the two uncertainty factors: turbine lifetime and electricity prices. In some simulations, the turbine will live longer than expected. Simulations with shorter turbine lifetimes, however, will force earlier investments. As hydropower producers face great variation in turbine lifetimes, the wide range in investment timing is a good representation of reality. Turbine experts have confirmed that the great uncertainty and variation in turbine lifetimes are due to the uniqueness of each turbine, such as design and operational pattern. In Section 4.3.2 we elaborate on how the lifetime varies with changing parameters for the gamma distributions.

Another cause of the range in investment timing is the uncertainty in electricity prices. This effect on investment timing from price uncertainty becomes clear when comparing our LSM-based approach with the benchmark rule, as the benchmark does not consider electricity price levels when determining investment timing. We will further discuss this in Section 4.2.

4.1.2 Electricity Price Level at the Time of Investment

In addition to examine the investment times, the electricity price levels at time of investment for the different condition states are illustrated with a box plot in Figure 6. The orange line represents the median electricity price level for the investments in a given condition state. Then, the area above and below the orange line represents the upper and lower price quartiles. Maximum and minimum electricity price levels for investments in a given condition state are marked as black horizontal lines. Note that the electricity price levels are not determined for condition state 1, as we start to study our base case turbine in condition state 2. However, our model can be applied to study turbines starting in any condition state.

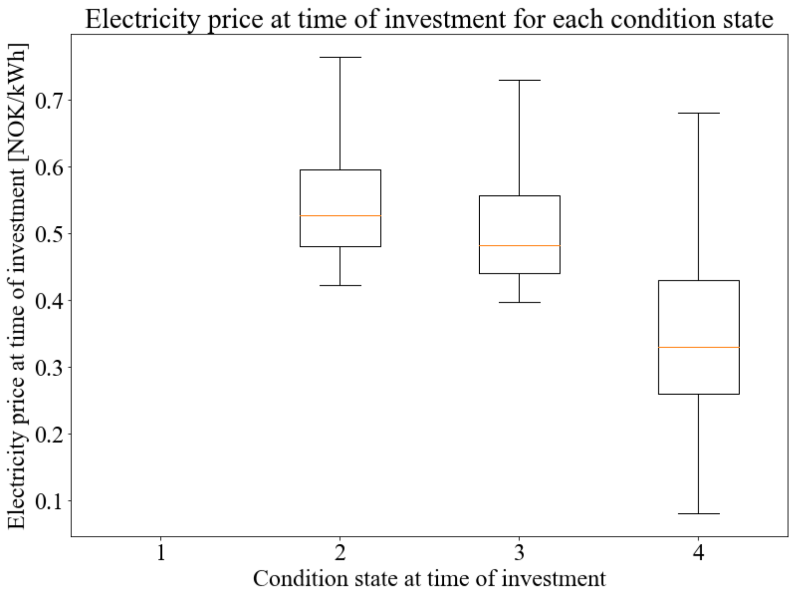

Figure 6: Electricity price levels at the time of investment for different condition states.

As observed in Figure 6, the median electricity price level is lowest for turbine replacements in condition state 4 and highest in condition state 2. This can be explained as turbines in higher condition states have a higher risk of failure than turbines in lower condition states. Thus, the hydropower producer will require a higher electricity price level if it should be optimal to invest in an early condition state, as we then expect that the old turbine will still live for some more years. On the other hand, the hydropower producer will be willing to invest at lower electricity price levels in condition state 4, as the producer wants to avoid a failure incurring a high cost.

There is a large spread in electricity price levels for investments within one condition state. Time spent in a condition state is assumed to be gamma distributed, and the data set used to estimate shape and scale parameters has large standard deviations. As we also argued in Section 4.1.2, this is, however, not inconsistent with real-world turbine lifetimes, as the uniqueness of each turbine causes large variations in lifetime. The electricity prices are modeled with positive drift, and we expect the long-term price level to increase slightly with time. In addition, volatility contributes to variations in electricity prices for different paths. Thus, large variations in time spent in a condition state, as well as electricity price uncertainty, can explain the large range in electricity price levels at the time of investment within a given condition state.

The spread in electricity price levels is greater for investments in condition state 4 than investments in condition state 2 and 3. This has two possible explanations. First, the average investment time in both condition state 2 and 3 are prior to that in condition state 4, respectively four, six and eight years into the investment horizon. Electricity price paths will differ more with time due to drift and randomness in the simulation. Thus, turbine replacements in condition state 4 will be executed under a wider range of price levels compared to investments in condition state 2 and 3. Second, 73% of the investments in condition state 4 are forced by a failure, and therefore replaced independently of the current price level. This explains the large difference between the maximum and the minimum price level for investments in condition state 4 compared to both condition state 2 and 3.

4.1.3 Electricity Price Level as Decision Rule

In order to give a more precise decision rule than what we can interpret from the box plot presented in the previous section, we have analyzed the electricity price level within the starting condition state of the base case. We set the condition state 2 as constant and examine the development of the required electricity price to make an investment favorable compared to waiting. The blue line in Figure 7 represents the required electricity price level for each time step. This price level is found numerically by finding the electricity price when the value of investing today equals the expected value of waiting to invest.

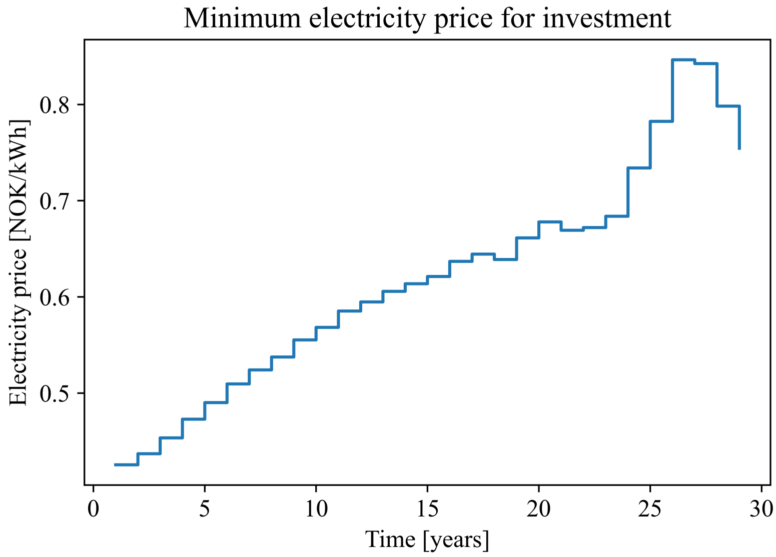

Figure 7: Minimum electricity price level for investment in condition state 2.

The initial required price level in condition state 2 is 0.40 NOK/kWh. The initial electricity price level in the base case is 0.30 NOK/kWh, which explains why none of the simulated paths invest in the first year, as can be seen from Figure 5. The minimum required electricity price to invest increases relatively linearly for the first 20 years. This can be explained by the expected annual growth of electricity prices, of 2% per year. An expected increase in electricity prices leads to an expected increase in the turbine system value, thus increasing the required electricity price for each year.

