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An Empirical Analysis of the US Stock Market.

Master's thesis in Financial Economics
Supervisor: Joakim Kvamvold
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Opening Remarks

This thesis marks the completion of the master's degrees in Financial Economics of the two authors, Tim Himle Levinh and Thomas Ombudstvedt, at the Institute of Economics (ISØ) at the Norwegian School of Science and Technology.

We want to thank our advisor, Joakim Kvamvold at Folketrygdfondet, for his excellent guidance and patience throughout the process of writing this thesis.

Abstract

This paper examines the development of the risk determinants for smaller portfolios that consists of 1 to 30 stocks and their risk performance. When we analyze the performance of simulated portfolios throughout the last 4 decades, we find that the diversification gains from adding stocks to a small portfolio have been stable throughout our sample. While 8-10 stocks suffice to remove most of the portfolios' idiosyncratic risk component, smaller portfolios typically have poor returns. We also use a disaggregated approach and parametric models to study the idiosyncratic volatility and co-movement between the returns of common stocks listed from 1980 to 2021 on the NYSE. We identify a regime shift where the average idiosyncratic volatility of common stocks has entered a lower volatility state from the late 1990s. In contrast, the co-movement between stock returns shifted to a higher correlation state from the early 2000s. The net effect of the regime shifts thus cancel each other out, causing the stable diversification gains.

Oppsummering

I denne avhandlingen undersøker vi utviklingen til risikokomponentene til mindre porteføljer bestående av 1 til 30 aksjer og deres risikonivå. Når vi analyserer simulerte porteføljer de siste fire tiårene finner vi at diversifiseringsgevinsten av å legge til aksjer i mindre porteføljer har vært stabil. Til tross for at 8 til 10 aksjer holder for å fjerne mesteparten av det idiosynkratiske risikoen i porteføljer, har mindre porteføljer generelt lav avkastning. Vi bruker også en dis-aggregert modell og parametriske modeller for å studere den idiosynkratiske volatiliteten og sambevegelsen til avkastningen til aksjene listet på NYSE fra 1980 til 2021. Vi identifiserer et regime bytte hvor den gjennomsnittlige idiosynkratiske volatiliteten til aksjer har skiftet til et lav-volatilitets regime fra slutten av 90-tallet. I kontrast skiftet sambevegelsen til aksjene til et høy-korrelasjonsregime fra 2000 tallet. Netto effekten av regimeendringene utligninger hverandre og forårsaker den stabile diversifiseringgevinsten.

1.0 Introduction

In this paper, we analyze how smaller portfolios consisting of 1 to 30 stocks have performed in different decades. We also explore how the underlying risk determinants of a portfolio; the co-movement of stock returns, and common stocks' idiosyncratic risk levels, have developed over time. We conduct our analysis and find that the diversification benefit for smaller portfolios has not changed through our sample from 1980 to 2021.

The results from our simulations of randomly selected portfolios consisting of 1 to 30 stocks every decade from 1980 to 2020, further indicate that 8-10 stocks eliminate most of a portfolio's idiosyncratic risk. In general, the simulated portfolios estimate total risk levels to have peaked in the 1980s and the 2000s. While the risk of our portfolios mostly flattens after adding 10 stocks to the portfolio, performing t-tests on the change in mean volatilities shows that there is still a 90% statistically significant change in volatility when adding the 30^{th} stock. This is true for all decades in our sample. The reduction in the portfolio risk gained by an additional stock in your portfolio is equal at the start and the end of our sample.

We also find that industry emplacement restrictions on the randomly drawn portfolios does not improve their risk performance. Finally, we calculate the reward to volatility ratio of each batch of simulated portfolios. When we study the Sharpe ratios it is evident that smaller portfolios are largely inefficient compared to an index benchmark. This corresponds with Bessembinder (2018) finding positive skewness in the return distribution of the stocks listed on the NYSE.

To explore the development in the risk determinants of smaller portfolios; the stock return co-movement and the idiosyncratic risk we create several time series for the two determinants. We analyze the series to see whether their development were consistent with stable diversification benefits.

We estimate two idiosyncratic risk series, one with a disaggregated model where we treat the volatility as observable, and one with a Fama- French (1993) regression model. We then use Markov-switching models to obtain our next empirical results. The models reveal that the firm-specific risk component of common stock returns has entered a regime with lower mean values from the turn of the century. The switching models estimate that a low idiosyncratic risk state with mean variance values of 0.002 and 0.004 has persisted with 99% probability for most of the sample past 2002.[1] The latter presents a significant shift from a high idiosyncratic risk regime with mean variance values of 0.009 and 0.011 that persisted with 99% probability for most of the sample from 1980 to 1992.

We also employ a Markov-switching model on a time series of the average realized pairwise return correlation of the stocks listed on the NYSE. From this, we find that the co-movement between individual stock returns reached a higher mean state from the early 2000s to the late 2010s. The Markov model identifies that a low co-movement state with mean values of 0.105 persisted with 98% probability for most of the sample from 1980-2004. For the sample past 2004 the Markov model estimates several switches to a high co-movement state with a mean of 0.279 that was estimated to persist with 94% probability.

Our VAR model finds that the level of industry specific volatility now influences the level of market volatility in a Granger causality analysis of our disagreed volatility series. This represents a change from previous studies where the level of industry specific volatility does not influence the level of market volatility in the short term.

The transition to a low-level idiosyncratic risk regime and higher return co-movement regime represents a break from Campbell, Lettau, Malkiel, and Xu (2001) research that identified the opposite tendencies prior to the turn of the century. However, our identified return correlation trends are consistent with what Sullivan and Xiong (2012) and LaCasce, Lillethun, Rynning-Tønnesen and Gaivoronski (2019) find in previous studies of stock co-movement. We also find a decrease in the firm-specific risk component in common stock is consistent with Brandt, Brav and Kumar (2010) findings. The transformation to high return correlation and low idiosyncratic risk regimes within similar time frames does explain our main finding of stable diversification gains from adding stocks to a portfolio in our samples.

One implication of our analysis is that investors with limited diversified portfolios can expect to remove most idiosyncratic risk from their portfolio with 8-10 stocks. This has been the case for the simulated portfolios in all our samples. This is consistent with Evans and Archer (1968) finding that 8-10 stocks alone will diversify away most of a stock portfolio's idiosyncratic risk, commonly mentioned in several finance textbooks (Bodie, Kane & Marcus, 2018). In terms of returns, a naively diversified portfolio of up to 30 stocks shouldn't be expected to be competitive with an index.

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[1] The estimated 99 % probability for the low variance states was briefly abrupted during the recessions of 2008 and 2019. Despite the latter, the 99% probability of the low mean state holds for 96% of the sample past 2002.

Most of our simulations indicate that the smaller portfolios are vastly outperformed by index benchmarks when returns are accounted for relative to risk. Considerable time variation in the total risk level of our simulated portfolios also represent a divergence from Tang’s (2004) findings.

This thesis first review previous research on the average stock volatility and co-movement between stock returns in the U.S. markets. After that, we review the results of simulation studies on the risk efficiency of smaller portfolios. The next section of the paper is a methodical guide to the various numerical methods we use in our analysis. After that, we also devote a brief section to describe the data we employ in our analysis.

The results section contains the risk performance of simulated portfolios, a discussion of the results from our study on stock volatility, co-movement development, and finally, a look at the risk performance of our simulations relative to returns. Our thesis culminates in a review of our main findings and tips for future research.

2.0 Literature Review

2.1 Small Portfolio's Risk Determinants

Diversification is the practice of spreading your investment capital across different assets to reduce overall risk exposure. Studies of the diversification efficiency of smaller portfolios are categorically separated into two types of studies. The first type of study analyzes diversification efficiency through simulations of randomly selected portfolios. The other type of study typically employs more technical models to analyze the development of a stock's firm-specific risk component and the co-movement of stock returns.

While the non-simulation studies do not directly analyze diversification performance, such analyzes of a portfolio's risk determinants will implicitly suggest how diversification benefits have varied over time. The latter is demonstrated here for a 2-stock portfolio:

$$Var_{portfolio} = W_1^2\sigma_x^2 + W_2^2\sigma_y^2 + 2W_1W_2\rho\sigma_x\sigma_y\ (x, y). \tag{1}$$

Where the covariance is equal to:

$$CoVar(x, y) = \frac{\Sigma(x-\underline{x})*\Sigma\left(\left(y-\underline{y}\right)\right)}{n} \tag{2}$$

and will reflect the linear dependence between two stocks. Moreover, it can be reformulated as

$$CoVar(x, y) = \rho\sigma_x\sigma_y, \tag{3}$$

where ρ is the correlation coefficient between the return of the two stocks. For a larger portfolio, the risk level will converge towards the average covariance between the stocks in the portfolio. The average covariance is a function of the average pairwise correlation coefficients and the average standard deviation of each stock's return.

A higher average pairwise return correlation thus leads to higher co-movement in the returns between underlying assets in a portfolio and a weaker diversification gains. The reason for the latter is that less of a volatility increase in one stock is canceled out by weaker or negative volatility development in the other portfolio constitutes. Similarly, a higher idiosyncratic risk level as a proportion of a stock's total volatility, increases the number of stocks required to attain an efficient portfolio. A single stock's return volatility or standard deviation can be composed into a systematic and firm-specific component . The firm-specific risk component: the idiosyncratic risk, steadily decline when we add less than perfectly correlated stocks to a portfolio. A higher initial idiosyncratic risk level implies that we need more stocks in our portfolios to remove the idiosyncratic risk (Copeland, Weston & Shastri, 2013).

2.2 The Development of a Typical Stock's Firm-Specific Volatility Component

Studies of a "typical stock's" idiosyncratic risk level in the U.S. markets have yielded varying results. Campbell, Lettau, Malkiel, and Xu (2001), henceforth referred to as CLMX, found that the idiosyncratic risk level in individual stocks listed on the NYSE, AMEX and NASDAQ, had a significant upward drifting deterministic trend from 1962 to 1997. Deviating from previous studies' use of factor models and parametric models, they treated firm volatility series as observable. They estimate an idiosyncratic risk series with a uniquely specified model and use daily stock return data from the Center for Research of Security Prices' (CRSP) database.

Interestingly, research based on more recent data discredits the possibility of an upward trend in idiosyncratic volatility in individual stocks. Brandt, Brav and Kumar (2010) replicate the CLMX study and identify the 1987 market crash and the build-up to the 2001 dot-com recession as the keys to finding any deterministic trend.

Bekaert, Hodrick, and Zhang's (2012) analysis of the idiosyncratic risk in individual stocks also identifies a clear break from the trend CLMX found, with newer data. By estimating an idiosyncratic volatility series with an alternative Fama-French (1993) model, Bekaert et al., (2012) dismiss the possibility of a deterministic trend in their series. Instead, they find idiosyncratic volatility in individual stocks best described as a stationary autoregressive process. A process that sporadically switches to higher variance regimes for short periods during recessions. Bekaert, Hodrick, and Zhang (2009) also criticize the CLMX practice for limiting the variability in their volatility series. However, their alternative estimation method differs little from the CLMX method in terms of results.

Lebedinsky and Wilmes (2017) replicate the CLMX study with newer data and further identify the level of idiosyncratic risk to have several spikes in the early 21st century due to the dot-com bubble and the 2008 financial crisis. However, they find that the level of risk is mean reverting throughout their sample from 1962 to 2014. Interestingly, their volatility series for idiosyncratic risk showcase slightly lower mean levels in the 21st century compared to the last two decades of the 20th century.

2.3 The Development of the Average Pairwise Correlation of Stock Returns

Although the CLMX study finds a rising level of idiosyncratic risk, the researchers also find that the average pairwise correlation of stock returns decreased from 1962 to 1997. As with the volatility series, they employ realized return and a non-parametric model to estimate the correlation coefficients. Their study further concludes that the volatility effects are stronger than the correlation effect, leading to less diversification efficiency in smaller portfolios.

More recent research on average pairwise stock return correlations differs from the work of CLMX. Both LaCasce, Lillethun, Rynning-Tønnesen, and Gaivoronski (2019) and Sullivan and Xiong (2012) identify the increasing popularity of mutual funds and exchange-traded funds (ETFs) as having a positive effect on the pairwise average return correlations. They cited how institutional investor's more extensive stock market ownership might lead to more trading and return commonality due to more trades being "automated" based on fundamentals and volume. Sullivan and Xiong (2012) graphed the average pairwise correlation of several stock indices from 1997 to 2012 using realized data. They find a clear upward correlation trend from the end of the CLMX sample period, and a possible regime shift to a higher mean state. A possible shift that corresponded with a 3-fold increase in the market share of institutional investors.

Sullivan and Xiong (2012) also employ regression models. An independent variable for the percentage of passive assets had a positive and 5% statistically significant effect on both dispersion in volume changes and the pairwise correlation of price changes. LaCasce et al., (2019) built on Sullivan and Xiong's work by employing a Markov switching model, which identified a bear market regime that positively affected stock return correlations. Benhmad (2013) also has similar findings with slightly different model specifications. LaCasce et al., (2019) also find that the number of ETFs has significant explanatory power for the average pairwise return correlations in larger indices. Kearney and Poti (2008) have in contrast attributed the increased worldwide stock return co-movement to stronger economic integration. Lebedinsky and Wilmes (2017) also find the stock return co-movement to increase from 2001 to 2017. However, they did not check for the possibility of different regimes in the data generation process.

In summary, newer research identifies an apparent change in how the idiosyncratic risk and pairwise average stock correlations have developed over the last 20 years. The trends that Campbell et al., (2001) identified with decreasing pairwise correlations and increasing idiosyncratic risk likely have reversed in the past 20 years. This is also consistent with our own findings in this paper.

2.4 Previous Findings in Simulation Studies

Like risk determinates studies, studies that have employed simulations to access the performance of smaller portfolios that consists of 1 to 30 stocks have produced varying results. As mentioned earlier in the paper, the notion that ten or fewer stocks is enough to diversify away idiosyncratic risk, dates from research conducted in the 60s. Evans and Archer (1968) simulate the risk performance of several portfolios that consists of 10 randomly selected stocks through time. They find equivalent risk levels in their randomly simulated portfolios and several market indices (Evans & Archer,1968).

Bernstein (2000) and Tang's (2004) simulations yield that 8 to 15 stocks is sufficient to obtain an efficient risk level. Interestingly, Tang's (2004) estimates imply that the number of stocks needed to attain a low-risk level was independent of which subsamples he employed in his time series analysis from 1962 to 2000. Tang (2004) also find that a less naïve diversification strategy, with enforced restrictions on which sector each stock is picked from, does not improve diversification.

Bernstein (2000) also find that the average pairwise stock correlation decreased from the 70s to the late 90s, which he attributed to the low number of stocks needed to achieve desired diversification effects. Bernstein do, however, find a substantial problem with the mentioned portfolios, that the Sharpe ratio of the created portfolios is significantly lower than that of a well-diversified portfolio. Several recent studies, such as Domain, Louton, and Racine (2007) and Bessembinder (2018), have cited positive skewness in the return distribution for individual stocks as the main reason why smaller portfolios have poor Sharpe ratios. They attribute the excellent performance of indices such as the S&P 500 to the top 4 percentile of stocks in the index with abnormal returns. Domain et al., (2007) simulations show that you need more than 100 stocks to acquire a Sharpe ratio comparable to their benchmark index.

The low percentage chance of picking a stock with strong returns in smaller portfolios, where most stocks rarely outperform even the 1- or 3-month T-bill returns, thus implies that we do not expect a good Sharpe ratio for smaller portfolios.

In summary, the results of the number of stocks needed to remove the level of idiosyncratic risk in a portfolio have remained rather stable through time, while accounting for returns raises the number of stocks needed for an efficient portfolio. Our own results largely correspond with the results from previous studies.

3.0 Methodology

3.1 Methodology for Idiosyncratic Volatility

In our analysis of whether the firm-specific volatility component of the volatility of stocks has risen, we first use the methodology employed by CLMX.

As CLMX, we estimate unconditional estimates of variances based on sums and averages of return and cross products. The CLMX approach assumes that the variance of the method is observable. According to Merton (1980), such variance estimates yield high accuracy and are robust provided that the squared deviations from the process realization are at a high frequency.

The CLMX approach separates the return of a "typical" stock into three different components: the market-wide return, the industry-specific residual, and the firm-specific residual.

We decompose the weighted average return volatilities in the following way:

$$\sum_i w_{it} \sum_{j\in i} w_{jit}\, Var(R_{ijt}) = \sum_i w_{it}\, Var(R_{it}) + \sum_i w_{it} \sum_{j\in i} w_{jit}\, Var(\eta_{it})$$

$$= Var(R_{mt}) + \sum_i w_{it}\, Var(\epsilon_{it}) + \sum_i w_{it}\sigma^2_{\eta it} = \sigma^2_{\eta it} + \sigma^2_{\epsilon t} + \sigma^2_{\eta t}. \quad (4)$$

As in the CLMX methodology, R_{jit} denotes the period t return of firm j in industry i, R_{it} denotes the weighted average return of industry i, and finally, R_{mt} is the average weighted market return. w_{it} is the weight of industry i in the whole market and we derive it by employing its market capitalization at time t. Similarly, w_{jit} is the weight of firm j in the industry i also at time t. ϵ_{it} and η_{it} are specified as $R_{it} = R_{mt} + \epsilon_{it}$ and $R_{jit} = R_{it} + \eta_{jit}$.

Per the CLMX methodology, we obtain the firm's weight by dividing its market capitalization by the sum of the market capitalization for all firms each month in the sample. Firms are re-weighted each month based on market capitalization, to gain the market-weighted return for each firm. Later the monthly weights are obtained with daily return data; then we calculate the market-weighted average return for each day in the sample. We calculate the daily excess return by subtracting the daily return from holding 3-month T-bill from the firm return series.

We use daily average excess returns to calculate the mean excess return of the market over the entire sample period and use it to estimate the market variance as:

$$Market_t = \widehat{\sigma_{mt}}^{\,2} = \sum_{s \in t} (R_{ms} - \mu_m)^2. \qquad (5)$$

The weight of each firm within its respective industry is the firm's market capitalization divided by the total industry market cap. The weight of firm j in industry i is equal to firm j's weight within the market divided by the sum of the weights of all firms within industry i. We use the industry weights and returns obtained from each firm in each industry, and we obtain the average returns for each industry. We employ 12 industry classifications defined by Fama and French (1993) and sort companies in their respective industries by their Standard Industrial Codes (SIC) in the CRSP database.

The volatility from each industry is obtained by subtracting excess market returns from the excess industry returns $R_{it} = Rm + \epsilon_{it}$ and squaring the differences $\hat{\sigma}_{e_{it}}{}^{2} = \sum_{s \in t} \epsilon_{is}^{2}$

We calculate the average industry volatility with individual industry volatilities and the weights of the industries in the overall market w_{it}

$$\sum_{i} w_{it} \hat{\sigma}_{e_{it.}} \qquad (6)$$

Lastly, to obtain a measure of the idiosyncratic firm volatility, we subtract the average industry return from each firm's excess returns, $R_{jit} = R_{it} + \eta_{jit}$ and square the differences to estimate $\hat{\sigma}_{\eta_{jit}}{}^{2}$

$$\hat{\sigma}_{\eta_{jit}}{}^{2} = \sum_{j \in t} \eta_{jis.}^{2} \qquad (7)$$

Then we sum the squared differences for each firm by month and multiply each firm’s total squared differences by the firm’s weight within its industry w_{jit}

$$\hat{\sigma}_{\eta_{jit}}{}^{2} = \sum_{j \in i} w_{jit} \widehat{\sigma_{\eta_{jit}}}^{\,2}. \qquad (8)$$

Finally, we calculate the average idiosyncratic volatility for all stocks as a weighted average of $\hat{\sigma}_{\eta_{it}}{}^2$.

When CLMX estimate the return volatility components of a typical firm, their methodology has the significant benefit of mitigating problems related to estimating individual betas and tracking covariances for individual firms and industries. Using realized returns in volatility estimates with the CLMX methodology, produce more robust results than what would be achieved with a parametric model (Schwert, 1989). We assume, as CLMX, that industry returns have unit betas relative to the market portfolio. An issue with the CLMX methodology described by Bekaert et al., (2009) is that the unit beta restriction potentially limits their models' ability to emulate stock's return variability. While Brandt et al., (2010) has criticized trend tests based on the estimation method for being sensitive to the choice of time sample.

To get an alternative measure of the trend in idiosyncratic risk and compensate for the aforementioned critique, we employ the Fama-French (1993) three-factor model and the methodology employed by Fu (2009). We use the regression:

$$R_{j,t} = \alpha_{0j,m} + \beta_{1,j,m}MKT_t + \beta_{2,j,m}SMB_t + \beta_{3,j,m}HML_t + u_{j,t}^{FF}. \qquad (9)$$

Of the Fama-French (1993) factors, MKT represents the excess return on the market portfolio, SMB is the small size factor, and HML represents the value factor. To estimate idiosyncratic risk, we first estimate the Fama-French (1993) regression for each stock with daily data with a rolling window of 42 trading days.[4] Mor details behind the 3-factor model can be found in Appendix 9.

We calculate the daily regression residuals for each stock. Then we obtain the average monthly standard deviations of the residuals for the stocks as a measure of the average monthly idiosyncratic risk level. We also enforce the restriction that all the stocks in each regression window have at least 15 observations, to mitigate issues with missing data.

We believe the described models suits the purpose of estimating the idiosyncratic risk and total risk level of the common stocks better than a parametric model such as the GARCH model or its derivatives. While GARCH models would have provided the possibility to forecast risk developments, our analysis is concerned with the historical development of the idiosyncratic risk in common stocks. Furthermore, the choice of the optimal parametric model

[4] We use the window of 42 days as a proxy for a two-month window, when the average and median number of trading days each month in our sample was 21

has significant implications for forecasts quality but also to lesser extent on how realized risk patterns are described according to Nelson (1992). Fitting the best parametric model for the underlying stock on the NYSE would be complicated. Schwert (1989) has also argued that using realized data in similar manner to the CLMX methodology yields more robust results. We also counter the main criticism of the CLMX methodology in that its underlying assumptions possibility restrict stock return variability with our alternative estimation method for the idiosyncratic risk series.

3.2 Method for Co-Movement Estimation

To document the development of co-movement in the return of individual stocks, we estimate the pairwise return correlation among stocks listed on the NYSE. We use a similar method as CLMX to estimate the average pairwise return correlations using daily and monthly data. We calculate the monthly pairwise correlations using the previous 12 months of daily data. We then estimate an equally weighted average of the correlation estimates. We obtain annual correlations estimates based on monthly data for the last 60 months of observations.

We also estimate a proxy for the correlation coefficients with an alternative methodology employed by CLMX. We obtain the average monthly R^2 from a 60-month rolling CAPM regression based on each stock, each month, with the following equation:

$$R_{j,t} = \alpha_{0j,m} + \beta_{1,j,m} MKT_t + u_{j,t}^{CAPM}. \quad (10)$$

The reason why the average R^2 of the capital asset pricing model (CAPM) regression serves as a proxy for the development of the pairwise co-movement in stock returns are twofold. Firstly, an increased R^2 implies that more of a stock's return can be attributed to systematic market risk. Thus, a higher R^2 implicitly leads to higher stock return co-movement when a higher proportion of a typical stock's return is influenced by a common factor. More theoretically, if all stocks shared the same characteristics and are equally correlated in returns, the variance of the market portfolio would be ρ times the variance of any individual stock. The latter scenario thus implies that the average R^2 of the market model would be ρ or the average pairwise correlation coefficient between the stocks (Campbell et al.,2001). While different stocks clearly will have unidentical characteristics, both the CLMX and Lebedinsky and Wilmes (2017) study have proven that the average R^2 of the market model serves as a good proxy for co-movement in stock returns.

3.3 Method for Simulations

To analyze how the diversification effect has varied across our different risk and correlation regimes, we employ simulations like those in Evans and Archer (1968). We create 10,000 portfolios that consists of 1 to 30 randomly selected stocks and compare their average variance to selected indices using historical return data. In total, we create 300,000 randomly selected portfolios in each of our main samples. We diversify the portfolios naively, with equal weights in each stock.

To test for significant change in the risk levels by adding an additional stock to our portfolio, we use the t-test on our t-values, given by:

$$t = \frac{\mu_2 - \mu_1}{\frac{\sigma_{diff}}{\sqrt{N-1}}}. \tag{11}$$

We chose the S&P 500 as one of our benchmarks since the index functions as an easily obtainable way for smaller investors to acquire a well-diversified portfolio. This does not imply that we see the index as a market portfolio proxy or want to benchmark against an actual market portfolio. We also benchmark against an equally weighted NYSE index so we consistently could compare our findings with the results obtained from previous studies that used the benchmark.

To analyze whether the different variance and correlations regimes affect the portfolio's risk level, we conduct our simulations in four different subsamples. We conduct our simulations with monthly return data through holding periods from 1980 to 1999, 1990 to 1999, 2000 to 2009, and finally from 2010 to 2019. The different sample periods allow us to see if the change to a lower idiosyncratic risk regime, but higher return co-movement regime has impacted the efficiency of smaller portfolios.

We restrict the firms used in our simulations to be firms that existed both at the start and end of each sample period. Since absent observations or including the stock prices as zero will lead to downward biased or incomplete variance estimates. While positive survivorship bias is not an ideal property either, we believe that our limited sample periods mitigate some of the issues related to this bias.

We also estimate the average Sharpe ratio for each batch of randomly selected portfolios by calculating the excess return of each portfolio and dividing it by its estimated standard deviation

$$Sharpe = (R_{portfolio} - Rf_{Tbill})/\sigma_{portfolio}. \quad (12)$$

We derive the excess return by subtracting the individual stocks' return from the monthly return from holding the 3-month T-bill. We estimate each stock return with the simple holding period return formula:

$$HPR = \frac{p_{t+1} - p_t}{p_t} - 1. \quad (13)$$

We account for the Sharpe ratio to check if Bessembinder's (2018*)* findings still hold with newer data in different subsamples with direct simulation studies.

To obtain the average variance and return of our randomly selected portfolios, we use matrix calculation in the programming language Python. Employing the power of programming further allows to conduct simulations at higher frequency than in previous research. Our method should yield a lower standard error in the simulations and more robust results.

We calculate the return of each portfolio as the equally weighted average return from the underlying stocks in the portfolio. We calculate the variance of the portfolios as the equally weighted sum of variances, and covariances of the stock's constituents in the portfolio

$$\sigma^2_{portfolio} = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i\, w_j \sigma_{ij} \quad (14)$$

where $i, j = 1,2,,,,n$.

We derive the covariance between the portfolio constituents using a correlation matrix and a vector with the product of the individual stock's investment weight and return variance

$$\sigma^2 = W * \sum * W^t, \quad (15)$$

where W = $(w_1\sigma_1 \quad w_2\sigma_2 \quad ..w_n\sigma\_n)$, $\Sigma = \begin{bmatrix} 1 & \rho_{12} & \cdots & \rho_{1n} \\ \rho_{21} & 1 & \cdots & \rho_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{n1} & \rho_{n2} & \cdots & 1 \end{bmatrix}$ and $W^t = \begin{vmatrix} w_1\sigma_1 \\ w_2\sigma_2 \\ \vdots \\ w_n\sigma_n \end{vmatrix}$

w_n denotes the investment weight in each individual stock which is equal to $\frac{1}{n}$ where n is the number of stocks in the portfolio. σ_{ij} denotes the covariance between pair of stocks in the portfolio. Where $w_n = \frac{1}{n}$. Our matrix and vectors are stepwise expanded according to number of stocks each portfolio had to contain.

Finally, we also test Tang's (2004) finding that a less naïve diversification strategy where the stocks are spread across different sectors does not improve the risk performance of smaller portfolios. We test Tang's (2004) finding by employing a restriction on the simulations where the portfolios must contain stocks spread across the Fama- French's (F.F) 12 identified industry classes. For the first simulations of portfolios that consists of 1-12 stocks, we employed the restriction where a maximum of only one stock can be in the portfolio per industry. For the portfolios that consists of 13-26 stocks, we employ the restriction of a maximum of two stocks from each industry. The final portfolios of 26-30 stocks have a restriction of 3 stocks maximum in each industry.

4.0 Data

We collect stock returns, shares outstanding and prices for firms listed on the New York Stock Exchange, from the CRSP database. We chose the time frame for our analysis to check if the last half of the CLMX sample led to biased results and whenever their identified trend in stock volatility had continued. The CRSP return data is adjusted for dividend payouts as well as stock-splits.

We also employ the F.F industry classification of 12 industries and sort the individual firms into the industries according to their SIC codes. We multiply shares outstanding with price to obtain total market cap and market cap per industry. We calculate the excess return for each stock by subtracting the 3-month T-bill rate from the individual stock return. We download the 3-month T-bill rate from the FRED Economic Database. We gathered the daily and monthly return associated with different factor loadings for the Fama-French (1993) model and the CAPM from Kenneth French's own website.

4.1 About the Data

Daily returns data from the NYSE between 1980-2021 gives us 23,939,904 observations in our main dataset. Throughout our dataset, there are 8,208 different firms. The total number of firms at any given time between 1980-2021 varies greatly throughout. The year with the most firms listed on the exchange was 1998, when 3,121 different firms are listed, and the fewest number of firms was in 1984, when 1,580 firms are listed. Throughout our series, there are somewhere between 4,868,760 and 1,274,410 unique pairs of stocks, which is relevant for our correlation analysis. We see rapid growth in firms from the late 80s until the dot-com bubble at the change of the century. After that, the number of firms remain stable. There are enough firms in our dataset to do valid empirical work at all points in our sample.

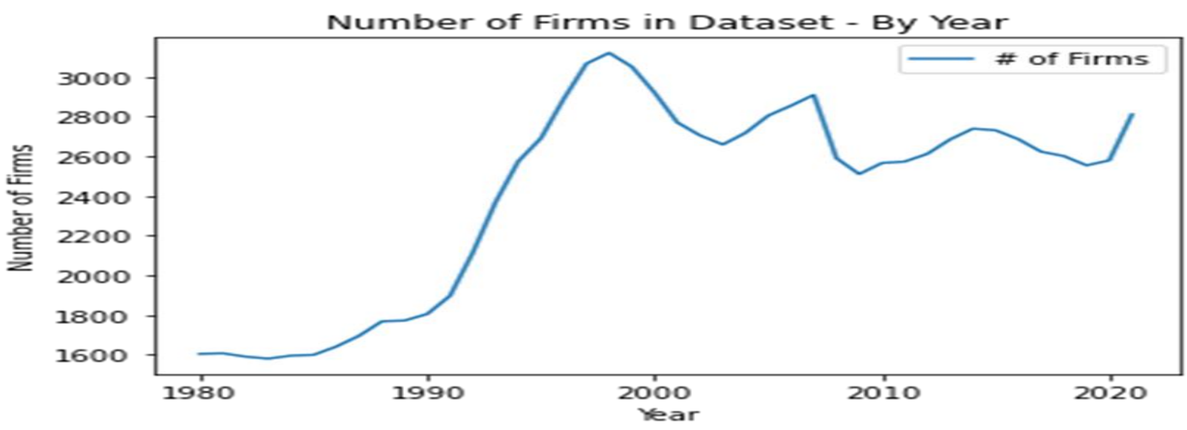

Figure 1. The number of firms listed on the NYSE stock exchange by year.

The 12 industries we break the data into are: durable goods, nondurable goods, manufacturing, energy, chemicals, business equipment, telecom, utilities, shops, healthcare, finance, and others. You can find a detailed description of our industry separation scheme in Table A.1 in Appendix 1.

4.2 Data Statistics

The average daily return in our series is 0.006%, and the standard deviation of returns is 2.851%, the highest observed return is 300% and lowest is -96.3%.[5]

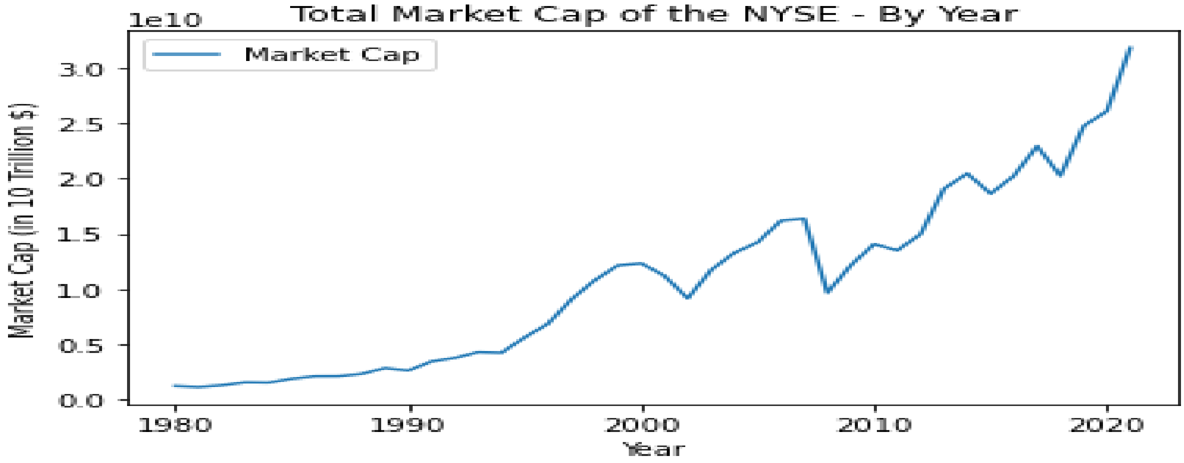

Figure 1. Total market capitalization of the NYSE, by year.

We also note that the market capitalization of the NYSE has steadily increased over the sample, and that the last 2 years have seen exceptional growth.

[5] We purged the dataset of any observations with returns over 300% or with zero returns for more than 6 months in a row, to remove bias from missing data or highly illiquid stocks with abnormal returns.

5.0 Results

5.1 Results of the Portfolio Simulations

The results from our simulations of the risk of portfolios tells us that the marginal risk reduction from adding additional stocks to smaller portfolios have been consistent through the last four decades. Our analysis does not contain any findings that suggest that the diversification efficiency in smaller portfolios today has improved or declined significantly. This is despite that the markets and investing landscape obviously having changed since the 1980s. The flow of information and the ability to place trades have increased rapidly, and international markets have gotten more integrated. Despite this, Figure 3 shows us that the effect of additional stocks on your portfolio has had the same effect on volatility for each decade.