There are, however, some price spikes after the first 20 years. The reason for this is the lack of data points to do an adequate regression to obtain the continuation function, as most of the turbines have failed after 20 years. Disregarding the price spikes at the end of the investment horizon, the decision rule is set by the plotted price level. In other words, a replacement should be performed the first time the real electricity price exceeds the level of the blue line.

4.1.4 Performance of Model

To evaluate our LSM-based model, performance measures are determined and listed in Table 4. In the table, our model is referred to as the LSM. The average value of the turbine system at the time of investment with LSM is found to be 1,166 MNOK. If all investments were executed at the true optimal time, the average value of the turbine system would have been 1,607 MNOK. Thus, our model has a performance of 72.50% in terms of obtaining the maximal possible value.

Table 4: Value of turbine system at the time of investment and performance of LSM.

Performance measures	**Results**
Average value of turbine system at the time of investment with LSM	1,166 MNOK
Average value of turbine system at the true optimal time of investment	1,607 MNOK
Percentage of paths investing at the true optimal time with LSM	33.66%
Number of investments at time of failure with LSM	11.98%

The true optimal investment time is defined as the point in time where an investment yields the highest possible turbine system value for a given path. Our model manages to determine the optimal investment time for 33.66% out of all simulated paths. However, in 42.35% of all cases, our model will lead to an investment with a value within 90% of the optimal value.

Our results show that 11.98% of all investments are forced by a failure. The hydropower producer will, in those cases, exploit the total lifetime of the old turbine but experience a turbine failure. As much as 92% of these failure-induced investments occur before the average failure time, which is 15 years into the investment horizon. The regression function only includes average values when constructing the expected values, and are not able to grasp the unforeseen early failures.

Due to our assumption, a failure will result in a full stop in production over a long period of time. In practice, this is a scenario the hydropower producer strives to avoid. The share of investments forced by a failure can be reduced if there is more certainty in the turbine lifetime. Also, our results have shown that a lower electricity price volatility will decrease the number of investments forced by a failure. This will be more thoroughly discussed in the sensitivity analysis in Section 4.3.1.

4.2 Comparing Base Case Results with Benchmark

In order to compare the results from our model with the benchmark, we study differences in investment timing and the value of the turbine system. As described in Section 3.2, the benchmark is only dependent on the technical condition of the old turbine when deciding on the replacement timing. The benchmark rule is to replace the turbine five years into condition state 3. Figure 8 shows that the LSM approach (blue bars) spans the investment timing compared to the benchmark (orange bars). There are hardly any benchmark investments for the first five years. As the turbine is in condition state 2 in the beginning of the investment period, and has to transition into condition state 3 and stay there for 5 years before an investment, the lack of early investments are comprehensible. As much as 33% of the benchmark investments occur in year seven, as seen in Figure 8. This indicates that a large proportion of the simulated turbines reaches the fifth year of condition state 3 or are forced to be replace due to a failure in this year. The spread of investment timing in the following years are caused by the uncertainty in time in each condition state.

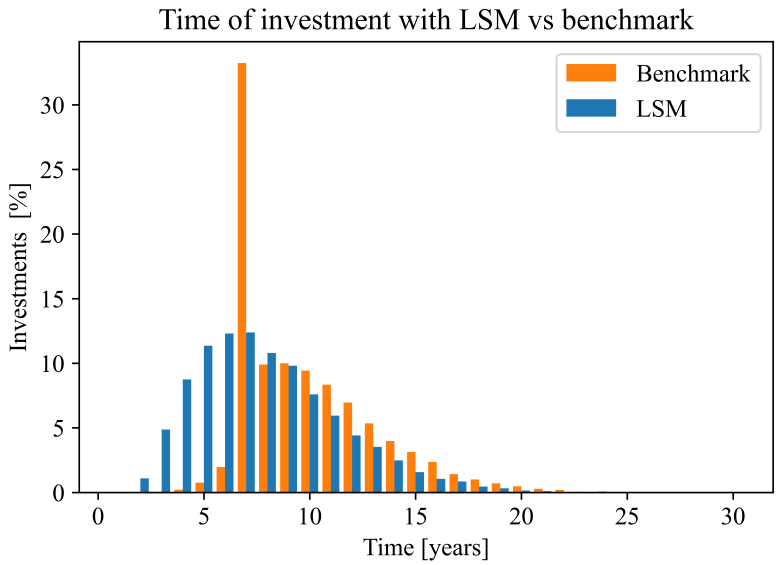

Figure 8: Comparison of the LSM model and the benchmark in terms of investment timing.

Our LSM-based model performs better than the benchmark in terms of choosing the time of investment. The average value of the turbine system at the time of investment found by the LSM and the benchmark, is respectively 1,166 MNOK and 813 MNOK. Thus, our modeling framework contributes to improved decision rules compared to the benchmark. The average investment time with the LSM is after seven years, which is earlier than that of the benchmark. However, the variance in timing is a lot larger with the LSM. As the electricity price is expected to increase with time, an initial assumption could be that a later average investment would lead to a higher value. However, as the investment timing of the benchmark is later and has a lower value than the LSM, this assumption would be incorrect. This shows a major value in considering both the economical and technological aspects when deciding when to replace. Finding the optimal time for each turbine gives a higher value than making a general rule only dependent on the condition state.

4.3 Sensitivity Analysis

To determine how much changes in the uncertainty factors influence the timing and value of a hydro turbine investment, we conduct a sensitivity analysis of both the electricity prices and time spent in each condition state. To analyze the electricity prices, we increase and decrease the volatility by 10% and 30%. While for the uncertainty in condition states, we change the scale parameter in all states, likewise with a 10% and 30% increase and decrease.

4.3.1 Change in Electricity Price Volatility

The changes in electricity price volatility are shown in Table 5 and in Figure 9. In Figure 9 we see that an increase in volatility slightly increases the span in investment timing, while a decrease reduces the span. The investments are delayed when the volatility increases and brought forward when the volatility decreases, with an average investment time of 7.5 years for a 30% increase and 6.4 years for a 30% decrease. Further, the value of postponing an investment to wait for more information increases with higher volatility, which explains the changes in investment timing.

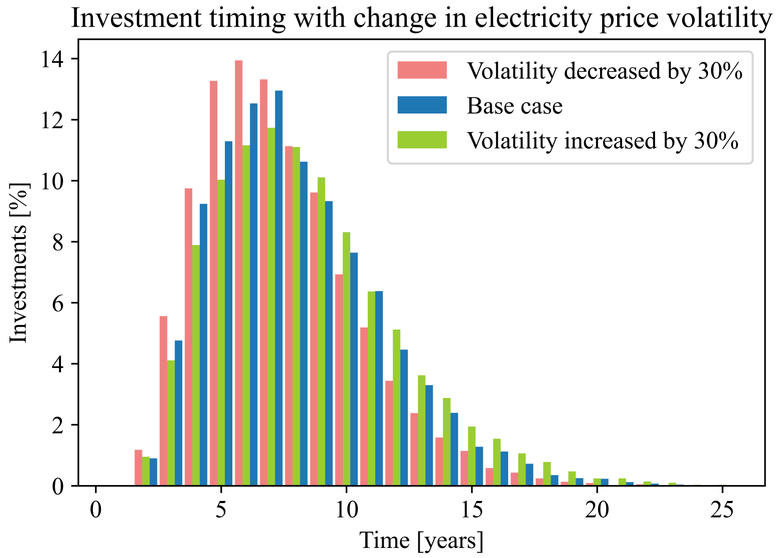

Figure 9: Sensitivity analysis of investment timing comparing base case with changes in electricity price volatility.

From Table 5 we can see that an increase in volatility will result in a higher average value of the investment with LSM, while lower volatility will decrease the value. This is reasonable as the volatility changes the dispersion of the prices, and the prices are expected to increase with time. As increased volatility also results in delayed investments, the average investment value is also consequently higher but now involves higher risk.

For the benchmark, the average value is virtually unchanged. The reason for this is that the investment timing of the benchmark does not consider electricity prices. We only see minimal changes ($\leq 1\%$) in the average investment value for the benchmark. The number of investments forced by a failure changes with approximately the same magnitude as the volatility. Higher volatility increases the uncertainty in investment timing and thus the probability of facing an unforeseen failure before the investment.

Table 5: Sensitivity analysis with changes in electricity price volatility compared to base case (BC).

Change in electricity price volatility	**-30%**	**-10%**	**BC**	**10%**	**30%**
Change in average V_{inv} for LSM (%)	-10.29	-4.03	0	3.88	15.93
Change in average V_{inv} for benchmark (%)	0.00	1.01	0	0.63	-0.83
Change in number of investments at failure (%)	-33.85	-9.80	0	15.03	25.51

4.3.2 Change in Condition State Scale Parameter

Time in each condition state is a stochastic variable; thus, there is uncertainty in the turbine lifetime. To examine this uncertainty, we have changed the scale parameter β_i of the gamma distribution for each condition state. The scale parameter represents the mean time between events; therefore, increasing the scale parameter expands the spread of the lifetime distribution (Kremelberg 2010). By increasing this, both the expected lifetime and standard deviation increase, as seen from (13) and (14) (Gertsbakh and Kordonskiy 1969).

$$E[\theta_i] = \beta_i k_i \tag{13}$$

$$SD[\theta_i] = \beta_i \sqrt{k_i} \tag{14}$$

The results from increasing and decreasing the scale parameter with 10% and 30% are shown in Table 6. Figure 10 illustrates the investment timing when the scale parameter is changed 30% up and down. We observe that the distributions of the investment timing have similar shapes as the gamma distribution when changing the scale parameter. This implies that the scale parameter has a major impact on the investment times.

As the scale parameter is increased, the total expected lifetime is extended. This results in delayed investments, illustrated by the green bars in Figure 10. Similarly, the decreased scale parameter, illustrated by pink bars, results in earlier investment timing than the base case. The average investment times are 7.0 years into the investment horizon for the base case, 3.4 years for a 30% decrease and 12.1 years for a 30% increase. The modification of the scale parameter also affects the total span in investment times, with an increase in the scale parameter resulting in a greater span. This is in line with reality, as a hydropower producer facing more uncertainty in turbine lifetime will find it challenging to pinpoint the optimal time of investment.

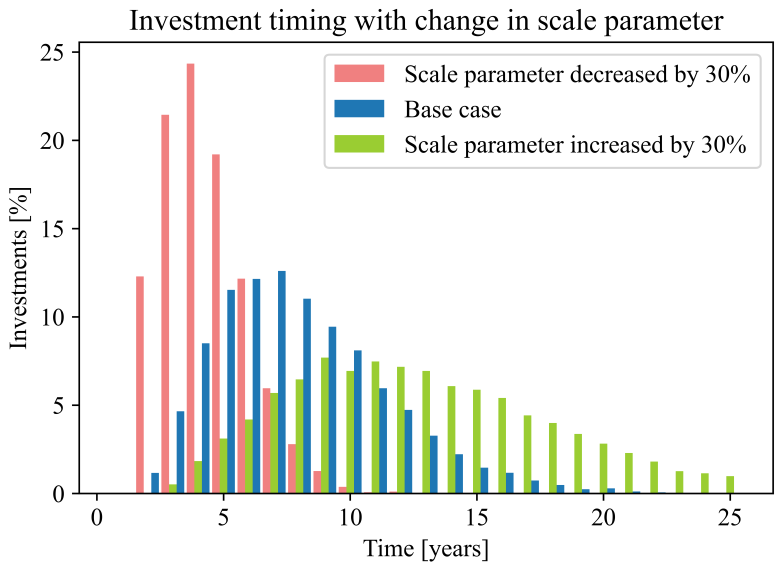

Figure 10: Sensitivity analysis of investment timing comparing base case with changes in scale parameter.

To analyze the impact the positive drift has on the investment value, we defined the drift to be 0% and ran the simulations again. From this, we found an increase of 22% in the investment value with a 30% higher scale parameter and a 16% decrease in investment value with a 30% reduction of the scale parameter. By comparing this to the investment values with 2% drift in Table 6, we observed that drift has a large impact on the investment values with a positive change in scale parameter. The later investments caused by an increased scale parameter will, on average, have a higher electricity price, as the prices are expected to increase with time. This leads to a higher average value of an investment with the LSM. For the negative change in scale parameter, the effect from the positive drift is less. This can be explained by the early investment times, where the positive drift has a limited impact on the expected increase in price level.

Only the prolonged time in condition state 2 affects the investment timing of the benchmark, since the investments are set to a fixed point in condition state 3. The increase in average value of investment for the benchmark with a higher scale parameter is therefore not as substantial as for the LSM. For the reduction in scale parameter, the change in investment value for the benchmark is, however, indistinguishable to that of the LSM. With a narrow investment span and short expected lifetime, the investment timing with the benchmark rule will deviate less from what we obtain with the LSM.

With a 30% lower scale parameter we see a significant reduction in investments forced by a failure. There is also a small improvement for a 10% decrease. For the higher scale parameters, the change in number of investments at failure is, however, ambiguously, and most likely due to randomness. With a low scale parameter, the great reduction of investments forced by failure can be explained by the small span in expected turbine lifetimes. Therefore, the regression function will better predict time of failure. In real life, this means less uncertainty in turbine lifetime for the hydropower producer. This property can be utilized by dividing the condition states of the turbine into multiple sub-states. With more continuous monitoring, this could mean that each sub-state would have a smaller scale parameter, which both leads to a shorter expected time in the condition state, but also less uncertainty. By keeping the total lifetime unchanged, the hydropower producer could then reduce the number of investments at failure, but also increase the investment values with better predictions.

Table 6: Sensitivity analysis with change in scale parameter compared to base case (BC).