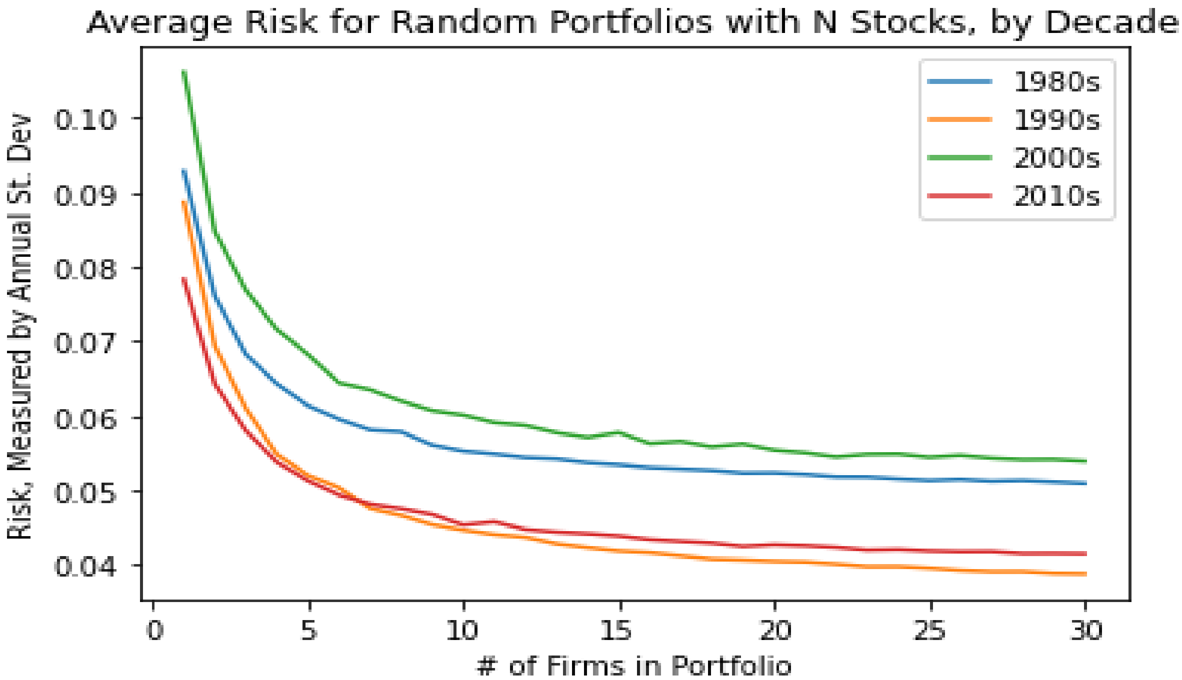

Figure 3. Throughout different decades, the average annual standard deviation of random portfolios consisting of 1-30 stocks.

Figure 3 contains the average annualized standard deviation of 10,000 simulated portfolios consisting of 1 to 30 stocks. We calculate the standard deviation with monthly return data. There is a clear tendency where the marginal reduction in the portfolio standard deviation largely decays after 8-10 stocks are included in the portfolio. The latter is consistent with Bernstein (2000) and Tang's (2004) findings where they found that 8-15 stocks would remove most of the portfolios idiosyncratic risk.

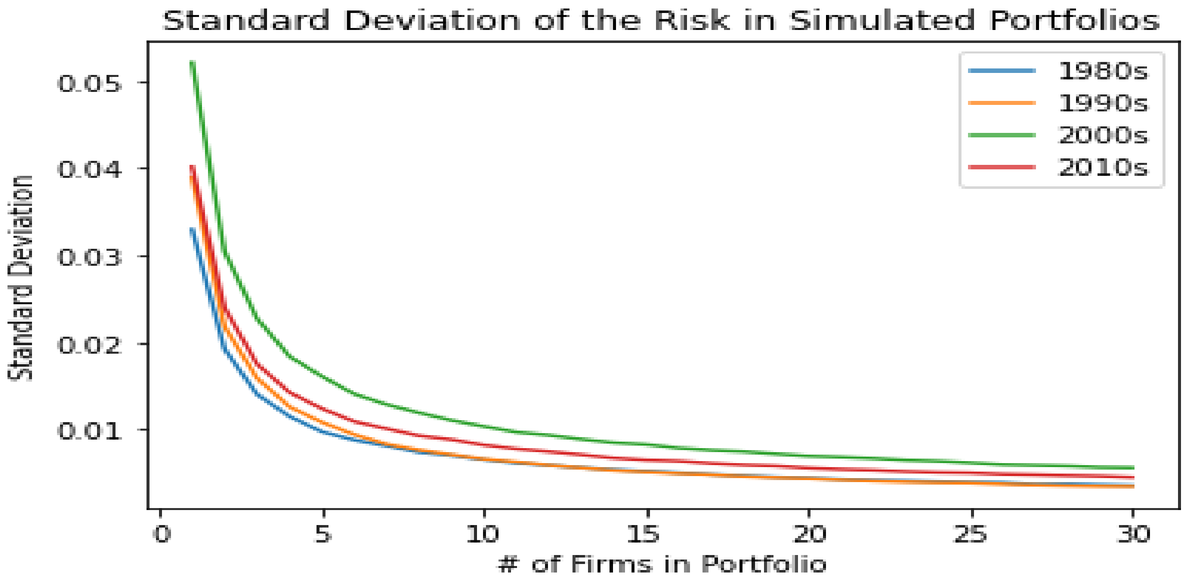

Figure 4. Standard deviation of risk in our simulated portfolios for each decade.

To analyze whether there is a reason to add more stocks than the 8-10 to the portfolio in terms of risk efficiency, we graphed the standard deviation of the 10,000 simulations for each portfolio by decade. The results can be seen in Figure 4. Although the decline in the expected risk of the portfolios is higher for the first ten portfolios, we expect that the smaller risk reductions from adding ten or more stocks should be more certain. The standard deviations showed the same trend as the risk, with an effect that decays rapidly after adding the first few stocks. However, there is still a steady decrease in the standard deviation of the simulations when adding the 30th stock, as opposed to in our risk result from Figure 3. The lower risk reductions of adding more than 10 stocks are thus partly compensated for with more certain risk reductions. Thus, it is hard to argue that there is no effect from adding up to 30 stocks and more when you reduce the uncertainty of your risk. Both the simulated risks and the standard deviation of these simulations can be found in Table A.4 and A.5 in Appendix 4.

Firms	1980s	1990s	2000s	2010s
8 -> 9	8.483	9.242	8.106	6.272
11 -> 12	4.994	6.485	2.94	5.434
14 -> 15	2.576	4.314	4.79	2.377
17 -> 18	4.458	5.976	2.692	2.009
20 -> 21	2.525	4.871	3.495	3.212
23 -> 24	3.225	3.019	1.087	1.606
26 -> 27	3.153	1.907	2.017	1.237
29 -> 30	3.02	3.185	1.937	3.036

Table 1: T-values for change in risk levels.

We perform t-tests on our volatility series from Table A.3 in Appendix 4. The results are reported in Table 1. We see that for all decades, adding up to 20 stocks to your portfolio produces a significant change, at the 95% confidence level, in your risk levels. After adding the 20th stock to our portfolio, we see that for certain portfolios the change in risk becomes insignificant. However, for three of the four decades, adding a 30th stock proves to be significant, and in the 1990s it is barely not significant at a 95% confidence level. Our critical t-value is 1.96.

The significant risk reductions beyond when the 8th stock is added to the portfolio aren't consistent with Evans and Archer (1968) finding that adding the 9th stock in a portfolio doesn't yield reductions in the portfolio risk level at a reasonable level of statistical significance. It would still be faulty to conclude that the diversification effect has improved since the 60s. The T statistic is relatively sensitive to the frequency of our simulations, and Evans and Archer (1968) employed 60 simulations per portfolio consisting of 1 to 40 stocks. When we turn down the simulation frequency from 10,000 portfolios to 1,000 portfolios the marginal reductions in the portfolio risk from adding the 12th stock are insignificant at the 5% level of statistical significance. See Table A.7 in Appendix 7.

According to our tests, it's evident that it's still possible to obtain marginal risk reductions from expanding the portfolio stepwise with more than 8-10 stocks. Similar risk reduction and t-test statistic patterns in the samples imply that there has been limited variation in the diversification gains in smaller portfolios in the last four decades. We can see that for every decade, the same pattern appears, where we get high t-values when adding up to 12 stocks before the t-values settle around a value of 2-3 in most of the remaining observations. From a pure risk perspective, we still find that Evans and Archer's (1968) main finding still holds. In practice, the marginal reduction in the portfolio risk abruptly declines after the 8-10th stock is added to a portfolio in all our samples, implying that 8-10 stocks are sufficient to reduce most of portfolios idiosyncratic risk component.

Although the marginal percentage reduction in the portfolio volatility for adding stock in each portfolio is even for all decades, our simulated portfolios have varying risk levels each decade. The fact that the risk performance of the portfolios varies across the different decades is in contrast with Tang's (2004) claim that the risk performance of smaller portfolios was invariant in different periods. The risk level of naïve portfolios generally has a higher risk level in the 1980s and 2000s than in the 1990s and 2010s. A possible explanation for the latter is that the magnitude of the market crash of 1987, 2001, and 2008 lead to a higher level of systematic risk. We find that the last 10 years have seen the improved performance of smaller portfolios in total risk level. In terms of diversification efficiency, the marginal gains from adding stocks to the portfolio are close to identical in all our samples.

We also estimate the yearly excess standard deviation for simulated portfolios consisting of 2, 5, 20 and 50 stocks, relative to the standard deviation of a NYSE equally weighted index. To quantify the diversification benefits relative to a market benchmark on a yearly basis.

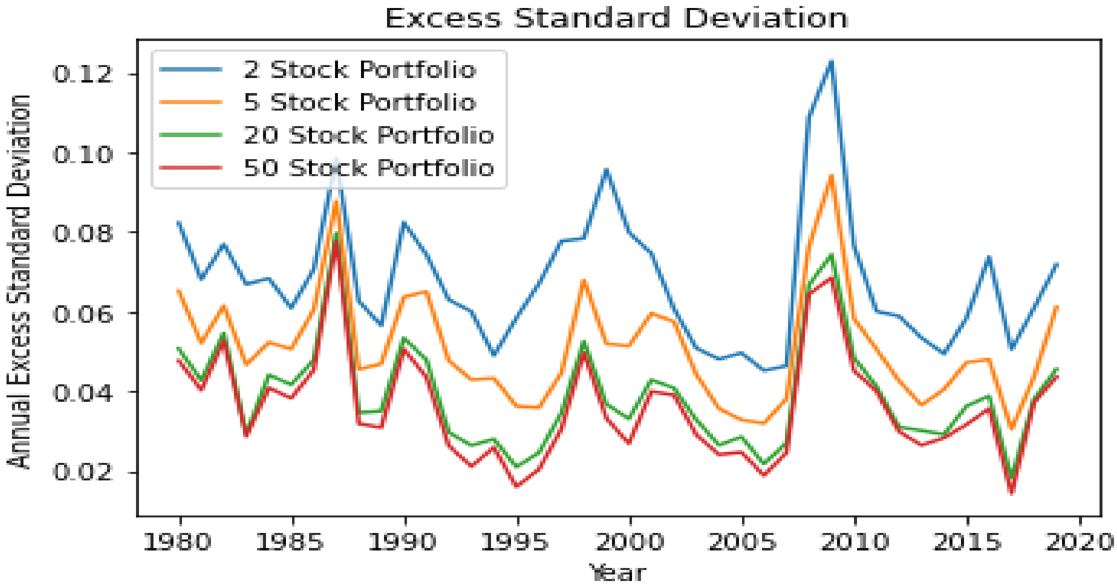

Figure 5. The yearly excess standard deviation of simulated portfolios consisting of 2, 5, 20 and 50 stocks.

Figure 5 shows a slightly lower average excess standard deviations the last decade compared to in the 2000s, despite the fact the pandemic likely led to an increase in excess standard deviation in the end of the sample. We note that the divergence in relative excess standard deviations across the decades seems to be lower than the divergence in risk alone. The excess standard deviation varies considerably throughout our sample. We still believe the series does not contain any trend which would suggest that diversification efficiency has improved or worsened considerably over our sample. While the excess standard deviation has improved slightly the last 10 years, the excess risk levels are similar at the start and the end of the sample. The relative performance of the differently seized portfolios also remains nearly identical at each end of the sample. We also note that recessions generally lead to lower diversification gains, which is consistent with the cyclical patterns found in stock return co-movement and firm specific risk volatility identified by Benhmad (2013) and Lebedinsky and

Wilmes (2017). An implication of this is that recessions could raise the number of stocks needed to achieve a better risk performance in smaller portfolios for short durations.

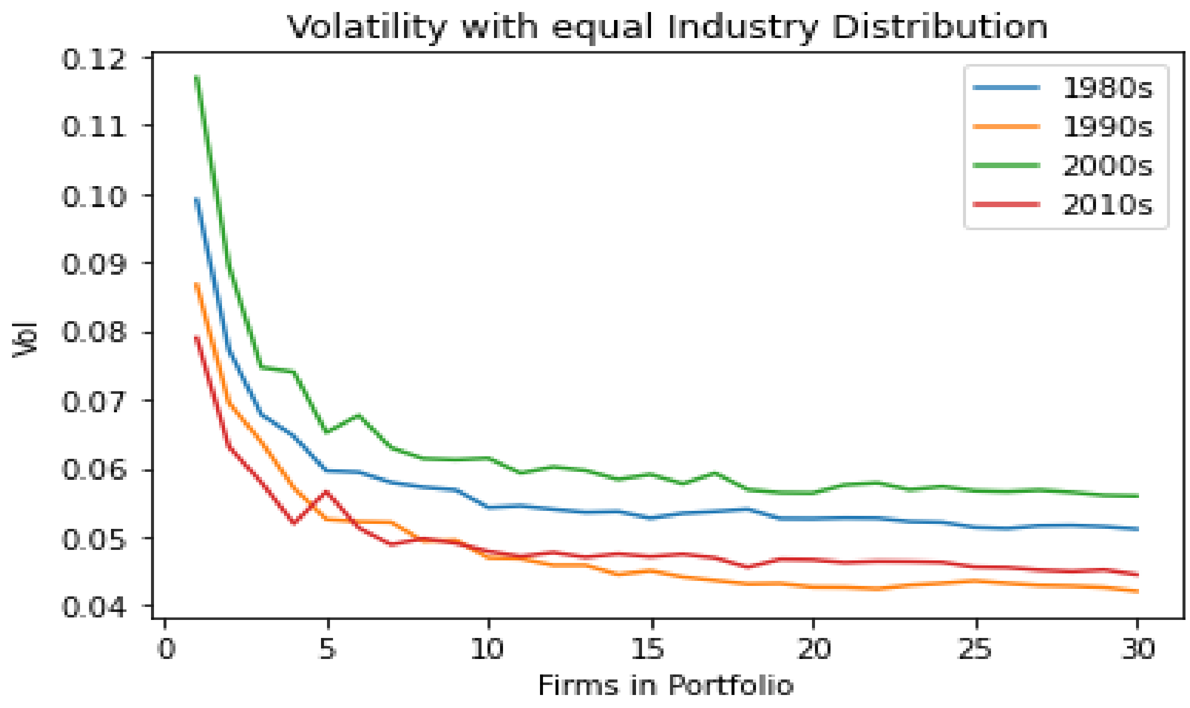

Figure 6. The average annual standard deviation of industry diversified portfolios consisting of 1-30 stocks, throughout different decades.

Figure 6 illustrates that enforcing industry allocation restrictions has little impact on the risk level of randomly drawn portfolios, which is consistent with Tang's (2004) work. The effect of adding a stock in each portfolio is somewhat more volatile with the industry restrictions in our simulations. For instance, the 2000s and 2010s restriction-based series contain sporadic increases in the simulated portfolios volatility when the portfolio expands with additional stocks. The latter could be due to the standard error of the simulations, or that restrictions make it more likely to pick stocks in a more volatile sector. The total risk of the portfolios also illustrates that smaller portfolio risk has fallen over the last decade. At the same time, the relative order of the volatility series arranged after its standard deviation has remained unaltered indicating little variation in the diversification effect.

The latter results are surprising when studies of the co-movement and idiosyncratic volatility of stock return have identified changes in the risk determinant structure originally described in the CLMX study. For instance, LaCasce et al., (2019) and Sullivan and Xiong (2012) found that the co-movement in stock return had increased significantly since the turn of the century. A change that should have yielded worse diversification benefits. Bekaert, Hodrick, and Zhang's (2012) and Brandt, Brav, and Kumar (2010) did however dismiss the positive trend in the idiosyncratic volatility CLMX categorized in the late 90s. The only possible explanation for why the marginal gains from adding stocks to a small portfolio have been consistent is that both trends in idiosyncratic risk and stock co-movement have changed since the CLMX study. Where a considerable decline in the firm-specific volatility component of stocks must have cancelled out the effect of increased stock return co-movement. This is exactly what we have found, and we will first summarize our analysis of the idiosyncratic volatility part.

5.2 Volatility Measures Graphical Analysis

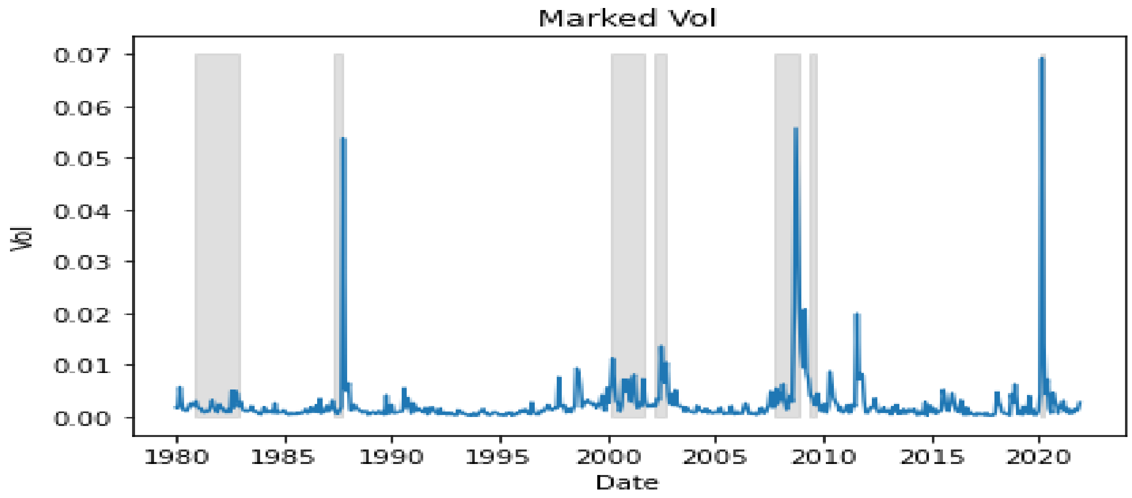

Figure 7. The monthly market volatility based on Equation (5) as a part of our three-component volatility decomposition in Equation (4). The figure contains the time series for the variance within each month of daily market returns. Bear-market periods are marked in grey to illustrate cyclical spikes in the series.