Change in scale parameter	**-30%**	**-10%**	**BC**	**10%**	**30%**
Change in average V_{inv} for LSM (%)	-22.16	-10.43	0	11.58	39.43
Change in average V_{inv} for benchmark (%)	-22.27	-6.29	0	7.11	17.92
Change in number of investments at failure (%)	-24.20	-4.57	0	3.64	-3.02

4.4 Model Discussion and Possible Extensions

In the continuation of discussions regarding our results, we elaborate on possible extensions to our model. Challenges faced when developing an investment decision model for the hydropower industry are lack of data as well as data sensitivity. The condition states are determined by visual inspections done several years apart, while our model would be more accurate with annual data. In addition, information regarding the operation, efficiency and costs are often hard to obtain due to sensitivity. As we can see from previous academic work within the hydropower industry, accepted results are often based on expert judgments. Thus, expert judgments are considered as the most reliable data possible to obtain in our case. The accuracy of the answers we got can be questioned as the numbers diverged significantly. However, as every turbine is uniquely customized to its specific location, it is natural that numbers suggested for a general turbine will diverge.

In addition to maintenance data, internal electricity price prediction models in Statkraft are also sensitive information. From conversations with Statkraft, it is clear that the company has departments that daily analyze and predict price developments. This includes not only electricity prices, but also prices essential to determine the total cost of a turbine investment, such as steel prices. Our model can be developed by implementing such advanced models for electricity price dynamics, as well as making the cost of investing in a new turbine to an uncertainty factor.

Another possible extension of our model, which is not dependent on access to sensitive information, is to allow for smaller damages to the turbine. In our model, we have assumed that the turbine must be replaced immediately after a failure. However, in a real-life situation where the turbine is damaged, it can, in many cases, be set into operation again after a relatively short period of time. Through extended maintenance work or repair, the turbine can operate almost as well as before or run temporarily until a new turbine is ready. The goal of this thesis has been to consider large turbine failures, but smaller damages can however be useful to include in an extended model.

Information on preventive maintenance work can further improve our model. By introducing preventive maintenance, a turbine which has been repaired can be reset to a lower condition state. Hence, the turbine is no longer forced to run through the condition states in a strict ascending order. Preventive maintenance will extend the total turbine lifetime. Implementing preventive maintenance time can therefore push the timing of investment to a later point in time. Presuming a positive electricity price drift, later investment timing will drive the price limits to a higher level. Preventive maintenance has however not been considered, because maintenance data is challenging to obtain.

In addition to preventive maintenance, lead time affects the time of investment. As mentioned in Section 1.2.2, replacing a turbine can take as long as six years from the planning phase until the new turbine is set into operation. During the lead time, there is a risk of failure. In this thesis, we assumed that the turbine is replaced immediately, and a possible extension is to include lead time. This will force an earlier investment timing, as the risk of failure remains even after the decision is made.

As an unforeseen failure includes large expenditures, we have included a major cost of failure in our model. Such costs are not included in an NPV analysis, which is the method commonly used in the hydropower industry. This makes our model advantageous to NPV. The operational pattern of Norwegian hydro turbines has changed over the recent years to include more intermittent operation, which tears more on the turbine. This increases the lifetime uncertainty, thus, the risk of failure. Therefore, it is even more crucial to include the cost of a potential failure to build a precise and relevant turbine investment decision model.

5 Conclusion

In this thesis, we have developed an investment model to determine the optimal timing of a major hydro turbine replacement. As opposed to the commonly used NPV method, we contribute to the field by making ROA investment model. ROA accounts for the value of flexible investment timing and uncertainty in turbine lifetime and electricity prices. We are also, to our knowledge, the first within the field of hydropower to include the lifetime uncertainty in a ROA. This can give important insight, as we have shown which impact lifetime uncertainty has on a turbine replacement's optimal investment timing and value.

We developed an LSM-based model that determined optimal investment timing and decision rules. Realistic values for a Francis turbine runner were used as a base case to test our algorithm. The results showed that the optimal investment time varied largely for each simulation path, with the average investment time being seven years into the investment horizon. The large variation is caused by both the lifetime uncertainty and the volatility in electricity prices, which reflects reality. Further, we found a decision rule for electricity price levels by setting the condition state constant. For each year in the investment horizon, we found the minimal electricity price level required for an investment to be preferred to postponing, and observed that the price level slightly increased with time. This is due to a positive drift in electricity prices.

We compared our LSM model to a benchmark defined as replacing the turbine five years into condition state 3, independent of electricity prices. The LSM approach gave a higher turbine system value than the benchmark, respectively 1,166 MNOK and 813 MNOK. This implies that including electricity prices provide better results than only considering the technical condition of the turbine.

Further, the sensitivity analysis showed how the results were influenced by changes in electricity price volatility and the scale parameter of time in each condition state. Both parameters' decrease gave a denser investment timing distribution with earlier average investments. Further, it gave a lower average investment value. Investigating the two parameters separately, we observed that the change in scale parameter had a greater influence on the investment value. Hydropower producers have advanced electricity price prediction models, but the focus on lifetime models is limited both in the industry and in academia.

5.1 Suggestions for Further Work

As a final comment in this thesis, we will make recommendations for further work on the topic. We encourage the hydropower industry to collect and systematize historical data on sojourn times in condition states for a large sample of hydro turbines. If this data includes information on the type of turbine, installed capacity, operational pattern, and previous maintenance work, these factors can be included in the model and make it more turbine specific. An extensive database on this will potentially improve our model's estimations of turbine lifetime compared to solely relying on a limited number of expert judgments. However, it must be emphasized that this work remains for personnel with access to maintenance and inspection reports, which are considered highly sensitive information.

Further, dividing the condition states into sub-states can give more precise turbine lifetime expectations. The lifetime is solely divided into four states, while a turbine can operate for 60-70 years. Each condition state will therefore correspond to several years in reality. Basing the decision rules and calculations on these condition states can consequently be imprecise. A solution to this can therefore be to expand the condition states. As the condition states are a part of a standard by the EBL, an expansion is a decision that must be made by the hydropower industry or done internally in each company. An example is to divide each state into sub-states, giving states like 1.1, 1.2, 2.1, etc. Expanding the number of condition states is easy to incorporate into our model. However, this requires more frequent condition monitoring from the hydropower producer.

Finally, our model assumes that turbine efficiency will decrease at a constant rate each year. Efficiency has been addressed as one of the keys to determining the optimal time of a replacement. There are incentives in the hydropower industry to develop methods for continuous efficiency measurements. When such measurements are installed, efficiency predictions based on historical data can be used to determine a turbine's actual efficiency accurately. Also, our model assumes independence between efficiency decay and condition state. We suggest that a possible correlation between the two is studied in more detail and implemented in the model if a clear correlation exists.

Bibliography

Abadie, Luis Maria and José Manuel Chamorro (2009). *Monte Carlo valuation of natural gas investments*. Review of Financial Economics, 18(1), 10-22. DOI: https://doi.org/10.1016/j.rfe.2008.10.002.

Andersson, Ane Marte, Morten Elverhøi, Stein-Erik Fleten, Sabine Fuss, Jana Szolgayová and Ole Christian Troland (2014). *Upgrading hydropower plants with storage: Timing and capacity choice*. Energy Systems, 5(2), 233-252.

Assereto, Martina and Julie Byrne (2021). *No real option for solar in Ireland: A real option valuation of utility scale solar investment in Ireland*. Renewable and Sustainable Energy Reviews, 143, 110892. DOI: https://doi.org/10.1016/j.rser.2021.110892.

Blanco, Gerardo, Fernando Olsina, Francisco Garces and Christian Rehtanz (2011). *Real option valuation of FACTS investments based on the least square Monte Carlo method*. IEEE Transactions on Power Systems, 26(3), 1389-1398.

Blanco, Gerardo, Daniel Waniek, Fernando Olsina, Francisco Garcés and Christian Rehtanz (2011). *Flexible investment decisions in the European interconnected transmission system*. Electric Power Systems Research, 81(4), 984-994. DOI: https://doi.org/10.1016/j.epsr.2010.12.001.

Bøckman, Thor, Stein-Erik Fleten, Erik Juliussen, Håvard J. Langhammer and Ingemar Revdal (2007). *Investment timing and optimal capacity choice for small hydropower projects*. European Journal of Operational Research, 190(1), 255-267. DOI: 10.1016/j.ejor.2007.05.044.

Denault, Michel, Jean Guy Simonato and Lars Stentoft (2013). *A simulation-and-regression approach for stochastic dynamic programs with endogenous state variables*. DOI: 10.1016/j.cor.2013.04.008.

Dixit, Robert K and Robert S Pindyck (1993). *Investment under uncertainty*. Princeton University Press. New Jersey.

Dønnestad, Eirik Magnus, Stein-Erik Fleten, Andreas Kleiven, Maria Lavrutich and Amalie Marie Teige (2022). *A real options analysis of existing green energy facilities: maintain or replace?* Energy Systems. DOI: 10.1007/s12667-022-00505-8.

Edenhofer, Ottmar, Ramón Pichs Madruga, Youba Sokona, Kristin Seyboth, Patrick Matschoss, Susanne Kadner, Timm Zwickel, Patrick Eickemeier, Gerrit Hansen, Steffen Schlömer and Cristoph von Stechow (2012). *Renewable Energy Sources and Climate Change Mitigation: Special Report of the Intergovernmental Panel on Climate Change*. Cambridge University Press.

Eggen, Arnt Ove (2021). *Modell for kjøremønsterrelaterte kostnader Prosjektrapport*. HydroCen rapport nr. 24.

Eggen, Arnt Ove and Eivind Solvang (2002). *The mapping and evaluation of condition parameters in hydropower plants [Norway]; Kartlegging og evaluering av tilstandsparametre i vannkraftverk*. SINTEF.

Fladen, Bjørnar, Erik Holmqvist and Dag Bachke (2010). *Veileder i planlegging, bygging og drift av små kraftverk*. NVE.

Georgievskaia, Evgeniia (2019). *Hydraulic turbines lifetime in terms of fracture mechanics*. Engineering Failure Analysis, 105, 1296-1305. DOI: 10.1016/j.engfailanal.2019.08.003.

Gertsbakh, Ilia Borukhovich and Khaim Borisovich Kordonskiy (1969). *Models of Failure*. Springer.

GrantThornton (2018). *Renewable energy discount rate survey results - 2017*. A Grant Thornton and Clean Energy Pipeline initiative. UK.

Haukeli, Ingrid Endresen, Ingrid Ueland, Henriette Birkelund, Fredrik Arnesen, Jarand Hole, Dag Spilde, Silje Jelsness, Frida Hugaas Aulie and Simon Oldani (2021). *Forsterket klimapolitikk påvirker kraftprisene*. Langsiktig kraftmarkedsanalyse 2021-2040. NVE, 29(1).

Henriksen, Mette Eltvik, Nanna Moland Wahl, Carl Andreas Veie and Fredrik Arnesen (2020). *Turbinopprustninger kan øke produksjonen i norske vannkraftverk med fire terawattimer*. NVE.

Horn, Anders, Frode Kjærland, Peter Molnár and Beate Wollen Steen (2015). *The use of real option theory in Scandinavia's largest companies*. International Review of Financial Analysis, 41, 74-81. DOI: 10.1016/j.irfa.2015.05.026.

Husak, Gregory J, Joel Michaelsen and Chris Funk (2007). *Use of the gamma distribution to represent monthly rainfall in Africa for drought monitoring applications*. International Journal of Climatology: A Journal of the Royal Meteorological Society, 27(7), 935-944.

IEA (2021). *Hydropower Special Market Report Analysis and Forecast to 2030*. International Energy Agency.

Jafarizadeh, Babak and Reidar Brumer Bratvold (2015). *Oil and Gas Exploration Valuation and the Value of Waiting*. Engineering Economist, 60(4), 245-262. DOI: 10.1080/0013791X.2015.1045647.

Kern, Jordan D, Gregory W Characklis and Benjamin T Foster (2015). *Natural gas price uncertainty and the cost-effectiveness of hedging against low hydropower revenues caused by drought*. DOI: 10.1002/2014WR016533.

Kozlova, Mariia (2017). *Real option valuation in renewable energy literature: Research focus, trends and design*. Renewable and Sustainable Energy Reviews, 80, 180-196. DOI: 10.1016/j.rser.2017.05.166.

Kremelberg, David (2010). *Practical statistics*. SAGE publications.

Kumar, Ravi and Sunil Kumar Singal (2015). *Operation and Maintenance Problems in Hydro Turbine Material in Small Hydro Power Plant*. Materials Today: Proceedings, 2(4-5), 2323-2331. DOI: 10.1016/j.matpr.2015.07.284.

Lambrecht, Bart M. (2017). *Real options in finance*. Journal of Banking and Finance, 81, 166-171. DOI: 10.1016/j.jbankfin.2017.03.006.

Longstaff, Francis and Eduardo Schwartz (2001). *Valuing American Options by Simulation: A Simple Least-Squares Approach*. The review of financial studies, 14(1), 113-147.

Lucia, Julio J and Eduardo Schwartz (2001). *Electricity Prices and Power Derivatives: Evidence from the Nordic Power Exchange*. Review of derivatives research, 5(1), 5-50.

Mauer, David C and Steven H Ott (1995). *Investment under Uncertainty: The Case of Replacement Investment Decisions*. The Journal of Financial and Quantitative Analysis, 30(4), 581-605.

Mo, Birger and Hans Olaf Hågenvik (2020). *Applying the ProdRisk-SHOP simulator for investment decisions*. HydroCen Report 19. Norwegian Research Centre for Hydropower Technology.