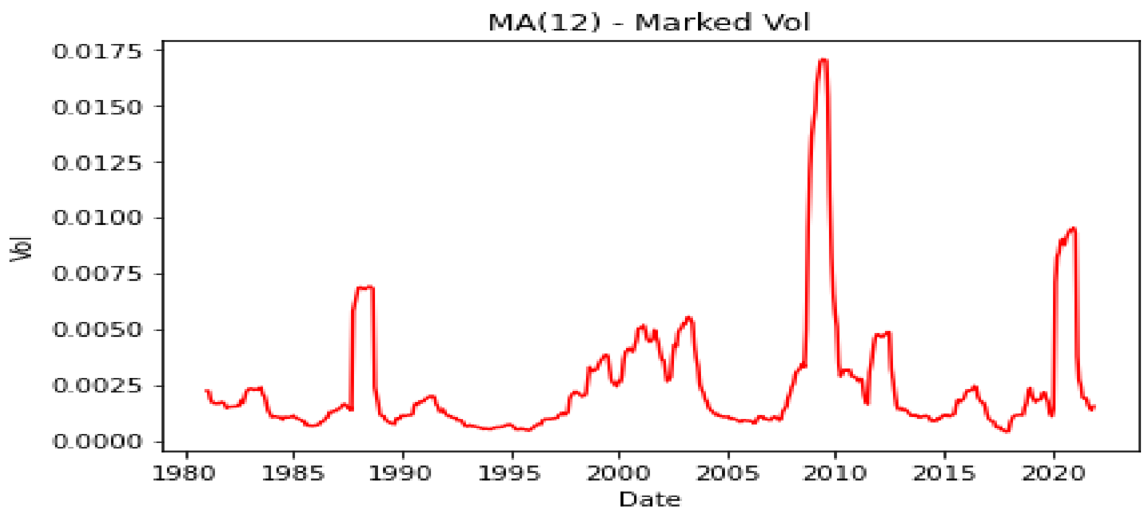

Figure 8. The MA (12) of the monthly market volatility based on Equation (5). The figure contains the backward-smoothed moving average (12) of the market volatility series.

Figure 7 shows the volatility of the value-weighted NYSE composite index from 1980 to 2021. Using daily data to obtain monthly estimates of the variance of the aggregated market.

We will now comment on the three individual volatility series based on the CLMX decomposition in Equation (4), to check whether the risk trends categorized in previous studies have persisted. We analyze whether a changed risk structure produced an outcome that would be consistent with the little variation in diversification benefits across our simulation samples. While the market has been described as volatile in the last 15 years with two major recessions, Figure 7 shows a clear tendency for mean reversals after the recessions for aggregated market volatility. Like countless previous studies, our graph for the estimated volatility of the American stock market identifies a tendency where recessions coincide with significant spikes in volatility. The volatility spikes are related to the market crash of 1987, the dot-com bubble, the 2008 financial crisis, and the recent pandemic. The graph does little to suggest any positive or negative linear trend in the market volatility.

Interestingly in comparison to previous studies, our graph also reveals the considerable impact of the pandemic recession. The mean of the volatility series from 1980- 2000 and 2000-2020 is 0.001 and 0.0013. A casual comparison of the original volatility series and its smoothed MA (12) alternative indicates that market volatility has a slower moving movement dimension and some noise related to the high frequency of the original estimate. The fact that aggregated risk in the market means reverting is consistent with limited time variation in the diversification efficiency of smaller portfolios.

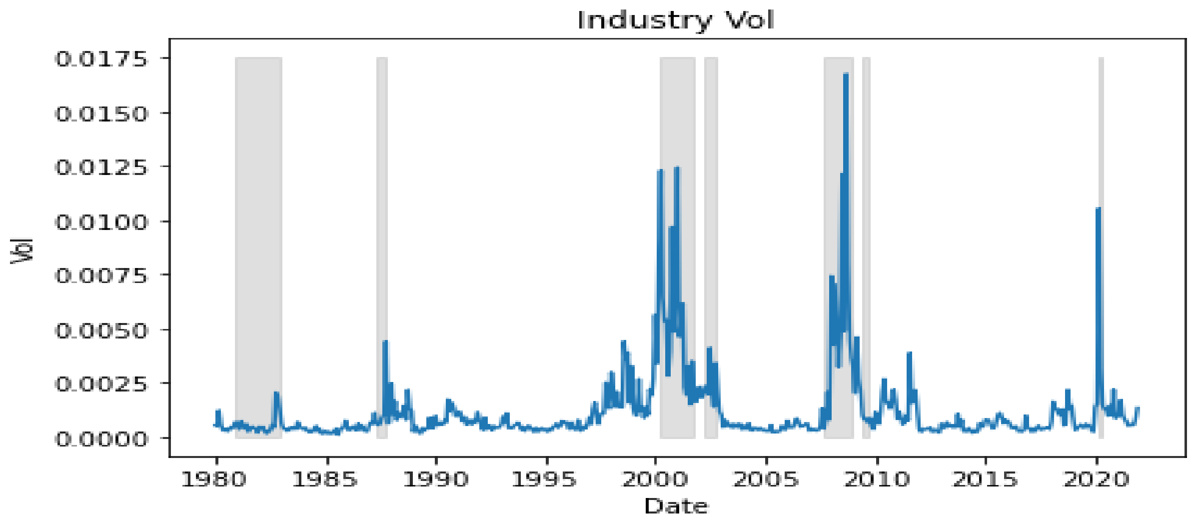

Figure 9. The monthly industry level volatility based on Equation (6) as part of our three component volatility decomposition in Equation (4). The figure summarizes the time series for variance within each month of daily industry returns relative to the market. We have marked bear-markets in grey.

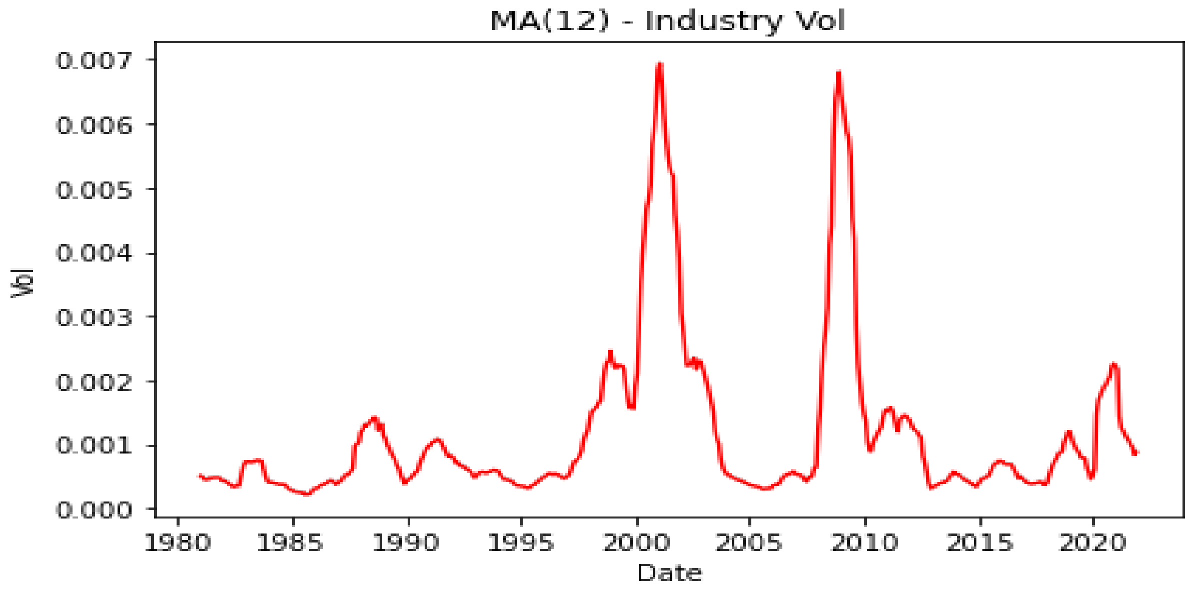

Figure 10. The MA(12) of the monthly industry volatility based on Equation (6). The figure is the backward 12-month moving average equivalent for the industry volatility series.

The volatility graph for the aggregated industries reveals a similar trend to that of the market graph, where spikes in volatility occur during recessions. When we compare volatility spikes of the industries' returns to the market return volatility spikes, it is evident that the dot-com bubble has a more significant relative impact on the industry series. In contrast, the 87-market crash has a less relative impact on the volatility of the industry returns. The industry returns volatility series does not suggest any trend. Its smoothed MA (12) equivalent reveals a slower-moving movement dimension, albeit to a lesser extent than for the market series.

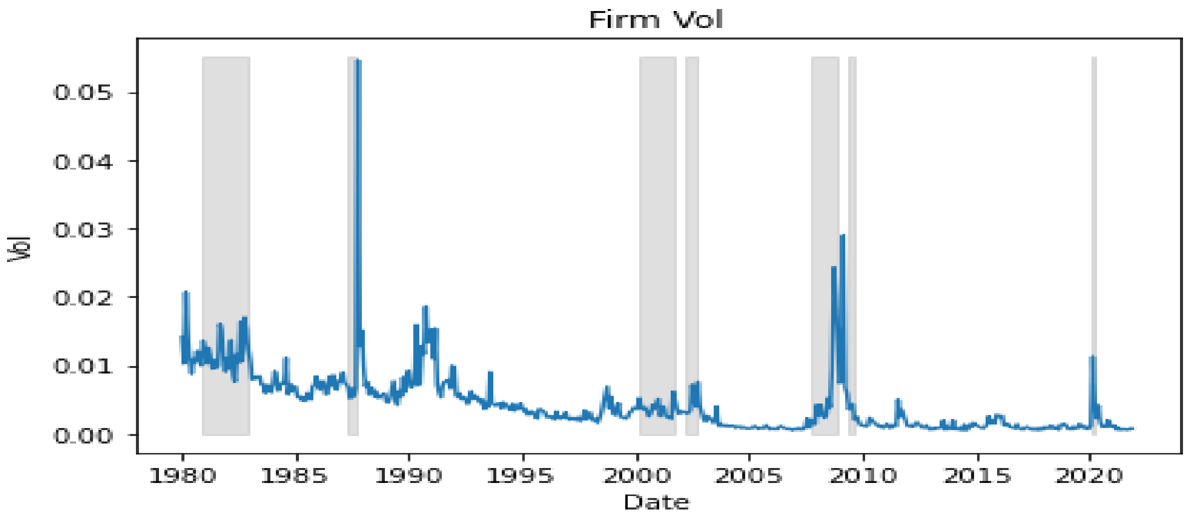

Figure 11. The monthly firm level volatility based on Equation (7) as a part of our three component volatility decomposition in Equation (4). The graph summarizes the time series for variance within each month of daily firm returns to the firm's industry. We have marked bear-markets in grey.

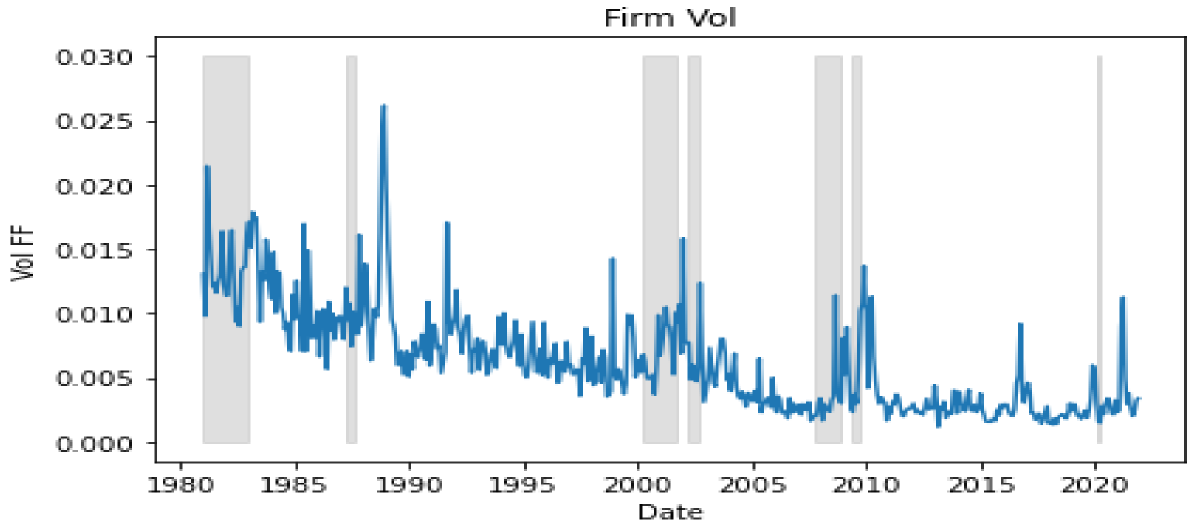

Figure 12. The monthly firm level volatility derived from Equation (9) a Fama French (1993) regression.

Figure 12 summarizes the average monthly standard deviations of the residuals obtained from the regression in Equation (9) for each stock in the sample with a 42-day window.

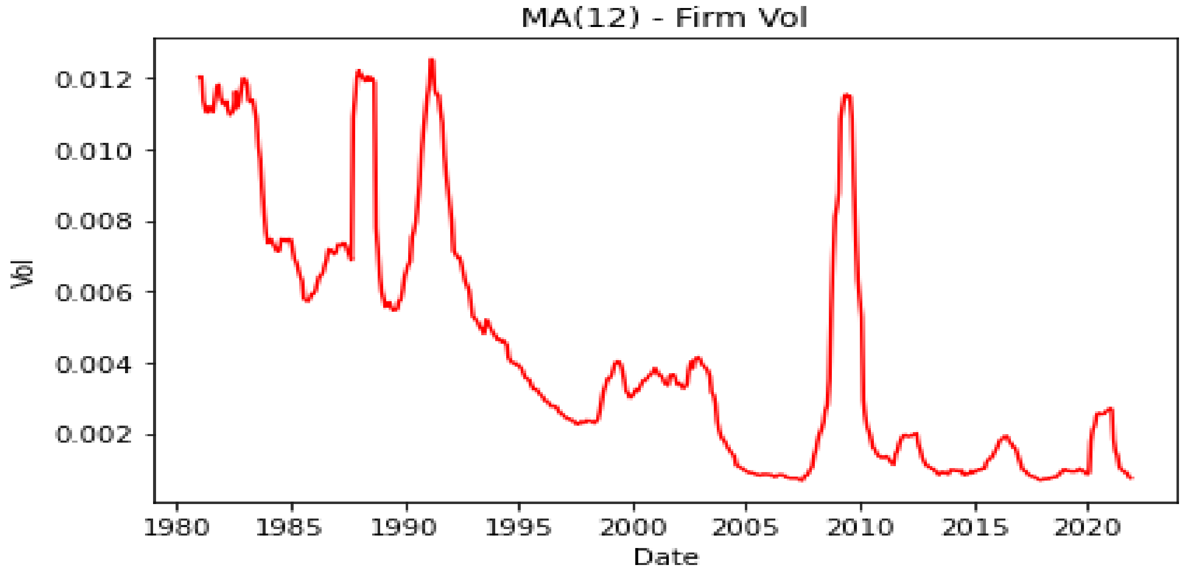

Figure 13. The MA (12) of the monthly firm volatility based on Equation (7). The figure contains the backward smoothed moving average (12) of the CLMX firm volatility series.

The CLMX volatility series for the firm-specific return volatility component has similar tendencies to the previous volatility graphs and one clear contrast. While there is a tendency for mean reversions after recessions, the dot-com bubble and the pandemic recession had a relatively limited impact on the firm volatility series compared to the other disaggregated volatility series. Furthermore, the positive trend CLMX categorized in idiosyncratic risk from 1962 to 1996 seems to revert from the middle of our sample. The latter is independent of how the risk series was estimated. A comparison of the Fama-French (1993) and CLMX volatility series seems to validate Bekaert et al., (2009) critique of the CLMX estimation method for limiting stock return variability. We find the F.F. series to be slightly more variable than the CLMX series, as confirmed when we compare the standard deviation of the two series. The divergence between the series is limited, on average the CLMX series predicts 22% lower idiosyncratic variance than the F.F series.

The typical idiosyncratic stock volatility has steadily declined over the last 20 years compared to the 20 years prior, except for short periods with volatility spikes related to the recessions of 2008 and 2020.

While several papers offer possible explanations for the rise in idiosyncratic risk in US markets from the 80s to the late 90s, there is limited research on what might have caused the recent decline in idiosyncratic risk levels. Brown and Kapadia (2007) and Fama and French (1993) attributed that firms listed in the 80s and 90s came from risker sectors to the rise in idiosyncratic risk. Thus, the purge of several risky tech companies during dot-com bubble graphed Figure 1, and the Dodd-Frank reforms after the 2008 financial crisis might have decreased the share of firms with high idiosyncratic risk levels. In terms of fundamentals, quantitative easing and a low-interest rate environment post-2008 have lowered firms' cost of debt and kept their discount rates to a low and stable level. The latter could have led to less firm exposure to idiosyncratic factors. We see the relative impacts of each volatility component in Figure 14.