Moghaddam, Kamran S. and John S. Usher (2011). *Preventive maintenance and replacement scheduling for repairable and maintainable systems using dynamic programming*. Computers and Industrial Engineering, 60(4), 654-665. DOI: 10.1016/j.cie.2010.12.021.

Myers, Stewart C (1977). *Determinants of Corporate Borrowing**. Journal of Financial Economics, 5, 147-175.

Nadarajah, Selvaprabu, François Margot and Nicola Secomandi (2017). *Comparison of least squares Monte Carlo methods with applications to energy real options*. European Journal of Operational Research, 256(1), 196-204. DOI: 10.1016/j.ejor.2016.06.020.

Nicolai, Robin P., Rommert Dekker and Jan M. van Noortwijk (2007). *A comparison of models for measurable deterioration: An application to coatings on steel structures*. Reliability Engineering and System Safety, 92(12), 1635-1650. DOI: 10.1016/j.ress.2006.09.021.

NVE (2017). *Veileder om elsertifikater ved oppgradering og utvidelse av vannkraftverk*. NVE.

Pindyck, Robert S (2001). *The Dynamics of Commodity Spot and Futures Markets: A primer*. The Energy Journal, 22(3).

Pringles, Rolando, Fernando Olsina and Francisco Garcés (2015). *Real option valuation of power transmission investments by stochastic simulation*. Energy Economics, 47, 215-226. DOI: 10.1016/j.eneco.2014.11.011.

Richardson, Steven, Adrian Kefford and Melinda Hodkiewicz (2013). *Optimised asset replacement strategy in the presence of lead time uncertainty*. International Journal of Production Economics, 141(2), 659-667. DOI: 10.1016/j.ijpe.2012.10.006.

Ruud, Einar Johan (2017). *Beslutningsstøtte med kontinuerlig virkningsgradsmåling i vannkraftverk*. MSc. Thesis. NTNU.

Schwartz, Eduardo and James E. Smith (2000). *Short-term variations and long-term dynamics in commodity prices*. DOI: 10.1287/mnsc.46.7.893.12034.

Solvang, Eivind, Maren Istad and Jørn Heggset (2011). *Vedlikeholdsstyring innen vannkraft: Status og beste praksis*. (317-2011). Energi Norge - EnergiAkademiet.

SSB (2021). *Electricity*. https://www.ssb.no/en/energi-og-industri/energi/statistikk/elektrisitet. Obtained: 13.02.2022.

Tian, Kuo, LQ Qiu and Ming Zeng (2012). *Flexibility investment decision-making model for transmission systems with flexible AC transmission equipments*. Power System Technology, 36(1), 230-236.

Trivedi, Chirag, Bhupendra Gandhi and Cervantes J. Michel (2013). *Effect of transients on Francis turbine runner life: A review*. Journal of Hydraulic Research, 51(2), 121-132. DOI: 10.1080/00221686.2012.732971.

Welte, Thomas (2008). *Deterioration and Maintenance Models for Components in Hydropower Plants*. Phd. Thesis. NTNU.

Welte, Thomas and Lars Eliasson (2011). *Start-stopp-kostnader for vannkraftverk: Beregningsverktøy og brukerveiledning*. (322-2011). Energi Norge - EnergiAkademiet.

Welte, Thomas, Jørn Heggset and Eivind Solvang (2011). *Sviktmodell for vannkraftverk: Skadetyper og tilstandskriterier*. (323-2011). Energi Norge - EnergiAkademiet.

Zhu, Lei (2012). *A simulation based real options approach for the investment evaluation of nuclear power*. Energy Management and Economics. Computers Industrial Engineering, 63(3), 585-593. DOI: https://doi.org/10.1016/j.cie.2012.02.012.

Zhu, Lei and Ying Fan (2011). *A real options–based CCS investment evaluation model: Case study of China's power generation sector*. Applied Energy, 88(12), 4320-4333. DOI: https://doi.org/10.1016/j.apenergy.2011.04.005.