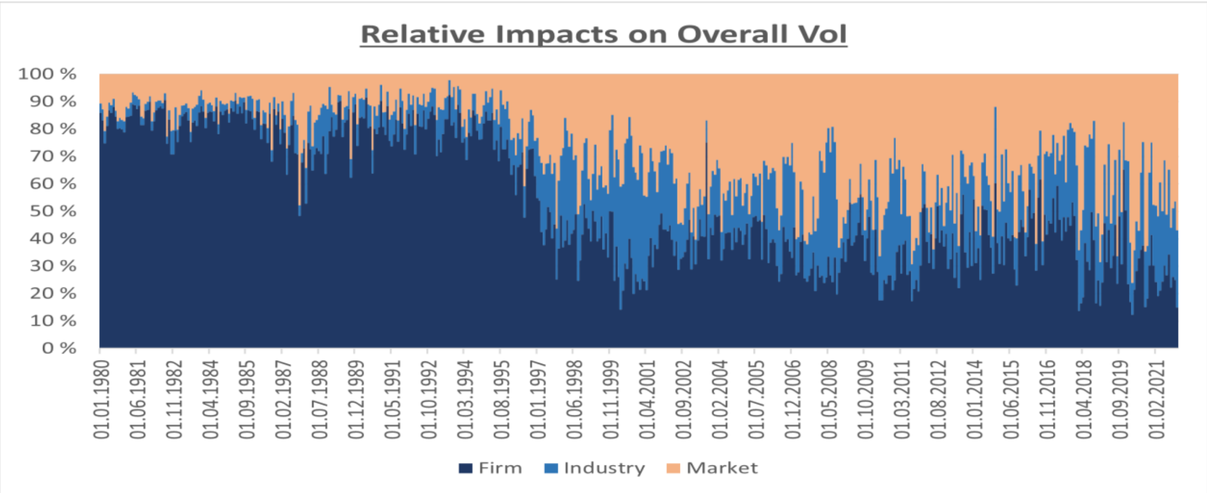

Figure 2. Relative impacts of firm, industry and market volatility.

We have now seen that one of the two factors that affect diversification gains, the idiosyncratic risk, indeed appears to have decreased since the CLMX study. We will later employ Markov-switching models to counter the possibility of the large recessionary spikes in the first part of the sample causing the trend. The models will allow us to identify possible different states of the data generating process behind the volatility series. Our models can thus identify whenever the recessions cause high variance states or whenever we genuinely see a different data generating process for the variance series after the end of the CLMX sample.

5.3 Test of the Volatility Series

We now analyze the autocorrelation structure of our 4-volatility series, to check for serial correlation and stochastic trends.

Autocorrelation Coefficient	Market Vol	Industry Vol	Firm Vol (CLMX)	Firm Vol (FF)
1	0.385	0.399	0.36	0.529
2	0.055	0.322	0.213	0.132
3	-0.05	0.115	0.067	0.025
4	0.003	-0.226	0.1	0.096
6	0.031	0.296	0.005	0.045
12	0.031	-0.051	0.114	0.087

Table 2. The autocorrelation structure of the volatility series.

Table 2 contains the estimated coefficients from the regression

$$Vol = \alpha_1 + \phi_1 vol_{t-1} + \phi_2 vol_{t-2} + \phi_3 vol_{t-3} + \phi_4 vol_{t-4} + \phi_5 vol_{t-6} + \phi_6 vol_{t-12} + u \quad (16)$$

for each volatility series.

All four series have significant autocorrelation structures; the series reveal considerable serial correlation. The latter is consistent with the somewhat enduring fluctuations in the volatility series. Due to the possible stochastic trend in either series, we employ a stochastic trend test for each series.

To test for the possibility of unit roots, we employ variants of the augmented Dickey-Fuller test. We run a regression with the first difference of the volatility series as the dependent variable, and a lagged version of the series and its lagged differences as explanatory variables, to test if the series contains unit rots. When we include the lagged first differences for either volatility series, we mitigate the issues related to the serial correlation in each series (Brooks, 2014).

We conduct several tests to determine the best fit for lag structure in the ADF-test. We eliminate the least statistically significant lag from each regression until all lags were statistically significant at the 5% level.

	MKT	IND	FIRM (CLMX)	FIRM (FF)
T-test	-9.19 (0.00)	-4.04 (0.00)	-4.04 (0.00)	-3.32 (0.01)
Z-test	-9.12(0.00)	-4.042 (0.0012)	-4.042 (0.0012)	-3.23 (0.02)
Lag order	2	6	4	6
For constant and trend				
T-test	-9.283 (0.00)	-3.71 (0.00)	-5.598 (0.00)	-5.82 (0.0)
Z-test	-9.280 (0.00)	-3.710 (0.0217)	-5,6 (0.00)	-5.81 (0.00)
Lag order	2	5	4	6

Table 3. Results from the augmented Dickey-Fuller tests.

Table 2 contains the estimated ψ_1 coefficient and its p-value from the regression

$$\Delta vol_t = \phi_0 + \psi_1 vol_{t-1} + \phi_1 \Delta vol_{t-1} +,,, \phi_n \Delta vol_{t-n} + u_t. \tag{17}$$

.

Where $\Delta vol_t = vol_t - vol_{t-1}$ is the first difference of vol and $\psi_1 = \phi_1 - 1$. Table 3 also contains the results of an equivalent model with a trend term and adjusted lag length.

We reject the zero hypothesis of a unit root for all the four series, independent of the inclusion of a trend term. Interestingly, of ADF tests, only the firm volatility series has a trend term at the 5% level of statistical significance. The trend coefficients of -0.00005 and -0.00006 for the CLMX and Fama- French (1993) series are negative but low in magnitude.

Since the series are stationary, we conduct further analyzes of the series in levels and not first differences. Since Figures 11 and 12 still indicate a negative trend in the idiosyncratic volatility series, we test for the alternative hypothesis of deterministic trends in the firm volatility series.

	Mean	Standard Deviation	Min	Max
Market Vol	0.002	0.0051	0.0001	0.0691
Industry Vol	0.001	0.002	0.0001	0.0167
Firm Vol (CLMX)	0.0044	0.005	0.0005	0.054
Firm Vol (FF)	0.0056	0.0056	0.001	0.026

Table 4. Descriptive statistics for the volatility series. The table contains the mean, standard deviation, maximum and minimum values for each volatility series.

The standard deviation of the volatility series further reveals that both the firm and market volatility series have similar variability. This result is consistent with the findings from the CLMX study.

We employ Vogelsang's (1998) test procedure to test if there is a deterministic linearity in either variance series. We select this test procedure due to the moderate persistence of the volatility series since the tests are resilient to different forms of serial correlation.

The test procedure is built on the following equations

$$v_t = \mu + \alpha_0 t + \rho v_{t-1} + u_t \tag{18}$$

$$u_t = \alpha_1 u_{t-1} + d(L)e_t, \tag{19}$$

where $v \in (Market\ vol, Industry\ vol\ , Firm\ vol^{CLMX}\ and\ Firm\ vol^{FF})$, μ is an intercept, α_0 is the trend coefficient finally ρ quantifies the series dependence of its own lag. The models error term is modelled as function of its own lag with the α_1 coefficient and have a term that contains the MA (∞) process of the white noise in the series. The Vogelsang test check whenever the parameter $\alpha_1 = 0$, to test for any time linearity. The test mitigates issues related to serial correlation since the conditions on the error terms which the T-distribution is derived from are weak and adjust to most covariance stationary process and I (1) processes (Vogelsang, 1998). The reported coefficients and their significance level are reported next.

	Market Vol	Industry Vol	Firm Vol (CLMX)	Firm Vol (FF)
Trend coefficient * 10^5	0.217	0.445	-1.01	-0.905
P-value trend coefficient	(0.151)	(0.264)	(0.00)	(0.00)
95% confidence intervals * 10^5	(-0.0081, 0.513)	(-0.034, 0.0012)	(-1.260, -0.768)	(-1.10, -0.69)

Table 5. Tests results from the Vogelsang test.

Table 5 contains the estimated coefficients for the trend term from each Vogelsang test, the p-value for each trend-coefficient and the 95% confidence interval for each trend coefficient.

Interestingly, we reject the zero hypotheses of no time linearity in the idiosyncratic risk series, while we fail to reject the zero hypotheses for the other series. The latter is a clear break from the CLMX study findings. The average return volatility of the stocks listed on the NYSE has a decreasing linear trend throughout our sample.

While a negative low magnitude trend coefficient is consistent with Figures 7 and 9, we want to check whether the trend is related to a regime change or varying data generating processes.

We have chosen to employ a Markov switching model to control for the possibility of different regimes for idiosyncratic variance. Based on Hamilton and Susmel's (1994) work, our model for the idiosyncratic variance follows an AR (1) model where all the estimated parameters can take one or two values dependent on the realization of a discrete regime variable. The model's primary attribute is its ability to identify when the time series switch from one regime to another when it is unclear when structural breaks occur, as is the case for our variance series (Hamilton & Susmel,1994).

Our Markov switching model has following transition probability matrix

$$\psi = \begin{pmatrix} p_{11} & 1 - p_{11} \\ 1 - p_{21} & p_{21} \end{pmatrix}. \qquad (20)$$

Where p_{11} denotes the probability of variance series staying in first identified regime at t_{+1} and $1 - \rho_{22}$ denotes the equivalent probability staying in second identified regime at t_{+1}.

Firm vol (CLMX)	Mean Coefficient (P-value)	Transition probabilities
State 1	0.0021 (0.00)	ρ_11 = 0.9952
State 2	0.00921 (0.00)	ρ_21 = 0.0088

Firm vol (FF)	Mean Coefficient (P-value)	Transition probabilities
State 1	0.0044 (0.00)	ρ_11 = 0.988
State 2	0.011 (0.00)	ρ_21 = 0.027

Table 6. The Dynamic Markov Switching model for the idiosyncratic volatility series.

Table 6 summarizes the estimated mean and its p-value for the two identified states, as well as the implied transition probabilities.

Our Markov model identifies two variance states for the CLMX idiosyncratic volatility series. For the CLMX based series the identified low variance state has a mean-variance of 0.0021 compared to the mean of 0.00921 of the high variance states. The implied transition probability from state 1 to 2 of 0.005 and state 2 to 1 of 0.002 also reflects persistent states. Graphing the transition probability for the first state indicates a clear shift from the higher variance state to the lower variance state from the mid-90s.

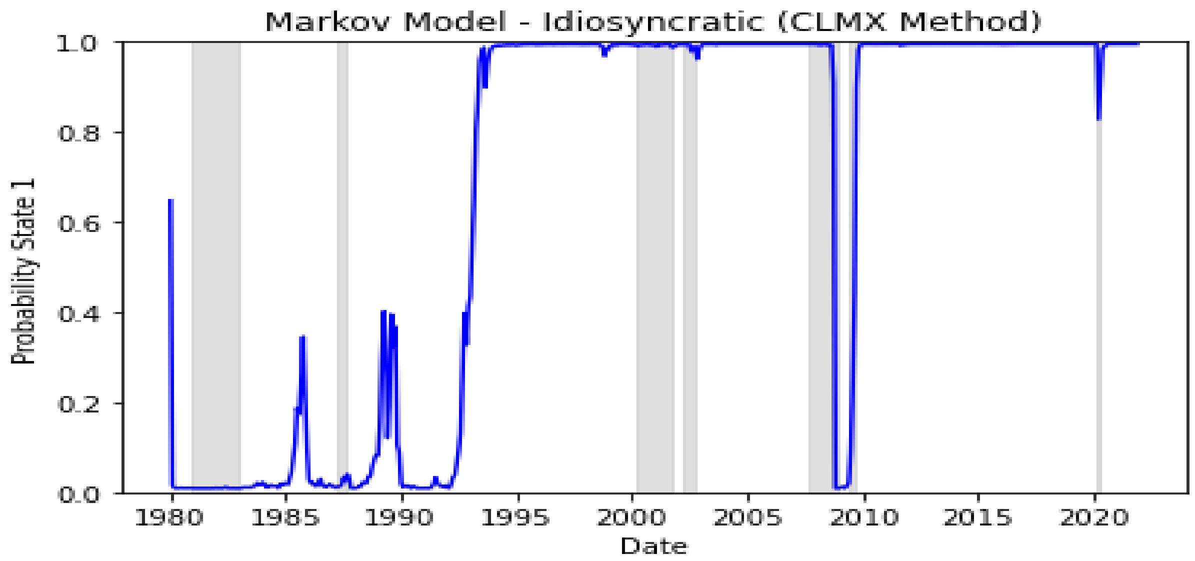

Figure 15. The time series of the estimated transition probabilities for the CLMX time series. The graph identifies the transition probability to the first identified variance regime over time for the CLMX volatility series. Recessions are marked in grey

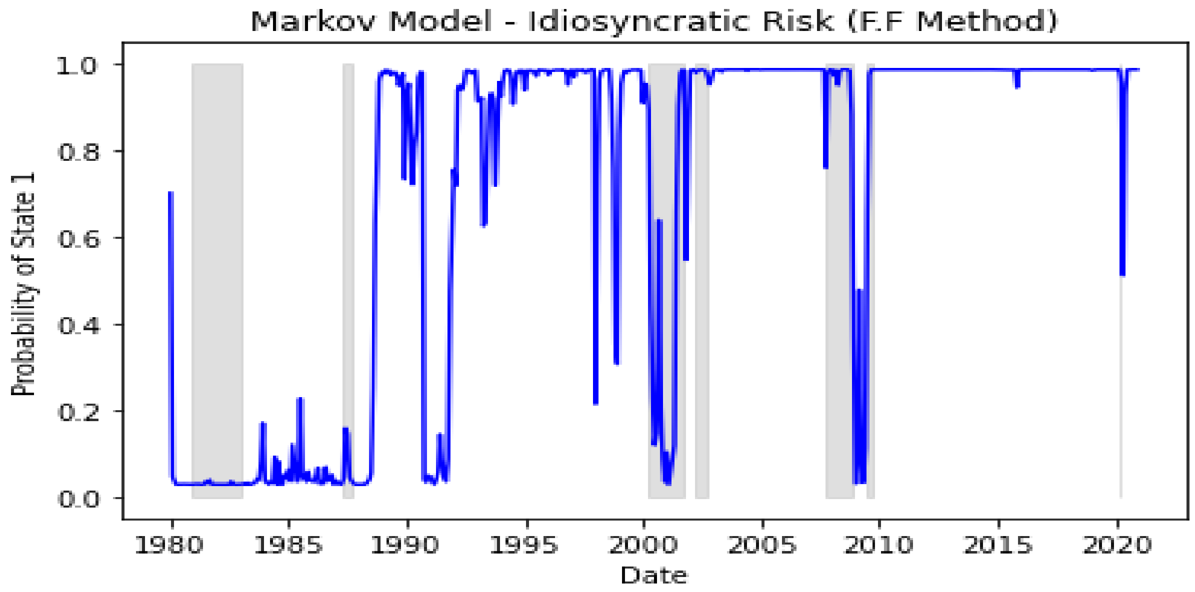

Figure 16. The time series for the transition probabilities from the Markov Switching model for the CLMX firm volatility series. The graph identifies the transition probability to the first identified variance regime over time for the F.F. volatility.

The results we obtain from the Markov model for the F.F idiosyncratic risk series are also relatively consistent with results from the previous Markov model in terms of transition probabilities. Both estimated states are highly persistent as in the previous model, and both models estimate a clear regime shift in the late 90s. The F.F based model also accounts for the dot-com bubble causing a temporary switch to a high variance state. The mean of the low variance state of 0.044 is twice as high as for the CLMX series, while the high variance state has a 22% higher estimated variance. Yet we still clearly identify a regime switch in both models.

The identification of two different variance regimes has several implications. Firstly, it represents a clear break from CLMX's main finding in a positive deterministic trend for the idiosyncratic volatility, with a highly persistent low variance state starting at the end of their sample. We also note that the 2008 recession implied a shift back to a high variance regime but that the series otherwise stayed in the low variance state. Secondly the transformation the low mean idiosyncratic variance regime supports our explanation to why our simulated portfolios produced stable diversification benefits across the samples. We will now see that the co-movement in stock returns have had a converse regime shift. With a stable diversification benefit, it is likely that the co-movement and firm specific risk effects have canceled each other out across the sample.

5.4 Analysis of the Development of the Pairwise Correlation Coefficients

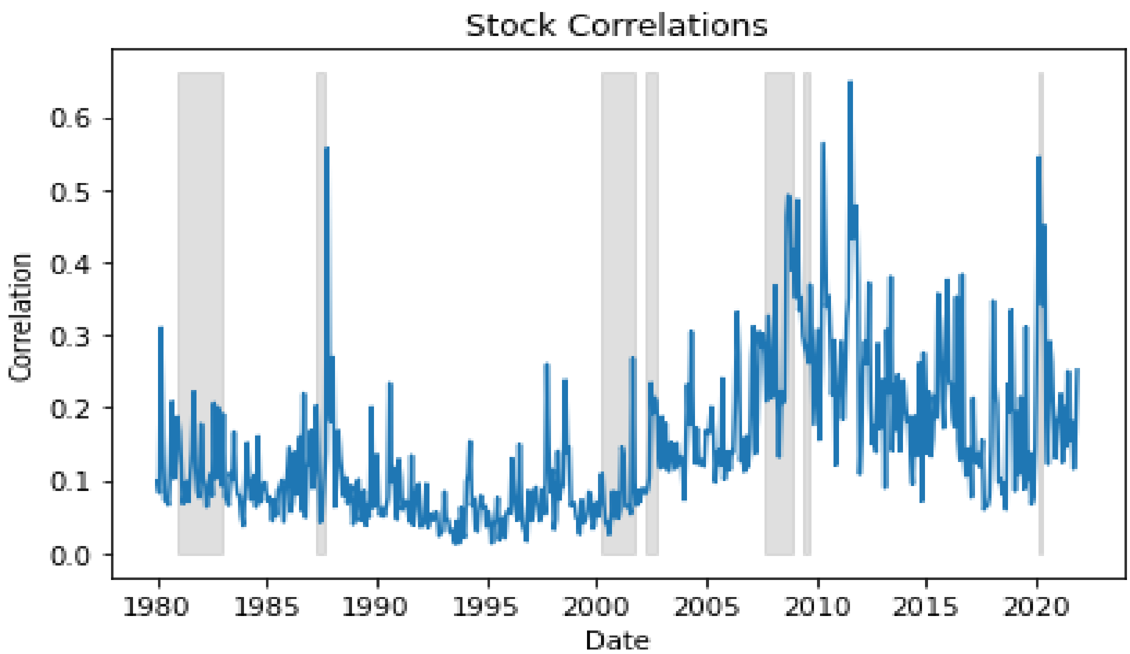

Figure 17. The average monthly pairwise correlation coefficient for stocks listed on the NYSE.

Figure 17 graphically summaries the average monthly pairwise correlation coefficient for the return of NYSE listed stocks. We obtain the monthly estimates using daily return data from 1980 to 2020.

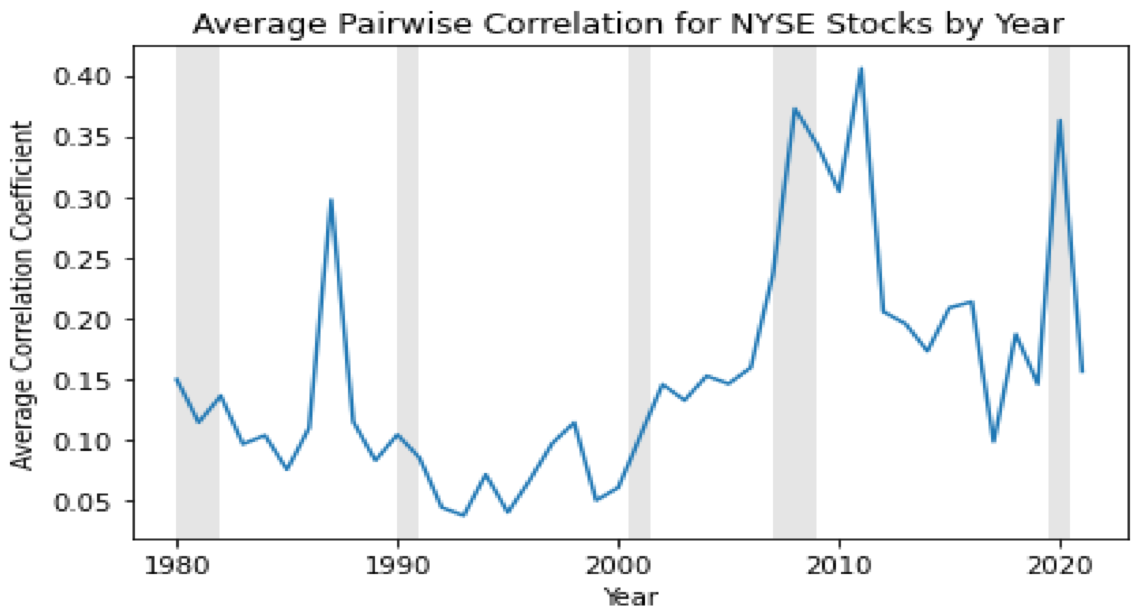

Figure 18. The average yearly pairwise correlation coefficient for stocks listed on the NYSE.

Figure 18 graphically summaries the average annual pairwise correlation coefficient for the return of NYSE listed stocks. We obtain the monthly estimates using monthly return data from 1980 to 2020.

We calculate an equally weighted average pairwise correlation coefficient for each month and year. The correlations estimates based on the daily data are lower than the monthly alternative. The likely reason for the latter is that daily stock returns typically contain negatively autocorrelated idiosyncratic components (Campbell et al., 2001).

Figure 17 and 18 is evidence that the average pairwise correlations increase significantly during recession or bear markets. Incidents such as the crash of 1987, the dot-com bubble, and the 2008 financial crisis leads to the average pairwise correlations coefficient more than doubling for short durations. Although the stocks tend to revert to lower average pairwise correlations after recessions, the tendency described by CLMX, where the average correlations decline as time progress, does not fit the data. As pointed out by previous research, the CLMX sample period ending right before the beginning of the dot-com bubble can lead to misleading results. The average pairwise correlation coefficient of 0.2 at the end of the monthly series created with daily data is four times higher than in 1997 . The last 20 years have generally involved higher correlation levels than in the other sample split for both series.

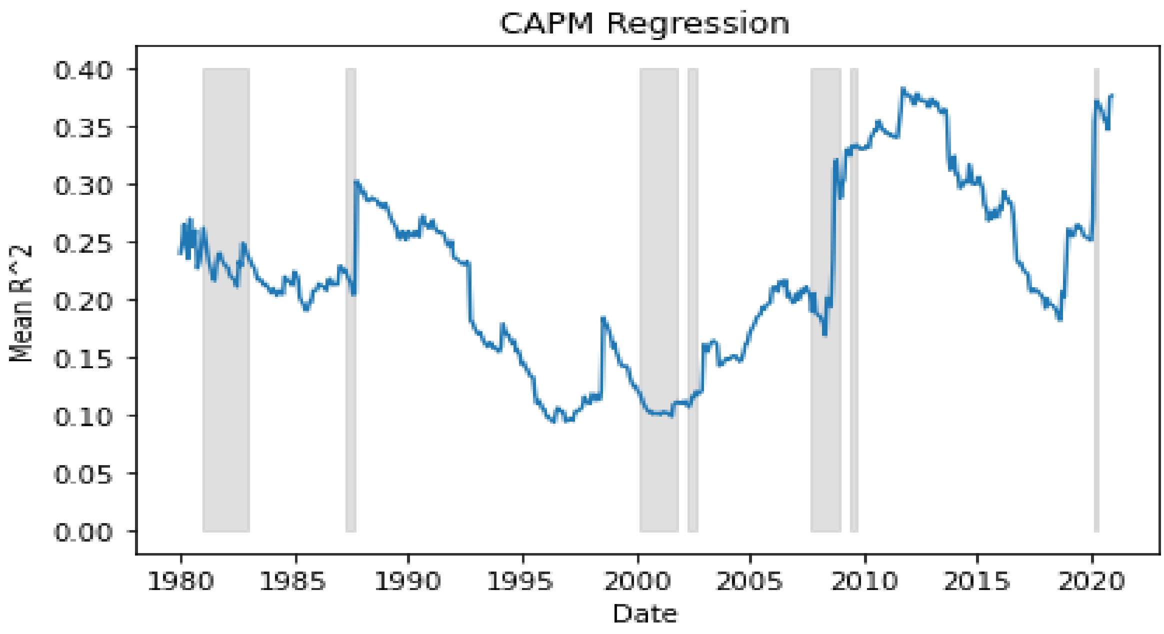

Figure 19. The average monthly R^2 from the CAPM regression on each stock.

Figure 19 contains the average monthly R^2 from the CAPM regression, based on Equation (10), on each stock using monthly return data and a 60-month rolling window.

The tendencies in our time series graphed in Figure 19 also largely corresponds with the results from our other co-movement model. We find that R^2 of the CAPM model still serves as a good proxy for the co-movement of stocks returns although to a lesser extent than in the CLMX sample.[7] A possible explanation for the divergence in the second half of the sample is Sullivan and Xiong (2012) claim that increased institutional trading or other factors has increased the baseline level of co-movement in stock returns. Yet our model still finds increased co-movement from 2005 to 2015 consistent with our previous findings, while the falling co-movement from 1990 to 1995 are consistent with CLMX findings which lead to their conclusion of declining stock return co-movement.

Possible explanations for the increased return co-movement vary from more economic integration, changed underlying fundamentals or the rising market share of institutional investors. The increase in the pairwise correlations post 2000 corresponds with the increasing

[7] The CLMX study found that their monthly R^2 series were close to identical to their pairwise correlation coefficient series based on daily data. The second split of our R^2 series significantly underpredicts the co-movement relative to in our co-movement series created with daily data.

popularity of ETFs and increasing markets shares of institutional ownership as described by Sullivan and Xiong (2012). However, we note that both the series show a tendency of falling pairwise correlations between 2014 to 2018 before the start of the pandemic recession.

We conduct our future tests with the monthly series due to the low frequency of the yearly series. A simple interpretation of Figure 17 also identifies a clear break in the data generating process from the middle of the sample. Thus, we once again employ the augmented Dickey Fuller test to test the possibility of a stochastic trend. Interestingly, while the data is stationary with a correctly specified lag structure, our Vogelsang trend test identifies a positive trend in the return co-movement. However, we believe the latter is due to different correlation regimes rather than a stochastic trend for the pairwise correlation coefficients. We observe a clear graphical break in the middle of the monthly co-movement series. We find it unlikely that the pairwise return correlation should converge towards its limit. The test results can be found in Appendix 6.

Employing a dynamic Markov switching model also confirms our suspicions. The model identifies two sets of states.

Correlation coefficients	Mean coefficient (P-value)	Transition probabilities
State 1	0.105 (0.00)	ρ_11=0.980
State 2	0.279 (0.00)	ρ_21=0.0639

Table 7. The Dynamic Markov Switching model for the pairwise correlation coefficient series.

Table 7 summarize the estimated mean and its p-value for the two identified states, as well as transition probabilities.

We identify a high and a low state regime for the correlation series like in our previous Markov estimations. Both states are also highly persistent, with implied transition probabilities of only 2% and 6% for the low and high mean state regimes. Transformations to the high correlation state primarily occur during recessions, while a high correlation state has lasted for most of the sample past 2005. The implied state 1 transition probability series also illustrate temporal shifts back to the low correlation state between 2015-2018 before the pandemic.

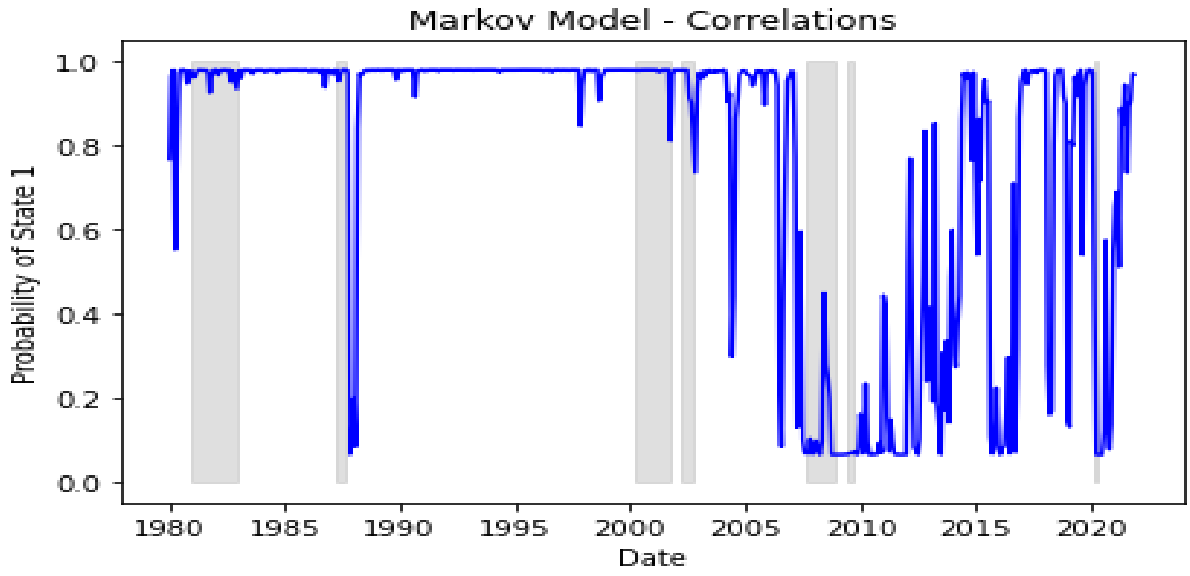

Figure 20. The time series of the estimated transition probabilities for the correlation series for the Dynamic Markov switching model.

Figure 20 identifies the transition probability to the first identified co-movement regime over time.

In summary, our analysis of the pairwise correlations in stock returns finds increased co-movement in stock returns the last 20 years with the turn to a higher mean regime in the early 2000s. The fact that the idiosyncratic volatility component of a typical stock in contrast has entered a lower mean regime during the last 20 years thus offer an explanation to the stable diversification benefits. Although the 80 and 90s where categorized by high levels of idiosyncratic risk and low co-movement the change to regimes with the opposite tendencies within similar time frames has produced stable diversification benefits.

The little time variation in diversification gains and equalizing regime shift effects can be due to the fact that idiosyncratic risk component in common stocks and their return co-movement could be viewed as two sides of the same coin. If a larger portion of different stocks volatility origins from idiosyncratic factors and less from systematic factors, their return will be less correlated due to less systematic influence. Conversely, a higher idiosyncratic risk level and less influence of systematic factors should produce the opposite outcome. The latter is consistent with the CAPM model in Figure 19 where a faltering R^2 corresponds with lower realized pairwise correlation coefficients. Thus, a scenario where both the co-movement and idiosyncratic risk rise could be viewed as unlikely. Recessions do nevertheless represent an exception to the canceling out argument. Recessions typically leads to high magnitude increases in both systematic and idiosyncratic risk as well as increased return co-movement as described by Benhmad (2013).

5.5 Short Term Volatility Dynamics

We also chose to analyze whether short term relationships between the volatility series have changed, due to the significant differences between the volatility and correlation estimates in our sample and the sample employed by CLMX.

We estimate a vector autoregressive model (VAR) to run Granger tests and a correlation matrix to see whether the short-term dynamics for the three levels of volatility has been altered.

Corrogram	Market Vol	Industry Vol	Firm Vol
Market Vol	1		
Industry Vol	0.563	1	
Firm Vol	0.482	0.163	1

Table 8. The correlation matrix for the 3 CLMX series.

From Table 8, we see a structure where the market series is more correlated with the industry than the firm series. The correlation between the industry and firm series is relatively low.

	Market Vol	Industry Vol	Firm Vol
MKT_t-1	-	0.00(6)	0.003(4)
IND_t-1	0.000(6)	-	0.000(6)
Firm_t-1	0.346(4)	0.006(6)	-

Table 9. The results from a Granger causality test with a bivariate VAR model.

Table 9 contains the p-value from an F-test where we test whether the lags from the different volatility series cause each other in the VAR system specified below for each pair volatility series. The optimal lag lengths are in parentheses.

$$Vol_1 = \alpha_1 + \phi_{11} vol_{t-1}^{1} \ldots + \phi_{1n} vol_{t-n}^{1} + \phi_{21}\, vol_{t-1}^{2} \; \ldots + \phi_{2n} vol_{t-n}^{2} + u_t \qquad (21)$$

$$Vol_2 = \alpha_2 + \phi_{21}\, vol_{t-1}^{2} \; \ldots + \phi_{2n} vol_{t-n}^{2} + \phi_{11} vol_{t-1}^{1} \ldots + \phi_{1n} vol_{t-n}^{1} + u_t. \qquad (22)$$

	Market Vol	Industry Vol	Firm Vol
MKT_t-1	-	0.00 (6)	0.03 (6)
IND_t-1	0.00 (6)	-	0.61 (6)
Firm_t-1	0.11 (6)	0.61 (6)	-

Table 10. The results from a Granger causality test with a tri-variate VAR model.

Table 10 contains the Granger causality tests results from a tri-variate VAR model, a model which expands on the model in Table 9 with all three-volatility series.

To specify the correct lag structure for our specification VAR models, we use the Akaike and Schwarz Bayesian information criteria to obtain the best fitted lags structure for each series. The results from our Granger causality tests reveal a slight divergence from the CLMX results. The firm series no longer Granger causes the market series in neither VAR model. The industry series now Granger cause the market and firm series in the bivariate model. In the tri-variate model the industry volatility only Granger causes the market series. The market series still leads to other volatility series. The fact that the industry series now affects the market series represents a new dynamic in the disaggregate volatility decomposition. Where industries rather than firm volatility now has a short-term effect on the volatility of the market. The enhanced short-term influence of the industries, and the declining influence of the firm specific risk component in our disaggregated model has coincided with a lower mean level of idiosyncratic.

In summary, our analysis of the average return volatility and co-movement in stocks listed on the NYSE has found increased co-movement and less idiosyncratic risk in stock returns over the last 20 years. In terms of short-term dynamics, we also find that the industry level volatility now has a short-term effect on the market series, representing a significant change in the volatility structure for a common stock. While the short-term influence of the firm specific volatility series on the other market series has become insignificant at the 95% level of statistical significance. The changes in the volatility and return co-movement structure thus offer an explanation to why the marginal gains from adding stock to small portfolios has remained largely time invariant. When the high level of idiosyncratic risk in stock returns during the 80s and early 90s eventually turned to lower mean state, the transformation was counterbalanced by the turn to a higher return co-movement regime within a similar time frame.

As a final robustness analysis, we also investigate our simulated portfolios reward to volatility ratio to test Bessembinder's (2018) finding that smaller portfolios are inefficient when returns are accounted for. We do this, as investors are not only interested in reducing risk, but the performance of their portfolios relative to risk.