Appendix

A Python Code for Hydro Turbine Investment Decision Tool

```python
# Hydro turbine investment decision model

# Import libraries
import math
import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import curve_fit
plt.rcParams["font.family"] = "Times New Roman"
plt.rcParams.update({'font.size': 12})

################################################################################
# PARAMETER VALUES

# Current condition state at t=0
state_start = 2

# Condition state for electricity price limit calculation
state_limit = 2

# Time in current condition state
t_state_start = 15

# Turbine age at t=0
age = 20

# Binary variable, set to 1 if one wants to calculate numerical limits for
# electricity price in condition state state_limit
limit_estimation = 1

# Annual drift of electricity prices
mu = 0.02

# Number of simulation paths
M =10000

# Initial electricity price
P0 = 0.3 # NOK/kWh

# Annual volatility of electricity price
sigma = 0.13

# Number of time steps per year
dt = 1
```

```python
# Annual discount rate
r = 0.06

# Annual inflow in therms of energy
Q = 150000000 # kWh

# Expected lifetime of new turbine
T_new = 30 # years

# Investment cost of new turbine
C_turbine = 20000000 # NOK

# Cost by unforeseen failure
C_fail =  1000000000  # NOK

# Benchmark rule, time we spend in condition 3 before we replace the turbine
t_cond3 = 5

# Efficiency of old turbine
eta_old_start = 0.9425

# Annual decrease in efficiency for old turbine
eta_old_decrease = 0.00073

# Efficiency of new turbine
eta_new_start = 0.952

# Annual decrease in efficiency for new turbine
eta_new_decrease = 0.00033

##############################################################################
# SHAPE AND SCALE PARAMETERS FOR GAMMA DISTRIBUTION

# Expert opinions on time in each condition state
observations = np.array([[3,5,8,5],[15,15,16,25], [12,15,5,5], [1,0.1,1,1]])

A = np.zeros(4)

# Calculate intermediate value A for each condition state
for i in range(4):
    ln_T_ij=0
    for j in range(4):
        ln_T_ij+=math.log(observations[i,j])
    A[i] = math.log(sum(observations[i,:]/4))-ln_T_ij/4

# Shape and scale parameters for the gamma distributions in state 1-4
shape=np.zeros(4)
scale = np.zeros(4)
```

```python
# Calculating shape and scale parameters for gamma distributions
for i in range(4):
    shape[i] = (1/(4*A[i]))*(1+math.sqrt(1+4*A[i]/3))
    scale[i] = sum(observations[i,:]/4)/shape[i]

#############################################################################
# TURBINE LIFETIME

# Turbine lifetime for each simulation
L = np.zeros(M)

# Time in each condition state for each simulation path
t_state= np.zeros((M,4))

for i in range (M):
        # Pick one random time t from the gamma distribution for each state
        # from starting state to state 4
        for j in range(state_start-1,4):
            t_state[i,j] =math.ceil( np.random.gamma(shape[j],scale[j]))

            # If time already spent in starting state exceeds simulated time,
            # we transition to the next state the following year
            if t_state[i,j]<=t_state_start and j ==state_start-1:
                t_state[i,j] = 1

            # Subtract time already spent in starting state
            elif j ==state_start-1:
                t_state[i,j]-=t_state_start

        # Find the total lifetime for one simulation
        L[i] = round(sum(t_state[i,:]))

# Defines T as investment horizon; the maximal lifetime of the simulations
T=int(max(L))

#############################################################################
# CONDITION STATES

# Condition state in each simulation path for each time step
x_state = np.zeros((M,T))

# Finds the current condition state in each simulation path at each time step
for i in range(M):
    for j in range(T):
        for k in range(4):
            if j<sum(t_state[i,0:k+1]):
                x_state[i,j] = k+1
                break
```

```
# Variable to save number of years spent in current condition state
# at each time step for each path
time_in_state = np.zeros((M,T))
time_in_state[:,0] = t_state_start

for i in range(M):
    for j in range(1,T):
        if x_state[i,j-1] ==x_state[i,j] and x_state[i,j]!=0:
            time_in_state[i,j] = time_in_state[i,j-1]+1

##############################################################################
# ELECTRICITY PRICE SIMULATION

# Electricity price in each path at each time
Pt = np.zeros((M,T))

# Simulate electricity price for all paths as GBM using Monte Carlo simulation
for i in range(M):
    Pt[i,0]=P0
    for j in range(1,T):
        Pt[i,j] = Pt[i,0]*math.exp((mu-sigma**2/2)*j+sigma*math.sqrt(j)
                                   *np.random.normal())

##############################################################################
# VALUE OF INVESTING AND PRODUCTION VALUE

# Value of investment as a function of time, electricity price,
# condition state and if a failure occurs
def value_invest(t, P_el_today, tilstandx, delta_fail):
    P_el_estimate = np.zeros(T_new)
    eta_new_v = np.zeros(T_new)
    prod_values = np.zeros(T_new)
    v=0

    # Value of production from new turbine for its expected lifetime
    for w in range(T_new):
        P_el_estimate[w]= P_el_today*math.exp(mu*w)
        eta_new_v[w] = eta_new_start-eta_new_decrease*w
        prod_values[w] = P_el_estimate[w]*eta_new_v[w]*Q
        v+= prod_values[w]*math.exp(-r*w)

    # Subtracts investment cost
    v-=C_turbine

    # Subtracts failure cost if an unforeseen failure has occurred
    if delta_fail == 1:
       v-=C_fail

    return(v)
```

```
# Function to define the value of production if the investment is postponed
# one year
def value_wait(path,year_decission):
    v_wait = Q*Pt[path,year_decission+1]*(eta_old_start-eta_old_decrease
                                         *(year_decission+1+age))*math.exp(-r)
    return v_wait

##############################################################################
# INVESTMENT VALUE

# Value of investing in new turbine for each path and time step
investment_value = np.zeros((M,T))

for i in range (M):
    for j in range (T):
        if j<L[i]-1:
            investment_value[i,j]= value_invest(j,Pt[i,j],x_state[i,j], 0)
        elif j == L[i]-1:
            investment_value[i,j]= value_invest(j,Pt[i,j],x_state[i,j], 1)

##############################################################################
# Turbine system value for each path and each time step
sys_value = np.zeros((M,T))

# Binary variable, 1 in time of investment for each path
investing = np.zeros((M,T))

# Defines turbine system value in year of failure for each path
for i in range(M):
         sys_value[i,int(L[i]-1)] = investment_value[i,int(L[i]-1)]
         investing[i,int(L[i]-1)] = 1

# Required electricity price for investment (numerical limit calculation)
min_invest_price = np.zeros((4,T))

# Regression functions
def func(X, a, b, c, d, e, f):
    x,y = X
    return a*x**2 + b*x + c*y**2 + d*y + e*x*y + f

def value_func(poly, el_price,state):
    a,b,c,d,e,f=poly
    return a*el_price**2+b*el_price+c*state**2+d*state+e*el_price*state+f

##############################################################################
# LEAST SQUARES MONTE CARLO

# Iterating backwards through the time periods
# (starts at the penultimate period)
for i in range(T-2,0,-1):
```

```
    x1_values = np.zeros(M)
    x2_values = np.zeros(M)

    # Defines x1 and x2 as variables for regression function
    for j in range(M):
            x1_values[j]=Pt[j,i]
            x2_values[j]=x_state[j,i]

    # Discounted values received at a later time if we do NOT invest at time i
    y_values = np.zeros(M)

    for j in range(M):
            y_values[j] = sys_value[j,i+1]*math.exp(-r) + value_wait(j,i)

    # Regression
    p, pcov = curve_fit(func, (x1_values, x2_values), y_values)

    # Finds the expected value of NOT investing by using the results from
    # the regression
    for j in range(M):
        if i<=L[j]-1:
            sys_value[j,i] = max(value_func(p,Pt[j,i],x_state[j,i]),
                                 investment_value[j,i])
            if investment_value[j,i]>value_func(p,Pt[j,i],x_state[j,i]):
                investing[j,i] = 1
                investing[j,i+1:T] = 0


    break_variable = 0

    # Calculates numerical price limits for investment with regression results
    if limit_estimation == 1:
            for q in np.arange(0.01,5,0.000001):
                v = value_invest(i,q,state_limit,0)

                if break_variable ==1:
                    break
                elif q>5-2*0.00001:
                    print("No match found")
                elif round(v,-6) ==round(value_func(p,q,state_limit),-6):
                    min_invest_price[2,i]=q
                    print("Match found for time ",i, ",electricity price: ", q)
                    break_variable = 1
                    break

# Plots minimum electricity price for investments
if limit_estimation ==1:
        t =range(0,30)
        plt.figure(dpi=1200)
        plt.plot(t[1:30],min_invest_price[2,1:30],drawstyle="steps-post")
```

```python
        plt.title("Minimum electricity price for investment")
        plt.ylabel("Electricity price [NOK/kWh]")
        plt.xlabel("Time [years]")
        plt.show()

###############################################################################
# DATA FOR INVESTMENT TIMING

# Time of investment for each path
invest_time = np.zeros(M)

# Electricity price at time of investment for each path
p_invest = np.zeros(M)

# Condition state at time of investment for each path
state_invest = np.zeros(M)

# Obtains investment data
for i in range(M):
    for j in range(T):
        if investing[i,j]!=0:
            p_invest[i] = Pt[i,j]
            invest_time[i] = j

    state_invest[i]=x_state[i,int(invest_time[i])]

###############################################################################
# PLOT FOR TIME OF INVESTMENTS WITH LSM

plot_time =np.zeros(30)
for i in range(0,30):
    plot_time[i] = i+1

num_invest = np.zeros(T)

for i in range(M):
    for j in range(T):
        if investing[i,j]>0:
            num_invest[j]+=1

num_invest_percent = num_invest/M*100

# Plotting time of investment with LSM
plt.figure(dpi=1200)
plt.bar(plot_time,num_invest_percent[0:30])
plt.ylabel('Investments [\%]')
plt.xlabel('Time [years]')
plt.title("Time of investments with LSM")
plt.show()
```

```
##############################################################################
# BENCHMARK RULE

# Value of investment with benchmark rule for each path
v_benchmark = np.zeros(M)

# Number of investments with benchmark for each year
num_invest_b = np.zeros(T)

# Determining time of invsestment for each path with benchmark
for i in range(M):
    for j in range (T):
        if x_state[i,j] == 3 and time_in_state[i,j] == t_cond3:
            num_invest_b[j]+=1
            v_benchmark[i] = value_invest(j,Pt[i,j],3,0)

        elif (x_state[i,j] == 4 and t_state[i,2] + time_in_state[i,j] ==
              t_cond3 and state_start != 3 and v_benchmark[i]==0):
            num_invest_b[j]+=1
            v_benchmark[i] = value_invest(j, Pt[i,j], 4, 0)

        elif (x_state[i,j] == 4 and t_state[i,2]+t_state_start +
              time_in_state[i,j] == t_cond3 and state_start== 3 and
              v_benchmark[i]==0):
            num_invest_b[j]+=1
            v_benchmark[i] = value_invest(j, Pt[i,j], 4, 0)

        elif j == L[i]-1 and v_benchmark[i]==0:
            num_invest_b[j]+=1
            v_benchmark[i] = value_invest(j, Pt[i,j], 4, 1)

##############################################################################
# COMPARISON OF LSM METHOD AND BENCHMARK

ttt=[]
tttt=[]
for i in range(0,30):
    ttt.append(float(i)+0.2+1)
    tttt.append(float(i)-0.2+1)

# Number of investments with benchmark for each year as percentage
num_invest_b_percent = num_invest_b/M*100

# Plotting time of iestment with LSM compared to benchmark
plt.figure(dpi=1200)
plt.bar(tttt,num_invest_b_percent[0:30], color="tab:orange", width= 0.4,
        label="Benchmark")
plt.bar(ttt,num_invest_percent[0:30], color="tab:blue", width= 0.4,
        label="LSM")
```

```
plt.legend(loc='upper right')
plt.ylabel('Investments  [\%]')
plt.xlabel('Time [years]')
plt.title("Time of investment with LSM vs benchmark")
plt.show()

##############################################################################
# ELECTRICITY PRICE AT TIME OF INVESTMENT FOR EACH CONDITION STATE

elprice_invest_1=[]
elprice_invest_2=[]
elprice_invest_3=[]
elprice_invest_4=[]

# Finds electricity prices at time of investment for each simulation sorted by
# condition state
for i in range(M):
        if state_invest[i] ==1:
            elprice_invest_1.append(p_invest[i])
        elif state_invest[i] ==2:
            elprice_invest_2.append(p_invest[i])
        elif state_invest[i] ==3:
            elprice_invest_3.append(p_invest[i])
        elif state_invest[i] ==4:
            elprice_invest_4.append(p_invest[i])

# Data for box plot
data=[elprice_invest_1,elprice_invest_2,elprice_invest_3,elprice_invest_4]
fig = plt.figure(figsize =(10, 7))

# Creating axes instance
ax = fig.add_axes([0, 0, 1, 1])

# Creating box plot
bp = ax.boxplot(data, showfliers=False, patch_artist=False)
plt.figure(dpi=1200)
ax.set_title("Electricity price at time of investment for each condition state",
             fontsize=28)
ax.set_xlabel("Condition state at time of investment", fontsize=24)
ax.tick_params(axis='y', labelsize=24)
ax.tick_params(axis='x', labelsize=24)
ax.set_ylabel("Electricity price at time of investment [NOK/kWh]", fontsize=24)
plt.show()

##############################################################################
# VALUE OF INVESTMENT

# Final investment value for each path with LSM
final_investment_value=np.zeros(M)
sum_investments = 0
```

```python
# Finds value of investment for each path
for i in range(M):
    for j in range(T):
        if investing[i,j] == 1:
            sum_investments+=sys_value[i,j]
            final_investment_value[i] = sys_value[i,j]

# Finds average investment value for LSM
avg_invest_value = sum_investments/M
print("Average investment value for LSM: ", round(avg_invest_value,0))

# Prints average time of investment
print("Average time of investment for LSM: ", round(sum(invest_time)/M))

# Finds average investment value for benchmark
avg_invest_benchmark=sum(v_benchmark)/M
print("Average investment value for benchmark: ",round(avg_invest_benchmark,0))

# True optimal investment value for each path
max_invest=np.zeros(M)

# Counters for number of optimal and 90\% of optimal investments for LSM
count_max= 0
count_90_max = 0

# Finds optimal investment value and number of optimal and 90\% of optimal
# investments for LSM
for i in range(M):
    max_invest[i] = max(investment_value[i,:])

    if max_invest[i] == final_investment_value[i]:
        count_max+=1
    elif 0.90*max_invest[i] <=final_investment_value[i]:
        count_90_max+=1

print("Percent of optimal investments with LSM: ", round(count_max/M*100))
print("Percent of investments with value within 90\% of optimal value: ",
      round(count_90_max/M*100))
print("Average of true optimal investment value: ", round(sum(max_invest)/M,0))

##############################################################################
# INVESTMENTS FORCED BY FAILURE

# Number of investments forced by failure per year
failure_invest_per_year=np.zeros(T)

# Finds number of investments forced by failure
for i in range(M):
    if L[i]-1 == invest_time[i]:
```

```
        failure_invest_per_year[int(invest_time[i])]+=1

print("Percentage of investments forced by failure: ",
      sum(failure_invest_per_year)/M*100)
print("Average remaining lifetime at t=0: ", round(sum(L)/M),2)
print("Number of investments forced by failure per year: " ,
      failure_invest_per_year)

###############################################################################
# INVESTMENT TIMING PER CONDITION STATE

# Time of investments in each condition state
time_invest_1 = []
time_invest_2 = []
time_invest_3 = []
time_invest_4 = []

# Number of investments for each condition state
num_invest_1 = 0
num_invest_2 = 0
num_invest_3 = 0
num_invest_4 = 0

# Finds number of investments and timing of investments for each condition state
for i in range(M):
        if state_invest[i] ==1:
            time_invest_1.append(invest_time[i])
            num_invest_1+=1
        elif state_invest[i] ==2:
            time_invest_2.append(invest_time[i])
            num_invest_2+=1
        elif state_invest[i] ==3:
            time_invest_3.append(invest_time[i])
            num_invest_3+=1
        elif state_invest[i] ==4:
            time_invest_4.append(invest_time[i])
            num_invest_4+=1

print("Percentage of investments per condition state: 1: ", num_invest_1/M*100,
      num_invest_2/M*100, num_invest_3/M*100, num_invest_4/M*100)
print("Average time of investment per condition state (2-4): ",
      int(sum(time_invest_2)/num_invest_2),
      int(sum(time_invest_3)/num_invest_3),
      int(sum(time_invest_4)/num_invest_4 ))
```

NTNU

Kunnskap for en bedre verden