5.6 Results of the Portfolio Simulation on Reward to Volatility Performance

To benchmark the performance of our simulated portfolios we summarize the average annual Sharpe ratios of the S&P 500 of the past four decades in Table 11. We also calculated the relative deviation between the Sharpe ratio of the S&P 500 and of our selected portfolios.

Decade	S&P 500 Sharpe Ratio	One Firm Sharpe vs S&P 500	Ten Firms	Twenty Firms	Thirty Firms
1980	0.6529	-78.04%	-71.62%	-70.46%	-70.21%
1990	1.099	-99.82%	-99.41%	-99.35%	-99.27%
2000	0.151	-7.38%	57.29%	70.51%	75.47%
2010	1.179	-64.93%	-40.01%	-35.18%	-33.70%

Table 11. The average annual Sharpe ratio of the S&P 500 in selected decades, and the relative deviation of the Sharpe ratios between the index and our selected portfolios.

We note that Domain et al., (2007) and Bessembinder (2018) claim smaller portfolios rarely outperform an index seems to hold in most of our samples where the S&P 500 significantly outperform most of the simulated portfolios. The 2000s does however represent a clear deviation from their findings. We thus turn individual commentary of each sample.

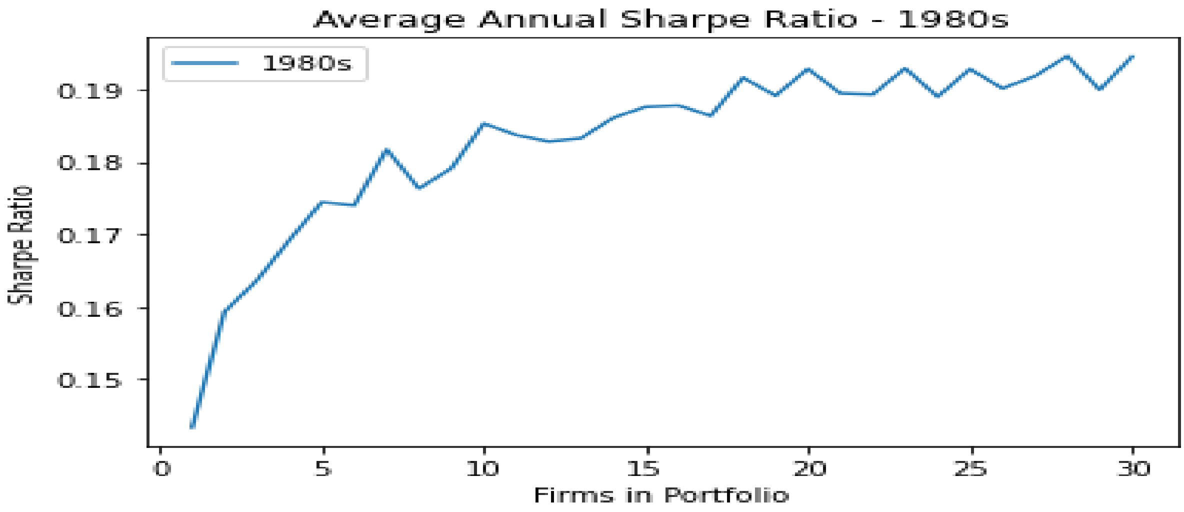

Figure 21. The average annual Sharpe ratio of randomly selected portfolios consisting of 1-30 stocks for our sample from 1980-1989.

For the 80s sample the graphed Sharpe ratios of portfolios consisting of 1 to 30 stocks reveals that the average simulated Sharpe ratio typically increases for each stock added to the portfolio. The efficiency gains for adding additional stock are however somewhat uneven but we believe the latter is due to the random nature of our simulations and its standard error and not any systematic deficiency in the diversification effect. The Sharpe ratio increases 23 out of 30 times when the portfolio expands with additional stocks. The Sharpe ratio's marginal increase is a decaying function of the stocks added to the portfolio.

In contrast to when we only focus the portfolio's risk, we observe a clear tendency that portfolios consisting of more than 8-10 stocks experience a significant improvement of the Sharpe ratio. We also note that the annualized Sharpe ratio of our randomly selected portfolios is poor when benchmarked against the Sharpe ratio of the S&P 500. The S&P 500 had a Sharpe ratio three times higher than the portfolios with 30 stocks and four times higher than the 2 stock portfolios.

The latter is consistent with previous work and implies that the number of stocks needed to obtain an efficient portfolio in terms of reward to volatility is far higher, than the number of stocks needed to achieve a low risk level.

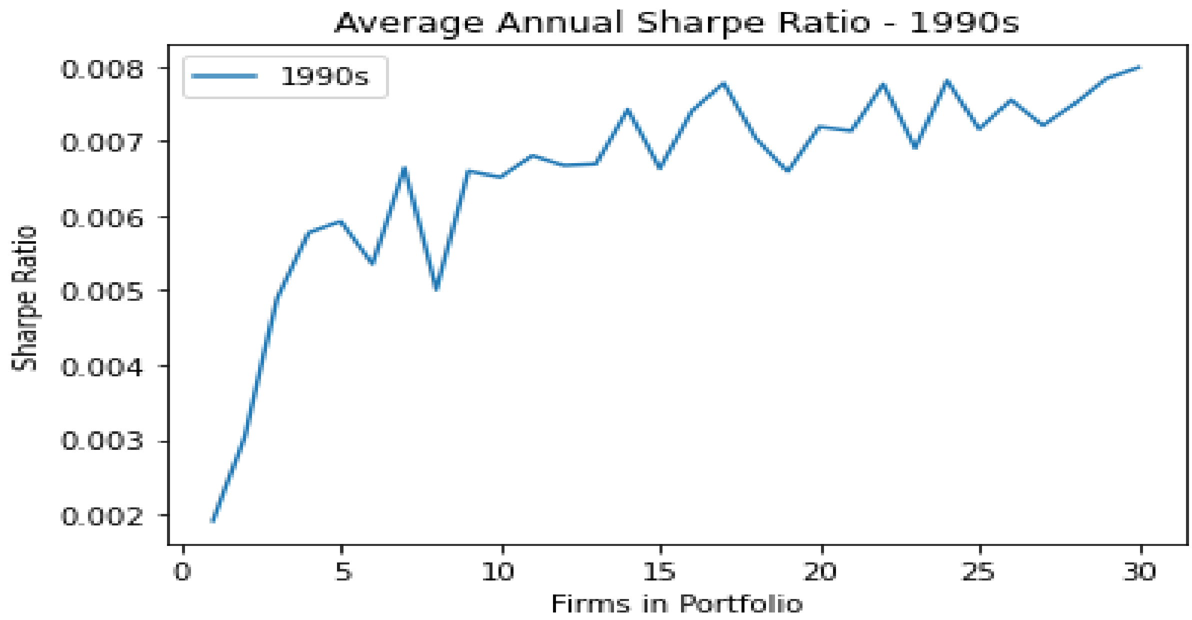

Figure 22. The average annual Sharpe ratio of randomly selected portfolios of 1-30 stocks for our sample from 1990-1999.

The results of Sharpe ratio analysis for the 90s correspond to the findings from the analysis for the previous sample. The Sharpe ratio improves relatively consistently when more stocks are added to the portfolio, although the effect still is somewhat unstable. While marginal Sharpe ratio gains are decaying as more stocks are added to the portfolio. The Sharpe ratio of the portfolios in the 90s sample were particularly poor. Few stocks likely generated returns higher than at the risk-free rate, even when the risk levels in the decade were lower than in the 80s. The S&P 500 outperform the 30 stocks portfolio by more than a factor of 10.

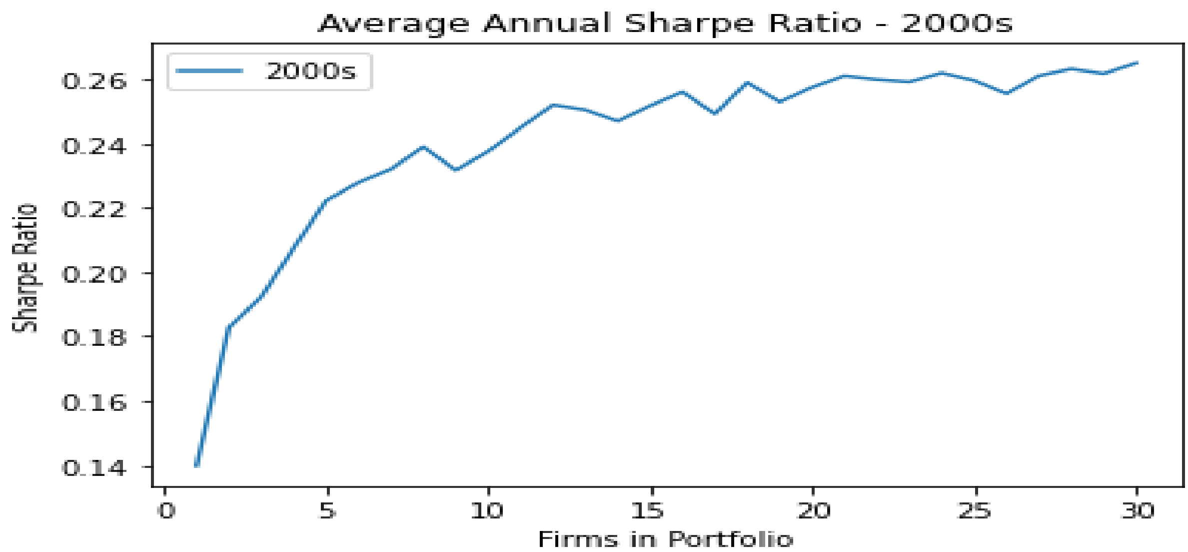

Figure 23. The average annual Sharpe ratio of randomly selected portfolios consisting of 1-30 stocks for our sample from 2000-2009.

Our simulation for the 2000s produce better Sharpe ratios than in the two previous decades. Despite having the highest risk level of any of our samples, it performed better in terms of reward to volatility than in any other sample. Surprisingly, we note that our simulated portfolios outperform the S&P 500. The 2000s has been described as a "lost decade" where the S&P 500 had poor annualized returns between 2000 and 2009. A possible explanation for the outperformance is that the smaller cap stocks not listed on the S&P 500 significantly outperformed the index that decade (Mahn, 2010). The survivorship bias in our sample is also a possible explanation for the outperformance. We chose to regard the results of the 2000s sample as an anomaly, as the other samples shows a clear tendency for poor return performance in the simulated portfolios. The maximum Sharpe ratio of 0.26 isn't particularly high either and the decade likely saw few stocks generating much excess return. We also note that adding stocks to the portfolio seems to yield more consistent benefits than in the previous samples.

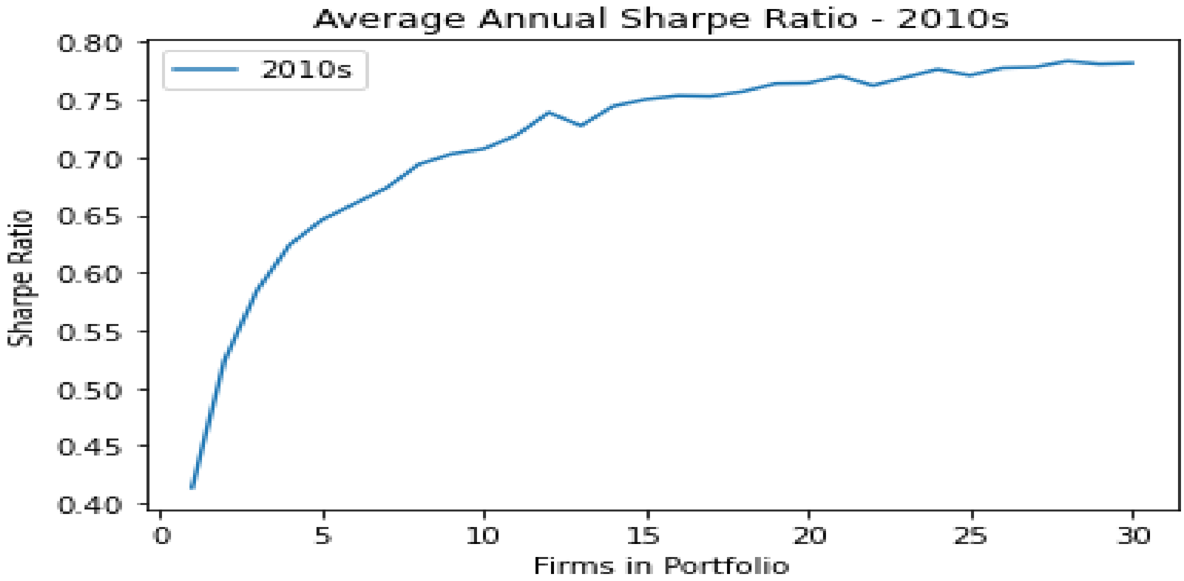

Figure 24. The average annual Sharpe ratio of randomly selected portfolios of 1-30 stocks for our sample from 2010-2019.

Finally, the 2010s produced the best Sharpe ratios yet, but not at the level of the S&P 500 index. The significantly better performance of the randomly portfolio relative to our benchmark is still significant. The marginal effect of increased stocks is the smoothest yet. Our analysis of the simulated portfolios thus primarily illustrate that smaller portfolios are largely inefficient when returns are accounted for. A Sharpe ratio below unity for any of our simulated portfolios implies that neither portfolio is efficient, when the standard deviation of each portfolio is higher than its excess return measured in percentages.

6.0 Conclusion

The answer to our original research question is that the diversification effect of adding stocks to smaller portfolios has not changed significantly. The additional benefit of adding stocks in smaller portfolios is at the same level today as has been over the past four decades. The overall risk level of portfolios consisting of 1 to 30 stocks has fallen over the last decade. While the risk level has fallen in absolute terms, the relative performance of smaller portfolios to the NYSE equally weighted index has remained relatively stable. The idiosyncratic component risk in common stocks has entered a lower mean state since the turn of the century. The co-movement in NYSE listed stocks returns has, in contrast, reached a higher mean state since the mid-2000s canceling out the effect of lower idiosyncratic volatility. The risk determinant regime shifts within a similar time frame serve as an explanation for why smaller portfolios have the same level of risk efficiency today as they had in the 80s.

According to our simulations, we first validate the common notion that 8 to 10 stocks suffice to remove most of the portfolio's exposure to firm-specific risk in our thesis. However, our simulation studies do find that such a portfolio is mainly inefficient in terms of returns. We note that the portfolio Sharpe ratio typically increases beyond when more than 8-10 stocks are added to the portfolio, yet 30 stocks are insufficient for a good Sharpe ratio. The implications of our findings are thus in line with the generally accepted notion that investors are better off holding passive index funds rather than creating smaller stock portfolios themselves.

Secondly, we find that recessions lead to increased stock return co-movement as well as higher levels of both systematic and idiosyncratic risk which will weaken the risk performance of the smaller portfolios. The latter could have a particularly adverse impact on leveraged traders, where the risk of their smaller portfolios increases at the same time as margin calls might increase due to falling returns.

Thirdly, the increased level of return co-movement and lower level of idiosyncratic risk could imply that small but less naively constructed portfolios where an investor pools together less correlated stocks have lower risk levels. However, our industry restrictions simulations do however somewhat contradict the efficiency of a variant of the described strategy. Although choosing stocks from different sectors will yield a lower average return correlation, industry diversification does not improve risk performance in our samples.

If Sullivan and Xiong (2012) findings hold, the larger market share holdings of financial institutions have increased the baseline level of stock return co-movement. The lower influence of systematic risk factors and higher levels of idiosyncratic risk might yield less risk efficiency in small portfolios in the future. It remains to be seen whether the trends in stock return co-movement and idiosyncratic risk develop so the diversification gains in smaller portfolios continue to be stable.

We have identified a trend and regime change from the CLMX study, but we could in the future see a return to the state identified by CLMX. When the increase in systematic risk following the recent pandemic and current inflation pressure eventually offsets, future research could identify whether the high level of stock return co-movement persists. A falling level of systematic risk and less government intervention in the financial markets could produce an outcome where more of the return of a stock is influenced by idiosyncratic factors and there is less co-movement in stock returns. The disruption of supply chains due to the pandemic and more focus on protectionism could further limit economic integration in the future, which could lower the co-movement of stock returns. The mentioned scenario could thus lead to a shift back to the risk determinant regimes originally described in the CLMX study.

It would also be interesting to see how the risk components have developed in different types of stocks in future research. It would, for instance, be interesting to see if recently listed stocks or stocks with specific market capitalization characteristics have a lower idiosyncratic risk component. The past two decades have seen tendencies where the top 4 percentile of firms have accrued a larger share of the total market capitalization than before. While the characteristics of newly listed companies were attributed to an increase in idiosyncratic risk in the 80s and 90s, more recently listed companies might have different characteristics. Other ways to explore the decline in idiosyncratic risk could be to focus on how improvements in information technology and more “efficient markets” could affect the firm-specific risk component.

Further research on the influence of ETFs and institutional ownership’s influence on the pairwise return coefficients could also focus on whether the mentioned factors have increased the baseline return co-movement or whether systematic risk factors mainly cause shifts.

Expanding our analysis to include other stock markets would also make it possible to analyze how US market volatility and return development affect other markets. One could also find whether increased globalization has led to increased co-movement in international markets.

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Appendix

1: Industry Data

Industry	Percent of Total Market	Average Daily Return	Average Standard Dev of Returns
Business Equipment	0.0734	0.000687	0.03407
Chemicals	0.0257	0.000683	0.02973
Durable Goods	0.0236	0.000556	0.02903
Energy	0.1069	0.000533	0.0317
Healthcare	0.0361	0.000669	0.0287
Manufacturing	0.0928	0.000592	0.0289
Non-durable Goods	0.0524	0.000592	0.02878
Other	0.1264	0.000636	0.03203
Wholesale	0.0843	0.000626	0.03242
Telecom	0.0262	0.000507	0.02944
Utilities	0.0344	0.000565	0.01901
Finance	0.3171	0.00047	0.02377

Table A.1: Industry statistics

2: SIC Codes for Industries in CRSP

Industry SICC Codes

Manufacturing	Consumer Non Durables	Consumer Durables	Business Equipment	Other	Healthcare	Wholesale	Energy	Chemicals	Utilities	TeleCom	Finance
2520-2589	2500-2519	0100-0999	3570-3579	4950-4959	2830-2839	5000-5999	1200-1399	2800-2829	4900-4949	4800-4899	6000-6999
2600-2699	2590-2599	2000-2399	3660-3692	4960-4961	3693-3693	7200-7299	2900-2999	2840-2899			
2750-2769	3630-3659	2700-2749	3694-3699	4970-4971	3840-3859	7600-7699					
3000-3099	3710-3711	2770-2799	3810-3829	4990-4991	8000-8099						
3200-3569	3714-3714	3100-3199	7370-7379								
3580-3629	3716-3716	3940-3989									
3700-3709	3750-3751										
3712-3713	3792-3792										
3715-3715	3900-3939										
3717-3749	3990-3999										
3752-3791											
3793-3799											
3830-3839											
3860-3899											

Table A.2: SIC Codes for each industry

3: The Assumptions of the CLMX Methodology

Complete decomposition of the assumptions we used in our disaggregated model is provided here.

Denoted by R_{jit} is the return on asset j in industry i included in portfolio p. The assets return can be separated into three components the risk-free rate, R_{ft}, a portfolio linked component and an asset-specific component

$$R_{jit} = R_{ft} + \beta_1\left(R_{pt} - R_{ft}\right) + u_{t,} \tag{23}$$

R_{pt} denotes the return of portfolio p, β_1 is a regression coefficient and u_t is the firm specific risk regression residual. The idiosyncratic residuals are assumed to be uncorrelated across all pair of firms and industries, with CAPM as the reference model. The residuals are however orthogonal on average. When the residuals origin from models that include the same set of regressors, their average correlation is zero.

We note that the unconditional variance of the asset i can also be decomposed into a systematic and idiosyncratic component

$$Var\left(R_{jit}\right) = (1-\beta_1)^2 Var\left(R_{f,t}\right) + {\beta_1}^2 + var(u_t). \tag{24}$$

By averaging across the assets, the variance of the typical asset can be approximately divided into a systematic and an idiosyncratic part.

$$Mean(var\left(R_{jit}\right) = Mean(1-\beta_1)^2 Var\left(R_{ft}\right) + Mean(\left({\beta_1}^2 Var\left(R_{pt}\right)\right) + Mean(Var(u_t)$$

$$\rightarrow$$

$$Mean(1-\beta_1)^2 Var\left(R_{ft}\right) + Mean(\beta_1^2) Var\left(R_{pt}\right) + Mean\left(Var(u_t)\right). \tag{25}$$

In Equation (25)-(28) the operator $Mean(\cdot)$ indicates a weighted mean across all assets included in the portfolio. Presuming that the cross-sectional variation of the β coefficients, CSV $(\beta_{i,p}^2)$ is limited, $Mean(\beta_1^2)$and $Mean((1-\beta_1, p)^2)$ can helpfully be approximated as follows:

$$Mean(\beta_1^2) = Mean(\beta_1) Avg\left(\beta_{i,p}\right) + CSV(\beta_1) = 1 + CSV\left(\beta_{i,p}\right) \cong 1 \tag{26}$$

$$Mean((1-\beta_1)^2 = \left(Mean(1-\beta_1)\right)^2 + CSV(1-\beta_1) = CSV(1-\beta_1) \cong 0. \tag{27}$$

Using Equation (26)-(27), the variance of average asset i_n converges towards the sum of the portfolio variance and of the average idiosyncratic variance:

$$Mean(Var(R_{i,t}) \cong Mean(R_{pt}) + Mean(Var(u_t)). \quad (28)$$

With a higher scale of analysis, the returns on the industry indices and on the individual stocks in the market portfolio can be described with the following equations

$$R_{it} = R_{ft} + \beta_{1i}(R_{mt} - R_{ft}) + \epsilon_{jt} \quad (29)$$

$$R_{ijt} = R_{f,t} + \beta_1(R_{m,t} - R_{f,t}) + \beta_{ij,j\epsilon_j,t} + \epsilon_{ij,t\,.} \quad (30)$$

R_{it} denotes the industry i return, while $R_{ijt}$is the return on the firm j in industry i, R_{mt} is the return on the market portfolio, β_{1m} , β_{1i} and β_1 are regressions coefficients and ϵ_{jt} ad ϵ_{ijt} are the industry and firm-level idiosyncratic regression residuals. By allowing

$$u_{ij,t} = \beta_{ij},j\ \epsilon_{j,t} + e_{ij,t} \quad (31)$$

$$R_{j,t} = R_{f,t} + \beta_1(R_{m,t} - R_{f,t}) + \beta_{ij,j\epsilon_j,t} + \epsilon_{ij,t} \quad (32)$$

can be rewritten as follows:

$$R_{ij1t} = R_{f,t} + \beta_1(R_{m,t} - R_{f,t}) + \beta_{ij,j\epsilon_j,t} + u_{ijt.} \quad (33)$$

As constructed $R_{m,t}$,$e_{ij,t}$and $\epsilon_{j,t}$ are orthogonal, yielding a u_{ijt}that is orthogonal with respects to $R_{m,t}$ and idiosyncratic regression residual. Equation (33) decomposes return into pure market and idiosyncratic components. While the equation

$$R_{j,t} = R_{f,t} + \beta_{j,m}(R_{m,t} - R_{f,t}) + \beta_{ij,j\epsilon_j,t} + \epsilon_{ij,t} \quad (34)$$

decomposes the idiosyncratic components into pure industry and firm level components.

Building on the decomposition of the variance of an asset as the sum of the portfolio variance and of the average idiosyncratic variance, we further separate the total stock variance into a systematic and idiosyncratic component $Var_t = MKT_t + IDIO_t$.

$$VAR_t = \sum_{j=1}^{n} w_{i,t} \sum_{i=1}^{k} w_{ij,t}\, Var(R_{ij,t}) \quad (35)$$

$$MKT_t = Var(R_{m,t}) \tag{36}$$

$$IDIO_t = \sum_{j=1}^{n} w_{i,t} \sum_{i=1}^{k} w_{ij,t} Var(u_{ijt}). \tag{37}$$

For Equations (36)-(38) k indicates the maximum number of stocks in each of the n industries, $w_{i,t}$ the weight of industry i in portfolio m n and $w_{ij,t}$ the weight of stock j in industry k, Var_t, is the weighted average total stock variance. $MKT_t$is the variance of the market portfolio, and $IDIO_t$ is the average idiosyncratic variance. VAR_t can be characterized as the variance of the typical stock, and $IDIO_t$ as the idiosyncratic variance.

Our methodology can be applied to any stock portfolio, an allows us to decompose the variance of typical industry and into a market and idiosyncratic component.

$$Var_t^{ind} = MKT_t + IND_t \tag{38}$$

Where

$$Var_t^{ind} = w_{j,t} Var(R_{j,t}), \tag{39}$$

$$IND_t = IND_t + FIRM_t. \tag{40}$$

Here, VAR_t^{ind} is average total industry variance and $IND_t$is the industry level average idiosyncratic variance. The idiosyncratic portion of average total variance can then be further decomposed into industry and firm level components

$$IDIO_t = IND_t + Firm_t \tag{41}$$

$$Firm = IDIO_t. \tag{42}$$

Our method chapter contains how we calculated the disagreed variance series, note that we have slightly changed some of CLMX original notation and expressions to make our method section more concise.

4: Simulation Data

# of Firms	1980s	1990s	2000s	2010s
1	0.092	0.088	0.106	0.078
2	0.076	0.069	0.084	0.064
3	0.068	0.061	0.076	0.058
4	0.064	0.054	0.071	0.053
5	0.061	0.052	0.068	0.051
6	0.059	0.05	0.064	0.049
7	0.058	0.047	0.063	0.048
8	0.057	0.046	0.062	0.047
9	0.056	0.045	0.06	0.046
10	0.055	0.044	0.06	0.045
11	0.054	0.044	0.059	0.045
12	0.054	0.043	0.058	0.044
13	0.054	0.042	0.057	0.044
14	0.053	0.042	0.057	0.044
15	0.053	0.041	0.057	0.043
16	0.053	0.041	0.056	0.043
17	0.052	0.041	0.056	0.043
18	0.052	0.04	0.055	0.042
19	0.052	0.04	0.056	0.042
20	0.052	0.04	0.055	0.042
21	0.052	0.04	0.055	0.042
22	0.051	0.04	0.054	0.042
23	0.051	0.039	0.054	0.042
24	0.051	0.039	0.054	0.042
25	0.051	0.039	0.054	0.041
26	0.051	0.039	0.054	0.041
27	0.051	0.039	0.054	0.041
28	0.051	0.039	0.054	0.041
29	0.051	0.038	0.054	0.041
30	0.05	0.038	0.053	0.041

Table A.3: Risk simulations

# of Firms	1980s	1990s	2000s	2010s
1	0.03297	0.03891	0.05209	0.04014
2	0.01923	0.0218	0.0304	0.02396
3	0.01407	0.01592	0.02276	0.01755
4	0.01153	0.01264	0.01841	0.01428
5	0.00977	0.01086	0.01612	0.01243
6	0.00881	0.00944	0.01409	0.01094
7	0.00813	0.00834	0.01289	0.01011
8	0.00742	0.00767	0.01194	0.00931
9	0.00704	0.00713	0.01107	0.00886
10	0.00657	0.00663	0.01038	0.00824
11	0.00618	0.0063	0.00972	0.0078
12	0.00592	0.00592	0.00937	0.00748
13	0.00564	0.00563	0.00891	0.00713
14	0.0054	0.00533	0.00851	0.00675
15	0.00523	0.00516	0.0083	0.00652
16	0.00506	0.00498	0.00789	0.0064
17	0.00486	0.00484	0.00764	0.00614
18	0.00471	0.00464	0.00748	0.00595
19	0.00456	0.00451	0.00722	0.00584
20	0.00445	0.0044	0.00698	0.0056
21	0.0043	0.00424	0.00686	0.00545
22	0.00419	0.00412	0.00671	0.00535
23	0.00413	0.00401	0.00651	0.0052
24	0.00405	0.00394	0.00639	0.0051
25	0.00396	0.00389	0.0062	0.00505
26	0.00392	0.00376	0.00599	0.0049
27	0.00372	0.00365	0.00591	0.00483
28	0.00375	0.00358	0.00583	0.00475
29	0.00366	0.00351	0.00568	0.00467
30	0.00359	0.0035	0.00564	0.00455

Table A.4: Standard deviation of risks

5: Correlation Tests

	Dickey Fuller test on Correlation
Z- test	-3.409 (0.0107)
T-test	-3.41 (0.001)

Table A.5: Dickey Fuller test

	Vogelsang test on Correlations
Trend coefficients	0.000141
P-value coefficients	0.00
95% Confidence Interval	(0.00009, 0.001921)

Table A.6: Vogelsang tests

6: Change in Volatility from Additional Stock

# of Firms	1980s	1990s	2000s	2010s
2	-0.01677	-0.01926	-0.0214	-0.014
3	-0.0078	-0.00838	-0.00781	-0.00625
4	-0.00396	-0.00616	-0.0053	-0.0042
5	-0.00293	-0.00281	-0.00332	-0.00248
6	-0.0018	-0.00165	-0.00383	-0.00191
7	-0.00143	-0.00276	-0.00091	-0.00121
8	-0.00025	-0.00093	-0.00152	-0.00061
9	-0.00178	-0.00123	-0.00127	-0.00077
10	-0.00081	-0.00074	-0.00059	-0.0014
11	-0.00038	-0.00059	-0.001	0.00041
12	-0.00047	-0.00037	-0.00038	-0.001
13	-0.00017	-0.00087	-0.00096	-0.00035
14	-0.0005	-0.0005	-0.00064	-0.00022
15	-0.00026	-0.00045	0.00065	-0.00029
16	-0.00039	-0.00022	-0.00146	-0.00048
17	-0.00018	-0.00042	0.00027	-0.00023
18	-0.00018	-0.00043	-0.00071	-0.00023
19	-0.00035	-0.00012	0.00039	-0.00045
20	3.23e-05	-0.00016	-0.00083	0.00022
21	-0.0002	-8.74e-05	-0.00036	-0.00012
22	-0.00033	-0.00028	-0.00056	-0.00022
23	-2.58e-05	-0.00033	0.00035	-0.00036
24	-0.00025	7.12e-06	4.18e-05	7.012e-05
25	-0.0002	-0.00021	-0.00044	-0.00019
26	0.00014	-0.0003	0.00026	-9.2068e-05
27	-0.00024	-0.00016	-0.00038	1.7e-05
28	9.41e-05	4.16e-06	-0.00019	-0.0002
29	-0.000198	-0.00024	2.94e-05	7.88376e-06
30	-0.00019	-6.45e-05	-0.00022	-3.31e-05

Table A.6: Change in volatility from adding one stock in portfolio, for each decade.

7: T-values for 1,000 Simulations

Firms	1980s	1990s	2000s	2010s
8 -> 9	3.108	2.879	2.175	2.236
11 -> 12	1.66	2.63	1.648	1.351
14 -> 15	1.819	0.109	1.576	0.754
17 -> 18	0.299	0.61	0.967	0.296
20 -> 21	0.365	1.872	0.986	0.623
23 -> 24	0.0072	0.148	0.215	0.827
26 -> 27	0.112	1.68	0.932	0.678
29 -> 30	0.268	0.385	1.039	0.516

Table A.7: T-values on change in risk levels, but with only 1,000 simulations ran.

8: Markov Model

For our Markov switching model, the number of possible occurrences is split into 2 states for the volatility and co-movement series.

It is assumed that the volatility or co-movement series switches regime according the unobserved variable s_t that takes integer values. s_t takes the value of one if the data generating process is in regime 1 at time t, if $s_t = 2$ the process is in regime 2 at time t.

Changes in the state variable $s_t$that causes regime changes are administered by a Markov process.

The Markov property is described with the equation:

$$P[a < y_t \leq b \,| y_1, y_2, \ldots . y_{t-1}] = P[a < y_t \leq b | y_{t-1}], \qquad (43)$$

where $y_t$is the variable suspect to switches between different mean states.

The Markov property implies that the probability distribution of the state at any time depends only on the state the previous period $t-1$ and not on the state the preceding periods. A Markov process is thus not path dependent.

The model is highly flexible, being capable of both capturing both changes in the variance and mean between state processes.

The Markov models compromises the unobserved state variable z_t, that is postulated to evaluate according to a first order Markov process

$$prob[z_t = 1|Z_t - 1 = 1] = p_{11} \tag{44}$$

$$prob[z_t = 2|Z_t - 1 = 1] = 1 - p_{11} \tag{45}$$

$$prob[z_t = 2|Z_t = 2] = p_{22} \tag{46}$$

$$prob[z_t = 1|Z_t = 2] = 1 - p_{22} \tag{47}$$

Where p_{11} denote the probability of being in regime 1, given that the process was in regime during the previous period. Conversely p_{22} denote the probability of being in regime 2 given that the process was in regime 2 to begin with. $p_{11} - 1$ and $1 - p_{22}$are thus the implied probabilities.[8] Under the first order Markov process Z_t evolves as an autoregressive AR(1) process $z_t = (1 - p_{11}) + \rho z_{t-1} + \eta_t$, where $\rho = p_{11} + p_{22} - 1$. z_t is the generalization of the dummy variables for the one time shifts in the previously described series. The model allows for several shifts from one mean state to the other.

According to our framework the realized risk series evolves according to the following equation:

$$Vol_t = \mu_1 + \mu_2 z_t + (\sigma_1^2 + \phi z_t)^{0,5} u_t. \tag{48}$$

Where $u_t \sim N(0,1)$. The expected values and variances of the series are $\mu_1$and σ_1^2 in state 1, and $(\mu_1 + \mu_2)$ and $\sigma_1^2 + \phi$. The unknown parameters $(\mu_1, \mu_2, \sigma_1^2, \phi, p_{11}, p_{22})$ are estimated with the maximum likelihood estimation procedure, where parameters are chosen as the ones who was most likely to produce the data (Enders, 2015).

9: Fama- French 3-Factor Model

The Fama- French 3-factor model is an extension of the classic CAPM model. The model is as follows:

$$R_{it} - R_{rt} = \alpha_{it} + \beta_1 (R_{Mt} - R_{ft}) + \beta_2 SMB_t + \beta_3 HML_t + \epsilon_{it}. \tag{49}$$

[8] Note that transition matrix referred to in text use a slightly altered notation to describe the transition probabilities, to make the results consistent with our estimated transition matrix output.

The left side of Equation (49) is the excess return for an asset. β_1 is the market premium coefficient, β_2 is the size premium coefficient, and $\beta_3$is the value premium coefficient.

The classical CAPM model only includes the market premium to estimate systematic risk, but the Fama- French 3-factor model includes SMB and HML to control for the fact that historically, firms with smaller market cap outperform bigger ones, and firms with higher book-to-market value typically outperform firms with lower book-to-market.

The data for $R_{Mt} - R_{ft},\ SMB_t$ and HML_t can be obtained from Kenneth French's website. Since the factors are representing the systematic risk of an asset, if we run a regression with these as the dependent variable, and the excess return of a security as the independent variable, ϵ_{it} is then the idiosyncratic of our asset (Fama & French, 1993).

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